MEASURING THE NATURAL RATE USING NATURAL EXPERIMENTS

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NBER Working Paper No. 31760
October 2023
JEL No. E4,E40,E5,G5

ABSTRACT

Every month, a fraction of UK property leases are extended for another 90 years or more. We use new data on thousands of these natural experiments from 2003 onwards to estimate the "natural rate of return on capital", r_K^*, which also represents the long-run dividend-price ratio. r_K^* stays around 4.8% between 2003 and 2006, but starts to fall at the onset of the Great Recession, reaching a low of 2.3% in 2022. Real-time monthly data shows a modest uptick in r_K^* in 2023 thus far. The natural experiment approach to measuring r_K^* is precise, avoids misspecification concerns and provides real-time estimates using publicly available data.

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A website accompanying this paper which displays real-time updates to our estimates,
in addition to data and replication instructions. is available at
https://sites.google.com/view/naturalrate/home

1 Introduction

The natural rate of return is an important diagnostic for the economy. Wicksell (1958) introduced this idea in the context of the natural rate of interest, which clears the market for saving and investment. More broadly, the natural rate of return is the required return on various assets, which balances the supply and demand for goods, employment, savings and investment. Measuring the natural rate of return is critical for detecting structural shifts in the economy, such as whether the era of "secular stagnation" has ended. As a result, central bankers and financial market participants pay a great deal of attention to this object.

However the natural rate of return is difficult to measure. Conceptually, the natural rate represents an economy's expected long run equilibrium, once transitory factors such as business cycles and adjustment costs have subsided. Therefore the correct measure must "look through" shorter run factors. An influential approach, pioneered by Laubach and Williams (2003) and Holston, Laubach and Williams (2017) (HLW), uses structural time series methods to measure the natural rate of return on safe assets (r^*). HLW make model-derived assumptions about the relationship between inflation, the output gap and interest rates, in order to filter out short run shocks and identify r^*.

This work and the subsequent literature are justly celebrated for tackling a difficult and important problem.[1] However HLW themselves point out that their "estimates of the natural rate of interest are inherently imprecise and potentially sensitive to model specification". To wit: the HLW estimates were discontinued during the pandemic due to misspecification (Williams, 2023), and structural models disagree about whether r^* rose after the pandemic (Baker, Casey, Del Negro, Gleich and Nallamotu, 2023). This misspecification may prevent central bankers from using r^* to guide policy, as the Bank of England's Chief Economist argued in a recent speech (Pill, 2023).

This paper takes a different approach to measuring the natural rate of return by using natural experiments and microdata instead. The natural experiment approach is less sensitive to misspecification issues and standard errors are orders of magnitude smaller. We measure r_K^*, which is the market's expectation of the natural rate of return on capital, or the the long run discount rate on capital, net of capital gains. Formally, r_K^* has three elements, with $r_K^* \equiv r^* + \zeta^* - g_P^*$, where ζ^* is the natural risk premium on capital, g_P^* is the long run expected capital gain and r^* is the natural rate of interest. r_K^* is also equivalent to the long-run (or natural) dividend-price ratio, as well as the Hall and Jorgenson (1967) user cost of capital, normalized by its price. The natural rate of return on capital thus reflects an

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[1]Related papers include Kiley (2015), Lubik and Matthes (2015), Johannsen and Mertens (2016), Crump et al. (2016), Hamilton et al. (2016), Rachel and Smith (2017), Christensen and Rudebusch (2017), Del Negro et al. (2017), Rachel and Summers (2019) and Del Negro et al. (2019).

important price that clears the market for private capital in the long run.

We develop a dynamic real-time estimate of r_K^* for housing using a natural experiment approach in the UK from 2003 to present. Most apartments in the U.K. are sold as "leaseholds"—long duration leases starting at ninety years or more, issued by the ultimate owner of the property, or "freeholder". The leaseholder can buy or sell the lease, giving each lease a series of market prices. Moreover leaseholders have the right to extend their lease conditional on paying freeholders the value of the lease extension. Lease extensions typically happen when the current lease has somewhere between 60 to 90 years left, and the typical extension is for an additional 90 years or more.

This paper assembles a new data set on leasehold extensions and transactions from 2003 to the present. We estimate the increase in the market value of a leasehold due to duration extension, by comparing its price before and after lease extension with an otherwise similar control group of leaseholds that do not get extended at the same time. We embed this difference-in-differences estimate of the market value of lease duration extension into a simple discounted-cash-flow pricing equation. The equation allows us to estimate *the market's expected* r_K^* via non-linear least squares, using over 130,000 lease extension experiments from 2003 onward.

An important empirical advantage of our natural experiment approach to estimating r_K^* is that extensions generate variation in lease duration for the *same property*, allowing us to "difference out" shorter run factors that should not influence the natural rate estimation. Intuitively, the price before extension measures short run value, whereas the price after extension measures both the short and long run value of the same property. Hence the difference in prices measures only the long run value, and short to medium run shocks to rate of return, such as monetary tightening, do not affect our estimate of r_K^*. Similarly, our estimate is unaffected by short run shocks to the service flow of housing or other hard-to-measure characteristics of any given property. For instance, a shock to demand for a particular segment of London property, which raises its service flow, does not affect our estimate.

The key assumption for our natural experiment to identify r_K^* is "parallel trends": the service flow of housing must grow similarly for extended properties and their control group. We support this identification assumption in four ways. First, there are no pre-trends, meaning prices of extending properties evolve similarly to the control before extension. Second, the treatment and control group are balanced on a rich set of hedonic characteristics. Third, market rents and hedonic characteristics evolve similarly for extenders and the control group. Fourth, the estimator is not sensitive to controlling for observed heterogeneity, suggesting bias from unobserved characteristics is small (Altonji, Elder and Taber, 2005; Oster, 2019).

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Figure 1: Time Series of the Natural Rate of Return of Capital
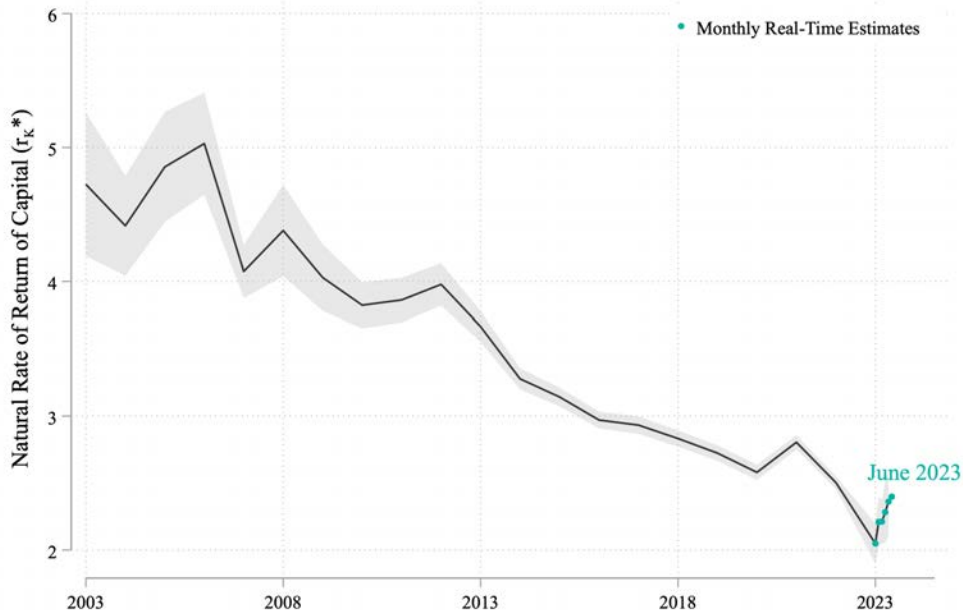
The figure shows estimates of r_K^* for each year of data and for the full sample of lease extensions. Monthly estimates for January 2023 through June 2023 are also reported and highlighted in teal. The shaded area shows 95% confidence intervals for the estimates. Standard errors are heteroskedasticity robust.

Figure 1 plots the average r_K^* for each year from 2003 to 2022, and the shaded area represents the 95% confidence interval. r_K^* stays around 4.8% between 2003 and 2006, but starts to fall at the onset of the Great Recession, reaching a low of 2.3% in 2022. The magnitude of this decline is large, equivalent to a doubling of the natural price-dividend ratio. There is a period in between, 2010 to 2012, when r_K^* stabilized for a while before continuing its downward trajectory. There is also a brief period during the early pandemic months when markets raise their expectation of r_K^*, but ultimately r_K^* returns to its longer-term downward trend through the end of 2022.

The number of lease extension experiments increases over time which results in more precise estimate of r_K^* in recent years. The standard error for our estimate of r_K^* for 2022 is 0.025pp, whereas standard errors from time series estimates of natural rates tend to be one to two orders of magnitude larger.[2] Time series estimates of natural rates are imprecise because there is relatively little variation in the aggregate time series that is informative about the long run (Farmer, Nakamura and Steinsson, 2021). By contrast our micro data based strategy exploits the large amount of variation available from the cross section, which turns out to be informative about natural rates.

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[2]For instance, the standard error for the HLW estimate of r^* in 2019 for the UK was 4.8 pp.

The lease extension data base put together in this paper is based on publicly available data, and is updated monthly in real time. Thus not only can our methodology be easily replicated, but our time-series estimates can be extended in real time. There are over a thousand leasehold extensions every month, which allows for precise estimates of r_K^* even at a monthly frequency. We illustrate this in Figure 1 by estimating r_K^* separately for each month for 2023, with the latest data point ending in June of 2023. Real-time monthly data shows a modest uptick in r_K^* in 2023 thus far.[3]

While our measure of the natural rate of return is from UK housing, a couple of observations suggest that our estimates should be relevant for other forms of capital as well. First, housing is a major asset class, so the natural rate of return for housing should relate to the broader economy. Moreover leaseholds are a significant share of the U.K. housing market, being 97% of apartments and 22% of all dwellings. Lease extension experiments occur throughout the U.K. in all major regions, and the U.K. housing market is deep and liquid.

Second, though there might be differences in the *level* of r_K^* for capital other than housing, the *trends* may still be similar. The level of $r_K^* \equiv r^* + \zeta^* - g_P^*$ might differ across assets due to different risk premia ζ^* or expected capital gains g_P^*. However, suppose that the fall in r_K^* for housing is due to factors that are not specific to housing. Then other assets should also experience similar falls in natural rates of return, because the usual arbitrage conditions equate risk-adjusted rates of return across assets, especially in the long run. We also present suggestive evidence that the decline in r_K^* for housing is due to common factors, rather than housing specific factors such as housing risk premia or capital gains.

Our paper is closely related to the seminal work of Giglio, Maggiori and Stroebel (2015), who were the first to observe that because UK apartments vary in duration, they are particularly well suited to estimating natural rates of return.[4] Giglio et. al. make a cross-sectional comparison of properties with different duration, and control for a rich set of hedonic characteristics, in order to estimate the level of r_K^*. Our paper studies the dynamics of r_K^*, and in doing so builds on their work in two ways. First, we use a quasi-experimental design based on lease extensions, which allows us to use variation in leasehold duration and price *within the same property*. We show that within-property variation is critical to accurately estimating r_K^*, due to unobserved heterogeneity in the service flow of housing. Our method can then estimate the dynamics of r_K^* reliably in real time, at monthly frequency. Second,

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[3]See our website for data, real-time estimates and replication instructions. As we discuss in Section 3.3, our estimates contain a lag of 3-5 months, so the June estimates capture market expectations of r_K^* at the start of 2023.

[4]See also Badarinza and Ramadorai (2015), Giglio, Maggiori and Stroebel (2016), and Bracke, Pinchbeck and Wyatt (2018).

the possibility of lease extension creates option value that might affect the price of UK lease-holds. We explicitly take this option value into account, and in Section 6 we develop a new bunching estimator to measure this option value.

Our paper also relates to a literature inferring the return to capital using data from national accounts (e.g. Gomme, Ravikumar and Rupert, 2015; Farhi and Gourio, 2018; Reis, 2022; Vissing-Jorgensen, 2022). This literature notes that the ratio of profits to the capital stock—which proxies for the return to capital when there is perfect competition and constant returns to scale—has been stable, whereas we find that the natural rate of return on capital has fallen. Amongst other explanations, rising monopoly power in goods markets can reconcile these two phenomena. As Farhi and Gourio (2018) show, given the natural rate of return on capital, rising monopoly power raises the profit-to-capital ratio.

Outline. The rest of the paper is structured as follows. Section 2 defines the natural rate of return on capital. Section 3 describes the data. Section 4 presents our empirical methodology. Section 5 presents our estimates of lease extension price changes and estimates of r_K^*. Section 6 measures the option value from lease extension and Section 7 concludes with a discussion of possible future work based on our new methodology.

2 Introducing The Natural Rate of Return on Capital

The natural rate of return on capital is markets' expectations of rates of return in the long run, after medium-run forces have subsided. Formally, consider the price of a unit of capital. The price at time t, P_t is given by the present value of its dividend, R_t, as

$$P_t = R_t \int_0^\infty e^{-\int_0^s (r(u)+\zeta(u)-g_R(u))du} ds. \tag{1}$$

In this equation, $r(u) + \zeta(u)$ is the discount rate of capital u periods in the future, which discounts the present value of dividends at a rate that sums the safe rate of return and the risk premium, all in real terms, and g_R is dividend growth.[5]

We adopt the compact notation for the natural rate of return on capital $y(u) \equiv r(u) + \zeta(u) - g_R(u)$, and will sometimes refer to $y(u)$ as the "yield". In defining the rate of return net of capital gains, we use a similar language to Farhi and Gourio (2018) and Reis (2022). We are interested in the *natural* rate of return on capital r_K^*, that is, rate of return of capital expected in the long run. We assume that r_K^* exists, in which case it is given by

$$\lim_{u \to \infty} r_K(u) = r_K^* = r^* + \zeta^* - g_P^*. \tag{2}$$

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[5]For simplicity this derivation omits a "rational bubble" term.

In the long run and on a balanced growth path, the dividend-price ratio converges to the the natural rate of return, since by Equation (1) we have

$$\frac{R_\infty}{P_\infty} = \lim_{u \to \infty} r(u) + \zeta(u) - g_R(u) = r^* + \zeta^* - g_P^*, \tag{3}$$

where the final equality exploits that in the long run, price and dividend growth are equal.

The natural rate of return on capital matters for at least two reasons. First, r_K^* contains information about the safe rate of return r^*, which matters greatly to policymakers. For instance, if risk premia and capital gains are stable over time, then Equation (2) shows that changes in r_K^* and changes in r^* are equal. Second, the natural rate of return on capital is an important variable in its own right, beyond information about the safe rate of return. In particular, since r_K^* is the natural dividend-price ratio, it is the key variable that clears the market for capital in the long run, and therefore the supply and demand for assets in the economy at large.

In Appendix A.3 we utilize a simple model of firm investment to show that under a no arbitrage assumption, r_K^* is equal to the user cost of capital, or equivalently the marginal product of capital, normalized by the price of capital. This equivalence is also present in Hall and Jorgenson (1967) and Farhi and Gourio (2018).

Crucially, r_K^* is independent of short run, transient shocks to rates of return or dividends. As such, the key challenge to estimating natural rates is to filter out these shorter run shocks. Current asset prices—even for very long duration assets—depend not only on r_K^* but also on shorter run rates of return, which include transitory factors such as monetary policy, credit booms, short run bubbles and adjustment costs. The reason is that, as Equation (1) shows, the price of a long duration asset depends on the integral of short and longer term discount rates. Therefore the current dividend-price ratio, R_t/P_t, is affected by these transitory shocks and may greatly differ from the natural ratio of dividends to prices—which by Equation (3) equals the natural rate of return. Moreover, the dividend of capital R_t is often difficult to observe. For instance, the service flow of owner-occupied housing cannot be observed from market transactions, and must be imputed. Therefore short run shocks to the dividend of capital—for instance, a temporary increase in demand for a certain segment of London housing—are an additional source of variation that confounds estimates of r_K^*.

Structural time series methods, such as Laubach and Williams (2003), make model-derived assumptions about the joint behavior of macroeconomic aggregates, in order to filter out these short run shocks and identify natural rates of return. The next section describes an alternative approach, based on microdata and natural experiments from the UK housing market.

3 Data & Lease Extension Details

This section introduces the setting—lease extensions for long duration leasehold properties in the United Kingdom—and explains the data sets we will use for our analysis.

3.1 Data

We use five data sets for the analysis of this paper. Most of the analysis uses data that are publicly available almost in real time. This is an important feature of our method, since it allows for real-time updates and replication. A website accompanying this paper contains data, replication instructions, and real time analysis. We now describe our five data sets.

(1) **Land Registry Transaction Data**: We obtain publicly available data on all property transactions registered in England and Wales between 1995 and June 2023 from the Land Registry. The data set includes the exact date, price and address for each transaction. Properties are also subdivided into two categories: freeholds and leaseholds. Freeholds are a perpetual claim to the ownership of a property. Leaseholds are long duration leases to the property that can be bought and sold, and which typically last for many decades at origination. As we will discuss in detail below, leasehold flat durations are periodically extended, after negotiation with the freeholder. The distinction between freeholds and leaseholds dates to medieval England, during which permanent ownership of land and property, known as "freehold" ownership, was available only to feudal nobility.[6] During this time, leasehold estates were granted to serfs who would work the land for a set period of time and in exchange would pay a portion of the harvest to the freehold landowner. During the 20th century, cash-poor landowners began to issue long leaseholds of 99 and 125 years, providing immediate liquidity without giving up ownership of the underlying land. To this day, leaseholds are very common in England and Wales, comprising approximately 5% of houses and 97% of apartments. The freeholds underlying UK flats are typically owned by landed estates (e.g. the Cadogan Estates) which are privately managed, and other private landlords, developers, and investment companies. A very small proportion of these freeholds are owned by the Crown or the Church of England.

(2) **Land Registry Lease Data**: Data on the length of lease terms, which vary significantly across leaseholds, are provided in a separate Land Registry data set, also publicly available. The Land Registry lease term data set includes the property address associated to the lease, the term length and origination date of the lease, as well as the date in which the lease was registered with the Land Registry. A majority of leases were originated and

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[6]The first known use of the term "freeholder" is in the Domesday Book published in 1086 under the reign of William the Conqueror.

registered after the 1950s. Starting in late 2003, lease registration became mandatory, so we capture all registered leases after this period. We will start our analysis in 2003, after mandatory registration.

Each entry in the lease term data set provides the length of the lease term, its registration date, and the start date of the lease. The Land Registry does not provide match keys to merge the transaction and lease term data sets, so we conduct a fuzzy merge based on provided addresses, as we detail in Appendix A.4.[7]

The Land Registry classifies properties into five types: flats, detached houses, semi-detached houses, terraced houses, and others. The three categories of houses are for the most part freehold properties. Even when they are leaseholds, houses tend to have very long lease terms relative to flats, with median remaining lease term over 800. Since leaseholds are mostly flats, we will drop non-flats for the bulk of our analysis.

(3) **Land Registry Extensions Data**: We obtain data on lease extensions from two sources, neither of which have been used in prior academic research to our knowledge. First, beginning in September 2021, the Land Registry began publicly releasing information on lease extensions. Second, we have obtained from the Land Registry all lease extensions before September 2021, which we have made available on the website accompanying this paper.

(4) **Rightmove Hedonics Data**: We obtain data on housing characteristics and rental prices from Rightmove, Inc. spanning 2006 to the present, which include the number of bedrooms, number of bathrooms, number of living rooms, floor area, property age, parking type, heating type and property condition (rated as Good, Average, or Poor) of listed properties. It also includes rents for rented properties. These data must be purchased from Rightmove, however, our main analysis can be carried out without these data.

(5) **Zoopla Hedonics Data**: We supplement this with data from Zoopla, Inc. which is provided for free to researchers by the Urban Big Data Centre. This data set also provides number of bedrooms and bathrooms and rents. Additionally, it includes the number of floors and receptions of the property. We are able to match approximately 80% of transactions to the Rightmove and Zoopla listing data. Rental data is available for about 40% of properties.

3.2 Lease Extension Details

This section provides additional details about leaseholds in England and Wales and the circumstances under which the duration of leases can be extended. As we described in the previous section, leaseholds are properties with very long but finite leases, moreover these

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[7]We exclude 0.02% of our transactions, which have implied negative lease terms at the time of transaction. We also exclude 0.6% of properties which are sold both as a leasehold and a freehold within our sample.

leases can be bought and sold. The lease length at the time it was issued is denoted its "initial lease term" and the lease length it has at any future point in time is denoted its "remaining lease term." The distribution of leases can be divided into two groups; about 70% of leasehold flats in our sample are *short leaseholds* with remaining terms of 250 years or less and the other 30% are *long leaseholds* with remaining terms of 700 years or more. There are practically no properties with remaining terms between 250 and 700.[8]. The most common initial terms for short leaseholds are for 99 and 125 years, which account for 77% of short leaseholds. The most common initial term for long leaseholds is 999 years, which account for 96% of all long leaseholds.

Figure 2: Diagram of Extension Time
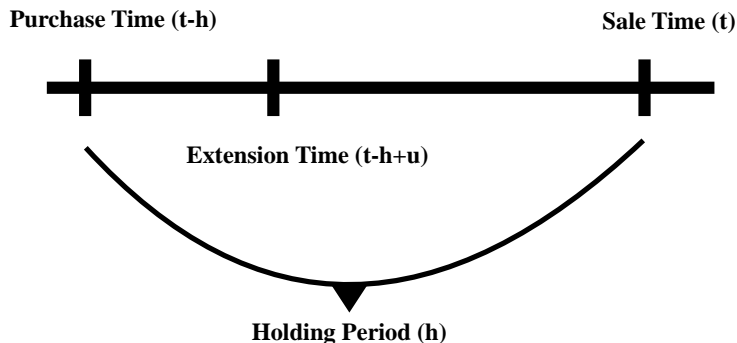
The figure is a diagrammatic representation of the notation we will use in the paper. We say a property is purchased at time $t - h$, sold at time t and held for an amount h of years. We say that a property extends at time $t - h + u$, where $u < h$.

Beginning in 1993, the Leasehold Reform, Housing and Urban Development Act (1993 Act) granted flat leasehold owners the right "to acquire a new lease" 90 years longer than the original lease, conditional on a one-off negotiated payment to the freeholder. This negotiation is potentially costly, since both the freeholder and the leaseholder may hire qualified surveyors in order to assess the value of the property, and the negotiation can be lengthy. We denote these cases as *lease extensions*. This option is particularly relevant for leasehold owners of short leaseholds for whom lease expiration may be more of a concern than for owners of long leaseholds.

Although the 1993 Act legally provides the option to extend, almost all extensions are negotiated out of court by leaseholders and their freehold landlords. Despite this, the court-recommended amount of 90 years is the most common extension length (accounting for about

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[8]This distribution is illustrated in Appendix Figure A.1

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30% of all extensions). A fraction of extensions, however, are for approximately 900 years, which effectively convert short leaseholds into long leaseholds.

We will now introduce some notation that we will use throughout the paper to refer to extended properties in our sample, which we show in a diagram in Figure 2. Consider a lease that transacts twice within the Land Registry Transaction Data Set. We say that a property was purchased at time $t - h$ and sold at time t, where h is the amount of time between purchase and sale. We are interested in properties which were extended at some time $t - h + u$ where $u < h$.

We denote the lease term to maturity, henceforth referred to interchangeably as lease duration, at purchase time as $T + h$ and its duration at sale time as $T + 90$ (notice that its duration would have been T at sale had the lease failed to be extended). We denote the price of a property i of duration T at time t by P_{it}^T. The transacted prices before and after lease extension, and the lease duration before extension, will be the key inputs into our estimation; however we do not observe the extension payment paid by leaseholders to freeholders.

As we see in Appendix Figure A.2, properties with a very short holding period, h, are very likely to be "flippers" who purchase, renovate and sell properties with the explicit intention of making a quick profit and therefore behave differently from other owners. Unless otherwise specified, we exclude properties with $h \leq 1$ year from our analysis.

Table 1: Number of Extensions

Extension Amount	90	700+	Other	Total
2003-2005	640	1,942	2,041	4,623
2006-2010	3,392	6,901	5,160	15,453
2011-2015	11,648	15,458	10,417	37,523
2016-2020	16,845	18,777	11,753	47,375
2021-2023	10,721	11,041	5,103	26,865
Total	43,246	54,119	34,474	131,839

The table reports the number of extended leases that have transaction data for each time period. The first column includes 90 year extensions, the next column includes 700+ year extensions, and the third column includes others, which are almost all non-90 under 200 year extensions.

In Table 1 we present the distribution of extensions for different amounts over time. In total, there are 131,839 lease extensions in our main sample, which is 5% of flats. How common are these lease extensions? In Appendix Figure A.3 we show the hazard rate of extension, defined as the conditional probability a property will extend given how long its remaining term to maturity is. We can see that almost no properties extended with more than 90 years remaining. After a property hits 80 years remaining, its extension probability

jumps to a probability of extension of about 5%, and then slowly falls back to 2% or so. We elaborate more on the particulars of this hazard rate in Appendix A.10.

For most of our analysis, we will focus on leases that extend by the typical amount of 90 years. For these leases, the median duration before extension is large, at around 70 years, as shown in Appendix Figure A.5. The median time between transactions is 10 years, with 7 years between purchase and lease extension (see Appendix Figure A.6 and Appendix Figure A.7). In Appendix A.1, we provide additional summary statistics about lease extensions, including a heatmap of the extension rate by region; and the distribution of transaction dates and extension amounts. We also show that leasehold owners are broadly representative of the overall UK housing market in terms of owner and mortgage characteristics and price-to-rent ratio business cycles in Appendix Table A.1 and Appendix Figure A.8, as well as the geographic distribution of extensions in Appendix Figure A.9.

3.3   What t do we observe?

In Section 5, we will estimate r^*_{Kt} over time, where t is the transaction date provided by the Land Registry. The date captured by the Land Registry is the date in which the form to transfer the property deed from the buyer and the seller is signed by both parties. This typically occurs shortly after the parties sign a contract to settle on the transaction price, and a few weeks before the actual move occurs.

However, the date that the transfer deed is signed can be several months after the parties agree on a price. After the seller accepts an offer from the buyer, both parties must undergo a lengthy process of conveyancing, which involves finalizing mortgage details, conducting an inspection of the property, and drafting the relevant contracts. This process can be particularly long when the property is sold on a housing chain, meaning that the seller is simultaneously purchasing a home or the buyer is simultaneously selling a home. Chains can be include several houses and a delay at any single part of the chain can slow down the sale process for all the other buyers and sellers on the chain.

We can bound the amount of time between the date in which the buyer and seller agree on a price and the date in which the price is recorded by the Land Registry by using property listing data from Rightmove. If we assume that sellers stop posting house listings after agreeing on a price with a buyer, then the last date that a listing is posted must precede the date in which the buyer and seller agree to a price. The median amount of time elapsed between the last property listing on Rightmove and the date recorded by the Land Registry for transactions which have an associated Rightmove listing is 3 months, with a mean of about 4.5 months. The full distribution of the time elapsed between the listing date and the

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Land Registry date recorded is presented in Figure A.10. This suggests that the estimates we present have a lag of around 4.5 months, so shocks to the economy may take this much time to materialize in our r_{Kt}^* estimates.

4 Empirical Methodology

This section explains how to use the price gain from lease extension to estimate the natural rate of return of capital, for the major asset class of UK property. We also explain how we select the control group for our estimator, and present evidence in support of our identification assumption.

4.1 Pricing Leaseholds

We start by deriving the price of a leasehold. The price of a leasehold P_t^T, with T years until expiration, and an option to extend by 90 years on expiration, is

$$P_t^T = R_t \int_0^T e^{-\int_0^s y(u)du}ds + \max\left[0, (1-\alpha)R_t \int_T^{T+90} e^{-\int_0^s y(u)du}ds + ...\right] \tag{4}$$

This equation starts with the asset pricing identity of Equation (1), since the first term is the present value of service flows from housing over the first T periods before the lease expires. The second term represents the option value of additional extensions. $(1-\alpha)$ is the share of the price gain from extension, after deducting the negotiated payment to the freeholder and various costs that this negotiation entails. These terms multiply the present value of service flows from the lease, over the 90 year period after the extension. The ellipsis refers to the value of future extensions after $T + 90$, which have a similar structure. The max operator acknowledges that option value is non-negative—instead of extending the lease, the leaseholder can choose not to extend and receives zero payoff.

   This equation clarifies that the option value of lease extension raises the value of a leasehold. Consider two cases. First, suppose $\alpha = 0$. Then, the leaseholder receives the entire value of a lease extension. Provided that the value of an extended lease is positive, the leaseholder will always choose to extend—the option of lease extension is always "in the money".[9] Then $P_t^T = R_t \int_0^\infty e^{-\int_0^s y(u)du}ds$, meaning the price of a finite duration leasehold is the same as the price of an equivalent, but infinite duration asset. Since the property always costlessly exercises its option to extend, then the effective duration of the asset is infinite. A second instructive case is $\alpha = 1$. In this case, the leaseholder receives none of the value from

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[9]For simplicity, we ignore uncertainty in the rate of return of the asset.

extension, and the price of a leasehold is $P_t^T = R_t \int_0^T e^{-\int_0^s y(u)du} ds$. This price is the same as an asset with a duration of exactly T periods. The service flows after T have no value to the leaseholder, since they go to the freeholder. With intermediate values $\alpha \in (0, 1)$, the price of a T duration leasehold is between the duration T price, and the infinite duration price.

In the main analysis, we will assume that $\alpha = 1$, so that the value of extension goes entirely to freeholders and not leaseholders. Therefore there is no option value from lease extension. This assumption is appropriate given the institutional features of the UK property market. As we discuss at length in Section 6, the law recommends that leaseholders pay freeholders the entire value of lease extensions. In Section 6 we will also consider a bunching estimator, which identifies α using discontinuities in legally mandated lease extension payments when $T = 80$. Our estimates of natural rates change by little when we estimate α directly.

We remind the reader that at long maturities, the yield $y(t)$ converges to the natural rate of return of capital r_K^*, as Section 2 discusses. That is, by Equation (3) of Section 2, we have $\lim_{t \to \infty} y(t) = r_K^*$. Therefore we will use information from long term yields in order to estimate r_K^*.

4.2   A Difference-in-Differences Estimator of r_K^*

Next, we embed the formula for the lease extension price into a difference-in-differences estimator, in order to identify r_K^* for UK housing. We will study a difference-in-differences estimator Δ_{it} for the percent price change after lease extension:

$$\Delta_{it} \equiv \left[\log P_{it}^{T+90} - \log P_{i,t-h}^{T+h}\right] - \left[\log P_{jt}^T - \log P_{j,t-h}^{T+h}\right].$$

In this equation, $\log P_{it}^{T+90} - \log P_{i,t-h}^{T+h}$ is the price change for a property i bought h periods previously, which extends by 90 years from a duration of T years remaining, at time t. Property j is a suitably chosen control property, bought and sold in the same periods, with the same duration as property i prior to extension. Substituting the formula (4) for the price of a leasehold, with $\alpha = 1$, into the difference-in-differences estimator implies

$$\Delta_{it} = \log \left(\int_0^{T+90} e^{-\int_0^s y(u)du} ds\right) - \log \left(\int_0^T e^{-\int_0^s y(u)du} ds\right) + \Delta_{t,t-h} \left(\log R_{it} - \log R_{jt}\right). \quad (5)$$

In this equation, the first two terms represent discounting of the extended lease versus its control property. The final term is the difference between the growth rate of the service flow of housing, for the treatment versus the control group.

The identification assumption of our estimator is a form of "parallel trends". The growth

in service flows for the treatment and control properties, before versus after extension, must be the same. If so, then the final term from Equation (5) vanishes. Helpfully, in this case the difference-in-differences estimator no longer depends on the holding period h, meaning the estimator automatically corrects for the size of the holding period.[10]

Finally, in order to implement the estimator we parameterize the shape of the yield curve $y(s)$. We make the simple assumption that $y(s) = r_K^*$, so that the yield curve is horizontal and equal to its terminal value, which is the natural rate of return on capital. Shortly, we will see that this assumption accurately prices leaseholds in our quasi-experiment, even when there is significant variation in the slope of the yield curve at short horizons. The reason is that when T is large, the difference-in-differences estimator is primarily identified from long duration flows between T and $T + 90$. With this parameterization and the parallel trends assumption, we arrive at the final form of our estimator

$$\Delta_{it} = \log\left(1 - e^{-r_K^*(T+90)}\right) - \log\left(1 - e^{-r_K^* T}\right). \tag{6}$$

4.3 Advantages of the Estimator

The key advantage of our estimator is that it "differences out" short run variation in property prices, from either short term yields or in the service flow of housing. This differencing out applies regardless of the precise specification of the short run shocks affecting property prices. Therefore our estimator is able to estimate long run rates of return without relying heavily on a particular, potentially mis-specified, structural model.

In more detail, a first advantage of the estimator is that it differences out the service flow of housing, provided that that parallel trends assumption holds. Terms related to service flow do not appear in the value of the difference-in-differences estimator (6). The service flow includes taxes, depreciation, as well as the utility from consuming housing. This property of the estimator is appealing because the service flow is difficult to measure directly, especially for owner occupied housing. Moreover there may well be significant unobserved heterogeneity in this service flow across properties, which may vary across time. For instance, consider a temporary increase in demand for a narrow segment of London property. This shock does not affect the natural return but does affect service flows and prices for certain properties in the short run—our estimator eliminates this variation.

The second advantage of our estimator is that it differences out the effect of short term yields on asset prices, in order to estimate the natural rate. Therefore a transitory shock to yields, such as a monetary tightening, does not affect our estimate of the natural rate.

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[10]Appendix Figure A.11 shows that estimates of Δ_{it} are uncorrelated with h.

Intuitively, suppose that the duration before extension T is large. The price before extension capitalizes flows over the first T periods. The price after extension capitalizes flows over $T + 90$ periods. Therefore the price change at extension identifies the price of cashflows *after* T, only in the far future. However in practice, this argument is hard to demonstrate analytically.[11]

We illustrate the success of our estimator in differencing out short term yields numerically. In general, the success of our estimator depend on the shape of the yield curve before it converges to the natural rate r_K^*, and on the duration of the property before it extends. Numerically, our estimator successfully estimates r_K^* under an empirically reasonable yield curve, and when it is applied to long duration properties. The black solid line in Figure 3 presents one possible parameterization of the forward yield $y(s)$, where the forward yield curve $y(s)$ flattens out to r_K^* for $s \geq 40$ years, with r_K^* equal to 3.0 pp.[12] Given our pa-

Figure 3: A Parameterization of the Housing Yield Curve
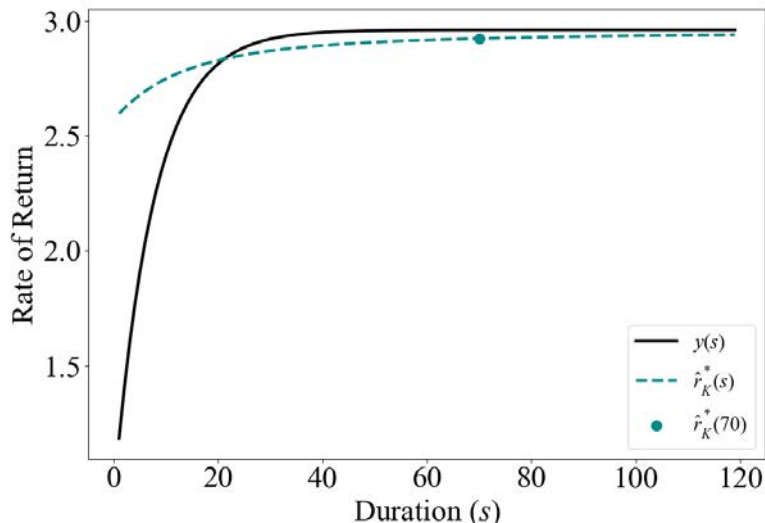
The black line presents one parameterization of the forward yield curve, $y(s)$, which is chosen so that its shape matches the forward curve implied by the UK 3-month LIBOR, the 10 year gilts and the 30 year gilts. $\hat{r}_K^*(s)$ is our estimator of r_K^* for each T, described below.

rameterization of $y(s)$, we can solve for $\log P_{it}^{T+90} - \log P_{it}^T$ for all T. Then, for each T, we can solve for our estimator of r_K^* as a function of T numerically. The resulting values of

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[11]In effect, we are studying the price of a positive coupon bond, which is not analytically tractable.

[12]We choose a flexible functional form $y(s) = \beta_1 - \beta_2 \cdot \beta_3^{-\beta_4(s-\beta_5)}$ and estimate the β parameters such that $y(0)$ is equal to the 3-month London Interbank Offered Rate (LIBOR), $y(10)$ is equal to the 10-year gilt yield, and the average of $y(s)$ for $10 \leq s \leq 30$ is equal to the 10 Year 20 Year gilt forward yield. For all the bond yields, we use the mean yield for our sample period. We present other possible parameterizations of $y(s)$ in Appendix A.5.

our estimator, which we term $\hat{r}_K^*(T)$, are plotted in blue in Figure 3. Our estimator $\hat{r}_K^*(T)$ closely approximates the true natural rate r_K^* for durations T after which the forward curve has flattened. We also plot the point estimate of $\hat{r}_K^*$ at $T = 70$, which is approximately the median duration of leaseholds at extension.

Our estimator produces tight estimates for a wide range of yield curves. In Figure 4a we present a time-varying yield curve for which the short-end fluctuates tremendously over time but the long end (r_K^*) is constant. For instance, the yield curve labelled d is very downward sloping, whereas yield curve g is very upward sloping. Then, the solid line in Figure 4b shows how our estimator $\hat{r}_K^*$ reacts to changes in the short-end of the yield curve, where $\hat{r}_K^*$ is estimated for a lease with 70 years remaining. The points corresponding to each instance of the yield curve in Figure 4a are labelled accordingly. For instance, point d in Figure 4b corresponds to estimates of r_K^* for the downward sloping yield curve d of Figure 4a. Our estimator is relatively stable despite the fluctuations in the short end of the yield curve, and remains within 0.1% of r_K^*. This logic suggests that our estimator can successfully estimate the dynamics of r_K^*, even in the presence of volatile shocks to short term rates.[13]

The reason why our estimator is able to provide a close approximation of r_K^* is because T is large, which effectively differences out most of the yield curve $y(s)$ for $s < T$. As T becomes smaller, the effect of the short-end on $\hat{r}_K^*(T)$ increases, meaning estimates of r_K^* become increasingly biased. To see this, we can consider an alternative estimator: the rent-to-price ratio of a freehold property, $\frac{R_{it}}{P_{it}^\infty}$. Like our estimator, the price-to-rent estimator cancels out the flow value of housing; it does not, however, difference out the short-end of the yield curve and is therefore far more susceptible to changes in short-term forward rates. The dashed line in Figure 4b indicates the value of the rent-to-price ratio, as the short-end of the yield curve shifts. Changes in the short-end of the yield curve affect the rent-to-price ratio by almost an order of magnitude more than they affect $\hat{r}_K^*(70)$. These results demonstrate that to effectively capture r_K^*, we must take the difference between *two* long duration assets; one does not suffice.

Our approach does not require a particular structural model of why short run yields vary. Short run yields may fluctuate due to cyclical movements in housing risk premia, safe interest rates or liquidity conditions. Bubbles of the form studied by Harrison and Kreps (1978) also manifest in short run yields, provided that these bubbles disproportionately affect short duration valuations. Regardless, our approach differences out this short run volatility in order to estimate the natural rate of return. As an additional, third advantage, our estimator also differences out any variation associated with "rational bubbles".[14]

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[13]In Appendix A.5 we show that we can use $\hat{r}_K^*(s)$ to bound the approximation error of our estimator, and therefore get a lower and upper bound for r_K^*.

[14]Our approach is related to how forward yields are calculated on financial assets such as zero coupon

Figure 4: "Differencing Out" the Short End

(a) $y(s)$ **(b)** $\hat{r}_K^*$ and Rent-to-Price
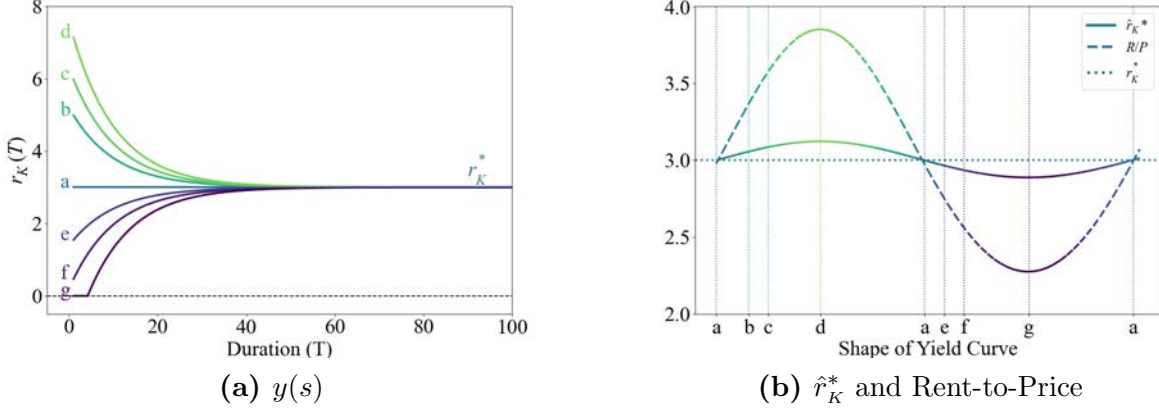
The figure illustrates the effect of fluctuations at the low-end of the yield curve on $\hat{r}_K^*$ and the rent-to-price rate, the rent-to-price ratio. Panel (a) presents several instances of the yield curve, $y(s)$ over time. Panel (b) indicates the estimates $\hat{r}_K^*$ and the rent-to-price ratio at each of these instances. For instance, point d on the right panel corresponds to the true value of r_K^*, the estimated value of r_K^*, and the rent-to-price ratio; given a yield curve d on the left panel. The the rent-to-price ratio is estimated such that $P_{it}^\infty / R_{it} = \int_t^\infty e^{-\int_t^s y(u)du} ds \equiv \frac{1}{R/P}$.

4.4   Implementing the Estimator and Selecting a Control Group

We now describe how to implement our estimator via nonlinear least squares, as well as the procedure to identify "control" properties. We also present evidence supporting our main identification assumption of parallel trends.

According to Equation (6), for each individual property i our difference-in-differences estimator is

$$\Delta_{it} = \log\left(1 - e^{-r_{Kt}^*(T_{it}+90)}\right) - \log\left(1 - e^{-r_{Kt}^* T_{it}}\right). \tag{7}$$

Here, we have generalized the expression of the estimator to allow a time varying natural rate, and to acknowledge that the duration of the property before extension, T_{it}, can vary. Equation (7) shows that we can estimate r_{Kt}^* by nonlinear least squares. The estimator is valid at any point in time, hence we can estimate the dynamics of r_{Kt}^*. Two statistics inform r_{Kt}^* in the estimator. First, the difference-in-difference Δ_{it} can be calculated for every property i, as the difference in price growth between the extending property and its control. Second, the covariance between Δ_{it} and the duration before extension T_{it} also helps to identify r_K^*.

We select a control group separately for each extending property, from neighboring properties of a similar duration that did not extend.[15] Let property p be a flat which

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bonds, but is available at far longer horizons than are normally available for financial assets.

[15]Therefore control properties are not treated, avoidiong the "forbidden comparisons" problem (Borusyak et al., 2021).

was purchased at time $t - h$, sold at time t, and extended for 90 years at some time $t - h < t - h + u \leq t$. Suppose p has duration $T + h$ at purchase and duration $T + 90$ at sale. The control pool for property p is the set, $Q^{(p)} = \{q : \text{Haversine Distance}(p, q) \leq x\text{km}\}$ where $x \in \{0.1, 0.5, 1, 5, 10, 20\}$, using the traditional Haversine Distance formula based on latitude and longitude coordinates.[16] In general, we choose x to be the smallest possible value such that both the control sets described below are non-empty. The controls for the purchase transaction are $Q_{t-h}^{(p)} = \left\{ q \in Q_p : t_q - h_q = t - h \text{ and } \left| \frac{(T+h) - (T_q + h_q)}{T+h} \right| \leq 10\% \right\}$ where $t_q - h_q$ is the purchase date and $T_q + h_q$ is the duration at purchase for property q. The controls for the sale transaction are $Q_t^{(p)} = \left\{ q \in Q_p : t_q = t \text{ and } \left| \frac{T - T_q}{T} \right| \leq 10\% \right\}$ where t_q is the sale date and T_q is the sale duration for property q. Note that we choose controls that have a lease term close to what the extended property p would have had at sale had it had *not* been extended.

Then, the control purchase price for extended property p is given by $P_{p,t-h} = \text{mean}\{P_{q,t_q - h_q} : q \in Q_{t-h}^{(p)}\}$ and the control sale price is given by $P_{p,t} = \text{mean}\{P_{q,t_q} : q \in Q_t^{(p)}\}$. Under this methodology, we are able to identify controls for 130,430 of our 131,839 lease extension experiments. This serves as our primary control measure.

Our identification assumption is parallel trends: growth in the service flow of housing should not differ for extending properties and controls. We provide two tests in support of this assumption. First, our treatment and control groups are close to being balanced on observable characteristics. We use the property characteristics data provided by Rightmove and Zoopla. The results of regressing the same hedonic characteristics on an extension dummy are presented in Panel A of Table 2. Flats that have been extended at some point tend to have slightly better amenities than neighbouring flats of similar lease length. The differences in magnitude are small, however. For instance, the mean number of bedrooms, after controlling for geographical region and lease length, in an extended property is 1.8, as opposed to 1.7 in a property that has not been extended. Average floor area is 68.5 vs 64.4 square meters for extended and non-extended properties, respectively. As a whole, rental prices, which ought to be representative of the aggregate quality of the property, are about 8% higher in extended properties.[17]

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[16]The Haversine formula is given by $d(p, q) = 2r \arcsin \left(\sqrt{\sin^2 \left(\frac{\phi_q - \phi_p}{2} \right) + \cos \phi_p \cdot \cos \phi_q \cdot \sin^2 \left(\frac{\lambda_q - \lambda_p}{2} \right)} \right)$ where (ϕ_p, λ_p) and (ϕ_q, λ_q) are latitude-longitude pairs for points p and q respectively and r is the radius of the Earth.

[17]In Appendix Figure A.12 we show how our main hedonic variables vary with log price, controlling for Local Authority fixed effects. Notice that property size, bedroom number, and the log of rental prices all vary approximately linearly with log price. We also regress our hedonic characteristics on an experiment fixed effect and plot the density distribution of the residuals in Appendix Figure A.13. These residual plots visually confirm how similar the amenities distribution is between extensions and controls.

Table 2: Hedonic Characteristics in Extended vs Non-Extended Flats

Panel A: Levels

	(1) Num Bedrooms	(2) Num Bathrooms	(3) Num Living Rooms	(4) Floor Area	(5) Age	(6) Log Rental Price
Extension	0.08***	0.04***	0.01***	4.08***	7.46***	7.89***
	(0.005)	(0.004)	(0.001)	(0.246)	(0.428)	(0.239)
Fixed Effects	✓	✓	✓	✓	✓	✓
N	1,353,712	1,066,889	940,298	1,076,266	839,631	747,548

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Panel B: Differences

	(1) Δ Num Bedrooms	(2) Δ Num Bathrooms	(3) Δ Num Living Rooms	(4) Δ Floor Area	(5) Δ Log(Rent)
Extension	0.00	-0.00	-0.00*	-0.03	-0.71
	(0.00)	(0.00)	(0.00)	(0.12)	(0.64)
Fixed Effects	✓	✓	✓	✓	✓
N	200,251	154,605	137,590	151,345	76,772

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Panel A reports level of hedonic characteristics for extended properties relative to their non-extended counterparts. It presents estimates of β for the specification, $X_{it} = \alpha + \beta \mathbb{1}(\text{Extended}_{it}) + \Gamma_{it} + \epsilon_{it}$, for several hedonic characteristics X_{it} where Γ_{it} are 10-year duration bin × Local Authority fixed effects. The sample is leasehold flats which appear at least once in the Land Registry Transaction Data Set. Panel B reports renovation rates in extended properties relative to their non-extended counterparts. It presents estimates from regression equation (8). For Columns 1-4, the sample includes properties with two separate Rightmove or Zoopla listings corresponding to two separate transactions. For Column 5, the sample includes properties that transact twice and have distinct rental price data around each transaction. In both panels, log rent is multiplied by 100. Standard errors are clustered at the purchase year, sale year and Local Authority level.

As a second test of parallel trends, we show that extending properties are no more likely to renovate than controls; nor do rents grow by more for extensions. As such, the service flow of housing seems to behave similarly for control and treated groups, in support of the identification assumption. We study Rightmove and Zoopla data for properties that have two distinct listings. For these properties we estimate the following specification,

$$\Delta_h \text{Hedonic Control}_{it} = \alpha + \beta \times \mathbb{1}\text{Extension}_{it} + \Gamma_{it} + \epsilon_{it} \tag{8}$$

where Γ_{it} are Local Authority × duration × year of hedonic characteristics listings fixed effects. The regression results are presented in Panel B of Table 2. We find no significant evidence that extended properties renovate at a different rate than other properties. Importantly, Column (5) also indicates that the rental value of the property, an observable proxy for the service flow, does not change before and after extension.

We also construct a second control measure, that explicitly accounts for hedonic differences between the treatment and the control group. In particular, we run the following

regression,

$$P_{it} = \alpha + \Gamma \times \text{Hedonic Controls}_{it} + \epsilon_{it}$$

where the hedonic controls are number of bedrooms, number of bathrooms and floor area. Then, we create a residualized price variable, $P_{it}^{(res)} = \overline{P} + \epsilon_{it}$ where $\overline{P}$ is the mean price level across all flats. $P_{it}^{(res)}$ represents the price of property i at time t, taking out the average effect of its hedonic characteristics. We can use the previously described procedure to condition on hedonic characteristics, by residualizing prices for both extensions and their controls.

The advantage of this second measure is that it controls for potentially confounding sources of heterogeneity in price. The disadvantage is that we only observe hedonic characteristics for a fraction of our experiments (e.g. we have data on bedroom count for 109 thousand and data on floor area for 87 thousand of our 131 thousand experiments), so it reduces our already limited sample. As we will see in Section 5, hedonic controls do not significantly affect our estimates, so the majority of our analysis is conducted using the first control measure. We use the hedonic control measure for robustness on all our results.[18]

5 Empirical Results

This section presents our main empirical results. Using our difference-in-differences approach, we estimate the level of the natural rate for UK property to be 3%. We also investigate the dynamics of the natural rate between 2003 and 2023. We find a decline from about 4.8% in 2006 to 2.3% in 2023. Moreover the fall takes place before 2020 while the natural rate is relatively stable afterwards—despite rising yields on safe assets over the same period.

5.1 Event Study Representation of Lease Extension

The duration of properties when they extend is long. Leaseholds that extend by 90 years have a median duration at sale of 157 years—meaning had they not extended, they would have had a duration of 67 years at sale.[19]

What effect does lease extension have on property valuations? To visualize this, we create a pseudo event study for the leases that extend by 90 years. We calculate the mean difference in log price between the extending property and its controls. We can calculate this price difference twice: once before and once after extension. We plot these price differences for all of the extension experiments, using time to extension, u, as our x-axis variable. The result

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[18]In much of our analysis, we focus on extensions of 90 years. Appendix Table A.2 repeats the analysis of Table 2 on that subsample only.

[19]Appendix Figure A.14 displays this information in a histogram, also including the duration of the control observations that did not extend.
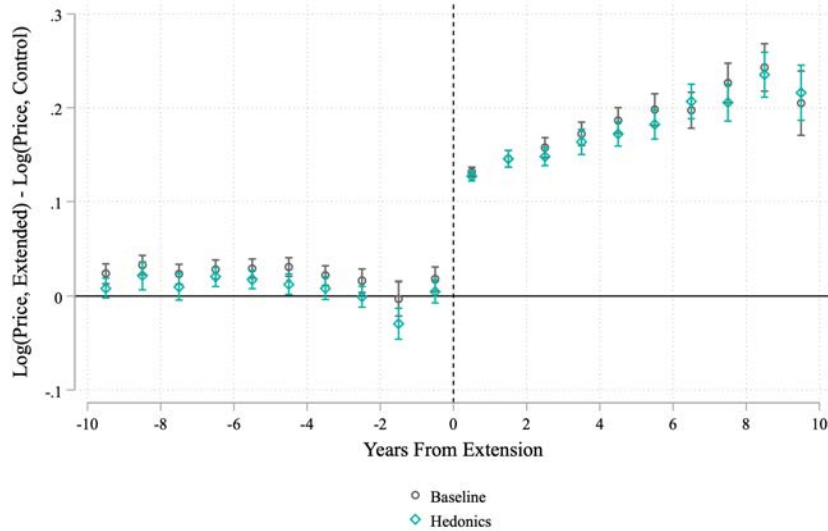
Figure 5: Event Study Representation of Lease Extension

The figure presents the difference in transaction price between extended properties and their controls before and after extension. The x-axis is the difference between transaction time and extension time, which we identify using the lease registration date for extended properties. The grey circles use mean log(price) as the y-axis variable and the blue diamonds use log(price) residualized on hedonic characteristics as the y-axis variable. We include 95% confidence intervals for each point estimate.

is in gray in Figure 5. Though we only observe each extension experiment once on either side of extension, we have stacked multiple experiments together according to their distance from extension. Therefore we can study the trends in prices versus the control before and afterward. The result is similar to an event study. Upon extension, the difference in log price between extended properties and their controls jumps by 0.12 log points, or about 12.7%. The difference continues to grow after this point because as time from extension increases, the lease term falls more, which results in a greater predicted difference in price between extension and control according to our simple asset pricing model. Moreover, consistent with our identification assumption, there are small and statistically insignificant "pre-trends" before lease extension. The difference-in-difference price increase at extension, Δ_{it}, is a key input into our estimation of r_K^*.

As we saw in Section 4.4, extended properties tend to have slightly better amenities than their non-extended counterparts. We can see this in Figure 5, where extended properties consistently sell at a time-invariant premium relative to controls before being extended. Given the parallel trends assumption and the absence of a pre-trend, this level shift is differenced out by our difference-in-difference estimate. However, the estimates plotted in blue in Figure 5 assuage any lingering worries by showing that the premium is largely accounted for by differences in property characteristics; i.e. once we control for hedonic variables, even small

pre-extension differences in price between extensions and their controls disappear. Most importantly, our difference-in-difference estimates are essentially identical with and without hedonics, as we shall see formally in Section 5.2.[20]

5.2 Estimating the Level of r_K^*

We present estimates of the level of r_K^*. We estimate Equation (7),

$$\Delta_{it} = \log\left(1 - e^{-r_K^*(T_{it}+90)}\right) - \log\left(1 - e^{-r_K^* T_{it}}\right)$$

using all lease extension experiments i. There are two sources of variation identifying r_K^*. First, the estimating equation predicts that the price gain is larger when r_K^* is lower, because a lower value of r_K^* raises the value of duration. Second, the gain from extension varies with duration, T_i. Leaseholds with shorter remaining terms will receive the benefit of extension sooner, which leads to greater price growth upon extension.

Figure 6: Duration Before Extension vs. Price Gain After Extension
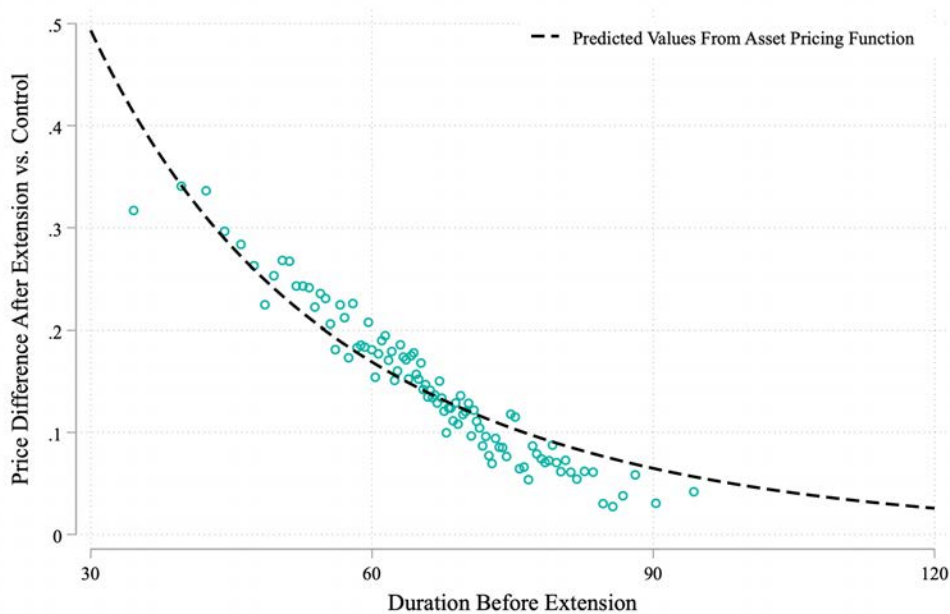
The figure is a binscatter of our difference-in-difference estimator against duration before extension, T, with 100 bins. The sample includes leases that were extended for 90 years. The black line shows fitted estimates of Equation (7).

Now we plot the variation from the data that identifies r_K^* according to our estimator. Figure 6 binscatters the difference-in-difference estimates Δ_{it} for various durations T_i, for

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[20]Figure A.15 and Figure A.16 show the event studies for lease extensions of sizes other than 90 years.

the sample of leaseholds that extend by 90 years. As predicted by the estimating equation, the percent increase in property value as a result of extension increases in the duration before extension—ranging from only 7% for 90-year duration properties to more than 30% for properties with duration of 40 years at extension. The black curve in Figure 6 presents fitted estimates from our nonlinear estimator. Inspection of Figure 6 indicates that our estimate of r_K^* is a good fit across both the average price gain after an extension, and variation in the price gain across T—the two statistics that identify r_K^*.[21]

Table 3: Estimated r_K^*

	Constant	Flexible				N
	(1)	(2)	(3)	(4)	(5)	(6)
		$T = 50$	$T = 60$	$T = 70$	$T = 80$	
No Hedonics	3***	2.93***	3***	3.06***	3.12***	43,083
	(0.02)	(0.03)	(0.02)	(0.02)	(0.03)	
No Hedonics (Hedonics Sample)	2.95***	2.85***	2.94***	3.03***	3.11***	27,636
	(0.02)	(0.03)	(0.02)	(0.02)	(0.04)	
Hedonics	2.94***	2.82***	2.93***	3.04***	3.16***	27,636
	(0.02)	(0.03)	(0.02)	(0.02)	(0.04)	

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 01$

Column 1 presents estimates of r_K^* from Equation (7) for 90 year extensions. Column 2-5 present estimates of $r_K^*(T)$ for $T \in \{50, 60, 70, 80\}$, where we parameterize r_K^* linearly as a function of T, $r_K^*(T) = \gamma + \beta \cdot T$. The first row presents estimates using the baseline procedure to find controls described in Section 4.4, which utilize the raw transaction price variable as the main outcome variable. The second row presents estimates using the raw transaction price variable, but restricted to the sample of observations which are not missing hedonic characteristics. The third row presents estimates of r_K^* and $r_K^*(T)$ produced using transaction price residualized on hedonic characteristics as the main outcome variable, as described in Section 4.4. Standard errors are heteroskedasticity robust.

Next, we present estimates of the r_K^* implied by Figure 6. We estimate as $r_K^* = 3.0\%$, indicated in Column 1 of Table 3. The second and third rows of Table 3 show that these estimates are largely unchanged by inclusion of hedonic controls, as already demonstrated by the event study plot in Figure 5.[22]

As we discussed in Section 4.3, our estimator is not perfectly accurate when the short end of the yield curve for housing is upward sloping. In this case, our estimated r_K^* will vary with duration T (i.e. $y(T)$ will be upward sloping if $y(T)$ is upward sloping, and vice versa if $y(T)$ is downward sloping). Therefore, we can estimate the degree to which the short-end of $y(s)$ affects our estimates by parameterizing our estimate of the natural rate of housing as $y(T) = \gamma + \beta \cdot T$. We exploit variation in duration at the time of extension to estimate both γ and β. In Columns 2-5 of Table 3, we present estimates of $y(T)$ under this more flexible

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[21]In Figure A.17 we present a binscatter using properties that were extended for more than 700 years.

[22]In Appendix Table A.3, we estimate r_K^* using all extension lengths, which produces very similar results.

form for our sample range of T. We can see that $y(T)$ varies little over the range we observe it, suggesting our estimates are close to the true r_K^*.[23,24]

One concern about our estimate may be that short duration leaseholds have an additional liquidity premium because banks are less willing to issue a mortgage against shorter duration leaseholds. We investigate this concern in Appendix A.6 and find that liquidity premia do not seem to affect our results: from survey data, short duration leaseholds seem to have similar characteristics to other leaseholders and seem to be similarly liquid. Explicitly incorporating liquidity frictions into our estimator affects the value of r_K^* little. In Appendix A.7, we investigate the seasonality of our estimate of r_K^*. Spot UK house prices are highly seasonal (Ngai and Tenreyro, 2014). Reassuringly, we find that our estimate of r_K^* is not seasonal, consistent with its interpretation as a long run rate of return.

5.3   Trend Dynamics of r_K^*

Figure 7: Price Change From Extension, Over Time
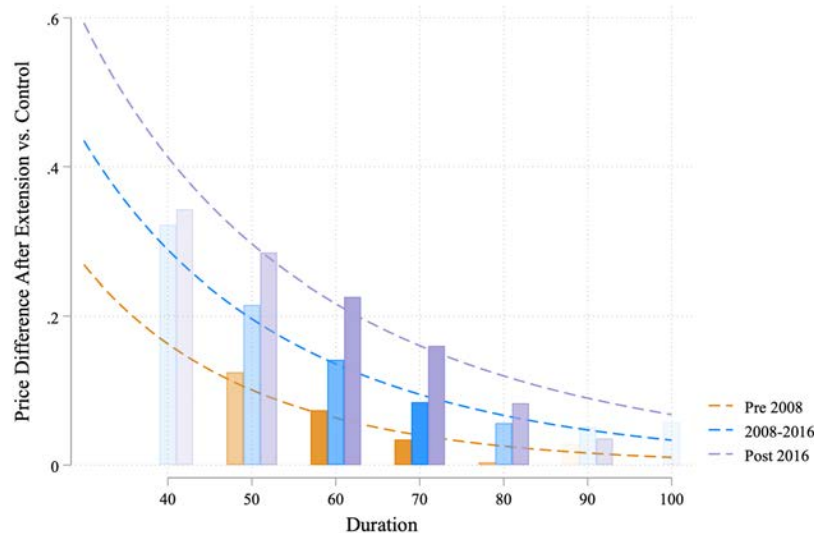
The figure shows the mean difference-in-difference estimate for various durations in the pre-2008, 2008-2016 and post-2016 periods. The bars show the level of Δ_{it} for each bin and the dashed lines shows fitted estimates. The sample includes only 90-year extensions. Bars are shaded proportionally to the number of observations that make up the bar.

Now we present our first key result: estimates of the trend dynamics of r_K^*. Interest rates

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[23]In Figure A.18 we estimate the event study of Figure 5 separately for different durations, which also documents the lack of pre-trends at all durations.

[24]We can also choose more flexible parameterizations of r_K^* which capture its curvature. For example, the forward sloping parameterization of $y(T) = \gamma + \beta \frac{(1-e^{-T})}{T}$ estimates that $y(T)$ is even more flat that the linear parameterization.

have been falling in the UK, as in much of the rest of the world, for at least four decades. Can the same be said for r_K^*? We find a sizable fall in r_K^* across the entire yield curve between the start and end of our sample period. This decline in r_K^* is depicted in Figure 7. In this figure we show that at all durations, the price growth associated with lease extension Δ_{it}, has increased by approximately 20pp between the beginning and end of our sample. The opacity of the bar graph is shaded by the number of observations in the bar, and contains only 90 year extensions. An increase in the price growth from lease extension implies that the value of long duration cashflows has increased—that is, r_K^* must have fallen.[25]

Given our large data set, we are able to estimate the dynamics of r_{Kt}^* with precision. We start in 2003, at the beginning of our sample. We have already presented the estimates in Figure 1 in the Introduction, which contains estimates of r_{Kt}^* at annual frequency. Our time varying estimate includes all lease extensions, instead of only 90 year extensions, in order to raise the precision of our estimates; the shaded region is 95% confidence intervals for the full sample of leases.[26]

The estimates show that the fall in r_K^* has been relatively consistent throughout this entire sample period. Between 2003 and 2023, we estimate r_K^* has fallen from around 4.8% to 2.3% — a more than 50% decline. The magnitude of this decline is large, corresponding to a doubling of the long run expected price-rent ratio. Notably, our estimates are stable during the 2020 pandemic, despite considerable volatility in shorter term asset prices during this period.

Our estimate of a declining *capital* yield before 2022 is related to the well known fact that *government bond* yields fell around the world before 2022 (e.g. Holston et al., 2017). However bond yields could have fallen due to factors that are specific to safe assets, such as quantitative easing or rising demand for the safety and liquidity of government bonds (e.g. Krishnamurthy and Vissing-Jorgensen, 2012; Caballero and Farhi, 2018). These safe asset-specific factors may be unrelated to the yield on capital in the real economy. To the contrary, our estimates show that capital yields also experienced large trend declines in yields before 2022, similar to bonds.

5.4   Real Time Dynamics of r_K^*

As mentioned earlier, one useful feature of our methodology is that it can be implemented in real-time, with publicly available data. Figure 8 plots our estimate of the natural rate of

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[25]In Figure A.19 we plot the event study of Figure 5 separately for each period, which also documents the lack of pre-trends at all times.

[26]Appendix Figure A.20 plots the time varying estimate r_{Kt}^* using various extension amounts shows similar results.

return, at monthly frequency until June 2023, the last month for which UK housing data are available. The shaded area represents the 95% confidence interval of our estimate of natural rates and shows how precise our standard errors are, even when estimating r^*_{Kt} at a monthly frequency. The tight standard errors result from having a reasonably large sample size of around 900 lease extensions per month.

Figure 8: Monthly estimates of r^*_K and rate of return on government bonds
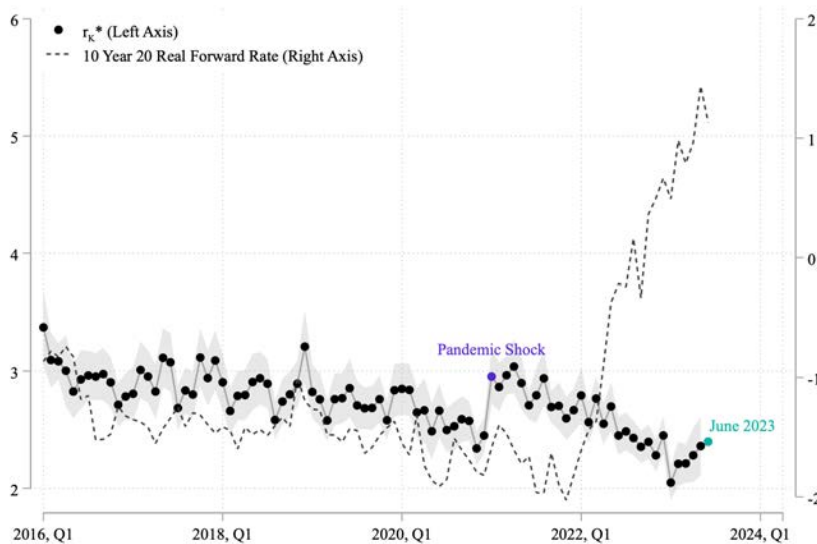
The solid line presents r^*_{Kt} estimated at a monthly frequency. The dotted line presents the real 10 year 20 year forward rate. The lines are plotted from January 2016 to June 2023. 95% confidence interval is shaded.

As discussed in Section 3.3, due to the natural lag between the agreement on terms of the contract and the formal recording of a transaction, our r^*_{Kt} estimates should be seen as reflective of the market expectation of the natural rate of capital with a lag of about 5 months. Two observations from the monthly time-series are worth mentioning: the first is that there is a spike in r^*_{Kt} at the start of the Pandemic Recession which is entirely driven by London. This likely represents a housing-specific shift in markets' expectations of possibly long lasting effects of the COVID-19 pandemic (e.g. a shift in ζ^*). However, this shift appears to have reversed with the end of the pandemic. In the future, our estimates should help policymakers to understand in real time whether other shocks have also affected long run expectations.

Second, the recent post-pandemic tightening cycle of monetary policy starting in January 2022 has had a large effect on real yields on safe assets, even at relatively long durations. The dashed line in Figure 8 indicates that the 10-year-20 real forward rate on government bonds has risen by about 300 basis points since the beginning of 2022. An uptick in r^*_{Kt} has

only recently started to materialize and is modest in magnitude, though it is important to remember that these estimates include a lag of about 5 months. The divergence in trends between the two series, thus far, suggests that a wedge has appeared between long-term bond rates and r_K^*. Time will tell how this wedge will close. There are two possibilities. On one hand, high interest rates may gradually lead to a rising natural rate of return, and a collapse in house prices. On the other, long-run interest rates will fall. According to this second possibility, the current rise in the long term forward rate is a temporary deviation from r^*, which has remained low. Our real-time estimates of r_K^* will allow us to track this convergence process on a monthly basis, with robust standard errors of about 0.09pp.

Our real-time estimates will therefore contribute to the active debate about whether the current increase in yields around the world will prove transitory. For instance, Blanchard (2023) argues that after the current period of monetary tightening, yields will revert to the low levels of the "secular stagnation" era. The real time dynamics of our measure of r_K^* will help to inform this debate.

Overall, we have estimated a trend fall in the natural rate of return, for the specific asset of UK housing, so there may be a question of how representative our estimates are of other natural rates, such as the natural rate of interest. We believe that other kinds of capital likely experienced a similar trend fall in the natural rate of return. Recall the definition of the natural rate of return, $r_K^* \equiv r^* + \zeta^* - g_P^*$. The level of $r_K^* \equiv r^* + \zeta^* - g_P^*$ might differ across assets due to different risk premia ζ^* or expected capital gains g_P^*. However, suppose that the fall in r_K^* for UK housing is due to factors that are not specific to housing, i.e. not due to housing-related changes in capital gains or risk premia. In that case, other forms of capital should also experience a similar trend decline in natural rates of return, because the standard arbitrage conditions mean that risk-adjusted rates of return are equal across different forms of capital, especially in the long run.

In Appendix A.8, we present suggestive evidence that the decline in r_K^* for housing is not due to housing specific factors. In particular, we estimate long run housing risk premia and expected capital gains using a standard Vector Autoregression approach (e.g. Campbell and Shiller, 1988), and show that neither long run UK housing risk premia, nor capital gains for housing, can account for the trend decline of r_K^* for housing. As such, we infer that the trend decline may also be relevant for other assets, though this exercise is tentative due to the uncertainty of the VAR based procedure.

5.5 Advantages of Natural Experiments and Microdata

Our microdata and quasi-experimental approach complements the prevailing approach to estimating natural rates, which uses structural time series methods (Laubach and Williams, 2003; Holston et al., 2017). This approach is rightly celebrated for tackling a key problem in macroeconomics, but is subject to concerns about model mis-specification. For instance, the Laubach & Williams estimate of r^* broke down after the sharp decline in GDP at the start of the Pandemic Recession. The reason appears to have been that the structural model of short run shocks to the economy was not well specified during the Pandemic (Williams, 2023). Our method is potentially more robust to model mis-specification, because our natural experiment approach "differences out" short run shocks to asset prices, without requiring a structural model of these shocks—meaning, for instance, that our estimates are available through the Pandemic.

Another benefit of micro-data is precision. Even before the Pandemic Recession, the HLW standard errors were relatively large. For the UK the mean standard error of r^* between 2000 and 2019 was 4.3pp — an order of magnitude greater than our real-time quarterly standard errors. The difference in precision is an intrinsic advantage of using microdata. Time series data is relatively uninformative about the long run, meaning inference about long run parameters is uncertain (Farmer et al., 2021). The micro-data approach instead uses informative cross-sectional variation to make inferences about long run parameters. We elaborate on the differences between our estimates and those of Laubach & Williams in Appendix A.9.[27]

Our microdata-based approach to estimating r^*_K builds on the key insight of Giglio et al. (2015), which was the first paper to observe that UK properties are uniquely well suited to estimating long term housing yields, because of their varying duration. Giglio et al use a cross-sectional comparison of freeholds and leaseholds with different duration to estimate the level of r^*_K. Building on their insight, our approach uses the quasi-experiment of lease extensions to estimate the dynamics of r^*_K. We now elaborate on some advantages of quasi-experiments compared with cross-sectional variation.

An important difference between the cross-sectional approach and the quasi-experimental approach is that in the former, the primary source of variation for duration is across properties rather than within properties. Long duration properties might have differences in the service flow of housing. For example, freehold flats might have higher quality construction. As such, the cross-sectional approach relies on detailed hedonic characteristics to control sources of variation in property price associated with the service flow. However with either

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[27]Also, the structural approach requires a model of inflation expectations. Our estimates are automatically in real terms.

the cross sectional or the quasi-experimental approach, unobserved heterogeneity may bias the estimates.

To gauge the effect of unobserved heterogeneity on the quasi-experimental and the cross sectional estimates, we study the sensitivity of the estimates to *observed* heterogeneity, in the spirit of Altonji et al. (2005) and Oster (2019). We therefore estimate r_K^* using both the quasi-experimental and the cross-sectional approach, controlling for over 100 different variations of hedonic characteristics. In one variation we do not include any controls. In another, we allow price to vary linearly with number of bedrooms and floor area, and in yet another we allow price to vary quadratically with these same controls. In the most extreme case, we control for fixed effects of the following seven characteristics: number of bedrooms, number of bathrooms, floor area, age, heating type, property condition rating, and availability of parking.[28] The other variations include all possible subsets of these seven characteristics.

We then plot our estimates of r_K^* under each variation for both the quasi-experimental and cross-sectional methodologies in Figure 9. Under the cross-sectional approach, the estimates vary tremendously from 1.3% (in the case of no hedonic controls) to more than 7%. In contrast, our quasi-experimental estimates of r_K^* are highly stable around 3%. These results provide tentative evidence that our quasi-experimental methodology offers estimates of the natural rate of return on housing that are relatively robust to unobserved heterogeneity.

6 Measuring the Option Value of Lease Extension

So far, we have assumed that there is no option value from lease extension, because leaseholders pay the market value of extension to freeholders when they extend. This section studies option value and its consequences for estimates of r_K^*, in three steps. First, we present a simple framework that encompasses a key feature of the UK law on extensions, namely discontinuities in the cost of lease extensions when leaseholds have 80 years remaining. Second, we use this framework to derive discontinuity based tests about whether there is option value. These tests indicate that the baseline estimate of the fall in r_K^*, which ignores option value, is a lower bound for the true fall in r_K^*. Third, we generalize our the difference-in-differences estimator to point identify the size of option value. We find that our baseline estimate of

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[28]The fixed effects controls are the same as the main specification Giglio et al. (2015). Giglio et. al. add an indicator variable for properties with missing hedonics and includes them in the main sample — whereas the current exercise restricts the sample with controls to properties that have hedonic characteristics. We remove properties without hedonics because these properties will not be affected by different ways of adding controls. Moreover, if controls are important, then including properties with missing control information may lead to omitted variable bias.

Figure 9: Stability of r_K^*, Controlling for Observables
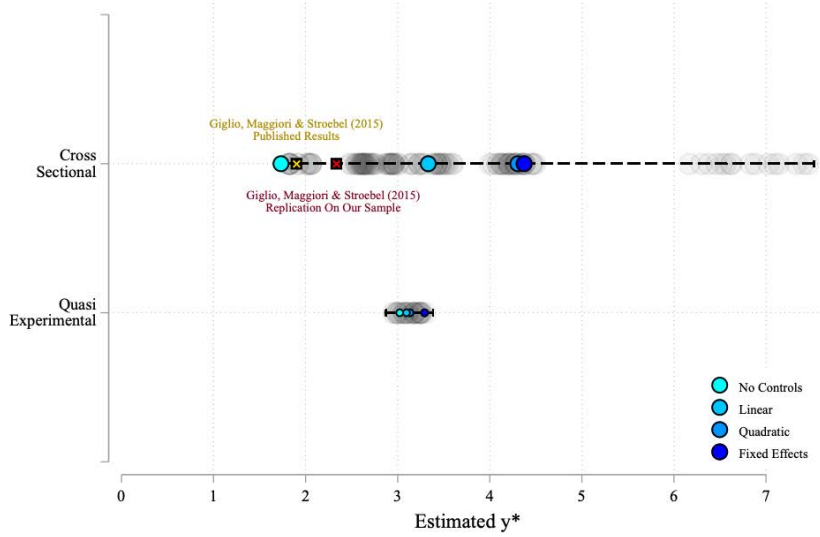
The figure presents estimates of r_K^* under various choices of hedonic controls. For the cross-sectional estimates, we estimate $\log P_{it}^T - \log P_{it}^\infty = \log(1 - e^{-r_K^* T})$ by NLLS, where P_{it}^∞ is the price of a freehold transacted in the same quarter and Local Authority as a T duration leasehold. For the quasi-experimental estimates, we follow the methodology described in Section 4. For each methodology, we perform over 100 estimations, controlling for different combinations of hedonic characteristics. We indicate in various shades of blue four important sets of controls: no controls, linear controls, quadratic controls, and the full set of hedonic fixed effects. The gold cross presents the r_K^* estimate from Giglio et al. (2015), and the red cross presents our replication of their estimate, using the full data from 2003-2023.

r_K^*, which ignores option value, is an excellent approximation.

6.1 A Framework for Lease Extension Costs

We now summarize the institutional framework of lease extensions and develop a simple model of this framework.

Tribunals. Leaseholders are legally entitled to a 90-year extension by the 1993 Leasehold Reform, Housing and Urban Development Act. According to the act, lease extensions ought to be priced at their market value, the present value of service flows from the lease. However the leaseholder and freeholder negotiate the size of the market value, by independently hiring surveyors. If there is no agreement, a Residential Property Tribunal determines the value of the extension after a costly legal process. Tribunals require that the extension is 90 years long, and price extensions using a two part formula, which requires an estimate of *reversion value* and *marriage value.*

Reversion value. The reversion value is the value of the lease extension according to a yield assumed by the tribunal. Therefore reversion value of property i at time t satisfies the

formula

$$RV_t^T = \frac{R_{it}}{r_{RV}} \left(e^{-r_{RV}T} - e^{-r_{RV}(T+90)} \right),$$ (9)

which is the value of the lease extension according to a Gordon Growth model, with a discount rate of r_{RV} and a current service flow R_{it}. In practice, the service flow R_{it} is imputed from the price of an observably similar freehold property. The discount rate is fixed by the tribunal at $r_{RV} = 5\%$.

Marriage value. The marriage value is the tribunal's estimate of the market value of the lease extension, given by $MV_{it}^T = P_{it}^{T+90} - P_{it}^T$, which is the difference in price between property to be extended, with duration T, and the price of the same property with duration $T+90$. The price of the property with $T+90$ duration is imputed from the transacted prices of observably similar properties. Provided that the tribunal imputes correctly, the marriage value is the market value of the lease extension, and satisfies

$$MV_{it}^T = \frac{R_{it}}{r_{Kt}^*} \left(e^{-r_{Kt}^* T} - e^{-r_{Kt}^*(T+90)} \right).$$ (10)

Here, we have written the marriage value as the market value of a lease extension according to a Gordon Growth model, where the natural rate expected by the market, r_{Kt}^*, enters the equation for market value.

Reversion value vs. marriage value. Our approach makes extensive use of the following observation. Compare Equation (9) and Equation (10) for reversion and market value. Reversion value calculated by the tribunal is greater than market value, if and only the natural rate is greater than the 5% yield assumed by the tribunal.

Discontinuity at 80 years duration. There is a discontinuity in the tribunal assessed cost of the lease extension when 80 years remain on the lease. For leases with more than 80 years remaining, the tribunal dictates that only the reversion value must be paid, whereas for leases with less than 80 years remaining, the cost is the average of the reversion value and the marriage value.

Tribunal costs to leaseholder. There are significant costs to the leaseholder of appealing to the tribunal. These costs include time and information costs, uncertainty from the outcome of the tribunal, costs of hiring a survey to value the lease, and legal costs associate with the tribunal. Moreover the law dictates that leaseholders must cover all of the freeholder's legal and surveyor fees associated with extension. We will denote these costs for property i by γR_{it}, which we assume for simplicity scales with the current service flow R_{it}.

Leaseholder's participation constraint. The leaseholder also has a participation constraint—if the tribunal assessed costs of extending are greater than the value of extending, the leaseholder can opt not to extend. Therefore the freeholder should reduce costs down

to the market value of extension, so that a transaction can occur. The converse is not true: if the tribunal associated costs are less than the market value of extending, the leaseholder will choose the tribunal costs instead of paying the freeholder the market value of extension.

Total extension costs. We can summarize the cost of extension κ_{it}^T, of a property i with T years remaining at time t, as

$$\kappa_{it}^T = \begin{cases} \min\left\{ RV_{it}^T + \gamma R_{it}, MV_{it}^T \right\} & T \geq 80 \\ \min\left\{ \frac{RV_{it}^T + MV_{it}^T}{2} + \gamma R_{it}, MV_{it}^T \right\} & T < 80 \end{cases} \tag{11}$$

Equation (11) recognizes that the cost of extension κ_{it}^T is the minimum of the market value of extension from the present value of rents; and the tribunal recommended value of extension plus the costs of a tribunal settlement. For simplicity, we have equated the market value of extension to MV_{it}^T, the marriage value. The tribunal recommended settlement varies discontinuously at 80 years, provided that the market value and the reversion value are not equal.[29]

Price of a T duration property. We assume property prices satisfy a simple no arbitrage assumption. Therefore the price of a property P_{it}^T, that has not extended, must equal the price of an otherwise similar property that has extended, after deducting lease extension costs. That is, prices satisfy

$$P_{it}^T = P_{it}^{T+90} - \kappa_{it}^T. \tag{12}$$

We denote $\alpha_t^T = \kappa_{it}^T / MV_{it}^T \leq 1$ as the share of the extension value that is lost by the leaseholder, noting that under our assumptions α does not depend on the service flow i.

Remark on option value. When $\alpha_t^T = 1$ we say there is *zero option value* and when $\alpha_t^T < 1$ there is *positive option value*. This terminology acknowledges that when $\alpha_t^T = 1$ for all T, then all of the gains of lease extension are lost to the leaseholder. As a result, the option to extend the lease has no value to the leaseholder ex ante.

6.2   A Discontinuity Based Test for Option Value

We now use our framework for lease extension costs to derive a discontinuity based test of whether there is positive option value. We summarize our predictions in the following proposition.

Proposition 6.1. *There exists some value $\bar{r}_K \leq r_{RV}$ such that:*

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[29]For exposition, we assume that the tribunal, the leaseholder and the freeholder all agree on the market value of the extension. Our qualitative and quantitative results are not affected by this assumption.

1. *If the natural rate satisfies $r^*_{Kt} \geq \bar{r}_K$ then: (i) there is zero option value at all years of duration remaining, that is, $\alpha^T_t = 1$ for all T; (ii) the price of a leasehold is continuous in duration as the property's duration falls below 80 years, so $\lim_{T \to 80^-} P^T_{it} = \lim_{T \to 80^+} P^T_{it}$.*

2. *If the natural rate satisfies $r^*_{Kt} < \bar{r}_K$ then: (i) there is positive option value above 80 years in duration, that is, $\alpha^T_t < 1$ for all $T > 80$ and option value discontinuously falls at 80 years, so that α^T_t discontinuously increases at $T = 80$; (ii) the price of a leasehold discontinuously falls as the property's duration falls below 80 years, so $\lim_{T \to 80^-} P^T_{it} < \lim_{T \to 80^+} P^T_{it}$.*

This proposition, which we prove in Appendix A.13, has two implications. First, we should expect zero option value at all durations, including above 80 years remaining, only if the natural rate is relatively high. Second, we can test for the presence of zero option value by searching for discontinuities in the price of leaseholds at 80 years.

Part (1) of the proposition shows that when natural rates are high, there is zero option value at all durations. Intuitively, suppose that natural rates are greater than the yield assumed by the tribunal to calculate the reversion value of the extension. Then, the reversion value of the lease extension calculated by the tribunal is greater than the market value. The freeholder will only require the leaseholder to pay the market value, given their participation constraint. Beneath 80 years, the tribunal assessed value remains above the market value of the lease extension—again, by the participation constraint, the freeholder can only force the leaseholder to pay the market value. Part (1) of the proposition also shows how to detect whether the economy has zero option value everywhere—in that case prices are continuous around 80 years of duration. Note that we can extrapolate from the 80 year threshold to conclude that there is no option value at any durations, because we know the functional form of lease extension costs.

Part (2) of the proposition shows that when natural rates are low, there will be positive option value when lease durations are greater than 80 years. Suppose that the natural rate is significantly lower than the yield used by the tribunal to calculate the reversion value of the lease extension. Then the cost paid by leaseholders to extend via the tribunal, if more than 80 years remain, is less than the market value of extension plus time and legal costs—meaning positive option value. Beneath 80 years, the tribunal assessed cost of lease extension discontinuously increases, since the tribunal assessed extension cost is now a weighted average of the reversion value and market value plus additional costs, and market value is greater than reversion value. As a result, the price of leases must discontinuously fall. Importantly, Part (2) of the proposition does not rule out full holdup for leases with less than 80 years

remaining.

We use Proposition 6.1 to test for whether there is option value. The natural rate of return for housing seems to have been declining over time. As a result, our proposition suggests that there should be zero option value at all durations, only in the early part of the sample. Later on, there should be positive option value, at least for long duration leases. Our discontinuity based test confirms these predictions.

We test for holdup by estimating whether there is a discontinuity in prices at 80 years, before and after 2010. To determine whether there is a discontinuity in property price at $T = 80$, we can estimate,

$$\frac{\Delta_h \log P_{it}^T}{h} = \alpha + \beta \cdot \mathbb{1}\text{Crossed } 80 + \Gamma_{i,t,t-h} \tag{13}$$

where the left hand side variable is the annualized log change in price of a property i between time $t - h$ and t and the right hand side variable is a dummy which checks whether i fell below 80 between time $t - h$ and t. More precisely, we say that a property crossed 80 if at time t, $T < 80$ and at time $t - h$, $T > 82$. We choose a cutoff of 82 because leaseholders cannot extend through the tribunals during the first two years of ownership, so any property purchased with less than 82 years remaining must pay the marriage value upon extension. $\Gamma_{i,t,t-h}$ are Purchase Year x Sale Year x Local Authority fixed effects. We restrict the sample to properties with duration between 70 and 90, to get the local effect around 80.

Table 4 reports the estimated coefficient from Equation (13). The first column presents estimates before 2010. There is no statistically significant discontinuity in price at 80. In fact, Appendix Figure A.21 shows that in the pre-2010 period, the estimated coefficient is approximately in the middle of a distribution of "placebo" experiments using 30 placebo cutoffs between 70-100. Column (2) of Table 4 presents estimates after 2010. There is a significant discontinuity in the price of properties that fall below 80 years duration remaining. This fall is much greater in magnitude than any placebo test with cutoffs ranging from 70-100, shown in Appendix Figure A.21. Taken together, the price discontinuity results show that there is zero option value at all durations before 2010, and positive option value for long duration leases after 2010. However the results do not pin down whether there is full holdup for leases with less than 80 years remaining.

Table 4: Test for Discontinuity at 80

	(1)	(2)
Crossed Cutoff	-0.05	-0.92***
	(0.10)	(0.12)
Sale Year x Purchase Year x LA FE	✓	✓
Period	Pre 2010	Post 2010
N	65,871	12,459

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

The table provides an estimate of the discontinuity in log price at $T = 80$. The sample includes properties with duration between 70 and 90. The first column is run on the pre-2010 data, and the second column is run on post-2010 data. Standard errors are clustered at the purchase year, sale year and Local Authority (LA) level.

Proposition 6.1 also suggests there should be time varying bunching of lease extensions. When the natural rate is relatively high, there is no gain to extending slightly before 80 years remaining; whereas when the yield is low there can be large gains to extending before 80 years. Therefore lease extensions should bunch in a time varying fashion—leases should be more likely to extend shortly before 80 years remain, only if natural rates are relatively low.

Figure 10 shows precisely this pattern of bunching. In the figure, we plot the likelihood that a lease extends when it has T years remaining, separately before and after 2010. Before 2010, the likelihood that a lease extends smoothly increases as leases cross the 80 year threshold. After 2010, the likelihood jumps just before 80 years, and there is a missing mass after 80 years. This time varying bunching strongly suggests that there is a difference in option value above versus below 80 years, only when natural rates are low—consistent with Proposition 6.1. Our results on option value so far imply an informative bound— the estimates of the fall in r_K^* from the main analysis of Section 5 , which assumes zero option value at all durations and times, are a lower bound on the magnitude of the true fall. Intuitively, if there is positive option value then the *true* duration of non-extended leases is higher than their *notional* duration. As such the price gain from lease extension is associated with a smaller increase in duration after lease extension. Therefore incorrectly assuming zero option value biases estimates of the natural rate of return upward. Given that option value emerges later in the sample, this bias increases over time, meaning the estimates that ignore this source of bias will under-estimate the fall in r_K^*. We now introduce more structure to show that this bias is small.
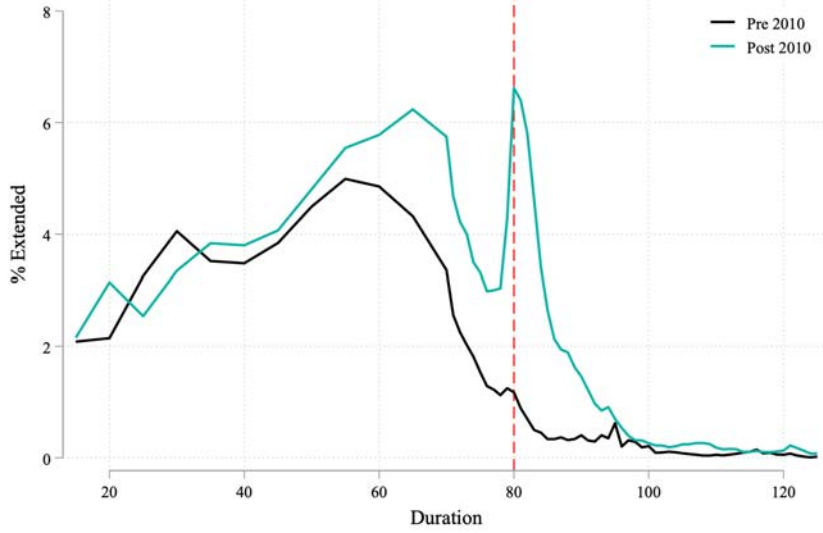
Figure 10: Hazard Rate of Extension, Before and After 2010

The figure shows the conditional probability of extension for each T. The black line shows the probability before 2010 and the blue line shows the probability after 2010. We exclude the post-2020 pandemic period due to disruptions to the lease registration process, which resulted in lower extension rates than usual.

6.3 A Difference-in-Differences Estimator of Option Value

This subsection directly estimates the degree of option value before and after 2010, both above and below the 80 year threshold, and explores the implications for r_K^*. To do so, we introduce more structure by embedding the framework for lease extension costs into our difference-in-differences estimator of r_K^*. Doing so lets us point identify option value, using a different source of variation from the discontinuity based tests of the previous subsection.

In order to incorporate option value and the threshold in a simple fashion, we take as given the probability that a lease of duration T extends at time t. We also assume that the share of extension value lost by leaseholders is piecewise constant in duration and discontinuous at 80 years remaining, which captures the discontinuities imposed by the tribunal. Therefore we have $\alpha_t^T = \alpha_t^H$ if $T > 80$ and $\alpha_t^T = \alpha_t^L$ otherwise. In this case, Appendix A.11 shows that the difference-in-differences estimator of the price gain upon lease extension becomes

$$
\begin{aligned}
\Delta_{it}^T = \log\left(1 - e^{-r_{Kt}^*(T_{it}+90)}\right) - \log\Big(&\left(1 - e^{-r_{Kt}^*T}\right) \\
&+ \left[\Pi_{Tt}^H\left(1 - \alpha_t^H\right) + \Pi_{Tt}^L\left(1 - \alpha_t^L\right)\right]e^{-r_{Kt}^*T_{it}}\left(1 - e^{-r_{Kt}^*90}\right)\Big)
\end{aligned}
\tag{14}
$$

For simplicity, this derivation makes the additional assumption that leases extend only

once.[30] In this equation, Π_{Tt}^H is the probability that a lease with $T > 80$ years remaining extends with more than 80 years remaining. Π_{Tt}^L is the probability that a lease extends with less than 80 years remaining. The cumulative probability of extension is derived from the observed extension hazard rate, as shown in Figure 10. Equation (14) is the same as the baseline estimator Equation (6) either when $\alpha_H = \alpha_L = 1$, or $\Pi_H^T = \Pi_L = 0$. In either case, there is no option value from lease extension and the final term in square brackets vanishes.

We jointly estimate r_{Kt}^*, α_t^H and α_t^L, by using the difference-in-differences estimator with option value, Equation (14), as an input into a non-linear least squares estimation similar to the baseline procedure. Relative to the baseline estimation, we also add information on the probabilities of extension Π_H^T and Π_L, which helps to identify α_t^H and α_t^L.

The estimated r_K^*, α_t^H and α_t^L parameters are presented in Table 5. In the estimation, we constrain the α parameters to lie between zero and one. The results suggest, once again, that $\alpha = 1$ for all T in the pre-2010 period. In the post-2010 period, there is positive option value when leases have more than 80 years remaining, but there is zero option value below 80 years remaining.

Table 5: Estimating Alpha

	(1)	(2)
r_K^*	4.27***	2.92***
	(0.05)	(0.01)
α_t^H	1.00***	0.72***
	(0.00)	(0.04)
α_t^L	1.00***	1.00***
	(0.00)	(0.01)
Period	Pre 2010	Post 2010
N	19,926	110,504

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

The table presents estimates of α_t^H and α_t^L, estimated jointly with r_K^* from a nonlinear least squares estimate of Equation (14). Estimates of α_t^L and α_t^H are constrained to lie within [0,1]. Standard errors are bootstrapped.

Our difference-in-difference estimates of option value in this subsection are consistent with the discontinuity-based results from the previous subsections—even though the two subsections use different sources of variation. In both subsections, there is zero option value for leases with less than 80 years remaining, at all times; and positive option value after 2010 for leases that extend with more than 80 years remaining. Our estimates of the change in option value before versus after 2010 for leases above 80 years, is also similar across the

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[30]The value of subsequent extensions, in the very far future, is quantitatively small but complicates the algebra.

two subsections. In this section, we estimate a change of 0.28. In the previous subsection we estimate a change of 0.44.[31]

6.4 Estimates of r_K^* Corrected for Option Value

Finally, we present estimates of r_K^* that correct for option value using the estimates of option value. Figure 11 presents a version of our r_K^* timeseries which corrects for $\alpha_t^H = 72\%$ in the post-2010 period. The solid line is the estimate of r_K^* from Section 5, using our baseline assumption of zero option value at all times and durations. The dashed line is the estimate of r_K^* using our estimates of the degree of option value.

Figure 11: Estimate of r_K^* Correcting for Option Value
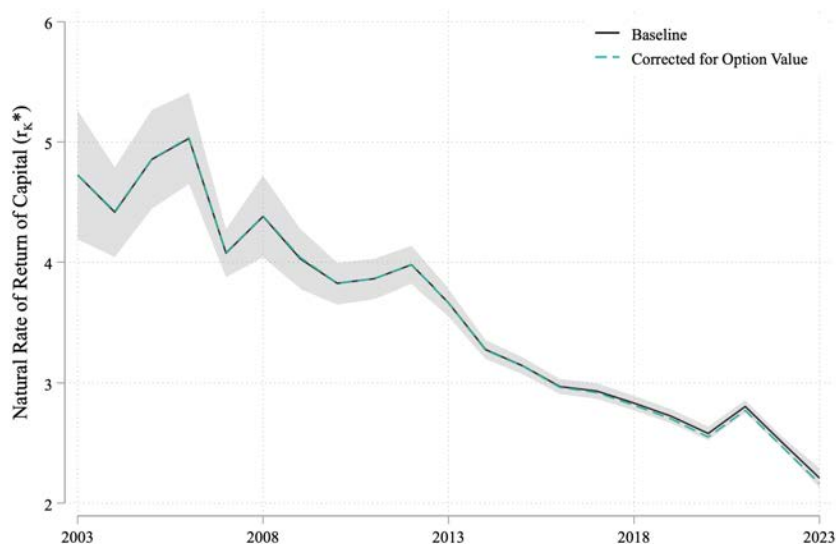
The figure presents a corrected r_K^* timeseries, adjusting for the fact that $\alpha_t^H = 0.69$ for the post-2010 period. The black line presents the unadjusted estimates from Section 5.3.

The estimate of r_K^* that corrects for option value is very similar to the baseline estimates that assume no option value. The reason is that most leases do not extend with more than 80 years remaining, as Figure 10 shows. Therefore the possibility of extending with more than 80 years remaining has a small effect on equilibrium prices, meaning departures from zero option value are quantitatively small.

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[31]The details of this calculation are presented in Appendix A.12.

7 Concluding Remarks

This paper estimates the natural rate of return on capital and its dynamics from 2003 to present for the UK property market. We exploit a natural experiment — extensions of long duration property leases in the British property market. Our findings show that r_K^* fell from an average of 4.8% between 2003 to 2006 to 2.3% in 2022.

Natural rates are valuable because they "look through" the short term factors affecting asset prices in real time. There has been a growing gap between natural rates and real forward yields in 2022-23. This gap will narrow either by a fall in spot real yields, as the "secular stagnation view" suggest, or a rise in natural rates if post-covid era reflects a structural regime shift. The consequences of these two scenarios for asset prices and the real economy could of course be very different. In recent months, there has been a modest uptick in r_K^*. The real time estimates of r_K^* should be helpful to determine the trajectory of the natural rate of return going forward.

The focus of this paper has been the measurement of the natural rate of capital, which as we have discussed, is difficult to do with precision and minimal assumptions. However, our paper introduces several questions for future research: Why has the natural rate fallen? And what does this imply about the state of the economy? Although these questions are out of the scope of this paper, we believe that there is much that can be learned about the economy from studying r_K^* in the cross section. In particular, we believe that the dynamics of r_K^* encode useful information about the elasticity of capital supply. We leave this work to future research.

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A   Appendix

A.1   Additional Figures

Figure A.1: Distribution of Lease Term for Leasehold Flats
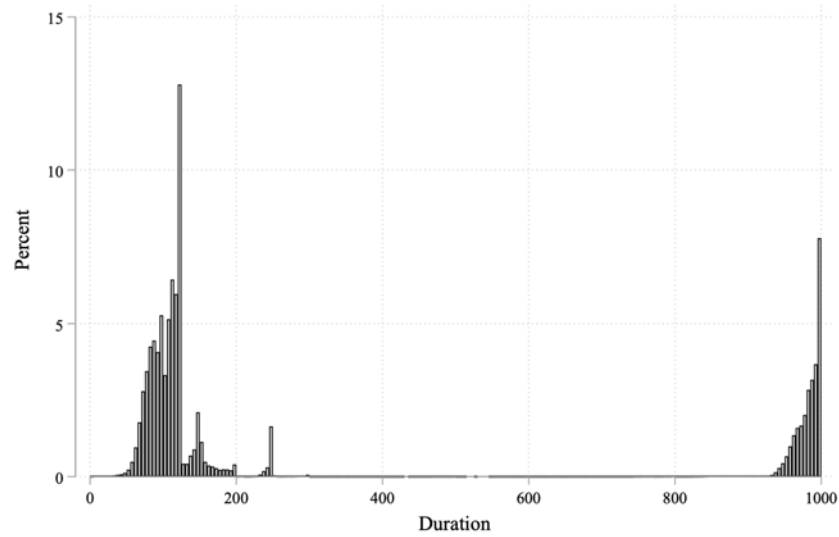
The figure is a histogram of the remaining lease term at the time of transaction. The sample is all leasehold flats that transact at least once in the Land Registry Transaction Data Set.
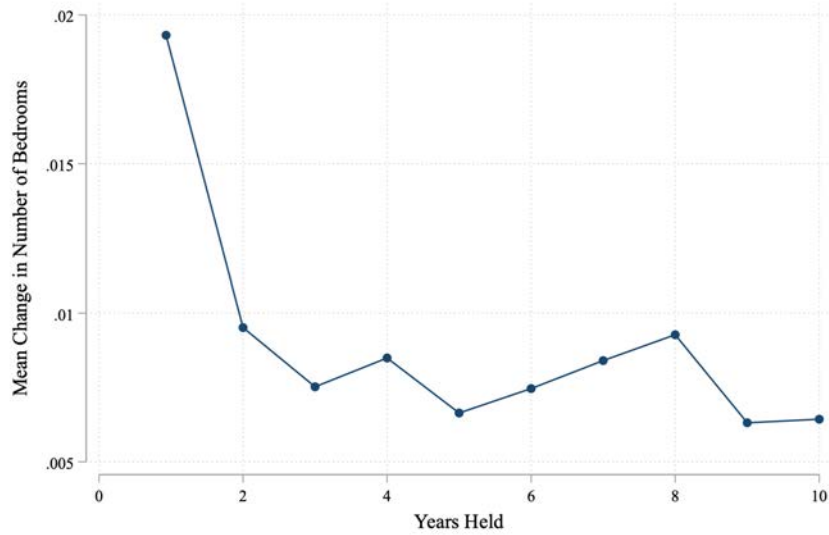
**Figure A.2:** Renovations By Holding Period

The figure shows the change in number of bedrooms reported relative to property holding period, where very short holds have a disproportionately high renovation rate. We take this as evidence that many of these are "flippers" who buy properties to re-sell them. The sample is all flats for which we observe two different Rightmove or Zoopla listings associated with two different property transactions.

Figure A.3: Hazard Rate of Lease Extension

(a) Raw   **(b)** Corrected
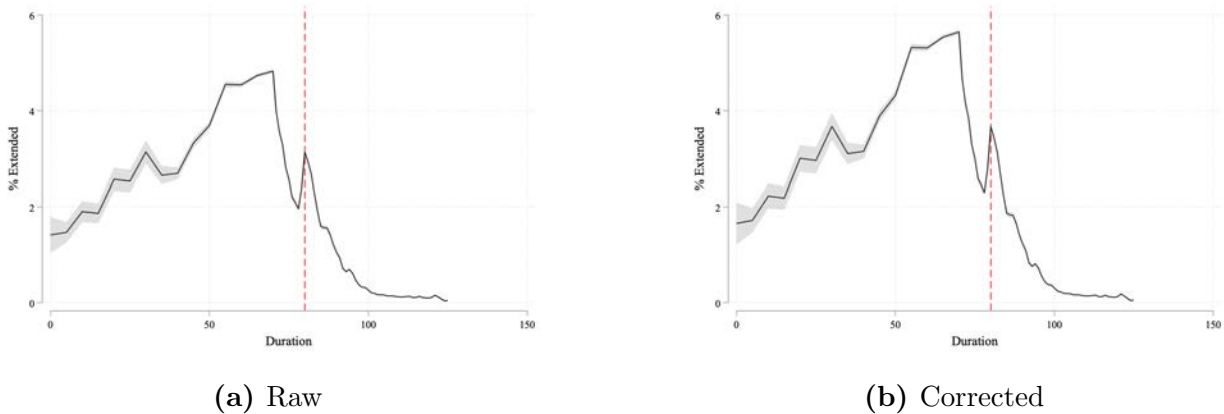
The figure shows the conditional probability of extension, $\theta(T)$ given that a property has duration T. In the first panel, the conditional probability of extension is given by $\theta_1(T) = \frac{N_T^{Ext}}{N_T}$ where N_T^{Ext} is the number of properties which extended with duration T and N_T is the number of properties that reached duration T. In the second panel, the conditional probability of extension is $\theta_2(T) = \gamma \frac{N_T^{Ext}}{N_T}$ where $\gamma = 1.17$ adjusts for the fact that our primary method does not identify properties which never transact before extension. The shaded area shows the 95% confidence interval.

Figure A.4: Cumulative Hazard Rate
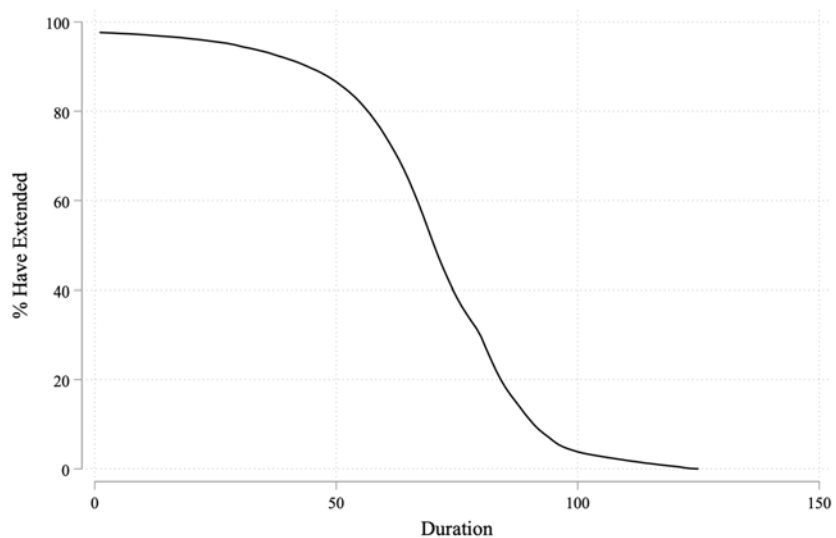
The figure shows the cumulative probability of extending over a property's lifetime. The sample includes all leases with at least one transaction and covers the 2003-2020 period. We exclude the pandemic period due to abnormally low extension rates.

Figure A.5: Histogram of Duration Before Extension
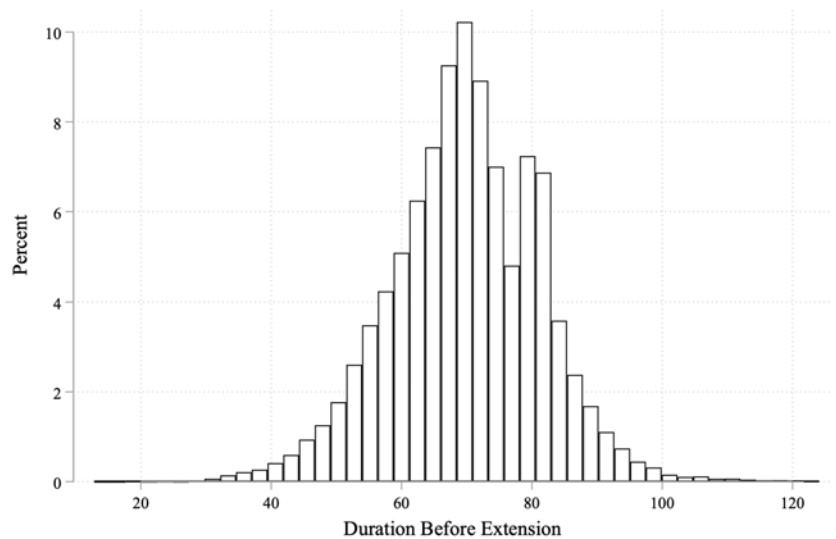
The figure presents a histogram of lease duration immediately before extension. The sample is all extended flats.

Figure A.6: Histogram of Years Between Transaction
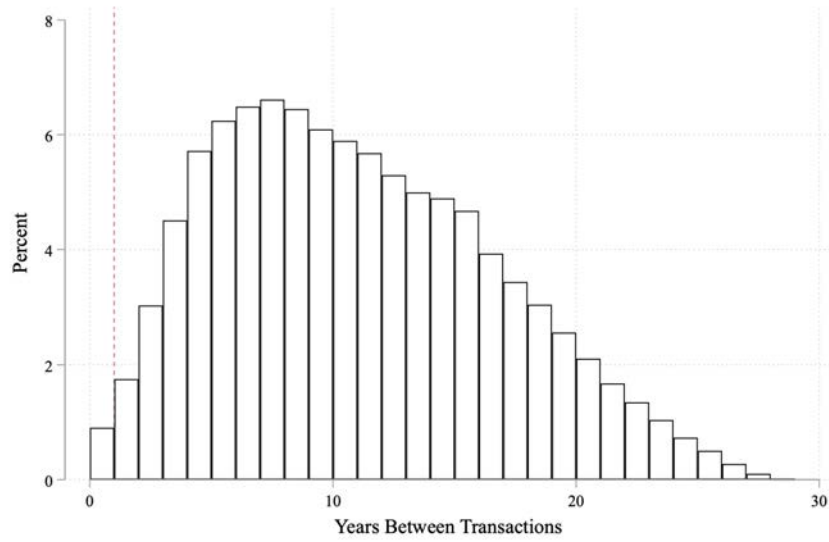
The figure shows a histogram of the holding period, h, for lease extensions which have a recorded transaction before and after extension. The dotted line shows the $h = 1$ cutoff; properties below the cutoff are not included in our primary sample. The sample is all extended flats.

Figure A.7: Histogram of Years Between Transaction and Extension
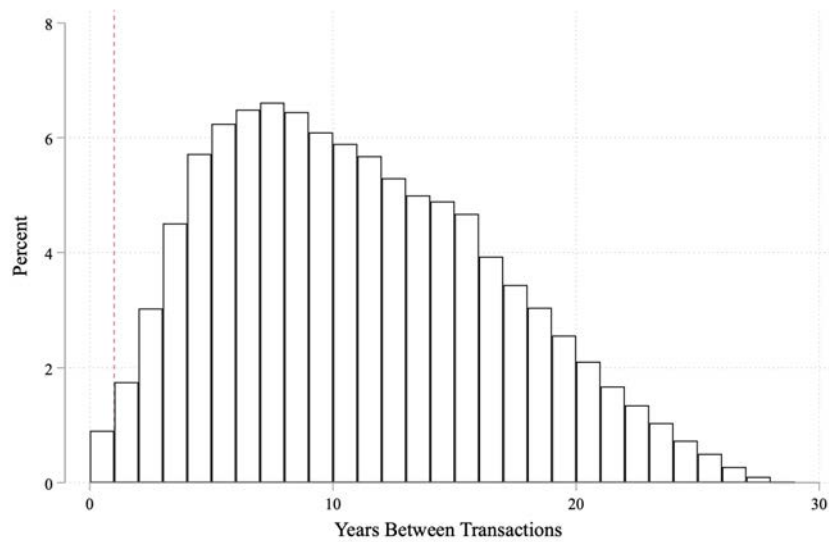
The figure presents a histogram of the number of years between purchase and extension time. The sample is all extended flats.

Figure A.8: Price to Rent (Freeholds vs Leaseholds)
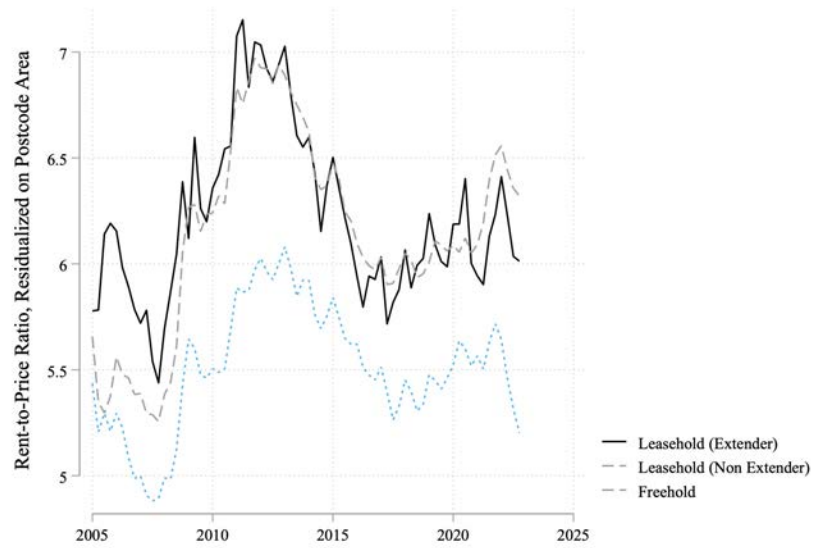
The figure shows the property-level price to rent ratio for leasehold and freehold properties. Leaseholds are subdivided into those which extend during our sample and those which do not. Property-level rental price data is collected from Rightmove and Zoopla.

Figure A.9: Heat Map of Extension Rate

●	0.00, 10.00
●	10.00, 50.00
●	50.00, 100.00
●	100.00, 200.00
●	200.00, 300.00
●	300.00, 500.00
●	500.00, 1000.00
●	1000.00, 5668.00
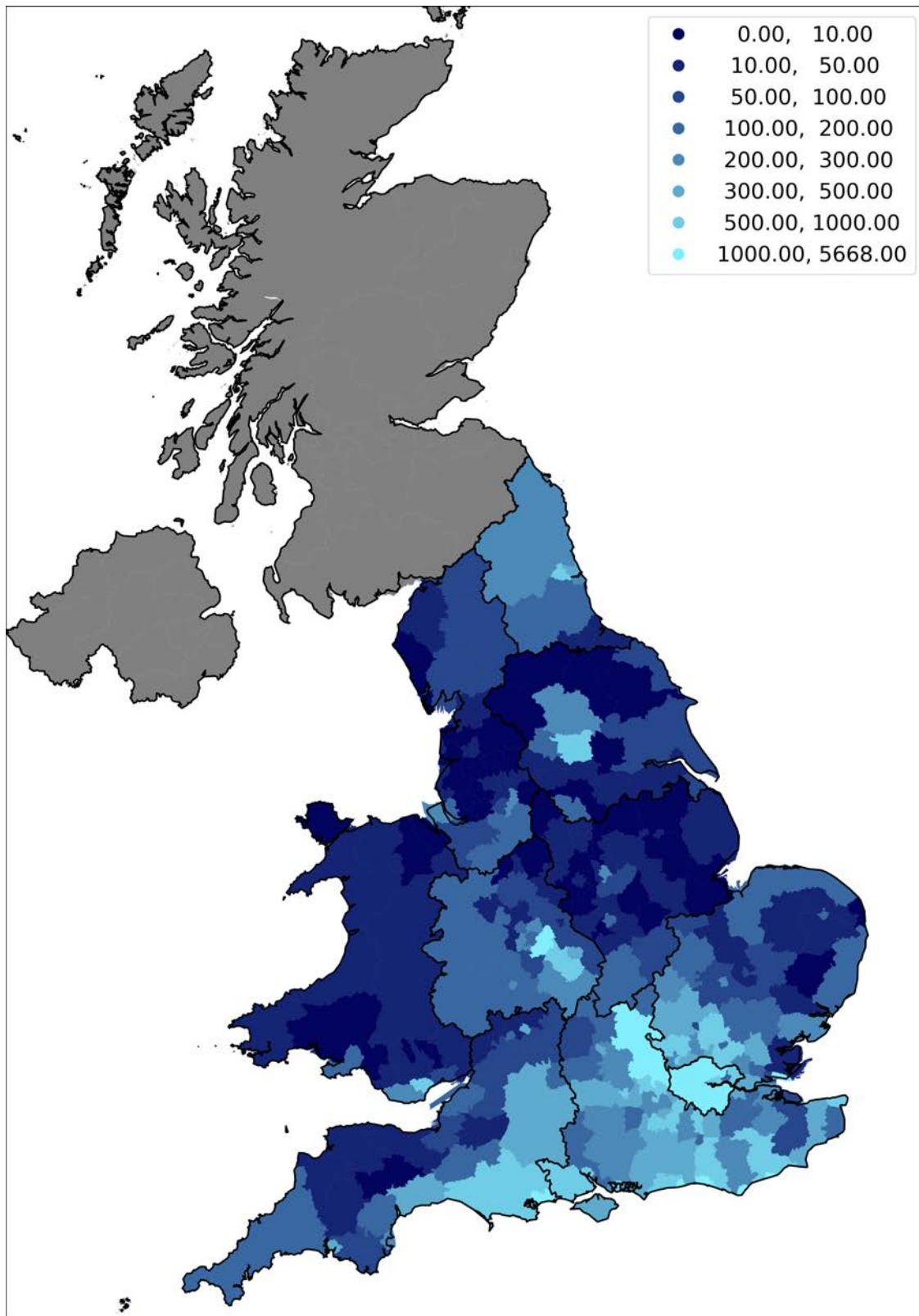
The figure shows a heatmap of the number of properties extended in each Local Authority in England and Wales.

Figure A.10: Time Elapsed Between Rightmove Listing and Land Registry Transaction Date
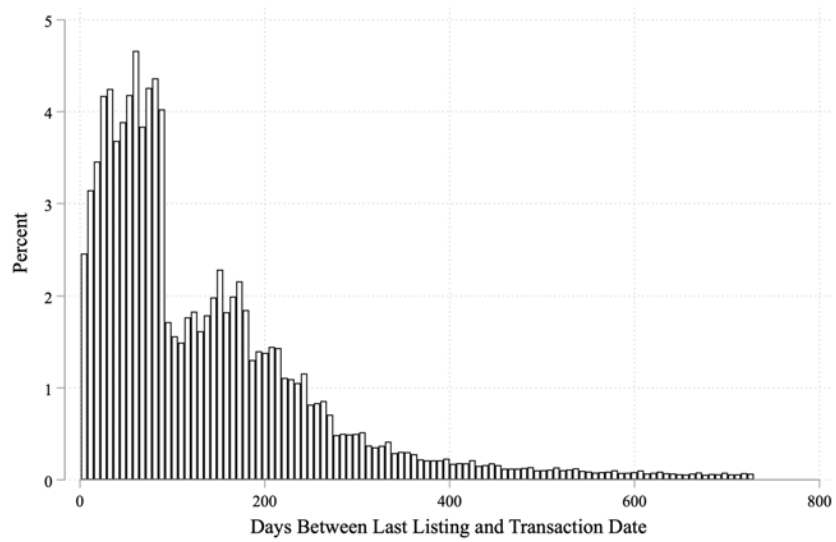
The figure shows a histogram of the amount of time (in days) between the last property listing on Rightmove and the date in which the transaction is recorded by the Land Registry. Each bin refers to one week. The sample is transactions for properties which have Rightmove listings within two years of the Land Registry transaction date.

Figure A.11: Relation Between Holding Period and Difference-in-Difference
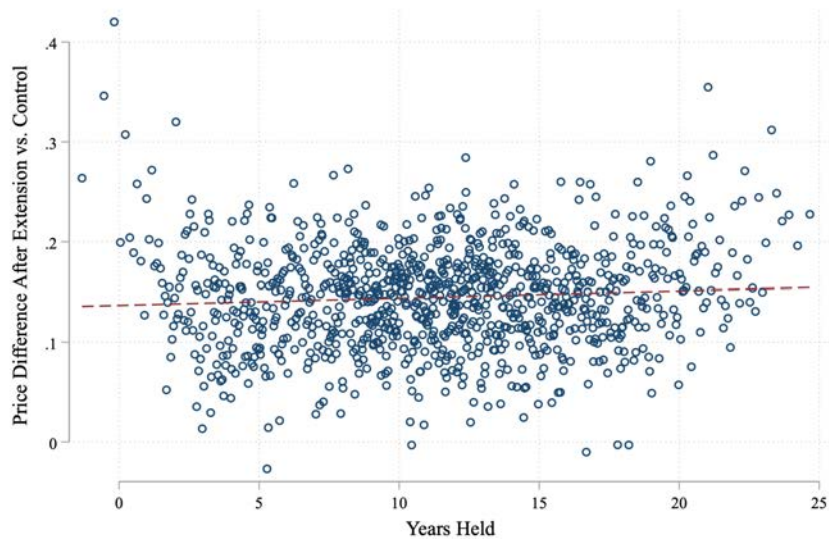
The figure shows a binscatter of the difference-in-difference estimator, Δ_{it}, by holding period, controlling for sale year t fixed effects. The sample is all 90 year extensions.

Figure A.12: Binscatter Log(Price) on Hedonics

(a) Bedrooms

(b) Bathrooms

(c) Living Rooms

(d) Floor Area

(e) Year Built

(f) Log(Rent)
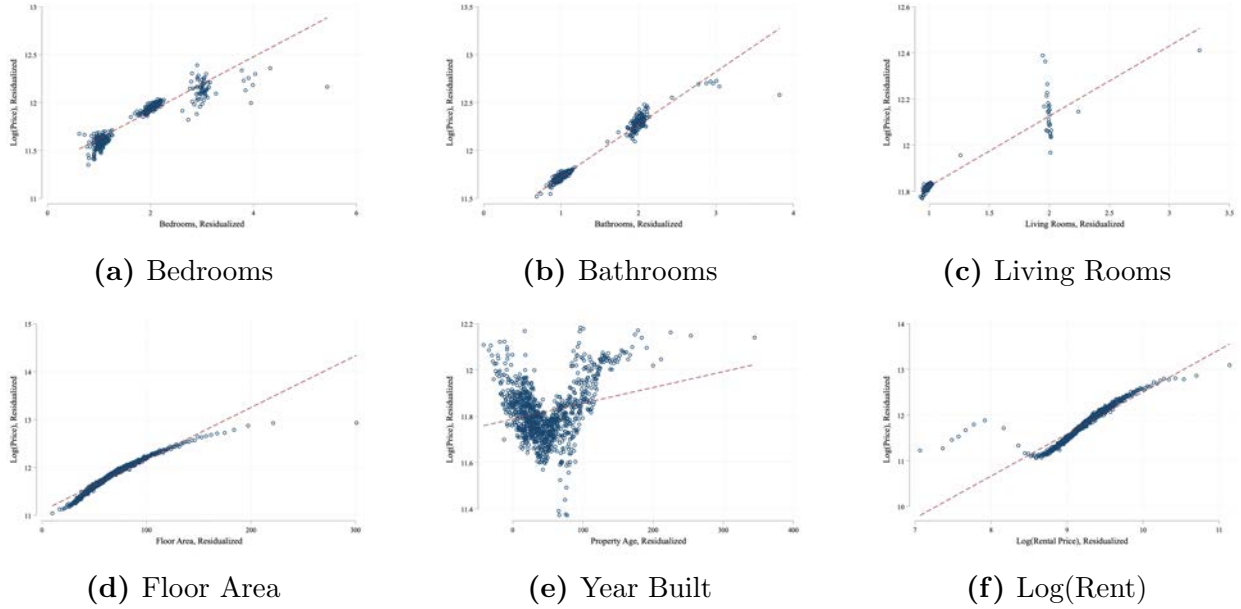
The figures are binscatters of log transaction price against the following hedonic characteristics: number of bedrooms, number of bathrooms, number of living rooms, floor area (sq. meters), year that the property was built, and log yearly rental price. Both the x and y-axis variables are residualized by Local Authority fixed effects, Γ_{it}. In particular, the y-axis variable is $\overline{\log(P_{it})} + \epsilon_{it}$ where $\overline{\log(P_{it})}$ is the mean log transaction price and ϵ_{it} is the residual from the following specification: $\log(P_{it}) = \Gamma_{it} + \epsilon_{it}$. The x-axis variable for each hedonic characteristic X_{it} is $\overline{X}_{it} + \eta_{it}$ where $\overline{X}_{it}$ is the mean level of X_{it} and η_{it} is the residual from the following specification: $X_{it} = \Gamma_{it} + \eta_{it}$. The sample is leasehold flats which appear at least once in the Land Registry Transaction Data Set.
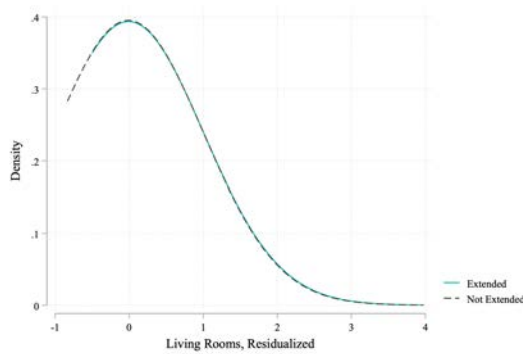
Figure A.13: Hedonics Residuals Density Curve for Extended and Non-Extended Properties
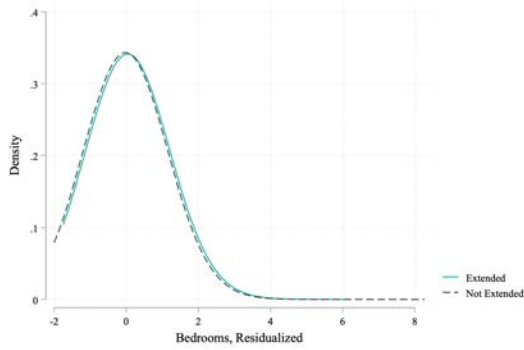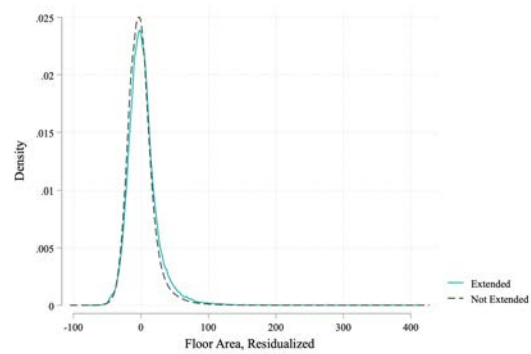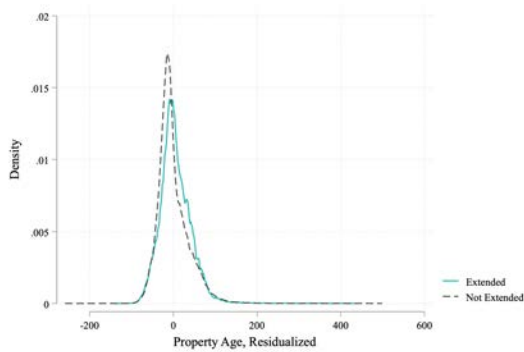

(a) Bedrooms
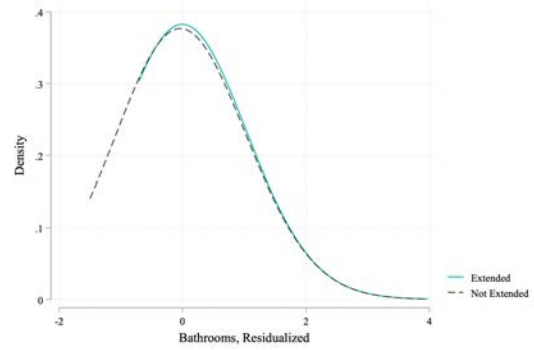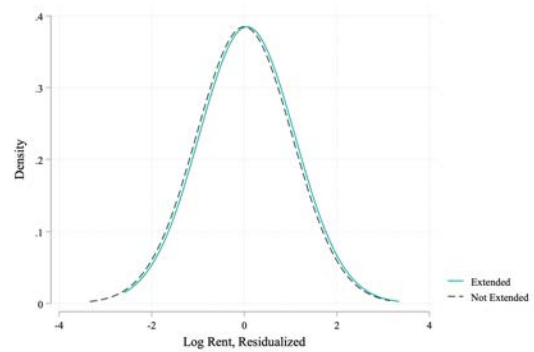
(b) Bathrooms

**(c)** Living Rooms

(d) Floor Area

(e) Age

(f) Log(Rent)

The figures show the distribution of residuals for extended and non-extended flats after regressing hedonic characteristics on 5-year duration bin × Local Authority × year fixed effects. The sample is leasehold flats which appear at least once in the Land Registry Transaction Data Set.

**Figure A.14:** Histogram of Remaining Lease Term At Sale
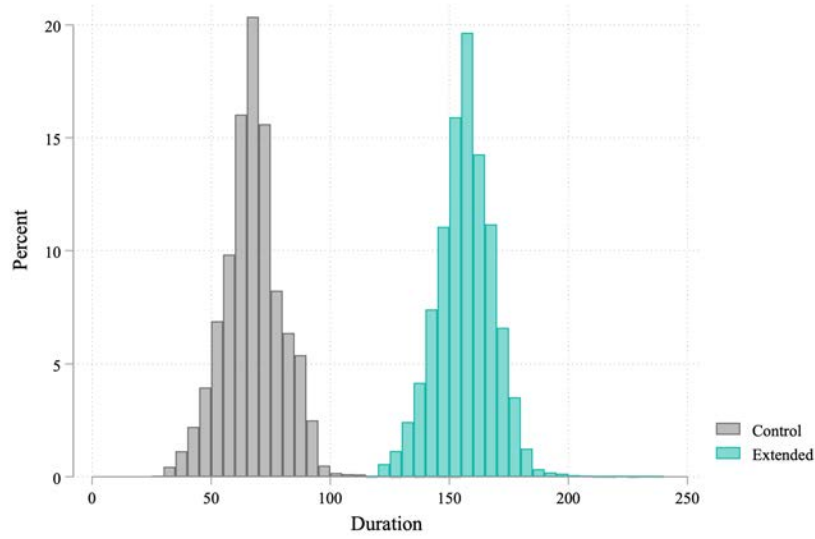
The figure shows a histogram of remaining duration at sale time t for the approximately 40 thousand experiments which were extended for 90 years. The blue-shaded histogram includes the sale duration for the controls in the experiment and the grey-shaded histogram includes the sale duration for the extensions.

Figure A.15: Event Study for Extension of Extension Length Less than 200 Years (Excluding 90 Year Extensions)
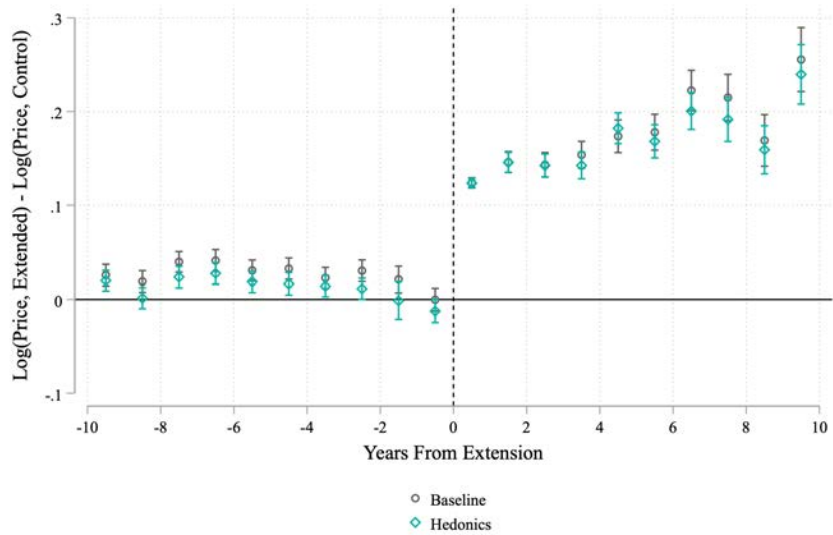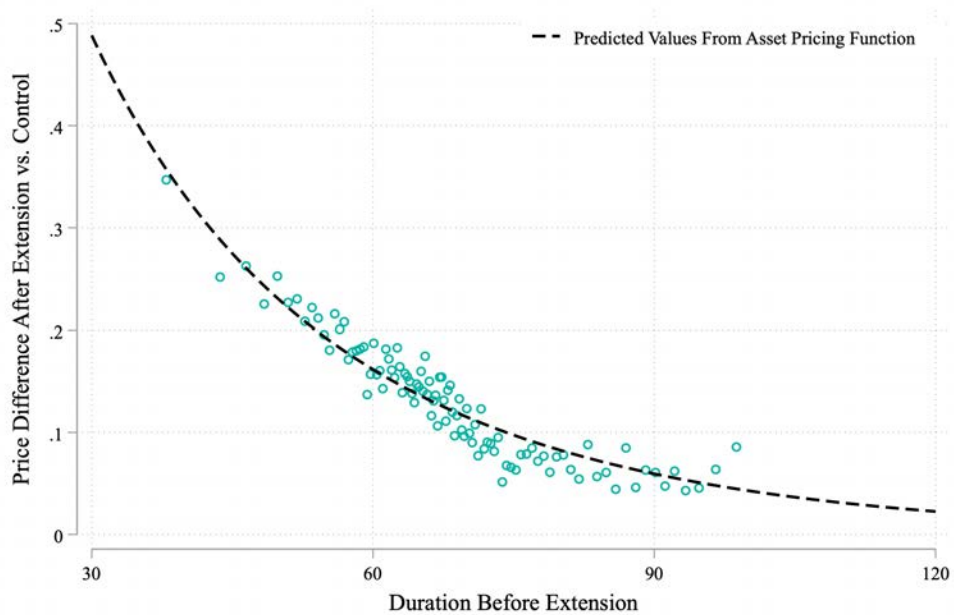

The figure replicates Figure 5 using extensions of lengths of less than 200 years, excluding 90 year extensions.

**Figure A.16:** Event Study for Extension of Extension Length More than 700 Years
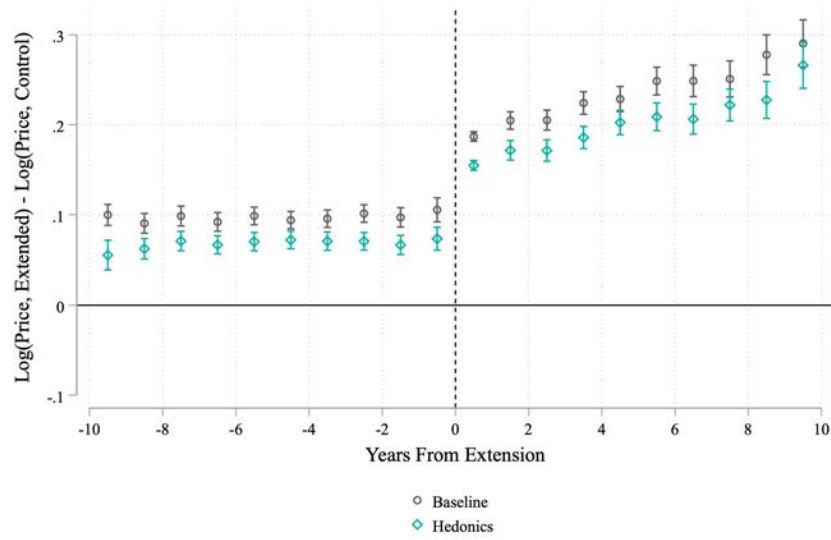
The figure replicates Figure 5 using extensions for lengths of more than 700 years.

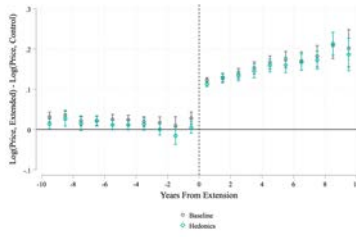**Figure A.17:** Duration vs. Price Gain After Extension, Leases with 700+ Years

The figure is a binscatter of our difference-in-difference estimator against duration before extension, T, with 100 bins. The sample includes leases that were extended for more than 700 years. The black line shows fitted estimates of Equation (7).

Figure A.18: Event Study By Duration
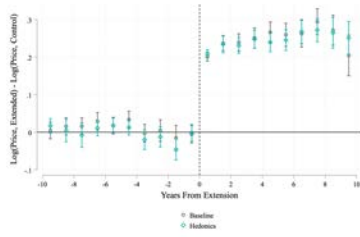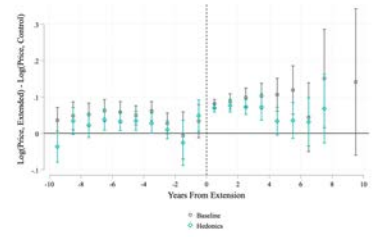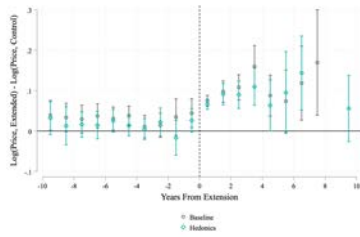

(a) $T \leq 60$ **(b)** $60 < T \leq 80$ **(c)** $T > 80$

The figure replicates Figure 5 for properties that were extended with low, medium and high durations.

Figure A.19: Event Study By Time Period
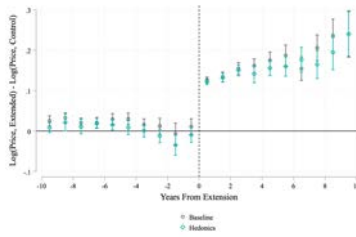
(a) 2003-2010   **(b)** 2010-2018   **(c)** 2018-2023
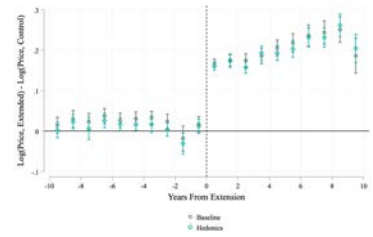
The figure replicates Figure 5 for three different sub-periods: 2003-2010, 2010-2018, and 2018-2023.

Figure A.20: r^*_{Kt} Estimates For Various Extension Amounts
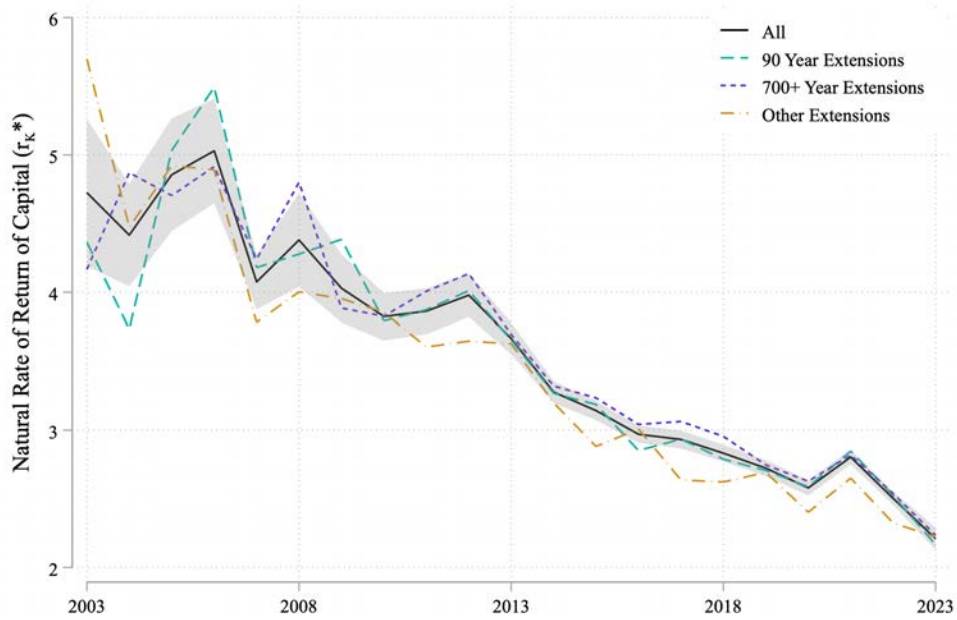
The figure presents estimates of r^*_{Kt} for every year of our sample period for lease extensions that were extended for 90 years, for more than 200 years, or for another amount, separately.

Figure A.21: Test for Discontinuity at 80 Years (Placebos)
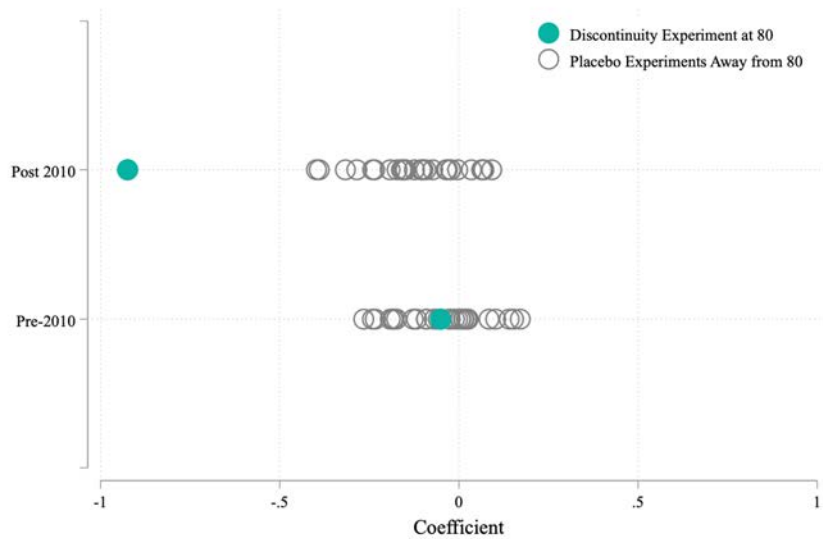
The figure presents the estimates from Table 4 in red. In grey, we run 30 placebo tests using alternative cutoffs between 70-100. In each case, the sample includes properties within 10 years above and below each cutoff.

A.2 Additional Tables

Table A.1: Freehold vs Leasehold Statistics (English Housing Survey)

	Freehold	Leasehold
Income	29,628.73	25,653.20
	(52.95)	(138.48)
Age	53.95	51.49
	(0.03)	(0.10)
% Have Mortgage	54.82	59.07
	(0.10)	(0.28)
LTV	72.17	76.16
	(0.14)	(0.39)
N	305,135	

This table reports the mean characteristic for freehold owners, in column 1, and leasehold owners, in column 2. The standard error of the mean is in parentheses.

Table A.2: Hedonic Characteristics in Extended vs Non-Extended Flats, Only 90 Year Extensions

Panel A: Levels

	(1)	(2)	(3)	(4)	(5)	(6)
	Num Bedrooms	Num Bathrooms	Num Living Rooms	Floor Area	Age	Log Rental Price
Extension	0.04***	0.01***	-0.00	1.18***	2.79***	6.24***
	(0.006)	(0.003)	(0.002)	(0.284)	(0.763)	(0.371)
Fixed Effects	✓	✓	✓	✓	✓	✓
N	1,280,574	1,008,459	889,345	1,018,774	793,599	708,613

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Panel B: Differences

	(1)	(2)	(3)	(4)	(5)
	Δ Num Bedrooms	Δ Num Bathrooms	Δ Num Living Rooms	Δ Floor Area	Δ Log(Rent)
Extension	-0.00	-0.00	-0.00***	0.06	-0.62
	(0.00)	(0.00)	(0.00)	(0.08)	(0.95)
Fixed Effects	✓	✓	✓	✓	✓
N	183,586	141,871	126,320	138,884	68,721

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

The table repeats the results from Table 2 for the sub-sample of 90 year extensions.

Table A.3: Estimated r_K^* for All Extensions

	Constant	Flexible				N
	(1)	(2)	(3)	(4)	(5)	(6)
		$T = 50$	$T = 60$	$T = 70$	$T = 80$	
No Hedonics	3.08***	3.05***	3.07***	3.10***	3.13***	132,552
	(0.01)	(0.02)	(0.01)	(0.01)	(0.02)	
No Hedonics (Hedonics Sample)	3.03***	2.98***	3.02***	3.06***	3.09***	79,603
	(0.01)	(0.02)	(0.01)	(0.01)	(0.02)	
Hedonics	3.06***	2.99***	3.05***	3.12***	3.18***	79,603
	(0.01)	(0.02)	(0.01)	(0.01)	(0.02)	

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 01$

The table repeats the estimates from Table 3 for all extensions.

A.3 Equivalence of r_K^* and User Cost of Capital

First consider a static model in which a firm has profits $\Pi(K, X)$, where X is the cost of other inputs and K is capital. The firm maximizes profits, i.e.

$$\max_{K,X} \Pi(K, X) - R_K \cdot K$$

where R_K is the cost of renting one unit of capital. Then, the first order condition for capital is,

$$\Pi_K(K, X) = R_K$$

so the marginal product of capital is equal to the user cost of capital.

Now, consider a dynamic problem in which capital depreciates at rate δ. Suppose that the firm purchases capital at price P_K. Then, the firm faces the following intertemporal maximization problem,

$$V(K_t) = \max_{I_t} \int_t^\infty e^{-(r_t^* + \zeta_t^*)s}(\Pi(K_s) - P_{K_s} I_s)ds$$

where I_t is investment at time t and $\dot{K}_t = I_t - \delta K_t$, such that the change in the capital stock is equal to investment minus depreciation. As in the main text, $r^* + \zeta^*$ is the required rate of return of the firm, which can be separated in a risk-free rate and a risk-premium. The

Hamiltonian for this problem is,

$$\mathcal{H} = \Pi(K_t) - P_{K_t}I_t + \lambda_t(I_t - \delta K_t)$$

where the optimality conditions are,

$$P_{K_t} = \lambda_t$$

$$\Pi_K(K_t) - \delta\lambda_t = (r_t^* + \zeta_t^*)\lambda_t - \dot{\lambda}_t$$

Substituting the optimality conditions and reorganizing yields,

$$\Pi_K(K_t) = P_{K_t}\left(r_t^* + \zeta_t^* + \delta - \frac{\dot{P}_{K_t}}{P_{K_t}}\right)$$

$$R_{K_t} = P_{K_t}\left(r_t^* + \zeta_t^* + \delta - \frac{\dot{P}_{K_t}}{P_{K_t}}\right)$$

where in the last equation, we utilize the fact that the marginal product of capital is equal to the user cost of capital. Normalizing both sides by the price of capital we see that,

$$\frac{R_{K_t}}{P_{K_t}} - \delta = r_t^* + \zeta_t^* - \frac{\dot{P}_{K_t}}{P_{K_t}}$$

$$= r_{Kt}^*$$

so the user cost of capital normalized by the price of capital and net of depreciation is equal to the natural rate of return of capital.

A.4   Details on Merge Between Lease and Transaction Data

This section briefly describes our procedure to merge Land Registry data on lease lengths, and house price transaction data. In the UK, every property can be uniquely identified by three items: the first address number, the second address number and the 6-digit postcode. Therefore, we merge according to the following procedure: First, we conduct a perfect merge using address as our merge key. This methods accounts for 93% of our matches. Second, we conduct a fuzzy merge on all observations not matched by step 1. The fuzzy merge matches observations in which (1) all numeric elements of the address are the same, (2) all single letters (e.g. A, B, C, etc.) are the same, (3) a select set of identifying terms (e.g. first floor, second floor, basement, etc.) are the same and (4) the postcode is the same in both addresses. For example, this allows for the property "3 Swan Court 59-61 TW13 6PE" in the transaction data set to be matched to "Flat 3 Swan Court 59-61 Swan Road Feltham

TW13 6PE" in the lease term data set. This method accounts for 7% of our matches.

Additionally, we purchase the leasehold titles from the Land Registry for approximately 20 thousand transactions for which we are unable to identify a lease term based on the fuzzy merge. These include cases where the lease term address has typos, or has been accidentally omitted from the main public data set.

A.5   Simulation Results For Flexible Forward Curve

In Section 4.3 we presented one possible parameterization of $y(T)$. In this section, we explore other parameterizations and present several insights from our simulation. We assume that for $T \geq 40$, $y(T) = r_K^*$ is constant. Hence, for $T < 40$ we assume $y(\cdot)$ can have any shape as long as it asymptotes to r_K^* as $T \to 40$.

Figure A.22a shows several possible choices of $y(T)$, all of which asymptote to the same value. Figure A.22b presents the yield curves associated with each of these forward curves, which we denote by $\rho(T)$; and Figure A.22c shows the corresponding $\hat{r}_K^*(T)$ curves that we estimate by Equation (7) using simulation data. We also plot the point estimate of $\hat{r}_K^*$ we obtain at the median of our true distribution.

Figure A.22: Long Run Discount Rates Using Simulation Data

(a) $y(T)$ **(b)** $\rho(T)$ **(c)** $r_K^*(T)$
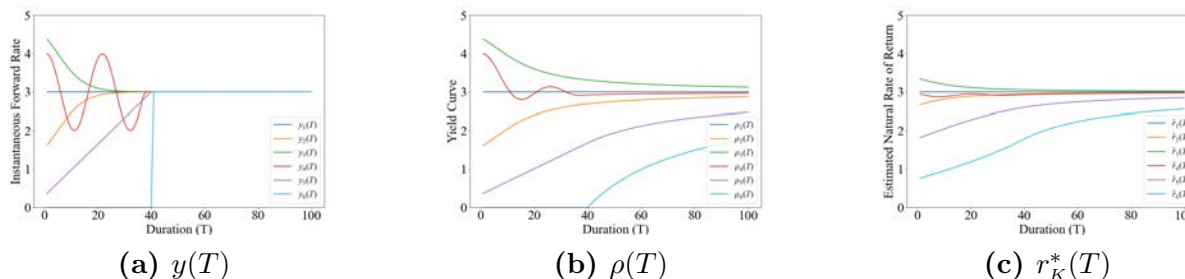
Panel (a) presents multiple possible parameterizations of $y(T)$, all of which asymptote to 3% by $T = 40$. Panel (b) presents the corresponding yield curve, $\rho(T)$, for each choice of $y(T)$. Panel (c) presents estimates of $r_K^*(T')$ obtained by NLLS corresponding to each choice of $y(T)$, where T' is the average between the control and extension sale duration. The point presents the estimate of r_K^* we would obtain at the median of our distribution.

These simulation results yield several key insights. First, when the yield curve is flat, $y(T) = \rho(T) = \hat{r}_K^*(T) = r_K^*$, as exemplified with the by the dark blue line in Figure A.22. When the yield curve is not flat, however, our estimate will differ from the true asymptotic value of $y(T)$ by some amount $\eta \equiv r_K^* - \hat{r}_K^*$. When the yield curve is upwards sloping, $\eta > 0$ and when the yield curve is downwards sloping, $\eta < 0$.

Notice that for $T \geq 40$, $\hat{r}_K^*(T)$ converges to r_K^* much more quickly than $\rho(T)$. The reason for this is that our difference-in-difference estimate differences out a large portion of the

short-end of the yield curve. To see this, consider a property with duration T that extends by k years to a total of 160 years ($T + k = 160$). The shorter T is, the less of the short-end that will be differenced out by our estimate. We present simulation evidence for this in Figure A.23. For this reason, it is important that our experiments take the difference between two long-duration properties, as opposed to one long duration property and one short duration property.

Figure A.23: Estimated r_K^* When Extending From T to 160
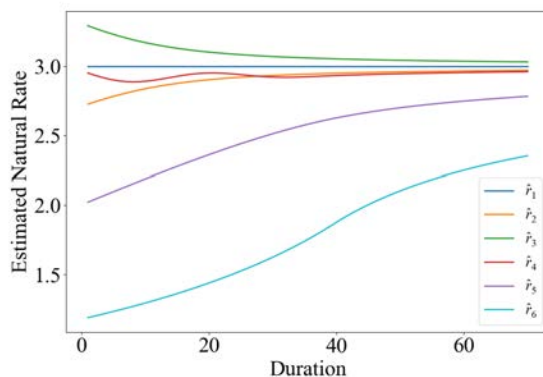
The figure indicates the point estimate of $\hat{r}_K^*$ we obtain by NLLS for an extension from duration T to duration 160. As the duration before extension increases, the estimate of $\hat{r}_K^*$ approaches the limit of the forward curve. We repeat this for each example forward curve, $y(T)$, from Figure A.22a.

A.6   Liquidity Premium

One institutional factor which could raise concerns about our estimator is the difficulty for owners of short leases to obtain a mortgage. If short leases have more limited access to financing than longer leases, we might worry that part or all of the observed price gain upon extension is a result of increased access to financing opportunities; in other words, we may wonder if the effect is driven by a "liquidity premium." Indeed, important lenders, such as Barclays, Halifax and The Co-Operative Bank, refuse to lend to leaseholds with less than 70 years remaining. Others have different thresholds, such as 55 years, and some have a preference for longer leases but allow for case-by-case exceptions.[32]

Reassuringly, using detailed micro-data from the English Housing Survey 1993-2019, we find that mortgage access and conditions are not vastly different for shorter and longer leaseholds, especially for those with more than 30 years remaining, as indicated in Table A.4. Approximately 60% of short (under 80) duration leaseholds were purchased with a mortgage,

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[32]A comprehensive list of lease length policies for banks in England can be found in the UK Finance Lenders' Handbook For Conveyancers.

relative to 58% of longer (over 80) duration leaseholds. The typical mortgage length and Loan-To-Value (LTV) ratios are similar across the duration spectrum, at around 23 years and 75-80%, respectively. Additionally, short leaseholds have similar interest rate types as long leaseholds, with around 30% choosing adjustable rate mortgages (as opposed to mortgages with a fixed interest rate for a number of years, or tracker mortgages which are indexed to the Bank of England bank rate). These results suggest that financing constraints are unlikely to drive the very large extension price changes we observed in Section 5.

Table A.4: Mortgage Statistics For Short Leaseholds

	Less Than 50 Years	50-60 Years	60-70 Years	70-80 Years	80-99 Years	100+ Years	Total
Mortgage Length	22.1	22.1	23.9	23.0	23.9	23.1	23.3
	(0.6)	(0.5)	(0.5)	(0.3)	(0.2)	(0.1)	(0.1)
LTV	76.3	80.8	81.4	77.7	73.3	76.5	76.2
	(3.3)	(2.6)	(1.8)	(1.7)	(1.0)	(0.6)	(0.5)
% Have Mortgage	59.9	60.4	62.1	58.1	63.9	55.6	58.2
	(2.4)	(2.4)	(1.6)	(1.4)	(0.8)	(0.5)	(0.4)
% Adjustable Rate	24.2	40.0	38.3	32.8	25.3	31.0	30.2
	(5.3)	(5.2)	(4.1)	(3.4)	(1.5)	(1.0)	(0.8)
N	18,292						

mean reported; standard error of mean in parentheses

The table reports several summary statistics for leases of various durations. The first row presents the average mortgage length; the second presents the average Loan-To-Value (LTV) ratio for the mortgage, calculated as the initial mortgage value divided by the market price of the property; the third row presents the percent of properties of that duration which have a mortgage; the fourth row presents the percent of properties with a mortgage that have a fully adjustable rate mortgage.

Another common way to test for the existence of a liquidity premium resulting from financing frictions is to use the amount of time a property was listed on the market (i.e. sale time minus the time of the first listing) as a proxy for its liquidity (Lippman and McCall, 1986; Lin and Vandell, 2007; Genesove and Han, 2012). The intuition is that properties for which the buyer cannot obtain a mortgage have a smaller pool of potential buyers, which ought to increase the amount of time that the property is on the market. We find limited evidence of this in the data, as illustrated in Figure A.24. After controlling for quarter × 3-digit postcode fixed effects and hedonics, the typical listing time for a 50-70 year lease is only 3-4 days longer than for a long lease of more than 100 years. This is negligible given that the average listing time is 5 months. For shorter leases, the listing period actually decreases further; all else equal, a leasehold with less than 50 years remaining will sell about a week faster than a leasehold with more than 50 years.

Figure A.24: Time on Market by Lease Duration
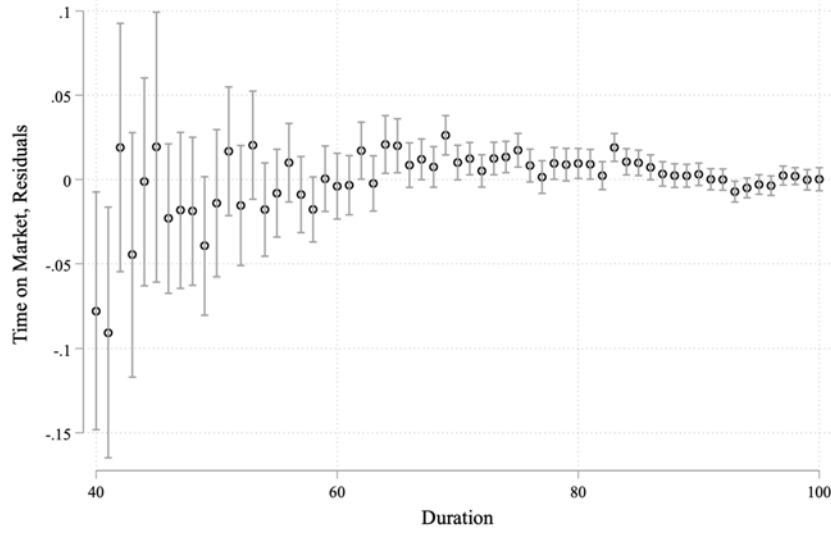
The figure shows the mean time on market for every duration under 125, de-meaned by quarter × Local Authority fixed effects and controls for bedroom count, floor area and property age.

To further test for the existence of a liquidity premium, we are able to model the case of discontinuous financing frictions and reject the existence of a liquidity premium using the data. Consider that under the threshold of 70 years–which is the most prominent bank mortgage cutoff duration—it becomes significantly more difficult to finance a leasehold property. Then the price of a $T > 70$ duration leasehold is given by,

$$P_t^T = R_t \left[\int_t^{t+(T-70)} e^{-(r_K^*)(s-t)} ds + e^{-(r_K^*)(T-70)} \int_{t+(T-70)}^{t+T} e^{-(r_K^*+\sigma)(s-(t+(T-70)))} ds \right]$$

$$= R_t \left[\frac{1 - e^{-(r_K^*)(T-70)}}{r_K^*} + \frac{(1 - e^{-(r_K^*+\sigma)\cdot 70})}{r_K^* + \sigma} (e^{-(r_K^*)(T-70)}) \right]$$

where rents below the cutoff of 70 are discounted at an additional rate σ. One way to think about σ is as the difference between the return on housing and outside investment options. Therefore, our difference in difference will yield the following equation:

$$\log P_t^{T+90} - \log P_t^T = \log \left[\frac{1 - e^{-(r_K^*)(T+90-70)}}{r_K^*} + \frac{1 - e^{-(r_K^*+\sigma)70}}{r_K^* + \sigma} (e^{-(r_K^*)(T+90-70)}) \right]$$

$$- \log \left[\frac{1 - e^{-(r_K^*)(\max\{0,T-70\})}}{r_K^*} + \frac{1 - e^{-(r_K^*+\sigma)(\min\{70,T\})}}{r_K^* + \sigma} (e^{-(r_K^*)(\max\{0,T-70\})}) \right]$$

$$\tag{15}$$

When $\sigma > 0$, the price change for extending a lease will have a kink at 70 as shown in Figure A.25. The reason for this is that as $T \to 70$, there are two incentives to extend: first is the value of 90 additional years of discounted rents and the second is the value of postponing the liquidity discount, σ. Once $T < 70$, the first incentive continues to grow, as we have discussed in earlier sections, but the second incentive becomes increasingly weaker, since there are less periods at which rents will be discounted at rate σ. Moreover, as $T \to 0$, $T + 90$ grows closer to 70, so the value of the extended lease also starts to decrease as it approaches the liquidity premium cutoff. This kink is not present in the data, which can be verified visually in Figure 6, and the existence of a discontinuous liquidity premium at $T = 70$ is rejected by estimating Equation (15) by NLLS.

Figure A.25: Liquidity Premium Example
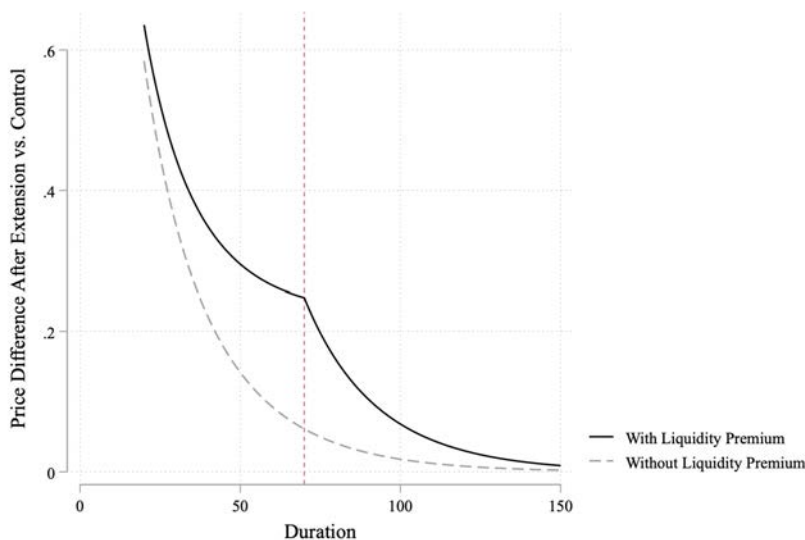
The figure shows the effect of a liquidity premium on the value of extending for 90 years ($P_t^{T+90} - P_t^T$). The dark line plots Equation (15) when $\sigma = 1\%$ and the dashed light line plots the same equation absent a liquidity premium, i.e. $\sigma = 0\%$. The liquidity premium is assumed to start at $T = 70$. We can see that when there is a liquidity premium, the value of extension will exhibit a kinked shape, with a kink at $T = 70$.

A.7   House Price Seasonality

An important feature of the UK housing market, as well as in other countries including the United States, is that it is highly seasonal, with systematically higher sale prices in the second and third quarter (the "hot months") and lower prices in the first and fourth quarters (Ngai and Tenreyro, 2014). This may be associated with a number of factors, including mortgage conditions and market tightness, which we would not expect to affect the long-run and therefore ought to be differenced out by our estimator.

To determine whether our estimates are seasonal, we must select control properties which transact in the same quarter as extended properties. Then, we can estimate r^*_{Kt} at a quarterly frequency, as described in the main text. We then test for seasonality by regressing our time series of r^*_{Kt} on quarter dummy variables, controlling for year fixed effects. The regression results are presented in Table A.5, which indicate that there is no statistically significant difference in the level of r^*_{Kt} across quarters.

Table A.5: Test for Estimate Seasonality

	(1)
2nd Quarter	0.09
	(0.07)
3rd Quarter	-0.02
	(0.06)
4th Quarter	-0.06
	(0.06)
Year FE	✓
N	82

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

The table indicates the regression results for a regression of quarter dummies on r^*_{Kt}. We control for year fixed effects and weight by the inverse variance of the r^*_{Kt} estimate. Standard errors are heteroskedasticity robust.

A.8   Components of r^*_K for UK Housing—A VAR Analysis

This section uses a VAR to estimate long run risk premia and expected capital gains for UK housing, in order to show that neither of these can account for the trend fall in the natural rate of return for UK housing that we have documented. We stress that this exercise is tentative, given the inherent uncertainty of any VAR based procedure.

To estimate risk premia and capital gains, we follow the standard practice of Campbell and Shiller (1988). The inputs into the VAR are: the growth rate of rents, the rent to price ratio, and the GDP growth rate. To estimate long run expected capital gains, in every quarter, we estimate forecasts of the rental growth for the subsequent 30 years and take the mean, recalling that long run expected rent growth is equal to long run expected capital gains. To estimate long run housing risk premia, we calculate the 30-year ahead predicted Rent-to-Price ratio (again using forecasts from the VAR). Then $\zeta = [\text{Predicted RTP}] + g - r$, where r is the 10 year 15 year real forward rate.

Figure A.26 displays the results. The estimates for long run expected capital gains, g^*_P, are stable. Estimates of the long run housing risk premium, ζ^* have been rising in the

post-Great Recession era. Therefore the estimates suggest that neither capital gains nor risk premia for UK housing can account for the trend fall in the natural rate of return, $r_K^* \equiv r^* + \zeta^* - g_P^*$, that we have documented.

Figure A.26: Risk Premia and Capital Gains for UK Housing in the Long Run


The figure presents estimates of long-run housing risk premia and capital gains estimated using a VAR. Estimates are produced using data on the UK rent-to-price ratio from the OECD Housing Price Index, as well as the mean level of house prices from Land Registry data and the mean level of rents from the Valuation Office Agency. UK GDP data is obtained from the St. Louis Fed, Federal Reserve Economic Data and long-run UK interest rates are obtained from the Bank of England.

A.9   Comparison with Holston et al. (2017) (HLW)

Our natural experiment and microdata approach to estimating natural rates complements the structural time series approach to estimating the natural rate of interest in real-time. The most well-known approach to estimate r^* is the method proposed by Laubach & Williams (2003) and Holston, Laubach & Williams (2017). This method assumes that the output gap is an autoregressive, and natural output is a random walk with drift. This output gap is then linked to the real natural rate of interest via an Euler equation and a Phillips curve.

When estimated using US data, HLW estimate that r^* has fallen by 2.9pp between 2000 and 2020, with a mean standard error on r^* of 1.35pp per quarter. When estimated with UK data, the decline of r^* over this time period is only 0.81pp, with average standard error of 4.3pp, as seen in Figure A.28a. The reason why the UK estimate has much larger standard errors has to do with higher inflation volatility in the UK, which is a small open economy and is therefore more responsive to exchange rate fluctuations (Figure A.27).

Figure A.27: Inflation Rate In US and UK
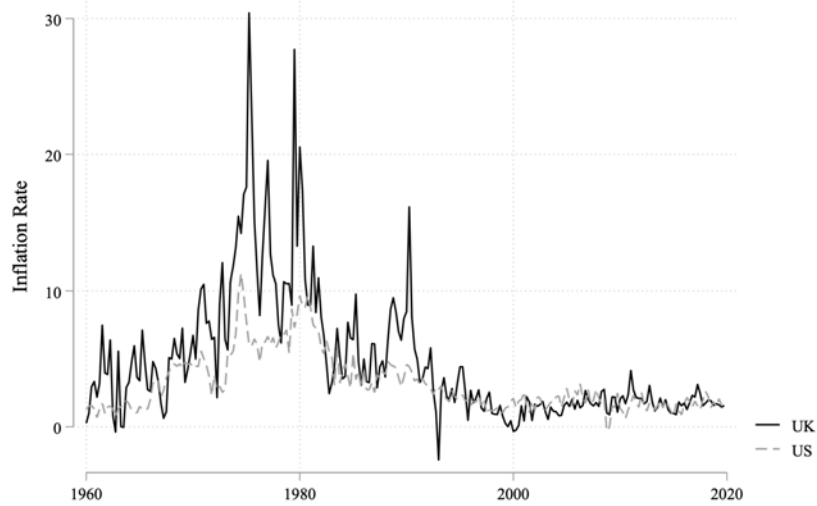
The solid dark grey line shows inflation in the UK between 1960-2020. The dashed light grey line shows inflation in the US between 1960-2020. The data is the same as that used by HLW.

Our estimates are a useful addition to those of HLW for several reasons. First, our estimate relies on minimal structure and is largely model-free. Moreover, our standard errors are on average 0.2 percentage points over our sample period — an order of magnitude smaller than the HLW r^* estimate for the UK.

Most importantly, our estimates persist throughout the pandemic and post-pandemic era and provide valuable insight on the growing wedge between discount rates of housing and of long-term government bonds. In contrast, when post-2020 data is included, the HLW methodology is de-stabilized by the steep decline in GDP in the second quarter of 2020. In the case of the US, the model is incapable of converging when post 2020 data is included. In the UK, inclusion of post 2020 data results in implausible estimates; ranging from -32.2% in the third quarter of 2020 to 25.9% in the second quarter of 1975. This error not only affects the pandemic era, but also propagates to previous to previous decades (Figure A.28b).

Figure A.28: HLW estimate of r^*

(a) Pre COVID-19 Pandemic   **(b)** Post COVID-19 Pandemic
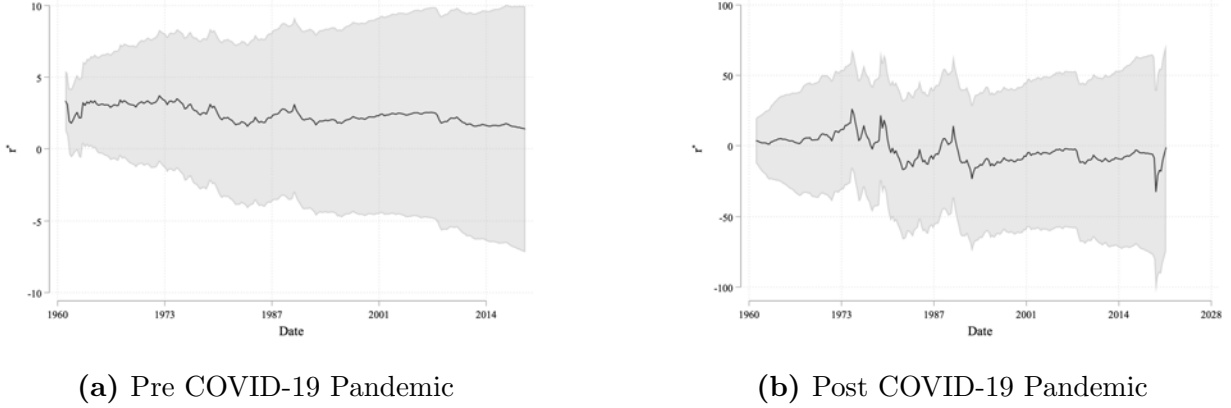
Panel (a) shows HLW's estimates of r^* in the UK using pre-2020 data. Panel (b) shows HLW's estiamtes of r^* using post-2020 data. The shaded area represents the 95% confidence interval.

A.10 Calculating the Extension Hazard Rate

In this section, we explain how we calculate the extension hazard rate, shown in Figure A.3 and Figure A.4. We define the conditional probability that a property i extends given that it has duration T as $\theta_i(T) = P(\text{Extended At } S|\text{Duration} = S)$. To get the total cumulative probability that a T duration property i will extend over the course of its lifetime, we must convert our conditional probabilities to unconditional probabilities as follows,

$$
\begin{aligned}
\pi_i(S) &= P_i(\text{Extended At } S) \\
&= P_i(\text{Extended At } i|\text{Duration} = S)P_i(\text{Duration} = i) \\
&= \theta_i(S) \prod_{U=S+1}^{T} (1 - \theta_i(U))
\end{aligned}
$$

The cumulative probability that property i extends over its lifetime is then given by $\Pi_i(T) = \sum_{S=1}^{T} \pi_i(S)$. In Figure A.4, we scale the hazard rate up by a factor of 1.17. This is because our method to identify lease extensions does not capture extensions that have no transactions before extension. We estimate that there are about 17% more extensions that have been extended but do not have pre-extension transaction data.

Then, the price of a T duration property at time t is given by the following recursive formula,

$$
P_{i,t}^T = \frac{R_{i,t+1} + \theta_i(T)(P_{i,t+1}^{T+90-1} - \kappa_{t+1}^{T-1}) + (1 - \theta_i(T))P_{t+1}^{T-1}}{1 + r^* + \zeta^*} \tag{16}
$$

Intuitively, the price of a T duration asset is the discounted dividends it yields next period,

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plus with probability $1 - \theta_i(T)$, the price of a $P^{T-1}_{i,t+1}$ asset, and with probability $\theta_i(T)$, the price of a $P^{T+90-1}_{i,t+1}$ duration asset minus the cost of extending, all appropriately discounted.

A.11 Difference-in-Differences Estimator with Option Value

This section derives our differences-in-differences estimator of r_K^* in the presence of option value from lease extensions. Let Π^H_{Tt} be the likelihood that a lease, with $T > 80$ years of duration remaining, extends before its duration reaches 80 years. Let Π^L_{Tt} be the likelihood that a lease with $T \leq 80$ years of duration remaining is extended at some point before expiration. Assume a constant discount rate r_K^*, and also assume that the event of extending is uncorrelated with the stochastic discount factor of the extender. Then the price of a leasehold is the present value of its cashflows, i.e.

$$P_t^T = \int_0^T e^{-r_K^* s} R_{t+s} ds + \Pi^H_{Tt} \left(1 - \alpha_t^H\right) \int_T^{T+90} e^{-r_K^* s} R_{t+s} ds + \Pi^L_{Tt} \left(1 - \alpha_t^L\right) \int_T^{T+90} e^{-r_K^* s} R_{t+s} ds.$$

In this equation, the first term is the present value of the first T years of service flow. The second term is the next 90 years of service flow, scaled by the share going to the leasholder, $\left(1 - \alpha^H\right)$; and the likelihood that the lease extends at any time before it falls below 80 years remaining, Π^T_H. The third term is the analogous option value if the lease extends with less than 80 years remaining. Rearranging this expression implies

$$
\begin{aligned}
P_t^T &= \int_0^T e^{-r_{Kt}^* s} R_{t+s} ds + \Pi^H_{Tt} \left(1 - \alpha_t^H\right) \int_T^{T+90} e^{-r_{Kt}^* s} R_{t+s} ds + \Pi^L_{Tt} \left(1 - \alpha_t^L\right) \int_T^{T+90} e^{-r_{Kt}^* s} R_{t+s} ds \\
&= \int_0^T e^{-r_{Kt}^* s} R_t ds + \Pi^H_{Tt} \left(1 - \alpha_t^H\right) e^{-r_{Kt}^* T} R_t \int_0^{90} e^{r_{Kt}^* s} ds + \Pi^L_{Tt} \left(1 - \alpha_t^L\right) e^{-r_{Kt}^* T} R_t \int_0^{90} e^{-r_{Kt}^* s} ds \\
&= \int_0^T e^{-r_{Kt}^* s} R_t ds + e^{-r_{Kt}^* T} R_t \int_0^{90} e^{r_{Kt}^* s} ds \left[\Pi^H_{Tt} \left(1 - \alpha_t^H\right) + \Pi^L_{Tt} \left(1 - \alpha_t^L\right)\right] \\
&= \int_0^T e^{-r_{Kt}^* s} R_t ds + e^{-r_{Kt}^* T} R_t \int_T^{T+90} e^{r_{Kt}^* s} ds \left[\Pi^H_{Tt} \left(1 - \alpha_t^H\right) + \Pi^L_{Tt} \left(1 - \alpha_t^L\right)\right] \\
&= \frac{1 - e^{-r_{Kt}^* T}}{r_{Kt}^*} R_t + e^{-r_{Kt}^* T} R_t \frac{1 - e^{-r_{Kt}^* 90}}{r_{Kt}^*} \left[\Pi^H_{Tt} \left(1 - \alpha_t^H\right) + \Pi^L_{Tt} \left(1 - \alpha_t^L\right)\right].
\end{aligned}
$$

Then for two properties i and j with identical service flow growth, where property i extends and j does not, we have

$$
\begin{aligned}
\Delta_{it}^T &= \log\left(\frac{1 - e^{-r_{Kt}^* (T+90)}}{r_{Kt}^*}\right) - \log\left(\frac{1 - e^{-r_{Kt}^* T}}{r_{Kt}^*} + e^{-r_{Kt}^* T} \frac{1 - e^{-r_{Kt}^* 90}}{r_{Kt}^*} \left[\Pi^H_{Tt} \left(1 - \alpha_t^H\right) + \Pi^L_{Tt} \left(1 - \alpha_t^L\right)\right]\right) \\
&= \log\left(1 - e^{-r_{Kt}^* (T+90)}\right) - \log\left(\left(1 - e^{-r_{Kt}^* T}\right) + e^{-r_{Kt}^* T} \left(1 - e^{-r_{Kt}^* 90}\right) \left[\Pi^H_{Tt} \left(1 - \alpha_t^H\right) + \Pi^L_{Tt} \left(1 - \alpha_t^L\right)\right]\right),
\end{aligned}
$$

which is the expression in the main text.

A.12 Estimating Change in Option Value Using Discontinuities

As before, assume that $\alpha_t^H = \alpha_t^T$ for $T \geq 80$ and $\alpha_t^L = \alpha_t^T$ for $T < 80$, such that the share of holdup in a given time period t is fixed above and below 80, separately. In this section, we aim to estimate $\alpha_t^L - \alpha_t^H$ for the post-2010 period, using the discontinuity in prices at $T = 80$ observed in Table 4. From the preceding subsection, the price of a property i with duration T is

$$P_{it}^T = \frac{R_{it}}{r_K^*} \left(1 - e^{-r_K^* T} + \left[\Pi_{Tt}^H \left(1 - \alpha_t^H \right) + \Pi^L \left(1 - \alpha_t^L \right) \right] e^{-r_{Kt}^* T_{it}} \left(1 - e^{-r_{Kt}^* 90} \right) \right) \qquad (17)$$

To condense notation, denote the option value term

$$\Omega(T) \equiv \left[\Pi_{Tt}^H \left(1 - \alpha_t^H \right) + \Pi^L \left(1 - \alpha_t^L \right) \right] e^{-r_{Kt}^* T_{it}} \left(1 - e^{-r_{Kt}^* 90} \right).$$

Then, the difference in change in price between time $t - h$ and t of properties i and j is,

$$\Delta \log P_{it}^T - \Delta \log P_{jt}^T = \log(1 - e^{-r_K^* T_i} + \Omega(T_i)) - \log(1 - e^{-r_K^*(T_i+h)} + \Omega(T_i + h))$$
$$- (\log(1 - e^{-r_K^* T_j} + \Omega(T_j)) - \log(1 - e^{-r_K^*(T_j+h)} + \Omega(T_j + h_j))).$$

This equation acknowledges that option value will discontinuously change around the 80 year threshold, via changes in the Ω terms. We can then estimate α_t^H by nonlinear least squares on the same sample as for regression Equation (13), by setting $\alpha_t^L = 1$, which is what we estimate in Table 5, and r_K^* to its mean for that period. We obtain an estimate of $\alpha_t^H = 0.56$ in the post-2010 period.

A.13 Proofs

Proposition A.1. *There exists some value $\bar{r}_K < r_{RV}$ such that:*

1. *If the natural rate satisfies $r_{Kt}^* \geq \bar{r}_K$ then*

   (a) *There is zero option value at all years of duration remaining, that is, $\alpha_t^T = 1$ for all T.*

   (b) *The price of a leasehold is continuous in duration as the property's duration falls below 80 years, so*
   $$\lim_{T \to 80^-} P_{it}^T = \lim_{T \to 80^+} P_{it}^T.$$

2. *If the natural rate satisfies $r^*_{Kt} < \bar{r}_K$ then*

   (a) *There is positive option value above 80 years in duration, that is, $\alpha^T_t < 1$ for all $T > 80$ and option value discontinuously falls at 80 years, so that α^T_t discontinuously increases at $T = 80$.*

   (b) *The price of a leasehold discontinuously falls as the property's duration falls below 80 years, so*

   $$\lim_{T \to 80^-} P^T_{it} < \lim_{T \to 80^+} P^T_{it}.$$

Proof. From Equation (11) and Equation (12), the price of a property is

$$P^T_{it} = \begin{cases} P^{T+90}_{it} - \min\left[RV^T_{it} + \gamma R_{it}, MV^T_{it}\right] & T \geq 80 \\ P^{T+90}_{it} - \min\left[\frac{RV^T_{it} + MV^T_{it}}{2} + \gamma R_{it}, MV^T_{it}\right] & T < 80. \end{cases} \tag{18}$$

Recall the definitions of reversion value and marriage value

$$MV^T_{it} = \frac{R_{it}}{r^*_{Kt}}\left(e^{-r^*_{Kt}T} - e^{-r^*_{Kt}(T+90)}\right) \tag{19}$$

$$RV^T_{it} = \frac{R_{it}}{r_{RV}}\left(e^{-r_{RV}T} - e^{-r_{RV}(T+90)}\right) \tag{20}$$

We will define $\bar{r}_K$ as the value of r^*_{Kt} such that $RV^T_{it} + \gamma R_{it} = MV^T_{it}$, that is, the tribunal costs for a lease above 80 years are exactly the market value. The value of $\bar{r}_K$ satisfies

$$\frac{R_{it}}{r_{RV}}\left(e^{-r_{RV}T} - e^{-r_{RV}(T+90)}\right) + \gamma R_{it} = \frac{R_{it}}{\bar{r}_K}\left(e^{-\bar{r}_K T} - e^{-\bar{r}_K(T+90)}\right)$$

$$\implies \frac{e^{-r_{RV}T} - e^{-r_{RV}(T+90)}}{r_{RV}} + \gamma = \frac{e^{-\bar{r}_K T} - e^{-\bar{r}_K(T+90)}}{\bar{r}_K} \tag{21}$$

where in the first line we have substituted in the definitions of marriage value (Equation (19)) and reversion value (Equation (20)). The right hand side of Equation (21) is strictly decreasing in $\bar{r}_K$. Therefore there is a unique value of $\bar{r}_K$ satisfying the equation.

Now we will prove part (1) of the proposition, in which $r^*_{Kt} \geq \bar{r}_K$. Equation (21) implies that for all $r^*_{Kt} \geq \bar{r}_K$ we must have

$$RV^T_{it} + \gamma R_{it} \geq MV^T_{it}. \tag{22}$$

Equation (22) implies

$$RV^T_{it} + \gamma R_{it} \geq MV^T_{it}$$

$$\implies RV_{it}^T + \gamma R_{it} + MV_{it}^T \geq 2MV_{it}^T$$

$$\implies \frac{RV_{it}^T + \gamma R_{it} + MV_{it}^T}{2} \geq MV_{it}^T$$

$$\implies \frac{RV_{it}^T + MV_{it}^T}{2} + \gamma R_{it} \geq MV_{it}^T \tag{23}$$

Therefore for $r_{Kt}^* \geq \bar{r}_K$, prices satisfy

$$P_{it}^T = \begin{cases} P_{it}^{T+90} - MV_{it}^T & T \geq 80 \\ P_{it}^{T+90} - MV_{it}^T & T < 80, \end{cases} \tag{24}$$

where we have substituted Equation (22) and Equation (23) into Equation (18) for $r_{Kt}^* \geq \bar{r}_K$. Recall the definition of α_t^T as the ratio of the lease extension cost to MV_{it}^T. Equation (24) shows that $\alpha_t^T = 1$ for all t, which proves part (1a) of the proposition. Since the top and bottom of Equation (24) are equal at $T = 80$, prices are continuous at $T = 80$, which proves part (1b) of the proposition.

Now we will prove part (2) of the proposition, in which $r_{Kt}^* < \bar{r}_K$. Equation (21) implies that for all $r_{Kt}^* < \bar{r}_K$ we must have

$$RV_{it}^T + \gamma R_{it} < MV_{it}^T. \tag{25}$$

Then by Equation (18), the price of a property with more than 80 years duration remaining is

$$P_{it}^T = P_{it}^{T+90} - \left(RV_{it}^T + \gamma R_{it} \right).$$

The price of a property with less than 80 years remaining is

$$P_{it}^T = P_{it}^{T+90} - \min\left[\frac{RV_{it}^T + MV_{it}^T}{2} + \gamma R_{it}, MV_{it}^T \right]. \tag{26}$$

Also, note that

$$RV_{it}^T + \gamma R_{it} < \min\left[\frac{RV_{it}^T + MV_{it}^T}{2} + \gamma R_{it}, MV_{it}^T \right], \tag{27}$$

since $RV_{it}^T + \gamma R_{it} < MV_{it}^T$ by inequality (25) and also by inequality (Equation (25)) we have

$$RV_{it}^T + \gamma R_{it} < MV_{it}^T$$

$$\implies RV_{it}^T < MV_{it}^T$$

$$\implies 2RV_{it}^T < RV_{it}^T + MV_{it}^T$$

$$\implies RV_{it}^T < \frac{RV_{it}^T + MV_{it}^T}{2}$$

$$\implies RV_{it}^T + \gamma R_{it} < \frac{RV_{it}^T + MV_{it}^T}{2} + \gamma R_{it}.$$

Equation (26) and Equation (27) imply that for $T < 80$

$$P_{it}^T < P_{it}^{T+90} - \left(RV_{it}^T + \gamma R_{it}\right)$$

Therefore prices discontinuously fall when T falls below 80 which proves part (2b) of the proposition. Since lease extension costs rise when T falls below 80, α_t^T also discontinuously rises when T falls below 80, which is part (2a) of the proposition.

$\square$