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ABSTRACT

We estimate a model of firm export dynamics featuring cross-country complementarities. The firm decides where to export by solving a dynamic combinatorial discrete choice problem, for which we develop a solution algorithm that overcomes the computational challenges inherent to the large dimensionality of its state space and choice set. According to our estimated model, firms enjoy cost reductions when exporting to countries geographically or linguistically close to each other, or that share deep trade agreements; and countries, especially small ones, sharing these traits with attractive destinations receive significantly more exports than in the absence of complementarities.

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1 Introduction

Since Baldwin (1988), a large literature studies the dynamics of firms in export markets. This literature nearly unanimously assumes a firm's export status in a country is unaffected by its status in other countries. There is however growing evidence questioning this assumption, supporting instead the thesis that there are cross-country complementarities, such that exporting to a country makes a firm more likely to export to other countries (e.g. Chaney, 2014; Morales et al., 2019).

The notion of cross-country export complementarities has important policy implications. E.g., it underpins claims that preferential trade agreements (PTAs) serve as gateways to markets beyond those of the agreements' signatories.[1] Also, the belief that the regulatory convergence deep PTAs impose on their members is a source of complementarities between them (Grossman et al., 2021) supports claims that these agreements attract imports from third countries (Baldwin, 2011; Mattoo et al., 2022), thus affecting policy evaluations of the creation and breakup of such agreements.[2]

The evidence questioning the hypothesis that a firm's export choices in a country are uninfluenced by its choices in other countries, along with the potential policy implications of cross-country export complementarities, together raise the question of how quantitatively important these complementarities are in determining firm exports, particularly in reaction to trade policy changes. To provide a first answer to this question, we build on Das et al. (2007) and extend a dynamic multi-country partial equilibrium model of firm export choices to incorporate cross-country complementarities. In our model, the firm may enjoy cost reductions in a country when jointly exporting to other countries, and chooses its per-period set of export destinations as the solution to a single-agent dynamic combinatorial discrete choice problem. We build on Jia (2008) and Arkolakis et al. (2021) to develop an algorithm to solve such problems, and estimate our model using a SMM estimator and firm-country-year data on the universe of exports from Costa Rica during 2005-2015.

The estimated model predicts cross-country complementarities increase the number of sample firm-country-year triplets with positive exports in 12%, and total exports in 7%. When evaluating the impact on Costa Rica of a Brexit-driven hypothetical UK-EU regulatory divergence, we predict total exports and the number of exporters to the UK to decrease in around 5% in the ten-year window post Brexit. Analogous predictions for the EU as a whole are below 0.5%, reflecting that complementarities tend to have a larger impact on exports to smaller markets. Finally, motivated by Costa Rica's recent request to join the CPTPP, we predict the impact of Costa Rica signing PTAs that eliminate tariffs on its exports to different trade areas, and show that researchers using models analogous to ours but that exclude the possibility of complementarities would predict an increase in Costa Rican exports similar to that implied by our model when the destinations eliminating tariffs

[1] In an example involving Costa Rica, whose data we use in this paper, its government has argued that its PTA with Singapore would increase its exports all throughout Asia (Ruiz, 2013). Similar claims have been made in relation to PTAs between Australia and Peru (Australian Govt., 2020) or Canada and Morocco (Canadian Govt., 2022).

[2] Deep PTAs differ from shallow ones in that they constrain members' regulations. Standard trade models (e.g., Eaton and Kortum, 2002; Anderson and van Wincoop, 2003) predict shallow PTAs will impact negatively third-country imports. In relation to Brexit, e.g., UNCTAD (2020) claims this third-country effect will be mitigated by increased UK-EU regulatory divergence as "trade costs rise for third countries due to production process adjustment costs or duplication of proofs of compliance."

exhibit weak complementarities with other countries (e.g., the CPTPP members), but significantly lower than that implied by our model when these destinations have strong complementarities among themselves and with other countries (e.g., the EU members).

Consistent with findings in the prior literature, firms in our sample tend to export to countries geographically or linguistically close to, or that share a deep PTA with, their other concurrent export destinations. This correlation in export choices decreases only marginally when controlling for sector-country-year and firm-year fixed effects and, thus, is mostly due to factors varying at the firm-country level. Although within-firm cross-country export complementarities could explain this correlation pattern, it may be caused instead by firm- and country-specific unobserved export profit (e.g., demand) shifters that are positively correlated across countries. To guide the separate identification of export complementarities and correlation in unobserved export profit shifters, and to quantify the role the former play in determining firm exports, we build a model of firm export dynamics that allows for both sources of cross-country correlation in firm export choices.

In our model, monopolistically competitive firms featuring constant marginal production costs face country- and period-specific variable, fixed, and sunk export costs. Variable costs are "iceberg" costs and, building on Roberts and Tybout (1997), firms face sunk costs when exporting to countries to which they did not export in the previous period. All export costs in a country are allowed to depend on its geographic and linguistic distance to, and the deepness of its PTAs with, the firm's home country. The fixed cost a firm faces in a country and period may additionally depend on the firm's other export destinations in the same period. Specifically, a firm may face smaller fixed costs in a country if it concurrently exports to another country, and the size of this cost reduction may depend on these countries' geographic and linguistic proximity, as well as the deepness of the PTAs of which both are members. To discipline the estimation of the parameters determining the extent to which fixed costs in a country depend on the firm's export choices in other countries, we allow this cost to also depend on a term unobserved to the researcher that is potentially correlated across countries according to a correlation coefficient that may similarly vary with the countries' geographic or linguistic proximity, or the deepness of the PTAs of which they are members.

The inclusion of sunk costs, and our modeling of fixed costs, make a firm's static export profits in a country and period weakly larger if the firm exported to the same country in the previous period, or if it exports to other countries in the same period. The firm internalizes the impact its export choice in a country and period has on profits in other countries and periods. It thus chooses each period's set of export destinations as the solution to a dynamic combinatorial discrete choice problem. We assume for tractability that the firm has perfect foresight on several payoff-relevant variables, but allow for firm uncertainty about future realizations of a country- and period-specific "blocking" shock that, if realized, prevents the firm from exporting to a country in a period. As in Eaton et al. (2016) and Caliendo et al. (2019), we assume all payoff-relevant variables on which firms have perfect foresight are constant after a terminal period.

Given commonly available computational capabilities, the optimization problem determining the firm's export path cannot be solved using standard algorithms. The reason is that the cardinalities of the per-period choice set and state space grow exponentially in the number of possible destinations: given J foreign countries, the choice set includes 2^J elements (each one a J-dimensional vector of binary variables indicating the set of countries to which the firm exports) and the state space includes 2^{2J} elements (each one indicating the firm's export bundle in the previous period and the current realization of the blocking shocks in every country). To compute the firm's optimal export path, we develop an algorithm that solves a series of increasingly complex problems that place gradually tighter bounds on the firm's optimal choice. Our algorithm exploits the supermodularity of the firm's objective function: exporting to a country in a period and state weakly increases the returns to exporting in every other country, future period, and possible state. It thus builds on prior work that has leveraged the supermodularity of the objective function to solve otherwise intractable static optimization problems (see Jia, 2008, Antràs et al., 2017, Arkolakis et al., 2021), and it extends the set of problems that are computationally feasible to solve to a family of supermodular problems featuring dynamics and firms' uncertainty about future payoffs.

The problem of separately identifying the parameters governing the sensitivity of a firm's country-specific fixed export costs to its concurrent export destinations from those determining the cross-country correlation in fixed costs' unobserved determinants is an instance of the general problem of separately identifying "path" (or group) dependence from correlated unobservables; in our case, across countries within a period. For any proximity measure between countries, be it geographic or linguistic, or whether they share a deep PTA, we use two types of moments to separately identify these parameters. First, moments capturing how the covariance in firm export choices in any two countries depends on their proximity. Second, moments capturing how the probability firms export to a destination depends on exogenous export profit shifters of *other* countries close to it.[3] While the first type of moments is particularly sensitive to the parameters determining the correlation in unobserved fixed cost shocks, the second type is especially sensitive to the parameters determining the impact exporting to a country has on fixed costs in other countries. In our model, both types of moments jointly identify the parameters of interest.

Our estimates reveal a large heterogeneity across country pairs in the impact exporting to one of them has on fixed costs in the other one. This heterogeneity reflects their geographic and linguistic proximity, as well as the deepness of the PTAs tying together their regulations; e.g., exporting to Korea reduces fixed costs in China in 0.3%, exporting to Canada brings down costs in the US in 3.5%, and exporting to France reduces costs in Germany in 9%. These cost savings accumulate as the firm adds destinations; e.g., for a firm exporting to France, adding Switzerland to its export bundle increases the reduction in fixed costs in Germany from 9% to 16%. Generally, EU members, being geographically close to each other and sharing a deep PTA, have fixed costs that are particularly sensitive to the firm export choices in the other members.

We use our model to perform three types of analysis. First, to quantify the role complementarities play in determining firm exports, we compare the in-sample predictions of a version of our

[3]Specifically, the second type of moments relates firm export choices in a country to the aggregate export potential of the countries close to it. We measure a country's export potential as the importer fixed effect in a sectoral gravity equation estimated using data on all country pairs that do not include Costa Rica as importer or exporter.

model in which we set to zero the parameters determining the strength of complementarities to those of alternative versions in which some or all of these parameters take their estimated values. Overall, complementarities increase the number of firm-country-periods with positive exports in 12%, and total exports in 7%. Of the three sources of complementarities we allow for, geographical proximity plays the largest role, causing by itself a 4% increase in exports, while allowing deep PTAs to generate complementarities increases exports in 2%, and linguistic proximity does so in 1%. These numbers mask a large heterogeneity across countries: most EU members see Costa Rican exports increase in at least 10% (with countries in Central Europe experiencing increases above 25%), while exports to large countries such as the US or China are largely unaffected.

Second, to measure the third-country effect of cross-country complementarities due to deep PTAs, we quantify the impact of Brexit on Costa Rican exports to the UK and the EU. We use our model to compare predicted exports in a setting in which the UK and the EU share no deep PTA post Brexit to those in a counterfactual setting in which the UK still belongs to the EU and, thus, shares a deep PTA with its members. Trade barriers between Costa Rica and all destinations are kept the same in both scenarios; thus, our analysis isolates the third-country effect of Brexit. In our model, in the four years between the Brexit referendum and the UK withdrawal from the EU, firms anticipate the future reduction in UK-EU complementarities, causing the number of firm-periods with positive exports and total exports to the UK to decrease in 1.6% and 0.8%, respectively. In the ten years following the withdrawal, the number of firm-periods with positive exports and total exports to the UK drop in close to 5%. Conversely, the impact on exports to the EU is minimal. Given the partial-equilibrium nature of our model, these predictions reflect the impact of cross-country complementarities alone.

Third, and last, we study the impact of Costa Rica signing PTAs that bring its export tariffs with different trade areas to zero, and compare our model's predictions to those of a re-estimated model similar to ours except it rules out the possibility of complementarities. Our model predicts eliminating Costa Rican export tariffs with the EU would increase the number of firm-country-years with positive exports and total exports to its members in 65% and 83%, respectively. Although tariffs with non-EU countries do not change, exports to some of them are affected and, e.g., exports to Iceland and to the UK increase in close to 7%, reflecting that the former shares a deep PTA with the EU, and the latter is geographically close to several of its members. Researchers using a model that excludes the possibility of complementarities would have predicted smaller increases of 55% and 80% in export participation and total exports to EU members, respectively, and no change in exports to non-members. When eliminating tariffs with CPTPP members instead, our model predicts an export growth to these countries that is less than 1 pp. higher than that predicted by the model without complementarities, with no significant change in exports to non-member countries predicted by either model. The reason for the larger difference in model predictions when studying a change in trade policy with EU members than when doing so with CPTPP members is that the former exhibit stronger complementarities among themselves and with non-member countries than the latter. Thus, whether models that allow for complementarities yield predictions similar to those of models that do not depends on the policy change being studied.

Our paper relates to several strands of the literature. First, it relates to the literature on export dynamics which, as reviewed in Alessandria et al. (2021a), has largely studied the firm's export decision in an aggregate market (Roberts and Tybout, 1997; Das et al., 2007; Alessandria and Choi, 2007; Arkolakis, 2016; Ruhl and Willis, 2017) or in independent markets (Fitzgerald et al., 2023).[4] Exceptions are Schmeiser (2012), Chaney (2014), Albornoz et al. (2016), and Morales et al. (2019), which allow for cross-market firm export complementarities. Relative to this work, our contribution is twofold: first, we solve a canonical partial-equilibrium model of firm export dynamics extended to allow for complementarities across many markets; second, we use the estimated model to quantify the role complementarities play in determining the reaction of firm exports to policy changes.[5]

Second, our paper also relates to a reduced-form literature identifying cross-market interdependencies in firm exports. While there is a large literature documenting correlation patterns in firm sales across markets (Lawless, 2009; Albornoz et al., 2012, 2023), there is a more recent literature using instruments to separately identify cross-market interdependencies from correlation in unobserved determinants of firm sales (Defever et al., 2015; Berman et al., 2015; Almunia et al., 2021; Albornoz et al., 2021; Mattoo et al., 2022). Our contribution is to allow for complementarities in an export dynamics model, to estimate the model parameters that determine the strength of these complementarities using an approach that builds on the literature using instruments to identify these complementarities, and to quantify the role complementarities play in firm exports.

Third, our paper relates to the work solving combinatorial discrete choice problems. This literature has focused nearly exclusively on static problems, and has implemented several approaches: evaluating all choices (Tintelnot, 2017); modeling combinatorial choices as an aggregation of multinomial ones (Hendel, 1999); approximating the discrete problem as a choice over a continuous variable (Oberfield et al., 2023; Castro-Vincenzi, 2022); using simulation-based global optimization algorithms that converge to the solution as the number of simulations grows to infinity (Houde et al., 2023; Castro-Vincenzi et al., 2023); or, devising algorithms that exploit the super- or sub-modularity of the objective function (Jia, 2008; Antràs et al., 2017; Arkolakis et al., 2021).[6] Building on this last approach, we introduce an algorithm to solve rational-expectations single-agent combinatorial dynamic discrete choice problems in which all choices are complements.

The rest of the paper proceeds as follows. Section 2 describes our data. Section 3 documents correlation patterns in firm exports. Section 4 introduces our model, and sections 5 and 6 explain how we solve and estimate it, respectively. In Section 7, we present the model estimates, and we discuss counterfactual results in Section 8. Section 9 concludes.

[4] Other work on export dynamics in a single market or independent markets includes Eaton et al. (2008, 2021a,b); Alessandria and Choi (2014a,b); Albornoz et al. (2016); Fitzgerald and Haller (2018); Dickstein and Morales (2018); Gumpert et al. (2020); Alessandria et al. (2021b). Work on dynamics in imports or multinational production with independent markets includes Conconi et al. (2016); Ramanarayanan (2017); Garetto et al. (2021); Lu et al. (2022).

[5] The paper closest to ours is Morales et al. (2019), which partially identifies export complementarities under weak restrictions on firm expectations, choice sets, and planning horizons, without solving the resulting model. A similar methodology to identify complementarities in firm imports has been used in Hoang (2022).

[6] For work incorporating dynamics, see Zheng (2016), who groups choices in clusters such that each choice affects choices in other clusters only through cluster-specific aggregates.

2 Data

Our analysis uses two types of data: data on characteristics of firms located in Costa Rica, and data on characteristics of foreign countries as destinations of Costa Rican exports.

Our firm-level data covers the period 2005-2015 and comes from three sources. First, the Costa Rican customs database, which provides information on export revenues by firm, foreign country, and year for the universe of Costa Rican firms. Second, an administrative dataset that, for all firms located in Costa Rica, contains information on their sector, total sales, and expenditure in labor and materials. Using information in these datasets, we construct a measure of firm domestic sales by subtracting total export revenue from total sales. Third, a dataset built by Alfaro-Ureña et al. (2022), which identifies the Costa Rican firms that belong to a foreign multinational corporation. We merge the three datasets using firm identifiers provided by Alfaro-Ureña et al. (2022), and restrict our sample to include only manufacturing firms (i.e., whose main activity is in sectors 10 to 33 according to ISIC Rev. 4) that are not part of a foreign multinational corporation.

The resulting dataset includes 7,203 firms. Approximately 8% of them export in a typical year. While exporting firms often export to a single destination (this being the case for approximately 40% of exporters), approximately 25% of them export to at least four destinations, 10% of them export to at least seven, and 5% of them export to at least ten. By sector, most export participation events are concentrated in the manufacturing of other food products (sector 1079 in the ISIC Rev. 4 classification) and of plastic products (sector 2220). The most popular destinations are either countries that are geographically close to Costa Rica (e.g., Nicaragua) or relatively large (e.g., the United States). We provide additional descriptive statistics in Appendix B.1.

We complement our firm-level data with data on country characteristics. We obtain information on the geographical distance between countries from CEPII's GeoDist (Mayer and Zignago, 2011), on the languages spoken in each country from Ethnologue (Eberhard et al., 2021), on the content of PTAs from Hofmann et al. (2019), on the tariffs applied to exports from Costa Rica from Barari and Kim (2022), and on countries' GDP from the World Bank. Among other purposes, we use these data to build geographical, linguistic, and regulatory distances between countries.

We denote the geographical distance between countries j and j' as $n^g_{jj'}$. As in Head and Mayer (2002), we measure $n^g_{jj'}$ as a population-weighted harmonic mean of distances between cities located in j and j'. Two features of this measure are worth noting. First, it accounts for the location of population within a country; e.g., according to this measure, Russia is closer to Germany (2,290 km) than to China (4,984 km). Second, large countries tend to be isolated; e.g., while the distance between Switzerland and the UK is 872 km, that between the US and Canada is 1,154 km.

We denote the linguistic distance between countries j and j' as $n^l_{jj'}$, and measure it as the probability two randomly selected individuals respectively drawn from j and j' do not speak a common language. To compute this probability, we use country-specific data on the population shares that speak any given language. Relative to measures based on the commonality of official languages between countries, $n^l_{jj'}$ reflects the actual prevalence of each language in each country, and thus accounts for the fact that certain languages are popular in countries in which they are not official; e.g., although the UK and Denmark share no official language, they are linguistically close according to our measure, as a large share of the Danish population reports speaking English.[7]

Our third distance measure between countries j and j' in a year t is an inverse measure of the breadth of the regulatory harmonization imposed by the PTAs of which j and j' are members in t, if any. We denote this measure as $n^a_{jj't}$, refer to it as the regulatory distance between j and j' in t, and build it using the data in Hofmann et al. (2019), which indicates whether a PTA contains provisions in each of 52 policy areas. We focus on the seven (out of the 52) areas that concern regulatory harmonization, and count in how many of them a PTA includes some provision.[8] When two countries are cosignatories of more than one PTA in a year t, we consider only the agreement containing provisions in the largest number of harmonization-focused policy areas, and compute:

$$n^a_{jj't} = 1 - \frac{1}{7}\left\{ \begin{array}{c} \text{number harmonization-focused policy areas in which} \\ \text{the PTA between } j \text{ and } j' \text{ in } t \text{ includes some provision} \end{array} \right\}. \tag{1}$$

This measure is between zero and one. E.g., EU members are bound by an agreement containing provisions in all seven harmonization areas of interest and, thus, $n^a_{jj't} = 0$ between them; NAFTA contains provisions in five of the seven areas and, thus, $n^a_{jj't} = 0.29$ between their members. In Appendices B.2 to B.4, we provide more information on the three distances introduced above.

3 Cross-country Correlation in Export Participation Decisions

If geographical, linguistic, or regulatory proximity are sources of cross-country complementarities in firm exports, a firm's export probability in a country j and year t will, all else equal, be larger if it concurrently exports to countries close to j according to any of these three distance measures. To explore whether firm exports in our sample exhibit these correlation patterns, for each firm i, country j, and year t, and for each of the three distance measures we consider, we compute a dummy variable that equals one if firm i exports in year t to at least one country close to j; e.g., for the case of geographical distance, we compute

$$Y^g_{ijt} = \mathbb{1}\Big\{ \sum_{j' \neq j} \mathbb{1}\{n^g_{jj'} \leqslant \bar{n}_g\} y_{ij't} > 0 \Big\}, \tag{2}$$

where $\mathbb{1}\{\cdot\}$ is an indicator function, $n^g_{jj'}$ is introduced in Section 2, $\bar{n}_g$ is a threshold determining whether we classify two countries as geographically close to each other, and $y_{ij't}$ is a dummy variable that equals one if firm i exports to country j' in year t. Thus, Y^g_{ijt} is a dummy that equals one if i exports in t to at least one country whose geographical distance to j is smaller than $\bar{n}_g$. In our baseline analysis, we set $\bar{n}_g$ such that we classify two countries as close if their distance is less than 790 km, which is the 2.5 percentile of the distribution of distances across all country pairs.

[7]The linguistic distance between the UK and Denmark is 0.11; i.e., we measure the probability that a randomly selected individual from Denmark does not understand a randomly selected individual from the UK to be 11%.

[8]These areas cover the harmonization of: sanitary or phytosanitary measures; technical barriers to trade; intellectual property rights; environmental standards; consumer protection laws; statistical methods; competition laws.

Table 1: Conditional Export Probabilities

	Panel A: No Controls				*Panel B: Controlling for Firm-Year Fixed Effects*			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
Y^g_{ijt}	0.2622^a (0.0092)			0.2082^a (0.0079)	0.2226^a (0.0089)			0.1957^a (0.0081)
Y^l_{ijt}		0.1617^a (0.0076)		0.0752^a (0.0054)		0.1220^a (0.0067)		0.0718^a (0.0055)
Y^a_{ijt}			0.0857^a (0.0037)	0.0386^a (0.0021)			0.0517^a (0.0026)	0.0259^a (0.0018)
Obs.	3,859,618				3,859,618			

	Panel C: Controlling for Sector-Country-Year Fixed Effects				*Panel D: Controlling for Firm-Year & Sector-Country-Year Fixed Effects*			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
Y^g_{ijt}	0.2462^a (0.0089)			0.1955^a (0.0076)	0.2043^a (0.0086)			0.1809^a (0.0078)
Y^l_{ijt}		0.1572^a (0.0074)		0.0764^a (0.0052)		0.1160^a (0.0066)		0.0720^a (0.0054)
Y^a_{ijt}			0.0809^a (0.0035)	0.0363^a (0.0019)			0.0473^a (0.0026)	0.0207^a (0.0018)
Obs.	3,859,618				3,859,618			

Note: ^{a} denotes 1% significance. Standard errors are clustered by firm. The dependent variable is a dummy that equals 1 if firm i exports to country j in year t. The covariates of interest are $Y^x_{ijt} = \mathbb{1}\{\sum_{j' \neq j} \mathbb{1}\{n^x_{jj'} \leqslant \bar{n}_x\} y_{ij't} > 0\}$ for $x \in \{g, l\}$, and $Y^a_{ijt} = \mathbb{1}\{\sum_{j' \neq j} \mathbb{1}\{n^a_{jj't} \leqslant \bar{n}_a\} y_{ij't} > 0\}$, with $\bar{n}_g = 790$ km, $\bar{n}_l = 0.11$ and $\bar{n}_a = 0.43$.

We use expressions analogous to that in equation (2) to define two dummy variables, Y^l_{ijt} and Y^a_{ijt}, that equal one if firm i exports in year t to at least one country sufficiently close to j according to the distance measures $n^l_{jj'}$ and $n^a_{jj't}$, respectively. In our baseline analysis, we classify two countries as linguistically close if the probability two randomly selected individuals from both countries speak a common language is at least 0.89 (i.e., if $n^l_{jj'} < 0.11$, where 0.11 is the 2.5 percentile of the distribution of linguistic distances across all country pairs), and we classify two countries as regulatory close if they are cosignatories of a PTA including provisions in at least four of the seven areas listed in footnote 8 (i.e., if $n^a_{jj't} < 0.43$).[9] In Appendix B.5, we present analogous results that rely on looser thresholds for classifying countries as close to each other.

Table 1 presents OLS estimates of regressions of a dummy variable that equals one if firm i exports to a country j in a year t on Y^g_{ijt}, Y^l_{ijt}, and Y^a_{ijt}. Panel A includes estimates of specifications without fixed effects. The results in column (1) show exporting in year t to a destination geographically close to a country j increases in 0.26 the probability the firm exports to j in t. The results in columns (2) and (3) indicate this probability increase is 0.16 when the destination is linguistically close to j, and 0.09 when it shares a deep PTA with j. These estimates reveal a strong correlation in firm export choices across countries close to each other, as the average probability a firm exports to a country in a year is below 0.01.

[9] According to these thresholds, e.g., Argentina and Spain (but not France and Switzerland) are linguistically close; and all members of the EU, NAFTA, CAFTA, or Mercosur are close in their regulations.

In panels B to D, we present estimates analogous to those in Panel A but for specifications that control for firm-year fixed effects, sector-country-year fixed effects, or both. The point estimates in these panels are only moderately smaller than those in Panel A. The results in Table 1 thus show that firms' export participation decisions in countries geographically or linguistically close to each other, or cosignatories of a deep PTA, are positively correlated, and that factors varying at the firm-year level (e.g., firm productivity) or at the sector-country-year level (e.g., market size, or total number of exporters in a destination) are not the main drivers of this correlation.

Although consistent with them, the patterns described in Table 1 are not evidence of the presence of cross-country complementarities in firm exports, as they may be due instead to firm-country specific export profit shifters that are positively correlated across countries geographically or linguistically close to each other, or that are cosignatories of a deep PTA. To guide the identification of cross-country complementarities, and to quantify the role these play in determining firm exports, we present below a model that accounts both for potential cross-country complementarities and for cross-country correlation in unobserved export determinants.

4 Dynamic Export Model With Complementarities

We present here a partial-equilibrium model in which forward-looking firms choose every period the bundle of countries they export to among a large set of potential destinations. When exporting to a country, firms face variable, fixed, and sunk costs. We allow the fixed costs a firm faces in a destination and period to be smaller if the firm also exports to other countries in the same period. This creates static cross-country complementarities: a firm's profits when exporting to multiple countries in a period are weakly larger than the sum of the profits of exporting to each of them individually. Guided by the patterns documented in Section 3, we allow the complementarities between any two countries to depend on the geographical and linguistic proximity between them, and on the deepness of the PTAs of which they are both members. Sunk costs make a firm's export choice in a country and period impact export profits in that country in the subsequent period. This creates dynamic within-country complementarities: a firm's profits when exporting to a country in two consecutive periods are weakly larger than the sum of the profits of exporting in each of the two periods individually. In the presence of static and dynamic complementarities, a firm's export choice in a country and period impacts its export profits in other countries and periods. Firms take this into account when choosing where to export. Specifically, firms determine their export bundle in a period after solving an infinite-horizon dynamic combinatorial discrete-choice problem.

We incorporate into our model several shocks that allow export profits to vary flexibly across firms, countries and periods. To make the optimization problem of potential exporters computationally tractable, we assume firms have perfect foresight on some (but not all) of these shocks, and follow Eaton et al. (2016) and Caliendo et al. (2019) in assuming all payoff-relevant variables on which firms have perfect foresight stay constant after a terminal period T.[10]

[10]This approach is similar to that in Kehoe et al. (2018), who assume that, after a terminal period, all variables

4.1 Setup

Firms produce in country h. Time and locations are discrete. We index periods by $t \geqslant 0$, firms by i, and foreign countries by j. Firm i is born exogenously at period $\underline{t}_i$ and, once born, is active forever. We denote the first and last sample periods as $\underline{t}$ and $\bar{t}$, respectively, and assume $T > \bar{t}$.

4.2 Marginal Costs, Demand Function, and Market Structure

Firm i has constant marginal production costs w_{it}. Exporting requires incurring in extra variable "iceberg" costs; specifically, firm i must ship τ_{ijt} units of output for a unit to reach j, and its marginal cost of selling in j at t is thus $\tau_{ijt} w_{it}$. The marginal cost of selling at home is $\tau_{ht} w_{it}$.

Conditional on firm i exporting to j at t, the quantity sold q_{ijt}, depends on the price p_{ijt} it sets, the price index P_{jt}, and the market expenditure Y_{jt}, according to the function $q_{ijt} = p_{ijt}^{-\eta} P_{jt}^{\eta-1} Y_{jt}$. Firms face a similar demand at home. Firms set optimal prices in all markets taking as given the market's expenditure and price index and, thus, fix a markup $\eta/(\eta-1)$ over their marginal cost.

4.3 Potential Export Revenues

The assumptions in Section 4.2 imply the potential export revenue of firm i in country j at t is

$$r_{ijt} = \left[\frac{\eta}{\eta - 1} \frac{\tau_{ijt} w_{it}}{P_{jt}} \right]^{1-\eta} Y_{jt}. \tag{3}$$

We model the impact of variable trade costs on potential export revenues as

$$(\tau_{ijt})^{1-\eta} = \exp(\xi_y y_{ijt-1} + \xi_s + \xi_{jt} + \xi_a \ln(a_{sjt}) + \xi_w \ln(w_{it})), \qquad \text{with} \qquad \xi_y \geqslant 0, \tag{4}$$

where y_{ijt-1} is a dummy variable that equals one if firm i exports to country j at period $t-1$, ξ_s is a term specific to the sector s to which firm i belongs, ξ_{jt} is a country-period term that accounts for trade barriers common to all firms located in country h, a_{sjt} equals one plus the average tariffs country j imposes at t on exports from h in sector s, and, as indicated above, w_{it} denotes marginal production costs. By allowing $(\tau_{ijt})^{1-\eta}$ to depend on w_{it}, we account for determinants of variable trade costs that may vary with firm productivity in a systematic way. Equations (3) and (4) imply

$$r_{ijt} = \exp(\alpha_y y_{ijt-1} + \alpha_s + \alpha_{jt} + \alpha_a \ln(a_{sjt}) + \alpha_r \ln(r_{iht})), \qquad \text{with} \qquad \alpha_y \geqslant 0, \tag{5}$$

where α_s and α_{jt} are sector and country-period specific terms, respectively, and r_{iht} is firm i's domestic sales at t. The positive dependency of r_{ijt} on the export participation dummy y_{ijt-1} accounts for the limited sales firms often obtain upon entering a new market.[11] The term α_s accounts for the impact of the sector-specific trade cost term ξ_s, and the country-period term α_{jt}

on which agents have perfect foresight converge deterministically to a balance growth path.

[11]These may be due to a lack of information or limited customer capital in a destination (Albornoz et al., 2012; Ruhl and Willis, 2017; Fitzgerald et al., 2023) or partial-year effects (Bernard et al., 2017; Gumpert et al., 2020).

accounts for the impact of the foreign price index P_{jt}, market size Y_{jt}, and variable trade cost component ξ_{jt}. The term $\alpha_a \ln(a_{sjt})$ accounts for the impact of tariff barriers, and domestic sales r_{iht} proxy for the impact of the firm's marginal production cost, w_{it}. See Appendix C for details.

According to equation (5), potential revenues in a country and period depend on the lagged export participation dummy y_{ijt-1} and four exogenous terms: the time-invariant term α_s and three time-varying terms comprising the country-period component α_{jt}, log domestic sales $\ln(r_{iht})$, and tariff barriers a_{sjt}. The time-invariant term and the in-sample values of the time-varying ones are observed or consistently estimated; see sections 2 and 6.1. Out-of-sample, we impose the following restrictions on the distribution of the time-varying exogenous determinants of export revenues.[12]

We assume α_{jt} and $\ln(r_{iht})$ are constant after period T and, for all $t \leqslant T$, follow stationary AR(1) processes with *iid* normal shocks and intercepts that may vary by country and firm, respectively. Formally, for all j and $t \leqslant T$, we assume $\alpha_{jt} = (X^{\alpha}_{jt})'\beta_\alpha + \rho_\alpha \alpha_{jt-1} + e^{\alpha}_{jt}$, with X^{α}_{jt} a vector including a constant, market j's log GDP at $\underline{t}$, and the geographic, linguistic, and regulatory distances between h and j; β_α and ρ_α are parameters with $|\rho_\alpha| < 1$; and, e^{α}_{jt} is *iid* normally distributed with mean zero and variance σ^2_α. Similarly, for all i and $t \leqslant T$, $\ln(r_{iht}) = (X^r_i)'\beta_r + \rho_r \ln(r_{iht-1}) + e^r_{iht}$, with X^r_i a vector including dummies for firm i's sector and location within country h; β_r and ρ_r are parameters with $|\rho_r| < 1$; and e^r_{it} is *iid* normally distributed with mean zero and variance σ^2_r. Additionally, we assume a_{sjt} is constant out-of-sample; i.e., for all j and s, $a_{sjt} = a_{sj\underline{t}}$ if $t \leqslant \underline{t}$, and $a_{sjt} = a_{sj\bar{t}}$ if $t \geqslant \bar{t}$. Finally, we assume the time series of these three time-varying determinants of revenues are independent of each other and of any other determinant of firm export profits.

4.4 Fixed and Sunk Export Costs

Firms may face fixed and sunk costs, which differ from variable costs in that, conditional on selling in a market, they are independent of the quantity sold. Fixed and sunk costs differ in that the former are paid every period a firm exports to a country, and the latter are only paid if the firm did not export to it in the previous period. We model fixed costs as the sum of four terms:

$$f_{ijt} = g_{jt} - eg_{ijt} + \nu_{ijt} + \omega_{ijt}. \tag{6}$$

The first term captures the impact of all distance measures between countries h and j,

$$g_{jt} = \gamma^F_0 + \sum_{x=\{g,l\}} \gamma^F_x n^x_{hj} + \gamma^F_a n^a_{hjt}. \tag{7}$$

The second term captures static complementarities in export destinations:

$$eg_{ijt} = \sum_{j' \neq j} y_{ij't} c_{jj't}, \tag{8}$$

[12]The need to restrict the out-of-sample distribution of the exogenous determinants of export revenues is due to our model featuring sunk costs and forward-looking firms with rational expectations, which implies firms' optimal export choices in-sample depend on their expected potential export revenues out-of-sample (see Section 4.6).

where the complementarities between countries j and j' are modeled as

$$c_{jj't} = \sum_{x=\{g,l\}} \gamma_x^E (1 + \varphi_x^E n_{hj}^x) \exp(-\kappa_x^E n_{jj'}^x) + \gamma_a^E (1 + \varphi_a^E n_{hjt}^a) \exp(-\kappa_a^E n_{jj't}^a), \tag{9}$$

with $(\gamma_x^E, \varphi_x^E) \geqslant 0$ for $x = \{g, l, a\}$. For all three distance measures we consider, equation (9) allows the fixed cost reduction a firm enjoys in a market j if it also exports to a market j' to depend on the distance between j and j' and between j and the firm's home market h. E.g., for $x = g$, a firm exporting to country j' experiences a reduction in fixed costs in country j equal to the product of a constant γ_g^E, a function $1 + \varphi_g^E n_{hj}^g$ of the distance between countries h and j, and a function $\exp(-\kappa_g^E n_{jj'}^g)$ of the distance between j and j'.

Imposing $(\gamma_x^E, \varphi_x^E) \geqslant 0$ for $x = \{g, l, a\}$ implies $c_{jj't} \geqslant 0$ for all (j, j', t), ruling out the possibility that adding an export destination may increase fixed costs in other countries. Along with the rest of the model, this sign restriction on $c_{jj't}$ implies the firm's country-specific export participation decisions are not substitutable, and is a necessary assumption for our algorithm to correctly solve the optimization problem determining firms' export bundles (see Section 5.1).

The third determinant of fixed export costs, ν_{ijt}, is a an unobserved (to the researcher) firm-country-period variable whose distribution in all periods prior to terminal period T is independent of all other determinants of firms' export profits, and satisfies the following restrictions:

$$\nu_{ijt} \sim \mathbb{N}(0, \sigma_\nu^2), \qquad \text{for all } i,\ j, \text{ and } t, \tag{10a}$$

$$\nu_{ijt} \perp\!\!\!\perp \nu_{i'j't'}, \qquad \text{if } i \neq i' \text{ or } t \neq t', \tag{10b}$$

$$\rho_{jj't} = \sum_{x=\{g,l\}} \gamma_x^N \exp(\kappa_x^N n_{jj'}^x) + \gamma_a^N \exp(\kappa_a^N n_{jj't}^a), \qquad \text{if } j \neq j', \tag{10c}$$

where $\rho_{jj't}$ is the correlation coefficient between ν_{ijt} and $\nu_{ij't}$. From T onwards, ν_{ijt} is constant: $\nu_{ijt} = \nu_{ijT}$ for $t \geqslant T$. By allowing for a firm-specific unobserved fixed cost term potentially correlated across countries, we allow for the correlation patterns in firm exports shown in Section 3 to be due not to complementarities but to correlated unobserved determinants of export profits.

The fourth term in equation (6), ω_{ijt}, is an *iid* unobserved (to the researcher) variable whose distribution is independent of all other determinants of profits and has two points of support, $\underline{\omega}$ and $\overline{\omega}$. Formally,

$$\omega_{ijt} \perp\!\!\!\perp \omega_{i'j't'} \qquad \text{if } i \neq i',\ j \neq j' \text{ or } t \neq t', \tag{11a}$$

$$P(\omega_{ijt} = \omega) = \begin{cases} p & \text{if } \omega = \underline{\omega}, \\ 1 - p & \text{if } \omega = \overline{\omega}. \end{cases} \tag{11b}$$

To simplify the model estimation, we set $(\underline{\omega}, \overline{\omega}) = (0, \infty)$ and, thus, ω_{ijt} is a "blocking" shock preventing firm i from exporting to country j in period t. Equation (11) characterizes the distribution of $\omega_{it} \equiv (\omega_{i1t}, \ldots, \omega_{iJt})$ in all periods; thus, ω_{it} may vary over time even after T.

We model sunk export costs in a more parsimonious way than fixed costs. Specifically, sunk costs in a market j and period t may only depend on the distance between countries h and j:

$$s_{jt} = \gamma_0^S + \sum_{x=\{g,l\}} \gamma_x^S n_{hj}^x + \gamma_a^S n_{hjt}^a, \qquad \text{with} \qquad s_{jt} \geqslant 0, \text{ for all } (j,t). \tag{12}$$

Sunk costs allow for dynamic complementarities in firm export decisions within a country.

4.5 Static Export Profits

The assumptions in Section 4.2 imply potential export revenues net of variable trade costs equal $\eta^{-1} r_{ijt}$. Netting out also fixed and sunk export costs, and using the expressions in equations (5), (6) and (8), the potential export profits of firm i in country j at period t may be written as

$$\pi_{ijt}(y_{it}, y_{ijt-1}, \omega_{ijt}) = u_{ijt}(y_{ijt-1}, \omega_{ijt}) + \sum_{j' \neq j} y_{ij't} c_{jj't}, \tag{13}$$

with

$$\begin{aligned} u_{ijt}(y_{ijt-1}, \omega_{ijt}) = \eta^{-1} &\exp(\alpha_y y_{ijt-1} + \alpha_s + \alpha_{jt} + \alpha_a \ln(a_{sjt}) + \alpha_r \ln(r_{iht})) \\ &- (g_{jt} + \nu_{ijt} + \omega_{ijt}) - (1 - y_{ijt-1}) s_{jt}, \end{aligned} \tag{14}$$

and where, denoting by J the number of foreign countries, the vector $y_{it} \equiv (y_{i1t}, \ldots, y_{iJt})$ identifies the bundle of export destinations of firm i at period t. Total export profits of i at t thus are

$$\pi_{it}(y_{it}, y_{it-1}, \omega_{it}) = \sum_{j=1}^{J} y_{ijt} \pi_{ijt}(y_{it}, y_{ijt-1}, \omega_{ijt}). \tag{15}$$

4.6 Optimal Export Choice

The firm chooses every period a set of export destinations maximizing its expected discounted sum of current and future profits. At any t, we assume firm i knows the distance measures between all countries, the true value of all model parameters, and the information set

$$\mathcal{J}_{it} = (\{x_{it'}\}_{t' \geqslant t}, y_{it-1}, \omega_{it}) \qquad \text{with} \qquad x_{it'} = (\nu_{it'}, \alpha_{t'}, a_{st'}, r_{iht'}), \tag{16}$$

where, for any variable z_{ijt}, z_{it} denotes the vector of values of z_{ijt} for every j; $z_{it} \equiv (z_{i1t}, \ldots, z_{iJt})$.[13] Notice the vector $x_{it'}$ includes all period-t' realized export profit shocks known to firm i at any period $t \leqslant t'$. Every firm i thus knows at any t the value of all exogenous determinants of current and future potential export profits except for the future fixed costs shocks $\{\omega_{it'}\}_{t'>t}$.

At any t, the problem firm i solves when choosing its period t export bundle may be written as

$$V_{it}(y_{it-1}, \omega_{it}) = \max_{y_{it} \in \{0,1\}^J} \mathbb{E}_{it}\Big[\pi_{it}(y_{it}, y_{it-1}, \omega_{it}) + \delta V_{it+1}(y_{it}, \omega_{it+1})\Big], \tag{17}$$

[13] Similarly, $z_t \equiv (z_{1t}, \ldots, z_{Jt})$ for any variable z_{jt} that is common across firms.

where $\mathbb{E}_{it}[\cdot]$ is the expectation operator with respect to the data generating process conditional on $\mathcal{J}_{it}$ (i.e., expectations are rational); the function $V_{it}(\cdot)$, firm i's value function at t, implicitly conditions on a path of shocks $\{x_{it'}\}_{t' \geqslant t}$ (i.e., $V_{it}(y_{it-1}, \omega_{it}) = V(\{x_{it'}\}_{t' \geqslant t}, y_{it-1}, \omega_{it})$); and $\delta < 1$ is the discount factor. Given equations (13), (16), and (17), we can rewrite

$$V_{it}(y_{it-1}, \omega_{it}) = \max_{y_{it} \in \{0,1\}^J} \Big\{ \sum_{j=1}^{J} y_{ijt}(u_{ijt}(y_{ijt-1}, \omega_{ijt}) + \sum_{j' \neq j} y_{ij't} c_{jj't}) + \delta \mathbb{E}_{it} V_{it+1}(y_{it}, \omega_{it+1}) \Big\}. \quad (18)$$

For all values of $\{x_{it'}\}_{t' \geqslant t}$, the function $V_{it}(\cdot)$ is bounded and, thus, a solution to the problem in equation (18) exists; see Appendix E.2.2. We denote firm i's optimal policy function at t as

$$o_{it}(y_{it-1}, \omega_{it}) = (o_{i1t}(y_{it-1}, \omega_{it}), \dots, o_{iJt}(y_{it-1}, \omega_{it})) \quad (19)$$

where $o_{ijt}(\cdot)$ is a function that equals one if firm i exports to country j at t, and zero otherwise. As x_{it} is constant from period T onwards (see sections 4.3 and 4.4), it holds that $V_{it}(\cdot) = V_{iT}(\cdot)$ for all $t \geqslant T$ and, consequently, $o_{it}(\cdot) = o_{iT}(\cdot)$ for all $t \geqslant T$. The firm's problem is thus non-stationary until terminal period T, and stationary henceforth.

5 Solution Algorithm

We describe here an algorithm to solve the problem in equation (18). We discuss the algorithm's properties in Appendix A, and illustrate in Appendix D.2 how it works in a simple setting.

Given a period t and a sequence of shocks $\{x_{it'}\}_{t' \geqslant t}$, the firm's optimization problem in equation (18) has three properties that make solving for the value of the policy function $o_{it}(y_{it-1}, \omega_{it})$ at every state (y_{it-1}, ω_{it}) computationally challenging:

- **P.1** *Large discrete choice set.* The choice set $\{0,1\}^J$ is discrete and has cardinality 2^J.
- **P.2** *Integration over a discrete random variable with many points of support.* For any choice y_{it}, evaluating the firm's objective function requires integrating numerically next period's value function, $V_{it+1}(y_{it}, \omega_{it+1})$, over ω_{it+1}, whose support includes 2^J points.
- **P.3** *Large state space.* As y_{it-1} and ω_{it} may each take 2^J values, the state space has 2^{2J} points.

These properties imply the choice set, the support of the random variable one must integrate over, and the state space grow exponentially in J. Allowing firms to export to a reasonable set of countries thus makes their optimization problem computationally challenging to solve. Specifically, as the firm's problem is non-stationary for all $t \leqslant T$, property P.3 implies one must solve $2^{2J}(T - \underline{t}_i + 1)$ optimization problems to compute firm i's export choices in all periods in which it is active and in all points in the state space. Properties P.1 and P.2 make finding the solution to each of these problems computationally challenging.

To overcome the challenges posed by properties P1 to P3, we develop a new solution algorithm. We consider each firm i independently and, given a sequence of shocks $\{x_{it'}\}_{t' \geqslant \underline{t}_i}$, we compute the value of the policy function $o_{it}(y_{it-1}, \omega_{it})$ for each $t \geqslant \underline{t}_i$ at a single state, which we mark with a "check" and write as $(\check{y}_{it-1}, \check{\omega}_{it})$. At a period t, the state $(\check{y}_{it-1}, \check{\omega}_{it})$ is that reached when the shocks $\{\omega_{it}\}_{t'=\underline{t}_i}^{t}$ follow a particular path of interest $\{\check{\omega}_{it'}\}_{t'=\underline{t}_i}^{t}$ and the firm makes the choices determined by the optimal policy function at all periods prior to t. Formally, for a firm i and period t, we compute the value $o_{it}(\check{y}_{it-1}, \check{\omega}_{it})$, where $\check{y}_{it-1}$ is determined by the following procedure:

$$\check{y}_{it'} = o_{it'}(\check{y}_{it'-1}, \check{\omega}_{it'}), \qquad \text{for } t' = \underline{t}_i, \ldots, t-1, \text{ with initial value } \check{y}_{i\underline{t}_i-1} = 0_J. \tag{20}$$

Note that, according to this procedure, $\check{y}_{i1} = o_{i1}(0_J, \check{\omega}_{i1})$, $\check{y}_{i2} = o_{i2}(\check{y}_{i1}, \check{\omega}_{i2})$, $\check{y}_{i3} = o_{i3}(\check{y}_{i2}, \check{\omega}_{i3})$, and so on.[14] In practice, the sequence of shocks $\{x_{it'}\}_{t' \geqslant \underline{t}_i}$ and $\{\check{\omega}_{it'}\}_{t' \geqslant \underline{t}_i}$ defining the path of interest at which we solve for firm i's choices correspond to either the values of these shocks observed in the data (or fixed to counterfactual values) or, when the corresponding variables are unobserved, to values randomly drawn from their distribution.

As our model is dynamic and firms are forward-looking, solving the optimization problem of firm i at period t and state $(\check{y}_{it-1}, \check{\omega}_{it})$ requires *some knowledge* of how the firm will subsequently behave at any state that may be reached from $(\check{y}_{it-1}, \check{\omega}_{it})$. However, it may not require knowing *exactly* the firm's optimal export bundle in all states that may subsequently be reached; e.g., if firm i's potential export profits in a country j at period t and state $(\check{y}_{it-1}, \check{\omega}_{it})$ are sufficiently high, its optimal decision may be to export to j at this state regardless of its optimal decision in subsequent periods. Our algorithm uses this idea and computes the optimal choice of a firm i at a period t and state $(\check{y}_{it}, \check{\omega}_{it})$ using *bounds* on the firm's optimal choice at future states.

Our algorithm has several steps. In each one, we obtain upper and lower bounds on the solution to the firm's problem at the path of interest. If the bounds coincide, they equal the solution as well. If they do not, we move to the next step. As we advance through steps, our bounds get tighter but harder to compute. We describe here the first two steps, and the remaining ones in Appendix D.1.

Step 1. For a country j and period t, assume we know for all $j' \neq j$ and $t' \geqslant t$ a constant upper bound $\bar{b}_{ij't'}$ such that $\bar{b}_{ij't'} \geqslant o_{ij't'}(y_{it'-1}, \omega_{it'})$ for all $(y_{it'-1}, \omega_{it'})$. We can then solve the firm's problem in j at t while conditioning on the constant upper bound $\bar{b}_{ij't'}$ for all $j' \neq j$ and $t' \geqslant t$:

$$\begin{gathered}\bar{V}_{ijt}(y_{ijt-1}, \omega_{ijt}) = \\ \max_{y_{ijt} \in \{0,1\}} \Big\{ y_{ijt}(u_{ijt}(y_{ijt-1}, \omega_{ijt}) + \sum_{j' \neq j} \bar{b}_{ij't}(c_{jj't} + c_{j'jt})) + \delta \mathbb{E}_{it} \bar{V}_{ijt+1}(y_{ijt}, \omega_{ijt+1}) \Big\}.\end{gathered} \tag{21}$$

The static and dynamic complementarities in our model imply that the solution to this problem is an upper bound on the firm's optimal choice in j at t; i.e., the solution is a function $\bar{o}_{ijt}(\cdot)$ such that $\bar{o}_{ijt}(y_{ijt-1}, \omega_{ijt}) \geqslant o_{ijt}(y_{ijt-1}, \omega_{ijt})$ for all $(y_{ijt-1}, \omega_{ijt})$. Importantly, the problem in equation (21) does not have any of the three properties that make solving the original problem in equation (18) computationally challenging: the control variable is binary, one only needs to integrate over

[14] For any z, we use z_J to denote a $J \times 1$ vector whose elements all equal z. Thus 0_J in equation (20) denotes a J dimensional vector of zeros.

the binary variable ω_{ijt+1}, and the vector $(y_{ijt-1}, \omega_{ijt})$ only takes four values.[15]

Given constant upper bounds $\bar{b}_{it} = (\bar{b}_{i1t}, \ldots, \bar{b}_{iJt})$ for all $t \geqslant \underline{t}_i$, we may solve the problem in equation (21) for all countries and periods, obtaining in this way upper-bound policy functions

$$\bar{o}_{it}(y_{it-1}, \omega_{it}) = (\bar{o}_{i1t}(y_{i1t-1}, \omega_{i1t}), \ldots, \bar{o}_{iJt}(y_{iJt-1}, \omega_{iJt})), \qquad \text{for all } \underline{t}_i \leqslant t \leqslant T. \tag{22}$$

More specifically, we use value function iteration to solve for period T value and policy functions $\bar{V}_{ijT}(\cdot)$ and $\bar{o}_{ijT}(\cdot)$, and backward induction to solve for $\{\bar{V}_{ijt}(\cdot)\}_{t=\underline{t}_i}^{T-1}$ and $\{\bar{o}_{ijt}(\cdot)\}_{t=\underline{t}_i}^{T-1}$.

The upper-bound policies $\{\bar{o}_{it}(\cdot)\}_{t \geqslant \underline{t}_i}$ we obtain depend on the constant upper bounds $\{\bar{b}_{it}\}_{t \geqslant \underline{t}_i}$ we use: the tighter these are, the tighter the resulting upper-bound policies will be. To initialize our algorithm, we use constant upper bounds implying the firm exports in all countries and periods. We denote these with a zero superscript (i.e., $\bar{b}_{ijt}^{[0]} = 1$ for all j and t) and use them to solve the problem in equation (21) for every country and period, obtaining in this way upper-bound policies $\bar{o}_{it}^{[0]}(\cdot)$ for all $t \geqslant \underline{t}_i$. Using these policies, we compute new constant upper bounds, which we use to solve again the problem in equation (21) and obtain new upper-bound policies. Generally, we implement an iterative algorithm computing each iteration's constant upper bounds using the policies obtained in the prior iteration. More specifically, to compute the period-t iteration-$(n+1)$ constant upper bound, we evaluate the period-t iteration-n upper-bound policy at the state compatible with the firm's optimization behavior at which the firm is most likely to export at t. This is the state reached when, for all $t' \leqslant t$, the blocking shocks equal the smallest value in their support and the firm chooses the bundle prescribed by $\bar{o}_{it'}^{[n]}(\cdot)$. That is, for a firm i, we compute the period-t iteration-$(n+1)$ constant upper bound by implementing the following iterative procedure:

$$\bar{b}_{it'}^{[n+1]} = \bar{o}_{it'}^{[n]}(\bar{b}_{it'-1}^{[n+1]}, \underline{\omega}_J), \qquad \text{for } t' = \underline{t}_i, \ldots, t, \text{ with initial value } \bar{b}_{i\underline{t}_i-1}^{[n+1]} = 0_J, \tag{23}$$

As shown in Appendix A, these bounds get tighter with every iteration and converge in a finite number of iterations.

We denote the converged upper-bound policies as $\{\bar{o}_{it}^{*}(\cdot)\}_{t \geqslant \underline{t}_i}$, obtain lower-bound policies $\{\underline{o}_{it}^{*}(\cdot)\}_{t \geqslant \underline{t}_i}$ in a similar way, and use both to obtain bounds on the firm choices along the path of interest $\{\breve{\omega}_{it}\}_{t \geqslant \underline{t}_i}$. Formally, denoting the upper and lower bounds at t along the path of interest as $\bar{\breve{y}}_{it}$ and $\underline{\breve{y}}_{it}$, respectively, we compute $\bar{\breve{y}}_{it}$ through the following iterative procedure:

$$\bar{\breve{y}}_{it'} = \bar{o}_{it'}^{*}(\bar{\breve{y}}_{it'-1}, \breve{\omega}_{it'}), \qquad \text{for } t' = \underline{t}_i, \ldots, t, \text{ with initial value } \bar{\breve{y}}_{i\underline{t}_i-1} = 0_J, \tag{24}$$

and compute $\underline{\breve{y}}_{it}$ analogously. If $\bar{\breve{y}}_{it} = \underline{\breve{y}}_{it}$ for all $t \geqslant \underline{t}_i$, these bounds identify the firm's optimal choices along the path of interest. If they differ for at least one period, we proceed to step 2.

Step 2. Denote by τ the smallest t with $\bar{\breve{y}}_{it} > \underline{\breve{y}}_{it}$. In this step, we tighten our bounds at τ. The procedure differs from that in step 1 in that we now solve the problem in equation (21) only for

[15] These are $(0, \underline{\omega})$, $(0, \overline{\omega})$, $(1, \underline{\omega})$, and $(1, \overline{\omega})$. As $\overline{\omega} = \infty$ in our application, $o_{ijt}(0, \overline{\omega}) = o_{ijt}(1, \overline{\omega}) = 0$ for all i, j and t, and we only need to compute $o_{ijt}(0, \underline{\omega})$ and $o_{ijt}(1, \underline{\omega})$.

$t = \tau$ at the state the firm reaches at τ at the path of interest, $(\breve{y}_{i\tau-1}, \breve{\omega}_{i\tau})$, and do so using constant upper bounds that condition on this state. The new initial constant upper bounds equal the firm's choices implied by the upper-bound policies $\{\bar{o}^*_{it}(\cdot)\}_{t \geqslant \underline{t}_i}$ when the state at τ is $(\breve{y}_{i\tau-1}, \breve{\omega}_{i\tau})$ and the blocking shocks for all $t > \tau$ equal the smallest value in their support. Formally, the new initial constant upper bound for period τ is $\bar{o}^*_{i\tau}(\breve{y}_{i\tau-1}, \breve{\omega}_{i\tau})$, and, for all $t > \tau$, we compute these through the following iterative procedure

$$\bar{b}^{[0]}_{it'|\tau} = \bar{o}^*_{it'}(\bar{b}^{[0]}_{it'-1|\tau}, \underline{\omega}_J), \qquad \text{for } t' = \tau + 1, \ldots, t, \text{ with initial value } \bar{b}^{[0]}_{i\tau|\tau} = \bar{o}^*_{i\tau}(\breve{y}_{i\tau-1}, \breve{\omega}_{i\tau}). \tag{25}$$

Solving the problem in equation (21) with these new constant upper bounds, we obtain new upper-bound policies for all $t \geqslant \tau$. As in step 1, we use these policies and a procedure analogous to that in equation (25) to compute new constant upper bounds, which we use to solve again the problem in equation (21) and obtain in this way new upper-bound policies. We implement this procedure until the guaranteed convergence (see Appendix A), denoting as $\bar{o}^*_{it|\tau}(\cdot)$ the resulting upper-bound policy for any $t \geqslant \tau$. We use these policies, in combination with similarly computed lower-bound policies $\underline{o}^*_{it|\tau}(\cdot)$, to obtain bounds on the firm's optimal choice at period τ at the path of interest:

$$\bar{\breve{y}}_{i\tau|\tau} = \bar{o}^*_{i\tau|\tau}(\breve{y}_{i\tau-1}, \breve{\omega}_{i\tau}), \qquad \text{and} \qquad \underline{\breve{y}}_{i\tau|\tau} = \underline{o}^*_{i\tau|\tau}(\breve{y}_{i\tau-1}, \breve{\omega}_{i\tau}). \tag{26}$$

If these bounds coincide, they also equal the optimal choice at τ at $(\breve{y}_{i\tau-1}, \breve{\omega}_{i\tau})$. If so, we proceed to the next period τ' at which the bounds computed in step 1 differ, implementing again the step 2 procedure to tighten the bounds at τ'. If the bounds in equation (26) do not coincide, we implement additional steps that we describe in Appendix D.1.

5.1 Discussion

Two model features are necessary for the algorithm introduced in Section 5 to provide valid and computationally feasible bounds on the firm's optimal choices at a path of interest.

First, the function the firm maximizes when making choices at any period and state is supermodular; i.e., the objective function in the optimization problem in equation (18) is supermodular. Supermodularity of the objective function implies we can compute upper and lower bounds on the firm's optimal policy function by iteratively solving for the firm's optimal policy in a subset of countries and periods while conditioning on upper and lower bounds, respectively, on the firm's optimal choices in all other countries and subsequent periods. In our model, the objective function is supermodular because of possible complementarities in export choices across countries within a period (due to fixed costs being weakly smaller when firms concurrently export to several destinations) and across periods within a country (due to weakly positive sunk costs). The specific source of complementarities is however irrelevant for the validity of the solution algorithm.

Second, given bounds on the firm's optimal choices in all other countries, the firm's dynamic optimization problem for one country (or a small set of them) is computationally tractable. For this, the dimensionality of the state vector in the country-specific problem in equation (21) must be small. In our model, this vector takes only four values, as $y_{ijt-1} \in \{0,1\}$ and $\omega_{ijt} \in \{\underline{\omega}, \bar{\omega}\}$ for all i, j, and t. Conditional on the state space of the country-specific problem being small, our solution algorithm is however still feasible if, e.g., ω_{ijt} has a distribution with more than two points of support; per-period profits in a country j depend on multiple lags of the firm's export participation choice in j; or, the firm's information is more limited than assumed in equation (16).

As discussed in Appendix D.3, in our sample, the share of export choices solved in each step of the algorithm, and the associated computing time, depend on the model parameter values. When these equal the baseline estimates (see Section 7), our algorithm finds in less than 13 minutes the solution to 99.89% of the more than 22 million choices we solve for when computing the model's sample predictions.[16] The unsolved choices are concentrated in countries sharing complementarities with a large number of other destinations; i.e., according to our estimates, those sharing deep PTAs with many other countries (e.g., members of the EU). At each step of the algorithm, the share of choices solved increases, and the computing time decreases, as the gravity component in fixed or sunk costs gets larger (i.e., as the value of the parameters entering equations (7) or (12) increase) and as complementarities get smaller (i.e., as γ_x^E or φ_x^E decrease, or as κ_x^E increases, for $x = \{g, l, a\}$).

6 Estimation Procedure

We estimate the model in two steps. In the first step, we estimate the demand elasticity and time series process of potential export revenues. In the second step, we estimate fixed and sunk costs.

6.1 First Step

We assume $r_{ijt}^{obs} = (r_{ijt} + \epsilon_{ijt})y_{ijt}$, where r_{ijt}^{obs} denotes observed export revenues, ϵ_{ijt} accounts for measurement error and, as a reminder, r_{ijt} is the potential export revenue of firm i in country j at t, and y_{ijt} is a dummy variable that equals one if i exports to j at t. Using d_s and d_{jt} to denote vectors of sector and country-year dummies, respectively, we assume $\mathbb{E}[\epsilon_{ijt} | y_{ijt-1}, d_s, d_{jt}, a_{sjt}, r_{iht}, y_{ijt} = 1] = 0$ and use a Poisson pseudo-maximum likelihood estimator and data on the sample of firms, countries, and years for which $y_{ijt} = 1$ to obtain estimates of the parameters entering the expression for potential export revenues in equation (5); i.e., $(\alpha_y, \alpha_a, \alpha_r, \{\alpha_{jt}\}_{jt}, \{\alpha_s\}_s)$.[17]

We also assume $r_{it}^{obs} = r_{it} + \varepsilon_{it}$, where r_{it}^{obs} denotes the observed total sales of firm i in year t, r_{it} is this variable's true value, and ε_{it} accounts for measurement error. As firms are monopolistically competitive and face in all markets a demand function with constant elasticity equal to η, it holds that $r_{it} = (\eta/(\eta - 1))vc_{it}$, where vc_{it} is the total variable costs of firm i in year t, which we measure as the sum of the wage bill and total expenditure in materials. Assuming $\mathbb{E}[\varepsilon_{it} | vc_{it}] = 0$, we use a

[16] The algorithm takes close to two minutes to find the solution to 99.72% of all choices. These times are measured at Princeton University's Della cluster using 44 processors with 20 GB of memory each.

[17] Out estimation procedure is compatible with interpreting ϵ_{ijt} as also capturing revenue components unknown to firms when choosing where to export. Assuming instead firms make this choice on the basis of such unobserved terms would force us (for computational reasons) to limit the number of parameters entering revenues; e.g., we may need to substitute the fixed effects $\{\alpha_{jt}\}_{jt}$ and $\{\alpha_s\}_s$ by functions of observed covariates and few parameters.

non-linear least squares estimator to obtain a consistent estimate of η.

Finally, given estimates of α_{jt} for all sample countries and years, and data on domestic sales for all sample firms and years, we compute OLS estimates of the parameters of the first-order autoregressive models for α_{jt} and $\ln(r_{iht})$; see Section 4.3.

6.2 Second Step

Given first-step estimates, we use a Simulated Method of Moments (SMM) estimator to obtain estimates of the fixed and sunk cost parameters; see equations (7) to (12). In Section 6.2.1, we use a simple example to illustrate the approach we follow to separately identify the parameters that, according to equation (9), determine the strength of cross-country complementarities in fixed costs from those that, according to equation (10c), determine the strength of the cross-country correlation in unobserved fixed cost determinants. In Section 6.2.2, we describe our SMM estimator.

6.2.1 Identification of Cross-Country Export Complementarities

Consider a simple setting with three destinations $j = \{1, 2, 3\}$. Crucially, countries 1 and 2 are identical except in their connection to country 3, which is "connected" to country 2 but not to country 1. Complementarities and the correlation coefficient in the fixed cost term ν_{ijt} thus equal zero between all country pairs except possibly between countries 2 and 3; i.e., $c_{jj't} = \rho_{jj't} = 0$ if $j = 1$ or $j' = 1$, $c_{23t} = c_{32t} = \bar{c}$, and $\rho_{23t} = \rho_{32t} = \bar{\rho}$ for $(\bar{c}, \bar{\rho}) \geqslant 0$. See Appendix F.1 for details.

To focus on the identification of $\bar{c}$ and $\bar{\rho}$, consider a researcher that knows the value of all other parameters and, in addition to the variables described in Section 2, observes potential export revenues for all firms, countries, and periods. Then, $\bar{c}$ and $\bar{\rho}$ are identified by the moment functions

$$\mathbb{m}_1 = \mathbb{E}[y_{i2t} - y_{i1t}] \qquad \text{and} \qquad \mathbb{m}_2 = \mathbb{C}[y_{i2t}, y_{i3t}], \tag{27}$$

where, generally, $\mathbb{m}_1$ captures the difference in export probabilities across destinations that differ only in the size of the countries "connected" to them (country 2 is connected to country 3 while country 1 is not; countries 1 and 2 are otherwise identical), and $\mathbb{m}_2$ captures the covariance in firm choices across "connected" countries (countries 2 and 3). As Table F.1 in Appendix F.1 shows, both moments functions equal zero when there are no complementarities and ν_{ijt} is independent across countries; i.e., when $\bar{c} = \bar{\rho} = 0$. Correlation in unobservables in the absence of complementarities (i.e., $\bar{\rho} > 0$ and $\bar{c} = 0$) yields correlated export choices without affecting the difference in export probabilities between connected and isolated countries (i.e., $\mathbb{m}_2 > 0$ and $\mathbb{m}_1 = 0$). Complementarities alone (i.e., $\bar{c} > 0$ with $\bar{\rho} = 0$) make both moment functions positive. This suggests an identification strategy in which $\mathbb{m}_1$ identifies the strength of the complementarities and, given these, $\mathbb{m}_2$ identifies the correlation in unobserved determinants of export profits. This logic is however incorrect, as $\mathbb{m}_1$ is also affected by the correlation in unobserved determinants of profits whenever complementarities are non-zero; i.e., $\mathbb{m}_1$ is also affected by $\bar{\rho}$ when $\bar{c} > 0$. What is true is that $\mathbb{m}_1$ and $\mathbb{m}_2$ are differentially affected by $\bar{c}$ and $\bar{\rho}$, and jointly identify them; see Figure F.1 in

Appendix F.1. To estimate our model, we use moments built using moment functions analogous to $\mathbb{m}_1$ and $\mathbb{m}_2$, but adjusted to account for the fact that no two countries in the data are identical in every dimension except the size of their "connected" countries, and that the degree in which any two countries are connected in our model (i.e., their proximity measures) is continuous.

6.2.2 Details on SMM Estimator

Consider a vector z_i that includes all first-step estimates (see Section 6.1) and all observed (to the researcher) firm i's payoff-relevant variables. That is, besides the first-step estimates, z_i includes, for all sample years, firm i's domestic sales and exports by destination, tariffs by destination for i's sector, and, for all country pairs, the distance measures introduced in Section 2. Consider also a vector χ_i including all firm i's payoff-relevant variables unobserved to the researcher: the fixed cost shocks $\nu_{it} \equiv (\nu_{i1t}, \ldots, \nu_{iJt})$ and $\omega_{it} \equiv (\omega_{i1t}, \ldots, \omega_{iJt})$ for all years, and, for non-sample years, the foreign countries' export revenue shifters $\alpha_t \equiv (\alpha_{1t}, \ldots, \alpha_{Jt})$ and firms' domestic sales. Finally, consider vectors y_i^{obs} and $y_i^s(\theta)$ of observed and model-implied, respectively, export choices in all countries and sample years. Specifically, $y_i^s(\theta)$ includes the model-implied choices given the vector of observed covariates and first-step estimates z_i, a vector θ of values for all parameters estimated in the second step (see Section 6.2), and a draw χ_i^s from the distribution of χ_i conditional on z_i. We can then write each of the $k = 1, \ldots, K$ moments we use in our SMM estimator as

$$\frac{1}{M} \sum_{i=1}^{M} \{m_k(y_i^{obs}, z_i, x) - \frac{1}{S} \sum_{i=1}^{S} m_k(y_i^s(\theta), z_i, x)\} = 0, \tag{28}$$

where M is the number of sample firms, $m_k(\cdot)$ is k's moment function, and x is a vector of market size measures for every sector, destination, and sample year. Specifically, x is a vector of export potentials, which we estimate for every sector, destination, and year as the corresponding importer fixed effect in a gravity equation estimated using sectoral trade data between all countries other than Costa Rica. In Appendix F.2, we summarize the distribution of export potentials and show that, controlling for the export potential of a destination, firms are more likely to export to destinations whose (geographical, linguistic or regulatory) neighbors' export potential is larger.

We use 89 moments that, for expositional purposes, we organize in three blocks. In the first block, with the goal of identifying the parameters that determine the level of fixed and sunk costs and how these vary with the distance between the firm's home country and each destination (i.e., γ_0^F, γ_0^S, and $\{(\gamma_x^F, \gamma_x^S)\}_{x=\{g,l,a\}}$), we use as moments the firm's export participation and survival probabilities by groups of destinations that differ in their distances to the firm's home country. In a second block, to identify the parameters determining the strength of export complementarities (i.e., $\{(\gamma_x^E, \psi_x^E, \kappa_x^E)\}_{x=\{g,l,a\}}$), we use moments that, similar to $\mathbb{m}_1$ in equation (27), capture firm export probabilities by groups of destinations that are similar in their size and distances to the firm's home country but different in the export potential of the countries close to them geographically or linguistically, or that share with them a deep PTA. Finally, to identify the parameters of the distribution of the unobserved terms ν_{it} and ω_{it} (i.e., σ_ν, p, and $\{(\gamma_x^N, \kappa_x^N)\}_{x=\{g,l,a\}}$), we combine moments that, similar to $\mathbb{m}_2$ in equation (27), capture the cross-country correlation in firm choices, with moments that capture the correlation in exports across firms and moments that capture the frequency with which we observe short-lived changes in a firm's export status in a destination.

We include in Appendix F.3 the full list of moments we use in our estimation. We provide in Appendix F.4 additional details on our SMM estimator. In Appendix F.5, we explore the robustness of our estimates to alternative realizations of the simulation draws χ_i^s we use in our moments.

7 Estimation Results

We summarize here our parameter estimates. Additional details are presented in Appendix F.6.

7.1 First-step Estimates: Potential Export Revenue Parameters

We estimate the parameters entering equation (5) using the 13,293 firm-country-year sample observations with positive exports. The estimate of α_y is 1.86 (robust s.e. equal to 0.07), implying firm potential export revenues grow significantly between the first and second year of exports to a destination. The estimate of α_a, which equals the elasticity of potential export revenues to tariffs, is -3.83 (s.e. equal to 0.07). If trade costs moved one-to-one with tariffs, this estimate would imply a demand elasticity η equal to 4.83. When estimating η as described in Section 6.1 (i.e., using information on total revenues and variable costs for all 44,785 firm-year sample observations), we obtain an estimate of 5.71 (s.e. equal to 0.49). As this estimate does not rely on assuming a perfect passthrough of tariffs to trade costs, we adopt it as our baseline. The estimate of α_r, the elasticity of potential export revenues to domestic sales, is 0.29 (s.e. equal to 0.04), reflecting that firms that are larger in the domestic market also tend to have larger potential export revenues.

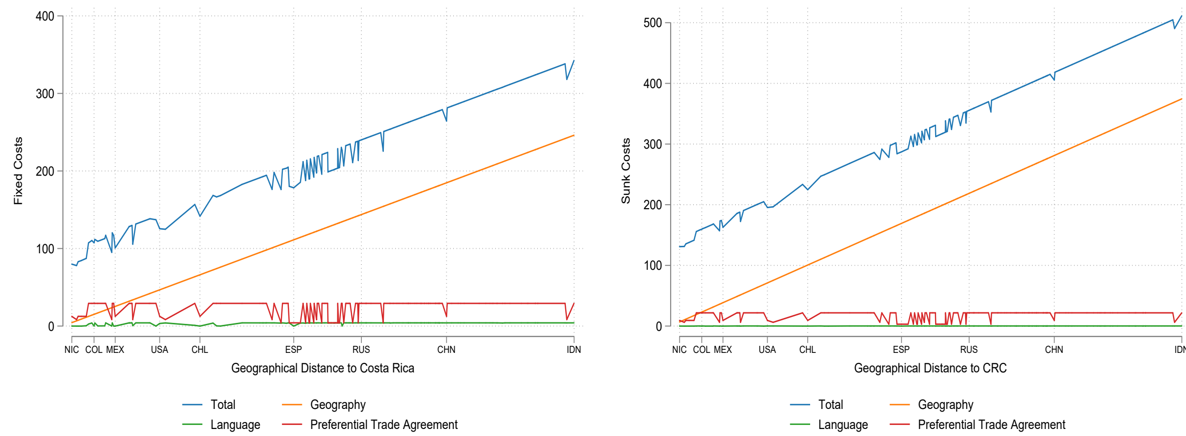
In Figure F.8, we summarize the estimates of the country-year fixed effects: countries with large estimated values of α_{jt} tend to be geographically close to Costa Rica (e.g., Guatemala) or large (e.g., the US), and countries with small estimated values tend to be geographically far from Costa Rica (e.g., Russia) or small (e.g., Oman). When using the 467 estimated values $\{\hat{\alpha}_{jt}\}_{jt}$ to estimate the parameters of the stochastic process of α_{jt} (see Section 4.3), we obtain an estimate of its autocorrelation parameter ρ_α equal to 0.69 (s.e. clustered by destination equal to 0.06), an estimate of the standard deviation σ_α of its innovations equal to 0.63, and estimates implying the mean of α_{jt} increases in country j's GDP and geographical proximity to Costa Rica (with the effect of linguistic and regulatory distances not significant at the 5% level). Similarly, when estimating the parameters of the autoregressive process for the firm's log domestic sales (see Section 4.3), we obtain an estimate of its autocorrelation parameter ρ_r equal to 0.86 (s.e. clustered by firm equal to 0.01), and an estimate of the standard deviation σ_r of its innovations equal to 0.87.

7.2 Second-step Estimates: Fixed and Sunk Costs Parameters

As shown in Figure 1, the estimates of the fixed and sunk cost parameters (see Table F.4) imply the gravity component of fixed costs (see equation (7)), and sunk costs, are well approximated by

Figure 1: Estimates of Fixed and Sunk Export Costs

(a) Fixed Export Costs (b) Sunk Export Costs



Note: In both figures, countries are identified by their ISO 3166-1 alpha-3 code, and placed in the horizontal axis by their distance to Costa Rica. The vertical axis indicates the estimated cost in thousands of 2010 USD.

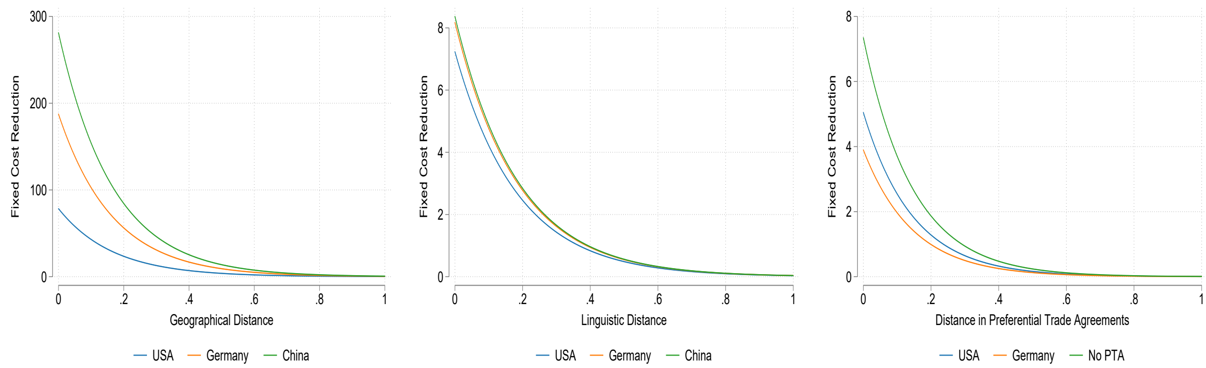
a constant (which equals $63,000 in the case of fixed costs and $115,000 in the case of sunk costs) plus a term that increases in the geographical distance between the firm's home country and each destination. The estimated impact of linguistic distance is small and not statistically significant, while the differences in fixed and sunk costs between a destination with whom Costa Rica has a deep PTA and one with whom it has no agreement are only $29,000 and $22,000, respectively. Adding all terms, the gravity terms in fixed costs in, e.g., Mexico, the US, and China, are close to $100,000, $125,000, and $180,000, respectively. For the US and Mexico, these are between the median and the 75 percentile (and below average) of the distribution of observed export revenues in those countries; for China, they are between the 75 and the 95 percentile (and close to the mean). Similarly, the sunk costs estimates in Mexico, the US, and China are close to $175,000, $200,000, and $400,000, respectively, and, thus, larger than the corresponding fixed cost estimates.

The actual fixed costs a firm faces in a country will however differ from the fixed cost gravity component due to the unobserved terms ν_{ijt} and ω_{ijt}, and to the effect of export complementarities; see equation (6). As ν_{ijt} is normal and its estimated standard derivation is close to $81,000, our estimates reveal a large cross-firm heterogeneity in fixed costs in any given country and period. All else equal, firms exporting to country j at period t will have on average low values of ν_{ijt} and, thus, actual exporters to a destination will likely face fixed costs that are well below what is implied by the fixed cost gravity component, even if they do not export anywhere else in the same period.

In Figure 2, we represent the estimated export complementarities. In each panel, we plot, for the corresponding index x in $\{g, l, a\}$, the function $\hat{\gamma}_x^E(1+\hat{\varphi}_x^E n_{hj}^x)\exp(-\hat{\kappa}_x^E n_{jj'}^x)$ for three countries j (the US, Germany, and China) against their distance to any other country j', $n_{jj'}^x$. Panel (a) shows that complementarities arising from geographic proximity are large between countries close to each other (e.g., these may reach $90,000 for countries that are 200 km apart) but decrease quickly,

Figure 2: Estimates of Sources of Cross-Country Complementarities

(a) Due to Geographical Proximity (b) Due to Linguistic Proximity (c) Due to Regulatory Proximity



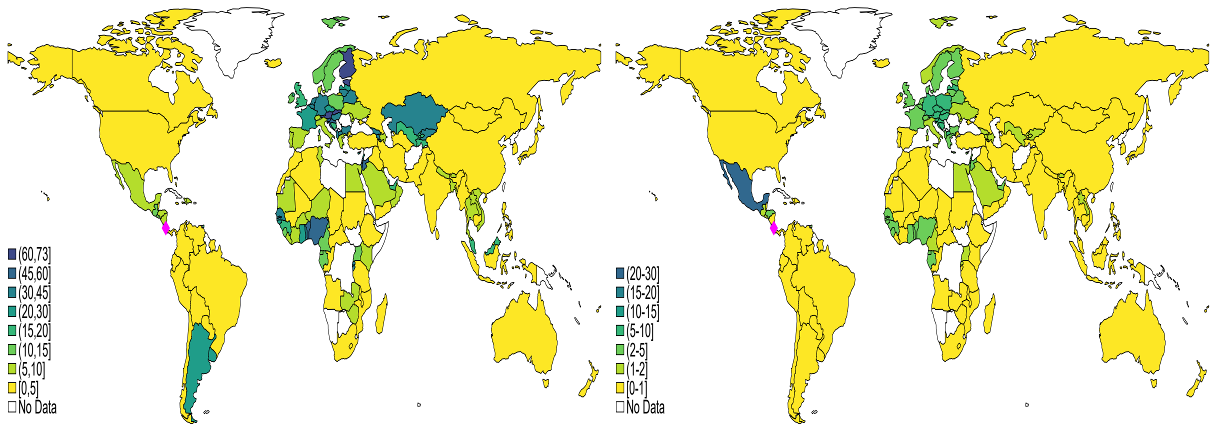
Note: In panels (a) to (c), the horizontal axis corresponds to the distance measures defined in equations (B.1), (B.2), and (1), respectively. The vertical axis indicates the estimated reduction in fixed export costs in thousands of 2010 USD.

being close to zero between countries whose bilateral distance is 800 km or more. The size of the geographical complementarities is heterogeneous across destinations depending on their distance to Costa Rica: for any given $n^g_{jj'}$, complementarities are larger for China than for Germany, and for Germany than for the US, reflecting their ranking in terms of the value of n^g_{hj}. Panel (b) shows that linguistic complementarities are always small, reaching a maximum of close to \$8,000 for country pairs whose linguistic distance is zero; i.e., whose residents understand each other with probability one. Finally, panel (c) shows that complementarities due to common participation in PTAs are close to zero unless these agreements are sufficiently deep. Among common members of deep PTAs, the fixed cost reduction in one of them for a firm that exports to the other varies between \$4,000 and close to \$8,000 depending on whether the country shares a PTA with Costa Rica.

In Figure 3, we quantify the cost reductions implied by the estimates reported in Figure 2. In panel (a), we show for each destination the cost reduction (relative to the gravity component of fixed costs) a firm experiences if it also exports to the country with whom its complementarities are the largest. This reduction is below 5% for countries such as the US or China, but is on average much larger for EU members, being above 45% for several of them. These estimates are due to EU members both sharing a deep PTA and being geographically close to each other. In panel (b), we show there are countries (e.g., Mexico) that, although do not share strong complementarities with any one country in particular (as shown in panel (a), exporting to Mexico's closest neighbor reduces fixed costs in it in less than 10%), benefit from sharing a moderate level of complementarities with many other countries (Mexico shares common language and membership in deep PTAs with many other countries). Thus, a firm exporting simultaneously to several countries that share common language or deep PTAs with, e.g., Mexico, may ultimately face small fixed costs in it. Linguistic and regulatory proximity may thus impact firm exports even if, as shown in Figure 2, linguistic and regulatory complementarities between any two countries are never large. In Figure F.9, we illustrate the complementarities of the US, China, Germany, and Spain, with all other countries.

Figure 3: Implications of Estimated Cross-Country Complementarities

(a) Fixed Costs Reduction from Closest Neighbor (b) Num. Neighbors Reducing Fixed Costs in $\geqslant 5\%$

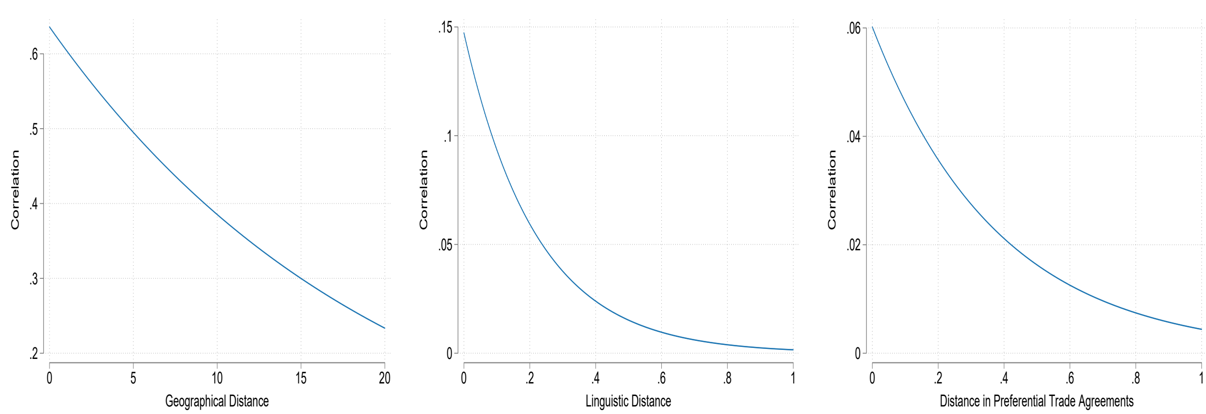


Note: In panel (a), we illustrate for each country j the value $\max_{j'}\{c_{jj't}/g_{jt}\}$. In panel (b), we illustrate for each j the number of other foreign countries $j' \neq j$ for whom $c_{jj't}/g_{jt} \geqslant 5\%$.

In Figure 4, we represent the estimated cross-country correlation in the fixed cost term ν_{ijt} within a firm-period. In each panel, we plot, for the corresponding index x in $\{g, l, a\}$, the function $\gamma_x^N \exp(\kappa_x^N n_{jj'}^x)$ against the distance $n_{jj'}^x$. The figure shows there is a large correlation in ν_{ijt}, and the key determinant of the correlation coefficient between any two countries is their geographic proximity, although their linguistic proximity also plays a role. It is thus potentially important to allow for correlated unobserved export profit shifters when estimating cross-country export complementarities. For the US, China, Germany and Spain, we illustrate in Figure F.9 the correlation coefficient of ν_{ijt} vis-a-vis any other country.

Figure 4: Estimates of Correlation Coefficient in Fixed Export Cost Shock

(a) Due to Geographical Proximity (b) Due to Linguistic Proximity (c) Due to Regulatory Proximity



Note: In panels (a) to (c), the horizontal axis indicates the distance measures defined in equations (B.1), (B.2), and (1), respectively. The vertical axis indicates the estimated correlation in ν_{ijt}.

8 Counterfactual Analysis

We implement three counterfactual exercises. In Section 8.1, we quantify the importance of export complementarities by comparing the predictions of versions of the estimated model in which some or all of the cross-country complementarities are set to zero. In Section 8.2, we use the estimated model to compute the impact on Costa Rican arm's length exports of a Brexit-induced increase in the regulatory distance between the UK and current EU members. In Section 8.3, for different counterfactual changes in Costa Rican export barriers, we compare the predictions of our model to those of a re-estimated model that assumes away the presence of complementarities.

8.1 Quantitative Importance of Cross-country Complementarities

To quantify the impact of complementarities, we compute model-implied export choices for each firm and year in the sample using 200 simulations of the vector χ_i of unobserved payoff-relevant variables (see Section 6.2.2). We do so for a baseline model that sets to zero all of the parameters that, according to equation (9), determine the strength of the complementarities, and compare the predictions of such model to that of alternative models that set some or all of these parameters to their estimated values. We report in Table 2 the cross-model differences in the predicted number of firm-country-years with positive exports (export events) and total export revenues. The results in column "All" show that including all complementarities causes the number of export events and total exports to increase in 11.8% and 7.1%, respectively. According to the remaining columns, the most important source of complementarities is spatial proximity: setting $(\gamma_g^E, \psi_g^E, \kappa_g^E)$ at their estimated values, while keeping complementarities due to linguistic and regulatory proximity equal to zero, causes export events and total exports to increase in 6.7% and 3.8%, respectively. Complementarities due to linguistic and regulatory proximity each cause a close to 2.5% increase in the number of the export events, and a 1.1% and 2.1%, respectively, increase in total exports.

Table 2: Impact of Cross-country Complementarities

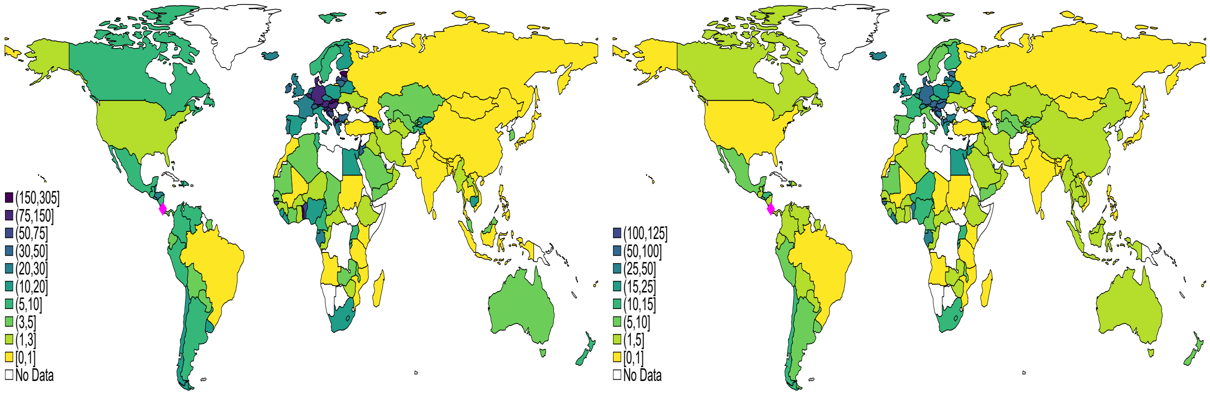
	Sources of Complementarities Included:			
Percentage Increase in:	All	Geographic Proximity	Linguistic Proximity	Common Deep PTA
Number of Export Events:	11.84%	6.65%	2.35%	2.58%
Export Revenues:	7.06%	3.79%	1.14%	2.07%

Note: In column *All*, we report the percentage difference in the number of export events and export revenues between a model in which the parameters $\{(\gamma_x^E, \psi_x^E, \kappa_x^E)\}_{x=\{g,l,a\}}$ are all set to zero and our estimated model. In the other columns, we compare models in which only the subset of these parameters indicated by the corresponding column label is set to their estimated values, while the other ones are kept at zero.

The smaller impact of complementarities on total exports relative to its impact on the number of export events is partly due to complementarities having, all else equal, a larger impact on less attractive destinations. To gain intuition on this model property, consider a setting with two destinations A and B such that, in the absence of complementarities, every firm's potential export

Figure 5: Impact of Eliminating Cross-country Complementarities

(a) On Number of Export Events (b) Export Revenues



Note: In Panel (a), we illustrate, for each destination and all firms and years in the sample, the percentage reduction in the total number of firm-year pairs with positive exports predicted by our model when we set the parameters $\{(\gamma_x^E, \psi_x^E, \kappa_x^E)\}_{x=\{g,l,a\}}$ to zero. In Panel (b), we provide analogous information for the total predicted export revenues.

profits in A are larger than in B. As shown in Appendix G, introducing complementarities in this context increases exports to B more than to A. The reason is that, without complementarities, exports to A are larger than to B and, with complementarities, firms benefit from a fixed cost reduction in B only if they also export to A. Thus, complementarities push firms to export to both countries, but this implies exports must grow more in the least attractive destination, as it had a lower level of exports in the setting without complementarities. As large markets are, all else equal, more attractive destinations, complementarities tend to have a larger impact on smaller markets.

Besides size, the geographical, linguistic, and regulatory proximity of each country to every other country also matters for the impact complementarities have on exports to it. As a result, as shown in Figure 5, there is a large heterogeneity across countries in the impact of complementarities. In many of them, these play a minimal role; conversely, for some, several of which are located in Central Europe, complementarities increase the number of export events and total exports from Costa Rica in more than 50%. These countries most affected by complementarities are typically small, geographically close to many other destinations, and members of deep PTAs that also include many other countries.

8.2 Third-Market Effects of Regulatory Differences Due to Brexit

A potential Brexit implication is that regulations in the UK and in the EU will drift apart. To quantify the third-country effect of this Brexit implication, we use our estimated model to evaluate the impact on Costa Rican arm's length exports of a permanent increase in 2021 (expected since the 2017 referendum, but unexpected before) in the regulatory distance, $n_{jj't}^a$, between the UK and all EU members from zero (its pre-Brexit value) to one (its maximum value). Specifically, for all sample firms and these two sets of values of the regulatory distances, we compute model-implied export choices for 200 simulations of the vector χ_i, and report in Table 3 the relative differences in

Table 3: Impact of Regulatory Differences Due to Brexit

Countries:	Percentage Reduction in: Export Events		Export Revenues	
	2017-20	*2021-30*	*2017-20*	*2021-30*
United Kingdom	-1.63%	-4.57%	-0.82%	-5.34%
European Union	-0.17%	-0.44%	-0.06%	-0.39%
In particular:				
Belgium	-0.48%	-1.62%	-0.10%	-1.33%
Ireland	-0.22%	-0.95%	-0.10%	-1.01%

Note: For the geographic area indicated in the column "Countries," we report the relative change for the periods 2017-20 and 2021-30 in the number of export events and total exports of all sample firms caused by a permanent change in 2021 (expected since 2017) in the regulatory distances between the UK and every EU member from zero to one.

the expected number of export events and total exports for the periods 2017-20 and 2021-30.

Our model predicts exports to the UK to fall as a consequence of the increased regulatory distance between the UK and the EU. Specifically, the predicted fall in export events and total exports in the 10 years after Brexit is around 5%. In the four years between the Brexit referendum and the UK's effective EU withdrawal, firms anticipate the policy change, and the number of export events and total exports to the UK fall in 1.6% and 0.8%, respectively. Although the reduction in complementarities between the UK and the EU is symmetric, the effect on exports to the UK is larger than that on exports to the EU, where the drop is always below 0.5%.

Zooming in on individual EU members, our model predicts that the countries geographically close to the UK will be more affected than those further away; e.g., in comparison to the 2021-30 0.4% reduction in overall exports to the EU, exports fall in 1.3% and 1% in Belgium and Ireland, respectively. To understand these effects, one should bear in mind that the estimated cross-country complementarities embedded in our model imply that the reduction in exports to the UK as a result of its regulatory isolation from the EU will have subsequent effects on countries geographically close to the UK, such as Belgium and Ireland. Similarly, exports to countries with large English-speaking populations will also be affected by the increase in the UK-EU regulatory distance, but these effects are small as linguistic complementarities are estimated to be small (see Section 7.2).

In the absence of complementarities, a partial-equilibrium model (such as ours) would predict Costa Rican exports to be unaffected by changes in trade barriers (regulatory or otherwise) between two destinations such as the UK and the EU. Standard general equilibrium models à la Eaton and Kortum (2002) or Anderson and van Wincoop (2003) imply exports of different origins are substitutes and, thus, predict Costa Rican exports to the UK and the EU to *increase* in reaction to the increase in the UK-EU trade barriers.[18] The third-market effects implied by cross-country complementarities in our model are thus of opposite sign to those in standard trade models.

[18] Adão et al. (2017) and Lind and Ramondo (2023) allow for more flexible elasticities of substitution across export countries, but maintain the assumption that different export countries are substitutes in any given destination. For a framework that allows for positive third-market effects, see Fajgelbaum et al. (2023).

8.3 Impact of Reductions in Export Tariffs

In 2022, Costa Rica applied for CPTPP membership. Motivated by it, we evaluate the effect on Costa Rican exports of a reduction in export tariffs to this trade bloc. Specifically, for the period 2022-37, all sample firms, and 200 simulations of the vector χ_i, we compute model-implied exports in a setting in which tariffs do no change and in one in which, from 2022 onwards, Costa Rican export tariffs to CPTPP members are zero. We do so using our estimated model and a re-estimated model analogous to ours but in which complementarities are assumed away (see Appendix F.7). To provide some guidance on when the model without complementarities will generate significantly different predictions from our estimated model, we also evaluate the effect on Costa Rican exports of eliminating export tariffs to a different trade area, the EU.

As shown in columns (1) and (2) in Table 4, the estimated model predicts the number of firm-year pairs with positive exports and total exports to CPTPP members to increase in 16% and 30%, respectively. Columns (5) and (6) reveal that a researcher using a model analogous to ours but in which cross-country complementarities are assumed away would have predicted a growth in Costa Rican exports to CPTPP members only slightly smaller than that predicted by our model. Furthermore, the model with complementarities predicts very minimal import growth in non-CPTPP countries, matching thus very close the zero import growth on these destinations predicted by the model without complementarities. The reason why cross-country complementarities play a small role in determining the impact of Costa Rica becoming a CPTPP member is that current members exhibit small complementarities both with each other and with non-members. Thus, the growth in exports in any member country has small spillovers on other countries.

In other contexts, the predictions of a model that assumes away cross-country complementarities may differ from those of our estimated model. To illustrate this point, we compute the impact of Costa Rica signing a PTA with the EU that sets its export tariffs to zero in all member countries. In this case, while the estimated model predicts the number of export events and total exports in

Table 4: Impact on Trade Area Members of Eliminating Export Tariffs to Them

Model With Cross-Country Complementarities				Model Without Cross-Country Complementarities			
Tariff Changes With CPTPP		Tariff Changes With The EU		Tariff Changes With CPTPP		Tariff Changes With The EU	
Export Events (1)	Export Revenues (2)	Export Events (3)	Export Revenues (4)	Export Events (5)	Export Revenues (6)	Export Events (7)	Export Revenues (8)
15.69%	29.93%	65.33%	83.08%	14.57%	28.89%	54.90%	79.58%

Note: The results in columns (1) to (4) are computed using our estimated model; those in columns (5) to (8) are computed using the model described in Appendix F.7. The results in columns (1), (2), (5), and (6) report the impact of eliminating Costa Rican export tariffs to all CPTPP members; those in columns (3), (4), (7), and (8) evaluate the impact of eliminating tariffs with all EU members. Results aggregate predictions for all sample firms, the period 2022-37, and 200 draws of χ_i.

current member countries to grow in 65% and 83%, respectively, the re-estimated model without complementarities predicts these growth rates to be 55% and 80%. The smaller difference in model predictions for total exports than for the total number of export events reflects that the extra export events predicted by the model with complementarities take place in EU members with small market sizes. Importantly, the model with complementarities differs from the model without complementarities in that the former predicts significant export growth to countries that are not EU members (thus, whose tariffs with Costa Rica did not change in the counterfactual exercise) but that are geographically close to some EU members, or that share a deep PTA with them; e.g., the predicted export growth among the Balkan countries that do not belong to the EU is above 10%, the export growth in Great Britain, Switzerland, and Iceland, is close to 7%, and that in Lebanon and Tunisia is around 3%. The reason for the significant disparity in model predictions in this case is that EU members exhibit strong complementarities between themselves and with other countries and, thus, a growth in exports in an EU member may have important spillovers on other destinations. The model without complementarities assumes away these spillovers and, thus, predicts much smaller changes in exports to many destinations.

9 Conclusion

We estimate and solve a partial-equilibrium firm export dynamics model featuring cross-country complementarities. In our model, the firm has rational expectations and chooses every period the bundle of export destinations that maximizes its expected discounted sum of current and future profits. We introduce a novel algorithm to solve the firm's combinatorial dynamic discrete choice optimization problem. Our estimates reveal substantial heterogeneity in complementarities across country pairs. Fixed export costs in several Central European countries are reduced in more than 50% if the firm also exports to these countries' closest neighbor. Conversely, for the US or China, exporting to their closest neighbor reduces fixed costs in these countries in less than 5%.

The impact of the estimated cross-country complementarities on export flows is non-negligible. We predict Costa Rica's total arm's length exports are approximately 7% larger due to these complementarities, reflecting a 12% increase in the number of firm-country-period triplets with positive exports. We use our estimated model to quantify the impact Brexit has on Costa Rican exports to the UK and the EU as a result of both countries no longer sharing a deep PTA: although bilateral trade barriers between Costa Rica and every foreign country are held constant in this counterfactual exercise, exports to the UK and the EU drop in 5% and 0.4%, respectively, illustrating that deep PTAs may give rise to positive trade *creation* effects. Finally, using Costa Rica's request to join CPTPP as motivating example, we show that researchers that assume away the presence of complementarities when predicting the impact of counterfactual changes in trade policy will obtain predictions similar to those of our estimated model when the policy changes affect isolated countries, and potentially quite different predictions when the policy changes affect countries that exhibit important complementarities with other destinations.

We provide a first quantification of the impact of cross-country complementarities on firms' optimal export decisions in the context of a dynamic framework, and develop tools that may be used to quantify the relevance of complementarities across alternatives in other dynamic discrete-choice settings. Our paper is an early step towards merging two literatures, the literature on firm export dynamics, which has a long tradition within international trade, and the more recent literature exploring interdependencies across choices in firm decisions. Natural next steps in this literature are to allow for sources of cross-choice interdependencies beyond those in our framework (e.g., increasing marginal production costs), or to study the impact complementarities have in a general-equilibrium framework. In the context of dynamic models, these extra steps involve substantial methodological contributions beyond those in our paper.

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A General Optimization Problem and Solution Algorithm

In Appendix A.1, we characterize an optimization problem encompassing that in equation (18). In Appendix A.2, we propose a solution algorithm covering those used in the steps described in Section 5 and Appendix D.1, and state several of its properties when applied to problems of the kind characterized in Appendix A.1. To simplify notation and without loss of generality, we focus on an agent born at period $t = 0$.

A.1 General Optimization Problem

Consider an agent that, in every period $t \geqslant 0$, makes J simultaneous binary choices with the goal of maximizing the expected discounted sum at birth of infinite per-period (static) payoffs.

Per-period payoffs in any t depend on a shock ω_t taking values in a set Ω_t according to a distribution $Q_t(\omega_t|\omega_{t-1}, \ldots, \omega_0)$. We denote as $z^t = \{\omega_{t'}\}_{t'=0}^{t}$ the history of shocks in all periods $t' \leqslant t$, and as $Z^t = \times_{t'=0}^{t}\Omega_{t'}$ the set of all possible period-t histories. We denote as $y_j(z^t) \in \{0,1\}$ a generic choice at z^t for alternative j, as $y(z^t) \in \{0,1\}^J$ a generic vector of choices at z^t for all J alternatives, and as $\boldsymbol{y} \in Y$ a generic vector of choices for all $t \geqslant 0$, all $z^t \in Z^t$, and all alternatives; i.e.,

$$Y = \times_{t=0, z^t \in Z^t}^{\infty} \{0,1\}^J. \tag{A.1}$$

Considering only optimization problems where the solution exists and is unique, we can write

$$\boldsymbol{o} = \operatorname*{argmax}_{\boldsymbol{y} \in Y} \Pi_0(\boldsymbol{y}), \tag{A.2}$$

where $\Pi_0(\boldsymbol{y})$ is the agent's objective function and $\boldsymbol{o}$ is the optimal choice for all $t \geqslant 0$ and all $z^t \in Z^t$.[19] Thus, using $o(z^t)$ to denote the agent's optimal choice at z^t, it holds

$$\boldsymbol{o} = \{o(z^t)\}_{t=0, z^t \in Z^t}^{\infty}. \tag{A.3}$$

The following assumption establishes a list of conditions on the objective function $\Pi_0(\cdot)$.

Assumption 1 *Assume:*

1. *(Additive separability of static profits) The function $\Pi_0(\cdot)$ satisfies*

$$\Pi_0(\boldsymbol{y}) = \pi_0(y(z^0), 0_J, \omega(z^0)) + \sum_{t=1}^{\infty} \delta^t \mathbb{E}\big[\pi_t(y(z^t), y(z^{t-1}), \omega(z^t))\big], \tag{A.4}$$

where the expectation is over $\{z^t\}_{t=1}^{\infty}$, 0_J is a $J \times 1$ vector of zeros, $\delta \in (0,1)$ and, for all $t \geqslant 0$,

$$\pi_t(y(z^t), y(z^{t-1}), \omega(z^t)) = \sum_{j=1}^{J} \big(\hat{\pi}_{jt}(y_j(z^t), y_j(z^{t-1}), \omega(z^t)) + \tilde{\pi}_{jt}(y(z^t), y(z^{t-1}))\big) \tag{A.5}$$

where $\hat{\pi}_{jt} \colon \{0,1\} \times \{0,1\} \times \Omega_t \longrightarrow \mathbb{R} \cup \{-\infty\}$ and $\tilde{\pi}_{jt} \colon \{0,1\}^J \times \{0,1\}^J \longrightarrow \mathbb{R}$.

2. *(Supermodularity) For all $t \geqslant 0$ and $\omega_t \in \Omega_t$, π_t is supermodular in $(y(z^t), y(z^{t-1}))$ on $\{0,1\}^J \times \{0,1\}^J$.*
3. *(Inaction) For all $j = 1, \ldots, J$, $t \geqslant 0$, and $z^t \in Z^t$, there exists $y_j(z^t) \in \{0,1\}$ such that, defining the set $X_t \equiv \{0,1\} \times \Omega_t$, it holds that $\hat{\pi}_{jt}(y_j(z^t), x) \geqslant -K$ for all $x \in X_t$ and a real number $K \geqslant 0$.*
4. *(Markov with finite state space) For all $t \geqslant 0$, Ω_t is finite and $Q_t(\omega_t|\omega_{t-1}, \ldots, \omega_0) = Q_t(\omega_t|\omega_{t-1})$.*
5. *(Stationarity) There exists T such that, for all $t \geqslant T$ and all $j = 1, \ldots, J$, it holds that $\Omega_t = \Omega_T$, $Q_t(\cdot) = Q_T(\cdot)$, $\hat{\pi}_{jt} = \hat{\pi}_{jT}$ and $\tilde{\pi}_{jt} = \tilde{\pi}_{jT}$.*

As shown in Appendix E.1, equating agents to firms and alternatives to potential export destinations, the model described in Section 4 satisfies all restrictions in Assumption 1.

[19] The restrictions in Section 4 imply the solution to the problem in equation (18) exists and is unique almost surely.

A.2 General Solution Algorithm

We describe here an iterative algorithm that yields upper bounds on the solution to the problem in equation (A.2) if the function $\Pi_0(\cdot)$ satisfies the restrictions listed in Assumption 1. An algorithm that yields lower bounds may be devised in an analogous fashion.

As a preliminary step, partition the J alternatives into U groups indexed by u. Denote as $M_u \subseteq \{1, \ldots, J\}$ the set of alternatives included in group u, and denote as M_u^c the complement of M_u; i.e., the set including all alternatives not in M_u. E.g., if $J = 4$ and $U = 3$, we can form the subsets $M_1 = \{1, 2\}$, $M_2 = \{3\}$, and $M_3 = \{4\}$, and the corresponding complements are $M_1^c = \{3, 4\}$, $M_2^c = \{1, 2, 4\}$, and $M_3^c = \{1, 2, 3\}$.

For each set M_u and each iteration $n = 1, 2, 3, \ldots$ of the algorithm, we solve

$$\bar{\boldsymbol{o}}_{M_u}^{(n)} = \underset{y_{M_u} \in Y_{M_u}}{\operatorname{argmax}} \, \Pi_0(\boldsymbol{y}_{M_u}, \bar{\boldsymbol{y}}_{M_u^c}^{(n)}), \tag{A.6}$$

where $\boldsymbol{y}_{M_u}$ is a generic vector of export choices for every alternative in the set M_u, all periods $t \geqslant 0$, and every history z^t that may be reached at t, and the set Y_{M_u} includes all feasible values of $\boldsymbol{y}_{M_u}$; i.e.,

$$Y_{M_u} = \times_{t=0, z^t \in Z^t}^{\infty} \{0, 1\}^{J_u},$$

where J_u is M_u's cardinality. The second argument of the function $\Pi_0(\cdot)$ in equation (A.6) is an upper bound on the firm's optimal choice in every alternative not in M_u, all periods $t \geqslant 0$, and all histories $z^t \in Z^t$; i.e.,

$$\bar{\boldsymbol{y}}_{M_u^c}^{(n)} = \{\bar{y}_{M_u^c}^{(n)}(z^t)\}_{t=0, z^t \in Z^t}^{\infty}, \qquad \text{with} \qquad \bar{y}_{M_u^c}^{(n)}(z^t) \geqslant o_{M_u^c}(z^t) \text{ for all } t \geqslant 0 \text{ and } z^t \in Z^t,$$

where $o_{M_u^c}(z^t)$ is the vector of optimal choices at period t and history z^t in all alternatives not in M_u.

Solving the problem in equation (A.6) for any group u at any iteration n requires specifying first the upper-bounds included in the vector

$$\bar{\boldsymbol{y}}_{M_u^c}^{(n)}.$$

For computational reasons, we set the upper bound corresponding to any country j, period t, and history z^t, to a value that does not vary across histories; i.e., we set

$$\bar{y}_j^{(n)}(z^t) = \bar{b}_{jt}^{(n)} \text{ for all } z^t \in Z^t. \tag{A.7}$$

In the first iteration (i.e., for $n = 1$), we set each of these upper bounds to its largest value within the feasible choice set; i.e., for every j and $t \geqslant 0$, we set

$$\bar{b}_{jt}^{(1)} = 1. \tag{A.8}$$

In all subsequent iterations (for all $n > 1$), we set

$$\bar{b}_{jt}^{(n)} = \max_{z^t \in Z^t} \bar{o}_j^{(n-1)}(z^t), \tag{A.9}$$

where $\bar{o}_j^{(n-1)}(z^t)$ is the element corresponding to alternative j, period t, and history z^t of the vector $\bar{\boldsymbol{o}}_{M_u}^{(n-1)}$ for the set of alternatives M_u including j. Equation (A.9) shows that, to compute the iteration-n upper bound on the firm's optimal choice in alternative j at history z^t, we use the outcome of the optimization problem in equation (A.6) at iteration $n-1$ for the set M_u including j. Specifically, as shown in equation (A.9), we assign to every j, t, and z^t, the max of the outcomes obtained for j and t across every $z^t \in Z^t$.

Theorem 1 establishes certain properties of the iterative algorithm defined in equations (A.6) to (A.9)

Theorem 1 *Let $\bar{b}_{jt}^{(n)}$ be defined by equations* (A.6) *to* (A.9)*, and let $o_j(z^t)$ be the element of the vector $\boldsymbol{o}$ defined in equation* (A.2) *that corresponds to alternative j and history z^t. Then, for all $j = 1, \ldots, J$, $t = 1, 2, \ldots$, $z^t \in Z^t$, and $n = 1, 2, 3, \ldots$, it holds that*

1. $\bar{b}_{jt}^{(n)} \geqslant o_j(z^t)$.

2. $\bar{b}_{jt}^{(n)} \leqslant \bar{b}_{jt}^{(n-1)}$.

3. There exists $N < \infty$ *such that* $\bar{b}_{jt}^{(n)} = \bar{b}_{jt}^{(n-1)}$ *for all* $n \geqslant N$.

Theorem 1 establishes that the values $\{\bar{b}_{jt}^{(n)}\}_{j=1,t=0}^{J,\infty}$ computed according to equations (A.6) to (A.9) are an upper bound on the firm's optimal choice at any feasible history, get tighter with every iteration, and converge after a finite number of iterations. See Appendix E for a proof of Theorem 1.

Property 3 of Theorem 1 does not imply that the upper bound defined by equations (A.6) to (A.9) (nor the analogous lower bound) converges to the solution of the firm's optimization problem in equation (A.2). However, as the partition of the J alternatives into U subgroups gets coarser, the upper bound defined by equations (A.6) to (A.9) (and the analogous lower bound) gets tighter. In the limiting case in which $U = 1$ and, therefore, $M_u = \{1, 2, \ldots, J\}$, the optimization problem in equation (A.6) coincides with that in equation (A.1) and, thus, solving this optimization problem is equivalent to solving the full firm's problem.

The algorithms implemented in each of the steps described in Section 5 and Appendix D.1 are special cases of the algorithm defined in equations (A.6) to (A.9). E.g., the algorithm implemented in step 1 is a case in which: (a) $U = J$ and, for $u = 1, \ldots, J$, the set M_u is a singleton; and (b) period $t = 0$ corresponds to the birth year of the firm (i.e., $t = \underline{t}_i$). The algorithm implemented in step 2 is a case in which: (a) $U = J$ and, for $u = 1, \ldots, J$, M_u is a singleton; and, (b) period $t = 0$ corresponds to the first period at which the step 1 upper and lower bounds differ. The algorithm implemented in step 5 is a case in which: (a) $U < J$, and for some $u = 1, \ldots, U$, the set M_u includes more than one country; and, (b) period $t = 0$ corresponds to the first period at which the upper and lower bounds computed in previous step differ.

Online Appendix for "Firm Export Dynamics in Interdependent Markets"

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Contents

B Additional Reduced-Form Results

B.1 Firm-level Data: Sample Descriptive Statistics

We provide here descriptive statistics for the firm-level data introduced in Section 2. In Table B.1, we report information for every sample year on total manufacturing exports, total number of exporting firms, and total number of foreign countries to which Costa Rican manufacturing firms exported in the corresponding year. While the total number of exporters remained stable at a number between approximately 400 and 450, and the total number of export destinations remained stable at around 90 destinations, the total export volume grew significantly in real terms between 2005 and 2015.

Table B.1: Aggregate Statistics

Years	Total Exports	Number of Exporters	Number of Destinations
2005	262,549.6	400	95
2006	303,344.6	415	96
2007	332,929.1	422	91
2008	371,202.9	419	91
2009	328,435.2	438	87
2010	347,235.1	432	96
2011	431,820.7	456	91
2012	479,806.0	459	90
2013	450,472.3	437	84
2014	494,083.5	436	84
2015	479,485.1	395	90

Notes: *Total Exports* are reported in thousands of 2013 US dollars.

In Table B.2, we report the mean and median domestic sales across all firms and across exporters. As it is common in datasets similar to ours, the distribution of domestic sales is skewed to the right (mean domestic sales are larger than median domestic sales), and exporters are larger on average than non-exporters (mean domestic sales in the subpopulation of exporters is larger than in the overall population). We also report in Table B.2 export revenues for the mean and median exporters in each sample year. Consistently with the fact that, between 2005 and 2015, total exports grew significantly while the number of exporters remained roughly constant, we observe the aggregate export revenue of the mean exporting firm also grew during the

Table B.2: Firm-level Statistics

Years	Domestic Sales (All Firms)		Domestic Sales (Exporters)		Exports		Number of Destinations (Exporters)		
	Average	Median	Average	Median	Average	Median	Average	Median	95th/99th perc.
2005	684.4	119.4	3,312.0	822.9	656.4	63.4	3.38	2	10/17
2006	695.4	118.4	3,553.2	772.6	731.0	63.1	3.28	2	10/18
2007	782.4	131.7	3,864.6	904.3	788.9	63.7	3.35	2	10/16
2008	889.6	147.0	4,693.6	1,160.0	885.9	66.4	3.30	2	9/18
2009	839.1	126.4	4,682.5	1,033.4	749.9	43.4	3.19	2	10/18
2010	937.2	139.2	5,256.7	1,161.1	803.8	56.7	3.28	2	9/18
2011	1,031.9	147.4	5,601.4	1,201.7	947.0	56.3	3.25	2	9/19
2012	1,067.5	154.1	5,663.2	1,091.7	1,045.3	65.9	3.22	2	9/19
2013	1,098.9	158.1	5,922.9	1,178.6	1,030.8	78.2	3.35	2	10/17
2014	1,043.8	147.4	5,793.3	1,208.3	1,133.2	59.7	3.28	2	10/18
2015	1,166.0	155.8	6,809.5	1,566.5	1,213.9	80.5	3.62	2	11/20

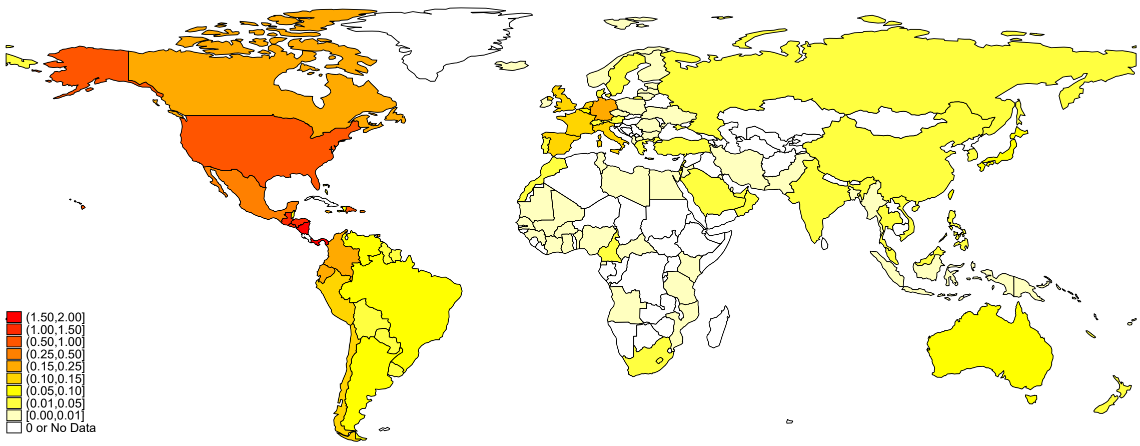
Notes: *Domestic sales* and *Exports* are reported in thousands of 2013 US dollars. We measure domestic sales by subtracting total export revenue (from the Customs dataset) from total revenue.

same period. Specifically, while total exports grew by 82% between 2005 and 2015, total export revenues for the average exporter grew at nearly the same rate, 85%.

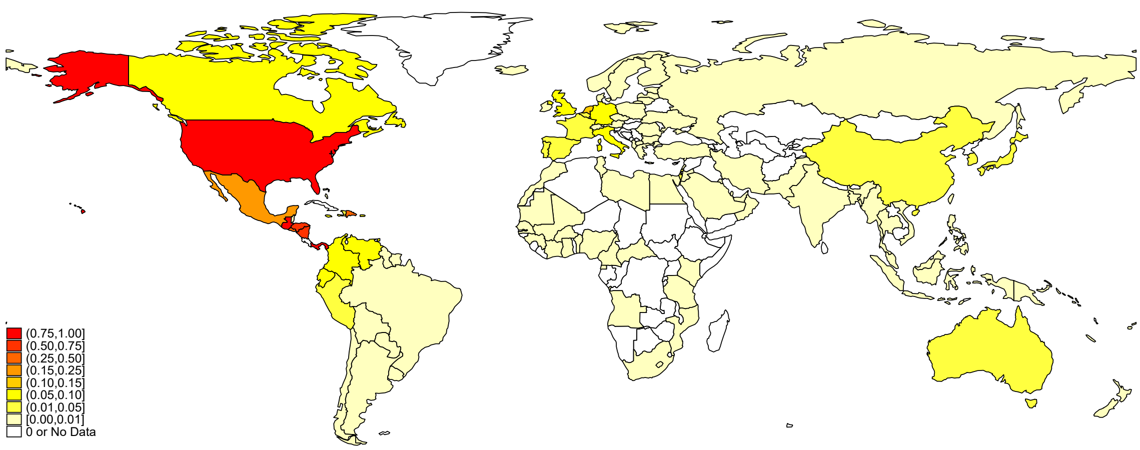
The last three columns in Table B.2 report statistics of the distribution of the number of export destinations across firms. Three features of this distribution are apparent. First, it is very skewed: the difference in the number of destinations between the median exporter and that at the 95th percentile (approximately 8 destinations) is the same as the difference between the exporter at the 95th percentile and that at the 99th percentile. Second, some firms export to a large number of destinations; the 95% percentile is close to 10, and the 99th percentile oscillates between 17 and 20. Third, the distribution is stable over time. Thus, the growth in average and median exports documented in Table B.2 is not due to a hypothetical growth in the number of destinations.

Figure B.1: Export Activity by Destination Country During Period 2005-2015

(a) Total Number of Export Events



(b) Total Volume of Exports



Notes: Panel (a) shows the total number of firm-year pairs with positive exports relative to that in the United States. Panel (b) shows the total volume of manufacturing exports relative to that in the United States.

In terms of the distribution of export activity across destinations, the maps in Figure B.1 reflect the total number of export events (i.e., firm-year pairs with positive exports) and the total volume of exports by destination for the period 2005-2015, in both cases relative to the corresponding magnitude in the United States. Both maps show that the most popular export destinations are countries in North and Central America, followed by China, Australia, and countries in Europe. Specifically, the top 5 destinations by total volume of exports are the United States, Guatemala, Panama, Nicaragua and Honduras.

In Table B.3, we present the mean and several percentiles of the distribution of annual firm-level exports to several countries over the period 2005-2015. The distribution of annual firm-level exports by market is both disperse and skewed to the right. The dispersion is reflected on the fact that, for all destinations considered in Table B.3, while the 25th percentile of the distribution of annual firm-level exports is below \$10,000, the 95th percentile is either above \$1,000,000 or close to it. The skewness is reflected in the fact that, while median exports to the US are approximately \$28,000, mean exports are close to \$600,000.

Table B.3: Distribution of Export Sales in Several Markets

Country	Average	Percentile					
		5	25	50	75	95	99
United States	597.6	0.4	5.0	28.1	227.4	3,477.9	9,615.9
Panama	271.4	1.2	7.4	32.5	138.6	1,013.6	5,022.9
Germany	350.8	0.3	6.3	54.0	419.5	1,844.9	3,015.5
Nicaragua	209.8	1.2	8.7	37.6	134.5	879.5	3,013.9
Mexico	295.4	0.4	9.0	51.0	284.2	1,224.8	2,637.1
China	128.8	0.2	3.9	21.8	68.9	713.7	1,584.0

Notes: All numbers in this table are reported in thousands of 2013 dollars.

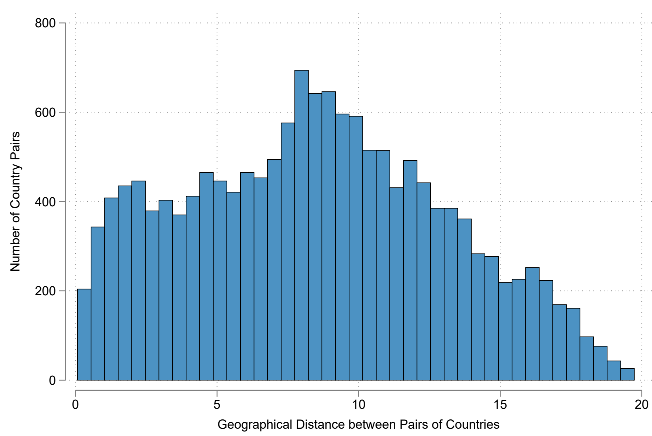
B.2 Geographical Distance

We measure the geographic distance $n^g_{jj'}$ between any two countries j and j' as

$$n^g_{jj'} \equiv \Big(\sum_{k \in j} \sum_{k' \in j'} \frac{pop_k}{pop_j} \frac{pop_{k'}}{pop_{j'}} (dist_{kk'})^{-1} \Big)^{-1}, \tag{B.1}$$

where k and k' respectively index cities in countries j and j', pop_k and $pop_{k'}$ denote the population of cities k and k', pop_j and $pop_{j'}$ denote the total population of the cities in countries j and j' used to calculate $n^g_{jj'}$, and $dist_{kk'}$ is the distance between k and k' in thousands of kilometers. In Figure B.2, we present a histogram of the geographical distance, computed according to the formula in equation (B.1), between any pair of countries. As Figure B.2 reveals, there is wide disparity in geographical distance across country pairs.

Figure B.2: Histogram of Bilateral Geographic Distances

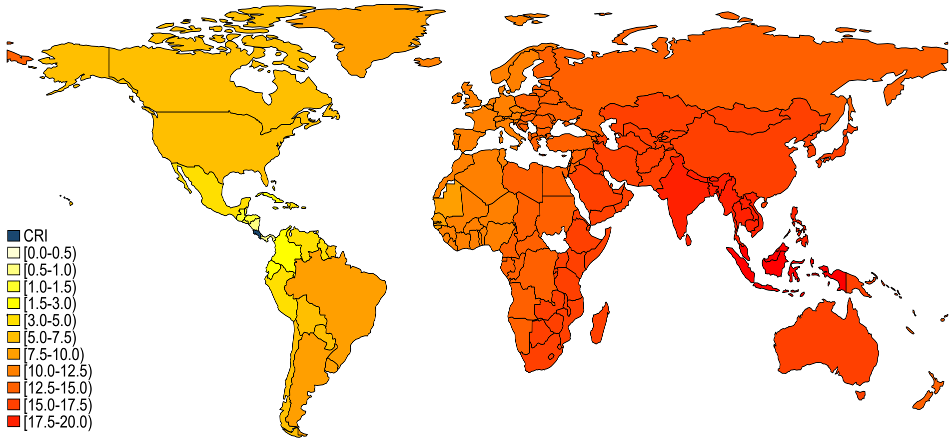


Notes: The vertical axis indicates the number of country pairs whose geographical distance according to equation (B.1) falls in the corresponding bin. The horizontal axis denotes geographical distance in thousands of kilometers.

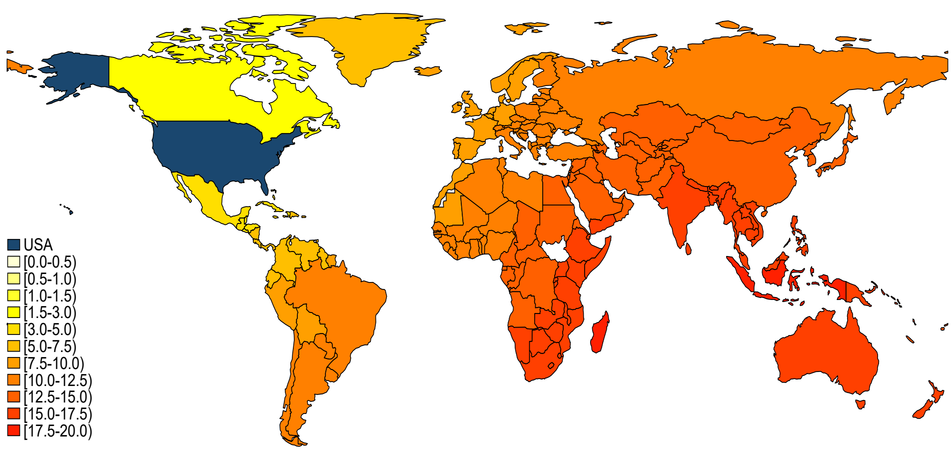
In Figure B.3, we represent in maps the geographical distance from Costa Rica (in Figure B.3a), the United States (in Figure B.3b), France (in Figure B.3c) and China (in Figure B.3d), respectively, to any other country of the world.

Figure B.3: Geographical Distances From Certain Countries

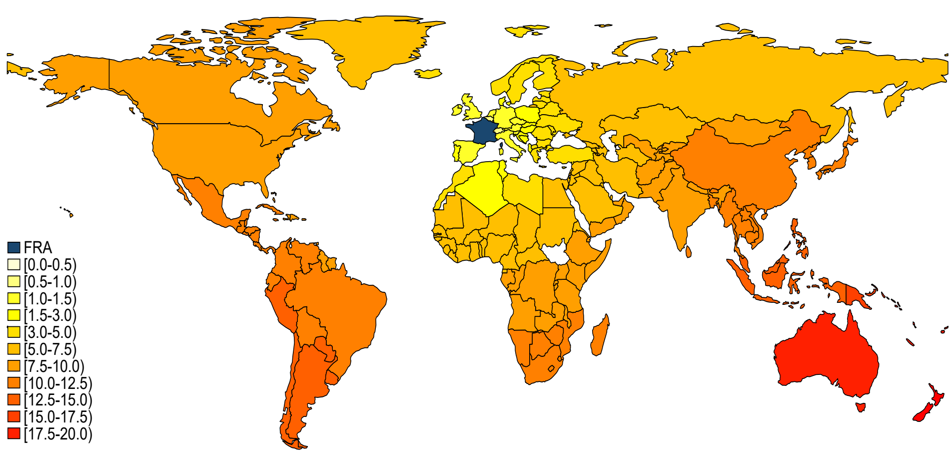
(a) From Costa Rica



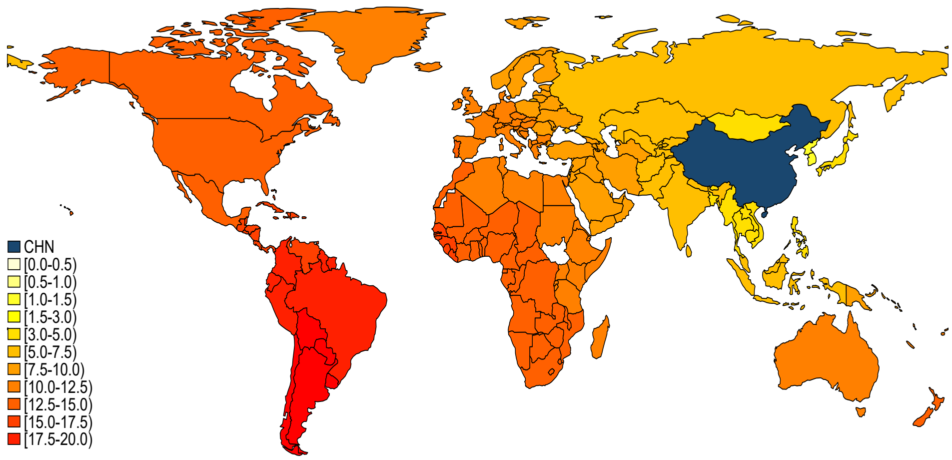
(b) From the United States



(c) From France



(d) From China



Notes: Each of the panels indicate the geographical distance (computed according to the expression in equation (B.1)) between a particular country (Costa Rica in Panel (a), the US in Panel (b), France in Panel (c), and China in Panel (d)) and any other country in the world. All distances are reported in thousands of kilometers.

B.3 Linguistic Distance

We measure the linguistic distance $n^l_{jj'}$ between any two countries j and j' as

$$n^l_{jj'} \equiv \max\Big\{0, 1 - \sum_{k=1}^{K} s_{jk}s_{j'k}\Big\}, \tag{B.2}$$

where s_{jk} is the share of country j's population that speak language $k = 1, \ldots, K$.

To obtain a list of languages and country-specific information on the population shares s_{jk} that speak any given language $k = 1, \ldots, K$, we use the information in Ethnologue (see Desmet et al., 2012, for another application of Ethnologue data).

Ethnologue defines languages according to 15 aggregation levels; e.g., at the 1st level, all Indo-European languages are considered the same language; at the 15th level, Spanish and Extremaduran are distinct. We use the 9th aggregation level, the first one classifying Portuguese and Spanish as distinct.

Ethnologue provides information by country on the population shares that speak any given language as first and second language, but it does not provide information on the distribution of second language speakers conditional on their first language. The measure in equation (B.2) assumes a joint distribution of first and second languages spoken in each country such that the linguistic distance between any two countries is minimized. To illustrate this point, consider a setting with only two languages, k_1 and k_2. In this setting, the probability that two individuals i and i' randomly selected from two countries j and j', respectively, speak a common language is:

$$P((\{i \text{ speaks } k_1\} \cap \{i' \text{ speaks } k_1\}) \cup (\{i \text{ speaks } k_2\} \cap \{i' \text{ speaks } k_2\}))$$
$$=$$
$$P(\{i \text{ speaks } k_1\} \cap \{i' \text{ speaks } k_1\}) + P(\{i \text{ speaks } k_2\} \cap \{i' \text{ speaks } k_2\}) -$$
$$P((\{i \text{ speaks } k_1\} \cap \{i' \text{ speaks } k_1\}) \cap (\{i \text{ speaks } k_2\} \cap \{i' \text{ speaks } k_2\})).$$

Using the notation in equation (B.2), we can rewrite this expression as

$$P((\{i \text{ speaks } k_1\} \cap \{i' \text{ speaks } k_1\}) \cup (\{i \text{ speaks } k_2\} \cap \{i' \text{ speaks } k_2\})) =$$
$$s_{jk_1}s_{j'k_1} + s_{jk_2}s_{j'k_2} - P((\{i \text{ speaks } k_1\} \cap \{i' \text{ speaks } k_1\}) \cap (\{i \text{ speaks } k_2\} \cap \{i' \text{ speaks } k_2\})),$$

and we can thus write the probability that two randomly selected individuals from countries j and j' do not speak a common language as

$$1 - s_{jk_1}s_{j'k_1} - s_{jk_2}s_{j'k_2} + P((\{i \text{ speaks } k_1\} \cap \{i' \text{ speaks } k_1\}) \cap (\{i \text{ speaks } k_2\} \cap \{i' \text{ speaks } k_2\})).$$

As the Ethnologue data does not contain information on the joint distribution of first and second languages spoken within a country, we cannot compute

$$P((\{i \text{ speaks } k_1\} \cap \{i' \text{ speaks } k_1\}) \cap (\{i \text{ speaks } k_2\} \cap \{i' \text{ speaks } k_2\})).$$

Given information on s_{jk_1}, s_{jk_2}, $s_{j'k_1}$, and $s_{j'k_2}$, we can however obtain a lower bound on this probability; denoting this lower bound as $LB_{jj'}$, it holds that

$$LB_{jj'} = \begin{cases} 0 & \text{if } s_{jk_1}s_{j'k_1} - s_{jk_2}s_{j'k_2} \leqslant 1, \\ s_{jk_1}s_{j'k_1} + s_{jk_2}s_{j'k_2} - 1 & \text{if } s_{jk_1}s_{j'k_1} + s_{jk_2}s_{j'k_2} > 1, \end{cases}$$

or, equivalently,

$$LB_{jj'} = \max\{0, s_{jk_1}s_{j'k_1} + s_{jk_2}s_{j'k_2} - 1\}.$$

Consequently, we can obtain a lower bound on the probability that two randomly selected individuals from countries j and j' do not speak a common language as

$$1 - s_{jk_1}s_{j'k_1} - s_{jk_2}s_{j'k_2} + LB_{jj'} = 1 - s_{jk_1}s_{j'k_1} - s_{jk_2}s_{j'k_2} + \max\{0, s_{jk_1}s_{j'k_1} + s_{jk_2}s_{j'k_2} - 1\}$$

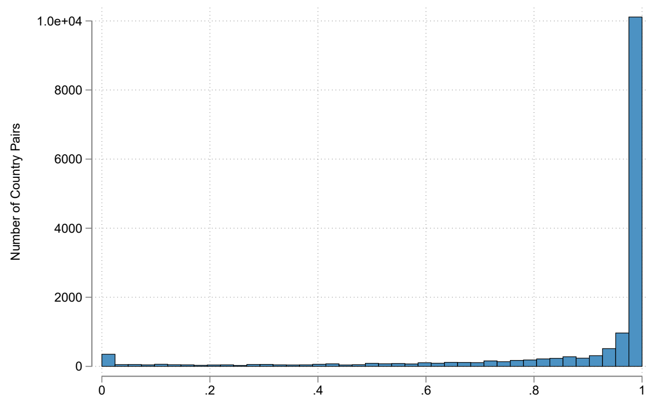
or, equivalently,

$$\max\{0, 1 - s_{jk_1}s_{j'k_1} - s_{jk_2}s_{j'k_2}\}.$$

This expression corresponds to that in equation (B.2) for the case with two languages, k_1 and k_2.

In Figure B.4, we present a histogram of bilateral linguistic distances. As Figure B.4 reveals, for most country pairs, a randomly selected resident of one of the two countries will not share any language with a randomly selected resident of the other country. Thus, for most country pairs, their linguistic distance equals one, which is the maximum possible value of the distance measure introduced in equation (B.2).

Figure B.4: Histogram of Bilateral Linguistic Distances

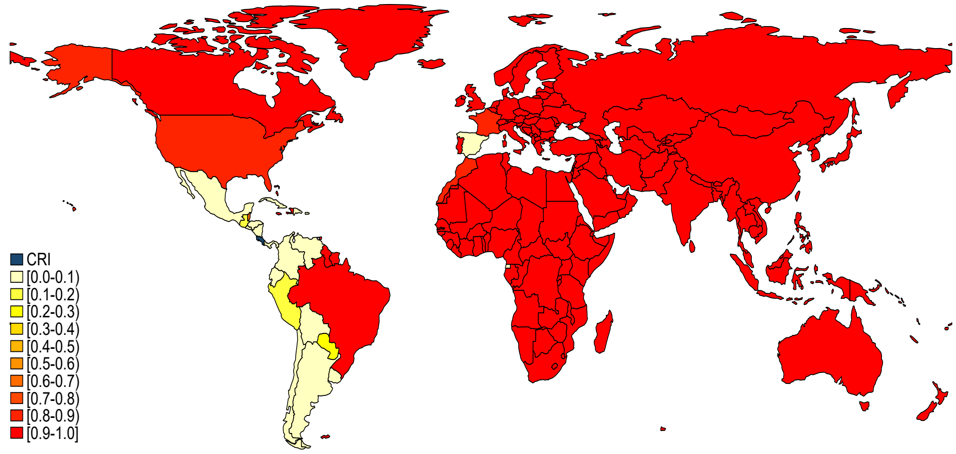


Notes: The vertical axis indicates the number of country pairs whose linguistic distance according to the formula in equation (B.2) falls in the corresponding bin. The horizontal axis denotes the corresponding linguistic distance.

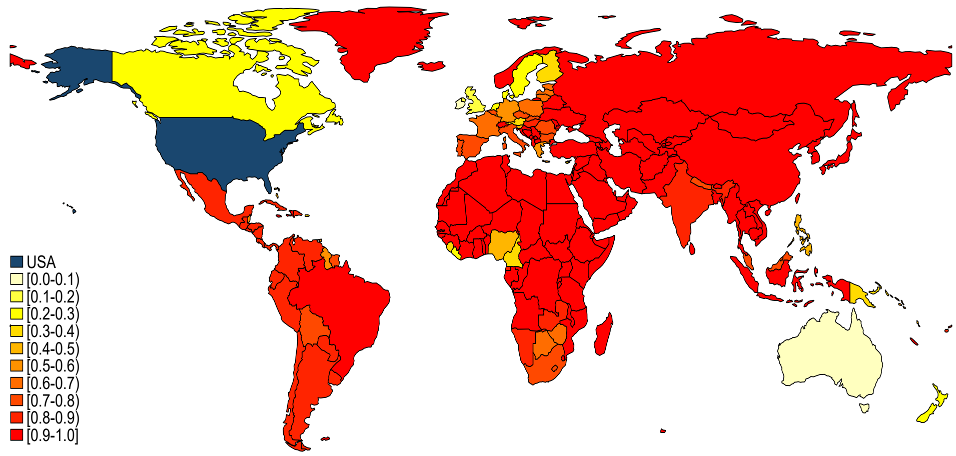
In Figure B.5, we represent bilateral linguistic distance measures from Costa Rica (in Figure B.5a), the US (in Figure B.5b), France (in Figure B.5c) and China (in Figure B.5d) to any other country of the world.

Figure B.5: Bilateral Linguistic Distances From Certain Origin Countries

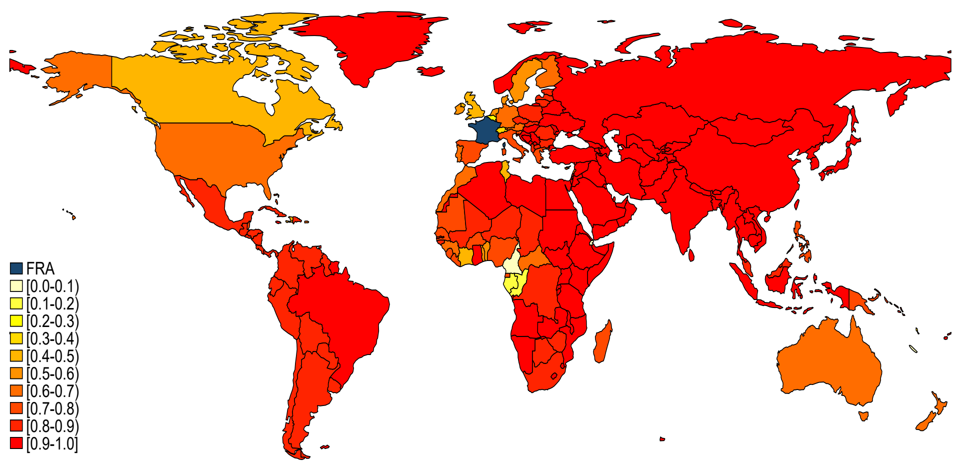
(a) From Costa Rica



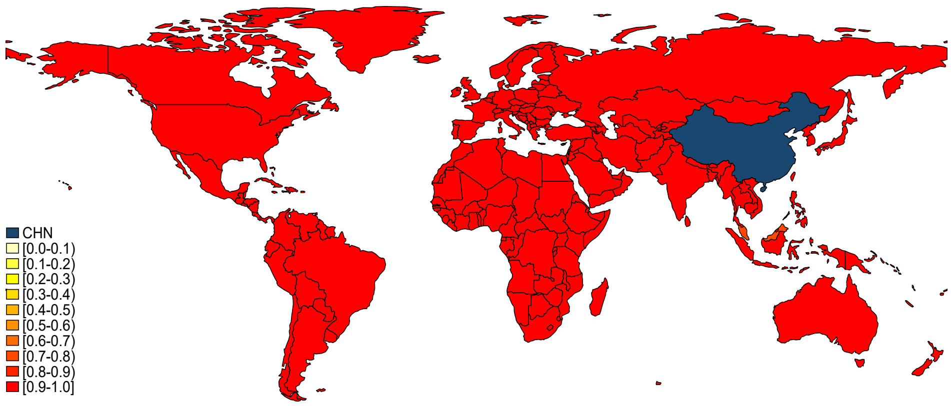
(b) From the United States



(c) From France



(d) From China



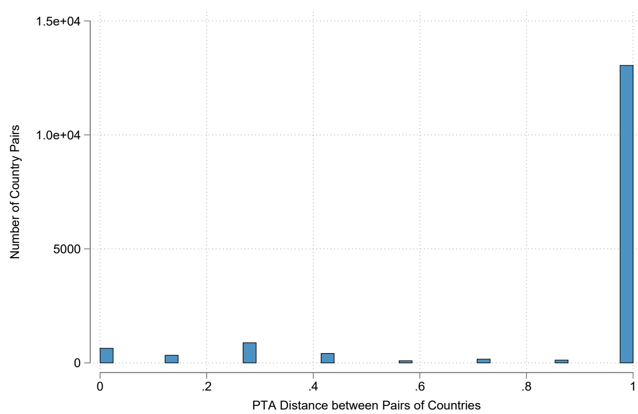
Notes: Each of the four panels in this figure indicate the linguistic distance (computed according to the expression in equation (B.2)) between a particular country (Costa Rica in Panel (a), the US in Panel (b), France in Panel (c), and China in Panel (d)) and any other country in the world.

The distance measures in Figure B.5a reflect the extent of the network of countries where Spanish is the most commonly spoken language. The measures in Figure B.5b reveal that the popularity of the English language as second language in many European countries implies that, according to the distance measure in equation (B.2), countries such as the Netherlands, Denmark, or Sweden, are linguistically very close to the US. Interestingly, as Figure B.5c reveals, the popularity of the English language as second language makes that certain countries that do not have English as official language (e.g., France and Sweden, France and Denmark) are linguistically close, the main reason being that many of their residents report speaking English. Finally, Figure B.5d reveals that China, in which a large share of their residents speak neither English nor Spanish, is generally isolated from a linguistic perspective.

B.4 Measures of Regulatory Distance

In Figure B.6, we present a histogram of an inverse measure of the breadth of the regulatory harmonization imposed by PTAs, computed according to the formula in equation (1). As Figure B.6 reveals, most country pairs do not share any PTA containing a provision in at least one of the policy areas listed in footnote 8.

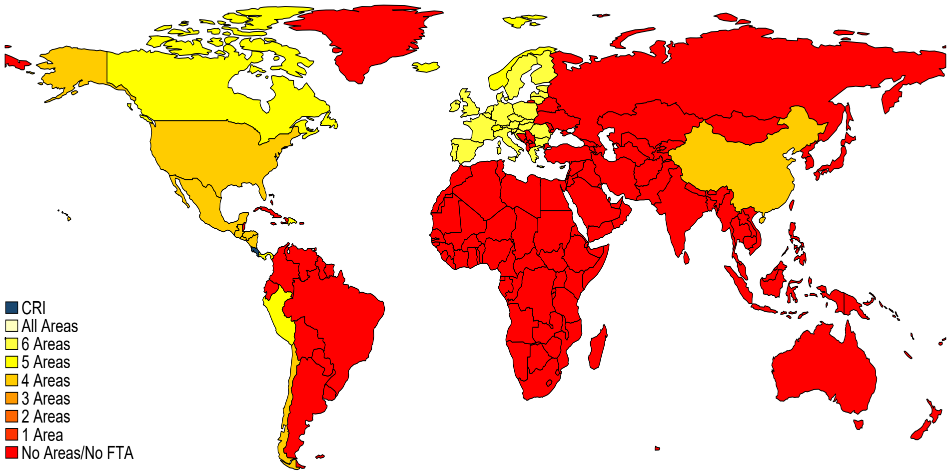
Figure B.6: Histogram of Bilateral Distances in PTAs



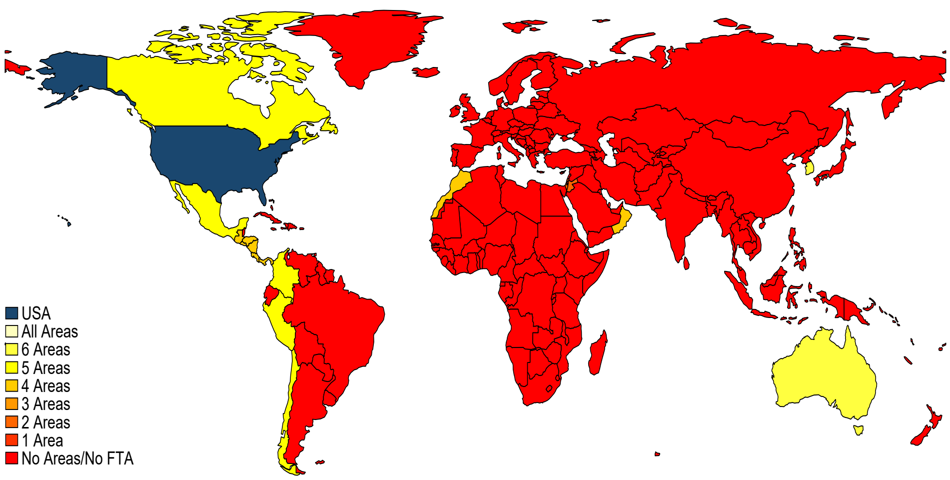
Notes: The vertical axis indicates the number of country pairs whose distance according to the formula in equation (1) falls in the corresponding bin. The horizontal axis denotes the value of the distance measure in equation (1).

Figure B.7: Bilateral Regulatory Distances From Certain Origin Countries

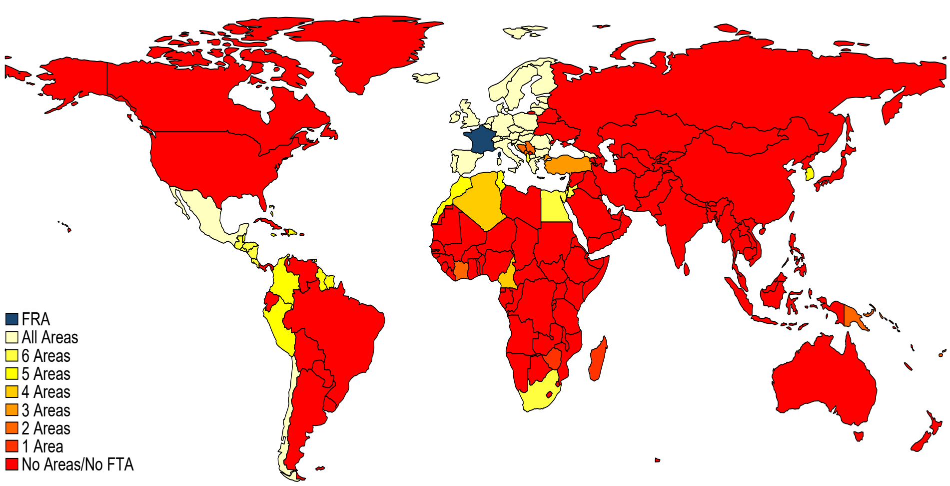
(a) From Costa Rica



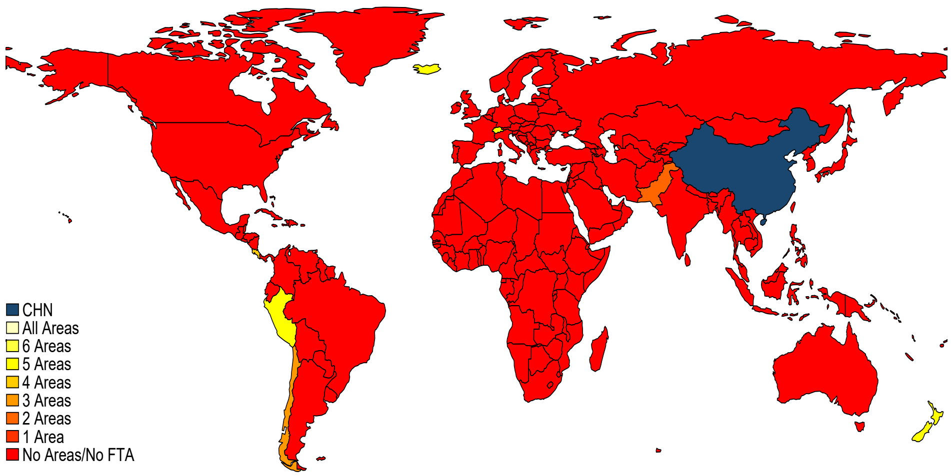
(b) From the United States



(c) From France



(d) From China



Notes: Each of the four panels in this figure illustrate the countries with which Costa Rica (in panel (a)), the United States (in panel (b)), France (in panel (c)), and China (in panel (d)) share in 2015 a PTA containing provisions in at least one of the seven policy areas listed in footnote 8. If it does, it indicates in how many of the seven policy areas listed in footnote 8 the corresponding preferential trade agreement contains some provision.

In Figure B.7, we illustrate the countries with which Costa Rica, the United States, France, and China, respectively, share in 2015 a PTA containing provisions in at least one of the policy areas listed in footnote 8. Whenever two countries had signed a PTA with a provision in one of these areas, Figure B.7 also indicates in how many of these areas the corresponding PTA includes a provision.

Figure B.7a reveals that Costa Rica has very deep integration agreements with Canada, members of the European Common Market, Panama, the Dominican Republic, and Peru, and less deep agreements with China, Chile, and other Central and North American countries. Figure B.7b shows that the US has a relatively deep PTA with Canada and Mexico (NAFTA), as well as with Colombia, Peru, Chile and Australia (these four are bilateral trade agreements), and a more shallow agreement with Central American countries (CAFTA). In the case of France, Figure B.7c illustrates that it has deep trade integration agreements not only with the other members of the European Common Market, but also with countries in North America (Mexico), Central America (e.g., Guatemala, Honduras, or Costa Rica), South America (e.g., Colombia, Peru, or Chile), Africa (e.g., Morocco, Tunisia, Egypt, or South Africa), and Asia (South Korea). Conversely, Figure B.7d illustrates that China has deep trade integration agreements with comparatively few and smaller countries (e.g., Iceland, Switzerland, Peru, or New Zealand).

In sum, the four panels in Figure B.7 show that countries differ significantly in the number and identity of the potential trade partners with whom they have signed deep PTAs. Furthermore, it is common for countries to sign deep PTAs with other countries that are neither geographically nor linguistically close to them (e.g., Costa Rica and China, the US and South Korea, France and South Africa, or China and Iceland).

B.5 Correlation in Export Participation Decisions: Additional Results

We present here estimates analogous to those in Section 3, but for alternative threshold values $\bar{n}_g$, $\bar{n}_l$, and $\bar{n}_a$. While we set $\bar{n}_g = 0.79$ (or 790 km), $\bar{n}_l = 0.11$, and $\bar{n}_a = 0.43$ in the main text, we set here instead $\bar{n}_g = 1.153$ (or 1,153 km), $\bar{n}_l = 0.5$ and $\bar{n}_a = 0.78$. The values of $\bar{n}_g$ and $\bar{n}_l$ we use here equal the 5th percentile of the distribution of the corresponding distance measure between any pair of countries in our sample; the value $\bar{n}_a = 0.72$ is equivalent to characterizing as deep any PTA that contains a provision in at least two of the seven policy areas listed in footnote 8.

In Table B.4, we present OLS estimates analogous to those in Table 1. A comparison of the estimates in these two tables reveals that, as we increase the set of countries classified as being geographically or linguistically close to a destination j, or as being cosignatories of a deep PTA with j, the impact that

Table B.4: Conditional Export Probabilities

	Panel A: No Controls				*Panel B: Controlling for Firm-Year Fixed Effects*			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
Y^g_{ijt}	0.1904^a			0.1345^a	0.1529^a			0.1217^a
	(0.0072)			(0.0059)	(0.0068)			(0.0060)
Y^l_{ijt}		0.1334^a		0.0733^a		0.1091^a		0.0760^a
		(0.0057)		(0.0038)		(0.0050)		(0.0041)
Y^a_{ijt}			0.0825^a	0.0297^a			0.0517^a	0.0222^a
			(0.0037)	(0.0016)			(0.0026)	(0.0018)
Obs.	3,859,618				3,859,618			

	Panel C: Controlling for Sector-Country-Year Fixed Effects				*Panel D: Controlling for Firm-Year & Sector-Country-Year Fixed Effects*			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
Y^g_{ijt}	0.1785^a			0.1269^a	0.1384^a			0.1116^a
	(0.0069)			(0.0057)	(0.0065)			(0.0057)
Y^l_{ijt}		0.1277^a		0.0706^a		0.1013^a		0.0721^a
		(0.0054)		(0.0035)		(0.0048)		(0.0039)
Y^a_{ijt}			0.0779^a	0.0283^a			0.0431^a	0.0169^a
			(0.0035)	(0.0015)			(0.0025)	(0.0017)
Obs.	3,859,618				3,859,618			

Note: ^{a} denotes 1% significance. Standard errors clustered by firm. The dependent variable in all specifications is a dummy that equals one if firm i exports to country j in year t. The covariates are $Y^x_{ijt} = \mathbb{1}\{\sum_{j' \neq j} \mathbb{1}\{n^x_{jj'} \leqslant \bar{n}_x\} y_{ij't} > 0\}$ for $x \in \{g, l\}$, and $Y^a_{ijt} = \mathbb{1}\{\sum_{j' \neq j} \mathbb{1}\{n^a_{jj't} \leqslant \bar{n}_a\} y_{ij't} > 0\}$, with $\bar{n}_g = 1.153$, $\bar{n}_l = 0.5$ and $\bar{n}_a = 0.78$.

exporting to at least one of these countries has on the probability of exporting to j decreases.

Comparing the estimate of the parameter on Y^g_{ijt} in column (4) of Panel D in Table 1 to that in Table B.4, we observe that the difference in the predicted export probability to any given destination is 0.18 when comparing firms that export to at least one country that is less than 790 km away from it to those that do not, but only 0.11 when comparing firms that export to at least one country that is less than $1,153$ km away from it to those that do not. This is consistent with the correlation in a firm's export participation decisions in any two countries decreasing in the geographical distance between both countries.

Similarly, comparing the estimate of the parameter on Y^a_{ijt} in column (4) of Panel D in Table 1 to that in Table B.4, we observe that the difference in the predicted export probability to any given destination between firms that export to at least one country that shares a deep PTA with it and those that do not decreases from 0.021 to 0.017 as we loosen the requirements a PTA must satisfy for us to classify it as "deep." This is consistent with the correlation in a firm's export participation decisions in any two countries increasing in the deepness of the PTAs linking both countries.

Finally, the estimate of the parameter on Y^l_{ijt} in column (4) of Panel D in Table 1 is very similar to that in Table B.4. In this case, the correlation in a firm's export participation decisions in any two countries seems not to vary much depending on whether the probability that two randomly chosen individuals, one from each country, understand each other is at least 0.89 (i.e., $\bar{n}_l = 0.11$, the threshold imposed in Table 1) or at least 0.5 (i.e., $\bar{n}_l = 0.5$, the threshold imposed in Table B.4). A possible explanation for this fact is that exporters select into their workforce workers knowledgeable of the languages spoken in the countries where they export and, consequently, the general prevalence of a language in a country is an imperfect predictor of the language barriers that exporting firms experience.

C Equation for Potential Export Revenues: Details

We derive the expression in equation (5) in three steps.

First Step. As firm i's marginal cost of selling in the home market h at period t is $\tau_{ht} w_{it}$ (see Section 4.2), the revenue firm i obtains in h at t is

$$r_{iht} = \Big[\frac{\eta}{\eta - 1} \frac{\tau_{ht} w_{it}}{P_{ht}} \Big]^{1-\eta} Y_{ht}. \tag{C.1}$$

Combining equations (3) and (C.1), we rewrite the potential export revenues of firm i in country j at period t as a function of its revenue in the domestic market:

$$r_{ijt} = \Big[\frac{\tau_{ijt}}{\tau_{ht}} \frac{P_{ht}}{P_{jt}} \Big]^{1-\eta} \frac{Y_{jt}}{Y_{ht}} r_{iht}. \tag{C.2}$$

Second Step. Substituting $(\tau_{ijt})^{1-\eta}$ in equation (C.2) by its expression in equation (4), we obtain

$$r_{ijt} = \exp(\xi_y y_{ijt-1} + \check{\xi}_{jt} + \alpha_s + \alpha_a \ln(a_{sjt}) + \xi_w \ln(w_{it}) + \ln(r_{iht})), \tag{C.3}$$

with

$$\check{\xi}_{jt} = \xi_{jt} + (1 - \eta) \ln(P_{ht}/P_{jt}) + \ln(Y_{jt}/Y_{ht}) - (1 - \eta) \ln(\tau_{ht}). \tag{C.4}$$

Third Step. Taking the logarithm of both sides of equation (C.1) and rearranging terms, we obtain

$$\ln(w_{it}) = \frac{1}{1 - \eta} (\ln(r_{iht}) - \ln(Y_{ht})) + \ln(\eta - 1) - \ln(\eta) + \ln(P_{ht}) - \ln(\tau_{ht}).$$

Plugging this equality into equation (C.3), we obtain equation (5) with $\alpha_s = \xi_s$, $\alpha_a = \xi_a$, and

$$\alpha_{jt} = \check{\xi}_{jt} + \xi_w(-(1/(1 - \eta)) \ln(Y_{ht}) + \ln(\eta - 1) - \ln(\eta) + \ln(P_{ht}) - \ln(\tau_{ht})), \tag{C.5a}$$
$$\alpha_r = 1 + \xi_w/(1 - \eta). \tag{C.5b}$$

D Solution Algorithm: Additional Details

D.1 Additional Steps

We discuss here how we tighten the upper bounds on firm choices at a period τ; the procedure for the lower bound being analogous.

Step 3. In this step, we tighten further the bounds at period τ. To do so, for every country j for which the bounds in equation (26) do not coincide, we solve a problem that differs from that in equation (21) in that, for period $\tau+1$ and a subset of countries M that does not include j, we condition on *functional* (instead of constant) upper bounds. Specifically, for any j such that $\bar{\check{y}}_{ij\tau|\tau} > \underline{\check{y}}_{ij\tau|\tau}$, we find the solution to

$$\max_{y_{ij\tau}} \Big\{ y_{ij\tau}(u_{ij\tau}(\check{y}_{ij\tau-1}, \check{\omega}_{ij\tau}) + \sum_{j' \neq j} \bar{\check{y}}_{ij'\tau|\tau}(c_{jj't} + c_{j'jt})) + \delta \mathbb{E}_{i\tau} \tilde{V}_{ij\tau+1}(y_{ij\tau}, \omega_{ij\tau+1}, \{\omega_{ij'\tau+1}\}_{j' \in M}) \Big\}, \tag{D.1}$$

with

$$\begin{aligned} \tilde{V}_{ij\tau+1}(y_{ij\tau}, \omega_{ij\tau+1}, \{\omega_{ij'\tau+1}\}_{j' \in M}) = \max_{y_{ij\tau+1}} \Big\{ & y_{ij\tau+1}(u_{ij\tau+1}(y_{ij\tau}, \omega_{ij\tau+1}) + \delta \mathbb{E}_{i\tau+1} \bar{V}_{ijt\tau+2}(y_{ij\tau+1}, \omega_{ij\tau+2}) \\ & + \sum_{j' \in M} \bar{b}_{ij'\tau+1|\tau}(\omega_{ij'\tau+1})(c_{jj'\tau+1} + c_{j'j\tau+1}) + \sum_{j' \notin M} \mathbb{1}\{j' \neq j\} \bar{b}^*_{ij'\tau+1|\tau}(c_{jj'\tau+1} + c_{j'j\tau+1})) \Big\}. \end{aligned} \tag{D.2}$$

The function $\tilde{V}_{ijt\tau+2}(y_{ij\tau+1}, \omega_{ij\tau+2})$ is country j's value function when the firm's choice in every period $t \geqslant \tau+2$ and every country other than j is set to the constant upper bounds obtained in the last iteration of the step 2 procedure. For every country j' other than j, equation (D.2) imposes the upper bounds

$$\bar{b}_{ij'\tau+1|\tau}(\omega_{ij'\tau+1}) = \bar{o}^*_{ij'\tau+1|\tau}(\bar{\check{y}}_{ij'\tau|\tau}, \omega_{ij'\tau+1}), \qquad \text{if } j' \in M, \tag{D.3a}$$

$$\bar{b}^*_{ij'\tau+1|\tau} = \bar{o}^*_{ij'\tau+1|\tau}(\bar{\check{y}}_{ij'\tau|\tau}, \underline{\omega}), \qquad \text{if } j' \notin M, \tag{D.3b}$$

where $\bar{o}^*_{ij'\tau+1|\tau}(\cdot)$ and $\bar{\check{y}}_{ij'\tau|\tau}$ are computed in step 2. By definition, $\bar{b}_{ij'\tau+1|\tau}(\omega_{ij'\tau+1}) \leqslant \bar{b}^*_{ij'\tau+1|\tau}$ and, thus, the bounds computed in step 3 are tighter than those computed in step 2, and they will be tighter the larger the set M. However, solving the problem in equation (D.1) requires computing an expectation over the vector $(\omega_{ij\tau+1}, \{\omega_{ij'\tau+1}\}_{j' \in M})$, a step that is computationally more complicated the larger the cardinality of M. In our application, for each country j, we choose M as the 16 countries that are geographically closer to j. If the step 3 upper and lower bounds do not coincide at $(\check{y}_{i\tau-1}, \check{\omega}_{i\tau})$ at τ, we proceed to step 4.

Step 4. In this step, we tighten further the bounds at period τ. To do so, we solve an optimization problem that differs from those solved in steps 1 to 3 in that, instead of computing policy functions iteratively country by country, we do so for several countries simultaneously.

Consider a set M of countries for which step 3 upper and lower bounds on the firm's optimal choices at the path of interest do not coincide at τ. For any $t \geqslant \tau$, define vectors y_{iMt} and ω_{iMt} that, for t and all countries j in M, include firm i's export choice y_{ijt} and blocking shock ω_{ijt}, respectively. Define also

$$\bar{V}_{iM\tau+h}(y_{iM\tau+h-1}, \omega_{iM\tau+h}) = \sum_{j \in M} \bar{V}_{ij\tau+h}(y_{ij\tau+h-1}, \omega_{ij\tau+h}), \tag{D.4}$$

where $\bar{V}_{ij\tau+h}(\cdot)$ is the country j's value function that results from equating the firm's choice in all periods $t \geqslant \tau+h$ and all countries other than j to the constant upper bounds obtained in the last iteration of the step 2 procedure. In step 4, we solve by backward induction for all $t \in [\tau, \tau+h-1]$ the problem

$$\begin{aligned} \bar{V}_{iMt}(y_{iMt-1}, \omega_{iMt}) = \max_{y_{iMt} \in \{0,1\}^M} \Big\{ \sum_{j \in M} \big\{ & y_{ijt}(u_{ijt}(y_{ijt-1}, \omega_{ijt}) + \sum_{j' \in M} y_{ij't} c_{jj't} + \\ & \sum_{j' \notin M} \mathbb{1}\{j \neq j'\} \bar{b}^*_{ij'\tau|\tau}(c_{jj't} + c_{j'jt})) \big\} + \delta \mathbb{E}_{it} \bar{V}_{iMt+1}(y_{iMt}, \omega_{iMt+1}) \Big\}, \end{aligned} \tag{D.5}$$

with $\bar{b}^*_{ij'\tau|\tau}$ and $\bar{V}_{iM\tau+h}(\cdot)$ defined as in equations (D.3b) and (D.4), respectively. Solving this problem is computationally more complicated the larger the set M and the horizon h are. In our application, if there are less than ten countries for which step 3 upper and lower bounds on the optimal choice at the path of interest at period τ differ, we include them all in M. If there are more than ten countries for which the step 3 bounds differ, we solve the problem in equation (D.5) repeatedly for different sets of countries, grouping together in these sets those countries that are geographically close to each other. Concerning h, we solve first the problem for $h = 1$, and increase progressively the value of h until $h = 10$.

Step 5. In this step, we tighten further the bounds at τ. To do so, we compute the firm's optimal export paths in a set M of countries fixing the firm's choices in all countries not in M to constant upper bounds. Specifically, in step 5, we first solve the following period-T problem for every value of $(y_{iMT-1}, \omega_{iMT})$:

$$\bar{V}_{iMT}(y_{iMT-1}, \omega_{iMT}) = \max_{y_{iMT} \in \{0,1\}^M} \Big\{ \sum_{j \in M} \{ y_{ijT}(u_{ijT}(y_{ijT-1}, \omega_{ijT}) + \sum_{j' \in M} y_{ij'T} c_{jj'T} + \sum_{j' \notin M} \mathbb{1}\{j \neq j'\} \bar{b}^{*}_{ij'T|\tau}(c_{jj'T} + c_{j'jT}))\} + \delta \mathbb{E}_{iT} \bar{V}_{iMT+1}(y_{iMT}, \omega_{iMT+1}) \Big\}. \tag{D.6}$$

As this problem is stationary, we use value-function iteration to solve for the value function $\bar{V}_{iMT}(\cdot)$. Given $\bar{V}_{iMT}(\cdot)$, we use backward induction to solve for the optimal policy function in M for all $t \in [\tau, T]$.

If M includes all J foreign countries, the problem in equation (D.6) coincides with that in equation (18) and, thus, its solution yields the firm's optimal policy function. Solving the problem in equation (18) for a large set M is however computationally infeasible. In our application, we choose M according to the following rules. If there are less than six countries for which step 4 upper and lower bounds on the optimal choice at the path of interest at period τ differ, we include them all in M. If there are more than six countries for which the step 4 bounds differ, we implement the step 5 algorithm repeatedly for different sets of six countries grouping together countries that are geographically close to each other.

Closing the algorithm. If there are countries for which the upper and lower bound on the optimal choice at the path of interest at period τ differ after step 5, we assume the optimal choice is to not export to those countries at τ at the state of interest.

D.2 Illustration of Algorithm in a Two-Country and Three-Period Setting

We illustrate here our algorithm in an example with two countries (A and B) and three periods. We use trees to represent graphically all possible paths of ω_{ijt}. With the letters L (with stands for low) and H (which stands for high), we denote the events in which the blocking shock respectively equals the smallest, $\underline{\omega}$, and largest, $\bar{\omega}$, values in their support. E.g., in Figure D.1, the orange path is one in which blocking shocks in A are low in all three periods while, in B, these are low in periods 1 and 3, and high in period 2.

Figure D.1: Possible Paths of Fixed Cost Shocks

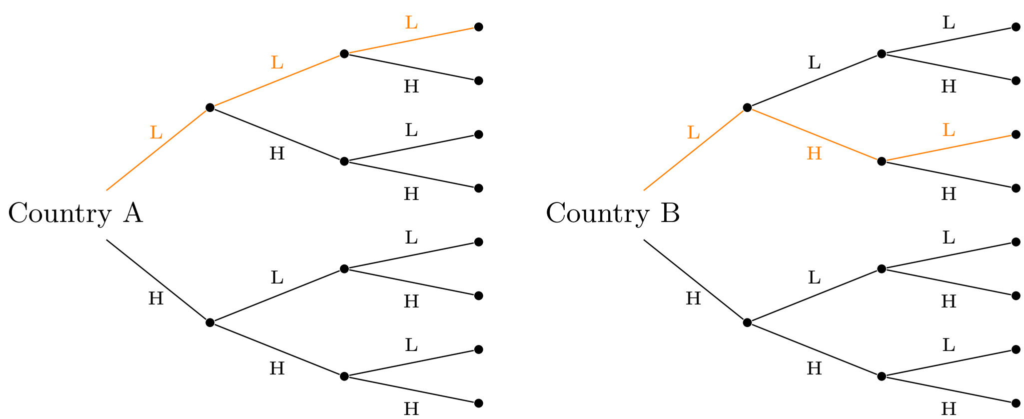
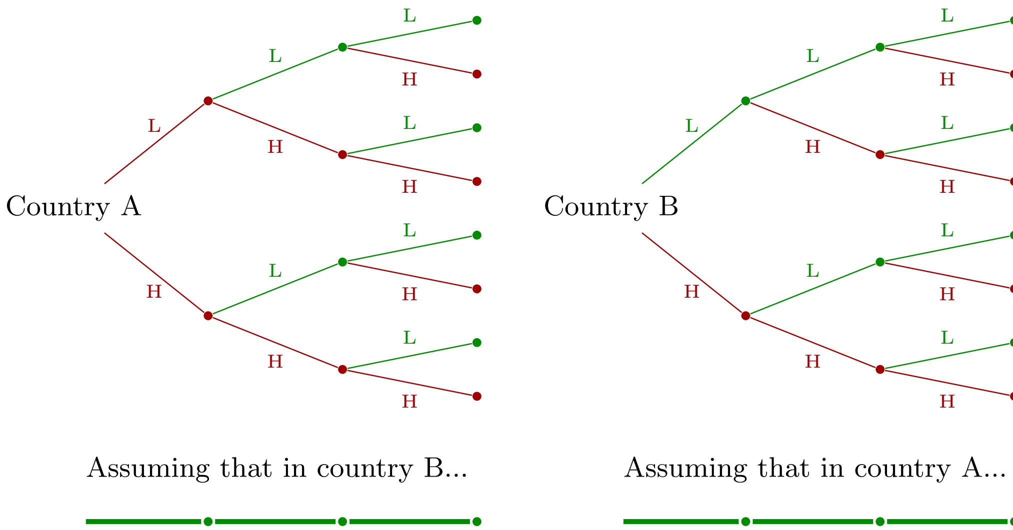


Figure D.2: Initial Upper-Bound Policy Functions



Step 1. In Figure D.2, we illustrate the first iteration of step 1 of the algorithm (see Section 5). The left panel illustrates the solution to the optimization problem in equation (21) for country A when setting $\bar{b}_{iBt} = 1$ for all three time periods; the right panel is analogous but for country B. Using the notation in Section 5, Figure D.2 thus illustrates the upper-bound policy function

$$\bar{o}^{[0]}_{it}(y_{it-1}, \omega_{it}) = (\bar{o}^{[0]}_{iAt}(y_{iAt-1}, \omega_{iAt}), \bar{o}^{[0]}_{iBt}(y_{iBt-1}, \omega_{iBt})), \qquad \text{for all } t = \{1, 2, 3\}. \tag{D.7}$$

Specifically, in all figures in this section, we use green to identify branches at which the firm exports, and red to identify branches at which it does not. The left panel in Figure D.2 thus shows that, conditional on the firm exporting to B in all periods and states (as reflected by the three green segments under "Assuming that in country B...”), the firm chooses not to export to A at $t = 1$ regardless of whether ω_{iA1} is high or low (as reflected by the two red segments branching out from the "Country A" label), and chooses to export to A at $t = 2$ and $t = 3$ if and only if ω_{iAt} in the corresponding period t is low (as reflected by the L-segments being green and the H-segments being red). Similarly, the right panel in Figure D.2 shows that, if the firm exports to A in all periods and states (as reflected by the three green segments under "Assuming that in country A...”), the firm chooses to export to B in any given period if and only if ω_{iBt} in the corresponding period t is low (as reflected by the L-segments being green and the H-segments being red).

In Figure D.3, we evaluate the upper-bound policy in equation (D.7), as represented in Figure D.2, at the path of shocks in which these equal their lowest possible value in every country and period (i.e., the path marked by thick lines in each tree's top branch). Doing so, we obtain new constant upper bounds on the firm's choice in all countries and periods. E.g., as the upper-bound policy represented in Figure D.2 prescribes the firm not to export to A at $t = 1$ even $\omega_{iA1} = \underline{\omega}$, we update from one to zero the constant upper bound in A at $t = 1$ (as reflected in the change in color of the segment labeled "Update"). Using the notation in Section 5, it is thus the case that

$$(\bar{b}^{[1]}_{iA1}, \bar{b}^{[1]}_{iA2}, \bar{b}^{[1]}_{iA3}) = (0, 1, 1) \qquad \text{and} \qquad (\bar{b}^{[1]}_{iB1}, \bar{b}^{[1]}_{iB2}, \bar{b}^{[1]}_{iB3}) = (1, 1, 1). \tag{D.8}$$

We represent in Figure D.4 the new upper-bound policy function we obtain by solving again the optimization problem in equation (21) but now conditioning on the constant upper bounds illustrated at the bottom of Figure D.4, and listed in equation (D.8). Comparing figures D.2 and D.4, we observe that the change in the constant upper bound in country A at period $t = 1$ drives a change in the upper-bound policy function in country B at $t = 1$ at the low fixed cost shock segment, whose color switches from green to red. As country B's constant upper bounds in figures D.2 and D.4 coincide, the upper-bound policy function in country A remains the same.

Figure D.3: Updated Constant Upper Bounds

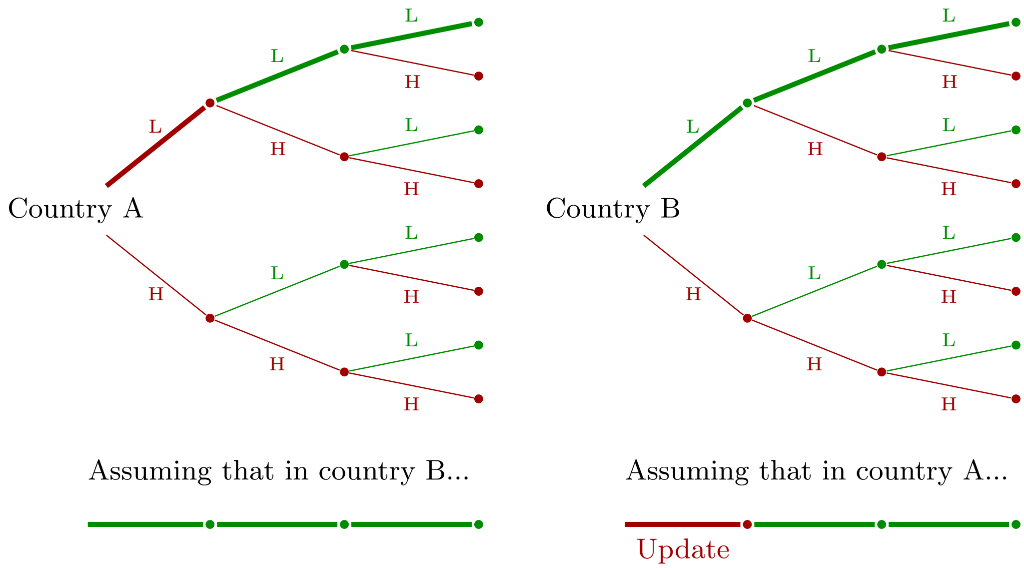
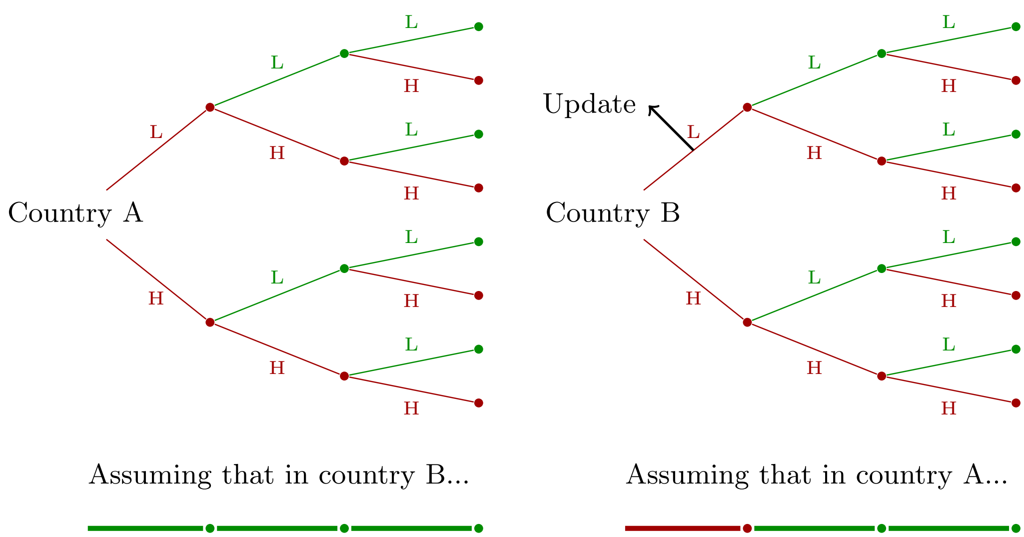


Figure D.4: Updated Upper-Bound Policy Functions



In Figure D.5, we evaluate the updated upper-bound policy illustrated in Figure D.4 at the path of shocks in which these equal their lowest possible value in every country and period, represented in Figure D.3 by the thick lines in each tree's top branch. Comparing figures D.3 and D.5, we observe that the update in the upper-bound policy in Figure D.4 with respect to that in Figure D.2 allows to update from one to zero the constant upper bound in B at $t = 1$ (as reflected in the change in color of the segment labeled "Update"). Using the notation in Section 5, it is then the case that

$$(\bar{b}^{[2]}_{iA1}, \bar{b}^{[2]}_{iA2}, \bar{b}^{[2]}_{iA3}) = (0, 1, 1) \qquad \text{and} \qquad (\bar{b}^{[2]}_{iB1}, \bar{b}^{[2]}_{iB2}, \bar{b}^{[2]}_{iB3}) = (0, 1, 1). \tag{D.9}$$

Continuing with the iterative process, we solve again the optimization problem in equation (21) but now conditioning on the updated constant upper bounds illustrated at the bottom of Figure D.5 and listed in equation (D.9). The solution is an upper-bound policy function identical to that obtained in the previous

Figure D.5: Updated Constant Upper Bounds

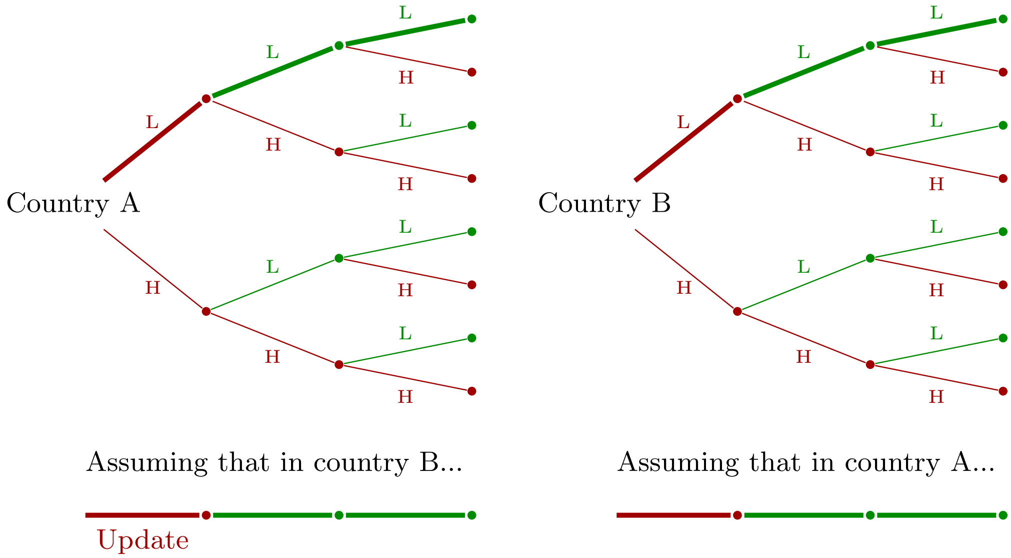


Figure D.6: Upper-Bound Policy Functions After Convergence

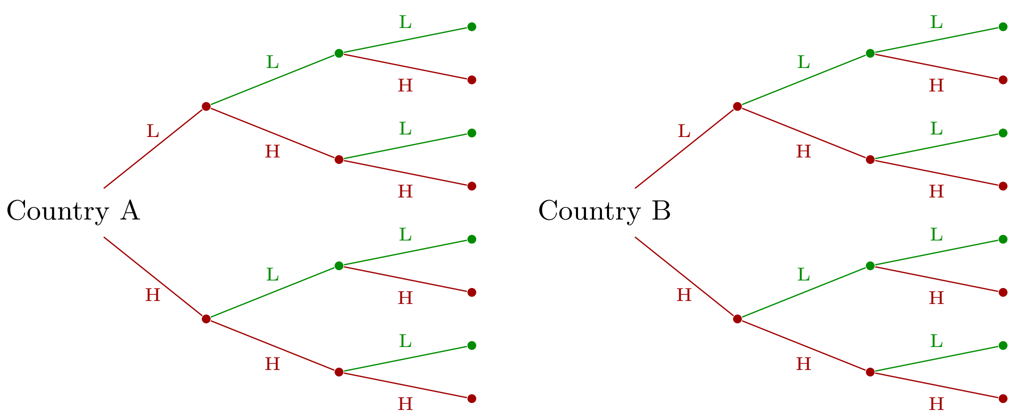


Figure D.7: Lower-Bound Policy Functions After Convergence

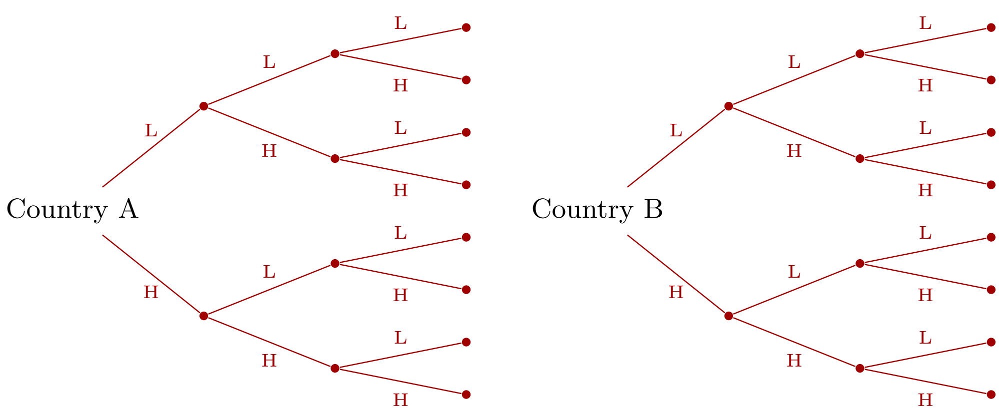
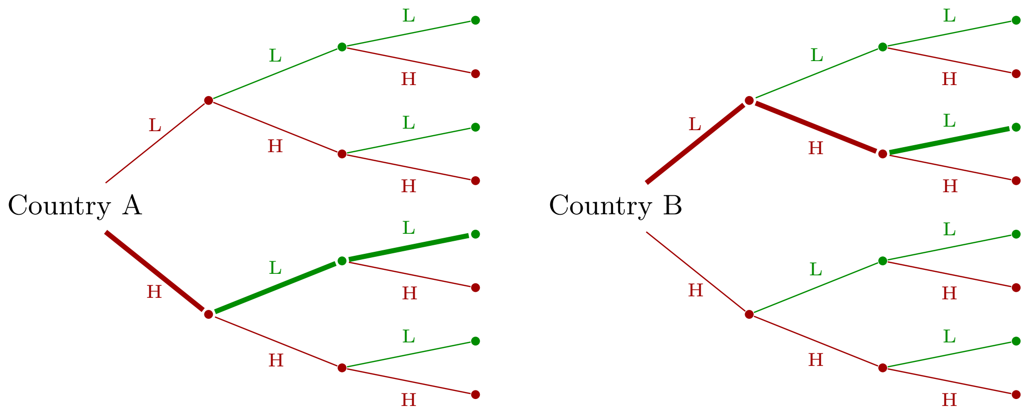


Figure D.8: Evaluating Upper-Bound Policy Functions at Path of Interest



iteration; i.e., that in Figure D.4. Intuitively, as the upper-bound policy in Figure D.4 already prescribes the firm not to export to A at $t = 1$ regardless of the value of ω_{iA1}, the update in the constant upper bound in B at $t = 1$ does not change the upper-bound policy function in A. Thus, after two iterations, the step 1 upper-bound policy function has converged to that represented in Figure D.6.

We follow analogous steps to compute lower-bound policy functions. Assume for simplicity the converged lower-bound policies prescribe the firm not to export to any country in any period regardless of the value of ω_{ijt} for any j and t. The converged lower-bound policy thus corresponds to that in Figure D.7.

The final stage in step 1 of our algorithm is to evaluate the converged lower- and upper-bound policy functions at a specific path of interest. Assume, e.g., this path is:

$$(\hat{\omega}_{iA1}, \hat{\omega}_{iA2}, \hat{\omega}_{iA3}) = (\bar{\omega}, \underline{\omega}, \underline{\omega}) \qquad \text{and} \qquad (\hat{\omega}_{iB1}, \hat{\omega}_{iB2}, \hat{\omega}_{iB3}) = (\underline{\omega}, \bar{\omega}, \underline{\omega}), \tag{D.10}$$

where, as a reminder, $\underline{\omega}$ and $\bar{\omega}$ are represented by L and H, respectively, in all figures in this section.

Figure D.8 is identical to Figure D.6 except that the path of interest is highlighted. The colors of the highlighted branches indicate the upper bounds on the firm's optimal choices at the path of interest; i.e.,

$$(\check{\bar{y}}_{iA1}, \check{\bar{y}}_{iA2}, \check{\bar{y}}_{iA3}) = (0, 1, 1) \qquad \text{and} \qquad (\check{\bar{y}}_{iB1}, \check{\bar{y}}_{iB2}, \check{\bar{y}}_{iB3}) = (0, 0, 1). \tag{D.11}$$

Similarly, given the converged lower-bound policy function in Figure D.7, the lower bounds on the firm's optimal choices at the path of interest are

$$(\check{\underline{y}}_{iA1}, \check{\underline{y}}_{iA2}, \check{\underline{y}}_{iA3}) = (0, 0, 0) \qquad \text{and} \qquad (\check{\underline{y}}_{iB1}, \check{\underline{y}}_{iB2}, \check{\underline{y}}_{iB3}) = (0, 0, 0). \tag{D.12}$$

Upper and lower bounds coincide at $t = 1$ for both countries; thus, the optimal choices at $t = 1$ at the path of interest are $(\check{y}_{iA1}, \check{y}_{iB1}) = (0, 0)$. At $t = 2$, both bounds differ in their prescribed choice in country A.

Step 2. In this step, we tighten the bounds at $t = 2$. To do so, we first compute new constant upper bounds that condition on the state reached at $t = 2$ at the path of interest; i.e., we evaluate the policy function in Figure D.6 along a path that, for $j = \{A, B\}$, sets $\omega_{ijt} = \check{\omega}_{ijt}$ for $t \leqslant 2$, and $\omega_{ijt} = \underline{\omega}$ for $t > 2$. In Figure D.9, we recover the upper-bound policy in Figure D.6, fade all branches that cannot be reached from the path of interest at $t = 2$ and mark with a wide line the relevant path. Conditioning on the path of interest up to $t = 2$ permits updating the constant upper bound in B at $t = 2$ (as reflected in the change in color of the segment labeled "Update" in Figure D.9). Using the notation in Section 5, it then holds that

$$(\bar{b}^{[0]}_{iA2|2}, \bar{b}^{[0]}_{iA3|2}) = (1, 1) \qquad \text{and} \qquad (\bar{b}^{[0]}_{iB2|2}, \bar{b}^{[0]}_{iB3|2}) = (0, 1). \tag{D.13}$$

We represent in Figure D.10 the upper-bound policy function obtained by solving the optimization problem in equation (21) for $t \geqslant 2$ with the new constant upper bounds represented at the bottom of Figure D.9 and listed in equation (D.13). Figure D.10 shows that the upper-bound policy in A at $t = 2$ is updated.

Figure D.9: Initial Constant Upper Bounds That Condition on Path of Interest for $t \leqslant 2$

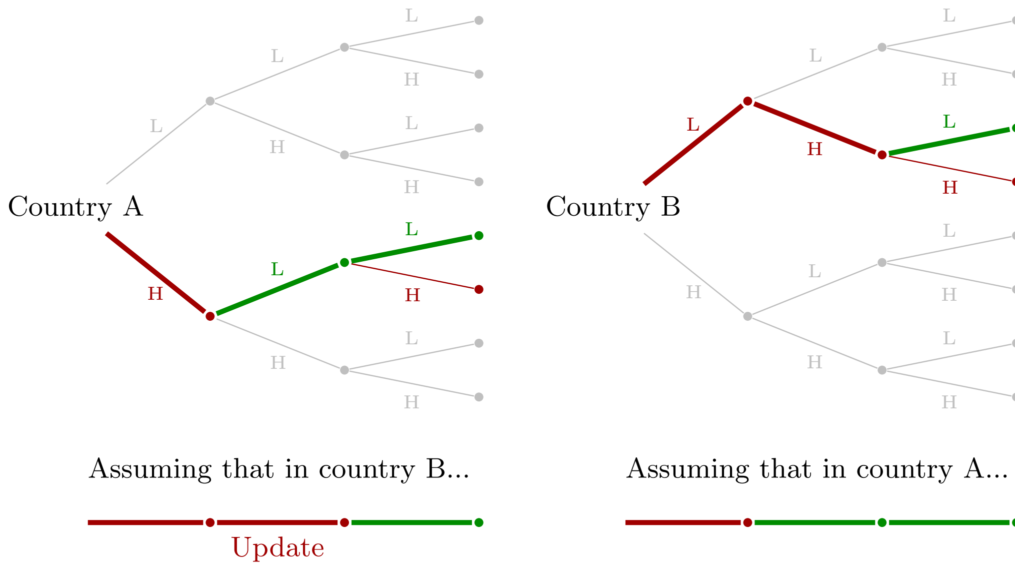
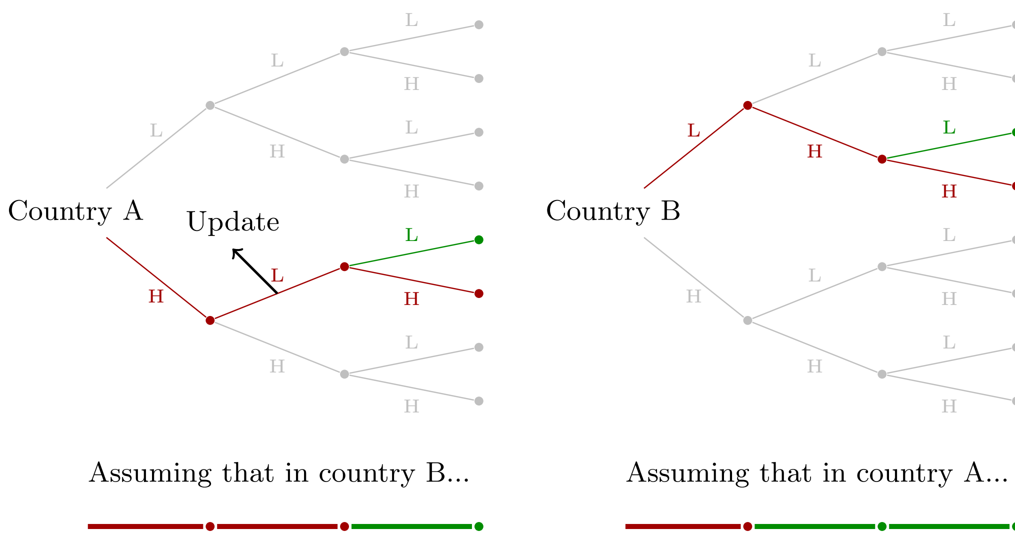


Figure D.10: Upper-Bound Policy Functions That Condition on Path of Interest for $t \leqslant 2$



Next, we evaluate the updated upper-bound policies in Figure D.10 along the path that, for $j = \{A, B\}$, sets $\omega_{ijt} = \check{\omega}_{ijt}$ for $t \leqslant 2$ and $\omega_{ijt} = \underset{\sim}{\omega}$ for $t > 2$, represented in Figure D.11 by thick lines. Comparing figures D.9 and D.11, we observe that the update in the upper-bound policy in Figure D.10 relative to that in Figure D.8 allows us to update the constant upper bound in A at $t = 2$ (see the red segment over the label "Update" in Figure D.11). In the notation introduced in Section 5, it is then the case that

$$(\bar{b}^{[1]}_{iA2|2}, \bar{b}^{[1]}_{iA3|2}) = (0, 1) \qquad \text{and} \qquad (\bar{b}^{[1]}_{iB2|2}, \bar{b}^{[1]}_{iB3|2}) = (0, 1). \tag{D.14}$$

Continuing with this iterative procedure, we solve again the optimization problem in equation (21) for periods $t \geqslant 2$, but now conditioning on the new constant upper bounds in equation (D.14) (see also bottom of Figure D.11). The solution to this problem yields upper-bound policy functions identical to those obtained in the previous iteration. Intuitively, as the upper-bound policy in Figure D.10 already prescribes the firm

Figure D.11: Updated Constant Upper Bounds That Condition on Path of Interest for $t \leqslant 2$

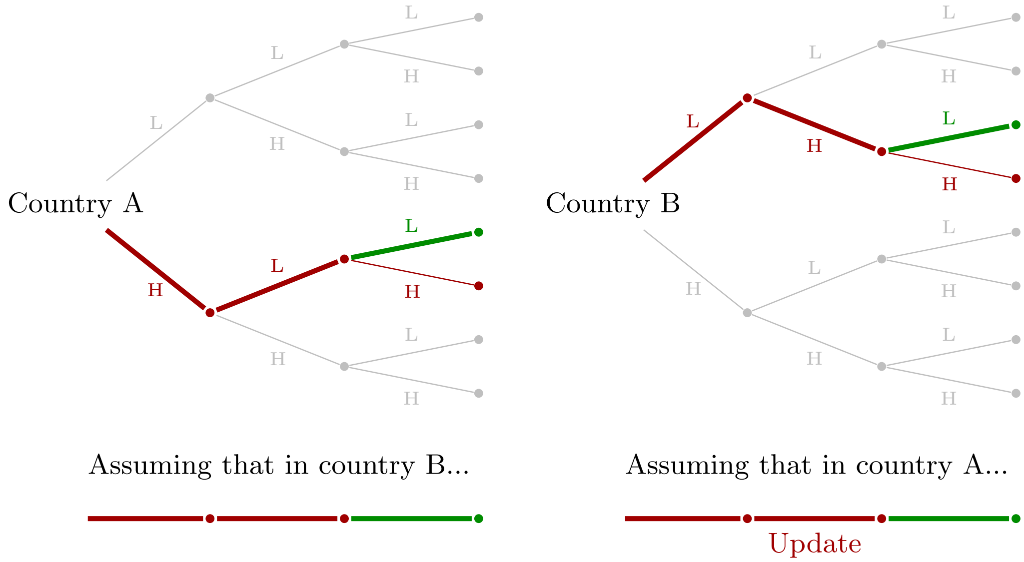
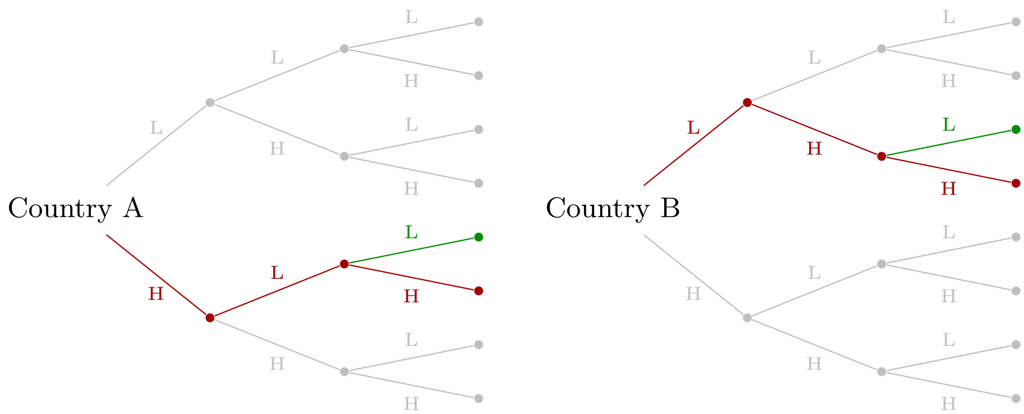


Figure D.12: Upper-Bound Policy Functions That Condition on Path for $t \leqslant 2$ After Convergence



not to export to B at $t = 2$ at the path of interest, the change in the constant upper bound in A at $t = 2$ does not change the upper-bound policy function. Thus, at this point, the step 2 upper-bound policy functions has converged; we represent it in Figure D.12.

We follow similar steps to compute a lower-bound policy function that conditions on the path of interest at $t = 2$. As the lower-bound policy that converged in step 1 (see Figure D.7) prescribe the firm not to export to any country at any period or state, the resulting constant lower bounds are

$$(\underline{b}^{[0]}_{iA2|2}, \underline{b}^{[0]}_{iA3|2}) = (0, 0) \qquad \text{and} \qquad (\underline{b}^{[0]}_{iB2|2}, \underline{b}^{[0]}_{iB3|2}) = (0, 0). \tag{D.15}$$

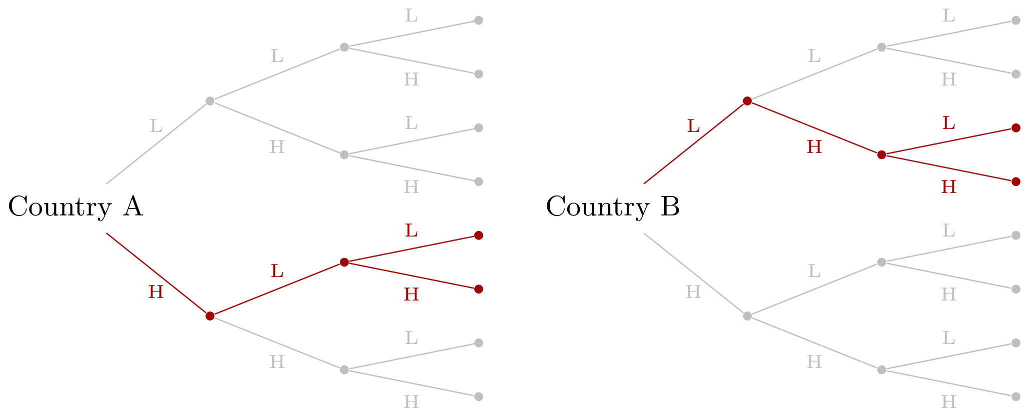
Given these, the lower-bound policy function cannot be updated further; we represent it in Figure D.13.

Evaluating the lower- and upper-bound policy functions in figures D.12 and D.13 at the path of interest at period $t = 2$, we obtain the following bounds on the firm's optimal export choices

$$\bar{\check{y}}_{iA2|2} = \underline{\check{y}}_{iA2|2} = 0 \qquad \text{and} \qquad \bar{\check{y}}_{iB2|2} = \underline{\check{y}}_{iB2|2} = 0. \tag{D.16}$$

As the bounds coincide, the firm's optimal choice at $t = 2$ at the path of interest is $(\check{y}_{iA2}, \check{y}_{iB2}) = (0, 0)$.

Figure D.13: Lower-Bound Policy Functions That Condition on Path at $t = 2$ After Convergence



Additional steps. At this point in the algorithm, we have computed the firm's optimal choice at the path of interest for $t \leqslant 2$. However, the step 1 bounds, described in equations (D.11) and (D.12), also differ at the path of interest at $t = 3$. Our algorithm thus proceeds by trying to tighten these bounds. To do so, we first implement a step 2 procedure analogous to the one just described, but now conditioning on the state reached at $t = 3$ along the path of interest. To save on space, we do not describe here how the step 2 algorithm is applied at $t = 3$. It suffices to say that it is not successful at tightening further the bounds on the firm's optimal choice along the path of interest at $t = 3$. Thus, we proceed to implement the extra steps described in Appendix D.1. Specifically, computing the firm's optimal choice at the state of interest at $t = 3$ requires solving jointly for the firm's optimal choices in A and B at this period.

D.3 Performance of the Algorithm

We present here summary statistics of the performance of the algorithm described in Section 5 and Appendix D.1. For all 4,709 firms in the sample, all 74 foreign countries we use in our estimation, 13 periods, and 5 simulation draws of ω_{ijt} for each i, j and t, we measure at the end of each step of the algorithm the percentage of all 22,650,290 ($4,709 \times 74 \times 13 \times 5$) choices solved and the cumulated running time (measured at Princeton University's Della cluster using 44 processors with 20 GB of memory per processor).

The statistics in Table D.1 are computed setting all parameter values to the baseline estimates reported in tables F.3 and F.4 in Appendix F.6. As reported in the first row in Table D.1, the step 1 of the algorithm (see Section 5 for a description) runs in slightly over two minutes, and provides the solution to 99.72% of the 22,650,290 choices considered. The 0.28% of choices that remain unsolved after step 1 of the algorithm are concentrated in a few countries but dispersed across firms and simulation draws; thus, the number of firms and draws whose choices in every country and period are solved in step 1 is only 78.51%.

Steps 2 and 3 increase the overall share of choices solved to 99.85%, and the share of firms and draws whose complete set of choices is solved to 93.07%. Furthermore, this is attained with a relatively small cost in terms of computing time, as step 3 is completed after less than 4 minutes of running time. In steps 4 and 5, we solve optimization problems that consider multiple countries simultaneously. As the last column in Table D.1 reveals, these steps are the slowest ones: approximately 70% of the 741 seconds it takes to run completely our algorithm are spent in steps 4 and 5. These steps are however useful at raising the share of choices solved to nearly 99.9%, and the share of firms and simulations entirely solved to nearly 96%.

The choices that remain unsolved after step 5 of the algorithm is finished are concentrated in countries that share cross-country complementarities with a large set of other potential export destinations. E.g., of all unsolved choices, nearly 7% are for Mexico, close to 6.5% are for Belgium, between 5% and 6% correspond to The Netherlands and Germany, and between 4% and 5% correspond to Sweden and France. These are all countries that share deep PTA (or regulatory proximity) with a number of other countries larger than the cardinality of the sets of destinations that we solve jointly in steps 4 and 5 of our algorithm: while we consider sets of 10 and 6 destinations in steps 4 and 5, respectively, both Mexico and all members of the EU

Table D.1: Performance of Algorithm at Baseline Estimates

	Percentage of Firms Solved	Percentage of Choices Solved	Time (in seconds)
Step 1	78.51%	99.72%	131
Step 2	82.74%	99.75%	163
Step 3	93.07%	99.85%	218
Steps 4 & 5	95.80%	99.89%	741

share deep PTA with more than 10 destinations.

In Table D.2, we present statistics analogous to those presented in the last row of Table D.1, but for alternative parameterizations in which we change the value of the model parameters each one at a time. Specifically, we present results for parameterizations in which we increase in 20% the value of the parameter indicated in the column labeled "Parameter," leaving all other parameters at their baseline estimates.

The results in Table D.2 show the performance of the algorithm improves (i.e., the percentage of firms and simulations for which all choices are solved increases, and the running time decreases) as we increase the value of those parameters that have a positive impact on the gravity component of fixed and sunk costs; i.e., the parameters entering the expressions in equations (7) and (12). Conversely, the performance of the algorithm worsens as we increase the value of the parameters that have a positive impact on the magnitude of the complementarities between countries (i.e., $(\gamma^E_x, \varphi^E_x)$ for $x = \{g, l, a\}$), and improves as we increase the value of the parameters that determine the speed at which the complementarities between any two countries decay in the distance between them (i.e., κ^E_x for $x = \{g, l, a\}$). The performance of the algorithm varies very little with the value of the parameters that determine the cross-country correlation in the fixed cost shock ν_{ijt}; i.e., the parameters entering the expression in equation (10c). Finally, when we increase the standard deviation of ν_{ijt} or the probability that ω_{ijt} equals $\underset{\sim}{\omega} = 0$ (i.e., when we increase σ_ν or p), the performance of the algorithm worsens.

Table D.2: Performance of Algorithm at Estimates 20% Higher than Baseline Ones

Parameter	Percentage of Firms Solved	Time (in seconds)	Parameter	Percentage of Firms Solved	Time (in seconds)
γ^F_0	97.18%	606	κ^E_l	96.03%	703
γ^F_g	97.25%	479	γ^E_a	91.28%	1256
γ^F_l	95.89%	710	φ^E_a	94.70%	935
γ^F_a	96.21%	628	κ^E_a	96.35%	647
γ^S_0	96.77%	582	γ^N_g	95.67%	795
γ^S_g	96.59%	569	κ^N_g	95.86%	742
γ^S_l	95.80%	719	γ^N_l	95.67%	687
γ^S_a	95.96%	692	κ^N_l	95.83%	689
γ^E_g	93.27%	1119	γ^N_a	95.77%	702
φ^E_g	93.59%	1070	κ^N_a	95.81%	686
κ^E_g	97.33%	479	σ_ν	93.88%	841
γ^E_l	95.52%	790	p	82.29%	2841
φ^E_l	95.65%	749			

Note: The *Percentage of Firms Solved* and *Time* are measured after Step 5 of the algorithm has concluded.

E General Optimization Problem: Mapping to Model and Proofs

E.1 Mapping Between Framework in Appendix A.1 and Model in Section 4

We show in this section that, equating agents to firms and alternatives to potential export destinations, the model described in Section 4 satisfies all restrictions in Assumption 1.

As part of the *first restriction*, equation (A.4) assumes agents maximize the expected infinite-horizon discounted sum of a sequence of static payoffs that exhibit one-period dependence. Equation (A.5) restricts these payoffs to be additively separable across alternatives and, in every alternative j, additively separable in the vector of shocks $\omega(z^t)$ and in the vector of choices in every alternative other than j. Finally, the restriction that the domain of the functions $\hat{\pi}_{jt}$ and $\tilde{\pi}_{jt}$ is finite and that these never equal infinity in their domain implies both $\hat{\pi}_{jt}$ and $\tilde{\pi}_{jt}$ are bounded from above. Additionally, $\tilde{\pi}_{jt}$ is also bounded from below.

Our model satisfies the first restriction in Assumption 1. Specifically, equation (A.4) is satisfied as equation (17) implies firms maximize the infinite-horizon expected discounted sum of static profits. Equation (A.5) is also satisfied as equations (13) to (15) imply that model-implied static profits are

$$\pi_t(y(z^t), y(z^{t-1}), \omega(z^t)) = \sum_{j=1}^{J} \{ y_j(z^t) u_{jt}(y_j(z^{t-1}), \omega_j(z^t)) + \sum_{j'=1}^{J} y_j(z^t) y_{j'}(z^t) c_{jj't} \}, \tag{E.1}$$

where $\omega(z^t)$ equals a vector $(\omega_1(z^t), \ldots, \omega_J(z^t))$, $c_{jj't}$ is defined in equation (9) for $j' \neq j$ (with $c_{jjt} = 0$), and u_{jt} is defined in equation (14). Static profits may thus be written as in equation (A.5) with

$$\begin{aligned} \hat{\pi}_{jt}(y_j(z^t), y_j(z^{t-1}), \omega(z^t)) = {} & y_j(z^t)(\eta^{-1} \exp(\alpha_y y_j(z^{t-1}) + \alpha_{jt} + \alpha_s + \alpha_a \ln(a_{sjt}) + \alpha_r \ln(r_{iht})) \\ & - (g_{jt} + \nu_{ijt} + \omega_j(z^t)) - (1 - y_j(z^{t-1}))s_{jt}), \end{aligned} \tag{E.2a}$$

$$\tilde{\pi}_{jt}(y(z^t), y(z^{t-1})) = \sum_{j'=1}^{J} y_j(z^t) y_{j'}(z^t) c_{jj't}. \tag{E.2b}$$

Finally, these model-implied functions $\hat{\pi}_{jt}$ and $\tilde{\pi}_{jt}$ satisfy the restrictions on their domain and range imposed in Assumption 1. Specifically, as $y_j(z^t) \in \{0,1\}$, $y_j(z^{t-1}) \in \{0,1\}$ and $\omega_j(z^t) \in \{0, \infty\}$ for all j and t, $\hat{\pi}_{jt}$ and $\tilde{\pi}_{jt}$ are bounded from above for any realization of ν_{jt} as long as the parameter space is finite.[20]

The *second restriction* in Assumption 1 imposes the function π_t is supermodular on the sets of choices at $t-1$ and t. As these sets are finite, Corollary 2.6.1 in Topkis (1998) implies one can prove π_t is supermodular by proving it has increasing differences in $y(z^t)$ and $y(z^{t-1})$. For any alternative j and period t, we denote as D_{jt} the change in π_t when changing the value of the choice in j at t, y_{jt}, from zero to one. Given equations (E.1) and (E.2), the expression for D_{jt} in the model described in Section 4 is

$$\begin{aligned} D_{jt} = {} & \eta^{-1} \exp(\alpha_y y_j(z^{t-1}) + \alpha_{jt} + \alpha_s + \alpha_a \ln(a_{sjt}) + \alpha_r \ln(r_{ht})) \\ & - (g_{jt} + \nu_{jt} + \omega_j(z^t)) - (1 - y_j(z^{t-1}))s_{jt} + \sum_{j' \neq j} y_{j'}(z^t)(c_{jj't} + c_{j'jt}). \end{aligned}$$

Since $\alpha_y \geqslant 0$ and $s_{jt} \geqslant 0$ for every j and t, D_{jt} is increasing in $y_j(z^{t-1})$. Since $c_{jj't} \geqslant 0$ for any j, j', and t, D_{jt} is also increasing in $\{y_{j'}(z^t)\}_{j' \neq j}$. Finally, D_{jt} is invariant to $y_{j'}(z^{t-1})$ if $j' \neq j$. Thus, π_t has increasing differences on the sets of export choices at $t-1$ and t and, consequently, π_t is supermodular on these sets. The second restriction in Assumption 1 is thus satisfied by the model described in Section 4.

The *third restriction* in Assumption 1 imposes that there exists a feasible strategy such that, if chosen by the agent, the functions $\{\hat{\pi}_{jt}\}_j$ entering static profits are bounded from below no matter the value of the shock ω_t. In the model in Section 4, not exporting to country j ensures $\hat{\pi}_{jt}$ equals zero; i.e., $\hat{\pi}_{jt}(0, x, \omega) = 0$ for any $x \in \{0,1\}$ and $\omega \in \Omega_t$. Thus, the third restriction in Assumption 1 is satisfied.

The *fourth restriction* imposes Ω_t is finite and the sequence of shocks $\{\omega_{jt}\}_{t \geqslant 0}$ is Markovian. In the

[20] As equation (E.2a) shows, the model-implied function $\hat{\pi}_{jt}$ depends on $\omega(z^t)$ only through a scalar $\omega_j(z^t)$. While this is not relevant for the algorithm's theoretical properties (and, thus, is not imposed in Assumption 1), it is critical for its computational tractability.

model in Section 4, Ω_t includes only two elements and ω_t is independent over time (see equation (11)); thus, this fourth restriction is satisfied.

Finally, the *fifth restriction* imposes that the firm's problem becomes stationary after a terminal period T; i.e., the functions $\{\hat{\pi}_{jt}\}_j$ and $\{\tilde{\pi}_{jt}\}_j$, the distribution of ω_t, and the set Ω_t become constant at T. In the model described in Section 4, Ω_t and the distribution of ω_t are time-invariant, and the functions $\hat{\pi}_{jt}$ and $\tilde{\pi}_{jt}$ become constant at T for every country j. Thus, the fifth restriction in Assumption 1 is satisfied.

E.2 Proof of Theorem 1: Preliminary Results

We prove here two preliminary results that we use in Appendix E.3 as part of the proof of Theorem 1. First, we show that restrictions 1 and 2 in Assumption 1 imply that the solution to the optimization problem in equation (A.6) for any given set of alternatives M_u is increasing in the second argument of the objective function Π_0; i.e., increasing in the upper bounds on the firm's optimal choice in every alternative not in M_u. Second, we show restrictions 1 and 3 to 5 in Assumption 1 imply there exists a solution to the optimization problem in equation (A.6), and that it attains the maximum. Additionally, we provide an algorithm to compute this solution. Finally, as a corollary, we show the solution of the optimization problem in equation (A.2) exists and the maximum is attained.

In our proofs, we use Lemma 2.6.1 and Theorem 2.8.1 in Topkis (1998), which we re-state here.

Lemma E.1 *(Topkis, 1998, Lemma 2.6.1) Suppose X is a lattice. Then,*

1. *If $f(x)$ is supermodular on X and $\alpha > 0$, then $\alpha f(x)$ is supermodular on X.*
2. *If $f(x)$ and $g(x)$ are supermodular on X, then $f(x) + g(x)$ is supermodular on X.*
3. *If $f_k(x)$ is supermodular on X for $k = 1, 2, \ldots$ and $\lim_{k\to\infty} f_k(x) = f(x)$ for each $x \in X$, then $f(x)$ is supermodular on X.*

Theorem E.1 *(Topkis, 1998, Theorem 2.8.1) If X is a lattice, T is a partially ordered set, S_t is a subset of X for each t in T, S_t is increasing in t on T, $f(x,t)$ is supermodular in x on X for each t in T, and $f(x,t)$ has increasing differences in (x,t) on $X \times T$, then $argmax_{x\in S_t} f(x,t)$ is increasing in t on $\{t : t \in T, argmax_{x\in S_t} f(x,t) \text{ is non-empty}\}$.*

E.2.1 First Preliminary Result

We prove here that, for any set of alternatives M_u and iteration n, if it exists, the solution $\bar{\boldsymbol{o}}^{(n)}_{M_u}$ to the optimization problem in equation (A.6) is increasing in the set of upper bounds on alternatives not in M_u; i.e., the solution to the optimization problem in equation (A.6) is increasing in

$$\bar{\boldsymbol{y}}^{(n)}_{M_u^c} = \{\bar{y}^{(n)}_{M_u^c}(z^t)\}^{\infty}_{t=0, z^t\in Z^t}, \qquad \text{with} \qquad \bar{y}^{(n)}_{M_u^c}(z^t) \geqslant o_{M_u^c}(z^t) \text{ for all } t \geqslant 0 \text{ and } z^t \in Z^t.$$

The proof has two steps. First, we show the agent's objective function according to equation (A.2), $\Pi_0(\boldsymbol{y})$, is supermodular in $\boldsymbol{y}$ on Y; see equation (A.1) for the definition of Y. Second, we show this implies that the solution to the optimization problem in equation (A.6) is increasing in the set of upper bounds on alternatives not in M_u.

Lemma E.2 *Assumption 1 implies $\Pi_0(\boldsymbol{y})$ is supermodular in $\boldsymbol{y}$ on Y.*

Proof. The second restriction in Assumption 1 in Appendix A.1 states that, for every period t and every feasible history z^t, $\pi_t(y(z^t), y(z^{t-1}), \omega(z^t))$ is supermodular in $(y(z^t), y(z^{t-1}))$ on $\{0,1\}^J \times \{0,1\}^J$. Define $\breve{\pi}_t(\boldsymbol{y}, z^t) = \pi_t(y(z^t), y(z^{t-1}), \omega(z^t))$, where, as indicated in Appendix A.1, $\boldsymbol{y}$ is a generic vector of agent choices at every history $z^t \in Z^t$ and every period $t \geqslant 0$. Therefore, $\breve{\pi}_t(\cdot)$ is identical to $\pi_t(\cdot)$, but written as a function of the whole vector of choices in every period and feasible history.

First, we show $\breve{\pi}_t(\boldsymbol{y}, z^t)$ is supermodular in $\boldsymbol{y}$. Specifically, we show that, for all $\boldsymbol{y}' \in Y$ and $\boldsymbol{y}'' \in Y$, it holds $\breve{\pi}_t(\boldsymbol{y}', z^t) + \breve{\pi}_t(\boldsymbol{y}'', z^t) \leqslant \breve{\pi}_t(\boldsymbol{y}' \vee \boldsymbol{y}'', z^t) + \breve{\pi}_t(\boldsymbol{y}' \wedge \boldsymbol{y}'', z^t)$, where the "join" $\vee$ takes the maximum element by element, and the "meet" $\wedge$ takes the minimum element by element. To prove this result, note that

$$\begin{aligned}\breve{\pi}_t(\boldsymbol{y}', z^t) + \breve{\pi}_t(\boldsymbol{y}'', z^t) &= \pi_t(y'(z^t), y'(z^{t-1}), \omega(z^t)) + \pi_t(y''(z^t), y''(z^{t-1}), \omega(z^t)) \\ &\leqslant \pi_t(y'(z^t) \vee y''(z^t), y'(z^{t-1}) \vee y''(z^{t-1}), \omega(z^t)) \\ &+ \pi_t(y'(z^t) \wedge y''(z^t), y'(z^{t-1}) \wedge y''(z^{t-1}), \omega(z^t)) \\ &= \breve{\pi}_t(\boldsymbol{y}' \vee \boldsymbol{y}'', z^t) + \breve{\pi}_t(\boldsymbol{y}' \wedge \boldsymbol{y}'', z^t),\end{aligned}$$

where the two equalities follow from the relationship between the functions π_t and $\breve{\pi}_t$, and the inequality follows from the supermodularity of $\pi_t(y'(z^t), y'(z^{t-1}), \omega(z^t))$ in $\{y(z^t), y(z^{t-1})\}$ on $\{0,1\}^J \times \{0,1\}^J$.

Second, we define a function $\Pi_0^\tau(\boldsymbol{y})$ as the expected discounted sum of static profits between periods $t = 0$ and $t = \tau$, and show that the supermodularity of $\breve{\pi}_t(\boldsymbol{y}, z^t)$ in $\boldsymbol{y}$ on Y implies $\Pi_0^\tau(\boldsymbol{y})$ is supermodular in $\boldsymbol{y}$ on Y. As the set Ω_t is finite for every period t (see restriction 4 in Assumption 1), we can write

$$\begin{aligned}\Pi_0^\tau(\boldsymbol{y}) &= \pi_0(y(z^0), 0_J, \omega(z^0)) + \sum_{t=1}^{\tau} \sum_{z^t \in Z^t} \delta^t \pi_t(y(z^t), y(z^{t-1}), \omega(z^t)) \Pr(z^t), \\ &= \breve{\pi}_0(\boldsymbol{y}, z^0) + \sum_{t=1}^{\tau} \sum_{z^t \in Z^t} \delta^t \breve{\pi}_t(\boldsymbol{y}, z^t) \Pr(z^t).\end{aligned}$$

Since $\breve{\pi}_t(\boldsymbol{y}, z^t)$ is supermodular in $\boldsymbol{y}$ on Y for every period t and history z^t, and the finite sum of supermodular functions is supermodular (see part 2 of Lemma E.1), then $\Pi_0^\tau(\boldsymbol{y})$ is supermodular in $\boldsymbol{y}$ on Y.

Finally, noting restriction 1 in Assumption 1 implies $\Pi_0(\boldsymbol{y}) = \lim_{\tau \to \infty} \Pi_0^\tau(\boldsymbol{y})$, we apply part 3 in Lemma E.1 to conclude that the supermodularity of $\Pi_0^\tau(\boldsymbol{y})$ in $\boldsymbol{y}$ on Y implies $\Pi_0(\boldsymbol{y})$ is supermodular in $\boldsymbol{y}$ on Y. ■

Lemma E.3 *Assumption 1 implies that, for every set of alternatives M_u and every iteration n of the algorithm described in Appendix A.2, if the solution to the optimization problem in equation* (A.6) *exists, it is increasing in the export strategy in every alternative not in M_u.*

Proof. This lemma states that, if it exists, $\bar{\boldsymbol{o}}_{M_u}^{(n)}$ is increasing in $\bar{\boldsymbol{y}}_{M_u}^{(n)}$. This lemma is implied by Theorem E.1 and the supermodularity of $\Pi(\boldsymbol{y})$ in $\boldsymbol{y}$ on Y. ■

E.2.2 Second Preliminary Result

We prove here that, for every subset of alternatives M_u and iteration n, the solution $\bar{\boldsymbol{o}}_{M_u}^{(n)}$ to the optimization problem in equation (A.6) exists and the maximum is attained. Specifically, Lemma E.4 below establishes the existence of the solution to the problem in equation (A.6), and that the maximum is attained, for every $t \geqslant T$; that, is, for all periods after the terminal period T, when the problem of the firm becomes stationary according to the restriction 5 in Assumption 1. Given Lemma E.4, establishing the existence of the solution to the problem in equation (A.6), and that the maximum is attained, for every $0 \leqslant t < T$ is straightforward by backward induction, as there are a finite number of feasible choices.

For any set of alternatives M_u and any vector $\bar{b}_{M_u^c} \in \{0,1\}^{J-J_u}$, we define the firm's expected discounted sum of static payoffs at T conditional on setting $\bar{y}_{M_u^c}(z^t) = \bar{b}_{M_u^c}$ for all $t \geqslant T$ and all $z^t \in Z^t$ as

$$\begin{aligned}\Pi_T\big(\boldsymbol{y}_{M_u}, \bar{b}_{M_u^c}, y(z^{T-1}), \omega(z^T)\big) &= \pi_T\big((y_{M_u}(z^T), \bar{b}_{M_u^c}), (y_{M_u}(z^{T-1}), y_{M_u^c}(z^{T-1})), \omega(z^T)\big) \\ &+ \sum_{t=T+1}^{\infty} \delta^{t-T} \mathbb{E}_T\Big[\pi_T\big((y_{M_u}(z^t), \bar{b}_{M_u^c}), (y_{M_u}(z^{t-1}), \bar{b}_{M_u^c}), \omega(z^t)\big)\Big],\end{aligned}$$

where $\pi_T(\cdot)$ equals the payoff function in equation (A.5) for $t = T$, $y(z^{T-1}) = (y_{M_u}(z^{T-1}), y_{M_u^c}(z^{T-1}))$, and $\boldsymbol{y}_{M_u}$ includes a generic set of choices for all alternatives in M_u, all $t \geqslant T$, and all $z^t \in Z^t$. We can then define the period-T value function

$$V_{TM_u}\big(\bar{b}_{M_u^c}, y(z^{T-1}), \omega(z^T)\big) = \sup_{\boldsymbol{y}_{M_u}} \Pi_T\big(\boldsymbol{y}_{M_u}, \bar{b}_{M_u^c}, y(z^{T-1}), \omega(z^T)\big). \tag{E.3}$$

Lemma E.4 *For any set of alternatives M_u and any vector $\bar{b}_{M_u^c} \in \{0,1\}^{J-J_u}$, Assumption 1 implies the solution to the problem in equation* (E.3) *exists and the maximum is attained.*

Proof. For any set of alternatives M_u and any vector $\bar{b}_{M_u^c} \in \{0,1\}^{J-J_u}$, we define the payoff function

$$\check{\Pi}_T\big(\boldsymbol{y}_{M_u}, \bar{b}_{M_u^c}, y(z^{T-1}), \omega(z^T)\big) =$$

$$\sum_{j \in M_u} \hat{\pi}_{jT}\big(y_j(z^T), y_j(z^{T-1}), \omega(z^T)\big) + \sum_{j=1}^{J} \tilde{\pi}_{jT}\big((y_{M_u}(z^T), \bar{b}_{M_u^c}), (y_{M_u}(z^{T-1}), y_{M_u^c}(z^{T-1}))\big) +$$

$$\sum_{t=T+1}^{\infty} \delta^{t-T} \mathbb{E}_T\Big[\sum_{j \in M_u} \hat{\pi}_{jT}\big(y_j(z^t), y_j(z^{t-1}), \omega(z^t)\big) + \sum_{j=1}^{J} \tilde{\pi}_{jT}\big((y_{M_u}(z^t), \bar{b}_{M_u^c}), (y_{M_u}(z^{t-1}), \bar{b}_{M_u^c})\big)\Big],$$

and the associated value function

$$\check{V}_{TM_u}\big(\bar{b}_{M_u^c}, y(z^{T-1}), \omega(z^T)\big) = \sup_{\boldsymbol{y}_{M_u}} \check{\Pi}_T\big(\boldsymbol{y}_{M_u}, \bar{b}_{M_u^c}, y(z^{T-1}), \omega(z^T)\big). \tag{E.4}$$

The function $\check{\Pi}_T(\cdot)$ differs from $\Pi_T(\cdot)$ in that $\check{\Pi}_T(\cdot)$ only includes those terms entering $\Pi_T(\cdot)$ that depend on $\boldsymbol{y}_{M_u}$. Thus, $\check{\Pi}_T(\cdot)$ and $\Pi_T(\cdot)$ differ in a term that is invariant to the choice of $\boldsymbol{y}_{M_u}$ and, consequently, a vector $\boldsymbol{y}_{M_u}$ will solve the optimization problem in equation (E.4) if and only if it also solves the optimization problem in equation (E.3).

Restriction 1 in Assumption 1 implies the functions $\hat{\pi}_{jT}(\cdot)$ and $\tilde{\pi}_{jT}(\cdot)$ are bounded from above. As $\delta < 1$, we can then conclude that the value function $\check{V}_{TM_u}(\cdot)$ in equation (E.4) is bounded from above. Restriction 3 in Assumption 1 implies there is a feasible value of the choice vector $\boldsymbol{y}_{M_u}$ such that $\hat{\pi}_{jT}(\cdot)$ is bounded from below for all $j \in M_u$. As restriction 1 in Assumption 1 also implies that the function $\tilde{\pi}_{jT}(\cdot)$ is bounded from below, we can then conclude that the value function $\check{V}_{TM_u}(\cdot)$ in equation (E.4) is bounded from below. In sum, restrictions 1 and 3 in Assumption 1 imply that $\check{V}_{TM_u}(\cdot)$ is bounded from above and from below.

Theorem 4.2 in Stokey et al. (1989) implies we can write $\check{V}_{TM_u}(\cdot)$ as the solution to the following functional equation,

$$\check{V}_{TM_u}(\bar{b}_{M_u^c}, (y_{M_u}, y_{M_u^c}), \omega) =$$

$$\sup_{y'_{M_u}} \Big\{ \sum_{j \in M_u} \hat{\pi}_{jT}(y'_j, y_j, \omega) + \sum_{j=1}^{J} \tilde{\pi}_{jT}\big((y'_{M_u}, \bar{b}_{M_u^c}), (y_{M_u}, y_{M_u^c})\big) + \delta \mathbb{E}\big[\check{V}_{TM_u}(\bar{b}_{M_u^c}, (y'_{M_u}, \bar{b}_{M_u^c}), \omega)\big]\Big\} \tag{E.5}$$

Since $\check{V}_{TM_u}(\cdot)$ is bounded from above and from below, equation (E.5) maps bounded functions into bounded functions. Additionally, it also satisfies the monotonicity and discounting properties of Blackwell's sufficient conditions for a contraction of modulus δ. Therefore, there is a unique bounded function $\check{V}_{TM_u}(\cdot)$ that solves the problem in equation (E.5); see Theorem 3.3 in Stokey et al. (1989). Since the solution to the problem in equation (E.5) is unique, then it must also be a solution to the sequence problem in equation (E.4). Furthermore, as the solution to the sequence problems in equations (E.3) and (E.4) coincide, we can conclude that the solution to the optimization problem in equation (E.3) exists. Finally, as the choice variable y'_{M_u} in equation (E.5) may only take finitely many values, the maximum is attained. ∎

Lemma E.5 *Assumption 1 implies the solution to the problem in equation* (A.2) *exists and the maximum is attained.*

Proof. It is an implication of Lemma E.4 when applied to the specific set M_u that includes all possible alternatives; i.e., $M_u = \{1, \ldots, J\}$. ∎

E.3 Proof of Theorem 1

E.3.1 Proof of Part 1 of Theorem 1

We prove part 1 of Theorem 1 by induction.

As the *base case*, note that, according to equation (A.8), $\bar{b}_{jt}^{(1)} = 1$ for all $j = 1, \ldots, J$ and, therefore,

$$\bar{b}_{jt}^{(n)} \geqslant o_j(z^t) \qquad \text{for } n = 1,\ j = 1, \ldots, J,\ t \geqslant 0, \text{ and } z^t \in Z^t.$$

As the *step case*, suppose that, for some arbitrary n, $\bar{b}_{jt}^{(n)} \geqslant o_j(z^t)$ for all $j = 1, \ldots, J$, $t \geqslant 0$, and $z^t \in Z^t$. For any group of alternatives M_u, denote as

$$\bar{\boldsymbol{b}}_{M_u}^{(n)}$$

the vector that assigns the value of $\bar{b}_{jt}^{(n)}$ to every alternative j in M_u, every $t \geqslant 0$, and every $z^t \in Z^t$; i.e.,

$$\bar{\boldsymbol{b}}_{M_u}^{(n)} = \{\bar{y}_j^{(n)}(z^t)\}_{t=0, z^t \in Z^t, j \in M_u}^{\infty}, \quad \text{with} \quad \bar{y}_j^{(n)}(z^t) = \bar{b}_{jt}^{(n)} \text{ for all } t \geqslant 0, \text{ all } j \in M_u, \text{ and all } z^t \in Z^t.$$

Thus, $\bar{\boldsymbol{b}}_{M_u}^{(n)} \geqslant \boldsymbol{o}_{M_u}$, where $\boldsymbol{o}_{M_u}$ is the vector containing the agent's optimal choice for every $j \in M_u$, every $t \geqslant 0$, and every $z^t \in Z^t$. For any alternative j and period t, equations (A.6) and (A.9) further imply that

$$\bar{b}_{jt}^{(n+1)} = \max_{z^t \in Z^t} \bar{o}_j^{(n)}(z^t),$$

where, for a set M_u including alternative j, $\bar{o}_j^{(n)}(z^t)$ is the corresponding element of $\bar{\boldsymbol{o}}_{M_u}^{(n)}$, defined as

$$\bar{\boldsymbol{o}}_{M_u}^{(n)} = \underset{\boldsymbol{y}_{M_u} \in Y_{M_u}}{\operatorname{argmax}}\ \Pi_0(\boldsymbol{y}_{M_u}, \bar{\boldsymbol{b}}_{M_u^c}^{(n)}).$$

To prove that $\bar{b}_{jt}^{(n+1)} \geqslant o_j(z^t)$ for all $j = 1, \ldots, J$, $t \geqslant 0$, and $z^t \in Z^t$, it is thus enough to prove that

$$\bar{\boldsymbol{o}}_{M_u}^{(n)} \geqslant \boldsymbol{o}_{M_u}. \tag{E.6}$$

For any group of destinations M_u, we can write $\boldsymbol{o}_{M_u}$ as

$$\boldsymbol{o}_{M_u} = \underset{\boldsymbol{y}_{M_u} \in Y_{M_u}}{\operatorname{argmax}}\ \Pi_0(\boldsymbol{y}_{M_u}, \boldsymbol{o}_{M_u^c}). \tag{E.7}$$

Lemma E.4 implies $\bar{\boldsymbol{o}}_{M_u}^{(n)}$ and $\boldsymbol{o}_{M_u}$ exist, and Lemma E.3 implies $\bar{\boldsymbol{o}}_{M_u}^{(n)} \geqslant \boldsymbol{o}_{M_u}$. Thus, it holds that

$$\bar{b}_{jt}^{(n+1)} \geqslant o_j(z^t),$$

for all $j = 1, \ldots, J$, $t \geqslant 0$, and $z^t \in Z^t$. ■

E.3.2 Proof of Part 2 of Theorem 1

We prove part 2 of Theorem 1 by induction.

As *base case*, note that equation (A.8) implies $\bar{b}_{jt}^{(1)} = 1$ for every alternative j and period t. As, naturally,

$$\bar{o}_j^{(1)}(z^t) \in \{0, 1\}$$

for every alternative j, period $t \geqslant 0$, and history $z^t \in Z^t$, it must be the case that $\bar{b}_{jt}^{(2)}$, defined according to equation (A.9), is also either 0 or 1 for every alternative j and period t. Consequently,

$$\bar{b}_{jt}^{(2)} \leqslant \bar{b}_{jt}^{(1)}, \qquad \text{for all } j = 1, \ldots, J \text{ and } t \geqslant 0.$$

As the *step case*, suppose that, for some arbitrary n, $\bar{b}_{jt}^{(n)} \leqslant \bar{b}_{jt}^{(n-1)}$ for all $j = 1, \ldots, J$ and $t \geqslant 0$. Given the definition of $\bar{\boldsymbol{y}}_{M_u}^{(n)}$ in equation (A.7), it is then the case that, for any set of alternatives M_u, it holds that

$$\bar{\boldsymbol{y}}_{M_u}^{(n)} \leqslant \bar{\boldsymbol{y}}_{M_u}^{(n-1)}. \tag{E.8}$$

Given the definition of $\bar{\boldsymbol{o}}_{M_u}^{(n)}$ in equation (A.6), Lemma E.4 guarantees $\bar{\boldsymbol{o}}_{M_u}^{(n)}$ and $\bar{\boldsymbol{o}}_{M_u}^{(n-1)}$ exist. Given equations (A.6) and (E.8), Lemma E.3 implies that

$$\bar{\boldsymbol{o}}_{M_u}^{(n)} \leqslant \bar{\boldsymbol{o}}_{M_u}^{(n-1)}.$$

Since, according to equation (A.9), $\bar{b}_{jt}^{(n+1)} = \max_{z^t \in Z^t} \bar{o}_j^{(n)}(z^t)$ for every t, j, and z^t, it then holds that

$$\bar{b}_{jt}^{(n+1)} \leqslant \bar{b}_{jt}^{(n)},$$

for all $j = 1, \ldots, J$, $t \geqslant 0$, and $z^t \in Z^t$. ■

E.3.3 Proof of Part 3 of Theorem 1

As shown in the proof of Lemma E.4, Assumption 1 implies that, for any arbitrary iteration n, $\bar{b}_{jt}^{(n)} = \bar{b}_{jT}^{(n)}$ for every alternative j and period $t \geqslant T$; this is a consequence of the agent's optimization problem becoming stationary after period T. Therefore, we can summarize the infinite set of upper bounds

$$\{\bar{b}_{jt}^{(n)}\}_{j=1,t\geqslant T}^{J}$$

in a vector that belongs to the set $\{0,1\}^J$; i.e., in a vector with a finite number of coordinates. For every period $t < T$ and an arbitrary iteration n, it is the case that

$$\bar{b}_{jt}^{(n)} \in \{0,1\}^J.$$

Therefore, for any arbitrary iteration, computing the full set of upper-bounds $\{\bar{b}_{jt}^{(n)}\}_{j=1,t\geqslant 0}^{J}$ implies computing the value of $(T+1)J$ unknowns, each of whom may equal either 0 or 1.

Part 2 of Theorem 1 indicates that, at every iteration n, the value of each of these upper bounds either decreases or remains constant. As there is a finite number $(T+1)J$ of upper bounds to solve for at each iteration n, and each of these upper bounds may equal either 0 or 1 (i.e., they are bounded from below by 0), it must then be the case that these bounds converge in a finite number of steps.

F Estimation: Additional Details

F.1 Identification of Cross-Country Complementarities: Details

Consider a simplified version of the model described in Section 4 in which we impose the following restrictions. First, there are only three foreign countries. Second, in terms of the parameters entering the expression for potential export revenues in equation (5), assume that $\alpha_y = \alpha_a = \alpha_r = 0$, $\alpha_s = 0$ for every s, and, for every period t, $\alpha_{1t} = \alpha_{2t} = \bar{\alpha} = 1.05$ and $\alpha_{3t} = \alpha_3 = 1.15$. Third, in terms of the fixed export costs determined in equations (6) to (11), assume that

$$\begin{aligned} f_{i1t} &= \gamma_0^F + \nu_{i1t} + \omega_{i1t}, \\ f_{i2t} &= \gamma_0^F - y_{i3t}\bar{c} + \nu_{i2t} + \omega_{i2t}, \\ f_{i3t} &= \gamma_0^F - y_{i2t}\bar{c} + \nu_{i3t} + \omega_{i3t}, \end{aligned}$$

with $\gamma_0^F = 80$, ν_{ijt} drawn according to the distribution in equation (10) with $\sigma_\nu = 80$ and, for every t,

$$\rho_{12t} = \rho_{13t} = 0, \qquad \text{and} \qquad \rho_{23t} = \bar{\rho},$$

and ω_{ijt} drawn according to the distribution in equation (11) with $p = 0.7$. Fourth, in terms of the sunk export cost determined in equation (12), assume that, for every $j \in \{1, 2, 3\}$ and period t, $s_{jt} = \gamma_s^0 = 120$.

In this simplified framework, we first show how the values of the moment functions $\mathbb{m}_1$ and $\mathbb{m}_2$ in equation (27) change as we change the value of the parameter determining the strength of the complementarities between countries 2 and 3 (i.e., $\bar{c}$), and the correlation coefficient in ν_{ijt} between countries 2 and 3 (i.e., $\bar{\rho}$). With that goal in mind, for any given value of $(\alpha_3, \bar{c}, \bar{\rho})$, we simulate the model for 500 simulations of each of the 4,709 firms in our sample, set terminal period $T = 120$ and, to obtain results robust to initial conditions and the terminal period, compute $\mathbb{m}_1$ and $\mathbb{m}_2$ using the information on y_{ijt} only for periods $50 \leqslant t \leqslant 64$.

In Table F.1, we set $\alpha_3 = \bar{\alpha}$ and compute $\mathbb{m}_1$ and $\mathbb{m}_2$ for four different values of $(\bar{c}, \bar{\rho})$. In the first row, we set $(\bar{c}, \bar{\rho}) = (0, 0)$, obtaining in this case that $\mathbb{m}_1 = \mathbb{m}_2 = 0$. Intuitively, as countries 1 and 2 are identical in every respect except in their potential export complementarities with country 3, export probabilities in both countries must be equal when the parameter that determines the strength of those complementarities is set to 0; i.e., when $\bar{c} = 0$. Similarly, as all firms are identical in every respect except in the fixed cost unobserved terms ν_{ijt} and ω_{ijt}, the within-firm covariance in export choices in countries 2 and 3 will equal zero when the parameter determining the potential correlation in these unobserved terms for these two countries equals zero; i.e., when $\bar{\rho} = 0$.

In the second row in Table F.1, we introduce complementarities between countries 2 and 3 by setting $\bar{c} = 30$. These complementarities increase the export probability in country 2 (and in country 3, although this is not relevant for Table F.1), while they do not affect the export probability in country 1 (as country 1 is isolated from any other potential export destination); therefore, $\mathbb{m}_1$ increases as $\bar{c}$ increases. As $\bar{c} > 0$, firms enjoy a reduction in fixed costs in country 2 if and only if they export in the same period to country 3 (and vice versa); therefore, an increase in the strength of the export complementarities, as determined by the value of $\bar{c}$, makes firms more likely to simultaneously export to countries 2 and 3 and, consequently, $\mathbb{m}_2$ also increases as $\bar{c}$ increases.

In the third row in Table F.1, we set the value of $\bar{c}$ back to zero (as in the first row) but introduce a positive correlation in ν_{ijt} between countries 2 and 3; i.e., $\bar{\rho} = 0.8$. When there are no cross-country complementarities, the within-firm positive correlation in fixed export costs in countries 2 and 3 does not affect the (marginal) export probability in any country; thus, $\mathbb{m}_1$ does not depend on the value of $\bar{\rho}$ when $\bar{c} = 0$. However, the within-firm positive correlation in fixed costs in countries 2 and 3 affects the probability that firms export simultaneously to those two countries, which increases; thus, $\mathbb{m}_2$ increases in the value of $\bar{\rho}$ when $\bar{c} = 0$.

In the fourth row in Table F.1, we set both $\bar{c}$ and $\bar{\rho}$ to positive values. When comparing the results in the second and fourth rows, we observe that introducing a positive correlation in ν_{ijt} between countries 2 and 3 in a baseline setting with cross-country complementarities (i.e., in a baseline setting with $\bar{c} > 0$) affects not only the joint probability that firms export simultaneously to countries 2 and 3 (i.e., the value of $\mathbb{m}_2$) but also the difference in the export probabilities between countries 2 and 3 (i.e., the value of $\mathbb{m}_1$).

In unreported results, we observe that the patterns in Table F.1 hold generally as we change the values of $\bar{c}$ and $\bar{\rho}$ between different numbers and as we set the size of country 3, α_3, to different values.

Table F.1: Impact of Complementarities and Correlation in Unobservables on Moment Conditions

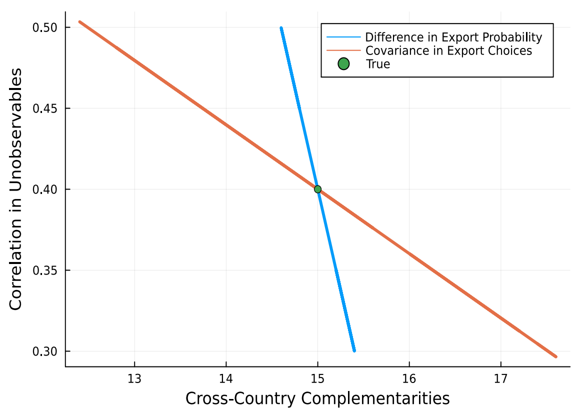
Parameters		Moments	
$\bar{c}$	$\bar{\rho}$	$\mathbb{E}[y_{i2t} - y_{i1t}]$	$\mathbb{C}[y_{i2t}, y_{i3t}]$
0	0	0	0
Positive	0	0.15	0.05
0	Positive	0	0.02
Positive	Positive	0.17	0.07

Note: by the label "Positive" in the first column, we denote cases in which $\bar{c} = 30$. By the label "Positive" in the second column, we denote cases in which $\bar{\rho} = 0.8$.

In Figure F.1, we perform a different exercise that more directly illustrates the capacity of moments $\mathbb{m}_1$ and $\mathbb{m}_2$ to identify the parameters $\bar{c}$ and $\bar{\rho}$. We simulate data from a "true" model in which we set $\alpha_3 = \bar{\alpha}$, $\bar{c} = 15$, and $\bar{\rho} = 0.4$, and we then compare how the values of moments $\mathbb{m}_1$ and $\mathbb{m}_2$ corresponding to the "true" model compare to those generated under alternative values of $\bar{c}$ and $\bar{\rho}$. More specifically, the green dot represents the true values of $\bar{c}$ and $\bar{\rho}$, and the blue and orange lines represent all values of $(\bar{c}, \bar{\rho})$ for which $\mathbb{m}_1$ and $\mathbb{m}_2$, respectively, equal their respective values in the "true" model. The slope of the orange line, e.g., shows we can keep moment $\mathbb{m}_2$ at its true value as we increase the value of the parameter $\bar{\rho}$ if we simultaneously decrease the value of the parameter $\bar{c}$. The blue line indicates the same is true for moment $\mathbb{m}_1$. Thus, neither moment alone allows to identify the parameter vector $(\bar{\rho}, \bar{c})$, but the fact that the orange and blue lines have different slopes implies that both moments jointly identify $(\bar{\rho}, \bar{c})$.

In unreported results, we observe that the patterns in Figure F.1 hold generally as we change the true values of $\bar{c}$ and $\bar{\rho}$ and as we set the size of country 3, α_3, to different values.

Figure F.1: Impact of Complementarities and Correlation in Unobservables on Moment Conditions



Notes: The axis labeled "Correlation in Unobservables" includes values of the parameter $\bar{\rho}$. The axis labeled "Cross-country Complementarities" includes values of the parameter $\bar{c}$. The green dot represents the true values of the parameters $\bar{c}$ and $\bar{\rho}$; i.e., $(\bar{c}, \bar{\rho}) = (15, 0.4)$. The blue and orange lines represent all values of $(\bar{c}, \bar{\rho})$ for which $\mathbb{m}_1$ and $\mathbb{m}_2$, respectively, equal their respective values in the "true" model.

F.2 Export Potential Measures

We define export potential in Appendix F.2.1. In Appendix F.2.2, we present summary statistics on the gravity equation estimates used to compute these export potentials, on the resulting export potential measures, and on the aggregate export potential of the countries geographically or linguistically close to each destination j, or that share a deep PTA with it. In Appendix F.2.3, we present reduced-form evidence showing firm export choices in a destination correlate with the aggregate export potential of the other countries that are geographically or linguistically close to it, or that share a deep PTA with it.

F.2.1 Definiton and Estimation of Export Potential Measures

We use country-to-country sector-specific trade flows, and the distance measures in Section 2, to compute measures of the *export potential* of Costa Rica in each sector, destination and year.[21] Specifically, we first compute Poisson pseudo-maximum-likelihood estimates of the parameters of the gravity equation

$$X^s_{odt} = \exp(\Psi^s_{ot} + \Xi^s_{dt} + \lambda^s_g n^g_{od} + \lambda^s_l n^l_{od} + \lambda^s_a n^a_{odt}) + u^s_{odt}, \tag{F.1}$$

where X^s_{odt} denotes the export volume from origin o to destination d in sector s and year t; Ψ^s_{ot} and Ξ^s_{dt} are sector-origin-year and sector-destination-year unobserved effects, respectively; n^g_{od}, n^l_{od}, and n^a_{odt} are the distance measures described in Section 2; λ^s_g, λ^s_l, and λ^s_l are sector-specific parameters; and u^s_{odt} is an unobserved term. Denoting parameter estimates with a hat, we measure Costa Rica's export potential in a sector s, destination j, and year t as

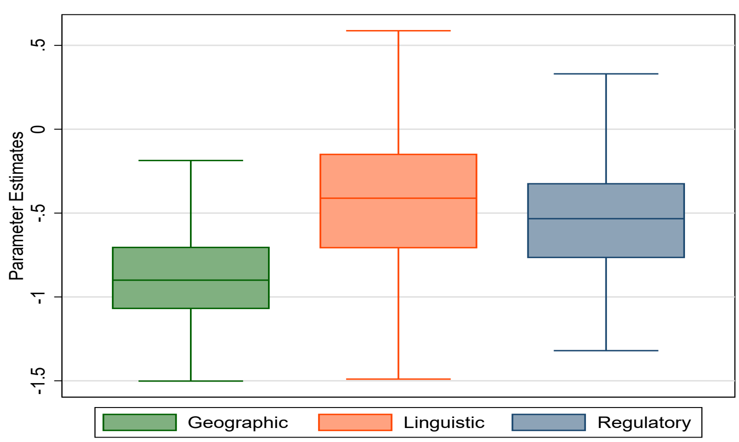
$$E^s_{jt} = \exp(\hat{\Xi}^s_{jt} + \hat{\lambda}^s_g n^g_{hj} + \hat{\lambda}^s_l n^l_{hj} + \hat{\lambda}^s_a n^a_{hjt}), \tag{F.2}$$

where n^g_{hj}, n^l_{hj}, and n^a_{hjt} denote distances between Costa Rica and country j.[22]

F.2.2 Gravity-Equation Estimates and Export Potential Measures: Statistics

In Figure F.2, we include boxplots summarizing the distribution across sectors of the parameter estimates $\hat{\lambda}^s_g$ (in green), $\hat{\lambda}^s_l$ (in orange), and $\hat{\lambda}^s_a$ (in blue). The estimates of λ^s_g are negative for all sectors and centered around -1. The estimates of λ^s_l and λ^s_a are also nearly always negative, although they tend to be smaller in absolute value than the estimates of λ^s_g.

Figure F.2: Estimates of Gravity Equation Parameters



Notes: These boxplots represent the distribution of $\hat{\lambda}^s_g$ (geographic), $\hat{\lambda}^s_l$ (linguistic) and $\hat{\lambda}^s_a$ (regulatory) across sectors.

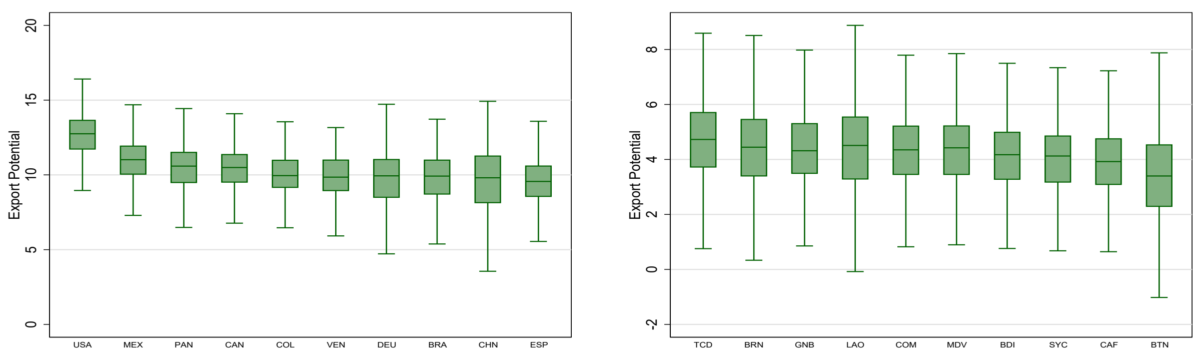
[21] The BACI data by CEPII reports country-to-country trade flows at the HS-6 level; see Gaulier and Zignago (2010) for details. Using a concordance provided by WITS (`https://wits.worldbank.org/product_concordance.html`), we aggregate this product-level data to generate sector-level flows, with sectors defined at the four-digit level according to ISIC Rev. 3. We use a concordance provided by UNSD (`https://unstats.un.org/unsd/classifications/Econ/ISIC.cshtml`) to further convert the data to four-digit sectors defined according to the ISIC Rev. 4.

[22] In estimating equation (F.1), we exclude all observations in which Costa Rica is the origin or destination country.

In Figure F.3, we present boxplots summarizing the distribution across sectors and years of the export potential measures E^s_{jt} for the ten destination countries with the largest (in Figure F.3a) and smallest (in Figure F.3b) mean export potentials. The US is the country with the largest mean value of E^s_{jt}. The distribution of E^s_{jt} for the US is actually distinctively different from that corresponding to all other destinations, with the first quartile of the distribution for the US being similar to the third quartile of the distribution of export potentials in Mexico, which is the country with the second largest mean export potential. Other destinations with large mean export potentials are countries that are geographically or linguistically close to Costa Rica (e.g., Panama, Colombia, Venezuela, Spain), or countries that are large importers (e.g., Canada, Germany, Brazil, China). As Figure F.3b shows, the ten destination countries with the smallest mean export potentials (e.g., Bhutan, the Central African Republic, Seychelles, or Burundi) are all small, distant from Costa Rica geographically and linguistically, and do not share any PTA with Costa Rica.

Figure F.3: Export Potential - Distributions by Country for Top 10 Destinations

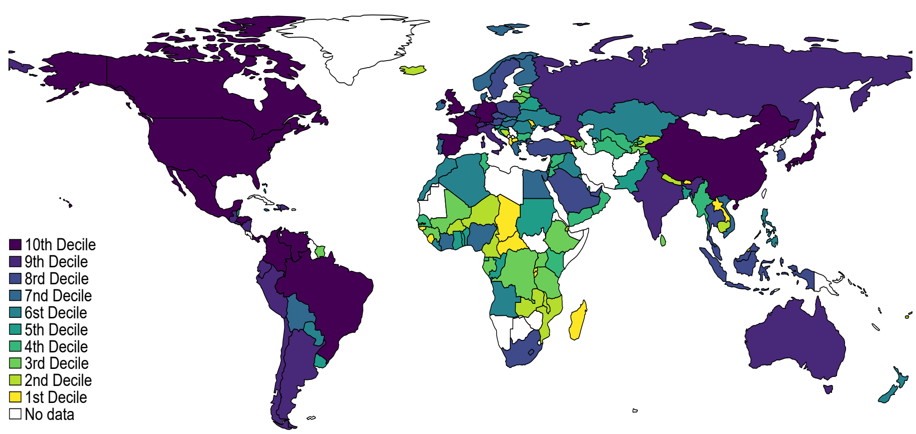
(a) Top 10 Destinations (b) Bottom 10 Destinations



Notes: These boxplots summarize the distribution of E^s_{jt} (see equation (F.2)) for the 10 destination countries with the largest (Figure F.3a) and smallest (Figure F.3b) mean export potentials, where the mean is computed across sectors and years in the period 2005-2015. Countries are listed according to their alpha-3 ISO code.

In Figure F.4, we show a map displaying, for each country j, the mean value of E^s_{jt} across the sectors and years in the sample. Most countries in North, Central, and South America, and in Europe, are in the top three deciles. Also in the top three deciles are Australia, Russia, China and India. On the contrary, most countries in Africa, several in South Asia, and the former Soviet republics are in the bottom deciles.

Figure F.4: Mean Export Potential by Destination Country



Notes: Map of the mean (across sectors and years in the period 2005-2015) E^s_{jt} by country.

In terms of the distribution of export potentials E^s_{jt} across sectors, we find that the five sectors with the largest mean export potentials are the manufacturing of pharmaceuticals, medicinal chemical and botanical products (sector 2100 according to the 4-digit ISIC Rev. 4), the building of ships and floating structures (sector 3011), the manufacturing of computers and peripheral equipment (sector 2620), the manufacturing of motor vehicles (sector 2910), and the manufacturing of basic iron and steel (sector 2410). The manufacturing of plastic products (sector 2220), which is one of the top sectors by aggregate volume of exports from Costa Rica during the period 2005-2015, is also in the top 10 of sectors by their mean export potential.

For each foreign country j, sector s, and year t, we use the export potential measures E^s_{jt} of countries other than j to construct the aggregate export potential of the countries geographically or linguistically close to j, or that share a deep PTA with it. Denoting the aggregate export potential of the countries that, e.g., are geographically close to a destination j as $AE^s_{jt,g}$, we compute it as the sum of the sector- and year-specific export potentials of all countries whose geographical distance to j is smaller than some threshold $\bar{n}_g$:

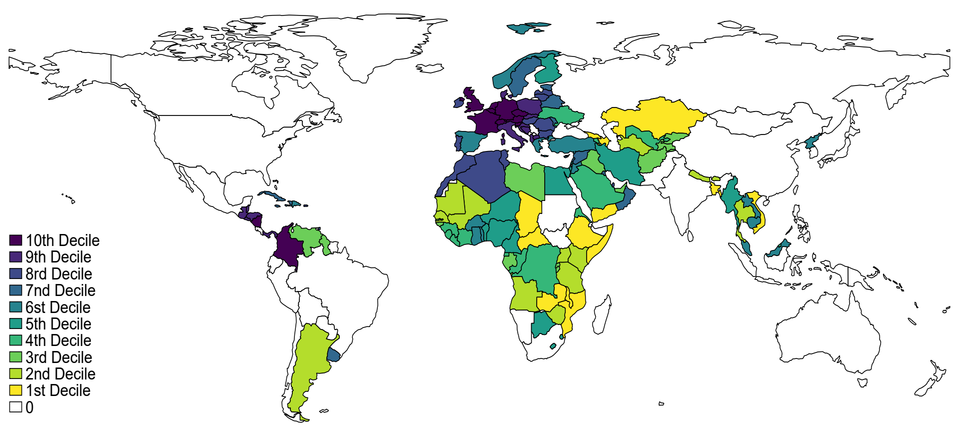
$$AE^s_{jt,g} = \sum_{j' \neq j} \mathbb{1}\{n^g_{jj'} \leqslant \bar{n}_g\} E^s_{j't}. \tag{F.3}$$

We build similar measures for countries linguistically close to j, or cosignatories of a deep PTA with j, denoted respectively as $AE^s_{jt,l}$ and $AE^s_{jt,a}$. We use as thresholds $\bar{n}_g = 0.79$ (790km), $\bar{n}_l = 0.11$, or $\bar{n}_a = 0.43$.

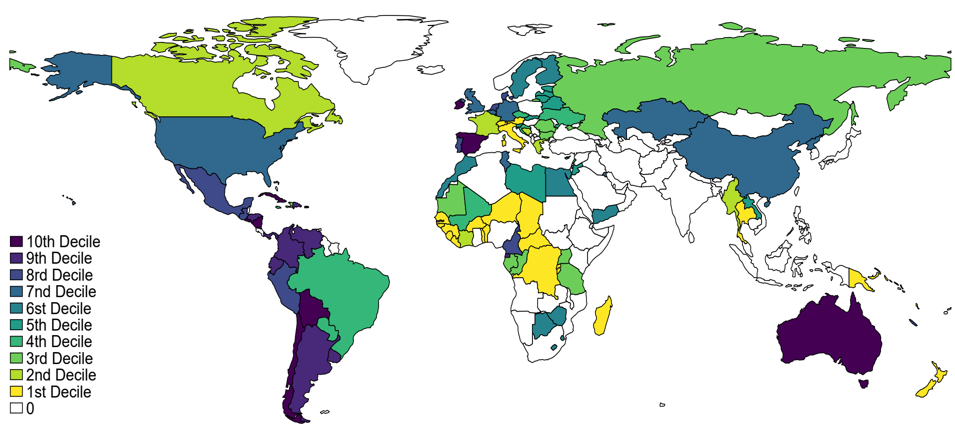
We describe in Figure F.5 the mean (across sectors and years in the period 2005-2015) value of $AE^s_{jt,g}$ (in Panel (a)), $AE^s_{jt,l}$ (in Panel (b)), and $AE^s_{jt,a}$ (in Panel (c)), for every destination in the sample.

Figure F.5: Aggregate Export Potential Measures

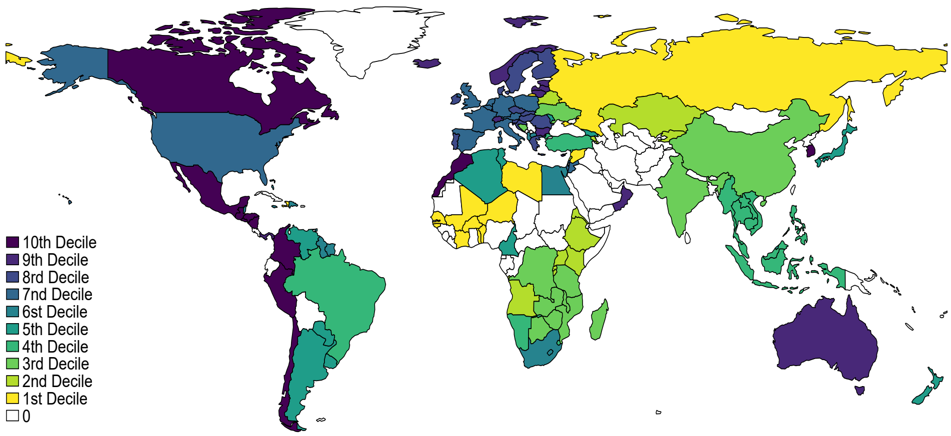
(a) Based on Geographic Distance



(b) Based on Linguistic Distance



(c) Based on Common Membership in a Deep PTA



Notes: Each panel in this figure displays the mean (across sectors and sample years) for each destination country of the aggregate export potential measures $AE^s_{jt,g}$ (in Panel (a)), $AE^s_{jt,l}$ (in Panel (b)), and $AE^s_{jt,a}$ (in Panel (c)).

As discussed in Section 2, our measure of the geographical distance between any two countries j and j' is a weighted average of the distances between cities located in j and j' and, thus, large countries tend to be geographically isolated. This explains why the US, Canada, Russia, or China have a zero value of the aggregate export potential measure $AE^s_{jt,g}$; these countries have no other country such that their bilateral geographic distance $n^g_{jj'}$ is below the threshold $\bar{n}_g = 790$ km we use in this figure to classify two countries as neighbors. Conversely, as illustrated in Figure F.5a, countries located in Central America and in Central Europe have many geographic neighbors with relatively large export potentials and, thus, their value of $AE^s_{jt,g}$ is large. The aggregate export potential of geographic neighbors is smaller for countries in Africa (which tend to have many neighbors, but small in terms of their own export potential).

The map in Figure F.5b shows that countries with a large share of Spanish speakers (e.g., Spain and several countries in South and Central America) and countries with a large share of English speakers (e.g., countries such as Australia and the UK in which English is an official language, but also countries in which English is not an official language such as Germany or Denmark) exhibit large values of $AE^s_{jt,l}$.

Finally, Figure F.5c shows that countries belonging to the EU, NAFTA or CAFTA, and countries that have deep PTA with one or more of these blocs (e.g., Morocco and Australia) have large values of $AE^s_{jt,a}$.

F.2.3 Correlation Between Export Potential Measures and Firms' Export Choices

As illustrated in Section 6.2.1, if geographical or linguistic proximity, or common participation in a deep PTA, are a source of cross-country complementarities in export participation decisions, a firm's export probability in a country j and year t will, all else equal, increase in the aggregate market size of the countries geographically or linguistically close to j, or that share a deep PTA with it. To test this implication, we use the aggregate export potential measures introduced above as proxy for the aggregate market size of the countries close to j, and compute OLS estimates of a regression of a dummy variable that equals one if firm i exports to country j in year t on flexible functions of country j's log export potential (introduced only as a control variable) and the log of the aggregate export potential of the countries geographically or linguistically close to j, or that share a deep PTA with it. Specifically, given the estimating equation

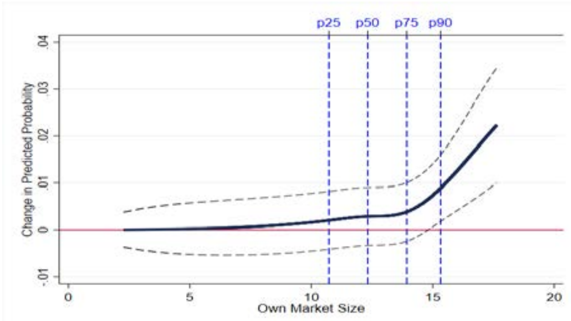
$$y_{ijt} = h_o(\ln(E_{sjt})) + \sum_{x=\{g,l,a\}} \mathbb{1}\{AE^s_{jt,x} > 0\} h_x(\ln(AE^s_{jt,x})) + \beta_{it} + u_{ijt}, \tag{F.4}$$

where $h_x(\cdot)$ for $x = \{o, g, l, a\}$ are cubic splines, and β_{it} is a firm-year fixed effect, panels (a) to (d) in Figure F.6 respectively show OLS estimates of the functions $h_o(\cdot)$, $h_g(\cdot)$, $h_l(\cdot)$, and $h_a(\cdot)$.

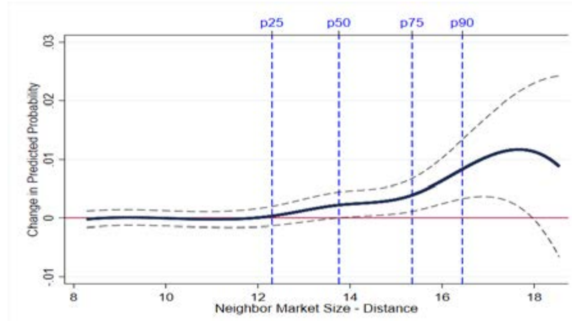
The estimates in Figure F.6 imply that the effect of a country's own export potential as well as the effect of the aggregate export potential of a country's neighbors is highly non-linear, with effects being generally

Figure F.6: Impact of Own and Neighbors' Export Potential

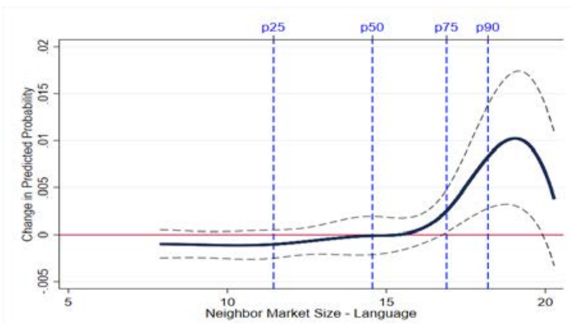
(a) Own - $h_o(\cdot)$



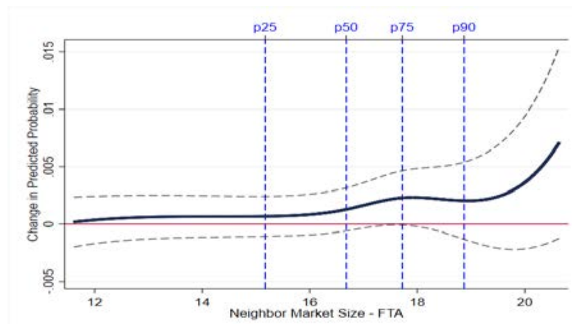
(b) Neighbors - Geography - $h_g(\cdot)$



(c) Neighbors - Language - $h_l(\cdot)$



(d) Neighbors - Deep PTA - $h_a(\cdot)$



Notes: Panels (a), (b), (c), and (d) show the point estimate and 95% confidence intervals for the cubic splines $h_o(\cdot)$, $h_g(\cdot)$, $h_l(\cdot)$, and $h_a(\cdot)$, respectively, in equation (F.4). The marks *p25*, *p50*, *p75*, and *p90* correspond to the 25th, 50th, 75th, and 90th percentiles of the corresponding covariate; i.e., E_{sjt} for panel (a), $AE^s_{jt,g}$ for panel (b), $AE^s_{jt,l}$ for panel (c), and $AE^s_{jt,a}$ for panel (d). Standard errors are clustered by country.

not statistically different from zero until we reach the destination that is at the 75th percentile of the distribution of the corresponding variable. From the 75th percentile onwards, the firm's export probability in a destination increases in the destination's own export potential and in the aggregate export potential of the countries geographically or linguistically close to it.

To test the robustness of the findings in Figure F.6, we also compute estimates of a regression similar to that in equation (F.4), but in which we capture the effect of E_{sjt}, $AE^s_{jt,g}$, $AE^s_{jt,l}$, and $AE^s_{jt,a}$ on y_{ijt} through step functions (instead of through cubic splines). Given the estimating equation

$$y_{ijt} = \tilde{h}_o(E_{sjt}) + \sum_{x=\{g,l,a\}} \tilde{h}_x(AE^s_{jt,x}) + \beta_{it} + u_{ijt}, \tag{F.5}$$

where $\tilde{h}_x(\cdot)$ for $x = \{o, g, l, a\}$ are step functions, panels (a) to (d) in Figure F.7 respectively show OLS estimates of the functions $\tilde{h}_o(\cdot)$, $\tilde{h}_g(\cdot)$, $\tilde{h}_l(\cdot)$, and $\tilde{h}_a(\cdot)$. More specifically, the function $\tilde{h}_o(E_{sjt})$ is defined as

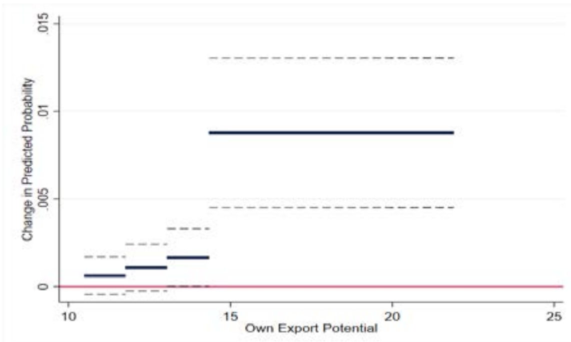
$$\begin{aligned}\tilde{h}_o(E_{sjt}) = {} & \beta_{o,1}\mathbb{1}\{0 \leqslant E_{sjt} \leqslant q_{o,25}\} + \beta_{o,2}\mathbb{1}\{q_{o,25} \leqslant E_{sjt} \leqslant q_{o,50}\} \\ & + \beta_{o,3}\mathbb{1}\{q_{o,50} \leqslant E_{sjt} \leqslant q_{o,75}\} + \beta_{o,4}\mathbb{1}\{q_{o,75} \leqslant E_{sjt}\},\end{aligned} \tag{F.6}$$

where $(\beta_{o,1}, \beta_{o,2}, \beta_{o,3}, \beta_{o,4})$ is a vector of unknown parameters, and $q_{o,Q}$ is the Qth percentile of the distribution of E_{sjt}. Similarly, for any $x = \{g, l, a\}$, the function $\tilde{h}_x(AE^s_{jt,x})$ is defined as

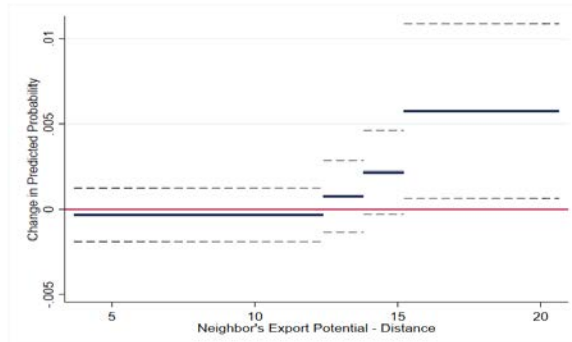
$$\tilde{h}_x(AE^s_{jt,x}) = \beta_{x,1}\mathbb{1}\{0 \leqslant AE^s_{jt,x} \leqslant q_{x,25}\} + \beta_{x,2}\mathbb{1}\{q_{x,25} \leqslant AE^s_{jt,x} \leqslant q_{x,50}\}$$

Figure F.7: Impact of Own and Neighbors' Export Potential

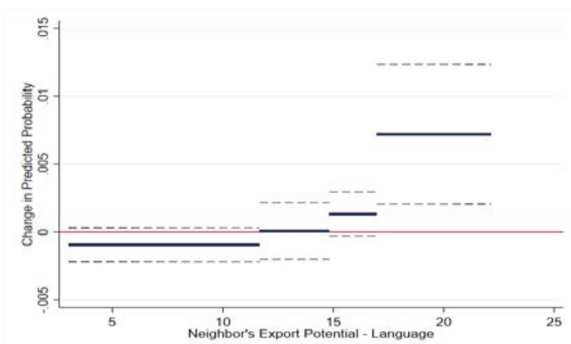
(a) Own - $h_o(\cdot)$



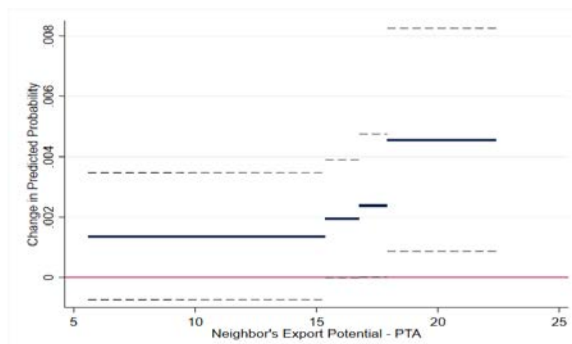
(b) Neighbors - Geography - $h_g(\cdot)$



(c) Neighbors - Language - $h_l(\cdot)$



(d) Neighbors - Deep PTA - $h_a(\cdot)$



Notes: Panels (a), (b), (c), and (d) show the point estimate and 95% confidence intervals for the step functions $\tilde{h}_o(\cdot)$, $\tilde{h}_g(\cdot)$, $\tilde{h}_l(\cdot)$, and $\tilde{h}_a(\cdot)$, respectively, in equation (F.5). Standard errors are clustered by country.

$$+ \beta_{x,3}\mathbb{1}\{q_{x,50} \leqslant AE^s_{jt,x} \leqslant q_{x,75}\} + \beta_{x,4}\mathbb{1}\{q_{x,75} \leqslant AE^s_{jt,x}\}, \tag{F.7}$$

where $(\beta_{x,1}, \beta_{x,2}, \beta_{x,3}, \beta_{x,4})$ are unknown parameters, and $q_{x,Q}$ is the Qth percentile of the distribution of $AE^s_{jt,x}$ conditional on $AE^s_{jt,x} > 0$. The estimates displaued in Figure F.7 are similar to those in Figure F.6.

F.3 List of Moment Conditions

As discussed in Section 6.2, our SMM estimator uses moment conditions that take the form

$$\frac{1}{M}\sum_{i=1}^{M}\{m_k(y_i^{obs}, z_i, x) - \frac{1}{S}\sum_{i=1}^{S} m_k(y_i^s(\theta), z_i, x)\} = 0, \tag{F.8}$$

where y_i^{obs} includes the observed firm i's export participation decisions in every country j and in every sample period t in which the firm is active; z_i includes all observed payoff-relevant variables and all estimates computed in the first step of our estimation procedure (see Section 6.1); x includes the export potential measures in equation (F.2) for all foreign countries and sample periods; and $y_i^s(\theta)$ includes all model-implied export participation decisions for given values of z_i and the parameter vector θ, and a draw χ_i^s from the distribution of χ_i conditional on z_i. Specifically, we can write z_i, x, and χ_i as

$$\begin{aligned} z_i \equiv (&\hat{\alpha}_y, \hat{\alpha}_a, \hat{\alpha}_r, \hat{\beta}_\alpha, \hat{\rho}_\alpha, \hat{\sigma}_\alpha, \hat{\beta}_r, \hat{\rho}_r, \hat{\sigma}_r, \{\hat{\alpha}_{jt}\}_{j=1,t=\underline{t}}^{J,\bar{t}}, \hat{\alpha}_s, \{r_{iht}\}_{t=t_i}^{\bar{t}}, \{a_{st}\}_{t=\underline{t}}^{\bar{t}}, \{(n^g_{jj'}, n^l_{jj'})\}_{j=1,j'=1}^{J,J}, \\ &\{n^a_{jj't}\}_{j=1,j'=1,t=\underline{t}}^{J,J,\bar{t}}, \{(n^g_{hj}, n^l_{hj})\}_{j=1}^{J}, \{n^a_{hjt}\}_{j=1,t=\underline{t}}^{J,\bar{t}}), \end{aligned} \tag{F.9}$$

$$x = \{E^s_{jt}\}^{J,\bar{t}}_{j=1,t=\underline{t}}, \tag{F.10}$$

$$\chi_i \equiv (\{\alpha_{jt}\}^{J,\underline{t}-1}_{j=1,t=\underline{t}_i}, \{\alpha_{jt}\}^{J,T}_{j=1,t=\bar{t}+1}, \{r_{iht}\}^{t-1}_{t=\underline{t}_i}, \{r_{iht}\}^{T}_{t=\bar{t}+1}, \{\nu_{ijt}\}^{J,T}_{j=1,t=\underline{t}_i}, \{\omega_{ijt}\}^{J,\bar{t}}_{j=1,t=\underline{t}_i}), \tag{F.11}$$

where s is firm i's sector, $\underline{t}$ and $\bar{t}$ are the first and last sample years, $\underline{t}_i$ is firm i's birth year, and $t_i = \max\{\underline{t}, \underline{t}_i\}$.

Each moment function $m_k(\cdot)$ is an average over foreign countries and periods of a function $\tilde{m}_{k,jt}(\cdot)$. Specifically, both for $y_i = y_i^{obs}$ and for $y_i = y_i^s(\theta)$, it holds that

$$m_k(y_i, z_i, x) \equiv \frac{1}{J(\bar{t} - t_i)} \sum_{j=1}^{J} \sum_{t=t_i}^{\bar{t}} \tilde{m}_{k,jt}(y_i, z_i, x). \tag{F.12}$$

We use 89 moments of the type defined by equations (F.8) and (F.12). We classify them in three blocks.

The first block includes moments targeted to identify the parameters determining the level of fixed and sunk costs as well as the impact on them of the distance between the firm's home country and each potential destination. Specifically, the first block of moments targets the identification of the parameters

$$(\gamma_0^F, \gamma_0^S, \{(\gamma_x^F, \gamma_x^S)\}_{x=\{g,l,a\}}),$$

which enter the model through the expressions in equations (7) and (12). A first set of moments in this block captures firms' export participation choices by groups of destinations that differ in their distances to the firm's home country. More specifically, these moments are defined by the functions

$$\tilde{m}_{k,jt}(y, z, x) = y_{ijt}\mathbb{1}\{n^{x_1}_{hjt} < \bar{n}_{x_1}\}\mathbb{1}\{n^{x_2}_{hjt} < \bar{n}_{x_2}\}n^{x_1}_{hjt}n^{x_2}_{hjt}, \tag{F.13a}$$

$$\tilde{m}_{k,jt}(y, z, x) = y_{ijt}\mathbb{1}\{n^{x_1}_{hjt} \geqslant \bar{n}_{x_1}\}\mathbb{1}\{n^{x_2}_{hjt} < \bar{n}_{x_2}\}n^{x_1}_{hjt}n^{x_2}_{hjt}, \tag{F.13b}$$

$$\tilde{m}_{k,jt}(y, z, x) = y_{ijt}\mathbb{1}\{n^{x_1}_{hjt} < \bar{n}_{x_1}\}\mathbb{1}\{n^{x_2}_{hjt} \geqslant \bar{n}_{x_2}\}n^{x_1}_{hjt}n^{x_2}_{hjt}, \tag{F.13c}$$

$$\tilde{m}_{k,jt}(y, z, x) = y_{ijt}\mathbb{1}\{n^{x_1}_{hjt} \geqslant \bar{n}_{x_1}\}\mathbb{1}\{n^{x_2}_{hjt} \geqslant \bar{n}_{x_2}\}n^{x_1}_{hjt}n^{x_2}_{hjt}, \tag{F.13d}$$

for all (x_1, x_2) in $\{(g, l), (g, a), (l, a)\}$. As a reminder, $n^g_{hjt} = n^g_{hj}$ and $n^l_{hjt} = n^l_{hj}$ for all t, and n^g_{hj}, n^l_{hj}, and n^a_{hjt} respectively denote the geographic, linguistic and regulatory distances between the firm's home country h and the foreign country j. The constants $\bar{n}_{x_1}$ and $\bar{n}_{x_2}$ are thresholds that split destination countries into two groups depending on whether their distance to the firm's home market h is larger or smaller than the corresponding threshold; specifically, we set $\bar{n}_g = 6$ (i.e., 6,000 km), $\bar{n}_l = 0.5$, and $\bar{n}_a = 1$. According to these thresholds, we split countries roughly depending on whether they are in the Americas (in which case $n^g_{hj} < 6$), on whether at least 50% of their population speak Spanish (in which case $n^l_{hj} < 0.5$), and on whether they have any sort of deep PTA with Costa Rica (in which case $n^a_{hjt} < 1$). E.g., the moment defined by the function in equation (F.13a) for $(x_1, x_2) = (g, l)$ is

$$\frac{1}{M}\sum_{i=1}^{M}\Big\{\frac{1}{J(\bar{t} - t_i)}\sum_{j=1}^{J}\sum_{t=t_i}^{\bar{t}}(y^{obs}_{ijt} - \frac{1}{S}\sum_{i=1}^{S} y^s_{ijt}(\theta))\mathbb{1}\{n^g_{hj} < 6\}\mathbb{1}\{n^l_{hj} < 0.5\}n^g_{hj}n^l_{hj}\Big\} = 0. \tag{F.14}$$

For the foreign countries less than 6,000 km away from Costa Rica and with linguistic distance to Costa Rica below 0.5, this moment captures the average (across firms, countries and periods) difference between the observed firm export participation choices and the average (across S simulated samples) choices implied by our model. When computing this average, each observation is weighted by the product of the geographic and linguistic distances of each destination to the firm's home country.

A second set of moments still within this first block are defined by the following functions:

$$\tilde{m}_{k,jt}(y_i, z_i, x) = y_{ijt}y_{ijt-1}\mathbb{1}\{n^{x_1}_{hjt} < \bar{n}_{x_1}\}\mathbb{1}\{n^{x_2}_{hjt} < \bar{n}_{x_2}\}n^{x_1}_{hjt}n^{x_2}_{hjt}, \tag{F.15a}$$

$$\tilde{m}_{k,jt}(y_i, z_i, x) = y_{ijt}y_{ijt-1}\mathbb{1}\{n^{x_1}_{hjt} \geqslant \bar{n}_{x_1}\}\mathbb{1}\{n^{x_2}_{hjt} < \bar{n}_{x_2}\}n^{x_1}_{hjt}n^{x_2}_{hjt}, \tag{F.15b}$$

$$\tilde{m}_{k,jt}(y_i, z_i, x) = y_{ijt}y_{ijt-1}\mathbb{1}\{n^{x_1}_{hjt} < \bar{n}_{x_1}\}\mathbb{1}\{n^{x_2}_{hjt} \geqslant \bar{n}_{x_2}\}n^{x_1}_{hjt}n^{x_2}_{hjt}, \tag{F.15c}$$

$$\tilde{m}_{k,jt}(y_i, z_i, x) = y_{ijt}y_{ijt-1}\mathbb{1}\{n^{x_1}_{hjt} \geqslant \bar{n}_{x_1}\}\mathbb{1}\{n^{x_2}_{hjt} \geqslant \bar{n}_{x_2}\}n^{x_1}_{hjt}n^{x_2}_{hjt}, \tag{F.15d}$$

for all (x_1, x_2) in $\{(g,l),(g,a),(l,a)\}$. These functions differ from those in equation (F.13) in that they depend not on whether a firm i exports to a country j at a period t (as captured by the dummy y_{ijt}) but on whether a firm i continues exporting at t to a country j to which it was exporting at $t-1$ (as captured by the dummy $y_{ijt}y_{ijt-1}$). E.g., the moment given the function in equation (F.15a) for $(x_1, x_2) = (g, l)$ is

$$\frac{1}{M}\sum_{i=1}^{M}\Big\{\frac{1}{J(\bar{t}-t_i)}\sum_{j=1}^{J}\sum_{t=t_i}^{\bar{t}}(y_{ijt}^{obs}y_{ijt-1}^{obs} - \frac{1}{S}\sum_{i=1}^{S}y_{ijt}^{s}(\theta)y_{ijt-1}^{s}(\theta))\mathbb{1}\{n_{hj}^{g} < 6\}\mathbb{1}\{n_{hj}^{l} < 0.5\}n_{hj}^{g}n_{hj}^{l}\Big\} = 0. \quad \text{(F.16)}$$

The interpretation of this moment is analogous to that in equation (F.14), with the only difference that it focuses in export survival events instead of export participation events.

Equations (F.13) and (F.15) list four moments each for each (x_1, x_2) in $\{(g,l),(g,a),(l,a)\}$. Thus, the first block of moments includes 24 moments in total.

The second block includes moments targeted to identify the parameters determining the strength of export complementarities. Specifically, this block of moments targets the identification of the parameters

$$\{(\gamma_x^E, \psi_x^E, \kappa_x^E)\}_{x=\{g,l,a\}},$$

which enter the model through the expression in equation (9). The functions defining the moments included in this second block capture firms' export probabilities by groups of destinations that differ in the aggregate export potential of the countries that are at a given geographical, linguistic, or regulatory distance to them. A key variable in these moments is thus the aggregate export potential of the countries that are within certain distance thresholds of each potential destination; we define these as

$$AE_{jt,x_1}^{s,x_2} = \sum_{j'\neq j}\mathbb{1}\{\underline{n}_{x_1}^{x_2} \leqslant n_{jj't}^{x_1} < \bar{n}_{x_1}^{x_2}\}E_{j't}^{s}, \quad \text{(F.17)}$$

where the index x_1 identifies the distance measure, and the index x_2 identifies the distance interval over which we are summing the export potential measures $E_{j't}^s$. The index x_1 takes values in the set $\{g,l,a\}$, with $x_1 = g$ denoting the geographical distance in equation (B.1), $x_1 = l$ denoting the linguistic distance in equation (B.2), and $x_1 = a$ denoting the regulatory distance in equation (1). The index x_2 takes values in the set $\{1,2,3\}$, and it determines the distance thresholds according to the following rules. For the case in which $x_1 = g$, the distance thresholds are

$$(\underline{n}_g^{x_2}, \bar{n}_g^{x_2}) = \begin{cases} (0, 426) & \text{if } x_2 = 1, \\ (426, 790) & \text{if } x_2 = 2, \\ (790, 1153) & \text{if } x_2 = 3. \end{cases} \quad \text{(F.18)}$$

For the case in which $x_1 = l$, the distance thresholds are

$$(\underline{n}_l^{x_2}, \bar{n}_l^{x_2}) = \begin{cases} (0, 0.01) & \text{if } x_2 = 1, \\ (0.01, 0.11) & \text{if } x_2 = 2, \\ (0.11, 0.50) & \text{if } x_2 = 3. \end{cases} \quad \text{(F.19)}$$

Finally, for the case in which $x_1 = a$, the distance thresholds are

$$(\underline{n}_a^{x_2}, \bar{n}_a^{x_2}) = \begin{cases} (0, \frac{1}{7}) & \text{if } x_2 = 1, \\ (\frac{1}{7}, \frac{3}{7}) & \text{if } x_2 = 2, \\ (\frac{3}{7}, \frac{6}{7}) & \text{if } x_2 = 3. \end{cases} \quad \text{(F.20)}$$

Then, for example, the variables $AE_{jt,g}^{s,1}$, $AE_{jt,g}^{s,2}$, and $AE_{jt,g}^{s,3}$ denote the aggregate export potential in sector s and year t of all countries j' other than country j which are less than 426 km away from j, between 426 km and 790 km away from j, and between 790 km and 1153 km away from j, respectively. There is a connection between the variables defined in equations (F.17) to (F.20) and those used as regressors in equation (F.4). Specifically, for any x_1 in $\{g,l,a\}$, the thresholds $\bar{n}_{x_1}^2$ defined in equations (F.18) to (F.20) coincide with the thresholds $\bar{n}_{x_1}$ used to compute the aggregate export potentials displayed in Figure F.5. Thus,

$$AE^{s}_{jt,x_1} = AE^{s,1}_{jt,x_1} + AE^{s,2}_{jt,x_1} \qquad \text{for } x_1 = \{g, l, a\}.$$

Given AE^{s,x_2}_{jt,x_1} for $x_1 = \{g, l, a\}$ and $x_2 = \{1, 2, 3\}$, the moments in this second block are defined by

$$\tilde{m}_{k,jt}(y_i, z_i, x) = y_{ijt}\mathbb{1}\{n^{x_1}_{hjt} < \bar{n}_{x_1}\}\mathbb{1}\{AE^{s,x_2}_{jt,x_1} = 0\}, \tag{F.21a}$$
$$\tilde{m}_{k,jt}(y_i, z_i, x) = y_{ijt}\mathbb{1}\{n^{x_1}_{hjt} < \bar{n}_{x_1}\}\mathbb{1}\{0 < AE^{s,x_2}_{jt,x_1} \leqslant p_{66}(AE^{s,x_2}_{jt,x_1})\}, \tag{F.21b}$$
$$\tilde{m}_{k,jt}(y_i, z_i, x) = y_{ijt}\mathbb{1}\{n^{x_1}_{hjt} < \bar{n}_{x_1}\}\mathbb{1}\{p_{66}(AE^{s,x_2}_{jt,x_1}) < AE^{s,x_2}_{jt,x_1}\}, \tag{F.21c}$$
$$\tilde{m}_{k,jt}(y_i, z_i, x) = y_{ijt}\mathbb{1}\{n^{x_1}_{hjt} \geqslant \bar{n}_{x_1}\}\mathbb{1}\{AE^{s,x_2}_{jt,x_1} = 0\}, \tag{F.21d}$$
$$\tilde{m}_{k,jt}(y_i, z_i, x) = y_{ijt}\mathbb{1}\{n^{x_1}_{hjt} \geqslant \bar{n}_{x_1}\}\mathbb{1}\{0 < AE^{s,x_2}_{jt,x_1} \leqslant p_{66}(AE^{s,x_2}_{jt,x_1})\}, \tag{F.21e}$$
$$\tilde{m}_{k,jt}(y_i, z_i, x) = y_{ijt}\mathbb{1}\{n^{x_1}_{hjt} \geqslant \bar{n}_{x_1}\}\mathbb{1}\{p_{66}(AE^{s,x_2}_{jt,x_1}) < AE^{s,x_2}_{jt,x_1}\}, \tag{F.21f}$$

where $p_{66}(\cdot)$ denotes the 66th percentile of the random variable in parenthesis. As a reminder, $n^{x_1}_{hjt}$ denotes for any x_1 in $\{g, l, a\}$ the corresponding distance between the firm's home country h and the foreign country j, and $\bar{n}_{x_1}$ is a threshold value we use to split foreign countries into two groups depending on whether their distance to the home market is larger or smaller than the corresponding threshold; specifically, we set $\bar{n}_g = 6$, $\bar{n}_l = 0.5$, and $\bar{n}_a = 1$, which are the same threshold values we use to define the moments in equations (F.13) and (F.15). E.g., the moment given by the function in equation (F.21a) for $(x_1, x_2) = (g, 1)$ is

$$\frac{1}{M}\sum_{i=1}^{M}\Big\{\frac{1}{J(\bar{t}-t_i)}\sum_{j=1}^{J}\sum_{t=t_i}^{\bar{t}}(y^{obs}_{ijt} - \frac{1}{S}\sum_{i=1}^{S} y^{s}_{ijt}(\theta))\mathbb{1}\{n^{g}_{hj} < 6\}\mathbb{1}\{AE^{s,1}_{jt,g} = 0\}\Big\} = 0. \tag{F.22}$$

This moment captures, for those foreign countries that are less than 6,000 km away from Costa Rica and have no country closer than 426 km to them, the difference between the export probability in the observed sample and the average export probability across S simulated samples. Similarly, the moment given by the function in equation (F.21b) for $(x_1, x_2) = (g, 1)$ is

$$\frac{1}{M}\sum_{i=1}^{M}\Big\{\frac{1}{J(\bar{t}-t_i)}\sum_{j=1}^{J}\sum_{t=t_i}^{\bar{t}}(y^{obs}_{ijt} - \frac{1}{S}\sum_{i=1}^{S} y^{s}_{ijt}(\theta))\mathbb{1}\{n^{g}_{hj} < 6\}\mathbb{1}\{0 < AE^{s,1}_{jt,g} \leqslant p_{66}(AE^{s,1}_{jt,g})\}\Big\} = 0. \tag{F.23}$$

This moment captures, for foreign countries that are less than 6,000 km away from Costa Rica and have countries located less than 426 km away from them whose aggregate export potential is positive but below the 66th percentile of the corresponding distribution, the difference between the export probability in the observed sample and the average export probability across S simulated samples.

Equation (F.21) lists six moments for each x_1 in $\{g, l, a\}$ and each x_2 in $\{1, 2, 3\}$. Thus, this block of moments could include 54 moments in total, each of them defined as the difference between the observed and simulated export probabilities in a subset of countries selected on the basis of their geographic, linguistic, or regulatory, distance to Costa Rica and of the aggregate export potential of the other potential destinations located within some pre-specified distance interval from those countries. However, two of these 54 moments select empty subsets of countries. As a result, the second block includes 52 moments in total.

The third block includes moments targeted to identify the parameters determining the distribution of the unobserved (to the researcher) terms ν_{it} and ω_{it}. Specifically, this block targets the identification of

$$(\sigma_\nu, p, \{(\gamma^N_x, \kappa^N_x)\}_{x=\{g,l,a\}}),$$

which enter the model through the expressions in equations (10) and (11). With the aim of identifying the variance of the fixed cost shock ν_{ijt}, σ^2_ν, we use moments defined by the following two functions

$$\tilde{m}_{k,jt}(y_i, z_i, x) = y_{ijt}\sum_{i' \neq i} y_{i'jt}\mathbb{1}\{Q(r_{iht}) = Q(r_{i'ht})\}, \tag{F.24a}$$

$$\tilde{m}_{k,jt}(y_i, z_i, x) = \mathbb{1}\{\sum_{j=1} y_{ijt} > 0\}, \tag{F.24b}$$

where $Q(\cdot): \mathbb{R}^+ \to \{1,2,3,4\}$ is a function that maps the firm's domestic revenue level into its corresponding quartile. The moment defined by the function in equation (F.24a) captures, on average across periods and pairs of firms i and i' whose domestic sales belong to the same quartile of the distribution, the similarity in the sets of export destinations of these two firms in the corresponding period. The function in equation (F.24b) captures whether firm i is an exporter at period t. These two moments help identify σ_ν.

With the aim of identifying p, we use moments defined by the following two functions

$$\tilde{m}_{k,jt}(y_i, z_i, x) = y_{ijt}(1 - y_{ijt-1})y_{ijt-2} + y_{ijt}(1 - y_{ijt-1})(1 - y_{ijt-2})y_{ijt-3}, \tag{F.25a}$$

$$\tilde{m}_{k,jt}(y_i, z_i, x) = (1 - y_{ijt})y_{ijt-1}(1 - y_{ijt-2}) + (1 - y_{ijt})y_{ijt-1}y_{ijt-2}(1 - y_{ijt-3}). \tag{F.25b}$$

The function in equation (F.25a) captures short (that last one or two periods) spells outside of an export market. The function in equation (F.25b) captures short export spells. As our model features firms that have perfect foresight on all payoff-relevant variables other than the fixed cost shock ω_{ijt}, short-lived transitions in and out of an export market will be largely driven by unexpected realizations of this fixed cost shock. The functions in equation (F.25) measure the frequency with which these short-lived transitions take place.

Finally, with the aim of identifying $\{(\gamma_x^N, \kappa_x^N)\}_{x=\{g,l,a\}}$, we use moments defined by the following functions

$$m_k(y, z, x) = y_{ijt} \sum_{j'=1}^{J} y_{ij't} \mathbb{1}\{y_{ijt-1} = y_{ij't-1}\} \mathbb{1}\{Q(E_{ijt}) = Q(E_{ij't})\} \mathbb{1}\{\underline{n}_{x_1}^{x_2} \leqslant n_{jj't}^{x_1} < \bar{n}_{x_1}^{x_2}\} \tag{F.26}$$

for any value of x_1 in $\{g, l, a\}$ and any value of x_2 in $\{1, 2, 3\}$, where $Q(\cdot): \mathbb{R}^+ \to \{1,2,3,4\}$ is a function that maps a country's export potential into its corresponding quartile. For any value of x_1 in $\{g, l, a\}$ and any value of x_2 in $\{1, 2, 3\}$, the thresholds $\underline{n}_{x_1}^{x_2}$ and $\bar{n}_{x_1}^{x_2}$ are determined as in equations (F.18) to (F.20). E.g., the moment built using the function in equation (F.26) for $(x_1, x_2) = (g, 1)$ captures, on average across firms and time periods, the frequency with which firms simultaneously export to any two countries j and j' in which they had the same export status in the previous period (as imposed by the condition that y_{ijt-1} and $y_{ij't-1}$ should coincide), that have similar export potentials (as imposed by the condition that E_{ijt} and $E_{ij't}$ should fall in the same quartile), and that are less than 426 km apart from each other. Intuitively, the function in equation (F.26) for $(x_1, x_2) = (g, 1)$ captures the correlation in firms' export participation decisions across countries of similar market size that are geographically very close to each other.

The function in equation (F.26) for $(x_1, x_2) = (g, 2)$ is analogous to that for $(x_1, x_2) = (g, 1)$, differing only in that, instead of focusing on pairs of countries that are less than 426 km apart, it focuses on pairs of countries whose bilateral distance is larger than 426 km and smaller than 790 km. Similarly, the function in equation (F.26) for $(x_1, x_2) = (g, 3)$ focuses instead on pairs of countries whose bilateral distance is larger than 790 km and smaller than 1,153 km. Thus, the functions in equation (F.26) for $x_1 = g$ and all three possible values of x_2 allow us to identify the parameters determining the correlation between ν_{ijt} and $\nu_{ij't}$ as a function of the geographical distance between countries j and j'.

Equations (F.24) and (F.25) list two moments each. Equation (F.26) lists one moment for each x_1 in $\{g, l, a\}$ and each x_2 in $\{1, 2, 3\}$. Thus, the third block of moments includes 13 moments in total.

F.4 Additional Details on SMM Estimator

We provide here additional details on two aspects of our SMM estimator. In Appendix F.4.1, we describe how we compute the vector of simulated choices $y_i^s(\theta)$ that enter the moment conditions; see equation (28). In Appendix F.4.2, we describe how we compute our SMM estimates given the vector of moment conditions.

F.4.1 Computing Vector of Simulated Choices

Given a value of the vector θ of fixed and sunk cost parameters, we describe here the steps we follow to compute each of the moment conditions we use in our estimation.

First step. For each firm i in the sample, we take $S = 5$ draws of the vector of unobserved payoff-relevant variables χ_i defined in equation (F.11). Specifically, for each draw, we implement the following procedure.

First, if we observe firm i in the first sample year, $\underline{t}$, then we treat its birth year, $\underline{t}_i$, as unknown, and we draw it randomly from the empirical distribution of firm ages in Costa Rica in 2010, as reported in World Bank (2012). If we do not observe firm i in $\underline{t}$, then we assume its birth year coincides with the first year it appears in the sample. The firm's birth year is thus observed, and not randomly drawn, in this case.[23]

Second, we simulate $\ln(r_{iht})$ for every out-of-sample period in which the firm is active; i.e., for all t in $[\underline{t}_i, \underline{t}) \cup (\bar{t}, T]$. If $\underline{t}_i < \underline{t}$, we simulate $(\ln(r_{ih\underline{t}_i}), \dots, \ln(r_{ih\underline{t}}))$ from a jointly normal distribution as determined by the corresponding AR(1) process for $\ln(r_{iht})$ specified in Section 4.3, conditioning on the firm's observed domestic sales in the first sample year, $r_{ih\underline{t}}$, as terminal condition, and on the the unconditional mean of this process as initial condition. To simulate $(\ln(r_{ih\bar{t}}), \dots, \ln(r_{ihT}))$, we first draw $T - \bar{t} + 1$ independent standard normal variables, which we then multiply by σ_r. We then use these draws of e^r_{iht} for every t in $[\bar{t} + 1, T]$, together with the firm's observed domestic sales in the last sample year, $r_{ih\bar{t}}$, to generate values of the firm's log domestic sales for every t in $[\bar{t} + 1, T]$. In this case, $\ln(r_{ih\bar{t}})$ operates as an initial condition of the corresponding process.

Third, we draw firm i's fixed cost shocks ν_{ijt} and ω_{ijt} for every country $j = 1, \dots, J$ and every t in $[\underline{t}_i, T]$. To obtain these draws of ν_{ijt}, we first draw $J(T - \underline{t}_i + 1)$ independent standard normal random variables, which we then multiply by the Cholesky decomposition of the variance matrix in equation (10). To obtain these draws of ω_{ijt}, we first draw $J(T - \underline{t}_i + 1)$ independent random variables distributed uniformly between 0 and 1; we then set $\omega_{ijt} = \underline{\omega}$ if the draw corresponding to country j and period t is smaller than the parameter p introduced in equation (11), and $\omega_{ijt} = \bar{\omega}$ otherwise.

Fourth, for each country j, we draw α_{jt} for every t between the earliest birth year in the corresponding simulated sample and the initial sample year, and for every t between the last sample year and the terminal period; i.e., for all t in $[\min_i\{\underline{t}_i\}, \underline{t}) \cup (\bar{t}, T]$. We simulate α_{jt} for all t in $[\min_i\{\underline{t}_i\}, \underline{t})$ from a jointly normal distribution as determined by the corresponding AR(1) process for α_{jt} specified in Section 4.3, conditioning on the unconditional mean of this process as initial condition, and on the observed value of α_{jt} in the first sample year, $\alpha_{j\underline{t}}$, as terminal condition. To simulate $(\alpha_{j\bar{t}+1}, \dots, \alpha_{jT})$, we first draw $T - \bar{t} + 1$ independent standard normal variables, which we then multiply by σ_α. We then use these draws of e^α_{jt} for every t in $[\bar{t} + 1, T]$, together with the value of α_{jt} observed in the last sample year, $\alpha_{j\bar{t}}$, to generate values of α_{jt} for every t in $[\bar{t} + 1, T]$. In this case, $\alpha_{j\bar{t}}$ operates as an initial condition of the corresponding AR(1) process.

Second step. For each firm i in the sample, we use the S draws of χ_i generated according to the procedure described above, the vector z_i of observed payoff-relevant variables, and a value of the parameter vector θ, to compute the vector of model-implied firm i's optimal export choices $y^s_i(\theta)$ for all $s = 1, \dots, S$ simulated samples. We do so implementing the algorithm described in Section 5.

F.4.2 Computing SMM Estimates

Denote the vector of moment conditions as $\mathcal{M}(y^{obs}, Z, x; \theta) = (\mathbb{m}_1(y^{obs}, Z, x; \theta), \dots, \mathbb{m}_K(y^{obs}, Z, x; \theta))'$ where

$$\mathbb{m}_k(y^{obs}, Z, x; \theta) = \frac{1}{M} \sum_{i=1}^{M} \Big\{ \frac{1}{J(\bar{t} - t_i)} \sum_{j=1}^{J} \sum_{t=t_i}^{\bar{t}} \{m_k(y^{obs}_i, z_i, x) - \frac{1}{S} \sum_{i=1}^{S} m_k(y^s_i(\theta), z_i, x)\} \Big\},$$

with $y^{obs} = \{y^{obs}_i\}^M_{i=1}$ and $Z = \{z_i\}^M_{i=1}$. Given $\mathcal{M}(y^{obs}, Z, x; \theta)$ and a $K \times K$ positive semi-definite matrix W, we compute our SMM estimate of θ as the solution to the following minimization problem

$$\min_\theta \mathcal{M}(y^{obs}, Z, x; \theta) W \mathcal{M}(y^{obs}, Z, x; \theta)'. \tag{F.27}$$

To solve this minimization problem numerically, we use a two-step algorithm: first, we use the TikTak global optimizer proposed in Arnoud et al. (2019) with 5,000 starting points, using BOBYQA as the local optimizer; second, we polish the outcome of the global optimizer using a Subplex local optimizer.

In practice, we compute a two-stage SMM estimate. In the first stage, we obtain estimates of θ, which we denote as $\hat{\theta}_1$, minimizing the objective function in equation (F.27) for a diagonal weight matrix $W = W_1$

[23] A firm will appear in our dataset as long as it has positive domestic sales, regardless of whether it exports.

in which every diagonal element $k = 1, \ldots, K$ equals

$$W_{1,k} = \frac{1}{(\mathbb{m}_k^{obs}(y^{obs}, Z, x))^2}, \qquad \text{with} \qquad \mathbb{m}_k^{obs}(y^{obs}, Z, x) \equiv \frac{1}{M}\sum_{i=1}^{M}\Big\{\frac{1}{J(\bar{t}-t_i)}\sum_{j=1}^{J}\sum_{t=t_i}^{\bar{t}} m_k(y_i^{obs}, z_i, x)\Big\}. \tag{F.28}$$

In the second stage, we obtain estimates of θ, which we denote as $\hat{\theta}_2$, minimizing the function in equation (F.27) for a diagonal weight matrix $W = W_2$ in which every diagonal element $k = 1, \ldots, K$ equals $W_{2,k} = (\hat{V}_k(y^{obs}, Z, x; \hat{\theta}_1))^{-1}$, with $\hat{V}_k(y^{obs}, Z, x; \hat{\theta}_1)$ the clustered-robust variance of the moment $\mathcal{M}_k(y^{obs}, Z, x; \hat{\theta}_1)$, with each cluster defined as a firm-year combination (see Section 11 in Hansen and Lee, 2019, for details). We present heteroskedasticity-robust, clustered at the firm-year level, and clustered at the firm level, standard error estimates. We compute each of these applying the formulas in Section 11 of Hansen and Lee (2019), with the adjustment for simulation noise in Gourieroux et al. (1993).

F.5 Alternative Simulation Draws

We evaluate here the sensitivity of our estimates of the vector θ of fixed and sunk cost parameters to the set of $S = 5$ draws of χ_i (see equation (F.11)) we use to compute those estimates. We take 50 independent sets of 5 draws of χ_i and, for each of them, we compute a new SMM estimate of θ. For each parameter in θ, we compute a non-parametric density of the estimates obtained in the 50 sets of simulations, and report in Table F.2 the mode of this density as well as our baseline estimate; see Table F.4 for our baseline estimates. Our baseline estimates are generally close to the mode of the distribution of the estimates obtained for different draws of χ_i, the only exception being the estimate of γ_g^F, which is 25% smaller than the mode of the density of the corresponding estimates.

Table F.2: Sensitivity of Baseline SMM Estimates to Alternative Simulation Draws

Parameters	Baseline Estimates	Alternative Estimates	Parameters	Baseline Estimates	Alternative Estimates
γ_0^F	62.92	63.53	κ_l^E	5.40	5.53
γ_g^F	13.11	17.68	γ_a^E	3.32	3.29
γ_l^F	4.14	2.79	φ_a^E	1.21	1.26
γ_a^F	29.28	28.99	κ_a^E	6.85	6.68
γ_0^S	114.76	115.09	γ_g^N	0.64	0.66
γ_g^S	19.95	19.88	κ_g^N	0.05	0.10
γ_l^S	0.23	0.26	γ_l^N	0.15	0.15
γ_a^S	21.83	21.07	κ_l^N	4.54	4.60
γ_g^E	9.83	10.79	γ_a^N	0.06	0.06
φ_g^E	1.96	1.98	κ_a^N	2.61	2.57
κ_g^E	6.02	6.03	σ_ν	80.04	79.98
γ_l^E	0.98	1.06	p	0.72	0.72
φ_l^E	2.74	2.76			

Note: the number in the "Baseline Estimates" column is the estimate reported in Table F.4; that in the "Alternative Estimates" column is the mode of the non-parametric density of the estimates obtained when reestimating our model using 50 alternative sets of draws of χ_i^s.

F.6 Estimation Results: Additional Details

F.6.1 First-step Estimates: Potential Export Revenue Parameters

In Table F.3, we present point estimates and standard errors for all parameters affecting the evolution over time of potential export revenues (see Section 4.3).

Table F.3: Estimates of Potential Export Revenue Parameters and Their Process

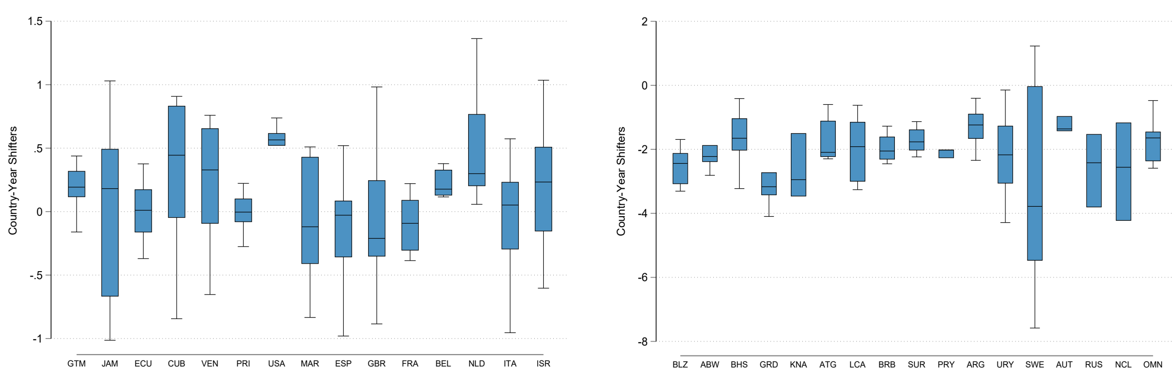
Potential Export Revenue Parameters		Process for Country- and Year-Specific Rev. Shifter		Process for Log Domestic Sales	
Parameter	Estimate (Standard Error)	Parameter	Estimate (Standard Error)	Parameter	Estimate (Standard Error)
α_y	1.856^a (0.066)	$\beta_{\alpha,g}$	-0.117^b (0.037)	ρ_r	0.857^a (0.012)
α_a	-3.832^a (0.066)	$\beta_{\alpha,l}$	-0.047 (0.071)	σ_r	0.865
α_r	0.285^a (0.041)	$\beta_{\alpha,a}$	-0.109 (0.079)		
		$\beta_{\alpha,gdp}$	0.079^a (0.019)		
		ρ_α	0.686^a (0.059)		
		σ_α	0.630		
Observations	13,293	Observations	467	Observations	43,300

Note: ^{a} denotes significance at 1%, ^{b} denotes significance at 5%. In parenthesis, standard error estimates. The results for *Potential Export Revenue Parameters* include country-year and sector fixed effects, and the displayed standard errors are heteroskedasticity robust standard errors. The results for *Process for Country- and Year-Specific Rev. Shifter* include no fixed effects, and the displayed standard errors are clustered by country. The results for *Process for Log Domestic Sales* include fixed effects for the firm's sector and province of location, and the displayed standard errors are clustered by firm.

In Figure F.8, we present box plots summarizing the distribution of the estimated values of α_{jt} across all sample periods for several specific countries. Specifically, panels (a) and (b) contain information for the 15 countries with the largest and smallest median estimates of α_{jt}, respectively.

Figure F.8: Estimates of Country- and Year-Specific Export Revenue Shifters

(a) Top-15 Destinations (b) Bottom-15 Destinations



Note: In both figures, countries are identified by their ISO 3166-1 alpha-3 code, and ordered in the horizontal axis by their distance to Costa Rica. For each country, the corresponding box plot represents (from top to bottom) the max, third quartile, median, first quartile and min of the estimated values of α_{jt} across all sample periods. Panel (a) displays box-plots of the estimates of $\{\alpha_{jt}\}_t$ for the 15 countries with the largest median estimates. Panel (b) displays analogous information for the 15 countries with the lowest median estimates.

F.6.2 Second-Step Estimates: Fixed and Sunk Costs Parameters

In Table F.4, we present point estimates and standard errors for all parameters entering fixed and sunk export costs (see Section 4.4).

Table F.4: SMM Estimates of Fixed and Sunk Cost Parameters

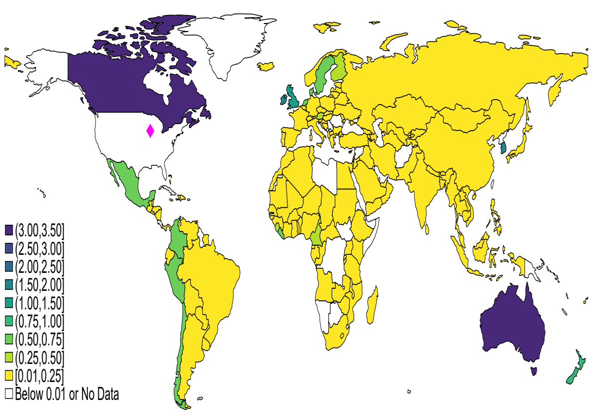
Parameter	Estimate (Standard Errors)	Parameter	Estimate (Standard Errors)
γ_0^F	62.92^a (1.11)(1.34)(2.77)	κ_l^E	5.40 (6.05)(7.84)(19.56)
γ_g^F	13.11^a (0.38)(1.17)(3.43)	γ_a^E	3.32^a (0.04)(0.04)(0.06)
γ_l^F	4.14^a (0.99)(1.71)(4.71)	φ_a^E	1.21 (0.52)(0.73)(1.51)
γ_a^F	29.28^a (0.78)(0.62)(1.09)	κ_a^E	6.85^a (1.02)(1.48)(3.18)
γ_0^S	114.76^a (3.18)(3.09)(6.03)	γ_g^N	0.64^a (0.00)(0.00)(0.01)
γ_g^S	19.95^a (0.92)(1.10)(2.80)	κ_g^N	0.05^a (0.00)(0.00)(0.01)
γ_l^S	0.23 (3.56)(4.43)(8.36)	γ_l^N	0.15^a (0.00)(0.00)(0.01)
γ_a^S	21.83^a (1.04)(0.83)(1.46)	κ_l^N	4.54^a (0.29)(0.31)(0.50)
γ_g^E	9.83^a (2.33)(2.85)(6.42)	γ_a^N	0.06^a (0.01)(0.01)(0.01)
φ_g^E	1.96^a (0.50)(0.66)(1.55)	κ_a^N	2.61^a (0.00)(0.00)(0.00)
κ_g^E	6.02^a (0.28)(0.49)(0.66)	σ_ν	80.72^a (0.51)(0.79)(2.05)
γ_l^E	0.98^a (0.08)(0.07)(0.11)	p	0.72^a (0.00)(0.00)(0.00)
φ_l^E	2.74 (2.88)(3.79)(7.16)		

Note: ^{a} denotes significance at 1%. In parenthesis, robust standard errors, standard errors clustered by firm-year, and standard errors clustered by firm, respectively. Displayed markers of statistical significance are determined on the basis of the standard errors clustered by firm-year.

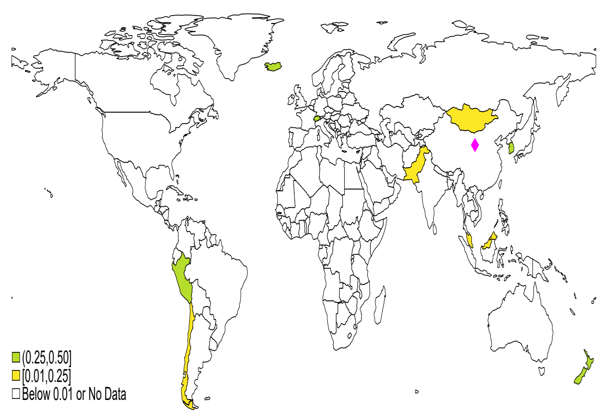
In Figure F.9, for the case of the US, China, Germany and Spain, we plot the value of $c_{jj't}/g_{jt}$ multiplied by 100 for all other destinations j'.

Figure F.9: Estimated Static Complementarities

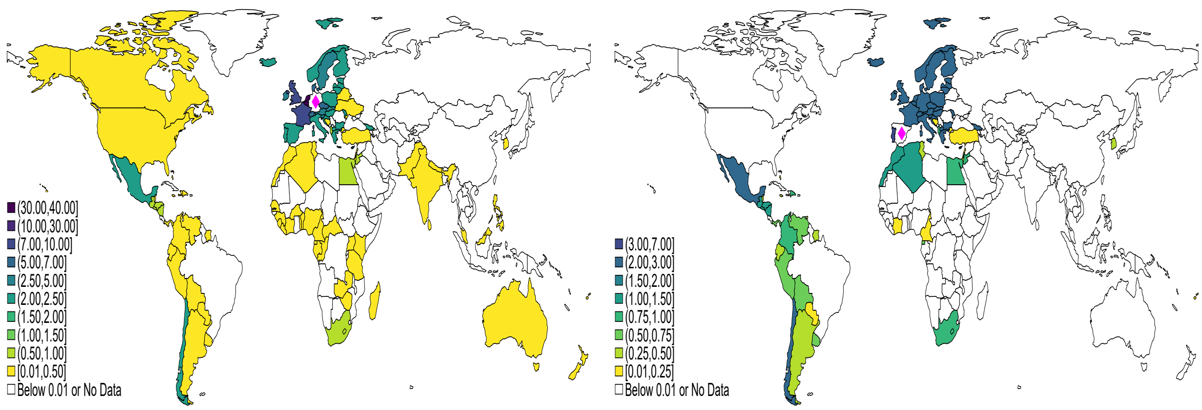
(a) The United States



(b) China



(c) Germany (d) Spain

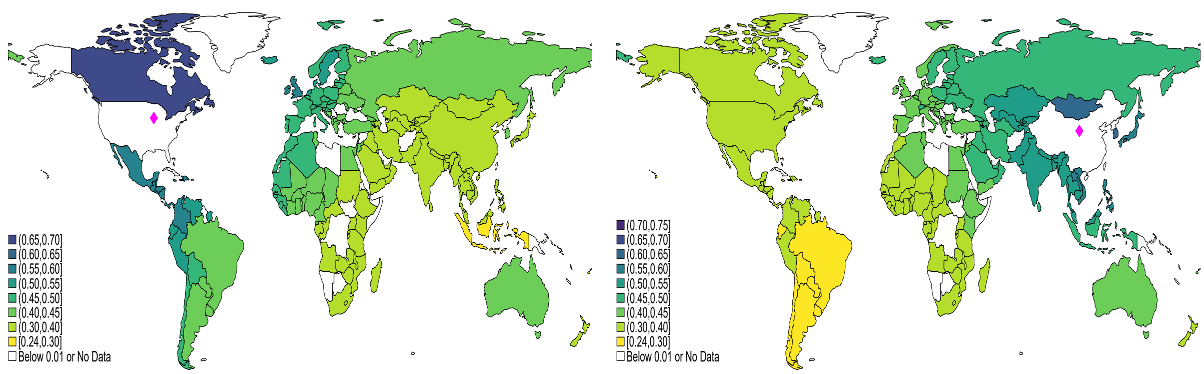


Note: In Panels (a), (b), (c) and (d) we illustrate, for the cases of the US, China, Germany, and Spain, respectively, the percentage reduction in fixed costs of exporting to these countries if the firm simultaneously also exports to each of the other possible export destinations.

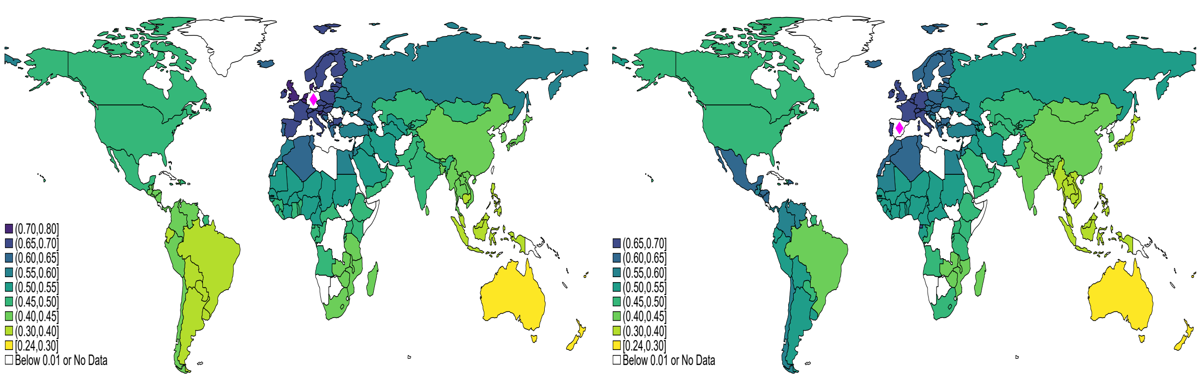
In Figure F.10, for the case of the US, China, Germany and Spain, we plot the value of $\rho_{jj't}$ for all other destinations j'.

Figure F.10: Estimated Pairwise Correlation Coefficients in Fixed Cost Shocks

(a) The United States (b) China



(c) Germany (d) Spain



Note: In Panels (a), (b), (c) and (d) we illustrate, for the cases of the US, China, Germany, and Spain, respectively, the correlation coefficient in the fixed cost shock ν_{ijt} between the corresponding country and every other country in the world.

F.7 Model Without Cross-Country Complementarities

We present here the estimates of a model analogous to that in Section 4 except for the additional restriction that the term in equation (9) equals zero for all countries and periods. Fixed and sunk costs in this restricted model thus only depend on the parameters $\theta_R \equiv (\gamma_0^F, \gamma_0^S, \sigma_\nu, p, \{(\gamma_x^F, \gamma_x^N, \kappa_x^N, \gamma_x^S)\}_{x=\{g,l,a\}})$.

In this restricted model, the estimation approach in Section 7.1 is still valid; thus, the estimates of the demand elasticity and the parameters entering potential export revenues coincide with those described in Section 7.1. Concerning the estimation of θ_R, we follow an approach analogous to that in Section 6.2, using the same moments described in Section F.3. We present in Table F.5 the resulting estimates.

Table F.5: Estimates of Fixed and Sunk Cost Parameters in Model Without Complementarities

Parameter	Estimate (Standard Errors)	Parameter	Estimate (Standard Errors)
γ_0^F	35.81^a (4.78)(7.93)(19.89)	γ_g^N	0.64^a (0.01)(0.01)(0.01)
γ_g^F	4.97^a (0.41)(0.75)(1.77)	κ_g^N	0.04^a (0.00)(0.00)(0.01)
γ_l^F	0.96 (2.64)(3.87)(9.59)	γ_l^N	0.18^a (0.03)(0.03)(0.07)
γ_a^F	6.32 (3.62)(6.05)(16.11)	κ_l^N	0.38 (0.52)(0.70)(1.59)
γ_0^S	70.70^a (6.17)(9.24)(21.09)	γ_a^N	0.10^a (0.01)(0.01)(0.03)
γ_g^S	36.21^a (0.22)(0.31)(0.31)	κ_a^N	0.42^a (0.05)(0.04)(0.10)
γ_l^S	0.16 (5.25)(9.93)(25.32)	σ_ν	41.59^a (0.76)(1.33)(3.36)
γ_a^S	27.39^a (4.48)(8.92)(24.36)	p	0.65^a (0.00)(0.00)(0.00)

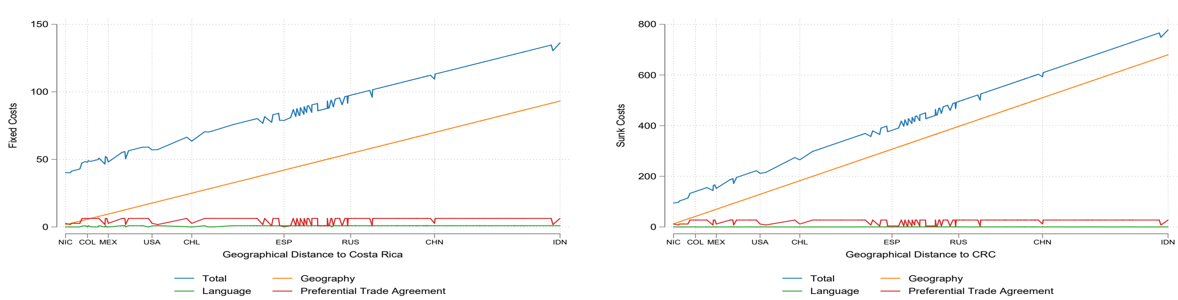
Note: [a] denotes significance at 1%. In parenthesis, robust standard errors, standard errors clustered by firm-year, and standard errors clustered by firm, respectively. Markers of statistical significance are determined on the basis of the standard errors clustered by firm-year.

Figure F.11 is analogous to Figure 1. The mean fixed cost function implied by the estimates in Table F.5 is smaller than the estimated mean fixed cost function for single-destination exporters displayed in panel (a) of Figure 1 for our general model with complementarities. This is to be expected, as the estimated mean fixed export costs in the restricted model without cross-country complementarities likely approximate a weighted average of the mean fixed export costs faced by different firms depending on their export bundles, with weights given by the frequency with which different export bundles are observed in the data.

Figure F.11: Fixed and Sunk Costs Estimates in Model Without Cross-Country Complementarities

(a) Fixed Export Costs

(b) Sunk Export Costs



Note: In both figures, countries are identified by their ISO 3166-1 alpha-3 code, and placed in the horizontal axis by their distance to Costa Rica. The vertical axis indicates the estimated cost in thousands of 2010 USD.

G Properties of Model With Complementarities

We consider here a simplified version of the model in Section 4 with the goal of understanding the role cross-country complementarities play on firm choices. Specifically, we impose on the model in Section 4 the following additional restrictions: (a) there are two markets, A and B; (b) for both markets, the fixed cost gravity term g_{jt} and sunk costs s_{jt} are constant over time; (c) the complementarity term in fixed costs c_{ABt} is constant over time; (d) $\omega_{ijt} = 0$ for every i, j and t; (e) $\alpha_y = 0$ and all other determinants of export revenues are constant over time, implying that r_{ijt} is constant over time for every firm i and market j.

Dropping the t subscript from all constant variables, and denoting the complementarities between markets A and B as c, firm i will thus solve the following optimization problem at $t = 0$:

$$\max_{\{y_{jt}\}_{jt}} \sum_{t \geqslant 0} \left\{ \delta^t (y_{iAt}\pi_{iA} - (1 - y_{iAt-1})s_A + y_{iBt}\pi_{iB} - (1 - y_{iBt-1})s_B + y_{iAt}y_{iBt}c) \right\} \tag{G.1}$$

where, for any country j, $\pi_{ij} = \eta^{-1}r_{ij} - g_j - \nu_{ij}$ is the potential export profits of firm i in j net of all components of fixed export costs other than the complementarity term; i.e., net of g_j and ν_{ij}. As no firm can export before the first period of activity, it holds that $y_{iAt-1} = y_{iBt-1} = 0$ when $t = 0$.

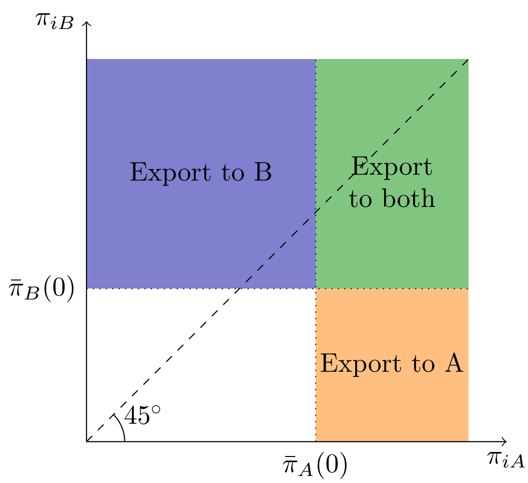
To understand the role complementarities play on firm choices, we consider two cases: one in which $c = 0$, and one in which $c > 0$. Without loss of generality, we keep all throughout the assumption that sunk export costs are lower in country B than in country A; i.e., $s_B < s_A$.

Case 1: no complementarities. In this case, $c = 0$ and the firm's export decision is independent across countries. As the problem in equation (G.1) is stationary, a firm exports to any country $j = \{A, B\}$ at any period $t \geqslant 0$ if and only if $\pi_{ij} \geqslant \bar{\pi}_j(0)$, for $\bar{\pi}_j(0) \equiv (1 - \delta)s_j$. Thus, as shown in panel (a) in Figure G.1, firms with $\pi_{iA} < \bar{\pi}_A$ and $\pi_{iB} \geqslant \bar{\pi}_B$ export only to B; firms with $\pi_{iA} \geqslant \bar{\pi}_A$ and $\pi_{iB} < \bar{\pi}_B$ export only to A; and, firms with $\pi_{iA} \geqslant \bar{\pi}_A$ and $\pi_{iB} \geqslant \bar{\pi}_B$ export to both countries. Consistently with the parametrization that $s_B < s_A$, the plot in panel (a) of Figure G.1 assumes that $\bar{\pi}_B(0) < \bar{\pi}_A(0)$.

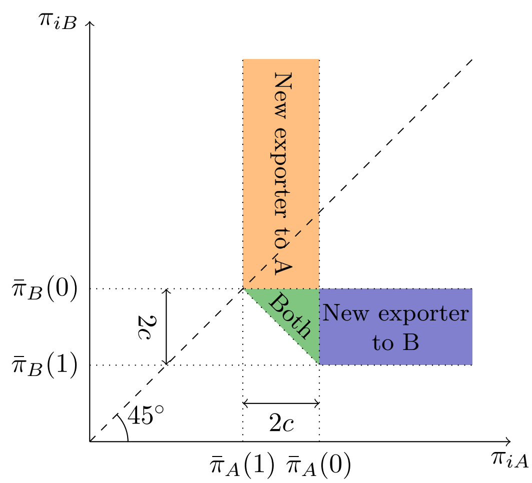
Case 2: positive complementarities. In this case, $c > 0$ and the firm's export decision is not independent across countries. Conditional on exporting to country $j' \neq j$, exporting to j is optimal if and only if $\pi_{ij} \geqslant \bar{\pi}_j(1)$ with $\bar{\pi}_j(1) = (1 - \beta)s_j - 2c$. Note that $\bar{\pi}_j(1) < \bar{\pi}_j(0)$ for any $c > 0$. Panel (b) in Figure G.1 illustrates the new exporters that emerge when c becomes positive. These new exporters are of two kinds. First, "natural exporters" to one of the markets (i.e., firms that export to one of the markets even when $c = 0$) and that, as complementarities become more important (i.e, as the value of c increases), start exporting to the other one. These are firms whose value of (π_{iA}, π_{iB}) falls in the orange and blue areas in

Figure G.1: Export Choices Models With and Without Complementarities

(a) Model Without Complementarities



(b) Model With Complementarities

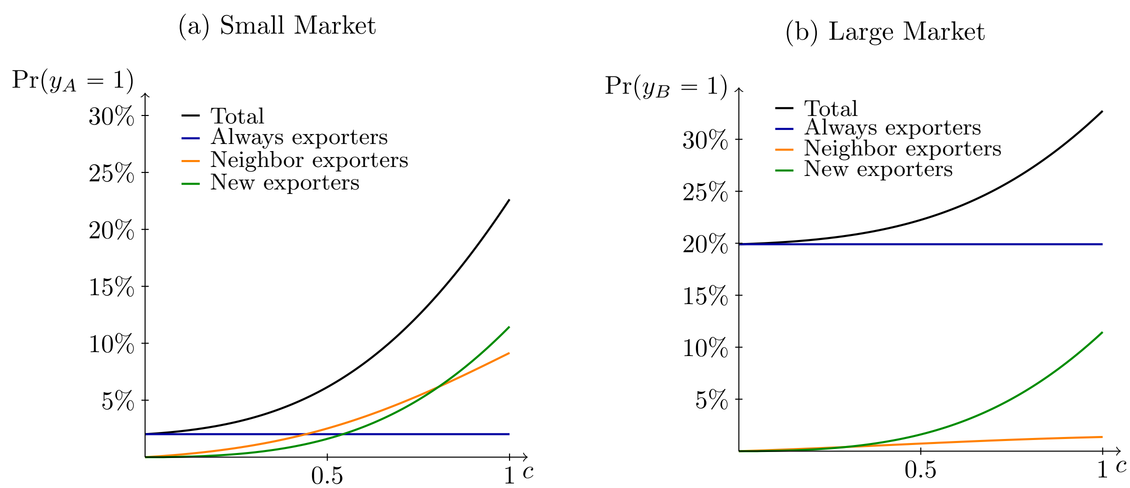


panel (b). Second, firms that do not export when $c = 0$, but export to both markets at the new level of c. These are firms whose value of (π_{iA}, π_{iB}) falls in the green area in panel (b).

Panel (b) in Figure G.1 shows how a firm i, depending on the values of (π_{iA}, π_{iB}), changes its set of destinations when c switches from being equal to zero to being positive. To determine how the share of firms exporting to either country changes as we change the value of c, we need to impose assumptions on the distribution of (π_{iA}, π_{iB}). In Figure G.2, we show how country-specific export shares change as we change the value of c when, for $j = \{A, B\}$, π_{ij} is normally distributed with mean μ (common in both markets) and variance equal to 1. We further assume that π_{iA} and π_{iB} are independent of each other. We impose values of μ, δ, s_A and s_B such that, when $c = 0$, the export share to A equals 2%, and the export share to B equals 20%. Thus, we can characterize markets A and B as being "small" and "large", respectively.

We extract several conclusions from Figure G.2. First, as reflected in the black lines in both panels, the effect on export shares of changes in c is non-linear: export shares are convex in c. Second, when comparing the export shares for positive values of c to those for $c = 0$, both the absolute and the relative increase in the export share is larger in the "small" export market (i.e., country A) than in the large one (i.e., country B). More specifically, when measuring the change in export shares as the value of c switches from zero to one, we observe that the percentage point increase in export shares in markets A and B is 21 pp. and 13 pp., respectively, and the relative increase in export shares in markets A and B is 11.5 (which equals 23%/2%) and 1.65 (which equals 33%/20%), respectively. Third, the reason for the larger impact of changes in c on export shares in A than in B is that there are many more firms that exported only to B in the case with $c = 0$ and add market A as export destination when c increases, than there are firms that exported only to A in the case with $c = 0$ and add market B as export destination when c increases; i.e., the probability that the vector (π_{iA}, π_{iB}) is in the area painted in orange in panel (b) of Figure G.1 is larger than the probability that it is in the area painted in blue in the same graph.

Figure G.2: Export Share and Cross-Country Complementarities



Note: In panel (a), for each value of c, "Total" denotes the share of firms that export to A at that value of c; "Always exporters" denotes the share of firms that export to A at that value of c *and* also export to A when $c = 0$; "Neighbor exporters" denotes the share of firms that export to A at that value of c, do not export to A when $c = 0$, *and* export to B when $c = 0$; and "New exporters" denotes the share of firms that export to A at that value of c and export neither to A nor to B when $c = 0$. The interpretation of the labels for panel (b) is analogous.

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