NBER WORKING PAPER SERIES

MACHINE FORECAST DISAGREEMENT

Turan G. Bali
Bryan T. Kelly
Mathis Mörke
Jamil Rahman

Working Paper 31583
http://www.nber.org/papers/w31583

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
August 2023, Revised August 2023

We thank Ian Dew-Becker, David McLean, and Xiaofei Zhao for their insightful and constructive comments. We also benefited from discussions with seminar participants at Georgetown University. AQR Capital Management is a global investment management firm, which may or may not apply similar investment techniques or methods of analysis as described herein. The views expressed here are those of the authors and not necessarily those of AQR or the National Bureau of Economic Research.

Machine Forecast Disagreement
Turan G. Bali, Bryan T. Kelly, Mathis Mörke, and Jamil Rahman
NBER Working Paper No. 31583
August 2023, Revised August 2023
JEL No. C15,C4,C45,C58,G1,G10,G17,G4,G40

ABSTRACT

We propose a statistical model of differences in beliefs in which heterogeneous investors are represented as different machine learning model specifications. Each investor forms return forecasts from their own specific model using data inputs that are available to all investors. We measure disagreement as dispersion in forecasts across investor-models. Our measure aligns with extant measures of disagreement (e.g., analyst forecast dispersion), but is a significantly stronger predictor of future returns. We document a large, significant, and highly robust negative cross-sectional relation between belief disagreement and future returns. A decile spread portfolio that is short stocks with high forecast disagreement and long stocks with low disagreement earns a value-weighted alpha of 15% per year. A range of analyses suggest the alpha is mispricing induced by short-sale costs and limits-to-arbitrage.

Turan G. Bali
McDonough School of Business
Georgetown University
Between 37th and O Streets
Washington, DC 20057
turan.bali@georgetown.edu

Mathis Mörke
University of St.Gallen
Unterer Graben 21
CH-9000 St. Gallen
Switzerland
mathis.moerke@unisg.ch

Bryan T. Kelly
Yale School of Management
165 Whitney Ave.
New Haven, CT 06511
and NBER
bryan.kelly@yale.edu

Jamil Rahman
Yale School of Management
jamil.rahman@yale.edu

1 Introduction

Belief disagreement is a primary motivation for trade; thus understanding disagreement is critical to understanding the behavior of financial markets. A theoretical literature seeks to understand how differences of opinion among investors impact market prices and volumes. A prominent example is Miller (1977), who predicts that stock prices are upward biased when there is a divergence of opinion among investors about stock value and pessimistic investors face short-sale constraints.[1]

Empirical work on disagreement is more limited due to the difficulty in measuring investor beliefs. A seminal study by Diether et al. (2002) proxies for belief heterogeneity using data on equity research analyst earnings forecasts. They show that higher analyst forecast dispersion (AFD) predicts lower future return in the cross section of individual stocks. Johnson (2004) questions the interpretation of Diether et al. (2002) and argues that AFD proxies for firm-specific risk.[2]

In this paper, we propose a new measure of investor disagreement. We then investigate whether our measure predicts cross-sectional variation in future returns of individual stocks, in general and also in light of heterogeneous short-sale frictions. We contribute to the literature in three ways.

Our first contribution is a new measure of belief disagreement at the asset level. Because the distribution of investor beliefs is not directly observable, we propose a statistical surrogate. Each investor is a prediction model from which beliefs about future returns are formed. These hypothetical investors have access to a common set of predictive information, but different investors use available information in different ways. We simulate the distribution of beliefs by endowing each investor with a machine learning model but introduce random variation in model specification across investors. By randomizing the set of model specifications, we capture the idea that investors have a distribution of prior beliefs, information frictions, and biases. Yet all investors in our model are sophisticated, though imperfect, optimizers. They are sophisticated in the sense that each investor's model is a random forest model that uses large predictor sets in flexible and nonlinear ways. But they are imperfect in that no investor has a correctly specified model; instead they have a variety of models that are heterogeneous approximations of the true data generating process. They are optimizers in that, given their model endowment, each investor uses the available data to estimate model parameters and form predictions.

A key question is whether the distribution from which we simulate model specifications (and

[1]Other important theoretical contributions include Harrison and Kreps (1978), Hong and Stein (1999), Chen et al. (2002), Hong and Stein (2003), Scheinkman and Xiong (2003) and Atmaz and Basak (2018).

[2]Other empirical papers studying the effect of belief disagreement include Anderson et al. (2005), Barber and Odean (2008), and Yu (2011).

hence investors' belief formation processes) is plausible. A significant portion of our analysis is dedicated to this question. We argue that (i) the calibration of our simulation distribution is reasonable and (ii) our results are robust to a range of distributions for simulating investors' models.

Given our construction of investor beliefs, we then measure stock-level disagreement as dispersion in investors' return forecasts, which we refer to as machine forecast disagreement (MFD). MFD has several attractive attributes. By sidestepping the difficult problem of directly and reliably surveying investor beliefs, the data coverage of MFD is much better than prior literature, which is essentially constrained by the availability of analyst forecasts from I/B/E/S. In contrast, MFD is available for all stocks at all times. Also, MFD is arguably a more objective measure of disagreement than AFD. While analysts are undoubtedly important information intermediaries in financial markets, evidence points to biases in their recommendations driven, for example, by incentives to secure underwriting and other investment banking business (see, e.g., Dugar and Nathan, 1995; Michaely and Womack, 1999; Chan et al., 2007). While we argue that machine learning models suffer less from behavioral biases or conflicts of interest, one may counter that our distribution of model specifications is biased in other ways. An attractive feature of MFD is that it constitutes a complete methodology for modeling and measuring disagreement. Shortcomings or biases in our specific implementation can be reformulated by other researchers to incorporate richer and more realistic belief simulations, and our results can in turn be re-analyzed in light of such model improvements.

Our second main contribution is documenting the strong explanatory power of MFD for the cross-sectional pricing of individual stocks. We find that stocks with higher MFD earn significantly lower future returns than otherwise similar stocks. In particular, a value-weighted (equal-weighted) portfolio of stocks in the highest MFD decile underperforms a portfolio of stocks in the lowest MFD decile by 1.17% (1.59%) per month with a Newey-West t-statistic of 3.51 (5.45), after controlling for common risk factors. We also present evidence that validates MFD as an effective measure of belief disagreement. While MFD has on average a 42% cross-sectional correlation with analyst disagreement, AFD is a notably weaker predictor of stock returns. The analogous value-weighted (equal-weighted) return of an AFD-based portfolio is 0.50% (0.99%) per month with a t-statistic of 1.98 (5.07). In Fama-MacBeth regressions, MFD is among the most statistically significant predictors of returns after controlling for other commonly studied characteristics, including value, investment, profitability, momentum, reversal, illiquidity, and volatility. We also show that the cross-sectional return prediction power of MFD extends to international equity markets (excluding the US) with the magnitudes and significance of international prediction effects closely in line with those for our main US sample.

Our third contribution is investigating the economic sources of MFD alpha. First, we condition our analysis on short-sale constraints. The overpricing of high-MFD stocks is especially pronounced among stocks with high short-sale costs. Stocks in the highest quintile of indicative borrowing fees experience an alpha spread of -2.54% per month, versus -0.74% for stocks in the lowest quintile. The difference of -1.80% (t-stat. $= -4.29$) is strongly supportive of the hypothesis that disagreement results in assets being more overpriced in the presence of more severe short-sale constraints. We find similar supportive evidence based on institutional ownership. The alpha spread on MFD-sorted portfolios of stocks with high retail ownership is -2.48% per month (t-stat.$=-8.88$), much larger than the alpha spread on MFD-sorted portfolios of stocks largely held by institutional investors of -0.61% per month (t-stat.$=-2.09$). The difference between these two alpha spreads, 1.87% per month (t-stat.=9.13), is highly significant and further supports the Miller (1977) hypothesis.

Next, we find supportive evidence that the MFD premium is associated with high-MFD stocks being mispriced, measured by the stock-level mispricing (MISP) definition of Stambaugh et al. (2015). We find an MFD alpha spread of -1.39% per month (t-stat.$= -4.45$) for stocks in the highest MISP quintile, compared to a spread of 0.02% (t-stat.$= 0.06$) for stocks in the lowest MISP quintile. The difference of these spreads is statistically significant (-1.40% with t-stat. $= -6.51$), suggesting that high-MFD stocks have a significantly higher mispricing score than low-MFD stocks.

We document additional support for the interpretation that MFD alpha is driven by stock-level mispricing by examining stock price reactions around earnings announcements. Assuming that investors exhibit biased expectations and are overly optimistic about high-MFD stocks, they update their beliefs in the presence of new information leading to a stock price correction (Engelberg et al., 2018). Hence, the return prediction during earnings announcements should exceed that of non-earnings periods. In line with this intuition, the return spread for the hedged MFD strategy is 66% (50%) higher during a one-day (three-day) earnings announcement window than on non-announcement days. We also find that the MFD alpha is significantly stronger for stocks with more severe limits-to-arbitrage, consistent with limits-to-arbitrage exacerbating asset mispricing.

The remainder of the paper is organized as follows. Section 2 introduces a belief-generating model from which we build a statistical measure of investor disagreement. Section 3 describes the data and variables. Section 4 presents the main empirical results on the predictability of cross-sectional equity returns. Section 5 runs a series of robustness checks. Section 6 investigates the sources of return predictability. Section 7 concludes the paper.

2 An empirical model of disagreement

Gu et al. (2020) consider a general conditional risk premium formulation

$$E_t[r_{i,t+1}] = g(z_{i,t})$$

where $z_{i,t} \in \mathbb{R}^d$ is data comprising the time t information set about asset i that is available to investors, and $g(\cdot)$ is a general (likely non-linear) function mapping that information into risk premia.

In order to model disagreement, we consider a collection of investors $k = 1, ..., K$. Each investor k has access to the complete information set $z_{i,t}$. However, investors differ in how they form expectations based on $z_{i,t}$. In particular, an investor k forms beliefs according to

$$E_{k,t}[r_{i,t+1}] = g_k(z_{i,t}).$$

That is, investor beliefs can disagree.

The idea that investors disagree is uncontroversial. As outlined by Barberis (2018), disagreement lies at the heart of many behavioral models of financial markets and is critical for generating the large trading volumes observed in many markets. Although the precise sources of disagreement are not well understood, "if two people are to disagree, one of three things must be true: (i) they have different prior beliefs; (ii) they observe different information; or (iii) one or both of them is not fully rational" (Barberis, 2018).

We propose a belief-generating model from which we build an empirical measure of investor disagreement. In particular, we simulate differences in beliefs across investors by endowing them with different models for forecasting returns from the common inputs $z_{i,t}$. We assume investor k forecasts returns according to

$$\begin{aligned} g_k(z_{i,t}) =& RF_k([\sin(z'_{i,t}w_k^1), \cos(z'_{i,t}w_k^1), \ldots, \sin(z'_{i,t}w_k^{p/2}), \cos(z'_{i,t}w_k^{p/2})]), \\ & w_k^j \in \mathbb{R}^d \sim iidN(0, \eta^2 I) \quad \forall j = 1, \ldots, p/2, \end{aligned} \tag{1}$$

where $RF_k(\cdot)$ denotes random forest regression. The investor-specific beliefs in Equation (1) have two main components. In the first component, information is processed through a non-linear Fourier operation using random linear combinations (w_k^j) of the common data. This component is taken from the "random features" methodology, developed in the machine learning literature by Rahimi and Recht (2007) and analyzed in the context of return prediction by Kelly et al. (2021). Random features provide a statistical mechanism for generating a distribution of data representations across investors. All investors are exposed to the same input data, $z_{i,t}$, but investor k processes the raw information in their own idiosyncratic way, transforming it into a feature set that is unique to k (though, naturally, correlated with other investors' views as well). These features summarize investor k's perception of the world around them, and can be interpreted as capturing differences in investors' access to information, information processing ability, or perceptive biases.

In the second stage, investors estimate $g_k(\cdot)$ using a random forest regression. Our motivation for this component is two-fold. First, the regression represents optimizing behavior on the part of investors as they learn how to best use their individual feature sets in a flexible specification. Random forest regression is known to be effective for capturing non-linearities and interaction effects in financial forecasting problems with high-dimensional predictor sets (see Gu et al., 2020; van Binsbergen et al., 2023, for applications to return and earnings prediction, respectively). Second, random forest introduces a further layer of heterogeneity in investor beliefs by randomizing regression specifications across investors (through the use of bootstrapping and dropout), which can be interpreted as heterogeneous model priors across investors.

In summary, our specification of $g_k(z_{i,t})$ is a reduced-form representation that accommodates the three potential sources of disagreement outlined by Barberis (2018). Once investors are endowed with a model and estimate the model subject to their respective data sets, they construct return forecasts for each stock in each month, $E_{k,t}[r_{t+1}]$. We measure disagreement, MFD, for stock i as the standard deviation of $E_{k,t}[r_{i,t+1}]$ across investors.

3 Data and variables

We use the dataset from Jensen et al. (2022b), a publicly available dataset of stock returns and characteristics.[3] The underlying return data are sourced from the Center for Research in Security Prices (CRSP) and accounting data from Compustat. We restrict our sample to common stocks

[3]The data, replication code, and documentation can be found at https://github.com/bkelly-lab/ReplicationCrisis/tree/master/GlobalFactors.

trading at the NYSE, AMEX, and NASDAQ. We exclude financial and utilities firms. To reduce the effect of small and illiquid stocks, we also exclude the low-priced stocks trading below $5 per share. As some of the 153 characteristics have a significant fraction of missing values, we select 130 characteristics with the smallest percentage of missing values to guarantee a greater homogeneity in the composition of characteristics over time (Didisheim et al., 2023).

To predict returns, we use the 130 stock characteristics as the complete information set z. We cross-sectionally rank all stock characteristics period-by-period and map these ranks into the $[-1, 1]$ interval following Kelly et al. (2019), Gu et al. (2020), and Freyberger et al. (2020).

Our sample covers the period from July 1966 to December 2022. Our approach utilizes a 10-year rolling window to estimate the random forest regressor. We calculate the month-t MFD using characteristics from the previous month $(t-1)$. Subsequently, we conduct out-of-sample cross-sectional asset pricing tests for the period August 1976 to December 2022.

3.1 MFD construction

The specific procedure for constructing our measure of disagreement, MFD, is as follows. We set the number of investors, K, to 100. For each investor k, we draw a random collection of linear combination weights $w_{k,j}, j = 1, \ldots, p/2$ from a normal distribution with standard deviation η. We set the number of random features, p, to 128 and the standard deviation of random weights, η, to 0.1. The random forest regression model has several hyper-parameters. We choose the maximum tree depth to be 6, the number of trees in the ensemble to be 2000, the fraction of features and of the sample to be taken as $\frac{\log 2(130)}{130}$ and to be 0.1, respectively. To ease the computational burden, we fit for each investor a random forest regression with the aforementioned hyperparameters every 12 months (similar approaches in stock and option return predictability have been used by Gu et al., 2020; Bali et al., 2023, respectively).

The set of eligible parameters for random forest regression and random features are chosen by the researchers and the choices are not necessarily innocuous. We base our choice for the parameters in random features on the findings of Jensen et al. (2022a). For random forest regression, our hyperparameter set is standard and similar to Gu et al. (2020). However, we conduct a range of sensitivity analyses in Section 5.1 that suggest our findings are robust to a variety of hyperparameter choices.

3.2 Control variables

In the cross-sectional regression analysis, we control for other firm characteristics that have been shown to predict future returns. All controls are taken from Jensen et al. (2022b). Specifically, SIZE is the firm's market capitalization computed as the market value of the firm's outstanding equity at the end of month $t-1$ (Fama and French, 2008). BM is the firm's book value of equity divided by its market capitalization, where the BM ratio is computed following Fama and French (2008). Short-term reversal (STR) is the stock's one-month lagged return. MOM is the cumulative return of the stock from month $t-12$ to month $t-1$ (omitting the STR month), following Jegadeesh and Titman (1993). Profitability (OP) is the ratio of the firm's operating profits to book equity following Fama and French (2015). Asset Growth (AG) is defined as the percent growth rate of total assets between two consecutive fiscal years, following Cooper et al. (2008). The turnover ratio (TURN) is the turnover of shares during the previous 126 trading days following Datar et al. (1998). The monthly illiquidity (ILLIQ) measure is calculated as the absolute daily return divided by the daily dollar trading volume, averaged over the last 126 trading days, following Amihud (2002). Idiosyncratic volatility (IVOL) is defined as the standard deviation of the daily residuals estimated from the regression of the daily excess stock returns on the daily market return over the previous year, following Ali et al. (2003). The standardized unexpected earnings (SUE) is constructed following Foster et al. (1984). The lottery payoff is proxied by the average of the five highest daily returns (MAX) in month $t-1$, following Bali et al. (2011).

3.3 Descriptive statistics

Table 1 presents summary statistics for the main cross-sectional variables. Concerning our key variable of interest, MFD, the time-series average of the cross-sectional mean is 16.04% with an average cross-sectional standard deviation of 5.74%. The average cross-sectional 10^{th} percentile of MFD is 9.61%, while the 90^{th} percentile is 24.39%, indicating a positively skewed distribution of MFD.

Figure 1 displays the annual time series plot of the aggregate MFD. It shows that the cross-sectional average MFD is generally higher during bad states of the economy and financial market downturns. Moreover, these states are also accompanied by a higher dispersion in cross-sectional MFD as depicted by the wider interquartile range. The positive skew of MFD is also visually confirmed, as the average MFD is closer to the third than the first quartile.

Table 2 includes the cross-sectional Spearman's rank correlation coefficient of MFD to the aforementioned control variables. The first column and the first row report a negative relation between MFD and one-month-ahead returns in excess of the risk-free rate. It further shows that smaller and less liquid stocks with higher analyst dispersion and higher idiosyncratic volatility also have higher MFD. This positive (negative) correlation of MFD with idiosyncratic volatility (size) suggests that the machine forecast disagreement is also a reasonably proxy for information uncertainty (see, e.g., Johnson, 2004; Zhang, 2006).

4 Empirical results

In this section, we conduct parametric and nonparametric tests to assess the predictive power of machine forecast disagreement (MFD) over future stock returns. First, we present results of the univariate portfolio-level analysis. Second, we compare the empirical performance of MFD and AFD in predicting future stock returns. Third, we report the average stock characteristics of the MFD-sorted decile portfolios. Fourth, we conduct bivariate portfolio-level analyses to assess the predictive power of MFD after controlling for well-known stock characteristics and risk factors. Finally, we present firm-level Fama-MacBeth cross-sectional regression results.

4.1 Univariate portfolio-level analysis

To construct the long-short portfolio for each month from August 1976 to December 2022, individual stocks are sorted by MFD into decile portfolios. We then compute the one-month-ahead value-weighted and equal-weighted average excess return of each decile portfolio. To examine the cross-sectional relation between MFD and future stock returns, we form a long-short portfolio that takes a long position in the lowest decile of MFD and a short position in the highest decile of MFD.

In Table 3, we report the average monthly excess returns (in excess of the one-month Treasury bill rate) of each decile portfolio, and the long-short portfolio. We also analyze abnormal returns (alphas) using different factor models. These include the capital asset pricing model (CAPM) with the market factor (MKT), the six-factor model (FF6) by Fama and French (2018) which includes MKT, size (SMB), value (HML), investment (CMA), profitability (RMW), and momentum (MOM) factors. Furthermore, we use the q4-factor model (HXZ) by Hou et al. (2015) with MKT, size (SMB_Q), investment (I/A), and profitability (ROE) factors. We also consider the mispricing factor model (SY) of Stambaugh and Yuan (2017) with MKT, SMB, management (MGMT), and performance (PERF) factors, along with the behavioral factor model (DHS) of Daniel et al. (2020) using MKT, post-earnings-announcement drift (PEAD), and financing (FIN) factors.

In general, the excess returns and the alphas of the MFD-sorted portfolios decrease from decile 1 to decile 10. The long-short portfolio that short-sells stocks in the highest 10^{th} percentile of MFD (decile 10) and buys stocks in the lowest 10^{th} percentile of MFD (decile 1) earns a value-weighted (equal-weighted) average return of 1.17% (1.59%) per month with a t-statistic of 3.51 (5.45), translating into an annualized return of 14.04% (19.08%).[4] Controlling for the robust risk and mispricing factors does not change the magnitude and statistical significance of the return spreads on the MFD-sorted portfolios for most of the factor models. Notably, we observe a reduction in the alphas of the long-short portfolio under the mispricing (Stambaugh et al., 2015) and behavioral (Daniel et al., 2020) factor models. For value-weighted portfolio sorts, the alpha decreases from 1.26% (CAPM) to 0.78% under the mispricing factor model, whereas for equal-weighted portfolio sorts the alpha decreases from 1.67% (CAPM) to 1.27% for the behavioral factor model. This suggests that the return predictability is potentially driven by mispricing rather than compensation for risk.

Next, we examine the persistence of the rank of MFD and the persistence of MFD-based return predictability. Table 4 presents stocks' transition probabilities across MFD groups in the next year. Specifically, we present the average probability that a stock in decile i (defined by the rows) in month t will be in decile j (defined by the columns) in month $t+12$. All the probabilities in the transition matrix should be approximately 10% (ten portfolios) if the evolution for MFD for each stock is random and the relative magnitude of MFD in one period has no implication about the relative MFD values next year. However, Table 4 shows that 50% of stocks in the lowest MFD decile (decile 1) in month t continue to be in the same decile in month $t+12$. Similarly, 49% of the stocks in the highest MFD decile (decile 10) in month t continue to be in the same decile in month $t+12$. Evidently, investor disagreement proxied by MFD is a highly persistent stock characteristic.

Prompted by this persistence, we investigate the longer-term predictive power of MFD by calculating the Fama and French (2018) six-factor (FF6) alphas of MFD-sorted portfolios from 2 to 12 months after portfolio formation. The results are presented in Table 5. For both the value- and equal-weighted portfolios, the six-factor alpha spread nearly monotonically decrease during the 2^{nd} to 12^{th} month after portfolio formation. For the value-weighted portfolios, the FF6 alpha spread remains economically large and highly significant during the second, third, and fourth month after

[4] The t-statistics reported in our tables are Newey and West (1987) adjusted with six lags to control for heteroskedasticity and autocorrelation.

portfolio formation, with respective alphas of −0.73% (t-stat=−2.18), −0.72% (t-stat=−2.12), and −0.64% (t-stat=−1.92) per month. The predictive power of MFD on future returns diminishes as one moves further away from the portfolio formation month and becomes insignificant after the fourth month; at 12 months, the FF6 alpha spread is −0.39% with a t-statistic of −1.20. Equal-weighted portfolio alphas remain statistically significant during this period; the alphas are −1.24% (t-statistic = −4.53) and −0.70% (t-statistic = −2.87) during the second month and 12 months after portfolio formation, respectively. These results show that the negative cross-sectional relation between MFD and future returns is relatively long-lived.

4.2 Comparing MFD to analyst-based disagreement

In this section, we compare the return predictive ability of analyst earnings forecast disagreement (AFD) versus MFD. Figure 2 shows the distribution of the monthly cross-sectional correlations between MFD and AFD based on Spearman's ρ. Generally, we observe a positive correlation between MFD and AFD. The average monthly cross-sectional correlation is 0.42 and is positive in all months. Further evidence on the positive correlation is given in Panel A of Table 6. It depicts the average AFD per MFD decile in univariate portfolio sorts on MFD. AFD is monotonically increasing in MFD deciles and the spread in AFD between MFD decile 10 and 1 is 0.29 with a t-statistic of 24.22.

Prior empirical evidence on the cross-sectional association between AFD and stock returns is mixed. Diether et al. (2002), Chen et al. (2002), Goetzmann and Massa (2005), Berkman et al. (2009), and Yu (2011) find a negative cross-sectional association between AFD and average stock returns. Others present evidence that the negative relation holds only for a sample of stocks with certain characteristics, e.g., small, illiquid, low credit quality, or short sale constrained. In particular, Malkiel and Cragg (1970), Qu et al. (2003), Doukas et al. (2006), Avramov et al. (2009), and Carlin et al. (2014) find either a positive or no significant relation between AFD and future stock returns.

We revisit the evidence on AFD using our longer time period. We analyze AFD-sorted portfolios in the same way we did for MFD. Table A1 in Appendix shows the equal-weighted and value-weighted decile portfolio returns as well as the return and alpha spreads between high-AFD and low-AFD decile portfolios. The Fama and French (2018) six-factor alpha spread is −0.99% (t-stat.=−4.99) and −0.50% (t-stat.=−1.97) for equal- and value-weighted portfolios, respectively. The evidence for AFD aligns with our findings for MFD, but the effect is notably weaker in both magnitude and statistical significance.

To further investigate the strength of the cross-sectional predictions from MFD and AFD, we use a bivariate portfolio analysis. Specifically, we first sort stocks into quintile portfolios every month based on AFD. Subsequently, we divide each AFD quintile into deciles based on MFD. Panel B of Table 6 reports the bivariate portfolio results. The MFD decile return spread is statistically significant in all AFD quintiles. The MFD return spread becomes larger in magnitude and more significant when analyst disagreement is more severe; in AFD quintile five, the equal-weighted MFD return spread is -1.27% (t-stat.$=-3.35$). Moreover, the corresponding FF6 alpha is statistically significant in these AFD quintiles except for the lowest quintile. The difference in alpha spread between the highest and lowest AFD quintile is economically large and highly statistically significant. Collectively, these results stress the relatively higher predictive power of MFD with respect to AFD and shows that MFD is much stronger for equities with high dispersion in analysts' earnings forecasts.

However, MFD is not just a stronger form of AFD. MFD comes with various advantages over AFD. First, MFD can be constructed for many more U.S. stocks. In our setting, we can construct MFD for about 67% more stocks on average compared to AFD.[5] Second, MFD can be calculated backwards in history before analyst coverage became available. I/B/E/S has been mainly used after 1983, whereas data on firm fundamentals date back to at least 1950 from easy-to-access databases. Third, MFD can cover international stocks. This might be of particular relevance for countries with little to no analyst coverage. Fourth, MFD might be less prone to behaviorial biases or conflicts of interest that have been found in literature (see, e.g., Dugar and Nathan, 1995; Michaely and Womack, 1999; Chan et al., 2007). Finally, MFD can be updated at the discretion of the researcher as it does not rely on the update cycle of analyst recommendations.

4.3 Average portfolio characteristics

We investigate if other firm characteristics can explain the negative relation between MFD and future stock returns. We sort stocks by MFD into decile portfolios each month and report the time-series averages of the cross-sectional medians of various firm-specific characteristics for each decile. Table 7 presents the average stock characteristics of each MFD-sorted decile portfolio and the long-short portfolio. The characteristics include the machine forecast disagreement (MFD), log market capitalization (SIZE), log book-to-market ratio (BM), asset growth (AG), operating

[5]The number is calculated for the time-period for which AFD is available.

profitability (OP), medium-term stock momentum (MOM), short-term reversal (STR), illiquidity (ILLIQ), turnover (TURN), standardized unexpected earnings (SUE), idiosyncratic volatility (IVOL), and lottery demand (MAX).

Earlier studies find that small, illiquid, lottery-like stocks with high idiosyncratic volatility exhibit high information uncertainty (e.g., Zhang, 2006; Kumar, 2009; Bali et al., 2011). Consistent with the literature, Table 7 shows that the stocks with higher MFD (or higher firm-specific uncertainty) are indeed smaller, less liquid, and have higher idiosyncratic volatility and stronger lottery features.

The literature shows that the firm characteristics considered in Table 7 are useful in explaining the cross-section of expected stock returns. Stocks with higher asset growth, lower profitability, lower past-12 month (momentum) returns, lower earnings surprise, higher idiosyncratic volatility, and higher MAX tend to have lower future returns. Considering the prior findings in the literature and the fact that these firm characteristics vary across MFD deciles, it is important to control for the effects of investment, profitability, momentum, post-earnings-announcement drift, idiosyncratic volatility, and/or the lottery demand effect when studying the cross-sectional relation between MFD and future stock returns. Thus, in the next two sub-sections, we control for these well-known return predictors in bivariate portfolio sorts and in cross-sectional regressions to further test whether the significant relation between MFD and future stock returns remains intact.[6]

4.4 Bivariate portfolio-level analysis

Next, we investigate the negative association between MFD and future stock returns while controlling for the established equity return predictors. We conduct 5x10 dependent double sorts based on firm characteristics and MFD. Each month, we first sort stocks into quintile portfolios based on a given control. Then, we further sort stocks by MFD into decile portfolios within each control variable quintile. This bivariate portfolio analysis provides 50 conditionally double-sorted portfolios. Portfolio 1 (10) is the combined portfolio of stocks with the lowest (highest) MFD in each control variable quintile.

Table 8 presents the return spreads and Fama and French (2018) six-factor (FF6) alphas on the

[6] In Table 3, we have already controlled for the market, size, value, momentum, investment, and profitability factors of Fama and French (2018) and Hou et al. (2015) as well as the mispricing and behavioral factors of Stambaugh and Yuan (2017) and Daniel et al. (2020) constructed based on earnings surprise (post-earnings-announcement drift) and a number of other well-known return predictors. As discussed in Section 4.1, the alpha spreads on MFD-sorted portfolios remain negative and highly significant in both value-weighted and equal-weighted portfolios after controlling for this large set of equity market factors.

bivariate portfolios. For brevity, we do not report the alphas for all 50 (5x10) portfolios. Instead, we report the abnormal returns on the portfolios of MFD averaged across the 5 control variable quintiles to produce the MFD-sorted decile portfolios while accounting for the impact of control variables. Panel A (Panel B) reports results for the equal-weighted (value-weighted) portfolio sorts. The last row in Panels A and B of Table 8 shows that the cross-sectional relation between MFD and future returns remains economically large and highly significant after controlling for a large set of well-known return predictors. The six-factor FF6 alpha spreads on the equal-weighted MFD-sorted portfolios are in the range of -0.72% per month (t-stat.=-4.37) and -1.36% per month (t-stat.=-5.46) and ranging from -0.39% per month (t-stat.=-1.97) to -1.20% per month (t-stat.=-4.49) for value-weighted bivariate sorts. These results indicate that even after controlling for various firm characteristics and risk factors in bivariate portfolios, there is a strong negative relation between MFD and future equity returns. In other words, the predictive power of MFD is not explained by other cross-sectional return predictors, including the existing measures of investor disagreement and information uncertainty.

4.5 Fama-MacBeth cross-sectional regressions

In this section, we conduct firm-level Fama-MacBeth regression analysis to test if MFD predicts the cross-section of future stock returns while controlling for other known predictors simultaneously. Each month, we run a cross-sectional regression of stock returns in that month on past MFD as well as a number of control variables, including the one-month lagged market beta, size, book-to-market, momentum, operating profitability, asset growth, earnings surprise, short-term return reversal, illiquidity, turnover ratio, idiosyncratic volatility, and lottery demand. The stock-level cross-sectional regressions are run each month and the standard errors of the average slope coefficients are corrected for heteroskedasticity and autocorrelation following Newey and West (1987).

Table 9 reports the results of stock-level Fama-MacBeth regressions. In column (1), we include MFD as well as beta, size, book-to-market, and momentum as additional cross-sectional predictors. Consistent with the portfolio results, we find a negative and significant relation between MFD and one-month-ahead returns. The average slope coefficient on MFD is -0.49 with a t-statistic of -6.93. In columns (2) and (3) we include additional return predictors in the cross-sectional regressions. Even in the presence of 12 well-known predictors, the average slope coefficient is -0.26 and statistically significant with a t-statistic of -4.41. MFD is also highly economically significant. The spread in the average standardized MFD between deciles 10 and 1 is approximately 3.37, and multiplying this spread by the average slope of -0.26 yields a return difference of -0.88% per month, controlling for all else. In most cases, the slope coefficients on the control variables are consistent with prior literature; short term reversal (STR), turnover (TURN), idiosyncratic volatility (IVOL), and MAX are negatively correlated with the future return, whereas momentum (MOM), profitability (OP), and earnings surprise (SUE) are positively related to the next month's return.

In column (4), we include the industry-adjusted return in month $t + 1$ to account for the industry effect. Specifically, we adjust the dependent variable by subtracting the firm's value-weighted Fama-French 48-industry return from the firm's current month return. Doing so allows us to tease out the return predictive power from MFD rather than the one-month industry momentum effect (Moskowitz and Grinblatt, 1999). The coefficient of MFD remains similar after controlling for the industry return directly. In column (5), we further control for the common characteristics that are shown to affect stock returns systematically. Specifically, we follow Daniel et al. (1997) and compute the characteristics-adjusted returns as the difference between the firm's return and the corresponding DGTW benchmark portfolio returns. We replace the firm's raw return with this characteristics-adjusted return as the dependent variable and run the same set of monthly cross-sectional regressions. Again, the magnitude of the slope coefficient on MFD becomes slightly weaker, but it remains highly significant, both economically and statistically.

Overall, these results indicate that MFD provides incrementally value-relevant information. The predictive power of MFD is distinct and robust to the inclusion of other well-known return predictors. In Table A2 in the Appendix, we additionally add AFD to the list of control variables and repeat stock-level Fama-MacBeth regressions. The inclusion of AFD does not influence the statistical and economic significance of MFD. Moreover, MFD exhibits a three times larger effect in economic terms compared to AFD.

5 Robustness check

In this section, we run a series of robustness checks to challenge the negative cross-sectional association between MFD and average stock returns. Section 5.1 discusses alternative constructions of MFD, and Section 5.2 discusses international evidence with the main MFD specification.

5.1 Different measures of MFD

We consider three alternations to the construction of MFD. First, we vary the hyper-parameters in the random forest regression as described in Equation (1). In Table B3 in the Appendix, we report average returns and alphas of MFD spread portfolios for various choices of hyper-parameters. The cross-sectional association between MFD and average stock returns is extraordinarily robust to these choices. The lowest FF6 equal-weighted alpha spread that we find across meta-parameters is -1.28% per month with a t-statistic of -4.52, while the largest alpha spread is -1.40% with a t-statistic of -4.93. Similar findings are obtained for the value-weighted portfolio sorts.

Second, Table B3 in the Appendix also includes average return and alpha spreads on MFD-sorted portfolios for an alternative model of generating MFD. The alternative model now estimates $g_k(\cdot)$ using ridge regressions, instead of using random forest regressions. For this model, we change Equation (1) so that investors forecast returns according to

$$g_k(z_{i,t}) = b_{k,0} + \sum_{m=1}^{M=p/2} [\beta_{k,2m-1} \sin(z'_{i,t} w_k^m) + \beta_{k,2m} \cos(z'_{i,t} w_k^m)] \tag{2}$$

with ridge parameter λ. w_k^j and $z_{i,t}$ are the same as in Equation (1). The association between MFD and average returns remains robust even with this alternate specification.

Finally, we vary the first stage of our belief generating mechanism. Instead of endowing investors with the full, but transformed information set, each investor k is equipped now with an incomplete information set of the non-transformed characteristics. Attention is a limited resource (Kahneman, 1973). As investors differ in their financial sophistication, they differ in their capabilities to acquire and process information. For example, retail investors are assumed to have a smaller attention capacity than institutional investors. Moreover, less sophisticated investors such as retail investors might not be able to access all information because they are not aware of their existence or do not have the tools and knowledge to acquire them.[7] Additionally, dedicated investment mandates may require the use of specific information even though the investor might have access to information beyond her use for investment decisions. Following this logic, we vary the

[7] Hong and Stein (1999) propose a theoretical model in which gradual diffusion of information among investors explains the observed predictability of stock returns. In their model, at least some investors can process only a subset of publicly available information because either they have limited information-processing capabilities or searching over all possible forecasting models using publicly available information itself is costly (Hirshleifer and Teoh, 2003), and there are limits to arbitrage (Shleifer and Vishny, 1997). Due to investors' limited attention and costly arbitrage, new informative signals are incorporated into stock prices partially because at least some investors do not adjust their demand by recovering informative signals from firm fundamentals or observed prices. As a result of this failure on the part of some investors, stock returns exhibit predictability.

fraction of raw characteristics each investor uses to range from 25% to 75% in Table B3. We also consider different hyper-parameter sets for the random forest regression. MFD keeps its strong cross-sectional explanation on average stock returns.

5.2 International evidence

The evidence presented so far relied on data for U.S. stocks. In this section, we test external validity of our results using individual stocks trading in international equity markets. We source stock returns and characteristics for a large global panel of 93 countries from Jensen et al. (2022b). We begin our sample in January 1986, which is the earliest start date for equity data for most developed countries. In line with our main analyses, we retain the 130 characteristics with the fewest number of missing observations. We apply the same data filters and methodology as in Section 3 to construct MFD for international stocks.[8] We divide global stock data into geographical regions. First, we focus on developed countries excluding the USA, following the classification in Jensen et al. (2022b).[9] Second, we examine individual stocks trading only in emerging markets. Third, we investigate the Group of 10 (G10 ex USA) and Group of 7 (G7 ex USA), excluding the USA in both cases. Finally, we examine individual stocks that trade in European countries (Europe).

Table 10 presents results from univariate portfolio-level analysis over the time period from February 1996 to December 2022. It reports the return of the long-short portfolio where individual stocks are sorted by MFD into decile portfolios. Additionally, Table 10 documents the alpha with respect to the international five-factor model of Fama and French (2017) augmented by the momentum factor (FF6).[10] Both the average return and FF6 alpha spreads are statistically and economically significant in equal- and value-weighted portfolio sorts regardless of the geographical region. Moreover, the results from alternative samples of international stocks are quantitatively similar to those obtained from the U.S. stocks. Therefore, Table 10 provides strong evidence that the negative cross-sectional relation between MFD and future returns is not confined to the US data, but also holds internationally.

[8] In contrast to return data for U.S. stocks, we follow Jensen et al. (2022b) and winsorize international stock returns at 0.1% in both tails each month.

[9] The classification in Jensen et al. (2022b) is based on the MSCI classification of each country as of January 7th 2021 and presented in Table J.3 in Jensen et al. (2022b).

[10] We obtain data for the international five-factor model of Fama and French (2017) as well as for the international momentum factor from Kenneth French's website (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french).

6 Sources of return predictability

Having established a robust negative cross-sectional association between MFD and average stock returns, we next investigate the potential economic mechanisms giving rise to this pattern. Motivated by the theoretical literature and the evidence presented in this paper so far, we explore mispricing versus risk in general, and more specifically investigate limits to arbitrage in the form of short sale constraints and information frictions.

6.1 Mispricing versus risk

In our results so far, we have documented significant alphas controlling for established factor models, which is a first indication that systematic risks (of the form captured by those models) do not explain the MFD pattern in average returns. Nor can other well-known firm-level risk measures (like idiosyncratic volatility or illiquidity) explain the MFD effect.

If the MFD pattern is indeed associated with mispricing, we expect it to be correlated with other known mispricing phenomena in the literature. In this vein, we compare MFD to the mispricing measure (MISP) of Stambaugh et al. (2015). We report the time-series average of the cross-sectional mispricing score for stocks in each MFD decile portfolio.[11] We also conduct dependent double sorts based on individual stock's MISP and MFD; that is, stocks are first grouped into 5 quintile portfolios on ascending sorts of MISP. Subsequently, stocks are grouped into 10 decile portfolios on ascending sorts of MFD within each MISP quintile. We then compute the return spreads and alphas with respect to the Fama and French (2018) six-factor model for MFD high-minus-low portfolios within each MISP quintile.

Table 11, Panel A, shows that the high MFD stocks indeed have a higher average mispricing score than the low MFD stocks. Furthermore, as reported in the last column of Panel A, Table 11, the 10-1 difference in the average mispricing score is 24.73 and statistically significant at the 1% level with a t-statistic of 42.34. Thus, we conclude that high-MFD stocks are more likely to be overvalued.

Next, we investigate whether the cross-sectional relation between MFD and future returns is stronger for overvalued vs. undervalued stocks. Specifically, we calculate the return spreads

[11] As discussed in Stambaugh et al. (2015), each month individual stocks are ranked independently based on 11 prominent equity return predictors (net stock issues, composite equity issues, accruals, net operating assets, asset growth, investment-to-assets, distress, O-score, momentum, gross profitability, and return on assets) in such an order that a higher rank is associated with lower one-month-ahead stock returns. The mispricing measure (MISP) is defined as the arithmetic average of the ranks of the 11 return predictors, and higher (lower) MISP indicates overvaluation (undervaluation).

and Fama and French (2018) six-factor alpha spreads of MFD-sorted portfolios within each MISP quintile. Panel B shows results for the equal-weighted bivariate portfolios of MISP and MFD. The last column in Panel B presents the FF6 alpha spreads between the high MFD and low MFD decile portfolios along with the Newey-West t-statistics. A notable point in Table 11 is that the return and alpha spreads on MFD-sorted portfolios increase monotonically (in absolute magnitude) moving from low-MISP to high-MISP quintile, and the FF6 alpha spread is highest at -1.39% per month with a t-statistic of -4.45 for overvalued stocks, i.e., in the high MISP quintile. Moreover, the alpha spread on MFD-sorted portfolios of overvalued stocks is economically and statistically greater than the alpha spread on MFD-sorted portfolios for undervalued stocks.

To further differentiate the negative cross-sectional association between MFD and future stock returns from a risk-based explanation, we study stock price reactions around earnings announcements. If the return predictability were explained by underlying risk, we would expect the returns to be evenly affected in subsequent periods. On the contrary, if the effect is consistent with mispricing, then the returns must be disproportionaly affected around earnings announcements, i.e., the return prediction around earnings announcements should be stronger than that around non-earnings announcement periods if investors are surprised by the good or bad news during that period and revise their expectations. Our approach is widely used in the literature (see, e.g., Bernard and Thomas, 1989; Porta et al., 1997; Engelberg et al., 2018). We follow Engelberg et al. (2018) and conduct a panel regression analysis of daily stock returns (Ret_t^d) on the previous month MFD, an earnings announcement window dummy (EDAY), and the interaction term between the two variables. We also include a set of control variables, consisting of the lagged values for each of the past ten days for stock returns, stock returns squared, and trading volume. We also control for day fixed effects and cluster the standard errors by day.

The date of the earnings announcement is defined as in Engelberg et al. (2018). Specifically, we compute the firm's trading volume scaled by market trading volume for the day before, the day of, and the day after the reported earnings announcement date, which is obtained from Compustat quarterly database. We then define the day with the highest scaled trading volume as the day of the earnings announcement. We select one-day or three-day earnings announcement windows centered on the earnings announcement date in our analysis. Panel A of Table 12 reports the regression results for the one-day window, whereas Panel B presents the results for the three-day window. In all cases and in line with the findings of Engelberg et al. (2018), the coefficients on the EDAY are positive and significant. Additionally, the coefficients on MFD are negative and highly statistically significant, corroborating the previously documented negative cross-sectional relation between MFD and future stock returns. More importantly, and consistent with the mispricing explanation, the coefficient for the interaction term between MFD and EDAY is negative and statistically significant, meaning that the negative cross-sectional relation is stronger on earnings announcement days. The coefficient is also economically significant. In column 2 of Panel A, the coefficient on MFD is -0.35 (t-stat. $= -5.43$), while the coefficient of MFD $\times$ EDAY interaction term is -0.58 (t-stat. $= -2.75$), meaning that the return spread for the hedged MFD strategy is 66% higher during an earnings announcement window than on non-announcement days. Analogously, based on column 4, the MFD premium is 50% higher during a three-day earnings announcement window than on non-announcement days. Thus, the evidence supports our mispricing argument that as investors appear to be surprised by the content of new information and subsequently update their beliefs, leading to an elevated MFD-return spread on earnings announcement days.

6.2 Short-selling costs

In light of the preceding evidence of MFD's association with mispricing, and that this mispricing is most prominent among high disagreement stocks, we investigate the Miller (1977) hypothesis that disagreement combined with short-sale constraints produces overpricing of high MFD stocks. We use two datasets that measure short sale frictions: the indicative borrowing fee provided by IHS Markit, and institutional ownership.

The indicative borrowing fee is calculated from proprietary data by IHS Markit. It is an estimate of the current costs for a hedge fund to borrow shares. Hence, we regard it as a good proxy for short-sale constraints. Besides borrowing costs between share lenders and prime brokers, its computation uses also rates from hedge funds to produce an indication of the current market rate. Panel A of Table 13 presents the time-series averages of cross-sectional medians for the indicative fee for equity deciles formed via a univariate MFD sort. Equities with higher MFD have higher indicative fees (or more binding short-sale constraints), and the difference between the high and the low MFD deciles is highly significant.

We next analyze the strength of the MFD return spread within indicative fee quintiles. Panel B of Table 13 shows that the abnormal return (six-factor alpha) to the zero-cost portfolio that buys stocks with the highest MFD and sells stocks with the lowest MFD increases in magnitude from low indicative fee to high indicative fee. For stocks within the lowest quintile (BORROWFEE Low), the FF6 alpha to the zero-cost portfolio is -0.74% per month with a t-statistic of -2.92, while the

MFD alpha more than triples in absolute terms to -2.54% per month with t-statistic of -5.80 if the indicative fee is highest (BORROWFEE High). The difference in MFD alpha spreads across indicative fee quintiles is economically and statistically significant; -1.80% per month (t-stat.= -4.29). These results indicate that the MFD premium is stronger among stocks with more severe short sale costs as measured by the indicative fee.

Table 14 repeats this analysis with an alternative measure of short sale constraints: institutional ownership (Nagel, 2005).[12] In Panel A of Table 14, we present the time-series averages of cross-sectional means for percentage institutional ownership (INST) for equity deciles formed via a univariate sort based on MFD. The results show that equities with higher MFD are more likely to be held by individual investors. The percentage institutional ownership is equal to 59% for decile 1. In contrast, for decile 10 which includes the equities with the highest MFD, the percentage institutional ownership drops to 36%. The difference in institutional holdings between the extreme MFD deciles is highly significant with a t-statistic of 30.85.

Next, we analyze the strength of the disagreement premium across institutional ownership portfolios using a dependent double sort analysis. Specifically, we first sort stocks into quintile portfolios every month based on the level of institutional ownership. Then, we divide each institutional ownership quintile into deciles based on MFD. In Panel B of Table 14, we present the Fama-French (2018) six-factor alpha for each of the 50 (5×10) resulting INST&MFD sorted portfolios as well as the six-factor alpha spread between the extreme MFD deciles, and associated t-statistics. A notable point in Table 14, Panel B, is that the magnitude of the abnormal return (FF6 alpha) to the zero-cost portfolio that buys stocks with the highest MFD and sells stocks with the lowest MFD increases monotonically in absolute value as one moves towards the stocks for which the level of institutional holdings is lowest (INST Low). For stocks in which institutional investors are most active (INST High), the FF6 alpha to the zero-cost portfolio is negative at -0.61% per month (t-stat.=-2.09), whereas the corresponding alpha spread on MFD-sorted portfolios is much higher at -2.48% (t-stat.=-8.88) for stocks in which retail investors are most active (INST Low). The diff-in-diff analysis of the FF6 alpha spreads of the stocks with high vs. low institutional holdings also generates an economically and statistically significant difference. Specifically, the difference between the six-factor alphas of the zero-cost MFD-sorted portfolios among the extreme institutional

[12] Institutional holdings data are obtained from Thompson-Reuters' Institutional Holdings (13F) database. To measure a stock's institutional holdings (INST), we define month-t INST to be the fraction of total shares outstanding that are owned by institutional investors as of the end of the last fiscal quarter during or before month t. Values of INST are available for the period from January 1980 to December 2022.

ownership quintiles (INST High – INST Low) is 1.87% with a t-statistic of 9.13.

6.3 Limits to arbitrage

In this section, we further explore the role of limits-to-arbitrage. If the predictive power of MFD is driven by mispricing to some extent, then we should expect the return predictability to be more pronounced for stocks with high arbitrage costs. In our next test, we use three proxies of limits-to-arbitrage that are prevalent in the literature.

The prior literature singles out idiosyncratic risk as the primary arbitrage cost (e.g., Pontiff, 2006). We rely on Ang et al. (2006) and measure the monthly IVOL as the standard deviation of the daily residuals from the regression of daily excess stock returns on the three factors of Fama and French (1993) over the past one month. Moreover, following Amihud (2002), we use the monthly illiquidity measure as our second proxy, computed as the absolute daily return divided by the daily dollar trading volume, averaged over the last 126 trading days. Finally, we rely on the market capitalization (size) as our third proxy, which is another widely used measure to capture costly arbitrage (e.g., Cohen and Lou, 2012; Lee et al., 2019). Instead of using a single proxy for limits-to-arbitrage, we follow Atilgan et al. (2020) and construct a composite index out of the three aforementioned proxies. The arbitrage cost index is created by arranging stocks in ascending order according to their idiosyncratic volatility and their illiquidity. Likewise, stocks are arranged in descending order based on their size. Each stock is assigned a score corresponding to its position in the decile rank for each variable. Finally, the stock-level arbitrage cost index is computed as the sum of these three scores, ranging from 3 to 30. A higher index value indicates more stringent limits-to-arbitrage.

We test the limits-to-arbitrage hypothesis using dependent bivariate portfolios. Specifically, we first sort stocks into quintile portfolios every month based on the arbitrage cost index. Then, we divide each arbitrage cost quintile into deciles based on MFD. Consistent with the limits-to-arbitrage hypothesis, Table 15 shows that the return and alpha spreads on MFD-sorted portfolios are negative and larger in absolute magnitude, and statistically more significant for stocks with high arbitrage costs, compared to the return and alpha spreads on MFD-sorted portfolios for stocks with low arbitrage costs. The difference of the return and alpha spreads of the stocks with high vs. low arbitrage costs also generates a highly significant difference in the MFD premium; the difference in alpha spreads is -1.67% with a t-statistic of -6.83. Thus, we conclude that the slow diffusion of information into stock prices due to limits-to-arbitrage provides a complementary explanation to the predictive power of MFD.[13]

7 Conclusion

This paper introduces a statistical model of investor beliefs from which we build a novel measure of investor belief disagreement. In particular, we simulate differences in beliefs across investors by endowing them with different machine learning models for forecasting returns from the same set of inputs. Thus, differences in beliefs across investors emerge from differences in the way they perceive and use data. Investor disagreement is measured as the standard deviation of expected return forecasts across investors.

We find a significantly negative and highly robust cross-sectional relation between this newly proposed measure, MFD, and future stock returns. In particular, the value-weighted arbitrage portfolio that takes a short position in the 10^{th} percentile of stocks with the highest MFD and takes a long position in the 10^{th} percentile of stocks with the lowest MFD yields a monthly alpha of 1.17% on a value-weighted basis. We also examine the long-term predictive power of MFD and find that the negative relation between MFD and future equity returns persists up to four months for the value-weighted portfolios. Finally, we find corroborative evidence for the significance of MFD from bivariate portfolio sorts and multivariate Fama–MacBeth regressions when we control for a large number of firm characteristics and risk factors.

We investigate the source of the MFD spread portfolio's alpha. We conduct comprehensive analyses to differentiate the risk versus mispricing explanations, and present evidence that the alpha for high-MFD stocks is driven primarily by mispricing. To better understand the economic mechanisms behind MFD-based return predictability, we test if the predictive power of MFD is explained by short-sale constraints and/or other limits to arbitrage. We show that the disagreement premium is significantly stronger for stocks with higher short-sale constraints. Relatedly, the negative relation between MFD and future returns is most pronounced for stocks with high arbitrage costs and high retail ownership. Therefore, our findings support the mispricing explanation of the disagreement premium, consistent with Miller (1977).

[13] Table C4 in the Appendix presents the univariate portfolio results obtained from long-short portfolios of 1,000 largest stocks. Although the alphas of MFD-sorted portfolios of 1,000 largest stocks in Table C4 are somewhat smaller than the corresponding alphas from all stocks in Table 3, the alpha spreads from big and liquid stocks remain economically large and statistically significant for all risk factor models. The fact that the alpha spreads are lower economically and weaker statistically for the mispricing factor model of Stambaugh et al. (2012) and the behavioral factor model of Daniel et al. (2020) support the mispricing-based explanation of the MFD premium induced by short-selling costs and limits-to-arbitrage.

References

Ali, A., Hwang, L.-S., and Trombley, M. A. (2003). Arbitrage risk and the book-to-market anomaly. *Journal of Financial Economics*, 69(2):355–373.

Amihud, Y. (2002). Illiquidity and stock returns: cross-section and time-series effects. *Journal of Financial Markets*, 5:31–56.

Anderson, E. W., Ghysels, E., and Juergens, J. L. (2005). Do heterogeneous beliefs matter for asset pricing? *Review of Financial Studies*, 18:875–924.

Ang, A., Hodrick, R. J., Xing, Y., and Zhang, X. (2006). The cross-section of volatility and expected returns. *Journal of Finance*, 61:259–299.

Atilgan, Y., Bali, T. G., Demirtas, K. O., and Gunaydin, A. D. (2020). Left-tail momentum: Underreaction to bad news, costly arbitrage and equity returns. *Journal of Financial Economics*, 135(3):725–753.

Atmaz, A. and Basak, S. (2018). Belief dispersion in the stock market. *Journal of Finance*, 73:1225–1279.

Avramov, D., Chordia, T., Jostova, G., and Philipov, A. (2009). Dispersion in analysts' earnings forecasts and credit rating. *Journal of Financial Economics*, 91(1):83–101.

Bali, T. G., Beckmeyer, H., Moerke, M., and Weigert, F. (2023). Option return predictability with machine learning and big data, forthcoming. *Review of Financial Studies*.

Bali, T. G., Cakici, N., and Whitelaw, R. F. (2011). Maxing out: Stocks as lotteries and the cross-section of expected returns. *Journal of Financial Economics*, 99:427–446.

Barber, B. M. and Odean, T. (2008). All that glitters: The effect of attention and news on the buying behavior of individual and institutional investors. *Review of Financial Studies*, 21:785–818.

Barberis, N. (2018). Psychology-based models of asset prices and trading volume. In *Handbook of Behavioral Economics: Applications and Foundations 1*, volume 1, pages 79–175. Elsevier.

Berkman, H., Dimitrov, V., Jain, P. C., Koch, P. D., and Tice, S. (2009). Sell on the news: Differences of opinion, short-sales constraints, and returns around earnings announcements. *Journal of Financial Economics*, 92(3):376–399.

Bernard, V. L. and Thomas, J. K. (1989). Post-earnings-announcement drift: delayed price response or risk premium? *Journal of Accounting Research*, 27:1–36.

Carlin, B. I., Longstaff, F. A., and Matoba, K. (2014). Disagreement and asset prices. *Journal of*

Financial Economics, 114:226–238.

Chan, L. K., Karceski, J., and Lakonishok, J. (2007). Analysts' conflicts of interest and biases in earnings forecasts. *Journal of Financial and Quantitative Analysis*, 42:893–913.

Chen, J., Hong, H., and Stein, J. C. (2002). Breadth of ownership and stock returns. *Journal of Financial Economics*, 66:171–205.

Cohen, L. and Lou, D. (2012). Complicated firms. *Journal of Financial Economics*, 104:383–400.

Cooper, M. J., Gulen, H., and Schill, M. J. (2008). Asset growth and the cross-section of stock returns. *Journal of Finance*, 63:1609–1651.

Daniel, K., Grinblatt, M., Titman, S., and Wermers, R. (1997). Measuring mutual fund performance with characteristic-based benchmarks. *Journal of Finance*, 52:1035–1058.

Daniel, K., Hirshleifer, D., and Sun, L. (2020). Short-and long-horizon behavioral factors. *Review of Financial Studies*, 33:1673–1736.

Datar, V. T., Naik, N. Y., and Radcliffe, R. (1998). Liquidity and stock returns: An alternative test. *Journal of Financial markets*, 1(2):203–219.

Didisheim, A., Ke, S., Kelly, B. T., and Malamud, S. (2023). Complexity in factor pricing models. *Swiss Finance Institute Research Paper*, (23-19).

Diether, K. B., Malloy, C. J., and Scherbina, A. (2002). Differences of opinion and the cross section of stock returns. *Journal of Finance*, 57:2113–2141.

Doukas, J. A., Kim, C. F., and Pantzalis, C. (2006). Divergence of opinion and equity returns. *Journal of Financial and Quantitative Analysis*, 41(3):573–606.

Dugar, A. and Nathan, S. (1995). The effect of investment banking relationships on financial analysts' earnings forecasts and investment recommendations. *Contemporary Accounting Research*, 12:131–160.

Engelberg, J., McLean, R. D., and Pontiff, J. (2018). Anomalies and news. *Journal of Finance*, 73:1971–2001.

Fama, E. F. and French, K. R. (1993). Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics*, 33:3–56.

Fama, E. F. and French, K. R. (2008). Dissecting anomalies. *Journal of Finance*, 63:1653–1678.

Fama, E. F. and French, K. R. (2015). A five-factor asset pricing model. *Journal of Financial Economics*, 116:1–22.

Fama, E. F. and French, K. R. (2017). International tests of a five-factor asset pricing model. *Journal of Financial Economics*, 123(3):441–463.

Fama, E. F. and French, K. R. (2018). Choosing factors. *Journal of Financial Economics*, 128:234–252.

Foster, G., Olsen, C., and Shevlin, T. (1984). Earnings releases, anomalies, and the behavior of security returns. *Accounting Review*, pages 574–603.

Freyberger, J., Neuhierl, A., and Weber, M. (2020). Dissecting characteristics nonparametrically. *Review of Financial Studies*, 33:2326–2377.

Goetzmann, W. N. and Massa, M. (2005). Dispersion of opinion and stock returns. *Journal of Financial Markets*, 8:324–349.

Gu, S., Kelly, B., and Xiu, D. (2020). Empirical asset pricing via machine learning. *Review of Financial Studies*, 33:2223–2273.

Harrison, J. M. and Kreps, D. M. (1978). Speculative investor behavior in a stock market with heterogeneous expectations. *Quarterly Journal of Economics*, 92:323–336.

Hirshleifer, D. and Teoh, S. H. (2003). Limited attention, information disclosure, and financial reporting. *Journal of Accounting and Economics*, 36:337–386.

Hong, H. and Stein, J. C. (1999). A unified theory of underreaction, momentum trading, and overreaction in asset markets. *Journal of Finance*, 54:2143–2184.

Hong, H. and Stein, J. C. (2003). Differences of opinion, short-sales constraints, and market crashes. *Review of Financial Studies*, 16:487–525.

Hou, K., Xue, C., and Zhang, L. (2015). Digesting anomalies: An investment approach. *Review of Financial Studies*, 28:650–705.

Jegadeesh, N. (1990). Evidence of predictable behavior of security returns. *Journal of Finance*, 45(3):881–898.

Jegadeesh, N. and Titman, S. (1993). Returns to buying winners and selling losers: Implications for stock market efficiency. *Journal of Finance*, 48:65–91.

Jensen, T. I., Kelly, B. T., Malamud, S., and Pedersen, L. H. (2022a). Machine learning and the implementable efficient frontier. *Swiss Finance Institute Research Paper*, (22-63).

Jensen, T. I., Kelly, B. T., and Pedersen, L. H. (2022b). Is there a replication crisis in finance?, forthcoming. *Journal of Finance*.

Johnson, T. C. (2004). Forecast dispersion and the cross section of expected returns. *Journal of Finance*, 59:1957–1978.

Kahneman, D. (1973). *Attention and effort*. Englewood Cliffs, NJ: Prentice-Hall.

Kelly, B. T., Malamud, S., et al. (2021). The virtue of complexity in machine learning portfolios.

Swiss Finance Institute Research Paper, (21-90).

Kelly, B. T., Pruitt, S., and Su, Y. (2019). Characteristics are covariances: A unified model of risk and return. *Journal of Financial Economics*, 134:501–524.

Kumar, A. (2009). Who gambles in the stock market? *Journal of Finance*, 64:1889–1933.

Lee, C. M. C., Sun, S. T., Wang, R., and Zhang, R. (2019). Technological links and predictable returns. *Journal of Financial Economics*, 132:76–96.

Malkiel, B. G. and Cragg, J. G. (1970). Expectations and the structure of share prices. *American Economic Review*, 60(4):601–617.

Michaely, R. and Womack, K. L. (1999). Conflict of interest and the credibility of underwriter analyst recommendations. *Review of Financial Studies*, 12:653–686.

Miller, E. M. (1977). Risk, uncertainty and divergence of opinion. *Journal of finance*, 32:1151–1168.

Moskowitz, T. J. and Grinblatt, M. (1999). Do industries explain momentum? *Journal of Finance*, 54:1249–1290.

Nagel, S. (2005). Short sales, institutional investors and the cross-section of stock returns. *Journal of Financial Economics*, 78:277–309.

Newey, W. K. and West, K. D. (1987). A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. *Econometrica*, 55:703–708.

Pontiff, J. (2006). Costly arbitrage and the myth of idiosyncratic risk. *Journal of Accounting and Economics*, 42:35–52.

Porta, L., Rafael, J. L., Shleifer, A., and Vishny, R. (1997). Good news for value stocks: Further evidence on market efficiency. *Journal of Finance*, 52:859–874.

Qu, S., Starks, L., and Yan, H. (2003). Risk, dispersion of analyst forecasts and stock returns.

Rahimi, A. and Recht, B. (2007). Random features for large-scale kernel machines. In *Proceedings of the 20th International Conference on Neural Information Processing Systems*, pages 1177–1184.

Scheinkman, J. A. and Xiong, W. (2003). Overconfidence and speculative bubbles. *Journal of Political Economy*, 111:1183–1220.

Shleifer, A. and Vishny, R. W. (1997). The limits of arbitrage. *Journal of Finance*, 52:35–55.

Stambaugh, R. F., Yu, J., and Yuan, Y. (2012). The short of it: Investor sentiment and anomalies. *Journal of Financial Economics*, 104:288–302.

Stambaugh, R. F., Yu, J., and Yuan, Y. (2015). Arbitrage asymmetry and the idiosyncratic volatility puzzle. *Journal of Finance*, 70:1903–1948.

Stambaugh, R. F. and Yuan, Y. (2017). Mispricing factors. *Review of Financial Studies*, 30:1270–

1315.

van Binsbergen, J. H., Han, X., and Lopez-Lira, A. (2023). Man vs. machine learning: The term structure of earnings expectations and conditional biases, forthcoming. *Review of Financial Studies*.

Yu, J. (2011). Disagreement and return predictability of stock portfolios. *Journal of Financial Economics*, 99:162–183.

Zhang, X. F. (2006). Information uncertainty and analyst forecast behavior. *Contemporary Accounting Research*, 23:565–590.

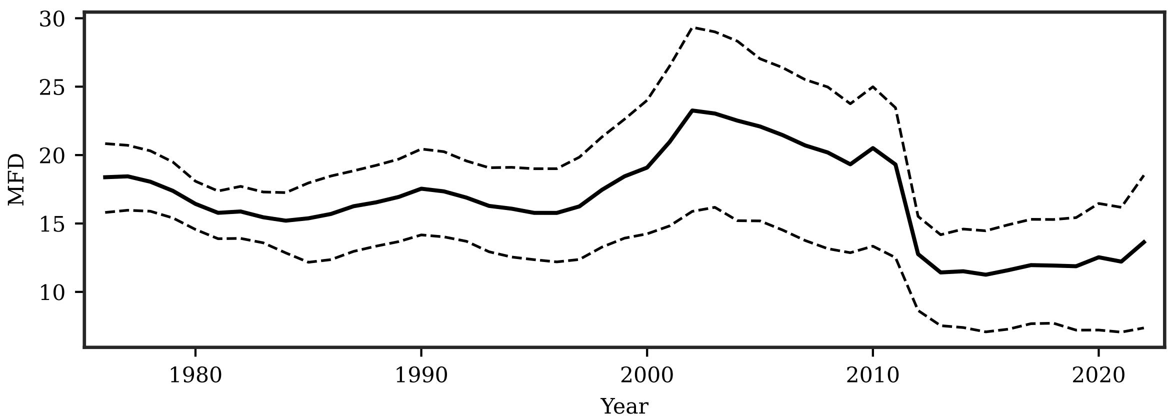

Figure 1: MFD Over Time

The figure shows the yearly time-series plot of the average stock-level MFD. The dashed lines show the time-series of the interquartile range. The sample period is from August 1976 to December 2022.

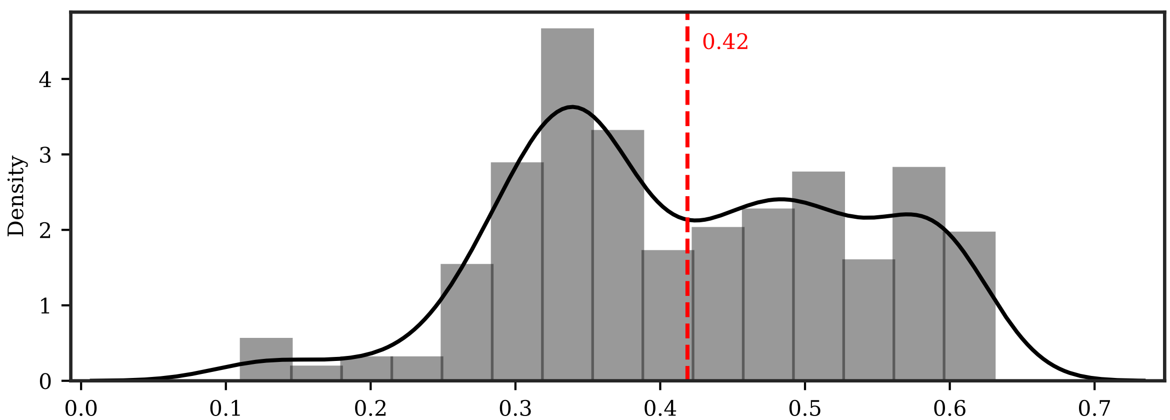

Figure 2: Cross-Sectional Correlation Between AFD and MFD

The figure shows the histogram and density of the monthly cross-sectional correlation between analyst forecast dispersion (AFD, Diether et al., 2002) and MFD. Each month t, the correlation between MFD and AFD is measured using Spearman's ρ. The vertical dashed red line shows the average monthly cross-sectional correlation between MFD and AFD. The sample period is from January 1983 to March 2022.

Table 1: Descriptive Statistics

	Mean	Sd	10^{th}	Q1	Q2	Q3	90^{th}
RET_{t+1}	0.60	10.70	-12.21	-5.93	0.11	6.53	13.96
MFD	16.04	5.74	9.61	11.66	14.91	19.38	24.39
SUE	-0.08	1.76	-1.89	-0.76	0.01	0.77	1.77
AG	0.28	0.71	-0.07	0.01	0.10	0.26	0.68
MOM	0.23	0.52	-0.27	-0.09	0.12	0.40	0.81
ILLIQ	0.90	2.57	0.00	0.02	0.10	0.55	2.17
OP	0.23	0.38	-0.06	0.12	0.23	0.34	0.50
IVOL	0.03	0.01	0.01	0.02	0.03	0.03	0.04
BETA	1.20	0.62	0.48	0.77	1.12	1.54	2.03
SIZE ($\times 10^{-9}$)	3.81	17.26	0.07	0.17	0.52	1.77	6.25
BM	0.62	0.46	0.16	0.29	0.52	0.83	1.22
MAX	0.04	0.02	0.02	0.02	0.03	0.04	0.06
TURN ($\times 10^{3}$)	6.13	8.47	1.24	2.51	4.42	7.35	11.86
STR	0.02	0.12	-0.11	-0.05	0.01	0.08	0.16
AFD	0.16	0.42	0.01	0.02	0.05	0.11	0.30

The table reports the summary statistics for the cross-sectional variables. The sample consists of all common stocks that are listed on NYSE, Amex, and Nasdaq. Financial firms (with one-digit SIC = 6), utility firms (with two-digit SIC = 49), and stocks trading below $5/share are excluded from the analysis. RET_{t+1} is the one-month-ahead return in excess of the risk-free rate of individual stocks. MFD is the machine forecast disagreement variable. SIZE is the firm's market capitalization at the end of month $t-1$ (Fama and French, 2008). BM is the ratio of the firm's book value of equity divided by its market capitalization, following Fama and French (2008). Asset Growth (AG) is a percentage of total asset growth between two consecutive fiscal years, following Cooper et al. (2008). Operating profits (OP) is the ratio of operating profits to book equity, following Fama and French (2015). Short-term reversal (STR) is the stock's one-month lagged return, following Jegadeesh (1990). MOM is the stock's cumulative return from the start of month $t-12$ to the end of month $t-1$, skipping STR, following Jegadeesh and Titman (1993). ILLIQ is the Amihud (2002) illiquidity measure computed using daily data over the last 126 trading days. TURN is the share turnover computed over the last 126 trading days, following Datar et al. (1998). SUE is the standardized unexpected earnings, following Foster et al. (1984). IVOL is the standard deviation of daily residuals estimated from the daily regression of excess stock returns on the excess market return over the previous year, following Ali et al. (2003). MAX is the average of the five highest daily returns of each stock in month $t-1$, following Bali et al. (2011). All variables are winsorized at the 1% level for both tails to mitigate the effect of outliers. The mean, standard deviation (Sd), 10^{th} percentile (10^{th}), first up to third quartil, and the 90^{th} percentile (90^{th}) are shown. The sample is from August 1976 to December 2022.

Table 2: Cross-Sectional Correlations to MFD

	Mean	Sd	10^{th}	Q1	Q2	Q3	90^{th}
RET_{t+1}	-0.07	0.13	-0.22	-0.14	-0.06	0.01	0.09
SUE	-0.05	0.07	-0.14	-0.10	-0.05	-0.01	0.02
AG	0.21	0.12	0.07	0.14	0.22	0.29	0.35
MOM	-0.15	0.17	-0.34	-0.26	-0.18	-0.05	0.08
ILLIQ	0.39	0.12	0.22	0.33	0.40	0.47	0.52
OP	-0.44	0.18	-0.61	-0.58	-0.53	-0.31	-0.13
IVOL	0.74	0.05	0.68	0.71	0.74	0.77	0.81
BETA	0.30	0.12	0.15	0.20	0.28	0.38	0.49
SIZE	-0.42	0.12	-0.54	-0.51	-0.44	-0.36	-0.25
BM	-0.07	0.15	-0.27	-0.17	-0.07	0.06	0.12
MAX	0.65	0.06	0.56	0.61	0.65	0.69	0.72
TURN	0.19	0.11	0.06	0.11	0.18	0.26	0.34
STR	0.03	0.14	-0.16	-0.05	0.04	0.12	0.20
AFD	0.39	0.15	0.23	0.32	0.38	0.50	0.59

The table reports summary statistics on the cross-sectional correlations of various stock characteristics with MFD. Correlation is measured using Spearman's ρ. The stock characteristics are defined in Table 1. The mean, standard deviation (Sd), 10^{th} percentile (10^{th}), first, second, and third quartil, and the 90^{th} percentile (90^{th}) are shown. The sample is from August 1976 to December 2022.

Table 3: Univariate Portfolio Sorts on MFD

Panel A: Equal-Weighted Portfolios

	Excess Return	t-stat	CAPM	t-stat	FF6	t-stat	HXZ	t-stat	SY	t-stat	DHS	t-stat
Low	0.95***	(5.40)	0.90***	(5.13)	1.01***	(5.50)	1.01***	(5.14)	1.05***	(5.42)	0.94***	(5.12)
2	1.03***	(5.45)	0.97***	(5.19)	1.08***	(5.48)	1.08***	(5.02)	1.12***	(5.46)	1.02***	(5.12)
3	1.02***	(4.89)	0.93***	(4.67)	1.08***	(5.12)	1.07***	(4.64)	1.12***	(5.21)	1.02***	(4.67)
4	0.96***	(4.35)	0.87***	(4.11)	1.02***	(4.50)	0.98***	(3.89)	1.04***	(4.43)	0.97***	(4.12)
5	0.94***	(3.92)	0.83***	(3.66)	0.99***	(4.05)	0.92***	(3.42)	0.99***	(3.93)	0.91***	(3.66)
6	0.83***	(3.28)	0.73***	(3.03)	0.93***	(3.59)	0.86***	(3.03)	0.92***	(3.38)	0.85***	(3.26)
7	0.67**	(2.46)	0.56**	(2.16)	0.75***	(2.73)	0.69**	(2.26)	0.76**	(2.52)	0.72***	(2.58)
8	0.35	(1.13)	0.23	(0.80)	0.48	(1.55)	0.42	(1.24)	0.45	(1.31)	0.48	(1.57)
9	-0.06	(-0.19)	-0.18	(-0.57)	0.15	(0.46)	0.11	(0.29)	0.20	(0.55)	0.17	(0.50)
High	-0.64*	(-1.73)	-0.77**	(-2.16)	-0.35	(-0.95)	-0.40	(-1.03)	-0.30	(-0.72)	-0.34	(-0.93)
H-L	-1.59***	(-5.45)	-1.67***	(-5.76)	-1.36***	(-4.83)	-1.42***	(-4.83)	-1.35***	(-4.22)	-1.27***	(-4.37)

Panel B: Value-Weighted Portfolios

	Excess Return	t-stat	CAPM	t-stat	FF6	t-stat	HXZ	t-stat	SY	t-stat	DHS	t-stat
Low	0.75***	(5.09)	0.75***	(4.69)	0.83***	(4.83)	0.83***	(4.78)	0.81***	(4.32)	0.75***	(4.35)
2	0.80***	(4.54)	0.80***	(4.29)	0.88***	(4.56)	0.88***	(4.27)	0.78***	(3.82)	0.82***	(4.15)
3	0.75***	(3.88)	0.72***	(3.66)	0.85***	(4.17)	0.85***	(3.94)	0.83***	(3.70)	0.73***	(3.52)
4	0.77***	(3.44)	0.73***	(3.21)	0.86***	(3.60)	0.89***	(3.51)	0.94***	(3.58)	0.78***	(3.22)
5	0.84***	(3.33)	0.78***	(3.08)	1.00***	(3.90)	0.92***	(3.36)	0.98***	(3.45)	0.88***	(3.24)
6	0.67***	(2.63)	0.66**	(2.52)	0.82***	(3.00)	0.89***	(3.04)	0.82***	(2.77)	0.79***	(2.86)
7	0.55*	(1.79)	0.50	(1.59)	0.77**	(2.27)	0.71**	(2.04)	0.73*	(1.85)	0.68**	(2.06)
8	0.41	(1.22)	0.34	(1.03)	0.66*	(1.92)	0.58	(1.59)	0.72*	(1.83)	0.64*	(1.78)
9	0.12	(0.34)	0.06	(0.16)	0.39	(1.05)	0.40	(0.99)	0.52	(1.24)	0.38	(0.97)
High	-0.42	(-1.06)	-0.50	(-1.27)	-0.03	(-0.07)	-0.02	(-0.04)	0.03	(0.05)	-0.09	(-0.22)
H-L	-1.17***	(-3.51)	-1.26***	(-3.79)	-0.86***	(-2.59)	-0.85**	(-2.33)	-0.78**	(-2.05)	-0.84**	(-2.40)

The table reports the average monthly excess returns and alphas on univariate portfolios of stocks sorted by MFD. Each month t, stocks are sorted into decile portfolios by MFD constructed using data up to month $t-1$. Panel A reports equal-weighted portfolio sorts whereas Panel B reports value-weighted portfolio sorts. Excess Return is the return in excess of the risk-free rate. Alpha is the intercept from a time-series regression of monthly excess returns on the factors of alternative models: the CAPM, Fama and French (2018) six-factor model (FF6), Stambaugh and Yuan (2017) mispricing factor model (SY), Hou et al. (2015) q-factor model (HXZ), and the Daniel et al. (2020) behavioral factor model (DHS). t-stat denote Newey and West (1987) adjusted t-statistics. *, **, and *** denote statistical significance at the 10%, 5%, and 1% level, respectively. The sample period is from August 1976 to December 2022 (December 2016 in case of SY).

Table 4: Transition Matrix

	Low	2	3	4	5	6	7	8	9	High
Low	50	23	11	6	4	2	1	1	1	0
2	24	26	18	12	8	5	3	2	1	1
3	12	20	20	16	12	8	6	4	2	1
4	6	14	17	17	15	12	9	6	3	1
5	3	9	14	16	16	14	12	9	5	2
6	2	6	10	13	16	16	15	12	8	4
7	1	3	6	10	13	16	17	16	12	6
8	0	2	4	7	10	14	17	19	17	10
9	0	1	2	4	6	9	14	19	24	22
High	0	0	1	1	2	4	7	12	24	49

The table reports transition probabilities for MFD at a lag of 12 months from August 1976 to December 2022. For each month t, all stocks are sorted into deciles on an ascending ordering of the MFD. The procedure is repeated in month $t + 12$. Low is the portfolio of stocks with the lowest MFD and High is the portfolio of stocks with the highest MFD. For each decile MFD in month t, the percentage of stocks that fall into each of the month $t + 12$ MFD decile is calculated. Transition probabilities are averaged across time. Each row corresponds to a different month t MFD portfolio and each column corresponds to a different month $t + 12$ MFD portfolio.

Table 5: Long-Term Predictive Power

Panel A: Equal-Weighted Portfolios

	$t+2$	$t+3$	$t+4$	$t+5$	$t+6$	$t+7$	$t+8$	$t+9$	$t+10$	$t+11$	$t+12$
Low	0.91***	0.91***	0.92***	0.86***	0.88***	0.88***	0.87***	0.85***	0.87***	0.87***	0.84***
	(5.13)	(5.19)	(5.22)	(4.81)	(4.92)	(4.85)	(4.84)	(4.69)	(4.74)	(4.66)	(4.60)
2	0.97***	0.96***	0.95***	1.00***	0.90***	0.91***	0.93***	0.86***	0.90***	0.88***	0.84***
	(5.11)	(4.99)	(4.91)	(5.16)	(4.65)	(4.69)	(4.75)	(4.32)	(4.54)	(4.49)	(4.23)
3	1.02***	0.92***	0.92***	0.89***	0.91***	0.92***	0.93***	0.94***	0.92***	0.91***	0.89***
	(4.95)	(4.52)	(4.40)	(4.26)	(4.38)	(4.44)	(4.46)	(4.47)	(4.44)	(4.21)	(4.16)
4	1.00***	0.94***	0.97***	0.89***	0.88***	0.86***	0.91***	0.85***	0.90***	0.89***	0.83***
	(4.56)	(4.21)	(4.34)	(4.05)	(3.90)	(3.81)	(4.06)	(3.77)	(3.96)	(3.98)	(3.66)
5	0.90***	0.94***	0.91***	0.85***	0.83***	0.85***	0.78***	0.82***	0.80***	0.80***	0.86***
	(3.69)	(3.91)	(3.82)	(3.56)	(3.48)	(3.54)	(3.28)	(3.38)	(3.31)	(3.30)	(3.53)
6	0.81***	0.78***	0.76***	0.74***	0.77***	0.75***	0.77***	0.73***	0.72***	0.71***	0.78***
	(3.19)	(3.06)	(3.02)	(2.93)	(3.03)	(2.90)	(3.01)	(2.86)	(2.85)	(2.79)	(3.02)
7	0.61**	0.65**	0.60**	0.63**	0.61**	0.62**	0.59**	0.67**	0.63**	0.61**	0.60**
	(2.21)	(2.39)	(2.21)	(2.28)	(2.27)	(2.34)	(2.16)	(2.50)	(2.32)	(2.26)	(2.18)
8	0.38	0.46	0.50	0.49*	0.43	0.50*	0.50*	0.52*	0.49*	0.50*	0.50*
	(1.26)	(1.51)	(1.64)	(1.65)	(1.46)	(1.69)	(1.71)	(1.80)	(1.69)	(1.75)	(1.76)
9	0.02	0.09	0.06	0.05	0.22	0.21	0.23	0.23	0.26	0.33	0.36
	(0.06)	(0.27)	(0.19)	(0.16)	(0.68)	(0.64)	(0.70)	(0.71)	(0.82)	(1.02)	(1.14)
High	-0.56	-0.52	-0.46	-0.38	-0.33	-0.28	-0.25	-0.16	-0.11	-0.14	-0.07
	(-1.53)	(-1.42)	(-1.26)	(-1.04)	(-0.90)	(-0.79)	(-0.70)	(-0.44)	(-0.32)	(-0.39)	(-0.19)
H-L	-1.47***	-1.44***	-1.38***	-1.23***	-1.20***	-1.16***	-1.12***	-1.00***	-0.98***	-1.00***	-0.90***
	(-5.13)	(-4.99)	(-4.88)	(-4.35)	(-4.37)	(-4.30)	(-4.14)	(-3.77)	(-3.77)	(-3.88)	(-3.46)
FF6	-1.24***	-1.20***	-1.16***	-1.01***	-1.00***	-0.96***	-0.94***	-0.79***	-0.79***	-0.80***	-0.70***
	(-4.53)	(-4.42)	(-4.27)	(-3.82)	(-3.82)	(-3.79)	(-3.66)	(-3.20)	(-3.24)	(-3.33)	(-2.87)

Panel B: Value-Weighted Portfolios											
	$t+2$	$t+3$	$t+4$	$t+5$	$t+6$	$t+7$	$t+8$	$t+9$	$t+10$	$t+11$	$t+12$
Low	0.74***	0.75***	0.76***	0.76***	0.76***	0.75***	0.76***	0.76***	0.78***	0.75***	0.73***
	(4.80)	(4.97)	(5.01)	(4.90)	(4.88)	(4.83)	(5.05)	(4.98)	(4.99)	(4.87)	(4.80)
2	0.79***	0.78***	0.78***	0.73***	0.65***	0.73***	0.76***	0.74***	0.76***	0.76***	0.75***
	(4.65)	(4.44)	(4.60)	(4.06)	(3.71)	(4.13)	(4.21)	(4.20)	(4.27)	(4.16)	(4.16)
3	0.72***	0.70***	0.65***	0.72***	0.77***	0.77***	0.78***	0.74***	0.70***	0.82***	0.83***
	(3.42)	(3.60)	(3.14)	(3.71)	(3.74)	(3.82)	(3.89)	(3.64)	(3.55)	(4.08)	(4.25)
4	0.88***	0.68***	0.81***	0.76***	0.79***	0.81***	0.81***	0.87***	0.82***	0.72***	0.69***
	(4.05)	(3.06)	(3.56)	(3.38)	(3.63)	(3.67)	(3.41)	(3.90)	(3.61)	(3.01)	(2.93)
5	0.70***	0.85***	0.75***	0.72***	0.83***	0.78***	0.73***	0.80***	0.71***	0.71***	0.75***
	(2.88)	(3.37)	(3.00)	(2.86)	(3.70)	(3.16)	(2.90)	(3.16)	(2.85)	(2.85)	(2.98)
6	0.69**	0.63**	0.73**	0.67**	0.53**	0.66**	0.72***	0.70***	0.69***	0.64**	0.82***
	(2.57)	(2.47)	(2.57)	(2.42)	(1.98)	(2.41)	(2.76)	(2.73)	(2.67)	(2.49)	(3.09)
7	0.56*	0.66**	0.60**	0.48	0.67**	0.59**	0.51*	0.65**	0.70**	0.72***	0.51*
	(1.79)	(2.05)	(2.05)	(1.60)	(2.13)	(2.06)	(1.72)	(2.16)	(2.36)	(2.68)	(1.72)
8	0.37	0.44	0.42	0.51	0.59*	0.46	0.48	0.60**	0.44	0.52	0.63**
	(1.09)	(1.34)	(1.30)	(1.51)	(1.81)	(1.40)	(1.47)	(2.01)	(1.41)	(1.57)	(2.11)
9	0.12	0.24	0.31	0.25	0.39	0.47	0.46	0.37	0.45	0.53	0.58*
	(0.32)	(0.64)	(0.87)	(0.73)	(1.07)	(1.34)	(1.31)	(1.04)	(1.32)	(1.51)	(1.65)
High	-0.32	-0.29	-0.18	-0.02	-0.05	0.05	-0.00	0.12	0.12	0.11	0.10
	(-0.80)	(-0.73)	(-0.46)	(-0.05)	(-0.13)	(0.14)	(-0.01)	(0.31)	(0.31)	(0.27)	(0.24)
H-L	-1.06***	-1.04***	-0.94***	-0.78**	-0.81**	-0.70**	-0.76**	-0.64*	-0.66**	-0.64*	-0.64*
	(-3.15)	(-3.00)	(-2.82)	(-2.27)	(-2.45)	(-2.11)	(-2.33)	(-1.95)	(-2.05)	(-1.95)	(-1.83)
FF6	-0.73**	-0.72**	-0.64*	-0.45	-0.50	-0.44	-0.50	-0.34	-0.42	-0.36	-0.39
	(-2.18)	(-2.12)	(-1.92)	(-1.41)	(-1.55)	(-1.33)	(-1.57)	(-1.06)	(-1.33)	(-1.18)	(-1.20)

The table reports the long-term predictive power of MFD. For each month $t+n$, where $n \in \{2, \ldots, 12\}$, individual stocks are sorted into decile portfolios based on month-t MFD. Panel A reports equal-weighted portfolio sorts. Panel B reports value-weighted portfolio sorts. Returns are average monthly excess returns. The table also shows the Fama and French (2018) six-factor alphas for each of the MFD-sorted high-minus-low portfolios. Newey and West (1987) adjusted $t-$statistics are reported in parentheses. *, **, and *** denote statistical significance at the 10%, 5%, and 1% level, respectively. The sample period is from August 1976 to December 2022.

Table 6: Analyst Forecast Dispersion and MFD

Panel A: Average AFD in MFD Decile Portfolio

	Low	2	3	4	5	6	7	8	9	High	H-L	t-stat
AFD	0.04	0.06	0.08	0.10	0.13	0.16	0.21	0.26	0.31	0.33	0.29***	24.22

Panel B: Bivariate Portfolio Sort on AFD

	Low	2	3	4	5	6	7	8	9	High	H-L	t-stat	FF6	t-stat
AFD Low	1.02	1.00	0.96	1.08	1.09	1.15	1.05	1.07	0.89	0.46	-0.56**	-2.01	-0.40	-1.44
AFD 2	1.03	0.94	0.92	0.89	0.94	0.93	0.78	0.71	0.53	-0.03	-1.06***	-3.33	-0.96***	-2.98
AFD 3	0.95	0.91	0.91	0.84	1.00	0.79	0.88	0.59	0.35	-0.06	-1.00***	-3.03	-0.81**	-2.30
AFD 4	0.85	0.91	0.90	0.62	0.85	0.75	0.65	0.32	0.03	-0.49	-1.34***	-3.43	-0.99***	-2.64
AFD High	0.73	0.80	0.65	0.47	0.31	0.16	-0.11	-0.01	-0.34	-0.54	-1.27***	-3.35	-0.95**	-2.42
AFD H-L	-0.29	-0.20	-0.30	-0.61	-0.78	-0.99	-1.16	-1.08	-1.23	-0.99	-0.71***	-2.97	-0.56**	-2.15

Panel A reports the average analyst forecast dispersion (AFD) of the MFD-sorted univariate decile portfolios. Low (high) AFD indicates a lower (higher) average forecast dispersion. Panel B reports 5x10 dependent bivariate equal-weighted portfolio sorts. First, quintile portfolios are formed every month using AFD. Next, decile portfolios are formed based on MFD within each firm-specific AFD quintile. Newey and West (1987) adjusted $t-$statistics are reported in parentheses. *, **, and *** denote statistical significance at the 10%, 5%, and 1% level, respectively. The sample period is from January 1983 to December 2022.

Table 7: Average Stock Characteristics of MFD-sorted Portfolios

	Low	2	3	4	5	6	7	8	9	High	H-L	t-stat
MFD	8.72	10.34	11.66	12.92	14.22	15.63	17.29	19.38	22.33	27.22	18.49***	(24.29)
SUE	0.08	0.05	0.05	0.03	0.04	0.03	0.01	-0.02	-0.07	-0.16	-0.24***	(-7.22)
AG	0.06	0.07	0.07	0.08	0.09	0.11	0.12	0.14	0.18	0.40	0.34***	(9.65)
MOM	0.17	0.16	0.16	0.15	0.14	0.13	0.11	0.08	0.04	0.01	-0.15***	(-5.46)
ILLIQ	0.03	0.03	0.05	0.08	0.12	0.16	0.21	0.26	0.31	0.33	0.30***	(5.06)
OP	0.31	0.29	0.27	0.26	0.25	0.23	0.21	0.17	0.10	-0.09	-0.39***	(-16.05)
IVOL	0.01	0.02	0.02	0.02	0.02	0.03	0.03	0.03	0.04	0.04	0.03***	(30.28)
BETA	0.92	0.97	1.02	1.07	1.12	1.18	1.25	1.34	1.44	1.60	0.68***	(16.35)
SIZE ($\times 10^{-9}$)	2.71	1.57	1.08	0.74	0.53	0.40	0.33	0.28	0.26	0.23	-2.48***	(-7.98)
BM	0.51	0.53	0.55	0.56	0.57	0.56	0.56	0.53	0.48	0.38	-0.13***	(-5.20)
MAX	0.02	0.02	0.02	0.03	0.03	0.03	0.04	0.04	0.05	0.05	0.04***	(28.12)
TURN ($\times 10^{3}$)	3.57	3.91	4.12	4.26	4.42	4.58	4.84	5.16	5.53	6.10	2.53***	(10.83)
STR ($\times 10^{3}$)	4.75	7.36	7.18	8.50	9.35	11.48	12.58	14.05	15.48	17.63	12.88***	(3.47)

The table reports the time-series averages of the monthly cross-sectional median for stock characteristics of univariate decile portfolios formed based on MFD. Low (High) denotes the portfolio of stocks with the lowest (highest) MFD. The last two columns show the differences between the High and Low (H-L) and the associated Newey and West (1987) adjusted $t-$statistics (t-stat). *, **, and *** denote statistical significance at the 10%, 5%, and 1% level, respectively. The sample period is from August 1976 to December 2022.

Table 8: Bivariate Portfolio Sorts

Panel A: Equal-Weighted Portfolios												
	SUE	AG	MOM	ILLIQ	OP	IVOL	BETA	SIZE	BM	MAX	TURN	STR
1	0.97***	1.02***	1.02***	1.07***	0.97***	1.03***	1.03***	1.10***	0.97***	0.85***	0.97***	0.96***
	(5.50)	(5.62)	(5.39)	(5.79)	(5.16)	(4.69)	(5.22)	(5.79)	(5.31)	(4.14)	(5.37)	(5.30)
2	0.95***	1.01***	0.98***	1.01***	0.90***	0.96***	1.05***	1.04***	1.00***	0.85***	0.99***	1.00***
	(5.04)	(4.97)	(4.92)	(5.11)	(4.63)	(4.30)	(5.19)	(5.17)	(5.10)	(3.96)	(5.00)	(5.19)
3	1.00***	0.93***	0.95***	0.97***	0.89***	0.90***	0.98***	0.97***	1.03***	0.91***	0.98***	0.98***
	(4.98)	(4.36)	(4.41)	(4.64)	(4.17)	(3.82)	(4.70)	(4.61)	(4.86)	(4.05)	(4.67)	(4.66)
4	0.97***	0.93***	0.89***	0.93***	0.91***	0.85***	1.04***	0.94***	0.98***	0.78***	0.96***	0.94***
	(4.56)	(4.14)	(4.00)	(4.16)	(4.02)	(3.53)	(4.75)	(4.17)	(4.35)	(3.39)	(4.33)	(4.23)
5	0.95***	0.90***	0.84***	0.83***	0.79***	0.78***	0.90***	0.87***	0.91***	0.76***	0.90***	0.86***
	(4.16)	(3.79)	(3.59)	(3.55)	(3.32)	(3.10)	(3.95)	(3.68)	(3.83)	(3.10)	(3.85)	(3.58)
6	0.88***	0.74***	0.82***	0.75***	0.67***	0.64**	0.86***	0.66***	0.84***	0.68***	0.81***	0.75***
	(3.64)	(2.87)	(3.28)	(2.98)	(2.67)	(2.56)	(3.50)	(2.62)	(3.30)	(2.67)	(3.22)	(2.98)
7	0.78***	0.61**	0.66**	0.59**	0.68***	0.51**	0.75***	0.61**	0.67**	0.52*	0.60**	0.67**
	(3.02)	(2.23)	(2.52)	(2.24)	(2.60)	(1.97)	(3.02)	(2.23)	(2.51)	(1.93)	(2.24)	(2.41)
8	0.52*	0.42	0.47*	0.35	0.52*	0.37	0.52**	0.35	0.39	0.51*	0.38	0.36
	(1.82)	(1.45)	(1.70)	(1.18)	(1.89)	(1.39)	(1.99)	(1.20)	(1.36)	(1.81)	(1.31)	(1.23)
9	0.18	0.12	0.14	0.04	0.31	0.19	0.25	-0.02	0.00	0.32	-0.01	0.03
	(0.58)	(0.38)	(0.46)	(0.11)	(1.05)	(0.73)	(0.89)	(-0.06)	(0.01)	(1.10)	(-0.03)	(0.08)
High	-0.42	-0.49	-0.35	-0.52	-0.37	-0.03	-0.33	-0.45	-0.62*	-0.04	-0.62*	-0.58
	(-1.20)	(-1.43)	(-1.00)	(-1.42)	(-1.09)	(-0.10)	(-1.06)	(-1.24)	(-1.75)	(-0.14)	(-1.77)	(-1.60)
H-L	-1.39***	-1.52***	-1.37***	-1.58***	-1.33***	-1.06***	-1.35***	-1.55***	-1.59***	-0.89***	-1.60***	-1.54***
	(-5.03)	(-6.03)	(-5.15)	(-5.71)	(-6.11)	(-7.82)	(-6.77)	(-5.61)	(-6.02)	(-4.74)	(-6.16)	(-5.74)
FF6	-1.17***	-1.34***	-1.18***	-1.31***	-1.22***	-0.98***	-1.25***	-1.27***	-1.40***	-0.72***	-1.39***	-1.36***
	(-4.24)	(-5.51)	(-4.78)	(-4.94)	(-5.17)	(-7.45)	(-6.58)	(-4.86)	(-5.36)	(-4.37)	(-5.64)	(-5.46)

Panel B: Value-Weighted Portfolios

	SUE	AG	MOM	ILLIQ	OP	IVOL	BETA	SIZE	BM	MAX	TURN	STR
1	0.79***	0.84***	0.79***	1.00***	0.77***	0.96***	0.84***	1.09***	0.79***	0.74***	0.84***	0.80***
	(5.21)	(5.20)	(4.67)	(5.89)	(4.52)	(4.63)	(4.51)	(5.99)	(4.80)	(3.89)	(5.04)	(5.12)
2	0.75***	0.82***	0.77***	0.93***	0.72***	0.84***	0.83***	1.02***	0.78***	0.76***	0.79***	0.75***
	(4.24)	(4.41)	(4.33)	(5.02)	(4.00)	(3.68)	(4.15)	(5.20)	(4.46)	(3.91)	(4.31)	(4.05)
3	0.72***	0.69***	0.84***	0.88***	0.70***	0.87***	0.85***	0.95***	0.83***	0.72***	0.79***	0.80***
	(3.79)	(3.33)	(4.15)	(4.41)	(3.41)	(3.60)	(4.19)	(4.64)	(4.10)	(3.21)	(4.11)	(3.81)
4	0.78***	0.73***	0.76***	0.85***	0.68***	0.81***	0.87***	0.92***	0.80***	0.77***	0.83***	0.78***
	(3.74)	(3.45)	(3.49)	(4.05)	(3.11)	(3.20)	(4.04)	(4.10)	(3.67)	(3.33)	(3.98)	(3.56)
5	0.89***	0.77***	0.73***	0.75***	0.59**	0.87***	0.73***	0.86***	0.87***	0.77***	0.72***	0.71***
	(4.01)	(3.26)	(3.07)	(3.37)	(2.44)	(3.13)	(3.15)	(3.70)	(3.70)	(3.15)	(3.15)	(2.99)
6	0.75***	0.65**	0.65***	0.74***	0.59**	0.62**	0.81***	0.65***	0.71***	0.53**	0.60**	0.71***
	(3.10)	(2.45)	(2.66)	(3.01)	(2.26)	(2.28)	(3.41)	(2.63)	(2.85)	(2.08)	(2.55)	(2.78)
7	0.63**	0.55*	0.49*	0.55**	0.60**	0.49*	0.61**	0.62**	0.59**	0.30	0.57**	0.63**
	(2.21)	(1.89)	(1.87)	(2.14)	(2.27)	(1.74)	(2.40)	(2.28)	(2.14)	(1.08)	(2.23)	(2.25)
8	0.56*	0.41	0.48	0.30	0.56*	0.30	0.47*	0.33	0.33	0.44	0.24	0.36
	(1.80)	(1.34)	(1.64)	(1.04)	(1.93)	(1.08)	(1.73)	(1.14)	(1.14)	(1.60)	(0.82)	(1.14)
9	0.18	0.19	0.13	0.01	0.36	0.16	0.32	-0.03	0.13	0.35	0.00	0.18
	(0.53)	(0.59)	(0.39)	(0.04)	(1.17)	(0.55)	(1.13)	(-0.08)	(0.39)	(1.12)	(0.01)	(0.52)
High	-0.27	-0.21	-0.27	-0.40	-0.07	-0.00	-0.15	-0.40	-0.54	0.10	-0.47	-0.24
	(-0.73)	(-0.56)	(-0.72)	(-1.09)	(-0.19)	(-0.01)	(-0.48)	(-1.08)	(-1.45)	(0.30)	(-1.28)	(-0.63)
H-L	-1.06***	-1.05***	-1.06***	-1.40***	-0.84***	-0.96***	-0.98***	-1.49***	-1.33***	-0.64***	-1.31***	-1.04***
	(-3.42)	(-3.53)	(-3.67)	(-4.92)	(-2.92)	(-4.80)	(-4.91)	(-5.28)	(-4.32)	(-2.88)	(-4.64)	(-3.37)
FF6	-0.81**	-0.78**	-0.82***	-1.12***	-0.59*	-0.80***	-0.81***	-1.20***	-1.02***	-0.39**	-1.04***	-0.78**
	(-2.55)	(-2.55)	(-2.93)	(-4.19)	(-1.88)	(-4.31)	(-4.23)	(-4.49)	(-3.36)	(-1.97)	(-3.90)	(-2.46)

The table reports results from bivariate portfolios based on dependent double sorts of various firm-specific characteristics and MFD. First, quintile portfolios are formed every month based on a firm-specific characteristic. Next, additional decile portfolios are formed based on MFD within each firm-specific characteristic quintile. Subsequently, we average returns for each MFD decile across the characteristic quintiles, yielding ten quintile-mean decile returns. The stock characteristics are described in Table 1. Panel A reports results from equal-weighted portfolio double sorts. Panel B reports results from value-weighted portfolio double sorts. Newey and West (1987) adjusted $t-$statistics are reported in parentheses. *, **, and *** denote statistical significance at the 10%, 5%, and 1% level, respectively. The sample period is from August 1976 to December 2022.

Table 9: Fama-MacBeth Cross-Sectional Regressions

	Excess Return	Excess Return	Excess Return	Industry-adj. Return	DGTW-adj. Return
Const	0.64***	0.63***	0.63***	-0.11	-0.17***
	(2.71)	(2.70)	(2.84)	(-1.22)	(-4.73)
MFD	-0.49***	-0.47***	-0.26***	-0.24***	-0.21***
	(-6.93)	(-7.14)	(-4.41)	(-4.53)	(-4.55)
BETA	0.09*	0.09*	0.09*	0.11***	0.09*
	(1.65)	(1.66)	(1.79)	(3.48)	(1.89)
SIZE	-0.06***	-0.06***	-0.06***	-0.05***	-0.05***
	(-3.05)	(-2.90)	(-3.05)	(-3.60)	(-4.16)
BM	0.06	0.08*	0.04	0.05	-0.07**
	(1.47)	(1.72)	(0.79)	(1.56)	(-2.04)
MOM	0.28***	0.28***	0.31***	0.25***	0.17***
	(5.28)	(5.06)	(4.78)	(4.86)	(3.69)
AG		-0.03	-0.09	-0.05	-0.06
		(-0.74)	(-1.63)	(-1.25)	(-1.42)
OP		0.08***	0.08**	0.07**	0.09**
		(2.80)	(2.49)	(2.42)	(2.48)
SUE			0.04**	0.04**	0.03
			(1.99)	(2.42)	(1.57)
ILLIQ			-0.01	0.01	-0.05
			(-0.27)	(0.23)	(-1.04)
IVOL			-0.10**	-0.15***	-0.10*
			(-2.18)	(-2.95)	(-1.91)
MAX			-0.13***	-0.10***	-0.12***
			(-3.15)	(-2.82)	(-2.71)
TURN			-0.07**	-0.06*	-0.06*
			(-2.10)	(-1.94)	(-1.74)
STR			-0.33***	-0.40***	-0.37***
			(-6.89)	(-8.82)	(-7.41)
Observations	1,149,551	1,085,655	964,438	922,472	922,472

The table reports Fama-MacBeth cross-sectional regressions for MFD. MFD and the control variables in month $t-1$ are matched to stock returns in month t. The dependent variable is the firm's future excess return in the first three columns (Excess Return), the firm's future return over its value-weighted industry peers' return (Industry-adj. Return), or the firm's DGTW adjusted return (DGTW-adj. Return). All dependent variables are given in percent. The control variables are described in Table 1, winsorized at 0.5% in both tails, and standardized. Newey and West (1987) adjusted $t-$statistics are reported in parentheses. *, **, and *** denote statistical significance at the 10%, 5%, and 1% level, respectively. The sample period is from August 1976 to December 2022.

Table 10: International Evidence

Panel A: Equal-Weighted Portfolios														
Region	Low	2	3	4	5	6	7	8	9	High	H-L	t-stat	FF6	t-stat
Developed ex USA	0.41*	0.55**	0.55**	0.50*	0.47	0.47	0.35	0.23	-0.10	-0.82*	-1.24***	(-4.47)	-1.08***	(-3.88)
Emerging	0.88**	0.76*	0.64	0.47	0.42	0.42	0.32	0.21	-0.25	-0.66	-1.53***	(-4.33)	-1.18***	(-3.19)
G7 ex USA	0.32	0.42*	0.47*	0.41	0.41	0.36	0.29	0.11	-0.19	-0.98**	-1.30***	(-4.15)	-1.11***	(-3.53)
G10 ex USA	0.35	0.48*	0.49*	0.47*	0.46	0.40	0.32	0.15	-0.11	-0.88*	-1.22***	(-4.19)	-1.05***	(-3.56)
Europe	0.72**	0.88***	0.81**	0.87***	0.78**	0.62*	0.55	0.30	-0.09	-0.82	-1.54***	(-5.04)	-1.34***	(-5.30)
Panel B: Value-Weighted Portfolios														
Region	Low	2	3	4	5	6	7	8	9	High	H-L	t-stat	FF6	t-stat
Developed ex USA	0.48*	0.54**	0.56**	0.63**	0.44	0.68**	0.52	0.39	0.22	-0.27	-0.75**	(-2.20)	-0.62*	(-1.75)
Emerging	0.76**	0.49	0.54	0.51	0.42	0.39	0.51	0.47	0.09	-0.34	-1.10**	(-2.45)	-0.88*	(-1.85)
G7 ex USA	0.38	0.47*	0.44*	0.50*	0.57*	0.54*	0.56	0.09	0.13	-0.53	-0.91**	(-2.43)	-0.77*	(-1.86)
G10 ex USA	0.40	0.60**	0.51*	0.44*	0.57**	0.55*	0.46	0.34	0.09	-0.47	-0.87**	(-2.47)	-0.75**	(-1.98)
Europe	0.78***	0.67**	0.75***	0.67**	0.66**	0.66*	0.53	0.40	-0.01	-0.05	-0.84**	(-2.10)	-0.74**	(-1.97)

The table reports the average monthly spread returns and alphas on univariate portfolios of international stocks sorted by MFD. Each month t, international stocks are sorted into decile portfolios by MFD constructed using data up to month $t-1$. Panel A reports equal-weighted portfolio sorts whereas Panel B reports value-weighted portfolio sorts. The table represents spread returns and alphas with respect to the international five-factor model of Fama and French (2017) augmented by the international momentum factor (FF6) for MFD sorted portfolios. The international stock sample comprises 93 countries and is taken from Jensen et al. (2022b). Stocks are classified into emerging and developing countries following the MSCI classification as of January 7th 2021 (see Table J.3 in Jensen et al., 2022b). Data on the international FF6 model is taken from Kenneth French's website. t-stat denote Newey and West (1987) adjusted t-statistics. *, **, and *** denote statistical significance at the 10%, 5%, and 1% level, respectively. The sample period is from February 1996 to December 2022.

Table 11: Mispricing and MFD

Panel A: Average MISP in MFD Decile Portfolio

	Low	2	3	4	5	6	7	8	9	High	H-L	t-stat
MISP	39.24	41.75	43.94	45.87	47.70	49.58	51.58	53.93	57.54	63.97	24.73***	42.34

Panel B: Bivariate Portfolio Sort on MISP

	Low	2	3	4	5	6	7	8	9	High	H-L	t-stat	FF6	t-stat
MISP Low	1.00	1.08	1.00	1.20	1.24	1.28	1.24	1.20	1.29	0.89	-0.11	-0.42	0.02	0.06
MISP 2	0.95	1.03	1.09	1.05	1.02	1.12	1.03	0.87	0.99	0.43	-0.53**	-2.20	-0.44*	-1.72
MISP 3	0.92	0.88	1.05	0.89	1.04	0.89	0.78	0.89	0.44	0.11	-0.80***	-2.84	-0.62**	-2.18
MISP 4	0.85	0.82	0.94	0.83	0.88	0.56	0.44	0.39	0.11	-0.01	-0.87***	-2.79	-0.64**	-1.99
MISP High	0.73	0.60	0.43	0.27	0.13	-0.06	-0.30	-0.40	-0.56	-0.93	-1.66***	-5.28	-1.39***	-4.45
MISP H-L	-0.27	-0.48	-0.57	-0.92	-1.11	-1.34	-1.55	-1.60	-1.85	-1.82	-1.56***	-7.18	-1.40***	-6.51

Panel A reports the time-series averages of the monthly cross-sectional median of the stock-level mispricing score (MISP) of Stambaugh et al. (2015) for each of the MFD-sorted univariate decile portfolios. Low (high) MISP indicates a lower (higher) mispricing score. Panel B reports 5x10 dependent bivariate equal-weight portfolio sorts. First, quintile portfolios are formed every month using MISP. Next, decile portfolios are formed based on MFD within each firm-specific MISP quintile. Newey and West (1987) adjusted $t-$statistics are reported in parentheses. *, **, and *** denote statistical significance at the 10%, 5%, and 1% level, respectively. The sample period is from August 1976 to December 2016.

Table 12: Earnings Announcement Returns Prediction

	Panel A: One-day Window		Panel B: Three-day Window	
Dep. variable	Ret_t^d	Ret_t^d	Ret_t^d	Ret_t^d
MFD	-0.29***	-0.35***	-0.28***	-0.34***
	(-4.65)	(-5.43)	(-4.41)	(-5.21)
MFD × EDAY	-0.61***	-0.58***	-0.54***	-0.51***
	(-2.90)	(-2.75)	(-4.72)	(-4.42)
EDAY	0.31***	0.32***	0.19***	0.19***
	(9.72)	(9.87)	(10.94)	(11.34)
Lagged Controls?	No	Yes	No	Yes
Day Fixed Effects?	Yes	Yes	Yes	Yes

The table reports results from the panel regressions of daily returns (Ret_t^d) on the previous month's MFD, an earnings announcement window dummy variable (EDAY), an interaction between MFD and EDAY, day-fixed effects, and other lagged control variables (coefficients unreported). Ret_t^d, the dependent variable, is multiplied by 100. An earnings announcement window is defined analogously to Engelberg et al. (2018) as the one-day or three-day window centered on an earnings release, i.e., days $t-1$, t, and $t+1$. EDAY is a dummy variable equalling one if the daily observation is during an announcement window, and zero otherwise. Following Engelberg et al. (2018), we obtain earnings announcement dates from the Compustat quarterly database and examine the firm's trading volume scaled by market trading volume for the day before, the day of, and the day after the reported earnings announcement date. An earnings announcement day is defined as the day with the highest scaled trading volume. MFD is by construction at the monthly frequency and its previous month value is merged to daily stock returns Ret_t^d. Control variables include lagged values for each of the past ten days for stock returns, squared stock returns, and trading volume. Standard errors are clustered by day. t-statistics are in parentheses and coefficients marked with *, **, and *** statistical significance at the 10%, 5%, and 1% level, respectively. The sample period is from August 1976 to December 2022.

Table 13: Short-Sale Contraints and MFD

Panel A: Average BORROWFEE in MFD Decile Portfolio

	Low	2	3	4	5	6	7	8	9	High	H-L	t-stat
BORROWFEE	0.54	0.51	0.54	0.61	0.71	0.89	1.14	1.48	2.34	5.00	4.45***	10.51

Panel B: Bivariate Portfolio Sort on BORROWFEE

	Low	2	3	4	5	6	7	8	9	High	H-L	t-stat	FF6	t-stat
BORROWFEE Low	1.01	0.98	1.01	0.89	0.99	0.97	0.87	0.75	0.58	0.19	-0.82***	-2.66	-0.74***	-2.92
BORROWFEE 2	1.23	0.67	0.74	0.56	0.33	0.56	0.57	0.62	0.45	-0.14	-1.36***	-2.99	-1.22***	-2.75
BORROWFEE 3	0.83	1.07	0.65	0.66	0.43	0.54	0.18	-0.17	-0.09	-0.42	-1.26***	-2.58	-1.18***	-2.64
BORROWFEE 4	0.97	0.71	0.71	0.43	-0.66	0.34	-0.26	-0.40	-0.97	-1.04	-2.01***	-3.68	-1.87***	-4.16
BORROWFEE High	0.23	0.01	0.27	-0.07	-0.32	-1.50	-1.78	-1.29	-2.17	-2.65	-2.88***	-5.78	-2.54***	-5.80
BORROWFEE H-L	-0.78	-0.97	-0.74	-0.97	-1.31	-2.47	-2.65	-2.04	-2.75	-2.84	-2.06***	-4.80	-1.80***	-4.29

Panel A reports the time-series averages of the monthly cross-sectional median of the stock-level indicative borrowing fee (BORROWFEE) taken from IHS Markit for each of the MFD-sorted univariate decile portfolios. Low (high) BORROWFEE indicates a lower (higher) indicate borrowing fee. Panel B reports 5x10 dependent bivariate equal-weight portfolio sorts. First, quintile portfolios are formed every month using BORROWFEE. Next, decile portfolios are formed based on MFD within each firm-specific BORROWFEE quintile. Newey and West (1987) adjusted $t-$statistics are reported in parentheses. *, **, and *** denote statistical significance at the 10%, 5%, and 1% level, respectively. The sample period is from January 2004 to April 2022.

Table 14: Institutional Ownership and MFD

Panel A: Average INST in MFD Decile Portfolio

	Low	2	3	4	5	6	7	8	9	High	H-L	t-stat
INST	0.59	0.58	0.55	0.53	0.51	0.49	0.47	0.44	0.41	0.36	-0.23***	-30.85

Panel B: Bivariate Portfolio Sort on INST

	Low	2	3	4	5	6	7	8	9	High	H-L	t-stat	FF6	t-stat
INST Low	1.02	1.00	0.83	0.67	0.59	0.09	-0.15	-0.59	-1.05	-1.64	-2.66***	-8.89	-2.48***	-8.88
INST 2	1.04	1.11	1.13	0.95	0.81	0.68	0.54	0.11	-0.04	-0.72	-1.75***	-4.92	-1.55***	-4.61
INST 3	1.11	1.07	1.15	1.03	1.02	0.87	0.79	0.52	0.18	-0.06	-1.16***	-3.59	-0.99***	-2.95
INST 4	1.10	1.07	1.08	0.98	1.00	0.97	0.91	0.84	0.63	0.24	-0.86***	-2.80	-0.66**	-2.14
INST High	0.98	0.91	0.92	0.85	0.91	1.16	0.86	0.85	0.67	0.24	-0.73**	-2.40	-0.61**	-2.09
INST H-L	-0.04	-0.09	0.08	0.17	0.32	1.07	1.01	1.44	1.73	1.88	1.92***	9.71	1.87***	9.13

Panel A reports the time-series averages of the monthly cross-sectional median of the stock-level institutional ownership (INST) for each of the MFD-sorted univariate decile portfolios. Low (high) INST indicates a lower (higher) institutional ownership. Panel B reports 5x10 dependent bivariate equal-weight portfolio sorts. First, quintile portfolios are formed every month using INST. Next, decile portfolios are formed based on MFD within each INST quintile. Newey and West (1987) adjusted $t-$statistics are reported in parentheses. *, **, and *** denote statistical significance at the 10%, 5%, and 1% level, respectively. The sample period is from January 1980 to December 2021.

Table 15: Limits-to-Arbitrage and MFD

Panel A: Average ARB in MFD Decile Portfolio

	Low	2	3	4	5	6	7	8	9	High	H-L	t-stat
ARB	8.59	10.71	12.59	14.21	15.68	16.99	18.21	19.38	20.63	22.27	13.69***	70.51

Panel B: Bivariate Portfolio Sort on ARB

	Low	2	3	4	5	6	7	8	9	High	H-L	t-stat	FF6	t-stat
ARB Low	0.85	0.87	0.86	0.78	0.79	0.77	0.83	0.68	0.73	0.52	-0.33**	-2.38	-0.35**	-2.44
ARB 2	1.13	1.06	1.02	1.00	0.93	0.88	0.85	0.73	0.68	0.28	-0.86***	-4.20	-0.77***	-3.73
ARB 3	1.25	1.09	1.06	0.99	0.93	0.88	0.81	0.58	0.36	-0.06	-1.32***	-5.15	-1.09***	-4.42
ARB 4	1.25	1.14	1.09	0.88	0.60	0.39	0.28	-0.03	-0.32	-0.60	-1.85***	-6.78	-1.54***	-5.49
ARB High	0.81	0.43	0.15	0.16	-0.31	-0.31	-0.69	-0.98	-1.28	-1.40	-2.21***	-8.13	-2.02***	-8.25
ARB H-L	-0.04	-0.45	-0.71	-0.63	-1.10	-1.07	-1.51	-1.65	-2.01	-1.92	-1.88***	-7.05	-1.67***	-6.83

Panel A reports the time-series averages of the monthly cross-sectional median of a limits-to-arbitrage score (ARB) for each of the MFD-sorted univariate decile portfolios. Low (high) ARB indicates a lower (higher) average arbitrage cost index. Panel B reports 5x10 dependent bivariate equal-weight portfolio sorts. First, quintile portfolios are formed every month using ARB. Next, decile portfolios are formed based on MFD within each firm-specific ARB quintile. The arbitrage cost index on the stock-level is constructed using firm size, firm age, idiosyncratic volatility and illiquidity of the stock. To construct it, we sort stocks in increasing order according to their idiosyncratic volatility and illiquidity. Similarly, we sort stocks into decreasing order of firm age and size. Each stock is given the corresponding score of its decile rank for each variable. Finally, the arbitrage cost index on the stock-level is the sum of the four scores such that it ranges from 4 to 40. Newey and West (1987) adjusted $t-$statistics are reported in parentheses. *, **, and *** denote statistical significance at the 10%, 5%, and 1% level, respectively. The sample period is from August 1976 to December 2022.

Appendix

A Analyst forecast dispersion

Table A1: Univariate Portfolio Sorts on AFD

Panel A: Equal-Weighted Portfolios

	Excess Return	t-stat	CAPM	t-stat	FF6	t-stat	HXZ	t-stat	SY	t-stat	DHS	t-stat
Low	1.11***	(5.17)	1.04***	(4.75)	1.21***	(4.89)	1.24***	(4.75)	1.27***	(4.93)	1.17***	(4.90)
2	0.79***	(3.71)	0.73***	(3.36)	0.90***	(3.68)	0.90***	(3.54)	0.91***	(3.60)	0.84***	(3.71)
3	0.80***	(3.42)	0.71***	(3.01)	0.91***	(3.43)	0.90***	(3.24)	0.93***	(3.24)	0.86***	(3.46)
4	0.73***	(2.90)	0.63**	(2.52)	0.82***	(2.85)	0.83***	(2.70)	0.83***	(2.70)	0.79***	(2.94)
5	0.74***	(2.80)	0.63**	(2.43)	0.87***	(2.94)	0.86***	(2.80)	0.87***	(2.83)	0.79***	(2.83)
6	0.69**	(2.50)	0.57**	(2.13)	0.79***	(2.60)	0.79**	(2.43)	0.78**	(2.49)	0.74**	(2.49)
7	0.56*	(1.90)	0.44	(1.52)	0.68**	(2.13)	0.61*	(1.79)	0.62*	(1.85)	0.63**	(2.06)
8	0.51	(1.64)	0.39	(1.28)	0.64*	(1.88)	0.60	(1.64)	0.56	(1.57)	0.63*	(1.88)
9	0.30	(0.93)	0.18	(0.58)	0.44	(1.24)	0.34	(0.92)	0.37	(0.98)	0.43	(1.24)
High	0.12	(0.37)	-0.03	(-0.09)	0.23	(0.64)	0.16	(0.42)	0.11	(0.29)	0.21	(0.59)
H-L	-0.99***	(-4.99)	-1.07***	(-5.66)	-0.98***	(-4.92)	-1.08***	(-5.30)	-1.16***	(-5.66)	-0.96***	(-4.51)

Panel B: Value-Weighted Portfolios

	Excess Return	t-stat	CAPM	t-stat	FF6	t-stat	HXZ	t-stat	SY	t-stat	DHS	t-stat
Low	0.89***	(5.35)	0.90***	(4.79)	0.99***	(4.73)	1.05***	(4.93)	1.01***	(4.43)	0.96***	(4.66)
2	0.75***	(4.03)	0.73***	(3.59)	0.85***	(3.91)	0.90***	(4.17)	0.82***	(3.39)	0.73***	(3.47)
3	0.69***	(3.08)	0.67***	(2.81)	0.83***	(3.25)	0.87***	(3.23)	0.76***	(2.63)	0.74***	(2.96)
4	0.67***	(2.75)	0.64**	(2.50)	0.81***	(3.20)	0.88***	(3.39)	0.80***	(2.76)	0.75***	(2.90)
5	0.78***	(3.05)	0.73***	(2.71)	0.94***	(3.31)	0.93***	(3.07)	0.81**	(2.45)	0.90***	(3.18)
6	0.77***	(3.34)	0.74***	(3.10)	0.96***	(3.82)	0.97***	(3.86)	0.94***	(3.31)	0.91***	(3.63)
7	0.72***	(2.89)	0.71***	(2.66)	0.86***	(3.05)	0.81***	(2.68)	0.88***	(2.61)	0.80***	(2.71)
8	0.63**	(2.16)	0.58*	(1.85)	0.81**	(2.41)	0.68*	(1.91)	0.69*	(1.86)	0.73**	(2.14)
9	0.68**	(2.26)	0.63**	(2.01)	0.89***	(2.64)	0.91**	(2.38)	0.68*	(1.89)	0.87**	(2.47)
High	0.39	(1.17)	0.32	(0.94)	0.50	(1.31)	0.40	(0.98)	0.50	(1.21)	0.48	(1.24)
H-L	-0.50**	(-1.97)	-0.58**	(-2.34)	-0.48*	(-1.75)	-0.66**	(-2.27)	-0.50	(-1.61)	-0.48*	(-1.73)

The table reports the average monthly excess returns and alphas on univariate portfolios of stocks sorted by AFD. Each month t, stocks are sorted into decile portfolios by month $t-1$'2 AFD. Panel A reports equal-weight portfolio sorts whereas Panel B reports value-weight portfolio sorts. Excess Return is the return in excess of the risk-free rate. Alpha is the intercept from a time-series regression of monthly excess returns on the factors of alternative models: the CAPM, Fama and French (2018) six-factor model (FF6), Stambaugh and Yuan (2017) mispricing factor model (SY), Hou et al. (2015) q-factor model (HXZ), and the Daniel et al. (2020) behavioral factor model (DHS). t-stat denote Newey and West (1987) adjusted t-statistics. *, **, and *** denote statistical significance at the 10%, 5%, and 1% level, respectively. The sample period is from January 1983 to December 2022.

Table A2: Fama-MacBeth Cross-Sectional Regressions on MFD Controlling for AFD

	Excess Return	Industry-adj. Return	DGTW-adj. Return
Const	0.38	-0.30	-0.25
	(1.13)	(-1.44)	(-1.43)
MFD	-0.26***	-0.21***	-0.21***
	(-3.96)	(-3.86)	(-3.93)
SUE	-0.01	-0.00	-0.02
	(-0.64)	(-0.28)	(-0.97)
AG	-0.14**	-0.11**	-0.13**
	(-1.99)	(-2.20)	(-2.51)
MOM	0.31***	0.26***	0.20***
	(4.02)	(4.78)	(3.85)
ILLIQ	-1.45	-0.90	-1.14
	(-1.47)	(-0.79)	(-0.99)
OP	0.07	0.05	0.09**
	(1.48)	(1.40)	(1.97)
IVOL	0.00	-0.05	0.09
	(0.00)	(-0.74)	(1.35)
BETA	0.05	0.07**	0.05
	(0.92)	(2.08)	(0.97)
SIZE	-0.03	-0.02*	-0.03**
	(-1.46)	(-1.70)	(-2.46)
BM	0.03	0.05	-0.04
	(0.46)	(1.30)	(-1.01)
MAX	-0.06	-0.06	-0.07
	(-0.91)	(-1.14)	(-1.14)
TURN	-0.02	-0.01	-0.04
	(-0.59)	(-0.27)	(-0.91)
STR	-0.37***	-0.40***	-0.38***
	(-5.77)	(-7.95)	(-6.43)
AFD	-0.08***	-0.07***	-0.08***
	(-3.22)	(-3.60)	(-3.57)
Observations	595,890	584,832	584,832

The table reports Fama-MacBeth cross-sectional regressions for MFD while additionally controlling for AFD. MFD, AFD and the control variables in month $t-1$ are matched to stock returns in month t. The dependent variable is the firm's future excess return in the first column (Excess Return), the firm's future return over its value-weighted industry peers' return (Industry-adj. Return), or the firm's DGTW adjusted return (DGTW-adj. Return). All dependent variables are given in percent. The control variables are described in Table 1, winsorized at 0.5% in both tails, and cross-sectionally standardized each month to have zero mean and unit standard deviation. Newey and West (1987) adjusted $t-$statistics are reported in parentheses. *, **, and *** denote statistical significance at the 10%, 5%, and 1% level, respectively. The sample period is from January 1983 to March 2022.

B Univariate portfolio sorts for different measures of MFD

Table B3: Univariate MFD-spread Portfolio Returns for Different Hyper-parameters

Panel A: Equal-Weighted Portfolios

Nbr. Investors	Model	Random Features	Nbr. features	Max. depth	Nbr. trees	Max. features	Max. samples	λ	H-L	t-stat	FF6	t-stat
250	Random Forest	Yes	128	6	2000	log2	0.10		-1.64***	(-5.55)	-1.40***	(-4.93)
100	Random Forest	Yes	128	6	5000	log2	0.10		-1.57***	(-5.34)	-1.34***	(-4.70)
100	Random Forest	Yes	128	6	5000	log2	0.05		-1.52***	(-5.27)	-1.28***	(-4.52)
100	Random Forest	Yes	128	7	1000	log2	0.10		-1.59***	(-5.57)	-1.34***	(-4.75)
100	Random Forest	Yes	128	6	1000	log2	0.10		-1.63***	(-5.52)	-1.40***	(-4.90)
100	Random Forest	Yes	128	6	500	log2	0.10		-1.54***	(-5.35)	-1.30***	(-4.60)
100	Random Forest	Yes	128	6	2000	log2	0.25		-1.61***	(-5.43)	-1.39***	(-4.95)
100	Random Forest	Yes	128	6	2000	1	0.10		-1.57***	(-5.49)	-1.33***	(-4.95)
100	Random Forest	Yes	128	5	1000	1	0.20		-1.57***	(-5.50)	-1.35***	(-4.94)
100	Random Forest	Yes	128	5	1000	sqrt	0.20		-1.51***	(-5.19)	-1.28***	(-4.57)
100	Random Forest	Yes	128	6	2000	sqrt	0.10		-1.54***	(-5.26)	-1.31***	(-4.61)
100	Random Forest	Yes	128	6	1000	sqrt	0.10		-1.58***	(-5.30)	-1.32***	(-4.58)
100	Random Forest	Yes	128	6	2000	sqrt	0.25		-1.58***	(-5.40)	-1.36***	(-4.90)
100	Random Forest	Yes	128	6	2000	sqrt	0.05		-1.59***	(-5.37)	-1.35***	(-4.72)
250	Ridge	Yes	256			0.25	1.00	2.72	-1.23***	(-5.59)	-1.02***	(-4.50)
250	Ridge	Yes	128			0.50	0.25	2.72	-1.04***	(-5.75)	-0.83***	(-4.51)
250	Ridge	Yes	128			0.25	1.00	2.72	-1.22***	(-5.44)	-1.00***	(-4.42)
100	Random Forest	No	65	6	3000	1	0.10		-1.68***	(-6.47)	-1.45***	(-5.74)
250	Random Forest	No	65	6	2000	1	0.10		-1.69***	(-6.18)	-1.46***	(-5.58)
100	Random Forest	No	65	7	1000	1	0.10		-1.71***	(-6.58)	-1.50***	(-5.96)
100	Random Forest	No	98	6	2000	1	0.10		-1.66***	(-6.01)	-1.42***	(-5.25)
100	Random Forest	No	33	6	2000	1	0.10		-1.56***	(-6.44)	-1.34***	(-5.89)
100	Random Forest	No	65	6	2000	1	0.25		-1.70***	(-6.39)	-1.47***	(-5.89)
100	Random Forest	No	65	6	2000	1	0.20		-1.70***	(-6.40)	-1.48***	(-5.81)

Panel B: Value-Weighted Portfolios

Nbr. Investors	Model	Random Features	Nbr. features	Max. depth	Nbr. trees	Max. features	Max. samples	λ	H-L	t-stat	FF6	t-stat
250	Random Forest	Yes	128	6	2000	log2	0.10		-1.14***	(-3.44)	-0.84**	(-2.57)
100	Random Forest	Yes	128	6	5000	log2	0.10		-1.12***	(-3.30)	-0.81**	(-2.38)
100	Random Forest	Yes	128	6	5000	log2	0.05		-1.07***	(-3.24)	-0.77**	(-2.24)
100	Random Forest	Yes	128	7	1000	log2	0.10		-1.13***	(-3.56)	-0.84**	(-2.55)
100	Random Forest	Yes	128	6	1000	log2	0.10		-1.25***	(-3.82)	-0.96***	(-2.89)
100	Random Forest	Yes	128	6	500	log2	0.10		-1.04***	(-3.14)	-0.76**	(-2.26)
100	Random Forest	Yes	128	6	2000	log2	0.25		-1.13***	(-3.33)	-0.84**	(-2.52)
100	Random Forest	Yes	128	6	2000	1	0.10		-1.12***	(-3.26)	-0.79**	(-2.34)
100	Random Forest	Yes	128	5	1000	1	0.20		-1.00***	(-3.07)	-0.75**	(-2.28)
100	Random Forest	Yes	128	5	1000	sqrt	0.20		-1.11***	(-3.33)	-0.80**	(-2.45)
100	Random Forest	Yes	128	6	2000	sqrt	0.10		-1.01***	(-2.98)	-0.71**	(-2.11)
100	Random Forest	Yes	128	6	1000	sqrt	0.10		-1.06***	(-3.24)	-0.77**	(-2.29)
100	Random Forest	Yes	128	6	2000	sqrt	0.25		-1.05***	(-3.16)	-0.75**	(-2.24)
100	Random Forest	Yes	128	6	2000	sqrt	0.05		-1.15***	(-3.30)	-0.83**	(-2.45)
250	Ridge	Yes	256			0.25	1.00	2.72	-0.75***	(-2.76)	-0.58**	(-2.13)
250	Ridge	Yes	128			0.50	0.25	2.72	-0.60**	(-2.40)	-0.41*	(-1.77)
250	Ridge	Yes	128			0.25	1.00	2.72	-0.70**	(-2.54)	-0.50*	(-1.74)
100	Random Forest	No	65	6	3000	1	0.10		-1.22***	(-3.95)	-0.87***	(-2.86)
250	Random Forest	No	65	6	2000	1	0.10		-1.30***	(-3.93)	-0.95***	(-2.87)
100	Random Forest	No	65	7	1000	1	0.10		-1.30***	(-4.11)	-1.01***	(-3.14)
100	Random Forest	No	98	6	2000	1	0.10		-1.20***	(-3.62)	-0.90***	(-2.65)
100	Random Forest	No	33	6	2000	1	0.10		-1.17***	(-4.06)	-0.88***	(-3.04)
100	Random Forest	No	65	6	2000	1	0.25		-1.38***	(-4.44)	-1.08***	(-3.51)
100	Random Forest	No	65	6	2000	1	0.20		-1.36***	(-4.38)	-1.07***	(-3.43)

The table reports the average monthly spread return between the highest and lowest decile of univariate portfolios based on MFD for various hyper-parameter sets for random forest regression. Each month t, stocks are grouped into decile portfolios based on their month $t-1$'s MFD. Then, the return of the portfolio going long into the highest and shorting the lowest decile portfolio is computed (H-L). The table depicts also the alpha of the high-minus low decile portfolio with respect to the Fama and French (2018) six-factor model (FF6). Nbr. Investors indicates the number of investors for which beliefs are modeled. Model denotes the model each investors uses to forecast expected returns. Random Features indicates if characteristics are processed through a non-linear Fourier operation. Nbr. features specifies the number of features for each investor. Max. depth denotes the maximum tree depth. Nbr. trees specifies how many trees are built. Max. features is the ratio of the number of features to consider. If it is set to "sqrt", then the maximal features are $1/\text{sqrt}(\cdot)$. If it is set to "log2", then the maximal features are $\log2(\cdot)/(\cdot)$. In the remaining cases it denotes the raw fraction. For random forest regression, it is the ratio for node splits in each tree. For ridge regression, it is the ratio of all features each investor considers. Max. samples is the fraction of observations used while fitting each tree in random forest and ridge regression, respectively. λ is the penalty term in ridge regression. t-stat denote Newey and West (1987) adjusted t-statistics. *, **, and *** denote statistical significance at the 10%, 5%, and 1% level, respectively. The sample period is from August 1976 to December 2022.

C Univariate portfolio sorts on MFD for 1000 largest stocks

Table C4: Univariate Portfolio Sorts on MFD for 1000 Largest Stocks

	Excess Return	t-stat	CAPM	t-stat	FF6	t-stat	HXZ	t-stat	SY	t-stat	DHS	t-stat
Low	0.93***	(5.29)	0.90***	(4.99)	1.02***	(5.35)	1.02***	(4.96)	1.01***	(5.14)	0.92***	(4.82)
2	0.90***	(5.02)	0.88***	(4.73)	1.00***	(5.12)	0.99***	(4.73)	1.02***	(4.95)	0.90***	(4.57)
3	0.91***	(4.87)	0.88***	(4.57)	1.02***	(5.05)	1.02***	(4.76)	1.03***	(4.89)	0.91***	(4.51)
4	0.94***	(4.65)	0.90***	(4.38)	1.03***	(4.78)	1.07***	(4.58)	1.07***	(4.78)	0.96***	(4.38)
5	0.85***	(4.08)	0.79***	(3.79)	0.95***	(4.24)	0.92***	(3.75)	0.96***	(4.07)	0.86***	(3.74)
6	0.82***	(3.68)	0.77***	(3.43)	0.94***	(3.86)	0.92***	(3.52)	0.96***	(3.66)	0.86***	(3.54)
7	0.82***	(3.30)	0.76***	(3.08)	0.95***	(3.55)	0.86***	(2.96)	0.94***	(3.23)	0.86***	(3.22)
8	0.80***	(2.79)	0.75***	(2.62)	0.97***	(3.08)	0.90***	(2.62)	0.95***	(2.64)	0.88***	(2.76)
9	0.52	(1.52)	0.45	(1.31)	0.78**	(2.19)	0.72*	(1.86)	0.78*	(1.93)	0.75**	(2.07)
High	0.01	(0.02)	-0.06	(-0.15)	0.34	(0.88)	0.34	(0.80)	0.38	(0.86)	0.31	(0.80)
H-L	-0.93***	(-3.02)	-0.96***	(-3.08)	-0.68**	(-2.29)	-0.68**	(-2.12)	-0.63*	(-1.82)	-0.61*	(-1.86)

The table reports the average monthly excess returns and alphas on univariate portfolios of stocks sorted by MFD. For each month t, only the as-of month t 1000 largest stocks by market capitalization are sorted into decile portfolios by MFD. All returns and alphas are expressed in percentage. Excess Return is the return in excess of the risk-free rate. Alpha is the intercept from a time-series regression of monthly excess returns on the factors of alternative models: the CAPM, Fama and French (2018) six-factor model (FF6), Stambaugh and Yuan (2017) mispricing factor model (SY), Hou et al. (2015) q-factor model (HXZ), and the Daniel et al. (2020) behavioral factor model (DHS). t-stat denote Newey and West (1987) adjusted t-statistics. *, **, and *** denote statistical significance at the 10%, 5%, and 1% level, respectively. The sample period is from August 1976 to December 2022 (December 2016 in case of SY).