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ABSTRACT

We introduce on-the-job search frictions in an otherwise standard monetary DSGE New-Keynesian model. Heterogeneity in productivity across jobs gives rise to a job ladder. Firms Bertrand-compete for employed workers according to the Sequential Auctions protocol of Postel-Vinay and Robin (2002). Outside job offers to employed workers, when accepted, reallocate employment up the productivity ladder; when declined, because matched by the current employer, they raise production costs and, due to nominal price rigidities, compress mark-ups, building inflationary pressure. When employment is concentrated at the bottom of the job ladder, typically after recessions, the reallocation effect prevails, aggregate supply expands, moderating marginal costs and inflation. As workers climb the job ladder, reducing slack in the employment pool, the inflation effect takes over. The model generates endogenous cyclical movements in the Neo Classical labor wedge and in the New Keynesian wage mark-up. The economy takes time to absorb cyclical misallocation and features propagation in the response of job creation, unemployment and inflation to aggregate shocks. The ratio between job-finding probabilities from job-to-job and from unemployment, a measure of the “Acceptance rate” of job offers to employed workers, predicts negatively inflation, independently of the unemployment rate.

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1 Introduction

The Phillips curve, an inverse short-run relationship between the rates of unemployment and inflation, has long been a guiding principle of monetary policy. Microfoundations of this relationship build on price-setting frictions, due either to explicit costs of price adjustment or to incomplete information about the nature of demand shocks faced by producers. In this body of work, the labor market is typically modeled as competitive, and features no unemployment; the relevant measure of slack is an output gap. Nominal wage rigidity can generate classical unemployment associated with such a gap (Erceg, Henderson and Levine (2000)). But the canonical model of unemployment, supported by a vast body of empirical evidence on labor market flows, builds on search frictions, a primitive feature of the trading environment, rather than on exogenously imposed sources of wage rigidity. In the so-called DMP framework, wages are set by Nash Bargaining, the value of unemployment being the worker's outside option. When the economy is expanding and firms post many vacancies, unemployed workers quickly find a new job, hence unemployment declines, while employed workers have a strong threat and bargaining power, and real wages rise (although this may happen gradually due to infrequent bargaining — Gertler and Trigari (2009)). This view seems to capture well the original idea behind the Phillips curve: low unemployment signals scarcity of labor, hence pressure on its price.[1].

In this paper, we advocate shifting emphasis away from unemployment alone, as the relevant indicator of slack to predict inflation, and towards the (mis)allocation of employment on a "job ladder". We embrace an alternative view of wage determination in a frictional labor market, where firms are able to unilaterally offer wages, and commit to their promises. Workers derive bargaining strength not from their outside option of unemployment, which is irrelevant as long as it does not bind, but from their (ability to receive) outside offers while employed.[2] An employed worker can either accept an outside offer, and move up the

[1]Krause, Lopez-Salido and Lubik (2008) show that modeling the labor market according to this DMP tradition does not have much additional explanatory power for inflation dynamics in an otherwise standard monetary DSGE model, but Christiano, Eichenbaum and Trabandt (2016) find that it significantly improves the overall empirical fit the model.

[2]Consistent with this view, Jaeger *et al.* (2018) find in administrative Austrian data that sudden and large changes in Unemployment Insurance benefit size and duration by age have no discernible effect on the continuing wages of UI eligible workers. In Moscarini and Postel-Vinay (2017), using microdata from the Survey of Income and Program Participation to control for composition effects in employment, we document that neither the unemployment rate nor the job-finding rate from unemployment have any significant comovement over time with nominal wage inflation. In contrast, the rate at which workers move from job to job (or employer to employer, EE) has a significant positive relationship with contemporaneous nominal wage inflation, even for stayers who do not switch jobs. Hall and Milgrom (2008) replace Nash Bargaining with another protocol that also insulates wages from outside options at high frequency. They do not feature on-the-job search and outside offers, which we emphasize in this paper as the main transmission channel of aggregate labor demand shocks to wages.

job ladder, or see it matched and decline it. In the latter case, the wage rises towards the marginal product on the current job and represents, for the employer, a cost-push shock. This outcome is more likely after a sufficiently long aggregate expansion, when workers have been moving up the ladder for a while and, therefore, have become difficult to poach away. In that case, cost pressure builds and, with a lag due to price rigidities, eventually manifests itself as price inflation.[3]

Our claim is that competition for employed, not (only) unemployed, workers transmits aggregate shocks to wages, and that the distinction is important, for two reasons.

First and foremost, these two types of competition point to different observable proxies of slack to guide monetary policy. Unemployment measures the *quantity* of unused human resources. Due to the same frictions that cause unemployment, however, employed workers are misallocated on a job ladder. The more severe this misallocation, i.e. the worse the *quality* of employment, the easier it is to poach workers away from competitors, and the stronger the incentives to create new jobs, independently of unemployment. Conversely, with less misallocation, most outside job offers will be either rejected or matched, causing little employment gains and large wage gains, which are cost-push shocks from the point of view of the employer. Unemployment can be seen as the bottom rung of a high job ladder. As such, it is at best an incomplete measure of slack, which must be supplemented by measures of employment misallocation, or symptoms thereof.

Second, unemployment and misallocation of employment have different cyclical patterns. As Shimer (2012) showed, cyclical movements in the unemployment rate are driven to a large extent by those in the job-finding rate from unemployment, which in turn reflect closely the vacancy/unemployment rate, thus job creation. The latter is a very volatile variable, but the unemployment rate tracks it very closely, because the job-finding probability in the US is high, over 25% per month, ruling out significant transitional dynamics and shock propagation. In contrast, the EE transition probability is low, about 2.5% per month,

[3] Ashley and Verbrugge (2018) show that, in a forecasting, reduced-form sense, the statistical relationship between the rates of inflation and unemployment is highly non linear, and characterized by two distinct measures of slack or unemployment gap, "bust" and "boom", and three distinct phases. The first ("bust") relationship is the one highlighted by Stock and Watson (2010): there is a sharp reduction in inflation that occurs as the unemployment rate is rising rapidly. The second ("null") relationship occurs as the unemployment rate subsequently begins to fall; during this phase, inflation is unrelated to any conventional unemployment gap. The final ("overheating") relationship begins once the unemployment rate drops below its natural rate. In our view, the transmission channel of aggregate demand to inflation is employment misallocation, which is, in the overheating phase, modest and highly correlated with the "boom" unemployment gap. Crump et al. (2019) combine microdata on labor market transition rates, to control for the effects of demographics on the unemployment rate, with data on inflation expectations, in a standard New Keynesian Phillips Curve framework without on-the-job search. They estimate the time series of the natural rate of unemployment, where inflation remains stable. They find that the unemployment gap from their estimated natural rate declined very slowly after the Great Recession.

so the reallocation of employment up the job ladder unfolds slowly, and the propagation of aggregate shocks through the poaching/outside offers channel can be strong. Because firms cannot perfectly target their pool of job applicants, they create more jobs and post more vacancies either when many job applicants are unemployed or when the employed are mismatched and easy to poach (or both). Thus, independently of the state of unemployment, the distribution of employment on the job ladder, a slow-moving state variable, determines job creation and, indirectly, also the pace of job-finding from unemployment.

To formalize and quantitatively investigate this hypothesis, we introduce search in the labor market, both on- and off-the job, and endogenous job creation into an otherwise standard monetary DSGE model with a representative risk-averse household and Calvo pricing. As in Postel-Vinay and Robin (2002)'s Sequential Auctions protocol, firms make unilateral offers, that can be renegotiated only by mutual consent, when outside offers arrive. We are interested in business cycles and in monetary policy, hence we allow for aggregate uncertainty. Accordingly, we allow firms to offer and commit to contracts that prescribe state-contingent wages, and Bertrand-compete in such contracts for already employed workers. Essentially, we replace the household's neoclassical labor supply of the standard DSGE models, where the labor market is perfectly competitive, with an on-the-job-search model of the labor market, where both (un)employment and the distribution of job quality within employment are state variables, the dynamics of which are determined by labor demand, through endogenous job creation.[4]

Our model features an endogenous Neoclassical labor wedge, which maps into an endogenous New-Keynesian wage mark-up, both measuring the deviation from the efficient benchmark where the marginal rate of substitution of consumption for leisure equals the marginal product of labor. This wedge captures the ex post profits from hiring an unemployed worker, necessary to cover upfront recruitment costs. In our model, however, the impact of aggregate shocks on employment or wages also depends on a new object, that we dub the *Mismatch wedge*. This is the shortfall in average labor productivity due to employment misallocation on the job ladder. Because workers can search on the job, they can upgrade their position, climb the job ladder and close the Mismatch wedge. Firms take advantage of this wedge to profitably poach from their competitors.

To evaluate the quantitative properties of the model at business cycle frequencies, we extend the model along several dimensions, including an intensive margin of labor supply and one of recruiting. We log-linearize the dynamic equilibrium equations, find the fundamental Rational Expectations Equilibrium solution, and calibrate "internally" some parameters,

[4]In Moscarini and Postel-Vinay (2018) we analyze the risk-neutral real version of this model, a business cycle search model with Sequential Auctions, but no price rigidities and no monetary authority.

including those of the structural aggregate shock processes themselves, by the Simulated Method of Moments. We find that the model provides significant propagation and amplification of aggregate shocks on the job-finding probability and on the unemployment rate. We propose a new, revealed-preference based summary statistic of employment misallocation,: the "Acceptance" ratio between the observed transition probabilities employer-to-employer (EE) and unemployment-to-employment (UE). When this AC = EE/UE ratio is high, employed workers quit often, relative to general job availability, so they must be mismatched, containing wage pressure. In the model, AC comoves inversely with inflation. In separate work in progress (Moscarini and Postel-Vinay (2023)), we find empirical support for this relationship, after estimating a regional price Phillips curve following the methodology of Hazell et al. (2022)

The rest of the paper is organized as follows. Section 2 illustrates motivating facts and lays out the conceptual framework behind our model, that we formally introduce and analyze in Section 3. There, we also discuss our model's equilibrium implications and relate to those of the DSGE literature with search and pricing frictions. In Section 4 we extend our model to make it more flexible for quantitative analysis, that we perform in Section 5. The Appendix presents derivations of the extended model equilibrium.

2 Conceptual Framework and Motivating Evidence

Aggregate "slack", the extent to which employment and production can ramp up to accommodate an increase in the demand for goods without putting significant pressure on wages and prices, is a key concept in macroeconomic analysis and in policymaking, in Phillips curves old and new. The literature and policy practice have focused on empirical measures of slack based entirely on stock (time-free) variables, which are meant to gauge the *quantity* of (non) employment: unemployment rate, output gap, labor share, non-participation (Krause and Lubik (2007); Gertler and Trigari (2009); Christiano, Eichenbaum and Trabandt (2016); Crump, Eusepi, Sahin and Giannoni (2019); Del Negro, Lenza, Primiceri and Tambalotti (2020)).

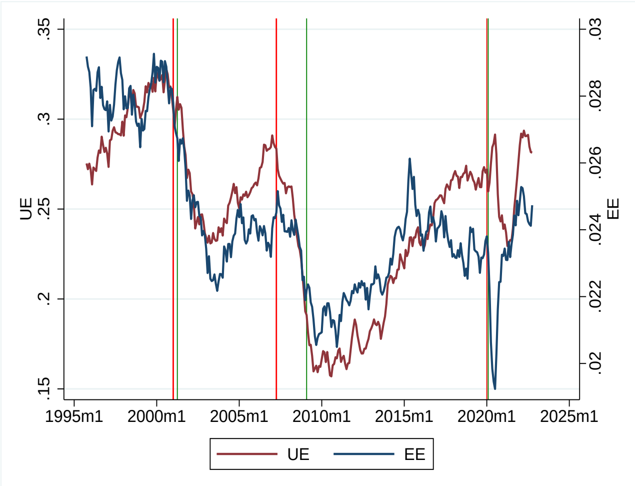
Our starting point is that these traditional measures do not capture another, potentially important dimension of slack: the *quality* of employment. If employed workers are mismatched, their efficiency units in production can grow, and output expand, by simply moving them to a different type of employment (tasks, company, occupation, industry, location). Therefore, slack can be significant even when unemployment is low. We need to supplement quantity measures with quality measures of employment.

Scholars and policymakers have long viewed mismatch through the lens of demand and supply of different skills. These are thought to be somewhat inflexible, thus a natural target for supply-side policies (such as retraining), rather than for aggregate demand management policies (monetary policy). At any rate, empirically, evidence for cyclical mismatch has been elusive (e.g. Sahin, Song, Topa and Violante (2014)). We take a different, "match-specific" view of misallocation, which *can* be mitigated by labor demand, because job scarcity leads workers to join and to remain in suboptimal matches during recessions (Moscarini (2001)). Our contributions is a business cycle model that makes the notion of cyclical mismatch mathematically precise and measurable.

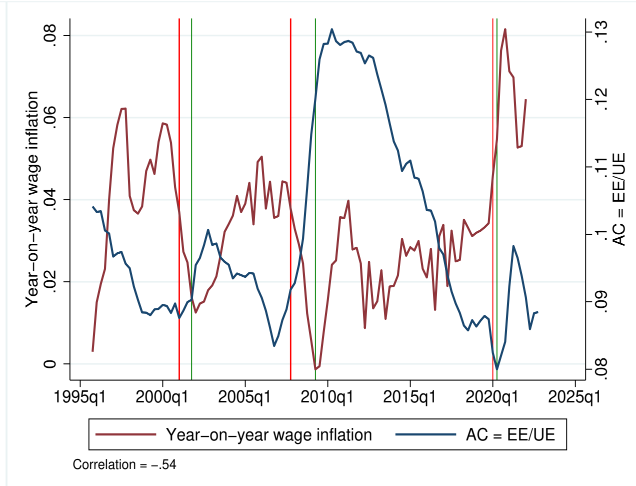
Our main argument is simple. If employed workers are willing to entertain outside offers, then, by revealed preferences, they must be mismatched, so they can be poached relatively easily, and any wage raise that they receive will likely reflect an improved allocation. If instead these workers are already well-matched, their employers will often successfully counter outside offers, causing wages to rise with no reallocation — a classic wage-push shock. Hence, mismatch and willingness to accept outside offers are negative predictors of inflation.

How can we make this insight operational? The UE probability accurately measures job availability, as the unemployed are unlikely to decline many offers. In fact, the empirical UE probability from the monthly Current Population Survey (CPS) is almost exactly log-linear in the ratio between vacancy counts from the Job Openings and Labor Turnover Survey (JOLTS) and unemployed counts from the CPS. In standard random job search models, the employed receive offers at a rate that comoves with that of the unemployed, but they are choosier. The AC = EE/UE ratio isolates how choosy the employed are: we refer to this ratio as the "Acceptance rate" (of outside offers), and we explore its predictive power for price and wage inflation.

A key piece of empirical information, and the binding data constraint, is the EE transition probability. A time series for the US is available at monthly frequency since 1994 from the CPS, and since 2000 at quarterly frequency from "Job-to-job Flows" series of the Census's Longitudinal Employer Household Dynamics (LEHD) data set. The CPS is monthly, is published promptly, which makes it uniquely useful for policy making, and provides a wealth of detailed information on labor market states (including the official unemployment rate, and related flows in and out of it, such as UE). The LEHD is quarterly, a low frequency for labor market transitions that causes time aggregation, is published with significant delay, does not distinguish between unemployment and non-participation, but has comprehensive coverage and huge sample size, which allow to precisely estimate the EE probability also at the state level by demographics. In this section, we present the basic facts from the CPS. In work in progress (Moscarini and Postel-Vinay (2023)), we exploit the LEHD-based evidence to perform econometric analysis of the causal effects of the AC ratio on nominal wage growth.



(a) Monthly UE (left scale) and EE (right scale) prob.



(b) Acceptance rate EE/UE and wage growth

Fig. 1: Labor market transition probabilities from the Monthly Current Population Survey. Source: Fujita, Moscarini and Postel-Vinay (2022)

Figure 1(a) plots time series of the monthly EE and UE transition probabilities estimated by Fujita, Moscarini and Postel-Vinay (2022). Figure 1(b) plots their AC = EE/UE ratio, averaged at quarterly frequency, and an index of aggregate nominal wage growth, drawn from the Quarterly Census of Employment and Wages. Both EE and UE probabilities are visibly procyclical, and show a decline in the late 1990s, with no trend thereafter. UE is more volatile, mostly because it keeps growing throughout expansions. In contrast, the cyclical rebound in the EE probability fizzles after a few years, and EE is relatively flat, if not sightly declining, in the mature phases of the last three completed business cycles: 1996-2000, 2004-2007 and 2015-2019. As an implication, the AC = EE/UE ratio shows no appreciable trend over the entire period, is strongly countercyclical, and, unlike the unemployment rate, accelerates downward as the expansion continues.

Finally, the AC ratio shows a clear negative association with aggregate nominal wage growth. In Moscarini and Postel-Vinay (2023)) we go beyond this simple association, and apply to US wage data the identification strategy of Hazell et al. (2022), based on a regional Phillips curve, which allows to control for common unobserved economy-wide factors, such as inflation expectations. We define a labor market by state and demographics, and augment the regional unemployment specification with local labor market flows. We find that the AC ratio has a significant negative causal impact on nominal wage growth in the nontradable sectors. These observations suggest that the AC ratio contains independent, valuable statistical information to gauge the extent of labor market slack within employment. We now formalize this hypothesis.

3 The Baseline Model

In this section, we introduce and analyze a baseline version of our model, that we use to illustrate the main mechanism. Our model nests a standard New Keynesian model with a competitive labor market; a New Keynesian model with only unemployed job search (Krause and Lubik 2007); a flexible-price business cycle search and matching model with (Robin 2011, Lise and Robin 2017, Moscarini and Postel-Vinay 2018) or without (Andolfatto 1996, Merz 1996) on-the-job search. In Section 4, we extend the model to make it more flexible for quantitative analysis.

3.1 The Economy

Agents, goods, endowments and technology. Time $t = 0, 1, 2, \cdots$ is discrete. Two vertically integrated sectors produce two kinds of non-storable output: an intermediate input, that we call Service, and differentiated varieties of a Final good. In the Service sector, firms produce Service with linear technology using only labor. Each unit of labor ("job match") produces y units of the Service, which is then sold on a competitive market at nominal price ω_t. Productivity y is specific to each match and is drawn once and for all, when the match forms, in a i.i.d. manner from a cdf Γ with support $[\underline{y}, \overline{y}]$. In the Final good sector, each variety $i \in [0, 1]$ is produced by a single firm, also indexed by i, with a linear technology that turns each unit of the homogeneous Service into z_t units of variety i, which are then sold in a monopolistic competitive market. TFP z_t follows a first-order Markov process.

A representative household is a collection of agents indexed by $j \in [0, 1]$. Each household member has an indivisible unit endowment of time per period, and the household is collectively endowed with ownership shares of all firms in both sectors. We indicate whether household member j is employed at time t by $\mathrm{e}_{t-1}(j) \in \{0, 1\}$. As explained below, employment is an individual state variable, predetermined at the end of the previous period. Members of each household provide perfect mutual insurance against idiosyncratic income shocks. We will discuss later the importance of this complete market assumption.

Preferences. The household has preferences represented by a per-period utility function

$$U\left(C_t, 1 - \int_0^1 \mathrm{e}_{t-1}(j)dj\right)$$

over a CES consumption aggregator of Final good varieties

$$C_t = \left(\int_0^1 c_t(i)^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}} \tag{1}$$

where $\eta > 1$, and total leisure enjoyed by all household members, $1 - \int_0^1 \mathrm{e}_{t-1}(j)dj$. U is increasing and concave, strictly concave in consumption alone. By large numbers, each representative household includes a share of jobless members equal to the national unemployment rate u, so we can also write $U(C, u)$ with $u = 1 - \int_0^1 \mathrm{e}(j)dj$. The household maximizes the present value of expected utility discounted with factor β. For simplicity, we assume that the preference weight on unemployment/leisure is low enough that all matches are preferable to unemployment at all points in time, and separations into unemployment are only exogenous.[5]

Search frictions in the labor market. Service sector firms advertise vacancies that randomly meet jobseekers. Advertising a vacancy costs κ_v units of the Final good aggregator C_t per period. Once a vacancy and a jobseeker meet, they draw match quality y; if the match is acceptable, production can begin in the following period.

In period t, the u_{t-1} previously unemployed workers search for open job vacancies. The $1 - u_{t-1}$ previously employed workers are separated from their jobs with time-invariant probability $\delta \in (0, 1]$ and become unemployed, to enjoy leisure for one period, along with those previously unemployed who did not find a job this period. The remaining $(1 - \delta)(1 - u_{t-1})$ employed workers receive this period, with time-invariant probability $s_1 \in (0, 1]$, an opportunity to search for a vacant job (a new match). Let

$$\theta_t = \frac{v_t}{u_{t-1} + s_1(1 - \delta)(1 - u_{t-1})}$$

be effective job market tightness, the ratio between total vacancies posted v_t and total search effort. A homothetic meeting function determines the probability $\phi(\theta) \in [0, 1]$, increasing in θ, that a searching worker locates an open vacancy, thus the probability $\phi(\theta)/\theta$, decreasing in θ, that an open vacancy meets a worker. The output of the Service sector can be thought of as a bundle of efficiency units of labor, assembled by Service sector firms in a frictional labor market, and leased to Final good producers in a competitive market at unit price ω_t. Service sector firms are essentially labor market intermediaries, solving the hiring problems of Final good producers.

[5] In Moscarini and Postel-Vinay (2018) we allow for endogenous separations in the flexible price, risk-neutral version of this economy.

Price and wage setting. In the Final good sector, each monopolistic competitive producer i of variety $i \in [0, 1]$ draws every period with probability $\nu \in (0, 1)$ in an i.i.d. fashion an opportunity to revise its nominal price $p_t(i)$. Given this price, either newly revised or not, the firm serves all the resulting demand $q_t^d(i)$ by buying the required quantity $q_t^d(i)/z_t$ of Service in a competitive market at nominal unit price ω_t.

In the labor market, a Service-producing firm can commit to guarantee each worker a state-contingent expected present value of payoffs in utility terms (a "contract"), implemented by state-contingent wage payments, until the match separates. The contract can be renegotiated by mutual consent only. The firm's commitment is limited, in that it can always unilaterally separate, so firms' profits cannot be negative (in expected PDV). Same for the worker: if the utility value from staying in the contract falls below the value of unemployment, the worker will quit. When an employed worker contacts an open vacancy, the recruiting firms and the current employer observe each other's match qualities with the worker and engage in Bertrand competition over contracts. The worker chooses the contract that delivers the larger value. When indifferent, the worker quits.

Financial markets, fiscal and monetary policy. Nominal bonds B_{t+1} issued by the government trade at price $(1+R_t)^{-1}$ units of numéraire, in exchange for one unit of it for sure one period later. A monetary authority is the monopolist of the numéraire and controls the nominal interest rate R_t on bonds following some (typically Taylor) rule. Bonds are traded by households and monetary authority, and can be in positive net supply. The Government levies a tax/pays a subsidy T_t to balance its budget, including interest on new bonds.

Households also trade ownership shares of all firms, producers of Final goods and of Service intermediate input, in competitive financial markets. Producers of Final good varieties earn, due to product differentiation, pure profits (or losses), which change randomly with infrequent Calvo pricing. Producers of the Service input, in order to cover upfront hiring costs due to search frictions in the labor market, must also earn pure profits, which fluctuate randomly, depending on the outcome of job posting. Both profit flows are rebated to shareholders as net dividends. To eliminate idiosyncratic risk in these dividends, the household combines these shares in mutual funds that own a representative cross-section of all firms.

Timing of events within a period:

1. all agents observe random innovations to aggregate TFP in the Final good sector and to monetary policy; the monetary authority chooses the nominal interest rate;

2. each producer of a Final good variety receives with probability ν an opportunity to change its price, independent over time and across varieties;

3. firms and previously employed workers produce Service and Final good varieties, immediately sold at the posted prices and used/consumed; firms in the Service sector pay their workers to fulfill the wage contracts they are currently committed to; firms in both sectors pay dividends to mutual fund owners; unemployed workers receive utility from leisure;[6]

4. households trade nominal bonds with the monetary authority and mutual fund shares with each other;

5. some existing job matches break up exogenously, moving workers into unemployment;

6. firms in the Service sector post vacancies;

7. all previously unemployed workers and a fraction s_1 of the remaining employed workers search for those vacancies;

8. upon meeting a job applicant, a vacancy-posting firm makes the worker a new offer; if the worker is already employed, his current employer makes a counteroffer;

9. if the worker, whether initially employed or not, receives and accepts a job offer, he becomes employed in the new match, otherwise he remains in his current state, either unemployed or employed in the current match.

3.2 Household Optimization

The household chooses stochastic processes for Final good consumption varieties $c_t(i)$, holdings of bonds B_t^d and ownership shares of (mutual funds of) firms in both sectors $(\vartheta_t^F, \vartheta_t^S)$, given their prices, resp. $p_t(i), R_t, p_t^F, p_t^S$. The household does not freely choose its member j's labor supply at time t, because employment status $\mathrm{e}_{t-1}(j)$ is predetermined by search frictions: rather, the household chooses the probability $a_t(j)$ that member j accept any new job offer they might receive at the end of period t. Given wages and prices, to be determined in equilibrium, the household then solves

$$\max_{\left\{c_t(i), B_t^d, \vartheta_t^F, \vartheta_t^S, a_t(j)\right\}} \mathbb{E}_0 \sum_{t=0}^{+\infty} \beta^t U\left(C_t, 1 - \int_0^1 \mathrm{e}_{t-1}(j)dj\right)$$

[6]We assume that production occurs after observing aggregate shocks, but before hiring new workers, who take one period to join the production line, just like newly unemployed workers must wait at least one period to rejoin employment. Because hiring (vacancy posting) activity absorb Final goods, the alternative timing could give rise to multiple equilibria: expecting high economic activity, Service firms hire aggressively, expand immediately their production of the intermediate input, whose price falls, stimulating the production of final goods, whose price also falls, making hiring cheap and justifying aggressive hiring in the first place.

where C_t is the CES consumption aggregator (1), subject to:

- the budget constraint (in nominal terms);

$$\int_0^1 p_t(i)c_t(i)di+\frac{B^d_{t+1}}{1+R_t}+\sum_{\mathcal{I}=S,F}\vartheta^{\mathcal{I}}_{t+1}p^{\mathcal{I}}_t \leq \sum_{\mathcal{I}=S,F}\vartheta^{\mathcal{I}}_t\left(\Pi^{\mathcal{I}}_t+p^{\mathcal{I}}_t\right)+\int_0^1 \mathrm{e}_{t-1}(j)w_t(j)dj+B^d_t-T_t$$

  where $\Pi^F_t = \int_0^1 \Pi^F_t(i)di$ are the total nominal profit flows earned by all Final good producers, $\Pi^F_t(i)$ by the only firm producing Final good variety i, and Π^S_t by each Service producer (after paying any hiring costs ex ante), while $\int_0^1 \mathrm{e}_{t-1}(j)w_t(j)dj$ are the household's nominal earnings, the sum of wages $w_t(j)$ paid to those workers $j\in[0,1]$ within the household who are currently employed by Service producers; because of search frictions, different workers receive different wages;

- the stochastic processes for each member j's employment status, $\mathrm{e}_t(j)\in\{0,1\}$:

$$\mathrm{e}_t(j)=\begin{cases}\mathrm{e}_{t-1}(j) & \text{with prob. } \mathrm{e}_{t-1}(j)(1-\delta)+\left(1-\mathrm{e}_{t-1}(j)\right)\left(1-\phi(\theta_t)a_t(j)\right)\\ 1-\mathrm{e}_{t-1}(j) & \text{otherwise}\end{cases} \tag{2}$$

- a No Ponzi Game condition

$$\Pr\left(\lim_{t\to\infty} B^d_t\prod_{\tau=0}^{t-1}(1+R_\tau)^{-1}=0\right)=1.$$

We solve the household's maximization problem in steps: consumption and financial portfolio allocation first, then labor market turnover decisions. The demand for each Final good variety is standard

$$c_t(i)=C_t\left(\frac{p_t(i)}{P_t}\right)^{-\eta} \tag{3}$$

where the ideal price index is

$$P_t=\left(\int_0^1 p_t(i)^{1-\eta}di\right)^{\frac{1}{1-\eta}} \tag{4}$$

and minimum expenditure equals $P_tC_t=\int_0^1 p_t(i)c_t(i)di$.

Divide the time-t budget constraint by P_t and attach to it a Lagrange multiplier λ_t, which then equals $U_C\left(C_t,1-\int_0^1 \mathrm{e}_{t-1}(j),dj\right) = U_C(C_t,u_{t-1})$, so it converts units of the consumption aggregator C_t into utils.[7] The demand for bonds gives rise to the standard

[7]Given our timing assumption and notation, total employment at time t is $\int_0^1 \mathrm{e}_{t-1}(j)=1-u_{t-1}$, pre-

Euler equation, where we use the fact that household leisure equals the unemployment rate:

$$(1 + R_t)\,\mathbb{E}_t\left[\mathcal{D}_t^{t+1}\frac{P_t}{P_{t+1}}\right] = 1 \tag{5}$$

which discounts the real interest rate $(1 + R_t)\,\mathbb{E}_t\left[P_t/P_{t+1}\right]$ with the pricing kernel

$$\mathcal{D}_t^{t+\tau} = \prod_{s=t}^{t-1+\tau}\left(\beta\frac{\lambda_{s+1}}{\lambda_s}\right) = \beta^{\tau}\frac{\lambda_{t+\tau}}{\lambda_t} = \beta^{\tau}\frac{U_C\left(C_{t+\tau}, u_{t-1+\tau}\right)}{U_C\left(C_t, u_{t-1}\right)}$$

For each sector $\mathcal{I} \in \{S, F\}$, optimal portfolio allocations and market clearing, ruling asset price bubbles out, imply a standard asset pricing formula:

$$\frac{p_t^{\mathcal{I}}}{P_t} = \mathbb{E}_t\left[\sum_{\tau=0}^{+\infty}\mathcal{D}_t^{t+\tau}\frac{\Pi_{t+\tau}^{\mathcal{I}}}{P_{t+\tau}}\right]$$

Firms maximize the value to their owners, namely, the utility value of the share price of each mutual fund, which is the present value of real profits, discounted by the pricing kernel, the representative household's stochastic discount factor.

We now turn to the new part of our analysis, labor market turnover decisions $a_t(j)$. The only objects in the household's maximization problem that depend on those decisions are the flow utility $U(C_t, \int_0^1 \mathrm{e}_{t-1}(j)dj)$, and nominal labor income $\int_0^1 \mathrm{e}_{t-1}(j)w_t(j)dj$, through the stochastic laws of motion of each member's employment status $\mathrm{e}_t(j)$, namely (2). Thus, when deciding job offer acceptance $a_t(j)$, the household solves the following dynamic sub-problem, subject to the constraint (2):

$$\max_{\{a_t(j)\}} \mathbb{E}_0\sum_{t=0}^{+\infty}\beta^t\left[U\left(C_t, 1 - \int_0^1 \mathrm{e}_{t-1}(j)dj\right) + \lambda_t\int_0^1 \mathrm{e}_{t-1}(j)\frac{w_t(j)}{P_t}dj\right] \tag{6}$$

Household members can take job acceptance decisions independently, because they do not affect each other's employment prospects: the household is one of many, and does not internalize congestion externalities in the search market, not even the externalities that its own members create on each other. The only interaction between household members is through income pooling, which explains the common Lagrange multiplier λ_t on earnings, independent of each member's identity j and current employment status $\mathrm{e}_{t-1}(j)$. Therefore, the household's labor turnover problem (6) separates into two types: one for each currently unemployed member ($\mathrm{e}_{t-1}(j) = 0$), yielding an optimal real value of unemployment $V_{0,t}^j = V_{0,t}$ independent of the worker's identity j, as all the unemployed are identical, and one for each

determined at the end of $t-1$.

employed member j ($\mathrm{e}_{t-1}(j) = 1$), with $V_{1,t}\left(w^t(j), y_t(j)\right)$ the value of working in a match of quality $y = y_t(j)$ and holding a contract $w^t(j)$ specifying a continuation stream of promised state-contingent wages, starting with a current wage $w_t(j)$. These values $V_{\mathrm{e}_{t-1},t}$ can be written in recursive form as follows. Let $U_u\left(C_t, 1 - \int_0^1 \mathrm{e}_{t-1}(j)dj\right) = U_u(C_t, u_{t-1})$ denote the marginal utility of having one more household member unemployed. The Bellman equations in utils read (still subject to (2)):

$$\lambda_t V_{0,t} = U_u(C_t, u_{t-1}) + \beta \mathbb{E}_t \Bigg[\max_{\{a_t(j)\}} \Big\langle \left(1 - \phi\left(\theta_t\right) a_t(j)\right) \lambda_{t+1} V_{0,t+1} + \phi\left(\theta_t\right) a_t(j) \lambda_{t+1} V_{1,t+1}\left(w^{t+1}(j), y_{t+1}(j)\right) \Big\rangle \mid \mathrm{e}_{t-1}(j) = 0 \Bigg]$$

$$\lambda_t V_{1,t}\left(w^t(j), y_t(j)\right) = \lambda_t \frac{w_t(j)}{P_t} + \beta \mathbb{E}_t \Bigg[\max_{\{a_t(j)\}} \Big\langle \delta \lambda_{t+1} V_{0,t+1} + (1-\delta) \lambda_{t+1} V_{1,t+1}\left(w^{t+1}(j), y_{t+1}(j)\right) \Big\rangle \mid \mathrm{e}_{t-1}(j) = 1, w^t(j), y_t(j) \Bigg]$$

The time index t of the values subsumes their dependence on payoff-relevant aggregate variables that are exogenous to the employment relationship but endogenous to the economy, such as job market tightness θ_t and price level P_t.

3.3 Service Producers' Optimization and Labor Market Equilibrium

Match values. Dividing the two equations throughout by λ_t and using our notation for the pricing kernel, $\mathcal{D}_t^{t+1} = \beta \lambda_{t+1}/\lambda_t$, yields a representation of those two problems in real terms. Further recalling that employers extract the full match rent from unemployed workers, the value $V_{1,t+1}\left(w^{t+1}(j), y_{t+1}(j) \mid \mathrm{e}_{t-1}(j) = 0\right)$ they offer them is just $V_{0,t+1}$. Thus, $V_{0,t}$ solves

$$V_{0,t} = \frac{U_u(C_t, u_{t-1})}{\lambda_t} + \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} V_{0,t+1} \right] = \frac{1}{\lambda_t} \mathbb{E}_t \sum_{\tau=0}^{\infty} \beta^\tau U_u\left(C_{t+\tau}, u_{t-1+\tau}\right) \tag{7}$$

We assume throughout that the relative weight in preferences on leisure vis-a-vis consumption is low enough that no match will ever break up endogenously, so all separations will be exogenous, with probability δ.

Next, we turn to the value of employment. Let $\overline{V}_{1,t+1}(y)$ denote the maximum value that a firm is willing to promise at the time offers are made in period t (i.e. at stage 8 of the within-period timing outlined earlier) to a worker with whom it can produce a flow y

of Service, without violating its own participation constraint (the firm has a zero outside option by free entry). In auction theory parlance, this is the firm's willingness to pay for a match y.

Consider a worker who is currently employed in a match of quality y, is not hit by a separation shock, and is promised an expected continuation value $\mathbb{E}_t\left[\mathcal{D}_t^{t+1}V_{1,t+1}\left(w^{t+1},y\right)\right]$, namely a wage w_{t+1}/P_{t+1} next period should he stay in the current match and produce y units of Service, and then a continuation contract from $t+2$ on. When such a worker meets at time t, with probability $s_1\phi(\theta_t)$, an open vacancy and draws a new match quality y', Bertrand competition produces one of three possible outcomes:

1. $\mathbb{E}_t\left[\mathcal{D}_t^{t+1}V_{1,t+1}\left(w^{t},y\right)\right] \geq \mathbb{E}_t\left[\mathcal{D}_t^{t+1}\overline{V}_{1,t+1}(y')\right]$, in which case the incumbent employer needs to do nothing to retain the worker, and the offer is irrelevant as the poacher cannot profitably match the value already promised by the incumbent to the worker;

2. $\mathbb{E}_t\left[\mathcal{D}_t^{t+1}V_{1,t+1}\left(w^{t+1},y\right)\right] < \mathbb{E}_t\left[\mathcal{D}_t^{t+1}\overline{V}_{1,t+1}(y')\right] < \mathbb{E}_t\left[\mathcal{D}_t^{t+1}\overline{V}_{1,t+1}(y)\right]$, in which case the incumbent employer raises its offer from $\mathbb{E}_t\left[\mathcal{D}_t^{t+1}V_{1,t+1}\left(w^{t+1},y\right)\right]$ to $\mathbb{E}_t\left[\mathcal{D}_t^{t+1}\overline{V}_{1,t+1}(y')\right]$ and profitably retains the worker;

3. $\mathbb{E}_t\left[\mathcal{D}_t^{t+1}\overline{V}_{1,t+1}(y)\right] \leq \mathbb{E}_t\left[\mathcal{D}_t^{t+1}\overline{V}_{1,t+1}(y')\right]$, in which case the worker is poached with an offer worth $\mathbb{E}_t\left[\mathcal{D}_t^{t+1}\overline{V}_{1,t+1}(y)\right]$.

The worker changes job only in the last case, and turnover decisions depend solely on the expected continuation value under full rent extraction $\mathbb{E}_t\left[\mathcal{D}_t^{t+1}\overline{V}_{1,t+1}(y)\right]$. Thus, in period t, the maximum expected continuation value $\mathbb{E}_t\left[\mathcal{D}_t^{t+1}\overline{V}_{1,t+1}(y')\right]$ that the worker can receive in a type-y match includes a future wage at $t+1$, as well as a continuation value which equals the discounted expected value of unemployment $\mathbb{E}_t\left[\mathcal{D}_{t+1}^{t+2}V_{0,t+2}\right]$ in case the worker is laid off at stage 5 of period $t+1$ (probability δ), and otherwise equals the (expected future) willingness to pay $\mathbb{E}_t\left[\mathcal{D}_{t+1}^{t+2}\overline{V}_{1,t+2}(y)\right]$ of the current employer, that the worker receives either from the incumbent employer itself, as part of the current contract, or from a poacher. This is because the incumbent firm y is already promising the maximum it can in period $t+1$, namely $\overline{V}_{1,t+1}(y')$, so it will not match any outside offers: the worker either stays at the same value, or leaves and receives the same value from a more productive poacher. Hence, the only remaining choice is the flow wage, and the maximum wage thaty the firm can pay in the next period in each state, without making a loss, is full revenues. Therefore, at each time t, the full rent-extraction contract solves recursively:

$$\overline{V}_{1,t}(y) = \frac{\omega_t}{P_t}y + \delta\mathbb{E}_t\left[\mathcal{D}_t^{t+1}V_{0,t+1}\right] + (1-\delta)\mathbb{E}_t\left[\mathcal{D}_t^{t+1}\overline{V}_{1,t+1}(y)\right]$$

Here the complete market assumption comes into play. Since households value a marginal dollar of profit as much as a dollar of labor income (at λ_t/P_t), value is perfectly transferable between individual workers and firms. Therefore, a worker's value $\overline{V}_{1,t}(y)$ of extracting full rents from a type-y job is also the value of said job to the firm-worker pair under any sharing rule, and we can define a type-y job *surplus* $S_t(y) = \overline{V}_{1,t}(y) - V_{0,t}$ at the offer-making stage of period t. Subtracting the expression (7) for $V_{0,t}$ from both sides of the last equation and then solving forward, we find a very simple expression for this surplus:

$$\begin{aligned} S_t(y) = \overline{V}_{1,t}(y) - V_{0,t} &= \frac{\omega_t}{P_t} y - \frac{U_u(C_t, u_{t-1})}{\lambda_t} + (1-\delta)\mathbb{E}_t\left[\mathcal{D}_t^{t+1} S_{t+1}(y)\right] \\ &= \mathbb{E}_t \sum_{\tau=0}^{+\infty} (1-\delta)^\tau \mathcal{D}_t^{t+\tau}\left(\frac{\omega_{t+\tau}}{P_{t+\tau}} y - \frac{U_u(C_{t+\tau}, u_{t-1+\tau})}{\lambda_{t+\tau}}\right) = W_t y - \mathcal{L}_t \end{aligned}$$

where we define recursively two expected PDV until match separation, expressed in units of the consumption aggregator C_t: one of a unit of Service ω_t/P_t — an object that can be interpreted as the "average real wage rate"

$$W_t = \frac{\omega_t}{P_t} + (1-\delta)\mathbb{E}_t\left[\mathcal{D}_t^{t+1} W_{t+1}\right] \tag{8}$$

and the other one of the Marginal Rate of Substitution between consumption and leisure

$$\mathcal{L}_t = \frac{U_u(C_t, u_{t-1})}{U_C(C_t, u_{t-1})} + (1-\delta)\mathbb{E}_t\left[\mathcal{D}_t^{t+1} \mathcal{L}_{t+1}\right] = \frac{\sum_{\tau=0}^{\infty}(1-\delta)^\tau \beta^\tau \mathbb{E}_t[U_u(C_{t+\tau}, u_{t-1+\tau})]}{U_C(C_t, u_{t-1})} \tag{9}$$

Crucially, the surplus $S_t(y)$ is affine increasing in y. Because the willingness to pay in the auction can be written as $\mathbb{E}_t\left[\mathcal{D}_t^{t+1}\overline{V}_{1,t+1}(y)\right] = \mathbb{E}_t\left[\mathcal{D}_t^{t+1}\left(V_{0,t+1} + W_{t+1} y - \mathcal{L}_{t+1}\right)\right]$, this too is affine increasing in y, with intercept and slope that vary stochastically over time. Therefore, the firm with the higher match quality y wins the auction, and we draw the main conclusion of this subsection: the equilibrium is Rank Preserving (RP), and the direction of reallocation is efficient, always from less to more productive matches. That the auction outcome depends only on the match surplus is a common result in Sequential Auctions theory, going back to Postel-Vinay and Robin (2002), which enormously simplifies the analysis (for example, allowing for complex patterns of heterogeneity on both sides of the market, as in Lise and Robin (2018)). We are the first to extend it to a proper general equilibrium environment, by observing that a well-known implication of complete markets, the equality of the marginal rates of substitution, also implies perfectly transferable utility at the margin.

Evolution of worker stocks: employment distribution on the job ladder and unemployment. Let $L_t(y)$ be the measure of workers employed in matches less productive than y at the end of period t, after separations and hiring. Given the timing of events and the RP property of equilibrium, this evolves as:

$$L_t(y) = (1-\delta)\left[1 - s_1\phi(\theta_t)\overline{\Gamma}(y)\right]L_{t-1}(y) + \phi(\theta_t)\Gamma(y)u_{t-1} \tag{10}$$

For $y = \overline{y}$, this gives the familiar law of motion of unemployment $u_t = 1 - L_t(\overline{y})$:

$$u_t = \left[1 - \phi(\theta_t)\right]u_{t-1} + \delta(1 - u_{t-1}) \tag{11}$$

Free entry and labor demand. By the time a firm and a worker who have met on the search market decide whether or not to form a match of quality y', they know the expected surplus $\mathbb{E}_t\left[\mathcal{D}_t^{t+1}S_{t+1}(y')\right]$ from it, as well as from the worker's current state: zero if the worker is unemployed, and $\mathbb{E}_t\left[\mathcal{D}_t^{t+1}S_{t+1}(y)\right]$ if the worker is employed in a type-y match.

The free entry condition equates the per period cost of vacancy posting to the vacancy contact probability times the expected return from a successful contact. The firm appropriates the entire expected discounted surplus $\beta\mathbb{E}_t\left[\mathcal{D}_t^{t+1}S_{t+1}(y)\right]$ from unemployed job applicants, and the difference in willingness to pay, thus expected surplus, between own match y and existing match y' from employed job applicants, $\beta\mathbb{E}_t\left[\mathcal{D}_t^{t+1}\left(\overline{V}_{t+1}(y) - \overline{V}_{t+1}(y')\right)\right] = \beta\mathbb{E}_t\left[\mathcal{D}_t^{t+1}\left(S_{t+1}(y) - S_{t+1}(y')\right)\right]$. Using $S_t(y) = W_t y - \mathcal{L}_t$, the free entry condition is then:

$$\begin{aligned}\kappa_v\frac{\theta_t}{\phi(\theta_t)} = {} & \frac{u_{t-1}}{u_{t-1} + (1-\delta)s_1(1-u_{t-1})}\left\{\mathbb{E}_t\left[\mathcal{D}_t^{t+1}W_{t+1}\right]\int_{\underline{y}}^{\overline{y}} y\,d\Gamma(y) - \mathbb{E}_t\left[\mathcal{D}_t^{t+1}\mathcal{L}_{t+1}\right]\right\} \\ & + \frac{(1-\delta)s_1(1-u_{t-1})}{u_{t-1} + (1-\delta)s_1(1-u_{t-1})}\mathbb{E}_t\left[\mathcal{D}_t^{t+1}W_{t+1}\right]\int_{\underline{y}}^{\overline{y}}\int_{\underline{y}}^{y}(y - y')\frac{dL_{t-1}(y')}{1-u_{t-1}}d\Gamma(y)\end{aligned} \tag{12}$$

In units of the Final good aggregate, on the LHS are vacancy posting costs times the expected duration of a vacancy, on the RHS are the expected profits earned by Service sector producers, namely, the average of the expected profits from hiring unemployed and an employed job applicants, weighted by the respective shares of the two types of job applicants in the pool of job searchers. Unemployed hires are homogeneous, while employed hires are distributed, at the time of vacancy posting, according to the p.d.f. $dL_{t-1}(y')/(1-u_{t-1})$ of match quality y' in their current job, a job that gives them bargaining power in wage negotiations. Note that the input into the meeting process is L_{t-1}, the pre-determined, beginning-of-period employment (distribution), while the cost of hiring unemployed workers is the (epected PDV of the) MRS $\mathcal{L}_{t+1}$, which depends on leisure u_t because the alternative for the prospective unemployed job applicant is to remain unemployed and to add marginal utility of leisure to the household, after hiring takes place.

3.4 Final Good Producers' Optimization

Final goods are demanded by households, for consumption, and by Service sector firms, to advertise vacancies, according to the same demand distribution (basket) as households. Keeping one vacancy open for one period of time requires using $\kappa_{v,t} c_t(i)$ units of each Final good variety i, proportional to household demand $c_t(i)$, so that $\kappa_{v,t} C_t = \kappa_v \theta_t [u_{t-1} + (1-\delta) s_1 (1 - u_{t-1})]$ are total vacancy costs in units of the numeraire Final good. Adding up, total demand for each variety i by households and Service firms is

$$q_t^d(i) = (1 + \kappa_{v,t}) c_t(i) = (1 + \kappa_{v,t}) C_t \left(\frac{p_t(i)}{P_t} \right)^{-\eta} = Q_t \left(\frac{p_t(i)}{P_t} \right)^{-\eta}$$

where, using Eq. (1), total output in the Final good sector equals

$$Q_t = (1 + \kappa_{v,t}) C_t = (1 + \kappa_{v,t}) \left(\int_0^1 c_t(i)^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}} = \left(\int_0^1 q_t(i)^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}}. \tag{13}$$

Each Final good producer $i \in [0,1]$ quotes a price and stands ready to serve total demand $q_t^d(i) = (1+\kappa_{v,t}) c_t(i)$, given the technology that turns each unit of the homogeneous Service, purchased at given unit price ω_t, into z_t units of Final good variety i. Serving the demand thus requires producing $q_t(i) = q_t^d(i)$ and paying a nominal input cost $\omega_t q_t(i)/z_t$. Producer i quotes price $p(i)$ and earns nominal profits

$$\widetilde{\Pi}_t^F(p(i)) = Q_t \left(\frac{p(i)}{P_t} \right)^{-\eta} \left(p(i) - \frac{\omega_t}{z_t} \right)$$

This producer is allowed to revise its price with probability ν each period. When this opportunity arises at time t, firm i chooses a price, that will be in effect until the future random time $t+\tau > t$ of the next opportunity, to maximize the expected PDV of real profits:

$$\max_{p(i)} \mathbb{E}_t \left[\sum_{\tau=0}^{+\infty} (1-\nu)^\tau \mathcal{D}_t^{t+\tau} \frac{\widetilde{\Pi}_{t+\tau}^F(p(i))}{P_{t+\tau}} \right].$$

Taking a FOC and rearranging, the optimal reset price is the same for all firms i:

$$p_t^* = \frac{\eta}{\eta - 1} \frac{\sum_{\tau=0}^{+\infty} (1-\nu)^\tau \mathbb{E}_t \left[\mathcal{D}_t^{t+\tau} Q_{t+\tau} P_{t+\tau}^{\eta-1} \frac{\omega_{t+\tau}}{z_{t+\tau}} \right]}{\sum_{\tau=0}^{+\infty} (1-\nu)^\tau \mathbb{E}_t \left[\mathcal{D}_t^{t+\tau} Q_{t+\tau} P_{t+\tau}^{\eta-1} \right]} \tag{14}$$

Because the selection of firms that get to reset their prices is random, using (4) the Final good price index P_t then solves:

$$P_t^{1-\eta} = \nu \left(p_t^*\right)^{1-\eta} + (1-\nu) P_{t-1}^{1-\eta} \tag{15}$$

The total profits that firm i rebates to its shareholders at time t equal $\Pi_t^F(i) = \widetilde{\Pi}_t^F \left(p_{t-\tau(i)}^*\right)$ where $\tau(i) \in \mathbb{N}$ is the age of firm i's price, distributed with probability $\nu(1-\nu)^\tau$.

3.5 Market-Clearing

Financial markets. The representative household holds all shares of all firms, $\vartheta_t^F = \vartheta_t^S = 1$, and purchases (redeems) all bonds B_t (previously) issued by the Government, so bond demand B_t^d equal supply B_t. The government balances its budget every period: $T_t = B_t - B_{t+1}/(1+R_t)$, so the households pay back to the Government in taxes all the net surplus of bonds redemptions (including interest) minus new bond purchases, neither borrow nor save, but spend all their income on the Final good.

Good markets. Market-clearing in each Final good variety i requires the supply $q_t(i)$ to equal the isoelastic demand $c_t(i)$ by households and the demand by Service firms to cover vacancy costs. Aggregating over varieties:

$$Q_t = C_t + \kappa_v \theta_t \left[u_{t-1} + (1-\delta)(1-u_{t-1})s_1\right] \tag{16}$$

Market-clearing in the Service market requires the supply of the input by its producers to equal its demand by Final good producers, which is $q_t(i)/z_t = Q_t \left(p_t(i)/P_t\right)^{-\eta}/z_t$ units of the input for each variety i. Aggregating over varieties:

$$\int_{\underline{y}}^{\overline{y}} y dL_{t-1}(y) = \frac{Q_t}{z_t} \left(\frac{\tilde{P}_t}{P_t}\right)^{-\eta}, \quad \text{where } \tilde{P}_t = \left(\int_0^1 p_t(i)^{-\eta} di\right)^{-1/\eta}. \tag{17}$$

3.6 General Equilibrium

The economy enters period t with a set of pre-determined aggregate objects: the employment distribution $L_{t-1}(\cdot)$, hence unemployment $u_{t-1} = 1 - \int_{\underline{y}}^{\overline{y}} dL_{t-1}(y)$, the distribution of Final good variety prices $p_{t-1}(\cdot)$ and, at the beginning of the period, the new realizations of TFP z_t and interest rate R_t. The first two are endogenous, infinitely-dimensional state variables. TFP has an exogenous law of motion. Monetary policy is assumed to follow a rule that makes R_t a stochastic function of the other aggregate states.

A key observation is that the price distribution $p_{t-1}(\cdot)$ enters equilibrium conditions only through the two price indexes $P_{t-1}, \tilde{P}_{t-1}$, which have known laws of motion: by the standard "random selection" property of Calvo pricing, P_t follows (15) and $\tilde{P}_t$ follows

$$\tilde{P}_t^{-\eta} = \nu\,(\,p_t^*)^{-\eta} + (1-\nu)\tilde{P}_{t-1}^{-\eta} \tag{18}$$

where we note that the reset price p_t^* that updates these two price indexes only depends on the aggregate variables C_t, P_t, ω_t and z_t through (14).

Definition 1 *Fix a process for z and a monetary policy rule, a measurable function from the aggregate state $\mathcal{S} = \left\langle P_{-1}, \tilde{P}_{-1}, L_{-1}(\cdot), R_{-1}, z \right\rangle$ to the current nominal interest rate R. A **Recursive Rational Expectations Equilibrium** is a collection of measurable functions $\{C, Q, \theta, \omega, T, W, \mathcal{L}\}$ of the aggregate state $\mathcal{S}_t$ that solve the consumption Euler equation* (5)*, the present value equations for marginal cost* (8) *and consumption/leisure trade-off* (9)*, the free entry condition* (12)*, the optimal reset price equation* (14)*, Government budget balance, and market-clearing* (16)-(17)*, and which determine a first-order Markov process for each endogenous component of the state vector:* (10) *for $L(\cdot)$, with $u = 1 - L(\overline{y})$;* (15) *for P; and* (18) *for $\tilde{P}$.*

The model in log-linearized form features a standard New-Keynesian structure, with Euler (IS) demand, Phillips Curve, and monetary policy rule, where, as usual, price levels are replaced by the inflation rate π_t. Our innovation is to generate the nominal marginal cost ω_t as the market-clearing price of an input, which is itself produced in a frictional labor market with mismatch and reallocation.

3.7 Discussion: Old and New Wedges

From the free entry condition (12), vacancy creation, thus θ_t, depends on the average of two expected returns, from unemployed and employed hires, weighted by the shares of these two groups in the job searching pool. The first term, the returns from the unemployed, is related to labor market slack as measured by the *quantity* of employment, and is traditionally measured by the unemployment rate. Our main result is that a second, independent form of slack originates from the *quality* of employment, as measured by the returns to hire employed workers. We examine these two terms in turn.

The Neoclassical labor wedge and the New Keynesian wage mark-up. The expected return from hiring an unemployed hire worker equals $\mathbb{E}_t\left[\mathcal{D}_t^{t+1}\left(W_{t+1}\mathbb{E}y - \mathcal{L}_{t+1}\right)\right]$, where

$$W_t\mathbb{E}y - \mathcal{L}_t = \mathbb{E}_t\sum_{\tau=0}^{+\infty}(1-\delta)^{\tau}\mathcal{D}_t^{t+\tau}\left[\underbrace{\frac{\omega_{t+\tau}}{P_{t+\tau}}\mathbb{E}y}_{\text{MPL}_{t+\tau}} - \underbrace{\frac{U_u(C_{t+\tau}, u_{t-1+\tau})}{U_C(C_{t+\tau}, u_{t-1+\tau})}}_{\text{MRS}_{t+\tau}}\right] \tag{19}$$

is the expected PDV of the difference between the Marginal Product of Labor on the extensive margin (from the last household member joining employment), in units of the Final good, and the Marginal Rate of Substitution between consumption of the Final good and leisure. Because hiring is an investment decision, MPL and MRS are taken in expected PDV.

Chari, Kehoe and McGrattan (2007) define the "labor wedge" as the ratio between the MRS and the MPL. Measured in the data through the lens of a neoclassical growth model with balanced growth preferences, this labor wedge is procyclical, that is, the implicit "tax" rate on labor income is countercyclical, and plays a central role in amplifying business cycles. In our model, a procyclical labor wedge translates into procyclical returns to hiring unemployed workers. Unlike in standard linear-utility search model, here the standard income effect on labor supply (the marginal utility of consumption in the denominator of the MRS) contributes to a procyclical wedge.

An alternative interpretation of the "Service" in our model is a composite quantity of labor, with Service producers acting as labor market intermediaries, or temp-agencies, that hire workers in a frictional labor market and sell their services to Final good producers in a competitive market. Therefore, ω_t/P_t is the average cost of efficiency units of labor to good producers, and the firm discounts the difference between this real wage index (scaled by average efficiency units $\mathbb{E}y$) and the MRS between consumption and leisure. Estimated New-Keynesian models (Smets and Wouters (2007)) define the "wage markup" as the ratio between the real wage and the MRS, and find that changes in this mark-up are key to explain inflation and output dynamics. Lacking a mechanism to generate endogenous changes in the wage mark-up, they attribute them to shocks, that they estimate to be procyclical. Erceg, Henderson and Levine (2000) generate wage mark-ups by assuming sticky nominal wages. Galí (2011) calls for a theory of an endogenous wage mark-up. Our model delivers just that.

If the markets for both input (labor) and output (Service) were competitive, both the labor wedge and the wage mark-up would be identically equal to one, with workers on their labor supply curve and firms on their labor demand curve. With our frictional labor market, the labor wedge is smaller than one and the wage mark-up is larger than one, to compensate for hiring costs, and, crucially, both are endogenous and time-varying.

The Mismatch wedge: A new source of labor market slack. Our model contains an additional, novel transmission mechanism of aggregate shocks to job creation, absent in either of those two strands of the literature. Vacancy posting is sensitive to the expected return from an *employed* hire, the double integral in (12), namely

$$\int_{\underline{y}}^{\overline{y}} \int_{\underline{y}}^{y} (y - y') \frac{dL_{t-1}(y')}{1 - u_{t-1}} d\Gamma(y)$$

This is independent of household preferences, including the MRS, and depends entirely on the distribution of employment $L_t(\cdot)$, which is a slow-moving aggregate state variable. We call this object the "Mismatch wedge", because it is zero in the frictionless limit, where every worker is always employed in the best possible match $\overline{y}$, and it is larger the more misallocated is employment on the ladder.[8]

This term introduces an additional component to labor demand, with a complex cyclical pattern. At a cyclical peak, workers have had time and opportunities to climb the ladder, so it is both difficult and expensive to poach employees from other firms, and the expected returns from hiring employed workers are weak. After a recession, as the unemployed regain employment, they restart from random rungs on the match quality ladder, which are worse than the employment distribution at the cyclical peak. Hence, early in a recovery, many recent hires are easily "poachable". The transition of cheap unemployed job applicants into employment at low-quality jobs makes these workers only slightly more expensive, and still quite profitable, to hire. As time goes by, and unemployment declines, employment reallocation picks up job the ladder through job-to-job quits, employed workers grow harder and more expensive to hire, ultimately putting pressure on the real input price ω_t/P_t, until we are back to a cyclical peak. The Mismatch wedge implies a procyclical wage mark-up, or countercylical labor wedge, as long as employment is still misallocated and "poachable".

[8] Our "Mismatch wedge" is somewhat reminiscent — although clearly not identical to — the "OP covariance" term first introduced by Olley and Pakes (1996) in their study of productivity in the telecommunications equipment industry, and taken up as a measure of mismatch in Bartelsman et al. (2013). In the context of our model, the OP covariance would be defined as the covariance between match productivity and match employment share, formally $\omega_t \int_{\underline{y}}^{\overline{y}} (\ln y - \mathbb{E} \ln y) \left(\frac{L_t'(y)}{(1-u_{t-1})\Gamma'(y)} - 1 \right) d\Gamma(y)$.

Implications for the propagation of aggregate shocks. In the US economy, the EE transition probability between employers is about 2.5% monthly, an order of magnitude smaller than the UE transition probability from unemployment to employment, and of similar magnitude to the transition probability EU from employment to unemployment. Therefore, movements in the employment distribution up the job ladder are slow. An important implication that we will illustrate later quantitatively is that, in our model, job-market tightness, thus the unemployment rate, have sluggish transitional dynamics, even when applied to a highly dynamic labor market such as the U.S.'. This stands in contrast to the implications of the canonical model with only search from unemployment, where tightness has no transitional dynamics, and the unemployment rate converges very quickly to its new steady state.

This difference is important. Without on-the-job search, the slow, prolonged decline in the U.S. unemployment rate over 2009-2019 can only be explained in the canonical model by a long and improbable sequence of small, consecutive, favorable aggregate shocks. A slowly mean-reverting process for the aggregate driver of business cycles will not do, because the free entry condition is forward-looking and would incorporate the expected gradual recovery. In contrast, our model has a built-in, slow-moving, endogenous propagation mechanism of temporary aggregate shocks. The job-finding rate, thus unemployment, evolve slowly because of congestion created by on-the-job searchers.

The Acceptance rate. While the Mismatch wedge is not directly measurable, its expression in (12) suggests an observable proxy, that we call the "Acceptance rate" of outside offers. This is the probability that a randomly chosen employed worker accepts an outside offer, after receiving it. In the model, this can be expressed, up to multiplicative constants, as the ratio between two observables, the EE and UE transition probabilities:

$$\mathrm{AC} = \frac{\mathrm{EE}}{\mathrm{UE}} = \frac{(1-\delta)s_1\phi(\theta_t)\int_{\underline{y}}^{\overline{y}}[1-\Gamma(y)]\frac{dL_{t-1}(y)}{1-u_{t-1}}}{\phi(\theta_t)} = (1-\delta)s_1\int_{\underline{y}}^{\overline{y}}[1-\Gamma(y)]\frac{dL_{t-1}(y)}{1-u_{t-1}}$$

Intuitively, since the unemployed accept all job offers, the UE flow measures the meeting rate of workers with open vacancies. Since job search is random, the same meeting rate applies to employed job searchers, up to their relative search intensity s_1. Controlling for UE, the evolution of the EE probability can only be due to a changing probability of accepting offers, which, in turn, depends directly on mismatch. By inspecting their expressions, we can see that both Mismatch wedge and Acceptance rate decline as the employment distribution on the ladder $L_{t-1}(y)/(1-u_{t-1})$ improves (in a First Order Stochastic Dominance sense). Indeed, after integration by parts, and ignoring the multiplicative constants, we can write

Mismatch wedge and Acceptance rate as, respectively,

$$\int_{\underline{y}}^{\overline{y}} \int_{\underline{y}}^{y} L_{t-1}(y')dy' \frac{d\Gamma(y)}{1-u_{t-1}} \quad \text{and} \quad \int_{\underline{y}}^{\overline{y}} L_{t-1}(y) \frac{d\Gamma(y)}{1-u_{t-1}}$$

We propose AC as a revealed-preference, empirical measure of employment misallocation.

Relation to the literature. DSGE models with job search frictions (Andolfatto (1996), Merz (1996), Krause and Lubik (2007), Gertler and Trigari (2009), Christiano, Eichenbaum and Trabandt (2016)) abstract from on-the-job search. Within the linear-utility labor market search tradition, Robin (2011) adopts the Sequential Auction model of a labor market with on-the-job search, but stresses permanent worker heterogeneity. Firms are identical, thus the job ladder has only two steps. Only unemployed hires generate profits for firms; employed hires extract all rents from both incumbent and prospective employer, hence there is no Mismatch wedge in the returns to job creation. This appears in two equilibrium search models with aggregate shocks. Moscarini and Postel-Vinay (2013) assume wage-contract posting without renegotiation, which could not easily accommodate nominal price stickiness. Lise and Robin (2017) allow for ex ante worker and firm heterogeneity and sorting within the more tractable Sequential Auction/renegotiation framework. The latter is the closest comparison to the present paper. We assume a much simpler model of the job ladder, based on ex post match quality draws rather than on ex ante two-sided heterogeneity, in order to flesh out the propagation mechanism of aggregate shocks that poaching introduces, and to be embed it in a full-fledged general equilibrium framework, with sticky prices and savings, where we can study monetary policy.

Faccini and Melosi (2018) study a version of our model with two possible match qualities, bad and good jobs, and time-varying search effort by the employed. They calibrate the model in steady state, and then use empirical time series of EE and UE transition probabilities to estimate, by Maximum Likelihood, the implied time series for aggregate demand (preference) shocks and for on-the-job search effort. The real marginal cost $\omega_t/(P_t z_t)$ series predicted by their version of our model is much more in line with observed post-2008 inflation than estimates of marginal costs derived from either the labor share, as in standard New Keynesian models with competitive labor market, or from the UE transition probability, as in versions of the New Keynesian model that introduce only unemployed job search. In the US, following the 2008-2009 Great Recession, both the EE probability and inflation exhibited a profound decline and an even slower recovery than employment. But EE fell by much less than UE, as mismatched workers, eager to upgrade, raised on-the-job search effort, slowly expanded labor supply, and kept inflation in check. In our model, where on-the-job search effort is fixed at s_1 by assumption, the action originates from firms' vacancy posting and from the acceptance probability of outside offers by workers, both directly influenced by the extent of mismatch.

4 The Extended Model

We extend the model and parameterize it to make it more flexible for quantitative evaluation. We present the main changes in the equilibrium conditions, that we fully derive in Appendix.[9]

4.1 Intensive Margin of Labor Supply in the Service Sector

Each worker employed in the Service sector can spend and freely adjust each period work effort h, at some utility cost captured by household preferences:

$$U(C_t) + bu_{t-1} - \mathcal{B} \int_0^1 \frac{h_t(j)^{1+1/\Xi}}{1+1/\Xi} dj \tag{20}$$

with $U' > 0 > U''$ and $b, \mathcal{B}, \Xi \geq 0$, parameters that quantify the strength of preferences for leisure on both the extensive and the intensive margin. The u_{t-1} initially unemployed workers exert no effort, so they enjoy a flow value of leisure b per person and contribute nothing to the household's utility cost of effort (the integral). The $1 - u_{t-1}$ already employed workers can either mimic the unemployed, spend no effort and enjoy b, or freely choose positive effort, but to do so must give up b, which is then a fixed (opportunity) cost of working. A worker who spends effort h in a match y produces Service output hy. We abstract from decreasing returns to effort, because they cannot be separately identified from the Frisch elasticity Ξ.

Workers cannot commit to their work effort and to stay at a firm. Firms in the Service sectors can commit to wage contracts. Because effort is not contractible, firms can base their compensation only on realized revenues. As is standard in moral hazard situations, match surplus is maximized by "selling the firm to the worker", that is, paying the worker earnings equal to the total revenues ωhy he generates. The firm then chooses a two-part contract, which offers this efficient incentive scheme and extracts profits through a (negative) base wage, independent of output. A worker employed at time t in a match of quality y then chooses an optimal supply of effort on the intensive margin

$$h_t(y) = \left(\frac{U'(C_t)}{\mathcal{B}} \frac{\omega_t}{P_t} y \right)^{\Xi} \tag{21}$$

[9] The model admits further extensions, such as stochastic Government spending, discount factor, TFP in the Service sector. Details are available upon request.

The aggregate supply of the Service intermediate good now equals

$$\left(\frac{U'(C_t)}{\mathcal{B}}\frac{\omega_t}{P_t}\right)^{\Xi}\int_{\underline{y}}^{\overline{y}} y^{1+\Xi}dL_{t-1}(y)$$

and responds contemporaneously to the price ω_t/P_t on the intensive margin, with Frisch elasticity Ξ, and then later on the extensive margin, as turnover reshapes $L_{t+\tau}(y)$.

In the Appendix, we also show that the analysis of sequential auctions is essentially unchanged. Equilibrium is Rank-Preserving: the more productive match wins the auction for an employed worker. The recursion for the PDV of the real input price changes to:

$$W_t = \frac{1}{1+\Xi}\left(\frac{\omega_t}{P_t}\right)^{1+\Xi}\left(\frac{U'(C_t)}{\mathcal{B}}\right)^{\Xi} + (1-\delta)\mathbb{E}_t\left[\mathcal{D}_t^{t+1}W_{t+1}\right] \tag{22}$$

while the opportunity cost of work, adjusted for exogenous separations, simplifies to:

$$\mathcal{L}_t = \frac{b}{U'(C_t)} + (1-\delta)\mathbb{E}_t\left[\mathcal{D}_t^{t+1}\mathcal{L}_{t+1}\right] = \frac{b}{U'(C_t)[1-\beta(1-\delta)]} \tag{23}$$

4.2 Meeting Probability

With time-invariant probability $\delta s_0 \in [0,1]$, employed workers at time t but can search for a new job immediately. If hired, they produce immediately, before new shocks occur and discounting applies. Those who are laid off but do not find another job immediately enjoy leisure and wait at least until next period to produce. Accordingly, the denominator of date-t job market tightness, aggregate search effort, now equals $u_{t-1} + \delta s_0(1-u_{t-1}) + (1-\delta)(1-u_{t-1})s_1$.

We also introduce shocks φ_t to the meeting function, which now reads $\varphi_t\phi(\theta_t)$.

4.3 Intensive Margin of Recruiting

We introduce an additional *intensive* margin of labor adjustment costs. Each open vacancy can receive at most one application per period. We now assume that, *after* receiving the application and observing the employment status of the applicant, the firm must spend resources (in units of the intermediate Service input) to interview the applicant and identify the match quality draw; otherwise, the match is not viable. Specifically, the firm can scale up the probability of a successful match by a factor $r \geq 0$ by controlling "recruiting intensity" r at cost $\kappa_s r^{1+\iota}/(1+\iota)$, for $\kappa_s, \iota > 0$, expressed in units of Service. Because the firm can condition recruiting investment on the employment status of the job applicant, $\mathrm{e}_{t-1} \in \{0,1\}$, which in turns determines the expected returns from a hire (employed workers are harder and more expensive to hire), optimal recruiting intensity $r^*_{\mathrm{e},t}$ depends on that status e_{t-1}. These investments do not affect job market tightness, as they can be chosen only *after* receiving job applications, but do affect hiring and job-finding probabilities, as well add to the aggregate demand for the intermediate Service input.

Recruiting intensity summarizes a collection of activities that the firm can undertake on the intensive margin, such as screening job applications, in order to accelerate hiring. See Gavazza et al. (2018) for empirical evidence and implications of this activity. Recruiting intensity mirrors job search effort by workers; we assign this search effort choice to firms because they are ex ante (before drawing new match quality) homogeneous and will choose the same recruiting intensity (per type of job applicant: unemployed, employed). In contrast, employed job applicants differ by their current rungs on the job ladder, thus their search effort would depend on the evolving and heterogeneous values of their current jobs, and wage contracts would influence search effort ex post.

Unlike in Gavazza et al. (2018), we assume that recruiting costs are incurred by the firm only *after* receiving job applications. Therefore, they reduce the expected surplus from a hire and they partially insulate adjustment costs from cyclical labor market tightness. Both effects contribute to raise the volatility of vacancy postings and thus (un)employment. The latter effect was emphasized by Pissarides (2009). Also, note that the expected surplus to the firm from any received job application is always positive, as the firm can refuse to recruit job applicants ex post. Allowing firms to condition recruiting intensity on the employment status of the job applicant helps to loosen the tight link between the UE and EE average transition rates, and mirrors job search effort by unemployed and employed workers documented and modeled by Faberman et al. (2021).

Formally, let $\Omega_{i,t}$ denote the expected returns to the firm from hiring an unemployed ($i = 0$) or employed ($i = 1$) job applicant, *after* receiving the application but before observing the quality of the match:

$$\begin{aligned}
\Omega_{0,t} &= \mathbb{E}_t\left[\mathcal{D}_t^{t+1} W_{t+1}\right] \int_{\underline{y}}^{\overline{y}} y^{1+\Xi} d\Gamma(y) - \mathbb{E}_t\left[\mathcal{D}_t^{t+1}\mathcal{L}_{t+1}\right] \\
\Omega_{1,t} &= \mathbb{E}_t\left[\mathcal{D}_t^{t+1} W_{t+1}\right] \int_{\underline{y}}^{\overline{y}} \int_{\underline{y}}^{y} \left(y^{1+\Xi} - y'^{1+\Xi}\right) \frac{dL_{t-1}(y')}{1-u_{t-1}} d\Gamma(y).
\end{aligned} \tag{24}$$

Optimal recruiting intensity $r^*_{i,t}$ thus solves

$$\max_{r_i} \left\langle r_i \Omega_{i,t} - \frac{\omega_t}{P_t}\kappa_s \frac{r_i^{1+\iota}}{1+\iota} \right\rangle \Rightarrow r^*_{i,t} = \left(\frac{P_t \Omega_{i,t}}{\omega_t \kappa_s}\right)^{\frac{1}{\iota}}$$

and returns an optimized objective

$$r^*_{i,t} r^{*\iota}_{i,t} \kappa_s \frac{\omega_t}{P_t} - \frac{\omega_t}{P_t} \kappa_s \frac{r^{*1+\iota}_{i,t}}{1+\iota} = \frac{\omega_t}{P_t} \kappa_s \frac{\iota}{1+\iota} {r^*_{i,t}}^{1+\iota}$$

Using these expressions, and the immediate rehiring option, the free entry condition is

$$\kappa_v \frac{\theta_t}{\varphi_t \phi(\theta_t)} = \frac{\omega_t}{P_t} \kappa_s \frac{\iota}{1+\iota} \frac{[u_{t-1} + \delta(1-u_{t-1})s_0] \, {r^*_{0,t}}^{1+\iota} + (1-\delta)(1-u_{t-1}) s_1 {r^*_{1,t}}^{1+\iota}}{u_{t-1} + \delta(1-u_{t-1})s_0 + (1-\delta)(1-u_{t-1})s_1}$$

The demand for each final good variety continues to be defined by the usual isoelastic expression $q^d_t(i) = Q_t \left(p_t(i)/P_t\right)^{-\eta}$ where final good output covers, as before (see Equation 16), consumption and vacancy costs, but now accounting for immediate rehiring (s_0):

$$Q_t = C_t + \kappa_v \theta_t \left[u_{t-1} + \delta(1-u_{t-1})s_0 + (1-\delta)(1-u_{t-1})s_1\right]$$

Similarly, the dynamics of the employment distribution must now take into account immediate rehiring of some newly unemployed workers and optimal firm recruiting effort:

$$\begin{aligned} L_t(y) &= (1-\delta) \left[1 - s_1 \varphi_t \phi(\theta_t) r^*_{1,t} \overline{\Gamma}(y)\right] L_{t-1}(y) + \varphi_t \phi(\theta_t) r^*_{0,t} \Gamma(y) \left[u_{t-1} + \delta s_0 (1-u_{t-1})\right] \quad (25) \\ u_t &= \left[1 - \varphi_t \phi(\theta_t) r^*_{0,t}\right] u_{t-1} + \delta \left[1 - s_0 \varphi_t \phi(\theta_t) r^*_{0,t}\right] (1-u_{t-1}) \end{aligned}$$

4.4 Decreasing Returns in the Final Good Sector

Production in the Final good sector is now subject to decreasing returns: y units of Service produce $z_t y^\zeta$ units of a variety, for $\zeta \in (0,1]$. This parameter is separately identified from demand elasticity η because the CES consumption aggregator is homothetic. This change introduces an endogenous component to the marginal cost of producing Final good varieties in the optimal re-pricing equation, shown in the Appendix, and curvature in the demand for Services from their downstream Final good buyers. The Service market-clearing condition, which now incorporates also demand for recruiting purposes, reads

$$\begin{aligned} \left(\frac{U'(C_t)}{\mathcal{B}} \frac{\omega_t}{P_t}\right)^{\Xi} \int_{\underline{y}}^{\overline{y}} y^{1+\Xi} dL_{t-1}(y) &= \frac{Q_t^{\frac{1}{\zeta}}}{z_t} \left(\frac{P_t}{\tilde{P}_t}\right)^{\frac{\eta}{\zeta}} \\ &+ \kappa_s \varphi_t \phi(\theta_t) \left[(u_{t-1} + \delta(1-u_{t-1})s_0) \frac{{r^*_{0,t}}^{1+\iota}}{1+\iota} + (1-\delta)(1-u_{t-1}) s_1 \frac{{r^*_{1,t}}^{1+\iota}}{1+\iota} \right] \end{aligned}$$

In the Appendix, we show the new expression for the optimal reset price.

4.5 Solution Method

Due to competition for employed workers in a frictional labor market, the distribution of employment $L_t(\cdot)$ is an infinitely-dimensional state variable. This property of the model complicates the computation of equilibrium in the presence of aggregate shocks, just like the distribution of wealth does in incomplete market models. Two characteristics intrinsic to frictional labor markets simplify this task.

First, while in incomplete market equilibrium the borrowing constraint imposes an occasionally binding lower bound to (the endogenous distribution of) wealth, in our case, by inspecting the law of motion, either (10) or (25), the similar constraint $L_t(\cdot) \geq 0$ never binds, because outflows from a job ladder rung are proportional to their size, and inflows are not.

Second, while wealth is a continuous variable, and some compromise between discretization and interpolation must be made to implement computation of the incomplete market model, in our case the match quality distribution, the domain of $L_t(\cdot)$, is a primitive, so we can assume it lives on a discrete grid, with no approximation error. We specify Γ as a discrete distribution over $\underline{y} = y_1 < y_2 < \cdots < y_K = \overline{y}$, with $K \geq 2$, and with corresponding probability masses $\gamma_1, \cdots, \gamma_K$ with $\gamma_k = \Gamma(y_k) - \Gamma(y_{k-1}) = \Gamma_k - \Gamma_{k-1}$, and likewise in $L_k, t = L_t(y_k)$. The unconditional mean of match quality is $\mathbb{E}y = \sum_{k=1}^{K} y_k \gamma_k$.

Both facts allow to log linearize the system of equilibrium conditions around its deterministic steady state, where we treat employment on each of the discrete rungs of the job ladder $\{y_k\}$ as a scalar variable, so we reduce the dynamics of $L_t(\cdot)$ to a (potentially very large) set of linear stochastic difference equations in $L_{k,t}$. We illustrate here our approach and results. An online appendix[10] presents detailed derivations.

5 Calibration and Estimation

5.1 Functional Forms and Shock Processes

We build Γ_k by discretizing a truncated Pareto distribution with slope parameter λ. We specify a Cobb-Douglas meeting function per unit of matching efficiency φ_t, namely $\phi_t(\theta_t) = \varphi_t \cdot \theta_t^{\alpha}$, with $\alpha \in [0, 1]$.

We further restrict attention to isoelastic preferences for Final good consumption over time, and we allow for preference shocks Υ_t:

$$U_t(C) = \left(\frac{C}{\Upsilon_t}\right)^{\frac{\sigma-1}{\sigma}} \frac{\sigma}{\sigma - 1}.$$

[10] Available at https://campuspress.yale.edu/moscarini/working-papers/

The monetary policy rule is

$$\ln(1+R_t) = \rho_R \ln(1+R_{t-1}) + (1-\rho_R)\left[\psi_\pi \ln(1+\pi_t) + \psi_u \ln\frac{u_t}{u} - \ln\beta\right] + \ln\varsigma_t$$

where ς_t is a monetary policy shock. We assume that this Taylor rule targets the annual inflation rate (past 12 months) at monthly frequency, and we use this definition as our empirical counterpart of inflation. Since the model generates monthly inflation rates, we need to keep track of the last 12 months of monthly inflation, which adds 11 state variables (and as many obvious equations) to the system.

Finally, we specify the structural shock processes as AR(1) in logs. For any variable $\o \in \{z, \varphi, \Upsilon, \varsigma\}$ (TFP, meeting efficiency, preference shock, deviation from the Taylor rule) we assume:

$$\ln\o_t = \rho_\o \ln\o_{t-1} + \sigma_\o \varepsilon_t^\o$$

where $\rho_\o, \sigma_\o$ are positive scalars, $\varepsilon_t^\o \sim \mathbb{N}(0,1)$, and we normalized to one the steady state values of these processes.

5.2 Methodology

The model has 4 state (pre-determined) exogenous variables, the structural shock processes $z_t, \varphi_t, \Upsilon_t, \varsigma_t$; $13+K$ endogenous state variables, the interest rate R_t through the monetary policy rule, 12 lags of inflation, and the measures of employment $\ell_{k,t-1} = L_{k,t-1} - L_{k-1,t-1}$ on each of the $k = 1, 2\cdots K$ rungs of the ladder, where unemployment u_{t-1} can replace $\ell_{K,t-1}$ by the identity $u_{t-1} = 1 - \sum_{k=1}^{K} \ell_{k,t-1}$; three "jump" variables, aggregate consumption C_t, the inflation rate π_t, and the expected PDV of real input price W_t, which appear in the system dated at time t and, in expectation, at time $t+1$; and, finally, six "static" variables, which only appear dated at time t and can all be solved out in terms of the other variables: Final good output Q_t and consumption C_t, job market tightness θ_t, real input price $x_t = \omega_t/(z_t P_t)$, and recruiting intensities $r_{0,t}^*$, $r_{1,t}^*$. Note that $\mathcal{L}_t$, the expected PDV of the MRS between consumption and leisure on the extensive margin, is a function of aggregate consumption C_t only and does not constitute an additional, independent jump variable.

The model has 28 structural parameters: four governing labor market turnover $(\delta, s_0, s_1, \alpha)$; five preferences $(\beta, \sigma, b, \eta, \Xi)$; four production technology, $(y_1, y_K, \lambda, \zeta)$; three hiring technology $(\iota, \kappa_s, \kappa_v)$; one Calvo parameter for pricing technology (ν); three Taylor rule parameters $(\rho_R, \psi_\pi, \psi_u)$; eight persistence and standard deviations of innovations to the four structural shocks, $\left(\rho_{[\o]}, \sigma_{[\o]}\right)$ for $\o \in \{z_t, \varphi_t, \Upsilon_t, \varsigma_t\}$.

We log linearize the equilibrium conditions around the deterministic steady state and write the resulting system in a standard state-space representation. We report all the equations in the online Appendix. The system is set up for Maximum Likelihood/Kalman Filter estimation. Instead, we choose a minimum distance/Simulated Method of Moments criterion. The 39 empirical moments that we target are moments of the HP filtered logarithms of seven aggregates (consumption, inflation, unemployment, nominal interest rate, vacancies, and UE and EE transition probabilities; details below): variances, cross-correlations, and first-order autocovariances, all derived from simulated data from the Rational Expectations equilibrium of the model, and four long-run averages, matched to model steady state counterparts. The latter allows to directly calibrate the turnover parameters δ and s_0, as illustrated in Appendix B.

We estimate the Taylor rule parameters by a Generalized Method of Moments, using lags of inflation and unemployment rates as instruments. We calibrate externally several parameters to conventional values. For each configuration of the remaining parameter values, we solve numerically for the fundamental REE solution and check determinacy. We then use this solution to simulate aggregate time series, and then compute the desired covariance and autocovariance structure of the observable magnitudes, that we compare with empirical moments. We estimate the remaining parameters by minimizing the distance between the simulated and empirical covariance structure of the seven macroeconomic aggregates over time. For variances we take the log distance, while for auto- and cross-correlations, that are already standardized, we take the distance in levels. This part of the calibration is "internal" to the model, because it exploits its equilibrium restrictions.

5.3 Data and Moment Construction

We collect a variety of relevant time series, which differ in time span and frequency. For each series, we use all the available data to perform, when not already available from the data source, seasonal adjustment using the Census X13's procedure, and then HP-filter (in levels or logs, depending on the series). After obtaining the seasonally adjusted and filtered series, we focus on the period from October 1995 to March 2020. Because we are especially interested in the covariance structure of the macroeconomic aggregates, we have to restrict to a common sample period. The start date is when one key series, the monthly transition probability from employer to employer, first becomes continuously available in the monthly CPS, which is the longest available series for this particular statistic. We stop when the COVID-19 pandemic hit, because movements in labor market variables in 2020-2022 have been exceptional and, in some cases, of questionable interpretation. For example, the spike in unemployment in spring 2020 was absorbed quickly by a wave of recalls, which were clearly not mediated by a frictional meeting process. Because of the relatively short duration of our time series, the pandemic outlier would significantly alter the empirical moments. We do, however, exploit data in 2020-2023 for the purpose of HP filtering, because in our view the pandemic gyrations are less likely to affect the estimated trend in the preceding decade (2010s) than a sample that ends in early 2020 itself would.

We measure:

- real aggregate consumption with the quarterly Personal Consumption Expenditure chain-weighted index (2012 dollars), seasonally adjusted (series PCECC96 from FRED), 1947:Q1-2023:Q1, that we log and HP-filter with smoothing parameter 1,600;

- the unemployment rate with the BLS monthly series, seasonally adjusted (LNS14000000), 1948:M1-2023:M5, that we log and HP-filter with smoothing parameter 8.1E6;

- annual inflation at monthly frequency with the 12-month difference in the log of the seasonally adjusted monthly core PCE deflator (series PCEPILFE from FRED), 1959:M1-2023:M5; we HP filter the inflation series with smoothing parameter 8.1E6;

- the interest rate with the effective monthly average Federal Funds rate (series FEDFUNDS from FRED), 1954:M7-2023:M5, that we divide by 100 and HP-filter with smoothing parameter 8.1E6;

- vacancies with the spliced HWI-JOLTS monthly series estimated by Barnichon (2010) and later updated by the author, 1951:M1-2021:M8, which is when the series currently ends. We supplement this in 2021:M9-2023:M4 with data on job openings from JOLTS; specifically, we regress the HWI series on JOLTS job openings in 2000:M12-2021:M8, and then use the prediction from this regression on recent JOLTS data to extend the HWI series after 2021:M8; we note that the in-sample prediction is nearly perfect, but the two series have a level and amplitude difference that the regression reconciles; we then take this spliced series, seasonally adjust, log and HP-filter with smoothing parameter 8.1E6;

- the monthly transition probability from unemployment to employment with the ratio between the UE monthly gross flow estimated by the BLS from the CPS matched files, and published at `https://www.bls.gov/webapps/legacy/cpsflowstab.htm`, 1990:M1-2023:M4, and the number of unemployed a month before, ratio that we then seasonally adjust, log and HP-filter with smoothing parameter 8.1E6;

- the monthly transition probability from employer to employer with the EE estimate by Fujita, Moscarini and Postel-Vinay (2022) at monthly frequency from the monthly

CPS matched files, 1995:M10-2023:M4, that we seasonally adjust, log and HP-filter with smoothing parameter 8.1E6;

- the share of offers that employed workers accept with the evidence from Faberman at al. (2021), who estimate it at 30.9%;

Even after seasonal adjustment, the EE and UE series present significant noise, which strongly suggests sampling error, due to the relative rarity of these two transitions in the monthly CPS. Furthermore, aggregate consumption is only available quarterly. For both reasons, we take a 3-month rolling moving average of all monthly series, before computing their moments that provide the targets of our minimum distance estimation. For aggregate consumption, this implies an assumption of equal distribution of PCE within a quarter. We apply this smoothing procedure to both empirical and simulated data.

We also exploit the average values of EE, UE, and unemployment rates, thus also the EU rate, over the sample period, to directly calibrate a few parameters from steady state restrictions.

5.4 Identification

We can normalize the unconditional mean of the output distribution $\int y^{1+\Xi} d\Gamma(y) = 100$. This allows to pin down y_1, given the other two parameters y_K and λ of the distribution Γ. In the online Appendix, we show that the model identifies the vacancy costs per unit of job search $\kappa_v \theta$ in steady state, but not the scale of the unit vacancy cost κ_v separately from that of job market tightness θ, and identifies the steady state ratio of recruiting intensities r_0^*/r_1^*, but not the levels of recruiting intensities r_i^* in steady state and cost κ_s. We have one free normalization to determine the scale of all of these objects. For example, we can choose the scale of vacancies so that either $\theta = 1$ or $r_i^* = 1$ in steady state, or $\kappa_s = 1$. We choose the latter, but this normalization is irrelevant to our quantitative results. Therefore, empirical information on the *level* of vacancies and job market tightness is irrelevant, and we can interpret vacancies more broadly, as the extensive margin of recruiting effort, distinct from the intensive margin.

We fix several parameters at conventional values: the discount factor β to reflect a 5% annual discount; the intertemporal elasticity of substitution $\sigma = 1/3$, to capture a weak intertemporal substitution channel in our complete market model; the price elasticity of Final good demand $\eta = 6$ to generate a 20% average net mark-up in the Final good sector; the Calvo probability of price adjustment, at $\nu = 1/9$, to reflect an average price duration of nine months; and returns to scale $\zeta = 2/3$. We note that the results are not very sensitive to changes in the values of ν and ζ, but are to changes in σ, as is the case in any New Keynesian model.

We can match directly four empirical moments, averaged over the available time span, to steady state model objects: the UE transition probability from Unemployment to Employment (25%); the unemployment rate (5.7%), implying an average observed EU probability $\delta\,(1 - s_0 r_0^* \phi(\theta))$ =1.5%; the observed EE transition probability (2.4%), which takes into account immediate re-hiring in a new job through a very short unemployment spell (s_0); and the share of outside offers accepted (30.9%). In the Appendix, we show that, from these average moments and steady state equilibrium conditions, we can estimate δ and s_0 once and for all, independently of the rest. In addition, for any constellation of the remaining parameter values, we can estimate b and $\kappa_v \theta$ exactly from steady state restrictions alone.

As mentioned, we estimate the parameters of the Taylor rule ρ_R, ψ_π, ψ_u by GMM, using lags of unemployment and of 12-month inflation as instruments.

This leaves to estimate from the empirical (auto-co)variance structure of our seven aggregate time series the following six preference, turnover and technology parameters: $(s_1, \alpha, \Xi, y_K, \lambda, \iota)$, plus the four structural shocks, $\left(\rho_{[\o]}, \sigma_{[\o]}\right)$ for $\o \in \{z_t, \varphi_t, \Upsilon_t, \varsigma_t\}$.

Before showing our estimates, we report a first, important result. We estimate a negligible size σ^φ of innovations to meeting efficiency φ_t. If we rule φ_t out altogether, the model preserves essentially the same fit, and most parameter estimates from the minimum distance criterion remain unchanged. Importantly, we fit well the correlation structure of the labor market variables. This implies that the congestion caused by on-the-job search and by the acceptance of outside offers on the job ladder, both endogenous forces in the model, can explain the observed shifts in the empirical Beveridge curve, as well as a large share of the volatility of transition rates, as equilibrium outcomes, obviating the need for another unobservable shock. Accordingly, we present results from the model driven only by shocks to TFP, preferences for consumption, and monetary policy.

5.5 Results

Table 1 illustrates the model fit, in terms of targeted and untargeted moments. Because of the objective function that we adopted, the minimum distance gives implicitly more weight to variances, that the model fits quite well, except for the unemployment rate and the UE job-finding probability, which are on opposite sides of the empirical targets, presumably due to the lack of endogenous, countercyclical separations (forcing the UE job-finding probability to do all the heavy lifting in driving unemployment fluctuations). The general pattern of auto- and cross-correlations is also captured well by the model. Among cross-correlations in Table

(a) Unconditional variances and autocorrelations

	variance		**autocorrelation**	
	data	model	data	model
aggregate consumption C	8.008E−5	8.152E−5	0.887	0.820
12-month inflation $\bar{\pi}_t = \sum_{s=0}^{11} \pi_{t-s}$	2.164E−5	2.450E−5	0.956	0.977
nominal interest rate R	1.873E−4	1.448E−4	0.993	0.972
unemployment rate u	0.049	0.039	0.991	0.990
vacancies v	0.032	0.031	[0.959]	0.824
UE job-finding probability	0.018	0.023	0.978	0.922
EE job to job transition probability	0.003	0.002	0.894	0.683
[AC=UE/EE]	0.011	0.014	0.968	0.922

(b) Cross-correlations

	C	$\bar{\pi}$	R	u	v	UE	EE
$\bar{\pi}$	0.23 (0.01)						
R	0.72 (0.60)	0.25 (0.15)					
u	−0.72 (−0.81)	−0.02 (−0.15)	−0.74 (−0.94)				
v	0.68 (0.75)	0.09 (0.23)	0.77 (0.80)	−0.85 (−0.84)			
UE	0.76 (0.79)	0.22 (0.18)	0.72 (0.92)	−0.94 (−0.95)	0.80 (0.96)		
EE	0.64 (0.70)	−0.05 (0.08)	0.59 (0.68)	−0.71 (−0.75)	0.79 (0.75)	0.70 (0.81)	
[AC]	−0.68 (−0.75)	−0.29 (−0.20)	−0.65 (−0.92)	0.88 (0.93)	−0.66 (−0.95)	−0.95 (−0.97)	−0.44 (−0.67)

Table 1: Empirical, [untargeted], (simulated) second moments, seasonally adjusted, in logs, HP-filtered.

Technology		
(log) aggregate TFP:		
persistence	ρ_z	0.494
volatility	σ_z	0.0367
discretized and truncated Pareto distribution of match quality Γ*:*		
lower bound	y_1	24
upper bound	y_K	44
shape parameter	λ	1.01
Final good production:		
Calvo probability(*)	ν	0.11
returns to scale(*)	ζ	0.67
Preferences		
flow value of leisure (extensive margin)	b	8.07
weight on cost of effort (intensive margin)	$\mathcal{B}$	.37
Frisch elasticity of effort (intensive margin)	Ξ	.37
elasticity of substitution between varieties(*)	η	6
intertemporal elasticity of substitution(*)	σ	0.33
(log) preference shock:		
persistence	ρ_Υ	0.94
volatility	σ_Υ	0.023
Search Frictions		
prob. of exogenous job destruction	δ	0.0228
unemployed search efficiency at short duration	s_0	1.34
on-the-job search efficiency	s_1	0.5
recruiting cost elasticity	ι	2.35
meeting function elasticity	α	0.38
Taylor Rule		
inflation parameter	ψ_π	1.17
unemployment parameter	ψ_u	-0.05
shocks:		
persistence	ρ_ς	0.93
volatility	σ_ς	0.023

Table 2: Parameter values calibrated internally by minimum distance (* externally).

1, Panel (b), we highlight the weak correlation between unemployment and inflation (a naive, reduced-form Phillips Curve), the strong negative correlation between unemployment and vacancies (the Beveridge curve), and the strong positive correlation between consumption and vacancies which are, in this model, the only form of investment.

Table 2 reports the parameter values minimize our objective function, given those we calibrate externally. Monetary policy shocks, estimated by GMM, turn out to be very small. TFP and preference shocks drive the economy. TFP/supply shocks are very transitory, while preference/demand shocks are very persistent.

The match distribution has a thick tail, close to Zipf's law (to $\lambda = 1$), highlighting the importance of match heterogeneity.

The implied Frisch elasticity of labor supply on the intensive margin is estimated at about 0.4, in the ballpark of standard estimates.

The estimated cost elasticity ι of recruiting effort is quite high. The meeting function elasticity of vacancies α is estimated at a low 0.38, reflecting the role of recruiting effort.

The model also implies that a share $\delta s_0/[\delta s_0 + (1-\delta)s_1]$ of observed EE transitions are involuntary and do not climb the job ladder. The model estimates this share at about 5.9%. Faberman at al. (2021) estimate at 11% the share of employed workers actively engaged in job search who did so either because they had to relocate or because they received an advance notice of a layoff.

The complete calibrated model generates a standard deviation of nominal marginal cost ω_t equal to a low value of 0.006. Recruiting efforts and job ladder dynamics adjust to this price signal and generate extra hiring and output; price stickiness also anchors nominal marginal costs.

5.6 Impulse Responses of Structural Shocks

Our model features one type of supply side shock, TFP, and two types of demand side shocks, one real to the preferences (marginal utility of consumption), and one nominal to the interest rate target. Figure 2 shows the effects of once-and-for-all one-standard-deviation innovations to each of these three sources of aggregate uncertainty, which then revert to the long-run mean, as plotted in red in the first row. In each case, we choose the sign of the innovations to have a contractionary effect on aggregate consumption.

The impulse response functions have all the expected signs, with one exception that we comment on below. The endogenous reallocation of employment on the job ladder feeds back on the incentives to create new jobs, generating propagation in the response of the aggregate economy. Our main result is that inflation and the acceptance probability AC are negatively

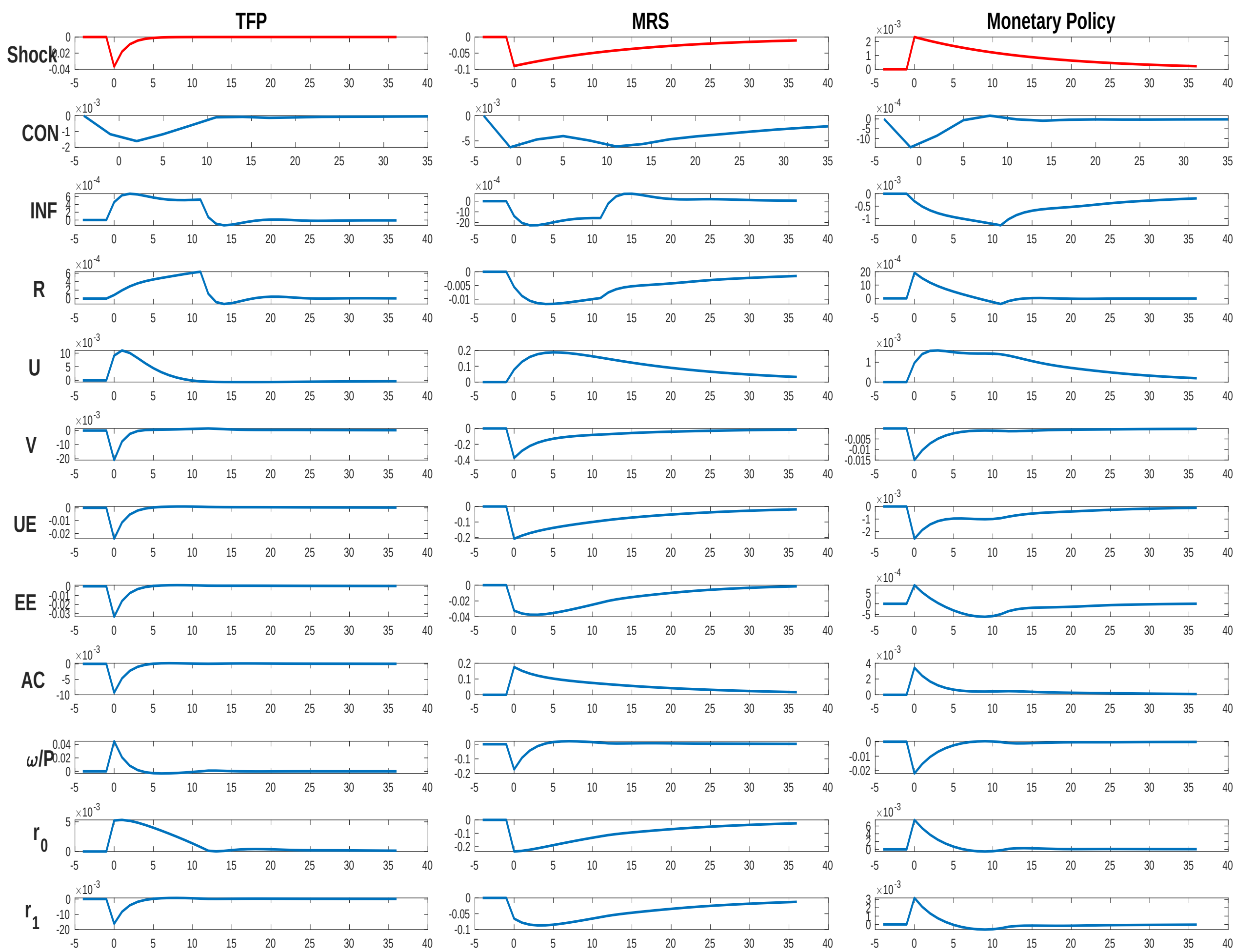


Fig. 2: Impulse Response Functions. Shocks in columns (red lines), variables in rows.

correlated in every case. The underlying mechanism differs by type of shock.

A negative TFP shock reduces job creation incentives, depressing both vacancy posting and recruiting effort of employed job applicants. Recruiting effort of the unemployed, somewhat surprisingly, rises, because of strong income effects that reduce the real opportunity cost of labor. Both UE and EE fall, but the former much farther, raising the AC ratio. Overall, the effects are contractionary and inflationary.

A negative demand (preference) shock to the MRS of consumption for leisure has a contractionary and deflationary effect across the board, and depresses UE more than EE, raising the AC ratio. The real input price ω_t/P_t falls on impact to clear the Service market, because employment and the production of Service input are predetermined.

Similarly, a positive shock to the nominal interest rate has contractionary and deflationary effects on the economy, except for the EE probability, which rises on impact, despite the decline in vacancy postings, to then decline and overshoot, before converging back to steady state. As the last line of Figure 2 illustrates, this is due to a positive response of effort spent by Service sector firms to recruit employed job applicants, which happens to more than offset the drop in vacancies. Recruiting effort $r^*_{1,t}$ rises because the fall in aggregate demand, given existing employment, causes the real input price ω_t/P_t to fall, as seen in the third row from the bottom. As vacancy costs are in units of final goods (price P_t) and recruiting costs are in units of service (price ω_t), firms take advantage of cheaper Service to divert resources away from vacancies and towards recruiting. In fact, recruiting effort rises also for unemployed job applicants, $r*_{0,t}$, although in this case not enough to compensate for the decline in vacancies, so the UE probability declines.[11]

As mentioned earlier, the model features internal propagation. After a supply-side, TFP shock, consumption (due to low intertemporal substitution and strong consumption smoothing), inflation and unemployment (due to the response of recruiting effort $r^*_{0,t}$) all revert to steady state more slowly than TFP. Aggregate demand shocks, to monetary policy and, especially, to preferences (MRS) are much more persistent than those to TFP. Preference shocks impact directly, not only through equilibrium prices, the incentives to open jobs and to recruit workers. They set in motion employment on the job ladder, thus the Mismatch

[11] Graves, Huckfeldt and Swanson (2023) estimate from US data the IRFs of various labor market flows to monetary policy shocks identified through high-frequency methods. The IRFs of the UE and EE probability (resp., their Figures 6 and 10) are qualitatively identical to ours, but the latter is not statistically significant. They interpret this finding as a lack of response of labor supply by the employed workers (on-the-job search effort, willingness to switch jobs) to the nominal interest rate shock, and cast doubt on our mechanism. Our estimated structural model, besides finding a negligible role of these monetary policy shocks in causing business cycles, clarifies that the response of EE is the result of both demand and supply forces. In this case of nominal interest rate shocks, even demand forces (vacancies and recruiting effort) run counter each other on impact, when the labor supply component of EE, misallocation, is pre-determined.

Wedge. Their effect is reduced on impact (compared to $r^*_{0,t}$ and UE) and propagated through recruiting effort $r^*_{1,t}$ and the EE probability.

6 Conclusions

We introduce on-the-job search in the labor market in an otherwise conventional monetary DSGE model. We emphasize the transmission of labor demand shocks to labor costs and, from there, to prices, through outside offers received by currently employed, but potentially mismatched, workers. We solve and estimate the model, despite the large dimensionality of the state space which includes the entire distribution of employment on a job ladder. We find that measures of labor market flows, by now central to the literature on labor markets but still absent from small-scale DSGE macro models, contain information on latent price pressure, above and beyond the conventional unemployment rate. A new summary statistics of these flow, the Acceptance rate, the ratio between the EE and UE transition probabilities, is an accurate proxy of employment misallocation and slack. Intuitively, when employed workers change jobs at healthy pace, while the unemployed re-join employment slowly due to scarcity of new job postings, the employed must be lenient in their standards, thus mismatched. The EE/UE ratio is available monthly and in real time to policy makers in the US. In Moscarini and Postel-Vinay (2023) we estimate a regional wage Phillips curve on disaggregated US data, and show that the EE/UE ratio gives rise to a much steeper Phillips curve than unemployment.

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APPENDIX

A Extended Model: Equilibrium Conditions

We first study the optimal choice of the intensive margin of production and the resulting equilibrium compensation. For ease of notation, we temporarily suppress time indices. WLOG, let $w(y, q, \omega)$ be the nominal earnings paid to a worker employed in a match of quality y who produces q units of the good , then sold at unit price ω. The household solves the static problem of choosing final good consumption C and effort $h(j)$ of each member j to maximize utility (20) subject to the budget constraint:

$$PC = \int_0^1 \mathrm{e}(j) w(q(j), y(j), \omega) dj + \text{savings and unearned income, independent of } h(j)$$

and to technology $q(j) = y(j)h(j)$. Dividing the budget constraint by P, forming a Lagrangian with multiplier λ, the FOC for C is $U'(C) = \lambda$, and the FOC for $h(j)$ by an employed worker ($\mathrm{e}(j) = 1$) is

$$\mathcal{B} h(j)^{1/\Xi} = \lambda \frac{1}{P} \frac{\partial w(q(j), y(j), \omega)}{\partial q} y(j)$$

so the supply of effort in a match of quality $y = y(j)$ is

$$h(y) = \left[\frac{U'(C)}{\mathcal{B}} \frac{1}{P} \frac{\partial w(y, q, \omega)}{\partial q} y \right]^{\frac{\Xi}{1+\Xi}}.$$

Firms in the Service sector choose the compensation package $w(y, q, \omega)$ to maximize profits. As is standard in moral hazard situations, match surplus is maximized by "selling the firm to the worker", that is, paying the worker total revenues from what they produce: $w(y, q, \omega) = \omega q$. The firm then chooses a two-part contract, which offers this efficient incentive scheme and extracts profits through a (negative) base wage $\underline{w}(y)$, independent of output. It follows that $\partial w(q(j), y(j), \omega)/\partial q = \omega$, so the employed worker supplies effort as in (21). This choice fully characterizes the intensive margin.

The indirect utility that the employed worker obtains from working and choosing effort optimally equals that from the base wage $\underline{w}(y)U'(C)$ plus the indirect utility from variable pay net of disutility of effort:

$$\mathcal{U}(y) = \frac{\omega}{P} y h(y) U'(C) - \mathcal{B} \frac{[h(y)]^{1+1/\Xi}}{1+1/\Xi}$$
$$= \frac{\omega}{P} y \left[\frac{U'(C)}{\mathcal{B}} \frac{\omega}{P} y \right]^{\Xi} U'(C) - \mathcal{B} \frac{\left[\frac{U'(C)}{\mathcal{B}} \frac{\omega}{P} y \right]^{1+\Xi}}{1+1/\tilde{\Xi}} = \left[\frac{U'(C)}{\mathcal{B}} \frac{\omega}{P} y \right]^{1+\Xi} \frac{\mathcal{B}}{1+\Xi} \tag{26}$$

Note that $\mathcal{U}(y) \geq 0$ for every $\Xi \geq 0$: the worker can always choose zero effort, produce zero output, bear no disutility from labor, and mimic unemployment, except that the unemployed also collect the value of leisure b. In general, the worker can reap higher utility than spending a negligible amount of effort, by either staying home or working harder.

We now reintroduce time indices to study dynamic choices. After handing over all revenues $\omega y h_t(y)$ to the worker as variable pay, the firm's value, in units of final good, is the expected PDV of minus base wages until separation, either to unemployment or to another firm. So, the firm must simply choose as low (and negative) a base wage as possible to still attract and retain the worker, in order to maximize profits. To find the incentives of the worker to accept an offer, we study again their Bellman equations.

On the extensive margin, for job acceptance and separation purposes, the household compares the utility of keeping a marginal worker unemployed with that of keeping the same worker employed in match y and choosing effort, to earn a nominal base wage $\underline{w}_t(y)$, and variable pay equal to nominal revenues $\omega y h_t(y)$, but pay the utility cost of effort. Because of search frictions, the extensive margin decision is dynamic. The utility value of the variable pay, i.e. ignoring the base wage, net of the disutility of effort, is $\mathcal{U}_t(y)$. Therefore, including the base wage, the Bellman value of working at firm y with this two-tier contract, in units of the Final good, is

$$V_{1,t}(y) = \frac{\underline{w}_t(y)}{P_t} + \frac{\mathcal{U}_t(y)}{U'(C_t)} + \beta \mathbb{E}_t \Big[\lambda_{t+1} \max_{\{a_t(j)\}} \Big\langle \delta s_0 r^*_{0,t} \varphi_t \phi(\theta_t) a_t(j) \hat{V}_{1,t+1}(y_{t+1})$$
$$+ \delta \left(1 - s_0 r^*_{0,t} \varphi_t \phi(\theta_t) a_t(j)\right) V_{0,t+1} + (1-\delta) V_{1,t+1}(y_{t+1}) \Big\rangle \Big]$$

where $\hat{V}$ denote the value of an employed worker who is displaced but draws immediately the opportunity to search within the period. Since a newly unemployed worker always receives the continuation value of unemployment, $\hat{V}_{1,t+1} = V_{0,t+1}$, the Bellman equation has the same form as in the baseline model:

$$V_{1,t}(y) = \frac{\underline{w}_t(y)}{P_t} + \frac{\mathcal{U}_t(y)}{U'(C_t)} + \delta \mathbb{E}_t \left[\mathcal{D}_t^{t+1} V_{0,t+1} \right] + (1-\delta) \mathbb{E}_t \left[\mathcal{D}_t^{t+1} V_{1,t+1}(y_{t+1}) \right]$$

where y_{t+1} may be either the same match ($y_{t+1} = y$) if the worker stays, or a new match if the worker receives and accepts an outside offer.

Under the additively separable specifications of preferences, unemployed household members enjoy expected present value, expressed in units of the Final good, equal to:

$$V_{0,t} = \frac{b}{U'(C_t)} + \mathbb{E}_t \left[\mathcal{D}_t^{t+1} V_{0,t+1} \right] = \frac{b}{U'(C_t)(1-\beta)}$$

and therefore

$$\mathbb{E}_t \left[\mathcal{D}_t^{t+1} V_{0,t+1} \right] = \mathbb{E}_t \left[\beta \frac{U'(C_{t+1})}{U'(C_t)} \frac{b}{U'(C_{t+1})(1-\beta)} \right] = \beta V_{0,t}$$

To extract all rents from *unemployed* job applicants, while offering all revenues as variable pay to maximize producer surplus, the firm promises a base wage to make the worker indifferent in each state next period, namely

$$\frac{\underline{w}_{t+1}(y)}{P_{t+1}} = \frac{b - \mathcal{U}_{t+1}(y)}{U'(C_{t+1})} - (1-\delta)\mathbb{E}_{t+1} \left[\mathcal{D}_{t+1}^{t+2} \left(V_{1,t+2}(y_{t+2}) - V_{0,t+2} \right) \right]$$

Adding variable pay to both sides, total compensation equals the opportunity cost of employment $b/U'(C_{t+1})$ plus the effort cost minus an option value that staying employed, at the firm or elsewhere, will pay off more than unemployment in the future (which is just the same as being unemployed today, times the discount factor β). Note that the option value cannot be negative, because workers can always quit to unemployment, but it has to be large enough for unemployed workers to participate. As before, this will require the minimum match quality $\underline{y}$ to be large enough compared to b.

When an *employed* worker receives an outside offer, the two firms Bertrand-compete in contracts. Variable pay remains equal to total revenues, in each match, to satisfy bilateral efficiency and to maximize the value of the contract. The current base wage that the worker was promised by the current employer may no longer be sufficient to beat the outside offer. Since firm profits equal the expected PDV of *minus* base wages, until separation, the maximum that a firm is willing to pay to employ a worker in a match y is a contract with a zero base wage, which (along with variable pay) breaks even: $\Pi_t(y) = 0$. This contract, zero base wage and full output variable pay, is the firm's willingness to pay, which determines the winner, and yields the worker utility $\overline{V}_{1,t}(y)$ in units of Final good. When the offer arrives at time t, whether the worker quits or not, they will be promised an expected value $\mathbb{E}_t \left[\mathcal{D}_t^{t+1} \overline{V}_{1,t+1}(y) \right]$, either by the same firm y or by the poaching firm, that will match firm y's bid in the second-price auction. Therefore, this contract, zero base wage and full output variable pay, yields the worker utility, in units of Final good, equal to:

$$\begin{aligned}\overline{V}_{1,t}(y) &= \frac{\mathcal{U}_t(y)}{U'(C_t)} + \delta\mathbb{E}_t\left[\mathcal{D}_t^{t+1}V_{0,t+1}\right] + (1-\delta)\mathbb{E}_t\left[\mathcal{D}_t^{t+1}\overline{V}_{1,t+1}(y)\right] \\ &= \frac{\mathcal{U}_t(y)}{U'(C_t)} + \beta V_{0,t} + (1-\delta)\mathbb{E}_t\left[\mathcal{D}_t^{t+1}\left(\overline{V}_{1,t+1}(y) - V_{0,t+1}\right)\right]\end{aligned}$$

Subtracting the value of unemployment from both sides and using its expression on the RHS

$$\overline{V}_{1,t}(y) - V_{0,t} = \frac{\mathcal{U}_t(y)}{U'(C_t)} - \frac{b}{U'(C_t)} + (1-\delta)\mathbb{E}_t\left[\mathcal{D}_t^{t+1}\overline{V}_{1,t+1}(y) - V_{0,t+1}\right].$$

and defining the match surplus:

$$\begin{aligned}S_t(y) &= \overline{V}_{1,t}(y) - V_{0,t} = \frac{\mathcal{U}_t(y) - b}{U'(C_t)} + (1-\delta)\mathbb{E}_t\left[\mathcal{D}_t^{t+1}S_{t+1}(y)\right] \\ &= \mathbb{E}_t\left[\sum_{\tau=0}^{+\infty}(1-\delta)^\tau\mathcal{D}_t^{t+\tau}\frac{\mathcal{U}_{t+\tau}(y) - b}{U'(C_{t+\tau})}\right] = \frac{1}{U'(C_t)}\mathbb{E}_t\left[\sum_{\tau=0}^{+\infty}(1-\delta)^\tau\beta^\tau\left(\mathcal{U}_{t+\tau}(y) - b\right)\right] \\ &= W_t y^{1+\Xi} - \mathcal{L}_t\end{aligned}$$

where, using the definition of $\mathcal{U}(y)$ in (26), we define W_t as in (22).

We obtain the expected returns from successfully recruiting an unemployed job applicant in a match y

$$\mathbb{E}_t\left[\mathcal{D}_t^{t+1}\left(\overline{V}_{1,t+1}(y) - \mathcal{L}_{t+1}\right)\right] = \mathbb{E}_t\left[\mathcal{D}_t^{t+1}W_{t+1}\right]y^{1+\Xi} - \beta\mathcal{L}_t$$

where we used (23), and a job applicant currently employed in a match $y' < y$

$$\begin{aligned}&\mathbb{E}_t\left[\mathcal{D}_t^{t+1}\left(\overline{V}_{1,t+1}(y) - \overline{V}_{1,t+1}(y')\right)\right] \\ &\qquad = \mathbb{E}_t\left[\mathcal{D}_t^{t+1}\left(S_{t+1}(y) - S_{t+1}(y')\right)\right] = \mathbb{E}_t\left[\mathcal{D}_t^{t+1}W_{t+1}\right]\left(y^{1+\Xi} - y'^{1+\Xi}\right)\end{aligned}$$

Taking expectations over y, y' yields the expected returns to recruiting effort (24).

Because the opportunity cost term $\mathcal{L}_t$ is independent of y, the match with the higher expected surplus $\mathbb{E}_t\left[\mathcal{D}_t^{t+1}S_{t+1}(y)\right]$ also has the higher willingness to pay $\mathbb{E}_t\left[\mathcal{D}_t^{t+1}\overline{V}_{t+1}(y)\right]$, and wins the auction. The surplus is increasing in y, hence the better match wins the auction and the equilibrium is Rank Preserving. The poacher y earns profits equal to its willingness to pay minus the winning bid, namely $\mathbb{E}_t\left[\mathcal{D}_t^{t+1}\max\left\langle\overline{V}_{t+1}(y) - \overline{V}_{t+1}(y'), 0\right\rangle\right] = \mathbb{E}_t\left[\mathcal{D}_t^{t+1}\max\left\langle S_{t+1}(y) - S_{t+1}(y'), 0\right\rangle\right]$. The firm hiring from unemployment earns profits equal to $\mathbb{E}_t\left[\mathcal{D}_t^{t+1}\max\left\langle S_{t+1}(y), 0\right\rangle\right]$. Higher consumption exerts a negative income effect on labor supply both on the intensive and extensive margins, thus on the match surplus.

In the Final good sector, the NFOC for an optimal reset price is

$$\mathbb{E}_t\left[\sum_{\tau=0}^{+\infty}(1-\nu)^{\tau}\mathcal{D}_t^{t+\tau}\frac{\Pi_{t+\tau}^{F\prime}(p(i))}{P_{t+\tau}}\right]=0$$

but marginal profits must now take into account curvature in the production function: $\Pi_t^{F\prime}(p)=(1-\eta)Q_tP_t^{\eta}p^{-\eta}+\frac{\eta}{\zeta}Q_t^{\frac{1}{\zeta}}P_t^{1+\eta/\zeta}p^{-1-\eta/\zeta}x_t$, implying an optimal reset price:

$$p_t^{\star\frac{\eta}{\zeta}+1-\eta}=\frac{\eta}{\eta-1}\frac{1}{\zeta}\cdot\frac{\sum_{\tau=0}^{+\infty}(1-\nu)^{\tau}\mathbb{E}_t\left[\mathcal{D}_t^{t+\tau}Q_{t+\tau}^{\frac{1}{\zeta}}P_{t+\tau}^{\frac{\eta}{\zeta}}x_{t+\tau}\right]}{\sum_{\tau=0}^{+\infty}(1-\nu)^{\tau}\mathbb{E}_t\left[\mathcal{D}_t^{t+\tau}Q_{t+\tau}P_{t+\tau}^{\eta-1}\right]}$$

B Steady State Calibration of Turnover Parameters

We now show how steady state moment conditions calibrate the turnover parameters δ, s_0, and possibly s_1 independently of the rest of the parameter vector and of the model's equilibrium conditions.

A useful property of any job ladder model is that the probability that an employed worker accepts an outside offer, in steady state equilibrium, does not depend directly on the sampling distribution Γ. If this distribution improves in a stochastic sense, workers on each rung of the ladder are more likely to accept outside offers, but there are fewer of them on lower rungs, and the two effects exactly cancel out, so the probability remains the same, and depends only on contact and separation rates. To prove this statement, proceed as follows. In steady state equilibrium, the employment distribution is

$$L(y)=\frac{\phi(\theta)r_0^*\Gamma(y)\left[u+\delta(1-u)s_0\right]}{\delta+(1-\delta)s_1\phi(\theta)r_1^*(1-\Gamma(y))}$$

and the probability that an employed worker accepts an outside offer equals:

$$\int_{\underline{y}}^{\overline{y}}[1-\Gamma(y)]\frac{dL(y)}{1-u}=\int_{\underline{y}}^{\overline{y}}\frac{L(y)}{1-u}d\Gamma(y)$$

after integration by parts. Using the expression for $L(y)$ and changing variable to $\tilde{\Gamma}=\Gamma(y)$:

$$\int_{\underline{y}}^{\overline{y}}\frac{L(y)}{1-u}d\Gamma(y)=\int_{\underline{y}}^{\overline{y}}\frac{u+\delta(1-u)s_0}{\delta+(1-\delta)s_1r_1^*\phi(\theta)\left[1-\Gamma(y)\right]}\phi(\theta)r_0^*\frac{\Gamma(y)}{1-u}d\Gamma(y)$$

$$= \int_0^1 \frac{u + \delta(1-u)s_0}{\delta + (1-\delta)s_1 r_1^* \phi(\theta)(1-\tilde{\Gamma})} \phi(\theta) r_0^* \frac{\tilde{\Gamma}}{1-u} d\tilde{\Gamma}$$
$$= \frac{\delta}{(1-\delta)s_1\phi(\theta)r_1^*} \left[-1 - \frac{\delta + (1-\delta)s_1\phi(\theta)r_1^*}{(1-\delta)s_1\phi(\theta)r_1^*} \ln \frac{\delta}{\delta + (1-\delta)s_1\phi(\theta)r_1^*} \right]$$

where the last line uses the formula for the steady-state unemployment rate and integrates in closed form to obtain the desired expression, independent of Γ.

Let $\mathsf{UE}, \mathsf{EU}, \mathsf{EE}$ denote the empirical average values, over some long time period, of the observed (thus, time-aggregated) transition probabilities; let $\mathsf{UR} = \mathsf{EU}/(\mathsf{UE} + \mathsf{EU})$ denote the steady state unemployment rate consistent with those two empirical transition rates; and AC denote the average share of outside offers accepted by employed worker. Set $\phi(\theta)r_0^* = \mathsf{UE}$ and $u = \mathsf{UR}$. Since $\delta = \mathsf{EU} + \delta s_0 \mathsf{UE}$, we can write $\delta s_0 = (\delta - \mathsf{EU})/\mathsf{UE}$. Since

$$\mathsf{EE} = \delta s_0 \mathsf{UE} + (1-\delta)s_1\phi(\theta)r_1^* \mathsf{AC} = \delta - \mathsf{EU} + (1-\delta)s_1\phi(\theta)r_1^* \mathsf{AC}$$

we can write

$$(1-\delta)s_1\phi(\theta)r_1^* = \frac{\mathsf{EE} - \delta + \mathsf{EU}}{\mathsf{AC}} \tag{27}$$

Then we can define the function

$$\mathcal{A}(\delta) = \frac{\delta}{(1-\delta)s_1\phi(\theta)r_1^*} \left[-1 - \frac{\delta + (1-\delta)s_1\phi(\theta)r_1^*}{(1-\delta)\phi(\theta)s_1 r_1^*} \ln \frac{\delta}{\delta + (1-\delta)s_1\phi(\theta)r_1^*} \right]$$

Replace $u = \mathsf{UR}$ and the expression for $(1-\delta)s_1\phi(\theta)r_1^*$ from (27). Then:

$$\mathcal{A}(\delta) = \frac{\delta \mathsf{AC}}{\mathsf{EE} - \delta + \mathsf{EU}} \left[-1 + \left(\frac{\delta \mathsf{AC}}{\mathsf{EE} - \delta + \mathsf{EU}} + 1 \right) \ln \left(\frac{\mathsf{EE} - \delta + \mathsf{EU}}{\delta \mathsf{AC}} + 1 \right) \right]$$

We can now find the root of $\mathcal{A}(\delta) = \mathsf{AC}$ to estimate the value of δ. As explained before, $\mathcal{A}$ does not depend on the functional form of the sampling distribution Γ and is only a function of δ, given the empirical moments $\mathsf{EE}, \mathsf{EU}, \mathsf{AC}$.

Next, we can compute the value of $s_0 = (1 - \mathsf{EU}/\delta)/\mathsf{UE}$.

Finally, an optional step exploits an empirical measure of a reallocation shock, which forces a measured EE transition. As mentioned in the text, Faberman et al. (2021) estimate at 11% the share of employed workers actively engaged in job search who either had to relocate or received an advance notice of a layoff. In the model, forced reallocation is a time-aggregated, short EUE spell. The share of all search effort on the job of this type is $\delta s_0/[\delta s_0 + (1-\delta)s_1]$. Using the estimated δ and s_0 and equating this expression to 0.11 backs up s_1. We instead estimate s_1 from dynamic moments, and use this statistic for validation.