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TEMPORAL AGGLOMERATION

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ABSTRACT

When economic activity is concentrated over space or over time, it is more efficient. Most production occurs in geographic hot spots, and most production occurs between 9 and 12 in the morning and 1 to 5 in the afternoon on weekdays. The thick-market efficiencies that encourage the concentration of activity in certain time periods may be internal to the firm, or they may be external to the firm. When they are internal, the firm can make efficient arrangements to take advantage of the effects. The firm should martial all its forces from time to time in bursts of activity. When thick-market effects are external to the firm, the possibility of indeterminacy can arise. Aggregate fluctuations may arise with either internal or external thick-market effects.

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The spatial concentration of economic activity is no great mystery to economists. Let there be costs of moving products from one place to another, and producers and consumers will locate themselves close to each other. Along a line drawn almost anywhere on land, economic activity (GNP per square foot) will vary by many orders of magnitude. Agglomeration tends to occur at natural ports and other salient geographic points. But the distribution of economic activity is probably close to indeterminate, so the location of economic hot spots is largely a matter of historical accident.

The distribution of economic activity over time is the subject matter of macroeconomics. Agglomeration over time involves the same principles as agglomeration over space. Agglomeration occurs at a wide range of frequencies. Over the day, activity reaches pronounced peaks in the morning and again in the afternoon. Over the week, the weekend is a trough. Over the year, activity gradually rises to a peak in the early fall and then collapses in the winter. Every two to ten years, there is a contraction of several percent in real GNP in the form of a recession. And the economy has important movements at even lower frequencies. In the U.S., activity was high in the late 1940s and early 1950s, low in the late 1950s and early 1960s, high in the late 60s and early 70s, low in the mid- and late 1970s and early 1980s, and high since then.

Until very recently, the models developed in formal macroeconomic analysis did not have properties conducive to temporal agglomeration. On the contrary, their assumptions about preferences and technology compelled smooth distributions of activity over time. Consumers prefer a smooth consumption stream to one with a higher average but significant variability over time. Workers have the same view favoring smooth work hours. A technology concave in labor input produces a higher average level of output for a given average of labor input if production occurs at the same level at all times.

All the facts about temporal agglomeration contradict the standard formal model. Until recently, macroeconomists had two ways to resolve the contradiction. One is to invoke a sufficiently powerful driving force within the neoclassical model so that it will predict realistic output volatility in spite of the forces tending toward smoothness over time. This strategy defines the real business cycle school. Their models posit large vibrations of the production function to explain observed agglomeration at business cycle frequencies. The natural tendency for concave preferences and technology to generate smooth output is overcome by the high productivity of work effort in boom periods compared to slack periods. The corresponding approach to spatial agglomeration is to explain higher output per square foot in Manhattan as against Iowa by assuming that land in Manhattan is hundreds of times more productive than land in Iowa.

The second view considers the standard neoclassical model to be a longer-term growth model. Deviations from the smooth path of output are the result of some kind of coordination failure or price rigidity. Efforts to put this type of model onto a firm theoretical basis have not appealed to a wide class of economists. The second view attracts many adherents among macroeconomists, however, because they find the alternative of the real business cycle model so unconvincing.

Recent thinking points in a direction that retains important parts of the neoclassical apparatus yet generate realistic outcomes. The essential idea is to drop the concavity of technology but to keep the concavity of preferences. New models of temporal agglomeration explain the irregularity of output as the victory of efficiencies of producing output in occasional batches over the cost of irregular work schedules. Unlike the real business cycle model, the new model based on increasing returns does not deny the cost of irregular work; that is, it does not assume that labor supply is highly elastic.

One important source of increasing returns and temporal agglomeration is the thick-market externality, described by Diamond [1982]. Under a thick-market externality, the costs of doing business are lower in places, or in times, of higher total activity. For example, it is cheaper to sell cameras in mid-town Manhattan than anywhere else in the country, because the density of camera-buyers is so high. Buyers are dense both because Manhattan is a major emporium for goods of all types and because there is a much wider selection of cameras at much lower prices than anywhere else. In the temporal setting, costs are lower in booms, because higher activity makes markets thicker.

Increasing returns need not derive from an externality in order to make temporal agglomeration efficient. Rogerson [1988] introduces a simple form of increasing returns—workers incur a fixed cost each day they go to work. Specialization of days into ones of intense work and ones of no work are optimal, even if preferences would favor spreading work over all available days absent the fixed cost. Rogerson's model explains an important form of temporal agglomeration, the concentration of work into the five-day workweek. His model also makes a contribution to the understanding of temporal agglomeration at lower frequencies, a topic I will return to later in this lecture.

Murphy, Shleifer, and Vishny [1988] offer a model, also founded on increasing returns, of once-and-for-all temporal agglomeration. In their model, if all industries adopt advanced technologies simultaneously, enough income is generated to cover the fixed costs of every industry. But income effects are not a good candidate to explain temporal agglomeration at business cycle or higher frequencies, because of permanent income considerations. Research in progress by the same authors considers episodic temporal agglomeration.

1. Simple models of agglomeration

Thick-market externalities

Consider an economy comprising many firms, indexed by i, in many units (spatial or temporal), indexed by t. Let y_{it} be the output of firm i in unit t, and let Y_t be the average output of firms in unit t. There is a single factor input, labor, and the common level of labor input for all firms in unit t is x_t. All firms have the same technology,

$$y_{it} = g(x_t, Y_t) \quad , \tag{1.1}$$

where the presence of Y_t recognizes a thick-market externality. The production function is increasing in x and Y and is concave in x and in Y separately, but not jointly; there is a region where $g(\theta x, \theta Y)$ has an elasticity greater than one with respect to θ. Then

$$Y_t = g(x_t, Y_t) \quad , \tag{1.2}$$

which can be solved for Y_t,

$$Y_t = f(x_t) \quad , \tag{1.3}$$

where $f(x_t)$ is the maximum level of output than can be produced with x_t.

In a spatial setting, where t runs over geographical units, it would be natural to put a constraint on the sum of the x_t:

$$\frac{1}{T}\sum x_t = \bar{x} \quad . \tag{1.4}$$

That is, whatever labor is available, $\bar{x}$, can be allocated across the geographical units. Each person would work in only one place; it is reasonable to exclude the possibility that somebody could achieve a higher level of satisfaction by spreading work over many places. In the temporal setting, just the opposite holds. Concentration of all work in one period might yield far less satisfaction than spreading the same amount of work over many periods. In principle, the analysis needs to deal with preferences defined over all periods, for goods and work. To keep the discussion simple, I will assume that individuals allocate hours of work according to a utility constraint,

$$\frac{1}{T}\sum u(x_t) = u(\bar{x}) \quad . \tag{1.5}$$

There is no substitutability between goods and work, but there is some substitutability between work in one period and work in another, depending on the concavity of u.

Because of the externality, a free-market economy will not generally achieve the optimal schedule of activity. If the production function $f(x)$ has sufficiently increasing returns to scale to offset the curvature of $u(x)$ (that is if the output-utility set $\{(y,v)|y\leq f(x) \text{ and } v\leq u(x) \text{ for some } x\in[0,1]\}$ is non-convex), then the optimum would involve switching back and forth between two levels of employment. Although the decentralized economy will not usually operate at either of these points, it may achieve some of the benefits of agglomeration through a regular cycle of activity.

I define a *periodic equilibrium point* as a level of employment that equates labor supply to labor demand at a level of employment different from the benchmark level, $\bar{x}$, in the labor supply constraint. That is, x is a periodic equilibrium point if

$$\frac{\partial g(x,f(x))}{\partial x} = -\lambda u'(x) \quad . \tag{1.6}$$

where λ is the shadow value of utility in terms of goods. If the economy cycles through a set of periodic equilibrium points, all sharing the same λ, and satisfies the labor supply constraint, then the points and associated frequencies make up a *periodic equilibrium.*

Absent the thick-market externality, equation 1.6 and the labor supply constraint will have the unique root $\bar{x}$ and $\lambda = -(\partial g/\partial x)/u'(\bar{x})$ and stable employment at this level will be optimal. The externality makes possible multiple periodic equilibrium points, because the marginal product of labor can be an increasing function of labor input, x. For example, if

$$g(x,Y) = xY^{\gamma} \tag{1.7}$$

and

$$u(x) = \frac{(1-x)^{1-1/\sigma}-1}{1\text{-}1/\sigma} \quad , \tag{1.8}$$

equilibrium requires

$$\frac{\partial g(x,f(x))}{\partial x} = x^{\frac{\gamma}{1-\gamma}} = -\lambda u'(x) = \lambda(1-x)^{-1/\sigma} \quad . \tag{1.9}$$

Figure 1–1 shows the three periodic equilibrium points. Shut-down, $x = 0$, is one point. The other two are roots of

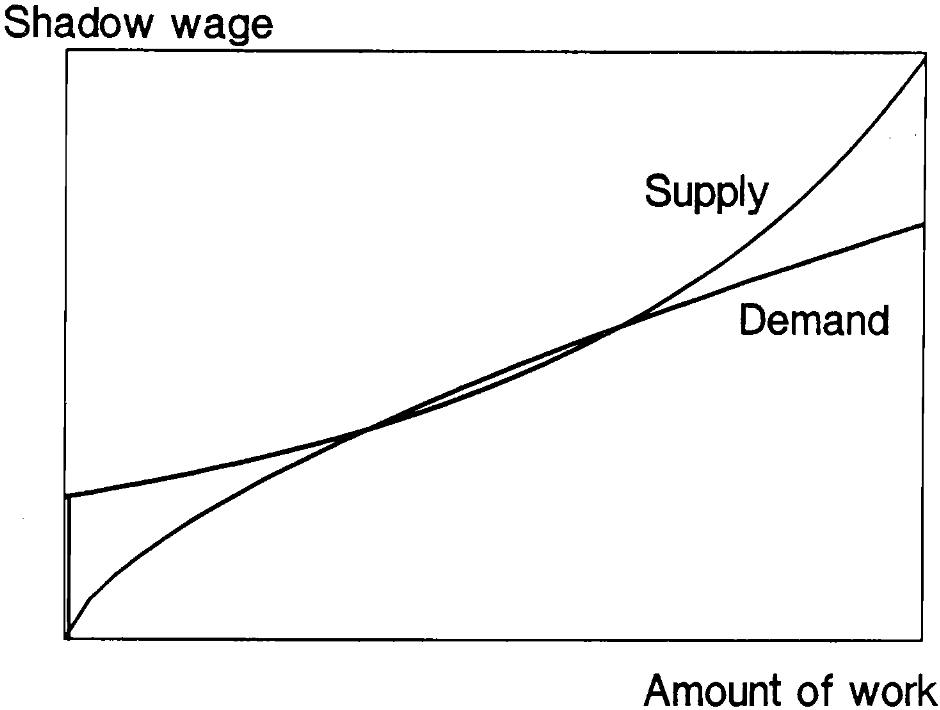
$$x^{\frac{\gamma}{1-\gamma}}(1-x)^{1/\sigma} = \lambda \quad . \tag{1.10}$$

As long as λ is small enough, there are two roots, x_L and x_H.

Let π_L and π_H be the fractions of time spent at the two positive periodic equilibrium points. The labor supply constraint requires

$$\pi_L u(x_L) + \pi_H u(x_H) = u(\bar{x}) \quad . \tag{1.11}$$

Figure 1-1. Periodic equilibrium Points



There is a two-dimensional subspace of values of λ, π_L, and π_H that correspond to three-point periodic equilibria (x=0, x_L, and x_H), three one-dimensional subspaces that correspond to two-point periodic equilibria (x=0 and x_L; x=0 and x_H; x= x_L and x_H), and two isolated points that correspond to unchanging non-periodic equilibria ($x_L=\hat{x}$ and $x_H=\hat{x}$).

The periodic equilibria with thick-market externalities can be ranked in welfare terms by the average amount of output they produce. The best periodic equilibrium is the one that comes closest to the optimal one that maximizes average output by switching back and forth from a low value (zero in the example) to a positive value. Although the optimum may be among the possible periodic equilibria, there is no guarantee that the economy will operate at or near the optimum. Instead, the economy can be in equilibrium in any of its periodic equilibria. If an inefficient cycle becomes established, it is self-replicating. In the framework developed here, all periodic equilibria have equally burdensome work schedules—workers are indifferent among them. But the superior equilibria produce more average output because they take better advantage of the economies of batch production, given increasing returns.

Thick-market economies internal to the firm

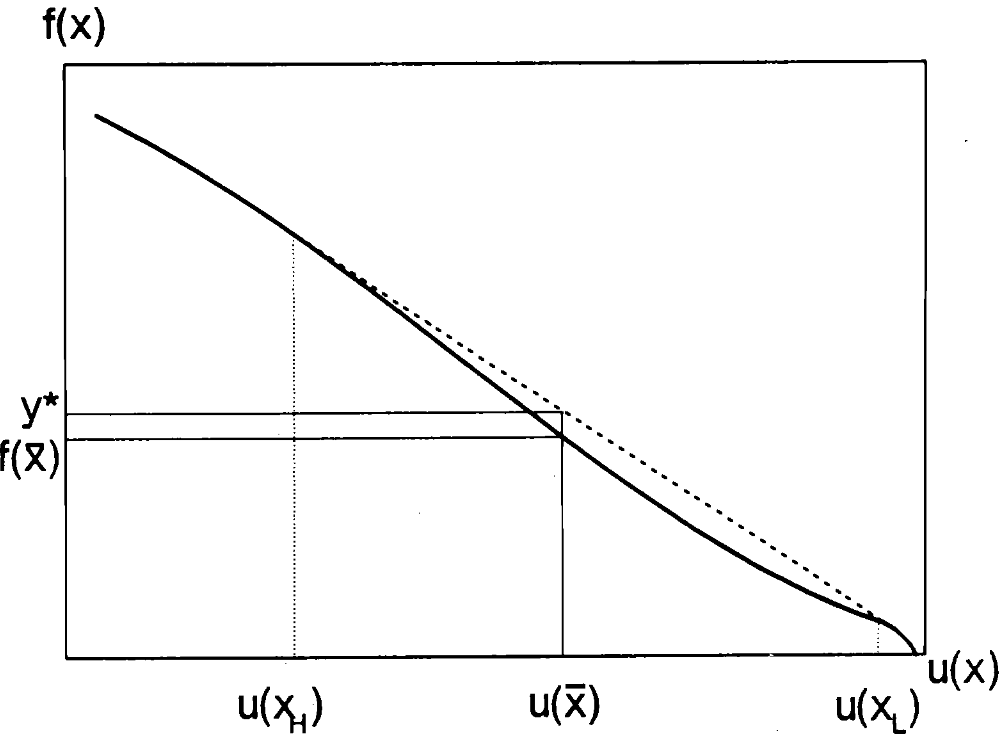
Thick-market economies need not be external to the firm. Indeed, in the spirit of Coase, the firm may extend to the point where at least the more important thick-market economies are within its boundaries. Simple increasing returns in the variable factors of production are one example of a thick-market economy.

A model similar to the one developed earlier in this section illustrates the optimal time schedule of activity in a firm with a dedicated group of workers. The firm maximizes its average level of output subject to a constraint on the utility achieved by the typical worker. Equivalently, it maximizes workers' utility subject to a constraint on average output. The exercise is similar to cost-minimization. No explicit assumption is made about the firm's output market, but implicitly the problem is reasonable only if the concavity of the firm's revenue function more than offsets the increasing returns of its technology. Absent this condition, there would be no bound on the size of the firm's work force.

In order to get temporal agglomeration within the firm, it is necessary that the firm operate in a region of increasing returns in those factors that can be varied. If capital services are costless once capital is in place (depreciation occurs over time but not in proportion to use), this condition is stronger than increasing returns in all factors including capital. If the entire cost of capital is incurred *ex post* (only odometer readings matter), then I am assuming increasing returns in the standard sense.

Figure 1–2 shows the utility-production frontier for the firm and its workers. The solid line maps $(u(x), f(x))$ for x between 0 and 1; at $x=1$, all available hours are devoted to work. For x between x_L and x_H, increasing returns in the technology, thanks to the thick-market effect, dominate the concavity of $u(x)$. If work is always at the level $\bar{x}$, the utility constraint is achieved, but output is only $f(\bar{x})$. With temporal agglomeration, on the other hand, the average level of output can be $y^* > f(\bar{x})$, by working the amount x_L during rest periods and the amount x_H during work spurts.

Figure 1-2. Utility-production frontier



A special case of Figure 1–2 applies to Rogerson's [1988] analysis of fixed costs of work. When the non-convexity has the form Rogerson assumes, x_L must be zero; the worker conserves on fixed costs of getting to work by taking entire days off. Hence Rogerson's model explains weekends and holidays; the firm and its workers achieve a superior combination of goods production and leisure by concentrating work in 235 work days out of the 365 available over the year.

Applications of agglomeration theory

The analysis of this section–either based on externalities or thick-market effects internal to the firm–has numerous geographical and temporal applications. Some examples are:

Low	High
Rural areas	Cities
Lobbies	Offices
Night	Day
Weekends	Weekdays
Winter	Summer
Slump	Boom
Weak decade	Strong decade

2. *Thick market economies*

Peter Diamond [1982] has developed the most thorough and rigorous analysis of a thick-market externality. In his model, the number of customers in the market determines each firm's inventory holding costs. In a thick-market equilibrium, production is high, so there are many customers (no producer consumes its own output). Costs of production and sales are low because the rapid arrival rate of customers means that inventories turn over frequently. Consequently high production levels are profitable. Another equilibrium with a thin market can also occur. Only the superior production opportunities are profitable because inventories turn over slowly.

Diamond's analysis is simpler than the one in the preceding section in one crucial respect: There is no variable factor to be allocated across time periods. The essential idea in section 1 is that the economy can take advantage of a mild thick-market effect by concentrating labor input in one time or place. Because the concentration of the variable input does not occur in Diamond's model, he needs a much stronger thick-market effect to get the result he seeks.

The other important differences is in the nature of the result. In Diamond's model, equation 1.2, without x, is

$$Y_t = g(Y_t) \tag{2.1}$$

and it has at least two roots. That is, the thick-market externality is so strong that there is a region where the elasticity of $g(\cdot)$ exceeds one; this property is an implication of Diamond's stochastic search model, not just an assumption. By contrast, in this lecture I assume only that $g(x,Y)$ has a region of increasing returns in x and Y together, which is consistent with only a mild thick-market effect. Diamond stresses the multiplicity of roots of equation 2.1, whereas I make the assumption that equation 1.2 has only one root, or, if there are multiple roots, that the economy chooses the largest one. It is important to note that the pattern of alternation between high and low levels of activity described in the previous section is *not* a movement between the two equilibria described by Diamond. Each is a high-level equilibrium in Diamond's sense. They differ because of different levels of labor input.

Other sources of thick-market effects

My discussion of thick-market effects at a practical level will deliberately blur the spatial-temporal distinction. I believe that it is useful to consider why activities in big cities might have lower costs in order to understand why activities in booms might have lower costs.

The inventory holding costs characterized very sharply in Diamond's model take many forms in real life. Inventory turnover rates probably differ by more than an order of magnitude between retailers in big cities and small towns. The magnitude of the corresponding holding costs varies widely. It always includes interest and storage costs. It may include depreciation for semi-durables and obsolescence for style and technological goods.

Selling and buying costs are lower in thick markets. A high density of buyers makes possible higher utilization of salespeople and facilities and also permits greater specialization. Compare the efficiency of 47th Street Photo to a camera store in a medium-sized city and to the camera department in a store in a small town. The efficiency probably varies over more than an order of magnitude.

On the buying side, high specialization of sellers and salespeople and lower search and transportation costs means greater efficiency in purchasing intermediate inputs in thick markets. For established purchasing relationships, thicker markets mean lower transportation costs. For example, in large cities, restaurants rent their knives from a service that delivers sharpened knives every few days. Such a service is much cheaper when the density of restaurants is high. Similarly, the pronounced increase in the use of overnight delivery services in the past decade has made these services much cheaper because customer density is so much greater.

Although the most obvious thick market economies apply to selling, buying, and distributing goods and services, the economies extend to actual production as well. In thick markets, components are available in much greater variety. Firms in remote locations or in undeveloped countries make parts which are available in the open market in locations with concentrated economic activity. Thick markets offer much more specialized workers and services. Firms outside of urban areas must make do with general practitioners for accounting, consulting, and legal services, or they must incur substantial travel costs to bring in specialists. Perhaps most important, workers and facilities in thick markets achieve higher utilization rates. Compare the number of sandwiches made per day by each worker in a big-city delicatessen to the output of similar workers in small towns.

Finally, a significant element of total cost in businesses arises from collecting and paying bills and from similar activities. In thick markets, workers are more specialized and are in closer contact with their counterparts in other firms.

3. Application of temporal agglomeration theory

Temporal agglomeration can occur whenever thick-market economies overcome the concavity of preferences. Both geographical and temporal agglomeration tend to occur at places or time favored by technology or preferences. Activity concentrates at natural ports, for example. Similarly, the extreme concentration of activity from 9 to 12 in the morning and from 1 to 5 in the afternoon reflects humankind's diurnal pattern of attentiveness.[1] On the other hand, the timing of weekends (but not the length of the week) is completely arbitrary.

The seasonal pattern of economic activity presents a particularly interesting example of temporal agglomeration. Although the third quarter (July, August, and September) is the time most favored for vacations in the U.S., that quarter has the highest level of labor input over the entire year (Barsky and Miron [1986]). The third-quarter peak in production is strong in many industries besides food processing and construction, where the role

[1]Of course, there is the deeper question of why humans are diurnal. In particular, neurophysiologists consider the fact that most mammals sleep quite a few hours a day an unsolved mystery. Temporal agglomeration may be the answer—the concentration of activity in daylight hours and the complete suppression of physical activity during sleep may have an evolutionary advantage.

of the weather is obvious.

The annual Christmas boom in the sale of durable goods is an excellent example of temporal agglomeration at work. In an economy without thick-market economies and other features conducive to agglomeration, the Christmas boom would have to be explained by a very strong preference on the part of most families to shop in December. The prices of goods would be higher in December, and the shopper would do better to buy earlier in the year or just after Christmas. Producers would put Christmas goods into inventory starting in January. In fact, the Christmas boom has a very different character. First, consumers gain by concentrating their annual shopping for some kinds of goods into a 6-week period. A long shopping list makes each visit to a store more productive. They also gain because the selection of products is wider than at other times of the year. Second, the selling process is much more productive during the Christmas boom. Inventories turn over faster than at other times of the year. Sales per salesperson and per square foot are substantially higher. One of the confirmations that thick-market economies are at work is that there is no systematic tendency toward higher prices during the Christmas boom—see Barsky *et al.* [1988].

The most interesting application of temporal agglomeration theory in macroeconomics is to booms and recessions. Is it reasonable to consider a several-year boom to be a cousin of the Christmas boom? Is a recession similar to the January contraction? An important difference between macroeconomic expansions and contractions, on the one hand, and diurnal, weekly, and seasonal movements, on the other hand, is the unpredictability of the former. Seasonal fluctuations occur in precise synchrony with the earth's revolution around the sun, but the business cycle is not in obvious synchrony with any single outside driving force. The only general statements that seem to hold for the U.S. economy are that a rise in interest rates often precedes a recession and that movements in GNP are correlated with movements in government purchases in goods and services. Not only is the business cycle irregular within any single country, but the intensity of the cycle varies across countries. Some countries, notably France, have almost no fluctuations in output or employment over 2 to 10 year periods, even though daily, weekly, and annual cycles are similar to other developed economies of the northern hemisphere.

Still, many of the same elements that make thick-market effects a plausible explanation of sharp seasonal fluctuations seem to apply to the business cycle as well. Just as New York is a good place to go shopping even though its stores sell a greater volume per square foot than those in other cities and towns, a boom is a good time for consumers to buy. And the efficiencies in buying, producing, distributing, and selling that offset the convexity of technology and preferences should apply just as much to the random fluctuations of the business cycle as to the systematic fluctuations by season.

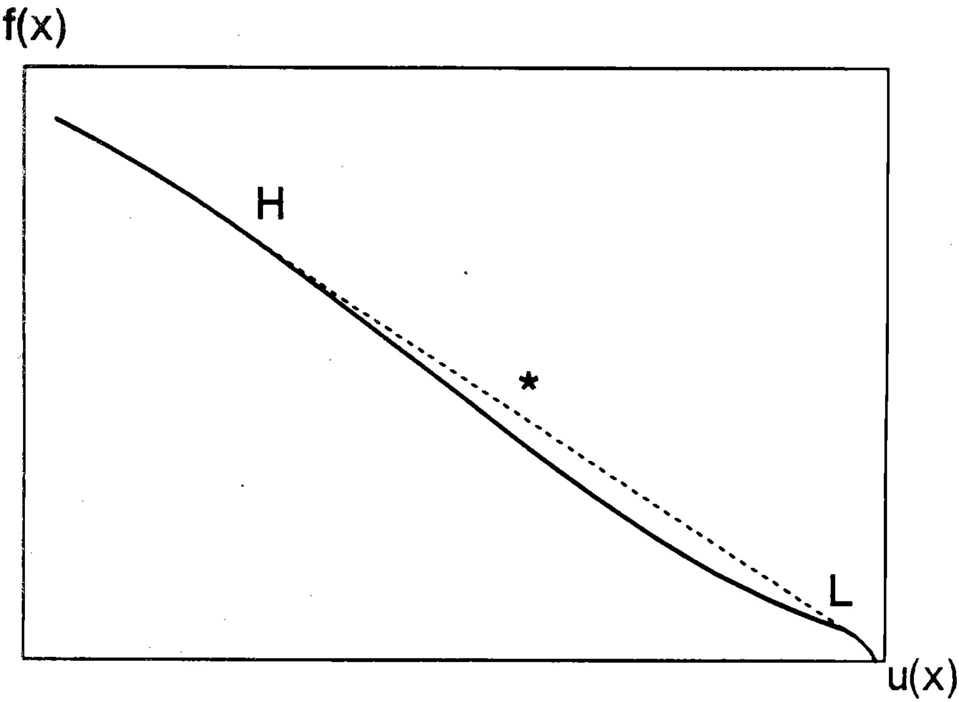
I think temporal agglomeration theory has a number of applications to business cycles. First, the optimal pattern of activity over time may be in synchrony with cyclical driving forces, such as government purchases. Second, the inefficient temporal pattern associated with thick-market externalities may also be in synchrony with irregular driving forces. Third, a boom or slump may represent the shift of the economy from one periodic equilibrium to another, as a result of some impulse.

Rogerson [1988] and, following Rogerson's suggestion, Prescott [1986] are the leading examples of papers in which the business cycle represents efficient temporal agglomeration. Figure 3–1, which repeats the gist of Figure 1–2, is helpful in understanding Rogerson's idea. By construction, points H and L are equivalent. First, they provide the double root $Y_H = g(x_H, Y_H)$ and $Y_L = g(x_L, Y_L)$. Second, H and L are points of indifference in the following sense: The extra output produced during a brief spell at H is just enough to compensate the worker for the foregone leisure, relative to L.

Optimality requires dividing time between H and L so as to achieve the point $*$. However, there may not be an exact prescription for the time pattern of the alternation—the only strong prescription is that the long-run average be at $*$. Hence a variable pattern, such as a random business cycle, that achieves $*$ as an average may involve little or no social cost relative to a stable daily, weekly, and annual schedule. When there is some cyclical driving force, such as changes in government purchases or in productivity, the pattern of temporal agglomeration will synchronize with the driving force. This is Rogerson's [1987] and Prescott's [1986] basic point.

Whereas the predictable pattern of daily, weekly, and annual movements between active and resting states of the economy synchronizes with the clock and the calendar, the business cycle synchronizes with random events. Wartime, with large increases in government purchases, invariably activates the economy. Financial crises are usually followed by recession and higher incidence of inactive periods. Fluctuations in the work

Figure 3-1. Utility-production frontier



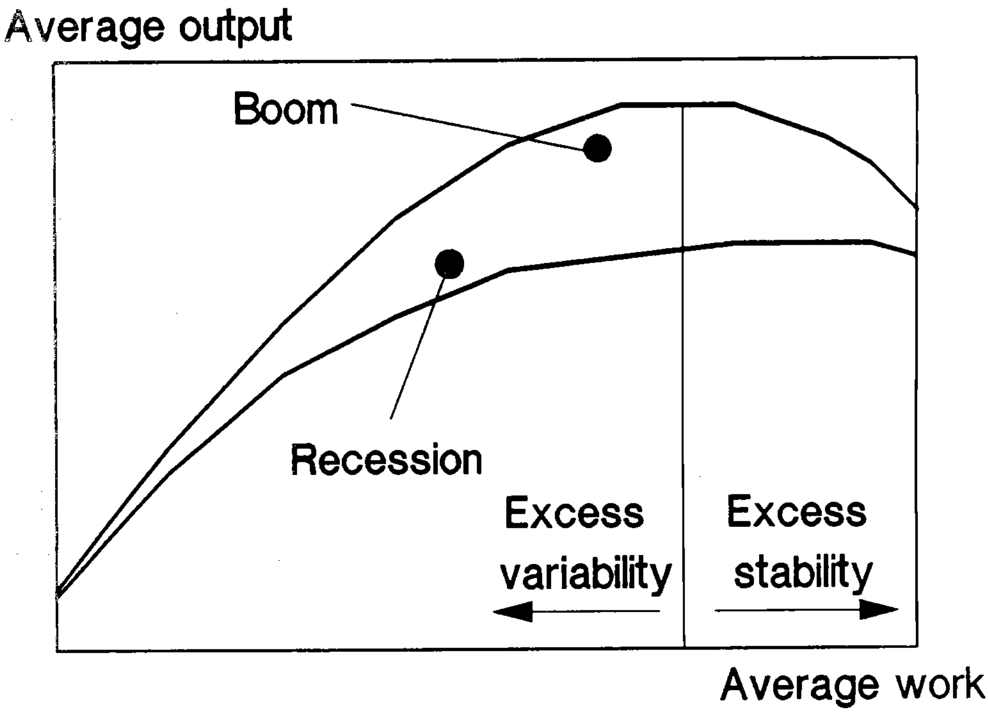
schedules of individual workers take the form of variations in annual weeks of work more than variations in hours per day.

The cyclical mechanism sketched here amplifies small disturbances. For example, a small increase in consumption will activate the economy, making it a more desirable time to consume (just as December is the best time to buy durables). In Prescott's real business cycle model, temporal agglomeration in the particular form proposed by Rogerson makes the economy activate itself when there is a favorable shift of technology.

In the real business cycle model, the synchrony of economic activity with a driving force is efficient, because the thick-market effect is internal to the firm and its workers. When a thick-market externality causes the economy to cycle through a periodic equilibrium, there is no presumption of efficiency. By extension, there is no presumption of efficiency if a periodic equilibrium synchronizes with a driving force. In that case, the times called booms are ones when the economy spends more than the usual time in the most active state. Later slack times enable the economy to satisfy the long-run requirement about the fraction of time in each periodic state.

A related idea is that a boom is a time when the economy shifts from one periodic equilibrium to another. Figure 3-2 illustrates how this works. The tricky part is to explain why employment as well as output rises in a boom. The curving band contains the average level of employment and the average level of output for each periodic equilibrium. All of the equilibria assign work schedules with the same level of utility. Schedules with low variability have higher average levels of employment; schedules with high variability have low average employment. To achieve

Figure 3-2. Output and employment variability



the same level of satisfaction, workers require compensation in the form of higher leisure in order to accept the disamenity of higher variability.

In the U.S. economy, employment and output move together. Hence, if the cycle involves movements from one periodic equilibrium to another, the leading case would be movements along the curve to the left of the optimum, which slopes upward. There, the economy always has too much instability in employment. A boom is when employment becomes more stable, which increases output. In a boom, workers give up leisure without sacrificing satisfaction from their work schedules. Although their total leisure is smaller, it is better timed.

The width of the band in Figure 3–2 depends on the extent to which the third and additional periodic equilibrium points create the possibility of work schedules with the same variability as the optimal schedules, but less output because the extra points are poorly placed. If the band is sufficiently wide, there is another story about booms and recessions–a boom occurs when the economy moves northeast in the diagram. In a boom, work schedules are rationalized, producing more output with less variability.

The temporal agglomeration view of the business cycle does not imply optimality of particular cyclical episodes, nor does it suggest that every episode of depressed economic activity is just a normal lull between episodes of concentrated output. In particular, I make no suggestion that the Great Depression was an extended rest. On the contrary, I think agglomeration theory makes an important contribution to the type of story told by Bernanke [1983]: The Depression was the result of a severe injury to the networks that make high levels of output possible. Recent thought has singled out the financial system—the network that provides credit and executes transactions. Other networks are essential as well. What would happen to U.S. GNP if the telephone system shut down for an extended period?

4. *Empirical implications of temporal agglomeration*

Most modern economies have volatile output and stable relative prices. Prices are not the shock absorbers described by neoclassical economics. This observation has led some macroeconomists to posit price rigidity as a starting point for theory. Temporal agglomeration theory derives output volatility from assumptions about technology and preferences. What about the stability of relative prices?

With respect to the real wage, Figure 1–1 shows that the shadow wage moves in tandem with employment in a periodic equilibrium with a thick-market externality. The same analysis applies to optimal time agglomeration as well. There are several reasons that measured wages might be stable in spite of Figure 1–1. First, wages may smooth the shadow wage over the periodic cycle—the wage paid at any given point may not be the allocative shadow wage. The same consideration applies to any macro model. Second, the labor supply schedule may fluctuate from one periodic equilibrium point to the next. Labor supply is greater during the day than at night, for example. Although supply shifts are not likely to be important for the business cycle, they could be important over the seasons. Third, the labor supply schedule may be highly elastic—again, this consideration stabilizes real wages in any macro model. Fourth, and most interesting and important, thick-market externalities may accrue directly to workers. If some of the benefits of thick markets take the form of higher take-home pay in relation to wages paid, or greater job satisfaction, then the shadow value of work will be higher during periods of high activity even if wages paid by employers remain the same. Akerlof, Rose, and Yellen [1988] argue that low-unemployment markets generate substantial improvements in job satisfaction because workers are better able to move to more satisfying jobs when the market is fluid. They demonstrate importance of voluntary job changes that do not raise cash wages.

For established, long-term workers, wage smoothing is probably the most important element of the explanation of constant real wages. For mobile workers, lower job-search cost and faster job-finding in thicker markets may be the most important element in the explanation of stable real wages. Wider selection of jobs with better matching and higher job satisfaction may also be significant.

The seasonal and cyclical stability of relative goods prices may be explained by considerations similar to those for real wages. Temporal agglomeration may be more pronounced for durable goods than for non-durables, for example. In the simplest model, the relative price of durables would fall along with the output of durables. But, just as with the real wage, there may be synchronization of the periodic cycle with shifts in preferences. For example, the Christmas boom in durables is the result of the suitability of these goods as presents. Second, it seems clear the customers share in thick-market benefits. As I mentioned in section 2, thick markets have wider selections and lower shopping costs. Seasonal and cyclical booms are times of lower shadow values of durables even if the relative price of durables is stable, because of the rise in consumer benefits not reflected in the price.

The intertemporal relative price of goods—the real interest rate—also obeys the principles just laid out. In the simplest model, the interest rate governing the deferral of the use of goods from the low state to the high state should be higher than the one from high to low. Over the year, in interest rate from March to September should be high and the rate from September to March should be low. In fact, there are almost no seasonal variations in real interest rates. As in the previous cases, movements in preferences and thick-market benefits accruing directly to consumers could explain the stability of real rates.

Productivity

The two sources of temporal agglomeration considered here—thick-market externalities and increasing returns from internal thick-market economies or other sources—both reveal themselves in productivity calculations. When some exogenous force stimulates output, measured productivity rises (Hall [1988a, 1988b]).

In the case of a thick-market externality, the production function is

$$Y = g(x, Y) \quad . \tag{4.1}$$

Solow's total factor productivity calculation starts with the finite difference approximation,

$$\Delta Y \doteq \frac{\partial g}{\partial x}\Delta x + \frac{\partial g}{\partial Y}\Delta Y \quad . \tag{4.2}$$

Under competition, the marginal product of labor is equated to the real wage,

$$\frac{\partial g}{\partial x} = \frac{w}{p} \quad . \tag{4.3}$$

Because $Y = g(x, Y)$, the marginal externality, $\partial g/\partial Y$, can be written as an elasticity, $(Y/g)\partial g/\partial Y$. Call this elasticity $\gamma/(1+\gamma)$. Then

$$\frac{\Delta Y}{Y} \doteq \frac{wx}{pY}\frac{\Delta x}{x} + \frac{\gamma}{1+\gamma}\frac{\Delta Y}{Y} \quad , \tag{4.4}$$

or

$$\frac{\Delta Y}{Y} \doteq (1+\gamma)\frac{wx}{pY}\frac{\Delta x}{x} \quad . \tag{4.5}$$

Solow's calculation assumes no externality ($\gamma = 0$). Consequently, it gives too little weight to the change in labor input, $\Delta x/x$, and the calculation of the productivity residual,

$$\frac{\Delta Y}{Y} - \frac{wx}{pY}\frac{\Delta x}{x} \quad . \tag{4.6}$$

records an increase in productivity any time some force causes an increase in employment.

For increasing returns, resulting from thick-market economies or other sources, the production function is

$$Y = f(x) \quad . \tag{4.7}$$

Again taking finite differences, I get

$$\Delta Y \doteq f'(x)\,\Delta x \quad . \tag{4.8}$$

Let $1+\gamma$ be the elasticity of $f(x)$ with respect to x. Then

$$\frac{\Delta Y}{Y} \doteq (1+\gamma)\frac{\Delta x}{x} \quad . \tag{4.9}$$

The Solow residual, calculated as in formula 4.6, will record false productivity growth when employment rises, because of the omission of the γ. The effect will be worsened if labor's share, wx/pY, is less than one.

5. *Conclusions*

Temporal agglomeration provides a workable theoretical framework for understanding the volatility of output, the stability of relative prices, and the correlation of output and productivity growth. Existing general equilibrium models fall short of satisfactory explanations of these phenomena. The neoclassical model predicts smooth evolution of activity over time; it cannot explain the concentration of activity over the day, the week, the year, or the business cycle. Temporal agglomeration makes endogenous the shifts in productivity that the real business cycle model invokes as an exogenous driving force.

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