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CURRENCY RISK PREMIUMS:
A MULTI-HORIZON PERSPECTIVE

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Currency Risk Premiums: A Multi-horizon Perspective
Mikhail Chernov and Magnus Dahlquist
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ABSTRACT

We review the literature on multi-horizon currency risk premiums. We show how the multi-horizon implications arise from the classic present-value relationship. We further show how these implications manifest themselves in the interaction between bond and currency risk premiums. This link is strengthened by explicitly accounting for stochastic discount factors. Information about currency risk premiums at different horizons presents a wealth of new evidence and challenges for existing models.

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1 Introduction

A long-standing issue in international finance is the source of variation in foreign exchange (FX) rates. While intuitive, the connection between macroeconomic fundamentals and exchange rates, also known as the FX macro disconnect, is weak, so an approach based on financial markets appears promising. This leads researchers to consider the classic present-value paradigm suggesting that exchange rates move either due to news in cash flows (i.e., the difference between foreign and domestic interest rates, driven by various fundamentals) or due to news in discount rates (i.e., currency risk premiums). In this survey, we argue that understanding currency risk premiums over different horizons offers a fresh view of movements in exchange rates.

The present-value relationship is typically used to decompose the variation in the current exchange rate into the cash-flow and discount-rate effects. In Section 2, we apply that same relationship to the currency depreciation rate over several horizons. The resulting expression reveals the connection between expected depreciation rates and the combination of cross-country differences between sovereign bond yields and their risk premiums as well as currency risk premiums. Thus, multi-horizon depreciation rates compel us to study the properties of international bonds. In turn, this endeavor could bring a wealth of useful spillovers from theories of bond risk premiums, which are motivated by violations of the expectations hypothesis (EH), to various theories of currency premium determination. Such theories are usually motivated by single-horizon evidence, such as violations of uncovered interest parity (UIP).

Studying bond risk premiums leads us to consider stochastic discount factors (SDFs) in Section 3. The price of an n -period bond equals the expectation of the n -period

SDF, which, in turn, is a product of n one-period SDFs. Thus, knowledge of the SDF delivers a whole collection of bond prices at different horizons and the corresponding yield curve. Knowledge of the yield curve is helpful in identifying empirically relevant properties of the SDF.

If financial markets are complete, exchange rates are closely connected to the SDFs as well. In logs, the currency depreciation rate is famously equal to the difference between the foreign and domestic SDFs. The result is labeled the asset market view (AMV) of exchange rates. If one does not believe that markets are complete, it is still possible to use a variant of this result to express the domestic SDF in terms of foreign currency and, therefore, use it to price foreign bonds (or other assets) or to evaluate the conditional implications of cross-sectional models of currency excess returns. Lastly, the SDF approach allows one to think about the long-run connection between the SDFs and the depreciation rates. In particular, if the exchange rate is stationary then UIP holds at the long horizon.

In light of the FX macro disconnect, the AMV offers a tantalizing potential to use financial market information to infer exchange rate dynamics. In Section 4, we discuss how one could use bonds to implement the AMV empirically. The key challenge is that models that successfully fit bond yields are far from generating anything resembling the actual exchange rate dynamics. This happens because bonds do not span exchange rates. While one can accommodate this feature in empirical models, it implies that the AMV cannot be implemented, at least not when using conventionally available bond data. This conclusion has important implications for how to go about modeling exchange rates in general equilibrium.

Another important aspect of multi-horizon currency properties is the pattern of UIP

violations. The pattern of coefficients relating the current interest rate differential to the currency risk premium over a given horizon is non-monotonic. It is referred to as a reversal because the correlation between the expected currency depreciation and the interest rate differential changes sign from negative to positive at intermediate horizons and then converges to zero at long horizons. Section 5 presents a bond-modeling framework that can capture this evidence by seriously considering the long-run purchasing power parity (PPP) and stationarity of the real exchange rate (RER). The RER should predict currency risk premiums in addition to the UIP's interest rate differential as a result of the cointegration between the nominal exchange rate (NER) and domestic and foreign price levels associated with its stationarity.¹ Furthermore, information in international bonds implies that risk premiums co-move with the RER.

Monetary policy has important implications for bond prices. Given the close connection between bonds and exchange rates, it is natural to wonder how monetary policy affects the latter. In Section 6, we review emerging attempts to connect the two. A natural starting point would be to complement an endowment economy with the Taylor rule. While such a framework can make useful progress in capturing UIP and EH violations, it runs into problems when capturing UIP departures at multiple horizons. Furthermore, it cannot accommodate unconventional monetary policy such as quantitative easing (QE). An intermediary-based approach appears to be more promising as it separates the properties of the marginal rate of substitution (of the intermediary) from those of aggregate consumption.

Most FX research focuses on the G10 currencies because of the high-quality data

¹Even if the RER is not stationary, a close connection between the NER and the domestic and foreign price levels still exists, leading to similar conclusions at the cost of additional modeling effort.

readily available for them. The natural question is whether the lessons from such studies carry over to emerging-economy currencies. In Section 7, we review research on UIP violations and develop novel evidence regarding the multi-horizon UIP. The evidence is intriguing, as the UIP coefficient has a different sign and the multi-horizon pattern is monotonic. We discuss possible explanations of these phenomena with a particular focus on the role of sovereign credit risk, which is basically nonexistent in the G10 world.

There are several reviews of exchange rates, currency risk premiums, and international bond risk premiums: see the collection of chapters in [James, Marsh, and Sarno \(2012\)](#) (in particular, the chapter by [Lustig and Verdelhan, 2012](#)) and [Engel \(2014\)](#) on exchange rates and currency risk premiums; and [Dahlquist and Hasseltoft \(2016\)](#) on international bond risk premiums. In this survey, we emphasize the relationship between currency and bond risk premiums across horizons. Note that multi-horizon effects are not limited to exchange rates only. [Binsbergen and Kojien \(2017\)](#) review multi-horizon patterns in other asset classes.

Appendix A contains a description of the data used in the survey to illustrate our research questions. In particular, it lists the economies corresponding to the G10 currencies and the emerging economies considered here.

2 Connection between exchange rates and bonds via the present-value approach

We start by applying the classic present-value approach to characterize how bonds of different maturities relate to exchange rates.

2.1 Present value of exchange rates

We denote the NER expressed as domestic per foreign currency by S_t . We assume that the US dollar (USD) is the domestic currency. The rate on a one-period forward currency contract (when buying one unit of the foreign currency) is F_t . We let $s_t = \log(S_t)$ and $f_t = \log(F_t)$. US and foreign nominal interest rates in logs between dates t and $t + 1$ are i_t and i_t^* , respectively.

A currency excess return can be achieved by trading either in the bond market or in the forward exchange rate market. The currency excess return in the bond market is:

$$rx_{t+1} = s_{t+1} - s_t + i_t^* - i_t, \tag{1}$$

which consists of the depreciation of the USD, $s_{t+1} - s_t$, and the interest rate differential, $i_t^* - i_t$. The currency excess return in the forward exchange rate market is:

$$rx_{t+1} = s_{t+1} - f_t. \tag{2}$$

If covered interest parity (CIP) holds (i.e., if $f_t - s_t = i_t - i_t^*$), the currency excess returns in (1) and (2) coincide.

We refer to rx_{t+1} as a currency excess return and to its conditional expectation, $E_t(rx_{t+1})$, as the currency risk premium (under rational expectations). Note that a correct log currency risk premium differs from $E_t(rx_{t+1})$ by a “convexity adjustment” due to Jensen’s inequality.

The log RER is:

$$q_t = s_t + p_t^* - p_t, \quad (3)$$

where p_t and p_t^* are the US and foreign log price levels, respectively. Let $\pi_{t+1} = p_{t+1} - p_t$ and $\pi_{t+1}^* = p_{t+1}^* - p_t^*$ be the US and foreign inflation rates, respectively.

As a result we can rewrite the currency excess return in (1) in terms of the RER:

$$q_t = (i_t^* - i_t) - (\pi_{t+1}^* - \pi_{t+1}) - rx_{t+1} + q_{t+1}. \quad (4)$$

Iterating this equation forward we obtain:

$$q_t = \sum_{j=1}^n (i_{t+j-1}^* - i_{t+j-1}) - \sum_{j=1}^n (\pi_{t+j}^* - \pi_{t+j}) - \sum_{j=1}^n rx_{t+j} + q_{t+n}. \quad (5)$$

Take the limit in n and apply conditional expectations to obtain the present value of the RER similar to [Campbell and Clarida \(1987\)](#):

$$q_t = \sum_{j=1}^{\infty} E_t(i_{t+j-1}^* - i_{t+j-1}) - \sum_{j=1}^{\infty} E_t(\pi_{t+j}^* - \pi_{t+j}) - \sum_{j=1}^{\infty} E_t(rx_{t+j}) + \omega_t^q, \quad (6)$$

where $\omega_t^q = \lim_{n \rightarrow \infty} E_t(q_{t+n})$. Following [Campbell and Clarida \(1987\)](#), we assume that this limit exists and refer to it as the expected long-run exchange rate. Note that ω_t^q is a martingale, as $E_t \omega_{t+1}^q = \omega_t^q$ by definition. Expression (6) says that the RER adjusted for its expected long-run level equals the sum of expected future real interest rate differentials (through nominal interest rate differentials and inflation differentials) and the sum of expected future currency excess returns (i.e., currency risk premiums). These sums are stationary variables, so the difference $q_t - \omega_t^q$ is

stationary even if the RER itself is not.

A similar expression can be used to obtain the present value of the NER:

$$s_t = \sum_{j=1}^{\infty} E_t(i_{t+j-1}^* - i_{t+j-1}) - \sum_{j=1}^{\infty} E_t(rx_{t+j}) + \omega_t^s, \quad (7)$$

where $\omega_t^s = \lim_{j \rightarrow \infty} E_t(s_{t+j})$.

Linking to present-value models of stocks (e.g., the classic version of [Campbell and Shiller, 1988](#)), we also refer to the interest rate differentials as cash flows and the expected excess returns as discount rates. The decomposition into expected future cash flows and discount rates may be helpful for understanding movements in exchange rates.

The accounting identities for the RER and NER can be filled with content by specifying the variables capturing the economic state and assuming their dynamics. However, as these relationships are identities, one cannot use them to test whether the chosen variables deliver correct conditional expectations. In general, the model-based right-hand side of these relationships delivers an exchange rate that is different from the observable one. That is because market prices reflect more information than is in an economist's information set. To avoid this pitfall, one must use the left-hand-side variable (i.e., the exchange rate) as one of the variables capturing the economic state.

Then, conditional on these choices, one can attribute variation in the exchange rate to the different components of these identities. In turn, these attributions could serve as helpful quantitative benchmarks for equilibrium models. For example, [Campbell and Clarida \(1987\)](#) fill the RER present value with empirical content by assuming a state space representation of the RER, real interest rate differential, and expected

long-run RER. Estimating the model allows them to attribute variation in the RER to innovations in the interest rate differential and in the long-run RER. They find that changes in the latter are not helpful in accounting for changes in the RER. See [Froot and Ramadorai \(2005\)](#), [Engel and West \(2005\)](#), and [Engel, Mark, and West \(2008\)](#) for other analyses and applications of the present-value approach.

If one is willing to assume that PPP holds in the long run (i.e., that the RER is stationary, also referred to as weak-form PPP), then ω_t^q is constant. Similarly, if the NER is stationary, then ω_t^s is constant. It is common in the literature to assume that the RER is stationary while the NER is not. However, there are notable exceptions: for example, [Itskhoki \(2021\)](#) argues for the non-stationarity of the RER and [Lustig, Stathopoulos, and Verdelhan \(2019\)](#) treat the NER as stationary. These disagreements prompt us to consider “finite horizon” versions of these present-value models whenever the long-term behavior of exchange rates is not relevant. Specifically, Equation (5) implies for the NER that:

$$s_t = \sum_{j=1}^n E_t(i_{t+j-1}^* - i_{t+j-1}) - \sum_{j=1}^n E_t(rx_{t+j}) + E_t(s_{t+n}). \quad (8)$$

In particular, it implies that the n -period expected depreciation rate is:

$$E_t(s_{t+n} - s_t) = - \sum_{j=1}^n E_t(i_{t+j-1}^* - i_{t+j-1}) + \sum_{j=1}^n E_t(rx_{t+j}). \quad (9)$$

This expression is consistent with the observation of [Greenwood, Hanson, Stein, and Sunderam \(2022\)](#) that “foreign exchange is conceptually similar to long-term bonds in that both are ‘interest-rate sensitive’ assets: they are heavily exposed to news about future short-term bonds.”

A special case of this expression has been of great interest in the literature. Setting $n = 1$ and assuming that the currency risk premium, $E_t(rx_{t+1})$, is constant, one obtains that the excess return, $rx_{t+1} = s_{t+1} - s_t + i_t^* - i_t$, in (1) is unpredictable. This is evaluated in the two predictability regressions of [Bilson \(1981\)](#), [Fama \(1984\)](#), and [Tryon \(1979\)](#):

$$s_{t+1} - s_t = \alpha + \beta(i_t - i_t^*) + \varepsilon_{t+1}, \quad (10)$$

and

$$rx_{t+1} = \alpha + (\beta - 1)(i_t - i_t^*) + \varepsilon_{t+1}. \quad (11)$$

Under the random walk hypothesis (RWH), depreciation rates are unpredictable and $\beta = 0$. Under UIP and the null hypothesis of constant currency risk premium, $\beta = 1$. When $\beta \neq 1$, the currency risk premium is time varying (it is linear in and perfectly correlated with the interest rate differential). According to Fama's estimates, $\beta \approx -1$.

We revisit the [Fama \(1984\)](#) evidence by extending the sample to more currencies and a longer period. The main purpose of the analysis is to investigate how the Fama estimates have changed over time. To that end, we re-estimate β every month in a panel of G10 currencies using an expanding window. [Figure 1](#) presents the pooled regression estimates allowing for currency fixed effects. The first estimate for the 1976–1985 sample period (roughly corresponding to the sample period studied by Fama) is around -1.5 . The estimate increases over time and is around -0.5 for the full 1976–2022 sample period. Statistically, β is indistinguishable from zero, which is consistent with the RWH evidence of [Meese and Rogoff \(1983\)](#). Despite

the increase, the β estimate is still far from the UIP-implied value of one. Hence, the UIP regression (11) suggests that the currency risk premium is related to the nominal interest rate differential and is thereby time varying. Relatedly, [Campbell and Clarida \(1987\)](#) assume that the currency risk premium is linear in the real interest rate differential and use such a specification in their analysis of the RER.

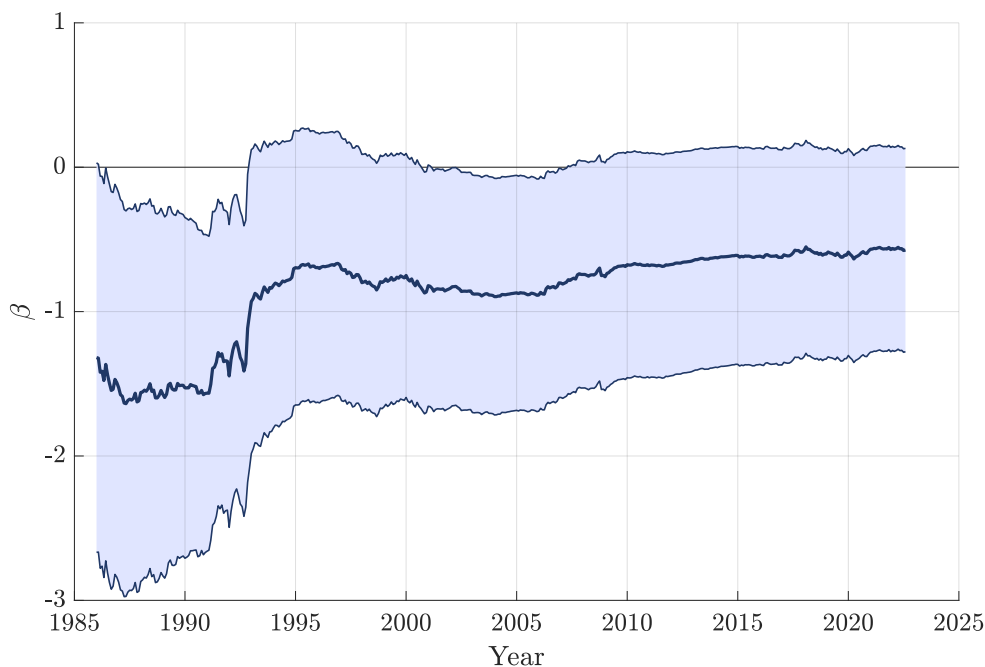


Figure 1: UIP β estimates over time

The figure shows β estimates from pooled expanding-window regressions for G10 currencies for the sample period January 1976 to July 2022. The pooled regressions include currency fixed effects. Appendix A describes the data used and lists the economies corresponding to the G10 currencies. The solid line (in blue) shows the point estimates and the shaded area (in light blue) shows the 95% confidence bands based on standard errors allowing for cross-sectional correlation in the errors as well as for heteroskedasticity in the errors. The first regression uses data from January 1976 to December 1985. The remaining regressions expand the sample period by one month at a time.

Assume that PPP holds in the long run (i.e., the RER is stationary), that the currency risk premium is linear in the nominal interest rate differential, which follows

an AR(1) process with mean μ_i and persistence coefficient ρ_i , and that the inflation differential is i.i.d. Then Equations (6) and (11) imply a simple present-value expression for the RER:

$$q_t = \beta \frac{\mu_i - (i_t - i_t^*)}{1 - \rho_i} + \omega^q, \quad (12)$$

where ω^q is the (constant) expected long-run exchange rate. See [Dahlquist and Pénasse \(2022\)](#) for a derivation.

This expression implies that movements in the RER are entirely due to movements in the interest rate differential. The β coefficient controls the sign of the correlation between the RER and the interest rate differential. When $\beta = 1$ and UIP holds, the currency risk premium (discount rate) is constant and the RER moves only because of cash flows; see Equation (6). In this case, the foreign currency is expensive when the interest rate is higher in the foreign country than in the domestic country. When $\beta \neq 1$, the interest rate differential predicts future currency excess returns and the RER also moves because of discount rates. Specifically, when $\beta < 0$, the discount rate effect dominates so that the foreign currency appears weak when its relative interest rate is high.

Empirically, the correlation between the RER and the interest rate differential is weak and typically sends conflicting signals: an increase in the interest rate differential today predicts a higher future excess return, but that tends to come with a higher current exchange rate, which predicts a lower future excess return. Thus, the relationship between exchange rates and interest rate differentials appears to be puzzling (see [Frankel and Rose, 1995](#), for a review). [Engel \(2016\)](#) notes that this tension relates to the challenge to the multiple horizon results we consider in Section 5. We will then consider a richer specification of the driving forces of the RER; in

particular, currency excess return predictions should include not only the interest rate differential but also the RER itself.

2.2 Exchange rates and bond yields

We can refine the observation of [Greenwood, Hanson, Stein, and Sunderam \(2022\)](#) about the similarity between exchange rates and long-term bonds by explicitly linking expectations about future short-term bonds to long-term bonds. We denote the price of an n -period domestic nominal zero-coupon bond by $B_t^{(n)}$. We let $b_t^{(n)} = \log(B_t^{(n)})$. The bond's log yield is $y_t^{(n)}$ with $y_t^{(1)} = i_t$. The bond excess return is:

$$rx_{t+1}^{(n)} = b_{t+1}^{(n-1)} - b_t^{(n)} + b_t^{(1)} = -(n-1)y_{t+1}^{(n-1)} + n y_t^{(n)} - y_t^{(1)}. \quad (13)$$

We refer to $E_t(rx_{t+1}^{(n)})$ as the bond risk premium. Again, it differs from a correct log bond risk premium by a “convexity adjustment” due to Jensen’s inequality. Formulations of the EH restrict bond risk premiums to being zero or at least constant over time. As such, the EH leaves no room for time-varying bond risk premiums.

Then, the long-term bond yield can be decomposed into expectations of short-term interest rates and bond risk premium components:

$$n y_t^{(n)} = \sum_{j=0}^{n-1} E_t(i_{t+j}) + \sum_{j=0}^{n-2} E_t(rx_{t+j+1}^{(n-j)}).$$

The first term reflects the EH; the second term is referred to as the term premium. Foreign bond yields are denoted by $y_t^{*(n)}$ with $y_t^{*(1)} = i_t^*$. A similar decomposition can be derived for foreign bonds.

The empirical literature on US bond risk premiums has centered on different variations of the EH (see, e.g., [Fama and Bliss, 1987](#); [Campbell and Shiller, 1991](#); [Cochrane and Piazzesi, 2005](#); [Backus, Foresi, Mozumdar, and Wu, 2001](#)). It has examined whether bond risk premiums indeed remain constant over time. The key result is that variables such as the level and slope of the yield curve or predictability factors summarizing the information in the yield curve predict bond excess returns. That is, there is evidence of time-varying bond risk premiums.

A large body of work applies an international perspective to bond risk premiums; see, for example, [Ilmanen \(1995\)](#), [Kessler and Scherer \(2009\)](#), and [Sekkel \(2011\)](#). [Dahlquist and Hasseltoft \(2013\)](#) find that both local and global factors predict bond excess returns. The global factor is closely linked to US bond risk premiums and to international business cycles, and predicts global economic growth. Moreover, correlations between international bond risk premiums have increased over time, suggesting increased integration between markets.

The bond yield decomposition allows us to link depreciation rates to bond yields. Equation (8) implies that:

$$E_t(s_{t+n} - s_t) = n(y_t^{(n)} - y_t^{*(n)}) + \sum_{j=0}^{n-1} E_t(rx_{t+j+1}) - \sum_{j=0}^{n-2} E_t(rx_{t+j+1}^{(n-j)} - rx_{t+j+1}^{*(n-j)}). \quad (14)$$

This relationship can be used in several different ways. For example, one can turn it around and interpret the n -period interest rate differential as reflecting expectations about future depreciation rates, currency risk premiums, and differences in foreign and domestic bond risk premiums. Such a perspective highlights that the bond differential does not depend on whether or not the expected long-run exchange rate,

ω_t^s , from Equation (7) is constant. Regardless of the perspective, the important conclusion is that there is a clear link between international currency and bond markets.

If both UIP and EH hold, then the currency and bond risk premiums are constant. [Bekaert, Wei, and Xing \(2007\)](#) test whether UIP and EH hold jointly in the data. If so, Equation (14) implies that the n -period excess return, $s_{t+n} - s_t + n(y_t^{*(n)} - y_t^{(n)})$, is unpredictable. This is a version of the multi-horizon UIP studied by [Alexius \(2001\)](#) and [Chinn and Meredith \(2004\)](#), who find support for no predictability at horizons of 60 and 120 months. The evidence does not necessarily imply that single-horizon UIP and EH hold, as the currency and bond risk premiums in Equation (14) could offset each other. When studying multi-horizon currency expectations, one must therefore understand the role of bonds and their risk premiums. This effort has the potential to impose additional restrictions on theories of currency risk premiums, which are usually motivated by single-horizon evidence.

2.3 Key takeaways

The present-value relationship is a powerful tool in characterizing the current exchange rate in terms of the multi-horizon expectations about future interest rates and currency and bond risk premiums. This relationship can be translated into the connection between expected multi-horizon currency depreciation rates, bond yields, and risk premiums.

3 Connection between exchange rates and bonds via SDFs

Relationship (14) connects expected depreciation rates and yield differentials, making good progress in emphasizing that both currency and bond risk premiums play a role. The relationship also articulates how different horizons come into the picture. What the relationship still does not explain is how the two types of risk compensation, i.e., those of currency and bond markets, relate to each other.

To make progress in that dimension, we engage another classical view that naturally connects international bonds to currencies: [Brandt, Cochrane, and Santa-Clara \(2006\)](#) coined it the asset market view (AMV) of exchange rates.

3.1 Exchange rates and SDFs

[Backus, Foresi, and Telmer \(2001\)](#) pioneered the AMV framework, since used in many papers. Suppose M_{t+1} is a one-period nominal SDF and R_{t+1} is the one-period nominal gross return on an asset, which could be foreign, both expressed in USD. Then, the n -period return on this asset satisfies the Euler equation (also referred to as the fundamental theorem of asset pricing):

$$1 = E_t(M_{t,t+n}R_{t,t+n}), \tag{15}$$

where $M_{t,t+n} = \prod_{j=1}^n M_{t+j}$ is the n -period SDF and $R_{t,t+n} = \prod_{j=1}^n R_{t+j}$ is the n -period gross return. One can change the numeraire and express this return in terms of the foreign currency: $R_{t,t+n} \cdot S_t/S_{t+n} \equiv R_{t,t+n}^*$. This foreign-currency-denominated

return satisfies (15) when the SDF is expressed in terms of the new numeraire as well:

$$1 = E_t(M_{t,t+n}^* R_{t,t+n}^*) = E_t(M_{t,t+n}^* \cdot S_t/S_{t+n} \cdot R_{t,t+n}). \quad (16)$$

Comparing expressions (15) and (16), it is tempting to conclude that:

$$\frac{S_{t+n}}{S_t} = \frac{M_{t,t+n}^*}{M_{t,t+n}}. \quad (17)$$

Indeed, when financial markets are complete, that is, there is a full set of Arrow–Debreu securities and both domestic- and foreign-currency investors can trade them, there is a unique SDF satisfying the fundamental theorem of asset pricing under each numeraire and Equation (17) holds. This is attractive, as one can infer properties of the currency depreciation rate from the properties of the SDFs. In this sense, the expression delivers the AMV.

However, when markets are not complete, there is an infinite number of SDFs satisfying the fundamental theorem of asset pricing under each numeraire. Thus, presumably, one can find a pair M, M^* for which Equation (17) holds. However, one must know the depreciation rate to be able to check whether a given SDF pair does satisfy the expression. Thus, the attractive AMV concept is no longer empirically operational: one cannot use it to identify the depreciation rate. [Chernov, Haddad, and Itskhoki \(2023\)](#) propose a general characterization of the relation between the currency depreciation rate and the corresponding SDFs when markets are incomplete.

In the following, we discuss how the Euler equation (15) can be used to evaluate,

using multiple horizons, whether a candidate model of the SDF prices currency risks correctly. Next, we discuss whether bond prices are connected to exchange rates via the AMV. We conclude by characterizing how the long-run properties of exchange rates are connected to those of SDFs.

3.2 Multiple horizons

The term structure of Euler equations or, equivalently, multi-horizon effects, tell us about conditional currency risk premiums, which is useful knowledge even if one is only interested in a single horizon. Specifically, [Chernov, Lochstoer, and Lundebj \(2022\)](#) demonstrate the connection between unconditional and conditional pricing via multiple horizons. [Backus, Boyarchenko, and Chernov \(2018\)](#) and [Backus, Chernov, and Zin \(2014\)](#) use the concept of entropy to make similar points.

Consider a level version of the currency excess return in [\(2\)](#):

$$R_{t+1}^e = (S_{t+1} - F_t)/F_t. \tag{18}$$

This definition implies that the amount of foreign currency bought is one “forward” USD. A return would be $R_{t+1} = R_{t+1}^e + \exp(i_t)$. Next, we consider the conditional and unconditional implications of the Euler equation [\(15\)](#) at the single- and n -period horizons.

First, suppose that $E_t(M_{t+1}R_{t+1}) = 1$, then, by the law of iterated expectations, $E(M_{t,t+n}R_{t,t+n}) = 1$ for any n . This means that a one-period conditional valuation conveys information about a collection of multi-period unconditional valuations. Second, the reverse of this result does not hold: one cannot establish correct conditional

valuations using unconditional values across many horizons. Still, multiple horizons do convey useful conditional information. If $E(M_{t,t+n}R_{t,t+n}) = 1$ for any n , then the conditional pricing error, $E_t(M_{t+1}R_{t+1}) - 1$, is zero mean and uncorrelated with the lagged errors, $M_{t-n,t}R_{t-n,t} - 1$, for any n . A collection of multi-period unconditional valuations conveys some information about conditional valuations. In particular, past conditional pricing errors $E_{t-n-1}(M_{t-n,t}R_{t-n,t}) - 1$ are not correlated with the current pricing errors $E_t(M_{t+1}R_{t+1}) - 1$.

It is instructive to see how that works in the case of $n = 2$. The conditional pricing errors, $E_t(M_{t+1}R_{t+1}) - 1$, have zero mean as $E(M_{t+1}R_{t+1}) - 1 = 0$. Next, consider two-period returns:

$$\begin{aligned} 1 &= E(M_{t-1,t+1} \cdot R_{t-1,t+1}) = E(M_t R_t \cdot E_t(M_{t+1} R_{t+1})) \\ &= E(M_t R_t) \cdot E(M_{t+1} R_{t+1}) + Cov(M_t R_t, E_t(M_{t+1} R_{t+1})). \end{aligned}$$

If the fundamental theorem of asset pricing holds at both one- and two-period horizons, then

$$Cov(M_t R_t - 1, E_t(M_{t+1} R_{t+1}) - 1) = 0.$$

This equation and its extension to any horizon tell us that conditional pricing errors are uncorrelated with errors for any horizon n . Thus, evaluating whether

$$E(M_{t,t+n}R_{t,t+n} - 1) = 0$$

for any horizon n is tantamount to evaluating the conditional properties of a candidate SDF and of conditional currency premiums, in particular. [Chernov](#), [Dahlquist](#),

and [Lochstoer \(2023\)](#) apply this idea to evaluating currency pricing models. In particular, they demonstrate that some models that pass a single-horizon unconditional pricing test are rejected when evaluated through the lens of multiple horizons.

3.3 AMV and bonds

The connection between depreciation rates and SDFs is attractive in light of a weak connection between movements in exchange rates and macroeconomic variables. This phenomenon is known as the exchange-rate disconnect puzzle ([Obstfeld and Rogoff, 2001](#)), or the FX macro disconnect. Given that it is a routine matter in asset pricing to extract SDFs from asset prices, the AMV offers a path towards connecting exchange rates to financial markets.

Indeed, a large literature assumes that the AMV equation, i.e., Equation (17), is valid and infers SDFs from bond prices, that is, it estimates M and M^* to match returns on domestic and foreign bonds (e.g., [Ahn, 2004](#); [Backus, Foresi, and Telmer, 2001](#); [Brennan and Xia, 2006](#); [Jotikasthira, Le, and Lundblad, 2015](#); [Kaminska, Meldrum, and Smith, 2013](#); [Sarno, Schneider, and Wagner, 2012](#)). Obviously, there is no reason to limit oneself to bonds only, but that is a first natural step. It also strengthens the present-value relationship between currencies and bonds.

Indeed, consider the valuation of a foreign bond:

$$B_t^{*(n)} = E_t(M_{t,t+n}^*) = E_t(M_{t,t+n} \cdot S_{t+n}/S_t). \quad (19)$$

Note that these expressions do not require assumptions about market completeness.

Continuing, under conditional log-normality, we get:

$$\begin{aligned} b_t^{*(n)} &= \log E_t \left(e^{\sum_{i=1}^n m_{t+i} + \Delta s_{t+i}} \right) \\ &= b_t^{(n)} + E_t \left(\sum_{i=1}^n \Delta s_{t+i} \right) + \frac{1}{2} \text{var}_t \left(\sum_{i=1}^n \Delta s_{t+i} \right) + \text{cov}_t \left(\sum_{i=1}^n m_{t+i}, \sum_{i=1}^n \Delta s_{t+i} \right), \end{aligned}$$

where $m_t = \log(M_t)$ and $\Delta s_{t+i} \equiv s_{t+i} - s_{t+i-1}$. Equivalently, we get the yield differential:

$$n(y_t^{(n)} - y_t^{*(n)}) = E_t(s_{t+n} - s_t) - \text{srp}_t^{(n)} + \text{vs}_t^{(n)}, \quad (20)$$

where $\text{srp}_t^{(n)} \equiv -\text{cov}_t(\sum_{i=1}^n m_{t+i}, \sum_{i=1}^n \Delta s_{t+i})$ is the multi-horizon version of the currency risk premium, $\text{srp}_t^{(1)} = E_t(rx_{t+1})$. As before, this notion of risk premium does not account for the convexity term $\text{vs}_t^{(n)} \equiv \frac{1}{2} \text{var}_t(\sum_{i=1}^n \Delta s_{t+i})$.

Comparing expressions (20) and (14), we see the connection of international bond risk premiums to currency risk premiums. As discussed above, the nature of the expected long-run exchange rate ω_t^s is irrelevant for bond prices. The explicit connection to the SDF emphasizes that this conclusion would be true even if m_t were subjected to the same shock as ω_t^s .

We can make the connection even more explicit by substituting Equation (20) into Equation (13) and its foreign counterpart, and by taking expectations:

$$E_t(rx_{t+1}^{(n)} - rx_{t+1}^{*(n)}) = E_t(\text{srp}_{t+1}^{(n-1)}) - \text{srp}_t^{(n)} + \text{srp}_t^{(1)} + \text{convexity}. \quad (21)$$

See [Chernov and Creal \(2023\)](#) for details. Therefore, ignoring convexity, differences in currency risk premiums across different horizons capture cross-country differences

in bond risk premiums.

Thus, we see that the cross-country difference in bond risk premiums are closely linked to cross-horizon differences in currency premiums. Indeed, [Gourinchas, Ray, and Vayanos \(2022\)](#) observe that as arbitrageurs operate in both domestic and foreign bond markets and in currency markets, bond and currency risk premiums are linked in equilibrium.

3.4 Long-run properties

The long-run properties of an SDF are connected to the properties of consol bonds. By extension, the long-run properties of domestic and foreign SDFs shed light on the long-run properties of depreciation rates.

Consider the [Alvarez and Jermann \(2005\)](#) and [Hansen and Scheinkman \(2009\)](#) SDF decomposition:

$$\begin{aligned} M_{t+1} &= \widehat{M}_{t+1} \cdot \widetilde{M}_{t+1}, \\ \widetilde{M}_{t+1} &= \gamma v_t / v_{t+1}, \end{aligned}$$

where γ is the largest eigenvalue of M_{t+1} and v_t is the corresponding eigenfunction, a stationary variable.² These components are commonly referred to as permanent and transitory, respectively. As $E_t(\widehat{M}_{t+1}) = 1$, [Hansen and Scheinkman \(2009\)](#) refer to \widehat{M}_{t+1} as a martingale component. [Bansal and Lehman \(1997\)](#) and [Hansen \(2012\)](#) argue that the martingale components of many equilibrium SDFs are the same as those of the power utility with constant relative risk aversion. \widetilde{M}_{t+1} equals the inverse

²That is, the pair γ, v_t is the solution to $E_t(M_{t+1}v_{t+1}) = \gamma v_t$ with the largest value of γ .

of the return on a consol bond. As a result, the long-run mean of the (log) consol bond return equals $-\log \gamma$. The yield of a consol bond is also equal to $-\log \gamma$.

Building on [Alvarez and Jermann \(2005\)](#), researchers like to discuss the size of the martingale component (i.e., how much it contributes to the SDF). The overall view is that this component is large and is used as a restriction that equilibrium models should match. In general, this interpretation is tenuous because the two components covary, although there are interesting special cases.

One example is when $\widehat{M}_{t+1} = 1$ and M_{t+1} then equals the inverse of the return on a consol bond. Thus, the consol bond is the asset with the highest possible return. Another example is when \widehat{M}_{t+1} can be further multiplicatively decomposed into $\widehat{M}_{t+1}^{[1]}$ and $\widehat{M}_{t+1}^{[2]}$, where $\widehat{M}_{t+1}^{[1]}$ is independent of \widetilde{M}_{t+1} . Because of this property, we refer to $\widehat{M}_{t+1}^{[1]}$ as a purely martingale component, having no impact on bond prices:

$$B_t^{(1)} = E_t(M_{t+1}) = E_t(\widehat{M}_{t+1}^{[1]}) \cdot E_t(\widehat{M}_{t+1}^{[2]} \cdot \widetilde{M}_{t+1}) = E_t(\widehat{M}_{t+1}^{[2]} \cdot \widetilde{M}_{t+1}).$$

While the first example appears to be unrealistic (see [Ross, 2015](#), and subsequent literature), the second example is consistent with many standard models. For example, an i.i.d. shock to consumption growth has a permanent effect on the marginal utility of the representative agent and does not affect bond prices (e.g., [Alvarez and Jermann, 2005](#); [Mehra and Prescott, 1985](#)). Alternatively, this could be the shock driving the expected long-run exchange rate, ω_t^s , as discussed around Equations (14) and (20).

Consider the same decomposition for the SDF denominated in the foreign currency. How are the components under the different numeraires related to each other? First,

one can obtain the same type of decomposition of the depreciation rate:

$$S_{t+1}/S_t = \widehat{S_{t+1}/S_t} \cdot \delta w_t/w_{t+1}.$$

Therefore, the long-run log one-period depreciation rate equals $\log \delta$. This constant reflects the trend in the level (i.e., the exchange rate itself). Second,

$$\gamma^* \widehat{M_{t+1}^*} v_t^*/v_{t+1}^* = M_{t+1}^* = M_{t+1} \cdot S_{t+1}/S_t = \gamma \delta \widehat{M_{t+1}^*} \widehat{S_{t+1}/S_t} \cdot (v_t w_t)/(v_{t+1} w_{t+1}). \quad (22)$$

This expression holds either under the AMV or if M_{t+1}^* is interpreted as simply the USD SDF M_{t+1} expressed in terms of the foreign currency. If and only if the NER is trend stationary, does $\widehat{S_{t+1}/S_t} = 1$ and $\widehat{M_{t+1}^*} = \widehat{M_{t+1}}$. Long-run consol bond yields are related via

$$y^{*(\infty)} = -\log \gamma^* = -\log \gamma - \log \delta = y^{(\infty)} - \lim_{n \rightarrow \infty} E(s_{t+n} - s_t)/n.$$

Therefore, long-run UIP holds, although all the objects involved are constant (Backus, Boyarchenko, and Chernov, 2018; Lustig, Stathopoulos, and Verdelhan, 2019).

Lustig, Stathopoulos, and Verdelhan (2019) study the log excess return on a strategy that borrows via a US n -period bond, converts the amount into foreign currency, invests in a foreign n -period bond, and then unwinds the position in one period. Using our notation, this would be:

$$\widetilde{r}_{t+1}^{(n)} \equiv \Delta s_{t+1} + (b_{t+1}^{*(n-1)} - b_t^{*(n)}) - (b_{t+1}^{(n-1)} - b_t^{(n)}).$$

So, their object of interest contains elements of both one-period depreciation rates

and one-period returns of n -period bonds.

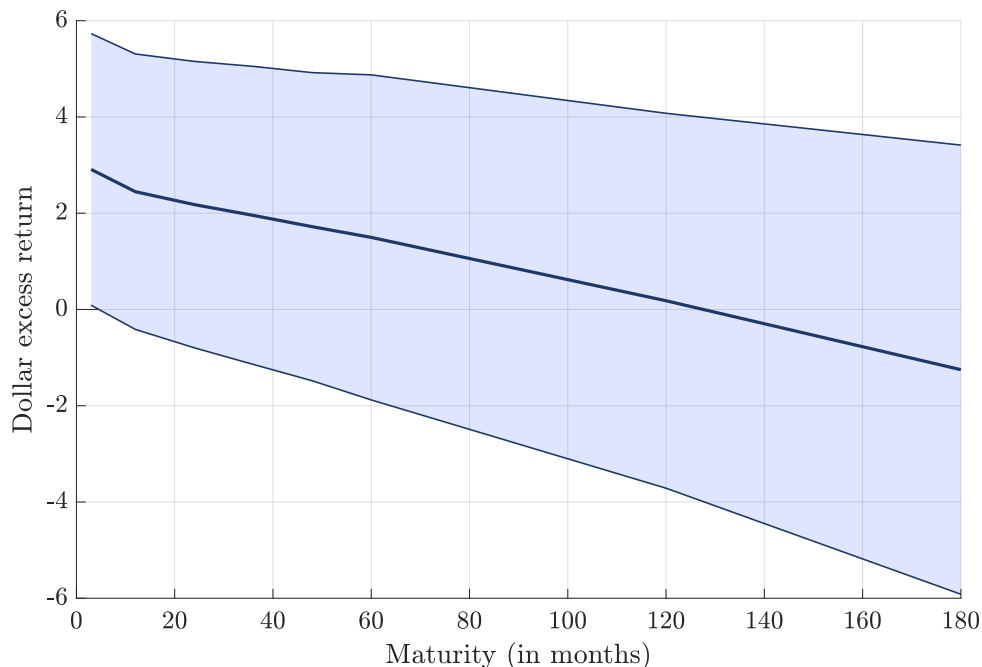


Figure 2: Slope carry returns

The figure shows the annualized mean dollar excess returns (expressed in %) with 95% confidence bands as a function of the bond maturity. Dollar excess returns correspond to the holding period returns, expressed in US dollars, of investment strategies that go long and short foreign bonds of different countries during the April 1985–July 2022 sample period.

Figure 2 updates the analysis of [Lustig, Stathopoulos, and Verdelhan \(2019\)](#) with more recent data. Consistent with their findings, we conclude that at long maturities, the average excess return on this “slope carry” strategy is negative, but not significantly different from zero. The long-horizon results then imply that:

$$E(\widetilde{r\bar{x}}_{t+1}^{(\infty)}) = \log \delta - \log \gamma^* + \log \gamma + E(\widehat{\Delta s}_{t+1}).$$

Note that $E(\widetilde{r\bar{x}}_{t+1}^{(\infty)}) = 0$ if and only if the NER is trend stationary (i.e., ω_t^s is

constant), as per our previous discussion of its long-run properties. Thus, [Lustig, Stathopoulos, and Verdelhan \(2019\)](#) argue on the basis of [Figure 2](#) that the NER must be stationary.

3.5 Key takeaways

The complete-market setting ties exchange depreciation rates to SDFs. The term structure of bond yields has a natural connection to the SDFs as well. Together, these observations elucidate the connection between expectations about future depreciation rates, bond yields, and risk premiums both at intermediate horizons and in the long run.

4 AMV and the evidence

AMV is frequently implemented in the context of affine models. The functional form of these models matters mostly for analytical tractability and empirical versatility, not for the conceptual issues explored here.

It is well known that affine models are exceptionally successful in capturing bond yield behavior. The challenge in the AMV literature is to match observed exchange rate dynamics with those implied by the two SDFs estimated using bond data. The main conclusion from this literature is that the depreciation rate inferred from the SDF ratio falls short of capturing the variation in depreciation rates and the UIP violations.

4.1 A typical implementation

Start with a state vector:

$$x_{t+1} = \mu_x + \Phi_x x_t + \Sigma_x \varepsilon_{t+1}.$$

To be concrete, let x_t be two-dimensional, consisting of a domestic and a foreign interest rate (i.e., $x_{1t} = i_t$ and $x_{2t} = i_t^*$). Next, we specify domestic and foreign SDFs:

$$-m_{t+1} = i_t + \frac{1}{2} \lambda_t^\top \lambda_t + \lambda_t \varepsilon_{t+1}, \quad \lambda_t = \lambda_0 + \lambda_1 x_t; \quad (23)$$

$$-m_{t+1}^* = i_t^* + \frac{1}{2} \lambda_t^{*\top} \lambda_t^* + \lambda_t^* \varepsilon_{t+1}, \quad \lambda_t^* = \lambda_0^* + \lambda_1^* x_t. \quad (24)$$

The conditional volatility of the SDF, i.e., λ_t or λ_t^* , is the market price of risk. The corresponding convexity terms ensure that $i_t = -\log E_t(\exp(m_{t+1}))$ and $i_t^* = -\log E_t(\exp(m_{t+1}^*))$.

Assuming complete financial markets, the inferred depreciation rate is:

$$\Delta s_{t+1} = m_{t+1}^* - m_{t+1} = i_t - i_t^* + \frac{1}{2} (\lambda_t^\top \lambda_t - \lambda_t^{*\top} \lambda_t^*) + (\lambda_t - \lambda_t^*) \varepsilon_{t+1}. \quad (25)$$

See, for example, [Brennan and Xia \(2006\)](#) and [Sarno, Schneider, and Wagner \(2012\)](#) for similar setups. The depreciation rate is driven by the same shocks as are yields, so, if the AMV were to hold, one could use bond data to infer the dynamics of the depreciation rate via Equation (25). However, in practice, it does not work. As [Chernov and Creal \(2023\)](#) show, such an approach implies that the dynamics of depreciation rates differ wildly from the observed ones. Figure 3 illustrates this for

the case of the GBP based on data from [Chernov and Creal \(2023\)](#).

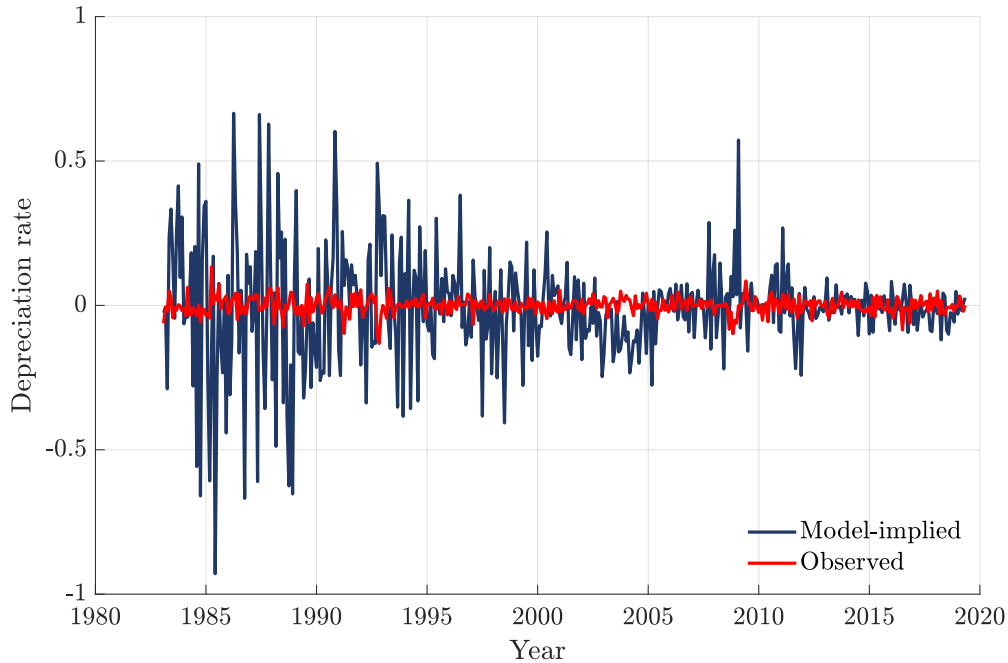


Figure 3: Model-implied and observed depreciation rates

The figure shows the depreciation rate implied by a model assuming that exchange rates are spanned by bonds (blue) against the observed depreciation rate (red) for the GBP during the January 1983–April 2019 sample period.

Alternatively, [Backus, Foresi, and Telmer \(2001\)](#) set λ_t to match US yields and λ_t^* to match the UIP regression. However, then the implied foreign yields are unrealistic: “The implied yield curve ... is hump shaped with long yields reaching as high as 80 percent per annum.”

4.2 A numeraire-based approach

Instead of working with two SDFs that may or may not deliver the right depreciation rate depending on the market environment, consider expressing the USD SDF in a different numeraire (i.e., currency). Replace Equations (24) and (25) with

$$\Delta s_{t+1} = \mu_s + \rho_s(x_{1t} - x_{2t}) + \Sigma_{sx}\varepsilon_{t+1}, \quad (26)$$

where μ_s and ρ_s could be exactly the α and β from the UIP regression (10). Given this specification, one can price foreign bonds using Equation (19). Note that this approach does not require any assumption about market completeness. One can of course use any other favorite model of the depreciation rate.

The remaining issue is that we still have the same shocks affecting yields and depreciation rates. Chernov and Creal (2023) emphasize that it is a problem. First they show, via a regression, that bonds with maturities of up to ten years cannot span depreciation rates, referring to this evidence as the FX bond disconnect. Chernov, Haddad, and Itskhoki (2023) update this evidence across more countries and more assets.

Second, Chernov and Creal (2023) demonstrate how to incorporate this lack of spanning into an affine model. Replace Equations (23) and (26) with

$$\begin{aligned} -m_{t+1} &= r_t + \frac{1}{2}\lambda_t^\top \lambda_t + \frac{1}{2}\gamma_t^\top \gamma_t + \lambda_t \varepsilon_{t+1} + \gamma_t \nu_{t+1}, & \gamma_t &= \gamma_0 + \gamma_1 x_t; \\ \Delta s_{t+1} &= \mu_s + \rho_s(x_{1t} - x_{2t}) + \Sigma_{sx}\varepsilon_{t+1} + \Sigma_s \nu_{t+1}. \end{aligned} \quad (27)$$

Here we are adding an extra (priced) shock, ν_t , to the depreciation rate. The price of its risk, γ_t , depends on x_t only and therefore only on the innovations affecting the

bond factors. Bond prices are therefore linear functions of x_t and cannot span the depreciation rate. This is consistent with the spanning evidence and also gives the model extra flexibility to match depreciation rates without sacrificing the ability to match bonds.

What does the extra shock represent? Quantitatively, it is important. Depending on the currency, the shock accounts for 89–96% of the variation in the depreciation rate. The maximal Sharpe ratio associated with the market prices of these innovations, $(\gamma_t^\top \gamma_t)^{1/2}$, averages 0.3 on a monthly basis.

Theoretically, the component of the SDF that is related to this shock, $\exp(-\gamma_t^\top \gamma_t/2 - \gamma_t \nu_{t+1})$, is the purely martingale component $\widehat{M}_t^{[1]}$ described in Subsection 3.4 or the expected long-run exchange rate ω_t^s . Comparing this model with a version in which bonds span depreciation rates in Equations (23) and (26), one can see that M_{t+1} and $M_{t+1}^* = M_{t+1} \cdot S_{t+1}/S_t$ do not have the same purely martingale components. Thus, according to Equation (22), the depreciation rate cannot have a constant martingale component. Hence, the evidence of the inability of bonds to span exchange rates is incompatible with a stationary NER.³

Finally, note that this model cannot be estimated using bonds only, as Σ_s and γ_t would not be identified; one also needs depreciation rate data. Thus, the model does not allow one to infer the properties of the exchange rate by observing bonds only, despite being able to capture their joint dynamics. The premise of the AMV fails.

³The bonds of longer maturities, the consol ones in particular, are not reliably available in a broad international dataset. There is a possibility that had consol bonds been available, they would span the depreciation rates. If that were the case, ν_t would have to be zero and one would conclude that NER was stationary.

4.3 Key takeaways

Attempts to implement the AMV empirically by inferring SDFs from bond prices fail. This is because bonds do not span exchange rates. A model that accommodates such a lack of spanning can capture the joint dynamics of bonds and exchange rates, but is incompatible with the AMV approach.

5 Implications for multi-horizon currency risk premiums

5.1 Motivating evidence

The starting point of a multi-horizon analysis is the single-horizon UIP regression (10). Extending the UIP hypothesis to multiple periods can be done in several ways.

One approach is based on the assumption that both UIP and EH hold; see the discussion in Subsection 2.2 around Equation (14). This approach implies that $s_{t+n} - s_t + n(y_t^{*(n)} - y_t^{(n)})$ is unpredictable. Recall from Subsection 3.4 that UIP holds at the infinite horizon if the exchange rate is trend stationary.

Another approach is to start with Equation (9), just as in the single-horizon UIP case, but to apply it to horizons of $n - 1$ and n . The difference between the two equations implies, under constant currency risk premiums, that $\Delta s_{t+n} - (i_{t+n-1} - i_{t+n-1}^*)$ is not predictable using variables in the information set at time t . Bacchetta and van Wincoop (2010) explore such predictability using the interest rate differential $i_t - i_t^*$, finding a striking non-monotonic pattern in the regression coefficient.

Figure 4 shows a similar non-monotonic pattern for empirically fitted currency risk premiums on the interest rate differential. This pattern is important because it is not obvious that existing mechanisms capable of explaining the single-horizon UIP violations would be able to capture the multi-horizon evidence. Indeed, Engel (2016) argues that capturing the evidence is challenging for the existing economic models. Chernov and Creal (2021) provide further arguments along these lines.

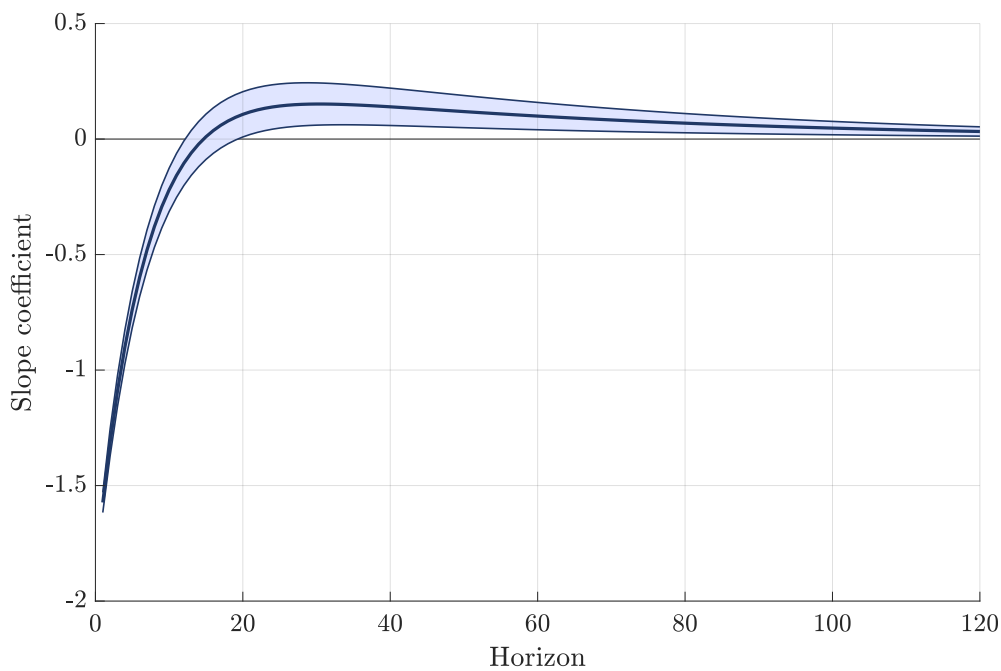


Figure 4: Multi-horizon predictability and interest rate differential

The figure shows the slope coefficients with 95% confidence bands from panel regressions of G10 currency risk premiums fitted on their nominal interest rate differentials at different horizons for the January 1976–July 2022 sample period. The pooled regressions include currency fixed effects. The one-month risk premiums are constructed by regressing the monthly currency excess return on the lagged currency excess return, lagged interest rate differential, and lagged real exchange rates. The risk premiums beyond one month are obtained by taking the power of a VAR model of the regression variables.

5.2 Modeling framework

Engel (2016), Chernov and Creal (2021), and Dahlquist and Pénasse (2022) develop a vector error correction model (VECM) framework based on weak-form PPP that implies that the RER must be one of the drivers of the currency risk premium. In turn, this dependency is capable of capturing the pattern in currency premiums across different horizons.

Consider the vectors of state variables $f_t = (\Delta s_t, \pi_t - \pi_t^*, i_t - i_t^*)^\top$. Stationarity of the RER q_t implies that levels of some of the variables in f_t , namely, NER s_t and price levels p_t and p_t^* , are cointegrated; see the definition in Equation (3). Therefore, we can specify the dynamics of f_t as a special case of a VECM:

$$f_{t+1} = \mu_f + \Phi_f f_t + \alpha_f q_t + \Sigma_f \varepsilon_{t+1}.$$

For example, a simple restricted version of this model is:

$$\Delta s_{t+1} = \mu_s + \rho_s(i_t - i_t^*) + \alpha_s q_t + \sigma_s \varepsilon_{s,t+1} \quad (28)$$

$$\pi_{t+1} - \pi_{t+1}^* = \mu_\pi + \alpha_\pi q_t + \sigma_\pi \varepsilon_{\pi,t+1}, \quad (29)$$

$$i_{t+1} - i_{t+1}^* = (1 - \rho_i)\mu_i + \rho_i(i_t - i_t^*) + \alpha_i q_t + \sigma_i \varepsilon_{i,t+1}. \quad (30)$$

The specification in Equation (28) takes inspiration from the UIP regression (10). It is complemented by the RER because of the VECM specification. The coefficients α_s , α_π , and α_i are the speed of adjustment parameters that determine how fast the system converges back to its long-run equilibrium after a shock to the RER.

The assumed dynamics of the state variables imply the dynamics of the RER. Ap-

plying Equation (3) to the differenced RER and substituting in the dynamics of the state variables in Equations (28)–(30), we obtain:

$$\begin{aligned} q_{t+1} &= q_t + \Delta s_{t+1} - (\pi_{t+1} - \pi_{t+1}^*) \\ &= \mu_s - \mu_\pi + \rho_s(i_t - i_t^*) + (1 + \alpha_s - \alpha_\pi)q_t + \sigma_s \varepsilon_{s,t+1} - \sigma_\pi \varepsilon_{\pi,t+1}. \end{aligned} \quad (31)$$

Dahlquist and Pénasse (2022) show that the weak form of PPP is not a serious constraint on the analysis. As we observed during our discussion of Equation (6), $q_t - \omega_t^q$ must be stationary, that is, the three variables s_t , $p_t - p_t^*$, and ω_t^q are cointegrated. Thus, the VECM framework would still be applicable if one complemented the state f_t with $\Delta \omega_t^q$ and replaced q_t with $q_t - \omega_t^q$. The long-run RER ω_t^q is unobservable and would have to be estimated with the Kalman filter.

Continuing with the stationary RER case, cointegration of s_t and $p_t - p_t^*$ requires that at least one of coefficients α_s , α_π , and α_i not equal zero. Empirically, $\alpha_s \neq 0$. Thus, the weak form of PPP implies that the depreciation rate is predictable from the RER and that the currency risk premium depends on the RER.

When $\alpha_s = \alpha_\pi = \alpha_i = 0$ in Equations (28)–(30), we get the present-value expression for the RER in Equation (12). In connection with that equation, we discussed the challenge of the model matching the correlation between the RER and the interest rate differential. When $\alpha_\pi = \alpha_i = 0$ but $\alpha_s \neq 0$, Dahlquist and Pénasse (2022) show that the RER can be represented as

$$q_t = \beta \frac{\mu_i - (i_t - i_t^*)}{1 - \rho_i} - \frac{\eta_t}{1 - \rho_\eta} + \omega_t^q, \quad (32)$$

where η_t reflects the new information in q_t relative to $i_t - i_t^*$ and is an AR(1) process

with the persistence coefficient ρ_η . [Dahlquist and Pénasse \(2022\)](#) refer to η_t as the missing risk premium. The new relationship shows that the RER and the interest rate differential do not need to be perfectly correlated and that the sign of the correlation is not determined by the sign of β , as it is in Equation (12).

5.3 Implications for forecasts

One can show that if the coefficient ρ_s equals zero in Equations (28) and (31), then the forecast of Δs_{t+1} n horizons ahead depends only on q_t with the coefficient $(1 + \alpha_s - \alpha_\pi)\alpha_s^{n-1}$. As a result, the pattern of forecasts would be monotonic. Thus, non-monotonicity of the multi-horizon UIP coefficients requires both $\alpha_s \neq 0$ and $\rho_s \neq 0$.

We of course know a lot about ρ_s because it is the slope coefficient β in the UIP regression, i.e., Equation (10). As discussed in connection with Figure 1, the most recent estimate of this coefficient is insignificantly different from zero. However, the VECM specification discussed implies that the standard UIP regression suffers from an omitted variable—the RER. The RER is negatively correlated with the interest rate differential, so its omission introduces an upward bias in the estimated β (or ρ_s).

Indeed, when we re-estimate the regression with the RER included, the estimated ρ_s is -0.72 with a standard error of 0.35 for the full sample (cf. the estimated β in Figure 1 is -0.57). Thus, this coefficient is significantly different from zero at the 95% confidence level and is responsible for generating the non-monotonicity in the UIP coefficients. Furthermore, Figure 4 is constructed for excess returns, so

the relevant coefficient is $\rho_s - 1$, which is even more significant. Lastly, in a more general model in [Chernov and Creal \(2021\)](#) and in the extension of the baseline model in [Dahlquist and Pénasse \(2022\)](#), the inflation differential is predictable from the interest rate differential with a positive sign. As a result, ρ_s in the RER dynamics (31) is replaced with an even more negative number, offering even stronger support for the documented non-monotonic pattern.

5.4 Implications for currency risk premiums

[Chernov and Creal \(2021\)](#) document that the pattern of slope coefficients from regressing the risk-neutral expectations of the depreciation rate, as measured by currency forward rates, on the interest differentials is monotonic across horizons (in sharp contrast to the true expectations). This means that the RER does not predict the depreciation rate under risk-neutral probability. Put differently, currency risk premiums must depend on the RER. To demonstrate this conclusion in the data, one must estimate the SDF by using, for instance, bond data.

Continuing with the VECM of the dynamics of f_t , one can construct a companion-form VAR for the vector $x_t = (f_t^\top, q_t)^\top$:

$$x_{t+1} = \mu_x + \Phi_x x_t + \Sigma_x \varepsilon_{t+1} \quad (33)$$

with

$$\mu_x = \begin{pmatrix} \mu_f \\ \mu_s - \mu_\pi \end{pmatrix} \quad \Phi_x = \begin{pmatrix} \Phi_f & \alpha_f \\ \Phi_s - \Phi_\pi & 1 + \alpha_s - \alpha_\pi \end{pmatrix} \quad \Sigma_x = \begin{pmatrix} \Sigma_f \\ \Sigma_s - \Sigma_\pi \end{pmatrix},$$

where, in the case of the simple model of Equation (28), $\Phi_s \equiv e_1^\top \Phi_f = (0, 0, \rho_s)$, $\Phi_\pi \equiv e_2^\top \Phi_f = (0, 0, 0)$, $\Sigma_s \equiv e_1^\top \Sigma_f = (\sigma_s, 0, 0)$, and $\Sigma_\pi \equiv e_2^\top \Sigma_f = (0, \sigma_\pi, 0)$.

This VAR is helpful in applying the VECM modeling approach to bond data. Complementing the VAR with the specification of risk prices via an SDF along the lines of Equation (27) allows us to connect the properties of currency risk premiums to PPP.

The basic model of Equation (28) must be extended to be able to fit bond yields. First, one needs to know the conditional mean of the SDF. As a result, there is a need to incorporate the level of the interest rate i_t , and not just its deviation from the foreign rate. Furthermore, as is customary in the term structure literature, one must introduce bond factors such as slope. For example, [Chernov and Creal \(2021\)](#) use the following extended state vector:

$$f_t = \left(\Delta s_t, \pi_t - \pi_t^*, i_t, \tau_t, i_t - i_t^*, \tau_t - \tau_t^* \right)^\top, \quad (34)$$

where $\tau_t = y_t^{(120)} - y_t^{(12)}$ is the slope of the yield curve. Depending on the research question and desired degree of accuracy, one could add other state variables, such as domestic inflation, π_t , or curvature of the yield curve, at the cost of loss in parsimony.

Using international bond prices as inputs, one can then identify the risk prices of each shock ε_t . Thus, estimating such a pricing model allows us to speak about the currency risk premiums associated with each variable of interest. In particular, all the state variables have the potential to affect currency risk premiums. [Chernov and Creal \(2021\)](#) find that the interest rate differential and the RER are significant at all horizons, as one would expect given the earlier discussion. Furthermore, the inflation differential and slope differential are significant at all horizons as well. The latter

result is consistent with the regression-based evidence presented by [Ang and Chen \(2010\)](#) and [Lloyd and Marin \(2021\)](#).

5.5 Equilibrium models

The reversal and non-monotonic pattern of the currency risk premium across horizons is challenging for leading equilibrium models to capture. [Engel \(2016\)](#) argues that this is difficult to match in a frictionless model and suggests an explanation based on liquidity risk. Similarly, [Valchev \(2020\)](#) proposes a model in which currency premiums arise as compensation for endogenous fluctuations in bond convenience yield differentials. When documenting the non-monotonic pattern, [Bacchetta and van Wincoop \(2010\)](#) also calibrated a model in which agents make infrequent portfolio decisions. Recently, [Dahlquist, Heyerdahl-Larsen, Pavlova, and Pénasse \(2023\)](#) develop a model with time-varying risk appetites (through deep habits) that produces asymmetric portfolios. The model is frictionless and the additional feature relied on is the stochastic volatility of the output growth process (calibrated to the real GDP growth of the G10 countries). The variation in risk appetite, together with the stochastic output volatility, is enough to capture the reversal and non-monotonic pattern.

5.6 Key takeaways

Currency risk premiums exhibit a non-monotonic pattern across horizons. If the interest rate differential is the sole driver of the currency risk premium, such a pattern is not feasible. The weak form of PPP (i.e., stationary RER) introduces RER as

an additional variable affecting expected currency excess returns. Together with the interest rate differential, it is responsible for the multi-horizon evidence of depreciation rate predictability. The non-monotonic pattern is challenging for equilibrium models to capture.

6 Monetary policy

The analysis of the interaction between monetary policy, exchange rates, and horizon effects is a small but important and rapidly developing field. [Backus, Gavazzoni, Telmer, and Zin \(2013\)](#) work in the tradition of consumption-based asset pricing, and incorporate conventional monetary policy via the Taylor rule. Given the equilibrium nature of such models, they have predictions for the whole yield curve in each country. [Gourinchas, Ray, and Vayanos \(2022\)](#) and [Greenwood, Hanson, Stein, and Sunderam \(2022\)](#) work with models in the tradition of demand-based asset pricing. Their models specify the demand of market participants for bonds of a specific maturity (i.e., a habitat), and market clearing connects this demand to bond prices and exchange rates. This feature allows one to speak about the impact of unconventional monetary policy (e.g., QE). In practice, this research primarily focuses on capturing UIP and EH violations.

6.1 Conventional monetary policy

[Backus, Gavazzoni, Telmer, and Zin \(2013\)](#) connect UIP to monetary policy in a particularly transparent way: “Why do countries with high interest rate policies have

currencies that tend to appreciate relative to those with low interest rate policies. The risk-premium interpretation of the UIP puzzle asserts that high interest rate currencies pay positive risk premiums. The question, therefore, can also be phrased in terms of currency risk: When a country pursues a relatively high-interest rate monetary policy, why does this make its currency risky?”

The authors combine endowment economies and Taylor rules in each of the two countries considered (as a result, monetary policy plays no role in the real economy). The representative agent in each country has Epstein–Zin preferences with identical values of preference parameters. The Taylor rule endogenizes inflation just as in the one-country setting of [Gallmeyer, Hollifield, Palomino, and Zin \(2007\)](#). The nominal interest rate is:

$$i_t = -\log E_t(M_{t+1} \cdot e^{-\pi_{t+1}}),$$

where M_{t+1} acts as the marginal rate of substitution. Now impose the Taylor rule:

$$i_t = \tau_0 + \tau_g g_t + \tau_\pi \pi_t,$$

where g_t is the exogenous endowment (i.e., consumption growth). Both equations for the nominal rate must hold at the same time, and that requirement delivers endogenous inflation:

$$\pi_t = -\tau_\pi^{-1}(\tau_0 + \tau_g g_t + \log E_t(M_{t+1} \cdot e^{-\pi_{t+1}})).$$

To translate this setup to two countries, it is common to simply place asterisks on shocks and parameters to denote a foreign country in an otherwise identical setup.

[Burnside and Graveline \(2020\)](#) point out that there is no trade in real assets in this setting. It could be viewed as partial equilibrium with the relationship between consumption and output unmodeled. The hope is that one could support the assumed consumption processes by using an explicit model of trade. Lastly, the exchange rate is obtained via the AMV under the assumption of complete markets.

Given that the yield-curve implications were successfully explored elsewhere, [Backus, Gavazzoni, Telmer, and Zin \(2013\)](#) focus on the conditions delivering the UIP violations. We observed in Section 4 that UIP violations are not hard to match in the context of affine models, when one is free to pick driving processes for all the relevant objects. In an equilibrium context, it does represent a modeling challenge because of the various cross-equation restrictions imposed by the theory.

Specifically, the authors focus on linking the differences in the SDFs, or marginal rates of substitution in this case, which are required to generate time-varying exchange rates, to the asymmetries in the Taylor rule coefficients τ_g and τ_π across the different countries. They refer to $\tau_g < \tau_g^*$ as foreign monetary policy being relatively procyclical, and to $\tau_\pi > \tau_\pi^*$ as foreign policy being relatively accommodative (of inflation).

The real UIP coefficient—the slope coefficient of a regression of the real depreciation rate on the real interest rate differential—is driven by the properties of the traditional endowment economy. The coefficient is unambiguously negative if the representative agents have a preference for the early resolution of uncertainty. The nominal UIP coefficient associated with the endogenous inflation is typically greater than its real counterpart.

Because inflation is a function of the Taylor rule coefficients, one is not free to

select it to match the empirical UIP regression coefficient. This is where the cross-country differences in these coefficients come into play. [Backus, Gavazzoni, Telmer, and Zin \(2013\)](#) show that, given their parametrization of the endowment, relatively procyclical and relatively accommodative policies generate relatively more currency risk. The combination of the two generates a foreign currency that is risky in that the unconditional expected return to “borrow domestic, lend foreign” is positive.

6.2 Unconventional monetary policy

[Gourinchas, Ray, and Vayanos \(2022\)](#) and [Greenwood, Hanson, Stein, and Sunderam \(2022\)](#) extend the one-country setting of [Vayanos and Vila \(2021\)](#). The single-country model captures the EH violations; it also speaks to the QE effects of an unexpected increase in bond demand (e.g., QE purchases reduce yields).

In a two-country setting, the authors consider economies with exogenously given risk-free short interest rates (encoding the conventional monetary policy) and demand functions of habitat investors, which depend on bond prices of a given maturity or exchange rate and on demand risk factors. Global arbitrageurs with quadratic preferences construct optimal portfolios of domestic and foreign bonds.

Markets must clear, that is, the demand of the habitat investor must balance the optimal portfolio positions of the arbitrageurs. As a result, bond prices and the exchange rate depend on the demand functions. In particular, shocks to demand risk factors could be interpreted as QE. We note that, because all the driving variables are stationary, the derived objects, including exchange rates, are stationary as well (i.e., there is no distinction between real and nominal in their models).

Greenwood, Hanson, Stein, and Sunderam (2022) consider a consol bond instead of the full term structure. They focus on UIP and EH violations, considering an illuminating example in which demand functions do not depend on prices. In this case, bond yields and the exchange rate depend on the exogenous short interest rates and demand risk factors. However, the levels of domestic and foreign short-term interest rates do not affect the currency and long-term-bond risk premiums. Put differently, the effects of exogenous shocks on long-term risk premiums are the same as in UIP and the EH.

This observation prompts them to introduce asset prices (i.e., bond prices or the exchange rate) into the demand functions for the corresponding assets. Such a specification forms the baseline case in Gourinchas, Ray, and Vayanos (2022). Thus, both papers are capable of matching the classic evidence.

Once the demand is linked to prices, changes in the foreign conventional monetary policy affect domestic term premiums and vice versa. This results in an attenuation effect: the impact on long-term risk premiums is smaller than that implied by UIP and the EH. In particular, this implies that the Friedman–Obstfeld–Taylor trilemma fails: In the absence of capital controls, foreign monetary policy affects domestic financial conditions despite floating exchange rates.

As Gourinchas, Ray, and Vayanos (2022) feature the full yield curve, they explore the multi-horizon implications of their model. They demonstrate that they could match the evidence of multi-horizon UIP regressions, the dependence of currency premiums on yield curve slopes, and the slope carry. In the estimated model, unconventional monetary policy has significant effects on bond yields, and these are transmitted almost one-to-one across countries. QE purchases (in either country),

which equal 10% of GDP with a half-life of seven years, reduce the domestic and foreign intermediate-maturity yields by about 50 basis points. By contrast, conventional policy has pronounced effects on domestic but not on foreign yields. The difference arises partly because positive correlation in short-term rates across countries magnifies the international transmission of unconventional policy and dampens that of conventional policy. Specifically, in response to QE in the domestic market, arbitrageurs reduce their positions in domestic bonds, and rebuild their exposure to short-term rate risk by buying positively correlated foreign bonds. The results suggest that flexible exchange rates have better insulation properties under conventional than unconventional monetary policy.

[Greenwood, Hanson, Stein, and Sunderam \(2022\)](#) also offer a nice comparison with the consumption-based models, such as the one presented in the previous subsection. The wealth of intermediaries in the global bond markets need not be closely tied to aggregate consumption or conditions in other financial markets. Thus, exchange rates move in response to shifts in the supply of and demand for assets in different currencies, which intermediaries must absorb. By contrast, in frictionless asset-pricing theories, the mere reshuffling of assets between different agents in the economy has no asset pricing implications.

Moreover, bad times for the marginal investors in global bond markets need not coincide with those for more broadly diversified investors or for representative households. In particular, while there is an SDF that prices risky assets in their model, it is not the case that short-term riskless rates satisfy the usual relationship, i.e., $i_t = -\log E_t(M_{t+1})$. Empirically, short-term real interest rates typically rise in economic expansions and fall in recessions. As a result, long-term real bonds are a macroeconomic hedge for the representative household, which leads most

consumption-based models to predict negative real term premiums. By contrast, as an implication of [Vayanos and Vila \(2021\)](#), long-term bonds are risky for specialized bond investors, who suffer capital losses when short-term rates rise, so real term premiums are positive.

Finally, in complete-market models in which the AMV holds, foreign currency counterfactually appreciates in bad times for foreign agents (i.e., the [Backus and Smith, 1993](#), puzzle). This appreciation occurs even though short-term foreign interest rates fall in bad foreign times. Furthermore, since long-term bonds are hedge assets in consumption-based models, foreign long-term bond yields fall in the same bad foreign times when foreign currency appreciates. As a result, foreign currency returns are positively correlated with long-term foreign bond returns and negatively correlated with long-term domestic bond returns. Thus, in most consumption-based models, the currency risk premium increases in the foreign-minus-domestic term premium differential.

By contrast, in the demand-based theory and in the data, foreign currency appreciates when short-term foreign interest rates rise relative to short-term domestic interest rates. Furthermore, the realized returns on foreign currency are negatively correlated with foreign bond returns and positively correlated with domestic bond returns. This is because the realized returns on exchange rates and long-term bonds are both driven by shocks to short-term interest rates. As a result, the expected return on foreign currency is negatively related to the foreign-minus-domestic term premium differential.

6.3 Key takeaways

Studying the impact of monetary policy on exchange rates necessitates modeling the latter jointly with domestic and foreign bond yield curves. Both consumption- and demand-based approaches are capable of capturing UIP and EH violations. Consumption-based models cannot speak to monetary policy affecting the long end of the bond yield curves (i.e., QE) and struggle to explain the [Backus and Smith \(1993\)](#) currency cyclical puzzle. Demand-based models show more promise by severing the close link between aggregate consumption and the wealth of the marginal agents (i.e., intermediaries).

7 Emerging economies

So far all the evidence discussed is based on the G10 currencies; in this section we discuss emerging economies. These economies are associated with more volatile macroeconomic fundamentals (e.g., output growth, inflation, and interest rates), sovereign credit risk, and more common economic and financial crises. Moreover, data are typically only available for shorter sample periods and are of limited quality. Taken together, this poses special challenges for researchers. With that in mind, we revisit the existing evidence of UIP violations in emerging economies and present new evidence regarding multi-horizon currency risk premiums. This evidence differs from that regarding developed economies. We offer interpretations of the differences but also call for more research on emerging economies.

7.1 UIP regressions

[Bansal and Dahlquist \(2000\)](#) provide evidence of the UIP regression for developed as well emerging economies. Starting with the developed economies, they report a negative β estimate of -0.32 for the 1976–1998 sample period. This is similar to our estimate of -0.57 for the G10 over the longer 1976–2022 sample period. In contrast, they report a pooled estimate of 0.19 for emerging economies whose data became available in the early 1990s. This is similar to our estimate of 0.15 for the emerging economies we consider over the 1996–2022 sample period. These results mean that the correlation between the expected currency depreciation and interest rate differential changes signs as one moves from developed to emerging economies.

Data for emerging economies are much sparser and typically of lower quality than those for developed ones. While the β s are imprecisely estimated and the null hypothesis of $\beta = 0$ (consistent with the RW hypothesis) cannot be rejected, the null hypothesis of $\beta = 1$ (consistent with UIP) can be rejected. Hence, a positive β for emerging economies still suggests a time-varying currency risk premium. [Frankel and Poonawala \(2010\)](#) and [Gilmore and Hayashi \(2011\)](#) provide further evidence in support of this.

Moreover, [Bansal and Dahlquist \(2000\)](#) link β estimates of developed and emerging economies to macroeconomic fundamentals; in particular, they find that economies with lower per capita income, higher average inflation and inflation volatility, and higher nominal interest rates have higher β s. This suggests that exchange rate models need to address the differences between developed and emerging economies. Below, we consider the relationship between β and credit risk.

7.2 Multi-horizon currency premiums

Following Subsection 5.1, we estimate the slope from regressing $E_t(\Delta s_{t+n}) - (i_{t+n-1} - i_{t+n-1}^*)$ on $i_t - i_t^*$ across the different horizons n for emerging economies. Figure 5 displays the results. For comparison, the figure also reports the G10 results over the same sample period for which data are available for emerging economies (thus, the G10 results differ between Figures 4 and 5).

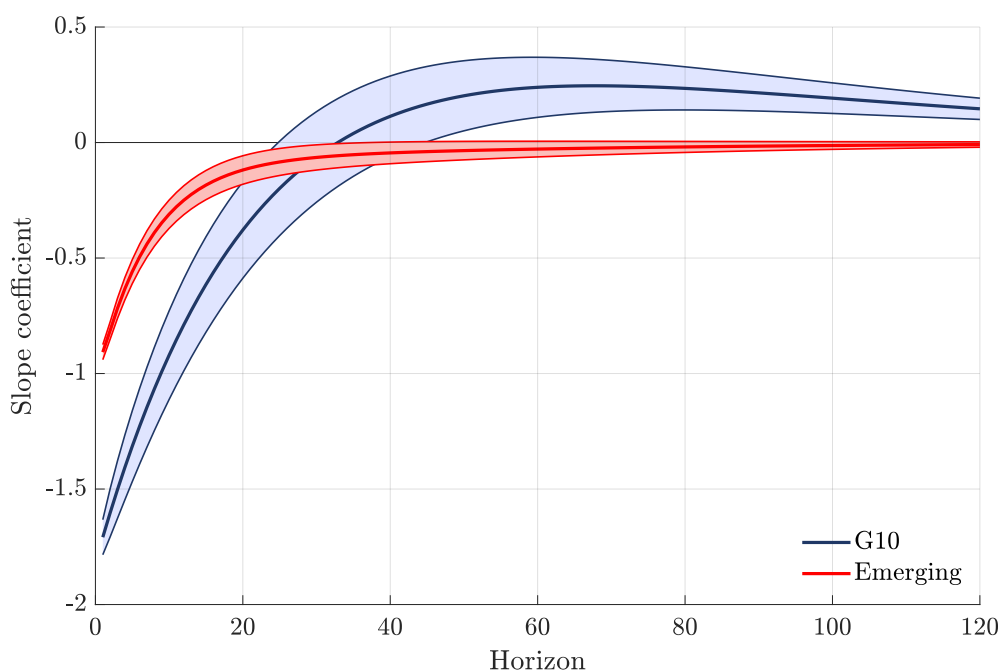


Figure 5: Multi-horizon predictability and interest rate differential

The figure shows the slope coefficients with 95% confidence bands from panel regressions of fitted G10 and emerging market currency risk premiums on their nominal interest rate differentials at different horizons for the January 1996–July 2022 sample period. The pooled regressions include currency fixed effects. The one-month risk premiums are constructed by regressing monthly currency excess return on the lagged currency excess return, lagged interest rate differential, and lagged real exchange rates. The risk premiums beyond one month are obtained by taking the power of a VAR model of the regression variables.

The first point on the plot corresponds to the traditional UIP regressions (i.e., $n = 1$), albeit using fitted expected as opposed to realized depreciation rates. The reported coefficient corresponds to $\beta - 1$. Moving on to $n > 1$, we see that, in stark contrast to the G10 case, the multi-horizon pattern of the emerging currency risk premiums is monotonic.

In light of the model presented in Subsection 5.2, the documented monotonic pattern of risk premiums suggests that the coefficient $\alpha_s = 0$ in Equation (28) equals zero. Indeed, both our evidence and that of [Kranmer \(2018\)](#) support this conclusion.

One possible implication of this evidence is that the RER is non-stationary for emerging economies. This possibility raises the question of whether it holds in the data. If so, it is unclear why the dynamic properties of the RER in emerging economies would differ from those of developed economies. Another possibility is that the RER is stationary and simply does not predict the currency depreciation rate. Such an interpretation is feasible because, although $\alpha_s = 0$, the other two speed-of-adjustment parameters in the VECM, i.e., α_π and α_i , may differ from zero. Alas, the evidence regarding the stationarity of the RER is mixed (see [Rogoff, 1996](#), and [Taylor and Taylor, 2004](#), for reviews). In particular, there seems to be a dearth of research on emerging economies.

In Section 6, we discuss equilibrium models featuring the role of monetary policy that seem to be capable of explaining the single- and multi-horizon UIP violations as quantified for the G10 countries. It would be interesting to see whether such models could be adjusted to explain the different evidence from emerging economies.

7.3 Sovereign credit risk

In this subsection we discuss the role of sovereign credit risk as the potential explanation for the evidence regarding emerging economies. We represent credit risk as a spread over the risk-free rate, consistent with [Duffie and Singleton \(1999\)](#):

$$i_{d,t}^* = i_t^* + c_t^*,$$

where i_t^* still denotes the risk-free interest rate, c_t^* is the credit spread, and $i_{d,t}^*$ is the short interest rate inferred from credit-risky sovereign bonds. All the evidence presented in this paper is relative to the USD, and we treat US sovereign debt as credit-risk free (i.e., $i_{d,t} = i_t$) for simplicity.

Because of this modification, we can rewrite the currency excess return from Equation (18) as:

$$rx_{t+1} = s_{t+1} - s_t + i_{d,t}^* - c_t^* - i_t,$$

which is unpredictable if the currency risk premium is constant. The UIP regression that one would want to implement in this case would be exactly as in Equation (10). Expressing it in terms of the credit spread, we obtain:

$$s_{t+1} - s_t = \alpha + \beta(i_t - i_{d,t}^*) + \beta c_t^* + \varepsilon_{t+1}.$$

However, one typically implements the following regression instead:

$$s_{t+1} - s_t = \alpha_d + \beta_d(i_t - i_{d,t}^*) + \varepsilon_{t+1}.$$

The standard omitted variable bias algebra implies that:

$$\beta_d = \beta \left(1 + \text{corr}(c_t^*, i_t - i_{d,t}^*) \cdot \frac{\text{std}(c_t^*)}{\text{std}(i_t - i_{d,t}^*)} \right).$$

The correlation between the credit spread and the interest rate differential should be negative, as a decline in the foreign interest rate should lead to an easing in foreign borrowing conditions. The volatility of the credit spread should be higher than that of the interest rate differential, implying that the ratio of volatilities is greater than one.

In the case of no credit risk, β_d coincides with β and is estimated to be negative, as we extensively discussed throughout this survey. The negative sign of the correlation makes $\beta_d > \beta$. The fact that the volatility ratio is greater than one may lead β_d to switch signs. Thus, depending on the specific magnitudes of $\text{corr}(c_t^*, i_t - i_{d,t}^*)$, $\text{std}(c_t^*)$, and $\text{std}(i_t - i_{d,t}^*)$, the sovereign credit risk may explain the higher values of the estimated β s reported for the emerging economies.

Moving on to the multi-horizon risk premiums, we consider how sovereign credit risk might be responsible for the monotonic pattern of slopes in Figure 5. Imagine a VECM-implied VAR just like the one in Equation (33), in which one of the state variables, f_t , is the credit spread c_t^* . If c_t^* affects the conditional expectation of Δs_{t+1} in addition to the interest rate differential and the RER, it contributes to the shape of the multi-horizon slope coefficients.

One scenario is that $\alpha_s = 0$, as discussed in Subsection 7.2. Then the predictability of Δs_{t+1} with c_t^* might generate a non-monotonic pattern. That would be a drawback rather than help in explaining the observed monotonic pattern. Put differently, if

$\alpha_s = 0$, then sovereign credit risk cannot explain the documented difference between the developed and emerging markets.

An alternative scenario is that $\alpha_s \neq 0$ and the pattern of slopes created through the predictability of Δs_{t+1} with c_t^* offsets the pattern generated by the RER. Such a scenario is possible if there is a connection between the RER and the credit spread. One form of connection is known as the “twin Ds” (i.e., default and devaluation), as studied by [Reinhart \(2002\)](#) and [Na, Schmitt-Grohe, Uribe, and Yue \(2018\)](#) in the context of emerging economies. While the twin-D effect is usually connected to the event of default, [Chernov, Creal, and Hördahl \(2023\)](#) show in the context of Asia-Pacific economies that there is a strong connection between the depreciation rate and the credit spread even outside of crises. It remains to be seen whether the twin-D effect is capable of offsetting the RER effect on the multi-horizon risk premium slope both qualitatively and quantitatively.

7.4 Key takeaways

Emerging economies have the potential to offer new insight into mechanisms underlying the UIP violations and multi-horizon currency patterns in risk premiums. That is because the evidence is drastically different from that from developed economies: the UIP coefficient is closer to the null and the risk premium pattern is monotonic, calling into question the role of the RER. Sovereign credit risk might offer an explanation for the differences between the developed and emerging economies.

8 Conclusion

In this survey, we present a case for studying currency risk premiums across multiple horizons. Such a perspective closely connects the properties of exchange rates and sovereign bonds. The critical takeaway from this analysis is that the bond-based evidence often clashes with the intuition about exchange rates developed based on single-horizon facts.

Common general equilibrium models, which explain single-horizon UIP violations, have difficulty explaining their multi-horizon versions. Empirical, or so-called no-arbitrage, models can capture the evidence jointly, but equilibrium mechanisms require departures from the representative agent or exogenous endowment economies. Emerging economies present further complications, as the single- and multi-horizon evidence differs from that from the G10 economies.

These challenges bring opportunities. While existing research focused on the empirical margin, the path forward is to systematize and conceptualize the accumulated evidence in a plausible theoretical framework. Here, we describe some of the early attempts to do so, but much remains to be done.

A Data

We collect daily spot and one-month forward exchange rates from Datastream. We follow [Chernov, Dahlquist, and Lochstoer \(2023\)](#) and use several data providers to construct a monthly dataset for the sample period from January 1976 to July 2022. The main results refer to countries for the G10 currencies (with currency codes in parentheses): Australia (AUD), Canada (CAD), Eurozone (EUR) spliced with Germany (DEM) before 1999, Japan (JPY), New Zealand (NZD), Norway (NOK), Sweden (SEK), Switzerland (CHF), United Kingdom (GBP), and the United States (USD). We collect consumer price indexes from the statistical database of the Organisation for Economic Co-operation and Development (OECD). For AUD and NZD, monthly consumer price indexes are not available. We therefore use quarterly indexes for AUD and NZD and forward fill the values in the months until the next quarter.

We complement the exchange rate and consumer price data with monthly G10 zero-coupon curves from [Wright \(2011\)](#) as they become available and up to May 2009. We follow [Lustig, Stathopoulos, and Verdelhan \(2019\)](#) and update these curves to July 2022 with zero-coupon curve data from Bloomberg.

We also consider a set of emerging market countries (as they are available) from December 1996 to July 2022: Brazil (BRL), Bulgaria (BGN), Chile (CLP), China (CNY), Colombia (COP), Croatia (HRK), Cyprus (CYP), Czech Republic (CZK), Greece (GRD), Hungary (HUF), Iceland (ISK), India (INR), Indonesia (IDR), Kuwait (KWD), Latvia (LVL), Lithuania (LTL), Mexico (MXN), Philippines (PHP), Poland (PLN), Russia (RUB), Saudi Arabia (SAR), Slovakia (SKK), Slovenia (SIT), South Africa (ZAR), South Korea (KRW), Taiwan (TWD), Thailand (THB), Turkey (TRY), and Ukraine (UAH). The selection is dictated by the availability and quality of the currency forward rate and consumer price index data.

We assume the USD to be the domestic currency. We express all currencies in USD per unit of foreign currency. An increase in the exchange rate implies an appreciation of the foreign currency and a depreciation of the USD.

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