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ABSTRACT

Global cities are attracting a growing number of tourists and foreign residents. This influx generates capital gains for property owners but adversely affects renters, creating potentially important production, congestion, and amenities externalities. We study the optimal policy regarding local and foreign residents in a model with key features emphasized in policy debates. Using this model, we provide sufficient statistics to calculate the optimal tax and transfer policies that internalize agglomeration, congestion, and other potential externalities. We find that it is not optimal to impose zoning regulations or to restrict, tax, or subsidize home purchases by foreign residents. However, it may be optimal to charge an entry fee to foreign residents.

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1 Introduction

In 1917, the American composer Cole Porter moved to Paris and acquired an opulent residence built in 1777 for the brother of Louis XVI. There he hosted luminaries like F. Scott Fitzgerald and Ernest Hemingway and composed classics like "Night and Day" and "Anything Goes."

Buying a home abroad was unusual at the beginning of the 20th century but has become increasingly common. As remote work expands (Dingel and Neiman, 2020, and Aksoy, Barrero, Bloom, Davis, Dolls, and Zarate, 2022), a growing number of people seek residence in foreign destinations. At the same time, rising incomes and falling air travel costs expanded tourism. According to the United Nations World Tourism Organization, international tourist arrivals grew by an average of 6.1 percent per year between 1950 and 2019.

These flows of foreign residents and tourists are reshaping housing markets across the globe. They create capital gains for property owners but reduce affordability for renters and create production, congestion, and amenity externalities. Policy responses vary widely–from laissez-faire and visa-for-investment programs to special taxes and ownership restrictions.[1] Tourism policies also vary, including laissez-faire approaches, subsidies for hotel and infrastructure investment, and per-diem taxes on visitors.[2]

Determining the optimal policy toward foreign residents is important for three reasons. Housing is the main asset in household portfolios (Cocco, 2005); its price and location determine commuting times and job access, shaping productivity and welfare; and most economic activity occurs in cities (Rossi-Hansberg and Wright, 2007).

We adopt a Mirrleesian framework (Mirrlees, 1971) that incorporates key insights from urban and economic geography to determine the optimal policy toward foreign residents. Rather than fixing policy instruments ex ante, we allow the planner to choose them subject to informational constraints–it cannot observe individuals' preferences over where to live and work. The resulting second-best allocations arise as general principles, independent of functional forms, preference distributions, or welfare weights. Optimal location-based transfers generally deviate from Pigouvian taxes to balance efficiency and equity.

[1] Examples include tax breaks and visa incentives in Portugal, Greece, and Spain (Kalin, Levy, and Muñoz (2024)); foreign-buyer taxes in Canada, Hong Kong, Israel, and Singapore; and purchase quotas in Switzerland.

[2] See Allen, Fuchs, Ganapati, Graziano, Madera, and Montoriol-Garriga (2020) for an insightful analysis of the effect of tourism on the welfare of the local population.

We show that it is optimal to use place-based taxes and transfers on locals and foreigners to internalize externalities. Taxing or subsidizing foreign home purchases is never optimal. This result may seem counterintuitive. The domestic economy has monopoly power over city-center housing, so why not impose an optimal tariff? When the number of foreigners is exogenous, it is indeed optimal to charge a lump-sum entry fee that can capture the gains from trade without distorting their housing choices. But, when the planner can also determine how many foreigners enter, subject to them being as well off as in their exogenous outside option, the optimal number of foreigners is reached when the marginal gain from trade is zero. So, the optimal entry fee is zero.

We derive sufficient statistics to evaluate the welfare effects of foreign inflows and the required place-based policies.[3] We emphasize how the equity-efficiency tradeoff is shaped by the presence of production, congestion, and other externalities, and how these considerations shape optimal transfers.

To motivate our analysis, we assemble a new dataset for Lisbon covering the 2011-21 period, including census data, tourism flows, millions of web-scrapped real estate listings, housing stock estimates, and commuting data. We focus on Lisbon because of data availability. However, anecdotal evidence suggests that Lisbon's experience is representative of broader trends in other global cities, such as Barcelona, Venice, and Vancouver.

We document five key facts:

1. A significant influx of foreign residents and tourists;
2. A small decline in the number of housing units in the city center;
3. A sharp rise in inflation-adjusted housing prices and rents in both the city center and peripheral municipalities;
4. A large outflow of domestic residents from the city center;
5. Rising commuting times due to congestion.

[3] We do not analyze the possibility of multiple equilibria. See Owens, Rossi-Hansberg, and Sarte (2020) for an analysis of how policy can also be used to implement a particular equilibrium.

Our model is consistent with these facts and provides a natural causal interpretation: the rise in the number of foreign residents and tourists drives Lisbon's urban dynamics during our sample period.

The baseline model features a central location and multiple peripheries with fixed stocks of housing and offices. Foreign residents prefer the center and have an exogenous outside option. Locals choose where to live and work, taking into account wages, amenities, commuting costs, and idiosyncratic preferences. Households are also heterogeneous in their ownership of houses and office buildings.

We begin by examining the competitive equilibrium and assessing the impact of a marginal increase in foreign residents on social welfare. Using recent advances in welfare analysis by Dávila and Schaab (2022), we decompose the impact on social welfare into three components. The first pertains to the housing capital gains that accrue to locals. An influx of foreign residents increases housing demand in the city center, raising rents. So, locals can make capital gains by selling houses to foreigners. This effect, which we call the foreign-resident surplus, is always positive. It is analogous to the immigration surplus discussed in the immigration literature (see, e.g., Borjas, 1995, and Guerreiro, Rebelo, and Teles, 2020). The second effect relates to the agglomeration or production externality emphasized by Jacobs (1969), Lucas (1988, 2001), Lucas and Rossi-Hansberg (2002), and Ahlfeldt, Redding, Sturm, and Wolf (2015). This effect can be negative if the arrival of foreigners leads to the relocation of workers from high- to low-productivity locations. The third effect is income inequality resulting from heterogeneity in work locations and real estate ownership. Next, we study the Mirrleesian policy toward local and foreign residents that maximizes the welfare of the local population.

We expand our model to incorporate additional features to address issues discussed in policy debates. We introduce congestion externalities by assuming that commuting time increases with the number of commuters. We show that, with endogenous commuting time, the sufficient statistics that describe the optimal place-based transfers to locals also take into account the correction of congestion externalities and the interaction between congestion and agglomeration externalities. This interaction arises because increased commuting time reduces agglomeration externalities.

In the extended model, we introduce the option of remote work, allowing workers to perform their jobs from home. In this scenario, locals can work for firms in the city center without commuting. Remote workers neither contribute to agglomeration externalities nor to commuting-related congestion. Optimal transfers consider the trade-off between the reduced impact of remote workers on agglomeration externalities and their positive effect in alleviating congestion.

We also assume that foreigners value authenticity, that is, they derive utility from having locals live and work in the city center. At first sight, one might think that this feature would not affect the social optimum. After all, the planner does not include foreigners' utility in the social welfare function. However, it is optimal to internalize this externality by providing transfers to locals who live and work in the city center. The rationale for this policy is that the externality affects foreigners' participation constraints and influences their relocation decisions.

We next consider settings in which foreigners directly affect the value locals place on amenities in the city center. We show that these amenity externalities do not affect the statistics for the optimal transfers to locals but introduce a reason to distort the entry of foreigners. If these externalities are negative, it is optimal to correct them by imposing a lump-sum tax on foreigners, similar to the per-diem or per-night tax levied on tourists by an increasing number of cities.[4] As in the baseline model, it is not optimal to distort foreigners' housing purchases relative to purchases of other goods.

We also analyze the case where the foreigners' outside option rises with the number of foreigners relocating to the city. As more foreigners move in, fewer settle elsewhere, lowering house prices or improving amenities abroad. This general-equilibrium effect arises only if the domestic city is large enough to influence conditions abroad. In this case, it is optimal to impose an entry fee to limit inflows and moderate the rise in the marginal foreigner's reservation utility.

Even though the extended model incorporates numerous ways in which the entry of foreign residents could affect the welfare of the local population, we find that distorting foreigners' housing purchases relative to purchases of other goods is never optimal.

Finally, we extend our framework to a dynamic setting to examine how inflows of foreign residents affect optimal policy in the long run, when offices can be converted into housing and vice versa. The model features overlapping generations of local workers who live for multiple periods, can relocate at a cost, and draw idiosyncratic preferences over residential-workplace pairs. This formulation highlights an important intergenerational conflict: current residents receive capital gains on housing, while future generations face higher housing costs. We show that the optimal allocation can be decentralized with simple instruments:

[4] In practice, these per-diem taxes can also be implemented by imposing a fixed fee on foreigners who rent or purchase a home. This fixed fee does not distort the choice between housing and consumption.

market-determined rents and lump-sum, history-contingent transfers. No additional zoning regulation or fiscal distortions are required. There are no distortions imposed on housing services, savings, or investment in structures, and foreigners face no entry restrictions beyond having to pay market rents. Redistribution, compensation for moving costs incurred by locals, and the internalization of agglomeration benefits are achieved through transfers rather than price or zoning interventions.

The paper is organized as follows. Section 2 briefly reviews the relevant literature. In Section 3, we document the key facts that motivate our model. Section 4 introduces the baseline model, characterizes the competitive equilibrium, and assesses the welfare impact of an increase in the number of foreign residents. Section 5 describes the optimal Mirrleesian policy for the baseline model. In Section 6, we extend the model to include elements such as traffic congestion, remote work, amenity effects, foreigners' preference for authenticity, and the influence of the number of incoming foreigners on the outside option of the marginal foreigner. Section 7 uses a dynamic model to explore the long-run implications of inflows of foreign residents. We summarize our conclusions in Section 8.

2 Related literature

We use a standard geography model which builds on an extensive literature that includes important contributions by Alonso (1964), Mills (1967), Muth (1969), Lucas and Rossi-Hansberg (2002), Rossi-Hansberg (2005), Desmet and Rossi-Hansberg (2013), Ahlfeldt et al. (2015), Allen, Arkolakis, and Li (2015), among others.

Our paper is most related to the analysis of the impact of foreign home buyers on welfare by Favilukis and Van Nieuwerburgh (2021). These authors build a quantitative model with two locations and assume that foreign residents disproportionally buy houses in the city center. Favilukis and Van Nieuwerburgh (2021) argue that the influx of foreign residents reduces local welfare. Their model does not feature the externalities we emphasize. In their framework, welfare losses stem from redistribution among people with different levels of home ownership. They conclude that policies that tax foreign home purchases may improve welfare. Our analysis also incorporates heterogeneity in home ownership. However, we do not place a priori restrictions on the set of available policy instruments. Instead, we impose Mirrlees (1971)-style information constraints. We show that distorting foreign home purchases relative to other goods is not optimal. In addition, absent amenity externalities and worldwide effects on the outside option of the marginal foreigner, we find that the entry of foreign residents should not be distorted.[5] The optimal policy is to tax the initial capital gains on houses to improve redistribution.

The paper is also closely related to work on migration policies by Duranton and Puga (2023). They develop a model with a continuum of cities and a rural area, in which individuals can migrate from the rural region into cities, generating both agglomeration and congestion externalities. To move into a city, migrants must pay rent and a permit fee. The permit entitles them to a pro rata share of all rents collected within the city. Incumbent residents set the permit fee to maximize their utility, weighing the positive effects of agglomeration against the negative impacts of congestion and the dilution of rental income. In our framework, foreign residents do not contribute to agglomeration externalities (in practice, they are primarily tourists, retirees, or remote workers with limited professional interaction with locals). They may, however, generate negative amenity externalities. Because foreign residents do not receive a share of rental income, their presence results in capital gains that accrue exclusively to locals.

Our analysis also relates to a growing body of literature on the optimal use of place-based policies to address local externalities. Notable contributions to this literature include Fu and Gregory (2019), Fajgelbaum and Gaubert (2020), Rossi-Hansberg, Sarte, and Schwartzman (2019), among others.[6] Fajgelbaum and Gaubert (2024) provide a recent survey on the optimal spatial-policy literature, emphasizing the use of place-based Pigouvian-type taxes to achieve efficiency. Because the migration elasticity is assumed to be arbitrarily large, there are no redistributive motives for those taxes, only an efficiency motive. Our welfare results are also related to recent work by Donald, Fukui, and Miyauchi (2024), which studies optimal spatial transfers and the welfare impact of shifts in technology, spatial dispersion of marginal utility, fiscal and technological externalities. They characterize the welfare gains from improving the U.S. highway network. Our paper focuses on the interplay between optimal spatial transfers and the optimal taxation of foreign residents.

Mongey and Waugh (2024) also study the efficiency properties of models with discrete choices and addi-

[5] Our findings echo themes discussed in Bhagwati (1971)'s work on international trade and welfare, particularly his "specificity rule," which states that policy interventions should address distortions as directly as possible.

[6] In general, the optimal policy depends on the distribution of the location preferences. Davis and Gregory (2021) argue that the distribution of these preferences cannot be identified using location-choice data. Rossi-Hansberg, Sarte, and Schwartzman (2019) show that the optimal policy in their environment is not significantly affected by the distribution of location preferences.

tive random utility, like those used in this paper. They calculate optimal Ramsey linear policies in a spatial equilibrium model and show how the results depend on efficiency vs. redistribution considerations. Our optimal transfer formulas also emphasize the distinct roles of these two forces.

Any analysis of optimal policy in a spatial setting brings to mind the Henry George theorem discussed by Arnott and Stiglitz (1979). This theorem states that a planner who cares about the welfare of both natives and immigrants should allow immigration to continue until the resulting increase in land rents is enough to finance the provision of pure public goods. This result does not apply in our setting because our model does not include the provision of an exogenous level of pure public goods financed by land taxation.

This paper is also related to the work of Gaubert, Kline, Vergara, and Yagan (2021), who study optimal Mirrleesian redistribution policies across space when individuals differ in their preferences for different locations and in their work productivity. These authors emphasize the importance of the responsiveness of workers' location decisions to fiscal policy. Like them, we find that the optimal redistributive policy depends on the elasticity of location choices in response to transfers. In addition, we study how externalities shape optimal transfers and characterize the optimal policy toward foreign residents. We do not consider differences in worker productivity or labor supply. However, our results regarding the optimal entry fees for foreigners and taxes on housing purchases, by foreigners and locals, would continue to hold in an extension of our model that includes those productivity and labor supply differentials, under the conditions discussed in Atkinson and Stiglitz (1976).

Ales and Sleet (2022) study optimal policy in an environment where individuals make discrete choices, including where to live and work. Our contribution relative to their work is to incorporate agglomeration and amenity externalities into the analysis and examine how these forces shape the optimal policy response to an inflow of foreign residents.

Section 6 explores how city amenities influence optimal policy. This discussion relates to a growing literature examining how changes in the composition of residents impact local amenities, see, e.g., Guerrieri, Hartley, and Hurst (2013), Diamond (2016), and Almagro and Dominguez-Iino (2022).

Our analysis in Section 7 is related to the literature on optimal city design. Allen, Arkolakis, and Li (2015) show that zoning policies mandating land use for housing or office buildings improve welfare in the presence of externalities. In contrast, we find that zoning policies are not optimal. This difference arises because we impose no restrictions on the set of policy instruments, allowing externalities to be addressed with policies that are less distortionary than zoning.

3 Empirical evidence

In this section, we present data for Lisbon to establish key facts that inform the design of our model.

We use data from the 2011 and 2021 Portuguese censuses to examine the location choices of both domestic and foreign residents, as well as commuting patterns. Tourism flow estimates are obtained from Statistics Portugal. Hotel occupancy is obtained from the Lisbon Tourism Association. We use the housing census (Census de Alojamento) to estimate changes in the housing stock. To calculate commuting times, we combine data from the Google Maps API with OpenStreetMap. We also assemble a new dataset of house rents and prices from web-scrapped real-estate listings. Appendix A provides a detailed description of our data sources.

Five key facts emerge from our empirical analysis.

Fact 1. *The Lisbon municipality experienced a significant influx of foreign residents and tourists between 2011 and 2022.*

The combined increase in foreign residents and the yearly-equivalent housing units occupied by tourists totals 45.5 thousand (20.6 + 24.9 thousand). With Lisbon's municipal population at 553 thousand in 2011, the influx of foreign residents represents 8.2 percent of the population. In summary, the decade since 2011 has seen a significant rise in the number of foreign residents in the center of Lisbon.

Figure 1 shows that the number of foreign residents in the Lisbon municipality (the city center) grew by 20.6 thousand people. Figure 2 shows a substantial increase in the number of tourists visiting Lisbon from 2011 to 2022, with tourists primarily concentrated in the city center. To estimate the number of yearly-equivalent tourists for each period, we divide the total number of tourist nights by 365×0.74, where 0.74 is the average hotel occupancy in Lisbon in 2023. This measure allows us to better quantify the impact of tourism on housing demand. The yearly-equivalent number of tourists increased by 24.9 thousand, rising from 23.8 thousand in 2011 to 48.7 thousand in 2022. Aggregate tourism data show that this upward trend continued through 2023 and 2024, suggesting that the high tourist volume in 2022 was not simply a post-COVID-19 rebound.

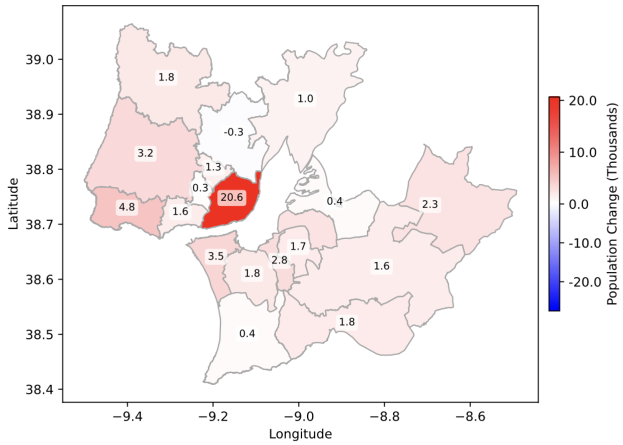

Figure 1: Inflows of foreigners into the Lisbon municipality and peripheries from 2011 to 2021

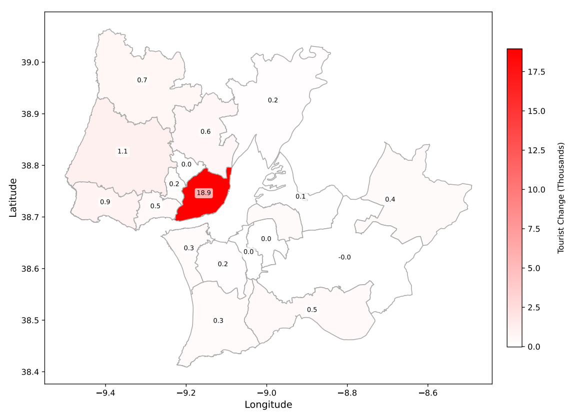

Figure 2: Tourism inflows into the Lisbon municipality and peripheries

Fact 2. *The number of effective housing units in the Lisbon municipality has slightly declined between 2011 and 2021.*

Figure 3 shows that there was a net decrease of 3 thousand family homes ("alojamentos familiares clássicos") in the Lisbon municipality between 2011 and 2021, representing a one percent reduction in total housing units. This decline is primarily due to the limited construction of new houses during this period, which was outpaced by the number of homes that became uninhabitable or were demolished, as well as the conversion of housing units into hotel rooms.

The Lisbon Tourism Association reports that the number of hotel rooms in Lisbon rose from 35.8 thousand in October 2016 to 43.8 thousand in September 2021. Although data before 2016 are unavailable, the increase of 8 thousand rooms is likely a reasonable estimate of capacity growth between 2011 and 2021. The expansion followed a period of sharply reduced investment due to the 2011 European debt crisis, which severely impacted the Portuguese economy (see Eichenbaum, Rebelo, and de Resende, 2017).

To convert the additional 8,000 hotel rooms into equivalent housing units, we divide by the average of 4.5 rooms per home (as reported by INE's 2021 Census on Population and Housing), resulting in an estimated 1,800 housing units. Combining the reduction in family homes with the increase in hotel rooms (measured in equivalent housing units) produces a small but negative net change in the housing stock (1.8 - 3.0 = -1.2 thousand).

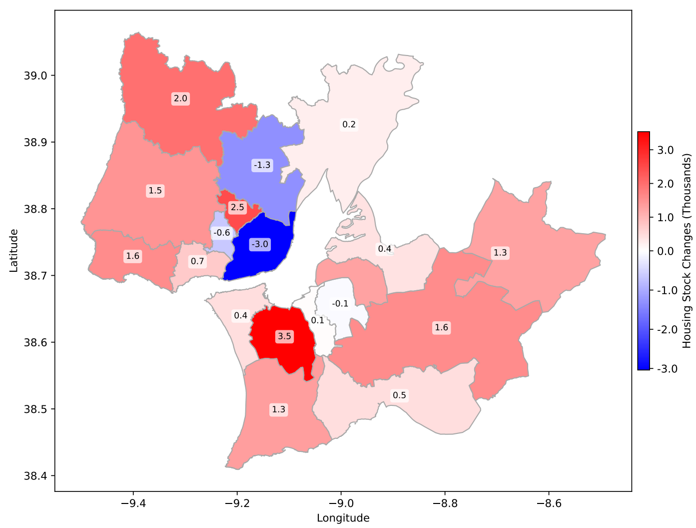

Figure 3: Changes in the stock of family house units between 2011 and 2021 in the municipalities Lisbon metropolitan area

Fact 3. *There was a significant rise in inflation-adjusted house prices and rents in Lisbon and peripheral municipalities.*

Figure 4 shows that our measure of inflation-adjusted rents in the Lisbon municipality increased by 41 percent, from 10.2 euros per square meter in 2011 to 14.4 euros per square meter in 2021. Figure 5 shows that, over the same period, our measure of inflation-adjusted housing prices per square meter increased by 25 percent, from 2,950 to 3,701 euros.

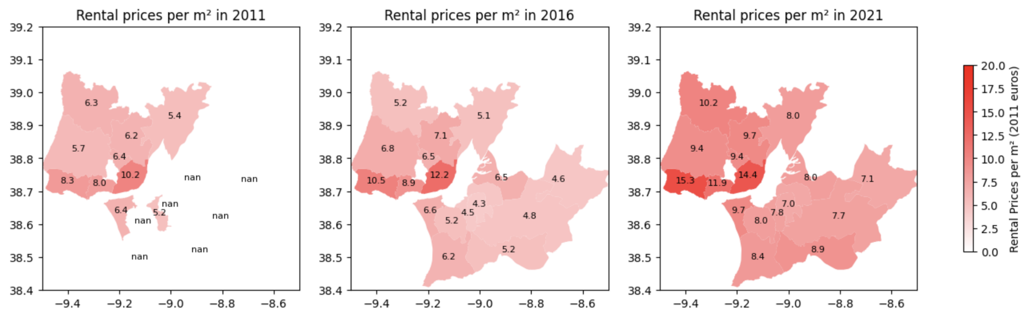

Figure 4: Inflation-adjusted house rents in the municipalities of the Lisbon metropolitan area

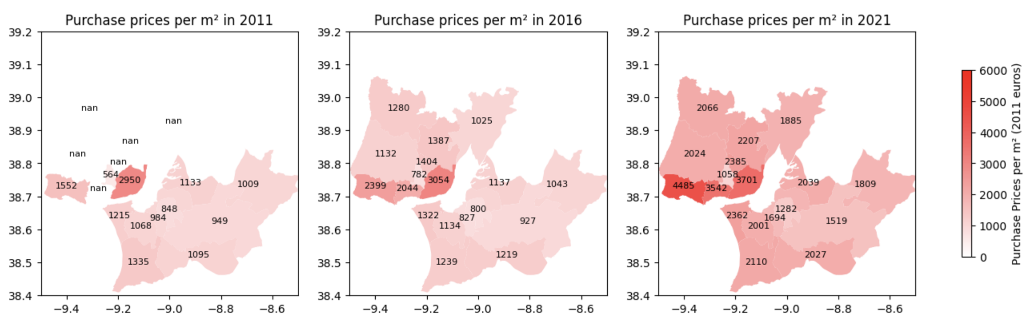

Figure 5: Inflation-adjusted house prices in the municipalities of the Lisbon metropolitan area

Fact 4. *There was a large outflow of domestic residents from the Lisbon municipality.*

Figure 6 shows that 27.5 thousand domestic residents left the Lisbon metropolitan area between 2011 and 2021. This figure also shows that the number of domestic residents living on the outskirts of Lisbon has increased.

Combining the inflow of foreign residents with the outflow of domestic residents, Lisbon's city center saw an overall population increase of almost 20,000 people.

Fact 5. *The number of commuters is substantial. These workers spend a significant amount of time commuting, and this time increases dramatically during rush hour due to traffic congestion.*

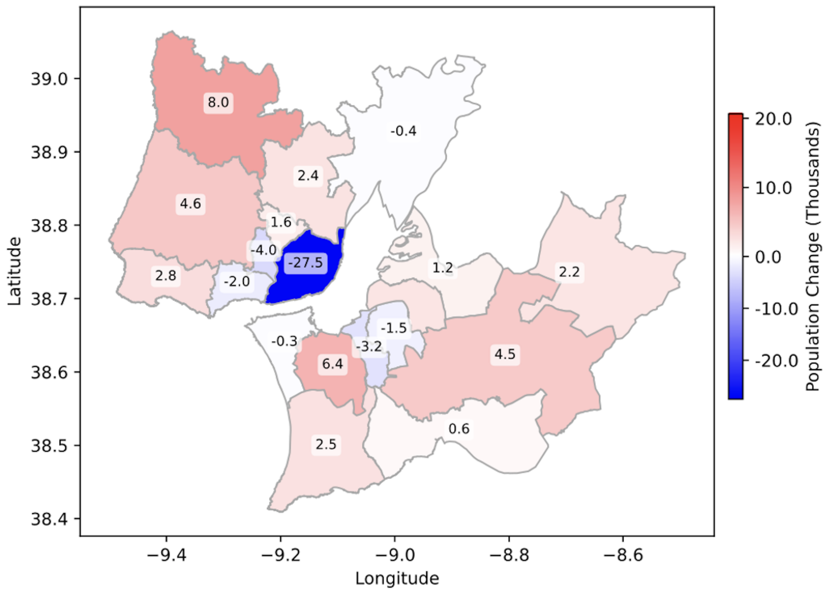

Figure 6: Domestic resident flows for the municipalities of the Lisbon metropolitan area

The city center remains the main hub of economic activity, drawing many daily commuters. Census data shows that nearly 250,000 workers commute to the Lisbon municipality on weekdays. Including non-working commuters, such as students, this number rises to 300,000. This daily influx increases the city center's population by almost 50 percent.

We use data from the Google Maps API to estimate the average weekday commuting time between the Lisbon municipality and the surrounding municipalities. First, we calculate the commuting times between Lisbon and each periphery at 8:00 AM and 5:00 PM. We then use commuting flow data from the 2021 census for the Lisbon metropolitan area to compute a weighted average. The resulting average round-trip commuting time is 81 minutes.

The average weekday round-trip commuting time between the Lisbon municipalities and the peripheries is 50 minutes at 3:00 AM and 81 minutes during rush hour, at 8:00 AM and 5:00 PM. So, commuting at 3:00 a.m., rather than during rush hour, reduces travel time by 38 percent.

Next, we introduce a model that is consistent with these key facts, in which the influx of foreign residents drives urban dynamics. Consistent with the data, we assume that the housing supply is constant. As the number of foreign residents increases, housing prices in the city center rise, leading some domestic residents to move to the periphery. This outflow raises housing prices in those areas. Some displaced residents continue working in the city center, enduring long commutes. For simplicity, our baseline model, presented in Section 4, abstracts from commuting congestion. This congestion effect, along with other extensions, is incorporated in the model presented in Section 6.

4 Competitive equilibrium in the baseline model

We consider a static city model composed of a city center, denoted by c, and a discrete number of peripheral locations, denoted by $p_1, p_2, ..., p_N$. Each location ℓ has an endowment of residential buildings, $\overline{H}_\ell$, and office buildings, $\overline{K}_\ell$, and produces a homogeneous and tradable good combining labor and office buildings. As described below, productivity in location ℓ depends on the local labor supply because of *production/agglomeration externalities*.

The economy is inhabited by a continuum of local workers (*locals*). These locals choose freely where to live and can commute to a different location for work. They derive utility from consuming a traded good and housing services. A large number of foreigners are willing to enter the city if the utility of entering exceeds their outside option. For simplicity, we assume that foreigners locate only in the city center.[7]

Locals Locals are heterogeneous along multiple dimensions. First, they have idiosyncratic taste preference $\xi_{\ell,j}$ for living in location ℓ and working in location j. Let $\boldsymbol{\xi} = \{\xi_{\ell,j}\}$ denote an individual's vector of taste preferences.

Second, individuals have heterogeneous ownership of houses and office buildings. Each individual is endowed with $\overline{h}_\ell \geq 0$ houses in location ℓ, and $\overline{k}_\ell \geq 0$ office buildings in location ℓ. Let $\mathbf{a} \equiv \{\overline{h}_\ell, \overline{k}_\ell\}$ denote the individual's vector of asset holdings in different locations.[8] The non-labor income of a local individual with assets $\mathbf{a}$ is $T_\mathbf{a} = \sum_\ell r_\ell \overline{h}_\ell + \sum_\ell r^K_\ell \overline{k}_\ell$.

Each individual is characterized by the vector $\{\mathbf{a}, \boldsymbol{\xi}\}$. We denote the distribution of asset holdings by $G(\mathbf{a})$. We assume that, for each $\mathbf{a}$, $\boldsymbol{\xi}$ is continuously distributed with support $\mathbb{R}^{(N+1)^2}$ and probabil-

[7]Our results would go through under the weaker assumption that foreign residents disproportionally locate in the city center relative to locals.

[8]Several papers in the spatial literature assume that real estate ownership in a given region is equally distributed among the people who choose to locate in that region. This assumption distorts people's spatial allocation (see Fajgelbaum and Gaubert, 2024). This distortion is absent in our model. However, since we allow for arbitrary correlations between asset holdings and location preferences, the model is rich enough to allow assets in a region to be disproportionately owned by individuals who live there.

ity density function $f_{\mathbf{a}}(\boldsymbol{\xi})$. These idiosyncratic location preferences ensure that all living-working location decision pairs are chosen by a non-zero mass of locals, thus eliminating corner solutions and simplifying the analysis. The distribution of asset holdings satisfies the aggregation equations: $\int \overline{h}_\ell dG(\mathbf{a}) = \overline{H}_\ell$ and $\int \overline{k}_\ell dG(\mathbf{a}) = \overline{K}_\ell$.

A local living in location ℓ and working in location j has utility $\mathcal{U}_{\mathbf{a},\xi} = U_{\mathbf{a},\ell,j} + \xi_{\ell,j}$, which is the sum of two components. The first component is:

$$U_{\mathbf{a},\ell,j} \equiv \overline{u}_{\ell,j} + u\left(c_{\mathbf{a},\ell,j}, h_{\mathbf{a},\ell,j}\right). \tag{1}$$

We refer to this component as "common utility" because it is common to all who live in location ℓ, work in location j, and have asset holdings $\mathbf{a}$. Common utility depends on the satisfaction people derive from consuming traded goods, $c_{\mathbf{a},\ell,j}$, buying housing services, $h_{\mathbf{a},\ell,j}$, and the amenity value of their location choices, $\overline{u}_{\ell,j}$. The second component is an idiosyncratic taste preference $\xi_{\ell,j}$ we describe above.

Locals have one unit of time to allocate to work and commuting. If they live in location ℓ and work in location j, they spend a fraction $t_{\ell,j} \geq 0$ of their time endowment commuting and therefore work only $1 - t_{\ell,j}$ hours. We normalize $t_{\ell,\ell} \equiv 0$ for all ℓ. The budget constraint of a local, with assets $\mathbf{a}$, living in ℓ and working in j is given by:

$$c_{\mathbf{a},\ell,j} + r_\ell h_{\mathbf{a},\ell,j} \leq w_j(1 - t_{\ell,j}) + T_{\mathbf{a}}, \tag{2}$$

where r_ℓ denotes the rental rate on houses and w_j denotes the hourly wage in location j. The price of the traded good is the numeraire.

The solution to the problem of the locals satisfies the budget constraint (2) with equality, as well as the condition:

$$\frac{u_h(c_{\mathbf{a},\ell,j}, h_{\mathbf{a},\ell,j})}{u_c(c_{\mathbf{a},\ell,j}, h_{\mathbf{a},\ell,j})} = r_\ell.$$

Given free mobility, individual i chooses to live in ℓ and work in j if $U_{\mathbf{a},\ell,j} + \xi_{\ell,j} \geq U_{\mathbf{a},\ell',j'} + \xi_{\ell',j'}$, for all ℓ' and j'.[9]

Foreign residents To simplify, we assume that foreign residents strictly prefer to live in the city center. Their problem is to choose consumption, c_f, and housing in the center, h_f, to maximize their utility:

[9]Because idiosyncratic location preferences follow a continuous distribution, the set of individuals who are indifferent between one or more locations has measure zero. Therefore, the way indifferences are resolved is inconsequential.

$$\mathcal{U}_f \equiv \overline{u}_f + u\left(c_f, h_f\right), \tag{3}$$

where $\overline{u}_f$ is the value foreign residents attach to the amenities in the center.

Foreigners bring a fixed endowment of the tradable good, y_f, which they use to pay for consumption and housing services. The foreign residents' budget constraint is:

$$c_f + r_c h_f \leq y_f. \tag{4}$$

The solution to the foreigners' problem satisfies equation (4) with equality, as well as the condition:

$$\frac{u_h(c_f, h_f)}{u_c(c_f, h_f)} = r_c. \tag{5}$$

When choosing not to enter the city, foreigners receive utility u_f^*. This outside option captures the utility they obtain from moving to an alternative city abroad. They only migrate if their participation constraint is satisfied:

$$\mathcal{U}_f \geq u_f^*. \tag{6}$$

Firms' problem Each location has a representative, perfectly-competitive firm. The firm in location j produces the homogeneous tradable good, y_j, by combining labor, l_j, and offices, k_j. The production function is given by

$$y_j = A_j\left(L_j\right) l_j^{\alpha} k_j^{1-\alpha}. \tag{7}$$

Productivity in region j is given by the function $A_j(L_j)$, which depends on total labor supply in location j due to agglomeration (or production) externalities. Locations with larger workforces are more productive because they offer more opportunities for workers to learn from one another. For concreteness, we assume that

$$A_j\left(L_j\right) = \overline{A}_j L_j^{\gamma}. \tag{8}$$

The parameter $\gamma \geq 0$ controls the strength of the agglomeration externality. If $\gamma = 0$, there are no production externalities. The higher γ is, the stronger these externalities are.

The firm hires workers at the wage rate w_j and rents office space at the rental rate r_j^K, earning profits $y_j - w_j l_j - r_j^K k_j$. The firm's optimality conditions are:

$$w_j = \alpha A_j\left(L_j\right)\left(\frac{k_j}{l_j}\right)^{1-\alpha} \quad \text{and} \quad r_j^K = (1-\alpha) A_j\left(L_j\right)\left(\frac{k_j}{l_j}\right)^{-\alpha}. \tag{9}$$

Aggregation and market clearing Let $\pi_{\mathbf{a},\ell,j}$ denote the share of locals who choose to live in ℓ and commute to j for work, conditional on asset level $\mathbf{a}$. So, $\sum_{\ell,j} \pi_{\mathbf{a},\ell,j} = 1$ for all $\mathbf{a}$. By the law of large numbers, the shares are given by $\pi_{\mathbf{a},\ell,j} \equiv \mathbb{P}[U_{\mathbf{a},\ell,j} + \xi_{\ell,j} \geq U_{\mathbf{a},\ell',j'} + \xi_{\ell',j'}, \quad \forall(\ell', j')|\mathbf{a}]$. The total share of individuals who live in region ℓ and work in j is given by $\overline{\pi}_{\ell,j} = \int \pi_{\mathbf{a},\ell,j} dG(\mathbf{a})$.

Using these definitions, we can write the housing-market clearing conditions as

$$\int \sum_j \pi_{\mathbf{a},\ell,j} h_{\mathbf{a},\ell,j} dG(\mathbf{a}) = \overline{H}_\ell \quad \ell \neq c, \quad \text{and} \quad \int \sum_j \pi_{\mathbf{a},c,j} h_{\mathbf{a},c,j} dG(\mathbf{a}) + N_f h_f = \overline{H}_c, \tag{10}$$

and the total labor supply in location j as $L_j = \sum_\ell \overline{\pi}_{\ell,j}(1 - t_{\ell,j})$.

Furthermore, market clearing conditions in the labor and office rental markets imply that $k_j = \overline{K}_j$ and $l_j = L_j$. So, output in location j is given by $y_j = Y_j = A_j \left(L_j\right) L_j^\alpha \overline{K}_j^{1-\alpha}$. The goods market clearing condition is:

$$\int \sum_{\ell,j} \pi_{\mathbf{a},\ell,j} c_{\mathbf{a},\ell,j} dG(\mathbf{a}) + N_f c_f = \sum_j Y_j + N_f y_f. \tag{11}$$

The left-hand side of this equation represents the total consumption of local residents across all living and working locations, combined with the total consumption of foreign residents, $N_f c_f$, where N_f denotes the number of foreign residents. On the right-hand side, we have the total production in each location, Y_j, along with the endowment of goods brought by the foreign residents, $N_f y_f$.

4.1 The welfare impact of an increase in foreign residents

We now use this model to examine the impact of an increase in the number of foreigners, $dN_f > 0$, on the welfare of the local population.[10] We show that the welfare effects of an influx of foreign residents can be decomposed into three terms: (1) the foreign-resident surplus, (2) the agglomeration-externality effect, and (3) the redistribution effect resulting from shifts in the cross-sectional distribution of wages and housing prices.

We define the social welfare function as:

$$\mathcal{W} \equiv \int \lambda_{\mathbf{a},\xi} \mathcal{U}_{\mathbf{a},\xi} f_{\mathbf{a}}(\boldsymbol{\xi}) d\boldsymbol{\xi} dG(\mathbf{a}), \tag{12}$$

[10] We assume that the participation constraint for foreigners (6) is slack, so that this change is feasible.

where $\lambda_{\mathbf{a},\xi} \geq 0$ denotes the welfare weight attributed to agents of type $\mathbf{a}, \boldsymbol{\xi}$. We derive the properties of the optimal policy without taking a stand on the weights $\lambda_{\mathbf{a},\xi}$ chosen by the planner. So, the optimal policy is independent of the planner's preferences for redistribution across the domestic population.

The influx of foreign residents increases the demand for housing in the city center, driving up property prices and leading the local population to reconsider where they live and work. In general equilibrium, these shifts in location choices affect not only housing prices across different regions but also wages and office building rents, through changes in labor supply and the impact of agglomeration externalities.

We now provide sufficient statistics that measure the impact of a marginal increase in foreigners on social welfare. For each variable x, we denote by dx the change associated with an infinitesimal increase in the number of foreign residents in the city center $dN_f > 0$.

Lemma 1. *The change in social welfare is given by*

$$d\mathcal{W} = \int \sum_{\ell,j} \pi_{\boldsymbol{a},\ell,j} \overline{\lambda}_{\boldsymbol{a},\ell,j} u_c\left(c_{\boldsymbol{a},\ell,j}, h_{\boldsymbol{a},\ell,j}\right) \left\{dw_j\left(1-t_{\ell,j}\right) - dr_\ell h_{\boldsymbol{a},\ell,j} + dT_{\boldsymbol{a}}\right\} dG(\boldsymbol{a}), \tag{13}$$

where $\overline{\lambda}_{\boldsymbol{a},\ell,j}$ denotes the conditional average welfare weight on individuals with assets $\boldsymbol{a}$ and location decisions (ℓ, j).

The variable $d\mathcal{W}$ is measured in units of utility, which does not have a natural interpretation. Following Dávila and Schaab (2022), we express welfare changes in a comparable unit by choosing the tradable consumption as the numeraire $d\mathcal{W}_{CE} \equiv \frac{d\mathcal{W}}{\int \sum_{\ell,j} \pi_{\mathbf{a},\ell,j} \overline{\lambda}_{\mathbf{a},\ell,j} u_c(c_{\mathbf{a},\ell,j}, h_{\mathbf{a},\ell,j}) dG(\mathbf{a})}$. We refer to this variable as the "consumption-equivalent" welfare change. Dávila and Schaab (2022) show that, in general, welfare changes can be decomposed into an *efficiency* and a *redistribution* component. In our model, this decomposition is given by

$$d\mathcal{W}_{CE} = \underbrace{\int \sum_{\ell,j} \pi_{\mathbf{a},\ell,j} \left\{dw_j\left(1-t_{\ell,j}\right) - dr_\ell \times h_{\mathbf{a},\ell,j} + dT_{\mathbf{a}}\right\} dG(\mathbf{a})}_{d\mathcal{W}_{CE}^{\text{Efficiency}}} + \underbrace{\mathbb{COV}^{\mathbf{\Pi}}\left(\omega_{\mathbf{a},\ell,j}\,,\, dw_j\left(1-t_{\ell,j}\right) - dr_\ell \times h_{\mathbf{a},\ell,j} + dT_{\mathbf{a}}\right)}_{d\mathcal{W}_{CE}^{\mathcal{R}}},$$

where $\omega_{\mathbf{a},\ell,j} \equiv \overline{\lambda}_{\mathbf{a},\ell,j} u_c(c_{\mathbf{a},\ell,j}, h_{\mathbf{a},\ell,j}) / \int \sum_{\ell,j} \overline{\lambda}_{\mathbf{a},\ell,j} \pi_{\mathbf{a},\ell,j} u_c(c_{\mathbf{a},\ell,j}, h_{\mathbf{a},\ell,j}) dG(\mathbf{a})$.[11]

[11] In this definition, the cross-sectional covariance between two variables is constructed as follows:

$$\mathbb{COV}^{\mathbf{\Pi}}(x,y) \equiv \int \sum_{\ell,j} \pi_{\mathbf{a},\ell,j} x_{\mathbf{a},\ell,j} y_{\mathbf{a},\ell,j} dG(\mathbf{a}) - \int \sum_{\ell,j} \pi_{\mathbf{a},\ell,j} x_{\mathbf{a},\ell,j} dG(\mathbf{a}) \int \sum_{\ell,j} \pi_{\mathbf{a},\ell,j} y_{\mathbf{a},\ell,j} dG(\mathbf{a}).$$

Analogously, for objects indexed by (ℓ, j) only, we write $\mathbb{COV}^{\overline{\mathbf{\Pi}}}(x,y) \equiv \sum_{\ell,j} \overline{\pi}_{\ell,j} x_{\ell,j} y_{\ell,j} - \left(\sum_{\ell,j} \overline{\pi}_{\ell,j} x_{\ell,j}\right)\left(\sum_{\ell,j} \overline{\pi}_{\ell,j} y_{\ell,j}\right)$.

The efficiency term is the sum of individuals' willingness-to-pay (which can be negative) for this change in the number of foreign residents. So, it corresponds to the Kaldor-Hicks efficiency criterion. A positive efficiency term implies that the new equilibrium is Kaldor-Hicks superior to the initial one.

The decomposition shows that the consumption-equivalent welfare change equals this Kaldor-Hicks efficiency component plus a correction for redistribution of resources across individuals. Among other effects, the redistribution term accounts for differences in individuals' exposure to capital gains on houses and office buildings. The effect of the increase in house rents of an individual's location ℓ is given by

$$dr_\ell \times \left[\overline{h}_{\mathbf{a},\ell} - h_{\mathbf{a},\ell,j}\right].$$

This expression shows the central importance of an individual's net position in assessing the welfare impact of increases in asset prices. Suppose individuals are net buyers of housing services, $h_{\mathbf{a},\ell,j} > \overline{h}_{\mathbf{a},\ell}$, then they are harmed by the increase in rents. If individuals are net sellers $h_{\mathbf{a},\ell,j} < \overline{h}_{\mathbf{a},\ell}$, then they benefit from the increase in house prices.[12]

Suppose that preferences are quasi-linear, $u(c,h) = c + v(h)$, and the planner assigns equal welfare weights to all individuals, $\lambda_{\mathbf{a},\xi} = 1$. Then, $\omega_{\ell,j} = 1$ for all ℓ, j and the redistribution component is equal to zero, $d\mathcal{W}^{\mathcal{R}}_{CE} = 0$. With equal welfare weights and quasi-linear utility, the distribution of consumption is irrelevant for social welfare because the marginal social value of consumption is identical for all individuals. So, in this extreme case, social welfare would only be summarized by the efficiency term.

We now take a closer look at the efficiency gains resulting from the entry of foreign residents. Proposition 1 states that these effects can be decomposed into two interpretable effects.

Proposition 1. *The efficiency welfare gains can be written as the sum of two terms: $d\mathcal{W}^{Efficiency}_{CE} = \mathcal{FS} + \mathcal{PE}$. The first term is the foreign-residents surplus, $\mathcal{FS}$, and it is given by:*

$$\mathcal{FS} \equiv dr_c \times N_f h_f.$$

The second term is the production or agglomeration externality term, $\mathcal{PE}$, and it is given by:

$$\mathcal{PE} \equiv \gamma \times \mathbb{COV}^{\overline{\Pi}}\left[\frac{Y_j}{L_j}\left(1 - t_{\ell,j}\right), \frac{d\overline{\pi}_{\ell,j}}{\overline{\pi}_{\ell,j}}\right] = \gamma \sum_{\ell,j} \overline{\pi}_{\ell,j} \frac{Y_j}{L_j}\left(1 - t_{\ell,j}\right) \frac{d\overline{\pi}_{\ell,j}}{\overline{\pi}_{\ell,j}}.$$

[12] This result is a special case of the asset-price redistribution channel, which is the focus of Fagereng, Gomez, Gouin-Bonenfant, Holm, Moll, and Natvik (2024). In a different setting, Dávila and Korinek (2018) also emphasize the role of pecuniary redistribution in equilibrium models.

See Appendix B.2 for the proof.

The foreign resident surplus equals the capital gains from selling houses to foreigners. As the arrival of new foreign residents increases the demand for housing in the city center, house rents rise ($dr_c > 0$), leading to higher rental income from foreign tenants and increasing the welfare of local homeowners. This effect is the foreign-resident surplus. This surplus is similar to the immigration surplus discussed in the immigration literature (e.g., Borjas, 1995 and Guerreiro et al., 2020), which results from an increase in the income accruing to fixed factors such as land from an increase in the labor force.

The interpretation of $\mathcal{PE}$ is as follows. In general equilibrium, the entry of foreign residents causes locals to relocate away from the city center. This shift in living arrangements is associated with changes in work location, which redistribute labor across different areas. If this relocation causes labor to shift from more productive regions to less productive ones, aggregate productivity declines, resulting in a welfare loss. This decrease in productivity may occur because labor moves toward less efficient peripheries or because locals now have to commute to the city center, reducing their total labor supply.

In summary, the entry of foreign residents has three key effects. First, it creates gains from trade associated with selling houses to foreign residents, generating the foreign resident surplus. Second, by reallocating labor supply to less productive regions or increasing commuting time, foreign residents' entry can negatively impact aggregate productivity via agglomeration externalities and reduce welfare. Finally, the general equilibrium effects of wages and house rents have redistributive consequences for residents across different parts of the city.

5 Mirrleesian optimal policy

Our analysis of the impact of foreign residents on the competitive equilibrium raises two questions. First, should the entry of foreigners be restricted when the foreign-resident surplus is smaller than the production externality? Second, should foreign home purchases be taxed to internalize the agglomeration externality? To address these questions, we now study the optimal public policy in our model and find that the answer to both questions is no.

In the spirit of Mirrlees (1971), we do not impose ex-ante restrictions on the set of instruments available to the government. Instead, we work directly with the informational constraints that arise because agent types are unobservable. We assume that the planner can differentiate between locals and foreigners, but cannot observe idiosyncratic tastes for locations. The planner can access information on individuals' home and work locations, asset holdings, and purchase decisions. The planner may find it optimal to condition allocations on asset holdings when either the welfare weights influencing the planner's objectives or the distribution of location preferences are correlated with those holdings (tagging). In other words, the planner can base an individual's allocations on their decisions and asset holdings, but not on their specific location preferences. If the distribution of asset holdings is independent of the location preferences and welfare weights, then it is optimal to treat all locals the same, conditional on their location choices.

Our results regarding the optimal treatment of foreigners would still hold even if the planner cannot condition allocations on initial asset holdings $\mathbf{a}$. However, in this case, the planner can condition locals' allocations only on their location decisions (ℓ, j), so the allocations and the transfers we describe below must be equal across all $\mathbf{a}$.

Incentive compatibility The planner designs allocations that give agent $(\mathbf{a}, \boldsymbol{\xi})$. However, since the planner cannot condition allocations on $\boldsymbol{\xi}$, all individuals with the same level of assets $\mathbf{a}$ who live and work in the same region can costlessly mimic this agent. It follows that, in any incentive compatible allocation, all individuals with assets $\mathbf{a}$, who live in ℓ, and work in j obtain the same level of common utility $U_{\mathbf{a},\ell,j}$ and have the same consumption, $c_{\mathbf{a},\ell,j}$, and housing allocations, $h_{\mathbf{a},\ell,j}$.

The incentive constraints resulting from this informational problem also require that location decisions be privately optimal based on the allocations determined by the planner. It follows that individual $(\mathbf{a}, \boldsymbol{\xi})$ chooses to live in location ℓ and work in location j if $U_{\mathbf{a},\ell,j} + \xi_{\ell,j} = \max_{\ell',j'}\{U_{\mathbf{a},\ell',j'} + \xi_{\ell',j'}\}$. In the Appendix, we show that these incentive compatibility constraints imply that

$$\pi_{\mathbf{a},\ell,j} = \mathbb{P}[U_{\mathbf{a},\ell,j} + \xi_{\ell,j} \geq U_{\mathbf{a},\ell',j'} + \xi_{\ell',j'} \quad \forall \ell', j' | \mathbf{a}], \tag{14}$$

plus the fact that welfare is given by $\mathcal{W}(\mathbf{U}) = \int \lambda_{\mathbf{a},\boldsymbol{\xi}} \max\{U_{\mathbf{a},\ell,j} + \xi_{\ell,j}\} f(\boldsymbol{\xi}) d\boldsymbol{\xi} dG(\mathbf{a})$. In words, incentive compatibility is equivalent to the population shares being induced by private optimality under free mobility.[13]

[13]This result is similar to the representation theorem in Donald et al. (2024), but it holds for general welfare functions.

We can think of the planner as directly designing allocations $\{c_{\mathbf{a},\ell,j}, h_{\ell,j}\}$ which determine $U_{\mathbf{a},\ell,j}$, subject to $\pi_{\mathbf{a},\ell,j}$ satisfying these incentive compatibility constraints. The Mirrleesian optimal allocations can be computed as follows. The planner maximizes the welfare function (12), subject to the resource constraints for goods, (11), where $L_j \equiv \sum_\ell \overline{\pi}_{\ell,j}(1-t_{\ell,j})$, the resource constraint for houses in each location, (10), the location-decisions constraints, (14), and the foreign-resident participation constraint, (6). We refer to this problem as the *Mirrleesian program*.

5.1 Decentralization with taxes

We present our main results in terms of the instruments that decentralize the optimal allocation. The decentralization we consider is a competitive equilibrium in which people may be taxed on their housing purchases and receive lump-sum taxes or transfers. For locals, these instruments are restricted to depend solely on their assets and observable location decisions, whereas for foreigners, the instruments can be chosen independently.

We let $(1+\tau^h_{\mathbf{a},\ell,j})r_\ell$ denote the effective rent paid by locals who live in location ℓ, j and $(1+\tau^h_f)r_c$ denote the effective rent paid by foreigners in the city center, which satisfy

$$(1+\tau^h_{\mathbf{a},\ell,j})r_\ell = \frac{u_h(c_{\mathbf{a},\ell,j}, h_{\mathbf{a},\ell,j})}{u_c(c_{\mathbf{a},\ell,j}, h_{\mathbf{a},\ell,j})} \quad \text{and} \quad (1+\tau^h_f)r_c = \frac{u_h(c_f, h_f)}{u_c(c_f, h_f)}.$$

The effective rent is an *after-tax price*, i.e., $\tau^h_{\mathbf{a},\ell,j}$ and τ^h_f denote the tax on housing purchases.

We define this fee as

$$\mathcal{T}_f \equiv y_f - c_f - (1+\tau^h_f)r_c h_f. \tag{15}$$

Foreigners pay an entry fee if their income exceeds their consumption and housing expenditures, i.e., if $\mathcal{T}_f > 0$. The total proceeds from taxing foreigners are $\Theta_f = N_f \tau^h_f r_c h_f + N_f \mathcal{T}_f$.

Finally, we define the transfers to individuals living in location ℓ and working in location j as

$$\mathcal{T}_{\mathbf{a},\ell,j} \equiv c_{\mathbf{a},\ell,j} + (1+\tau^h_{\mathbf{a},\ell,j})r_\ell h_{\mathbf{a},\ell,j} - w_j(1-t_{\ell,j}), \tag{16}$$

where $\mathcal{T}_{\mathbf{a},\ell,j}$ includes the rents from housing and office buildings owned by household **a**. Wages and office rents are given by equations (9), respectively, replacing $l_j = L_j$ and $k_j = \overline{K}_j$. By construction, adding up all transfers yields:

$$\int \sum_{\ell,j} \pi_{\mathbf{a},\ell,j} \mathcal{T}_{\mathbf{a},\ell,j} dG(\mathbf{a}) = \sum_{\ell} r_\ell \overline{H}_\ell + \sum_j r_j^K \overline{K}_j + \int \sum_{\ell,j} \pi_{\mathbf{a},\ell,j} \tau^h_{\mathbf{a},\ell,j} r_\ell h_{\mathbf{a},\ell,j} dG(\mathbf{a}) + \Theta_f.$$

The planner may also choose N_f subject to the foreigners' participation constraint (6). If $\mathcal{U}_f > u_f^*$, this allocation is only implementable if the planner forces a quantity restriction, since more foreigners would be willing to enter the city. We say that the equilibrium features no quota restrictions if the participation constraint is satisfied with equality.

Proposition 2. *If an allocation satisfies the constraints of the Mirrleesian program, then it can be decentralized as a competitive equilibrium with the appropriate choices of $\tau^h_{\mathbf{a},\ell,j}$, τ^h_f, $\mathcal{T}_{\mathbf{a},\ell,j}$, $\mathcal{T}_f$ and N_f.*

The proof follows directly from the observation that, by construction, the allocations satisfy the market clearing conditions and are consistent with the locals' location decisions. Under this tax system, the consumption and housing decisions, $c_{\ell,j}$ and $h_{\ell,j}$, are privately optimal for locals living in location ℓ, j, and the consumption and housing choices, c_f and h_f, are privately optimal for the foreigners that move to the city.

5.2 Optimal policy towards foreigners

We solve the Mirrleesian program in two steps. First, we take the number of foreigners N_f as given and solve for the remaining quantities. Then, we characterize the necessary conditions for the optimal number of foreign residents N_f.

Optimal policy for a fixed number of foreign residents Proposition 3 summarizes the optimal tax treatment of foreign residents in the Mirrleesian optimum, holding the number of foreign residents fixed.

Proposition 3. *Suppose that the number of foreign residents is fixed. In the decentralization of the optimal allocation, the following conditions hold:*

1. *Foreigners' housing purchases are not subject to taxes, $\tau^h_f = 0$.*

2. *There is an optimal entry fee on foreigners, $\mathcal{T}_f$, which sets their utility equal to their outside option,*

$$u_f^* = \overline{u}_f + u(y_f - r_c h_f - \mathcal{T}_f, h_f),$$

where h_f is the optimal housing choice for foreigners. So, in the optimum, there are no quota restrictions on the entry of foreigners.

The planner's optimal strategy is to ensure that the marginal rates of substitution between housing and consumption are equal for all locals and foreigners living in the same area. This condition implies that foreigners and locals in the city center pay the same house prices, r_c, so the optimal tax on foreign home purchases is zero. This result follows from standard public-finance principles: it is more efficient to implement a discriminatory lump-sum tax than to distort the allocation of goods through taxation.

Second, it is optimal to impose a lump-sum entry fee on foreigners that equates their utility to their outside option, making them indifferent about moving. The welfare function includes only the utility of the local population, so it is optimal to tax the gains foreigners derive from moving to the city center and redistribute them to the locals.

At the optimum, foreigners' utility always equals their outside option, making them indifferent about moving. This result implies that implementing the optimal policy is consistent with the free mobility of foreigners into the country. Consequently, it is not optimal to impose binding quotas on the number of foreigners who can enter the home country. The intuition for this result is that it is always better to control the inflow of foreign residents through an entry fee rather than a quota system. The entry fee generates additional tax revenue that can be redistributed toward locals.

Relation to the optimal tariff literature We can interpret the sales of houses to foreigners as exports paid for in units of the tradable consumption good. The home country is a monopolist on the sale of houses to foreigners, yet our model implies that the optimal trade tariff is zero. At first glance, this conclusion contradicts the classical result that it is optimal to use a trade tariff to manipulate terms of trade.

This apparent contradiction arises because, unlike the standard trade literature, we impose no exogenous restrictions on the policy instruments available to the home country. In particular, in our model, the government can impose a lump-sum tax on foreigners, a policy tool typically excluded from traditional trade models.

In Appendix E, we employ a standard international trade model to examine how our findings relate to the trade literature. We show that the optimal policy is to set tariffs to zero and levy a lump-sum tax on foreigners, which can be interpreted as a rights-to-trade fee. This fee captures the gains foreigners derive from trade. Additionally, we show that when a lump-sum tax is not feasible, it is optimal to impose a tariff.

This setup is analogous to a monopolist using a two-part tariff: it sets the price equal to marginal cost and charges a fixed fee that extracts all consumer surplus. Similarly, in our model, it is optimal to refrain from taxing foreigners' housing purchases and instead impose a lump-sum tax on foreigners.

The optimal number of foreign residents We now discuss the policies that optimize the number of foreign residents. Let $W^*(N_f)$ denote the welfare associated with the optimal number of foreign residents, N_f. Using an envelope argument and the results derived in the previous section, we find that the marginal effect of an additional foreigner on welfare is given by:

$$\frac{dW^*(N_f)}{dN_f} / \mu^C = y_f - c_f - r_c h_f = \mathcal{T}_f,$$

where μ^C denotes the planner's shadow value for consumption resources. The marginal effect of an additional foreigner on welfare is equal to the marginal value of selling h_f houses and buying $y_f - c_f$ additional consumption goods.

The difference between the value of additional consumption goods and the value of houses sold equals the entry fee $\mathcal{T}_f$. If the fee is positive, $\mathcal{T}_f > 0$, then letting in an additional foreigner strictly increases welfare. Conversely, if the fee is negative, $\mathcal{T}_f < 0$, allowing in an additional foreigner strictly decreases welfare. Intuitively, suppose the value of the consumption goods brought in by the marginal foreigner exceeds the value of the houses they purchase. In that case, it is optimal to let an additional foreigner enter the home country.

Following this logic, the planner allows additional foreigners to enter the economy until the entry fee, which sets their utility equal to the outside option, is zero:

$$\frac{dW^*(N_f)}{dN_f} = 0 \Leftrightarrow \mathcal{T}_f = 0.$$

This surprising result implies that the optimal policy toward foreigners is laissez-faire. From the previous section, we know that it is optimal not to tax foreign housing purchases, and there are no quotas limiting the entry of foreign residents. Here, we also show that the optimal number of foreigners is obtained when the entry fee is zero. In other words, foreign entry should be free and undistorted. These results are summarized in the following proposition.

Proposition 4. *In the decentralization of the optimal allocation, the policy towards foreign residents is laissez-faire:*

1. *Taxes on foreigners' housing purchases are zero,* $\tau_f^h = 0$.
2. *Entry fees are zero,* $\mathcal{T}_f = 0$.
3. *There are no quotas/restrictions on foreign entry.*

One important aspect of these optimal policies is that they do not depend on specific assumptions regarding the utility function or the distribution of location preferences.

International-trade interpretation From an international trade perspective, this result states that the optimal number of trading partners (foreigners) is such that the gains from trading with the marginal partner are zero. This policy maximizes the gains from trade in the home country and, therefore, maximizes welfare. In Appendix E, we further elaborate on the relation between our results and those obtained in a standard trade model.

Public-finance interpretation From a public-finance perspective, these results can be interpreted as the optimality of production efficiency (Diamond and Mirrlees, 1971). At an abstract level, foreigners can be interpreted as a technology that converts houses into consumption goods. In the previous section, we assumed that N_f was fixed, so the entry fee did not distort the number of entering foreigners. When the number of foreigners is endogenous, it is not optimal to distort their inflow, so the optimal entry fee is zero.

Surprisingly, production efficiency remains optimal even in the presence of externalities. This result arises because the externalities do not directly depend on the number of foreign residents but only on the local labor supply in each location. The Mirrleesian planner has enough instruments to get locals to internalize these agglomeration effects. As shown in the previous section, these instruments take the form of higher transfers for individuals with location decisions that yield above-average labor income and lower transfers for individuals with location decisions that yield below-average labor income. Production efficiency is no longer optimal when foreigners contribute directly to the externalities. In Section 6, we expand our analysis to examine a broader range of potential impacts from foreign residents and discuss how our baseline results are affected.

5.3 Optimal place-based transfers for locals

The following proposition provides sufficient statistics to calculate the optimal place-based tax/transfer policies required to implement the optimal solution.

Proposition 5. *In the decentralized optimal allocation, housing taxes are zero for all locals:* $\tau^h_{\boldsymbol{a},\ell,j} = 0$*. Transfers to locals have three components:*

$$\mathcal{T}_{\boldsymbol{a},\ell,j} = \Xi_{\boldsymbol{a}} + \Xi^{\mathcal{PE}}_{\boldsymbol{a},\ell,j} + \Xi^{\mathcal{R}}_{\boldsymbol{a},\ell,j}, \tag{17}$$

where

1. *the common term is* $\Xi_{\boldsymbol{a}}$ *is a common transfer to all individuals with asset holdings* $\boldsymbol{a}$ *which satisfies*

$$\int \Xi_{\boldsymbol{a}} dG(\boldsymbol{a}) = \underbrace{\sum_{\ell} r_\ell \overline{H}_\ell + \sum_j r^K_j \overline{K}_j}_{\text{Rents on houses and offices}} .$$

2. *the production-externality correction term is*

$$\Xi^{\mathcal{PE}}_{\boldsymbol{a},\ell,j} \equiv \gamma \left\{ \frac{Y_j}{L_j}\left(1 - t_{\ell,j}\right) - \sum_{\ell,j} \pi_{\boldsymbol{a},\ell,j} \frac{Y_j}{L_j}(1 - t_{\ell,j}) \right\}$$

3. *the redistribution-correction term*

$$\Xi^{\mathcal{R}}_{\boldsymbol{a},\ell,j} \equiv \mu^{IC}_{\boldsymbol{a},\ell,j} - \sum_{\ell,j} \pi_{\boldsymbol{a},\ell,j} \mu^{IC}_{\boldsymbol{a},\ell,j}$$

   is given by the difference of the Lagrange multiplier for each location choice $(\boldsymbol{a}, \ell, j)$ *on the incentive compatibility constraint relative to its average conditional on* $\boldsymbol{a}$*.*

Proposition 5 shows that locals' housing purchases are not taxed in the decentralization of the optimum. This result follows from the well-known public-finance principle that uniform commodity taxation is optimal (see Atkinson and Stiglitz, 1976).

The optimal transfers in this Mirrleesian setting can be decomposed into three terms: (1) a common transfer to all individuals with asset holdings **a**, (2) a production-externality correction term, and (3) a redistribution-correction term. We now discuss each term.

Common transfer The optimal transfers to natives consist of three components. The first component is a common non-distortionary transfer provided to all individuals with asset holdings $\mathbf{a}$. This common transfer, $\Xi_{\mathbf{a}}$, enables redistribution across groups with different asset holdings in a non-distortionary and unrestricted way. The entry of foreign residents generates three effects: (1) a surplus from foreign residents, (2) a production externality, and (3) a redistribution effect from capital gains on houses and offices. The first effect is always positive, while the second is corrected through the targeted transfers. The third effect can be positive or negative but averages to zero across groups. Therefore, the welfare gains from foreign entry can always be redistributed using $\Xi_{\mathbf{a}}$.

In the model, capital gains can be redistributed through lump-sum taxes and transfers. In practice, this redistribution can be implemented by taxing capital gains on housing and transferring the revenue to those with below-average property holdings. In a static model like ours, this tax does not distort individuals' decisions. In a dynamic setting, capital gain taxes remain non-distortionary as long as investment expenses can be deducted from the tax base (see Abel, 2007).

Production-externality correction term The second term represents the correction for production externalities. When agglomeration externalities are present, individual location choices are suboptimal from a social standpoint (see also Fajgelbaum et al., 2019, Rossi-Hansberg et al., 2019, and Fajgelbaum and Gaubert, 2020, 2024). The Mirrleesian planner implements transfers to correct these externalities, encouraging individuals to move to locations where their labor productivity exceeds the average. The magnitude of this second term is proportional to γ, the elasticity that controls the importance of the agglomeration effects.

Redistribution-correction term The third term reflects the correction associated with redistribution within a group with asset level $\mathbf{a}$. Due to its informational disadvantage, the planner can address the agglomeration externality only by designing transfers that alter the distribution of consumption across locations. As a result, the Mirrleesian planner faces an equity-efficiency trade-off: the transfers that correct the production externality induce regional variation in consumption. Because people differ in their marginal valuations of consumption, regional variation in consumption affects social welfare. This effect incentivizes the planner to deviate from the transfers that maximize efficiency.

To illustrate how this term relates to redistribution among locals, consider a scenario where the utility function is quasi-linear, $u(c,h) = c + v(h)$, and the welfare weights are equal, $\lambda_{\mathbf{a},\xi} = 1$. In this case, the redistribution term vanishes, since welfare is independent of how consumption is distributed among locals. The following corollary highlights that, under these conditions, the optimal Mirrleesian plan focuses entirely on maximizing economic efficiency, with no need for redistribution adjustments.

Corollary 1. *Assume that (i) preferences are quasi-linear, $u(c,h) = c + v(h)$, and (ii) welfare weights are homogeneous, $\lambda_{\boldsymbol{a},\xi} = 1$. Then, the optimal transfers to locals are given by (17), with $\Xi^{\mathcal{R}}_{\boldsymbol{a}} = 0$.*

In other settings, determining the precise value of the redistribution term can be more complex. This term, which depends on the shadow costs required to satisfy the incentive compatibility constraints, can be difficult to interpret.

We now provide a sufficient condition so that the redistribution component can be expressed in terms of the dispersion of marginal utilities of consumption. This condition requires restrictions on the migration elasticity, which are standard in the spatial economics literature. Suppose that for each $\mathbf{a}$, $\xi_{\ell,j}$ is distributed according to a type-I extreme value distribution with location parameter 0 and scale parameter $\eta_{\mathbf{a}}^{-1} > 0$ plus a constant $\delta_{\mathbf{a},\ell,j}$. In this case, the share of individuals who choose to live in location ℓ and work in j is given by

$$\pi_{\mathbf{a},\ell,j} = \frac{e^{\eta_{\mathbf{a}}(\delta_{\mathbf{a},\ell,j}+U_{\mathbf{a},\ell,j})}}{\sum_{\ell',j'} e^{\eta_{\mathbf{a}}(\delta_{\mathbf{a},\ell',j'}+U_{\mathbf{a},\ell',j'})}},$$

and the parameter $\eta_{\mathbf{a}}$ disciplines the migration elasticity. As $\eta_{\mathbf{a}} \to 0$ people become insensitive to utility differences across locations, i.e., the migration elasticity is zero. Conversely, as $\eta_{\mathbf{a}} \to \infty$, $\pi_{\mathbf{a},\ell,j} > 0$ if and only if $\delta_{\mathbf{a},\ell,j} + U_{\mathbf{a},\ell,j} \in \max_{\ell',j'}\{\delta_{\mathbf{a},\ell',j'} + U_{\mathbf{a},\ell',j'}\}$, i.e., people's location decisions are infinitely sensitive to utility differences.

Corollary 2. *Suppose that, for each $\boldsymbol{a}$, $\xi_{\ell,j}$ is i.i.d. type-I extreme value distribution with parameters $(0, \eta_{\boldsymbol{a}}^{-1})$ plus a constant $\delta_{\boldsymbol{a},\ell,j}$. Then,*

$$\Xi^{\mathcal{R}}_{\boldsymbol{a},\ell,j} = \eta_{\boldsymbol{a}}^{-1}\left(\frac{\overline{\lambda}_{\boldsymbol{a},\ell,j}}{\overline{\lambda}_{\boldsymbol{a}}\left[\sum_{\ell,j}\pi_{\boldsymbol{a},\ell,j}\left[u_c\left(c_{\boldsymbol{a},\ell,j},h_{\boldsymbol{a},\ell,j}\right)\right]^{-1}\right]^{-1}} - \frac{1}{u_c\left(c_{\boldsymbol{a},\ell,j},h_{\boldsymbol{a},\ell,j}\right)}\right). \tag{18}$$

Corollary 2 connects the redistribution term to the dispersion in the marginal utility of consumption. We see that people with location decisions (ℓ, j) receive positive transfers if the marginal valuation of resources is higher than the (harmonic) average $\overline{\lambda}_{\mathbf{a},\ell,j} u_c(c_{\mathbf{a},\ell,j}, h_{\mathbf{a},\ell,j}) > \overline{\lambda}_{\mathbf{a}} \left[\sum_{\mathbf{a},\ell,j} \pi_{\mathbf{a},\ell,j} u_c\left(c_{\mathbf{a},\ell,j}, h_{\mathbf{a},\ell,j}\right)^{-1}\right]^{-1}$.

Recall that $\Xi^{\mathcal{R}}_{\mathbf{a},\ell,j}$ measures the extent to which the planner deviates from the solution that maximizes efficiency. Corollary 2 emphasizes that this deviation depends on $\eta_{\mathbf{a}}$, the parameter that determines the elasticity of location choices. As $\eta_{\mathbf{a}} \to \infty$, the redistribution term goes to zero. When the migration elasticity is high, using location choices as a basis for redistribution is costly because it generates large demographic shifts. This is the benchmark studied in Fajgelbaum and Gaubert (2020), where the optimal transfers coincide with the Pigouvian principle. Instead, when the migration elasticity is low, it is less costly to implement location-based redistribution because the population's spatial distribution is less responsive to transfers.[14]

6 Extensions of the baseline model

In this section, we extend the baseline model to include four issues frequently discussed in policy debates.

The first extension makes commuting time endogenous. In the baseline model, the time spent commuting between two locations is constant. In practice, commuting time tends to increase with the number of commuters. This extension introduces a *commuting externality*, which affects both the welfare costs associated with the entry of foreign residents and the optimal transfers required to correct externalities.

The second extension incorporates the possibility of remote work, allowing locals to work either onsite at an office or remotely from home.[15] Since the Covid-19 pandemic, remote work has become ubiquitous, contributing to a significant increase in the number of foreign residents. Remote work allows individuals to work in the city center without incurring commuting costs. For this reason, this extension alters the welfare impact of foreign residents' entry and influences the design of optimal transfers.

The third extension involves endogenizing the amenity value that foreigners place on living in the city center. We assume that foreign residents derive utility from the authenticity of the city center, meaning they value the presence of locals in the area. This authenticity reflects various non-market attributes, such as

[14] This result echoes the findings in Gaubert et al. (2021), who emphasize the role of the migration elasticity in designing optimal redistribution policies.

[15] For a dynamic theory of remote work and city structure where agglomeration forces can lead to multiple equilibria, see Monte, Porcher, and Rossi-Hansberg (2023).

cultural heritage and traditions. While this authenticity externality does not directly affect the welfare costs of the foreign influx, it creates an additional incentive to encourage locals to reside in the city center.

The fourth extension regards the impact of foreign residents on the amenity value that locals experience by living in the city center. This amenity externality can be positive, for example, if locals appreciate the increased cultural diversity brought by foreigners. However, it can also be negative if the presence of foreigners makes the center less attractive to locals. This extension can also capture the effects of congestion on public goods caused by the influx of foreigners (see Guerreiro et al., 2020). As we discuss next, we can correct this externality by charging foreigners an entry fee.

Lastly, we consider the case in which the elasticity of the supply of foreign residents is finite by allowing the foreigners' outside options to vary with the number of foreigners entering the city.

6.1 The competitive equilibrium

In this section, we describe the environment and the competitive equilibrium.

6.1.1 Local households

As in the baseline model, locals choose where to live, ℓ, and where to work, j. They can also choose their work arrangement, e. This work arrangement can take two forms: o for office/onsite work or h for remote work from home. A local who lives in ℓ and works in j with work arrangement e, has utility $U_{\mathbf{a},\ell,j,e} + \xi_{\ell,j,e}$.

We assume that people's choices about the location of their residence, workplace, and whether to work remotely or onsite are influenced by idiosyncratic taste preferences, $\xi_{\ell,j,e}$. Their common utility is given by

$$U_{\mathbf{a},\ell,j,e} \equiv \overline{u}_{\ell,j,e} + u(c_{\mathbf{a},\ell,j,e}, h_{\mathbf{a},\ell,j,e}). \tag{19}$$

Amenities externality To model the effect that the entry of foreigners may have on the amenity value that locals derive from each location, we assume that the amenity value directly depends on N_f: $\overline{u}_{\ell,j,e} = \phi_{\ell,j,e}\left(N_f\right)$. If $\phi'_{\ell,j,e}\left(N_f\right) > 0$, then the entry of foreign residents increases the attractiveness of location choices ℓ, j for employment status e. If $\phi'_{\ell,j,e}\left(N_f\right) < 0$, then the entry of foreign residents reduces the attractiveness of location choices ℓ, j for employment status e.

Budget constraint in the competitive equilibrium A local living in ℓ and working in j with work arrangement e and asset level $\mathbf{a}$ faces the budget constraint:

$$c_{\mathbf{a},\ell,j,e} + r_\ell h_{\mathbf{a},\ell,j,e} = w_{j,e}\left(1 - t_{\ell,j,e}\right) + T_{\mathbf{a}}, \tag{20}$$

where the variables are analogous to the baseline model. The wage $w_{j,e}$ depends on both the work location and the work arrangement. As in the baseline model, if a local lives and works in the same place, they do not spend time commuting, $t_{\ell,\ell,e} = 0$ for all ℓ and e. Similarly, remote workers do not spend time commuting, so $t_{\ell,j,h} = 0$ for all ℓ and j.

Congestion externalities In the baseline model, commuting time between two locations is exogenous. However, as we show in the empirical section, commuting times rise with the number of commuters due to traffic congestion. We model this phenomenon by assuming that

$$t_{\ell,j,o} \equiv \bar{t}_{\ell,j,o}[1 + \delta(\overline{\pi}_{\ell,j,o})], \tag{21}$$

for $\ell \neq j$ and where $\overline{\pi}_{\ell,j,o} \equiv \int \pi_{\mathbf{a},\ell,j,o} dG(\mathbf{a})$. Commuting time consists of a fixed component, $\bar{t}_{\ell,j,o}$, and a variable part, $\bar{t}_{\ell,j,o}\delta(\pi)$, which increases with the number of commuters. We assume that the elasticity of additional commuting time is constant

$$\psi \equiv \frac{\delta'(\pi)\pi}{\delta(\pi)}.$$

Goods and housing consumption and location choices Consider a household residing in location ℓ and working in location j with work arrangement e. Their optimal consumption of goods and housing services satisfies

$$\frac{u_h(c_{\mathbf{a},\ell,j,e}, h_{\mathbf{a},\ell,j,e})}{u_c(c_{\mathbf{a},\ell,j,e}, h_{\mathbf{a},\ell,j,e})} = r_\ell. \tag{22}$$

along with the budget constraint, (20) , which must hold with equality.

The optimal location and work arrangement choices maximize $\mathcal{U}_{\mathbf{a},\ell,j,e} + \xi_{\ell,j,e}$. As in the baseline model, we let $\pi_{\mathbf{a},\ell,j,e}$ denote the share of locals that live in ℓ, work in j with employment arrangement e, conditional on an asset level $\mathbf{a}$. This share is given by $\pi_{\mathbf{a},\ell,j,e} = \mathbb{P}[U_{\mathbf{a},\ell,j,e} + \xi_{\ell,j,e} \geq U_{\mathbf{a},\ell',j',e'} + \xi_{\ell',j',e'}, \quad \forall(\ell', j', e')|\mathbf{a}]$. As in the baseline model, the overall share with choices (ℓ, j, e) is given by $\overline{\pi}_{\ell,j,e} \equiv \int \pi_{\mathbf{a},\ell,j,e} dG(\mathbf{a})$.

6.1.2 Foreign residents

The foreign residents face the same problem as in the baseline model. They choose consumption and housing to maximize utility $\mathcal{U}_f \equiv \overline{u}_f + u(c_f, h_f)$, subject to the budget constraint $c_f + r_c h_f = y_f$. Foreigners are willing to relocate if the utility gained from moving exceeds their outside option, $\mathcal{U}_f \geq u_f^*$.

Authenticity externalities We assume that foreign residents derive utility from the "authenticity" of the city center, which is fostered by a greater presence of locals living and working there. We model this effect by making the amenity value that foreigners experience depend on the number of locals residing and working in the city center, $\overline{u}_f = \phi_f(\boldsymbol{\pi})$, where $\boldsymbol{\pi} = \{\overline{\pi}_{\ell,j,e}\}_{\ell,j,e}$ represents the distribution of locals across different locations and work arrangements.

Finite elasticity of supply of foreign residents We extend the baseline model by allowing the foreigners' outside option to vary with the number of foreigners entering the city. We assume that the outside option is given by $u_f^* = \phi_f^*(N_f)$. One interpretation of this formulation is that an influx of foreigners into the city reduces the number of foreigners entering other cities worldwide, thereby increasing the relative attractiveness of those alternative locations, either through lower housing prices or improved amenities. This general equilibrium effect is only relevant if the domestic city is "large," in the sense that the inflow of foreigners can influence outcomes elsewhere. In Appendix D.1, we present an alternative formulation that allows for heterogeneity in foreigners' outside options or preferences for entering the city, and show that our results remain robust.

We assume that $\chi = \frac{d \log u_f^*}{d \log N_f} \geq 0$ measures the elasticity of the outside option with respect to the number of foreigners entering the city. If $\chi = 0$, the baseline model assumption of a "small" economy holds, implying that N_f does not affect the attractiveness of other locations. If $\chi > 0$, the outside option improves as more foreigners arrive, making additional entrants less inclined to join.

6.1.3 Firms' problem

The production function of the representative firm in location j is given by

$$Y_j = A_j \left(L_{j,o}\right) \left(l_{j,o}^{\alpha} k_j^{1-\alpha} + \zeta l_{j,h}\right),$$

where $l_{j,o}$ and $l_{j,h}$ denote the number of people working for the firm in the office and at home, respectively. The agglomeration or production externality, $A_j\left(L_{j,o}\right)$, depends on the total number of people who work in offices in location j, $L_{j,o}$. This externality increases the productivity of all workers. The parameter ζ determines the productivity of remote workers. The production function of the baseline model corresponds to the case of $\zeta = 0$.

As in the baseline model, we assume that $A_j\left(L_{j,o}\right) = \overline{A}_j L_{j,o}^{\gamma}$, where γ controls the strength of the production externality and $\overline{A}_j$ denotes a location specific productivity parameter.

A firm located in j maximizes its profits, equal to the value of its production minus the costs of hiring workers. The firm incurs the cost of hiring onsite workers, $w_{j,o}l_{j,o}$, where $w_{j,o}$ is the wage for onsite workers, and costs for hiring remote workers, $w_{j,h}l_{j,h}$, where $w_{j,h}$ represents the wage for remote workers. The firm also faces costs for renting office space, $r_j^K k_j$, with r_j^K representing the rental rate for office buildings in location j. The optimality conditions for the firm's problem are:

$$w_{j,h} = A_j\left(L_{j,o}\right)\zeta, \tag{23}$$

$$w_{j,o} = \alpha A_j\left(L_{j,o}\right) l_{j,o}^{\alpha-1} k_j^{1-\alpha}, \tag{24}$$

$$r_j^K = (1-\alpha) A_j\left(L_{j,o}\right) l_{j,o}^{\alpha} k_j^{-\alpha}. \tag{25}$$

Market clearing and equilibrium There are two labor market clearing conditions. The first pertains to onsite workers in location j, $l_{j,o} = L_{j,o} = \sum_\ell \overline{\pi}_{\ell,j,o}(1 - t_{\ell,j,o})$. The second relates to remote workers employed by firms in location j, $l_{j,h} = L_{j,h} = \sum_\ell \overline{\pi}_{\ell,j,h}$. The market clearing condition for office buildings in location j is given by $k_j = \overline{K}_j$.

The goods market clearing condition is $\int \sum_{\ell,j,e} \pi_{\mathbf{a},\ell,j,e} c_{\mathbf{a},\ell,j,e}\, dG(\mathbf{a}) + N_f c_f = \sum_j A_j(L_{j,o})(L_{j,o}^{\alpha}\overline{K}_j^{1-\alpha} + \zeta L_{j,h}) + N_f y_f$, where N_f denotes the number of foreign residents and y_f is their income. Lastly, the housing market clearing conditions are $\int \sum_{j,e} \pi_{\mathbf{a},c,j,e} h_{\mathbf{a},c,j,e}\, dG(\mathbf{a}) + N_f h_f = \overline{H}_c$, for the city center and $\int \sum_{j,e} \pi_{\mathbf{a},p,j,e} h_{\mathbf{a},p,j,e}\, dG(\mathbf{a}) = \overline{H}_p$, for the peripheries. The variables $\overline{H}_c$ and $\overline{H}_p$ represent the total available housing in the city center and periphery p, respectively.

It is useful to define $\Pi^{\text{office}} \equiv \sum_{\ell,j} \overline{\pi}_{\ell,j,o}$, the share of workers who are office-based, and $\Pi^{\text{remote}} = 1 - \Pi^{\text{office}}$ the share of workers who work remotely.

6.2 The welfare impact of increasing the number of foreigners

We now study the effect of an increase in the number of foreign residents, $dN_f > 0$, on the welfare of the local population. The change in social welfare, defined by (12), is given by

$$d\mathcal{W} = \int \sum_{\ell,j,e} \pi_{\mathbf{a},\ell,j,e} \overline{\lambda}_{\mathbf{a},\ell,j,e} \Big[d\overline{u}_{\ell,j} + u_c\left(c_{\mathbf{a},\ell,j,e}, h_{\mathbf{a},\ell,j,e}\right) \Big\{ dw_{j,e}\left(1 - t_{\ell,j,e}\right) \\ - w_{j,e} dt_{\ell,j,e} - dr_\ell \times h_{\mathbf{a},\ell,j,e} + dT_{\mathbf{a}} \Big\} \Big] dG(\mathbf{a}).$$

As in the baseline model, we decompose the overall change in welfare measured in consumption-equivalent units into efficiency and redistribution components,

$$d\mathcal{W}_{CE} = \underbrace{\int \sum_{\ell,j,e} \pi_{\mathbf{a},\ell,j,e} \left[\widetilde{\phi}'_{\mathbf{a},\ell,j,e}(N_f) + dw_{j,e}\left(1 - t_{\ell,j,e}\right) - w_{j,e} dt_{\ell,j,e} - dr_\ell \times h_{\mathbf{a},\ell,j,e} + dT_{\mathbf{a}} \right] dG(\mathbf{a})}_{d\mathcal{W}_{CE}^{\text{Efficiency}}} \\ + \underbrace{\mathbb{COV}^{\mathbf{\Pi}}\left(\omega_{\mathbf{a},\ell,j,e}, \widetilde{\phi}'_{\ell,j,e}(N_f) + dw_{j,e}\left(1 - t_{\ell,j,e}\right) - w_{j,e} dt_{\ell,j,e} - dr_\ell \times h_{\mathbf{a},\ell,j,e} + dT_{\mathbf{a}}\right)}_{d\mathcal{W}_{CE}^{\mathcal{R}}},$$

where $\widetilde{\phi}'_{\mathbf{a},\ell,j,e}(N_f) \equiv \frac{\phi'_{\ell,j,e}(N_f)}{u_c\left(c_{\mathbf{a},\ell,j,e}, h_{\mathbf{a},\ell,j,e}\right)}$ denotes the consumption-equivalent amenity effect.

Proposition 6. *The efficiency welfare gains can be decomposed into six terms:*

$$d\mathcal{W}_{CE}^{\textit{Efficiency}} = \mathcal{FS} + \mathcal{PE} - \mathcal{CE} - \mathcal{PCE} + \mathcal{AE} + \mathcal{RW},$$

where each term is constructed as follows.

1. *The foreign-residents surplus, $\mathcal{FS}$, is*

$$\mathcal{FS} = dr_c \times N_f h_f.$$

2. *The production-externality effect, $\mathcal{PE}$, is*

$$\mathcal{PE} \equiv \gamma \times \Pi^{\textit{office}} \times \mathbb{COV}^{\overline{\mathbf{\Pi}}_o}\left(\frac{Y_j}{L_{j,o}}\left(1 - t_{\ell,j,o}\right), \frac{d\overline{\pi}_{\ell,j,o}}{\overline{\pi}_{\ell,j,o}}\right).$$

3. *The congestion-externality effect, $\mathcal{CE}$, is*

$$\mathcal{CE} \equiv \psi \times \Pi^{\textit{office}} \times \mathbb{COV}^{\overline{\mathbf{\Pi}}_o}\left(w_{j,o}(t_{\ell,j,o} - \bar{t}_{\ell,j,o}), \frac{d\overline{\pi}_{\ell,j,o}}{\overline{\pi}_{\ell,j,o}}\right).$$

4. *The production-congestion-externalities complementarity effect, $\mathcal{PCE}$ is*

$$\mathcal{PCE} \equiv \gamma\psi \times \Pi^{office} \times \mathbb{COV}^{\Pi_o}\left(\frac{Y_j}{L_{j,o}}(t_{\ell,j,o} - \bar{t}_{\ell,j,o}), \frac{d\overline{\pi}_{\ell,j,o}}{\overline{\pi}_{\ell,j,o}}\right).$$

5. *The amenities-externality effect, $\mathcal{AE}$, is*

$$\mathcal{AE} \equiv \mathbb{E}^{\Pi}\left[\widetilde{\phi}'\left(N_f\right)\right] \equiv \int \sum_{\ell,j,e} \pi_{\boldsymbol{a},\ell,j,e} \widetilde{\phi}'_{\boldsymbol{a},\ell,j,e}\left(N_f\right) dG(\boldsymbol{a}).$$

6. *The remote-work effect, $\mathcal{RW}$, is*

$$\mathcal{RW} \equiv \left(\gamma \sum_j Y_j - \psi \sum_{\ell,j} \overline{\pi}_{\ell,j,o} w_{j,o}(t_{\ell,j,o} - \bar{t}_{\ell,j,o}) - \gamma\psi \sum_{\ell,j} \overline{\pi}_{\ell,j,o} \frac{Y_j}{L_{j,o}}(t_{\ell,j,o} - \bar{t}_{\ell,j,o})\right) \times \frac{d\Pi^{office}}{\Pi^{office}}.$$

Generically, the covariance terms in these formulas can be written as follows. For any two variables x and y, the covariance is given by:

$$\mathbb{COV}^{\Pi_o}(x_{\ell,j,o}, y_{\ell,j,o}) = \sum_{\ell,j} \frac{\overline{\pi}_{\ell,j,o}}{\Pi^{\text{office}}} x_{\ell,j,o} y_{\ell,j,o} - \left(\sum_{\ell,j} \frac{\overline{\pi}_{\ell,j,o}}{\Pi^{\text{office}}} x_{\ell,j,o}\right)\left(\sum_{\ell,j} \frac{\overline{\pi}_{\ell,j,o}}{\Pi^{\text{office}}} y_{\ell,j,o}\right).$$

This operator computes the covariance between two variables x and y in the cross-section of locals, conditional on working from the office.

These extensions introduce additional channels through which an influx of foreign residents influences welfare. Interestingly, despite the increased complexity, the welfare impacts can still be broken down into easily interpretable components. Below, we outline these components and provide the intuition for their structure.

Foreign-residents surplus The foreign-resident surplus takes the same form as in the baseline model. An increase in foreign residents drives up housing demand, leading to higher rents. As a result, local property owners benefit from increased rental income.

Production externalities The production- or agglomeration-externality effects are also similar to those in the baseline model. Labor is better allocated to places with higher average labor productivity because the contribution to the agglomeration externality becomes more significant. Suppose an influx of foreign residents displaces locals from high-productivity areas. In that case, three outcomes are possible: (1) locals may continue working in these high-productivity areas but incur commuting costs, or (2) they may relocate to lower-productivity areas and work locally. Both scenarios lead to a decline in the productivity gains from the agglomeration externality, resulting in negative cross-sectional covariance and a corresponding welfare loss.

The scale of this welfare loss is influenced by the strength of the agglomeration externality, captured by γ. When $\gamma = 0$, there is no production externality, so labor reallocation does not affect welfare. Conversely, a high value of γ amplifies the production-externality effect. Since only office workers contribute to the agglomeration externality, the magnitude of this effect is further scaled by Π^{office}, the share of workers employed in offices.

Congestion externalities The congestion externality arises because commuting times are endogenous. As the number of foreign residents rises, locals change their housing and work location decisions. If workers move to the peripheries but continue working in the city center, the number of commuters increases. Because of congestion, commuting time also increases, reducing labor income. The covariance term captures the welfare losses associated with the change in commuting time. The term $w_{j,o}(t_{\ell,j,o} - \bar{t}_{\ell,j,o})$ captures the labor income loss from commuting congestion. If the number of commuters increases for routes with high-income losses from commuting, the covariance term will be positive, leading to a welfare loss. Intuitively, if the rise in foreign residents leads to an increase in people living in the peripheries but working in a highly productive city center, then the rise in commuting times will lead to income losses proportional to the income value of that commuting time.

The magnitude of the welfare loss is influenced by the strength of the congestion externality represented by ψ. If $\psi = 0$, commuting times are exogenous, so there is no congestion externality effect. If ψ is high, commuting times are highly sensitive to the number of commuters, amplifying the effect. Since only office workers commute, the effect is multiplied by Π^{office}, the proportion of workers who commute to offices.

Complementarity between production and congestion externalities The production externality depends on the total number of hours worked in the city center. As commuting times increase, the overall labor supply decreases, thereby reducing the associated production externalities. So, there is a complementarity between the congestion and the production externalities, which is influenced by the product $\gamma\psi$.

Amenities externalities As described above, the influx of foreign residents affects the value of the amenities that locals enjoy in the city center. Unlike other externalities, this effect has a direct impact of dN_f on the local population's utility. The average of the amenities determines the strength of this effect $\widetilde{\varphi}'_{\mathbf{a},\ell,j,e}(N_f)$. If this average is positive, then the influx of foreign residents increases the attractiveness of the city center, thereby improving the welfare of locals. If this average is negative, then the influx of foreign residents decreases the attractiveness of the city center, thereby harming the welfare of locals.

Remote work The influx of foreign residents encourages locals to move to the peripheries and work remotely for firms in the city center. Because working arrangements are optimized, the increase in remote work does not affect welfare directly. However, it interacts with both the production externality and the congestion externality. Since remote workers do not contribute to the production externality, welfare falls because labor productivity declines. This effect is controlled by γ. Since remote workers do not commute, there are two additional positive effects. The first is the decrease in commuting times, which improves labor income for those who do not work remotely. This effect is controlled by ψ. The second is analogous to the production-congestion complementarity: a decrease in commuting times increases the labor supplied by non-remote workers and increases productivity through the agglomeration externality.

Redistribution In general equilibrium, the entry of foreign residents affects the value of amenities, wages, commuting times, and house rents throughout the city. The effects on welfare resulting from the spatial redistribution of resources is captured by $d\mathcal{W}^{\mathcal{R}}_{CE}$. Importantly, this is the only term influenced by the choice of welfare weights $\lambda_{\mathbf{a},\xi}$. The interpretation of this term is the same as in the baseline model.

6.3 Mirrleesian optimal policy

In this section, we analyze the Mirrleesian optimal policy. As in the baseline model, we introduce no ex-ante restrictions on the set of instruments but work directly from the informational constraints. The planner can distinguish between locals and foreigners and observe people's decisions and their asset endowments.

Therefore, allocations and the policy instruments used to implement them can only be conditioned on these observable factors.

As in the baseline model, to compute the optimum, we can summarize the incentive constraints using the implied shares of the local population that make each choice (14).

We first discuss the optimal policy towards foreigners and then the optimal treatment of the local population. As in the baseline model, we present the optimal policy results in terms of the instruments that decentralize that optimal allocation. The set of instruments include taxes on housing purchases for locals, $\tau^h_{\mathbf{a},\ell,j,e}$, and foreigners, τ^h_f, lump-sum transfers on locals, $\mathcal{T}_{\mathbf{a},\ell,j,e}$, and foreigners, $\mathcal{T}_f$, and potential quotas on foreign entry. Note that, in the extended model, the tax instruments for locals depend not only on location choices, but also on their work-arrangement choice and are constructed in an analogous way.

6.3.1 Optimal policy towards foreigners

The following proposition summarizes the optimal treatment of foreigners in this model. As in the baseline model, when the number of foreign residents is fixed, there exists an optimal positive entry fee that equates their utility to their outside option. This result implies that implementing the Mirrleesian optimal plan does not require quota restrictions on foreign entry.

In Proposition 7, we extend the analysis to include the optimal choice of the number of foreign residents. This proposition generalizes the results of Proposition 4.

Proposition 7. *In the decentralization of the optimal allocation:*

1. *There are no quotas/restrictions on foreign entry.*

2. *Taxes on foreigners' housing purchases are zero,* $\tau^h_f = 0$.

3. *There is an optimal entry fee on foreigners which satisfies*

$$\mathcal{T}_f = -\mathbb{E}^{\mathbf{\Pi}}\left[\widetilde{\phi}'\left(N_f\right)\right] + \chi \widetilde{u}^*_f,$$

   where $\widetilde{u}^*_f \equiv u^*_f / u_c(c_f, h_f)$.

Despite the presence of additional externalities, the conclusions from Proposition 4 remain largely valid. First, it is never optimal to impose quotas on the entry of foreign residents, because managing the flow of foreign residents through taxation is more efficient than imposing quantity restrictions. Second, it is also never optimal to tax foreign home purchases, distorting their housing choices.

The key difference with respect to Proposition 4 is that, in the extended model, the optimal entry fee is no longer zero.

First, the entry fee is designed to ensure that foreign residents internalize their impact on the amenity valuations of locals. The intuition for this result is as follows. In the extended model, foreigners impose a direct externality on the welfare of natives. Therefore, it is optimal for the planner to distort the entry margin using an entry fee. If the $\mathbb{E}^{\mathbf{\Pi}}\left[\widetilde{\phi}'\left(N_f\right)\right] > 0$, foreigners improve the amenity value of the city center, so the entry fee is lower, perhaps even negative, to encourage their entry. If $\mathbb{E}^{\mathbf{\Pi}}\left[\widetilde{\phi}'\left(N_f\right)\right] < 0$, foreigners deteriorate the amenity value of the city center, so the entry fee is higher to discourage their entry.

Second, the entry fee also depends on χ, which controls the elasticity of the foreign residents' outside option with respect to N_f. In the baseline model, χ is zero. When χ is positive, the optimal entry fee is larger than when $\chi = 0$. It is optimal to bring fewer foreigners to moderate the increase in the reservation utility of the marginal foreigner. Importantly, this effect depends on whether the city is large with respect to the rest of the world. A large city can affect the foreigners' outside option, so the planner charges a higher fee to keep this option low.

6.3.2 Optimal place-based transfers for locals

As in the baseline model, we define $\tau^h_{\mathbf{a},\ell,j,e}$ as the house tax on people who live in ℓ, work in j, have employment status e with asset holdings $\mathbf{a}$. The transfer to these individuals is

$$\mathcal{T}_{\mathbf{a},\ell,j,e} \equiv c_{\mathbf{a},\ell,j,e} + (1+\tau^h_{\mathbf{a},\ell,j,e}) r_\ell h_{\mathbf{a},\ell,j,e} - w_{j,e}(1-t_{\ell,j,e}). \tag{26}$$

We compute wages of remote and office workers and rents on offices using equations (23) and (24) and (25), respectively, replacing $l_{j,e}$ with $L_{j,e}$ and k_j with $\overline{K}_j$. The following proposition provides sufficient statistics to calculate the tax/transfer policies required to implement the optimal solution.

Proposition 8. *In the decentralized optimal allocation, housing purchases by locals are not taxed:* $\tau^h_{\boldsymbol{a},\ell,j,e} = 0$. *The total transfers implemented by the planner are the sum of six terms*

$$\mathcal{T}_{\boldsymbol{a},\ell,j,e} = \Xi_{\boldsymbol{a}} + \Xi^{\mathcal{PE}}_{\boldsymbol{a},\ell,j,e} + \Xi^{\mathcal{CE}}_{\boldsymbol{a},\ell,j,e} + \Xi^{\mathcal{PCE}}_{\boldsymbol{a},\ell,j,e} + \Xi^{\mathcal{AE}}_{\boldsymbol{a},\ell,j,e} + \Xi^{\mathcal{R}}_{\boldsymbol{a},\ell,j,e}, \tag{27}$$

where

1. *the common transfer $\Xi_{\boldsymbol{a}}$ is such that*

$$\int \Xi_{\boldsymbol{a}} dG(\boldsymbol{a}) = \sum_j r_j^K \overline{K}_j + \sum_\ell r_\ell \overline{H}_\ell + N_f \mathcal{T}_f,$$

2. *the production-externality correction term is*

$$\Xi^{\mathcal{PE}}_{\boldsymbol{a},\ell,j,o} \equiv \gamma \left\{ \frac{Y_j}{L_{j,o}} \left(1 - t_{\ell,j,o}\right) - \sum_{\ell,j} \pi_{\boldsymbol{a},\ell,j,o} \frac{Y_j}{L_{j,o}} \left(1 - t_{\ell,j,o}\right) \right\},$$
$$\Xi^{\mathcal{PE}}_{\boldsymbol{a},\ell,j,h} \equiv -\gamma \sum_{\ell,j} \pi_{\boldsymbol{a},\ell,j,o} \frac{Y_j}{L_{j,o}} \left(1 - t_{\ell,j,o}\right),$$

3. *the congestion-externality correction term is*

$$\Xi^{\mathcal{CE}}_{\boldsymbol{a},\ell,j,o} \equiv -\psi \left\{ w_{j,o}(t_{\ell,j,o} - \bar{t}_{\ell,j,o}) - \sum_{\ell,j} \pi_{\boldsymbol{a},\ell,j,o} w_{j,o}(t_{\ell,j,o} - \bar{t}_{\ell,j,o}) \right\},$$
$$\Xi^{\mathcal{CE}}_{\boldsymbol{a},\ell,j,h} \equiv \psi \sum_{\ell,j} \pi_{\boldsymbol{a},\ell,j,o} w_{j,o}(t_{\ell,j,o} - \bar{t}_{\ell,j,o}),$$

4. *the production-congestion-externalities-complementarity correction term is*

$$\Xi^{\mathcal{PCE}}_{\boldsymbol{a},\ell,j,o} \equiv -\psi\gamma \left\{ \frac{Y_j}{L_{j,o}} \left(t_{\ell,j,o} - \bar{t}_{\ell,j,o}\right) - \sum_{\ell,j} \pi_{\boldsymbol{a},\ell,j,o} \frac{Y_j}{L_{j,o}} \left(t_{\ell,j,o} - \bar{t}_{\ell,j,o}\right) \right\},$$
$$\Xi^{\mathcal{PCE}}_{\boldsymbol{a},\ell,j,h} \equiv \psi\gamma \sum_{\ell,j} \pi_{\boldsymbol{a},\ell,j,o} \frac{Y_j}{L_{j,o}} \left(t_{\ell,j,o} - \bar{t}_{\ell,j,o}\right)$$

5. *the authenticity-externality correction term is*

$$\Xi^{\mathcal{AE}}_{\boldsymbol{a},\ell,j,e} \equiv N_f \left\{ \frac{d\widetilde{\phi}_f(\boldsymbol{\pi})}{d\pi_{\boldsymbol{a},\ell,j,e}} - \sum_{\ell,j,e} \pi_{\boldsymbol{a},\ell,j,e} \frac{d\widetilde{\phi}_f(\boldsymbol{\pi})}{d\pi_{\boldsymbol{a},\ell,j,e}} \right\},$$

   where $\frac{d\widetilde{\phi}_f(\boldsymbol{\pi})}{d\pi_{\boldsymbol{a},\ell,j,e}} = \frac{d\phi_f(\boldsymbol{\pi})}{d\pi_{\boldsymbol{a},\ell,j,e}} / u_c(c_f, h_f)$,

6. *the redistribution-correction term is*

$$\Xi^{\mathcal{R}}_{\boldsymbol{a},\ell,j,e} \equiv \mu^{IC}_{\boldsymbol{a},\ell,j,e} - \sum_{\ell,j,e} \pi_{\boldsymbol{a},\ell,j,e} \mu^{IC}_{\boldsymbol{a},\ell,j,e}$$

   is given by the difference between the Lagrange multiplier on the incentive compatibility constraint and its average.

Not surprisingly, the additional features of this extended model increase the number of possible externalities. Still, we can continue to decompose the optimal transfers into several interpretable terms. We now describe each in turn.

Common transfer As in the baseline model, the planner redistributes the income generated from office and residential rents and taxes levied on foreigners among the local population.

Production-externality correction term As in the baseline model, the planner corrects for the production externality by giving higher transfers than average to office workers in locations where average labor productivity is above the cross-sectional mean. Since remote workers do not contribute to the production externality, the planner reduces transfers to remote workers to finance positive transfers to office workers. The magnitude of this transfer is determined by the elasticity of productivity to total office labor supply, γ.

Congestion-externality correction term The congestion-externality correction term captures the transfers necessary for locals to internalize their impact on commuting costs. Intuitively, commuters receive a lower transfer than non-commuters (workers who live and work in the same place or remote workers). The magnitude of this transfer is determined by the elasticity of commuting costs with respect to the number of commuters ψ.

Production-congestion-externalities-complementarity correction term As discussed in the previous section, the production and congestion externalities are complementary. All else equal, a decrease in commuting costs reduces labor supply, which in turn lowers average productivity. The term $\Xi^{\mathcal{PCE}}_{\mathbf{a},\ell,j,e}$ affects the transfers so that commuters also internalize their effects on total factor productivity.

Authenticity-externality correction term The presence of locals in the city center, either working or living, increases the amenity value for foreigners. The planner corrects this externality by providing higher transfers to locations and work choices that have a greater-than-average effect on the amenity value for foreigners.

Redistribution To correct the location and work location choices of locals, the planner must design transfers that alter the cross-sectional distribution of consumption. Because of concavity in utility and potential heterogeneity in welfare weights. $\lambda_{\mathbf{a},\xi}$, the planner has different marginal valuations for the consumption of different people. As a result, the optimal Mirrleesian plan deviates from the Pigouvian-corrective transfers to enhance redistribution across the population. The results for the baseline model regarding settings with quasi-linear preferences (Corollary 1) or extreme value ξ (Corollary 2) also hold in this extended model.

7 The long run: the future of global cities

In this section, we extend our framework to a dynamic setting to explore two long-run questions. First, how does an influx of foreign residents reshape the optimal city structure: where should offices and housing be located? Second, does achieving this optimal configuration require zoning regulation?

Regarding the first question, our results indicate that office space in the city center should be gradually converted to housing to accommodate the higher demand for central residential locations. As for the second, we show that the decentralized Mirrleesian allocation discussed earlier already provides the necessary incentives, implying that no additional zoning regulation or fiscal intervention is needed to attain the optimal city structure.

The model features overlapping generations. An influx of foreign residents creates inequality between the current generations, which receive the capital gains from their housing stock, and future generations that face higher housing costs. Workers can move across locations over their lifetimes, subject to mobility costs. We characterize how optimal transfers redistribute capital gains and address productive externalities and redistribution concerns throughout the life cycle.

We discuss the model and associated optimal policy in detail in Appendix F. Here, we briefly summarize the model's structure. The city has a center and several peripheral locations. In every location, there are stocks of residential and office buildings that evolve over time as developers invest and structures depreciate.

The population consists of overlapping generations of local workers and inflows of foreign residents. Each generation lives for $A+1$ periods and, for simplicity, has no bequest motive. Once born, they choose where to live and can commute to another location for work. They can move in future periods by paying a moving cost. Locals draw idiosyncratic tastes over residential–workplace pairs, so even with the same prices and wages, different people may choose different neighborhoods or commutes. We assume that people's idiosyncratic tastes are drawn at birth, but can evolve deterministically with age. This assumption captures the idea that location preferences evolve smoothly over time and simplifies the informational revelation problem faced by the planner, allowing us to focus on the dynamics arising from investment and city structure. Locals earn labor income that varies predictably with age and can also save in both financial assets and real estate. We assume that the initial old population owns the existing housing and office stock. The new generations are born with no assets.

A large pool of foreigners decides whether to enter the city. For simplicity, foreigners choose only whether to live in the center and make a static consumption and housing choice; they enter only if the city delivers at least as much utility as their outside option, and they bring an exogenous endowment of the traded good.[16]

Production is carried out by perfectly competitive firms in each location that use office space and local labor. Because of agglomeration externalities, a larger workforce in a location raises firms' productivity there, which in turn affects wages and the rental value of office space. Developers in each location transform the final traded good into new housing and office buildings; next period's stocks equal undepreciated structures plus new investment. New investment is subject to adjustment costs, capturing both construction of new buildings or conversion of buildings from one use to another. Our model allows for a general description of location and time-dependent adjustment costs to capture the idea that repurposing office buildings into housing may be easier in some locations than others, or easier in the long run than in the short run. These adjustment costs are paid in units of the final good. The time t resource constraint is given by

$$C_t + N_{f,t}c_{f,t} + \sum_{\ell} \Phi_{\ell,t}(I^h_{\ell,t}, I^k_{\ell,t}) = \sum_{\ell} Y_{\ell,t} + N_{f,t}y_{f,t},$$

where C_t is total consumption by locals, $N_{f,t}c_{f,t}$ is total consumption by foreigners, $Y_{\ell,t}$ is output in location ℓ, and $N_{f,t}y_{f,t}$ is the total endowment brought in by foreigners. The function $\Phi_{\ell,t}(I^h_{\ell,t}, I^k_{\ell,t})$ is the total cost of

[16]Since foreigners make static decisions, intertemporal trade between locals and foreigners cannot occur, requiring international trade to balance in each period.

investment including adjustment costs in location ℓ at time t, which depends on new investment in housing,

$$I^h_{\ell,t} = H_{\ell,t+1} - (1-\delta)H_{\ell,t},$$

and office buildings,

$$I^k_{\ell,t} = K_{\ell,t+1} - (1-\delta)K_{\ell,t},$$

where $H_{\ell,t}$ and $K_{\ell,t}$ are the stocks of housing and office buildings in location ℓ at time t, respectively, and δ is the depreciation rate.

A social planner can condition allocations on prior observables and individuals' location decisions. For the cohorts initially alive, these observables are their birth year, previous location choices, and asset holdings. For newborns, the only observable is their birth cohort. The planner can also screen the distribution of idiosyncratic tastes by offering menus of location choices, so realized choices reveal information.[17] Given this setting, the planner assigns consumption, housing, labor across work locations, and investment in structures, subject to four sets of constraints: (i) incentive compatibility, ensuring that individuals willingly follow the assigned residential–workplace paths given their private tastes; (ii) market clearing for housing, labor, offices, and goods; (iii) the laws of motion for structures; and (iv) foreigners' participation. We assume that the planner has full commitment.

Decentralization of the optimal allocation The optimal allocation can be decentralized with simple policy instruments. Individuals face location-specific housing rents and may be taxed on the purchase of their housing services: for an individual i living in location ℓ at time t, the tax on housing services satisfies:

$$(1+\tau^h_{i,t})r^h_{\ell,t} = \frac{u_h(c_{i,t}, h_{i,t})}{u_c(c_{i,t}, h_{i,t})},$$

where $r^h_{\ell,t}$ denotes the rental rate on housing services in location ℓ at time t. For each period and location, we normalize the housing tax for one individual to zero.

Individuals can save in a risk-free financial asset that yields the interest rate r_{t+1} or invest in houses or

[17] Our assumptions about idiosyncratic tastes imply that, for each generation, the information-revelation problem occurs only once, at birth. There are no dynamics from information revelation. These assumptions simplify the information problem, allowing us to focus on the dynamics arising from investment. Donald, Fukui, and Miyauchi (2025) study a dynamic Mirrleesian problem in which people draw i.i.d. tastes every period, abstracting from investment and city structure. In that sense, the two papers are complementary.

office buildings.[18] These savings can also be taxed at the person-specific rate $\tau^s_{i,t}$, which, for an individual i who lives both in periods t and $t+1$, satisfies

$$u_c(c_{i,t}, h_{i,t}) = (1 - \tau^s_{i,t+1})(1 + r_{t+1})\beta u_c(c_{i,t+1}, h_{i,t+1}).$$

For each period, we normalize the savings tax for one individual to zero. Finally, we define the transfers to a local as the excess expenditure over their total labor income, $\mathcal{T}_t$.

In the decentralization, we define the price of new buildings to be equal to their marginal cost of investment

$$p^h_{\ell,t} = \frac{\partial \Phi_{\ell,t}(I^h_{\ell,t}, I^k_{\ell,t})}{\partial I^h_{\ell,t}}, \quad \text{and} \quad p^k_{\ell,t} = \frac{\partial \Phi_{\ell,t}(I^h_{\ell,t}, I^k_{\ell,t})}{\partial I^k_{\ell,t}}.$$

The returns from investing in new buildings are given by

$$R^h_{\ell,t+1} \equiv \frac{r^h_{\ell,t+1} + (1-\delta)p^h_{\ell,t+1}}{p^h_{\ell,t}}, \quad \text{and} \quad R^k_{\ell,t+1} \equiv \frac{r^k_{\ell,t+1} + (1-\delta)p^k_{\ell,t+1}}{p^k_{\ell,t}},$$

where $r^h_{\ell,t}$ and $r^k_{\ell,t}$ are the rents on housing and office buildings, respectively.

The government may subsidize or tax investment in new buildings to affect the city structure.[19] We denote these subsidies/taxes by $\varsigma^h_{\ell,t}$ and $\varsigma^k_{\ell,t}$ for housing and office buildings, respectively:

$$(1 + \varsigma^h_{\ell,t})R^h_{\ell,t+1} = 1 + r_{t+1}, \quad \text{and} \quad (1 + \varsigma^k_{\ell,t})R^k_{\ell,t+1} = 1 + r_{t+1}.$$

If $\varsigma^h_{\ell,t} > 0$ ($\varsigma^k_{\ell,t} > 0$), the government subsidizes investment in new housing (office) buildings; if $\varsigma^h_{\ell,t} < 0$ ($\varsigma^k_{\ell,t} < 0$), the government taxes investment in new housing (office) buildings. The following proposition summarizes the optimal policy towards investment in new buildings.

Optimal policy towards foreigners The main optimal-policy results regarding foreign residents mirror those in the simple static model: there are no taxes on housing services for foreigners and no person-specific distortion to intratemporal choices, that is, no taxation of housing services. For simplicity, the dynamic model incorporates only production externalities as in Section 4. Under these assumptions, the optimal entry fee for foreign residents is zero. It is straightforward to extend the model to include amenities and congestion externalities, in which case the optimal entry fee would be similar to that in Proposition 7.

[18] Because individuals have finite lifespans, face no uncertainty, and have no bequest motive, they fully deplete their assets in the final period of life.

[19] Equivalently, the planner could impose quotas or zoning policies as in Allen et al. (2015).

Optimal policy towards locals In this dynamic model, the optimal policy towards local residents is more complex due to additional factors such as age heterogeneity, multiple migration decisions, and moving costs. We show that the analog of Proposition 5 takes the following form. First, as in the static model, there are no taxes on the purchase of housing services. In addition, the non-labor income received through transfers depends on whether individuals belong to the initially existing population or to newly born cohorts.

Proposition 9. *In the decentralization of the optimal allocation, housing services are not taxed. The savings tax is zero, i.e., in the optimal allocation, the intertemporal marginal rates of substitution are equalized across individuals. Transfers to the pre-existing cohorts who enter period zero with age a, prior location decisions* $\omega_{-1} \equiv (\ell_{-1}, j_{-1})$*, asset holdings* $\boldsymbol{a}$*, and who make further location decisions* $\boldsymbol{\omega} \equiv \{\ell_{a+t}, j_{a+t}\}_{t=0}^{A-a}$*, have three components:*

$$\mathcal{T}_{-a}(\omega_{-1}, \boldsymbol{a}, \boldsymbol{\omega}) = \Xi_{-a}(\omega_{-1}, \boldsymbol{a}) + \sum_{t=0}^{A-a} q_t \Xi^{\mathcal{PE}}_{-a,t}(\omega_{-1}, \boldsymbol{a}, \omega_{a+t}) + \Xi^{\mathcal{R}}_{-a}(\omega_{-1}, \boldsymbol{a}, \boldsymbol{\omega}),$$

where q_t *denotes the discount factor between time t and time* 0*, i.e.,* $q_t \equiv \prod_{s=1}^{t} \frac{1}{1+r_s}$*. Analogously, transfers to the newborn cohorts at time t are conditional on their birthyear and on their observable location decisions* $\boldsymbol{\omega} \equiv \{\ell_a, j_a\}_{a=0}^{A}$*, and have three components:*

$$\mathcal{T}_t(\boldsymbol{\omega}) = \Xi_t + \sum_{a=0}^{A} q_{t+a} \Xi^{\mathcal{PE}}_{t,a}(\omega_a) + \Xi^{\mathcal{R}}_t(\boldsymbol{\omega}).$$

where

- *The common terms* $\Xi_{-a}(\omega_{-1}, \boldsymbol{a})$ *and* Ξ_t *denote common transfers to all individuals with the same observables, which sum to total income generated from office and residential rents.*

- *The production-externality correction term for the initial old is given by*

$$\Xi^{\mathcal{PE}}_{-a,t}(\omega_{-1}, \boldsymbol{a}, \omega_{a+t}) \equiv \gamma \times \left(\frac{Y_{j_{a+t},t}}{L_{j_{a+t},t}} (1 - t_{\omega_{a+t}}) \theta_{a+t} - \mathbb{E}\left[\frac{Y_{j,t}}{L_{j,t}} (1 - t_\omega) \theta_{a+t} \middle| a, \omega_{-1}, \boldsymbol{a} \right] \right)$$

  and for the newborns at time t is given by:

$$\Xi^{\mathcal{PE}}_{t,a}(\omega_a) \equiv \gamma \times \left(\frac{Y_{j_a,t+a}}{L_{j_a,t+a}} (1 - t_{\omega_a}) \theta_a - \mathbb{E}\left[\frac{Y_{j,t+a}}{L_{j,t+a}} (1 - t_\omega) \theta_a \middle| t \right] \right),$$

  where θ_a *is the deterministic labor productivity of a person of age a,* $\mathbb{E}[\cdot|a, \omega_{-1}, \boldsymbol{a}]$ *denotes the cross-sectional average conditional on* $(a, \omega_{-1}, \boldsymbol{a})$*, and* $\mathbb{E}[\cdot|t]$ *the cross-sectional average conditional on a person in cohort t.*

- *The redistribution-correction term is given by the difference of the Lagrange multiplier on the incentive compatibility constraint relative to its cross-sectional average for this group of people:*

$$\Xi^{\mathcal{R}}_{-a}(\omega_{-1}, \boldsymbol{a}, \boldsymbol{\omega}) \equiv \frac{\beta^{-a}\eta^{IC}_{-a}(\omega_{-1}, \boldsymbol{\omega}^{a}, \boldsymbol{a})}{\eta^{C}_{0}} - \mathbb{E}\left[\frac{\beta^{-a}\eta^{IC}_{-a}(\omega_{-1}, \boldsymbol{\omega}, \boldsymbol{a})}{\eta^{C}_{0}}\middle| a, \omega_{-1}, \boldsymbol{a}\right],$$

  for the initial old, and

$$\Xi^{\mathcal{R}}_{t}(\boldsymbol{\omega}) \equiv \frac{\beta^{t}\eta^{IC}_{t}(\boldsymbol{\omega})}{\eta^{C}_{0}} - \mathbb{E}\left[\frac{\beta^{t}\eta^{IC}_{t}(\boldsymbol{\omega})}{\eta^{C}_{0}}\middle| t\right]$$

  for those born in cohort t.

As in the static model, the planner imposes no taxes on housing services. Furthermore, the planner imposes no taxes on savings. It follows that all individuals' marginal rates of substitution between consumption at time t and $t+1$ equal the market interest rate $1 + r_{t+1}$. As in the static model, these properties of the optimal policy follow from the principles in Atkinson and Stiglitz (1976).

Several properties of the optimal transfers are noteworthy. First, the planner uses information about individual asset holdings and birth year to design transfers that redistribute capital gains both within and across generations. Second, Proposition 9 generalizes Proposition 5 to a dynamic environment where people make multiple location choices over their lifetimes. In this setting, the optimal transfers depend on the sequence of location choices, which the planner uses as a screening device. However, Proposition 9 shows that the optimal transfers only depend on the history of location decisions through the redistribution motive. The externality-correction term is separable across ages and depends only on the individual's current location decisions. This result implies that two individuals with different location histories receive the same externality-correction transfer if they make the same current location decision. In other words, efficient location decisions in the dynamic model follow the same principles as in the static model. This result generalizes the findings in Fajgelbaum and Gaubert (2020) to a dynamic environment with overlapping generations and multiple location choices over the life cycle.

Optimal city structure The next proposition characterizes the optimal incentives for investing in houses and office buildings.

Proposition 10. *In the decentralization of the optimal allocation, investment in housing and office buildings is neither taxed nor subsidized; that is, $\varsigma^{h}_{\ell,t} = 0$ and $\varsigma^{k}_{\ell,t} = 0$ for all locations ℓ and times t.*

The dynamic model allows us to analyze the optimal city structure, accounting for time lags and investment in the construction of new or the reconversion of old buildings. The key result is that the optimal city structure emerges in equilibrium without the need for zoning regulations or taxes/subsidies on investment in buildings. The intuition for this result is as follows. The planner uses transfers to local residents to internalize the external effects of their location choices. Once these externalities are internalized, market prices reflect the social value of new housing and office buildings in each location. Consequently, it is never optimal to restrict the supply of housing or office buildings in any location.

Corollary 3. *In the long-run steady state that emerges after an influx of foreign residents, the optimal city structure is such that rent-to-price ratios are equalized across building types,* $\frac{r^h_{ss}}{p^h_{ss}} = \frac{r^k_{ss}}{p^k_{ss}}$.

The optimality conditions also shed light on how the city's spatial structure evolves over time. Suppose that, in steady state, the marginal cost of producing an additional unit of housing equals that of producing an additional unit of office space. Then, in the steady state before the influx of foreign residents, the rental rates on houses and offices are equalized. The arrival of foreign residents increases the demand for city-center housing, driving up local housing rents. As labor relocates to the peripheries, the marginal productivity of capital in the city center falls, lowering office rents. In the long run, these forces induce an expansion of housing supply and a contraction of office space in the city center.

In the periphery, there are two opposing forces. On the one hand, some locals move to the peripheries, raising housing demand and house prices. On the other hand, the expansion of labor supply in the peripheries increases capital's productivity, driving up rental rates for office buildings.

8 Conclusion

Many countries and urban areas are trying to devise policies to ensure that the local population benefits from a potentially large influx of foreign residents and tourists.

We show that public policy can play a crucial role in addressing agglomeration, congestion, amenities, and other externalities generated by this influx. Achieving the socially optimal outcome requires designing taxes and transfers for locals based on their residential and work-related choices. These transfers encourage workers to internalize the external effects of their living and work decisions.

When foreign residents directly affect local amenities, their entry should be regulated through an entry fee, similar to the per-diem taxes that some cities already impose on tourists. Suppose the city is large enough to influence rents in other global cities by adjusting the number of foreign residents it admits. In that case, it may also be optimal to impose an entry fee to keep the foreigners' outside options low.

If the ownership of housing and office buildings is unevenly distributed, the arrival of foreign residents may generate capital gains that accrue disproportionately to property owners. In this case, optimal policy calls for taxes or transfers that redistribute these gains across the population.

Looking ahead, the optimal long-run adjustment involves repurposing office spaces in the city center for residential use and relocating production facilities to the peripheries. This approach mirrors the urban plan implemented in Paris. In the 19th century, Napoleon III gave Baron Haussmann sweeping powers to redesign Paris. The result was the monumental city we know today, with wide boulevards, grand squares, and views of the Eiffel Tower unobstructed by towering skyscrapers. Over time, office buildings, production facilities, and much of the city's housing stock moved to La Défense and other peripheral areas. The ability of Paris to welcome foreign residents impressed Ernest Hemingway, who wrote, "There are only two places in the world where we can live happy—at home and in Paris."

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Online Appendix

A Data appendix

We use data from the 2011 and 2021 Portuguese census surveys to estimate population changes and their corresponding commuting flows. For population changes, we use the indicator that reports the resident population by gender, age group, nationality, and residency: "População residente por local de residência à data dos censos [2021] [NUTS - 2013], sexo, grupo etário e nacionalidade." For commuting flows, we use an indicator on work commute flows for residents, classified according to their place of residence, sex, employment status, and the duration of their commute to their place of work or study "População residente que vive no alojamento a maior parte do ano por local de residência à data dos sensos [2021] (NUTS - 2013), sexo, condição perante o trabalho, escalão de duração dos movimentos pendulares e local de trabalho ou estudo". These indicators are available for both census periods, allowing us to estimate demographic shifts and commuting flows.

We use the Statistics Portugal indicator "Dormidas nos estabelecimentos de alojamento turístico por localização geográfica (NUTS - 2024) e Local de residência (País - lista reduzida); Anual - INE, Inquérito à permanência de hóspedes na hotelaria e outros alojamentos" to estimate the number of tourist-equivalent residents in Portugal for the years 2011 and 2022. This indicator provides the number of nights tourists spend in accommodation establishments and the tourists' country of residence. These data are collected annually by Statistics Portugal (INE) through a survey of guest stays in hotels and other accommodations. We do not use this indicator for 2021 because of the impact of Covid-19 on tourism flows. To estimate the number of yearly-equivalent tourists for each period, we divide the total number of tourist nights by 365×0.74, where 0.74 is the average hotel occupancy in Lisbon in 2023. This calculation gives us an average daily number of tourists, which represents the equivalent number of residents if those tourists were to stay for an entire year. This method allows us to quantify the impact of tourism on the resident population by providing a comparable measure of "tourist-equivalent" yearly residents.

Estimates of the housing stock are based on data from the Census de Alojamento on the number of family home units (alojamentos familiares clássicos).

To estimate commute times for individuals between Lisbon's center and its periphery, we used the Google Maps API. We obtained geographic data for the Lisbon metropolitan area from the OpenStreetMap repository. These data provide coordinates and names of the various municipalities. We aggregated the geographic data to obtain mean coordinates for each location municipality in the Lisbon metropolitan region. Finally, we used the Google Maps API to define commute scenarios for peak (8 AM and 5 PM) and non-peak hours (3:00 AM) across weekdays. We calculated commute times for each pair of origin and destination coordinates, excluding identical pairs and accounting for variations in traffic conditions. We used a reference Monday in July to standardize departure times.

Housing stock estimates are based on the indicator "Alojamentos familiares clássicos (Parque habitacional) por Localização geográfica (NUTS - 2013); Anual". The definition of family accommodation is a room or a set of rooms, including any annexes, located within a permanent building or a structurally distinct part of one. These accommodations must have an independent entrance that provides direct access to a street, garden, or a shared passageway within the building, such as a staircase, corridor, or gallery, among others.

The API requests provide data on distance and duration for driving, both under normal and traffic conditions. The data we collected includes details on the origin, destination coordinates, time slots, day of the week, distance, and duration. Using the Google Maps API enables us to capture accurate real-world commute times that reflect temporal and spatial variations in traffic within Lisbon.

We used data from ArquivoPT, a web archive service that preserves content from Portuguese websites, to estimate regional rent and residential real estate prices. Like the Wayback Machine, ArquivoPT enables users to search and access historical snapshots of the web. The complete dataset contains housing prices in Portugal from 2001 to 2023. These prices are sourced from listings on websites of real estate agencies and aggregators operating in the Portuguese market, including BPI Imobili'ario, Casa Sapo, Era, Remax, Idealista, Trovit, and Imovirtual.

The listing registry varies over time due to changes in the technology and online presence of these platforms. For instance, the coverage and comprehensiveness of the listings can fluctuate based on changes in website design, data retention policies, and technological advancements. Specifically, for 2011, we have consistent listings from Idealista, while for 2016 and 2021, we have consistent listings from Era, Idealista, and Imovirtual.

Finally, the listings represent asking prices, not transaction prices. The data includes the sellers' asking price, which may differ from the final sale prices. We believe our measures are likely to underestimate price increases because the housing market has become tighter over time. As a result, it is more likely that asking prices were accepted toward the end of the sample period compared to the beginning.

B Appendix to Section 4

B.1 Proof of Lemma 1

An agent's common utility in ℓ, j for asset level $\mathbf{a}$ is given by:

$$U_{\mathbf{a},\ell,j} = \max_{c,h} \overline{u}_{\ell,j} + u(c,h), \quad \text{s. to } c + r_\ell h = w_j(1 - t_{\ell,j}) + T_{\mathbf{a}}.$$

Since individuals can freely choose where to live and work, for the equilibrium common utilities $\mathbf{U} \equiv \{U_{\mathbf{a},\ell,j}\}$, each individual's utility is given by:

$$\mathcal{U}_{\mathbf{a},\xi} = \max_{\ell_{\mathbf{a},\xi}, j_{\mathbf{a},\xi}} \left[U_{\mathbf{a},\ell_{\mathbf{a},\xi},j_{\mathbf{a},\xi}} + \xi_{\ell_{\mathbf{a},\xi},j_{\mathbf{a},\xi}} \right].$$

Social welfare can be written as:

$$\mathcal{W}(\mathbf{U}) = \int \lambda_{\mathbf{a},\xi} \max_{\ell_{\mathbf{a},\xi}, j_{\mathbf{a},\xi}} \left[U_{\mathbf{a},\ell_{\mathbf{a},\xi},j_{\mathbf{a},\xi}} + \xi_{\ell_{\mathbf{a},\xi},j_{\mathbf{a},\xi}} \right] f_{\mathbf{a}}(\boldsymbol{\xi}) d\boldsymbol{\xi} dG(\mathbf{a}),$$

which can equivalently be written as

$$\mathcal{W}(\mathbf{U}) = \max_{\{\ell_{\mathbf{a},\xi}, j_{\mathbf{a},\xi}\}_{\mathbf{a},\xi}} \int \lambda_{\mathbf{a},\xi} \left[U_{\mathbf{a},\ell_{\mathbf{a},\xi},j_{\mathbf{a},\xi}} + \xi_{\ell_{\mathbf{a},\xi},j_{\mathbf{a},\xi}} \right] f_{\mathbf{a}}(\xi) d\boldsymbol{\xi} dG(\mathbf{a}).$$

This result follows from the fact that, conditional on $\mathbf{U}$, the problem becomes separable for each individual.

Using the envelope theorem on each maximization problem, we find that the marginal effects are given by[20]

$$\begin{aligned} dU_{\mathbf{a},\ell,j} =& u_c(c_{\mathbf{a},\ell,j}, h_{\mathbf{a},\ell,j}) \left[dw_j(1 - t_{\ell,j}) - dr_\ell h_{\mathbf{a},\ell,j} + dT_{\mathbf{a}} \right], \\ d\mathcal{W} =& \int \lambda_{\mathbf{a},\xi} dU_{\mathbf{a},\ell_{\mathbf{a},\xi},j_{\mathbf{a},\xi}} f_{\mathbf{a}}(\boldsymbol{\xi}) d\boldsymbol{\xi} dG(\mathbf{a}). \end{aligned}$$

[20]Formally, the marginal effects we present hold almost everywhere, see Milgrom and Segal (2002). So, these marginal effects hold generically.

Now note that for each $\mathbf{a}$ we can define $\overline{\lambda}_{\mathbf{a},\ell,j} = \int_{(\ell_{\mathbf{a},\xi}, j_{\mathbf{a},\xi})=(\ell,j)} \lambda_{\mathbf{a},\xi} / \pi_{\mathbf{a},\ell,j} f_{\mathbf{a}}(\boldsymbol{\xi}) d\boldsymbol{\xi}$, and so

$$d\mathcal{W} = \int \sum_{\ell,j} \pi_{\mathbf{a},\ell,j} \overline{\lambda}_{\mathbf{a},\ell,j} dU_{\mathbf{a},\ell,j} dG(\mathbf{a})$$

and finally replacing $dU_{\mathbf{a},\ell,j}$ we obtain

$$d\mathcal{W} = \int \sum_{\ell,j} \pi_{\mathbf{a},\ell,j} \overline{\lambda}_{\mathbf{a},\ell,j} u_c(c_{\mathbf{a},\ell,j}, h_{\mathbf{a},\ell,j}) \left[dw_j(1 - t_{\ell,j}) - dr_\ell h_{\mathbf{a},\ell,j} + dT_{\mathbf{a}} \right] dG(\mathbf{a}).$$

B.2 Proof of Proposition 1

We seek to decompose:

$$d\mathcal{W}_{CE}^{\text{Efficiency}} = \int \sum_{\ell,j} \pi_{\mathbf{a},\ell,j} \left\{ dw_j \left(1 - t_{\ell,j} \right) - dr_\ell h_{\mathbf{a},\ell,j} + dT_{\mathbf{a}} \right\} dG(\mathbf{a}) \tag{28}$$

First, note that

$$\int \sum_{\ell} \pi_{\mathbf{a},\ell,j} (1 - t_{\ell,j}) dG(\mathbf{a}) = L_j, \qquad \int \sum_{j} \pi_{\mathbf{a},c,j} h_{\mathbf{a},c,j} dG(\mathbf{a}) = \overline{H}_c - N_f h_f,$$

$$\int \sum_{j} \pi_{\mathbf{a},p,j} h_{\mathbf{a},p,j} dG(\mathbf{a}) = \overline{H}_p, \qquad \int \sum_{\ell,j} \pi_{\mathbf{a},\ell,j} dT_{\mathbf{a}} dG(\mathbf{a}) = \sum_{\ell} dr_\ell \overline{H}_\ell + \sum_j dr_j^K \overline{K}_j.$$

Using these results, we can write

$$\begin{aligned} d\mathcal{W}_{CE}^{\text{Efficiency}} &= \sum_j dw_j L_j - \sum_\ell dr_\ell \overline{H}_\ell + dr_c \times N_f h_f + \sum_\ell dr_\ell \overline{H}_\ell + \sum_j dr_j^K \overline{K}_j \\ &= \sum_j d\log(w_j) w_j L_j + \sum_j d\log(r_j^K) r_j^K \overline{K}_j + dr_c \times N_f h_f \end{aligned}$$

Now using the fact that

$$w_j L_j = \alpha Y_j, \qquad r_j^K \overline{K}_j = (1-\alpha) Y_j,$$

$$d\log(w_j) = (\gamma + \alpha - 1) \frac{dL_j}{L_j}, \qquad d\log(r_j^K) = (\gamma + \alpha) \frac{dL_j}{L_j},$$

we can further simplify the expression above as follows,

$$d\mathcal{W}_{CE}^{\text{Efficiency}} = \gamma \sum_j \frac{Y_j}{L_j} dL_j + dr_c \times N_f h_f$$

and since $dL_j = \sum_\ell \overline{\pi}_{\ell,j}(1-t_{\ell,j})\frac{d\overline{\pi}_{\ell,j}}{\overline{\pi}_{\ell,j}}$, we can write

$$d\mathcal{W}_{CE}^{\text{Efficiency}} = \gamma \sum_{\ell,j} \overline{\pi}_{\ell,j} \frac{Y_j}{L_j}(1-t_{\ell,j})\frac{d\overline{\pi}_{\ell,j}}{\overline{\pi}_{\ell,j}} + dr_c \times N_f h_f.$$

Finally, by definition

$$\mathbb{COV}^{\overline{\Pi}}\left(\frac{Y_j}{L_j}(1-t_{\ell,j}), \frac{d\overline{\pi}_{\ell,j}}{\overline{\pi}_{\ell,j}}\right) = \sum_{\ell,j} \overline{\pi}_{\ell,j} \frac{Y_j}{L_j}(1-t_{\ell,j})\frac{d\overline{\pi}_{\ell,j}}{\overline{\pi}_{\ell,j}} - \sum_{\ell,j} \overline{\pi}_{\ell,j} \frac{Y_j}{L_j}(1-t_{\ell,j}) \underbrace{\sum_{\ell,j} \overline{\pi}_{\ell,j} \frac{d\overline{\pi}_{\ell,j}}{\overline{\pi}_{\ell,j}}}_{=0}.$$

So, we can define

$$\mathcal{PE} \equiv \gamma \times \mathbb{COV}^{\overline{\Pi}}\left(\frac{Y_j}{L_j}(1-t_{\ell,j}), \frac{d\overline{\pi}_{\ell,j}}{\overline{\pi}_{\ell,j}}\right),$$
$$\mathcal{FS} \equiv dr_c \times N_f h_f.$$

C Appendix to Section 5

C.1 Second-best problem and incentive compatibility

Let $c_{\mathbf{a},\xi}$, $h_{\mathbf{a},\xi}$, $\ell_{\mathbf{a},\xi}$ and $j_{\mathbf{a},\xi}$ denote, respectively, the consumption, housing, living location, and working location of each type. The utility net of location preferences ξ for this person is:

$$U_{\mathbf{a},\xi} \equiv \overline{u}_{\ell_{\mathbf{a},\xi}, j_{\mathbf{a},\xi}} + u(c_{\mathbf{a},\xi}, h_{\mathbf{a},\xi})$$

The incentive compatibility constraints of the direct revelation mechanism can be written as

$$U_{\mathbf{a},\xi} + \xi_{\ell_{\mathbf{a},\xi}, j_{\mathbf{a},\xi}} \geq U_{\mathbf{a},\xi'} + \xi_{\ell_{\mathbf{a},\xi'}, j_{\mathbf{a},\xi'}} \tag{29}$$

for all $\boldsymbol{\xi}$, $\boldsymbol{\xi}'$, and $\mathbf{a}$.

It follows from (29) that if two people with the same $\mathbf{a}$ have the same location choices, then they must have the same level of common utility, i.e., assuming $(\ell_{\mathbf{a},\xi}, j_{\mathbf{a},\xi}) = (\ell_{\mathbf{a},\xi'}, j_{\mathbf{a},\xi'})$, then

$$U_{\mathbf{a},\xi} = U_{\mathbf{a},\xi'}. \tag{30}$$

Let $U_{\mathbf{a},\ell,j}$ denote the level of common utility attained by individuals with assets $\mathbf{a}$ and location choices ℓ, j.

Incentive compatibility can now be equivalently written as

$$\{\ell_{\mathbf{a},\xi}, j_{\mathbf{a},\xi}\} = \arg\max_{\ell,j}\{U_{\mathbf{a},\ell,j} + \xi_{\ell,j}\}, \tag{31}$$

and $U_{\mathbf{a},\xi} = U_{\mathbf{a},\ell_{\mathbf{a},\xi},j_{\mathbf{a},\xi}}$.

It follows that, given $\mathbf{U} \equiv \{U_{\mathbf{a},\ell,j}\}$, incentive compatibility implies that the share of individuals with assets $\mathbf{a}$ and location choices ℓ, j is given by

$$\pi_{\mathbf{a},\ell,j} = \mathbb{P}\left[U_{\mathbf{a},\ell,j} + \xi_{\ell,j} \geq U_{\mathbf{a},\ell',j'} + \xi_{\ell',j'} \quad \forall \ell', j' \middle| \mathbf{a}\right], \tag{32}$$

and the social welfare function is

$$\mathcal{W}(\mathbf{U}) = \max_{\ell_{\mathbf{a},\xi}, j_{\mathbf{a},\xi}} \int \lambda_{\mathbf{a},\xi}[U_{\mathbf{a},\ell_{\mathbf{a},\xi},j_{\mathbf{a},\xi}} + \xi_{\ell_{\mathbf{a},\xi},j_{\mathbf{a},\xi}}] f(\boldsymbol{\xi}) d\boldsymbol{\xi} dG(\mathbf{a}). \tag{33}$$

These are the only restrictions on aggregate shares and social welfare implied by incentive compatibility. This means that if the planner chooses common utility levels $U_{\mathbf{a},\ell,j}$, location shares $\pi_{\mathbf{a},\ell,j}$, and welfare $\mathcal{W}$ which satisfy (32) and (33), then we can always find individual location choices which are consistent with incentive compatibility.

C.2 The Mirrleesian program

The Mirrleesian program is

$$\max \mathcal{W}(\mathbf{U}) \quad \text{s. to} \tag{34}$$

$$\int \sum_j \pi_{\mathbf{a},c,j} h_{\mathbf{a},c,j} dG(\mathbf{a}) + N_f h_f = \overline{H}_c \tag{35}$$

$$\int \sum_j \pi_{\mathbf{a},p,j} h_{\mathbf{a},p,j} dG(\mathbf{a}) = \overline{H}_p \tag{36}$$

$$U_{\mathbf{a},\ell,j} = \overline{u}_{\ell,j} + u\left(c_{\mathbf{a},\ell,j}, h_{\mathbf{a},\ell,j}\right) \tag{37}$$

$$\pi_{\mathbf{a},\ell,j} = \hat{\pi}_{\mathbf{a},\ell,j}(\mathbf{U}) \equiv \mathbb{P}\left[U_{\mathbf{a},\ell,j} + \xi_{\ell,j} \geq U_{\mathbf{a},\ell',j'} + \xi_{\ell',j'} \quad \forall \ell', j' | \mathbf{a}\right] \tag{38}$$

$$\int \sum_{\ell,j} \pi_{\mathbf{a},\ell,j} c_{\mathbf{a},\ell,j} dG(\mathbf{a}) + N_f c_f = \sum_j A\left(L_j\right) \overline{K}_j^{1-\alpha} L_j^{\alpha} + N_f y_f \tag{39}$$

$$\overline{u}_f + u\left(c_f, h_f\right) \geq u_f^*, \tag{40}$$

where $L_j \equiv \int \sum_\ell \pi_{\mathbf{a},\ell,j}(1 - t_{\ell,j}) dG(\mathbf{a})$.

We write the Lagrangian for optimization as

$$\mathcal{L} \equiv \mathcal{W}(\mathbf{U}) + \sum_{\ell} \mu_{\ell}^{H} \left(\overline{H}_{\ell} - \int \sum_{j} \pi_{\mathbf{a},\ell,j} h_{\mathbf{a},\ell,j} dG(\mathbf{a}) \right) - \mu_{c}^{H} N_f h_f$$
$$+ \int \sum_{\ell,j} \pi_{\mathbf{a},\ell,j} \mu_{\mathbf{a},\ell,j}^{U} \left[\overline{u}_{\ell,j} + u\left(c_{\mathbf{a},\ell,j}, h_{\mathbf{a},\ell,j}\right) - U_{\mathbf{a},\ell,j} \right] dG(\mathbf{a}) + \int \sum_{\ell,j} \mu^{C} \mu_{\mathbf{a},\ell,j}^{IC} \left[\pi_{\mathbf{a},\ell,j} - \hat{\pi}_{\mathbf{a},\ell,j}(\mathbf{U}) \right] dG(\mathbf{a})$$
$$+ \mu^{C} \left[\sum_{j} A\left(L_j\right) \overline{K}_j^{1-\alpha} L_j^{\alpha} + N_f y_f - \int \sum_{\ell,j} \pi_{\mathbf{a},\ell,j} c_{\mathbf{a},\ell,j} dG(\mathbf{a}) - N_f c_f \right] + \mu^{f} \left[\overline{u}_f + u\left(c_f, h_f\right) - u_f^{*} \right]$$

C.3 Proof of Proposition 3

Taking first-order conditions with respect to $c_{\mathbf{a},\ell,j}$, $h_{\mathbf{a},\ell,j}$, c_f, and h_f, we obtain

$$\begin{aligned}
[c_{\mathbf{a},\ell,j}] &\quad \mu_{\mathbf{a},\ell,j}^{U} u_c(c_{\mathbf{a},\ell,j}, h_{\mathbf{a},\ell,j}) = \mu^{C} \\
[h_{\mathbf{a},\ell,j}] &\quad \mu_{\mathbf{a},\ell,j}^{U} u_h(c_{\mathbf{a},\ell,j}, h_{\mathbf{a},\ell,j}) = \mu_{\ell}^{H} \\
[c_f] &\quad \mu^{f} u_c(c_f, h_f) = N_f \mu^{C} \\
[h_f] &\quad \mu^{f} u_h(c_f, h_f) = N_f \mu_c^{H}
\end{aligned}$$

These imply that

$$\frac{u_h(c_{\mathbf{a},\ell,j}, h_{\mathbf{a},\ell,j})}{u_c(c_{\mathbf{a},\ell,j}, h_{\mathbf{a},\ell,j})} = \frac{\mu_{\ell}^{H}}{\mu^{C}}, \quad \text{and} \quad \frac{u_h(c_f, h_f)}{u_c(c_f, h_f)} = \frac{\mu_c^{H}}{\mu^{C}}.$$

So, the marginal rates of substitution for houses and consumption are equalized for all individuals who live in location ℓ, including foreigners. This condition implies that $\tau_{\mathbf{a},\ell,j}^{h} = 0$ and $\tau_f^{h} = 0$.

Finally, note that at the optimum, the foreigners' participation constraint must bind

$$\overline{u}_f + u(c_f, h_f) = u_f^{*}. \tag{41}$$

since $c_f = y_f - r_c h_f - T_f$, then the entry fee satisfies

$$\overline{u}_f + u(y_f - r_c h_f - T_f, h_f) = u_f^{*}.$$

C.4 Proof of Proposition 5

The first-order conditions with respect to $\pi_{\mathbf{a},\ell,j}$ are given by

$$\mu^C \mu^{IC}_{\mathbf{a},\ell,j} - \mu^H_\ell h_{\mathbf{a},\ell,j} + \mu^C \left((\gamma+\alpha)\frac{Y_j}{L_j}(1-t_{\ell,j}) - c_{\mathbf{a},\ell,j} \right) = 0$$
$$\Leftrightarrow \mu^{IC}_{\mathbf{a},\ell,j} + \gamma \frac{Y_j}{L_j}(1-t_{\ell,j}) = c_{\mathbf{a},\ell,j} + r_\ell h_{\mathbf{a},\ell,j} - w_j(1-t_{\ell,j}) \equiv \mathcal{T}_{\mathbf{a},\ell,j}.$$

Let $\Xi_{\mathbf{a}} \equiv \sum_{\ell,j} \pi_{\mathbf{a},\ell,j}\mathcal{T}_{\mathbf{a},\ell,j}$, then we can write

$$\mathcal{T}_{\mathbf{a},\ell,j} = \Xi_{\mathbf{a}} + \gamma \left[\frac{Y_j}{L_j}(1-t_{\ell,j}) - \sum_{\ell,j} \pi_{\mathbf{a},\ell,j} \frac{Y_j}{L_j}(1-t_{\ell,j}) \right] + \mu^{IC}_{\mathbf{a},\ell,j} - \sum_{\ell,j} \pi_{\mathbf{a},\ell,j}\mu^{IC}_{\mathbf{a},\ell,j}.$$

Now define

$$\Xi^{\mathcal{PE}}_{\mathbf{a},\ell,j} \equiv \gamma \left[\frac{Y_j}{L_j}(1-t_{\ell,j}) - \sum_{\ell,j} \pi_{\mathbf{a},\ell,j} \frac{Y_j}{L_j}(1-t_{\ell,j}) \right]$$
$$\Xi^{\mathcal{R}}_{\mathbf{a},\ell,j} \equiv \mu^{IC}_{\mathbf{a},\ell,j} - \sum_{\ell,j} \pi_{\mathbf{a},\ell,j}\mu^{IC}_{\mathbf{a},\ell,j}.$$

By construction $\sum_{\ell,j} \pi_{\mathbf{a},\ell,j}\Xi^{\mathcal{PE}}_{\mathbf{a},\ell,j} = \sum_{\ell,j} \pi_{\mathbf{a},\ell,j}\Xi^{\mathcal{R}}_{\mathbf{a},\ell,j} = 0$. Finally,

$$\int \Xi_{\mathbf{a}} dG(\mathbf{a}) = \int \sum_{\ell,j} \pi_{\mathbf{a},\ell,j} c_{\mathbf{a},\ell,j} dG(\mathbf{a}) + \int \sum_{\ell,j} \pi_{\mathbf{a},\ell,j} r_\ell h_{\mathbf{a},\ell,j} dG(\mathbf{a}) - \int \sum_{\ell,j} \pi_{\mathbf{a},\ell,j} w_j(1-t_{\ell,j}) dG(\mathbf{a})$$
$$= \sum_j Y_j + N_f(y_f - c_f) + \sum_\ell r_\ell \overline{H}_\ell - N_f r_c h_f - \sum_j \alpha Y_j = \sum_\ell r_\ell \overline{H}_\ell + \sum_j (1-\alpha) Y_j,$$

where $r^K_j \overline{K}_j = (1-\alpha)Y_j$.

C.5 Proof of Corollary 1

With equal welfare weights, the first order condition with respect to $U_{\mathbf{a},\ell,j}$ becomes[21]

$$\pi_{\mathbf{a},\ell,j} - \pi_{\mathbf{a},\ell,j}\mu^U_{\mathbf{a},\ell,j} - \sum_{\ell',j'} \mu^C \mu^{IC}_{\mathbf{a},\ell',j'} \frac{d\hat{\pi}_{\mathbf{a},\ell',j'}(\mathbf{U})}{dU_{\mathbf{a},\ell,j}} = 0.$$

With quasi-linear utility, the first order condition with respect to $c_{\mathbf{a},\ell,j}$ is

$$\mu^U_{\mathbf{a},\ell,j} u_c(c_{\mathbf{a},\ell,j}, h_{\mathbf{a},\ell,j}) = \mu^C \Leftrightarrow \mu^U_{\mathbf{a},\ell,j} = \mu^C.$$

[21] We assume that the marginal condition for $\mathcal{W}(\mathbf{U})$ with respect to each $U_{\mathbf{a},\ell,j}$ holds at the optimum.

Combining these two conditions, we obtain

$$\pi_{\mathbf{a},\ell,j} - \pi_{\mathbf{a},\ell,j}\mu^C - \sum_{\ell',j'} \mu^C \mu^{IC}_{\mathbf{a},\ell',j'} \frac{d\hat{\pi}_{\mathbf{a},\ell',j'}(\mathbf{U})}{dU_{\mathbf{a},\ell,j}} = 0,$$

which summed across ℓ, j imply

$$1 - \mu^C - \sum_{\ell',j'} \mu^C \mu^{IC}_{\mathbf{a},\ell',j'} \sum_{\ell,j} \frac{d\hat{\pi}_{\mathbf{a},\ell',j'}(\mathbf{U})}{dU_{\mathbf{a},\ell,j}} = 0,$$

and since $\sum_{\ell,j} \frac{d\hat{\pi}_{\mathbf{a},\ell',j'}(\mathbf{U})}{dU_{\mathbf{a},\ell,j}} = 0$, then $\mu^C = 1$. Replacing μ^C in the first order condition with respect to $U_{\mathbf{a},\ell,j}$, we obtain

$$\sum_{\ell',j'} \mu^{IC}_{\mathbf{a},\ell',j'} \frac{d\hat{\pi}_{\mathbf{a},\ell',j'}(\mathbf{U})}{dU_{\mathbf{a},\ell,j}} = 0.$$

This equation must hold for all $\mathbf{a}, \ell, j$, which implies that it can only be satisfied if $\mu^{IC}_{\mathbf{a},\ell',j'} = \mu^{IC}_{\mathbf{a}}$ is constant across ℓ', j' since

$$\sum_{\ell',j'} \frac{d\hat{\pi}_{\mathbf{a},\ell',j'}(\mathbf{U})}{dU_{\mathbf{a},\ell,j}} = \frac{d\sum_{\ell',j'} \hat{\pi}_{\mathbf{a},\ell',j'}(\mathbf{U})}{dU_{\mathbf{a},\ell,j}} = \frac{d(1)}{dU_{\mathbf{a},\ell,j}} = 0.$$

C.6 Proof of Corollary 2

Under the conditions specified,

$$\hat{\pi}^{IC}_{\mathbf{a},\ell,j}(\mathbf{U}) = \frac{e^{\eta_{\mathbf{a}}(\delta_{\mathbf{a},\ell,j}+U_{\mathbf{a},\ell,j})}}{\sum_{\ell',j'} e^{\eta_{\mathbf{a}}(\delta_{\mathbf{a},\ell',j'}+U_{\mathbf{a},\ell',j'})}}.$$

It follows that

$$\frac{d\hat{\pi}^{IC}_{\mathbf{a},\ell,j}(\mathbf{U})}{dU_{\mathbf{a},\ell,j}} = \eta_{\mathbf{a}}\pi_{\mathbf{a},\ell,j} - \eta_{\mathbf{a}}\pi_{\mathbf{a},\ell,j}\pi_{\mathbf{a},\ell,j} \qquad \frac{d\hat{\pi}^{IC}_{\mathbf{a},\ell,j}(\mathbf{U})}{dU_{\mathbf{a},\ell',j'}} = -\eta_{\mathbf{a}}\pi_{\mathbf{a},\ell,j}\pi_{\mathbf{a},\ell',j'}.$$

The first order condition with respect to $U_{\mathbf{a},\ell,j}$ becomes

$$\pi_{\mathbf{a},\ell,j}\overline{\lambda}_{\mathbf{a},\ell,j} - \pi_{\mathbf{a},\ell,j}\mu^U_{\mathbf{a},\ell,j} - \mu^C\mu^{IC}_{\mathbf{a},\ell,j}\eta_{\mathbf{a}}\pi_{\mathbf{a},\ell,j} + \sum_{\ell',j'} \mu^C\mu^{IC}_{\mathbf{a},\ell',j'}\eta_{\mathbf{a}}\pi_{\mathbf{a},\ell',j'}\pi_{\mathbf{a},\ell,j} = 0$$

$$\Leftrightarrow \frac{\overline{\lambda}_{\mathbf{a},\ell,j}}{\mu^C} - \frac{\mu^U_{\mathbf{a},\ell,j}}{\mu^C} = \eta_{\mathbf{a}} \left(\mu^{IC}_{\mathbf{a},\ell,j} - \sum_{\ell',j'} \mu^{IC}_{\mathbf{a},\ell',j'}\pi_{\mathbf{a},\ell',j'} \right)$$

Taking a $\pi_{\mathbf{a},\ell,j}$-weighted sum over ℓ, j we obtain

$$\overline{\lambda}_{\mathbf{a}} = \sum_{\ell,j} \pi_{\mathbf{a},\ell,j}\mu^U_{\mathbf{a},\ell,j},$$

where $\overline{\lambda}_{\mathbf{a}} \equiv \sum_{\ell,j} \pi_{\mathbf{a},\ell,j} \overline{\lambda}_{\mathbf{a},\ell,j}$.

Using the first-order condition with respect to $c_{\mathbf{a},\ell,j}$ we obtain

$$\frac{1}{u_c(c_{\mathbf{a},\ell,j}, h_{\mathbf{a},\ell,j})} = \frac{\mu^U_{\mathbf{a},\ell,j}}{\mu^C}.$$

Again, taking a $\pi_{\mathbf{a},\ell,j}$-weighted sum over ℓ, j we obtain

$$\sum_{\ell,j} \pi_{\mathbf{a},\ell,j} \frac{1}{u_c(c_{\mathbf{a},\ell,j}, h_{\mathbf{a},\ell,j})} = \frac{\sum_{\ell,j} \pi_{\mathbf{a},\ell,j} \mu^U_{\mathbf{a},\ell,j}}{\mu^C} = \frac{\overline{\lambda}_{\mathbf{a}}}{\mu^C} \Leftrightarrow \mu^C = \overline{\lambda}_{\mathbf{a}} \left[\sum_{\ell,j} \pi_{\mathbf{a},\ell,j} \left(u_c(c_{\mathbf{a},\ell,j}, h_{\mathbf{a},\ell,j}) \right)^{-1} \right]^{-1}.$$

Combining these expressions, we obtain

$$\begin{aligned} \mu^{IC}_{\mathbf{a},\ell,j} - \sum_{\ell',j'} \mu^{IC}_{\mathbf{a},\ell',j'} \pi_{\mathbf{a},\ell',j'} &= \eta_{\mathbf{a}}^{-1} \left(\frac{\overline{\lambda}_{\mathbf{a},\ell,j}}{\mu^C} - \frac{\mu^U_{\mathbf{a},\ell,j}}{\mu^C} \right) \\ &= \eta_{\mathbf{a}}^{-1} \left(\frac{\overline{\lambda}_{\mathbf{a},\ell,j}}{\overline{\lambda}_{\mathbf{a}} \left[\sum_{\ell,j} \pi_{\mathbf{a},\ell,j} \left(u_c(c_{\mathbf{a},\ell,j}, h_{\mathbf{a},\ell,j}) \right)^{-1} \right]^{-1}} - \frac{1}{u_c(c_{\mathbf{a},\ell,j}, h_{\mathbf{a},\ell,j})} \right) \end{aligned}$$

D Appendix to Section 6

D.1 Heterogeneous Foreigners

Suppose foreigners differ in their preferences for entering the city. Let ξ_f denote a foreigner's idiosyncratic preference for entry. Then, the utility of a foreigner is given by

$$\underbrace{\overline{u}_f + u(c_f, h_f)}_{\mathcal{U}_f} + \xi_f.$$

The cross-sectional distribution of ξ_f is given by $\Phi_f(\xi_f)$, which means that there is a total number of $\overline{N}_f[1 - \Phi_f(\xi_f)]$ foreigners with preferences $\xi_f > \xi$, where $\overline{N}_f$ denotes the total number of foreigners (a large number). Foreigners are willing to move to the city if

$$\mathcal{U}_f + \xi_f \geq u^*_f.$$

Define $\xi^*_f(N_f)$ to be the threshold level of ξ_f such that

$$\xi^*_f(N_f) = \max\{\xi_f \,:\, \overline{N}_f[1 - \Phi(\xi_f)] = N_f\},$$

i.e., ξ_f^* denotes the individual preference of the marginal entrant when N_f people enter. Then, defining $\phi_f(N_f) \equiv u_f^* - \xi_f^*(N_f)$, we can write the participation constraint of the foreigners as

$$\mathcal{U}_f \geq \phi_f(N_f).$$

Note that ξ_f can equivalently be interpreted as a heterogeneous outside option.

D.2 Proof of Proposition 6

We seek to decompose:

$$d\mathcal{W}_{CE}^{\text{Efficiency}} = \int \sum_{\ell,j,e} \pi_{\mathbf{a},\ell,j,e} \left[\widetilde{\phi}'_{\mathbf{a},\ell,j,e}(N_f) + dw_{j,e}\left(1 - t_{\ell,j,e}\right) - w_{j,e} dt_{\ell,j,e} - dr_\ell \times h_{\mathbf{a},\ell,j,e} + dT_{\mathbf{a}}\right] dG(\mathbf{a})$$

First, note that

$$\int \sum_{\ell} \pi_{\mathbf{a},\ell,j,e}(1 - t_{\ell,j,e}) dG(\mathbf{a}) = L_{j,e}, \qquad \int \sum_{j,e} \pi_{\mathbf{a},c,j,e} h_{\mathbf{a},c,j,e} dG(\mathbf{a}) = \overline{H}_c - N_f h_f,$$

$$\int \sum_{j,e} \pi_{\mathbf{a},p,j,e} h_{\mathbf{a},p,j,e} dG(\mathbf{a}) = \overline{H}_p, \qquad \int \sum_{\ell,j,e} \pi_{\mathbf{a},\ell,j,e} dT_{\mathbf{a}} dG(\mathbf{a}) = \sum_\ell dr_\ell \overline{H}_\ell + \sum_j dr_j^K \overline{K}_j,$$

$$dt_{\ell,j,e} = \bar{t}_{\ell,j,e} \delta'(\overline{\pi}_{\ell,j,e}) d\overline{\pi}_{\ell,j,e} = \psi \underbrace{\bar{t}_{\ell,j,e} \delta(\overline{\pi}_{\ell,j,e})}_{t_{\ell,j,e} - \bar{t}_{\ell,j,e}} \frac{d\overline{\pi}_{\ell,j,e}}{\overline{\pi}_{\ell,j,e}},$$

with $dt_{\ell,j,h} = 0$. Using these results, we can write

$$\begin{aligned} d\mathcal{W}_{CE}^{\text{Efficiency}} &= \mathbb{E}^{\mathbf{\Pi}}\left[\widetilde{\phi}'_{\mathbf{a},\ell,j,e}(N_f)\right] + \sum_{j,e} dw_{j,e} L_{j,e} - \sum_\ell dr_\ell \overline{H}_\ell \\ &\quad - \psi \sum_{\ell,j} \pi_{\ell,j,o}(t_{\ell,j,o} - \bar{t}_{\ell,j,o}) \frac{d\overline{\pi}_{\ell,j,o}}{\overline{\pi}_{\ell,j,o}} + dr_c \times N_f h_f + \sum_\ell dr_\ell \overline{H}_\ell + \sum_j dr_j^K \overline{K}_j \\ &= \mathbb{E}^{\mathbf{\Pi}}\left[\widetilde{\phi}'_{\mathbf{a},\ell,j,e}(N_f)\right] + \sum_{j,e} d\log(w_{j,e}) w_{j,e} L_{j,e} + \sum_j d\log(r_j^K) r_j^K \overline{K}_j \\ &\quad - \psi \sum_{\ell,j} \overline{\pi}_{\ell,j,o} w_{j,o}(t_{\ell,j,o} - \bar{t}_{\ell,j,o}) \frac{d\overline{\pi}_{\ell,j,o}}{\overline{\pi}_{\ell,j,o}} + dr_c \times N_f h_f \end{aligned}$$

Now using the fact that

$$w_{j,o} L_{j,o} = \alpha Y_{j,o}, \qquad r_j^K \overline{K}_j = (1-\alpha) Y_{j,o},$$

$$w_{j,h} L_{j,h} = Y_{j,h}, \qquad d\log(w_{j,h}) = \gamma \frac{dL_{j,o}}{L_{j,o}},$$

$$d\log(w_{j,o}) = (\gamma + \alpha - 1) \frac{dL_{j,o}}{L_{j,o}}, \qquad d\log(r_j^K) = (\gamma + \alpha) \frac{dL_{j,o}}{L_{j,o}},$$

where $Y_{j,o} = A_j(L_{j,o})\overline{K}_j^{1-\alpha} L_{j,o}^{\alpha}$ and $Y_{j,h} = A_j(L_{j,o})\zeta L_{j,h}$, we can further simplify the expression above to

$$d\mathcal{W}_{CE}^{\text{Efficiency}} = \mathbb{E}^{\mathbf{\Pi}}\left[\widetilde{\phi}'_{\mathbf{a},\ell,j,e}(N_f)\right] + \gamma\sum_j \frac{Y_j}{L_{j,o}} dL_{j,o} - \psi\sum_{\ell,j}\pi_{\ell,j,o}w_{j,o}(t_{\ell,j,o} - \bar{t}_{\ell,j,o})\frac{d\overline{\pi}_{\ell,j,o}}{\overline{\pi}_{\ell,j,o}} + dr_c \times N_f h_f$$

Now, note that

$$dL_{j,o} = \sum_\ell \overline{\pi}_{\ell,j,o}(1-t_{\ell,j,o})\frac{d\overline{\pi}_{\ell,j,o}}{\overline{\pi}_{\ell,j,o}} - \sum_\ell \overline{\pi}_{\ell,j,o} dt_{\ell,j,o} = \sum_\ell \overline{\pi}_{\ell,j}(1-t_{\ell,j,o})\frac{d\overline{\pi}_{\ell,j,o}}{\overline{\pi}_{\ell,j,o}} - \psi\sum_\ell \overline{\pi}_{\ell,j,o}w_{j,o}(t_{\ell,j,o} - \bar{t}_{\ell,j,o})\frac{d\overline{\pi}_{\ell,j,o}}{\overline{\pi}_{\ell,j,o}}$$

and since $dL_{j,o} = \sum_\ell \overline{\pi}_{\ell,j,o}(1-t_{\ell,j,o})\frac{d\overline{\pi}_{\ell,j,o}}{\overline{\pi}_{\ell,j,o}}$, we can write

$$\begin{aligned} d\mathcal{W}_{CE}^{\text{Efficiency}} &= \mathbb{E}^{\mathbf{\Pi}}\left[\widetilde{\phi}'_{\mathbf{a},\ell,j,e}(N_f)\right] + \gamma\sum_{\ell,j}\overline{\pi}_{\ell,j,o}\frac{Y_j}{L_{j,o}}(1-t_{\ell,j,o})\frac{d\overline{\pi}_{\ell,j,o}}{\overline{\pi}_{\ell,j,o}} - \gamma\psi\sum_{\ell,j}\overline{\pi}_{\ell,j,o}\frac{Y_j}{L_{j,o}}(t_{\ell,j,o} - \bar{t}_{\ell,j,o})\frac{d\overline{\pi}_{\ell,j,o}}{\overline{\pi}_{\ell,j,o}} \\ &\quad - \psi\sum_{\ell,j}\pi_{\ell,j,o}w_{j,o}(t_{\ell,j,o} - \bar{t}_{\ell,j,o})\frac{d\overline{\pi}_{\ell,j,o}}{\overline{\pi}_{\ell,j,o}} + dr_c \times N_f h_f. \end{aligned}$$

Finally,

$$\begin{aligned} d\mathcal{W}_{CE}^{\text{Efficiency}} &= \mathbb{E}^{\mathbf{\Pi}}\left[\widetilde{\phi}'_{\mathbf{a},\ell,j,e}(N_f)\right] + \Pi^{\text{office}}\gamma\sum_{\ell,j}\frac{\overline{\pi}_{\ell,j,o}}{\Pi^{\text{office}}}\frac{Y_j}{L_{j,o}}(1-t_{\ell,j,o})\frac{d\overline{\pi}_{\ell,j,o}}{\overline{\pi}_{\ell,j,o}} \\ &\quad - \gamma\psi\Pi^{\text{office}}\sum_{\ell,j}\frac{\overline{\pi}_{\ell,j,o}}{\Pi^{\text{office}}}\frac{Y_j}{L_{j,o}}(t_{\ell,j,o} - \bar{t}_{\ell,j,o})\frac{d\overline{\pi}_{\ell,j,o}}{\overline{\pi}_{\ell,j,o}} \\ &\quad - \psi\Pi^{\text{office}}\sum_{\ell,j}\frac{\overline{\pi}_{\ell,j,o}}{\Pi^{\text{office}}}w_{j,o}(t_{\ell,j,o} - \bar{t}_{\ell,j,o})\frac{d\overline{\pi}_{\ell,j,o}}{\overline{\pi}_{\ell,j,o}} \\ &\quad + dr_c \times N_f h_f. \end{aligned}$$

$$\begin{aligned} \Leftrightarrow d\mathcal{W}_{CE}^{\text{Efficiency}} &= \overbrace{\mathbb{E}^{\mathbf{\Pi}}\left[\widetilde{\phi}'_{\mathbf{a},\ell,j,e}(N_f)\right]}^{\mathcal{AE}} + \overbrace{\Pi^{\text{office}}\gamma\mathbb{COV}^{\overline{\mathbf{\Pi}}_o}\left(\frac{Y_j}{L_j}(1-t_{\ell,j}), \frac{d\overline{\pi}_{\ell,j,o}}{\overline{\pi}_{\ell,j,o}}\right)}^{\mathcal{PE}} \\ &\quad \overbrace{- \gamma\psi\Pi^{\text{office}}\mathbb{COV}^{\overline{\mathbf{\Pi}}_o}\left(\frac{Y_j}{L_j}(t_{\ell,j,o} - \bar{t}_{\ell,j,o}), \frac{d\overline{\pi}_{\ell,j,o}}{\overline{\pi}_{\ell,j,o}}\right)}^{\mathcal{PCE}} \\ &\quad \overbrace{- \psi\Pi^{\text{office}}\mathbb{COV}^{\overline{\mathbf{\Pi}}_o}\left(w_{j,o}(t_{\ell,j,o} - \bar{t}_{\ell,j,o}), \frac{d\overline{\pi}_{\ell,j,o}}{\overline{\pi}_{\ell,j,o}}\right)}^{\mathcal{CE}} \\ &\quad \overbrace{+ \left(\gamma\sum_j Y_j - \psi\sum_{\ell,j}\overline{\pi}_{\ell,j,o}w_{j,o}(t_{\ell,j,o} - \bar{t}_{\ell,j,o}) - \gamma\psi\sum_{\ell,j}\overline{\pi}_{\ell,j,o}\frac{Y_j}{L_{j,o}}(t_{\ell,j,o} - \bar{t}_{\ell,j,o})\right) \times \frac{d\Pi^{\text{office}}}{\Pi^{\text{office}}}}^{\mathcal{RW}} \\ &\quad + dr_c \times N_f h_f. \end{aligned}$$

D.3 The Mirrleesian program

The Mirrleesian program is

$$\max \mathcal{W}(\mathbf{U}) \quad \text{s. to} \tag{42}$$

$$\int \sum_{j,e} \pi_{\mathbf{a},c,j,e} h_{\mathbf{a},c,j,e} dG(\mathbf{a}) + N_f h_f = \overline{H}_c \tag{43}$$

$$\int \sum_{j,e} \pi_{\mathbf{a},p,j,e} h_{\mathbf{a},p,j,e} dG(\mathbf{a}) = \overline{H}_p \tag{44}$$

$$U_{\mathbf{a},\ell,j,e} = \overline{u}_{\mathbf{a},\ell,j,e} + u\left(c_{\mathbf{a},\ell,j,e}, h_{\mathbf{a},\ell,j,e}\right) \tag{45}$$

$$\pi_{\mathbf{a},\ell,j,e} = \hat{\pi}_{\mathbf{a},\ell,j,e}(\mathbf{U}) \equiv \mathbb{P}\left[U_{\mathbf{a},\ell,j,e} + \xi_{\ell,j,e} \geq U_{\mathbf{a},\ell',j',e'} + \xi_{\ell',j',e'} \quad \forall \ell', j', e' | \mathbf{a}\right] \tag{46}$$

$$\int \sum_{\mathbf{a},\ell,j,e} \pi_{\mathbf{a},\ell,j,e} c_{\mathbf{a},\ell,j,e} dG(\mathbf{a}) + N_f c_f = \sum_j A\left(L_{j,o}\right)[\overline{K}_j^{1-\alpha} L_{j,o}^{\alpha} + \zeta L_{j,h}] + N_f y_f \tag{47}$$

$$\overline{u}_f + u\left(c_f, h_f\right) \geq \phi_f(N_f), \tag{48}$$

where $L_{j,e} \equiv \int \sum_\ell \pi_{\mathbf{a},\ell,j,e}(1 - t_{\ell,j,e}) dG(\mathbf{a})$.

We write the Lagrangian for optimization as

$$\begin{aligned}
\mathcal{L} \equiv & \mathcal{W}(\mathbf{U}) + \sum_\ell \mu_\ell^H \left(\overline{H}_\ell - \int \sum_{j,e} \pi_{\mathbf{a},\ell,j,e} h_{\mathbf{a},\ell,j,e} dG(\mathbf{a})\right) - \mu_c^H N_f h_f \\
& + \int \sum_{\ell,j,e} \pi_{\mathbf{a},\ell,j,e} \mu^U_{\mathbf{a},\ell,j,e} \left[\overline{u}_{\mathbf{a},\ell,j,e} + u\left(c_{\mathbf{a},\ell,j,e}, h_{\mathbf{a},\ell,j,e}\right) - \mathcal{U}_{\mathbf{a},\ell,j,e}\right] dG(\mathbf{a}) \\
& + \int \sum_{\ell,j,e} \mu^C \mu^{IC}_{\mathbf{a},\ell,j,e} \left[\pi_{\mathbf{a},\ell,j,e} - \hat{\pi}_{\mathbf{a},\ell,j,e}(\mathbf{U})\right] dG(\mathbf{a}) \\
& + \mu^C \left[\sum_j A\left(L_{j,o}\right)[\overline{K}_j^{1-\alpha} L_{j,o}^{\alpha} + \zeta L_{j,h}] + N_f y_f - \int \sum_{\mathbf{a},\ell,j,e} \pi_{\mathbf{a},\ell,j,e} c_{\mathbf{a},\ell,j,e} dG(\mathbf{a}) - N_f c_f\right] \\
& + \mu^f \left[\overline{u}_f + u\left(c_f, h_f\right) - \phi_f(N_f)\right]
\end{aligned}$$

D.4 Proof of Proposition 7

At the optimum, the foreigners participation constraint must bind

$$\overline{u}_f + u(c_f, h_f) = \phi_f(N_f), \tag{49}$$

so there are no quota restrictions on the entry of foreign residents.

Taking first-order conditions with respect to $c_{\mathbf{a},\ell,j,e}$, $h_{\mathbf{a},\ell,j,e}$, c_f, and h_f, we obtain

$$[c_{\mathbf{a},\ell,j,e}] \quad \mu^U_{\mathbf{a},\ell,j,e} u_c(c_{\mathbf{a},\ell,j,e}, h_{\mathbf{a},\ell,j,e}) = \mu^C$$

$$[h_{\mathbf{a},\ell,j,e}] \quad \mu^U_{\mathbf{a},\ell,j,e} u_h(c_{\mathbf{a},\ell,j,e}, h_{\mathbf{a},\ell,j,e}) = \mu^H_\ell$$

$$[c_f] \quad \mu^f u_c(c_f, h_f) = N_f \mu^C$$

$$[h_f] \quad \mu^f u_h(c_f, h_f) = N_f \mu^H_c$$

These imply that

$$\frac{u_h(c_{\mathbf{a},\ell,j,e}, h_{\mathbf{a},\ell,j,e})}{u_c(c_{\mathbf{a},\ell,j,e}, h_{\mathbf{a},\ell,j,e})} = \frac{\mu^H_\ell}{\mu^C}, \quad \text{and } \frac{u_h(c_f, h_f)}{u_c(c_f, h_f)} = \frac{\mu^H_c}{\mu^C}.$$

So, the marginal rates of substitution for houses and consumption are equalized for all individuals who live in location ℓ, including foreigners. This implies that $\tau^h_{\mathbf{a},\ell,j,e} = 0$ and $\tau^h_f = 0$.

Finally, the first order condition with respect to N_f is given by

$$\int \sum_{\ell,j,e} \pi_{\mathbf{a},\ell,j,e} \mu^U_{\mathbf{a},\ell,j,e} \phi'_{\ell,j,e}(N_f) dG(\mathbf{a}) - \mu^H_c h_f + \mu^C [y_f - c_f] - \mu^f \phi^{*,\prime}_f(N_f) = 0.$$

Using the fact that $\mu^U_{\mathbf{a},\ell,j,e}/\mu^C = 1/u_c(c_{\mathbf{a},\ell,j,e}, h_{\mathbf{a},\ell,j,e})$, $\mu^H_c/\mu^C = r_c$, and $\mu^f/\mu^C = N_f/u_c(c_f, h_f)$ we can write

$$\mathcal{T}_f \equiv y_f - c_f - r_c h_f = -\int \sum_{\ell,j,e} \pi_{\mathbf{a},\ell,j,e} \widetilde{\phi}'_{\mathbf{a},\ell,j,e}(N_f) dG(\mathbf{a}) + \chi \widetilde{u}^*_f.$$

D.5 Proof of Proposition 8

We have already established that $\tau^h_{\mathbf{a},\ell,j,e} = 0$. Taking first order conditions with respect to $\pi_{\mathbf{a},\ell,j,o}$ we get

$$\begin{aligned}
&\mu^C \mu^{IC}_{\mathbf{a},\ell,j,o} - \mu^H_\ell h_{\mathbf{a},\ell,j,o} \\
&+ \mu^C \left[\gamma \frac{Y_j}{L_{j,o}} (1 - t_{\ell,j,o}) + \alpha \frac{Y_{j,o}}{L_{j,o}} (1 - t_{\ell,j,o}) - \gamma \frac{Y_j}{L_{j,o}} \bar{t}_{\ell,j,o} \delta'(\overline{\pi}_{\ell,j,o}) - \alpha \frac{Y_{j,o}}{L_{j,o}} \bar{t}_{\ell,j,o} \delta'(\overline{\pi}_{\ell,j,o}) - c_{\mathbf{a},\ell,j,o} \right] \\
&+ \mu^f \frac{d\phi_f(\boldsymbol{\pi})}{d\pi_{\mathbf{a},\ell,j,o}} = 0 \\
\Leftrightarrow &\mu^{IC}_{\mathbf{a},\ell,j,o} - r_\ell h_{\mathbf{a},\ell,j,o} \\
&+ \gamma \frac{Y_j}{L_{j,o}} (1 - t_{\ell,j,o}) + w_{j,o}(1 - t_{\ell,j,o}) - \gamma \psi \frac{Y_j}{L_{j,o}} (t_{\ell,j,o} - \bar{t}_{\ell,j,o}) - \psi w_{j,o} (t_{\ell,j,o} - \bar{t}_{\ell,j,o}) - c_{\mathbf{a},\ell,j,o} \\
&+ \frac{d\widetilde{\phi}_f(\boldsymbol{\pi})}{d\pi_{\mathbf{a},\ell,j,o}} = 0
\end{aligned}$$

and so

$$\mathcal{T}_{\mathbf{a},\ell,j,o} = \mu^{IC}_{\mathbf{a},\ell,j,o} + \gamma \frac{Y_j}{L_{j,o}}(1 - t_{\ell,j,o}) - \gamma\psi \frac{Y_j}{L_{j,o}}(t_{\ell,j,o} - \bar{t}_{\ell,j,o}) - \psi w_{j,o}(t_{\ell,j,o} - \bar{t}_{\ell,j,o}) + \frac{d\widetilde{\phi}_f(\boldsymbol{\pi})}{d\pi_{\mathbf{a},\ell,j,o}}.$$

Similarly, the first order conditions with respect to $\pi_{\mathbf{a},\ell,j,h}$ are

$$\begin{aligned}
&\mu^C \mu^{IC}_{\mathbf{a},\ell,j,h} - \mu^H_\ell h_{\mathbf{a},\ell,j,h} + \mu^C \left[A(L_{j,o})\zeta - c_{\mathbf{a},\ell,j,h} \right] + \mu^f \frac{d\phi_f(\boldsymbol{\pi})}{d\pi_{\mathbf{a},\ell,j,h}} = 0 \\
\Leftrightarrow &\mu^{IC}_{\mathbf{a},\ell,j,o} - r_\ell h_{\mathbf{a},\ell,j,o} + w_{j,h} - c_{\mathbf{a},\ell,j,o} + \frac{d\widetilde{\phi}_f(\boldsymbol{\pi})}{d\pi_{\mathbf{a},\ell,j,h}} = 0
\end{aligned}$$

and so

$$\mathcal{T}_{\mathbf{a},\ell,j,h} = \mu^{IC}_{\mathbf{a},\ell,j,h} + \frac{d\widetilde{\phi}_f(\boldsymbol{\pi})}{d\pi_{\mathbf{a},\ell,j,h}}.$$

Let $\Xi_{\mathbf{a}} = \sum_{\ell,j,e} \pi_{\mathbf{a},\ell,j,e} \mathcal{T}_{\mathbf{a},\ell,j,e}$. Then,

$$\begin{aligned}
\Xi_{\mathbf{a}} = &\gamma \sum_{\ell,j} \pi_{\mathbf{a},\ell,j,o} \frac{Y_{j,o}}{L_{j,o}}(1 - t_{\ell,j,o}) - \psi \sum_{\ell,j} \pi_{\mathbf{a},\ell,j,o} w_{j,o}(t_{\ell,j,o} - \bar{t}_{\ell,j,o}) - \gamma\psi \sum_{\ell,j} \pi_{\mathbf{a},\ell,j,o} \frac{Y_j}{L_{j,o}}(t_{\ell,j,o} - \bar{t}_{\ell,j,o}) \\
&+ N_f \sum_{\ell,j,e} \pi_{\mathbf{a},\ell,j,e} \frac{d\widetilde{\phi}_f(\boldsymbol{\pi})}{d\pi_{\mathbf{a},\ell,j,e}} + \sum_{\ell,j,e} \pi_{\mathbf{a},\ell,j,e} \mu^{IC}_{\mathbf{a},\ell,j,e}.
\end{aligned}$$

So,

$$\begin{aligned}
\mathcal{T}_{\mathbf{a},\ell,j,o} = &\Xi_{\mathbf{a}} + \overbrace{\gamma \left(\frac{Y_j}{L_{j,o}}(1 - t_{\ell,j,o}) - \sum_{\ell,j} \pi_{\mathbf{a},\ell,j,o} \frac{Y_{j,o}}{L_{j,o}}(1 - t_{\ell,j,o}) \right)}^{\Xi^{\mathcal{PE}}_{\mathbf{a},\ell,j,o}} \overbrace{-\psi \left(w_{j,o}(t_{\ell,j,o} - \bar{t}_{\ell,j,o}) - \sum_{\ell,j} \pi_{\mathbf{a},\ell,j,o} w_{j,o}(t_{\ell,j,o} - \bar{t}_{\ell,j,o}) \right)}^{\Xi^{\mathcal{CE}}_{\mathbf{a},\ell,j,o}} \\
&\overbrace{-\gamma\psi \left(\frac{Y_j}{L_{j,o}}(t_{\ell,j,o} - \bar{t}_{\ell,j,o}) - \sum_{\ell,j} \pi_{\mathbf{a},\ell,j,o} \frac{Y_j}{L_{j,o}}(t_{\ell,j,o} - \bar{t}_{\ell,j,o}) \right)}^{\Xi^{\mathcal{PCE}}_{\mathbf{a},\ell,j,o}} + \overbrace{N_f \left(\frac{d\widetilde{\phi}_f(\boldsymbol{\pi})}{d\pi_{\mathbf{a},\ell,j,o}} - \sum_{\ell,j,e} \pi_{\mathbf{a},\ell,j,e} \frac{d\widetilde{\phi}_f(\boldsymbol{\pi})}{d\pi_{\mathbf{a},\ell,j,e}} \right)}^{\Xi^{\mathcal{AE}}_{\mathbf{a},\ell,j,o}} \\
&+ \overbrace{\mu^{IC}_{\mathbf{a},\ell,j,o} - \sum_{\ell,j,e} \pi_{\mathbf{a},\ell,j,e} \mu^{IC}_{\mathbf{a},\ell,j,e}}^{\Xi^{\mathcal{R}}_{\mathbf{a},\ell,j,o}}
\end{aligned}$$

and analogously

$$
\begin{aligned}
\mathcal{T}_{\mathbf{a},\ell,j,h} =& \Xi_{\mathbf{a}} \overbrace{-\gamma\left(\sum_{\ell,j}\pi_{\mathbf{a},\ell,j,o}\frac{Y_{j,o}}{L_{j,o}}(1-t_{\ell,j,o})\right)}^{\Xi^{\mathcal{PE}}_{\mathbf{a},\ell,j,h}} + \overbrace{\psi\left(\sum_{\ell,j}\pi_{\mathbf{a},\ell,j,o}w_{j,o}(t_{\ell,j,o}-\bar{t}_{\ell,j,o})\right)}^{\Xi^{\mathcal{CE}}_{\mathbf{a},\ell,j,h}} \\
&+ \overbrace{\gamma\psi\left(\sum_{\ell,j}\pi_{\mathbf{a},\ell,j,o}\frac{Y_j}{L_{j,o}}(t_{\ell,j,o}-\bar{t}_{\ell,j,o})\right)}^{\Xi^{\mathcal{PCE}}_{\mathbf{a},\ell,j,o}} + \overbrace{N_f\left(\frac{d\widetilde{\phi}_f(\boldsymbol{\pi})}{d\pi_{\mathbf{a},\ell,j,h}} - \sum_{\ell,j,e}\pi_{\mathbf{a},\ell,j,e}\frac{d\widetilde{\phi}_f(\boldsymbol{\pi})}{d\pi_{\mathbf{a},\ell,j,e}}\right)}^{\Xi^{\mathcal{AE}}_{\mathbf{a},\ell,j,h}} \\
&+ \overbrace{\mu^{IC}_{\mathbf{a},\ell,j,h} - \sum_{\ell,j,e}\pi_{\mathbf{a},\ell,j,e}\mu^{IC}_{\mathbf{a},\ell,j,e}}^{\Xi^{\mathcal{R}}_{\mathbf{a},\ell,j,h}}
\end{aligned}
$$

E Relation to the optimal tariff literature

We can interpret the sales of houses to foreigners as exports paid for in units of the tradable consumption good. So, there is a connection between our results and those in the trade literature (see, e.g., Dixit, 1985, Caliendo and Parro, 2022, and references therein). In this Appendix, we discuss this relation using a simple trade model.

Consider a world with a home country and a continuum $n \in \mathbb{R}_+$ of identical foreign countries. Countries are endowed with two consumption goods, 1 and 2. The home country has y_1 units of good 1 and y_2 units of good 2. Each foreign country has y_1^* and y_2^* units of goods 1 and 2, respectively (throughout, we use stars to denote foreign-country variables). The representative agent of the home country has utility $u(c_1, c_2)$, and the representative agent of each foreign country has utility $u^*(c_1^*, c_2^*)$.

Abstracting from location choices and goods production, this model is analogous to our baseline model if we interpret one good as houses and the other as consumption.

E.1 Why is the optimal tax on houses bought by foreigners zero?

To compute the optimal tariff, we assume that the home country can unilaterally impose a proportional tax τ on imports (or, equivalently, a subsidy to exports). The resulting tax revenue, T, is rebated back to the households of the home country. The budget constraints of home and foreign consumers are given by

$$c_1 - y_1 + (1+\tau)p(c_2 - y_2) - T = 0, \tag{50}$$

$$c_1^* - y_1^* + p(c_2^* - y_2^*) = 0, \tag{51}$$

where p denotes the relative price of good 2 in units of good 1. Two first-order conditions describe the equilibrium in this economy,

$$\frac{u_2}{u_1} = (1+\tau)p, \tag{52}$$

$$\frac{u_2^*}{u_1^*} = p, \tag{53}$$

the budget constraints (50) and (51), the resource constraints,

$$c_1 + nc_1^* = y_1 + ny_1^*, \tag{54}$$

$$c_2 + nc_2^* = y_2 + ny_2^*, \tag{55}$$

and the government budget constraint,

$$T = \tau p(c_2 - y_2). \tag{56}$$

We compute the optimal tariff using the primal approach developed by Lucas and Stokey (1983). This approach involves choosing $\{c_1, c_2, c_1^*, c_2^*\}$ to maximize the utility in the home country, subject to the resource constraints (54) and (55), the implementability condition

$$u_1^*(c_1^* - y_1^*) + u_2^*(c_2^* - y_2^*) = 0, \tag{57}$$

and a participation constraint for the foreign countries:[22]

$$u^*(c_1^*, c_2^*) \geq \overline{u}^*. \tag{58}$$

This constraint reflects the existence of unmodeled alternatives to trading with the home country that guarantee a level of utility $\overline{u}^*$.

Theorem 1. *Let φ and λ_p denote the Lagrange multipliers associated with (57) and (58), respectively. The optimal tariff is given by*

$$\tau = \varphi \frac{\left(\frac{u_{22}^*}{u_2^*} - \frac{u_{21}^*}{u_1^*}\right)(c_2^* - y_2^*) - \left(\frac{u_{11}^*}{u_1^*} - \frac{u_{12}^*}{u_2^*}\right)(c_1^* - y_1^*)}{\lambda_p + \varphi\left[1 + \frac{u_{11}^*}{u_1^*}\left(c_1^* - y_1^*\right) + \frac{u_{21}^*}{u_1^*}\left(c_2^* - y_2^*\right)\right]} \neq 0. \tag{59}$$

[22]These are necessary and sufficient conditions to solve for the equilibrium allocations. They are necessary because the equilibrium conditions imply them. Sufficiency can be proved as follows. Take a set of allocations $\{c_1, c_2, c_1^*, c_2^*\}$ that satisfies these conditions. These allocations can be equilibrium allocations for an appropriate choice of prices and policies. We can always find a tariff, τ, and a relative price, p, that satisfy the marginal rates of substitution (52) and (53), respectively. We can always find T that satisfies the domestic budget constraint (50). Using these values for p, τ, and T, the foreign budget constraint (51) is satisfied since the implementability condition (57) is also satisfied. The government budget constraint is satisfied by Walras' law. Finally, the resource constraints are also satisfied since they are imposed. It follows that we can always construct an equilibrium that implements the allocations $\{c_1, c_2, c_1^*, c_2^*\}$.

Suppose that $u^*(c_1^*, c_2^*) = [(c_1^*)^{1-\Xi} + (c_2^*)^{1-\Xi}]/(1-\Xi)$, then the optimal tariff takes the form

$$\tau = \Xi\varphi \frac{\left(\frac{c_1^*-y_1^*}{c_1^*}\right) - \left(\frac{c_2^*-y_2^*}{c_2^*}\right)}{\lambda_p + \varphi\left[1 - \Xi\left(\frac{c_1^*-y_1^*}{c_1^*}\right)\right]}.$$

Suppose $\varphi > 0$. If foreigners export good 2, then $c_1^* > y_1^*$ and $c_2^* < y_2^*$. The optimal tariff is positive ($\tau > 0$). If foreigners export good 1, then $c_1^* < y_1^*$ and $c_2^* > y_2^*$. The optimal tariff is negative ($\tau < 0$).

This is the classical result that a country has an incentive to unilaterally tax imports or subsidize exports to manipulate terms of trade and obtain monopolistic rents. In our baseline model, the home country exports houses and imports traded goods. So, why do we find that taxing the houses foreigners purchase is not optimal?

In deriving the optimal tariff, we have assumed that levying a lump-sum tax on foreigners is impossible. However, our baseline model does not preclude this possibility since the home country can impose an entry fee on foreign residents. Suppose that in our trade model, the home country can charge foreign countries a fee T^* for the right to trade. The foreigners' budget constraint is

$$c_1^* - y_1^* + p(c_2^* - y_2^*) + T^* = 0. \tag{60}$$

The domestic budget constraint takes the same form (50),

$$c_1 - y_1 + (1+\tau)p(c_2 - y_2) - T = 0$$

where the rebates to domestic households are now given by

$$T = \tau p(c_2 - y_2) + nT^*.$$

We do not need to impose the implementability condition (57), since it can always be satisfied by choosing an appropriate trade fee, T^*. So, the new planning problem is to maximize the home country's welfare, subject to (54), (55), and (58).

Proposition 11. *Suppose that the home country can impose a rights-to-trade fee, T^*. Then, the optimal tariff is zero*

$$\tau = 0. \tag{61}$$

The rights-to-trade fee is set so that foreign countries are indifferent between trading and not trading:

$$u^*(c_1^*, c_2^*) = \overline{u}^*. \tag{62}$$

When a lump-sum instrument is available, it is always better to use it to extract the gains from trade from foreign countries than to impose a distortionary tax on trade. The reason is as follows. A zero tariff maximizes the gains from trade. These gains are then taxed away by the home country using the lump-sum instrument. This scheme resembles the optimal use of a two-part tariff by a monopolist. It is optimal for the monopolist to set the price equal to marginal cost and charge a fixed fee to extract all consumer surplus.

In our model, we impose no exogenous restrictions on the available instruments. Instead, the set of feasible instruments is determined by the planner's or government's primitive informational constraints. Since the planner can observe the country of origin, it can design a tax system with a lump-sum tax on foreigners. The result above implies that taxing houses is not optimal.

In our model in the main text, for any fixed number of foreign countries N_f, it is optimal for the home country to set a non-zero entry fee $T_f \neq 0$ to extract gains from foreign countries relative to their outside option.

E.2 Why is a zero entry fee optimal in our model?

The third part of Proposition 3 states that the optimal entry fee is zero in our main model. This result reflects the fact that the planner can choose the optimal number of foreigners, N_f.

To discuss the optimal entry fee using the trade model presented in this section, we allow the home country to choose the number of trading partners, n. Let λ_1 and λ_2 denote the Lagrange multipliers on resource constraints for good 1 and 2, respectively. The first-order condition for n is[23]

$$\lambda_1(y_1^* - c_1^*) + \lambda_2(y_2^* - c_2^*) = 0. \tag{63}$$

This equation equates marginal benefits with marginal costs. The marginal benefit of an additional trading partner is the value of the goods they bring to the table $\lambda_1 y_1^* + \lambda_2 y_2^*$. The marginal cost is the value of goods they consume $\lambda_1 c_1^* + \lambda_2 c_2^*$.

Combining (63) with the implementability condition (57), we find that

$$\frac{\lambda_1(y_1^* - c_1^*)}{u_1^*(y_1^* - c_1^*)} = \frac{\lambda_2(y_2^* - c_2^*)}{u_2^*(y_2^* - c_2^*)} \Leftrightarrow \frac{u_2^*}{u_1^*} = \frac{\lambda_2}{\lambda_1} = \frac{u_2}{u_1}. \tag{64}$$

[23]We assume throughout that the solution is interior.

If the home country cannot levy a lump-sum tax, T^*, then the optimal tariff is $\tau = 0$ when the number of trading partners is chosen optimally.

If the home country can choose $T^* \neq 0$, then we already know that $\tau = 0$ and $p = u_2^*/u_1^* = \lambda_2/\lambda_1$. It then follows from (63) that

$$(y_1^* - c_1^*) + \frac{u_2^*}{u_1^*}(y_2^* - c_2^*) = 0 \Leftrightarrow (y_1^* - c_1^*) + p(y_2^* - c_2^*) = 0 \Leftrightarrow T^* = 0. \tag{65}$$

So, even if the home country can levy a lump-sum tax, then the optimal rights-to-trade fee is $T^* = 0$ when the number of trading partners is chosen optimally.

These results are summarized in the following proposition, which echoes Proposition 3.

Proposition 12. *Suppose the home country can choose the number of trading partners, n. Then, the optimal number of trading partners is such that:*

1. *If the home country cannot impose a rights-to-trade fee, then the optimal tariff is zero, $\tau = 0$.*
2. *If the home country can impose a rights-to-trade fee, then the optimal fee is zero, $T^* = 0$.*

It follows that the optimal number of trading partners is the same as in a laissez-faire solution. To explain why, we start with too few trading partners. As we increase n, each trading partner receives a smaller portion of the home country's exports. The relative price of the exported good rises, and the home country benefits more from exports.[24] To satisfy the participation constraint, the home country must reduce the rights-to-trade fee. The benefit from increasing the value of exports is strictly greater than the reduction in fee revenue.

For analogous reasons, in our model, optimizing the number of foreigners N_f requires setting the entry fee, T_f, to zero.

E.3 Numerical example

We illustrate the results described in propositions 11 and 12 with a numerical example. We assume that the utility function takes the form $u(c_1, c_2) = (c_1^{1-\Xi} + c_2^{1-\Xi})/(1-\Xi)$ and $u^*(c_1^*, c_2^*) = [(c_1^*)^{1-\Xi} + (c_2^*)^{1-\Xi}]/(1-$

[24] The home country also exports more in total, so it consumes a lower amount of the exported good and more of the imported good.

Ξ) and set $\Xi = 0.25$. We also set $y_1 = 1$, $y_2 = 0.3$, $y_1^* = 0.3$ and $y_2^* = 1$. We set the foreigner's outside option to $\overline{u}^* = 1.7371$.[25]

Figure 7 displays the optimal tariff as a function of the number of trading partners, n, when the rights-to-trade fee is restricted to zero. We also display the optimum under the additional assumption that trading partners are freely disposable, i.e., the home country can trade with fewer than the n countries. The dotted red line represents the results under this additional assumption. The panel in position (1,1) displays the welfare in the home country, the panel in (1,2) the optimal tariff, the panel in (2,1) the rights-to-trade fee (which in this case is restricted to zero), and finally the panel (2,2) the transfer of the tariff revenue to the domestic household, T.

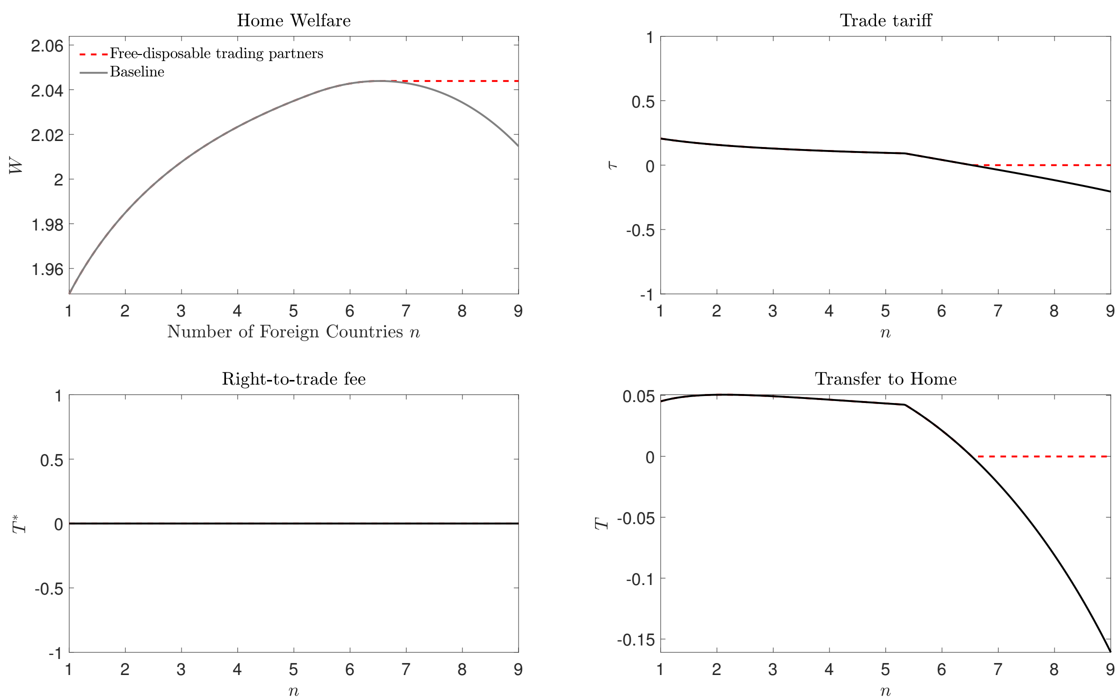

Figure 7: Optimal tariff

When the rights-to-trade fee is restricted to zero, it is optimal to impose a tariff, i.e., a tax on imports. As the number of trading partners increases, the optimal tariff falls. Home welfare rises for small n and

[25] In this numerical example, as the outside converges to the utility under autarky, $u^*(y_1^*, y_2^*)$, the optimal number of trading partners converges to infinity.

reaches a maximum when $n = n^* = 6.53$. As shown in Proposition 12, the optimal tariff when the country can choose the optimal number of trade partners is zero. Beyond this optimal number of trade partners, home welfare declines because the home country must subsidize imports. This subsidy transfers resources to foreign countries and helps satisfy their outside option.

So, when $n \geq n^*$ and trading partners are freely disposable, it is optimal to implement a laissez-faire policy in which tariffs are zero and foreign countries freely choose whether to trade with the home country.

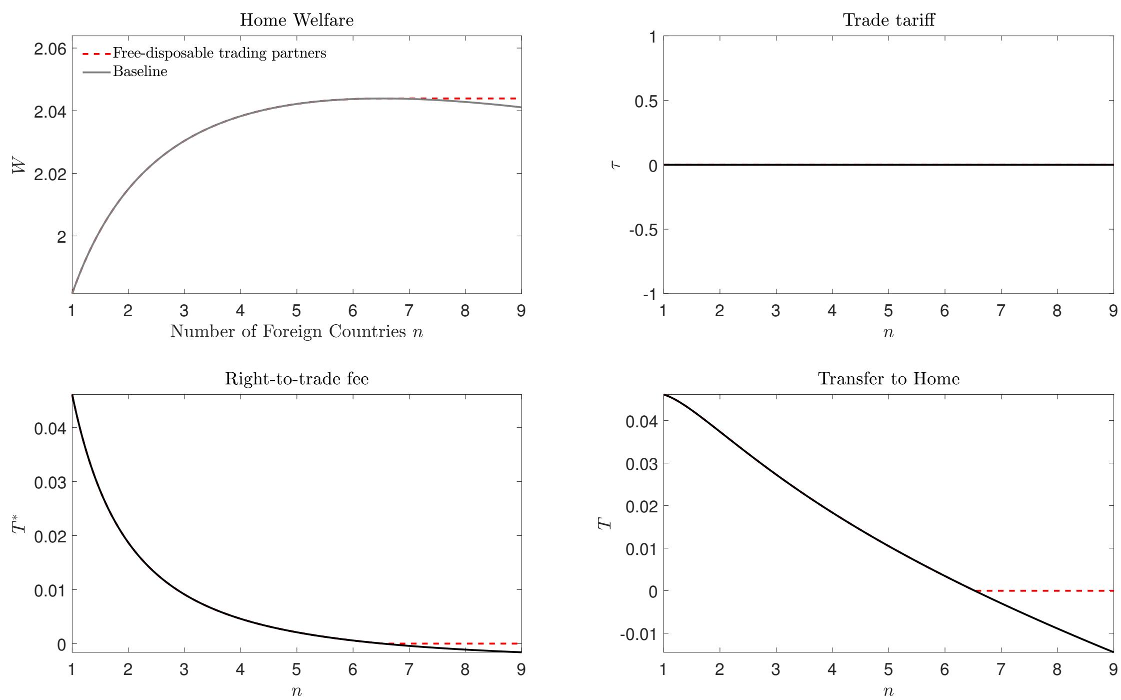

Figure 8: Optimal rights-to-trade fee

Figure 8 displays the results for the case of the optimal tariff and rights-to-trade fee as a function of the number of trading partners, n. As in Figure 7, we also display the optimum under the additional assumption that there is free-disposal of trading partners, i.e., the home country can trade with fewer than the n countries. The dotted red line represents these results. The panel in position (1,1) displays the welfare in the home country, the panel in (1,2) the optimal tariff, the panel in (2,1) the trade fee (which in this case is restricted to be zero), and finally the panel (2,2) the transfer of the tariff revenue to the domestic household,

T.

When the home country can impose a rights-to-trade fee, setting the tariff to zero is always optimal, as in Proposition 11. As the number of trading partners increases, the optimal rights-to-trade fee falls. Home welfare rises for small n and reaches a maximum when $n = n^* = 6.53$. If $n < n^*$, it is optimal to impose a positive rights-to-trade fee. As n increases, the optimal rights-to-trade fee falls and reaches zero when $n = n^*$, as shown in Proposition 12. If $n > n^*$, the optimal rights-to-trade fee becomes negative. This subsidy transfers resources to foreign countries and helps satisfy their outside option.

For $n \geq n^*$ and free-disposability of trading partners, it is optimal to implement a laissez-faire policy in which tariffs are zero and foreign countries freely choose whether to trade with the home country.

F Appendix to Section 7 – Optimal policy in a dynamic model

In this section, we discuss the dynamic model that we use to study optimal policy in the long run.

Time is discrete and infinite, $t = 0, 1, \ldots$. We consider a model of a city composed of a city center, denoted by c, and a discrete number of peripheral locations, denoted by $p_1, ..., p_N$. At time t, location $\ell \in L \equiv \{c, p_1, ..., p_N\}$ has $H_{\ell,t}$ residential buildings and $K_{\ell,t}$ office buildings. As described below, productivity in location ℓ depends on the local labor supply because of *production/agglomeration externalities*.

The economy is inhabited by overlapping generations of local workers (*locals*) who live for $A + 1$ periods. Each generation is composed of a unit continuum of workers. We index the generations by their age at time t: $a \in \{0, ..., A\}$. Newborns choose freely where to live and can commute to a different location for work. Later in life, workers can readjust their location decisions, subject to a moving cost. At each point in time, workers receive labor and savings income. At each date, a large number of foreigners are willing to enter the city if the utility of entering exceeds their outside option. For simplicity, we assume that foreigners locate only in the city center and that their location decision is static.[26] Both locals and foreigners derive utility from consuming traded goods and housing services.

[26]Our results would go through under the weaker assumption that foreign residents disproportionally locate in the city center relative to locals. It would also be easy to extend our results to foreigners who stay multiple periods in the city. We maintain the assumption that the foreigners' location decision is static to reflect the short-run nature of digital nomads and tourists.

F.1 Locals

Locals differ in their preferences for different locations. They have idiosyncratic taste preferences for living and working in different locations at each age. Formally, let $\xi^a_{\ell,j}$ denote the individual's preference for living in location ℓ and working in location j at age a. At age a, locals provide θ_a efficiency units of labor. This parameter allows us to model lifecycle income and productivity dynamics. We denote by $\boldsymbol{\xi} = \{\xi^a_{\ell,j}\}$ the vector of taste preferences. Individuals draw the preferences $\boldsymbol{\xi}$ at the start of their life from a distribution $F(\boldsymbol{\xi})$, with pdf $f(\boldsymbol{\xi})$. Agents draw an entire life-cycle vector at birth and these need not remain constant over an individual's lifetime, allowing for changing preferences as individuals age.

Let $\omega_a = (\ell_a, j_a) \in L^2$ denote the pair of residential and location choices made by an individual at age a. We denote by $\boldsymbol{\omega} \equiv \{\omega_a\}_{a=0}^A$ the collection of decisions that workers make throughout their lifetime. A worker who changes their location decisions from ω to ω' pays a utility moving cost equal to $mc_{\omega,\omega'}$, with the normalization $mc_{\omega,\omega} = 0$.

Initial Old The generation with age a at time 0 enters the period with initial location decisions ω_{-1}. They also have heterogeneous ownership of houses and office buildings. Each individual is endowed with $h_{\ell,0}$ and $k_{\ell,0}$ units of houses and office buildings in each location ℓ. Let $\mathbf{a} \equiv \{h_{\ell,0}, k_{\ell,0}\}$ denote the individual's vector of asset holdings in different locations. Each member of this generation is characterized by the vector $\{a, \omega_{-1}, \mathbf{a}, \boldsymbol{\xi}\}$. For each age a, we denote the distribution over initial living locations and assets by $G_{-a}(\omega_{-1}, \mathbf{a})$. We assume that for each ω_{-1} and $\mathbf{a}$, $\boldsymbol{\xi}$ is continuously distributed with probability density function $f_{a,\omega_{-1},\mathbf{a}}(\boldsymbol{\xi})$. We allow for arbitrary dependence of these taste preferences on initial location and asset holdings. The distribution of asset holdings satisfies the aggregation equations

$$\sum_a \sum_{\omega_{-1}} \int h_{\ell,0} dG_{-a}(\omega_{-1}, \mathbf{a}) = H_{\ell,0}, \quad \text{and} \quad \sum_a \sum_{\omega_{-1}} \int k_{j,0} dG_{-a}(\omega_{-1}, \mathbf{a}) = K_{j,0}. \tag{66}$$

The local's problem is to maximize lifetime utility

$$\sum_{\tilde{a}=a}^{A} \beta^{\tilde{a}} \left\{ \bar{u}_{\ell_{\tilde{a}}} + u\left(c^{\tilde{a}}_{\tilde{a}-a}, h^{\tilde{a}}_{\tilde{a}-a}\right) + \xi^{\tilde{a}}_{\omega_{\tilde{a}}} - mc_{\omega_{\tilde{a}-1},\omega_{\tilde{a}}} \right\} \tag{67}$$

Newborns at time t At each date t, a new generation of locals is born with zero assets. We assume that, in the first period of life, the household can freely choose where to locate. After the initial date, the household can change its location decisions, subject to a moving cost. As discussed above, the newborn households draw a vector of location preferences $\boldsymbol{\xi}$ from the distribution $F(\boldsymbol{\xi})$. The local's problem is to maximize lifetime utility

$$\sum_{a=0}^{A} \beta^a \left\{ \overline{u}_{\ell_a} + u\left(c_{t+a}^a, h_{t+a}^a\right) + \xi_{\omega_a}^a - mc_{\omega_{a-1},\omega_a} \right\} \tag{68}$$

Foreign residents To simplify, we assume that foreign residents strictly prefer to live in the city center and face a static decision problem. It is easy to extend the model to allow foreigners to have a dynamic choice problem. At time t, each foreigner chooses consumption $c_{f,t}$ and housing in the city center $h_{f,t}$, to maximize their utility:

$$\mathcal{U}_{f,t} \equiv \overline{u}_f + u\left(c_{f,t}, h_{f,t}\right), \tag{69}$$

where $\overline{u}_f$ is the value foreign residents attach to the amenities in the center.

Foreigners bring a fixed endowment of the tradable good, $y_{f,t}$, which they use to pay for consumption and housing services and potentially pay taxes. When choosing not to enter the city, foreigners receive utility $u_{f,t}^*$. This outside option captures the utility they obtain from moving to an alternative city abroad. They only migrate if their participation constraint is satisfied:

$$\mathcal{U}_{f,t} \geq u_{f,t}^*. \tag{70}$$

Firms' problem Each location has a representative, perfectly competitive firm. The firm in location j produces the homogeneous tradable good, $y_{j,t}$, by combining labor, $l_{j,t}$, and offices, $k_{j,t}$. The production function is given by

$$y_{j,t} = A_j\left(L_{j,t}\right) l_{j,t}^{\alpha} k_{j,t}^{1-\alpha}. \tag{71}$$

Productivity in region j is given by the function $A_j(L_{j,t})$, which depends on total labor supply in location j due to agglomeration (or production) externalities. Locations with larger workforces are more productive because they offer more opportunities for workers to learn from one another. For concreteness, we assume that

$$A_j\left(L_{j,t}\right) = \overline{A}_j L_{j,t}^{\gamma}. \tag{72}$$

The parameter $\gamma \geq 0$ controls the strength of the agglomeration externality. If $\gamma = 0$, there are no production externalities. The higher is γ, the stronger are these externalities.

The firm hires workers at the wage rate $w_{j,t}$ and rents office space at the rental rate $r^K_{j,t}$, earning profits $y_{j,t} - w_{j,t} l_{j,t} - r^K_{j,t} k_{j,t}$. The firm's optimality conditions are:

$$w_{j,t} = \alpha A_j \left(L_{j,t}\right) \left(\frac{k_{j,t}}{l_{j,t}}\right)^{1-\alpha} \quad \text{and} \quad r^K_{j,t} = (1-\alpha) A_j \left(L_{j,t}\right) \left(\frac{k_{j,t}}{l_{j,t}}\right)^{-\alpha}. \tag{73}$$

In equilibrium, total output in location j is $Y_{j,t} = y_{j,t}$.

House and Office Building Supply Representative developers in each location build houses and offices using the final traded good to maximize profits. At each point, developers invest $\Phi_{\ell,t}(I^h_{\ell,t}, I^k_{\ell,t})$ units of the final good to produce additional housing units

$$H_{\ell,t+1} = I^h_{\ell,t} + (1-\delta) H_{\ell,t}, \quad \text{and} \quad K_{\ell,t+1} = I^k_{\ell,t} + (1-\delta) K_{\ell,t}. \tag{74}$$

F.2 Optimal policy

As in the static model, we assume that the planner can condition allocations on observable characteristics. In this case, the observable characteristics of the initial old generations are their past location decisions, asset holdings, and age. For newborns, the observable characteristic is age/birth year. Furthermore, the planner can elicit their taste preferences $\boldsymbol{\xi}$ by offering a menu of location choices. As in the static model, this is equivalent to conditioning allocations on their realized location choices given the induced allocations.

It follows that the planner can design consumption and housing demand allocations

$$\{c^{\tilde{a}}_{\tilde{a}-a}(\omega_{-1}, \boldsymbol{\omega}^a, \mathbf{a}), h^{\tilde{a}}_{\tilde{a}-a}(\omega_{-1}, \boldsymbol{\omega}^a, \mathbf{a})\}$$

for the initial old and $\{c^a_{t+a}(\boldsymbol{\omega}), h^a_{t+a}(\boldsymbol{\omega})\}$ for the newborns, as well as housing supply allocations $\{H_{\ell,t}, K_{\ell,t}\}$ and labor supply allocations $\{L_{j,t}\}$. The planner chooses these allocations to maximize the social welfare function

For the initial old, the level of "common utility" associated with these choices if given by

$$\mathcal{U}_{-a}(\omega_{-1}, \boldsymbol{\omega}^a, \mathbf{a}) = \sum_{\tilde{a}=a}^{A} \beta^{\tilde{a}} \left\{ \overline{u}_{\ell_{\tilde{a}}} + u\left(c^{\tilde{a}}_{\tilde{a}-a}(\omega_{-1}, \boldsymbol{\omega}^a, \mathbf{a}), h^{\tilde{a}}_{\tilde{a}-a}(\omega_{-1}, \boldsymbol{\omega}^a, \mathbf{a})\right) - mc_{\omega_{\tilde{a}-1}, \omega_{\tilde{a}}} \right\} \tag{75}$$

We let $\pi_{-a}(\omega_{-1}, \boldsymbol{\omega}^a, \mathbf{a})$ denote the share of locals with age a at time 0, initial location ω_{-1}, and initial assets $\mathbf{a}$ who choose the sequence of location decisions ω^a. This is given by

$$\pi_{-a}(\omega_{-1}, \boldsymbol{\omega}^a, \mathbf{a}) = \mathbb{P}\left[\mathcal{U}_{-a}(\omega_{-1}, \boldsymbol{\omega}^a, \mathbf{a}) + \sum_{\tilde{a}=a}^{A} \beta^{\tilde{a}} \xi^{\tilde{a}}_{\omega_{\tilde{a}}} \geq \mathcal{U}_{-a}(\omega_{-1}, \tilde{\boldsymbol{\omega}}^a, \mathbf{a}) + \sum_{\tilde{a}=a}^{A} \beta^{\tilde{a}} \xi^{\tilde{a}}_{\tilde{\omega}_{\tilde{a}}} \quad \forall \tilde{\boldsymbol{\omega}}^a | a, \omega_{-1}, \mathbf{a}\right]. \tag{76}$$

We assume that $\boldsymbol{\xi}$ is continuously distributed such that $\pi_{-a}(\omega_{-1}, \boldsymbol{\omega}^a, \mathbf{a}) > 0$ for all $\boldsymbol{\omega}^a$.

For the newborns at time t, the level of "common utility" associated with these choices is given by

$$\mathcal{U}_t(\omega) = \sum_{a=0}^{A} \beta^a \left\{ \overline{u}_{\ell_a} + u\left(c^a_{t+a}(\boldsymbol{\omega}), h^a_{t+a}(\boldsymbol{\omega})\right) - mc_{\omega_{a-1},\omega_a} \right\}, \tag{77}$$

with the convention $mc_{\omega_{-1},\omega_0} = 0$.

We let $\pi_t(\boldsymbol{\omega})$ denote the share of newborns at time t who choose the sequence of location decisions $\boldsymbol{\omega}$. This is given by

$$\pi_t(\boldsymbol{\omega}) = \mathbb{P}\left[\mathcal{U}_t(\boldsymbol{\omega}) + \sum_{a=0}^{A} \beta^a \xi^a_{\omega_a} \geq \mathcal{U}_t(\widetilde{\boldsymbol{\omega}}) + \sum_{a=0}^{A} \beta^a \xi^a_{\widetilde{\omega}_a}, \quad \forall \widetilde{\boldsymbol{\omega}} \right]. \tag{78}$$

We assume that $\boldsymbol{\xi}$ is continuously distributed such that $\pi_t(\boldsymbol{\omega}) > 0$ for all $\boldsymbol{\omega}$. The conditions (76) and (78) constitute the planner's incentive-compatibility constraints.

Let C^a_t denote the aggregate consumption of individuals with age a at time t, i.e.,

$$C^a_t \equiv \begin{cases} \sum_{\boldsymbol{\omega}} \pi_t(\boldsymbol{\omega}) c^a_t(\boldsymbol{\omega}), & \text{if } t-a \geq 0 \text{ (newborns)}, \\ \sum_{\boldsymbol{\omega}^{a-t}} \int \pi_{-a}(\omega_{-1}, \boldsymbol{\omega}^{a-t}, \mathbf{a}) c^a_t(\omega_{-1}, \boldsymbol{\omega}^{a-t}, \mathbf{a}) dG_{-a}(\omega_{-1}, \mathbf{a}), & \text{if } t-a < 0 \text{ (initial old)}. \end{cases} \tag{79}$$

Analogously, we define house consumption of individuals with age a at time t as

$$H^{a,d}_{\ell,t} \equiv \begin{cases} \sum_{\boldsymbol{\omega}:\omega_a \in \{\ell\} \times L} \pi_t(\boldsymbol{\omega}) h^a_t(\boldsymbol{\omega}), & \text{if } t-a \geq 0 \text{ (newborns)}, \\ \sum_{\boldsymbol{\omega}^{a-t}:\omega_a \in \{\ell\} \times L} \int \pi_{-a}(\omega_{-1}, \boldsymbol{\omega}^{a-t}, \mathbf{a}) h^a_t(\omega_{-1}, \boldsymbol{\omega}^{a-t}, \mathbf{a}) dG_{-a}(\omega_{-1}, \mathbf{a}), & \text{if } t-a < 0 \text{ (initial old)}, \end{cases} \tag{80}$$

and labor supply:

$$L^{a,s}_{j,t} \equiv \begin{cases} \sum_{\boldsymbol{\omega}:\omega_a \in L \times \{j\}} \pi_t(\boldsymbol{\omega}) \theta_a (1 - t_{\omega_a}), & \text{if } t-a \geq 0 \text{ (newborns)}, \\ \sum_{\boldsymbol{\omega}^{a-t}:\omega_a \in L \times \{j\}} \int \pi_{-a}(\omega_{-1}, \boldsymbol{\omega}^{a-t}, \mathbf{a}) \theta_a (1 - t_{\omega_a}) dG_{-a}(\omega_{-1}, \mathbf{a}), & \text{if } t-a < 0 \text{ (initial old)}. \end{cases} \tag{81}$$

Using these definitions, the housing-market clearing conditions are given by

$$\sum_a H^{a,d}_{c,t} + N_{f,t} h_{f,t} = H_{c,t} \quad \text{and} \quad \sum_a H^{a,d}_{\ell,t} = H_{\ell,t} \quad \forall \ell \neq c. \tag{82}$$

Firms in location j hire labor and rent buildings to satisfy the market clearing conditions

$$l_{j,t} = L_{j,t} = \sum_a L^{a,s}_{j,t}, \quad \text{and} \quad k_{j,t} = K_{j,t}. \tag{83}$$

Structures evolve according to

$$K_{\ell,t+1} = (1-\delta) K_{\ell,t} + I^k_{\ell,t},$$

$$H_{\ell,t+1} = (1-\delta) H_{\ell,t} + I^h_{\ell,t}.$$

Finally, goods market clearing implies that

$$\sum_a C_t^a + N_{f,t}c_{f,t} + \sum_\ell \Phi_{\ell,t}(I_{\ell,t}^h, I_{\ell,t}^k) = \sum_j Y_{j,t} + N_{f,t}y_{f,t}. \tag{84}$$

The left-hand side of this equation represents the total consumption of local residents across all living and working locations, combined with the total consumption of foreign residents, $N_{f,t}c_{f,t}$, where $N_{f,t}$ denotes the number of foreign residents at time t. On the right-hand side, we have the total production in each location, $Y_{j,t}$, along with the endowment of goods brought by the foreign residents, $N_{f,t}y_{f,t}$.

The planning problem is to design allocations to maximize the social welfare function:

$$\mathcal{W} \equiv \sum_{t=-A}^{\infty} \beta^t \mathcal{W}_t(\mathbf{U}_t), \tag{85}$$

where

$$\mathcal{W}_t(\mathbf{U}_t) \equiv \begin{cases} \int \lambda_{t,\xi} \max\{\mathcal{U}_t(\boldsymbol{\omega}) + \sum_{a=0}^A \beta^a \xi_{\omega_a}^a\} dF(\boldsymbol{\xi}), & \text{if } t \geq 0 \text{ (newborns)}, \\ \int\int \lambda_{t,\omega_{-1},\mathbf{a},\xi} \max\{\mathcal{U}_{-a}(\omega_{-1}, \boldsymbol{\omega}^a, \mathbf{a}) + \sum_{\tilde{a}=a}^A \beta^{\tilde{a}} \xi_{\omega_{\tilde{a}}}^{\tilde{a}}\} dF_{\omega_{-1},\mathbf{a}}(\boldsymbol{\xi}) dG_{-a}(\omega_{-1}, \mathbf{a}), & \text{if } -A \leq t < 0 \text{ (initial old)}, \end{cases} \tag{86}$$

where $\lambda \geq 0$ denotes the welfare weight attributed to the different agents. We derive the properties of the optimal policy without taking a stand on the weights λ chosen by the planner. So, the optimal policy is independent of the planner's preferences for redistribution across the domestic population. The planning problem is constrained by the IC constraints above, plus the market clearing conditions for labor, housing, office buildings, and goods, plus the participation constraint of foreign residents (70).

The problem is therefore given by

$$\max \sum_{t=-A}^{\infty} \beta^t \mathcal{W}_t(\mathbf{U}_t) \quad \text{s.t.}$$

$$[\beta^{-a}\eta^{IC}_{-a}(\omega_{-1},\boldsymbol{\omega}^a,\mathbf{a})]: \quad \pi_{-a}(\omega_{-1},\boldsymbol{\omega}^a,\mathbf{a}) = \mathbb{P}\left[\mathcal{U}_{-a}(\omega_{-1},\boldsymbol{\omega}^a,\mathbf{a}) + \sum_{\tilde{a}=a}^{A} \beta^{\tilde{a}} \xi^{\tilde{a}}_{\omega_{\tilde{a}}} \geq \mathcal{U}_{-a}(\omega_{-1},\tilde{\boldsymbol{\omega}}^a,\mathbf{a}) + \sum_{\tilde{a}=a}^{A} \beta^{\tilde{a}} \xi^{\tilde{a}}_{\tilde{\omega}_{\tilde{a}}} : \quad \forall \tilde{\boldsymbol{\omega}}^a | a, \omega_{-1}, \mathbf{a}\right]$$

$$[\beta^{-a}\eta^{U}_{-a}(\omega_{-1},\boldsymbol{\omega}^a,\mathbf{a})]: \quad \mathcal{U}_{-a}(\omega_{-1},\boldsymbol{\omega}^a,\mathbf{a}) = \sum_{\tilde{a}=a}^{A} \beta^{\tilde{a}} \left\{ \overline{u}_{\ell_{\tilde{a}}} + u\left(c^{\tilde{a}}_{\tilde{a}-a}(\boldsymbol{\omega}^a,\mathbf{a}), h^{\tilde{a}}_{\tilde{a}-a}(\boldsymbol{\omega}^a,\mathbf{a})\right) - mc_{\omega^a_{t-1},\omega^a_{\tilde{a}-a}} \right\}$$

$$[\beta^t \eta^{IC}_t(\boldsymbol{\omega})]: \quad \pi_t(\boldsymbol{\omega}) = \mathbb{P}\left[\mathcal{U}_t(\boldsymbol{\omega}) + \sum_{a=0}^{A} \beta^a \xi^a_{\omega_a} \geq \mathcal{U}_t(\tilde{\boldsymbol{\omega}}) + \sum_{a=0}^{A} \beta^a \xi^a_{\tilde{\omega}_a} : \quad \forall \tilde{\boldsymbol{\omega}}\right]$$

$$[\beta^t \eta^{U}_t(\boldsymbol{\omega})]: \quad \mathcal{U}_t(\omega) = \sum_{a=0}^{A} \beta^a \left\{ \overline{u}_{\ell_a} + u\left(c^a_{t+a}(\boldsymbol{\omega}), h^a_{t+a}(\boldsymbol{\omega})\right) - mc_{\omega_{a-1},\omega_a} \right\}$$

$$[\beta^t \eta^C_t]: \quad \sum_{a=0}^{A} C^a_t + \sum_{\ell} \Phi_{\ell,t}(I^h_{\ell,t}, I^k_{\ell,t}) = \sum_j Y_{j,t} + N_{f,t}(y_{f,t} - c_{f,t})$$

$$[\beta^t \eta^H_{\ell,t}]: \quad \sum_a H^{a,d}_{\ell,t} + N_{f,\ell,t} h_{f,\ell,t} = H_{\ell,t},$$

$$[\beta^t \eta^F_t]: \quad \overline{u}_f + u(c_{f,t}, h_{f,t}) \geq u^*_{f,t},$$

where, for ease of notation, C^a_t and $H^{a,d}_{\ell,t}$ denote the average consumption and housing demand in location ℓ of individuals with age a. The coefficients in square brackets denote the multipliers associated with each constraint.

We present our main results in terms of the instruments that decentralize the optimal allocation. The decentralization we consider is a competitive equilibrium in which people may be taxed on their housing purchases and receive lump-sum taxes or transfers. For locals, these instruments are restricted to depend solely on their assets and observable location decisions, whereas for foreigners, the instruments can be chosen independently.

For an agent i, we denote by $(1+\tau^h_{i,t})r^h_{\ell,t}$ the effective rent paid at time t if they live in location ℓ. These satisfy

$$(1+\tau^h_{i,t})r^h_{\ell,t} = \frac{u_h(c_{i,t}, h_{i,t})}{u_c(c_{i,t}, h_{i,t})}. \tag{87}$$

This is a tax on house rents. For each period, we can normalize the tax for one individual to zero.

Furthermore, for an individual who values consuming both at time t and $t+1$, we denote by $\tau^s_{i,t+1}$ the savings tax faced by these individuals, which satisfies

$$u_c(c_{i,t}, h_{i,t}) = (1 - \tau^s_{i,t+1})(1 + r_{t+1})\beta u_c(c_{i,t+1}, h_{i,t+1}) \tag{88}$$

This is an Euler equation wedge. For each period, we can normalize the tax for one individual to zero. We also define $q_t \equiv \prod_{j=1}^{t} \frac{1}{1+r_j}$ as the price of a bond that pays one unit of the final good at time t, with $q_0 \equiv 1$.

Furthermore, we define the price of new buildings to be equal to their marginal cost

$$p^h_{\ell,t} = \frac{\partial \Phi_{\ell,t}(I^h_{\ell,t}, I^k_{\ell,t})}{\partial I^h_{\ell,t}}, \quad \text{and} \quad p^k_{\ell,t} = \frac{\partial \Phi_{\ell,t}(I^h_{\ell,t}, I^k_{\ell,t})}{\partial I^k_{\ell,t}},$$

and we say that there is an investment tax/subsidy $\tau^h_{\ell,t}$ and $\tau^k_{\ell,t}$ on building houses and offices, respectively, if

$$(1 - \tau^h_{\ell,t+1})\frac{r^h_{\ell,t+1} + (1-\delta)p^h_{\ell,t+1}}{p^h_{\ell,t}} = 1 + r_{t+1}, \quad \text{and} \quad (1 - \tau^k_{\ell,t+1})\frac{r^k_{\ell,t+1} + (1-\delta)p^k_{\ell,t+1}}{p^k_{\ell,t}} = 1 + r_{t+1}.$$

Foreigners pay an entry fee if their income exceeds their consumption and housing expenditures. We define this fee as

$$\mathcal{T}_{f,t} \equiv y_{f,t} - c_{f,t} - (1 + \tau^h_{f,t})r^h_{c,t}h_{f,t}. \tag{89}$$

So, the total proceeds from taxing foreigners are $\Theta_{f,t} = N_{f,t}\tau^h_{f,t}r^h_{c,t}h_{f,t} + N_{f,t}\mathcal{T}_{f,t}$.

Finally, we define the transfers to a local as the excess expenditure over their total labor income, $T_t(\omega_{-1}, \mathbf{a}, \boldsymbol{\omega}^a)$ for the initial old and $T_t(\boldsymbol{\omega})$ for the newborns.

F.3 Optimal policy towards foreigners

The first order conditions with respect to $c_{f,t}$ and $h_{f,t}$ are given by

$$\begin{aligned} [c_{f,t}]: &\quad \beta^t \eta^F_t u_c(c_{f,t}, h_{f,t}) = \beta^t \eta^C_t N_{f,t} \\ [h_{f,t}]: &\quad \beta^t \eta^F_t u_h(c_{f,t}, h_{f,t}) = \beta^t \eta^H_{c,t} N_{f,t}. \end{aligned}$$

which implies that

$$\frac{u_h(c_{f,t}, h_{f,t})}{u_c(c_{f,t}, h_{f,t})} = \frac{\eta^H_{c,t}}{\eta^C_t}.$$

Furthermore, the first order conditions with respect to consumption and housing services of locals are given by

$$[c_t^a(\omega_{-1}, \boldsymbol{\omega}^a, \mathbf{a})]: \quad \beta^t \eta_t^U(\omega_{-1}, \boldsymbol{\omega}^a, \mathbf{a}) u_c(c_t^a(\omega_{-1}, \boldsymbol{\omega}^a, \mathbf{a}), h_t^a(\omega_{-1}, \boldsymbol{\omega}^a, \mathbf{a})) = \beta^t \eta_t^C$$

$$[h_t^a(\omega_{-1}, \boldsymbol{\omega}^a, \mathbf{a})]: \quad \beta^t \eta_t^U(\omega_{-1}, \boldsymbol{\omega}^a, \mathbf{a}) u_h(c_t^a(\omega_{-1}, \boldsymbol{\omega}^a, \mathbf{a}), h_t^a(\omega_{-1}, \boldsymbol{\omega}^a, \mathbf{a})) = \beta^t \eta_{\ell_a,t}^H$$

which implies that

$$\frac{u_h(c_t^a(\omega_{-1}, \boldsymbol{\omega}^a, \mathbf{a}), h_t^a(\omega_{-1}, \boldsymbol{\omega}^a, \mathbf{a}))}{u_c(c_t^a(\omega_{-1}, \boldsymbol{\omega}^a, \mathbf{a}), h_t^a(\omega_{-1}, \boldsymbol{\omega}^a, \mathbf{a}))} = \frac{\eta_{\ell_a,t}^H}{\eta_t^C} \qquad \frac{u_h(c_t^a(\boldsymbol{\omega}), h_t^a(\boldsymbol{\omega}))}{u_c(c_t^a(\boldsymbol{\omega}), h_t^a(\boldsymbol{\omega}))} = \frac{\eta_{\ell_a,t}^H}{\eta_t^C}.$$

The combination of these equations implies that the optimal tax on houses is zero both for foreigners and all locals, i.e., $\tau_{f,t}^h = \tau_{i,t}^h = 0$ for all i.

The first-order conditions with respect to $N_{f,t}$ are given by

$$\beta^t \eta_t^C (y_{f,t} - c_{f,t}) - \beta^t \eta_{c,t}^H h_{f,t} = 0,$$

and using the fact that $r_{c,t}^h = \frac{\eta_{c,t}^H}{\eta_t^C}$, we obtain that the optimal entry fee is given by

$$\mathcal{T}_{f,t} = y_{f,t} - c_{f,t} - r_{c,t}^h h_{f,t} = 0.$$

F.4 Optimal policy towards locals

As noted above, the optimal tax on housing services is zero for all locals. Furthermore, note that

$$\frac{\beta u_c(c_{t+1}^{a+1}(\omega_{-1}, \boldsymbol{\omega}^{\tilde{a}}, \mathbf{a}), h_{t+1}^{a+1}(\omega_{-1}, \boldsymbol{\omega}^{\tilde{a}}, \mathbf{a}))}{u_c(c_t^a(\omega_{-1}, \boldsymbol{\omega}^{\tilde{a}}, \mathbf{a}), h_t^a(\omega_{-1}, \boldsymbol{\omega}^{\tilde{a}}, \mathbf{a}))} = \frac{\beta \eta_{t+1}^C}{\eta_t^C} = \frac{1}{1 + r_{t+1}} \qquad \frac{u_c(c_{t+1}^{a+1}(\boldsymbol{\omega}), h_{t+1}^{a+1}(\boldsymbol{\omega}))}{u_c(c_t^a(\boldsymbol{\omega}), h_t^a(\boldsymbol{\omega}))} = \frac{\beta \eta_{t+1}^C}{\eta_t^C} = \frac{1}{1 + r_{t+1}}.$$

This shows that there are no person-specific intertemporal distortions, i.e., the optimal savings tax is zero $\tau_{i,t+1}^s = 0$ for all i.

The first order condition with respect to $\pi_{-a}(\omega_{-1}, \boldsymbol{\omega}^a, \mathbf{a})$ is given by

$$\beta^{-a} \eta_{-a}^{IC}(\omega_{-1}, \boldsymbol{\omega}^a, \mathbf{a}) + \sum_{\tilde{a}=a}^{A} \beta^{\tilde{a}-a} \eta_{\tilde{a}-a}^C \left((\alpha + \gamma) \frac{Y_{j_{\tilde{a}}, \tilde{a}-a}}{L_{j_{\tilde{a}}, \tilde{a}-a}} (1 - t_{\omega_{\tilde{a}}}) \theta_{\tilde{a}} - c_{\tilde{a}-a}^{\tilde{a}}(\omega_{-1}, \boldsymbol{\omega}^a, \mathbf{a}) \right) - \sum_{\tilde{a}=a}^{A} \beta^{\tilde{a}-a} \eta_{\ell_{\tilde{a}}, \tilde{a}-a}^H h_{\tilde{a}-a}^{\tilde{a}}(\omega_{-1}, \boldsymbol{\omega}^a, \mathbf{a}) = 0$$

Dividing through by η_0^C we obtain

$$\frac{\beta^{-a} \eta_{-a}^{IC}(\omega_{-1}, \boldsymbol{\omega}^a, \mathbf{a})}{\eta_0^C} + \sum_{\tilde{a}=a}^{A} q_{\tilde{a}-a} \left((\alpha + \gamma) \frac{Y_{j_{\tilde{a}}, \tilde{a}-a}}{L_{j_{\tilde{a}}, \tilde{a}-a}} (1 - t_{\omega_{\tilde{a}}}) \theta_{\tilde{a}} - c_{\tilde{a}-a}^{\tilde{a}}(\omega_{-1}, \boldsymbol{\omega}^a, \mathbf{a}) - r_{\ell_{\tilde{a}}, \tilde{a}-a}^h h_{\tilde{a}-a}^{\tilde{a}}(\omega_{-1}, \boldsymbol{\omega}^a, \mathbf{a}) \right) = 0$$

The transfer to these agents is given by

$$T_{-a}(\omega_{-1},\mathbf{a},\boldsymbol{\omega}^a) = \sum_{\widetilde{a}=a}^{A} q_{\widetilde{a}-a}\left(c^{\widetilde{a}}_{\widetilde{a}-a}(\omega_{-1},\boldsymbol{\omega}^a,\mathbf{a}) - w_{j_{\widetilde{a}},\widetilde{a}-a}(1-t_{\omega_{\widetilde{a}}})\theta_{\widetilde{a}} + r^h_{\ell_{\widetilde{a}},\widetilde{a}-a}h^{\widetilde{a}}_{\widetilde{a}-a}(\omega_{-1},\boldsymbol{\omega}^a,\mathbf{a})\right)$$

so,

$$\begin{aligned}T_{-a}(\omega_{-1},\mathbf{a},\boldsymbol{\omega}^a) =&\overline{T}_{-a}(\omega_{-1},\mathbf{a}) + \gamma\times\sum_{\widetilde{a}=a}^{A} q_{\widetilde{a}-a}\left(\frac{Y_{j_{\widetilde{a}},\widetilde{a}-a}}{L_{j_{\widetilde{a}},\widetilde{a}-a}}(1-t_{\omega_{\widetilde{a}}})\theta_{\widetilde{a}} - \sum_{\omega_{-1}}\sum_{\boldsymbol{\omega}^a}\int \pi_{-a}(\omega_{-1},\boldsymbol{\omega}^a,\mathbf{a})\frac{Y_{j_{\widetilde{a}},\widetilde{a}-a}}{L_{j_{\widetilde{a}},\widetilde{a}-a}}(1-t_{\omega_{\widetilde{a}}})\theta_{\widetilde{a}}dG_{-a}(\omega_{-1},\mathbf{a})\right)\\&+\frac{\beta^{-a}\eta^{IC}_{-a}(\omega_{-1},\boldsymbol{\omega}^a,\mathbf{a})}{\eta^C_0} - \sum_{\omega_{-1}}\sum_{\boldsymbol{\omega}^a}\int \pi_{-a}(\omega_{-1},\boldsymbol{\omega}^a,\mathbf{a})\frac{\beta^{-a}\eta^{IC}_{-a}(\omega_{-1},\boldsymbol{\omega}^a,\mathbf{a})}{\eta^C_0}dG_{-a}(\omega_{-1},\mathbf{a})\end{aligned}$$

The first order condition with respect to $\pi_t(\boldsymbol{\omega})$ is given by

$$\beta^t\eta^{IC}_t(\boldsymbol{\omega}) + \sum_{a=0}^{A}\beta^a\eta^C_{t+a}\left((\alpha+\gamma)\frac{Y_{j_a,t+a}}{L_{j_a,t+a}}(1-t_{\omega_a})\theta_a - c^a_{t+a}(\boldsymbol{\omega})\right) - \sum_{a=0}^{A}\beta^a\eta^H_{\ell_a,t+a}h^a_{t+a}(\boldsymbol{\omega}) = 0$$

Dividing through by η^C_0 we obtain

$$\frac{\beta^t\eta^{IC}_t(\boldsymbol{\omega})}{\eta^C_0} + \sum_{a=0}^{A}q_{t+a}\left((\alpha+\gamma)\frac{Y_{j_a,t+a}}{L_{j_a,t+a}}(1-t_{\omega_a})\theta_a - c^a_{t+a}(\boldsymbol{\omega}) - r^h_{\ell_a,t+a}h^a_{t+a}(\boldsymbol{\omega})\right) = 0$$

The transfer to these agents is given by

$$T_t(\boldsymbol{\omega}) = \sum_{a=0}^{A}q_{t+a}\left(c^a_{t+a}(\boldsymbol{\omega}) - w_{j_a,t+a}(1-t_{\omega_a})\theta_a + r^h_{\ell_a,t+a}h^a_{t+a}(\boldsymbol{\omega})\right)$$

so,

$$\begin{aligned}T_t(\boldsymbol{\omega}) =&\overline{T}_t + \gamma\times\sum_{a=0}^{A}q_{t+a}\left(\frac{Y_{j_a,t+a}}{L_{j_a,t+a}}(1-t_{\omega_a})\theta^a - \sum_{\boldsymbol{\omega}}\pi_t(\boldsymbol{\omega})\frac{Y_{j_a,t+a}}{L_{j_a,t+a}}(1-t_{\omega_a})\theta_a\right)\\&+\frac{\beta^t\eta^{IC}_t(\boldsymbol{\omega})}{\eta^C_0} - \sum_{\boldsymbol{\omega}}\pi_t(\boldsymbol{\omega})\frac{\beta^t\eta^{IC}_t(\boldsymbol{\omega})}{\eta^C_0}\end{aligned}$$

F.5 Optimal investment policy

The first order conditions with respect to $H_{\ell,t+1}$ and $K_{\ell,t+1}$ are given by

$$\begin{aligned}[H_{\ell,t+1}]:&\quad -\beta^t\eta^C_t p^h_{\ell,t} + \beta^{t+1}\eta^H_{\ell,t+1} + \beta^{t+1}\eta^C_{t+1}(1-\delta)p^h_{\ell,t+1} = 0\\ [K_{\ell,t+1}]:&\quad -\beta^t\eta^C_t p^k_{\ell,t} + \beta^{t+1}\eta^C_{t+1}\left((1-\alpha)\frac{Y_{\ell,t+1}}{K_{\ell,t+1}} + (1-\delta)p^k_{\ell,t+1}\right) = 0\end{aligned}$$

The first equation shows that

$$-\beta^t \eta_t^C p_{\ell,t}^h = \beta^{t+1} \eta_{t+1}^C (r_{\ell,t+1}^h + (1-\delta) p_{\ell,t+1}^h),$$

since $r_{\ell,t+1}^h = \frac{\eta_{\ell,t+1}^H}{\eta_{t+1}^C}$. Furthermore, since $1 + r_{t+1} = \frac{\eta_t^C}{\beta \eta_{t+1}^C}$, we obtain that

$$1 + r_{t+1} = \frac{r_{\ell,t+1}^h + (1-\delta) p_{\ell,t+1}^h}{p_{\ell,t}^h},$$

i.e., the optimal investment tax on housing is zero $\tau_{\ell,t+1}^h = 0$ for all ℓ.

The second equation shows that

$$-\beta^t \eta_t^C p_{\ell,t}^k = \beta^{t+1} \eta_{t+1}^C \left(r_{\ell,t+1}^k + (1-\delta) p_{\ell,t+1}^k \right),$$

since $r_{\ell,t+1}^k = (1-\alpha) \frac{Y_{\ell,t+1}}{K_{\ell,t+1}}$. Furthermore, since $1 + r_{t+1} = \frac{\eta_t^C}{\beta \eta_{t+1}^C}$, we obtain that

$$1 + r_{t+1} = \frac{r_{\ell,t+1}^k + (1-\delta) p_{\ell,t+1}^k}{p_{\ell,t}^k},$$

i.e., the optimal investment tax on office buildings is zero $\tau_{\ell,t+1}^k = 0$ for all ℓ.