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ABSTRACT

This paper extends the canonical security competition one-shot game by developing a dynamic framework where states can invest in a technology to eliminate their rival. The model includes a settlement stage for negotiation, stochastic contestable resources, and it assumes diplomacy never fails–so payoff-dominated equilibria are not played. Within this setting, we fully characterize equilibrium behavior for all discount factors, comparing cases with high and low elimination costs. The dynamic structure reveals why, even when states coordinate on the best possible equilibrium outcome, long-lasting disarmed peace is rare. By combining the escalation of military capabilities with the constraining implications of effective diplomacy, the model rationalizes the persistent cycles of peace, arms races, and conflict observed in history. Our approach identifies strategic mechanisms that restrict the sustainability of disarmament and clarifies the conditions under which arms races or conflict are inevitable. These findings offer a deeper understanding of the recurrent nature of international conflict under repeated interaction.

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1 Introduction

In the economics of conflict and international relations literature, the standard game-theoretic model of conflict is a one-shot contest game, in which states simultaneously and independently make investments (e.g., weapons) to increase their probability of winning a prize (e.g., a disputable resource).[1] In the Nash equilibrium, even when partial resource destruction is modeled as a deterrent to conflict, there are positive levels of arming and open conflict, and part of the resource is destroyed in the fight, leading to an inefficient outcome. All states could be better off if they chose not to arm and peacefully split the resource. If the game is augmented with a bargaining and settlement stage after arming decisions, the destruction associated with open conflict can be mitigated, but unless such destruction is unrealistically high, inefficient arming persists. In other words, the equilibrium that is robust to changes in the destruction of resources associated with open conflict consists of armed peace and negotiated settlement, and only when open conflict destroys the vast majority of the resources can efficient equilibria be sustained. Thus, in realistic scenarios, there is positive arming, and the model has the structure of a prisoner's dilemma (often referred to as a security dilemma in international relations), which could open the door for more cooperative outcomes to be sustained if repeated interactions are allowed.[2]

Repeated interactions among states are a reasonable environment to explore for at least four reasons. First, in many cases, there are few relevant states or blocs of states involved in an international dispute. Drawing an analogy with firms' behavior, collusion tends to be easier to sustain with a smaller number of firms involved. Second, states are long-lived entities, which suggests the importance of considering dynamic interactions. Third, states tend to have powerful incentives to plan and act through diplomacy and coordination, given that mistakes could lead to serious consequences (e.g., war). Finally, states usually have the capabilities to make contingent plans and act strategically (e.g., diplomats

[1] See Hirshleifer (1989) for an important seminal work and Garfinkel and Skaperdas (2007) for a comprehensive survey of the literature in the economics of conflict. This is also the standard rent-seeking model (Tullock (1980) and Nitzan (1994)).

[2] The model's structure best fits disputes over resources or territories that are both highly valuable and insecurely held—situations where formal ownership or control is not universally recognized and remains subject to challenge. Historical examples include the Korean Peninsula, where both North and South claim legitimacy over the entire territory, the Falkland Islands dispute between the United Kingdom and Argentina, and oil-rich borderlands such as the Iran–Iraq frontier. These contests are sustained by the inability to institutionalize a permanent settlement and by the perception that control of the prize yields long-term strategic or economic advantage. Similar dynamics also appear in access disputes over strategic maritime routes like the Suez Canal or the South China Sea, where shifts in commercial and military importance keep the stakes alive.

and military experts). Thus, international relations should be an excellent candidate for applying the folk theorem and related results for infinitely repeated games to sustain more cooperative outcomes (see, for example, Keohane (1986), Oye (1986), and Kydd (2015), chapter 8).[3]

Indeed, if the standard model of conflict is repeated infinitely and if states are patient enough, disarmed peace can be sustained as a subgame perfect equilibrium, presenting us with a conundrum. In a one-shot contest game model of conflict, the equilibrium is either open conflict (when a negotiated settlement is not possible) or armed peace (when a negotiated settlement is possible). If conflict is modeled as a repeated contest game, the set of equilibrium payoffs can range from the efficient outcome of disarmed peace (assuming states are patient) to the same highly inefficient equilibria of the one-shot game. There is, therefore, a wider range of more cooperative equilibria in repeated interactions. In international relations, however, diplomacy is often viewed as the key mechanism to navigate these equilibria. Through diplomatic channels, states are expected to be capable of eliminating dominated equilibria and converging toward the most favorable equilibrium payoffs. Given these premises, one would anticipate that this dynamic version of the standard one-shot contest game would naturally lead to a state of disarmed peace, provided states are patient enough.

The empirical reality, however, presents a stark contrast to these theoretical expectations. Despite the theoretical possibility of achieving unarmed peace through repeated interactions and diplomatic efforts, the historical record is replete with instances of ongoing arming, arms races, and even outright war. If conflict is modeled as an infinite repetition of the standard one-shot contest game, the only way to rationalize these events is to attribute them to diplomatic failures. For example, disarmed peace in odd periods and open conflict in even periods can be sustained as a subgame perfect equilibrium, which can be interpreted as diplomacy failing in even periods. We do not deny the possibility of historical instances of diplomatic failures, but we think that the discrepancy between theory and reality suggests a more fundamental gap in the existing models of conflict. Therefore, in our analysis we give diplomacy the benefit of the doubt and assume that states always find a way to avoid payoff-dominated equilibria. Alternatively, our analysis can be seen as an effort to rationalize some of the historical record on conflict using an infinite horizon dynamic contest game, even when diplomacy fully exhausts all the possibilities of cooperation.

The paper introduces a comprehensive model that provides a nuanced explanation for the persistent occurrence of arming and conflict despite the theoretical

[3] One might even argue that if these results do not apply to interactions among states, there should be little hope that they will apply to other strategic interactions, such as firms trying to organize collusion or commoners solving the tragedy of the commons.

predictions of disarmament and peace under infinitely repeated interactions and fully effective diplomacy. Our model incorporates the foundational aspects of the standard contest game and extends it by introducing dynamic elements that more accurately reflect the complexities of international relations. First, we allow states to interact repeatedly over time. Second, after observing each other's military power, states can agree on a settlement rule such that each country gets a fraction of the resource. If no settlement is reached, open conflict occurs, an open conflict occurs, and a portion of the resource is destroyed. Third, there is an exogenously and randomly determined opportunity that the victor of an open conflict can eliminate the other state at an additional fixed cost.[4] Moreover, we distinguish between two cases, whether this elimination of the rival is permanent or temporary.[5] Finally, we introduce a randomly determined disputable resource; that is, the resource's value is stochastic and varies from period to period.[6]

With these technologies, we obtain several novel results. First, we establish an impossibility result: if the elimination of the rival is permanent, greater patience (high δ) makes unarmed peace harder to sustain, and in the limit ($\delta \to 1$), open conflict with extremely high arming becomes unavoidable. Second, the cost

[4]Elimination technologies vary in form but share the property of decisively removing a rival's ability to contest the resource in the near future. At the extreme, the nuclear bombings of Hiroshima and Nagasaki illustrate a near-permanent elimination capability, in which the destruction severely reduced Japan's industrial capacity for an extended period and effectively curtailed its ability to project military power thereafter. Other cases include large-scale forced deportations, as with post-World War II expulsions of ethnic Germans from Eastern Europe, or the Soviet dismantling of German industry in occupied zones. More commonly, elimination has been temporary: the destruction of Iraq's military capacity during the 1991 Gulf War, for example, was followed by a decade of rebuilding, or the temporary incapacitation of Argentina's navy after the Falklands War, which nonetheless did not end its claim to the islands.

[5]Temporary elimination occurs when the defeated side eventually recovers its strength and re-enters the contest. This was evident after Napoleon's exile to Elba in 1814, followed by his return during the Hundred Days; after the initial North Korean retreat in late 1950, which was reversed by Chinese intervention and a renewed offensive; and after the Iran–Iraq War's alternating phases of exhaustion and renewed fighting in the 1980s. In these cases, the resource—whether territorial control, political authority, or strategic advantage—remains contestable, and the defeated side invests in rearmament or strategic alliances to mount a comeback, producing cycles of peace and conflict rather than a single, decisive end to hostilities.

[6]When the value of the disputed resource fluctuates unpredictably, sudden increases can destabilize otherwise cooperative equilibria. The Suez Crisis of 1956 illustrates how geopolitical shifts and rising commercial importance of the canal spurred military escalation. The 1973 Yom Kippur War coincided with the global oil crisis, amplifying the stakes for Middle Eastern territory and control over oil supply routes. Similar patterns have emerged in sub-Saharan Africa's mineral-rich zones, where discoveries of valuable deposits—such as coltan in the Democratic Republic of Congo—triggered sudden increases in arming and conflict intensity. In these cases, the "random" shock to resource value creates strong short-term incentives to arm, even in environments otherwise conducive to sustained peace.

of elimination plays a central role: there are threshold values—dependent on other parameters–above which moderate levels of patience can sustain temporary or permanent cooperation, particularly when the elimination technology is only temporary. In this temporary elimination case, cooperation becomes cyclical, producing recurring patterns of peace, conflict, and, at times, heavy arming followed by temporary elimination. Finally, when the value of the contestable resource fluctuates, the model also generates arms races (e.g., a cold war) as part of the broader cycle of peace and conflict.

While most elements in our model have been individually examined in the literature, they have not previously been integrated into a single framework. Our contribution is to systematically combine repeated interactions, stochastic resource values, strategic settlement negotiations, and the possibility of a decisive elimination, yielding a more robust and multi-dimensional understanding of conflict dynamics. This unified approach recovers standard results as special cases while clarifying the mechanisms behind a variety of outcomes. For example, cycles of escalation and de-escalation with temporary elimination emerge when the discount factor is moderately high. Even without the elimination technology, the interaction between repeated play and random resource values can generate cycles of armed and unarmed peace without culminating in open conflict. As noted by Powell (1993), Skaperdas and Syropoulos (1996), and McBride and Skaperdas (2006), elimination opportunities can explain open conflict. Our model extends this insight: in an infinite dynamic game, permanent elimination with high patience produces a single conflict followed by perpetual unarmed peace, whereas non-permanent elimination allows for recurrent cycles of armed peace, conflict, temporary defeat, and eventual resurgence.

The paper contributes to two bodies of literature in two subjects of study: game-theoretic models of conflict in economics and security competition in international relations.

There are several papers that have formalized the idea that open conflict differs from settlement under the shadow of conflict because it changes the future bargaining power of the parties (see Fearon, 1995, for a seminal formalization of this idea). Settlement is rationalized by the idea that open conflict is destructive, so parties are better off not engaging in it. For instance, the notion that the victor of an open conflict can eliminate its enemy is one possible such mechanism. Skaperdas and Syropoulos (1996) and Garfinkel and Skaperdas (2000) introduce elimination opportunities but only consider a two-period model. Powell (1993) and McBride and Skaperdas (2006) explore an infinite game but restrict the analysis to Markov perfect equilibria. Our first contribution to this literature is to explore an environment in which the classical folk theorem has a serious chance—in the sense that without introducing the elimination technology, states can sustain unarmed peace equilibria. Our second contribution is to address one potentially unrealistic prediction of these models (including our version with permanent elimination), namely, that after one country eliminates its enemy, unarmed peace persists forever. McBride and Skaperdas (2006) consider that several battles might be necessary to finally eliminate an enemy, which explains arming after open conflict. Still, once the enemy has been finished off, unarmed peace persists. Our model with temporary elimination, in contrast, allows for recurrent periods of peace and conflict.

Other papers have also studied infinite horizon dynamic models of conflict. Yared (2010) develops an asymmetric repeated game in which an aggressive country seeks concessions from a non-aggressive country. Temporary wars might occur after a period of escalating demands to incentivize the non-aggressive country to make concessions. Asymmetric information plays a key role because when concessions are not made, the aggressive country does not know if the non-aggressive country is able to make them. Instead, we focus on large symmetric conflicts—i.e., when both countries have the capacity to attack and even annihilate each other—and consider environments with perfect information.[7] Acemoglu et al. (2012) study an infinite horizon dynamic model of resource war in which a resource-poor country might consider attacking a resource-rich country. The model can generate open conflict (i.e., war) but only in period $t = 0$. Acemoglu and Wolitzky (2014) generate an equilibrium with cycles of conflict but use a very different modeling approach. They consider an overlapping-generations model with incomplete information in which two groups play a coordination game, and each group does not know if the other consists of good or bad types. Cooperative actions might be wrongly misperceived, starting a cycle of conflict until agents realize that the conflict began by mistake and give cooperation a renewed opportunity. In contrast, in our model, there is complete information, and we assume that diplomacy can eliminate any misperception or coordination failure.

Our study is also related to international relations and the well-known debate between realists and liberals. A simple way of describing the realist school in international relations is that, for realists, international politics is predominantly a prisoner's dilemma among countries (e.g., Mearsheimer, 2001). From this perspective, the standard game-theoretic model of conflict as a one-shot contest game can be interpreted as a formalization of the fundamental realist view of international relations as a security dilemma. The problem, of course, is that one can start with a one-shot contest game/security dilemma and, through repeated interactions, end up with disarmed peace sustained, for example, by grim-trigger

[7] While imperfect information, say regarding the rival's arming, is possible, countries have intelligence systems that allow them to have a reasonably good idea of the capacity of others' military power.

strategies (e.g., Keohane, 1986). In other words, the folk theorem provides a theoretical link between a realist world and highly liberal cooperative outcomes. The conundrum is that this extreme liberal prediction is rarely observed in practice. Why?

Our model explores several answers to this question. Bargaining, even in a one-shot game, can prevent open conflict. Nevertheless, arming is typically part of the equilibrium. In fact, only extremely high destructive costs from open conflict could rationalize an unarmed equilibrium. In a repeated game, the destruction level required for disarmed peace interacts with the degree of patience, implying that war and inefficient arming can only be prevented when states are highly patient.

When the elimination technology is permanent, high patience makes long-standing disarmed peace unsustainable. Some cooperation is still possible in states of the world where elimination is not feasible, but as soon as the opportunity to eliminate arises, open conflict becomes inevitable for patient states. A drawback of this equilibrium is that, after the victor eliminates its enemy, disarmed peace persists forever. When elimination is only temporary, however, this no longer holds as $\delta \to 1$. In equilibrium, open conflict grants the victor only temporary control over the disputed resource. Eventually, the defeated country recovers, and the resource becomes contestable again. Thus, there is room for both temporary and permanent cooperation. Nevertheless, some arming is often necessary, meaning that security competition and war cannot be fully eradicated from the international system.

Another feature of our model—less damaging to the disarmed peace prediction when states are patient—is that, without the elimination technology, stochastic resource values can generate cycles of arming and disarming without open conflict, even for highly patient states. In this context, when the disputed resource's value is high, and thus the temptation to deviate from disarmed peace increases, arming escalation becomes necessary to sustain peace.[8] This feature reinstates some of the properties of the one-shot security dilemma in the infinite dynamic contest game. However, as states become sufficiently patient, disarmed peace can still be sustained. That is, the extreme liberal prediction does not fully disappear. More importantly, the fundamental logic of the folk theorem persists, even when stochastic fluctuations in the value of the disputed resource require countries to accept arming escalation when the stakes are high.

The rest of the paper is organized as follows. Section 2 introduces the stochastic model. Section 3 solves a simpler version of the model, without the elimination

[8] Bellicose rhetoric may also escalate in such periods. One possible explanation is that each country needs to signal to its rival that arming has been increased to meet the new circumstances, which is important to sustain the most cooperative possible equilibrium.

technology. We break down the study of the elimination technology in two main cases: we analyze permanent elimination in Section 4 and temporary elimination in Section 5. Section 6 adds stochastic resources to the analysis. Finally, Section 7 presents our conclusions.

2 A Dynamic Model of Conflict with Destruction

We depart from the standard model of conflict by adding several elements that will help explain more realistic features observed historically, especially in the last century. We pay special attention to an exogenous randomly determined opportunity that the victor of an open conflict can eliminate the other player at an additional fixed cost; moreover, we distinguish between two cases, whether elimination is permanent or not. Due to the random realization of states that affect the payoffs and actions, we study a stochastic game.[9]

In each period, countries $i = 1, 2$ seek access to a disputable resource R. These countries choose a level of military power denoted by $G_i \geq 0$ (for guns) that fully depreciates by next period. Given a profile of military powers (G_1, G_2), the probability of winning a conflict is:

$$\pi_i(G_i, G_j) = \begin{cases} \frac{G_i}{G_i + G_j}, G_i + G_j > 0 \\ \frac{1}{2}, G_i = G_j = 0 \end{cases}$$

The perishable resource will be replaced by a new stock R in each period. After countries observe the profile of military power (G_1, G_2), they decide whether to settle ($S_i \in \{0, 1\}$). If countries settle ($S = S_1 S_2 = 1$), they receive $\pi_i(G_i, G_j)R$ of the resource.[10] If countries do not settle ($S = 0$), this necessarily starts an open conflict, a fraction $1 - \theta$ of the resource is destroyed, and country i obtains R with probability $\pi_i(G_i, G_j)$ and nothing with probability $1 - \pi_i(G_i, G_j)$.[11]

At the beginning of each period, before deciding G_i, a random state is realized which gives the victor of an open conflict, say i, the opportunity to pay an additional cost X to eliminate the loser (i.e., country j). This exogenous opportunity

[9]This is a slight extension of a repeated game. Although there is repeated interaction, there is no unique stage game, since payoffs and actions depend on different realizations of state variables.

[10]Although there are other allocation rules, this particular one is the most natural. Moreover, we assume that the settlement rule S is decided independently and simultaneously and that it is fully enforceable. That is, we do not model the possible advantages of a unilateral unexpected aggression after settlement.

[11]Note that, conditional on R, payoffs are deterministic under settlement, while payoffs are stochastic under open conflict. Expected payoffs, nevertheless, look similar due to the assumption that the probability of winning a war equals the share obtained during settlement.

arrives with a probability ϕ, and it is common knowledge. The opportunity to eliminate is realized before the decision to spend on military power G_i, but the decision to eliminate is made after winning the conflict. Since the decision to eliminate may or may not be taken, we need to be explicit about this action being taken. We use a random variable Z to denote whether an opportunity to eliminate has arrived ($Z = 1$) or not ($Z = 0$). Moreover, we denote by $W_i = 1$ if a country decides to eliminate and $W_i = 0$ otherwise.

After an elimination occurs, the next period, the game transitions to a state in which the previous conflict's victor can claim all the resources at zero cost. More formally, when an elimination occurs, the game transitions to an uncontestable state ($C = 0$). Periods in which the resource can be contested will be labeled contestable states and denoted by $C = 1$. At the end of each period in an uncontestable state, with probability γ, the game will transition back to a contestable state and the loser of the previous conflict will be able to contest the resource again. With $1 - \gamma$, the game will remain in the uncontestable state next period.

Since no decisions are taken at an uncontestable state, to simplify the notation, we will not explicitly model the payoffs and actions in such states. Let $a_i = (G_i, S_i, W_i)$ be the actions taken by player i in contestable states and $\psi = (Z, C)$ the profile of states. Then, each country's instant payoff when $C = 1$ is:

$$u_i(a_i, a_j, \psi) = \pi_i(G_i, G_j)\big[SR + (1 - S)\theta R - (1 - S)ZW_iX\big] - G_i, \qquad (1)$$

for i and $j = 1, 2$. After the opportunity to eliminate is realized and used, as long as $C = 0$ remains, all subsequent payoffs will be R for the victor of the most recent open conflict and zero for the vanquished country. After the state returns to $C = 1$, the payoff will be given by (1) once again.

Countries discount future payoffs by a factor $0 \leq \delta < 1$. Given a stream of actions $a_i^t = (G_i^t, S_i^t, W_i^t)$ from each country at each period, $\{a_i^t\}_{t=0}^{\infty}$, and states $\psi^t = (Z^t, C^t)$, $\{\psi^t\}_{t=0}^{\infty}$, the (normalized) payoff of the supergame is:

$$(1 - \delta) \sum_{t=0}^{\infty} \delta^t E\left[u_i(a_i^t, a_j^t, \psi^t)\right],$$

where the expectation is taken over states and actions. In this supergame, a history at the beginning of period t is a sequence of all action profiles and states up to period t:[12]

$$h^t = \left\{(a_1^0, a_2^0, \psi^0), (a_1^1, a_2^1, \psi^1), \ldots, (a_1^{t-1}, a_2^{t-1}, \psi^{t-1})\right\},$$

[12] To simplify notation, we can say that even in state $C = 0$, countries can take actions. However, such actions do not affect the payoffs.

for $t \geq 1$ and $h^0 = \emptyset$ in the initial period, and we assume that the game starts at a contestable state.

Since all information is public in this game, the equilibrium concept is subgame perfect equilibrium (SPE). We restrict attention to symmetric equilibria.[13]

3 Equilibrium with No Elimination

To better understand the dynamics of the game, we build up in complexity by solving simpler scenarios first. In this section, we start by solving the one-shot game and then consider the repeated game without stochastic states. In both cases, we do not allow the winner of an open conflict to eliminate its rival.

3.1 One-shot Game

There are no actions to be taken in uncontestable states. Thus, we refer to the case when $\delta = 0$, the current state is contestable, and there is no opportunity to eliminate as the one-shot game.[14] Note that even in the one-shot game, we still need to use subgame perfection as the equilibrium concept, as the level of guns (G_1, G_2) is first decided and observed and then the settlement stage follows (S_1, S_2), which leads to either peace or conflict (i.e., no settlement necessarily translates into to open conflict). This, in turn, means that interesting equilibria can arise even in this simple case. In order to understand possible equilibria, we relate how countries react to observing anticipated or unanticipated arming by the opponent. These reactions have strong implications on the type of equilibria that can arise. After stating the results, we will also have a discussion of implications and interpretations.

First, we note that for any profile of guns choices (G_1, G_2), settlement (i.e., $S_1S_2 = 1$) and no settlement (i.e., $S_1S_2 = 0$) can both be Nash equilibrium outcomes. Moreover, settlement always Pareto dominates no settlement. In this sense, the post-arming subgame has the structure of a coordination problem. Two reasonable selection criteria are the following norms. Under the *non-cooperative norm*, $S_1S_2 = 0$ for all (G_1, G_2). Under the *cooperative norm*, $S_1S_2 = 1$ for

[13]Since there are effectively no actions being taken in uncontestable states, we consider that as part of a "symmetric" equilibrum, as long as actions and payoffs are ex-ante symmetric in contestable states. Moreover, there could be equilibria in which a country, say i, obtains a larger share of the resources. As long as the other country, j, gets a payoff higher than its minmax, that can be an equilibrium. We ignore such equilibria because they do not add relevance or insight to the study, but greatly increase the space of possible equilibria.

[14]Since there are no incentives to "invest" in the elimination of an enemy, it is trivial to see that the elimination option will never be taken, as long as the cost is positive $X > 0$.

all (G_1, G_2). That is, under the non-cooperative norm, there is always open conflict. In contrast, under the cooperative norm, there is never an open conflict as the countries always reach an agreement. Finally, it is possible to have arming-dependent selection criteria. In particular, under the *conditional cooperative norm*, $S_1 S_2 = 1$ for all (G_1, G_2) such that $G_i \leq \hat{G}$ for some equilibrium $\hat{G}$ and $S_1 S_2 = 0$, otherwise.

The following proposition summarizes the subgame perfect Nash equilibrium gun levels under each selection criterion.

Proposition 1. *Assume that* $\delta = 0$.

1. *Under the non-cooperative norm (i.e., players coordinate in* $S_1 S_2 = 0$ *for all* (G_1, G_2)*). Then, the equilibrium level of arming is* $G^N = \theta R/4$ *and the associated payoff is* $V^N = \theta R/4$.

2. *Under the cooperative norm (i.e., players coordinate in* $S_1 S_2 = 1$ *for all* (G_1, G_2)*). Then, the equilibrium level of arming is* $G^N = R/4$ *and the associated payoff is* $V^N = R/4$.

3. *Under the conditional cooperative norm:*

   (a) *If* $\theta \leq 1/2$*, then the most cooperative equilibrium level of arming is* $G^P = 0$ *and the associated payoff is* $V^P = R/2$.

   (b) *If* $\theta > 1/2$*, then the most cooperative equilibrium level of arming is* $G^{SB} \in (0, \theta R/4]$*, where* G^{SB} *is the unique solution to* $F(G) = -2G + 2\sqrt{G\theta R} + \left(\frac{1-2\theta}{2}\right) R = 0$*, and the associated payoff is* $V^{SB} = R/2 - G^{SB}$.

Proof. See the Appendix. □

The equilibrium described in Proposition 1.1 is identical to the Nash equilibrium of the standard one-shot contest game when no settlement stage is included. The non-cooperative norm can be interpreted as the total lack of trust as players expect that they will never be able to coordinate in the Pareto-superior settlement equilibrium, regardless of their guns choices. This makes the settlement stage completely irrelevant. For international relations, one possible interpretation is that this is a situation in which countries do not have a diplomatic channel that can be used to reach a negotiated settlement.

The equilibrium described in Proposition 1.2 is the equilibrium usually stressed when the standard one-shot contest game is augmented with a settlement stage. The idea is that regardless of gun choices, countries can always use diplomacy and peaceful negotiations to settle the dispute, divide the disputed resource according to the winning probabilities, avoid the destruction associated with open conflict, and enforce the agreement with guns. In other words, players will always coordinate in the Pareto-superior settlement equilibrium. Thus, armed peace can always be achieved. [15]

The equilibrium described in Proposition 1.3 is often overlooked in the literature, but it is interesting because it anticipates the opportunities for cooperation if the game is repeated infinitely and also suggests that diplomacy can be a double-edged sword. To see the opportunities for cooperation, note that the equilibrium in Proposition 1.3 involves a lower level of arming and a higher payoff than the equilibrium in Propositions 1.1 or 1.2. The reason is that the conditional cooperative norm leverages the multiplicity of equilibrium in the settlement stage to enact more cooperative behavior in gun choices. Specifically, the conditional cooperative norm 'punishes' high levels of arming with the Pareto-inferior no settlement equilibrium and 'rewards' low levels of arming with the Pareto-superior settlement equilibrium. When the destruction associated with open conflict is high (formally, $\theta \leq 1/2$), this is enough to induce disarmed peace. When the destruction associated with open conflict is low (formally, $\theta \leq 1/2$), fully disarmed peace is not an equilibrium, but a lower level of arming than in Proposition 1.1 can still be sustained.

To see the double-edged sword nature of diplomacy, note that the conditional cooperative norm can be interpreted as 'offering' a diplomatic solution when arming is low but also 'threatening' with open conflict when arming is high. From this perspective, the problem of the noncooperative norm in Proposition 1.1 is the total lack of diplomacy (no carrots), while the problem of the cooperative norm in Proposition 1.2 is an excessive attachment to diplomacy (no sticks).

Finally, note that when $\theta = 1$ (that is, if there is no destruction associated with open conflict), all the norms induce the same equilibrium, that is, $G^N = R/4$. Thus, when there is no advantage of a settled agreement over open conflict because conflict is not destructive, the conditional cooperative norm cannot use the destruction of open conflict as a threat to reduce arming.

[15] Note an odd prediction of the one-shot rent-seeking game. The equilibrium level of arming is lower under open conflict ($G^N = \theta R/4$) than under settlement ($G^N = R/4$). One possibility is to replace the rent-seeking model (1) by the following guns vs. butter model:

$$u_i(a_i, a_j, \psi) = \pi_i(G_i, G_j)\big[SR + (1-S)\theta - (1-S)ZW_iX\big]\,(B_1 + B_2)\,,$$

where $B_i = R_i - G_i$ for $i = 1, 2$. Under this specification and assuming that $1/3 < R_1/R_2 < 3$, the equilibrium level of arming is $G^N = R/4$, where $R = R_1 + R_2$, which does not depend on S. The associated equilibrium payoffs ($V^N = [S + (1-S)\theta]R/4$) are identical for both models. It is perfectly possible to redo all our analysis, replacing the rent-seeking model for this guns vs. butter model. Results will persist.

3.2 Repeated Game

Now we consider a standard repeated environment (i.e., without stochastic states). Thus, the opportunity to eliminate never arrives ($\phi = 0$). In addition, an open conflict destroys a proportion $1-\theta$ of the resources; as a consequence, since there is no opportunity to eliminate, a settlement equilibrium always dominates an open conflict equilibrium. Thus, we ignore equilibria without settlement on the equilibrium path.

Under the previous considerations, we can utilize a standard folk theorem to characterize the best attainable equilibrium. The mechanism that allows for such cooperative equilibrium is also standard: countries behave nicely to each other under the credible threat that if anyone ever deviates from such cooperative equilibrium, the next period both countries will revert to an inefficient one-period equilibrium allocation and remain there forever (i.e., grim-trigger strategies).

From Proposition 1, there are three possible equilibria to consider as punishment for a deviation. The most severe is the equilibrium described in Proposition 1.1. Under a non-cooperative norm, both countries play $G^N = \theta R/4$ and $S_1 = S_2 = 0$, which gives instant payoffs of $V^N = \theta R/4$. The equilibrium described in Proposition 1.2 is slightly less severe as players continue to avoid open conflict on the punishment path. Specifically, under a cooperative norm, both countries play $G^N = R/4$ and $S_1 = S_2 = 1$, which gives instant payoffs of $V^N = R/4$. It is also worth noting that Proposition 1.1 includes Proposition 1.2 (setting $\theta = 1$). Finally, the equilibrium described in Proposition 1.3 is the least severe one and, hence, the least effective in inducing cooperation. Consequently, we will not consider it as a punishment path. In conclusion, we will employ the non-cooperative norm equilibrium in Proposition 1.1 to punish deviations starting in the next period.

To further induce cooperation, a deviation can also be immediately punished by open conflict, which yields a payoff of θR. In summary, a deviation triggers not only a permanent reversion to the worst one-period non-cooperative equilibrium starting next period, but also an immediate switch to the non-cooperative norm.[16]

The best attainable allocation is the efficient allocation $G^P = 0$. The payoff from remaining in this efficient equilibrium is $R/2$, as nothing is wasted in guns and the resource is equally split. However, given that both countries have zero military power, an arbitrarily small deviation by one country would greatly improve its "bargaining power" at virtually zero cost. The other country will immediately react switching to open conflict. Thus, the one-shot deviation payoff would be θR. Finally, as prescribed by the equilibrium, starting in the next

[16] This immediate switch to the non-cooperative norm selects the worst possible Nash equilibrium, i.e., $S_1 = S_2 = 0$, in the settlement stage.

period, both countries will play the equilibrium in Proposition 1, which gives instant payoff of $\theta R/4$. Thus, to sustain the efficient equilibrium in a repeated game, the discount factor δ has to satisfy:

$$\frac{R}{2} \geq (1-\delta)\theta R + \delta\frac{\theta R}{4} \tag{2}$$

What if condition (2) is not satisfied? One can see, from Equation (2), that by investing virtually zero resources in guns, a country could secure all the resources when deviating from the cooperative equilibrium. A second best $G^{SB} \in (0, \theta R/4)$ can still be sustained in the repeated game. The reason is that if $G^{SB} > 0$, the potential deviation would not be as profitable during settlement talks. Following this idea, the second best payoff is: $R/2 - G^{SB}$, which is an equilibrium as long as this payoff is better than a one-shot deviation and then reverting to the equilibrium in Proposition 1.1 forever. Thus, this equilibrium requires:

$$\frac{R}{2} - G^{SB} \geq (1-\delta)\left(\frac{g^D(G^{SB},\theta R)}{G^{SB} + g^D(G^{SB},\theta R)}\theta R - g^D(G^{SB},\theta R)\right) + \delta\frac{\theta R}{4} \tag{3}$$

where $g^D(G,\theta R) = \sqrt{\theta R G} - G$ is the static best-response to G given resources θR.[17] Then:

Proposition 2. *Assume that $\delta \geq 0$.*

1. *If $\theta \leq \frac{2}{4-3\delta}$, then the first-best payoff $V^P = R/2$ characterized by unarmed peace $G^P = 0$ can be attained as an equilibrium of the repeated game.*
2. *If $\theta > \frac{2}{4-3\delta}$, then the second-best payoff $V^{SB} = R/2 - G^{SB} > R/4$ with military power $G^{SB} \in \left(0, \left(\frac{1-\delta}{2-\delta}\right)^2 \theta R\right)$ is sustainable as an equilibrium of the repeated game. Moreover, G^{SB} is the unique solution to $F(G) = -(2-\delta)G + 2(1-\delta)\sqrt{G\theta R} + \left[\frac{2-(4-3\delta)\theta}{4}\right]R = 0$ and $\left(\frac{1-\delta}{2-\delta}\right)^2 \theta R \leq \frac{\theta R}{4}$ (with strict inequality for $\delta > 0$).*

Proof. See the Appendix. □

It is interesting to compare Propositions 1 and 2. Not surprisingly, repeated interactions can support more cooperation than in the one-shot game when in the latter we consider either a non-cooperative norm (as in Proposition 1.1) or a fully cooperative norm (as in Proposition 1.2). More interesting is that repeated interactions also open the door for more cooperation than the one-shot game under

[17]Formally, the solution to $\max_g \pi_i(g,G)\theta R - g$ is $g^D(G,\theta R) = \sqrt{\theta R G} - G$.

the conditional cooperative norm. To illustrate this, assume that open conflict is not destructive (i.e., $\theta = 1$). Then, if $\delta \geq 2/3$, unarmed peace can be sustained as an equilibrium of the repeated game, while the conditional cooperative norm in Proposition 1.3 cannot improve in $V^N = R/4$.

A more systematic way to compare Propositions 1.3 and 2 is to observe that the higher the value of δ, the more likely the first-best payoff can be sustained as an equilibrium, and the higher the second-best payoff when the first-best cannot be sustained. Moreover, Proposition 2 is identical to Proposition 1.3 when $\delta = 0$. Thus, the equilibrium in Proposition 2 always dominates the equilibrium in Proposition 1.3 for $\delta = 0$.

Summarizing, depending on the parameters, in the best equilibrium, there is either eternal unarmed peace or a repeated settlement with small military spending. Thus, repeated interactions open the door for substantial international cooperation and even the possibility of a Utopian fully peaceful world. This hardly corresponds to the historical record. In reality, we observe cycles of peace, arm races, and serious instances of open conflict. In Section 6.1 we show that adding stochastic states can accommodate arm races within the best equilibrium path, which might help explain the rise of tensions in a cold war equilibrium, but not instances of open conflict. So far, the only possibility to obtain open conflict is to shut down repeated interactions and either assume that conflict is not destructive or that countries follow a strict non-cooperative norm, which are not reasonable assumptions. Nevertheless, under those assumptions, we obtain an unrealistic result, namely, open conflict always prevails.

4 Equilibrium with Permanent Elimination

In this section, we study the main feature of the model: the possibility of an elimination. Specifically, with probability $\phi > 0$, if countries go to open conflict, the winner has the opportunity to eliminate the loser at a cost X, who thereafter will not be able to contest the resource. This elimination opportunity greatly increases the complexity of the model. To better understand the basic interactions, in this section we consider the case in which the elimination is permanent ($\gamma = 0$) and the contested resource is not stochastic.

We consider two types of cooperative equilibria: *permanent* and *temporary* cooperation. In an equilibrium with permanent cooperation, countries always choose settlement no matter if open conflict allows the victor to eliminate the loser or not. In an equilibrium with temporary cooperation, settlement is sustained only when the victor of an open conflict will not be able to eliminate the loser.

For both types of equilibria, we characterize the best attainable equilibrium.

The mechanism that allows for such cooperative equilibrium is related but not identical to the grim-trigger strategy. Similarly to the grim-trigger strategy, countries stick to the cooperative path under the credible threat that if anyone ever deviates from such cooperative equilibrium, the next period both countries will revert to the worst equilibrium with open conflict. Moreover, as we stipulated in the repeated game, we consider that a deviation is immediately punished by open conflict. In contrast to the grim-trigger strategy, when the victor can eliminate the loser, not every deviation can be effectively punished. In particular, if a country deviates from $S_i = 1$ to $S_i = 0$ when $Z = 1$, it might be the case that one of the countries is wiped out and hence there is no possible future punishment. Nevertheless, it is useful to start the analysis characterizing the most severe equilibrium punishment (i.e., the worst possible equilibrium under open conflict) that countries can threaten to revert starting next period if the resource is still contestable.

4.1 Punishment Equilibria

To determine the lowest possible equilibrium payoffs under the possibility of open conflict, we note that the non-cooperative norm should be used as it leads to the lowest possible payoffs. Moreover, we also note that there is no unilateral profitable deviation from $S_1 = S_2 = 0$ because settlement requires that both players agree $S_1 = S_2 = 1$. Thus, under the non-cooperative norm, in contestable states (no player has been eliminated yet), there will be open conflict. The following lemma characterizes the worst possible equilibrium under the non-cooperative norm. When $Z = 0$, open conflict will be used in this equilibrium. In contrast, when $Z = 1$, depending on the parameters, sometimes the lowest equilibrium payoff is attained by using the elimination option after an open conflict and sometimes the lowest equilibrium payoff is attained by not using the elimination option. Since elimination is an absorbing state, we compute the most severe credible punishment starting on a contestable state.

Lemma 1. *Under the non-cooperative norm.*[18] *Let* $\bar{X} = \frac{[(4-\theta)(1-\delta)+3\phi\delta]\delta R}{(1-\delta)[4(1-\delta)+3\phi\delta]}$ *and* $R^{WO} = \theta R - X + \frac{\delta R}{1-\delta}$.

1. *Suppose that* $X > \bar{X}$. *Then, the worst equilibrium is* $G^N = \theta R/4$ *for* $Z = 0$ *and* $G^N = \theta R/4$ *and no elimination for* $Z = 1$. *The associated punishment expected payoff is* $V^{PU} = V^N = \frac{\theta R}{4}$.

2. *Suppose that* $X \leq \bar{X}$. *Then, the worst equilibrium is* $G^N = \theta R/4$ *for* $Z = 0$

[18] That is, assume that in every contestable state, $S_1 S_2 = 0$ for all (G_1, G_2) and Z.

and $G^{WO} = R^{WO}/4$ *and eliminate the loser for* $Z = 1$. *The associated punishment expected payoff is* $V^{PU} = V^{WO} = \frac{(1-\delta)\left[(1-\phi)\theta R+\phi R^{WO}\right]}{4(1-\delta+\phi\delta)}$.

Proof. See the Appendix. □

In words, there are two possible equilibria to consider. In one equilibrium, countries never use the elimination option. This equilibrium is just the eternal repetition of the equilibrium in Proposition 1.1 or, which is the same, the punishment used in Proposition 2 to sustain cooperation. In the second equilibrium, countries use the elimination option whenever it is available (that is, when $Z = 1$). In this equilibrium, the first time that $Z = 1$ the victor eliminates the loser, and the game permanently transitions to an uncontestable state in which the victor takes all the resource in every period. Moreover, there is a unique Nash equilibrium level of arming. To see this, assume that countries expect to use the elimination option. Then, country i's expected payoff is given by:

$$u_i\left(G_i, G_{-1}\right) = \pi_i\left(G_i, G_{-1}\right)\left[(1-\delta)\left(\theta R - X\right) + \delta R\right] - (1-\delta)\,G_i$$

Thus, the unique Nash equilibrium level of arming is $G_i = G^{WO} = R^{WO}/4 = \left(\theta R - X + \frac{\delta R}{1-\delta}\right)/4$ for $i = 1, 2$.[19]

Depending on the parameter values, only one or both of these equilibria exist. In particular, for X greater than a threshold, only the first equilibrium exists, for intermediate values of X both equilibria exist, and for X below a threshold only the second equilibrium exists. Moreover, we show that whenever both equilibria exist, the one that generates lower payoffs is the equilibrium in which countries use the elimination option.

The intuition behind the proof of Lemma 1 is simple. When the cost of using the elimination technology is high (formally $X > \bar{X}$), countries are not interested in using it, and, hence, the worst possible equilibrium is identical to the equilibrium punishment in Proposition 2. In contrast, when the cost of the elimination technology is low (formally $X \leq \bar{X}$), both countries are willing to use it if they expect the other country to do the same. Moreover, in such circumstances, the lower payoff equilibrium is always the one in which the victor eliminates the loser.

4.2 Cooperation Strategies and Payoffs

Next, we explore cooperative equilibria in contrast to those described in Lemma 1. We start with permanent cooperation, i.e., when countries choose to settle

[19] $X \leq \bar{X}$ ensures that $-X + \frac{\delta R}{1-\delta} > 0$ and, hence, $G^{WO} > G^N = \theta R/4 > 0$. See the Appendix for more details.

regardless of whether the elimination option is available or not. In particular, consider the following strategy to sustain permanent cooperation.

Definition 1. ***Permanent cooperation strategy.***

- *Cooperation phase: Suppose that no deviation has occurred. If* $Z = 0$*, play* $G(0)$ *and* $S = 1$*; while if* $Z = 1$*, play* $G(1)$ *and* $S = 1$*.*
- *Punishment phase for* $Z = 0$*: Any deviation from* $G(0)$ *and* $S = 1$ *triggers, starting in the next period, the worst equilibrium in Lemma 1. In addition, a deviation from* $G(0)$ *triggers an open conflict immediately.*
- *Punishment phase for* $Z = 1$*:*
  - *Deviation from* $S = 1$*: If this leads to an elimination, this deviation cannot be punished. If it does not lead to an elimination, it triggers, starting in the next period, the worst possible equilibrium in Lemma 1.*
  - *Deviation from* $G(1)$*: This is immediately punished with* $S = 0$ *and eliminate the loser if elimination is an optimal choice for the victor and, in case it is not, starting in the next period, with the worst possible equilibrium in Lemma 1.*
  - *To determine whether the use of the elimination technology is an optimal choice, the victor assumes that, starting in the next period, the worst possible equilibrium in Lemma 1 will be played.*

Suppose that countries are following the permanent cooperation strategy with $G(0) \in \left[0, \frac{\theta R}{4}\right)$ and $G(1) \in \left[0, \frac{\theta R}{4}\right)$ and that no country deviates. Then:

$$V^{PC}(0) = (1-\delta)\left(\frac{R}{2} - G(0)\right) + \delta\left[\phi V^{PC}(1) + (1-\phi) V^{PC}(0)\right]$$
$$V^{PC}(1) = (1-\delta)\left(\frac{R}{2} - G(1)\right) + \delta\left[\phi V^{PC}(1) + (1-\phi) V^{PC}(0)\right]$$

Thus, for $Z = 0$, each country obtains $R/2 - G(0)$ and in the next period with probability ϕ we have $Z = 1$ and with probability $1-\phi$ we have $Z = 0$. Similarly, for $Z = 1$, each country obtains $R/2 - G(1)$, and in the next period with probability ϕ we have $Z = 1$ and with probability $1-\phi$ we have $Z = 0$. Solving for the above equations, the expected payoffs for each state $Z = 0, 1$ are given by:

$$V^{PC}(0) = \frac{R}{2} - (1-\delta\phi) G(0) - \delta\phi G(1) \tag{4}$$
$$V^{PC}(1) = \frac{R}{2} - (1-\delta+\delta\phi) G(1) + \delta(1-\phi) G(0) \tag{5}$$

For some parameter values, it will not be possible to sustain permanent cooperation, but countries will still be able to sustain cooperation only when the victor will not be able to eliminate the loser. In particular, consider the following strategy to sustain temporary cooperation:

Definition 2. ***Temporary cooperation strategy.***

- *Cooperation phase for $Z = 0$: Suppose that no deviation has occurred. Then, play $G(0)$ and $S = 1$.*
- *Punishment phase for $Z = 0$: Any deviation from $G(0)$ and $S = 1$ triggers, starting in the next period, the worst equilibrium in Lemma 1. In addition, a deviation from $G(0)$ triggers an open conflict immediately.*
- *No cooperation for $Z = 1$: Play G^{WO}, $S = 0$ and eliminate.*

Suppose that countries are following the temporary cooperation strategy with $G(0) \in \left[0, \frac{\theta R}{4}\right)$ and that no country deviates. Then:

$$V^{TC}(0) = (1-\delta)\left(\frac{R}{2} - G(0)\right) + \delta\left[\phi V^{TC}(1) + (1-\phi)V^{TC}(0)\right]$$
$$V^{TC}(1) = \frac{1}{2}\left[(1-\delta)(\theta R - X) + \delta R\right] - (1-\delta)G^{WO}$$

Thus, for $Z = 0$, each country obtains $R/2 - G(0)$ and in the next period with probability ϕ we have $Z = 1$ and with probability $1-\phi$ we have $Z = 0$. For $Z = 1$, countries fight an open conflict, the victor obtains θR and pays the elimination cost X in the present; and obtains R forever. The loser gets nothing. Both countries invest G^{WO} in guns. Solving for the above equations, the expected payoffs for each state $Z = 0, 1$ are given by:

$$V^{TC}(0) = \frac{(1-\delta)\left(\frac{R}{2} - G(0) + \frac{\delta\phi R^{WO}}{4}\right)}{1-\delta+\delta\phi} \tag{6}$$

$$V^{TC}(1) = \frac{(1-\delta)R^{WO}}{4} \tag{7}$$

4.3 Optimal Deviations and Sustainability Conditions

The following lemma characterizes the optimal deviation and the associated deviation rewards for a country that chooses to stop following the cooperation strategies.

Lemma 2. *Suppose that countries are following either the permanent cooperation strategy with* $G(0) \in \left[0, \frac{\theta R}{4}\right)$ *and* $G(1) \in \left[0, \frac{\theta R}{4}\right)$ *or the temporary cooperation strategy with* $G(0) \in \left[0, \frac{\theta R}{4}\right)$.

1. *Assume that* $Z = 0$. *Then, the most profitable deviation is*

$$g^D(G(0), \theta R) = \sqrt{\theta R G(0)} - G(0)$$

The associated deviation payoff is given by:

$$V^D(G(0)) = (1-\delta)\, u^D(G(0), \theta R) + \delta V^{PU}$$

where $u^D(G(0), \theta R) = \frac{g^D(G(0),\theta R)}{G(0)+g^D(G(0),\theta R)}\theta R - g^D(G(0), \theta R)$ *and* V^{PU} *is given by Lemma 1.*

2. *Assume that* $Z = 1$. *Let* $\bar{X} = \frac{[(4-\theta)(1-\delta)+3\phi\delta]\delta R}{(1-\delta)[4(1-\delta)+3\phi\delta]}$ *and* $R^{WO} = \theta R - X + \frac{\delta R}{1-\delta}$.

   (a) *Suppose that* $X > \bar{X}$. *Then, the most profitable deviation is*

$$g^D(G(1), \theta R) = \sqrt{\theta R G(1)} - G(1),$$

   inducing countries to fight an open conflict, after which the victor will not eliminate the loser. The associated deviation payoff is given by:

$$V^D(G(1)) = (1-\delta)\, u^D(G(1), \theta R) + \delta \frac{\theta R}{4}$$

   where $u^D(G(1), \theta R) = \frac{g^D(G(1),\theta R)}{G(1)+g^D(G(1),\theta R)}\theta R - g^D(G(1), \theta R)$

   (b) *Suppose that* $X \leq \bar{X}$. *Then, the most profitable deviation is*

$$g^D\left(G(1), R^{WO}\right) = \sqrt{R^{WO} G(1)} - G(1),$$

   inducing countries to fight an open conflict, after which the victor will eliminate the loser. The associated deviation payoff is given by:

$$V^D(G(1)) = (1-\delta)\, u^D\left(G(1), R^{WO}\right)$$

   where $u^D\left(G(1), R^{WO}\right) = \frac{g^D\left(G(1),R^{WO}\right)}{G(1)+g^D\left(G(1),R^{WO}\right)} R^{WO} - g^D\left(G(1), R^{WO}\right)$

Proof. See the appendix. □

Note that depending on the cost X, the most profitable deviation may or may not include the use of the elimination technology. In particular, as X grows, deviating from a more efficient equilibrium will not lead to the elimination of the loser in that period or the punishment periods that follow.

Combining Lemmas 1 and 2, we obtain sustainability conditions for some degree of cooperation in equilibrium. In particular, to sustain as an equilibrium the permanent cooperation strategy with $G(0), G(1) \in \left[0, \frac{\theta R}{4}\right)$ it must be the case that:

$$F^{PC,Z}(G(0), G(1)) := V^{PC}(Z) - V^{D}(G(Z)) \geq 0 \tag{8}$$

for $Z = 0, 1$.

To sustain as an equilibrium the temporary cooperation strategy with $G(0) \in \left[0, \frac{\theta R}{4}\right)$ it must be the case that:

$$F^{TC}(G(0)) := V^{TC}(0) - V^{D}(G(0)) \geq 0 \tag{9}$$

4.4 Impossibility of Avoiding Open Conflict for $\delta \to 1$

Before we fully characterize cooperative equilibria, it is imperative to explore what happens with sustainability conditions for cooperative equilibria when $\delta \to 1$. After all, as we have shown in Proposition 2, when there is no technology that allows the elimination of the rival, unarmed peace can be sustained for $\delta \geq \frac{2}{3}$. The following proposition provides the answer.

Proposition 3. *Suppose that $\delta \to 1$. Then, it is impossible to sustain a permanent cooperative equilibrium, but it is always possible to sustain a temporary cooperative equilibrium. Moreover, a temporary cooperative equilibrium is not Pareto efficient (regardless of whether $\delta < 1$ or $\delta \to 1$).*

Proof. See the Appendix. □

The intuition behind Proposition 3 is as follows. When $\delta \to 1$, it is impossible to deter a deviation when $Z = 1$. The reason is that the elimination technology works as an investment that becomes more attractive as countries become more patient. At the same time, the certainty that cooperation will breakdown when there is a chance of eliminating the rival does not block cooperation while $Z = 0$. The reason is that there is no advantage in switching to open conflict when it is not possible for the victor to eliminate the loser.

It is worth recalling the path predicted by a temporary cooperative equilibrium. Countries cooperate, avoiding open conflict and possibly reducing arming levels while they cannot eliminate their enemy, but as soon as the elimination opportunity arrives, an instance of open conflict is inevitable. Thereafter, the victor takes full control of the disputed resource, and disarmed peace prevails forever. In other words, what might appear to be patient players cooperating forever (à la folk theorem) reveals only temporary cooperation that suddenly switches to full-fledged war to permanently settle the dispute.

Finally, to see that a temporary equilibrium is not Pareto efficient, note that in any temporary equilibrium whenever $Z = 0$ countries invest $G(0) \geq 0$; the first time that $Z = 1$ they invest $G^{WO} > 0$; and, thereafter, there is zero investment in guns. Thus, even if a temporary cooperative equilibrium with $G(0) = 0$ can be sustained, in such an equilibrium there is a positive investment in guns the first time that $Z = 1$. In contrast, for $\delta < 1$, efficiency requires zero investment in guns along the equilibrium path. For $\delta \to 1$, we have $\lim_{\delta\to 1} V^{TC}(0) = \lim_{\delta\to 1} V^{TC}(1) = R/4$, while efficiency allows each country to obtain $V_i = R/2$.

4.5 Full Characterization of Cooperative Equilibria

Beyond analyzing the limit case as $\delta \to 1$–arguably the most interesting and counterintuitive scenario–we also fully characterize the set of equilibria for all discount factors. To do so, we distinguish between two cases: high and low elimination costs. In addition, when multiple equilibria arise, we compare them using the ex-ante expected payoff induced by each equilibrium as our evaluation criterion. That is, we assume diplomacy is fully effective in the sense that, for any form of cooperation (whether permanent or temporary), it identifies and implements the best possible outcome that can be sustained as an equilibrium. However, even under this optimistic assumption, diplomacy may not always suffice to achieve ideal outcomes. Formally, in the case of permanent cooperation, we focus on the cooperative equilibrium that maximizes:

$$V^{PC} = \phi V^{PC}(1) + (1-\phi) V^{PC}(0)$$

subject to sustainability constraints $F^{PC,Z}(G(0), G(1)) \geq 0$ for $Z = 0, 1$, that is, equation (8).

And for temporary cooperation, we look for the equilibrium that maximizes:

$$V^{TC} = \phi V^{TC}(1) + (1-\phi) V^{TC}(0)$$

subject to sustainability constraint $F^{TC}(G(0)) \geq 0$ (that is, equation (9)).

It is useful to distinguish two possible situations, namely, high- and low-elimination cost. The following proposition considers the case in which the cost of elimination is high.

Proposition 4. *Suppose that $X > \bar{X}$.*

1. *If $\theta \leq \frac{2}{4-3\delta}$, then permanent complete cooperation, i.e., disarmed peace with $G(Z) = 0$ for $Z = 0, 1$ can be sustained.*

2. *If $\theta > \frac{2}{4-3\delta}$, then the best possible cooperative equilibria that can be sustained is permanent partial cooperation (i.e., armed peace) with $G^{PC} \in \left(0, \left(\frac{1-\delta}{2-\delta}\right)^2 \theta R\right)$ for $Z = 0, 1$, where G^{PC} is the unique solution to $F(G) = -(2-\delta) G + 2(1-\delta)\sqrt{G\theta R} + \left[\frac{2-(4-3\delta)\theta}{4}\right] R = 0$ and $\left(\frac{1-\delta}{2-\delta}\right)^2 \theta R \leq \frac{\theta R}{4}$ (with strict inequality for $\delta > 0$).*

Proof. See the Appendix. □

Proposition 4 is almost identical to Proposition 2. Perhaps not surprisingly, if the elimination cost is high enough, countries are not willing to use the elimination technology, and hence the most severe punishment they can use to deter a deviation from the cooperative path is to revert to the one-shot equilibrium under the non-cooperative norm forever. In other words, the logic of the folk theorem is reinstated. There is, however, a subtle difference between Propositions 4 and 2. Note that the additional condition $X > \bar{X}$ is required for Proposition 4. Crucially, this condition does not hold for δ high enough. In particular, note that it never holds when $\delta \to 1$.

Next, we study the more interesting case, that is, when the elimination cost is low. The following proposition summarizes the results.

Proposition 5. *Suppose that $X \leq \bar{X}$ and let $\bar{X}_{low} = \theta R + \frac{\left[2\delta - \frac{2-2\delta+\delta\phi}{1-\delta+\delta\phi}\right] R}{2(1-\delta)}$ and $\bar{X}_{high} = \theta R + \frac{(2\delta-1)}{2(1-\delta)} R$.*

1. *Suppose that either $\theta \leq \frac{1-2\delta}{2(1-\delta)}$ or $\frac{1-2\delta}{2(1-\delta)} < \theta \leq \frac{4(1-\delta)+3\phi\delta}{2(1-\delta)(4-3\delta+3\phi\delta)}$ and $X \geq \bar{X}_{high}$. Then, permanent complete cooperation, i.e., disarmed peace with $G(Z) = 0$ for $Z = 0, 1$, can be sustained.*

2. *Suppose that $\frac{1-2\delta}{2(1-\delta)} < \theta \leq \frac{4(1-\delta)+3\phi\delta}{2(1-\delta)(4-3\delta+3\phi\delta)}$ and $\bar{X}_{low} < X < \bar{X}_{high}$. Let $\bar{G}(0) = \left(\frac{1-\delta}{2-\delta-\phi\delta}\right)^2 \theta R < G^N$, $\bar{G}(1) = \left(\frac{1-\delta}{2-2\delta+\delta\phi}\right)^2 R^{WO} < G^{WO}$, and $\hat{G}(1) \in \left(0, \bar{G}(1)\right)$ be the unique solution to $F^{PC,1}(0, G(1)) = 0$.*

   (a) *If $\hat{G}(1) \leq F^{PC,0}(0,0)/\delta\phi$, then the best possible permanent cooperative equilibrium that can be sustained is partial cooperation with $G(0) = 0$ and $G(1) = \hat{G}(1)$.*

(b) *If* $\hat{G}(1) > F^{PC,0}(0,0)/\delta\phi$, *then the best possible permanent cooperative equilibrium that can be sustained is partial cooperation with* $G(0) \in (0, \bar{G}(0))$ *and* $G(1) \in \left(\hat{G}(1), \bar{G}(1)\right)$ *given by the unique solution to* $F^{PC,0}(G(0), G(1)) = F^{PC,1}(G(0), G(1)) = 0$ *that satisfies:*

$$\frac{\partial F^{PC,0}(G(0),0)}{\partial G(0)} \frac{\partial F^{PC,1}(0,G(1))}{\partial G(1)} > \delta^2(1-\phi)\phi.$$

3. *If* $\theta \leq \frac{2}{4-3\delta+3\phi\delta}$, *then the best temporary cooperative equilibrium that can be sustained is* $G(0) = 0$.

4. *If* $\theta > \frac{2}{4-3\delta+3\phi\delta}$, *then the best temporary cooperative equilibrium that can be sustained is* $G(0) \in \left(0, \left(\frac{1-\delta+\delta\phi}{2-\delta+\delta\phi}\right)^2 \theta R\right)$ *given by the unique solution to* $F^{TC}(G(0)) = 0$. *Moreover,* $\left(\frac{1-\delta+\delta\phi}{2-\delta+\delta\phi}\right)^2 \theta R < G^N = \frac{\theta R}{4}$.

Proof. See the Appendix. □

Proposition 5.1 states that even when $X \leq \bar{X}$ (from Lemma 1), it could still be possible to sustain the first-best allocation $G(Z) = 0$ for $Z = 0, 1$. Why is a country not willing to deviate and play open conflict, win the war with probability 1, and then eliminate the loser? There are two reasons. First, during open conflict, a fraction $1-\theta$ of the resource is destroyed. Second, the elimination costs X may still be too high to deviate from cooperation, even when it is not high enough to be an equilibrium under the non-cooperative norm. That is, when $\theta \leq \frac{1-2\delta}{2(1-\delta)}$, the destruction of open conflict is severe enough to discourage a deviation even if elimination was costless.[20] For $\frac{1-2\delta}{2(1-\delta)} < \theta \leq \frac{4(1-\delta)+3\phi\delta}{2(1-\delta)(4-3\delta+3\phi\delta)}$, if elimination were costless, a country would always be willing to deviate. However, if the cost of elimination is high enough, it is still the case that no country finds profitable to deviate from complete permanent cooperation in order to eliminate its rival.

What happens if complete permanent cooperation cannot be sustained? In the spirit of Propositions 2 and 4, Proposition 5.2 shows that a second-best allocation with permanent but partial cooperation can be sustained, provided that open conflict is destructive enough and the cost of elimination is not too low. An interesting feature of the equilibria described in Proposition 5.2 is that gun choices

[20] More formally, under complete permanent cooperation a country obtains $R/4$, while if it deviates to open conflict and then eliminate its rival it gets $(\theta R - X) + \delta R$. Thus, for $\theta \leq \frac{1-2\delta}{2(1-\delta)}$, deviation does not pay.

fluctuate to sustain cooperation. For example, when $\hat{G}(1) \leq F^{PC,0}(0,0)/\delta\phi$, in the best permanent cooperative equilibrium, there is disarmed peace when elimination is not possible but positive arming when the victor can eliminate the lower. Another interesting feature of Proposition 5.2 is that it is never optimal to organize permanent cooperation with positive arming for $Z = 0$ and no arming for $Z = 1$. In other words, some arming is always required to avoid open conflict when the victor has the chance to permanently eliminate the loser.

From Proposition 3 we already know that when $\delta \to 1$, it is not even possible to sustain the second-best allocation characterized in Proposition 5.2. More precisely, the following corollary characterizes when open conflict is unavoidable.

Corollary 1. *Open conflict is unavoidable if and only if*

$$X \leq \begin{cases} \bar{X} & \theta > \frac{4(1-\delta)+3\phi\delta}{2(1-\delta)(4-3\delta+3\phi\delta)} \\ \bar{X}_{low} & \frac{1-2\delta}{2(1-\delta)} < \theta \leq \frac{4(1-\delta)+3\phi\delta}{2(1-\delta)(4-3\delta+3\phi\delta)} \end{cases} \tag{10}$$

Moreover:

1. The lower the cost of elimination technology (X lower) and the higher the chances that countries can use it (ϕ higher), the less likely open conflict can be avoided.

2. The destructiveness of open conflict has a non-monotonic effect on the likelihood of open conflict. Initially, as destructiveness declines (θ higher), open conflict becomes more likely, but eventually further reductions in destructiveness make open conflict easier to avoid.

Proof. See the Appendix. □

That open conflict is unavoidable does not imply that there is no room for temporary cooperation. In fact, Propositions 5.3 and 5.4 fully characterize the best temporary cooperative equilibrium in which countries only cooperate when $Z = 0$ but switch to open conflict when the opportunity to eliminate the rival arrives (i.e., for $Z = 1$). It is worth mentioning that, unlike permanent cooperation, it is always possible to sustain some level of temporary cooperation. The intuition behind this result is simple. Even if there is no hope of avoiding open conflict when $Z = 1$, countries still have incentives to cooperate when $Z = 0$ because they can temporarily avoid the destruction associated with open conflict and the cost of high levels of arming.

An interesting exercise is how the expectation that cooperation will not survive when $Z = 1$ affects the level of cooperation when $Z = 0$. Note that the threshold in Proposition 5.3 (i.e., $\frac{2}{4-3\delta+3\phi\delta}$) is strictly decreasing in ϕ. Thus, as the probability that the elimination technology will be available increases (a higher ϕ), it is less likely that in a temporary cooperative equilibrium no arming can be sustained when $Z = 0$. It is also easy to prove that when $\theta > \frac{2}{4-3\delta+3\phi\delta}$, $G(0)$ is strictly increasing in ϕ. Thus, as the probability that the elimination technology will be available increases, less cooperation can be sustained when such technology is not available.[21]

Proposition 5 offers a much richer and realistic set of possibilities than Proposition 4. First, cooperation might require increasing and decreasing arming choices. Thus, the model is compatible with hot periods in which security competition intensifies and calmer periods in which countries deescalate tensions. Moreover, rising arming and tensions are not necessarily the prelude to war, but rather a mechanism to secure cooperation. Second, sometimes open conflict is unavoidable, because winning a decisive war that permanently eliminates the rival is extremely tempting. In these circumstances, countries cooperate until the elimination opportunity arrives, when they fight a full-fledged open war that permanently settles the dispute; thereafter, disarmed peace persists.

4.6 Permanent versus Temporary Cooperation

Proposition 5 fully characterizes the best possible permanent and temporary equilibrium, respectively, but it does not compare them with each other. There are three situations to consider: When complete permanent cooperation can be sustained, there is no need to compare it with a temporary cooperative equilibrium because complete permanent cooperation always induces a higher expected payoff than any other equilibrium. When neither complete nor partial permanent cooperation can be sustained, there is nothing to compare. The best possible equilibrium is the temporary cooperative equilibrium described in Propositions 5.3 and 5.4. Finally, the only non-trivial case where a comparison is meaningful is when partial permanent cooperation is possible, but complete permanent cooperation is not. In this setting, the comparison is not redundant, since temporary cooperation is always feasible. The following proposition undertakes this comparison.

Proposition 6. *Suppose that* $\frac{1-2\delta}{2(1-\delta)} < \theta \leq \frac{4(1-\delta)+3\phi\delta}{2(1-\delta)(4-3\delta+3\phi\delta)}$ *and* $\bar{X}_{low} < X < \bar{X}_{high}$. *Then, partial permanent cooperation induces higher ex-ante expected payoff than temporary cooperation if and only if*

$$\Delta = \left[\begin{array}{c} F^{PC,0}(0,0) - \delta\phi G^{PC}(1) - \delta(1-\phi) G^{PC}(0) \\ -\frac{(1-\delta)}{1-\delta+\delta\phi} \left[F^{TC}(0) - \delta(1-\phi) G^{TC}(0) \right] \end{array} \right] \geq 0$$

[21]This also implies that gun choices in Proposition 4 are always lower than gun choices in Propositions 5.3 and 5.4.

where

$$F^{PC,0}(0,0) = \frac{R}{2} - (1-\delta)\,\theta R - \frac{(1-\delta)\,\delta\left[\theta R + \phi\left(-X + \frac{\delta R}{1-\delta}\right)\right]}{4\,(1-\delta+\phi\delta)}$$
$$F^{TC}(0) = \frac{2-\theta\,(4-3\delta+3\phi\delta)}{4}R$$

$G^{PC}(0)$ and $G^{PC}(1)$ are given by Proposition 5.2 and $G^{T}(0)$ by Propositions 5.3 and 5.4. Moreover, if $\theta > \frac{2}{4-3\delta+3\phi\delta}$ and $\hat{G}(1) \leq F^{PC,0}(0,0)/\delta\phi$, $\Delta \geq 0$, while if $\theta \leq \frac{2}{4-3\delta+3\phi\delta}$ and $\hat{G}(1) > F^{PC,0}(0,0)/\delta\phi$, $\Delta < 0$.

Proof. See the Appendix. □

The more interesting result in Proposition 6 is that it is not always the case that partial permanent cooperation dominates temporary cooperation. In fact, if $\theta \leq \frac{2}{4-3\delta+3\phi\delta}$ and $F^{PC,0}(0,0) < 0$, it is better to cooperate only when $Z = 0$. The reason is that inducing cooperation for $Z = 0$ and $Z = 1$ might require very high levels of arming compared to those needed to induce cooperation only for $Z = 0$. Thus, avoiding open conflict, even when sustainable, is not necessarily the best alternative.

To summarize, the elimination technology offers a substantial departure from the results in more standard setups that allow for folk theorem rosy predictions. First, open conflict and associated destruction are unavoidable when $\delta \to 1$. Second, avoiding open conflict, when possible, might require a fluctuation in arming choices. Finally, even if countries can escape open conflict, they may choose not to do so because the benefits from post-war eternal peace outweigh the additional costs in arming needed to discourage open conflict.

There is, however, one very unrealistic feature in the temporary cooperative equilibrium described in Propositions 5 and 6. Because elimination is an absorbing state (permanent elimination), after the first instance of open conflict, the resource becomes non disputable, and hence there is disarmed peace forever after. In the next section, we relax the assumption of permanent elimination to obtain more realistic results. Since we are still interested in equilibria with open conflict, we will focus on situations in which the elimination cost is low.

5 Equilibrium with Temporary Elimination

Suppose that $\gamma > 0$. Then, there is a chance that countries return to contestable states even after a country has been eliminated. This does not affect choices in uncontestable states, where the victor still collects R without investing anything in guns, but makes the elimination technology less attractive (because now with probability γ the resource will again be contestable). Fortunately, to characterize the best possible equilibrium when $\gamma > 0$ we can employ the same approach as we used in Section 4. Thus, we start the analysis characterizing the most severe equilibrium punishment (i.e., the worst possible equilibrium under open conflict) that countries can threaten to revert to punish a deviation from the cooperative path.

Lemma 3. *Under the non-cooperative norm.*[22] *Let* $\bar{X}(\gamma) = \frac{[(4-\theta)(1-\delta+\delta\gamma)+3\delta\phi]\delta R}{(1-\delta+\delta\gamma)[4(1-\delta+\delta\gamma)+3\delta\phi]}$ *and* $R^{WO}(\gamma) = \theta R - X + \frac{\delta R}{1-\delta+\delta\gamma}$.

1. *Suppose that* $X > \bar{X}(\gamma)$. *Then, the worst equilibrium is* $G^N = \frac{\theta R}{4}$ *for* $Z = 0$ *and* $G^N = \frac{\theta R}{4}$ *and no elimination for* $Z = 1$. *The associated punishment expected payoff is* $V^{PU} = V^N = \frac{\theta R}{4}$.
2. *Suppose that* $X \leq \bar{X}(\gamma)$. *Then, the worst equilibrium is* $G^N = \frac{\theta R}{4}$ *for* $Z = 0$ *and* $G^{WO}(\gamma) = \frac{R^{WO}(\gamma)}{4}$ *and elimination for* $Z = 1$. *The associated punishment expected payoff is* $V^{PU} = V^{WO}(\gamma) = \frac{(1-\delta+\delta\gamma)\left[\phi R^{WO}(\gamma)+(1-\phi)\theta R\right]}{4[1-\delta(1-\phi-\gamma)]}$.

Proof. See the Appendix. □

Lemma 3 is a generalization of Lemma 1.[23] When the cost of using the elimination technology is greater than $\bar{X}(\gamma)$, the most severe punishment is the static Nash equilibrium under no settlement. In contrast, when the cost of using the elimination technology is less than $\bar{X}(\gamma)$, the most severe punishment includes the use of the elimination technology whenever available.

As in Section 4, better equilibria can be sustained than those in Lemma 3. It is straightforward to adapt the permanent and temporary cooperation strategies (definitions 1 and 2, respectively) to deal with the possibility that $\gamma > 0$. All we need to do is to use Lemma 3 instead of Lemma 1 in the punishment phases.

Computing expected payoffs under these cooperation strategies is also straightforward. Suppose that countries are following the permanent cooperation strategy with $G(0) \in \left[0, \frac{\theta R}{4}\right)$ and $G(1) \in \left[0, \frac{\theta R}{4}\right)$ and that no country deviates. Since under this strategy, the game never transitions to the non-contestable state, the expected payoffs for each contestable state are still given by (4) and (5). Thus:

$$V^{PC}(1,0) = \frac{R}{2} - (1-\delta\phi)\,G(0) - \delta\phi G(1)$$
$$V^{PC}(1,1) = \frac{R}{2} - (1-\delta+\delta\phi)\,G(1) + \delta(1-\phi)\,G(0)$$

[22]That is, assume that in every contestable state, $S_1 S_2 = 0$ for all (G_1, G_2) and Z.
[23]Formally, for $\gamma = 0$, we have $\bar{X}(0) = barX$ and $R^{WO}(0) = R^{WO}$.

Intuitively, if countries are cooperating in each contestable state, the elimination technology is never used, and hence, no country is ever temporarily eliminated, which implies that the probability that a defeated country comes back has no effect on expected payoffs.

In contrast, when countries are following a temporary cooperation strategy, the game will eventually transition to the uncontestable state, then move back again to the contestable state, and so on. Therefore, for $\gamma > 0$,(6) and (7) do not apply anymore. Suppose that countries are following the temporary cooperation strategy with $G(0) \in \left[0, \frac{\theta R}{4}\right)$ and that no country deviates. Let $V^{TC}(1, Z)$ denote the payoff in a contestable state $C = 1$ with $Z = 0, 1$, and $V^{TC}(0, w)$ and $V^{TC}(0, l)$ the payoffs in the uncontestable state $C = 0$ for the winner and the loser, respectively. Then:

$$\begin{aligned}
V^{TC}(1,0) &= (1-\delta)\left(\frac{R}{2} - G(0)\right) + \delta V^{TC}(1) \\
V^{TC}(1) &= \phi V^{TC}(1,1) + (1-\phi)V^{TC}(1,0) \\
V^{TC}(1,1) &= \frac{(1-\delta)(\theta R - X) + \delta V^{TC}(0,w)}{2} + \frac{\delta V^{TC}(0,l)}{2} - (1-\delta)G^{WO}(\gamma) \\
V^{TC}(0,w) &= (1-\delta)R + \delta\left[\gamma V^{TC}(1) + (1-\gamma)V^{TC}(0,w)\right] \\
V^{TC}(0,l) &= \delta\left[\gamma V^{TC}(1) + (1-\gamma)V^{TC}(0,l)\right]
\end{aligned}$$

Thus, when the resource is contestable and $Z = 0$, each country obtains $R/2 - G(0)$ and in the next period the resource will also be contestable. In particular, with probability ϕ the state will be $(C = 1, Z = 1)$ and with probability $1 - \phi$, it will be $(C = 1, Z = 0)$. When the resource is contestable and $Z = 1$, countries fight an open conflict; the winner obtains θR and pays the elimination cost X in the present; and, in the next period, the game transitions to the uncontestable state. The loser gets nothing in the present. Both countries invest G^{WO} in guns. If the resource is not contestable, the winner gets R at no cost in the present period, and in the next period the game switches to the contestable state with probability γ. The loser gets nothing in the present. Solving for the above equations, the expected payoffs for $C = 1$ and $Z = 0, 1$ are given by:

$$\begin{aligned}
V^{TC}(1,0) &= \frac{(1-\delta+\delta\gamma-\phi\delta^2\gamma)\left(\frac{R}{2} - G(0)\right) + \frac{\delta\phi R^{WO}(\gamma)}{4}}{1-\delta(1-\gamma-\phi)} \\
V^{TC}(1,1) &= \frac{\frac{(1-\delta+\delta\gamma)(1-\delta)R^{WO}(\gamma)}{4} + (1-\phi)\delta^2\gamma V^{TC}(1,0)}{1-\delta+\delta\gamma-\delta^2\gamma\phi}
\end{aligned}$$

The following lemma characterizes optimal deviations from cooperation strategies.

Lemma 4. *Suppose that countries are following either the permanent cooperation strategy with $G(0) \in \left[0, \frac{\theta R}{4}\right)$ and $G(1) \in \left[0, \frac{\theta R}{4}\right)$ or the temporary cooperation strategy with $G(0) \in \left[0, \frac{\theta R}{4}\right)$. Assume that $X \leq \bar{X}(\gamma)$.*

1. *Assume that $Z = 0$. Then, the most profitable deviation is $g^D(G(0), \theta R) = \sqrt{G(0)\theta R} - G(0)$. The associated deviation payoff is given by:*

$$V^D(G(0)) = (1-\delta)\, u^D(G(0), \theta R) + \delta V^{WO}(\gamma)$$

where $u^D(G(0), \theta R) = \frac{g^D(G(0),\theta R)}{G(0)+g^D(G(0),\theta R)}\theta R - g^D(G(0), \theta R)$.

2. *Assume that $Z = 1$. Then, the most profitable deviation is*

$$g^D\left(G(1), R^{WO}(\gamma)\right) = \sqrt{G(1)R^{WO}(\gamma)} - G(1),$$

which induces an open conflict, after which the victor eliminates the loser. The associated deviation payoff is given by:

$$V^D(G(1)) = (1-\delta)\, u^D\left(G(1), R^{WO}(\gamma)\right) + \frac{\gamma\delta^2 V^{WO}(\gamma)}{1-\delta+\delta\gamma}$$

where $u^D\left(G(1), R^{WO}(\gamma)\right) = \frac{g^D\left(G(1),R^{WO}(\gamma)\right)}{G(1)+g^D\left(G(1),R^{WO}(\gamma)\right)} R^{WO}(\gamma) - g^D\left(G(1), R^{WO}(\gamma)\right)$.

Proof. See the Appendix. □

Lemma 4 is a generalization of Lemma 2 when $X \leq \bar{X}(\gamma)$. That is, we focus on the more interesting case in which the most severe punishment uses the elimination technology.

Combining Lemmas 3 and 4 we can generalize sustainability constraints (8) for permanent cooperation and (9) for temporary cooperation. In particular, to sustain as an equilibrium the permanent cooperation strategy with $G(0), G(1) \in \left[0, \frac{\theta R}{4}\right)$ it must be the case that:

$$F_\gamma^{PC,Z}(G(0), G(1)) = V^{PC}(1, Z) - V^D(G(Z)) \geq 0 \tag{11}$$

for $Z = 0, 1$. To sustain as an equilibrium the temporary cooperation strategy with $G(0) \in \left[0, \frac{\theta R}{4}\right)$ it must be the case that:

$$F_\gamma^{TC}(G(0)) = V^{TC}(1, 0) - V^D(G(0)) \geq 0 \tag{12}$$

The following proposition characterizes sustainability constraints (11) when $\delta \to 1$.

Proposition 7. *Suppose that $\gamma > 0$ and $\delta \to 1$. Then, permanent complete cooperation, i.e., disarmed peace with $G(Z) = 0$ for $Z = 0, 1$, can be sustained.*

Proof. See the Appendix. □

Note the difference with Proposition 3. While for $\gamma = 0$, $\delta \to 1$ makes disarmed peace unsustainable, for $\gamma > 0$, we recover sustainability for the Pareto efficiency allocation. The reason behind this difference is that for $\gamma = 0$, if countries play $G(1) = 0$, it would be extremely tempting to deviate. A defector will win the open conflict with probability 1 and permanently eliminate its enemy at virtually no cost, obtaining R in perpetuity. In contrast, when $\gamma > 0$, the defector will collect R only until the game moves back to a contestable state, at which point the defector will be punished with the equilibrium described in Lemma 3.2. Moreover, this punishment equilibrium could be really severe given that for $Z = 1$, the victor of the open conflict always chooses to use the elimination technology.

However, Proposition 7 does not imply that for $\gamma > 0$, it is always possible to avoid open conflict and the use of elimination technology. In fact, we obtain a generalization of Proposition 5, which implies that although temporary cooperation can always be sustained, permanent cooperation might not be sustainable. Moreover, we also obtain a generalization of Proposition 6, which implies that even when partial permanent cooperation is sustainable, countries might obtain a higher expected payoff with temporary cooperation.

Proposition 8. *Suppose that $\gamma \geq 0$ and $X \leq \bar{X}(\gamma) = \frac{[(4-\theta)(1-\delta+\delta\gamma)+3\delta\phi]\delta R}{(1-\delta+\delta\gamma)[4(1-\delta+\delta\gamma)+3\delta\phi]}$. There are thresholds $\bar{\theta}^{PC}_{low}$, $\bar{\theta}^{PC}_{high}$, $\bar{\theta}^{TC}$, $\bar{X}_{low}(\gamma)$, and $\bar{X}_{high}(\gamma)$ such that:*

1. *Suppose that $\theta \leq \bar{\theta}^{PC}_{low}$, or $\bar{\theta}^{PC}_{low} < \theta \leq \bar{\theta}^{PC}_{high}$ and $X \geq \bar{X}_{high}(\gamma)$. Then, permanent complete cooperation, i.e., disarmed peace with $G(Z) = 0$ for $Z = 0, 1$, can be sustained as an equilibrium.*

2. *Suppose that $\bar{\theta}^{PC}_{low} < \theta \leq \bar{\theta}^{PC}_{high}$ and $\bar{X}_{low}(\gamma) < X < \bar{X}_{high}(\gamma)$. Let $\bar{G}(0) = \left(\frac{1-\delta}{2-\delta-\phi\delta}\right)^2 \theta R < G^N$, $\bar{G}(1) = \left(\frac{1-\delta}{2-2\delta+\delta\phi}\right)^2 R^{WO}(\gamma) < G^{WO}(\gamma)$, and $\hat{G}(1) \in \left(0, \bar{G}(1)\right)$ be the unique solution to $F^{PC,1}_{\gamma}(0, G(1)) = 0$.*

   (a) *If $\hat{G}(1) \leq F^{PC,0}_{\gamma}(0,0)/\delta\phi$, then the best possible permanent cooperative equilibrium that can be sustained is partial cooperation with $G(0) = 0$ and $G(1) = \hat{G}(1)$.*

   (b) *If $\hat{G}(1) > F^{PC,0}_{\gamma}(0,0)/\delta\phi$, then the best possible permanent cooperative equilibrium that can be sustained is partial cooperation with $G(0) \in$*

$\left(0, \bar{G}(0)\right)$ *and* $G(1) \in \left(\hat{G}(1), \bar{G}(1)\right)$ *given by the unique solution to* $F_{\gamma}^{PC,0}(G(0), G(1)) = F_{\gamma}^{PC,1}(G(0), G(1)) = 0$ *that satisfies:*

$$\frac{\partial F_{\gamma}^{PC,0}(G(0), 0)}{\partial G(0)} \frac{\partial F_{\gamma}^{PC,1}(0, G(1))}{\partial G(1)} > \delta^2 (1-\phi)\phi.$$

3. *If* $\theta \leq \bar{\theta}^{TC}$, *then the best temporary cooperative equilibrium that can be sustained is* $G(0) = 0$.

4. *If* $\theta > \bar{\theta}^{TC}$, *then the best temporary cooperative equilibrium that can be sustained is* $G(0) > 0$ *given by the unique solution to* $F_{\gamma}^{TC}(G(0)) = 0$. *Moreover,* $G(0) < G^N = \frac{\theta R}{4}$.

Proof. See the Appendix. □

As we have already mentioned, Proposition 8 is a generalization of Proposition 5 for any γ. Crucially, when $\gamma > 0$ it is still the case that for some parameter values it is not possible to sustain permanent cooperation, while it is always possible to sustain temporary cooperation. In other words, sometimes open conflict is unavoidable, but countries can always benefit from temporary cooperation when they cannot use the elimination technology.[24] More precisely, we have the following corollary.

Corollary 2. *Suppose that* $\gamma > 0$. *Open conflict is unavoidable if and only if*

$$X \leq \begin{cases} \bar{X}(\gamma) & \text{if } \theta > \bar{\theta}_{high}^{PC} \\ \bar{X}_{low}(\gamma) & \text{if } \bar{\theta}_{low}^{PC} < \theta \leq \bar{\theta}_{high}^{PC} \end{cases} \tag{13}$$

Moreover:

1. *The lower the cost of the elimination technology (X lower), the less likely open conflict can be avoided.*
2. *The destructiveness of open conflict has a non-monotonic effect on the likelihood of open conflict. Initially, as destructiveness declines (higher* θ*), open conflict becomes more likely, but eventually further reductions in destructiveness make open conflict easier to avoid.*
3. *If* $\theta > \bar{\theta}_{high}^{PC}$, *an increase in* γ *makes open conflict less likely.*

Proposition 8 does not compare permanent versus temporary equilibria. There are three situations to consider: When $\theta \leq \bar{\theta}_{low}^{PC}$, or $\bar{\theta}_{low}^{PC} < \theta \leq \bar{\theta}_{high}^{PC}$ and

[24] Of course that while for $\gamma = 0$, the region of the parameter space for which disarmed peace can be sustained never covers the case $\delta \to 1$; for $\gamma > 0$, it always includes it.

$X \geq \bar{X}_{high}(\gamma)$, disarmed peace can be sustained (Proposition 8.1), and this is clearly the best possible cooperative equilibrium. When $\bar{\theta}^{PC}_{low} < \theta \leq \bar{\theta}^{PC}_{high}$ and $X \leq \bar{X}_{low}(\gamma)$ or $\theta > \bar{\theta}^{PC}_{high}$, neither complete nor partial permanent cooperation can be sustained, and, therefore, the best possible equilibrium is the temporary cooperative equilibrium described in Propositions 8.3 and 8.4. Finally, when $\bar{\theta}^{PC}_{low} < \theta \leq \bar{\theta}^{PC}_{high}$ and $\bar{X}_{low}(\gamma) < X < \bar{X}_{high}(\gamma)$ only partial but not complete permanent cooperation can be sustained (Proposition 8.2). Temporary cooperation can always be sustained (Propositions 8.3 and 8.4). The following proposition characterizes this comparison.

Proposition 9. *Suppose that $\bar{\theta}^{PC}_{low} < \theta \leq \bar{\theta}^{PC}_{high}$ and $\bar{X}_L(\gamma) < X < \bar{X}_H(\gamma)$. Then, partial permanent cooperation induces higher ex-ante expected payoff than temporary cooperation if and only if*

$$\Delta = \begin{bmatrix} F^{PC,0}_{\gamma}(0,0) - \delta\phi G^{PC}(1) - \delta(1-\phi) G^{PC}(0) \\ -\frac{(1-\delta+\delta\gamma)}{1-\delta(1-\gamma-\phi)}\left[F^{TC}_{\gamma}(0) - \delta(1-\phi) G^{TC}(0)\right] \end{bmatrix} \geq 0$$

where

$$F^{PC,0}_{\gamma}(0,0) = \frac{R}{2} - (1-\delta)\theta R - \frac{\delta(1-\delta+\delta\gamma)\left[\phi R^{WO}(\gamma) + (1-\phi)\theta R\right]}{4\left[1-\delta(1-\phi-\gamma)\right]}$$

$$F^{TC}_{\gamma}(0) = \left[\frac{1-\delta(1-\gamma)-\phi\delta^2\gamma}{(1-\delta+\delta\gamma)}\right]\frac{R}{2} - \left[\frac{(1-\delta+\delta\gamma)\delta(1-\phi)+4(1-\delta)\left[1-\delta(1-\phi-\gamma)\right]}{4(1-\delta+\delta\gamma)}\right]\theta R$$

$G^{PC}(0)$ and $G^{PC}(1)$ are given by Proposition 8.2 and $G^{TC}(0)$ by Propositions 8.3 and 8.4. Moreover, if $\theta > \bar{\theta}^{TC}$ and $\hat{G}(1) \leq F^{PC,0}_{\gamma}(0,0)/\delta\phi$, $\Delta \geq 0$, while if $\theta \leq \bar{\theta}^{TC}$ and $\hat{G}(1) > F^{PC,0}_{\gamma}(0,0)/\delta\phi$, $\Delta < 0$.

Proof. See the Appendix. □

Proposition 9 is a generalization of Proposition 6 for any γ. In particular, even when $\gamma > 0$ (which reduces the incentives to use the elimination technology) and permanent cooperation can be sustained, countries might obtain a higher expected payoff engaging in open conflict and only restricting cooperation for $Z = 0$.

To summarize, if we introduce the possibility that a country can only temporarily eliminate its rival, the model can generate several instances of open conflict without compromising any of the key results in Section 4. There are instances in which open conflict is unavoidable. When avoidable, countries might need to modify arming choices to discourage deviations of the cooperative equilibrium. Finally, even when diplomacy is fully effective and countries coordinate in the best possible cooperative equilibrium, and it is possible to avert open conflict, countries may choose to periodically engage in open and only restrict cooperation to states in which the elimination technology is not available.

6 Stochastic Resource Values and Non-aggressive Escalation

When the contested resource is not stochastic, the model can capture two forms of escalations. First, in the permanent cooperation equilibrium described in Proposition 8.2, case (a), countries increase their arming choices when $Z = 1$ (i.e., when the elimination technology is available) to deter open conflict. This is an instance of non-aggressive escalation. Second, in the temporary cooperative equilibrium described in Propositions 8.2 and 8.3, countries increase their gun choices when $Z = 1$, in preparation for a full-fledged open war that temporarily settles the dispute. This is an instance of aggressive escalation. Note that both forms of escalation cannot simultaneously occur in equilibrium. The reason is simple. Either countries can and are willing to avoid open conflict when $Z = 1$ or they do not. In the first scenario, a non-aggressive escalation might take place when $Z = 1$. Otherwise, there will be an aggressive escalation for $Z = 1$.

This section explores an alternative mechanism for non-aggressive escalations. Specifically, we incorporate random fluctuations in resource levels, alternating between high-stakes and low-stakes realizations, $R = H, L$ with $q \in (0, 1)$ denoting the probability that $R = H$. By itself, this extension is not capable of inducing open conflict, but combined with the elimination technology it adds an interesting feature, namely, non-aggressive and aggressive escalations along the same temporary cooperation equilibrium path.

6.1 Stochastic Resources Without the Elimination Technology

As a benchmark, we start by assuming $\phi = 0$, that is, the opportunity to eliminate never arrives.

It is immediate to extend Proposition 1 for the case with stochastic resources. In particular, under the non-cooperative norm (i.e., when countries coordinate in $S_1 = S_2 = 0$ for all (G_1, G_2)), the unique static Nash equilibrium level of arming is $G^N(R) = \theta R/4$ and the associated payoff is $V^N(R) = \theta R/4$, where $R = L, H$.

Thus, the expected payoff from the static Nash equilibrium in this stochastic version of the game will be $V^N = \theta\mathbf{E}/4$, where $\mathbf{E}(R) = qH + (1-q)L$.

For $\delta > 0$ better equilibria than $G^N(L)$ when $R = L$ and $G^N(H)$ when $R = H$ can be sustained. To do so, assume that countries employ the following permanent cooperation strategy: If no deviation has occurred, the countries play $G(L) \in \left[0, G^N(L)\right)$ and settle the dispute when $R = L$ and $G(H) \in \left[0, G^N(H)\right)$ and settle the dispute when $R = H$. Any deviation triggers, starting in the next period, the static Nash equilibrium under the non-cooperative norm. In addition, a deviation in gun choices triggers an open conflict immediately.

If countries play the permanent cooperation strategy with $G(L)$ and $G(H)$ and nobody deviates, expected payoffs are given by:

$$V^{PC}(R) = (1-\delta)\left(\frac{R}{2} - G(R)\right) + \delta\left[qV^{PC}(H) + (1-q)V^{PC}(L)\right]$$

for $R = L, H$. Solving for the above equations, we obtain $V^{PC}(L)$ and $V^{PC}(H)$.

A deviation can occur in either state L or H. Optimal deviations payoffs are given by:

$$V^D(G(R)) = (1-\delta)\left(\frac{g(G(R), \theta R)}{G(R) + g(G(R), \theta R)}\theta R - g(G(R), \theta R)\right) + \delta V^N$$

for $R = L, H$.

Thus, to sustain the permanent cooperation strategy with $G(L)$ and $G(H)$ it must be the case that:

$$F^R(G(L), G(H)) = V^{PC}(R) - V^D(G(R)) \geq 0. \tag{14}$$

for $R = L, H$.

As in the two previous sections, we focus on the best sustainable cooperative equilibrium, that is, we look for the equilibrium that maximizes ex-ante expected payoff

$$V^{PC} = qV^{PC}(H) + (1-q)V^{PC}(L) = \frac{\mathbf{E}(R)}{2} - qG(H) - (1-q)G(L)$$

subject to sustainability constraints $F^R(G(L), G(H)) \geq 0$ for $R = L, H$, that is, equation (14). The following proposition summarizes the results.

Proposition 10. *Assume that the disputable resource is stochastic and* $\phi = 0$*.*

1. *Suppose that* $\theta \leq \bar{\theta}(H) = \frac{2(1-\delta)H + 2\delta[qH + (1-q)L]}{4(1-\delta)H + \delta[qH + (1-q)L]}$*. Then, permanent complete cooperation, i.e., disarmed peace with* $G(R) = 0$ *for* $R = H, L$*, is an equilibrium of the stochastic game. The associated first-best expected payoff is* $V^P = \mathbf{E}(R)/2$*.*

2. *Suppose that* $\theta > \bar{\theta}(H)$. *Let* $\hat{G}(H)$ *be the unique solution to*

$$F^{H}(0, G(H)) = 0$$

   (a) *If* $\hat{G}(H) \leq F^{L}(0,0)/\delta q$, *then the best possible cooperative equilibrium that can be sustained is partial cooperation with* $G(L) = 0$ *and* $G(H) = \hat{G}(H)$.

   (b) *If* $\hat{G}(H) > F^{L}(0,0)/\delta q$, *then the best possible cooperative equilibrium that can be sustained is partial cooperation with* $G(L) > 0$ *and* $G(H) > \hat{G}(H)$ *given by the unique solution to:*

$$F^{L}(G(L), G(H)) = F^{H}(G(L), G(H)) = 0$$

   that satisfies:

$$\frac{\partial F^{L}(G(L), 0)}{\partial G(L)} \frac{\partial F^{H}(0, G(H))}{\partial G(H)} > \delta^{2} q (1-q).$$

Proof. See the Appendix. □

Proposition 10 is a generalization of Proposition 2 for the case of stochastic resource values. In the best cooperative equilibrium, there is either eternal unarmed peace or armed peace with reduced military spending compared to one-shot Nash equilibrium arming levels. When $\theta \leq \bar{\theta}(H)$ (which always holds when countries are patient enough), there is unarmed peace regardless of the value of disputable resources. Otherwise, arming is required to sustain a peaceful settlement when the disputable resource is high.

The most interesting result is Proposition 10.2, case (a). Countries understand that when the stakes are high ($R = H$), temptation is also high and therefore they cannot commit to unarmed peace. However, countries also understand that "guns flexing" does not mean they must start an open conflict and become eternal enemies. Thus, when the stakes are low ($R = L$), countries can trust each other and follow the unarmed peace allocation. Crucially, this arms race when $R = H$ is necessary to avoid a deviation that would trigger open conflict. In other words, stochastic resource values provide a simple mechanism to capture non-aggressive escalations and de-escalations.

In summary, allowing for a stochastic disputable resource does not lead to open conflict, but it explains periods of more and less intense arming and security competition. However, note that the fundamental result of the folk theorem persists. With enough patience, unarmed peace can be sustained.

6.2 Stochastic Resources With the Elimination Technology

To finalize our analysis, we allow the combination of stochastic resources and stochastic opportunities to eliminate the rival. To characterize the best possible equilibrium, we employ a similar approach that we used in Sections 4 and 5. We use the worst possible equilibrium under open conflict as a threat to sustain more cooperative equilibria. As in Section 5, we focus on the case with low elimination cost, and hence the most severe punishment uses the elimination technology whenever it is available. A permanent cooperation strategy is a strategy in which countries settle in every contestable state, that is, for all $Z = 0, 1$ and $R = H, L$. A temporary cooperation strategy refers to a strategy in which countries always settle for $Z = 0$ (that is, when $Z = 0$ and $R = H, L$) but engage in open conflict for at least one state in which the elimination technology is available (that is, either for $(Z, R) = (1, H)$ or $(Z, R) = (1, L)$). If in state $(1, R)$ with $R \in \{H, L\}$ countries do not cooperate, they choose $G^{WO}(\gamma) = R^{WO}(\gamma)/4$ and the victor always eliminates the loser, where

$$R^{WO}(\gamma) = \theta R - X + \frac{\delta \mathbf{E}(R)}{1 - \delta + \delta\gamma}$$

. A full characterization of the best cooperative equilibrium is possible, but not particularly informative. Instead, the following proposition focuses on some of the most interesting results.

Proposition 11. *Assume that the disputable resource is stochastic, $\phi > 0$, and $X \leq \mathbf{E}\left[\bar{X}(\gamma)\right] = \frac{[(4-\theta)(1-\delta+\delta\gamma)+3\delta\phi]\delta\mathbf{E}(R)}{(1-\delta+\delta\gamma)[4(1-\delta+\delta\gamma)+3\delta\phi]}$.*

1. *Suppose that $\gamma = 0$ and $\delta \to 1$. Then, it is impossible to sustain a permanent cooperative equilibrium.*

2. *Suppose that $\gamma > 0$ and $\delta \to 1$. Then, permanent complete cooperation, i.e., disarmed peace with $G(Z, R) = 0$ for all $Z = 0, 1$ and $R = H, L$ can be sustained.*

3. *Suppose that $\gamma \geq 0$ and $\delta < 1$. There are thresholds $\theta^{PC}(H)$ and $X^{PC}(H)$ such that if $\theta > \theta^{PC}(H)$ and $X < X^{PC}(H)$, it is impossible to sustain permanent cooperation.*

4. *Let $F^{TC,0,R}(G(0, L), G(0, H))$ denote the sustainability constraint for state $Z = 0$ and R when countries use a temporary cooperation strategy (without cooperation for $Z = 1$). There is a threshold $\bar{\theta}^{TC}(H)$ such that:*

   (a) *Suppose that $\theta \leq \bar{\theta}^{TC}(H)$. Then, the best temporary cooperative equilibrium (without cooperation for $Z = 1$) that can be sustained is $G(0, R) = 0$ for $R = H, L$.*

(b) *Suppose that* $\theta > \bar{\theta}^{TC}(H)$ *and* $\hat{G}(0,H) \leq \frac{(1-\delta+\delta\gamma+\delta\phi)F^{TC,0,L}(0,0)}{\delta(1-\delta+\delta\gamma)(1-\phi)q}$. *Then the best temporary cooperative equilibrium (without cooperation for* $Z = 1$*) that can be sustained is* $G(0,L) = 0$ *and* $G(0,H) = \hat{G}(0,H)$*, where* $\hat{G}(0,H)$ *is the unique solution to*

$$F^{TC,0,H}(0, G(0,H)) = 0$$

(c) *Suppose that* $\theta > \bar{\theta}^{TC}(H)$ *and* $\hat{G}(0,H) > \frac{(1-\delta+\delta\gamma+\delta\phi)F^{TC,0,L}(0,0)}{\delta(1-\delta+\delta\gamma)(1-\phi)q}$. *Then the best temporary cooperative equilibrium (without cooperation for* $Z = 1$*) that can be sustained* $G(0,L) > 0$ *and* $G(0,H) > \hat{G}(0,H)$ *given by the unique solution to:*

$$F^{TC,0,H}(G(0,L), G(0,H)) = F^{TC,0,L}(G(0,L), G(0,H)) = 0$$

that satisfies

$$\left[\frac{\partial F^{TC,0,H}}{\partial G(0,H)}\right]\left[\frac{\partial F^{TC,0,L}}{\partial G(0,L)}\right] > \frac{(1-\delta+\delta\gamma)^2 \delta^2 (1-\phi)^2 q(1-q)}{(1-\delta+\delta\gamma+\delta\phi)^2}$$

Proof. See the Appendix. □

Proposition 11.1 considers the case in which the uncontestable state is an absorbing state ($\gamma = 0$). For this case, we can extend the result of Proposition 3. For $\delta \to 1$, it is impossible to sustain cooperation when the elimination option is available. To understand the logic behind this result, consider the problem faced by a country that chooses to deviate from cooperation in order to use the elimination technology.

$$\max_{G_i} u_i(G_i, G_{-i}) = \pi_i(G_i, G_j)\left[(1-\delta)(\theta R - X) + \delta \mathbf{E}(R)\right] - (1-\delta) G_i,$$

The solution to this problem is $G_i = g^D\left(G_{-i}, R^{WO}(0)\right)$ and the associated payoff is

$$u_i^D = (1-\delta)\left(R^{WO}(0) - 2\sqrt{R^{WO}(0)\, G_{-i}} + G_{-i}\right)$$

Thus,

$$\lim_{\delta\to 1} u_i^D = \lim_{\delta\to 1} (1-\delta) R^{WO}(0) = \mathbf{E}(R).$$

That is, when $\delta \to 1$, we have $u_i^D \to \mathbf{E}(R)$, which is an extremely tempting deviation. Intuitively, as countries become very patient, they have the opportunity to permanently eliminate the rival and fully capture the resource at virtually no cost.

In contrast, if the uncontestable state is not absorbing ($\gamma > 0$), Proposition 11.2 extends the result of Proposition 7. As $\delta \to 1$, it is possible to sustain disarmed peace as an equilibrium. Indeed, the payoff of a country that deviates from cooperation when the elimination opportunity is available is given by:

$$u_i^D = (1-\delta)\left(R^{WO}(\gamma) - 2\sqrt{R^{WO}(\gamma)\,G_{-i}} + G_{-i}\right) + \frac{\delta^2\gamma V^{PU}}{1-\delta+\delta\gamma}$$

where V^{PU} is the punishment payoff. Thus,

$$\lim_{\delta\to 1} u_i^D = \lim_{\delta\to 1}(1-\delta)\,R^{WO}(\gamma) + \lim_{\delta\to 1}\frac{\delta^2\gamma V^{PU}}{1-\delta+\delta\gamma} = 0 + \lim_{\delta\to 1} V^{PU}$$

That is, when $\delta \to 1$, there is no immediate reward for defecting and the deviation payoff converges to the punishment payoff, which is lower than the expected payoff under cooperation. Intuitively, as countries become very patient, the short-term gains from temporarily eliminating the rival are worthless, and all that matters is that in the long term countries will enter in an eternal war, only interrupted with sporadic episodes of disarmed peace after one of the countries temporarily eliminates the other.

The possibility of sustaining disarmed peace as an equilibrium when $\gamma > 0$ and $\delta \to 1$ should not be misinterpreted as implying that open conflict can always be avoided when $\delta > 0$. In fact, Proposition 11.3 shows that for $\delta > 0$ but sufficiently low, it might be impossible to avoid open conflict when $Z = 1$ and $R = H$. The idea is that for $\delta < 1$, countries might still have incentives to start a war to temporarily eliminate the rival if θ is high enough (that is, the destruction of war is not that severe) and X is low enough (that is, it is cheap to temporarily eliminate the rival country).

Even when permanent cooperation cannot be sustained, this does not fully eliminate the possibilities for cooperation. For example, Proposition 11.4 characterizes the best possible temporary cooperative equilibrium in which countries do not cooperate when the elimination technology is available (that is, when $Z = 1$). In particular, note the equilibrium described in Proposition 11.4 case (b), which is characterized by periods of unarmed peace (when $(Z, R) = (0, L)$), armed peace with settlement (when $(Z, R) = (0, H)$), and open conflict (when $Z = 1$). On this equilibrium path, there are two types of escalations. When $Z = 0$ and $R = H$, countries escalate their arming choices to discourage open conflict (an instance of nonaggressive escalation). In contrast, for $Z = 1$, countries escalate their arming choices in preparation for open conflict (an example of aggressive escalation).

Proposition 11.4 case (b) offers a more realistic perspective on international relations. First, when the elimination technology is not possible, there is room for some international cooperation. Second, when stakes are high arming must be scaled up, but this does not trigger a spiral of arming or open conflict (nonaggressive escalation). Third, when the elimination technology is available, open conflict cannot be avoided (aggressive escalation) but conflict does not settle disputes forever. Eventually, the defeated country reemerged and a new cycle of armed peace and open conflict starts all over again.

7 Conclusions

Departing from a standard static security dilemma game, we develop an infinitely repeated dynamic game that allows for more cooperative outcomes than the static Nash equilibrium. Since international relations–particularly through diplomacy– should be a prime setting for applying the folk theorem to reach payoff-dominated equilibria, one might expect nations to sustain optimal outcomes. This reasoning, however, leads to an overly optimistic and unrealistic prediction: that with sufficiently patient actors, eternal unarmed peace can be sustained as an equilibrium. To bring the model's predictions closer to the historical record, we introduce two additional features: an elimination opportunity and a randomly determined disputable resource. These modifications generate equilibrium paths that alternate periods of unarmed peace, armed peace with settlement, and open conflict. Importantly, in our dynamic model, opportunities for international cooperation never vanish entirely, but eternal unarmed peace becomes far less likely.

There are many possible extensions to our analysis; we highlight four.

First, we have ignored coalition formation, which plays a crucial role in international relations and could either expand or restrict the range of cooperative outcomes.

Second, we have not differentiated between offensive and defensive technologies or weapons, a distinction that could affect the likelihood of armed peace versus open conflict.

Third, we have modeled weapons as perishable goods, whereas in reality they often last for years. Relaxing this assumption would complicate the dynamics of conflict but could yield richer results, including more realistic arms races.

Finally, within our framework, there is room for improving the way we model interactions in non-contestable periods and the transition for returning to the contestable states. We have assumed that during non-contestable states, the victor has full power; it obtains R at no cost. We have also assumed that the transition back to the contestable state is completely random. Thus, there is nothing that the victor or the loser can do to affect outcomes during non-contestable states or the probability that the losers will come back.

Relaxing the first assumption is not that complicated. For example, we can assume that the loser faces a higher cost of guns and, hence, will keep a lower share of the disputed resource. This will add some complications to the computation of expected payoffs during non-contestable periods, but the main logic of our model will persist.

Relaxing the second assumption might be more complicated, but one simple possibility is to assume that by investing in guns the victor and/or the loser can affect the probability that the loser will come back: in fact, making γ endogenous. Provided that these investments do not fully eliminate the incentives to wipe out the rival, the main logic of our model will persist.

In summary, these changes will not seriously affect our results and add an important element of realism to our model, namely, arming during non-contestable states.

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Online Appendix: "The Elusive Quest for Disarmed Peace: Contest Games and International Relations"

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Abstract

This appendix presents the proofs of all lemmas and propositions in "The Elusive Quest for Disarmed Peace: Contest Games and International Relations".

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A.1 One-shot Game

This Appendix presents the proof of Proposition 1.

Proposition 1 *Assume that $\delta = 0$.*

1. *Under the non-cooperative norm (i.e., players coordinate in $S_1S_2 = 0$ for all (G_1, G_2)), the equilibrium level of arming is $G^N = \theta R/4$ and the associated payoff is $V^N = \theta R/4$.*
2. *Under the cooperative norm (i.e., players coordinate in $S_1S_2 = 1$ for all (G_1, G_2)), the equilibrium level of arming is $G^N = R/4$ and the associated payoff is $V^N = R/4$.*
3. *Under the conditional cooperative norm:*
   - (a) *If $\theta \leq 1/2$, then the most cooperative equilibrium level of arming is $G^P = 0$ and the associated payoff is $V^P = R/2$.*
   - (b) *If $\theta > 1/2$, then the most cooperative equilibrium level of arming is $G^{SB} \in (0, \theta R/4]$, where G^{SB} is the unique solution to $F(G) = -2G + 2\sqrt{G\theta R} + \left(\frac{1-2\theta}{2}\right) R = 0$, and the associated payoff is $V^{SB} = R/2 - G^{SB}$.*

Proof: Let

$$u_i(G_1, G_2, S_1, S_2) = \pi_i(G_1, G_2)[S_1S_2R + (1 - S_1S_2)\theta R] - G_i$$

Settlement stage: Suppose that players have already selected (G_1, G_2). If $\theta = 1$, then settlement decisions played no role. Formally, $u_i(G_1, G_2, S_1, S_2) = \pi_i(G_1, G_2)R - G_i$ for all (S_1, S_2). So, assume that $\theta < 1$. There are two possible Nash equilibria, namely, $S_1 = S_2 = 0$ (i.e., no settlement) and $S_1 = S_2 = 1$ (i.e., settlement). To prove that $S_1 = S_2 = 0$ is a Nash equilibrium, note that there is no unilateral deviation because even if one player deviates to $S_1 = 1$, there is no settlement. To prove that $S_1 = S_2 = 1$ is a Nash equilibrium, note that if a player deviates, there is no settlement, and hence, each player obtains $\pi_i(G_1, G_2)\theta R - G_i < \pi_i(G_1, G_2)R - G_i$. Finally, $S_i = 0$ and $S_{-i} = 1$ is not a Nash equilibrium. To prove this, note that if $-i$ deviates to, it obtains $\pi_i(G_1, G_2)R - G_i > \pi_i(G_1, G_2)\theta R - G_i$.

Guns choices stage: If $\theta = 1$, regardless of the cooperation norm, it is always the case that $u_i(G_1, G_2, S_1, S_2) = \pi_i(G_1, G_2)R - G_i$. Then, the unique Nash equilibrium profile of guns is $G_1 = G_2 = G^N = R/4$. The associated equilibrium payoff is $V^N = R/4$. If $\theta < 1$, there are three cases to consider:

Case 1: Suppose that the non-cooperative norm prevails. That is, players coordinate in $S_1S_2 = 0$ for all (G_1, G_2). Then, $u_i(G_1, G_2) = \pi_i(G_1, G_2)\theta R - G_i$. Thus, the unique Nash equilibrium profile of guns is $G_1 = G_2 = G^N = \theta R/4$. The associated equilibrium payoff is $V^N = \theta R/4$.

Case 2: Suppose that the cooperative norm prevails. That is, players coordinate in $S_1S_2 = 1$ for all (G_1, G_2). Then, $u_i(G_1, G_2) = \pi_i(G_1, G_2)R - G_i$. Thus, the unique Nash equilibrium profile of guns is $G_1 = G_2 = G^N = R/4$. The associated equilibrium payoff is $V^N = R/4$.

Case 3: Suppose that a conditional cooperative norm prevails. That is, players coordinate in $S_1S_2 = 1$ for all $(G_1, G_2) \in [0, G] \times [0, G]$ and $S_1S_2 = 0$ for all $(G_1, G_2) \in (G, R] \times (G, R]$, where $G \in [0, \theta R/4)$. Suppose that players select $G_1 = G_2 = G$. Then,

$$u_i^C = u_i(G, G, 1, 1) = \frac{R}{2} - G$$

If player i deviates a play $G_i > G$, its optimal deviation is given by

$$\arg\max_{G_i} \{u_i(G_i, G, 0, 0) = \pi_i(G_i, G)\theta R - G_i\}$$

Solving we obtain that the most profitable deviation for player i is $g^D(G, \theta R) = \sqrt{G\theta R} - G$. Note that $g^G(G, \theta R) > G$ if and only if $G < \theta R/4$, which always holds. Then, the associated deviation payoff is given by:

$$u_i^D = \frac{g^D(G, \theta R)}{g^D(G, \theta R) + G}\theta R - g^D(G, \theta R)$$

Player i does not have an incentive to deviate if and only if $u_i^C \geq u_i^D$ or, which is equivalent,

$$F(G) = -2G + 2\sqrt{G\theta R} + \left(\frac{1 - 2\theta}{2}\right)R \geq 0$$

To obtain the most cooperative equilibrium, we solve

$$\max_{G \in [0, \theta R/4)} \left\{u_i^C = \frac{R}{2} - G\right\}$$

$$s.t.: F(G) = -2G + 2\sqrt{G\theta R} + \left(\frac{1 - 2\theta}{2}\right)R \geq 0$$

There are two cases to consider

Case 3.a: Suppose that $\theta \leq 1/2$. Then, $F(0) \geq 0$ and, hence, the solution to the above optimization problem is $G^P = 0$.

Case 3.b: Suppose that $\theta > 1/2$. Then, $F(0) < 0$, $\lim_{G \to \theta R/4} F(G) = (1-\theta) R/2 > 0$ and $F'(G) = -2 + \sqrt{\theta R/G} > 0$. Thus, there is a unique $G^{SB} \in [0, \theta R/4)$ such that $F(0) < 0$ for all $G \in (0, G^{SB})$, $F(G^{SB}) = 0$, and $F(G) > 0$ for all $G \in (G^{SB}, \theta R/4)$. Therefore, the unique solution to the above optimization problem is G^{SB}.

This completes the proof of Proposition 1. ■

A.2 Repeated Game

This section presents the proof of Proposition 2.

Proposition 2 *Assume that $\delta \geq 0$.*

1. *If $\theta \leq \frac{2}{4-3\delta}$, then the first-best payoff $V^P = R/2$ characterized by unarmed peace $G^P = 0$ can be attained as an equilibrium of the repeated game.*

2. *If $\theta > \frac{2}{4-3\delta}$, then the second-best payoff $V^{SB} = R/2 - G^{SB} > R/4$ with military power $G^{SB} \in \left(0, \left(\frac{1-\delta}{2-\delta}\right)^2 \theta R\right)$ is sustainable as an equilibrium of the repeated game. Moreover, G^{SB} is the unique solution to*

$$F(G) = -(2-\delta) G + 2(1-\delta)\sqrt{G\theta R} + \left[\frac{2-(4-3\delta)\theta}{4}\right] R = 0$$

and $\left(\frac{1-\delta}{2-\delta}\right)^2 \theta R < \frac{\theta R}{4}$.

Proof: Consider the following cooperation strategy. Players play $G_1 = G_2 = G \in$ and $S_1 = S_2 = 1$, provided that no deviation has occurred. If a player deviates from this, starting in the next period, players reverse to the equilibrium described in Proposition 1.1 forever. Moreover, a deviation from $G_i = G$ is immediately punished with $S_1 = S_2 = 0$. If both player use this strategy, the payoff of player i is given by:

$$V_i^C = \frac{R}{2} - G$$

If player i deviates, it optimal deviation

$$\arg \max_{G_i \in [0, \theta R/4]} \left\{ (1-\delta) \left[\pi_i (G_i, G) \theta R - G_i\right] + \delta V^N \right\}$$

where $V^N = \theta R/4$. Solving, we obtain that the most profitable deviation for player i is $g^D(G, \theta R) = \sqrt{G\theta R} - G$. Thus, the associated deviation payoff is given by:

$$V_i^D = (1-\delta)\left[\frac{g^D(G,\theta R)}{g^D(G,\theta R) + G}\theta R - g^D(G,\theta R)\right] + \delta \frac{\theta R}{4}$$

Player i does not have an incentive to deviate if and only if $V_i^C \geq V_i^D$ or, which is equivalent,

$$F(G) = -(2-\delta)G + 2(1-\delta)\sqrt{G\theta R} + \left[\frac{2-(4-3\delta)\theta}{4}\right]R \geq 0$$

To obtain the most cooperative equilibrium, we solve

$$\max_{G\in[0,\theta R/4)} \left\{ V_i^C = \frac{R}{2} - G \right\}$$
$$s.t. : F(G) = -(2-\delta)G + 2(1-\delta)\sqrt{G\theta R} + \left[\frac{2-(4-3\delta)\theta}{4}\right]R \geq 0$$

There are two cases to consider:

Case 3.a: Suppose that $\theta \leq \frac{2}{4-3\delta}$. Then, $F(0) \geq 0$ and, hence, the solution to the above optimization problem is $G^P = 0$.

Case 3.b: Suppose that $\theta > \frac{2}{4-3\delta}$. Then, $F(0) = \frac{[2-(4-3\delta)\theta]R}{4} < 0$ and $F\left(\frac{\theta R}{4}\right) = \frac{(1-\theta)R}{2} > 0$. Moreover, $\frac{\partial F(G)}{\partial G} = -(2-\delta) + (1-\delta)\sqrt{\frac{\theta R}{G}}$, which implies that $\frac{\partial F(G)}{\partial G} > 0$ if $G < \left(\frac{1-\delta}{2-\delta}\right)^2 \theta R, \frac{\partial F}{\partial G}\left(\left(\frac{1-\delta}{2-\delta}\right)^2 \theta R\right) = 0$, and $\frac{\partial F(G)}{\partial G} < 0$ if $G > \left(\frac{1-\delta}{2-\delta}\right)^2 \theta R$. Finally, note that $\left(\frac{1-\delta}{2-\delta}\right)^2 \theta R < \frac{\theta R}{4}$. Thus, there is a unique $G^{SB} \in \left(0, \left(\frac{1-\delta}{2-\delta}\right)^2 \theta R\right)$ such that $F(0) < 0$ for all $G \in \left(0, G^{SB}\right)$, $F\left(G^{SB}\right) = 0$, and $F(G) > 0$ for all $G \in \left(G^{SB}, \theta R/4\right)$. Therefore, the unique solution to the above optimization problem is G^{SB}.

This completes the proof of Proposition 2. ■

A.3 Permanent Elimination

This section presents the proofs of Lemmas 1 and 2, Propositions 3, 4, and 5, and Corollary 1.

Lemma 1 *Under the non-cooperative norm.*[1] *Let* $\bar{X} = \frac{[(4-\theta)(1-\delta)+3\phi\delta]\delta R}{(1-\delta)[4(1-\delta)+3\phi\delta]}$ *and* $R^{WO} = \theta R - X + \frac{\delta R}{1-\delta}$.

1. *Suppose that* $X > \bar{X}$. *Then, the worst equilibrium is* $G^N = \frac{\theta R}{4}$ *for* $Z = 0$ *and* $G^N = \frac{\theta R}{4}$ *and no elimination for* $Z = 1$. *The associated punishment expected payoff is* $V^{PU} = V^N = \frac{\theta R}{4}$.

2. *Suppose that* $X \leq \bar{X}$. *Then, the worst equilibrium is* $G^N = \frac{\theta R}{4}$ *for* $Z = 0$ *and* $G^{WO} = \frac{R^{WO}}{4}$ *and eliminate the loser for* $Z = 1$. *The associated punishment expected payoff is* $V^{PU} = V^{WO} = \frac{(1-\delta)\left(\phi R^{WO}+(1-\phi)\theta R\right)}{4(1-\delta+\phi\delta)}$.

Proof: Suppose that both players will always select $S_1 = S_2 = 0$. If $S_{-i} = 0$, there will be open conflict regardless of S_i. Thus, in the settlement stage there is no unilateral profitable deviation from $S_1 = S_2 = 0$. Then, for $Z = 0$ we have

$$V_i^{PU}(0) = \max_{G_i^{PU}(0)>0} \left\{ (1-\delta)\left[\pi_i(0)\theta R - G_i^{PU}(0)\right] + \delta V_i^{PU} \right\}$$

[1] That is, assume that in every contestable state, $S_1 S_2 = 0$ for all (G_1, G_2) and Z.

where $\pi_i(0) = \frac{G_i^{PU}(0)}{G_1^{PU}(0)+G_2^{PU}(0)}$. For $Z = 1$ we have:

$$V_i^{PU}(1) = \max_{G_i(1)>0} \left\{\pi_i(1)\left[(1-\delta)\theta R + V_i^{PU}(1,w)\right] + (1-\pi_i(1))V_i^{PU}(1,l) - (1-\delta)G_i^{PU}(1)\right\}$$

$$V_i^{PU}(1,w) = \max_{W_i\in\{0,1\}} \left\{W_i(-(1-\delta)X + \delta R) + (1-W_i)\delta V_i^{PU}\right\}$$

$$V_i^{PU}(1,l) = (1-W_{-i})\delta V_i^{PU}$$

where $\pi_i(1) = \frac{G_i^{PU}(1)}{G_1^{PU}(1)+G_2^{PU}(1)}$ and V_i^{PU} is the expected value of the game, that is:

$$V_i^{PU} = \phi V_i^{PU}(1) + (1-\phi)V_i^{PU}(0)$$

We look for a symmetric equilibrium in which $V_i^{PU} = V^{PU}$ for both players. Thus, in equilibrium, there are only two possible situations to consider; either $-(1-\delta)X + \delta R \geq \delta V^{PU}$ and, hence, $W_i = 1$ for $i = 1, 2$, or $-(1-\delta)X + \delta R < \delta V^{PU}$ and, hence, $W_i = 0$ for $i = 1, 2$.

Equilibrium with elimination: Assume that, in equilibrium, $-(1-\delta)X + \delta R \geq \delta V^{PU}$. Then, we have:

$$V_i^{PU}(1) = \max_{G_i^{PU}(1)>0} \left\{(1-\delta)\left(\pi_i(1)R^{WO} - G_i^{PU}(1)\right)\right\}$$

$$V_i(0) = \max_{G_i^{PU}(0)>0} \left\{(1-\delta)\left[\pi_i(0)\theta R - G_i^{PU}(0)\right] + \delta V^{PU}\right\}$$

$$V^{PU} = \phi V_i(1) + (1-\phi)V_i(0)$$

where $R^{WO} = \theta R - X + \frac{\delta R}{1-\delta}$. For $Z = 1$, the unique Nash equilibrium gun profile is $G_1^{PU}(1) = G_2^{PU}(1) = G^{WO} = R^{WO}/4$, which implies $V_i^{PU}(1) = (1-\delta)R^{WO}/4$. For $Z = 0$, the unique Nash equilibrium gun profile is $G_1^{PU}(1) = G_2^{PU}(2) = G^N = \theta R/4$, which implies that $V_i^{PU}(0) = \frac{(1-\delta)\left(\theta R + \delta\phi R^{WO}\right)}{4[1-\delta(1-\phi)]}$. Therefore:

$$V^{PU} = V^{WO} = \frac{(1-\delta)\left(\phi R^{WO} + (1-\phi)\theta R\right)}{4(1-\delta+\phi\delta)}$$

Finally, we must verify that $-(1-\delta)X + \delta R \geq \delta V^{PU}$, which holds if and only if

$$X \leq \frac{[(4-\theta)(1-\delta) + 3\phi\delta]\,\delta R}{(1-\delta)[4(1-\delta) + 3\phi\delta]}$$

Moreover, note that $-(1-\delta)X + \delta R \geq \delta V^{PU}$ implies that $R^{WO} = \theta R - X + \frac{\delta R}{1-\delta} > 0$, and hence, $G^{WO} = R^{WO}/4 > 0$.

Equilibrium without elimination: Assume that, in equilibrium, $-(1-\delta)X + \delta R \leq \delta V^{PU}$. Then, we have:

$$V_i(Z) = \max_{G_i^{PU}(Z)>0} \left\{(1-\delta)\left[\pi_i(Z)\theta R - G_i^{PU}(Z)\right] + \delta V^{PU}\right\} \; for\; Z = 0, 1$$

$$V^{PU} = \phi V_i(1) + (1-\phi)V_i(0)$$

Solving, we have that the unique Nash equilibrium gun profile is $G_1^{PU}(Z) = G_2^{PU}(Z) = G^N = \theta R/4$ for $Z = 0, 1$, which implies,

$$V_i^{PU}(0) = V_i^{PU}(1) = V^{PU} = \frac{\theta R}{4}$$

Finally, we must verify that $-(1-\delta)X + \delta R \leq \delta V^{PU}$, which holds if and only if

$$X \geq \frac{(4-\theta)\delta R}{4(1-\delta)}$$

Note that $\frac{[(4-\theta)(1-\delta)+3\phi\delta]\delta R}{(1-\delta)[4(1-\delta)+3\phi\delta]} > \frac{(4-\theta)\delta R}{4(1-\delta)}$. Thus, we have three possible cases to consider:

Case 1: If $X < \frac{(4-\theta)\delta R}{4(1-\delta)}$, then the only equilibrium is $G_1^{PU}(0) = G_2^{PU}(0) = \theta R/4$ for $Z = 0$ and $G_1^{PU}(1) = G_2^{PU}(1) = R^{WO}/4$ and $W_1 = W_2 = 1$ for $Z = 1$. Thus, the equilibrium discounted expected payoff is $V^{PU} = V^{WO}$.

Case 2: If $X > \bar{X}$, then the only equilibrium is $G_1^{PU}(0) = G_2^{PU}(0) = \theta R/4$ for $Z = 0$ and $G_1^{PU}(1) = G_2^{PU}(1) = \theta R/4$ and $W_1 = W_2 = 0$ for $Z = 1$. Thus, the equilibrium discounted expected payoff is $V^{PU} = V^N = \frac{\theta R}{4}$.

Case 3: If $\frac{(4-\theta)\delta R}{4(1-\delta)} \leq X \leq \bar{X}$, then there are two possible equilibria, described in cases 1 and 2, respectively. $V^{WO} < V^N$ if and only if $X > \frac{\delta(1-\theta)R}{(1-\delta)}$. Moreover, note that $\frac{\delta(4-\theta)R}{4(1-\delta)} > \frac{\delta(1-\theta)R}{(1-\delta)}$, which implies that $V^{WO} < V^N$ for all $\frac{(4-\theta)\delta R}{4(1-\delta)} \leq X \leq \bar{X}$. Thus, whenever both equilibria exist, the equilibrium described in case 1 is more severe.

This completes the proof of Lemma 1. ■

Lemma 2 *Suppose that countries are following either the permanent cooperation strategy with* $G(0) \in \left[0, \frac{\theta R}{4}\right)$ *and* $G(1) \in \left[0, \frac{\theta R}{4}\right)$ *or the temporary cooperation strategy with* $G(0) \in \left[0, \frac{\theta R}{4}\right)$.

1. *Assume that* $Z = 0$*. Then, the most profitable deviation is* $g^D(G(0), \theta R) = \sqrt{G(0)\theta R} - G(0)$*. The associated deviation payoff is given by:*

$$V^D(G(0)) = \frac{g^D(G(0), \theta R)}{g^D(G(0), \theta R) + G(0)}\theta R - g^D(G(0), \theta R) + \delta V^{PU}$$

where V^{PU} *is given by Lemma 1.*

2. *Assume that* $Z = 1$.

   (a) *Suppose that* $X > \bar{X}$*. Then, the most profitable deviation is* $g^D(G(1), \theta R) = \sqrt{G(1)\theta R} - G(1)$*, inducing countries to fight an open conflict, after which the victor will not eliminate the loser. The associated deviation payoff is given by:*

$$V^D(G(1)) = \frac{g^D(G(1), \theta R)}{g^D(G(1), \theta R) + G(1)}\theta R - g^D(G(1), \theta R) + \frac{\delta \theta R}{4(1-\delta)}$$

(b) *Suppose that* $X \leq \bar{X}$*. Then, the most profitable deviation is* $g^D\left(G(1), R^{WO}\right) = \sqrt{G(1)R^{WO}} - G(1)$*, inducing countries to fight an open conflict, after which the victor will eliminate the loser. The associated deviation payoff is given by:*

$$V^D\left(G(1)\right) = \frac{g^D\left(G(1), R^{WO}\right)}{g^D\left(G(1), R^{WO}\right) + G(1)} R^{WO} - g^D\left(G(1), R^{WO}\right)$$

Proof: Suppose that $Z = 0$.

Deviation in the settlement stage: Suppose that $(G_1(1), G_2(1))$ is given and $S_1 = S_2 = 1$. Consider a unilateral deviation to open conflict, i.e., $S_i = 0$. Then, the expected payoff of player i under this deviation is given by:

$$V_i(0) = (1-\delta)\left(\pi_i(0)\theta R - G_i(0)\right) + \delta V^{PU}$$

where $\pi_i(1) = \frac{G_1(1)}{G_1(1)+G_2(1)}$. But if player i chooses $S_i = 1$, then it gets $V_i(0) = (1-\delta)\left(\pi_i(0) R - G_i(0)\right) + \delta V^C$, where $V^C \geq V^{PU}$ is the expected payoff under cooperation. Therefore, for $Z = 0$, players do not have an incentive to unilateraly deviate to open conflict.

Deviation in the gun choice: Suppose that $(G_1(0), G_2(0))$ is given. If i deviates, its expected payoff will be given by:

$$\arg\max_{G_i} \left\{ V_i^D = (1-\delta)\left(\frac{G_i}{G_i + G_{-i}(0)}\theta R - G_i\right) + \delta V^{PU} \right\}$$

where V^{PU} is the punishment payoff. Note that regardless of V^{PU}, the most profitable deviation for player i is

$$g^D\left(G_{-i}(0), R\right) = \sqrt{G_{-i}(0)\theta R} - G_{-i}(0)$$

Thus, the associated deviation payoff is given by:

$$V_i^D = (1-\delta)\left(\frac{g^D\left(G_{-i}(0), R\right)}{g^D\left(G_{-i}(0), R\right) + G_{-i}(0)}\theta R - g^D\left(G_{-i}(0), R\right)\right) + \delta V^{PU}$$

Suppose that $Z = 1$.

Deviation in the settlement stage: Suppose that $(G_1(1), G_2(1))$ is given and $S_1 = S_2 = 1$. Consider a deviation to open conflict, i.e., $S_i = 0$. Then, the expected payoff of player i under this deviation is given by:

$$V_i^D = \pi_i(1)\left[(1-\delta)\theta R + V_i(1,w)\right] + \left[1-\pi_i(1)\right]V_i(1,l) - (1-\delta)G_i(1)$$
$$V_i(1,w) = \max_{W_i \in \{0,1\}} \left\{ W_i\left[-(1-\delta)X + \delta R\right] + (1-W_i)\delta V^{PU} \right\}$$
$$V_i(1,l) = (1-W_{-i})\delta V^{PU}$$

where $\pi_i(1) = \frac{G_i(1)}{G_1(1)+G_2(1)}$ and V^{PU} is the punishment payoff. From Lemma 1, there are two cases to consider.

Case 1: Suppose that $X > \bar{X}$. Then, $V^{PU} = V^{N} = \frac{\theta R}{4}$. Therefore, $-(1-\delta)X + \delta R < \delta V^{PU}$ if and only if $-(1-\delta)X + \delta R < \delta \frac{\theta R}{4}$, which holds if and only if $X > \frac{(4-\theta)\delta R}{4(1-\delta)}$. Since $\bar{X} > \frac{(4-\theta)\delta R}{4(1-\delta)}$, $X > \frac{(4-\theta)\delta R}{4(1-\delta)}$ always holds. Thus, $W_i = W_{-i} = 0$ and, hence,

$$V_i^D = (1-\delta)\left[\pi_i(1)\,\theta R - G_i(1)\right] + \delta \frac{\theta R}{4}$$

Case 2: Suppose that $X \leq \bar{X}$. Then, $V^{PU} = V^{WO}$. Therefore, $-(1-\delta)X + \delta R \geq \delta V^{PU}$ if only if $-(1-\delta)X + \delta R \geq \delta \frac{(1-\delta)\theta R + \phi[-(1-\delta)X + \delta R]}{4(1-\delta+\phi\delta)}$, which always holds for $X \leq \bar{X}$. Thus, $W_i = W_{-i} = 1$ and, hence,

$$V_i^D = (1-\delta)\left[\pi_i(1)\left(\theta R - X + \frac{\delta R}{1-\delta}\right) - G_i(1)\right]$$

Deviation in the gun choice: Since any deviation in guns choices will be immediately punished with open conflict, we must consider the two cases above.

Case 1: Suppose that $X > \bar{X}$. Then, the optimal deviation for player i is given by:

$$\arg\max_{G_i} \left\{ V_i^D = (1-\delta)\left(\frac{G_i}{G_i + G_{-i}(1)}\theta R - G_i\right) + \delta \frac{\theta R}{4} \right\}$$

Solving, we obtain that the most profitable deviation for player i is

$$g^D(G_{-i}(1), R) = \sqrt{G_{-i}(1)\,\theta R} - G_{-i}(1)$$

Thus, the associated deviation payoff is given by:

$$V^D(G_{-i}(1)) = (1-\delta)\left[\frac{g^D(G_{-i}(1), R)}{g^D(G_{-i}(1), R) + G_{-i}(1)}\theta R - g^D(G_{-i}(1), R)\right] + \delta \frac{\theta R}{4}$$

Case 2: Suppose that $X \leq \bar{X}$. Then, the optimal deviation for player i is given by:

$$\arg\max_{G_i} \left\{ V_i^D = (1-\delta)\left(\frac{G_i}{G_i + G_{-i}(1)} R^{WO} - G_i\right) \right\}$$

where $R^{WO} = \theta R - X + \frac{\delta R}{1-\delta}$. Solving, we obtain that the most profitable deviation for player i is

$$g^D\left(G_{-i}(1), R^{WO}\right) = \sqrt{G_{-i}(1)\,R^{WO}} - G_{-i}(1),$$

Thus, the associated deviation payoff is given by:

$$V^D(G_{-i}(1)) = (1-\delta)\left[\frac{g^D\left(G_{-i}(1), R^{WO}\right)}{g^D\left(G_{-i}(1), R^{WO}\right) + G_{-i}(1)} R^{WO} - g^D\left(G_{-i}(1), R^{WO}\right)\right]$$

Selecting $G_1(Z) = G_2(Z)$ for $Z = 1, 2$, we obtain Lemma 2. This completes the proof of Lemma 2. ■

Proposition 3 *Suppose that $\delta \to 1$. Then, it is impossible to sustain a permanent cooperative equilibrium, but it is always possible to sustain a temporary cooperative equilibrium. Moreover, a temporary cooperative equilibrium is not Pareto efficient (regardless of whether $\delta < 1$ or $\delta \to 1$).*

Proof: In the proof of Proposition 5 we show that the permanent cooperation strategy with $G(0) \in \left[0, \frac{\theta R}{4}\right)$ and $G(1) \in \left[0, \frac{\theta R}{4}\right)$ can be sustained as an equilibrium if and only if $F^{PC,0}(G(0), G(1)) \geq 0$ and $F^{PC,0}(G(0), G(1)) \geq 0$. In particular, we need:

$$F^{PC,1}(G(0), G(1)) = \left[\begin{array}{c} -(2-2\delta+\delta\phi) G(1) + 2(1-\delta)\sqrt{G(1) R^{WO}} \\ -\delta(1-\phi) G(0) + \frac{R}{2} - (1-\delta) R^{WO} \end{array} \right] \geq 0$$

where $R^{WO} = \theta R - X + \frac{\delta R}{1-\delta}$.Note that:

$$\lim_{\delta \to 1} F^{PC,1}(G(0), G(1)) = -\left[\frac{R}{2} + \phi G(1) + (1-\phi) G(0)\right] < 0$$

Thus, when $\delta \to 1$, $F^{PC,1}(G(0), G(1)) < 0$ and, hence, it is not possible to sustain a permanent cooperative equilibrium.

In the proof of Proposition 5 we show that the temporary cooperation strategy with $G(0) \in \left[0, \left(\frac{1-\delta+\delta\phi}{2-\delta+\delta\phi}\right)^2 \theta R\right]$ can be sustained if and only if $F^{TC}(G(0)) \geq 0$, where

$$F^{TC}(G(0)) = \left[\begin{array}{c} -(2-\delta+\delta\phi) G(0) + (1-\delta+\delta\phi) 2\sqrt{G(0)\theta R} \\ + \left[\frac{2-\theta(4-3\delta+3\phi\delta)}{4}\right] R \end{array} \right] \geq 0$$

Note that:

$$\lim_{\delta \to 1} F^{TC}(G(0)) = -(1+\phi) G(0) + 2\phi\sqrt{G(0)\theta R} + \left[\frac{2-\theta(1+3\phi)}{4}\right] R$$

Hence:

$$\lim_{\delta \to 1} F^{TC}(0) = \left[\frac{2-\theta(1+3\phi)}{4}\right] R$$

$$\lim_{\delta \to 1} F^{TC}\left(\frac{\theta R}{4}\right) = \frac{(1-\theta) R}{2} > 0$$

Thus, if $\theta \leq \frac{2}{1+3\phi}$, then $\lim_{\delta \to 1} F^{TC}(0) \geq 0$ and, hence, it is possible to sustain a temporary cooperative equilibrium with $G(0) = 0$. On the contrary, if $\theta > \frac{2}{1+3\phi}$, then $\lim_{\delta \to 1} F^{TC}(0) < 0$. Since $\lim_{\delta \to 1} F^{TC}(G(0))$ is strictly increasing in $G(0)$ for all $G(0) \in \left[0, \frac{\theta R}{4}\right]$ and $\lim_{\delta \to 1} F\left(\frac{\theta R}{4}\right) > 0$, there must exist $G(0) \in \left(0, \frac{\theta R}{4}\right)$ such that $\lim_{\delta \to 1} F^{TC}(G(0)) > 0$. Thus, a temporary cooperative equilibrium with $G(0) \in \left(0, \frac{\theta R}{4}\right)$ can be sustained.

This completes the proof of Proposition 3. ■

Proposition 4 *Suppose that $X > \bar{X}$.*

1. *If $\theta \leq \frac{2}{4-3\delta}$, then permanent complete cooperation, i.e., disarmed peace with $G(Z) = 0$ for $Z = 0, 1$ can be sustained.*

2. *If* $\theta > \frac{2}{4-3\delta}$*, then the best possible cooperative equilibria that can be sustained is permanent partial cooperation (i.e., armed peace) with* $G^{PC} \in \left(0, \left(\frac{1-\delta}{2-\delta}\right)^2 \theta R\right)$ *for* $Z = 0, 1$*, where* G^{PC} *is the unique solution to*

$$F(G) = -(2-\delta)G + 2(1-\delta)\sqrt{G\theta R} + \left[\frac{2-(4-3\delta)\theta}{4}\right]R = 0$$

and $\left(\frac{1-\delta}{2-\delta}\right)^2 \theta R < G^N = \frac{\theta R}{4}$.

Proof: Suppose that countries are following the permanent cooperation strategy with $G(0) \in \left[0, \frac{\theta R}{4}\right)$ and $G(1) \in \left[0, \frac{\theta R}{4}\right)$ and that no country deviates. Then, the expected payoffs for each state $Z = 0, 1$ are given by:

$$V^{PC}(0) = (1-\delta)\left(\frac{R}{2} - G(0)\right) + \delta\left[\phi V^{PC}(1) + (1-\phi)V^{PC}(0)\right]$$
$$V^{PC}(1) = (1-\delta)\left(\frac{R}{2} - G(1)\right) + \delta\left[\phi V^{PC}(1) + (1-\phi)V^{PC}(0)\right]$$

Solving we have:

$$V^{PC}(0) = \frac{R}{2} - G(0) + \delta\phi\left[G(0) - G(1)\right]$$
$$V^{PC}(1) = \frac{R}{2} - G(1) + \delta(1-\phi)\left[G(1) - G(0)\right]$$

Sustainability for $Z = 0$. Suppose that $Z = 0$ and player i deviates from cooperation. From Lemma 2, the most profitable deviation for player i is $g^D(G(0), \theta R) = \sqrt{G(0)\theta R} - G(0)$ and the associated deviation payoff is given by:

$$V^D(G(0)) = (1-\delta)\left[\frac{g^D(G(0),\theta R)}{g^D(G(0),\theta R) + G(0)}\theta R - g^D(G(0),\theta R)\right] + \delta V^{PU}$$

From Lemma 1, $X > \bar{X}$, implies that $V^{PU} = V^N = \frac{\theta R}{4}$. Thus, player i does not have an incentive to deviate if and only if $V^{PC}(0) \geq V^D(G(0))$ or, which is equivalent,

$$F^{PC,0}(G(0), G(1)) =$$
$$= -(2-\delta-\delta\phi)G(0) + 2(1-\delta)\sqrt{G(0)\theta R} - \delta\phi G(1) + \frac{\left[2-(4-3\delta)\theta\right]R}{4} \geq 0$$

Sustainability for $Z = 1$. Suppose that $Z = 1$ and player i deviates from cooperation. From Lemma 2, the most profitable deviation for player i is $g^D(G(1), \theta R) = \sqrt{G(1)\theta R} - G(1)$ and the associated payoff is given by:

$$V^D(G(1)) = (1-\delta)\left[\frac{g^D(G(1),\theta R)}{g^D(G(1),\theta R) + G(1)}\theta R - g^D(G(1),\theta R)\right] + \delta V^{PU}$$

From Lemma 1, $X > \bar{X}$ implies that $V^{PU} = V^{N} = \frac{\theta R}{4}$. Thus, player i does not have an incentive to deviate if and only if $V^{PC}(0) \geq V^{D}(G(0))$ or, which is equivalent,

$$F^{PC,1}(G(0),G(1)) =$$
$$= -(2-2\delta+\delta\phi)\,G(1) + 2(1-\delta)\sqrt{G(1)\theta R} - \delta(1-\phi)\,G(0) + \frac{[2-(4-3\delta)\,\theta]\,R}{4} \geq 0$$

Best possible permanent cooperation equilibrium: To determine the best possible cooperative equilibrium, we solve

$$\max_{G(0)\geq 0, G(1)\geq 0} \left\{ \phi V^{PC}(1) + (1-\phi)\,V^{PC}(0) = \frac{R}{2} - \phi G(1) - (1-\phi)\,G(0) \right\}$$
$$s.t.:\ F^{PC,0}(G(0),G(1)) \geq 0 \ and\ F^{PC,1}(G(0),G(1)) \geq 0$$

We start proving the following three results.

Result 1: If a solution exists, it must satisfy $G(0) \in \left[0, \bar{G}(0)\right]$ and $G(1) \in \left[0, \bar{G}(1)\right]$, where $\bar{G}(0) = \left(\frac{1-\delta}{2-\delta-\phi\delta}\right)^2 \theta R < \frac{\theta R}{4}$ and $\bar{G}(1) = \left(\frac{1-\delta}{2-2\delta+\delta\phi}\right)^2 \theta R < \frac{\theta R}{4}$.

Proof: Take the derivative of $F^{PC,0}(G(0),G(1))$ with respect to $G(0)$ and $G(1)$:

$$\frac{\partial F^{PC,0}(G(0),G(1))}{\partial G(0)} = -(2-\delta-\delta\phi) + (1-\delta)\sqrt{\frac{\theta R}{G(0)}},\ \frac{\partial F^{0}(G(0),G(1))}{\partial G(1)} = -\delta\phi$$

Thus, $\lim_{G(0)\to 0} \frac{\partial F^{PC,0}(G(0),G(1))}{\partial G(0)} = \infty$, $\frac{\partial F^{PC,0}(G(0),G(1))}{\partial G(0)} > 0$ for all $G(0) \in \left(0, \bar{G}(0)\right)$, $\frac{\partial F^{PC,0}\left(\bar{G}(0),G(1)\right)}{\partial G(0)} = 0$, and $\frac{\partial F^{PC,0}(G(0),G(1))}{\partial G(0)} < 0$ for all $G(0) > \bar{G}(0)$. Thus, $F^{PC,0}(G(0),G(1))$ is strictly increasing in $G(0)$ for all $G(0) \in \left[0, \bar{G}(0)\right]$, strictly decreasing in $G(0)$ for all $G(0) \geq \bar{G}(0)$. Since $\frac{\partial F^{PC,0}(G(0),G(1))}{\partial G(1)} < 0$ for all $G(0)$, $F^{PC,0}(G(0),G(1))$ is strictly decreasing in $G(1)$.

Similarly, take the derivative of $F^{PC,1}(G(0),G(1))$ with respect to $G(0)$ and $G(1)$:

$$\frac{\partial F^{PC,1}(G(0),G(1))}{\partial G(0)} = -\delta(1-\phi),\ \frac{\partial F^{PC,1}(G(0),G(1))}{\partial G(1)} = -(2-2\delta+\delta\phi) + \sqrt{\frac{\theta R - X + \frac{\delta R}{1-\delta}}{G(1)}}$$

Thus, it is easy to verify that $F^{PC,1}(G(0),G(1))$ is strictly increasing in $G(1)$ for all $G(1) \in \left[0, \bar{G}(1)\right]$, strictly decreasing in $G(1)$ for all $G(1) \geq \bar{G}(1)$, and strictly decreasing in $G(0)$.

Suppose that $(G(0), G(1))$ satisfies $F^{PC,0}(G(0),G(1)) \geq 0$ and $F^{PC,1}(G(0),G(1)) \geq 0$ and $G(0) > \bar{G}(0)$. Then, for $G^{*}(0) \in \left[\bar{G}(0), G(0)\right)$ we also have $F^{PC,0}(G^{*}(0), G(1)) \geq 0$ and $F^{PC,1}(G^{*}(0), G(1)) \geq 0$. Thus, $(G(0), G(1))$ is not a solution. Similarly, suppose that $(G(0), G(1))$ satisfies $F^{PC,0}(G(0),G(1)) \geq 0$ and $F^{PC,1}(G(0),G(1)) \geq 0$ and $G(1) > \bar{G}(1)$. Then, for $G^{*}(1) \in \left[\bar{G}(1), G(1)\right)$ we also have $F^{PC,0}(G(0), G^{*}(1)) \geq 0$ and $F^{PC,1}(G(0), G^{*}(1)) \geq 0$. Thus, $(G(0), G(1))$ is not a solution. Therefore, any solution must satisfy $G(0) \in \left[0, \bar{G}(0)\right]$ and $G(1) \in \left[0, \bar{G}(1)\right]$. ■

Result 2: $F^{PC,0}$ and $F^{PC,1}$ are quasiconcave functions for all $(G(0), G(1)) \in \Re^2_+$.

Proof: Note that $F^{PC,0}(G(0),G(1)) \geq c$ can be written as $F^{PC,0}(G(0),G(1)) = \frac{1}{\delta\phi}\left[F^{PC,0}(G(0),0) - c\right] - G(1) \geq 0$, where

$$F^{PC,0}(G(0),0) = -(2-\delta-\delta\phi)\,G(0) + 2(1-\delta)\sqrt{G(0)\,\theta R} + \frac{[2-(4-3\delta)\,\theta]\,R}{4}$$

Note that $F^{PC,0}(G(0),0)$ is strictly concave in $G(0)$ for all $G(0)$, which implies that $\frac{1}{\delta\phi}\left[F^{PC,0}(G(0),0)-c\right]$ is also strictly concave in $G(0)$ for all $G(0)$ and c. Thus, it must be the case that $\left\{(G(0),G(1)) : G(1)\leq\frac{1}{\delta\phi}\left[F^{PC,0}(G(0),0)-c\right]\right\}$ is a convex set for every c. Therefore, $\left\{(G(0),G(1)) : F^{PC,0}(G(0),G(1))\geq c\right\}$ is a convex set for all c, which implies that $F^{PC,0}(G(0),G(1))$ is quasiconcave.

Similarly, $F^{PC,1}(G(0),G(1)) \geq c$ can be written as $F^{PC,1}(G(0),G(1)) \geq 0 = \frac{1}{\delta(1-\phi)}\left[F^{PC,1}(0,G(1))-c\right]-G(0)\geq 0$, where

$$F^{PC,1}(0,G(1))=-(2-2\delta+\delta\phi)G(1)+2(1-\delta)\sqrt{G(1)\theta R}+\frac{[2-(4-3\delta)\theta]R}{4}$$

Note that $F^{PC,1}(0,G(1))$ is strictly concave in $G(1)$ for all $G(1)$, which implies that $\frac{1}{\delta(1-\phi)}\left[F^{PC,1}(0,G(1))-c\right]$ is also strictly concave in $G(1)$ for all $G(1)$ and c. Thus, it must be the case that $\left\{(G(0),G(1)) : G(0)\leq\frac{1}{\delta(1-\phi)}\left[F^{PC,1}(0,G(1))-c\right]\right\}$ is a convex set for every c. Therefore, $\left\{(G(0),G(1)) : F^{PC,1}(G(0),G(1))\geq c\right\}$ is a convex set for all c, which implies that $F^{PC,1}(G(0),G(1))$ is quasiconcave. ■

Since the objective function is linear and the constraints are quasiconcave, the following Kuhn-Tucker conditions are sufficient for a global maximum.

$$-(1-\phi)+\lambda^0\left[-(2-\delta-\delta\phi)+(1-\delta)\sqrt{\frac{\theta R}{G(0)}}\right]-\lambda^1\delta(1-\phi)+\mu_L^0-\mu_H^0=0$$
$$-\phi-\lambda^0\delta\phi+\lambda^1\left[-(2-2\delta+\delta\phi)+(1-\delta)\sqrt{\frac{\theta R}{G(1)}}\right]+\mu_L^1-\mu_H^1=0$$
$$\lambda^0\geq 0,\ F^{PC,0}(G(0),G(1))\geq 0,\ \lambda^0F^{PC,0}(G(0),G(1))=0$$
$$\lambda^1\geq 0,\ F^{PC,1}(G(0),G(1))\geq 0,\ \lambda^1F^{PC,1}(G(0),G(1))=0$$
$$\mu_L^0\geq 0,\ G(0)\geq 0,\ \mu_L^0G(0)=0$$
$$\mu_H^0\geq 0,\ \bar{G}(0)-G(0)\geq 0,\ \mu_H^0\left[\bar{G}(0)-G(0)\right]=0$$
$$\mu_L^1\geq 0,\ G(1)\geq 0,\ \mu_L^1G(1)=0$$
$$\mu_H^1\geq 0,\ \bar{G}(1)-G(1)\geq 0,\ \mu_H^1\left[\bar{G}(1)-G(1)\right]=0$$

Case 1: Suppose that $G(0)=G(1)=0$. Solving we obtain: $\mu_H^0=\mu_H^1=0$, $\lambda^0=\lambda^1=0$, $\mu_L^0=(1-\phi)$, $\mu_L^1=\phi$, $F^{PC,0}(0,0)\geq 0$ and $F^{PC,1}(0,0)\geq 0$. Thus, we must check that $F^{PC,0}(0,0)=F^{PC,1}(0,0)=\frac{[2-(4-3\delta)\theta]R}{4}\geq 0$, which holds if and only if $\theta\leq\frac{2}{4-3\delta}$.

Case 2: Suppose that $G(0)\in\left(0,\bar{G}(0)\right)$ and $G(1)=0$. Solving we obtain: $\mu_L^0=\mu_H^0=\mu_H^1=0$, $\lambda^1=0$, $\lambda^0=(1-\phi)\left[-(2-\delta-\delta\phi)+(1-\delta)\sqrt{\frac{\theta R}{G(0)}}\right]^{-1}>0$, $\mu_L^1=\phi\left(1+\lambda^0\delta\right)>0$, $F^{PC,0}(G(0),0)=0$ and $F^{PC,1}(G(0),0)\geq 0$. Since $F^{PC,0}(G(0),0)$ is strictly increasing in $G(0)$ for all $G(0)\in\left[0,\bar{G}(0)\right]$, at most, there is one solution to $F^{PC,0}(G(0),0)=0$. Moreover, there is a solution that satisfies $G(0)\in(0,\bar{G}(0))$ if and only if $F^{PC,0}(0,0)<0$ and $F^{PC,0}(\bar{G}(0),0)>0$. Note, however, that $F^{PC,0}(0,0)<0$ implies $F^{PC,1}(0,0)<0$ and $F^{PC,1}(0,0)<0$ implies $F^{PC,1}(G(0),0)<0$ because $F^{PC,1}(G(0),0)$ is strictly decreasing in $G(0)$. Thus, $F^{PC,0}(0,0)<0$ is incompatible with $F^{PC,1}(G(0),0)\geq 0$. Summing up, there is no solution such that $G(0)\in(0,\bar{G}(0))$ and $G(1)=0$.

Case 3: Suppose that $G(0)=0$ and $G(1)\in(0,\bar{G}(1))$. Solving we obtain: $\mu_H^0=\mu_L^1=\mu_H^1=0$, $\lambda^0=0$, $\mu_L^0=(1-\phi)+\lambda^1\delta(1-\phi)>0$, $\lambda^1=\phi\left[-(2-2\delta+\delta\phi)+(1-\delta)\sqrt{\frac{\theta R}{G(1)}}\right]^{-1}>0$, $F^{PC,1}(0,G(1))=0$ and $F^{PC,0}(0,G(1))\geq 0$. Since $F^{PC,1}(0,G(1))$ is strictly increasing in $G(1)$ for all $G(1)\in[0,\bar{G}(1)]$, at most, there is one solution to $F^{PC,1}(0,G(1))=0$. Moreover, there is a solution that satisfies $G(1)\in(0,\bar{G}(1))$ if and only if $F^{PC,1}(0,0)<0$ and $F^{PC,1}(0,\bar{G}(1))>0$. Note, however, that $F^{PC,1}(0,0)<0$ implies $F^{PC,0}(0,0)<0$ and implies $F^{PC,0}(0,G(1))<0$ because $F^{PC,0}(0,G(1))$ is strictly decreasing in $G(1)$. Thus, $F^{PC,1}(0,0)<0$ is incompatible with $F^{PC,0}(0,G(1))\geq 0$. Summing up, there is no solution such that $G(0)=0$ and $G(1)\in(0,\bar{G}(1))$.

Case 4: Suppose that $G(0)\in(0,\bar{G}(0))$ and $G(1)\in(0,\bar{G}(1))$. Solving, we obtain: $\mu_L^0=\mu_L^1=\mu_H^0=\mu_H^1=0$, and

$$\lambda^0\left[-(2-\delta-\delta\phi)+(1-\delta)\sqrt{\frac{\theta R}{G(0)}}\right]-\lambda^1\delta(1-\phi)=(1-\phi)$$

$$-\lambda^0\delta\phi+\lambda^1\left[-(2-2\delta+\delta\phi)+(1-\delta)\sqrt{\frac{\theta R}{G(1)}}\right]=\phi$$

The above system of linear equations has a solution if and only if

$$\left[-(2-\delta-\delta\phi)+(1-\delta)\sqrt{\frac{\theta R}{G(0)}}\right]\left[-(2-2\delta+\delta\phi)+(1-\delta)\sqrt{\frac{\theta R}{G(1)}}\right]\neq\delta^2\phi(1-\phi)$$

A solution must satisfy [$\lambda^0>0$ and $\lambda^1>0$] or [$\lambda^0<0$ and $\lambda^1<0$]. $\lambda^0>0$ and $\lambda^1>0$ if and only if

$$\left[-(2-\delta-\delta\phi)+(1-\delta)\sqrt{\frac{\theta R}{G(0)}}\right]\left[-(2-2\delta+\delta\phi)+(1-\delta)\sqrt{\frac{\theta R}{G(1)}}\right]>\delta^2\phi(1-\phi)$$

$\lambda^0>0$ and $\lambda^1>0$ implies that $F^{PC,0}(G(0),G(1))=0$, and $F^{PC,1}(G(0),G(1))=0$, which implies that $G(1)-G(0)=\sqrt{\theta R}\left[\sqrt{G(1)}-\sqrt{G(0)}\right]$. Thus, it must be the case that $G(1)=G(0)=G^{PC}$. Therefore, we must find G^{PC} such that $F^{PC,0}(G^{PC},G^{PC})=F^{PC,1}(G^{PC},G^{PC})=F(G^{PC})=0$, where

$$F(G)=-(2-\delta)G+2(1-\delta)\sqrt{G\theta R}+\frac{[2-(4-3\delta)\theta]R}{4}$$

$F(0)=\frac{[2-(4-3\delta)\theta]R}{4}<0$ if and only $\theta>\frac{2}{4-3\delta}$. $\frac{\partial F(G)}{\partial G}=-(2-\delta)+(1-\delta)\sqrt{\frac{\theta R}{G}}$, which implies that $\frac{\partial F(G)}{\partial G}>0$ for $G<\left(\frac{1-\delta}{2-\delta}\right)^2\theta R$, $\frac{\partial F}{\partial G}\left(\left(\frac{1-\delta}{2-\delta}\right)^2\theta R\right)=0$, and $\frac{\partial F(G)}{\partial G}<0$ for $G>\left(\frac{1-\delta}{2-\delta}\right)^2\theta R$. Also note that

$\left(\frac{1-\delta}{2-\delta}\right)^2 \theta R < \bar{G}(Z) < \frac{\theta R}{4}$ for all Z. Finally, $F\left(\frac{\theta R}{4}\right) = \frac{(1-\theta)R}{2} > 0$, which implies that $F\left(\bar{G}(Z)\right) > 0$ and $F\left(\left(\frac{1-\delta}{2-\delta}\right)^2 \theta R\right) > 0$. Therefore, there are two possible situations to consider. If $\theta \leq \frac{2}{4-3\delta}$, $F(G) > 0$ for all $G \in \left(0, \frac{\theta R}{4}\right)$. Thus, there is no solution such that $G(0) \in \left(0, \bar{G}(0)\right)$ and $G(1) \in \left(0, \bar{G}(1)\right)$. On the contrary, if $\theta > \frac{2}{4-3\delta}$, then there is a unique $G^{PC} \in \left(0, \left(\frac{1-\delta}{2-\delta}\right)^2 \theta R\right)$ such that $F(0) < 0$ for all $G \in \left(0, G^{PC}\right)$, $F\left(G^{PC}\right) = 0$, and $F(G) > 0$ for all $G \in \left(G^{PC}, \bar{G}(Z)\right)$. Finally, we must check that G^{PC} satisfies

$$\left[-(2-\delta-\delta\phi) + (1-\delta)\sqrt{\frac{\theta R}{G^{PC}}}\right]\left[-(2-2\delta+\delta\phi) + (1-\delta)\sqrt{\frac{\theta R}{G^{PC}}}\right] > \delta^2\phi(1-\phi)$$

or, which is equivalent,

$$H\left(G^{PC}\right) = 2(2-\delta)G^{PC} - (4-3\delta)\sqrt{G^{PC}\theta R} + (1-\delta)\theta R > 0,$$

Since $H(G) > 0$ for all $G \in \left[0, \left(\frac{1-\delta}{2-\delta}\right)^2 \theta R\right]$, $H\left(G^{PC}\right) > 0$ always holds.

Summing up, $G(0) = G(1) = G^{PC}$, where $G^{PC} \in \left(0, \left(\frac{1-\delta}{2-\delta}\right)^2 \theta R\right)$ is the unique solution to $F(G) = 0$ is a solution if and only if $\theta > \frac{2}{4-3\delta}$.

Case 5: Suppose that $G(0) = \bar{G}(0)$ or $G(1) = \bar{G}(1)$. If $G(0) = \bar{G}(0)$, we have $\mu_L^0 = 0$ and, hence, $(1-\phi) + \lambda^1\delta(1-\phi) + \mu_H^0 = 0$, a contradiction because $\lambda^1 \geq 0$ and $\mu_H^0 \geq 0$. If $G(1) = \bar{G}(1)$, we have $\mu_L^1 = 0$ and, hence $\phi + \lambda^0\delta\phi + \mu_H^1 = 0$, a contradiction because $\lambda^0 \geq 0$ and $\mu_H^1 \geq 0$.

This completes the proof of Proposition 4. ■

Proposition 5 *Suppose that $X \leq \bar{X}$. Let*

$$\bar{X}_{low} = \theta R + \frac{\left[2\delta - \frac{2-2\delta+\delta\phi}{1-\delta+\delta\phi}\right] R}{2(1-\delta)} \text{ and } \bar{X}_{high} = \theta R + \frac{(2\delta-1)}{2(1-\delta)} R$$

1. *Suppose that $\theta \leq \frac{1-2\delta}{2(1-\delta)}$ or $\frac{1-2\delta}{2(1-\delta)} < \theta \leq \frac{4(1-\delta)+3\phi\delta}{2(1-\delta)(4-3\delta+3\phi\delta)}$ and $X \geq \bar{X}_{high}$. Then, permanent complete cooperation, i.e., disarmed peace with $G(Z) = 0$ for $Z = 0, 1$, can be sustained.*

2. *Suppose that $\frac{1-2\delta}{2(1-\delta)} < \theta \leq \frac{4(1-\delta)+3\phi\delta}{2(1-\delta)(4-3\delta+3\phi\delta)}$ and $\bar{X}_{low} < X < \bar{X}_{high}$. Let $\bar{G}(0) = \left(\frac{1-\delta}{2-\delta-\phi\delta}\right)^2 \theta R < G^N$, $\bar{G}(1) = \left(\frac{1-\delta}{2-2\delta+\delta\phi}\right)^2 R^{WO} < G^{WO}$, and $\hat{G}(1) \in \left(0, \bar{G}(1)\right)$ be the unique solution to $F^{PC,1}(0, G(1)) = 0$.*

   (a) *If $\hat{G}(1) \leq F^{PC,0}(0,0)/\delta\phi$, then the best possible permanent cooperative equilibrium that can be sustained is partial cooperation with $G(0) = 0$ and $G(1) = \hat{G}(1)$.*

(b) *If* $\hat{G}(1) > F^{PC,0}(0,0)/\delta\phi$, *then the best possible permanent cooperative equilibrium that can be sustained is partial cooperation with* $G(0) \in \left(0, \bar{G}(0)\right)$ *and* $G(1) \in \left(\hat{G}(1), \bar{G}(1)\right)$ *given by the unique solution to*

$$F^{PC,0}(G(0), G(1)) = F^{PC,1}(G(0), G(1)) = 0$$

that satisfies:

$$\frac{\partial F^{PC,0}(G(0), 0)}{\partial G(0)} \frac{\partial F^{PC,1}(0, G(1))}{\partial G(1)} > \delta^2 (1-\phi)\phi.$$

3. *If* $\theta \leq \frac{2}{4-3\delta+3\phi\delta}$, *then the best temporary cooperative equilibrium that can be sustained is* $G(0) = 0$.

4. *If* $\theta > \frac{2}{4-3\delta+3\phi\delta}$, *then the best temporary cooperative equilibrium that can be sustained is* $G(0) \in \left(0, \left(\frac{1-\delta+\delta\phi}{2-\delta+\delta\phi}\right)^2 \theta R\right)$ *given by the unique solution to* $F^{TC}(G(0)) = 0$. *Moreover,* $\left(\frac{1-\delta+\delta\phi}{2-\delta+\delta\phi}\right)^2 \theta R < G^N = \frac{\theta R}{4}$.

Proof Parts 1 and 2: Suppose that countries are following the permanent cooperation strategy with $G(0) \in \left[0, \frac{\theta R}{4}\right)$ and $G(1) \in \left[0, \frac{\theta R}{4}\right)$ and that no country deviates. Then, the expected discounted payoffs for each state $Z = 0, 1$ are given by:

$$V^{PC}(0) = \frac{R}{2} - G(0) + \delta\phi\left[G(0) - G(1)\right]$$
$$V^{PC}(1) = \frac{R}{2} - G(1) + \delta(1-\phi)\left[G(1) - G(0)\right]$$

Sustainability for $Z = 0$: Suppose that $Z = 0$ and player i deviates from cooperation. From Lemma 2, the most profitable deviation for player i is $g^D(G(0), \theta R) = \sqrt{G(0)\theta R} - G(0)$ and the associated deviation payoff is given by:

$$V^D(G(0)) = (1-\delta)\left[\frac{g^D(G(0), \theta R)}{g^D(G(0), \theta R) + G(0)}\theta R - g^D(G(0), \theta R)\right] + \delta V^{PU}$$

From Lemma 1, $X \leq \bar{X}$, implies that $V^{PU} = V^{WO}$. Thus, player i does not have an incentive to deviate if and only if $V^{PC}(0) \geq V^D(G(0))$ or, which is equivalent,

$$F^{PC,0}(G(0), G(1)) =$$
$$-(2-\delta-\delta\phi)G(0) + 2(1-\delta)\sqrt{G(0)\theta R} - \delta\phi G(1) + F^{PC,0}(0,0) \geq 0$$

where

$$F^{PC,0}(0,0) = \frac{R}{2} - (1-\delta)\theta R - \frac{(1-\delta)\delta\left[\phi R^{WO} + (1-\phi)\theta R\right]}{4(1-\delta+\phi\delta)}$$

Sustainability for $Z = 1$: Suppose that $Z = 1$ and player i deviates from cooperation. From Lemma 2, the most profitable deviation for player i is $g^D\left(G(1), R^{WO}\right) = \sqrt{G(1)R^{WO}} - G(1)$, which leads to open conflict and wipe-out. The associated deviation payoff is given by:

$$V^D(G(1)) = (1-\delta)\left[\frac{g^D\left(G(1), R^{WO}\right)}{g^D\left(G(1), R^{WO}\right) + G(1)}R^{WO} - g^D\left(G(1), R^{WO}\right)\right]$$

Thus, player i does not have an incentive to deviate if and only if $V^{PC}(1) \geq V^{D}(G(1))$ or, which is equivalent,

$$F^{PC,1}(G(0), G(1)) =$$
$$= -(2 - 2\delta + \delta\phi) G(1) + 2(1-\delta)\sqrt{G(1) R^{WO}} - \delta(1-\phi) G(0) + F^{PC,1}(0,0) \geq 0$$

where

$$F^{PC,1}(0,0) = \frac{R}{2} - (1-\delta) R^{WO}$$

Best possible permanent cooperation equilibrium: To determine the best possible permanent cooperative equilibrium, we solve

$$\max_{G(0)\geq 0, G(1)\geq 0} \left\{ \phi V^{PC}(1) + (1-\phi) V^{PC}(0) = \frac{R}{2} - \phi G(1) - (1-\phi) G(0) \right\}$$
$$s.t.: F^{PC,0}(G(0), G(1)) \geq 0 \ and \ F^{PC,1}(G(0), G(1)) \geq 0$$

Result 1: If a solution to exist, it must satisfy $G(0) \in \left[0, \bar{G}(0)\right]$ and $G(1) \in \left[0, \bar{G}(1)\right]$, where $\bar{G}(0) = \left(\frac{1-\delta}{2-\delta-\phi\delta}\right)^2 \theta R < \frac{\theta R}{4}$ and $\bar{G}(1) = \left(\frac{1-\delta}{2-2\delta+\delta\phi}\right)^2 R^{WO} < \frac{R^{WO}}{4}$.

Proof: Take the derivative of $F^0(G(0), G(1))$ with respect to $G(0)$ and $G(1)$:

$$\frac{\partial F^{PC,0}(G(0), G(1))}{\partial G(0)} = -(2 - \delta - \delta\phi) + (1-\delta)\sqrt{\frac{\theta R}{G(0)}}, \ \frac{\partial F^{PC,0}(G(0), G(1))}{\partial G(1)} = -\delta\phi$$

Thus, $\lim_{G(0)\to 0} \frac{\partial F^{PC,0}(G(0),G(1))}{\partial G(0)} = \infty$, $\frac{\partial F^{PC,0}(G(0),G(1))}{\partial G(0)} > 0$ for all $G(0) \in \left(0, \bar{G}(0)\right)$, $\frac{\partial F^{PC,0}\left(\bar{G}(0),G(1)\right)}{\partial G(0)} = 0$, and $\frac{\partial F^{PC,0}(G(0),G(1))}{\partial G(0)} < 0$ for all $G(0) > \bar{G}(0)$. Thus, $F^{PC,0}(G(0), G(1))$ is strictly increasing in $G(0)$ for all $G(0) \in \left[0, \bar{G}(0)\right]$, strictly decreasing in $G(0)$ for all $G(0) \geq \bar{G}(0)$. Since $\frac{\partial F^{PC,0}(G(0),G(1))}{\partial G(1)} < 0$ for all $G(0)$, $F^{PC,0}(G(0), G(1))$ is strictly decreasing in $G(1)$.

Similarly, take the derivative of $F^{PC,1}(G(0), G(1))$ with respect to $G(0)$ and $G(1)$:

$$\frac{\partial F^{PC,1}(G(0), G(1))}{\partial G(0)} = -\delta(1-\phi), \ \frac{\partial F^{PC,1}(G(0), G(1))}{\partial G(1)} = -(2 - 2\delta + \delta\phi) + \sqrt{\frac{R^{WO}}{G(1)}}$$

This, it is easy to verify that $F^{PC,1}(G(0), G(1))$ is strictly increasing in $G(1)$ for all $G(1) \in \left[0, \bar{G}(1)\right]$, strictly decreasing in $G(1)$ for all $G(1) \geq \bar{G}(1)$, and strictly decreasing in $G(0)$.

Suppose that $(G(0), G(1))$ satisfies $F^{PC,0}(G(0), G(1)) \geq 0$ and $F^{PC,1}(G(0), G(1)) \geq 0$ and $G(0) > \bar{G}(0)$. Then, for $G^*(0) \in \left[\bar{G}(0), G(0)\right)$ we also have $F^{PC,0}(G^*(0), G(1)) \geq 0$ and $F^{PC,1}(G^*(0), G(1)) \geq 0$. Thus, $(G(0), G(1))$ is not a solution. Similarly, suppose that $(G(0), G(1))$ satisfies $F^{PC,0}(G(0), G(1)) \geq 0$ and $F^{PC,1}(G(0), G(1)) \geq 0$ and $G(1) > \bar{G}(1)$. Then, for $G^*(1) \in \left[\bar{G}(1), G(1)\right)$ we also have $F^{PC,0}(G(0), G^*(1)) \geq 0$ and $F^{PC,1}(G(0), G^*(1)) \geq 0$. Thus, $(G(0), G(1))$ is not a solution. Therefore, any solution must satisfy $G(0) \in \left[0, \bar{G}(0)\right]$ and $G(1) \in \left[0, \bar{G}(1)\right]$. ■

Result 2: $F^{PC,0}(G(0), G(1))$ and $F^{PC,1}(G(0), G(1))$ are quasiconcave functions for all $(G(0), G(1)) \in \Re^2_+$.

Proof: Note that $F^{PC,0}(G(0),G(1)) \geq c$ can be written as $F^{PC,0}(G(0),G(1)) = \frac{1}{\delta\phi}\left[F^{PC,0}(G(0),0) - c\right] - G(1) \geq 0$, where

$$F^{PC,0}(G(0),0) = -(2-\delta-\delta\phi)G(0) + 2(1-\delta)\sqrt{G(0)\theta R} + F^{PC,0}(0,0)$$

Note that $F^{PC,0}(G(0),0)$ is strictly concave in $G(0)$ for all $G(0)$, which implies that $\frac{1}{\delta\phi}\left[F^{PC,0}(G(0),0) - c\right]$ is also strictly concave in $G(0)$ for all $G(0)$ and c. Thus, it must be the case that $\left\{(G(0),G(1)) : G(1) \leq \frac{1}{\delta\phi}\left[F^{PC,0}(G(0),0) - c\right]\right\}$ is a convex set for every c. Therefore, $\left\{(G(0),G(1)) : F^{PC,0}(G(0),G(1)) \geq c\right\}$ is a convex set for all c, which implies that $F^{PC,0}(G(0),G(1))$ is quasiconcave. Similarly, $F^{PC,1}(G(0),G(1)) \geq c$ can be written as $F^{PC,1}(G(0),G(1)) = \frac{1}{\delta(1-\phi)}\left[F^{PC,1}(0,G(1)) - c\right] - G(0) \geq 0$, where

$$F^{PC,1}(0,G(1)) = -(2-2\delta+\delta\phi)G(1) + 2(1-\delta)\sqrt{G(1)R^{WO}} + F^{PC,1}(0,0)$$

Note that $F^{PC,1}(0,G(1))$ is strictly concave in $G(1)$ for all $G(1)$, which implies that $\frac{1}{\delta(1-\phi)}\left[F^{PC,1}(0,G(1)) - c\right]$ is also strictly concave in $G(1)$ for all $G(1)$ and c. Thus, it must be the case that $\left\{(G(0),G(1)) : G(0) \leq \frac{1}{\delta(1-\phi)}\left[F^{PC,1}(0,G(1)) - c\right]\right\}$ is a convex set for every c. Therefore, $\left\{(G(0),G(1)) : F^{PC,1}(G(0),G(1)) \geq c\right\}$ is a convex set for all c, which implies that $F^{PC,1}(G(0),G(1))$ is quasiconcave. ■

Result 3: If $F^{PC,1}(0,0) \geq 0$, then $F^{PC,0}(0,0) \geq 0$.

Proof: $F^{PC,0}(0,0) \geq 0$ if and only if $\frac{R[1-2(1-\delta)\theta]}{2(1-\delta)} \geq \delta V^{WO} = \frac{\phi R^{WO}+(1-\phi)\theta R}{4(1-\delta+\phi\delta)}$. $F^{PC,1}(0,0) \geq 0$ if and only if $\frac{R[1-2(1-\delta)\theta]}{2(1-\delta)} \geq -X + \frac{\delta R}{1-\delta}$. $X \leq \bar{X}$ implies that $-X + \frac{\delta R}{1-\delta} \geq \delta V^{WO}$. Therefore, $F^{PC,1}(0,0) \geq 0$ implies $F^{PC,0}(0,0) \geq 0$. ■

Since the objective function is linear and the constraints are quasiconcave, the following Kuhn-Tucker conditions are sufficient for a global maximum.

$$-(1-\phi) + \lambda^0\left[-(2-\delta-\delta\phi) + (1-\delta)\sqrt{\frac{\theta R}{G(0)}}\right] - \lambda^1\delta(1-\phi) + \mu_L^0 - \mu_H^0 = 0$$

$$-\phi - \lambda^0\delta\phi + \lambda^1\left[-(2-2\delta+\delta\phi) + (1-\delta)\sqrt{\frac{R^{WO}}{G(1)}}\right] + \mu_L^1 - \mu_H^1 = 0$$

$$\lambda^0 \geq 0,\ F^0(G(0),G(1)) \geq 0,\ \lambda^0 F^0(G(0),G(1)) = 0$$

$$\lambda^1 \geq 0,\ F^1(G(0),G(1)) \geq 0,\ \lambda^1 F^1(G(0),G(1)) = 0$$

$$\mu_L^0 \geq 0,\ G(0) \geq 0,\ \mu_L^0 G(0) = 0$$

$$\mu_H^0 \geq 0,\ \bar{G}(0) - G(0) \geq 0,\ \mu_H^0\left[\bar{G}(0) - G(0)\right] = 0$$

$$\mu_L^1 \geq 0,\ G(1) \geq 0,\ \mu_L^1 G(1) = 0$$

$$\mu_H^1 \geq 0,\ \bar{G}(1) - G(1) \geq 0,\ \mu_H^1\left[\bar{G}(1) - G(1)\right] = 0$$

There are several cases to consider:

Case 1: Suppose that $G(0) = G(1) = 0$. Solving, we obtain: $\lambda^0 = \mu_H^0 = 0$, $\mu_L^0 = 1-\phi$, $\lambda^1 = \mu_H^1 = 0$, $\mu_L^1 = \phi$, $F^{PC,0}(0,0) \geq 0$, and $F^{PC,1}(0,0) \geq 0$. Since $F^{PC,1}(0,0) \geq 0$ implies $F^{PC,0}(0,0) \geq 0$, we must check that $F^{PC,1}(0,0) \geq 0$. $F^{PC,1}(0,0) \geq 0$ if and only if $\frac{R[1-2(1-\delta)\theta]}{2(1-\delta)} \geq -X + \frac{\delta R}{1-\delta}$ or, which is equivalent, $X \geq \bar{X}_{high} = \frac{[2\delta-1+2(1-\delta)\theta]R}{2(1-\delta)}$. If $\theta \leq \frac{1-2\delta}{2(1-\delta)}$, this constraint always holds because $X > 0$. If $\theta > \frac{1-2\delta}{2(1-\delta)}$ we need to check that $\frac{[2\delta-1+2(1-\delta)\theta]R}{2(1-\delta)} \leq \bar{X}$, which holds if and only if $\theta \leq \frac{4(1-\delta)+3\phi\delta}{2(1-\delta)(4-3\delta+3\phi\delta)}$. Summing up, $G(0) = G(1) = 0$ is a solution if and only if $[\theta \leq \frac{1-2\delta}{2(1-\delta)}]$ or $[X \geq \bar{X}_{high}$ and $\frac{1-2\delta}{2(1-\delta)} < \theta \leq \frac{4(1-\delta)+3\phi\delta}{2(1-\delta)(4-3\delta+3\phi\delta)}]$.

Case 2: Suppose that $G(0) \in \left(0, \bar{G}(0)\right)$ and $G(1) = 0$. Solving we obtain: $\mu_L^0 = \mu_H^0 = \mu_H^1 = 0$, $\lambda^1 = 0$, $\lambda^0 = (1-\phi)\left[-(2-\delta-\delta\phi) + (1-\delta)\sqrt{\frac{\theta R}{G(0)}}\right]^{-1} > 0$, $\mu_L^1 = \phi\left(1+\lambda^0\delta\right) > 0$, $F^{PC,0}(G(0),0) = 0$ and $F^{PC,1}(G(0),0) \geq 0$. Since $F^{PC,0}(G(0),0)$ is strictly increasing in $G(0)$ for all $G(0) \in \left[0, \bar{G}(0)\right]$, at most, there is one solution to $F^{PC,0}(G(0),0) = 0$. Moreover, there is a solution that satisfies $G(0) \in \left(0, \bar{G}(0)\right)$ if and only if $F^{PC,0}(0,0) < 0$ and $F^{PC,0}\left(\bar{G}(0),0\right) > 0$. Note, however, that $F^{PC,0}(0,0) < 0$ implies $F^{PC,1}(0,0) < 0$ (due to Result 3) and $F^{PC,1}(0,0) < 0$ implies $F^{PC,1}(G(0),0) < 0$ (because $F^{PC,1}(G(0),0)$ is strictly decreasing in $G(0)$). Thus, $F^{PC,0}(0,0) < 0$ is incompatible with $F^{PC,1}(G(0),0) \geq 0$. Summing up, there is no solution such that $G(0) \in \left(0, \bar{G}(0)\right)$ and $G(1) = 0$.

Case 3: Suppose that $G(0) = 0$ and $G(1) \in \left(0, \bar{G}(1)\right)$. Solving we obtain: $\mu_H^0 = \mu_L^1 = \mu_H^1 = 0$, $\lambda^0 = 0$, $\mu_L^0 = (1-\phi) + \lambda^1\delta(1-\phi) > 0$, $\lambda^1 = \phi\left[-(2-2\delta+\delta\phi) + (1-\delta)\sqrt{\frac{R^{WO}}{G(1)}}\right]^{-1} > 0$, $F^{PC,1}(0,G(1)) = 0$ and $F^{PC,0}(0,G(1)) \geq 0$. Since $F^{PC,1}(0,G(1))$ is strictly increasing in $G(1)$ for all $G(1) \in \left[0, \bar{G}(1)\right]$, at most, there is one solution to $F^{PC,1}(0,G(1)) = 0$. Moreover, there is a solution that satisfies $G(1) \in \left(0, \bar{G}(1)\right)$ if and only if $F^{PC,1}(0,0) < 0$ and $F^{PC,1}\left(0, \bar{G}(1)\right) > 0$. $F^{PC,1}(0,0) < 0$ if and only if $X < \theta R + \frac{(2\delta-1)}{2(1-\delta)}R$; while $F^{PC,1}\left(0, \bar{G}(1)\right) > 0$ if and only if $X > \theta R + \frac{\left(2\delta - \frac{2-2\delta+\delta\phi}{1-\delta+\delta\phi}\right)}{2(1-\delta)}R$. Therefore, we need $\theta R + \frac{\left(2\delta - \frac{2-2\delta+\delta\phi}{1-\delta+\delta\phi}\right)}{2(1-\delta)}R < X < \theta R + \frac{(2\delta-1)}{2(1-\delta)}R$. Thus, we must check that $\theta R + \frac{(2\delta-1)}{2(1-\delta)}R > 0$, which holds if and only if $\theta > \frac{1-2\delta}{2(1-\delta)}$. We must also check that $\theta R + \frac{(2\delta-1)}{2(1-\delta)}R \leq \bar{X}$, which holds if and only if $\theta \leq \frac{4(1-\delta)+3\phi\delta}{2(1-\delta)(4-3\delta+3\phi\delta)}$. Finally, we must verify that the unique solution to $F^{PC,1}(0,G(1)) = 0$ satisfies $F^{PC,0}(0,G(1)) \geq 0$ or, which is equivalent, that $G(1) \leq F^{PC,0}(0,0)/\delta\phi$. Summing up, $G(0) = 0$ and $G(1) = \hat{G}(1) \in \left(0, \bar{G}(1)\right)$, where $\hat{G}(1)$ is the unique solution to $F^{PC,1}(0,G(1)) = 0$ is a solution if and only if $\frac{1-2\delta}{2(1-\delta)} < \theta \leq \frac{4(1-\delta)+3\phi\delta}{2(1-\delta)(4-3\delta+3\phi\delta)}$, $\theta R + \frac{\left(2\delta - \frac{2-2\delta+\delta\phi}{1-\delta+\delta\phi}\right)}{2(1-\delta)}R < X < \theta R + \frac{(2\delta-1)}{2(1-\delta)}R$, and $\hat{G}(1) \leq F^{PC,0}(0,0)/\delta\phi$.

Case 4: Suppose that $G(0) \in \left(0, \bar{G}(0)\right)$ and $G(1) \in \left(0, \bar{G}(1)\right)$. Solving, we obtain: $\mu_L^0 = \mu_L^1 = \mu_H^0 = \mu_H^1 = 0$, and

$$\lambda^0\left[-(2-\delta-\delta\phi) + (1-\delta)\sqrt{\frac{\theta R}{G(0)}}\right] - \lambda^1\delta(1-\phi) = (1-\phi)$$

$$-\lambda^0\delta\phi + \lambda^1\left[-(2-2\delta+\delta\phi) + (1-\delta)\sqrt{\frac{R^{WO}}{G(1)}}\right] = \phi$$

The above system of linear equations has a solution if and only if

$$\left[-\left(2-\delta-\delta\phi\right)+\left(1-\delta\right)\sqrt{\frac{\theta R}{G\left(0\right)}}\right]\left[-\left(2-2\delta+\delta\phi\right)+\left(1-\delta\right)\sqrt{\frac{R^{WO}}{G\left(1\right)}}\right]\neq\delta^{2}\phi\left(1-\phi\right)$$

A solution must satisfy $[\lambda^0 > 0$ and $\lambda^1 > 0]$ or $[\lambda^0 < 0$ and $\lambda^1 < 0]$. $\lambda^0 > 0$ and $\lambda^1 > 0$ if and only if

$$\left[-\left(2-\delta-\delta\phi\right)+\left(1-\delta\right)\sqrt{\frac{\theta R}{G\left(0\right)}}\right]\left[-\left(2-2\delta+\delta\phi\right)+\left(1-\delta\right)\sqrt{\frac{R^{WO}}{G\left(1\right)}}\right]>\delta^{2}\phi\left(1-\phi\right) \tag{1}$$

$\lambda^0 > 0$ and $\lambda^1 > 0$ implies that $F^{PC,0}\left(G\left(0\right),G\left(1\right)\right)=0$, and $F^{PC,1}\left(G\left(0\right),G\left(1\right)\right)=0$. Thus, we must solve the following system of equations:

$$F^{PC,0}\left(G\left(0\right),0\right)-\delta\phi G\left(1\right)=0$$
$$F^{PC,1}\left(0,G\left(1\right)\right)-\delta\left(1-\phi\right)G\left(0\right)=0$$

We need $F^{PC,1}\left(0,\bar{G}\left(1\right)\right)>0$ or, otherwise, $F^{PC,1}\left(0,G\left(1\right)\right)<0$ for all $G\left(1\right)\in\left[0,\bar{G}\left(1\right)\right]$ and, hence, there is no solution to the above system of equations satisfying $G\left(0\right)>0$. Also assume that $F^{PC,1}\left(0,0\right)<0$. Otherwise, the solution is $G\left(0\right)=G\left(1\right)=0$ (see case 1). Since $F^{PC,1}\left(0,G\left(1\right)\right)$ is strictly increasing in $G\left(1\right)$ for all $G\left(1\right)\in\left[0,\bar{G}\left(1\right)\right]$, for a solution of the above system of equations to satisfy $G\left(0\right)>0$ it must be the case that $G\left(1\right)>\hat{G}\left(1\right)$, where $\hat{G}\left(1\right)$ is the unique solution to $F^{PC,1}\left(0,G\left(1\right)\right)=0$. Define

$$Q\left(G\left(1\right)\right)=F^{PC,0}\left(\frac{F^{PC,1}\left(0,G\left(1\right)\right)}{\delta\left(1-\phi\right)},0\right)-\delta\phi G\left(1\right)$$

Since $F^{PC,1}\left(0,G\left(1\right)\right)$ is strictly concave in $G\left(1\right)$ and $F^{PC,0}\left(G\left(0\right),0\right)$ is strictly concave in $G\left(0\right)$, $Q\left(G\left(1\right)\right)$ is strictly concave in $G\left(1\right)$. Moreover, note that

$$\begin{aligned}\lim_{G(1)\to\hat{G}(1)}\frac{\partial Q\left(G\left(1\right)\right)}{\partial G\left(1\right)}&=\lim_{G(1)\to\hat{G}(1)}\left[\frac{\partial F^{PC,0}\left(\frac{F^{PC,1}(0,G(1))}{\delta(1-\phi)},0\right)}{\partial G\left(0\right)}\frac{\partial F^{PC,1}\left(0,G\left(1\right)\right)}{\partial G\left(1\right)}\frac{1}{\delta\left(1-\phi\right)}-\delta\phi\right]\\&=\left(\lim_{G(0)\to0}\frac{\partial F^{PC,0}\left(0,0\right)}{\partial G\left(0\right)}\right)\frac{\partial F^{PC,1}\left(0,\hat{G}\left(1\right)\right)}{\partial G\left(1\right)}\frac{1}{\delta\left(1-\phi\right)}-\delta\phi\end{aligned}$$

where $\frac{\partial F^{PC,1}\left(0,\hat{G}(1)\right)}{\partial G(1)}>0$ and $\lim_{G(0)\to0}\frac{\partial F^{PC,0}(0,0)}{\partial G(0)}=\infty$. Thus, for $G\left(1\right)\to\hat{G}\left(1\right)$, $Q\left(G\left(1\right)\right)$ is strictly increasing, which implies that $Q\left(G\left(1\right)\right)$ is either strictly increasing for all $G\left(1\right)\in\left[\hat{G}\left(1\right),\bar{G}\left(1\right)\right]$ or, it is strictly increasing for all $G\left(1\right)\in\left[\hat{G}\left(1\right),G^{m}\right]$, adopts a maximum at $G\left(1\right)=G^{m}$, and it is strictly decreasing for all $G\left(1\right)\in\left[G^{m},\bar{G}\left(1\right)\right]$. Thus, there are 5 possible situations to consider:

Case 4.a: Suppose that $Q\left(\hat{G}\left(1\right)\right)>0$ and $Q\left(G\left(1\right)\right)$ is strictly increasing for all $G\left(1\right)\in\left[\hat{G}\left(1\right),\bar{G}\left(1\right)\right]$. Then, there is no solution to $Q\left(G\left(1\right)\right)=0$.

Case 4.b: Suppose that $Q\left(\hat{G}(1)\right) > 0$ and $Q(G(1))$ is strictly increasing for all $G(1) \in \left[\hat{G}(1), G^m\right]$, adopts a maximum at $G(1) = G^m$, and it is strictly decreasing for all $G(1) \in \left[G^m, \bar{G}(1)\right]$. Then, there is at most one solution to $Q(G(1)) = 0$, which exists if and only if $Q\left(\bar{G}(1)\right) \leq 0$. Note, however, that for such a solution it must be the case that $\frac{\partial Q(G(1))}{\partial G(1)} \leq 0$, which holds if and only if $\frac{\partial F^{PC,0}\left(\frac{F^{PC,1}(0,G(1))}{\delta(1-\phi)},0\right)}{\partial G(0)} \frac{\partial F^{PC,1}(0,G(1))}{\partial G(1)} \leq \delta^2(1-\phi)\phi$, a violation of (1).

Case 4.c: Suppose that $Q\left(\hat{G}(1)\right) = 0$. Assume that $Q(G(1))$ is strictly increasing for all $G(1) \in \left[\hat{G}(1), \bar{G}(1)\right]$. Then, the unique solution to $Q(G(1)) = 0$ is $G(1) = \hat{G}(1)$, which implies $G(0) = 0$, a contradiction. Assume that $Q(G(1))$ is strictly increasing for all $G(1) \in \left[\hat{G}(1), G^m\right]$, adopts a maximum at $G(1) = G^m$, and it is strictly decreasing for all $G(1) \in \left[G^m, \bar{G}(1)\right]$. If $Q\left(\bar{G}(1)\right) \leq 0$, then there are two solutions to $Q(G(1)) = 0$. One solution is $G(1) = \hat{G}(1)$, which implies, $G(0) = 0$, a contradiction. For the other solution, it must be the case that $\frac{\partial Q(G(1))}{\partial G(1)} \leq 0$, which violates (1). If $Q\left(\bar{G}(1)\right) > 0$, then the unique solution to $Q(G(1)) = 0$ is $G(1) = \hat{G}(1)$, which implies $G(0) = 0$, a contradiction.

Case 4.d: Suppose that $Q\left(\hat{G}(1)\right) < 0$ and $Q(G(1))$ is strictly increasing for all $G(1) \in \left[\hat{G}(1), \bar{G}(1)\right]$. Then, there is at most one solution to $Q(G(1)) = 0$, which exists if and only if $Q\left(\bar{G}(1)\right) > 0$. Moreover, for this solution it must be the case that $\frac{\partial Q(G(1))}{\partial G(1)} > 0$.

Case 4.e: Suppose that $Q\left(\hat{G}(1)\right) < 0$ and $Q(G(1))$ is strictly increasing for all $G(1) \in \left[\hat{G}(1), G^m\right]$, adopts a maximum at $G(1) = G^m$, and it is strictly decreasing for all $G(1) \in \left[G^m, \bar{G}(1)\right]$. If $Q(G^m) < 0$, then there is no solution to $Q(G(1)) = 0$. If $Q(G^m) = 0$. Then, the unique solution to $Q(G(1)) = 0$ is $G(1) = G^m$. If $Q(G^m) > 0$ and $Q\left(\bar{G}(1)\right) < 0$, then there are two solutions to $Q(G(1)) = 0$ one with $G(1) < G^m$ and another with $G(1) > G^m$. Note, however, that for the solution with $G(1) > G^m$ we have $\frac{\partial Q(G(1))}{\partial G(1)} < 0$, which violates (1). Thus, only the solution with $G(1) < G^m$ satisfies the Kuhn-Tucker conditions. Finally, if $Q(G^m) > 0$ and $Q\left(\bar{G}(1)\right) > 0$, there is a unique solution to $Q(G(1)) = 0$ for which $\frac{\partial Q(G(1))}{\partial G(1)} > 0$.

Summing up, $G(0) \in \left(0, \bar{G}(0)\right)$ and $G(1) \in \left(0, \bar{G}(1)\right)$ given by the unique solution to $F^{PC,0}(G(0), G(1)) = 0$, and $F^{PC,0}(G(0), G(1)) = 0$ that satisfies $\frac{\partial F^0(G(0),0)}{\partial G(0)} \frac{\partial F^1(0,G(1))}{\partial G(1)} > \delta^2(1-\phi)\phi$ is a solution if and only if $F^{PC,1}(0,0) < 0$, $F^{PC,1}\left(0, \bar{G}(1)\right) > 0$, and $Q\left(\hat{G}(1)\right) < 0$, where $\hat{G}(1) \in \left(0, \bar{G}(1)\right)$ is the unique solution to $F^{PC,1}(G(1), 0) = 0$. In case 3 we have already proved that $F^{PC,1}(0,0) < 0$ and $F^{PC,1}\left(0, \bar{G}(1)\right) > 0$ if and only if $\frac{1-2\delta}{2(1-\delta)} < \theta \leq \frac{4(1-\delta)+3\phi\delta}{2(1-\delta)(4-3\delta+3\phi\delta)}$, $\theta R + \frac{\left(2\delta - \frac{2-2\delta+\delta\phi}{1-\delta+\delta\phi}\right)}{2(1-\delta)} R < X < \theta R + \frac{(2\delta-1)}{2(1-\delta)} R$. Finally, $Q\left(\hat{G}(1)\right) = F^{PC,0}\left(\frac{F^{PC,1}\left(0,\tilde{G}(1)\right)}{\delta(1-\phi)}, 0\right) - \delta\phi\hat{G}(1) = F^{PC,0}(0,0) - \delta\phi\hat{G}(1)$. Thus, $Q\left(\hat{G}(1)\right) < 0$ if and only if $\hat{G}(1) > F^{PC,0}(0,0)/\delta\phi$.

Case 5: Suppose that $G(0) = \bar{G}(0)$ or $G(1) = \bar{G}(1)$. If $G(0) = \bar{G}(0)$, we have $\mu_L^0 = 0$ and, hence, $(1-\phi) + \lambda^1\delta(1-\phi) + \mu_H^0 = 0$, a contradiction because $\lambda^1 \geq 0$ and $\mu_H^0 \geq 0$. If $G(1) = \bar{G}(1)$, we have $\mu_L^1 = 0$ and, hence $\phi + \lambda^0\delta\phi + \mu_H^1 = 0$, a contradiction because $\lambda^0 \geq 0$ and $\mu_H^1 \geq 0$.

This completes the proof of Parts 1 and 2. ■

Proof of Parts 3 and 4: Suppose that countries are following the temporary cooperation strategy with $G(0) \in \left[0, \frac{\theta R}{4}\right)$ and that no country deviates. Then, the expected discounted payoffs for each state $Z = 0, 1$ are given by:

$$V^{TC}(0) = (1-\delta)\left(\frac{R}{2} - G(0)\right) + \delta\left[\phi V^{TC}(1) + (1-\phi)V^{TC}(0)\right]$$
$$V^{TC}(1) = (1-\delta)\frac{R^{WO}}{4}$$

Solving, we obtain:

$$V^{TC}(0) = \frac{(1-\delta)\left(\frac{R}{2} - G(0) + \frac{\delta\phi}{4}R^{WO}\right)}{1-\delta(1-\phi)}$$

Sustainability for $Z = 0$: Suppose that $Z = 0$ and player i deviates from cooperation. From Lemma 2, the most profitable deviation for player i is $g^D(G(0), \theta R) = \sqrt{G(0)\theta R} - G(0)$ and the associated payoff is given by:

$$V^D(G(0)) = (1-\delta)\left[\frac{g^D(G(0), \theta R)}{g^D(G(0), \theta R) + G(0)}\theta R - g^D(G(0), \theta R)\right] + \delta V^{PU}$$

From Lemma 1, if $X \leq \bar{X}$, implies that $V^{PU} = V^{WO}$. Thus, player i does not have an incentive to deviate if and only if $V^{TC}(0) \geq V^D(G(0))$ or, which is equivalent,

$$F^{TC}(G(0)) =$$
$$-(2-\delta+\delta\phi)G(0) + 2(1-\delta+\delta\phi)\sqrt{G(0)\theta R} + F^{TC}(0) \geq 0$$

where

$$F^{TC}(0) = \left[\frac{2-\theta(4-3\delta+3\phi\delta)}{4}\right]R$$

Best possible temporary cooperation equilibrium: To determine the best possible temporary cooperative equilibrium, we solve

$$\max_{G(0)\geq 0}\left\{\begin{array}{c}\phi V^{TC}(1) + (1-\phi)V^{TC}(0) = \\ \left(\frac{1-\delta}{1-\delta+\delta\phi}\right)\left[\frac{\phi R^{WO}}{4} + (1-\phi)\frac{R}{2} - (1-\phi)G(0)\right] \\ s.t.\text{: } F^{TC}(G(0)) \geq 0\end{array}\right\}$$

There are two cases to consider:

Case 1: Suppose that $\theta \leq \frac{2}{4-3\delta+3\phi\delta}$. Then, $F^{TC}(0) \geq 0$ and, hence, $G(0) = 0$ is the solution to the above maximization problem.

Case 2: Suppose that $\theta > \frac{2}{4-3\delta+3\phi\delta}$. Then, $F^{TC}(0) < 0$. Moreover, $\frac{\partial F^{TC}(G(0))}{\partial G(0)} = -(2-\delta+\delta\phi) + (1-\delta+\delta\phi)\sqrt{\frac{\theta R}{G(0)}}$. Thus, $F^{TC}(G(0))$ is strictly increasing in $G(0)$ for all $G(0) \in \left[0, \left(\frac{1-\delta+\delta\phi}{2-\delta+\delta\phi}\right)^2\theta R\right]$ and strictly decreasing in G for all $G \in \left[\left(\frac{1-\delta+\delta\phi}{2-\delta+\delta\phi}\right)^2\theta R, \frac{\theta R}{4}\right]$. Moreover, $\left(\frac{1-\delta+\delta\phi}{2-\delta+\delta\phi}\right)^2\theta R \leq \frac{\theta R}{4}$, with strict inequality for $\delta < 1$. Finally, note that $F^{TC}\left(\left(\frac{1-\delta+\delta\phi}{2-\delta+\delta\phi}\right)^2\theta R\right) = \frac{(1-\delta+\delta\phi)^2}{2-\delta+\delta\phi}\theta R + \left[\frac{2-\theta(4-3\delta+3\phi\delta)}{4}\right] R > 0$. Thus, the solution to the above maximization problem is $G(0) \in \left(0, \left(\frac{1-\delta+\delta\phi}{2-\delta+\delta\phi}\right)^2\theta R\right)$ given by the unique solution to $F^{TC}(G(0)) = 0$. ■

Corollary 1 *Open conflict is unavoidable if and only if*

$$X \leq \begin{cases} \bar{X} = \frac{[(4-\theta)(1-\delta)+3\phi\delta]\delta R}{(1-\delta)[4(1-\delta)+3\phi\delta]} & \text{if } \theta > \frac{4(1-\delta)+3\phi\delta}{2(1-\delta)(4-3\delta+3\phi\delta)} \\ \bar{X}_{low} = \theta R + \frac{\left[2\delta - \frac{2-2\delta+\delta\phi}{1-\delta+\delta\phi}\right]R}{2(1-\delta)} & \text{if } \frac{1-2\delta}{2(1-\delta)} < \theta \leq \frac{4(1-\delta)+3\phi\delta}{2(1-\delta)(4-3\delta+3\phi\delta)} \end{cases}$$

Moreover:

1. *The lower the cost of elimination technology (X lower) and the higher the chances that countries can use it (ϕ higher), the less likely open conflict can be avoided.*

2. *The destructiveness of open conflict has a non-monotonic effect on the likelihood of open conflict. Initially, as destructiveness declines (θ higher), open conflict becomes more likely, but eventually further reductions in destructiveness make open conflict easier to avoid.*

Proof: From Proposition 5.2, it is not possible to sustain a permanent cooperative equilibrium if and only if $[\frac{1-2\delta}{2(1-\delta)} < \theta \leq \frac{4(1-\delta)+3\phi\delta}{2(1-\delta)(4-3\delta+3\phi\delta)}$ and $X \leq \bar{X}_{low}]$ or $[\theta > \frac{4(1-\delta)+3\phi\delta}{2(1-\delta)(4-3\delta+3\phi\delta)}$ and $X \leq \bar{X}]$.

Clearly, the lower the cost of the elimination technology (the lower X), the more likely that both of these conditions hold.

Rearranging terms we have that we have that it is not possible to sustain a permanent cooperative equilibrium if and only if

$$X \leq \begin{cases} \bar{X} = \frac{[(4-\theta)(1-\delta)+3\phi\delta]\delta R}{(1-\delta)[4(1-\delta)+3\phi\delta]} & \text{if } \phi > \frac{2(1-\delta)(2-4\theta+3\theta\delta)}{2\theta(1-\delta)-1} \\ \bar{X}_{low} = \theta R + \frac{\left[2\delta - \frac{2-2\delta+\delta\phi}{1-\delta+\delta\phi}\right]R}{2(1-\delta)} & \text{if } \phi \leq \frac{2(1-\delta)(2-4\theta+3\theta\delta)}{2\theta(1-\delta)-1} \end{cases}$$

Note that $\bar{X}_{low}$ and $\bar{X}$ are both strictly increasing in ϕ. Moreover, $\bar{X}_{low} < \bar{X}$ for $\phi \leq \frac{2(1-\delta)(2-4\theta+3\theta\delta)}{2\theta(1-\delta)-1}$. Thus, an increase in ϕ makes the above inequality more likely to hold.

Finally, note that $\bar{X}_{low}$ is strictly increasing in θ, while $\bar{X}$ is strictly decreasing in θ. Moreover, $\bar{X}_{low} < \bar{X}$ for $\theta \leq \frac{4(1-\delta)+3\phi\delta}{2(1-\delta)(4-3\delta+3\phi\delta)}$. Thus, an increase in θ, makes open conflict becomes more likely if $\theta < \frac{4(1-\delta)+3\phi\delta}{2(1-\delta)(4-3\delta+3\phi\delta)}$ and less likely if $\theta \geq \frac{4(1-\delta)+3\phi\delta}{2(1-\delta)(4-3\delta+3\phi\delta)}$.

This completes the proof of Corollary 1. ■

Proposition 6 *Suppose that* $\frac{1-2\delta}{2(1-\delta)} < \theta \leq \frac{4(1-\delta)+3\phi\delta}{2(1-\delta)(4-3\delta+3\phi\delta)}$ *and* $\bar{X}_{low} < X < \bar{X}_{high}$. *Then, partial permanent cooperation induces higher ex-ante expected payoff than temporary cooperation if and only if*

$$\Delta = \left[\begin{array}{c} F^{PC,0}(0,0) - \delta\phi G^{PC}(1) - \delta(1-\phi)G^{PC}(0) \\ -\frac{(1-\delta)}{1-\delta+\delta\phi}\left[F^{TC}(0) - \delta(1-\phi)G^{TC}(0)\right] \end{array}\right] \geq 0$$

where

$$F^{PC,0}(0,0) = \frac{R}{2} - (1-\delta)\theta R - \frac{(1-\delta)\delta\left[\theta R + \phi\left(-X + \frac{\delta R}{1-\delta}\right)\right]}{4(1-\delta+\phi\delta)}$$
$$F^{TC}(0) = \frac{2-\theta(4-3\delta+3\phi\delta)}{4}R$$

$G^{PC}(0)$ and $G^{PC}(1)$ are given by Proposition 5.2 and $G^{T}(0)$ by Propositions 5.3 and 5.4. Moreover, if $\theta > \frac{2}{4-3\delta+3\phi\delta}$ and $\hat{G}(1) \le F^{PC,0}(0,0)/\delta\phi$, $\Delta \ge 0$, while if $\theta \le \frac{2}{4-3\delta+3\phi\delta}$ and $\hat{G}(1) > F^{PC,0}(0,0)/\delta\phi$, $\Delta < 0$.

Proof: The ex-ante expected payoff under partial permanent cooperation is given by:

$$V^{PC} = \frac{R}{2} - \phi G^{P}(1) - (1-\phi)G^{P}(0)$$

where $G^{PC}(0)$ and $G^{PC}(1)$ are given by Proposition 5.2.

The ex-ante expected payoff under temporary cooperation is given by:

$$V^{TC} = \phi V^{TC}(1) + (1-\phi)V^{TC}(0)$$

where $V^{TC}(0) = \frac{(1-\delta)\left(\frac{R}{2} - G^{TC}(0) + \frac{\delta\phi R^{WO}}{4}\right)}{1-\delta(1-\phi)}$ and $V^{TC}(1) = \frac{(1-\delta)R^{WO}}{4}$. Therefore, we obtain:

$$V^{TC} = \frac{(1-\delta)\left[\frac{\phi}{4}R^{WO} + (1-\phi)\left(\frac{R}{2} - G^{TC}(0)\right)\right]}{1-\delta+\delta\phi}$$

where $G^{TC}(0)$ is given by Proposition 5.3 or Proposition 5.4.

$V^{PC} \ge V^{TC}$ if and only if

$$F^{PC,0}(0,0) - \delta\phi G^{PC}(1) - \delta(1-\phi)G^{PC}(0) \ge \frac{(1-\delta)\left[F^{TC}(0) - \delta(1-\phi)G^{TC}(0)\right]}{1-\delta+\delta\phi}$$

where

$$F^{PC,0}(0,0) = \frac{R}{2} - (1-\delta)\theta R - \frac{(1-\delta)\delta\left[\theta R + \phi\left(-X + \frac{\delta R}{1-\delta}\right)\right]}{4(1-\delta+\phi\delta)}$$
$$F^{TC}(0) = \frac{2-\theta(4-3\delta+3\phi\delta)}{4}R$$

Note that $\frac{1-2\delta}{2(1-\delta)} \le \frac{2}{4-3\delta+3\phi\delta} \le \frac{4(1-\delta)+3\phi\delta}{2(1-\delta)(4-3\delta+3\phi\delta)}$ with strict inequalities if $\delta > 0$. So, there are several cases to consider:

Case 1: Suppose that $\frac{1-2\delta}{2(1-\delta)} < \theta \le \frac{2}{4-3\delta+3\phi\delta}$, $\bar{X}_{low} < X < \bar{X}_{high}$, and $\hat{G}(1) \le F^{PC,0}(0,0)/\delta\phi$, where $\hat{G}(1) \in (0, \bar{G}(1))$ is the unique solution to $F^{PC,1}(G(1),0) = 0$. Then, from Proposition 5.2.a, the best possible permanent cooperative equilibria that can be sustained is partial cooperation with $G^{PC}(0) = 0$

and $G^{PC}(1) = \hat{G}(1)$. From Proposition 5.3, the best temporary cooperative equilibria that can be sustained is $G^{TC}(0) = 0$. Therefore, $V^{PC} \geq V^{TC}$ if and only if

$$F^{PC,0}(0,0) - \delta\phi\hat{G}(1) \geq \frac{(1-\delta)F^{TC}(0)}{1-\delta+\delta\phi}$$

Moreover, note that $\theta \leq \frac{2}{4-3\delta+3\phi\delta}$ implies $F^{TC}(0) > 0$.

Case 2: Suppose that $\frac{2}{4-3\delta+3\phi\delta} < \theta \leq \frac{4(1-\delta)+3\phi\delta}{2(1-\delta)(4-3\delta+3\phi\delta)}$, $\bar{X}_{low} < X < \bar{X}_{high}$, and $\hat{G}(1) \leq F^{PC,0}(0,0)/\delta\phi$, where $\hat{G}(1) \in \left(0, \bar{G}(1)\right)$ is the unique solution to $F^{PC,1}(G(1),0) = 0$. Then, from Proposition 5.2.a, the best possible permanent cooperative equilibria that can be sustained is partial cooperation with $G^{PC}(0) = 0$ and $G^{PC}(1) = \hat{G}(1)$. From Proposition 5.4, the best temporary cooperative equilibria that can be sustained is $G^{TC}(0) \in \left(0, \left(\frac{1-\delta+\delta\phi}{2-\delta+\delta\phi}\right)^2 \theta R\right)$ given by the unique solution to $F^{TC}(G(0)) = 0$. Therefore, $V^{PC} \geq V^{TC}$ if and only if

$$F^{PC,0}(0,0) - \delta\phi\hat{G}(1) \geq \frac{(1-\delta)\left[F^{TC}(0) - \delta(1-\phi)G^{TC}(0)\right]}{1-\delta+\delta\phi}$$

Since $\hat{G}(1) \leq F^{PC,0}(0,0)/\delta\phi$, and $\theta > \frac{2}{4-3\delta+3\phi\delta}$ implies $F^{TC}(0) < 0$, the above inequality always holds.

Case 3: Suppose that $\frac{1-2\delta}{2(1-\delta)} < \theta \leq \frac{2}{4-3\delta+3\phi\delta}$, $\theta R + \bar{X}_{low} < X < \bar{X}_{high}$, and $\hat{G}(1) > F^{PC,0}(0,0)/\delta\phi$, where $\hat{G}(1) \in \left(0, \bar{G}(1)\right)$ is the unique solution to $F^{PC,1}(G(1),0) = 0$. Then, from Proposition 5.2.b, the best possible permanent cooperative equilibria that can be sustained is partial cooperation with $G^{PC}(0) > 0$ and $G^{PC}(1) > \hat{G}(1)$ given by the unique solution to $F^{PC,0}(G(0),G(1)) = F^{PC,1}(G(0),G(1)) = 0$ that satisfies $\frac{\partial F^{PC,0}(G(0),0)}{\partial G(0)} \frac{\partial F^{PC,1}(0,G(1))}{\partial G(1)} > \delta^2(1-\phi)\phi$. From Proposition 5.3, the best temporary cooperative equilibria that can be sustained is $G^{TC}(0) = 0$. Therefore, $V^{PC} \geq V^{TC}$ if and only if

$$F^{PC,0}(0,0) - \delta\phi G^{PC}(1) - \delta(1-\phi)G^{PC}(0) \geq \frac{(1-\delta)F^{TC}(0)}{1-\delta+\delta\phi}$$

Since $G^{PC}(1) > \hat{G}(1) > F^{PC,0}(0,0)/\delta\phi$, and $\theta \leq \frac{2}{4-3\delta+3\phi\delta}$ implies $F^{TC}(0) > 0$, the above inequality never holds.

Case 4: Suppose that $\frac{2}{4-3\delta+3\phi\delta} < \theta \leq \frac{4(1-\delta)+3\phi\delta}{2(1-\delta)(4-3\delta+3\phi\delta)}$, $\bar{X}_{low} < X < \bar{X}_{high}$, and $\hat{G}(1) > F^{PC,0}(0,0)/\delta\phi$, where $\hat{G}(1) \in \left(0, \bar{G}(1)\right)$ is the unique solution to $F^{PC,1}(G(1),0) = 0$. Then, from Proposition 5.2.b, the best possible permanent cooperative equilibria that can be sustained is partial cooperation with $G^{PC}(0) > 0$ and $G^{PC}(1) > \hat{G}(1)$ given by the unique solution to $F^{PC,0}(G(0),G(1)) = F^{PC,1}(G(0),G(1)) = 0$ that satisfies $\frac{\partial F^{PC,0}(G(0),0)}{\partial G(0)} \frac{\partial F^{PC,1}(0,G(1))}{\partial G(1)} > \delta^2(1-\phi)\phi$. From Proposition 5.4, the best temporary cooperative equilibria that can be sustained is $G^{T}(0) \in \left(0, \left(\frac{1-\delta+\delta\phi}{2-\delta+\delta\phi}\right)^2 \theta R\right)$ given by the unique solution to $F^{TC}(G(0)) = 0$. Therefore, $V^{PC} \geq V^{TC}$ if and only if

$$F^{PC,0}(0,0) - \delta\phi G^{PC}(1) - \delta(1-\phi)G^{PC}(0) \geq \frac{(1-\delta)\left[F^{TC}(0) - \delta(1-\phi)G^{TC}(0)\right]}{1-\delta+\delta\phi}$$

Moreover, $\theta > \frac{2}{4-3\delta+3\phi\delta}$ implies $F^{TC}(0) < 0$.

This completes the proof of Proposition 6. ■

A.4 Temporary Elimination

This section presents the proof of Lemmas 2 and 3 and Propositions 7, 8, and 9.

Lemma 3 *Under the non-cooperative norm.*[2] *Let* $\bar{X}(\gamma) = \frac{[(4-\theta)(1-\delta+\delta\gamma)+3\delta\phi]\delta R}{(1-\delta+\delta\gamma)[4(1-\delta+\delta\gamma)+3\delta\phi]}$ *and* $R^{WO}(\gamma) = \theta R - X + \frac{\delta R}{1-\delta+\delta\gamma}$.

1. *Suppose that* $X > \bar{X}(\gamma)$. *Then, the worst equilibrium is* $G^N = \frac{\theta R}{4}$ *for* $Z = 0$ *and* $G^N = \frac{\theta R}{4}$ *and no elimination for* $Z = 1$. *The associated punishment expected payoff is* $V^{PU} = V^N = \frac{\theta R}{4}$.

2. *Suppose that* $X \leq \bar{X}(\gamma)$. *Then, the worst equilibrium is* $G^N = \frac{\theta R}{4}$ *for* $Z = 0$ *and* $G^{WO}(\gamma) = \frac{R^{WO}(\gamma)}{4}$ *and eliminate the loser for* $Z = 1$. *The associated punishment expected payoff is* $V^{PU} = V^{WO}(\gamma) = \frac{(1-\delta+\delta\gamma)\left[\phi R^{WO}(\gamma)+(1-\phi)\theta R\right]}{4(1-\delta)[1-\delta(1-\phi-\gamma)]}$.

Proof: Suppose that when the resource is contestable both players always select $S_1 = S_2 = 0$. If $S_{-i} = 0$, there is open conflict regardless of S_i. Thus, in the settlement stage there is no unilateral profitable deviation from $S_1 = S_2 = 0$.

When the resource is contestable and elimination is not possible, the expected discounted payoff of player i is given by:

$$V_i^{PU}(C=1, Z=0) = \max_{G_i(0)>0} \left\{(1-\delta)\left[\pi_i(0)\theta R - G_i^{PU}(0)\right] + \delta V_i^{PU}(C=1)\right\}$$

where $\pi_i(0) = \frac{G_i^{PU}(0)}{G_1^{PU}(0)+G_2^{PU}(0)}$ and $V_i^{PU}(C=1)$ is the expected value of the game conditional on the resource being contestable, i.e.,

$$V_i^{PU}(C=1) = (1-\phi)V_i^{PU}(C=1, Z=0) + \phi V_i^{PU}(C=1, Z=1)$$

When the resource is contestable and elimination is possible, the expected discounted payoff of player i is given by:

$$V_i^{PU}(C=1, Z=1) = \max_{G_i(1)>0} \left\{\pi_i(1)\left[(1-\delta)\theta R + V_i^{PU}(w)\right] + (1-\pi_i(1))V_i^{PU}(l) - (1-\delta)G_i^{PU}(1)\right\}$$
$$V_i^{PU}(w) = \max_{W_i\in\{0,1\}} \left\{W_i\left[-(1-\delta)X + \delta V_i^{PU}(C=0, w)\right] + (1-W_i)\delta V_i^{PU}(C=1)\right\}$$
$$V_i^{PU}(l) = W_{-i}\delta V_i^{PU}(C=0, l) + (1-W_{-i})\delta V_i^{PU}(C=1)$$

where $\pi_i(1) = \frac{G_i^{PU}(1)}{G_1^{PU}(1)+G_2^{PU}(1)}$.

Finally, when the resource is not contestable, the expected discounted payoff of player i is given by:

$$V_i^{PU}(C=0, w) = (1-\delta)R + \delta\left[(1-\gamma)V_i^{PU}(C=0, w) + \gamma V_i^{PU}(C=1)\right]$$
$$V_i^{PU}(C=0, l) = \delta\left[(1-\gamma)V_i^{PU}(C=0, l) + \gamma V_i^{PU}(C=1)\right]$$

[2] That is, assume that in every contestable state, $S_1 S_2 = 0$ for all (G_1, G_2) and Z.

Solving for $V_i^{PU}(C=0,w)$ and $V_i^{PU}(C=0,l)$ as a function of $V_i^{PU}(C=1)$ we have:

$$V_i^{PU}(C=0,w)=\frac{(1-\delta)R+\gamma\delta V_i^{PU}(C=1)}{1-\delta(1-\gamma)},\ V_i(C=0,l)=\frac{\delta\gamma V_i^{PU}(C=1)}{1-\delta(1-\gamma)}$$

Also note that $W_i=1$ if and only if $-(1-\delta)X+\delta V_i^{PU}(C=0,w)\geq\delta V_i^{PU}(C=1)$. Introduing $V_i^{PU}(C=0,w)$ we have that $W_i=1$ if and only if

$$-X+\frac{\delta R}{1-\delta(1-\gamma)}\geq\frac{\delta V_i^{PU}(C=1)}{1-\delta(1-\gamma)}$$

We look for a symmetric equilibrium in which $V_i^{PU}(C=1)=V^{PU}$ for both players. Thus, in equilibrium, there are only two possible situations to consider; either $-X+\frac{\delta R}{1-\delta(1-\gamma)}\geq\frac{\delta V^{PU}}{1-\delta(1-\gamma)}$ and, hence, $W_i=1$ for $i=1,2$, or $-X+\frac{\delta R}{1-\delta(1-\gamma)}<\frac{\delta V^{PU}}{1-\delta(1-\gamma)}$ and, hence, $W_i=0$ for $i=1,2$.

Equilibrium with elimination: Assume that, in equilibrium, $-X+\frac{\delta R}{1-\delta(1-\gamma)}\geq\frac{\delta V^{PU}}{1-\delta(1-\gamma)}$. Then, we have:

$$V_i^{PU}(C=1,Z=0)=\max_{G_i(1)>0}\left\{(1-\delta)\left[\pi_i(0)\theta R-G_i^{PU}(0)\right]+\delta V^{PU}\right\}$$
$$V_i^{PU}(C=1,Z=1)=\max_{G_i(1)>0}\left\{(1-\delta)\left[\pi_i(1)R^{WO}(\gamma)-G_i^{PU}(1)\right]+\frac{\delta^2\gamma V^{PU}}{1-\delta+\delta\gamma}\right\}$$
$$V^{PU}=(1-\phi)V_i^{PU}(C=1,Z=0)+\phi V_i^{PU}(C=1,Z=1)$$

where $R^{WO}(\gamma)=\theta R-X+\frac{\delta R}{1-\delta+\delta\gamma}$. For $Z=1$, the unique Nash equilibrium guns profile is $G_1^{PU}(1)=G_2^{PU}(1)=G^{WO}(\gamma)=R^{WO}(\gamma)/4$, which implies $V_i^{PU}(C=1,Z=1)=\frac{(1-\delta)R^{WO}(\gamma)}{4}+\frac{\delta^2\gamma V^{PU}}{1-\delta+\delta\gamma}$. For $Z=0$, the unique Nash equilibrium guns profile is $G_1^{PU}(1)=G_2^{PU}(2)=G^N=\theta R/4$, which implies that $V_i^{PU}(C=1,Z=0)=\frac{(1-\delta)\theta R}{4}+\delta V^{PU}$. Therefore,

$$V^{PU}=V^{WO}(\gamma)=\frac{(1-\delta+\delta\gamma)\left[\phi R^{WO}(\gamma)+(1-\phi)\theta R\right]}{4\left[1-\delta(1-\phi-\gamma)\right]}$$

Finally, we must verify that $-X+\frac{\delta R}{1-\delta(1-\gamma)}\geq\frac{\delta V^{PU}}{1-\delta(1-\gamma)}$, which holds if and only if

$$X\leq\bar{X}(\gamma)=\frac{\left[(4-\theta)(1-\delta+\delta\gamma)+3\delta\phi\right]\delta R}{(1-\delta+\delta\gamma)\left[4(1-\delta+\delta\gamma)+3\delta\phi\right]}$$

Moreover, note that $-X+\frac{\delta R}{1-\delta+\delta\gamma}\geq\frac{\delta V^{PU}}{1-\delta(1-\gamma)}$ implies that $R^{WO}(\gamma)=\theta R-X+\frac{\delta R}{1-\delta+\delta\gamma}>0$, and hence, $G^{WO}(\gamma)=R^{WO}(\gamma)/4>0$.

Equilibrium without elimination: Assume that, in equilibrium, $-X+\frac{\delta R}{1-\delta(1-\gamma)}<\frac{\delta V^{PU}}{1-\delta(1-\gamma)}$. Then, we have:

$$V_i^{PU}(C=1,Z)=\max_{G_i(0)>0}\left\{(1-\delta)\left[\pi_i(Z)\theta R-G_i^{PU}(Z)\right]+\delta V^{PU}\right\}\text{ for }Z=0,1$$
$$V^{PU}=(1-\phi)V_i^{PU}(C=1,Z=0)+\phi V_i^{PU}(C=1,Z=1)$$

Solving, we have that the unique Nash equilibrium guns profile is $G_1^{PU}(Z) = G_2^{PU}(Z) = G^N = \theta R/4$ for $Z = 0, 1$, which implies,

$$V_i^{PU}(C = 1, Z) = V^{PU} = \frac{\theta R}{4}$$

Finally, we must verify that $-X + \frac{\delta R}{1-\delta(1-\gamma)} < \frac{\delta V^{PU}}{1-\delta(1-\gamma)}$, which holds if and only if

$$X > \frac{(4-\theta)\,\delta R}{4\,(1-\delta+\delta\gamma)}$$

Note that $\bar{X}(\gamma) = \frac{[(4-\theta)(1-\delta+\delta\gamma)+3\delta\phi]\delta R}{(1-\delta+\delta\gamma)[4(1-\delta+\delta\gamma)+3\delta\phi]} > \frac{(4-\theta)\delta R}{4(1-\delta+\delta\gamma)}$. Thus, we have three possible cases to consider:

Case 1: If $\bar{X}(\gamma) < \frac{(4-\theta)\delta R}{4(1-\delta+\delta\gamma)}$, then the only equilibrium is $G_1^{PU}(0) = G_2^{PU}(0) = \theta R/4$ for $Z = 0$ and $G_1^{PU}(1) = G_2^{PU}(1) = R^{WO}(\gamma)/4$ and $W_1 = W_2 = 1$ for $Z = 1$. Thus, the equilibrium expected payoff is $V^{PU} = V^{WO}(\gamma)$.

Case 2: If $X > \bar{X}(\gamma)$, then the only equilibrium is $G_1^{PU}(0) = G_2^{PU}(0) = \theta R/4$ for $Z = 0$ and $G_1^{PU}(1) = G_2^{PU}(1) = \theta R/4$ and $W_1 = W_2 = 0$ for $Z = 1$. Thus, the equilibrium discounted expected payoff is $V^{PU} = V^N = \frac{\theta R}{4}$.

Case 3: If $\frac{(4-\theta)\delta R}{4(1-\delta+\delta\gamma)} \leq X \leq \bar{X}(\gamma)$, then there are two possible equilibria, described in cases 1 and 2, respectively. $V^{WO}(\gamma) < V^N$ if and only if $X > \frac{\delta(1-\theta)R}{(1-\delta+\delta\gamma)}$. Moreover, note that $\frac{(4-\theta)\delta R}{4(1-\delta+\delta\gamma)} > \frac{\delta(1-\theta)R}{(1-\delta+\delta\gamma)}$, which implies that $V^{WO}(\gamma) < V^N$ for all $\frac{(4-\theta)\delta R}{4(1-\delta+\delta\gamma)} \leq X \leq \bar{X}(\gamma)$. Thus, whenever both equilibria exist, the equilibrium described in case 1 is more severe.

This completes the proof of Lemma 3. ■

Lemma 4 *Suppose that countries are following either the permanent cooperation strategy with* $G(0) \in \left[0, \frac{\theta R}{4}\right)$ *and* $G(1) \in \left[0, \frac{\theta R}{4}\right)$ *or the temporary cooperation strategy with* $G(0) \in \left[0, \frac{\theta R}{4}\right)$. *Assume that* $X \leq \bar{X}(\gamma)$.

1. *Assume that* $Z = 0$. *Then, the most profitable deviation is* $g^D(G(0), \theta R) = \sqrt{G(0)\theta R} - G(0)$. *The associated deviation payoff is given by:*

$$V^D(G(0)) = (1-\delta)\left[\frac{g^D(G(0), R)}{g^D(G(0), R) + G(0)}\theta R - g^D(G(0), R)\right] + \delta V^{WO}(\gamma)$$

2. *Assume that* $Z = 1$. *Then, the most profitable deviation is* $g^D\left(G(1), R^{WO}(\gamma)\right) = \sqrt{G(1)R^{WO}(\gamma)} - G(1)$, *which leads to open conflict and wipe-out. The associated deviation payoff is given by:*

$$V^D(G(1)) = (1-\delta)\left[\frac{g^D\left(G(1), R^{WO}(\gamma)\right)}{g^D(G(1), R^{WO}(\gamma)) + G(1)}R^{WO}(\gamma) - g^D\left(G(1), R^{WO}(\gamma)\right)\right] + \frac{\gamma\delta^2 V^{WO}(\gamma)}{1-\delta+\delta\gamma}$$

Proof: Suppose that countries are following either the permanent cooperation strategy with $G(0) \in \left[0, \frac{\theta R}{4}\right)$ and $G(1) \in \left[0, \frac{\theta R}{4}\right)$ or the temporary cooperation strategy with $G(0) \in \left[0, \frac{\theta R}{4}\right)$.

Suppose that $C = 1$ and $Z = 0$.

Deviation in the settlement stage: Consider a unilateral deviation to open conflict, i.e., $S_i = 0$. Then, the expected payoff of player i under this deviation is given by:

$$V_i\left(C=1,Z=0\right)=\left(1-\delta\right)\left[\frac{\theta R}{2}-G\left(0\right)\right]+\delta V^{PU}$$

where $V^{PU}=V^{WO}\left(\gamma\right)$. But if player i chooses $S_i = 1$, then it gets $V_i\left(0\right)=\left(1-\delta\right)\left[\frac{R}{2}-G\left(0\right)\right]+\delta V^{C}$, where $V^C \geq V^{PU}$ is the expected payoff under cooperation. Therefore, for $Z = 0$, players do not have an incentive to unilateraly deviate to open conflict.

Deviation in the gun choice: If i deviates, its expected payoff will be given by:

$$\arg\max_{G_i}\left\{V_i^D=\left(1-\delta\right)\left[\frac{G_i}{G_i+G\left(0\right)}\theta R-G_i\right]+\delta V^{WO}\left(\gamma\right)\right\}$$

The most profitable deviation for player i is $g^D\left(G\left(0\right),R\right)=\sqrt{G\left(0\right)\theta R}-G\left(0\right)$. Thus, the associated deviation payoff is given by:

$$V^D\left(G\left(0\right)\right)=\left(1-\delta\right)\left[\frac{g^D\left(G\left(0\right),\theta R\right)}{g^D\left(G\left(0\right),\theta R\right)+G\left(0\right)}\theta R-g^D\left(G\left(0\right),\theta R\right)\right]+\delta V^{WO}\left(\gamma\right)$$

Suppose that $Z = 1$.

Deviation in the settlement stage: Consider a deviation to open conflict, i.e., $S_i = 0$. Then, the expected payoff of player i under this deviation is given by:

$$\begin{aligned}
V_i^D\left(C=1,Z=1\right)&=\pi_i\left(1\right)\left[\left(1-\delta\right)\theta R+V_i\left(w\right)\right]+\left[1-\pi_i\left(1\right)\right]V_i\left(l\right)-\left(1-\delta\right)G_i\left(1\right)\\
V_i\left(w\right)&=\max_{W_i\in\{0,1\}}\left\{W_i\left[-\left(1-\delta\right)X+\delta V_i\left(C=0,w\right)\right]+\left(1-W_i\right)\delta V^{PU}\right\}\\
V_i\left(l\right)&=W_{-i}\delta V_i\left(C=0,l\right)+\left(1-W_{-i}\right)\delta V^{PU}\\
V_i\left(C=0,w\right)&=\frac{\left(1-\delta\right)R+\gamma\delta V^{PU}}{1-\delta+\delta\gamma}\\
V_i\left(C=0,l\right)&=\frac{\gamma\delta V^{PU}}{1-\delta+\delta\gamma}
\end{aligned}$$

where $\pi_i\left(1\right)=1/2$ and V^{PU} is the punishment payoff. Since $X \leq \bar{X}\left(\gamma\right)$, we have $V^{PU}=V^{WO}\left(\gamma\right)$. Therefore, $-\left(1-\delta\right)X+\delta V_i\left(C=0,w\right)\geq\delta V^{PU}$ if only if $-X+\frac{\delta R}{1-\delta+\delta\gamma}\geq\frac{\delta V^{WO}\left(\gamma\right)}{1-\delta+\delta\gamma}$, which always holds for $X\leq\bar{X}\left(\gamma\right)$. Thus, $W_i = W_{-i} = 1$ and, hence,

$$V_i^D\left(C=1,Z=1\right)=\left(1-\delta\right)\left[\pi_i\left(1\right)R^{WO}\left(\gamma\right)-G\left(1\right)\right]+\frac{\gamma\delta^2V^{WO}\left(\gamma\right)}{1-\delta+\delta\gamma}$$

Deviation in the gun choice: Since any deviation in guns choices will be immediately punished with open conflict and $X\leq\bar{X}\left(\gamma\right)$, the optimal deviation for player i is given by:

$$\arg\max_{G_i}\left\{V_i^D=\frac{G_i}{G_i+G\left(1\right)}R^{WO}\left(\gamma\right)-G_i+\frac{\gamma\delta^2V^{WO}\left(\gamma\right)}{1-\delta+\delta\gamma}\right\}$$

Solving, we obtain that the most profitable deviation for player i is $g^D\left(G(1), R^{WO}(\gamma)\right) = \sqrt{G(1)\, R^{WO}(\gamma)} - G(1)$. Thus, the associated deviation payoff is given by:

$$V^D(G(1)) = (1-\delta)\left[\frac{g^D\left(G(1), R^{WO}(\gamma)\right)}{g^D\left(G(1), R^{WO}(\gamma)\right) + G(1)} R^{WO}(\gamma) - g^D\left(G(1), R^{WO}(\gamma)\right)\right] + \frac{\gamma\delta^2 V^{WO}(\gamma)}{1-\delta+\delta\gamma}$$

This completes the proof of Lemma 4. ■

Proposition 7 *Suppose that $\gamma > 0$ and $\delta \to 1$. Then, permanent complete cooperation, i.e., disarmed peace with $G(Z) = 0$ for $Z = 0, 1$, can be sustained.*

Proof: In the proof of Proposition 8 we show that the permanent cooperation strategy with $G(0) \in \left[0, \frac{\theta R}{4}\right)$ and $G(1) \in \left[0, \frac{\theta R}{4}\right)$ can be sustained as an equilibrium if and only if

$$F^{PC,0}(G(0), G(1)) = \\ -(2-\delta-\delta\phi)\, G(0) + 2(1-\delta)\sqrt{G(0)\theta R} - \delta\phi G(1) + F^{PC,0}(0,0) \geq 0$$

and

$$F^{PC,1}(G(0), G(1)) = \\ -(2-2\delta+\delta\phi)\, G(1) + 2(1-\delta)\sqrt{G(1)\, R^{WO}(\gamma)} - \delta(1-\phi)\, G(0) + F^{PC,1}(0,0) \geq 0$$

where

$$F^{PC,0}(0,0) = \frac{R}{2} - (1-\delta)\,\theta R - \frac{\delta(1-\delta+\delta\gamma)\left[\phi R^{WO}(\gamma) + (1-\phi)\,\theta R\right]}{4\left[1-\delta(1-\phi-\gamma)\right]}$$

$$F^{PC,1}(0,0) = \frac{R}{2} - (1-\delta)\, R^{WO}(\gamma) - \frac{\gamma\delta^2\left[\phi R^{WO}(\gamma) + (1-\phi)\,\theta R\right]}{4\left[1-\delta(1-\phi-\gamma)\right]}$$

$$R^{WO}(\gamma) = \theta R - X + \frac{\delta R}{1-\delta+\delta\gamma}.$$

Take the limit of $F^{PC,0}(0,0)$ and $F^{PC,1}(0,0)$ when $\delta \to 1$:

$$\lim_{\delta\to 1} F^{PC,0}(0,0) = \frac{\left[\phi + (2-\theta)\,\gamma\right] R + \gamma\phi X}{4(\phi+\gamma)} > 0$$

$$\lim_{\delta\to 1} F^{PC,1}(0,0) = \frac{\left[\phi + (2-\theta)\,\gamma\right] R + \gamma\phi X}{4(\phi+\gamma)} > 0$$

Thus, when $\delta \to 1$, $F^{PC,0}(0,0) > 0$ and $F^{PC,1}(0,0) > 0$, which implies the permanent cooperation strategy with $G(0) = G(1) = 0$ can be sustained as an equilibrium. This completes the proof of Proposition 7. ■

Proposition 8 *Suppose that $\gamma \geq 0$ and $X \leq \bar{X}(\gamma) = \frac{[(4-\theta)(1-\delta+\delta\gamma)+3\delta\phi]\delta R}{(1-\delta+\delta\gamma)[4(1-\delta+\delta\gamma)+3\delta\phi]}$. There are thresholds $\bar{\theta}^{PC}_{low}$, $\bar{\theta}^{PC}_{high}$, $\bar{\theta}^{TC}$, $\bar{X}_{low}(\gamma)$, and $\bar{X}_{high}(\gamma)$ such that:*

1. *Suppose that* $\theta \leq \bar{\theta}^{PC}_{low}$ *or* $\bar{\theta}^{PC}_{low} < \theta \leq \bar{\theta}^{PC}_{high}$ *and* $X \geq \bar{X}_{high}(\gamma)$. *Then, permanent complete cooperation, i.e., disarmed peace with* $G(Z) = 0$ *for* $Z = 0, 1$, *can be sustained.*

2. *Suppose that* $\bar{\theta}^{PC}_{low} < \theta \leq \bar{\theta}^{PC}_{high}$ *and* $\bar{X}_{low}(\gamma) < X < \bar{X}_{high}(\gamma)$. *Let* $\bar{G}(0) = \left(\frac{1-\delta}{2-\delta-\phi\delta}\right)^2 \theta R < G^N$, $\bar{G}(1) = \left(\frac{1-\delta}{2-2\delta+\delta\phi}\right)^2 R^{WO}(\gamma) < G^{WO}(\gamma)$, *and* $\hat{G}(1) \in \left(0, \bar{G}(1)\right)$ *be the unique solution to* $F^{PC,1}(0, G(1)) = 0$.

   (a) *If* $\hat{G}(1) \leq F^{PC,0}(0,0)/\delta\phi$, *then the best possible permanent cooperative equilibrium that can be sustained is partial cooperation with* $G(0) = 0$ *and* $G(1) = \hat{G}(1)$.

   (b) *If* $\hat{G}(1) > F^{PC,0}(0,0)/\delta\phi$, *then the best possible permanent cooperative equilibrium that can be sustained is partial cooperation with* $G(0) \in \left(0, \bar{G}(0)\right)$ *and* $G(1) \in \left(\hat{G}(1), \bar{G}(1)\right)$ *given by the unique solution to*

$$F^{PC,0}(G(0), G(1)) = F^{PC,1}(G(0), G(1)) = 0$$

   that satisfies:

$$\frac{\partial F^{PC,0}(G(0), 0)}{\partial G(0)} \frac{\partial F^{PC,1}(0, G(1))}{\partial G(1)} > \delta^2 (1-\phi)\phi.$$

3. *If* $\theta \leq \bar{\theta}^{TC}$, *then the best temporary cooperative equilibrium that can be sustained is* $G(0) = 0$.

4. *If* $\theta > \bar{\theta}^{TC}$, *then the best temporary cooperative equilibrium that can be sustained is* $G(0) \in \left(0, \left\{\frac{(1-\delta)[1-\delta(1-\gamma-\phi)]}{(2-\delta+\delta\phi)(1-\delta+\delta\gamma)-2\delta^2\gamma\phi}\right\}^2 \theta R\right)$ *given by the unique solution to* $F^{TC}(G(0)) = 0$. *Moreover,* $\left\{\frac{(1-\delta)[1-\delta(1-\gamma-\phi)]}{(2-\delta+\delta\phi)(1-\delta+\delta\gamma)-2\delta^2\gamma\phi}\right\}^2 \theta R < G^N = \frac{\theta R}{4}$.

Proof of parts 1 and 2: Suppose that countries are following the permanent cooperation strategy with $G(0) \in \left[0, \frac{\theta R}{4}\right)$ and $G(1) \in \left[0, \frac{\theta R}{4}\right)$ and that no country deviates. Then, the expected discounted payoffs for each state $Z = 0, 1$ are given by:

$$V^{PC}(C=1, Z=0) = (1-\delta)\left[\frac{R}{2} - G(0)\right] + \delta\left[(1-\phi)V_i^{PC}(C=1, Z=0) + \phi V_i^{PC}(C=1, Z=1)\right]$$
$$V^{PC}(C=1, Z=1) = (1-\delta)\left[\frac{R}{2} - G(1)\right] + \delta\left[(1-\phi)V_i^{PC}(C=1, Z=0) + \phi V_i^{PC}(C=1, Z=1)\right]$$

Solving, we obtain:

$$V^{PC}(C=1, Z=0) = \frac{R}{2} - G(0) + \delta\phi\left[G(0) - G(1)\right]$$
$$V^{PC}(C=1, Z=1) = \frac{R}{2} - G(1) + \delta(1-\phi)\left[G(1) - G(0)\right]$$

Sustainability for $Z = 0$: Suppose that $Z = 0$ and player i deviates from cooperation. Then, from Lemma 4, the most profitable deviation for player i is $g^D(G(0), \theta R) = \sqrt{G(0)\theta R} - G(0)$ and the associated deviation payoff is given by:

$$V^D(G(0)) = (1-\delta)\left[\frac{g^D(G(0), \theta R)}{g^D(G(0), \theta R) + G(0)}\theta R - g^D(G(0), \theta R)\right] + \delta V^{WO}(\gamma)$$

Thus, player i does not have an incentive to deviate if and only if $V^{PC}(C = 1, Z = 0) \geq V^D(G(0))$ or, which is equivalent,

$$F^{PC,0}(G(0), G(1)) =$$
$$-(2 - \delta - \delta\phi) G(0) + 2(1-\delta)\sqrt{G(0)\theta R} - \delta\phi G(1) + F^{PC,0}(0,0) \geq 0$$

where

$$F^{PC,0}(0,0) = \frac{R}{2} - (1-\delta)\theta R - \frac{\delta(1-\delta+\delta\gamma)\left[\phi R^{WO}(\gamma) + (1-\phi)\theta R\right]}{4\left[1-\delta(1-\phi-\gamma)\right]}$$

Sustainability for $Z = 1$: Suppose that $Z = 1$ and player i deviates from cooperation. From Lemma 4, the most profitable deviation for player i is $g^D\left(G(1), R^{WO}(\gamma)\right) = \sqrt{G(1)R^{WO}(\gamma)} - G(1)$, which induces an open conflict, after which the victor eliminates the loser. The associated deviation payoff is given by:

$$V^D(G(1)) = (1-\delta)\left[\frac{g^D\left(G(1), R^{WO}(\gamma)\right)}{g^D\left(G(1), R^{WO}(\gamma)\right) + G(1)}R^{WO}(\gamma) - g^D\left(G(1), R^{WO}(\gamma)\right)\right] + \frac{\gamma\delta^2 V^{WO}(\gamma)}{1-\delta+\delta\gamma}$$

Thus, player i does not have an incentive to deviate if and only if $V^{PC}(C = 1, Z = 1) \geq V^D(G(1))$ or, which is equivalent,

$$F^{PC,1}(G(0), G(1)) =$$
$$-(2 - 2\delta + \delta\phi) G(1) + 2(1-\delta)\sqrt{G(1) R^{WO}(\gamma)} - \delta(1-\phi) G(0) + F^{PC,1}(0,0) \geq 0$$

where

$$F^{PC,1}(0,0) = \frac{R}{2} - (1-\delta) R^{WO}(\gamma) - \frac{\gamma\delta^2\left[\phi R^{WO}(\gamma) + (1-\phi)\theta R\right]}{4\left[1-\delta(1-\phi-\gamma)\right]}$$

Best possible permanent cooperation equilibrium: To determine the best possible permanent cooperative equilibrium, we solve

$$\max_{G(0)\geq 0, G(1)\geq 0}\left\{\begin{array}{c}\phi V^{PC}(C=1, Z=1) + (1-\phi) V^{PC}(C=1, Z=0) \\ = \frac{R}{2} - \phi G(1) - (1-\phi) G(0)\end{array}\right\}$$
$$s.t.: F^{PC,0}(G(0), G(1)) \geq 0 \; and \; F^{PC,1}(G(0), G(1)) \geq 0$$

Let $\bar{G}(0) = \left(\frac{1-\delta}{2-\delta-\delta\phi}\right)^2 \theta R$ and $\bar{G}(1) = \left(\frac{1-\delta}{2-2\delta+\delta\phi}\right)^2 R^{WO}(\gamma)$. Following the same precedure that we used in the proof of Proposition 5, we that there is no solution such that $G(0) > \bar{G}(0)$ or $\bar{G}(1) > \bar{G}(1)$, and that $F^{PC,0}$ and $F^{PC,1}$ are quasiconcave functions for all $(G(0), G(1))$.

Since the objective function is linear and the constraints are quasiconcave, the following Kuhn-Tucker conditions are sufficient for a global maximum.

$$-(1-\phi)+\lambda^0\left[-(2-\delta-\delta\phi)+(1-\delta)\sqrt{\frac{\theta R}{G(0)}}\right]-\lambda^1\delta(1-\phi)+\mu_L^0-\mu_H^0=0$$
$$-\phi-\lambda^0\delta\phi+\lambda^1\left[-(2-2\delta+\delta\phi)+(1-\delta)\sqrt{\frac{R^{WO}(\gamma)}{G(1)}}\right]+\mu_L^1-\mu_H^1=0$$
$$\lambda^0\geq 0,\ F^0(G(0),G(1))\geq 0,\ \lambda^0F^0(G(0),G(1))=0$$
$$\lambda^1\geq 0,\ F^1(G(0),G(1))\geq 0,\ \lambda^1F^1(G(0),G(1))=0$$
$$\mu_L^0\geq 0,\ G(0)\geq 0,\ \mu_L^0G(0)=0$$
$$\mu_H^0\geq 0,\ \bar{G}(0)-G(0)\geq 0,\ \mu_H^0\left[\bar{G}(0)-G(0)\right]=0$$
$$\mu_L^1\geq 0,\ G(1)\geq 0,\ \mu_L^1G(1)=0$$
$$\mu_H^1\geq 0,\ \bar{G}(1)-G(1)\geq 0,\ \mu_H^1\left[\bar{G}(1)-G(1)\right]=0$$

There are several cases to consider:

Case 1: Suppose that $G(0)=G(1)=0$. Solving, we obtain: $\lambda^0=\mu_H^0=0$, $\mu_L^0=1-\phi$, $\lambda^1=\mu_H^1=0$, $\mu_L^1=\phi$, $F^{PC,0}(0,0)\geq 0$, and $F^{PC,1}(0,0)\geq 0$. Since $F^{PC,1}(0,0)\geq 0$ implies $F^{PC,0}(0,0)\geq 0$, we must check that $F^{PC,1}(0,0)\geq 0$. $F^{PC,1}(0,0)\geq 0$ if and only if

$$X\geq\bar{X}_{hight}(\gamma)=\frac{(1-\delta+\rho)\theta R}{1-\delta+\phi\rho}+\frac{\delta R}{(1-\delta+\delta\gamma)}-\frac{R}{2(1-\delta+\phi\rho)}$$

where $\rho=\frac{\gamma\delta^2}{4[1-\delta(1-\phi-\gamma)]}$. If $\theta\leq\bar{\theta}_{low}^{PC}$, where

$$\bar{\theta}_{low}^{PC}=\frac{(1-\delta+\delta\gamma)-2\delta(1-\delta+\phi\rho)}{2(1-\delta+\rho)(1-\delta+\delta\gamma)}$$

this constraint always holds because $X>0$. If $\theta>\bar{\theta}_{low}^{PC}$ we need to check that $\bar{X}_{hight}(\gamma)\leq\bar{X}(\gamma)$, which holds if and only if $\theta\leq\bar{\theta}_{high}^{PC}$, where

$$\bar{\theta}_{high}^{PC}=\frac{4(1-\delta+\delta\gamma)+3\delta\phi}{2(1-\delta+\rho)\left[4(1-\delta+\delta\gamma)+3\delta\phi\right]+2\delta(1-\delta+\phi\rho)}$$

Summing up, $G(0)=G(1)=0$ is a solution if and only if $[\theta\leq\bar{\theta}_{low}^{PC}]$ or $[X\geq\bar{X}_{hight}(\gamma)$ and $\bar{\theta}_{low}^{PC}<\theta\leq\bar{\theta}_{high}^{PC}]$.

Case 2: Suppose that $G(0)\in\left(0,\bar{G}(0)\right)$ and $G(1)=0$. Solving we obtain: $\mu_L^0=\mu_H^0=\mu_H^1=0$, $\lambda^1=0$, $\lambda^0=(1-\phi)\left[-(2-\delta-\delta\phi)+(1-\delta)\sqrt{\frac{\theta R}{G(0)}}\right]^{-1}>0$, $\mu_L^1=\phi\left(1+\lambda^0\delta\right)>0$, $F^{PC,0}(G(0),0)=0$ and $F^{PC,1}(G(0),0)\geq 0$. Since $F^{PC,0}(G(0),0)$ is strictly increasing in $G(0)$ for all $G(0)\in\left[0,\bar{G}(0)\right]$, at most, there is one solution to $F^{PC,0}(G(0),0) = 0$. Moreover, there is a solution that satisfies $G(0) \in \left(0, \bar{G}(0)\right)$ if and only if $F^{PC,0}(0,0) < 0$ and $F^{PC,0}\left(\bar{G}(0),0\right) > 0$. Note, however, that $F^{PC,0}(0,0) < 0$ implies $F^{PC,1}(0,0) < 0$ (due to Result 3) and $F^{PC,1}(0,0) < 0$ implies $F^{PC,1}(G(0),0) < 0$ (because $F^{PC,1}(G(0),0)$ is strictly decreasing in $G(0)$). Thus, $F^{PC,0}(0,0) < 0$ is incompatible with $F^{PC,1}(G(0),0) \geq 0$. Summing up, there is no solution such that $G(0) \in \left(0, \bar{G}(0)\right)$ and $G(1) = 0$.

Case 3: Suppose that $G(0) = 0$ and $G(1) \in \left(0, \bar{G}(1)\right)$. Solving we obtain: $\mu_H^0 = \mu_L^1 = \mu_H^1 = 0$, $\lambda^0 = 0$, $\mu_L^0 = (1-\phi) + \lambda^1 \delta (1-\phi) > 0$, $\lambda^1 = \phi \left[-(2-2\delta+\delta\phi) + (1-\delta)\sqrt{\frac{R^{WO}(\gamma)}{G(1)}}\right]^{-1} > 0$, $F^{PC,1}(0,G(1)) = 0$ and $F^{PC,0}(0,G(1)) \geq 0$. Since $F^{PC,1}(0,G(1))$ is strictly increasing in $G(1)$ for all $G(1) \in \left[0, \bar{G}(1)\right]$, at most, there is one solution to $F^{PC,1}(0,G(1)) = 0$. Moreover, there is a solution that satisfies $G(1) \in \left(0, \bar{G}(1)\right)$ if and only if $F^{PC,1}(0,0) < 0$ and $F^{PC,1}\left(0,\bar{G}(1)\right) > 0$. $F^{PC,1}(0,0) < 0$ if and only if $X < \bar{X}_{hight}(\gamma)$; while $F^{PC,1}\left(0,\bar{G}(1)\right) > 0$ if and only if $X > \bar{X}_{low}(\gamma)$, where

$$\bar{X}_{low}(\gamma) = \left[\frac{(1-\delta+\delta\phi)(1-\delta+\rho)+\rho(1-\delta)}{(1-\delta+\delta\phi)(1-\delta+\rho\phi)+\rho\phi(1-\delta)}\right]\theta R + \\ + \frac{\delta R}{(1-\delta+\delta\gamma)} - \frac{(2-2\delta+\delta\phi)R}{2\left[(1-\delta+\delta\phi)(1-\delta+\rho\phi)+\rho\phi(1-\delta)\right]}$$

Therefore, we need $\bar{X}_{low}(\gamma) < X < \bar{X}_{hight}(\gamma)$. Thus, we must check that $\bar{X}_{hight} > 0$, which holds if and only if $\theta > \bar{\theta}_{low}^{PC}$. We must also check that $\bar{X}_{hight}(\gamma) \leq \bar{X}(\gamma)$, which holds if and only if $\theta \leq \bar{\theta}_{high}^{PC}$. Finally, we must verify that the unique solution to $F^{PC,1}(0,G(1)) = 0$ satisfies $F^{PC,0}(0,G(1)) \geq 0$ or, which is equivalent, that $G(1) \leq F^{PC,0}(0,0)/\delta\phi$.

Summing up, $G(0) = 0$ and $G(1) = \hat{G}(1) \in \left(0, \bar{G}(1)\right)$, where $\hat{G}(1)$ is the unique solution to $F^{PC,1}(0,G(1)) = 0$ is a solution if and only if $\bar{\theta}_{low}^{PC} < \theta \leq \bar{\theta}_{high}^{PC}$, $\bar{X}_{low}(\gamma) < X < \bar{X}_{hight}(\gamma)$, and $\hat{G}(1) \leq F^{PC,0}(0,0)/\delta\phi$.

Case 4: Suppose that $G(0) \in \left(0, \bar{G}(0)\right)$ and $G(1) \in \left(0, \bar{G}(1)\right)$. Solving, we obtain: $\mu_L^0 = \mu_L^1 = \mu_H^0 = \mu_H^1 = 0$, and

$$\lambda^0 \left[-(2-\delta-\delta\phi) + (1-\delta)\sqrt{\frac{\theta R}{G(0)}}\right] - \lambda^1 \delta (1-\phi) = (1-\phi)$$

$$-\lambda^0 \delta\phi + \lambda^1 \left[-(2-2\delta+\delta\phi) + (1-\delta)\sqrt{\frac{R^{WO}}{G(1)}}\right] = \phi$$

The above system of linear equations has a solution if and only if

$$\left[-(2-\delta-\delta\phi) + (1-\delta)\sqrt{\frac{\theta R}{G(0)}}\right]\left[-(2-2\delta+\delta\phi) + (1-\delta)\sqrt{\frac{R^{WO}}{G(1)}}\right] \neq \delta^2\phi(1-\phi)$$

A solution must satisfy $[\lambda^0 > 0$ and $\lambda^1 > 0]$ or $[\lambda^0 < 0$ and $\lambda^1 < 0]$. $\lambda^0 > 0$ and $\lambda^1 > 0$ if and only if

$$\left[-(2-\delta-\delta\phi) + (1-\delta)\sqrt{\frac{\theta R}{G(0)}}\right]\left[-(2-2\delta+\delta\phi) + (1-\delta)\sqrt{\frac{R^{WO}}{G(1)}}\right] > \delta^2\phi(1-\phi) \tag{2}$$

$\lambda^0 > 0$ and $\lambda^1 > 0$ implies that $F^{PC,0}(G(0), G(1)) = 0$, and $F^{PC,1}(G(0), G(1)) = 0$. Thus, we must solve the following system of equations:

$$F^{PC,0}(G(0), 0) - \delta\phi G(1) = 0$$
$$F^{PC,1}(0, G(1)) - \delta(1-\phi) G(0) = 0$$

We need $F^{PC,1}\left(0, \bar{G}(1)\right) > 0$ or, otherwise, $F^{PC,1}(0, G(1)) < 0$ for all $G(1) \in \left[0, \bar{G}(1)\right]$ and, hence, there is no solution to the above system of equations satisfying $G(0) > 0$. Also assume that $F^{PC,1}(0,0) < 0$. Otherwise, the solution is $G(0) = G(1) = 0$ (see case 1). Since $F^{PC,1}(0, G(1))$ is strictly increasing in $G(1)$ for all $G(1) \in \left[0, \bar{G}(1)\right]$, for a solution of the above system of equations to satisfy $G(0) > 0$ it must be the case that $G(1) > \hat{G}(1)$, where $\hat{G}(1)$ is the unique solution to $F^{PC,1}(0, G(1)) = 0$. Define

$$Q(G(1)) = F^{PC,0}\left(\frac{F^{PC,1}(0, G(1))}{\delta(1-\phi)}, 0\right) - \delta\phi G(1)$$

Since $F^{PC,1}(0, G(1))$ is strictly concave in $G(1)$ and $F^{PC,0}(G(0), 0)$ is strictly concave in $G(0)$, $Q(G(1))$ is strictly concave in $G(1)$. Moreover, note that

$$\lim_{G(1)\to\hat{G}(1)} \frac{\partial Q(G(1))}{\partial G(1)} = \lim_{G(1)\to\hat{G}(1)} \left[\frac{\partial F^{PC,0}\left(\frac{F^{PC,1}(0,G(1))}{\delta(1-\phi)}, 0\right)}{\partial G(0)} \frac{\partial F^{PC,1}(0, G(1))}{\partial G(1)} \frac{1}{\delta(1-\phi)} - \delta\phi\right]$$
$$= \left(\lim_{G(0)\to 0} \frac{\partial F^{PC,0}(0,0)}{\partial G(0)}\right) \frac{\partial F^{PC,1}\left(0, \hat{G}(1)\right)}{\partial G(1)} \frac{1}{\delta(1-\phi)} - \delta\phi$$

where $\frac{\partial F^{PC,1}\left(0,\hat{G}(1)\right)}{\partial G(1)} > 0$ and $\lim_{G(0)\to 0} \frac{\partial F^{PC,0}(0,0)}{\partial G(0)} = \infty$. Thus, for $G(1) \to \hat{G}(1)$, $Q(G(1))$ is strictly increasing, which implies that $Q(G(1))$ is either strictly increasing for all $G(1) \in \left[\hat{G}(1), \bar{G}(1)\right]$ or, it is strictly increasing for all $G(1) \in \left[\hat{G}(1), G^m\right]$, adopts a maximum at $G(1) = G^m$, and it is strictly decreasing for all $G(1) \in \left[G^m, \bar{G}(1)\right]$. Thus, there are 5 possible situations to consider:

Case 4.a: Suppose that $Q\left(\hat{G}(1)\right) > 0$ and $Q(G(1))$ is strictly increasing for all $G(1) \in \left[\hat{G}(1), \bar{G}(1)\right]$. Then, there is no solution to $Q(G(1)) = 0$.

Case 4.b: Suppose that $Q\left(\hat{G}(1)\right) > 0$ and $Q(G(1))$ is strictly increasing for all $G(1) \in \left[\hat{G}(1), G^m\right]$, adopts a maximum at $G(1) = G^m$, and it is strictly decreasing for all $G(1) \in \left[G^m, \bar{G}(1)\right]$. Then, there is at most one solution to $Q(G(1)) = 0$, which exists if and only if $Q\left(\bar{G}(1)\right) \leq 0$. Note, however, that for such a solution it must be the case that $\frac{\partial Q(G(1))}{\partial G(1)} \leq 0$, which holds if and only if $\frac{\partial F^{PC,0}\left(\frac{F^{PC,1}(0,G(1))}{\delta(1-\phi)},0\right)}{\partial G(0)} \frac{\partial F^{PC,1}(0,G(1))}{\partial G(1)} \leq \delta^2(1-\phi)\phi$, a violation of (2).

Case 4.c: Suppose that $Q\left(\hat{G}(1)\right) = 0$. Assume that $Q(G(1))$ is strictly increasing for all $G(1) \in \left[\hat{G}(1), \bar{G}(1)\right]$. Then, the unique solution to $Q(G(1)) = 0$ is $G(1) = \hat{G}(1)$, which implies $G(0) = 0$, a contradiction. Assume that $Q(G(1))$ is strictly increasing for all $G(1) \in \left[\hat{G}(1), G^m\right]$, adopts a maximum at $G(1) = G^m$, and it is strictly decreasing for all $G(1) \in \left[G^m, \bar{G}(1)\right]$. If $Q\left(\bar{G}(1)\right) \leq 0$, then there are two solutions to $Q(G(1)) = 0$. One solution is $G(1) = \hat{G}(1)$, which implies, $G(0) = 0$, a contradiction. For the other solution, it must be the case that $\frac{\partial Q(G(1))}{\partial G(1)} \leq 0$, which violates (2). If $Q\left(\bar{G}(1)\right) > 0$, then the unique solution to $Q(G(1)) = 0$ is $G(1) = \hat{G}(1)$, which implies $G(0) = 0$, a contradiction.

Case 4.d: Suppose that $Q\left(\hat{G}(1)\right) < 0$ and $Q(G(1))$ is strictly increasing for all $G(1) \in \left[\hat{G}(1), \bar{G}(1)\right]$. Then, there is at most one solution to $Q(G(1)) = 0$, which exists if and only if $Q\left(\bar{G}(1)\right) > 0$. Moreover, for this solution it must be the case that $\frac{\partial Q(G(1))}{\partial G(1)} > 0$.

Case 4.e: Suppose that $Q\left(\hat{G}(1)\right) < 0$ and $Q(G(1))$ is strictly increasing for all $G(1) \in \left[\hat{G}(1), G^m\right]$, adopts a maximum at $G(1) = G^m$, and it is strictly decreasing for all $G(1) \in \left[G^m, \bar{G}(1)\right]$. If $Q(G^m) < 0$, then there is no solution to $Q(G(1)) = 0$. If $Q(G^m) = 0$. Then, the unique solution to $Q(G(1)) = 0$ is $G(1) = G^m$. If $Q(G^m) > 0$ and $Q\left(\bar{G}(1)\right) < 0$, then there are two solutions to $Q(G(1)) = 0$ one with $G(1) < G^m$ and another with $G(1) > G^m$. Note, however, that for the solution with $G(1) > G^m$ we have $\frac{\partial Q(G(1))}{\partial G(1)} < 0$, which violates (2). Thus, only the solution with $G(1) < G^m$ satisfies the Kuhn-Tucker conditions. Finally, if $Q(G^m) > 0$ and $Q\left(\bar{G}(1)\right) > 0$, there is a unique solution to $Q(G(1)) = 0$ for which $\frac{\partial Q(G(1))}{\partial G(1)} > 0$.

Summing up, $G(0) \in \left(0, \bar{G}(0)\right)$ and $G(1) \in \left(0, \bar{G}(1)\right)$ given by the unique solution to $F^{PC,0}(G(0), G(1)) = 0$, and $F^{PC,0}(G(0), G(1)) = 0$ that satisfies $\frac{\partial F^0(G(0),0)}{\partial G(0)} \frac{\partial F^1(0,G(1))}{\partial G(1)} > \delta^2(1-\phi)\phi$ is a solution if and only if $F^{PC,1}(0,0) < 0$, $F^{PC,1}\left(0, \bar{G}(1)\right) > 0$, and $Q\left(\hat{G}(1)\right) < 0$, where $\hat{G}(1) \in \left(0, \bar{G}(1)\right)$ is the unique solution to $F^{PC,1}(G(1), 0) = 0$. In case 3 we have already proved that $F^{PC,1}(0,0) < 0$ and $F^{PC,1}\left(0, \bar{G}(1)\right) > 0$ if and only if $\bar{\theta}^{PC}_{low} < \theta \leq \bar{\theta}^{PC}_{high}$, $\bar{X}_{low}(\gamma) < X < \bar{X}_{hight}(\gamma)$. Finally, $Q\left(\hat{G}(1)\right) = F^{PC,0}\left(\frac{F^{PC,1}\left(0,\tilde{G}(1)\right)}{\delta(1-\phi)}, 0\right) - \delta\phi\hat{G}(1) = F^{PC,0}(0,0) - \delta\phi\hat{G}(1)$. Thus, $Q\left(\hat{G}(1)\right) < 0$ if and only if $\hat{G}(1) > F^{PC,0}(0,0)/\delta\phi$.

Case 5: Suppose that $G(0) = \bar{G}(0)$ or $G(1) = \bar{G}(1)$. If $G(0) = \bar{G}(0)$, we have $\mu^0_L = 0$ and, hence, $(1-\phi) + \lambda^1\delta(1-\phi) + \mu^0_H = 0$, a contradiction because $\lambda^1 \geq 0$ and $\mu^0_H \geq 0$. If $G(1) = \bar{G}(1)$, we have $\mu^1_L = 0$ and, hence $\phi + \lambda^0\delta\phi + \mu^1_H = 0$, a contradiction because $\lambda^0 \geq 0$ and $\mu^1_H \geq 0$.

Proof of parts 3 and 4: Suppose that countries are following the temporary cooperation strategy with $G(0) \in \left[0, \frac{\theta R}{4}\right)$ and that no country deviates. Then, the expected discounted payoffs for each state $Z = 0, 1$ are given by:

$$V^{TC}(C=1, Z=0) = (1-\delta)\left(\frac{R}{2} - G(0)\right) + \delta V^{TC}(C=1)$$

$$V^{TC}(C=1) = (1-\phi)V^{TC}(C=1, Z=0) + \phi V^{TC}(C=1, Z=1)$$

$$V^{TC}(C=1, Z=1) = (1-\delta)\frac{R^{WO}(\gamma)}{4} + \frac{\delta^2\gamma V^{TC}(C=1)}{1-\delta(1-\gamma)}$$

Solving we obtain:

$$V^{TC}(C=1,Z=0)=\frac{(1-\delta)\left[1-\frac{\phi\delta^2\gamma}{1-\delta(1-\gamma)}\right]\left(\frac{R}{2}-G(0)\right)}{\left[1-\frac{\phi\delta^2\gamma}{1-\delta(1-\gamma)}-\delta(1-\phi)\right]}+\frac{(1-\delta)\delta\phi\frac{R^{WO}(\gamma)}{4}}{\left[1-\frac{\phi\delta^2\gamma}{1-\delta(1-\gamma)}-\delta(1-\phi)\right]}$$

$$V^{TC}(C=1)=\frac{(1-\phi)V^{TC}(C=1,Z=0)+(1-\delta)\phi\frac{R^{WO}(\gamma)}{4}}{1-\frac{\phi\delta^2\gamma}{1-\delta(1-\gamma)}}$$

Sustainability for $Z=0$: Suppose that $Z=0$ and player i deviates from cooperation. From Lemma 4, the most profitable deviation for player i is $g^D(G(0),\theta R)=\sqrt{G(0)\theta R}-G(0)$ and the associated payoff is given by:

$$V^D(G(0))=(1-\delta)\left[\frac{g^D(G(0),\theta R)}{g^D(G(0),\theta R)+G(0)}\theta R-g^D(G(0),\theta R)\right]+\delta V^{WO}(\gamma)$$

Thus, player i does not have an incentive to deviate if and only if $V^{TC}(C=1,Z=0)\geq V^D(G(0))$ or, which is equivalent,

$$\begin{gathered}F^{TC}(G(0))=\\-\left[\frac{(2-\delta)(1-\delta+\delta\gamma)+(1-\delta-\delta\gamma)\delta\phi}{(1-\delta+\delta\gamma)}\right]G(0)\\+\frac{2(1-\delta)[1-\delta(1-\gamma-\phi)]}{(1-\delta+\delta\gamma)}\sqrt{G(0)\theta R}+F^{TC}(0)\end{gathered}$$

where

$$\begin{aligned}F^{TC}(0)=&\left[\frac{1-\delta(1-\gamma)-\phi\delta^2\gamma}{(1-\delta+\delta\gamma)}\right]\frac{R}{2}\\&-\left[\frac{(1-\delta+\delta\gamma)\delta(1-\phi)+4(1-\delta)[1-\delta(1-\phi-\gamma)]}{4(1-\delta+\delta\gamma)}\right]\theta R\end{aligned}$$

Best possible temporary cooperation equilibrium: To determine the best possible temporary cooperative equilibrium, we solve

$$\max_{G(0)\geq 0}\left\{\begin{array}{c}(1-\phi)V^{TC}(C=1,Z=0)+\phi V^{TC}(C=1,Z=1)=\\\frac{(1-\delta)(1-\delta+\delta\gamma)}{1-\delta(1-\gamma-\phi)}\left[(1-\phi)\left(\frac{R}{2}-G(0)\right)+\frac{\phi R^{WO}(\gamma)}{4}\right]\end{array}\right\}$$
$$s.t.\text{: } F^{TC}(G(0))\geq 0$$

There are two cases to consider:

Case 1: Suppose that $\theta\leq\bar{\theta}^{TC}=\frac{2\left[1-\delta(1-\gamma)-\phi\delta^2\gamma\right]}{(1-\delta+\delta\gamma)\delta(1-\phi)+4(1-\delta)[1-\delta(1-\phi-\gamma)]}$. Then, $F^{TC}(0)\geq 0$ and, hence, $G(0)=0$ is the solution to the above maximization problem.

Case 2: Suppose that $\theta>\bar{\theta}^{TC}$. Then, $F^{TC}(0)<0$. Moreover,

$$\frac{\partial F^{TC}(G(0))}{\partial G(0)} = -\left[\frac{(2-\delta)(1-\delta+\delta\gamma)+(1-\delta-\delta\gamma)\delta\phi}{(1-\delta+\delta\gamma)}\right] + \frac{(1-\delta)[1-\delta(1-\gamma-\phi)]}{(1-\delta+\delta\gamma)}\sqrt{\frac{\theta R}{G(0)}}$$

Thus, $F^{TC}(G(0))$ is strictly increasing in $G(0)$ for all $G(0) \in \left[0, \tilde{G}(0)\right]$ and strictly decreasing in G for all $G \in \left[\tilde{G}(0), \frac{\theta R}{4}\right]$, where

$$\tilde{G}(0) = \left\{\frac{(1-\delta)[1-\delta(1-\gamma-\phi)]}{[(2-\delta)(1-\delta+\delta\gamma)+(1-\delta-\delta\gamma)\delta\phi]}\right\}^2 \theta R$$

Moreover, $\tilde{G}(0) \leq \frac{\theta R}{4}$, with strict inequality for $\delta < 1$. Finally, note that

$$F^{TC}\left(\tilde{G}(0)\right) > 0$$

Thus, the solution to the above maximization problem is $G(0) \in \left(0, \tilde{G}(0)\right)$ given by the unique solution to $F^{TC}(G(0)) = 0$.

This completes the proof of Proposition 8. ■

Corollary 2 *Suppose that $\gamma > 0$. Open conflict is unavoidable if and only if*

$$X \leq \begin{cases} \bar{X}(\gamma) & \text{if } \theta > \bar{\theta}^{PC}_{high} \\ \bar{X}_{low}(\gamma) & \text{if } \bar{\theta}^{PC}_{low} < \theta \leq \bar{\theta}^{PC}_{high} \end{cases}$$

Moreover:

1. *The lower the cost of the elimination technology (X lower), the less likely open conflict can be avoided.*

2. *The destructiveness of open conflict has a non-monotonic effect on the likelihood of open conflict. Initially, as destructiveness declines (higher θ), open conflict becomes more likely, but eventually further reductions in destructiveness make open conflict easier to avoid.*

3. *If $\theta > \bar{\theta}^{PC}_{high}$, an increase in γ makes open conflict less likely.*

Proof: From Proposition 8, open conflict cannot be avoided if and only if $[\theta > \bar{\theta}^{PC}_{high}$ and $X \leq \bar{X}(\gamma)]$ or $[\bar{\theta}^{PC}_{low} < \theta \leq \bar{\theta}^{PC}_{high}$ and $X \leq \bar{X}_{low}(\gamma)]$.

Clearly, the lower the cost of the elimination technology (the lower X), the more likely that both of these conditions hold.

Note that $\bar{X}_{low}(\gamma)$ is strictly increasing in θ, while $\bar{X}(\gamma)$ is strictly decreasing in θ. Moreover, $\bar{X}_{low}(\gamma) < \bar{X}(\gamma)$ for $\theta \leq \bar{\theta}^{PC}_{high}$. Thus, an increase in θ, makes open conflict becomes more likely if $\theta < \bar{\theta}^{PC}_{high}$ and less likely if $\theta \geq \bar{\theta}^{PC}_{high}$.

Finally, suppose that $\theta > \bar{\theta}^{PC}_{high}$. Note that $\bar{X}(\gamma)$ is strictly decreasing in γ. Thus, for $\theta > \bar{\theta}^{PC}_{high}$, an increase in γ makes open conflict less likely.

This completes the proof of Corollary 2. ■

Proposition 9 *Suppose that $\bar{\theta}^{PC}_{low} < \theta \leq \bar{\theta}^{PC}_{high}$ and $\bar{X}_{low}(\gamma) < X < \bar{X}_{high}(\gamma)$. Then, partial permanent cooperation induces higher ex-ante expected payoff than temporary cooperation if and only if*

$$\Delta = \begin{bmatrix} F^{PC,0}(0,0) - \delta\phi G^{PC}(1) - \delta(1-\phi) G^{PC}(0) \\ -\frac{(1-\delta+\delta\gamma)}{1-\delta(1-\gamma-\phi)} \left[F^{TC}(0) - \delta(1-\phi) G(0)\right] \end{bmatrix} \geq 0$$

where

$$F^{PC,0}(0,0) = \frac{R}{2} - (1-\delta)\theta R - \frac{\delta(1-\delta+\delta\gamma)\left[\phi R^{WO}(\gamma) + (1-\phi)\theta R\right]}{4\left[1-\delta(1-\phi-\gamma)\right]}$$

$$F^{TC}(0) = \left[\frac{1-\delta(1-\gamma)-\phi\delta^2\gamma}{(1-\delta+\delta\gamma)}\right]\frac{R}{2} - \left[\frac{(1-\delta+\delta\gamma)\delta(1-\phi)+4(1-\delta)\left[1-\delta(1-\phi-\gamma)\right]}{4(1-\delta+\delta\gamma)}\right]\theta R$$

$G^{PC}(0)$ and $G^{PC}(1)$ are given by Proposition 8.2 and $G^{T}(0)$ by Propositions 8.3 and 8.4. Moreover, if $\theta > \bar{\theta}^{TC}$ and $\hat{G}(1) \leq F^{PC,0}(0,0)/\delta\phi$, $\Delta \geq 0$, while if $\theta \leq \bar{\theta}^{TC}$ and $\hat{G}(1) > F^{PC,0}(0,0)/\delta\phi$, $\Delta < 0$.

Proof: The ex-ante expected payoff under partial permanent cooperation is given by:

$$V^{PC} = \frac{R}{2} - \phi G^{PC}(1) - (1-\phi) G^{PC}(0)$$

where $G^{PC}(0)$ and $G^{PC}(1)$ are given by Proposition 8.2.

The ex-ante expected payoff under temporary cooperation is given by:

$$V^{TC} = \frac{(1-\delta)\left[(1-\phi)\left(\frac{R}{2} - G^{T}(0)\right) + \frac{\phi R^{WO}(\gamma)}{4}\right]}{1 - \frac{\phi\delta^2\gamma}{1-\delta(1-\gamma)} - \delta(1-\phi)}$$
$$= \frac{(1-\delta+\delta\gamma)}{\left[1-\delta(1-\gamma-\phi)\right]}\left[(1-\phi)\left(\frac{R}{2} - G^{TC}(0)\right) + \frac{\phi R^{WO}(\gamma)}{4}\right]$$

where $G^{TC}(0)$ is given by Proposition 8.3 or Proposition 8.4.

$V^{PC} \geq V^{TC}$ if and only if

$$F^{PC,0}(0,0) - \delta\phi G^{PC}(1) - \delta(1-\phi) G^{PC}(0) \geq \frac{(1-\delta+\delta\gamma)\left[F^{TC}(0) - \delta(1-\phi) G^{TC}(0)\right]}{1-\delta(1-\gamma-\phi)}$$

where

$$F^{PC,0}(0,0) = \frac{R}{2} - (1-\delta)\theta R - \frac{\delta(1-\delta+\delta\gamma)\left[\phi R^{WO}(\gamma) + (1-\phi)\theta R\right]}{4\left[1-\delta(1-\phi-\gamma)\right]}$$

and

$$F^{TC}(0) = \left[\frac{1-\delta(1-\gamma)-\phi\delta^2\gamma}{(1-\delta+\delta\gamma)}\right]\frac{R}{2} - \left[\frac{(1-\delta+\delta\gamma)\delta(1-\phi)+4(1-\delta)[1-\delta(1-\phi-\gamma)]}{4(1-\delta+\delta\gamma)}\right]\theta R$$

Note that $\bar{\theta}^{PC}_{low} < \bar{\theta}^{TC} < \bar{\theta}^{PC}_{high}$. So, there are several cases to consider:

Case 1: Suppose that $\bar{\theta}^{PC}_{low} < \theta \leq \bar{\theta}^{TC}$, $\bar{X}_{low}(\gamma) < X < \bar{X}_{high}(\gamma)$, and $\hat{G}(1) \leq F^{PC,0}(0,0)/\delta\phi$, where $\hat{G}(1) \in \left(0, \bar{G}(1)\right)$ is the unique solution to $F^{PC,1}(G(1),0) = 0$. Then, from Proposition 8.2.a, the best possible permanent cooperative equilibria that can be sustained is partial cooperation with $G^{PC}(0) = 0$ and $G^{PC}(1) = \hat{G}(1)$. From Proposition 8.3, the best temporary cooperative equilibria that can be sustained is $G^{TC}(0) = 0$. Therefore, $V^{PC} \geq V^{TC}$ if and only if

$$F^{PC,0}(0,0) - \delta\phi\hat{G}(1) \geq \frac{(1-\delta+\delta\gamma)F^{TC}(0)}{1-\delta(1-\gamma-\phi)}$$

Moreover, note that $\theta \leq \bar{\theta}^{TC}$ implies $F^{TC}(0) > 0$.

Case 2: Suppose that $\bar{\theta}^{TC} < \theta \leq \bar{\theta}^{PC}_{high}$, $\bar{X}_{low}(\gamma) < X < \bar{X}_{high}(\gamma)$, and $\hat{G}(1) \leq F^{PC,0}(0,0)/\delta\phi$, where $\hat{G}(1) \in \left(0, \bar{G}(1)\right)$ is the unique solution to $F^{PC,1}(G(1),0) = 0$. Then, from Proposition 8.2.a, the best possible permanent cooperative equilibria that can be sustained is partial cooperation with $G^{PC}(0) = 0$ and $G^{PC}(1) = \hat{G}(1)$. From Proposition 8.4, the best temporary cooperative equilibria that can be sustained is $G^{TC}(0) \in (0, \theta R)$ given by the unique solution to $F^{TC}(G(0)) = 0$. Therefore, $V^{PC} \geq V^{TC}$ if and only if

$$F^{PC,0}(0,0) - \delta\phi\hat{G}(1) \geq \frac{(1-\delta+\delta\gamma)\left[F^{TC}(0) - \delta(1-\phi)G^{TC}(0)\right]}{1-\delta(1-\gamma-\phi)}$$

Since $\hat{G}(1) \leq F^{PC,0}(0,0)/\delta\phi$, and $\theta > \bar{\theta}^{TC}$ implies $F^{TC}(0) < 0$, the above inequality always holds.

Case 3: Suppose that $\bar{\theta}^{PC}_{low} < \theta \leq \bar{\theta}^{TC}$, $\bar{X}_{low}(\gamma) < X < \bar{X}_{high}(\gamma)$, and $\hat{G}(1) > F^{PC,0}(0,0)/\delta\phi$, where $\hat{G}(1) \in \left(0, \bar{G}(1)\right)$ is the unique solution to $F^{PC,1}(G(1),0) = 0$. Then, from Proposition 8.2.b, the best possible permanent cooperative equilibria that can be sustained is partial cooperation with $G^{PC}(0) > 0$ and $G^{PC}(1) > \hat{G}(1)$ given by the unique solution to $F^{PC,0}(G(0), G(1)) = F^{PC,1}(G(0), G(1)) = 0$ that satisfies $\frac{\partial F^{PC,0}(G(0),0)}{\partial G(0)} \frac{\partial F^{PC,1}(0,G(1))}{\partial G(1)} > \delta^2(1-\phi)\phi$. From Proposition 8.3, the best temporary cooperative equilibria that can be sustained is $G^{TC}(0) = 0$. Therefore, $V^{PC} \geq V^{TC}$ if and only if

$$F^{PC,0}(0,0) - \delta\phi G^{PC}(1) - \delta(1-\phi)G^{PC}(0) \geq \frac{(1-\delta+\delta\gamma)F^{TC}(0)}{1-\delta(1-\gamma-\phi)}$$

Since $G^{PC}(1) > \hat{G}(1) > F^{PC,0}(0,0)/\delta\phi$, and $\theta \leq \bar{\theta}^{TC}$ implies $F^{TC}(0) \geq 0$, the above inequality never holds.

Case 4: Suppose that $\bar{\theta}^{TC} < \theta \leq \bar{\theta}^{PC}_{high}$, $\bar{X}_{low}(\gamma) < X < \bar{X}_{high}(\gamma)$, and $\hat{G}(1) > F^{PC,0}(0,0)/\delta\phi$, where $\hat{G}(1) \in \left(0, \bar{G}(1)\right)$ is the unique solution to $F^{PC,1}(G(1),0) = 0$. Then, from Proposition 8.2.b, the best possible permanent cooperative equilibria that can be sustained is partial cooperation with $G^{PC}(0) >$

0 and $G^{PC}(1) > \hat{G}(1)$ given by the unique solution to $F^{PC,0}(G(0), G(1)) = F^{PC,1}(G(0), G(1)) = 0$ that satisfies $\frac{\partial F^{PC,0}(G(0),0)}{\partial G(0)} \frac{\partial F^{PC,1}(0,G(1))}{\partial G(1)} > \delta^2(1-\phi)\phi$. From Proposition 8.4, the best temporary cooperative equilibria that can be sustained is $G^T(0) \in (0, \theta R)$ given by the unique solution to $F^{TC}(G(0)) = 0$. Therefore, $V^{PC} \geq V^{TC}$ if and only if

$$F^{PC,0}(0,0) - \delta\phi G^{PC}(1) - \delta(1-\phi) G^{PC}(0) \geq \frac{(1-\delta+\delta\gamma)\left[F^{TC}(0) - \delta(1-\phi) G^{TC}(0)\right]}{1-\delta(1-\gamma-\phi)}$$

Moreover, $\theta > \bar{\theta}^{TC}$ implies $F^{TC}(0) < 0$.

This completes the proof of Proposition 9. ■

A.5 Stochastic Resource Values

This section presents the proofs of Propositions 10 and 11.

Proposition 10 *Assume that* $\phi = 0$.

1. *Suppose that* $\theta \leq \bar{\theta}(H) = \frac{2(1-\delta)H + 2\delta[qH+(1-q)L]}{4(1-\delta)H + \delta[qH+(1-q)L]}$. *Then, permanent complete cooperation, i.e., disarmed peace with* $G(R) = 0$ *for* $R = H, L$, *is an equilibrium of the stochastic game. The associated first-best expected payoff is* $V^P = \mathbf{E}(R)/2$.

2. *Suppose that* $\theta > \bar{\theta}(H)$. *Let* $\bar{G}(L) = \left(\frac{1-\delta}{2-\delta-\delta q}\right)^2 \theta L < G^N(L)$, $\bar{G}(H) = \left(\frac{1-\delta}{2-2\delta+\delta q}\right)^2 \theta H < G^N(H)$ *and* $\hat{G}(H) \in (0, \bar{G}(H))$ *be the unique solution to* $F^H(0, G(H)) = 0$.

   (a) *If* $\hat{G}(H) \leq F^L(0,0)/\delta q$, *then the best possible cooperative equilibrium that can be sustained is partial cooperation with* $G(L) = 0$ *and* $G(H) = \hat{G}(H)$.

   (b) *If* $\hat{G}(H) > F^L(0,0)/\delta q$, *then the best possible cooperative equilibrium that can be sustained is partial cooperation with* $G(L) \in (0, \bar{G}(L))$ *and* $G(H) \in \left(\hat{G}(H), \bar{G}(H)\right)$ *given by the unique solution to:*

$$F^L(G(L), G(H)) = F^H(G(L), G(H)) = 0$$

   that satisfies:

$$\frac{\partial F^L(G(L), 0)}{\partial G(L)} \frac{\partial F^H(0, G(H))}{\partial G(H)} > \delta^2 q(1-q).$$

Proof: Suppose that countries are following the cooperation strategy with $G(L) \in \left[0, \frac{\theta L}{4}\right)$ and $G(H) \in \left[0, \frac{\theta H}{4}\right)$ and that no country deviates. Then, the expected payoffs are given by:

$$V^C(R) = (1-\delta)\left(\frac{R}{2} - G(R)\right) + \delta\left[qV^{PC}(H) + (1-q)V^{PC}(L)\right]$$

for $R = H, L$. Solving for $V^C(H)$ and $V^C(L)$ we have:

$$V^C(H) = (1-\delta)\frac{H}{2} + \frac{\delta \mathbf{E}(R)}{2} - (1-\delta+\delta q)G(H) - \delta(1-q)G(L)$$
$$V^C(L) = (1-\delta)\frac{L}{2} + \frac{\delta \mathbf{E}(R)}{2} - (1+\delta q)G(L) - \delta q G(H)$$

where $\mathbf{E}(R) = qH + (1-q)L$.

Sustainability: Suppose that the resource is $R = H, L$ and player i deviates from cooperation. To determine the most profitable deviation for player i we must solve:

$$\max_{G_i}\left\{V_i^D = (1-\delta)(\pi_i(G_i, G(R))\theta R - G_i) + \delta V^N\right\}$$

where $\pi_i(G_i, G(R)) = \frac{G_i}{G_i + G(R)}$ and $V^N = \theta \mathbf{E}(R)/4$. Thus, player i's most profitable deviation is

$$g^D(G(0), \theta R) = \sqrt{G(0)\theta R} - G(0)$$

and the associated deviation payoff is given by:

$$V^D(G(R)) = (1-\delta)\left(\theta R - 2\sqrt{G(R)\theta R} + G(R)\right) + \frac{\delta\theta \mathbf{E}(R)}{4}$$

Player i does not have an incentive to deviate if and only if $V^C(R) \geq V^D(G(R))$ or, which is equivalent, $F^R(G(L), G(H)) \geq 0$ for $R = H, L$, where

$$F^H(G(L), G(H)) = -(2-2\delta+\delta q)G(H) + 2(1-\delta)\sqrt{G(H)\theta H} - \delta(1-q)G(L) + F^{PC,H}(0,0)$$
$$F^L(G(L), G(H)) = -(2-\delta+\delta q)G(L) + 2(1-\delta)\sqrt{G(L)\theta L} - \delta q G(H) + F^{PC,L}(0,0)$$
$$F^R(0,0) = \frac{(1-\delta)R(1-2\theta)}{2} + \frac{\delta(2-\theta)\mathbf{E}(R)}{4} \textit{ for } R = H, L$$

Best possible permanent cooperation equilibrium: To determine the best possible permanent cooperative equilibrium, we solve

$$\max_{G(H),G(L)}\left\{\begin{array}{l} V^C = qV^C(H) + (1-q)V^C(L) \\ = \frac{\mathbf{E}(R)}{2} - qG(H) - (1-q)G(L) \end{array}\right\}$$
$$s.t.\text{: } F^H(G(L), G(H)) \geq 0 \text{ and } F^L(G(L), G(H)) \geq 0$$

Using the same procedure that we used in the proof of Proposition 5, we have the following results:

Result 1: If a solution exists, it must satisfy $G(L) \in \left[0, \bar{G}(L)\right]$ and $G(H) \in \left[0, \bar{G}(H)\right]$, where $\bar{G}(L) = \left(\frac{1-\delta}{2-\delta-\delta q}\right)^2 \theta L < \frac{\theta L}{4}$ and $\bar{G}(H) = \left(\frac{1-\delta}{2-2\delta+\delta q}\right)^2 \theta H < \frac{\theta H}{4}$.

Result 2: $F^H(G(L), G(H))$ and $F^L(G(L), G(H))$ are quasiconcave functions for all $(G(L), G(H)) \in \Re_+^2$.

Result 3: If $F^H(0,0) \geq 0$, then $F^L(0,0) \geq 0$. **Proof**: $F^H(0,0) \geq 0$ if and only if $\theta \leq \bar{\theta}(H) = \frac{2(1-\delta)H + 2\delta \mathbf{E}(R)}{4(1-\delta)H + \delta \mathbf{E}(R)}$, while $F^L(0,0) \geq 0$ if and only if $\theta \leq \bar{\theta}(L) = \frac{2(1-\delta)L + 2\delta \mathbf{E}(R)}{4(1-\delta)L + \delta \mathbf{E}(R)}$. Moreover, $\bar{\theta}(H) \leq \bar{\theta}(L)$ with strict inequality if $\delta \in (0,1)$.

Since the objective function is linear and the constraints are quasiconcave, the following Kuhn-Tucker conditions are sufficient for a global maximum.

$$-q+\lambda^{H}\left[-\left(2-\delta-\delta q\right)+\left(1-\delta\right)\sqrt{\frac{\theta H}{G\left(H\right)}}\right]-\lambda^{L}\delta\left(1-q\right)+\mu_{m}^{H}-\mu_{M}^{H}=0$$

$$-\left(1-q\right)-\lambda^{H}\delta q+\lambda^{L}\left[-\left(2-2\delta+\delta\phi\right)+\left(1-\delta\right)\sqrt{\frac{\theta L}{G\left(L\right)}}\right]+\mu_{m}^{L}-\mu_{M}^{L}=0$$

$$\lambda^{H}\geq 0,\ F^{H}\left(G\left(L\right),G\left(H\right)\right)\geq 0,\ \lambda^{H}F^{H}\left(G\left(L\right),G\left(H\right)\right)=0$$

$$\lambda^{L}\geq 0,\ F^{L}\left(G\left(L\right),G\left(H\right)\right)\geq 0,\ \lambda^{L}F^{L}\left(G\left(L\right),G\left(H\right)\right)=0$$

$$\mu_{m}^{H}\geq 0,\ G\left(H\right)\geq 0,\ \mu_{m}^{H}G\left(H\right)=0$$

$$\mu_{M}^{H}\geq 0,\ \bar{G}\left(H\right)-G\left(H\right)\geq 0,\ \mu_{M}^{H}\left[\bar{G}\left(H\right)-G\left(H\right)\right]=0$$

$$\mu_{m}^{L}\geq 0,\ G\left(L\right)\geq 0,\ \mu_{m}^{L}G\left(L\right)=0$$

$$\mu_{M}^{L}\geq 0,\ \bar{G}\left(L\right)-G\left(L\right)\geq 0,\ \mu_{M}^{L}\left[\bar{G}\left(L\right)-G\left(L\right)\right]=0$$

There are several cases to consider:

Case 1: Suppose that $G(H)=G(L)=0$. Then, $\lambda^L=\lambda^H=\mu_M^H=\mu_M^L=0$, $\mu_m^H=q$, $\mu_m^L=(1-q)$, $F^H(G(L),G(H))\geq 0$ and $F^L(G(L),G(H))\geq 0$. Since $F^H(G(L),G(H))\geq 0$ implies $F^L(G(L),G(H))\geq 0$, we mjust check $F^H(G(L),G(H))\geq 0$, which holds if and only if $\theta\leq\bar{\theta}(H)$. Summing up, $G(H)=G(L)=0$ is a solution if and only if $\theta\leq\bar{\theta}(H)$.

Case 2: Suppose that $G(L)\in\left(0,\bar{G}(L)\right)$ and $G(H)=0$. Then, $\mu_M^H=\mu_m^L=\mu_M^L=0$, $\lambda^H=0$, $\mu_m^H=\lambda^L\delta(1-q)+q>0$, $\lambda^L=(1-q)\left[-(2-2\delta+\delta\phi)+(1-\delta)\sqrt{\frac{\theta L}{G(L)}}\right]^{-1}>0$, $F^L(G(L),0)=0$, and $F^H(G(L),0)\geq 0$. Since $F^L(G(L),0)$ is strictly increasing in $G(L)$ for all $G(L)\in\left[0,\bar{G}(L)\right]$, at most, there is one solution to $F^L(G(L),0)=0$. Moreover, there is a solution that satisfies $G(L)\in\left(0,\bar{G}(L)\right)$ if and only if $F^L(0,0)<0$ and $F^L\left(\bar{G}(L),0\right)>0$. Note, however, that $F^L(0,0)<0$ implies $F^H(0,0)<0$ (due to Result 3) and $F^H(0,0)<0$ implies $F^H(G(L),0)<0$ (because $F^H(G(L),0)$ is strictly decreasing in $G(L)$). Thus, $F^L(0,0)<0$ is incompatible with $F^H(G(L),0)\geq 0$. Summing up, there is no solution such that $G(L)\in\left(0,\bar{G}(L)\right)$ and $G(H)=0$.

Case 3: Suppose that $G(L)=0$ and $G(H)\in\left(0,\bar{G}(H)\right)$. Then, $\mu_M^L=\mu_m^H=\mu_M^H=0$, $\lambda^L=0$, $\lambda^H=\left[-(2-\delta-\delta q)+(1-\delta)\sqrt{\frac{\theta H}{G(H)}}\right]^{-1}q>0$, $\mu_m^L=(1-q)+q\lambda^H\delta>0$, $F^H(0,G(H))=0$, and $F^L(0,G(H))\geq 0$. Since $F^H(0,G(H))$ is strictly increasing in $G(H)$ for all $G(H)\in\left[0,\bar{G}(H)\right]$, at most, there is one solution to $F^H(0,G(H))=0$. Moreover, there is a solution that satisfies $G(H)\in\left(0,\bar{G}(H)\right)$ if and only if $F^H(0,0)<0$ and $F^H\left(0,\bar{G}(H)\right)>0$. $F^H(0,0)<0$ if and only if $\theta>\bar{\theta}(H)$. $F^H\left(0,\bar{G}(H)\right)>0$ if and only if

$$\theta<\frac{2\left(1-\delta\right)H+2\delta\mathbf{E}\left(R\right)}{\frac{4\left(1-\delta+\delta q\right)\left(1-\delta\right)H}{\left(2-2\delta+\delta q\right)}+\delta\mathbf{E}\left(R\right)}$$

which always holds because the right hand side of the above inequality is always greater than or equal 1. Finally, we must verify that the unique solution to $F^H(0,G(H))=0$ satisfies $F^L(0,G(H))\geq 0$ or, which is equivalent, that $G(H)\leq F^L(0,0)/\delta q$.

Summing up, $G(L) = 0$ and $G(H) = \hat{G}(H) \in \left(0, \bar{G}(H)\right)$, where $\hat{G}(H)$ is the unique solution to $F^H\left(0, \bar{G}(H)\right) = 0$ is a solution if and only if $\theta > \bar{\theta}(H)$ and $\hat{G}(H) \leq F^L(0,0)/\delta\phi$.

Case 4: Suppose that $G(L) \in \left(0, \bar{G}(L)\right)$ and $G(H) \in \left(0, \bar{G}(H)\right)$. Then, $\mu_m^H = \mu_M^H = \mu_m^L = \mu_M^L = 0$

$$\lambda^H\left[-(2-\delta-\delta q) + (1-\delta)\sqrt{\frac{\theta H}{G(H)}}\right] - \lambda^L\delta(1-q) = q$$
$$-\lambda^H\delta q + \lambda^L\left[-(2-2\delta+\delta\phi) + (1-\delta)\sqrt{\frac{\theta L}{G(L)}}\right] = 1-q$$

The above system of linear equations has a solution if and only if

$$\left[-(2-\delta-\delta q) + (1-\delta)\sqrt{\frac{\theta H}{G(H)}}\right]\left[-(2-2\delta+\delta q) + (1-\delta)\sqrt{\frac{\theta L}{G(L)}}\right] \neq \delta^2 q(1-q)$$

A solution must satisfy [$\lambda^H > 0$ and $\lambda^L > 0$] or [$\lambda^H < 0$ and $\lambda^L < 0$]. $\lambda^H > 0$ and $\lambda^L > 0$ if and only if

$$\left[-(2-\delta-\delta q) + (1-\delta)\sqrt{\frac{\theta H}{G(H)}}\right]\left[-(2-2\delta+\delta q) + (1-\delta)\sqrt{\frac{\theta L}{G(L)}}\right] > \delta^2 q(1-q)$$

$\lambda^H > 0$ and $\lambda^L > 0$ implies that $F^H(G(L), G(H)) = 0$, and $F^L(G(L), G(H)) = 0$. Thus, we must solve the following system of equations:

$$F^H(0, G(H)) - \delta(1-q)G(L) = 0$$
$$F^L(G(L), 0) - \delta q G(H) = 0$$

Following the same procedure that we used in the proof of Proposition ..., we have: $G(L) \in \left(0, \bar{G}(L)\right)$ and $G(H) \in \left(\hat{G}(H), \bar{G}(H)\right)$ given by the unique solution to $F^H(G(L), G(H)) = 0$, and $F^L(G(L), G(H)) = 0$ that satisfies $\frac{\partial F^H(G(L),G(H))}{\partial G(H)}\frac{\partial F^L(G(L),G(H))}{\partial G(L)} > \delta^2(1-q)q$ is a solution if and only if $F^H(0,0) < 0$, $F^H\left(0, \bar{G}(H)\right) > 0$, and $\hat{G}(H) > F^L(0,0)/\delta q$, where $\hat{G}(H)$ is the unique solution to $F^H(0, G(H)) = 0$. Moreover, $F^H(0,0) < 0$ if and only if $\theta > \bar{\theta}(H)$ and $F^H\left(0, \bar{G}(H)\right) > 0$ always holds.

Case 5: Suppose that $G(L) = \bar{G}(0)$ or $G(H) = \bar{G}(H)$. If $G(L) = \bar{G}(0)$, we have $\mu_m^L = 0$ and, hence, $(1-q) + \lambda^H\delta q + \mu_M^L = 0$, a contradiction because $\lambda^H \geq 0$ and $\mu_M^L \geq 0$. If $G(H) = \bar{G}(H)$, we have $\mu_m^H = 0$ and, hence $q + (1-q)\lambda^L\delta + \mu_M^H = 0$, a contradiction because $\lambda^L \geq 0$ and $\mu_M^H \geq 0$.

This completes the proof of Proposition 10. ■

Proposition 11 *Assume that* $X \leq \mathbf{E}\left[\bar{X}(\gamma)\right] = \frac{[(4-\theta)(1-\delta+\delta\gamma)+3\delta\phi]\delta\mathbf{E}(R)}{(1-\delta+\delta\gamma)[4(1-\delta+\delta\gamma)+3\delta\phi]}$.

1. *Suppose that* $\gamma = 0$ *and* $\delta \to 1$. *Then, it is impossible to sustain a permanent cooperative equilibrium.*
2. *Suppose that* $\gamma > 0$ *and* $\delta \to 1$. *Then, permanent complete cooperation, i.e., disarmed peace with* $G(Z,R) = 0$ *for all* $Z = 0,1$ *and* $R = H, L$ *can be sustained.*

3. *There are thresholds* $\bar{\theta}^{PC}(H)$ *and* $\bar{X}^{PC}(H)$ *such that, if* $\theta > \bar{\theta}^{PC}(H)$ *and* $X < \bar{X}^{PC}(H)$*, it is impossible to sustain permanent cooperation.*

4. *There is a threshold* $\bar{\theta}^{TC}(H)$ *such that:*

   (a) *Suppose that* $\theta \leq \bar{\theta}^{TC}(H)$*. Then, the best temporary cooperative equilibrium (with no cooperation for* $Z = 1$*) that can be sustained is* $G(0,R) = 0$ *for* $R = H, L$*.*

   (b) *Suppose that* $\theta > \bar{\theta}^{TC}(H)$ *and* $\hat{G}(0,H) \leq \frac{(1-\delta+\delta\gamma+\delta\phi)F^{TC,0,L}(0,0)}{\delta(1-\delta+\delta\gamma)(1-\phi)q}$*. Then the best temporary cooperative equilibrium (with no cooperation for* $Z = 1$*) that can be sustained is* $G(0,L) = 0$ *and* $G(0,H) = \hat{G}(0,H) \in \left(0, \bar{G}(0,H)\right)$*, where* $\hat{G}(0,H)$ *is the unique solution to* $F^{TC,0,H}(0, G(0,H)) = 0$*.*

   (c) *Suppose that* $\theta > \bar{\theta}^{TC}(H)$ *and* $\hat{G}(0,H) > \frac{(1-\delta+\delta\gamma+\delta\phi)F^{TC,0,L}(0,0)}{\delta(1-\delta+\delta\gamma)(1-\phi)q}$*. Then the best temporary cooperative equilibrium (with no cooperation for* $Z = 1$*) that can be sustained* $G(0,L) \in \left(0, \bar{G}(0,L)\right)$ *and* $G(0,H) \in \left(\hat{G}(0,H), \bar{G}(0,H)\right)$ *given by the unique solution to:*

$$F^{TC,0,H}(G(0,L), G(0,H)) = F^{TC,0,L}(G(0,L), G(0,H)) = 0$$

   that satisfies

$$\left[\frac{\partial F^{TC,0,H}(G(0,L), G(0,H))}{\partial G(0,H)}\right]\left[\frac{\partial F^{TC,0,L}(G(0,L), G(0,H))}{\partial G(0,L)}\right] > \frac{(1-\delta+\delta\gamma)^2 \delta^2 (1-\phi)^2 q(1-q)}{(1-\delta+\delta\gamma+\delta\phi)^2}$$

Proof:

Punishment equilibrium**:** Suppose that in a contestable state countries always play $G^N(R) = \theta R/4$ and $S = 0$ when $Z = 0$ and $G^{WO}(R) = R^{WO}(\gamma)/4 = \left(\theta R - X + \frac{\delta \mathbf{E}(R)}{1-\delta+\delta\gamma}\right)/4$, $S = 0$ and $W = 1$ when $Z = 1$. Then, expected payoffs for each state are given by:

$$\begin{aligned}
V^{PU}(1,0,R) &= (1-\delta)\left(\frac{\theta R}{2} - G^N(R)\right) + \delta V^{PU}(1) \text{ for } R = H, L \\
V^{PU}(1,1,R) &= \frac{(1-\delta)(\theta R - X) + \delta V^{PU}(0,w)}{2} + \frac{\delta V^{PU}(0,l)}{2} - (1-\delta) G^{WO}(R) \text{ for } R = H, L \\
V^{PU}(1) &= \phi \mathbf{E}\left[V^{TC}(1,1,R)\right] + (1-\phi)\mathbf{E}\left[V^{TC}(1,0,R)\right] \\
V^{PU}(0,w) &= (1-\delta)\mathbf{E}(R) + \delta\left[\gamma V^{PU}(1) + (1-\gamma) V^{PU}(0,w)\right] \\
V^{PU}(0,l) &= \delta\left[\gamma V^{PU}(1) + (1-\gamma) V^{PU}(0,l)\right]
\end{aligned}$$

where $\mathbf{E}(R) = qH + (1-q)L$. Solving for the uncontestable states, we obtain:

$$V^{PU}(0,w) = \frac{(1-\delta)\mathbf{E}(R) + \delta\gamma V^{PU}(1)}{1-\delta+\delta\gamma} \text{ and } V^{PU}(0,l) = \frac{\delta\gamma V^{PU}(1)}{1-\delta+\delta\gamma}$$

Introducing these expressions into the contestable states value functions, we obtain:

$$\begin{aligned}
V^{PU}(1,0,R) &= (1-\delta)\frac{\theta R}{4} + \delta V^{PU}(1) \text{ for } R = H, L \\
V^{PU}(1,1,R) &= \frac{(1-\delta) R^{WO}(\gamma)}{4} + \frac{\delta^2 \gamma V^{PU}(1)}{1-\delta+\delta\gamma} \text{ for } R = H, L
\end{aligned}$$

Taking expectations, we have:

$$\mathbf{E}\left[V^{PU}(1,0,R)\right] = (1-\delta)\frac{\theta\mathbf{E}(R)}{4} + \delta V^{PU}(1) \;\; for\; R = H, L$$
$$\mathbf{E}\left[V^{PU}(1,1,R)\right] = \frac{(1-\delta)\mathbf{E}\left[R^{WO}(\gamma)\right]}{4} + \frac{\delta^2\gamma V^{PU}(1)}{1-\delta+\delta\gamma} \; for\; R = H, L$$
$$V^{PU}(1) = \phi\mathbf{E}\left[V^{TC}(1,1,R)\right] + (1-\phi)\mathbf{E}\left[V^{TC}(1,0,R)\right]$$

where $\mathbf{E}\left[R^{WO}(\gamma)\right] = \theta\mathbf{E}(R) - X + \frac{\delta\mathbf{E}(R)}{1-\delta+\delta\gamma}$. Solving we obtain:

$$V^{PU}(1) = \mathbf{E}\left[V^{WO}(\gamma)\right] = \frac{(1-\delta+\delta\gamma)\left[(1-\phi)\theta\mathbf{E}(R) + \phi\mathbf{E}\left[R^{WO}(\gamma)\right]\right]}{4(1-\delta+\delta\gamma+\delta\phi)}$$

For this to be an equilibrium we need:

$$-(1-\delta)X + \delta V^{PU}(0,w) \geq \delta V^{PU}(1)$$

which holds if and only if

$$X \leq \mathbf{E}\left[\bar{X}(\gamma)\right] = \frac{[(4-\theta)(1-\delta+\delta\gamma) + 3\delta\phi]\,\delta\mathbf{E}(R)}{(1-\delta+\delta\gamma)[4(1-\delta+\delta\gamma)+3\delta\phi]}$$

Proof of parts 1, 2 and 3:

Permanent cooperation: Suppose that countries play the permanent cooperation strategy with $G(Z,R) \in \left[0, \frac{\theta R}{4}\right)$ for $Z = 0, 1$, and $R = H, L$ and that no country deviates. Then, the expected payoffs for each state are given by:

$$V^{PC}(1,Z,R) = (1-\delta)\left(\frac{R}{2} - G(Z,R)\right) + \delta V^{PC}(1) \;\text{ for } R = H, L \text{ and } Z = 0, 1$$
$$V^{PC}(1) = \phi\mathbf{E}\left[V^{PC}(1,1,R)\right] + (1-\phi)\mathbf{E}\left[V^{PC}(1,0,R)\right]$$

Solving for $V^{PC}(1)$, we have:

$$V^{PC}(1) = \frac{\mathbf{E}(R)}{2} - \phi\mathbf{E}(G(1,R)) - (1-\phi)\mathbf{E}(G(0,R))$$

where $\mathbf{E}(G(Z,R)) = qG(Z,H) + (1-q)G(Z,L)$.

Optimal deviations from permanent cooperation:

Suppose that $Z = 0$. Then, the optimal deviation

$$g^D(G(0,R), \theta R) = \sqrt{G(0,R)\,\theta R} - G(0,R)$$

and the associated expected payoffs are:

$$V^D(G(0,R)) = (1-\delta)\left[\pi^D(G(0,R))\,\theta R - g^D(G(0,R), \theta R)\right] + \delta V^{PU}(1)$$

where $\pi^D(G(0,R)) = \frac{g^D(G(0,R),\theta R)}{G(0,R)+g^D(G(0,R),\theta R)}$ and $V^{PU}(1) = \mathbf{E}\left[V^{WO}(\gamma)\right]$. Therefore:

$$V^D(G(0,R)) = (1-\delta)\left[\theta R - 2\sqrt{G(0,R)\,\theta R} + G(0,R)\right] + \delta \mathbf{E}\left[V^{WO}(\gamma)\right]$$

Suppose that $Z = 1$. Then, in order to compute the optimal deviation we must solve the following problem:

$$V^D(G(1,R)) = \max_{G^D} \left\{ \begin{array}{c} \pi\left(G(1,R), G^D\right)\left[(1-\delta)(\theta R - X) + \delta V(0,w)\right] \\ +\left[1-\pi\left(G(1,R), G^D\right)\right]\delta V(0,l) - (1-\delta)G^D \end{array} \right\}$$
$$V(0,w) = (1-\delta)\mathbf{E}(R) + \delta\left[\gamma V^{PU}(1) + (1-\gamma)V(0,w)\right]$$
$$V(0,l) = \delta\left[\gamma V^{PU}(1) + (1-\gamma)V(0,l)\right]$$

where $V(0,w) = \frac{(1-\delta)\mathbf{E}(R)+\delta\gamma V^{PU}(1)}{1-\delta+\delta\gamma}$, $V(0,l) = \frac{\delta\gamma V^{PU}(1)}{1-\delta+\delta\gamma}$, and $V^{PU}(1) = \mathbf{E}\left[V^{WO}(\gamma)\right]$. Solving for G^D we have that the optimal deviation is given by:

$$g^D\left(G(1,R), R^{WO}(\gamma)\right) = \sqrt{G(1,R)\,R^{WO}(\gamma)} - G(1,R)$$

and the associated deviation payoffs are:

$$V^D(G(1,R)) = (1-\delta)\left[R^{WO}(\gamma) - 2\sqrt{G(1,R)\,R^{WO}(\gamma)} + G(1,R)\right] + \frac{\delta^2\gamma\mathbf{E}\left[V^{WO}(\gamma)\right]}{1-\delta+\delta\gamma}$$

Sustainability constraints for permanent cooperation: Suppose that players are playing a permanent cooperation strategy with guns choices $\mathbf{G} = (G(Z,R))$ for $Z = 0,1$ and $R = H, L$, where $G(Z,R) \in \left[0, \frac{\theta R}{4}\right)$. Then, for this strategy to be an equilibrium it must be the case that:

$$F^{PC,Z,R}(\mathbf{G}) = V^{PC}(1,Z,R) - V^D(G(Z,R)) \geq 0$$

for $Z = 0,1$ and $R = H, L$ or, which is equivalent,

$$F^{PC,0,R}(\mathbf{G}) = (1-\delta)\left(\frac{R}{2} - G(0,R)\right) + \delta V^{PC}(1)$$
$$-\left[\theta R - 2\sqrt{G(0,R)\,\theta R} + G(0,R)\right] - \delta\mathbf{E}\left[V^{WO}(\gamma)\right] \geq 0$$

and

$$F^{PC,1,R}(\mathbf{G}) = (1-\delta)\left(\frac{R}{2} - G(1,R)\right) + \delta V^{PC}(1)$$
$$-(1-\delta)\left[R^{WO}(\gamma) - 2\sqrt{G(1,R)\,R^{WO}(\gamma)} + G(1,R)\right] - \frac{\delta^2\gamma\mathbf{E}\left[V^{WO}(\gamma)\right]}{1-\delta+\delta\gamma} \geq 0$$

where

$$V^{PC}(1) = \frac{\mathbf{E}(R)}{2} - \phi\mathbf{E}(G(1,R)) - (1-\phi)\mathbf{E}(G(0,R))$$
$$\mathbf{E}\left[V^{WO}(\gamma)\right] = \frac{(1-\delta+\delta\gamma)\left[(1-\phi)\theta\mathbf{E}(R) + \phi\mathbf{E}\left[R^{WO}(\gamma)\right]\right]}{4(1-\delta+\delta\gamma+\delta\phi)}$$

$$R^{WO}(\gamma) = \theta R - X + \frac{\delta R}{1-\delta+\delta\gamma}$$

$$\mathbf{E}\left[V^{WO}(\gamma)\right] = \frac{(1-\delta+\delta\gamma)\left[(1-\phi)\,\theta\mathbf{E}(R) + \phi\mathbf{E}\left[R^{WO}(\gamma)\right]\right]}{4(1-\delta+\delta\gamma+\delta\phi)}$$

$$\frac{\delta^2\gamma}{(1-\delta+\delta\gamma)}\frac{\left[(1-\delta+\delta\gamma)\left[\theta\mathbf{E}(R) - \phi X\right] + \phi\delta\mathbf{E}(R)\right]}{4(1-\delta+\delta\gamma+\delta\phi)}$$

$$\frac{\gamma\theta\mathbf{E}(R) - \gamma\phi X + \phi\mathbf{E}(R)}{4(\gamma+\phi)}$$

$$\frac{\mathbf{E}(R)}{4}$$

$$\frac{\gamma\left[\theta\mathbf{E}(R) - \phi X + \frac{\delta\mathbf{E}(R)}{1-\delta+\delta\gamma}\right]}{4(\gamma+\phi)}$$

$$\frac{\gamma\theta\mathbf{E}(R) - \gamma\phi X + \phi\mathbf{E}(R)}{4(\gamma+\phi)}$$

Sustainability of permanent cooperation when $\delta \to 1$:
Suppose that $\gamma = 0$. Take the limit of $F^{PC,1,R}(\mathbf{G})$ when $\delta \to 1$:

$$\lim_{\delta\to 1} F^{PC,1,R}(\mathbf{G}) = V^{PC}(1) - \mathbf{E}(R) < 0$$

Thus, for $\gamma = 0$, when $\delta \to 1$, it is impossible to sustain a permanent cooperative equilibrium.

Suppose that $\gamma > 0$ and consider $\mathbf{G} = \mathbf{0}$, that is complete permanent cooperation. Take the limit of $F^{PC,0,R}(\mathbf{0})$ when $\delta \to 1$:

$$\begin{aligned}\lim_{\delta\to 1} F^{PC,0,R}(\mathbf{0}) &= \frac{\mathbf{E}(R)}{2} - \lim_{\delta\to 1}\mathbf{E}\left[V^{WO}(\gamma)\right] \\ &= \frac{\mathbf{E}(R)}{2} - \frac{(\theta\gamma+\phi)\,\mathbf{E}(R) - \gamma\phi X}{4(\gamma+\phi)} > 0\end{aligned}$$

Take the limit of $F^{PC,1,R}(\mathbf{0})$ when $\delta \to 1$:

$$\begin{aligned}\lim_{\delta\to 1} F^{PC,1,R}(\mathbf{0}) &= \frac{\mathbf{E}(R)}{2} - \lim_{\delta\to 1}\left[(1-\delta)\,R^{WO}(\gamma)\right] - \lim_{\delta\to 1}\frac{\delta^2\gamma\mathbf{E}\left[V^{WO}(\gamma)\right]}{1-\delta+\delta\gamma} \\ &= \frac{\mathbf{E}(R)}{2} - \frac{(\theta\gamma+\phi)\,\mathbf{E}(R) - \gamma\phi X}{4(\gamma+\phi)} > 0\end{aligned}$$

Thus, provided that $\gamma > 0$, $G(Z,R) = 0$ for $Z = 0,1$ and $R = H, L$ can be sustained as an equilibrium when $\delta \to 1$.

Imposibility of avoiding open conflict for $\gamma > 0$. Let

$$\bar{G}(0,H) = \left[\frac{1-\delta}{2(1-\delta)+\delta(1-\phi)q}\right]^2 \theta H$$
$$\bar{G}(0,L) = \left[\frac{1-\delta}{2(1-\delta)+\delta(1-\phi)(1-q)}\right]^2 \theta L$$
$$\bar{G}(1,H) = \left[\frac{1-\delta}{2(1-\delta)+\delta\phi q}\right]^2 H^{WO}(\gamma)$$
$$\bar{G}(1,L) = \left[\frac{1-\delta}{2(1-\delta)+\delta\phi(1-q)}\right]^2 L^{WO}(\gamma)$$

It is easy to prove that $F^{PC,1,H}(\mathbf{G})$ is strictly increasing in $G(1,H)$ for all $G(1,H) \leq \bar{G}(1,H)$ and strictly decreasing in $G(1,H)$ for all $G(1,H) \geq \bar{G}(1,H)$. It is also easy to prove that $F^{PC,1,H}(\mathbf{G})$ is strictly decreasing in $G(0,H)$, $G(0,L)$, and $G(1,L)$. Consider guns choices $\left(\bar{G}(1,H),\mathbf{0}\right)$, that is, $G(1,H) = \bar{G}(1,H)$ and $G(1,L) = G(0,L) = G(0,H) = 0$. Note that $F^{PC,1,H}\left(\bar{G}(1,H),\mathbf{0}\right) \geq F^{PC,1,H}(\mathbf{G})$ for all $\mathbf{G}$. Therefore, if $F^{PC,1,H}\left(\bar{G}(1,H),\mathbf{0}\right) < 0$, then $F^{PC,1,H}(\mathbf{G}) < 0$ for any $\mathbf{G}$ and, hence, if countries are playing a permanent cooperation strategy, it is impossible to avoid a deviation when $(Z,R) = (1,H)$. Moreover, $F^{PC,1,H}\left(\bar{G}(1,H),\mathbf{0}\right) < 0$ if and only if

$$X < \bar{X}^{PC}(H) = \frac{2[aH + b\mathbf{E}(R)]\theta - (1-\delta)H + \delta\mathbf{E}(R)}{2(a+b)} + \frac{\delta\mathbf{E}(R)}{1-\delta+\delta\gamma}$$

where

$$a = \frac{(1-\delta)(1-\delta+\delta\phi q)}{2(1-\delta)+\delta\phi q},\ b = \frac{\delta^2\gamma}{4(1-\delta+\delta\gamma+\delta\phi)}$$

and $\bar{X}(H) > 0$ if and only if

$$\theta > \bar{\theta}^{PC}(H) = \frac{(1-\delta+\delta\gamma)[(1-\delta)H + \delta\mathbf{E}(R)] - 2(a+b)\delta\mathbf{E}(R)}{2(1-\delta+\delta\gamma)[aH + b\mathbf{E}(R)]}$$

Summing up, if $\theta > \bar{\theta}^{PC}(H)$ and $X < \bar{X}^{PC}(H)$, it is impossible to sustain permanent cooperation.

Proof of part 4:

Temporary cooperation with no cooperation for $Z = 1$: Suppose that countries play the temporary cooperation strategy with $G(0,R) \in \left(0, \frac{\theta R}{4}\right)$ for $R = H, L$ and that no country deviates. Then, the expected payoffs for each state are given by:

$$V^{TC}(1,0,R) = (1-\delta)\left(\frac{R}{2} - G(0,R)\right) + \delta V^{TC}(1)$$
$$V^{TC}(1,1,R) = \frac{(1-\delta)(\theta R - X) + \delta V^{TC}(0,w)}{2} + \frac{\delta V^{TC}(0,l)}{2} - (1-\delta)G^{WO}(R)$$
$$V^{TC}(1) = \phi\mathbf{E}\left[V^{TC}(1,1,R)\right] + (1-\phi)\mathbf{E}\left[V^{TC}(1,0,R)\right]$$
$$V^{TC}(0,w) = (1-\delta)\mathbf{E}(R) + \delta\left[\gamma V^{TC}(1) + (1-\gamma)V^{TC}(0,w)\right]$$
$$V^{TC}(0,l) = \delta\left[\gamma V^{TC}(1) + (1-\gamma)V^{TC}(0,l)\right]$$

Solving for the uncontestable states, we obtain:

$$V^{TC}(0,w) = \frac{(1-\delta)\,\mathbf{E}(R) + \delta\gamma V^{TC}(1)}{1-\delta+\delta\gamma} \quad and \quad V^{TC}(0,l) = \frac{\delta\gamma V^{TC}(1)}{1-\delta+\delta\gamma}$$

Introducing these expressions into the contestable states value functions, we obtain:

$$V^{TC}(1,0,R) = (1-\delta)\left(\frac{R}{2} - G(0,R)\right) + \delta V^{TC}(1)$$
$$V^{TC}(1,1,R) = \frac{(1-\delta)\,R^{WO}(\gamma)}{4} + \frac{\delta^2\gamma V^{TC}(1)}{1-\delta+\delta\gamma}$$
$$V^{TC}(1) = \phi\mathbf{E}\left[V^{TC}(1,1,R)\right] + (1-\phi)\,\mathbf{E}\left[V^{TC}(1,0,R)\right]$$

Taking expectations, we obtain:

$$\mathbf{E}\left[V^{TC}(1,0,R)\right] = (1-\delta)\left[\frac{\mathbf{E}(R)}{2} - \mathbf{E}\left[G(0,R)\right]\right] + \delta V^{TC}(1)$$
$$\mathbf{E}\left[V^{TC}(1,1,R)\right] = \frac{(1-\delta)\,\mathbf{E}\left[R^{WO}(\gamma)\right]}{4} + \frac{\delta^2\gamma V^{TC}(1)}{1-\delta+\delta\gamma}$$

Solving for $V^{TC}(1)$ we have:

$$V^{TC}(1) = \frac{(1-\delta+\delta\gamma)}{1-\delta+\delta\gamma+\delta\phi}\left\{\phi\frac{\mathbf{E}\left[R^{WO}(\gamma)\right]}{4} + (1-\phi)\left[\frac{\mathbf{E}(R)}{2} - \mathbf{E}\left[G(0,R)\right]\right]\right\}$$

Optimal deviations from temporary cooperation with no cooperation for $Z=1$: Suppose that $Z=0$. Then, the optimal deviation is

$$g^D\left(G(0,R),\theta R\right) = \sqrt{G(0,R)\,\theta R} - G(0,R)$$

and the associated expected payoffs are:

$$V^D\left(G(0,R)\right) = (1-\delta)\left[\pi^D\left(G(0,R)\right)\theta R - g^D\left(G(0,R),\theta R\right)\right] + \delta V^{PU}(1)$$

where $\pi^D\left(G(0,R)\right) = \frac{g^D(G(0,R),\theta R)}{G(0,R)+g^D(G(0,R),\theta R)}$ and $V^{PU}(1) = \mathbf{E}\left[V^{WO}(\gamma)\right]$. Therefore:

$$V^D\left(G(0,R)\right) = (1-\delta)\left[\theta R - 2\sqrt{G(0,R)\,\theta R} + G(0,R)\right] + \delta\mathbf{E}\left[V^{WO}(\gamma)\right]$$

Sustainability constraints for temporary cooperation with no cooperation for $Z=1$: Suppose that countries play the temporary cooperation strategy with guns choices $G(0,L), G(0,H)$. For this strategy to be an equilibrium it must be the case that:

$$F^{TC,0,R}\left(G(0,L),G(0,H)\right) = V^{TC}(1,0,R) - V^D\left(G(0,R)\right) \geq 0$$

or, which is equivalent,

$$F^{TC,0,R}\left(G\left(0,L\right),G\left(0,H\right)\right)=\left(1-\delta\right)\left(\frac{R}{2}-G\left(0,R\right)\right)+\delta V^{TC}\left(1\right)$$
$$-\left(1-\delta\right)\left[\theta R-2\sqrt{G\left(0,R\right)\theta R}+G\left(0,R\right)\right]-\delta\mathbf{E}\left[V^{WO}\left(\gamma\right)\right]\geq 0$$

where

$$V^{TC}\left(1\right)=\frac{\left(1-\delta+\delta\gamma\right)}{1-\delta+\delta\gamma+\delta\phi}\left\{\phi\frac{\mathbf{E}\left[R^{WO}\left(\gamma\right)\right]}{4}+\left(1-\phi\right)\left[\frac{\mathbf{E}\left(R\right)}{2}-\mathbf{E}\left[G\left(0,R\right)\right]\right]\right\}$$
$$\mathbf{E}\left[V^{WO}\left(\gamma\right)\right]=\frac{\left(1-\delta+\delta\gamma\right)\left[\left(1-\phi\right)\theta\mathbf{E}\left(R\right)+\phi\mathbf{E}\left[R^{WO}\left(\gamma\right)\right]\right]}{4\left(1-\delta+\delta\gamma+\delta\phi\right)}$$

Best possible temporary cooperation equilibrium with no cooperation for $Z=1$. To determine the best possible temporary cooperation equilibrium with no cooperation for $Z=1$, we solve

$$\max_{\mathbf{G}}\left\{V^{TC}\left(1\right)\right\}\ \textit{subject to: } F^{TC,0,R}\left(G\left(0,L\right),G\left(0,H\right)\right)\geq 0\ \textit{for } R=H,L$$

Using the same procedure that we used in the proof of Proposition 5, we have the following results:

Result 1: If a solution exist, it must satisfy $G\left(0,L\right)\in\left[0,\bar{G}\left(0,L\right)\right]$ and $G\left(0,H\right)\in\left[0,\bar{G}\left(0,H\right)\right]$, where

$$\bar{G}\left(0,L\right)=\left[\frac{1-\delta}{2\left(1-\delta\right)+\frac{\delta\left(1-\delta+\delta\gamma\right)\left(1-\phi\right)\left(1-q\right)}{1-\delta+\delta\gamma+\delta\phi}}\right]^{2}\theta L<\frac{\theta L}{4}$$
$$\bar{G}\left(0,H\right)=\left[\frac{\left(1-\delta\right)}{2\left(1-\delta\right)+\frac{\delta\left(1-\delta+\delta\gamma\right)\left(1-\phi\right)q}{1-\delta+\delta\gamma+\delta\phi}}\right]^{2}\theta H<\frac{\theta H}{4}.$$

Result 2: $F^{TC,0,H}\left(G\left(0,L\right),G\left(0,H\right)\right)$ and $F^{TC,0,L}\left(G\left(0,L\right),G\left(0,H\right)\right)$ are quasiconcave functions for all $\left(G\left(0,L\right),G\left(0,H\right)\right)\in\Re_{+}^{2}$.

Result 3: If $F^{TC,0,H}\left(0,0\right)\geq 0$, then $F^{TC,0,L}\geq 0$. **Proof**: $F^{TC,0,R}\left(0,0\right)\geq 0$ if and only if $\theta\leq\bar{\theta}^{TC}\left(R\right)$, where

$$\bar{\theta}^{TC}\left(R\right)=\frac{\left(1-\delta\right)R+\frac{\delta\left(1-\delta+\delta\gamma\right)\left(1-\phi\right)\mathbf{E}\left(R\right)}{\left(1-\delta+\delta\gamma+\delta\phi\right)}}{2\left(1-\delta\right)R+\frac{\delta\left(1-\delta+\delta\gamma\right)\left(1-\phi\right)\mathbf{E}\left(R\right)}{2\left(1-\delta+\delta\gamma+\delta\phi\right)}}$$
$$=\frac{2\left(1-\delta\right)R\left(1-\delta+\delta\gamma+\delta\phi\right)+2\delta\left(1-\delta+\delta\gamma\right)\left(1-\phi\right)\mathbf{E}\left(R\right)}{4\left(1-\delta\right)R\left(1-\delta+\delta\gamma+\delta\phi\right)+\delta\left(1-\delta+\delta\gamma\right)\left(1-\phi\right)\mathbf{E}\left(R\right)}$$

Moreover, $\bar{\theta}^{TC}\left(H\right)\leq\bar{\theta}^{TC}\left(L\right)$ with strict inequality if $\delta\in\left(0,1\right)$.

Since the objective function is linear and the constraints are quasiconcave, the following Kuhn-Tucker conditions are sufficient for a global maximum.

$$-q+\lambda^{H}\left[-2\left(1-\delta\right)-\frac{\delta\left(1-\delta+\delta\gamma\right)\left(1-\phi\right)q}{1-\delta+\delta\gamma+\delta\phi}+\left(1-\delta\right)\sqrt{\frac{\theta H}{G\left(0,H\right)}}\right]$$
$$-\lambda^{L}\frac{\left(1-\delta+\delta\gamma\right)\delta\left(1-\phi\right)\left(1-q\right)}{1-\delta+\delta\gamma+\delta\phi}+\mu_{m}^{H}-\mu_{M}^{H}=0$$
$$-\left(1-q\right)+\lambda^{L}\left[-2\left(1-\delta\right)-\frac{\delta\left(1-\delta+\delta\gamma\right)\left(1-\phi\right)\left(1-q\right)}{1-\delta+\delta\gamma+\delta\phi}+\left(1-\delta\right)\sqrt{\frac{\theta L}{G\left(0,L\right)}}\right]$$
$$-\frac{\left(1-\delta+\delta\gamma\right)\delta\left(1-\phi\right)q}{1-\delta+\delta\gamma+\delta\phi}\lambda^{H}+\mu_{m}^{L}-\mu_{M}^{L}=0$$
$$\lambda^{H}\geq0,\ F^{TC,0,H}\left(G\left(0,L\right),G\left(0,H\right)\right)\geq0,\ \lambda^{H}F^{TC,0,H}\left(G\left(0,L\right),G\left(0,H\right)\right)=0$$
$$\lambda^{L}\geq0,\ F^{TC,0,L}\left(G\left(0,L\right),G\left(0,H\right)\right)\geq0,\ \lambda^{L}F^{TC,0,L}\left(G\left(0,L\right),G\left(0,H\right)\right)=0$$
$$\mu_{m}^{H}\geq0,\ G\left(H\right)\geq0,\ \mu_{m}^{H}G\left(H\right)=0$$
$$\mu_{M}^{H}\geq0,\ \bar{G}\left(H\right)-G\left(H\right)\geq0,\ \mu_{M}^{H}\left[\bar{G}\left(H\right)-G\left(H\right)\right]=0$$
$$\mu_{m}^{L}\geq0,\ G\left(L\right)\geq0,\ \mu_{m}^{L}G\left(L\right)=0$$
$$\mu_{M}^{L}\geq0,\ \bar{G}\left(L\right)-G\left(L\right)\geq0,\ \mu_{M}^{L}\left[\bar{G}\left(L\right)-G\left(L\right)\right]=0$$

There are several cases to consider:

Case 1: Suppose that $G(0,H)=G(0,L)=0$. Then, $\lambda^L=\lambda^H=\mu_M^H=\mu_M^L=0$, $\mu_m^H=q$, $\mu_m^L=(1-q)$, $F^{TC,0,H}(G(0,L),G(0,H))\geq 0$ and $F^{TC,0,L}(G(0,L),G(0,H))\geq 0$. Since $F^{TC,0,H}(G(0,L),G(0,H))\geq 0$ implies $F^{TC,0,L}(G(0,L),G(0,H))$, we mjust check $F^{TC,0,H}(G(0,L),G(0,H))\geq 0$, which holds if and only if $\theta\leq\bar{\theta}^{TC}(H)$. Summing up, $G(0,H)=G(0,L)$ is a solution if and only if $\theta\leq\bar{\theta}^{TC}(H)$.

Case 2: Suppose that $G(0,L)\in\left(0,\bar{G}(0,L)\right)$ and $G(0,H)=0$. Then, $\mu_M^H=\mu_m^L=\mu_M^L=0$, $\lambda^H=0$, $\mu_m^H=q+\lambda^L\frac{(1-\delta+\delta\gamma)\delta(1-\phi)(1-q)}{1-\delta+\delta\gamma+\delta\phi}>0$,

$$\lambda^{L}=\left(1-q\right)\left[-2\left(1-\delta\right)-\frac{\delta\left(1-\delta+\delta\gamma\right)\left(1-\phi\right)\left(1-q\right)}{1-\delta+\delta\gamma+\delta\phi}+\left(1-\delta\right)\sqrt{\frac{\theta L}{G\left(0,L\right)}}\right]^{-1}>0,$$

$F^L(G(L),0)=0$, and $F^{TC,0,H}(G(0,L),0)\geq 0$. Since $F^{TC,0,L}(G(0,L),0)$ is strictly increasing in $G(0,L)$ for all $G(0,L)\in\left[0,\bar{G}(0,L)\right]$, at most, there is one solution to $F^{TC,0,L}(G(0,L),0)$. Moreover, there is a solution that satisfies $G(0,L)\in\left(0,\bar{G}(0,L)\right)$ if and only if $F^{TC,0,L}(0,0)<0$ and $F^{TC,0,L}\left(\bar{G}(0,L),0\right)>0$. Note, however, that $F^{TC,0,L}(0,0)<0$ implies $F^{TC,0,H}(0,0)<0$ (due to Result 3) and $F^{TC,0,H}(0,0)<0$ implies $F^{TC,0,H}(G(0,L),0)<0$ (because $F^{TC,0,H}(G(0,L),0)$ is strictly decreasing in $G(0,L)$). Thus, $F^{TC,0,L}(0,0)<0$ is incompatible with $F^{TC,0,H}(G(0,L),0)\geq 0$. Summing up, there is no solution such that $G(0,L)\in\left(0,\bar{G}(0,L)\right)$ and $G(0,H)=0$.

Case 3: Suppose that $G(0,L)=0$ and $G(0,H)\in\left(0,\bar{G}(0,H)\right)$. Then, $\mu_M^L=\mu_m^H=\mu_M^H=0$, $\lambda^L=0$,

$$\lambda^{H}=\left[-2\left(1-\delta\right)-\frac{\delta\left(1-\delta+\delta\gamma\right)\left(1-\phi\right)q}{1-\delta+\delta\gamma+\delta\phi}+\left(1-\delta\right)\sqrt{\frac{\theta H}{G\left(0,H\right)}}\right]^{-1}q>0$$

$\mu_m^L = 1 - q + \frac{(1-\delta+\delta\gamma)\delta(1-\phi)q}{1-\delta+\delta\gamma+\delta\phi}\lambda^H > 0$, $F^{TC,0,H}(0, G(0,H)) = 0$, and $F^{TC,0,L}(0, G(0,H)) \geq 0$. Since $F^{TC,0,H}(0, G(0,H))$ is strictly increasing in $G(0,H)$ for all $G(0,H) \in \left[0, \bar{G}(0,H)\right]$, at most, there is one solution to $F^{TC,0,H}(0, G(0,H)) = 0$. Moreover, there is a solution that satisfies $G(0,H) \in \left(0, \bar{G}(0,H)\right)$ if and only if $F^{TC,0,H}(0,0) < 0$ and $F^{TC,0,H}\left(0, \bar{G}(0,H)\right) > 0$. $F^{TC,0,H}(0,0) < 0$ if and only if $\theta > \bar{\theta}^{TC}(H)$. $F^{TC,0,H}\left(0, \bar{G}(0,H)\right) > 0$ if and only if

$$\theta < \frac{(1-\delta)\frac{H}{2} + \frac{\delta(1-\delta+\delta\gamma)(1-\phi)2\mathbf{E}(R)}{4(1-\delta+\delta\gamma+\delta\phi)}}{\frac{\left[1-\delta+\frac{\delta(1-\delta+\delta\gamma)(1-\phi)q}{1-\delta+\delta\gamma+\delta\phi}\right](1-\delta)H}{2(1-\delta)+\frac{\delta(1-\delta+\delta\gamma)(1-\phi)q}{1-\delta+\delta\gamma+\delta\phi}} + \frac{\delta(1-\delta+\delta\gamma)(1-\phi)\mathbf{E}(R)}{4(1-\delta+\delta\gamma+\delta\phi)}}$$

which always holds because the right hand side of the above inequality is always greater than or equal 1. Finally, we must verify that the unique solution to $F^{TC,0,H}(0, G(0,H)) = 0$ satisfies

$$F^{TC,0,L}(0, G(0,H)) = F^{TC,0,L}(0,0) - \frac{\delta(1-\delta+\delta\gamma)(1-\phi)q}{1-\delta+\delta\gamma+\delta\phi}G(0,H) \geq 0$$

or, which is equivalent, that $G(0,H) \leq \frac{(1-\delta+\delta\gamma+\delta\phi)F^{TC,0,L}(0,0)}{\delta(1-\delta+\delta\gamma)(1-\phi)q}$.

Summing up, $G(0,L) = 0$ and $G(0,H) = \hat{G}(0,H) \in \left(0, \bar{G}(0,H)\right)$, where $\hat{G}(0,H)$ is the unique solution to $F^{TC,0,H}(0, G(0,H)) = 0$ is a solution if and only if $\theta > \bar{\theta}^{TC}(H)$ and $\hat{G}(0,H) \leq \frac{(1-\delta+\delta\gamma+\delta\phi)F^{TC,0,L}(0,0)}{\delta(1-\delta+\delta\gamma)(1-\phi)q}$.

Case 4: Suppose that $G(0,L) \in \left(0, \bar{G}(0,L)\right)$ and $G(0,H) \in \left(0, \bar{G}(0,H)\right)$. Then, $\mu_m^H = \mu_M^H = \mu_m^L = \mu_M^L = 0$,

$$\left[\frac{\partial F^{TC,0,H}(G(0,L), G(0,H))}{\partial G(0,H)}\right]\lambda^H - \frac{(1-\delta+\delta\gamma)\delta(1-\phi)(1-q)}{1-\delta+\delta\gamma+\delta\phi}\lambda^L = q$$
$$-\frac{(1-\delta+\delta\gamma)\delta(1-\phi)q}{1-\delta+\delta\gamma+\delta\phi}\lambda^H + \left[\frac{\partial F^{TC,0,L}(G(0,L), G(0,H))}{\partial G(0,L)}\right]\lambda^L = (1-q)$$

The above system of linear equations has a solution if and only if

$$\left[\frac{\partial F^{TC,0,H}(G(0,L), G(0,H))}{\partial G(0,H)}\right]\left[\frac{\partial F^{TC,0,L}(G(0,L), G(0,H))}{\partial G(0,L)}\right] \neq \frac{(1-\delta+\delta\gamma)^2\delta^2(1-\phi)^2 q(1-q)}{(1-\delta+\delta\gamma+\delta\phi)^2}$$

A solution must satisfy [$\lambda^H > 0$ and $\lambda^L > 0$] or [$\lambda^H < 0$ and $\lambda^L < 0$]. $\lambda^H > 0$ and $\lambda^L > 0$ if and only if

$$\left[\frac{\partial F^{TC,0,H}(G(0,L), G(0,H))}{\partial G(0,H)}\right]\left[\frac{\partial F^{TC,0,L}(G(0,L), G(0,H))}{\partial G(0,L)}\right] > \frac{(1-\delta+\delta\gamma)^2\delta^2(1-\phi)^2 q(1-q)}{(1-\delta+\delta\gamma+\delta\phi)^2}$$

$\lambda^H > 0$ and $\lambda^L > 0$ implies that

$$F^{TC,0,H}(G(0,L), G(0,H)) = F^{TC,0,L}(G(0,L), G(0,H)) = 0$$

Thus, we must solve the following system of equations:

$$F^{TC,0,H}\left(0,G\left(0,H\right)\right)-\frac{\delta\left(1-\delta+\delta\gamma\right)\left(1-\phi\right)\left(1-q\right)}{1-\delta+\delta\gamma+\delta\phi}G\left(L\right)=0$$
$$F^{TC,0,L}\left(G\left(0,L\right),0\right)-\frac{\delta\left(1-\delta+\delta\gamma\right)\left(1-\phi\right)q}{1-\delta+\delta\gamma+\delta\phi}G\left(H\right)=0$$

Following the same procedure that we used in the proof of Proposition ..., we have: $G\left(0,L\right)\in\left(0,\bar{G}\left(0,L\right)\right)$ and $G\left(0,H\right)\in\left(\hat{G}\left(0,H\right),\bar{G}\left(0,H\right)\right)$ given by the unique solution to $F^{TC,0,H}\left(G\left(0,L\right),G\left(0,H\right)\right)=0$, and $F^{TC,0,L}\left(G\left(0,L\right),G\left(0,H\right)\right)=0$ that satisfies

$$\left[\frac{\partial F^{TC,0,H}\left(G\left(0,L\right),G\left(0,H\right)\right)}{\partial G\left(0,H\right)}\right]\left[\frac{\partial F^{TC,0,L}\left(G\left(0,L\right),G\left(0,H\right)\right)}{\partial G\left(0,L\right)}\right]>\frac{\left(1-\delta+\delta\gamma\right)^{2}\delta^{2}\left(1-\phi\right)^{2}q\left(1-q\right)}{\left(1-\delta+\delta\gamma+\delta\phi\right)^{2}}$$

is a solution if and only if $F^{TC,0,H}\left(0,0\right)<0$, $F^{TC,0,H}\left(0,\bar{G}\left(0,H\right)\right)>0$, and $\hat{G}\left(0,H\right)>\frac{(1-\delta+\delta\gamma+\delta\phi)F^{TC,0,L}(0,0)}{\delta(1-\delta+\delta\gamma)(1-\phi)q}$, where $\hat{G}\left(0,H\right)$ is the unique solution to $F^{TC,0,H}\left(0,G\left(0,H\right)\right)=0$. Moreover, $F^{TC,0,H}\left(0,0\right)<0$ if and only if $\theta>\bar{\theta}^{TC}\left(H\right)$ and $F^{TC,0,H}\left(0,\bar{G}\left(H\right)\right)>0$ always holds.

Case 5: Suppose that $G\left(0,L\right)=\bar{G}\left(0,L\right)$ or $G\left(0,H\right)=\bar{G}\left(0,H\right)$. If $G\left(0,L\right)=\bar{G}\left(0,0\right)$, we have $\mu_{m}^{L}=0$ and, hence, $(1-q)+\lambda^{H}\frac{(1-\delta+\delta\gamma)\delta(1-\phi)q}{1-\delta+\delta\gamma+\delta\phi}+\mu_{M}^{L}=0$, a contradiction because $\lambda^{H}\geq0$ and $\mu_{M}^{L}\geq0$. If $G\left(0,H\right)=\bar{G}\left(0,H\right)$, we have $\mu_{m}^{H}=0$ and, hence $q+\lambda^{L}\frac{(1-\delta+\delta\gamma)\delta(1-\phi)(1-q)}{1-\delta+\delta\gamma+\delta\phi}+\mu_{M}^{H}=0$, a contradiction because $\lambda^{L}\geq0$ and $\mu_{M}^{H}\geq0$.

This completes the proof of Proposition 11. ■