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Why does the folk theorem do not seem to work when it is mostly needed?

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ABSTRACT. In International Relations the canonical model of inter-estate interactions is a one-shot security competition game. The model has the structure of a prisoners dilemma, which results in an equilibrium with two sources of inefficiency: excessive arming and possibly the destruction associated with open conflict. Standard arguments in game theory suggest that more cooperative outcomes should emerge given that states often engage in repeated interactions. Historical record, on the contrary, shows cycles of peace, arms races, and serious instances of open conflict. Long-lasting disarmed peace is rarely observed. The paper develops a unified model of conflict that reveals possible theoretical mechanisms to produce such historical outcomes.

Keywords: Conflict. Security competition. Folk theorem. Stochastic games.

1 Introduction

In economics of conflict and international relations the standard game theoretic model of conflict is a one-shot contest game in which several players simultaneously and independently make investments (e.g., weapons) to increase the probability of winning a prize (e.g., a disputable resource).[1] In the Nash equilibrium, even when resource destruction is modeled as a way to deter conflict, there are positive levels of arming, open conflict, and part of the resource is destroyed in

[1]See Hirshleifer (1989) for an important seminal work and Garfinkel and Skaperdas (2007) for a comprehensive survey of the literature in economics of conflict. This is also the standard rent seeking model (Tullock (1980) and Nitzan (1994).

the fight, which is of course an inefficient outcome. All players could be better off if they choose no arming and peacefully split the resource. If the game is augmented with a bargaining and settlement stage after arming decisions have been made, the destruction associated with open conflict can be averted, but inefficient arming persists. In other words, there is an equilibrium with armed peace and negotiated settlement. In any case, arming has the structure of a prisoner's dilemma (often referred as a security dilemma in international relations), which opens the door for more cooperative outcomes to be sustained if repeated interactions are allowed.

In international relations, repeated interactions among countries seem a reasonable environment to explore for at least four reasons. First, in many cases, there are few relevant countries or blocks of countries involved in an international dispute. Making an analogy with firms' behavior, collusion tends to be easier to sustain the fewer the number of firms. Second, countries are long-lived entities, which suggests that it is important to seriously consider dynamic interactions. Third, countries tend to have powerful incentives to plan and act strategically, given that mistakes could lead to serious consequences (e.g., war). Finally, countries usually count with the capabilities to make contingent plans and act strategically (e.g., diplomats and military experts). Thus, international relations should be an excellent candidate to apply the Folk Theorem and related results for infinitely repeated games in order to sustain more cooperative outcomes (see, for example, Keohane (1986), Oye (1986), and Kydd (2015), chapter 8).[2] Indeed, if the standard model of conflict is repeated infinitely and countries are patient enough, disarmed peace can be sustained as a subgame perfect equilibrium, which leaves us with a conundrum. If conflict is modeled as a one-shot contest game, the equilibrium is either open conflict (when negotiated settlement is not possible) or armed peace (when negotiated settlement is possible). If conflict is modeled as a repeated contest game, the equilibrium is either disarmed peace (when countries are patient) or armed peace with lower levels of arming than under the one-shot game (when countries are more impatient).

In reality, we rarely observe disarmed peace; under armed peace there are periods of arming escalation and periods of de-escalation; there are also events of open conflict, which often settle a dispute for a long period, but there are also examples of defeated countries that after they recover became revisionist once again. To make these patterns consistent with a game theoretic approach to conflict and better understand the mechanisms behind them, we introduce several elements to the standard model of conflict. First, we allow countries to interact repeatedly

[2] One might even argue that if these results do not apply to interactions among countries, there should be little hope that they will apply to other strategic interactions, such as firms trying to organize collusion or commoners solving the tragedy of the commons.

over time. Second, we introduce a randomly determined disputable resource; that is, the resource's value is stochastic and varies from period to period. Third, after observing each other's military power, countries can agree on a settlement rule such that each country gets a fraction of the resource. If no settlement is agreed, an open conflict happens, and a portion of the resource is destroyed. Finally, there is an exogenous randomly determined opportunity that the victor of an open conflict can wipe-out the other player at an additional fixed cost. Moreover, we distinguish between two cases, whether the wipe-out is permanent or not. Because of the random realization of states that affect the payoffs and actions, we study a stochastic game.[3]

Most of above elements have been, to some extent, individually explored in the literature on economics of conflict. To the best of our knowledge, however, there is no model that systematically incorporates all of them. Moreover, in our model it is very easy to shut down one or several elements, which allows us to recover standard results as special cases. More importantly, our model clearly shows the elements required to generate each result. For example, to obtain cycles of escalation and de-escalation but without reaching the point of open conflict, it suffices to introduce repeated interactions and a randomly determined disputable resource. As already shown by Powell (1993), Skaperdas and Syropoulos (1996), and McBride and Skaperdas (2006) wipe-out opportunities can explain open conflict. However, in an infinite dynamic game, if wipe-out is permanent, there is only one instance of open conflict, followed by unarmed peace forever. On the contrary, when wipe-out is not permanent, there might be cycles of armed peace, open conflict, temporary wipe-out, followed by the recovery of the defeated country and the beginning of a new cycle.

The rest of the paper is organized as follows. Section 2 develops the model. Section 3 characterizes the equilibrium for the simplest scenarios: the one-shot game and the repeated game without stochastic states. Section 4 studies the stochastic game without wipe-out opportunities. Section 5 considers the more complicated scenario: wipe-out opportunities in the stochastic game. Section 6 presents our conclusions.

1.1 Related literature

The paper relates to two bodies of literature in two subjects of study: game theoretic models of conflict in economics and security competition in international relations.

[3] This is a slight extension to a repeated game. Although there is repeated interaction, there is no unique "stage game," since the payoffs and actions depend on different realizations of states variables.

There are several papers that have formalized the idea that open conflict differs from settlement under the shadow of conflict because it changes the future bargaining power of the parties, which rationalizes an equilibrium with costly open conflict.[4] The notion that the victor of an open conflict can wipe out its enemy is one possible such mechanism. Skaperdas and Syropoulos (1996) and Garfinkel and Skaperdas (2000) introduce wipe-out opportunities, but they only consider a two-period model.Powell (1993); McBride and Skaperdas (2006) explore an infinite game, but they restrict the analysis to Markov perfect equilibria. Our first contribution to this literature is exploring an environment in which the classical Folk theorem has a serious chance in the sense that without introducing the wipe-out technology, countries can sustain unarmed peace.[5] Our second contribution is to deal with one potentially unrealistic prediction of these models (including our version with permanent wipe-out), namely, that after one country wipes out its enemy, unarmed peace persists forever. McBride and Skaperdas (2006) considers that several battles might be necessary to finally wipe-out an enemy, which explains arming after open conflict. Still, once the enemy has been finished off, unarmed peace persists. Our model with temporary wipe-out, on the contrary, opens the door for recurrent periods of peace and conflict. [6]

The paper is also related to international relations and the well-known debate between realists and liberals. One simple way of describing the realist school on international relations is that for realists, international politics is predominantly a prisoner's dilemma game among countries (e.g., Mearsheimer (2001)). From this perspective, the standard game theoretic model of conflict as a one-shot contest game can be interpreted as a formalization of the fundamental realist view of international relations as a security dilemma. The problem of course is that one can start with a one-shot contest game/security dilemma and, through repeated interactions, end up with disarmed peace sustained, for example, by grim-trigger strategies (e.g., Keohane (1986)). In other words, the Folk Theorem provides a theoretical link between a realist world and very liberal cooperative outcomes. The conundrum is that this extreme liberal prediction is rarely observed in the real world. Why does the Folk Theorem do not work when it is mostly needed?

The paper explores several answers. The Folk Theorem implies that, if countries are not patient enough, disarmed peace cannot be sustained. Still, an equilibrium with armed peace and lower levels of arming than in the one-shot game can be

[4]The seminal paper is Fearon (1995).

[5]Yared (2010) studies an asymmetric repeated game, where the aggressive country seeks concessions from a non-aggressive country.

[6]Acemoglu and Wolitzky (2014) also produce an equilibrium with cycles of conflict, but using a very different model. They consider an overlapping generations model with imperfect information.

sustained. Thus, in equilibrium, countries keep some level of arming to guarantee their share of the disputed resource, which they combine with a credible threat to escalate arming if the agreement is violated. If the resource is fixed, one problem is that, in equilibrium, there is never arming escalation because the threat of it acts as credible deterrence. We show that one simple change to remedy this issue is to assume that the value of the disputed resource is stochastic and varies from period to period. In this context, when the value of the disputed resource is high, arming escalation is necessary to sustain armed peace. Otherwise, countries will deviate, and cooperation will be stopped. [7] Summing up, our change reinstates some of the features of the one-shot security realist dilemma for the infinite dynamic contest game. However, as countries become patient enough, it is still the case that disarmed peace can be sustained. That is, the extreme liberal prediction does not fully disappear. More importantly, the fundamental logic behind the Folk theorem persists even when stochastic variations in the value of the disputed resource require that that countries accept arming escalation when the stakes are high.

Our second change in the model is more corrosive for the Folk Theorem and its disarmed peace prediction when countries are patient enough. When the wipe-out technology is permanent, unarmed peace is not an equilibrium anymore. It is still possible to sustain some cooperation in states in which wiping-out the enemy is not possible, but as soon as the wipe-out opportunity arrives, open conflict is inevitable if countries are patient enough. As we have already mentioned, one problem with this equilibrium is that, after the victor wipes-out its enemy, there is disarmed peace forever. When the wipe-out technology is only temporary, however, this is not the case anymore. In equilibrium, open conflict only confers to the victor temporary control over the disputed resource. Eventually, the defeated country recovers and the resource becomes disputable once again. There is room for some cooperation is states in which wipe-out is not possible even after countries have fought a war. There is temporary unarmed peace after a war settles a dispute. But security competition and war cannot be fully eradicated from the international system.

[7] Also note that the bellicose rhetoric in those periods might escalate as well. One possible explanation is that each country might need to remind its rival that arming has been increased accordingly to deal with the new circumstances, which is important to sustain the most cooperative possible equilibrium.

2 A dynamic model of conflict

We depart from the standard model of conflict by adding several elements that will help explain more realistic features observed historically, especially in the last century. First, we allow countries to interact repeatedly over time. Second, there is a randomly determined disputable resource; that is, the resource's value is stochastic and varies from period to period. Third, after observing each other's military power, countries can agree on a settlement rule such that each country gets a fraction of the resource. If no settlement is agreed, an open conflict happens and a portion of the resource is destroyed. Finally, there is an exogenous randomly determined opportunity that the victor of an open conflict can "wipe-out" the other player at an additional fixed cost; moreover, we distinguish between two cases, whether the wipe-out is permanent or not. Because of the random realization of states that affect the payoffs and actions, we study a stochastic game.[8]

In each period, countries $i = 1, 2$ decide a level of military power denoted by $G_i \geq 0$ (for guns) that perishes by next period. Given a profile of military powers (G_1, G_2), the probability of winning a war is:

$$\pi_i(G_i, G_j) = \begin{cases} \frac{G_i}{G_i+G_j}, G_i + G_j > 0 \\ \frac{1}{2}, G_i = G_j = 0 \end{cases}$$

The outcome of the war decides who has access to a disputable resource that can take a low or a high value: $R \in \{L, H\}$ with probabilities $1-q$ and q, respectively, and satisfying $0 < L < H$. Moreover, this resource is also perishable and replaced by a new realization of R in each period. After countries observe the profile of military power (G_1, G_2), they decide whether to settle or not $S_i \in \{0, 1\}$. If countries settle ($S = S_1 S_2 = 1$), they receive $\pi_i(G_i, G_j)R$ of the resource.[9] If countries do not settle ($S = 0$), this necessarily start an open conflict, a fraction $1 - \theta$ of the resource is destroyed, and country i expects to capture $\pi_i(G_i, G_j)\theta R$ of the resource. Finally, we assume that in order to start an open conflict, a country needs to have a strictly positive stock of military power.[10]

[8] This is a slight extension to a repeated game. Although there is repeated interaction, there is no unique "stage game," since the payoffs and actions depend on different realizations of states variables.

[9] Although there are other equilibrium allocation shares, this particular one is the most natural. Moreover, the settlement rule S is decided simultaneously, however, it is not profitable to deviate from a settlement in this environment. That is, we do not model the possible advantages of a unilateral "unexpected aggression."

[10] That is, $G_i = 0$ implies $S_i = 1$. This assumption rules out the frivolous action of starting a war with no army.

At the beginning of each period, before deciding G_i, a random state realizes that would give the victor of an open conflict, say i, the opportunity to pay an additional cost X to "wipe-out" country j. This exogenous opportunity arrives with a probability ϕ. The opportunity to wipe-out realizes before the decision to spend on military power G_i, but the decision to wipe-out or not is made after winning the conflict. Since the decision to wipe-out may or may not be taken, we need to be explicit about it. We will use a random variable Z to denote that an opportunity to wipe-out has arrived $Z = 1$ or not $Z = 0$. Moreover, we denote by $W_i = 1$ if a country decides to wipe-out and $W_i = 0$ if it does not happen.

After a wipe-out, the game transitions to a state in which the last war victor can claim all the resources at zero cost, and has a chance γ to remain in that state next period. Formally, when a wipe-out occurs, the game transitions to an "uncontestable state," ($C = 0$). The periods in which the resource can be contested will be labeled as "contestable state" and denoted by $C = 1$ (states in which G_i, S_i, W_i are decided). At the end of each period of an uncontestable state, $C = 0$ will repeat next period with probability γ. However, with probability $1-\gamma$, the state will transition back to a contestable state and the loser of the previous conflict can contest the resource again.

Since no decisions are taken at an uncontestable state, to simplify the notation, we will not explicitly model the payoffs and actions on such states. Let $a_i = (G_i, S_i, W_i)$ be the actions taken by player i on contested states and $\psi = (R, Z, C)$ the profile of states. Then, each country's instant payoff when $C = 1$ is:

$$u_i(a_i, a_j, \psi) = \pi_i(G_i, G_j)[S + (1 - S)\theta]R - (1 - S)ZW_iX - G_i, \tag{1}$$

for i and $j = 1, 2$, and $R = L, H$.

After the opportunity to "wipe-out" realizes and is used, as long as $C = 0$ remains, all subsequent payoffs will be R for the victor of the open conflict and zero for the vanquished country. After the state returns to $C = 1$, the payoff is given by (1) once again. Countries discount future payoff by a factor $0 \leq \delta < 1$. Given a stream of actions $a_i^t = (G_i^t, S_i^t, W_i^t)$ from each country at each period, $\{a_i^t\}_{t=0}^{\infty}$, and states $\psi^t = (R^t, Z^t, C^t)$, $\{\psi^t\}_{t=0}^{\infty}$, the (normalized) payoff of the supergame is:

$$(1 - \delta) \sum_{t=0}^{\infty} \delta^t E \left[u_i(a_i^t, a_j^t, \psi^t) \right]$$

where the expectation is taken over states and actions (that depend on and also induce states). In this supergame, a history at the beginning of period t is a sequence of all action profiles and states up to period t:[11]

$$h^t = \left\{ (a_1^0, a_2^0, \psi^0), (a_1^1, a_2^1, \psi^1), \ldots, (a_1^{t-1}, a_2^{t-1}, \psi^{t-1}) \right\},$$

for $t \geq 1$ and $h^0 = \emptyset$ in the initial period, and we assume that the game starts at a contestable state. Since all information is public in this game, the equilibrium concept is subgame perfect equilibrium (SPE). We restrict attention to on-path symmetric equilibria.[12]

3 Repeated game without stochastic states

To better understand the dynamics of the game, we build up in complexity by solving first simpler scenarios. As a first step, in this section we solve the one-shot game and then consider the repeated game without stochastic states (neither resources nor wipe-out opportunities). In section 4 we allow for stochastic resources. Finally, the last element to be included is the possibility to wipe-out in section 5.

3.1 One-shot game

As one would anticipate, the equilibrium of the one-shot game, which is equivalent to setting $\delta = 0$, is characterized by high levels of military power. There are two equilibria: one with settlement and one without settlement, which can be interpreted as open conflict.

Proposition 1. *In the equilibrium with settlement, the static Nash equilibrium is characterized by $G^N(R) = \frac{R}{4}$ and payoffs $V^N = qH/4 + (1-q)L/4$. In the equilibrium without settlement, the gun levels are $G^N(R) = \frac{\theta R}{4}$ and payoffs $V^N = q\theta H/4 + (1-q)\theta L/4$.*

Proof. See the Appendix. □

The decision to settle or not is a coordination sub-game, in which settlement and no-settlement are equilibria. The settlement equilibrium clearly dominates the open conflict equilibrium because countries avoid the destruction associated with

[11] To simplify notation, we can say that even in state $C = 0$, countries can take actions. However, such actions do not affect the payoffs.

[12] That is, there are equilibria in which one country, say i, could get a larger share of the resources. As long as the other country, j, gets a payoff higher than its minimax payoff, that can be an equilibrium. We ignore such equilibria because they do not add relevance or insight to the study, but greatly increase the space of possible equilibria.

war. Otherwise, both equilibria are very similar in the sense that arming has the structure of a prisoner's dilemma leading to an inefficient Nash equilibrium.[13]

3.2 Repeated game

Now we consider a standard repeated environment (i.e., without stochastic states). Thus, the opportunity to wipe-out never arrives ($\phi = 0$) and we set the resource to be a constant $\hat{R}$ throughout the entire repeated game. In addition, an open conflict destroys a proportion $1-\theta$ of the resources; as a consequence, since there is no opportunity to wipe-out, a settlement equilibrium always dominates an open conflict equilibrium. Thus, we ignore equilibria without settlement on-path.

Under the previous considerations, we can utilize a standard folk theorem to characterized the best attainable equilibrium as follows: the efficient allocations, $G^P = 0$ (from Pareto efficient), can be sustained as an equilibrium as long as δ is high enough. The mechanism that allows for such cooperative equilibrium is also standard: countries behave nicely to each other under the credible threat that if anyone ever deviates from such efficient equilibrium, both countries will immediately revert to the static Nash inefficient allocations and remain there forever (i.e. grim-trigger strategies).

More specifically, the payoff from remaining in the efficient equilibrium is $\hat{R}/2$, as nothing is wasted in guns and resources are equally split. However, given that both countries have zero military power, an arbitrarily small deviation by one of those countries would greatly improve its "bargaining power" during the settlement talks, at virtually zero cost. Thus, the one-shot deviation payoff would be $\hat{R}$.[14] Finally, as prescribed by the equilibrium, from that moment on, both countries play the static Nash equilibrium which gives instant payoffs of $\hat{R}/4$. Thus, in order to sustain the efficient equilibrium in a repeated game, the discount factor δ has to satisfy:

$$\frac{\hat{R}}{2} \geq (1-\delta)\hat{R} + \delta\frac{\hat{R}}{4} \tag{2}$$

[13]Note that for any profile of guns choices, settlement (i.e., $S_1 S_2 = 1$) and no settlement (i.e., $S_1 S_2 = 0$) are both Nash equilibrium outcomes. Moreover, settlement always Pareto dominates no settlement. In this sense, the settlement sub-game has the structure of a coordination problem. Two reasonable selection criteria are the following norms. Under a cooperative norm, $S_i = 1$ for all (G_1, G_2) and i. Under the non-cooperative norm, $S_i = 0$ for all (G_1, G_2) and i. Thus, under the cooperative norm there is never open conflict as countries always reach an agreement. On the contrary, under the non-cooperative norm, there is always open conflict.

[14]Here we use the assumption that, since $G_j = 0$, an open conflict is not possible to further deter a deviation.

Note that in the one-shot game with $H = L = \hat{R}$, the Nash payoff would be $\hat{R}/4$.[15] Thus, the efficient equilibrium is an improvement for both countries. Moreover, as δ increases, the right-hand side decreases. Thus, for δ high, the above condition holds.

What if condition (2) is not satisfied? One can see, from Equation (2), that by investing virtually zero resources in guns, a country could secure all the resources when deviating from the cooperative equilibrium. A second best $G^{SB}(\delta) \in (0, \hat{R}/4)$ can still be sustained in the repeated game. The reason is that if $G^{SB}(\delta) > 0$, the potential deviation would not be so profitable during settlement talks. Following this idea, the second best payoff is: $\hat{R}/2 - G^{SB}(\delta)$, which is an equilibrium as long as this payoff is better than a one-shot deviation and then reverting to the static Nash equilibrium forever.[16]

Thus, the equilibrium requires:

$$\frac{\hat{R}}{2} - G^{SB} \geq (1-\delta)\left(\frac{g(G^{SB}, R)}{G^{SB} + g(G^{SB}, R)}\hat{R} - g(G^{SB}, R)\right) + \delta\frac{\hat{R}}{4} \qquad (3)$$

where $g(G, R)$ is the static best-response to G given resources R.[17] Then:

Proposition 2. *Suppose that $H = L = \hat{R}$ and $\phi = 0$. The first best payoffs $\hat{R}/2$ characterized by unarmed peace $G^P = 0$ can be attained as an equilibrium of the repeated game as long as $\delta \geq \hat{\delta} = 2/3$. If $\delta < 2/3$, a second best that dominates the static Nash allocation is sustainable as an equilibrium with military power $G^{SB}(\delta) = \left(\frac{2-3\delta}{2(2-\delta)}\right)^2 \hat{R} < \hat{R}/4$ and payoffs $V^{SB} = \hat{R}/2 - G^{SB}(\delta) > \hat{R}/4$.*

Proof. See the Appendix. □

[15] Also note that an even more severe "punishment payoff" would be to start an open conflict in every period that follows, thus reaching a slightly smaller payoff: $\theta\hat{R}/4$. This would make a deviation from the equilibrium also slightly less attractive, while decreasing the requirements on δ to sustain the efficient zero-arming as an equilibrium. We ignore this feature to keep a clean threshold for $\hat{\delta} = 2/3$.

[16] Moreover, in order to even further deter deviations and since $G^{SB} > 0$, a deviation can also be punished by an open conflict, making the captured resources lower $\theta\hat{R}$. This was impossible in (2) because in equilibrium $G_i = 0$ was expected, thus making an open conflict impossible. Nevertheless, in (3), this additional action would not substantially change any of the results and will only make the δ threshold slightly lower at the cost of making the analysis much more complicated. Therefore, for simplicity, we omit this additional possibility.

[17] The solution to $\max_g \pi_i(g, G)R - g$ is $g(G) = -G + (RG)^{1/2}$. Also note that there are other off-path deviations that would end up on similar equilibria; for instance, open conflict on the first period and settlement forever after, or vice versa, settlement on the first period and then open conflict forever. Nevertheless, the on-path equilibria allocation of resources to guns would be nearly the same.

Summarizing, in the best equilibrium, either there is eternal unarmed peace or a repeated settlement with small military spending. Thus, repeated interactions open the door for substantial international cooperation and even the possibility of an Utopian fully peaceful world. This hardly corresponds with the historical record. In reality, we observe cycles of peace, arms races, and serious instances of open conflict.

4 Stochastic game without wipe-out

The next step is to study the case where the level of resources fluctuates randomly ($R = L, H$), while the opportunity to wipe-out never arrives ($\phi = 0$). As a first step, we also observe that the first best (eternal unarmed peace) can be an equilibrium when $\delta \to 1$. Let V^P be the ex-ante expected payoff from the efficient equilibrium with zero military power. That is equal to $V^P = qH/2 + (1-q)L/2$. Similarly, the expected payoff from the static Nash equilibrium in this stochastic version of the game will be $V^N = qH/4 + (1-q)L/4$. In principle, a deviation can happen in either state: L or H. Thus, analogous to before:

$$(1-\delta)\frac{H}{2} + \delta V^P \geq (1-\delta)H + \delta V^N \tag{4}$$

and

$$(1-\delta)\frac{L}{2} + \delta V^P \geq (1-\delta)L + \delta V^N \tag{5}$$

both would need to be satisfied. Nevertheless, the most tempting moment to deviate from an equilibrium is when the stakes are high, thus:

Lemma 1. *Assume* $\phi = 0$. *If the incentives condition* (4) *is satisfied, then* (5) *will also be satisfied.*

Proof. See the Appendix. □

Given the previous property, the first best of the stochastic game is an equilibrium as long as (4) is satisfied, say at a discount factor δ^P or higher. In addition, there are also equilibria that dominate the static Nash allocation even when $\delta < \delta^P$. Namely, we propose the following strategies: (i) when the disputable resources are high, some minimal level of "military collateral" is allowed without reverting to static Nash; (ii) when the disputable resources are low, unarmed peace is expected; and (iii) in both states, countries settle. That is, countries understand that when the stakes are high, temptation is high and therefore they cannot commit to unarmed peace. However, countries also understand that a minimal "guns flexing" does not mean they must start an open conflict and become eternal enemies. Thus, when the stakes are low, countries can trust each other and follow the unarmed peace allocation again.

Therefore, the proposed equilibrium must satisfy two conditions in order to guarantee a commitment payoff $V^C(\delta) = q(H/2 - G^C) + (1-q)L/2$. The first condition is that on low-stake periods, unarmed peace must be sustainable:

$$(1-\delta)\frac{L}{2} + \delta V^C(\delta) \geq (1-\delta)L + \delta V^N \tag{6}$$

The second condition is that on high-stake periods, a minimal arms race must be high enough to deter the temptation from attacking each other:[18]

$$(1-\delta)\left(\frac{H}{2} - G^C\right) + \delta V^C \geq (1-\delta)\left(\frac{g(G^C, H)}{G^C + g(G^C, H)}H - g(G^C, H)\right) + \delta V^N \tag{7}$$

The previous conditions characterize an equilibrium that can be easily computed. First, we set Equation (7) to be an equality and solve $G^C(\delta)$. Then, we compute $V^C(\delta)$ to find δ^C from Equation (6) that is the minimum discount factor that sustains this equilibrium. Indeed, by continuity, and since (4) binds before (5), as δ decreases, there is a range of discount factors in (δ^C, δ^P) such that this equilibrium exists.[19]

Finally, even if $\delta < \delta^C$, there is still room for an equilibrium that dominates the static Nash allocation. To attain it, however, unarmed peace will be impossible to sustain even in low-stake states. Then, the strategies would be as follows: build a level of guns $0 < \underline{G}(R) < R/4$ in state R and always settle. If a country ever deviates, both immediately revert to static Nash. The two levels of guns must satisfy:

$$(1-\delta)\left(\frac{H}{2} - \underline{G}(H)\right) + \delta \underline{V} \geq (1-\delta)\left(\frac{g(\underline{G}(H), H)}{\underline{G}(H) + g(\underline{G}(H), H)}H - g(\underline{G}(H), H)\right) + \delta V^N \tag{8}$$

and

$$(1-\delta)\left(\frac{L}{2} - \underline{G}(L)\right) + \delta \underline{V} \geq (1-\delta)\left(\frac{g(\underline{G}(L), L)}{\underline{G}(L) + g(\underline{G}(L), L)}L - g(\underline{G}(L), L)\right) + \delta V^N \tag{9}$$

[18] Note that for simplicity, we are ignoring a slightly harsher punishment that is to starting an open conflict. See Footnote 16.

[19] $G^C(\delta) = \frac{\left(\delta^2\left(q^2+4\right)-8\delta+4\right)H}{4(2-(2-q)\delta)^2} + \frac{(1-q)\delta L}{4(2-\delta(2-q))} - \frac{(1-\delta)\left(\left(\delta q^2 H-(q-1)((q-2)\delta+2)L\right)\delta H\right)^{\frac{1}{2}}}{(2-(2-q)\delta)^2}$. However, the closed form expression is not particularly insightful. The existence of this range of equilibria is what matters.

where $\underline{V} = E[R]/2 - E[\underline{G}(R)]$ is the equilibrium payoff and $\underline{G}(R)$ can be computed by solving both Equations with equality. Then:

Proposition 3. *Assume $\phi = 0$.*

i) If $\delta \geq \delta^P = 2H/((2+q)H + (1-q)L)$, $G^P = 0$ is an equilibrium of the stochastic game with payoff $V^P = qH/2 + (1-q)L/2$.

ii) If $\delta \geq \delta^C = \frac{2L^{\frac{1}{2}}}{2qH^{\frac{1}{2}} - (2q-3)L^{\frac{1}{2}}}$ and $\delta < \delta^P$, the best equilibrium is characterized by settlement with a positive military power $G^C(\delta)$ in high stake periods and there is unarmed peace in low stake periods.

iii) If $\delta < \delta^C$, there best equilibrium is characterized by settlement with a positive military power $\underline{G}(R)$ in both states.

Proof. See the Appendix. □

To summarize, when countries are patient enough, there is unarmed peace regardless of the value of disputable resources. For intermediate levels of patience, arming is required to sustain a peaceful settlement when the disputable resource is high. Finally, for low levels of patience, military spending fluctuate from period to period. Specifically, there is arming escalation when the value of disputable resources is high and deescalation when the value of disputable resources is low. Thus, allowing an stochastic disputable resource does not lead to open conflict but it explains periods of more and less intense arming and security competition. Note, however, that the fundamental result of the folk theorem persists. With enough patience, unarmed peace can be sustained.

5 Stochastic game with wipe-out

Finally, we consider the case in which $\phi > 0$; thus, at a cost X, after winning an open conflict, a country can wipe-out its rival.

5.1 Fixed resources with permanent wipe-out

The wipe-out opportunity greatly increases the complexity of the model. To better understand the basic interactions, we first study the case in which the resources are fixed at $\hat{R}$ and a wipe-out is permanent ($\gamma = 1$).
The first step towards constructing equilibria of this super game is to understand that if the wipe-out action is used in equilibrium, there is only one equilibrium

level of military power profiles in periods when the wipe-out opportunity arrives. The reason is that, given that the wipe-out is permanent, the continuation payoff cannot be contingent on "more coopeartive" actions in the future. Thus, we consider the equilibrium of best response functions to the following maximization problem:

$$\max_{G_i} \pi_i(G_i, G_j)\big((1-\delta)(\theta\hat{R} - X) + \delta\hat{R}\big) - (1-\delta)G_i. \tag{10}$$

That is, in states when the wipe-out option is available, a country may take into account that its actions will lead to itself being the unique claimant of the resources in the future (because $\gamma = 1$). Note that the maximization problem in Equation (10) is very similar to the static best response after defining a pseudo-resource $\overline{R}(\delta) = \theta\hat{R} - X + \delta\hat{R}/(1-\delta)$. That is, the objective function in problem (10) becomes $(1-\delta)u_i(a_i, a_j, \psi)$, evaluated at $\psi = (\overline{R}(\delta), 1, 1)$. Obviously, if resources were negative, the solution would be $G_i = 0$. Thus, assuming that the parameters are such that $\overline{R}(\delta) > 0$, the solution to (10) has the same functional form as the (static) best-response. Then,

Lemma 2. *The following statements characterize the solution to Equation* (10) *and its comparison to the static Nash allocations:*

i) The best response to G under Equation (10) *is:*

$$(\overline{R}(\delta)G)^{1/2} - G,$$

provided $G > 0$ and $\overline{R}(\delta) > 0$.

ii) If $\overline{R}(\delta) > 0$, the solution to mutual best-response functions from Equation (10) *is: $\overline{R}(\delta)/4$, with payoffs $(1-\delta)\overline{R}(\delta)/4$. Moreover, $\delta \to 1$ implies $\overline{R}(\delta) > 0$, regardless of how high is X.*

iii) Regardless of the sign of $\overline{R}(\delta)$, the static Nash payoff with settlement ($\hat{R}/4$) is always higher than $(1-\delta)\overline{R}(\delta)/4$. However, the static Nash payoff without settlement ($\theta\overline{R}(\delta)$) may be lower, for δ small.

iv) If $\overline{R}(\delta) > 0$, the most severe punishment equilibrium strategy is to play the static Nash when $Z = 0$ and to play the solution to Equation (10) *and wipe-out when $Z = 1$. This new punishment equilibrium strategy has ex-ante payoff equal to:*

$$V^W = (1-\delta)\frac{\phi\overline{R}(\delta) + (1-\phi)\theta\hat{R}}{4(1-(1-\phi)\delta)}$$

v) If $\overline{R}(\delta) \leq 0$ the repetition of the static Nash allocation without settlement is still the most severe punishment equilibrium payoff.

Proof. See the Appendix. □

Lemma 2 says that for δ large or X small, any equilibrium will end up with a wipe-out with probability one. Moreover, the most severe equilibrium punishment payoffs may not necessarily be the static Nash allocation. The previous result characterizes the "punishment payoffs." However, we note that when $\overline{R}(\delta) \geq 0$, the punishment payoff is also the only equilibrium payoff. This foreshadows an impossibility result that we will discuss below.
Next, in order to study other equilibria, it is useful to understand what possible deviations we must consider. Let us recall that there are three action-stages on each period: first, the level of military power $G_i \geq 0$, second, the decision to settle or not $S_i \in \{0, 1\}$, and third, after not settling, the decision to wipe-out or not $W_i \in \{0, 1\}$ which is only available to the victor of the open conflict (i.e. following no settlement). From the assumptions of the model, there are never profitable deviations on the second stage: we assumed that it is not possible to renege from a settlement agreement (e.g. the other country will still be able to respond instantaneously at no disadvantage). Thus, we consider two possible deviations: a level of military power that is different from the one prescribed by the equilibrium and/or wiping-out when it was not expected:

Lemma 3. *The following statements characterize what types of deviations would be feasible from a candidate to equilibrium:*

i) In a profile of strategies when wiping-out is not a prescribed action, and if wiping-out happens to be a profitable deviation on the last stage of the current period, then it is also profitable to deviate at the first stage by increasing G_i.

ii) In a profile of strategies when wiping-out is a prescribed action, not wiping-out is never a profitable deviation.

Proof. See the Appendix. □

What Lemma 2 and 3 show is that, once we introduce the wipe-out option, the static Nash allocation may not be used as a punishment equilibrium payoff, and to consider deviations from the equilibrium, we must look at deviations that necessarily involve changing the military power (i.e. not exclusively at the wiping-out stage). [20]

[20]These two results extend to the general stochastic game, as long as $\gamma = 1$. Nevertheless, we

We know that if $\overline{R}(\delta) > 0$, then any equilibrium will prescribe to wipe-out as soon as possible. However, in states when the wipe-out option is not available, perhaps some cooperation is possible, as there is a probability of $1-\phi$ that in next period, the wipe-out option will still be unavailable. Indeed, consider the following strategy: play $G^{WS} < \hat{R}/4$ when the wipe-out is not available and wipe-out as soon as the option becomes available. If someone ever deviates, play the strategy prescribed in part (iv) of Lemma 2. This will be an equilibrium as long as:

$$(1-\delta)\left(\frac{\hat{R}}{2} - G^{WS}\right) + \delta V^{WS} \geq (1-\delta)\left(\frac{g(G^{WS})}{G^{WS}+g(G^{WS})}\hat{R} - g(G^{WS})\right) + \delta V^{W} \tag{11}$$

where V^{WS} is the payoff induced by the proposed strategy which equals:

$$V^{WS} = \frac{\phi V^{W} + (1-\phi)(1-\delta)\left(\frac{\hat{R}}{2} - G^{WS}\right)}{1-(1-\phi)\delta}$$

Therefore, by solving G^{WS} as a function of δ from Equation (11), we can obtain the minimum military power that would sustain the equilibrium proposed.
Finally, the last step is to compare two thresholds for the discount factor. First, let $\overline{\delta}$ be the discount factor such that $\overline{R}(\delta) = 0$. From Proposition 2, we know that if $\overline{\delta} < 2/3$, the first best will never be implemented. However, if $\overline{\delta} \geq 2/3$, then any δ satisfying $2/3 \leq \delta \leq \overline{\delta}$ can implement the first best. Then,

Proposition 4. *When the wipe-out option is available, a wipe-out is an absorbing state, and resources are constant at $\hat{R}$, the equilibrium is characterized as follows:*

i) ***Large cost of wiping-out.*** $X \geq (2+\theta)\hat{R}$ *implies* $\overline{\delta} \geq 2/3$. *Then:*

a) If $\delta < 2/3$, then we can sustain the second best from Proposition 2 as equilibrium,

b) If $2/3 \leq \delta \leq \overline{\delta}$, then we can sustain the first from Proposition 2 as equilibrium.

c) If $\overline{\delta} < \delta$, then we can sustain G^{WS} from Equation (11) *as an equilibrium in states where the opportunity to wipe-out has not arrived.*

know from Proposition 3 that when the resources fluctuate, there are several cases to consider. Moreover, although the equilibrium will be characterized by thresholds in δ, those threshold are not necessarily monotonic in the resource $R = L, H$, as we will see later in Equations (21) and (22). Therefore, a full characterization is impractical for the general case. Thus, to finish this section, we provide a full characterization of the simple case.

*ii) **Small cost of wiping-out.** $X < (2+\theta)\hat{R}$ implies $\overline{\delta} < 2/3$. Then:*

a) If $\delta \leq \overline{\delta}$, then we can sustain the second best from Proposition 2 as equilibrium.

b) If $\overline{\delta} < \delta$, then we can sustain G^{WS} from Equation (11) as equilibrium in states where the opportunity to wipe-out has not arrived.

Proof. See the Appendix. □

The intuition behind Proposition 4 is as follows. When wipe-out is relatively costly (formally, $X \geq (2+\theta)\hat{R}$), the Folk theorem applies provided that countries are not excessively patient (formally, $\delta \leq \overline{\delta}$). The reason is that wipe-out works as an investment project, in the sense that it requires to incur a cost in the present for the prospect of a flow of benefits in the future. As a consequence, the more patient countries are the more likely they find open conflict profitable when there is an opportunity to wipe-out. In other words, when wipe-out is relatively costly, there is less room for an Utopian fully peaceful world than when wipe-out is not possible, but it is still the case that armed peace and even disarmed peace can be sustained as an equilibrium when repeated interactions are considered.

When wipe-out is relatively cheap (formally, $X < (2+\theta)\hat{R}$), the predictions of the model are more interesting. Disarmed peace is never an equilibrium outcome before one country is wiped-out.[21] When countries are not patient enough (formally, $\delta \leq \overline{\delta}$), there is armed peace, while when countries are patient enough (formally, $\delta > \overline{\delta}$), open conflict cannot be permanently avoided. In particular, in equilibrium, there is some room for international cooperation (i.e., lower arming than the static Nash) but only until the wipe-out opportunity arrives, when open conflict inevitably breaks out. Disarmed peace only emerges after open conflict permanently settle the dispute.

Proposition 4 finally offers a substantial departure from the Folk Theorem rosy predictions. There are, however, two unrealistic features in the equilibrium described in Proposition 4 part *ii*. First, because the disputable resource is assumed fixed there is no room for arming escalation/deescalation within armed peace (as we obtained in Proposition 3). Second, because wipe-out is an absorbing state, after open conflict, the resource becomes non disputable and, hence, there is disarmed peace forever after. In the two sections that follow we relax these assumptions and obtain more realistic results. Since we are still interested in equilibria with open conflict, we focus on high discount factors.

[21] More formally, for X sufficiently small, the first best is not an equilibrium for any value of δ.

5.2 Stochastic resources with permanent wipe-out

Let us allow resources to vary over time, but still assume that the uncontestable state is an absorbing state ($\gamma = 1$). The main question we want to answer is, can $\delta \to 1$ guarantee the efficient egalitarian equilibrium $G_1 = G_2 = 0$ even when X is arbitrarily small? In the case of $\gamma = 1$, the short answer is no, even for X large (but finite). The reason is that in this setup, the expected long term payoff from cooperation is $V^P = qH/2 + (1-q)L/2$ and the expected long term payoff from wiping-out the other country is $E[R] = qH + (1-q)L$, which doubles V^P. Thus, as δ increases, the wipe-out option indeed becomes more attractive.

Moreover, as studied before, since wiping-out is followed by an absorbing state, the static Nash allocation from Proposition 1 is not always an equilibrium payoff. That is, in standard repeated games, the static Nash equilibrium is also an equilibrium of the repeated game, but this may not be the case when $\phi > 0$ and $\gamma = 1$. Indeed, consider the extension to Equation (10) for $R = L, H$:

$$\max_{G_i} \pi_i(G_i, G_j)\big((1-\delta)(\theta R - X) + \delta E[R]\big) - (1-\delta)G_i \tag{12}$$

We check for what values of δ the static Nash allocation will no longer be an equilibrium in light of the possible deviation from (12). To do so, we note that for $\delta = 0$, the wipe-out option would not be used, as part of the resource is lost $\theta > 0$ and there is a fixed cost to wipe-out $X > 0$. Thus, we compare two payoffs: on the one hand, the eternal static Nash equilibrium is played, even if the opportunity to wipe-out arrives now or in the future, which has to be an equilibrium for δ small. On the other hand, a deviation from static Nash that is a best-response using (12), which should be profitable for δ large. Moreover, note that when the stakes are low, by defining $\hat{L}(\delta) = \theta L - X + \delta E[R]/(1-\delta)$, the objective function in problem (12) becomes $(1-\delta)u_i(a_i, a_j, \psi)$, evaluated at $\psi = (R, Z, C) = (\hat{L}(\delta), 1, 1)$. Similarly, the same applies by defining $\hat{H}(\delta) = \theta H - X + \delta E[R]/(1-\delta)$ when the stakes are high. Therefore, after redefining the resource, the solution to (12) has the same functional form as the (static) best-response. Then,

Proposition 5. *Consider the maximization problem in Equation* (12)*. Then:*

i) The solution to mutual best-response functions from Equation (12) *when resources are* $R = L, H$ *is* $\hat{G}(R) = (\theta R - X + \delta E[R]/(1-\delta))/4$*. For a given* $R = L, H$*, the maximum in* (12) *equals* $(1-\delta)\hat{G}(R)$*. Moreover, if* $\hat{L}(\delta) \geq 0$*, in the supergame, the symmetric equilibrium with the lowest payoff is to wipe-out as soon as possible. As* $\delta \to 1$*, this is always the case, regardless of how high is* X*. Finally, given* $R = L, H$*, the maximum attained to mutual best-responses with payoffs in* (12) *is increasing in* δ *if and only if* $X > \theta R - E[R]$*.*

ii) If we fix G_j, then, the best response to G_j is $(\hat{L}(\delta)G_j)^{1/2} - G_j$ when $R = L$ and $(\hat{R}(\delta)G_j)^{1/2} - G_j$ when $R = H$, assuming the expressions are positive, which is always the case for δ high enough. Moreover, for a fixed G_j, the maximum attained in Equation (12) *converges to $E[R]$ as $\delta \to 1$.*

Proof. See the Appendix. □

If each country expects that its rival is going to start an open conflict, it is a best-response to heavily invest in guns and go to war as well. $\hat{G}(R)$ is computed taking into account that the other country will not settle. Therefore, if $\delta \to 1$ and $\gamma = 1$ it does not matter how high is X, in any equilibrium, countries always go to war as soon as there is an opportunity to wipe-out. Thus, as in Lemma 2, depending on δ, the static Nash allocation is not always an equilibrium of the repeated game. In contrast, for a fixed G_j (in particular, one that would not depend on δ), a best response to it would make G_i arbitrarily large, but it would not increase as fast as the benefit for the future payoff when $\delta \to 1$.
Moreover, unlike Lemma 2, here the static Nash allocation has to satisfy two conditions, which are:

$$(1-\delta)\frac{L}{4} + \delta V^N \geq \frac{g(L/4, \hat{L}(\delta))}{L/4 + g(L/4, \hat{L}(\delta))}\hat{L}(\delta) - (1-\delta)g(L/4, \hat{L}(\delta)) \tag{13}$$

and

$$(1-\delta)\frac{H}{4} + \delta V^N \geq \frac{g(H/4, \hat{H}(\delta))}{H/4 + g(H/4, \hat{H}(\delta))}\hat{H}(\delta) - (1-\delta)g(H/4, \hat{H}(\delta)). \tag{14}$$

Note that (13) and (14) are (in general) satisfied with equality at different levels of δ. Let $\underline{\delta}^L$ be the discount factor that solves (13) with equality and and $\underline{\delta}^H$ be the discount factor that solves (14) with equality. We note that the right-hand side of Equations (13) and (14) is not necessarily monotonic in δ. In particular, it is not possible to guarantee that either $\underline{\delta}^L < \underline{\delta}^H$ or $\underline{\delta}^L > \underline{\delta}^H$. Nevertheless, for δ sufficiently high, the right-hand side of Equations (13) and (14) is increasing in δ and approaches $E[R]$, as $\delta \to 1$. Indeed, in the next two results, we focus on the case $\delta \to 1$.
Consider any equilibrium that Pareto-dominates the static Nash allocation. In particular, consider either the first best allocation or the fluctuating equilibrium from Proposition 3. In both cases, the military power is lower than that of the static Nash allocation. Thus, a deviation from such low levels of military power increases the probability to win war much more compared to the deviation from the static Nash allocations. Therefore,

Proposition 6. *If $\phi > 0$ and $\gamma = 1$, as $\delta \to 1$ any equilibrium prescribes that when $Z = 1$, countries always wipe-out.*

Proof. See the Appendix. □

We note that the equilibria in Proposition 3 were constructed thinking of high discount factors. Thus, the previous result greatly hinders the feasibility of the allocations in Proposition 3. That is, higher discount factors and lower levels of military power increase the feasibility of a wipe-out as an equilibrium.

Finally, it is worth noting that, in principle, the wipe-out action seems efficient when $\gamma = 1$. It is characterized by one period of heavy conflict followed by, although not egalitarian, eternal peace. This may seem good, especially when $\delta \to 1$. However, higher δ also means that countries are willing to sacrifice more to obtain a high future payoff. In turn, this implies that the initial conflict will be extremely costly. That is, let $\mathcal{W}^P = E[R]$ be expected surplus corresponding to the Pareto efficient allocation from Proposition 3 and $\hat{\mathcal{W}}$ be the ex-ante surplus from a symmetric best-response equilibrium to Equation (12) on the first period that the wipe-out option is available ($Z = 1$), followed by eternal peace in which only one country can claim all the resources. From Proposition 5, this implies that $\mathcal{W}^P - \hat{\mathcal{W}} = E[R]/2 + (1-\delta)(X + (1-\theta)E[R])/2$.

Therefore,

Proposition 7.

$$\lim_{\delta \to 1} \mathcal{W}^P - \hat{\mathcal{W}} = \frac{E[R]}{2}$$

which means that $\hat{\mathcal{W}}$ is bounded away from efficiency, even as $\delta \to 1$.

Proof. See the Appendix. □

5.3 Wipe-out is not permanent

If $\gamma < 1$, there is a chance to return to contestable states even after a country has been wiped-out. Because it does not make sense that a country can create a sizable army shortly after a wipe-out, we focus on γ high, but bounded away from 1.

Let us define three low-payoff (punishment) equilibria that will be useful to sustain better equilibria. After a wipe-out, countries transition to a state in which the resources cannot be contested. Let $\overline{V}^W$ and $\underline{V}^W$ be the punishment payoffs of the winner and the loser after a deviation wipe-out, respectively. Let V^{NW} be the "punishment" payoff in contestable states in which countries play the open conflict and wipe-out whenever possible, else they play the static Nash. Then:

$$\overline{V}^W = (1-\delta)E[R] + \delta(\gamma \overline{V}^W + (1-\gamma)V^{NW}) \tag{15}$$

$$\underline{V}^W = \delta(\gamma \underline{V}^W + (1-\gamma)V^{NW}) \tag{16}$$

$$\begin{aligned} V^{NW} &= (1-\phi)\left((1-\delta)\frac{\theta E[R]}{4} + \delta V^{NW}\right) \\ &+ \phi\left((1-\delta)\left(\frac{\theta E[R]-X}{2} - E[G^{NW}]\right) + \delta\frac{\overline{V}^W + \underline{V}^W}{2}\right) \end{aligned} \tag{17}$$

where $G^{NW}(R)$ is the symmetric solution to best response functions that, given R and G, maximize:

$$U^D(G,R) = (1-\delta)\max_g \left\{ \pi_i(g,G)\left(\theta R - X + \frac{\delta(\overline{V}^W - \underline{V}^W)}{1-\delta}\right) - g + \frac{\delta \underline{V}^W}{1-\delta} \right\} \tag{18}$$

Given problem (18), the following result is useful throughout the remainder of this paper:

Lemma 4. *Consider any equilibrium strategy σ_0 such that wipe-out is used whenever possible. Let V_0 be its expected payoff in contestable states, and $\overline{V}_0$ and $\underline{V}_0$ be its expected continuation payoffs in uncontestable states after a wipe-out for the winner and the loser, respectively. Then: $\overline{V}_0 - \underline{V}_0 = (1-\delta)E[R]/(1-\gamma\delta)$. Moreover, consider another equilibrium strategy σ_1 such that wipe-out is also used whenever possible (i.e. in the same states as σ_0). Let $V_0|_{Z=1}$ and $V_1|_{Z=1}$ be the payoff in contestable states conditional on an opportunity to wipe-out, for strategies σ_0 and σ_1, respectively. Then:*

$$V_0|_{Z=1} - \delta \underline{V}_0 = V_1|_{Z=1} - \delta \underline{V}_1$$

Proof. See the Appendix. □

In words, regardless of what the continuation payoff will be after returning to a contestable state (V^{NW} or something else), the difference between payoffs following a wipe-out (analogous to Equations 15 and 16) will still be $(1-\delta)E[R]/(1-\gamma\delta)$. This means that, up to a constant, the solution to the maximization problem defined in (18) will not depend on whether a cooperative or punishment equilibrium is being played.

From Lemma 4, $\overline{V}^W - \underline{V}^W = (1-\delta)E[R]/(1-\gamma\delta)$. As done previously, we define $L^{NW}(\delta) = \theta L - X + \delta E[R]/(1-\gamma\delta)$ and $H^{NW}(\delta) = \theta H - X + \delta E[R]/(1-\gamma\delta)$. Then, it is easy to see that:

Lemma 5. *When $\gamma < 1$ and $X > \theta H + E[R]/(1-\gamma)$, the strategy "wipe-out whenever possible" is not part of any equilibrium as $\delta \to 1$.*

Proof. See the Appendix. □

However, we note that we want to focus on γ high. Therefore, the previous result is only valid for wipe-out costs X that are very high. If we assume that $X < \theta L + \delta E[R]/(1-\gamma\delta)$, which is reasonable as we focus on γ high, wiping-out can be sustained as an equilibrium and the off-path (most severe) punishment payoff is characterized by Equations (15), (16) and (17).
Once the off-path equilibrium has been established, we can use grim-trigger strategies to find conditions for the eternal unarmed peace allocation as an equilibrium:

$$(1-\delta)\frac{L}{2} + \delta V^P \geq (1-\delta)L + \delta V^{NW} \tag{19}$$
$$(1-\delta)\frac{H}{2} + \delta V^P \geq (1-\delta)H + \delta V^{NW} \tag{20}$$
$$(1-\delta)\frac{L}{2} + \delta V^P \geq (1-\delta)(\theta L - X) + \delta \overline{V}^W \tag{21}$$
$$(1-\delta)\frac{H}{2} + \delta V^P \geq (1-\delta)(\theta H - X) + \delta \overline{V}^W \tag{22}$$

where we recall that $V^P = E[R]/2$. In order for this to be an equilibrium as $\delta \to 1$, Equations (19) and (20) need $V^P > V^{NW}$. This is always the case, since $X > 0$ and $\theta < 1$. Moreover, (20) satisfied is a sufficient condition for (19) to be satisfied.
However, comparing Equations (21) and (22) is not an straightforward task. On the one hand, we note that if the loss in resources due to conflict is more than a half ($\theta < 1/2$), Equation (21) is a sufficient condition for (22) to be satisfied. On the other hand, if the loss of resources due to conflict is less than a half ($\theta > 1/2$), Equation (22) is a sufficient condition for (21) to be satisfied.
Moreover, for a fixed δ, if the probability to remain in an uncontestable state, γ, is high, $\overline{V}^W$ can be larger than V^P. Indeed, from Equations (15) and (16), one can see that $\underline{V}^W < V^{NW}$ and $(\underline{V}^W + \overline{V}^W)/2$ is a convex combination of V^P and V^{NW}. Then, we have the following result:

Proposition 8. *Consider $\phi > 0$ and $\gamma < 1$. Assume that $X < \theta L + \delta E[R]/(1-\gamma\delta)$. Then: $1 < \delta(2-\gamma)$ is a sufficient condition for $V^P > \overline{V}^W$. Therefore, this implies that as $\delta \to 1$, Equations (19)-(22) are all satisfied, and eternal unarmed peace is attainable.*

Proof. See the Appendix. □

The key condition in the previous result is $1 < \delta(2-\gamma)$. Note that for this to be satisfied, broadly speaking, γ, although possibly high, needs to be bounded away from 1, and δ needs to be high. That is, note that if $\gamma = 1$, the expression above will never be satisfied. Thus, Proposition 6 can be seen as a special case of Proposition 8. Moreover, $1 < \delta(2-\gamma)$ is a sufficient condition. There are weaker conditions to satisfy $V^P > \overline{V}^W$; nevertheless, that would require explicitly computing both terms, and that would be an extremely tedious task to do.

What happens when the discount factor is not high enough? There are several possible parameters to consider, and a full characterization of all possible combinations does not seem practical. We, thus, focus on one particular equilibrium that calls our attention. If the discount factor does not satisfy the threshold for δ implied by Proposition 8, are there parameter conditions under which unarmed peace, armed peace, and destructive conflict are all observed on the equilibrium path?

We propose the following strategy: when the opportunity to wipe-out does not arrive ($Z = 0$), countries do not start a conflict. If the stakes are low, unarmed peace ensures. If the stakes are high, countries spend some resources in guns, but settle. When the opportunity to wipe-out is available ($Z = 1$), countries always start a conflict. If anyone ever deviates, countries play the previously used strategies implied by Equations (15)-(17). Note that the only difference between this equilibrium and (15)-(17) is that here countries do not play static Nash on states without the wipe-out opportunity. The strategy described above is more complex and, therefore, a formal description requires several steps. First, we define on-path payoffs $V(Z, R)$ as follows:

$$V(0, L) = (1-\delta)\frac{L}{2} + \delta V^* \tag{23}$$

$$V(0, H) = (1-\delta)\left(\frac{H}{2} - G(0, H)\right) + \delta V^* \tag{24}$$

$$V(1, L) = (1-\delta)\left(\frac{L}{2} - G(1, L)\right) + \frac{\delta}{2}\left(\overline{V}^{*W} + \underline{V}^{*W}\right) \tag{25}$$

$$V(1, H) = (1-\delta)\left(\frac{\theta H - X}{2} - G(1, H)\right) + \frac{\delta}{2}\left(\overline{V}^{*W} + \underline{V}^{*W}\right) \tag{26}$$

where:

$$V^* = E[V(Z,R)], \tag{27}$$

$$\overline{V}^{*W} = (1-\delta)E[R] + \delta\left(\gamma\overline{V}^{*W} + (1-\gamma)V^*\right), \text{ and} \tag{28}$$

$$\underline{V}^{*W} = \delta\left(\gamma\underline{V}^{*W} + (1-\gamma)V^*\right) \tag{29}$$

are the expected payoff in a contestable state, the continuation payoff after winning a conflict and the continuation payoff after losing a conflict, respectively. Since there are no incentives to be satisfied in uncontestable states, $\overline{V}^{*W}$ and $\underline{V}^{*W}$ do not impose any further restrictions on countries' behavior. The incentive constraints that must be satisfied are:

$$V(0,L) \geq (1-\delta)L + \delta V^{NW} \tag{30}$$

$$V(0,H) \geq (1-\delta)\left(\pi_i\left(g(G(0,H),H), G(0,H)\right)\theta H - g(G(0,H),H)\right) + \delta V^{NW} \tag{31}$$

We know that for δ large, both constraints are satisfied, and as δ decreases, the first one to bind will be (31). As a first step, we must establish that V^* is larger than V^{NW}:

Lemma 6. *$V^* > V^{NW}$.*

Proof. See the Appendix. □

This inequality is quite obvious, given the second part of Lemma 4. On states in which the wipe-out is possible, both equilibria take the same action. The only difference is that V^* plays a more efficient allocation on states in which wipe-out is not possible. Given Lemma 6, the following result is straightforward:

Proposition 9. *Consider $\phi > 0$ and $\gamma < 1$. Assume that $X < \theta L + \delta E[R]/(1-\gamma\delta)$. Then, for δ high enough, Equations (30) and (31) are satisfied. Thus, the payoffs defined in Equations (23)-(26) are attained in an equilibrium characterized by periods of unarmed peace, armed peace with settlement, and open conflict.*

Proposition 9 offers a more realistic perspective on international relations. First, when wipe-out is not possible, there is room for some international cooperation. Second, when stakes are high arming must be scaled up, but this does not trigger a spiral of arming or open conflict. Third, when wipe-out is possible, open conflict cannot be avoided, but conflict does not settle disputes forever. Eventually, the defeated country reemerged and a new cycle of armed peace and open conflict starts all over again.

6 Conclusions

Starting from an standard static security dilemma game we have developed an infinitely dynamic game in which there are serious opportunities to sustain more cooperative outcomes than the static Nash equilibrium. We have also argued that international relations should be an excellent environment to apply the Folk Theorem and related results from repeated games, but this leads to extremely rosy and unrealistic predictions (in the limit, eternal unarmed peace can be sustained as an equilibrium). Thus, we have introduced several features to the dynamic model in order to make its predictions more aligned with the historical record. In particular, introducing a randomly determined disputable resource and wipe-out opportunities that temporarily knock out the opponent we are able to generate an equilibrium path in which there are periods of unarmed peace, armed peace with settlement, and open conflict. Crucially, in our dynamic model opportunities for international cooperation do not completely disappear but eternal unarmed peace is much less likely to emerge.

There are numerous ways to extend our analysis. We will only mention three of them. First, we have completely ignored coalition formation, which is extremely important in international relations and could significantly expand and/or contract the scope of cooperative outcomes. Second, we have not distinguished between offensive and defensive technologies/weapons. This might also affect the likelihood of armed peace versus open conflict. Finally, we have treated weapons as a perishable good, while in reality they are capital goods that last several years. Relaxing this assumption most likely will seriously complicate the dynamics of conflict but will also open the door for interesting outcomes such as more realistic armed races.

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Appendix

Proof of Proposition 1. It is immediate from the definition of the payoffs. □

Proof of Proposition 2. It is immediate from Equations (2) and (3). □

Proof of Lemma 1. It is immediate from Equations (4) and (5). □

Proof of Proposition 3. The proof is standard. For part (i), given Lemma 1, it is enough to satisfy Equation (4). Then, the result follows. For part (ii), the result follows from Equations (6), (7) and the description of the equilibrium. Finally, part (iii) follows from Equations (8), (9), and the description of the equilibrium. □

Proof of Lemma 2. Parts (i) and (ii) are straightforward. Part (iii) just follows from the definition of $\overline{R}(\delta)$. Part (iv) follows from the definition of the strategies: play static Nash when $Z = 0$ and wipe-out when $Z = 1$. In this case, there is a probability of ϕ that the wipe-out opportunity arrives. Thus, the payoff satisfies:

$$V^W = \phi(1-\delta)\frac{\overline{R}(\delta)}{4} + (1-\phi)\left((1-\delta)\frac{\theta\hat{R}}{4} + \delta V^W\right).$$

Solving V^W yields the result. Finally, part (v) also follows immediately since G cannot be negative.

□

Proof of Lemma 3. For part (i), regarding a deviation exclusively during the wipe-out decision, we recall that G_i and no-settlement are already fixed at that stage. Thus, a deviation would be non-profitable if:

$$\delta V' \geq -(1-\delta)X + \delta\hat{R}, \tag{32}$$

where V' is an equilibrium payoff without wipe-out.
If Equation (32) does not holds, consider the decision at the first stage, when choosing G_i. The ex-ante payoff from deviating to a different army size $\tilde{G}_i$ and then wipe-out would be:

$$\pi_i(\tilde{G}_i, G_j)\left((1-\delta)(\theta\hat{R} - X) + \delta\hat{R}\right) + (1 - \pi_i(\tilde{G}_i, G_j))\delta V' - (1-\delta)\tilde{G}_i$$

The derivative of the above Equation with respect to G_i is:

$$\frac{\partial}{\partial G_i}\pi_i(\tilde{G}_i, G_j)\left((1-\delta)\theta\hat{R} + \delta(\hat{R} - V') - (1-\delta)X\right) - (1-\delta)$$

If we evaluate the expression above at the original G_i, we see that the expression is positive. The reason is that since $\delta(\hat{R} - V') - (1-\delta)X > 0$ by assumption, a sufficient condition for the term to be positive is that $\frac{\partial \pi_i}{\partial G_i}\theta R \geq 1$. Since we restrict attention to symmetric equilibria, $\frac{\partial \pi_i}{\partial G_i} = 1/4G$, and since any efficient equilibrium will have $G \leq \theta R/4$ (which is the static Nash allocation in the case of no settlement), the result follows.

Part (*ii*) is straightforward from Equation (10).

□

Proof of Proposition 4. The relationship between X and $\overline{\delta}$ comes directly from the definition of $\overline{R}(\delta)$. Then, for part (*i*), that means that there is a range of discount factors in $(2/3, \overline{\delta})$ such that the first best from Proposition 2 can be an equilibrium as the discount factor is larger than 2/3 but it is not large enough to make the wipe-out option profitable. Then, the result follows. Finally, for part (*ii*), $\overline{\delta} < 2/3$ means that there is no range of discount factors such that the first best from Proposition 2 can be an equilibrium. Then, the result follows. □

Proof of Proposition 5. For part (*i*), the properties of $\hat{G}(R)$ come from a slight extension to Lemma 2 and 3. The proof is straightforward. Finally, the monotonicity in the value of Problem (12) comes from computing the maximum: $(1-\delta)\hat{G}(R)$. For part (*ii*), the best response, call it G_i^*, to any fixed G_j with any resources R^* is always $G_i^* = (R^* G_j)^{1/2} - G_j$. Finally, the maximum is attained by replacing this expression into the payoff, to obtain:

$$(1-\delta)\left(\pi_i(G_i^*, G_j)\left(\theta R - X - \frac{\delta}{1-\delta}E[R]\right) - G_i^*\right)$$

By using either $\hat{L}(\delta)$ or $\hat{H}(\delta)$ in the best response G_i^*, the above expression reduces to:

$$(1-\delta)\left((R^*)^{1/2} - (G_j)^{1/2}\right)^2$$

where R^* is either $\hat{L}(\delta)$ or $\hat{H}(\delta)$, for $R = L, H$, respectively. Finally, applying the limit as $\delta \to 1$, the expression above converges to $E[R]$. □

Proof of Proposition 6. From part (*ii*) of Proposition 5, we know that the right hand side of Equations (13) and (14) converges to $E[R]$, as $\delta \to 1$. Therefore, the result follows. □

Proof of Proposition 7. At the Pareto efficient allocation, there is never war and countries share the resources. Thus, $\mathcal{W}^P = E[R]$. On the other hand, from Proposition 5, we know that the maximum attained at Equation (12) is $(1-\delta)\hat{G}(R) = ((1-\delta)(\theta R - X) + \delta E[R])/4$. Thus, the ex-ante surplus when the wipe-out opportunity is know to be taken is twice as much the expectation of that expression: $\hat{\mathcal{W}} = ((1-\delta)(\theta E[R] - X) + \delta E[R])/2$. After some rearrangements, we obtain: $\mathcal{W}^P - \hat{\mathcal{W}} = E[R]/2 + (1-\delta)(X + (1-\theta)E[R])/2$. Then, the result follows.

□

Proof of Lemma 4. For any equilibrium strategy σ_0, the payoffs following a wipe-out will be:

$$\overline{V}_0 = (1-\delta)E[R] + \delta(\gamma\overline{V}_0 + (1-\gamma)V_0) \quad (33)$$
$$\underline{V}_0 = \delta(\gamma\underline{V}_0 + (1-\gamma)V_0) \quad (34)$$

Therefore, the difference $\overline{V}_0 - \underline{V}_0$ follows from the definition, regardless of V_0. Now, consider the analogous to the maximization problem in Equation (18), but with continuation payoffs $\underline{V}_0$ and $\overline{V}_0$. Note that by replacing $\overline{V}_0 - \underline{V}_0$ for its value $(1-\delta)E[R]/(1-\gamma\delta)$, the solution (but not the value) of the problem does not depend on $\underline{V}_0$ and $\overline{V}_0$. Indeed, it becomes:

$$(1-\delta)\max_g \left\{ \pi_i(g,G)\left(\theta R - X + \frac{\delta E[R]}{1-\gamma\delta}\right) - g + \frac{\delta \underline{V}_0}{1-\delta} \right\}$$

The last term: $\frac{\delta\underline{V}_0}{1-\delta}$ does not depend on g, therefore the solution does not depend on the continuation payoffs. Finally, assume that there is another equilibrium strategy σ_1 such that it also wipes-out when $Z=1$ and the state is contestable. Then, the equilibrium G in that state is the same for σ_0 and σ_1. Therefore, up to a constant, the maximization problems are identical at that state. Thus:

$$V_0|_{Z=1} - \delta\underline{V}_0 = V_1|_{Z=1} - \delta\underline{V}_1$$

□

Proof of Lemma 5. It is immediate to see that if $X > \theta H + E[R]/(1-\gamma)$, even if δ is arbitrarily close to 1, $L^{NW}(\delta)$ and $H^{NW}(\delta)$ are both negative, meaning that wipe-out is never a part of any equilibrium. □

Proof of Proposition 8. Instead of proving that $V^P > \overline{V}^W$, we prove an even stronger condition. We show that the difference $V^P - \overline{V}^W$ is larger than $V^{NW} - \underline{V}^W$, which we will show is strictly positive.

First, from Equation (16), we find that $\underline{V}^W = \delta(1-\gamma)V^{NW}/(1-\delta\gamma)$, with the factor $\delta(1-\gamma)/(1-\delta\gamma) < 1$. Moreover, $V^{NW} \geq 0$; otherwise a country can just not participate in any conflict and secure zero.

Then, $X < \theta L + \delta E[R]/(1-\gamma\delta)$ implies that the solution to (18) has $G^{NW}(R) > 0$ for $R = L, H$, and that $U^D(G,R) > 0$. This in turn implies that V^{NW} and $\underline{V}^W$ are both positive. Thus, $V^{NW} - \underline{V}^W > 0$.

Finally, we find a sufficient condition for $V^P - \overline{V}^W > V^{NW} - \underline{V}^W > 0$. We rearrange terms as as follows:

$$
\begin{aligned}
V^P - V^{NW} &> \overline{V}^W - \underline{V}^W \\
\frac{1}{2}V^P - \frac{1}{2}V^{NW} + V^{NW} &> \frac{1}{2}\overline{V}^W - \frac{1}{2}\underline{V}^W + \underline{V}^W \\
\frac{1}{2}V^P + \frac{1}{2}V^{NW} &> \frac{1}{2}\overline{V}^W + \frac{1}{2}\underline{V}^W \\
\frac{1}{2}V^P + \frac{1}{2}V^{NW} &> \frac{1-\delta}{1-\gamma\delta}V^P + \frac{\delta(1-\gamma)}{1-\gamma\delta}V^{NW} \\
\left(\frac{1}{2} - \frac{1-\delta}{1-\gamma\delta}\right)V^P &> \left(\frac{\delta(1-\gamma)}{1-\gamma\delta} - 1\right)\frac{V^{NW}}{2} \qquad (35)
\end{aligned}
$$

where, the fist line is just a rearrangement of the desired inequality; the second line multiplies both sides by 1/2 and adds $V^{NW} > \underline{V}^W$; the third line simplifies; the fourth line replaces $(\overline{V}^W + \underline{V}^W)/2$ that is obtained from Equations (15) and (16); and the fifth line rearranges the terms.

On Equation (35), the right-hand side is negative, since $\delta(1-\gamma)/(1-\gamma\delta) < 1$ and $V^{NW} > 0$. The left-hand side will be positive as long as $1/2 > (1-\delta)/(1-\gamma\delta)$, or rearranging: $\delta(2-\gamma) > 1$.

□

Proof of Lemma 6. To simplify notation, let $u^* = E[R]/2 - qG(0, H)$ and $u^{NW} = E[R]/4$ be the expected instant payoffs conditional on a contestable state without the option to wipe-out, for the equilibria with payoff V^* and V^{NW}, respectively. We note that the equilibrium with payoff V^* is not playing static Nash. Thus, $G(0, L) = 0 < L/4$ and $G(0, H) \leq R/4$. Thus, $u^* > u^{NW}$.

Then, using the notation from Lemma 4, $V^* = (1-\phi)\big((1-\delta)u^* + \delta V^*\big) + \phi V^*|_{Z=1}$ and $V^{NW} = (1-\phi)\big((1-\delta)u^{NW} + \delta V^{NW}\big) + \phi V^{NW}|_{Z=1}$. By bringing V^* and V^{NW} to the left-hand side of both expressions, respectively, and then subtracting the later from the former:

$$
\begin{aligned}
(1-\delta(1-\phi))(V^* - V^{NW}) &= (1-\phi)(1-\delta)(u^* - u^{NW}) + \phi(V^*|_{Z=1} - V^{NW}|_{Z=1}) \\
(1-\delta(1-\phi))(V^* - V^{NW}) &= (1-\phi)(1-\delta)(u^* - u^{NW}) + \phi\delta(\underline{V}^{*W} - \underline{V}^W) \\
(1-\delta(1-\phi))(V^* - V^{NW}) &= (1-\phi)(1-\delta)(u^* - u^{NW}) + \phi\delta^2\frac{1-\gamma}{1-\gamma\delta}(V^* - V^{NW})
\end{aligned}
$$

where Lemma 4 was used from the fist to the second line, and Equations (16) and (29) were used from the second to the third line. This last expression implies that, since $u^* > u^{NW}$, $1-\delta(1-\phi) > \phi\delta^2(1-\gamma)/(1-\gamma\delta)$ implies $V^* > V^{NW}$. Rearranging terms, this desired condition is equivalent:

$$1 > \delta \left(1 - \phi + \phi \frac{\delta - \delta\gamma}{1 - \delta\gamma} \right)$$

which always holds.

□

Proof of Propositon 9. The left-hand side of Equation (30) and (31) will approach V^* as $\delta \to 1$. Since $V^* > V^{NW}$, the inequalities in (30) and (31) will be satisfied for δ high enough. Therefore, the described strategies with payoffs in Equations (23)-(26) are an equilibrium. □