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ABSTRACT

This paper shows that the outcome of trade wars for tariffs and welfare will be affected by the monetary policy regime. The key message is that trade policy interacts with monetary policy in a way that magnifies the welfare costs of discretionary monetary policy in an international setting. If countries follow monetary policies of flexible inflation targeting, trade wars are relatively mild, with low equilibrium tariffs and small welfare costs. Discretionary monetary policies imply much higher tariffs, high inflation rates, and substantially larger welfare costs. We quantify the effects of a global trade war among major economies using estimates of trade elasticities, economic size, net foreign assets and trade openness. We find large welfare benefits of an inflation targeting monetary policy for all countries.

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1 Introduction

This paper studies the consequences of trade wars in an open-economy macroeconomic model with sticky prices. The key message of the paper is that trade policy interacts with monetary policy in a way that magnifies the welfare costs of discretionary monetary policy in an international setting, both in terms of inflation and in average tariff levels. By contrast, we find that a policy of flexible inflation targeting delivers substantial welfare gains, not just in low inflation rates, but also lower tariffs.

The recent rise in protectionist policies among major countries has led to fears of a global trade war. A steep escalation in tariffs between the U.S. and China in 2018, and the more recent domestic protectionism in U.S. legislation suggest an increasing breakdown of the system of rules-based international trading arrangements. In a global trade war, countries would pursue optimal trade policies independently, restricting trade to gain national advantage. A trade war would reduce global welfare, and may leave all countries worse off than under a cooperative trading system.

The study of optimal trade policy in non-cooperative environments has a long history in the literature on international trade. Johnson (1953)'s classic paper pictures a trade war as a non-cooperative equilibrium where each country chooses an optimal tariff to improve its terms of trade. This basic theory has been extended in many directions to incorporate features such as variations in country size, intra-industry trade, imperfect competition, and firm heterogeneity.[1] Other papers have reported quantitative estimates of the size of tariffs and the welfare costs of trade wars in calibrated models of global trade. For instance, Ossa (2014) estimates optimal tariffs in a trade war using a quantitative multi-country, multi-industry model, and finds that average tariffs in an all-out trade war would be greater than 60 percent.[2]

All of these studies are based on models with fully flexible prices, where there is no effective role for monetary policy. By contrast, our paper constructs a model of trade wars in a setting where monetary policy matters.[3] In our model, optimal tariffs are determined by traditional terms-of-trade motives, but tariffs are set within an open-economy macroeconomic model with nominal rigidities. In this environment, the incentive to impose tariffs and the outcome of trade wars will depend critically on the monetary policies being followed by each country.

At first glance, it may not seem obvious that monetary policy would have an appreciable effect on the incentive to engage in trade protection. However, the open-economy macroeconomics

[1]See for instance, Kennan and Riezman (1988), Syropoulos (2002), Gros (1987), Bagwell and Staiger (2003), Opp (2010), Felbermayr, Jung, and Larch (2013) and Campolmi, Fadinger, and Forlati (2014). For recent surveys, see Bagwell and Staiger (2016) and Caliendo and Parro (2021). We note also that there is a large literature that models optimal tariff setting based on motivations that go beyond the terms of trade manipulation. See for instance Grossman and Helpman (1995).

[2]See also Perroni and Whalley (2000), Broda, Limao, and Weinstein (2008), and Ossa (2011) for other quantitative studies of optimal tariffs.

[3]As discussed below, a growing recent literature explores the impact of trade policy shocks in open economy macro models. See Barattieri, Cacciatore, and Ghironi (2021), Erceg, Prestipino, and Raffo (2023), Furceri et al. (2018), and Lindé and Pescatori (2019) among others. A key difference with our paper is that these studies take tariffs as exogenous.

literature has long shown that the terms of trade represents one of the key channels through which monetary policy operates.[4] Further, in second-best environments, most policies – including both trade and monetary policies – will be interconnected. Since a key driver of trade policy in our model is the extent to which tariffs can affect the terms of trade, it is likely that the stance of monetary policy will have implications for the choice of trade policy if there is a limited set of policy instruments, since both policies affect the terms of trade.

We develop an open-economy macroeconomic model with sticky prices where countries specialize in a range of goods. Households consume and supply labor, trade goods and bonds, and monopolistically competitive firms maximize profits subject to price adjustment costs. For the baseline model, prices are set in producer currencies. Benevolent governments set tariffs to maximize social welfare in their own country, but we explore different assumptions about how monetary policy is implemented. Policymakers impose tariffs to gain strategic advantage relative to trading partners through the terms-of-trade channel, but the incentive to levy tariffs is strongly affected by the degree of price rigidity and the stance of monetary policy.

Our key comparison is the outcome of a trade war when monetary policy consists of flexible inflation targeting, which embeds a degree of commitment, and the outcome under a discretionary policy, where the central bank determines the inflation rate in each period without commitment. We ask how different monetary policies affect the equilibrium degree of protection and the welfare costs of a trade war.

It is well known that a rules-based monetary policies dominate discretionary policies in an economy with production distortions, due to the inflation bias stemming from discretionary policy. But in an open economy, the inflation bias is tempered by the incentive to use contractionary monetary policy to improve the terms of trade, thus acting so as to reduce the welfare costs of discretion (see for instance Corsetti and Pesenti (2001)). However, with a trade war, a large gap opens up between a rules-based monetary policy and monetary discretion. With an inflation targeting rule, both inflation and tariffs are low – in fact tariffs are lower than they would be under flexible prices – since the rule dampens the incentive to impose tariffs. But with discretion, both inflation and tariffs are significantly higher. Inflation rates are higher because monetary policy now focuses solely on the production distortion. Tariffs are higher because, with a discretion monetary policy, the tariff focuses solely on manipulating the terms of trade. As a result, the welfare costs of monetary discretion are significantly magnified in a global economy where countries engage in trade wars.

Two assumptions are central to our results. First, we assume that both tariffs and monetary policy operate in economies with pre-existing distortions tied to monopoly markups on local varieties, implying an inefficient steady state. This ensures that the choice of optimal tariffs interacts with the stance of monetary policy. Second, prices are sticky so that monetary policy plays a role in equilibrium outcomes. These assumptions act together to ensure that optimal tariffs depend on the stance of monetary policy, yet depending on which type of monetary policy,

[4] This has been referred to as the 'terms-of-trade externality'. See Corsetti and Pesenti (2001), Benigno and Benigno (2003) or Ferrero (2020) and references therein for a recent survey.

they may be either higher or lower than that of a flexible price economy.

The main body of the paper is organized in three parts. The first part explains the central intuition in a simple analytical model of a small open economy with sticky prices that sets a tariff to exploit monopoly power in its export good. The second part extends the analysis to a two-country setting in which countries engage in a trade war under alternative assumptions about monetary policy. In a third part, the results are applied to a multi-country calibrated world-economy model accounting for estimated trade elasticities, net foreign assets, trade openness, and economic size.

In the first part (Section 3), we build on a simple small open economy model. If the steady state is inefficient and monetary policy set by an inflation targeting rule, the optimal tariff is lower under sticky prices than under flexible prices. The reason is that a tariff hike reduces the natural real interest rate and creates deflationary pressures. With sticky prices, deflation reduces output. Given that output is already inefficiently low, this reduces the incentive to levy a tariff.

If monetary policy is set under discretion instead of an inflation targeting rule, the presence of sticky prices leads to the opposite result. Tariffs are higher than under flexible prices and increasing in price stickiness. As described above, this comes from the dilemma between the terms-of-trade externality and the monopoly distortions faced by the discretionary monetary authority. On the one hand, a restrictive monetary policy (deflation) reduces output and appreciates the terms of trade. On the other hand, an expansionary monetary policy (inflation) brings output closer to its socially efficient level. Acting alone, monetary policy balances these two objectives and may choose either a positive or a negative inflation rate. But tariffs offer a separate instrument to improve the terms of trade, which allows monetary policy to focus solely on the monopoly distortion. The end result is an inflation rate that is higher than that without a tariff, and a tariff rate that is higher than that under flexible prices.

The second part (Section 4) extends the model to a full-blown two-country setting. Output is produced with labor and intermediate good inputs from both countries. Capital markets are open and households trade nominal one-period bonds internationally. In this case, tariffs are determined in a Nash equilibrium of a non-cooperative game between policymakers. In a symmetric equilibrium, both countries are worse off than under free trade. But again, the tariff outcome depends critically on the monetary policy stance. Under flexible inflation targeting, tariffs are 2.5 percentage point lower than they would be in a flexible price equilibrium and both countries are better off under sticky prices. Inflation targeting thus has welfare benefits over and above the benefits of low inflation, effectively fostering reduced protection and expanded trade. On the contrary, under discretionary monetary policy, policymakers pursue a more aggressive trade policies, and tariffs jump by 6 percentage points. At the same time, both countries choose positive inflation rates aimed at reducing the domestic output distortion. As a result, welfare is much lower than under flexible prices. The costs of discretionary monetary policy in this case are two-fold. The first is the usual cost of high *ex-post* inflation, but it is compounded by the costs of lower trade induced by higher tariffs.

In the baseline case, we assume zero steady-state net foreign assets (NFA). In Section 5, we relax this assumption and allow for steady-state net external indebtedness. In this case, a key additional determinant of tariff levels is the currency in which external assets are denominated. This is due to an additional 'wealth effect' that governs the incentive to raise tariffs in the presence of non-zero NFA. If NFA are denominated in a country's currency, this country will be more protectionist if it is a net creditor, since in this case it will attempt to further improve its terms of trade to raise the returns on its net external assets. If alternatively the country is a net debtor it will set lower tariffs to reduce the payments on its net external debt. But our main result continues to hold since, with an inflation targeting monetary policy, tariffs are lower in both countries than they would be under flexible prices.

The third part of the paper (Section 6) undertakes a data-driven quantitative analysis of optimal non-cooperative tariffs in a calibrated multi-country model. We consider a sample of major countries, using country-specific calibrations exploiting estimates of economic size, trade openness, trade elasticities, population size, and net external debt positions. The model is solved based on the assumption that NFA are denominated in U.S. dollars. Overall, the sample covers more than 60% of world GDP. We compute the equilibrium tariff rates in a Nash game where each country chooses its welfare-maximizing tariff taking as given the tariff rates of all other countries. In the baseline case, we assume that each country follows a monetary policy of flexible inflation targeting. This gives the simulated outcome of a full-scale global trade war. The average tariff rate among all countries is 13.7 percent, but tariffs differ considerably across countries, ranging from 5.6 percent to 17.2 percent, depending on economic size, estimated trade elasticities and trade openness.

In a purely flexible price version of the model, tariffs would be higher for all countries. The average tariff rate would be 18 percent, and welfare would be lower for all countries. We then solve the model under the assumption that all countries follow a purely discretionary monetary policy. In that case, average tariffs rise to 19 percent, ranging from 11 to 24.4 percent and all countries have positive rates of inflation. Consistent with our theoretical findings, relative to the flexible price equilibrium, welfare is lower for all countries when monetary policy is discretionary. We finally perform a counterfactual exercise and compute the implied tariff rates in the absence of trade imbalances. We find that the U.S. would be significantly more protectionist under zero NFA compared to the baseline case, as their tariff rate would increase by 50 percent. Intuitively, as a large net debtor in its own currency, the U.S. tends to soften its tariff rate relative to a zero NFA benchmark, so as to reduce its foreign interest payments.

Last, Section 7 returns to the basic two-country model to focus on cases where monetary policy is constrained in certain ways beyond the baseline model. We first look at the situation where one country maintains an exchange rate peg against the other. This leads to an equilibrium with significantly lower tariffs than in the model with flexible inflation targeting. With a fixed exchange rate, raising the tariff rate puts upward pressure on inflation and thus diminishes the incentive to engage in a tariff war. We then analyze a situation of 'dominant currency pricing', where the currency of one country is used for price setting in all traded goods. This leads to a significant asymmetry in the trade war, with the dominant currency country being significantly less protectionist than its trading partner. The reason is that, with all prices set in its own currency, it is more difficult for the dominant currency country to manipulate terms of trade in its favor. But again, this result depends on the stance of monetary policy. If we alternatively consider an environment with dominant currency pricing and discretionary monetary policy, we find the opposite – the dominant currency country would be more protectionist.

2 Literature

Our paper draws inspiration from a number of different areas. First, our paper builds on a long tradition of macroeconomic models dealing with monetary policy in open economies. Using a two-country model with monopolistic competition, Corsetti and Pesenti (2001) show how national welfare may depend on a terms-of-trade externality. There are many subsequent papers analyzing optimal monetary policy in different open-economy frameworks, among them Benigno and Benigno (2003), Galì and Monacelli (2005), Faia and Monacelli (2008), de Paoli (2009), Bhattarai and Egorov (2016), Groll and Monacelli (2020), Fujiwara and Wang (2017), or more recently Egorov and Mukhin (2023). Most if not all of the above contributions highlight the importance of the terms-of-trade externality for the design and effects of monetary policy in open economies. We prolong this tradition in characterizing the interaction between the strategic choice of tariff rates and the stance of monetary policy.

Second, there is trade literature on optimal tariffs and trade wars, as discussed above, and surveyed in Bagwell and Staiger (2016) and Caliendo and Parro (2021). Johnson (1953) is the most celebrated early paper showing graphically an equilibrium where each country sets an optimal tariff in a non-cooperative game. Kennan and Riezman (1988) and Syropoulos (2002) note that large countries are more likely to gain in tariff wars. Gros (1987) derives optimal tariffs in a model of intra-industry trade and shows that even small countries can gain from imposing a tariff. Opp (2010) considers a trade war in a Ricardian model with a continuum of goods, Felbermayr, Jung, and Larch (2013) compute the optimal tariffs in a trade war using the Melitz (2003) model, and Campolmi, Fadinger, and Forlati (2014) offer a detailed analysis of optimal non-cooperative policies with a large set of instruments, including tariffs.[5]

The surveys on optimal trade taxes provided by Bagwell and Staiger (2016) and Caliendo and Parro (2021) also discuss a large body of empirical works on the effects of tariffs. An early paper by Perroni and Whalley (2000) provides quantitative estimates of non-cooperative tariffs in a simple Armington model. Ossa (2011) provides such estimates in a Krugman model which features only new-trade production relocation effects. Broda, Limao, and Weinstein (2008) test the optimal tariff formula using estimates of the inverse export supply elasticities faced by a number of non-WTO member countries. Ossa (2014) explicitly models a trade war in a quantitative

[5] More generally, our paper also relates to the literature on tax and structural reforms to manipulate the real exchange rate, which includes Correia, Nicolini, and Teles (2008), Hevia and Nicolini (2013), Farhi, Gopinath, and Itskhoki (2014), Eggertsson, Ferrero, and Raffo (2014), Cacciatore et al. (2016), Auray, Eyquem, and Ma (2017) or Barbiero et al. (2019).

multi-country model.[6,7] As noted above, all of these papers focus on trade models without a sticky prices or a role for monetary policy, which is the focus of our study.

By contrast, a more recent literature has explored the impact of trade policy within the modern macroeconomic toolkit. Barattieri, Cacciatore, and Ghironi (2021) investigate empirically the impact of exogenous changes in tariffs in an SVAR framework, and develop a small open economy model with firm entry and endogenous tradability that successfully rationalizes the empirical evidence. We adopt an alternative approach, consider tariffs as endogenous and explore the consequences of alternative strategic settings for both monetary policy and tariffs. Another paper by Erceg, Prestipino, and Raffo (2023) looks at the impact of trade policies in the form of import tariffs and export subsidies. They find that the effects critically depend on the response of the real exchange rate, and that in turn depends on the expectations about future policies and potential retaliation from trade partners. A recent paper by Furceri et al. (2018) examines the macroeconomic consequences of tariff shocks, and shows that these shocks are generally contractionary. Lindé and Pescatori (2019) study the conditions under which Lerner symmetry holds, and how this affects the macroeconomic costs of a trade war. However, none of these papers model the endogenous determination of trade policy. Our work also relates to a large policy-focused literature on the relationship between exchange rates and trade policy. Eichengreen (1981) and Krugman (1982) represent early contributions while more recent studies include Oatley (2010), Gunnar and Francois (2006), Bown and Crowley (2013).

Finally, there is a literature in a similar vein to our paper, modeling endogenous trade policy in a macro framework. Bergin and Corsetti (2020) also consider tariffs as policy instruments in addition to monetary policy, but their focus is rather on the implications of monetary policy on the building of comparative advantages. Jeanne (2021) investigates the interaction between 'currency wars' and 'trade wars' in an analytical framework of a continuum of small open economies with downward nominal wage rigidity and, in some cases, a global liquidity trap, and explores the benefits of international cooperation. By contrast, our study is focused on a two-country model where countries are large. Further, it focuses on a discretionary Ramsey non-cooperative approach to policymaking.[8] We abstract from considering cooperative policies but instead look at how the monetary policy framework affects the intensity of trade wars. Bergin and Corsetti (2023) is also closely related to our paper. They develop a multi-country DSGE model with trade in intermediate goods and firms entry. They look at the optimal response of monetary policy to exogenous tariff shocks, which they find to be expansionary given the deflationary effects of tariff hikes. While relying on related mechanisms, our paper focuses on the endogenous formation

[6]See also Lashkaripour (2021) and Fajgelbaum and Khandelwal (2022) for more recent empirical approaches estimating the cost of trade wars.

[7]We note also that there is a large literature on modeling implicit or explicit trade agreements between countries. See for instance Bagwell and Staiger (2003) and subsequent literature. We abstract from the possibility of sustainable trade agreements in the current paper. See Auray, Devereux, and Eyquem (2022) for an example of sustainable trade policy in a macroeconomic model with sticky prices.

[8]This type of approach echoes the approach of Chari, Nicolini, and Teles (2018) or Auray, Eyquem, and Gomme (2018), although these papers focus on flexible price environments.

of non-cooperative tariffs.[9] Costinot, Lorenzoni, and Werning (2014) argue that optimal capital controls may arise from differences in output growth across countries. We focus on steady-state outcomes, so their argument does not apply.[10] Finally, Korinek (2016) analyzes the conditions under which international policy cooperation can be Pareto-improving, and shows that tariffs may be used to complete international financial markets. While our paper speaks to the same theme and analyzes international policy interaction in distorted environments, we abstract from discussing the interesting issue of cooperation.[11]

3 An Example Model

We first describe a simplified special case of our general model. Consider a small economy model where trade is balanced every period and the Foreign demand curve for the Home export good is given. The full two-country model is described in Section 4.

3.1 Equilibrium Conditions

Households. Preferences over consumption and hours are given by:

$$\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j U\left(C_{ht+j}, C_{ft+j}, H_{t+j}\right), \tag{1}$$

where

$$U\left(C_{ht}, C_{ft}, H_t\right) = u\left(C_{ht}, C_{ft}\right) - \ell\left(H_t\right). \tag{2}$$

Here, $\beta < 1$ is the discount factor and u is continuous, twice differentiable, and satisfies $u_{c_{ii}} < 0$ and $u_{c_{ij}} \geq 0$, for $i = \{h, f\}$, and $i \neq j$. Consumption of the Home export good is C_{ht}, and consumption of the Foreign imported good is C_{ft}.[12] Function $\ell(.)$ is a continuous and twice differentiable function of hours worked, satisfying $\ell'(.) > 0$, and $\ell''(.) > 0$. The Home country budget constraint is:

$$B_t + P_{ht}C_{ht} + (1+\tau_t) S_t P^*_{ft} C_{ft} = R_{t-1}B_{t-1} + W_t H_t + \Pi_t + TR_t, \tag{3}$$

where P_{ht} (P^*_{ft}) is the Home (Foreign) goods price in Home (Foreign) currency, and B_t the stock of local nominal bonds returning R_t between t and $t+1$. Variable S_t is the nominal exchange rate, τ_t is an import tariff imposed by the Home government, W_t is the Home nominal wage, Π_t represents the profits of Home firms and TR_t is a lump-sum transfer from the Home government.

[9] See also Lechthaler (2017), who focuses on the effect of monetary policy and the policymakers planning horizon on tariff setting in a model with sticky prices, but in quite a different setting.

[10] As put by Costinot, Lorenzoni, and Werning (2014) themselves, in the long run, if endowments converge to a steady state, then taxes on international capital flows converge to zero. A dynamic version of our model would feature dynamic terms-of-trade effects, especially under incomplete financial markets.

[11] See Auray, Devereux, and Eyquem (2020) for a related framework in which the issue of monetary policy cooperation is discussed in details.

[12] For simplicity we will assume that the cross derivative of the utility function is zero, so that $u_{c_{hf}} = 0$. This makes no difference to the results but simplifies the exposition of the example model.

Optimal choices over consumption and hours lead to the following conditions:

$$\beta \mathbb{E}_t \left\{ \frac{R_t}{\pi_{ht+1}} \frac{u_{c_{ht+1}}}{u_{c_{ht}}} \right\} = 1, \tag{4}$$

$$u_{c_{ft}} = u_{c_{ht}} \left(1 + \tau_t\right) \frac{S_t P^*_{ft}}{P_{ht}}, \tag{5}$$

$$\ell' \left(H_t\right) = u_{c_{ht}} \frac{W_t}{P_{ht}}, \tag{6}$$

where $\pi_{ht} = P_{ht}/P_{ht-1}$ is the PPI Home inflation rate.

Firms. Home firms produce differentiated goods. The aggregate good is a composite of these differentiated goods, where the elasticity of substitution between individual goods is $\epsilon > 1$. For now, assume that the firm's production depends only on labor. Output of firm i is:

$$Y_t(i) = A_t H_t(i), \tag{7}$$

where A_t is a measure of aggregate productivity. The profits of firm i are then:

$$\Pi_t(i) = \left[(1+s) P_{ht}(i) - \frac{W_t}{A_t} - \frac{\phi}{2} \left(\frac{P_{ht}(i)}{P_{ht-1}(i)} \right)^2 P_{ht}\left(i\right) \right] Y_t\left(i\right), \tag{8}$$

where $P_{ht}(i)$ is the price set by firm i and s is a sales subsidy. Firm i chooses its price to maximize the present value of its expected profits subject to the demand function for individual goods $Y_t(i) = \left(P_{ht}(i)/P_{ht}\right)^{-\epsilon} Y_t$:

$$\mathbb{E}_t \sum_{j=0}^{\infty} \omega_{t+j} \Pi_{t+j}\left(i\right), \tag{9}$$

where ω_t is the firm's nominal stochastic discount factor, and ϕ captures the importance of price adjustment costs. Assuming symmetry among individual good producers, profit maximization produces the following Phillips curve:

$$\mathbb{E}_t \left\{\Omega_{t,t+1}\right\} = \mathcal{W}_t A_t^{-1} = \mathbb{E}_t \left\{ \theta + \phi \epsilon^{-1} \left(\pi_{ht} \left(\pi_{ht} - 1\right) - \beta \pi_{ht+1} \left(\pi_{ht+1} - 1\right) \right) \right\}, \tag{10}$$

where $\mathcal{W}_t = W_t/P_{ht}$ is the real wage and $\theta = \left(1+s\right)\left(\epsilon - 1\right)/\epsilon \leq 1$ is a subsidy-adjusted measure of monopolistic distortions – the inverse of the subsidy-adjusted markup.[13] If an optimal subsidy $s = 1/\left(\epsilon - 1\right)$ is in place, then $\theta = 1$ and the markup is zero. Equilibrium wages are not distorted. If current and future inflation is zero and the optimal subsidy is in place, then $\mathbb{E}_t \left\{\Omega_{t,t+1}\right\} = 1$ and $\mathcal{W}_t = A_t$. In the absence of a subsidy, $\theta < 1$ which implies a positive markup distortion, and $\mathcal{W}_t < A_t$. Then $\mathbb{E}_t \left\{\Omega_{t,t+1}\right\}$ measures the overall distortion bearing on the real wage, whether stemming from nominal rigidities under sticky prices ($\phi > 0$) – in which case it depends on the inverse of the slope of the Phillips curve $\phi\epsilon^{-1}$ – and/or from monopolistic distortion through the inverse of the subsidy-adjusted markup $\theta < 1$ – in which case it depends on the elasticity of

[13]Here we simplify by assuming the firm's discount factor for the expected future inflation cost is constant at β. This makes little difference to the example model.

substitution between varieties ϵ.

The first distortion is clearly not specific to the Rotemberg formulation of nominal rigidities or to the fact that rigidities apply to prices rather than nominal wages. Any impediment on free price or wage adjustment delivers a Phillips curve in which real wage adjustments are inefficient. Appendix D.2 shows that the results of the model extend easily to Calvo pricing. Further, Appendix B shows that nominal wage rigidity results in almost identical equilibrium conditions so our results would be unchanged. The second distortion has nothing to do with the framework assumed for nominal rigidities, as it only depends on the presence of monopolistic competition.

For the main results of the paper, we assume that the sales subsidy s is absent. The presence of an output distortion is critical to generate an interaction between monetary policy and the level of optimal tariffs, both in the case of inflation targeting and monetary policy discretion. Also of course, the presence of an output distortion is a critical ingredient in the literature on monetary policy under discretion versus commitment.

Government and Foreign sector. The government balances its budget. Tariffs generate revenues, while monopoly subsidies paid to firms represent a cost. The difference is rebated back to households and the budget constraint of the government writes:

$$TR_t = \tau_t S_t P^*_{ft} C_{ft} - sP_{ht}Y_t, \tag{11}$$

where the last expression on the right-hand side represents total subsidies paid to firms. Regarding the Foreign sector, we make the simple assumption that Foreign demand for the Home good depends only on the relative price of the Foreign good to the Home good. The latter equals to the terms of trade of the Home country, defined as $\mathcal{S}_t = S_t P^*_{ft}/P_{ht}$. Thus, the small open economy faces the following Foreign demand for its exported goods:

$$C^*_{ht} = \Lambda \mathcal{S}_t^{\eta}, \tag{12}$$

where Λ is a constant and η the elasticity of Foreign demand.

Monetary policy. To begin with, we assume that the monetary authority follows a simple Taylor rule:

$$R_t = \beta^{-1} \pi_{ht}^{\mu_\pi}. \tag{13}$$

In this case, the stance of monetary policy is measured by the reaction coefficient of the Taylor rule μ_π. We take μ_π as given, and show below how it affects the equilibrium degree of protection when the Home tariff is chosen optimally by the Home authority under discretion. We contrast this case with a situation where both the Home tariff and the rate of inflation are chosen optimally under discretion.[14]

Equilibrium. Conditional on the following goods market clearing condition:

$$A_t H_t \Phi_t = C_{ht} + C^*_{ht},$$

[14] We note that the main point of Taylor (1993) was to contrast a rule such as Equation (13) with a purely discretionary monetary policy.

where $\Phi_t = 1 - \frac{\phi}{2}(\pi_{ht} - 1)^2$, and assuming balanced trade every period, the full equilibrium reduces to:

$$\begin{aligned}
\text{Balanced trade} &: \quad \Lambda \mathcal{S}_t^{\eta} = \mathcal{S}_t C_{ft}, &(14)\\
\text{Market clearing} &: \quad A_t H_t \Phi_t = C_{ht} + \Lambda \mathcal{S}_t^{\eta}, &(15)\\
\text{Labor market} &: \quad \ell'(H_t) = A_t u_{c_{ht}} \mathbb{E}_t \{\Omega_{t,t+1}\}, &(16)\\
\text{Optimal spending} &: \quad u_{c_{ht}} (1+\tau_t) \mathcal{S}_t = u_{c_{ft}}, &(17)\\
\text{Inflation} &: \quad \mathbb{E}_t \left\{ \frac{\pi_{ht}^{\mu_\pi}}{\pi_{ht+1}} \frac{u_{c_{ht+1}}}{u_{c_{ht}}} \right\} = 1. &(18)
\end{aligned}$$

The last equation stems from combining the Euler equation with the monetary policy rule. In the next paragraphs, we assume that the Home government chooses tariffs to maximize the current-period argument of Equation (1) subject to Equations (14), (15), (16) and (18). Equation (17) is ignored since it determines the tariff rate given the equilibrium of the real economy. We assume that trade policy is made under discretion, whereby the government takes the actions of its successors as given. Since this simplified economy features balanced trade, the government essentially faces a static problem in each period. Appendix A gives the details and proofs of the following results that focus on steady-state outcomes.

3.2 Results

3.2.1 Inflation rate with an inflation targeting rule

We first note an important result when monetary policy is conducted through an inflation targeting rule.

Result 1. *With an inflation targeting rule:*

1. *The steady-state inflation rate is independent of the steady state tariff rate,*
2. *A temporary tariff reduces the inflation rate.*

Proof. See Appendix A.

Result 1.1 is straightforward because in the steady state, the inflation rate converges to $\pi_h = 1$ under the monetary policy rule (13). Result 1.2 is also intuitive. A temporary increase in the small economy's tariff rate leads to a temporary increase in the consumption of the Home good. This implies a lower expected growth rate of Home good consumption, leading to a fall in the effective real interest rate. The monetary policy rule (13) then implies that the nominal interest rate falls along with the current-period inflation rate.

3.2.2 Optimal tariff under flexible prices

Let us now derive the optimal tariff for the small economy under flexible prices, *i.e.* when $\phi = 0$.

Result 2. *With flexible prices ($\phi = 0$), the steady state optimal tariff rate for the small open economy is given by:*

$$1 + \tau = \frac{\eta}{\eta - 1} \frac{1 - \theta\Delta}{1 - \Delta} \leq \frac{\eta}{\eta - 1}, \tag{19}$$

where $\Delta = \frac{A^2 u_{c_{hh}}}{\ell''(H)} \theta < 0$.

Proof. See Appendix A.

In the case where $\theta = 1$, Result 2 indicates that $\tau = \frac{1}{\eta - 1}$ which is is the classic monopoly tariff formula, where trade policy exploits the market power of Home firms over Foreign demand. But in the general case where $\theta < 1$, the optimal tariff falls below the classic monopoly tariff rate. Intuitively, Home output is inefficiently low due to the monopoly distortion. A tariff leads to an increase in consumption of the Home good, C_h, which reduces Home output further, due to the income effect on labor supply. This reduces the welfare gains from a marginal increase in the tariff rate, leading the policymaker to reduce its desired tariff below the tariff implied by the classic monopoly formula.

3.2.3 Optimal tariff with sticky prices and an inflation targeting monetary rule

How does the presence of sticky prices affect the incentive to impose a tariff? Under sticky prices ($\phi > 0$), let us first assume that monetary policy is determined by the inflation targeting rule (13). The optimal tariff policy chosen under discretion takes into account the monetary rule (13), but since the policymaker acts with discretion taking all future variables as given, it follows from Result 1.2 that it must take account of its tariff choice on the inflation rate, and hence on employment and output through the labor market equilibrium condition (16). Then we show the following result.

Result 3. *With sticky prices ($\phi > 0$) and an inflation targeting monetary policy rule, the steady-state equilibrium inflation rate is zero, $\pi_h = 1$, and the optimal tariff is given by:*

$$1 + \tau = \frac{\eta}{\eta - 1} \frac{1 - \theta\Delta_1}{1 - \Delta_1} \leq \frac{\eta}{\eta - 1}, \tag{20}$$

where $\Delta_1 = \frac{A^2 u_{c_{hh}}}{\ell''(H)} \left(\theta + \frac{\phi}{\mu_\pi \epsilon} \right) < 0$. *When $\theta < 1$, the tariff rate is below the optimal tariff under flexible prices.*

Proof. See Appendix A.

This extends Result 2 to take account of the influence of ϕ on the optimal tariff rate. Again, when $\theta = 1$, we see that the tariff rate is the same as Result 2 and the classic monopoly tariff formula applies. But when $\theta < 1$, the presence of sticky prices under the monetary rule (13) leads to a lower optimal tariff than that under flexible prices. The logic is as follows. As before, the tariff will shift Home consumption away from Foreign imports towards Home goods. A rise in C_{ht}, given C_{ht+1}, reduces the natural interest rate $u_{c_{ht}} / u_{c_{ht+1}}$, which through the policy rule (13), pushes down inflation. When prices are sticky, a fall in inflation reduces current output through the Phillips curve (10). Given that output is already inefficiently low because $\theta < 1$, this further raises the welfare cost of the tariff and leads the policymaker to set an equilibrium tariff below the flexible price tariff.

We also see from Result 3 that the optimal tariff is decreasing in the degree of price stickiness ϕ, but increasing in the strength of the monetary policy rule μ_π. Formal proofs are given in Appendix A. When prices are stickier, the deflationary effect of a rise in the tariff is magnified and raises the associated welfare losses, which pushes policymakers to apply a lower tariff rate. Conversely, a tighter monetary policy rule reduces the (negative) impact of a tariff on inflation, and thus reduces the policymaker's perceived distortionary impacts of a tariff on output. In the limit, as μ_π rises arbitrarily high, the price level is fully stabilized and the tariff approaches its flexible price level.[15]

3.2.4 Optimal tariff with sticky prices and discretionary monetary policy

Condition (13) specifies a monetary policy rule. We now contrast this situation with a discretionary monetary policy, and allow both the tariff rate and the inflation rate to be chosen under discretion. In this case, the monetary policy rule (13) no longer applies, since the policymaker chooses inflation optimally. Specifically, the government chooses both τ_t and π_{ht} to maximize current-period utility subject to Equations (14), (15), and (16). Under these conditions, we establish the following result.

Result 4. *When both the inflation rate and the tariff rate are chosen under discretion and $\theta < 1$:*

1. *The discretionary inflation rate is positive, $\pi_h > 1$, and the optimal tariff exceeds the optimal tariff under flexible prices. The two conditions characterizing the optimal tariff rate and inflation rate are:*

$$1 + \tau = \frac{\eta}{\eta - 1} \frac{1 - \Omega^2 \Delta_2}{1 - \Delta_2 \Omega \Phi} \leq \frac{\eta}{\eta - 1}, \tag{21}$$

$$\Omega = \frac{\left(\Phi - \Delta_3 \frac{(\pi_h - 1)}{(2\pi_h - 1)} \right)}{\left(1 - \Delta_4 \frac{(\pi_h - 1)}{(2\pi_h - 1)} \Omega \right)}, \tag{22}$$

   where $\Delta_2 = \frac{A^2 u_{c_{hh}}}{\ell''(H)} < 0$, $\Delta_3 = \frac{\ell''(H) H \epsilon}{A u_{c_h}}$, $\Delta_4 = \frac{u_{c_{hh}} A H \epsilon}{u_{c_h}} < 0$, and $\Omega = \theta + \frac{\phi}{\epsilon}(1 - \beta)\pi_h(\pi_h - 1)$, $\Phi = 1 - \frac{\phi}{2}(\pi_h - 1)^2$.

2. *The inflation rate exceeds that under discretionary monetary policy and a zero tariff.*

Proof 4.1. When $\theta < 1$, note from Equation (22) that $\pi_h = 1$ would imply that the left-hand side is less than unity while the right-hand side is unity. Since the right-hand side of Equation (22) is

[15] It is also important that the tariff is chosen under discretion. Under commitment, the monetary rule and the degree of price stickiness would have no effect on the tariff rate, although the presence of pricing distortions would still lead the tariff to fall below the classic monopoly tariff rate.

decreasing in π_h, the equilibrium inflation rate must be positive, *i.e.* $\pi_h > 1$. With $\pi_h > 1$, since $\Omega > \Phi$, and $\Delta_2 < 0$, it follows from Equation (21) that the tariff rate will be greater than that under flexible prices.

Proof 4.2. Appendix A shows that the inflation rate under monetary discretion is characterized as the solution to the condition:

$$\Omega = \frac{\left(\Phi - \Delta_3 \frac{\phi(\pi_h - 1)}{(2\pi_h - 1)}\right)}{\left(1 - \Delta_4 \frac{\phi(\pi_h - 1)}{(2\pi_h - 1)} \Omega - \tilde{\lambda} u_{c_{hh}} \mathcal{S}\right)},$$

where $\tilde{\lambda}$ is a Lagrange multiplier on the policy constraint which is shown to be positive in Appendix A. Since the denominator of this expression is greater than that of Equation (22), the same argument as that above establishes that the inflation rate in the discretionary equilibrium without tariffs is lower than that in the case where the optimal tariff is chosen.

Result 4.1 shows that the implications of monetary policy and price stickiness for tariffs is fundamentally dependent on the the monetary policy framework. As shown by Result 3, with an inflation targeting rule, the steady-state inflation rate is zero ($\pi_h = 1$) and the tariff rate is lower than what it would be in an environment with purely flexible prices as long as $\theta < 1$. However, under a discretionary monetary policy, the policymaker will actively employ two instruments in combination to exploit market power in trade, but also to eliminate the monopoly distortion in price setting. While the inflation targeting rule limits the desire to exploit monopoly power in trade with sticky prices, as described above, the discretionary monetary policy allows policymakers to exploit another instrument to target the pricing distortion, and increases the incentive to direct the tariff towards the objective of improving terms of trade. As a result, the discretionary rate of inflation is positive ($\pi_h > 1$), and the tariff rate exceeds the rate that would apply under flexible prices. By the same token, because, in the absence of tariff choice, the discretionary inflation rate has to target both inefficiencies, the discretionary inflation rate would be lower with in the absence of tariffs.

In the quantitative model of Section 4, we extend the scope to multiple countries and strategic interaction between countries in tariffs and inflation rates. We show that this difference between rules and discretion has similarly large implications for the equilibrium degree of protection, inflation and welfare outcomes in a more general framework.

Note again from Result 4.1 that if $\theta = 1$, the tariff rate would equal that given by the classic monopoly tariff formula, and the inflation rate would be zero, so that Results 2, 3 and 4.1 are identical. In this case, neither sticky prices nor monetary policy would have any implications for the optimal tariff rate. But as long as the steady state is distorted, we find that price stickiness and monetary policy have key implications for the equilibrium rate of protection. Depending on the monetary stance, nominal rigidities can imply either lower or higher tariffs. Because of this, in the subsequent analysis, we focus our discussion exclusively on the case where $\theta < 1$. A complete analysis of the results for all values of θ is presented in Appendix G.

This simple partial equilibrium model can only illustrate the motives for tariff setting on the part of a single economy facing an exogenous foreign demand with a constant elasticity. In the next section, we extend the model and allow two countries to choose tariffs in a Nash game where demand elasticities are determined by the joint choices of the Home and Foreign economy consumers and firms. Despite that, the key implications of Result 3 and Result 4.1 continue to hold.

4 An Extended Model

4.1 Model Summary

We now extend the analysis to a two country general equilibrium model, where both countries set optimal tariffs, engaging in a trade war, but allowing for alternative monetary policy settings. We also generalize the analysis, allowing for CES preferences, production using intermediate goods, trade in intermediate goods, home bias, potential differentials in country size and international capital mobility. The full description of the competitive equilibrium is set out in Appendix C but here we sketch out the most important differences from the example model before discussing the results. In particular, we assume that period utility is now:

$$U(C_t, H_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{H_t^{1+\psi}}{1+\psi}, \tag{23}$$

with:

$$C_t = \left(\gamma^{\frac{1}{\lambda}} C_{ht}^{1-\frac{1}{\lambda}} + (1-\gamma)^{\frac{1}{\lambda}} C_{ft}^{1-\frac{1}{\lambda}} \right)^{\frac{1}{1-\frac{1}{\lambda}}}, \tag{24}$$

Here $\gamma \geq n$ captures the possibility of home bias in preferences and λ the elasticity of substitution between Home and Foreign goods.[16] Given this, the true price index for the Home consumer becomes:

$$P_t = \left(\gamma P_{ht}^{1-\lambda} + (1-\gamma)((1+\tau_t) S_t P_{ft}^{*})^{1-\lambda} \right)^{1/(1-\lambda)}. \tag{25}$$

Households now have access to both local and international bonds subject to small portfolio adjustment costs, which relaxes the assumption of balanced trade. Further, firms now use domestic and imported intermediate goods in production, so the production function for Home firm i becomes:

$$Y_t(i) = A_t H_t(i)^{1-\alpha} X_t(i)^{\alpha}, \tag{26}$$

where $X_t(i)$ represents the use of intermediate goods by Home firm i and $H_t(i)$ its use of labor. We also allow that intermediate good inputs are composed of Home and Foreign goods in a

[16] Letting $0 \leq x \leq 1$ represents the degree of home bias in preferences, where $x = 0$ ($x = 1$) represents zero (full) home bias, we can define $\gamma = n + x(1-n)$.

different composition than that of the consumption good, *i.e.*

$$X_t(i) = \left(\gamma_x^{\frac{1}{\lambda}} X_{ht}(i)^{1-\frac{1}{\lambda}} + (1-\gamma_x)^{\frac{1}{\lambda}} X_{ft}(i)^{1-\frac{1}{\lambda}}\right)^{\frac{1}{1-\frac{1}{\lambda}}}. \tag{27}$$

Let us define $P_{ht}/P_t = 1/\mathcal{P}_t$, where $\mathcal{P}_t = \left(\gamma + (1-\gamma)((1+\tau_t)\mathcal{S}_t)^{1-\lambda}\right)^{\frac{1}{1-\lambda}}$, and likewise $\mathcal{P}_{x,t} = \left(\gamma_x + (1-\gamma_x)((1+\tau_t)\mathcal{S}_t)^{1-\lambda}\right)^{\frac{1}{1-\lambda}}$. Then, Appendix C shows that, conditional on monetary policies $\{R_t, R_t^*\}$ and tariff policies $\{\tau_t, \tau_t^*\}$, the equilibrium can be written in the form of 9 equations in the 9 variables π_{ht}, π_{ft}^*, Y_t, Y_t^*, C_t, C_t^*, b_t, b_t^* and $\mathcal{S}_t$. These are expressed as follows:

$$\theta + \phi\epsilon^{-1}\left(\pi_{ht}(\pi_{ht}-1) - \mathbb{E}_t\left\{\omega_{t+1}\pi_{ht+1}(\pi_{ht+1}-1)\frac{Y_{t+1}}{Y_t}\right\}\right) = \mathcal{MC}_t, \tag{28}$$

$$\theta + \phi\epsilon^{-1}\left(\pi_{ft}^*\left(\pi_{ft}^*-1\right) - \mathbb{E}_t\left\{\omega_{t+1}^*\pi_{ft+1}^*\left(\pi_{ft+1}^*-1\right)\frac{Y_{t+1}^*}{Y_t^*}\right\}\right) = \mathcal{MC}_t^*, \tag{29}$$

$$Y_t\left(1-\frac{\phi}{2}(\pi_{ht}-1)^2\right) - D_t - D_{xt}^* = 0, \tag{30}$$

$$Y_t^*\left(1-\frac{\phi}{2}\left(\pi_{ft}^*-1\right)^2\right) - D_t^* - D_{xt} = 0, \tag{31}$$

$$\mathbb{E}_t\left\{\frac{R_t\omega_{t+1}}{\pi_{ht+1}}\right\} = 1, \tag{32}$$

$$\mathbb{E}_t\left\{\frac{R_t^*\omega_{t+1}^*}{\pi_{ft+1}^*}\right\} = 1, \tag{33}$$

$$nb_t + (1-n)\frac{\mathcal{S}_t\mathcal{P}_t^*}{\mathcal{P}_t}b_t^* = 0, \tag{34}$$

$$\mathbb{E}_t\left\{\frac{\mathcal{S}_{t+1}\omega_{t+1}}{\mathcal{S}_t\omega_{t+1}^*(1+\nu(b_t-b))} - 1\right\} = 0, \tag{35}$$

$$b_t - \frac{\mathcal{S}_t\mathcal{P}_{t-1}}{\mathcal{S}_{t-1}\mathcal{P}_t\omega_t^*}b_{t-1} - \mathcal{P}_t^{-1}\left(D_{xt}^* - \frac{1-n}{n}\mathcal{S}_t D_{xt}\right) = 0, \tag{36}$$

where $\omega_t = \beta\frac{C_{t-1}^{\sigma}\mathcal{P}_{t-1}}{C_t^{\sigma}\mathcal{P}_t}$ and $\omega_t^* = \beta\frac{C_{t-1}^{*\sigma}\mathcal{P}_{t-1}^*}{C_t^{*\sigma}\mathcal{P}_t^*}$ are the stochastic discount factors and $\theta = \frac{(1+s)(\epsilon-1)}{\epsilon} \leq 1$ is the inverse of the steady-state subsidy-corrected markup. Further, D_t, D_{xt}, D_t^*, D_{xt}^* are the demands for local and imported final and intermediate goods, and $\mathcal{MC}_t$ and $\mathcal{MC}_t^*$ are the real marginal production costs.[17] Equations (28) and (29) are the Home and Foreign Phillips curves. Equations (30) and (31) are the goods market clearing conditions for the two countries. Equations (32) and (33) are the Home and Foreign Euler equations. Equation (34) represents the equilibrium on international bonds market, Equation (35) is the modified uncovered interest rate parity condition while Equation (36) is the balance of payment equation expressing the dynamics of net foreign assets. With a flexible nominal exchange rate, the model is closed using

[17] The expressions of D_t, D_{xt}, D_t^*, D_{xt}^*, $\mathcal{MC}_t$ and $\mathcal{MC}_t^*$ as functions of the above endogenous variables are presented in Appendix C.

the following monetary policy rules:

$$R_t = \beta^{-1} \pi_{ht}^{\mu_\pi}, \tag{37}$$
$$R_t^* = \beta^{-1} \pi_{ft}^{*\mu_\pi^*}. \tag{38}$$

4.2 Parameter Values

The model is parametrized to an annual frequency. The discount factor of households is $\beta = 0.96$, consistent with a real interest rate of 4% *per annum*. Both countries are of equal size in the baseline calibration so that $n = 0.5$. Further, we assume a home bias parameter $x = 0.4$ which implies $\gamma = \gamma_x = (1 - \gamma^*) = (1 - \gamma_x^*) = 0.7$. Under free trade (zero tariffs), this number is associated with a 60% total trade openness ratio. We consider a baseline value of $\sigma = 1$, implying a log utility for consumption, but also examine alternative values of σ. The Frisch elasticity is $\psi^{-1} = 2.5$ following Chetty et al. (2011) and we normalize $\chi = 1$. The elasticity of substitution between varieties is $\epsilon = 6$, consistent with a 20% steady-state price-cost markup. The (annual) Rotemberg parameter is $\phi = 40$ and the baseline monetary policy rule inflation parameter is $\mu_\pi = 1.5$, in line with empirical estimates. Following Bergin and Corsetti (2023), we consider the share of intermediate goods in production to be $\alpha = 0.4$. Last, the trade elasticity is $\lambda = 5$. This is on the high end of the range estimated by Feenstra et al. (2018), but is more appropriate for the evaluation of trade policy. The bond adjustment cost parameter suggested by Ghironi and Melitz (2005) is 0.0025 in a quarterly set-up which, in our annual set-up, implies $\nu = 0.01$. Finally, the baseline results are derived under the assumption that trade is balanced in the steady state, *i.e.* $b = b^* = 0$, an assumption that is relaxed in Section 5.

4.3 Baseline Case

Our baseline case considers a trade war with sticky prices and the monetary policy rules (37) and (38). In a discretionary Nash equilibrium of trade policy, the Home government solves:

$$\max_{\{C_t, C_t^*, H_t, H_t^*, \mathcal{S}_t, \pi_{ht}, \pi_{ft}^*, \tau_t\}} V(b_{t-1}) = U(C_t, H_t) + \beta \mathbb{E}_t \{V(b_t)\}, \tag{39}$$

subject to Equations (28)-(36) and monetary policy rules (37)-(38), while the Foreign government solves:

$$\max_{\{C_t, C_t^*, H_t, H_t^*, \mathcal{S}_t, \pi_{ht}, \pi_{ft}^*, \tau_t^*\}} V^*(b_{t-1}) = U(C_t^*, H_t^*) + \beta \mathbb{E}_t \{V^*(b_{t-1})\}, \tag{40}$$

subject to the same constraints. The resulting first-order conditions determine optimal discretionary Nash tariffs.

Table 1 illustrates the resulting allocations and welfare effects under sticky prices. It also reports the case of flexible prices, and the case where tariffs and inflation rates are jointly determined by a Nash discretionary game, *i.e.* which we label discretionary monetary policy.[18]

Table 1: Trade wars.

	Base.	Flex. Prices	Discr. MP	Unil. Discr. MP	Large H.	Unil.
τ	0.204	0.229	0.264	0.261	0.239	0.212
τ^*	0.204	0.229	0.264	0.208	0.174	0.000
π_h	1.000	1.000	1.033	1.033	1.000	1.000
π_f^*	1.000	1.000	1.033	1.000	1.000	1.000
$\mathcal{S}$	1.000	1.000	1.000	0.977	0.954	0.900
C	0.284	0.282	0.276	0.278	0.290	0.297
C^*	0.284	0.282	0.276	0.281	0.274	0.282
L	0.910	0.909	0.922	0.921	0.913	0.905
L^*	0.910	0.909	0.922	0.911	0.907	0.921
Home welfare loss (%)	3.071	3.513	6.651	5.681	1.088	−1.880
Foreign welfare loss (%)	3.071	3.513	6.651	4.052	6.120	4.449

"Base.": Nash equilibrium with sticky prices. "Flex. Prices": Nash equilibrium with flexible prices. "Discr. MP": inflation target chosen under discretion. "Unil. Discr. MP": Nash tariffs with Home country unilaterally choosing its inflation target under discretion and the Foreign country follows an inflation targeting rule. "Large H.": Home country size is $n = 0.7$ (against $n = 0.5$ in the baseline case). "Unil.": Home tariff setter unilaterally chooses its tariff while the Foreign tariff is $\tau_t^* = 0$. Welfare losses are computed against the zero-tariff equilibrium with price stability.

The first column of Table 1 reports the steady-state allocation in the baseline symmetric case with a distorted steady state.[19] The trade war leads to mutual tariff rates of 20.4 percent. In a symmetric equilibrium, there is no change in the terms of trade, but the rise in domestic prices leads to a shift back in labor supply, which reduces equilibrium employment and output. At the same time, the fall in consumption of imported goods distorts the composition of consumption and leads to a fall in aggregate consumption in both countries. As a result, a trade war has large negative effects on real activity and implies higher labor effort combined with lower consumption levels, inducing large welfare losses. Compared to the free-trade equilibrium with price stability, the baseline equilibrium generates symmetric 3.1 percent welfare losses.

The second column of Table 1 documents the equivalent case with flexible prices. The trade war is more intense and results in 2.5 percentage point higher tariffs. Welfare losses are half a percentage point higher under flexible prices than under sticky prices. These numbers are qualitatively consistent with Result 2 *vs* Result 3. Hence, the presence of price stickiness can have a quantitatively important dampening effect on the tariff rates and welfare costs of a trade war when monetary policy is characterized by a flexible inflation targeting policy.

The third column of Table 1 illustrates the outcome of the trade war when combined with discretionary monetary policy. Tariff rates are 6 percentage points higher than in the baseline

[18] In this case, the policy rules (37)-(38) are replaced by $R_t = \beta^{-1}(\pi_{ht}/\bar{\pi}_h)^{\mu_\pi}$ and $R_t^* = \beta^{-1}(\pi_{ft}^*/\bar{\pi}_f^*)^{\mu_\pi^*}$, and inflation targets $\bar{\pi}_h$ and $\bar{\pi}_f^*$ are chosen strategically along with tariffs by Nash policymakers instead of being implicitly set to unity to achieve price stability.

[19] See Appendix G for a comparison of the results when the steady state is not distorted.

case, and inflation rates are 3.3 percent annually. The welfare costs of a trade war under discretionary monetary policy are more than doubled compared to those under inflation targeting. This mirrors perfectly the comparison of Results 3 and 4.1 in the example model of Section 3. In the presence of a distorted allocation ($\theta < 1$), the addition of another discretionary policy instrument in the form of an inflation choice allows policymakers to separately target the terms-of-trade externality and the monopoly distortion. In a symmetric Nash equilibrium of the trade war, both countries are worse off. But the welfare losses are compounded, since the trade war is more intense when policymakers do not have to balance the benefits of terms-of-trade manipulation against the costs of exacerbating production distortions, given that these are now targeted by monetary policy. At the same time, the pursuit of discretionary monetary policy in the presence of monopoly distortions leads to an inefficiently high rate of inflation in both countries, as in the classic model of Barro and Gordon (1983). Thus we have a simple conclusion: under a trade war with monetary discretion, tariffs are higher than with monetary rules, and the inflation bias is higher than in the absence of the trade war.[20]

What if countries follow differential monetary policy stances? The fourth column of Table 1 show the results when the Home country follows a discretionary monetary policy while the Foreign countries follows an inflation targeting rule. In this case, both Home and Foreign tariffs are higher than the baseline case with inflation targeting, but the Home tariff rate is much higher, and the Home country sets a positive rate of inflation. The higher tariff benefits the Home country due to an appreciated terms of trade, but the higher inflation rate is costly in terms of welfare. The Home country is thus worse off than in the baseline case, as also is the Foreign country, although the cost of discretionary inflation leads Home welfare to fall by more.

The previous results assume equally-sized countries. But in reality there are large differences in size between the countries potentially engaged in trade wars. It is natural to think that large countries would tend to either gain or lose less from a trade war, relative to a free trade outcome. In the baseline model without an endogenous trade policy choice, country size is actually irrelevant for real outcomes or welfare.[21] But size will matter when countries engage in trade wars. The fifth column of Table 1 illustrates the importance of large versus small countries in the case of trade wars, assuming the Home economy now represents 70 percent of the world economy instead of 50 percent. Relative to the equal-size Nash equilibrium, the Home tariff rises and the Foreign tariff falls. Because the larger country's consumption basket is more weighted towards its own goods, the cost of a tariff on domestic consumption is less, while conversely, that for the Foreign country is greater. The result is that the (large) Home country is more protectionist, obtains a significant terms-of-trade advantage, and gains in welfare relative to the Foreign country. Country size is thus an advantage in the trade war environment.

The last column of Table 1 reports the unilateral case, where the Home country chooses an optimal tariff unilaterally while the Foreign country chooses a zero import tariff. Under the

[20] Although not shown in Table 1, the symmetric inflation rate under monetary discretion but without a trade war (zero tariffs) would be 2.5 percent, consistent with Result 4.2.

[21] This is because as country size varies, so also does the range of goods that each country produces, so size has no implications for the terms of trade.

baseline calibration, the Home country chooses a tariff rate of 21.2 percent (against 20.4 percent in the symmetric trade war). This generates a 10 percent appreciation of its terms of trade and raises Home welfare at the expense of Foreign welfare. In addition, the tariff raises relative Home consumption, but reduces relative Home output. Table 1 thus report welfare gains (instead of losses) for the Home country with respect to the symmetric free-trade equilibrium, while Foreign losses are magnified. Thus, as we would guess, any one country would gain from imposing a tariff starting from an initial situation of free trade.

In a trade war, the presence of sticky prices leads to lower tariffs and higher welfare, given an inflation targeting monetary policy. By the same token, as shown in the next subsection, a more aggressive monetary rule will lead to higher tariffs, and this will actually reduce steady-state welfare, since equilibrium tariffs move closer to the flexible price outcome. But when a country imposes a unilateral tariff, this conclusion is reversed. Appendix A shows that when the Home country chooses a tariff unilaterally, without Foreign retaliation, the tariff rate will be higher, the higher the elasticity of the policy rate to the inflation rate. In this case, the Home country, setting tariffs on its own, sets a higher tariff (again when $\theta < 1$, so there is an initial output distortion), and closer to the rate that would apply with fully flexible prices. And in the absence of retaliation, Home welfare is higher (and Foreign welfare lower) under flexible prices.

Table 7 in Appendix D illustrates the outcome under alternative parameter values for the trade war. Not surprisingly, the most important parameter is the trade elasticity. Our calibration uses $\lambda = 5$, which is on the high side of the trade elasticities used in the aggregate macro literature but well in the range of the values considered in the trade literature. For a value of $\lambda = 8$, we find that the symmetric Nash equilibrium of the trade war implies a tariff rate of 11.1 percent, substantially lower than that of Table 1. The consequent welfare effects of a trade war compared to a free-trade equilibrium are then less, but the main qualitative implications are robust. Appendix D also investigates the implications of Calvo pricing instead of Rotemberg pricing, and shows that Nash tariffs are basically identical for a comparable calibration of price rigidity parameters (as seen in Table 8).

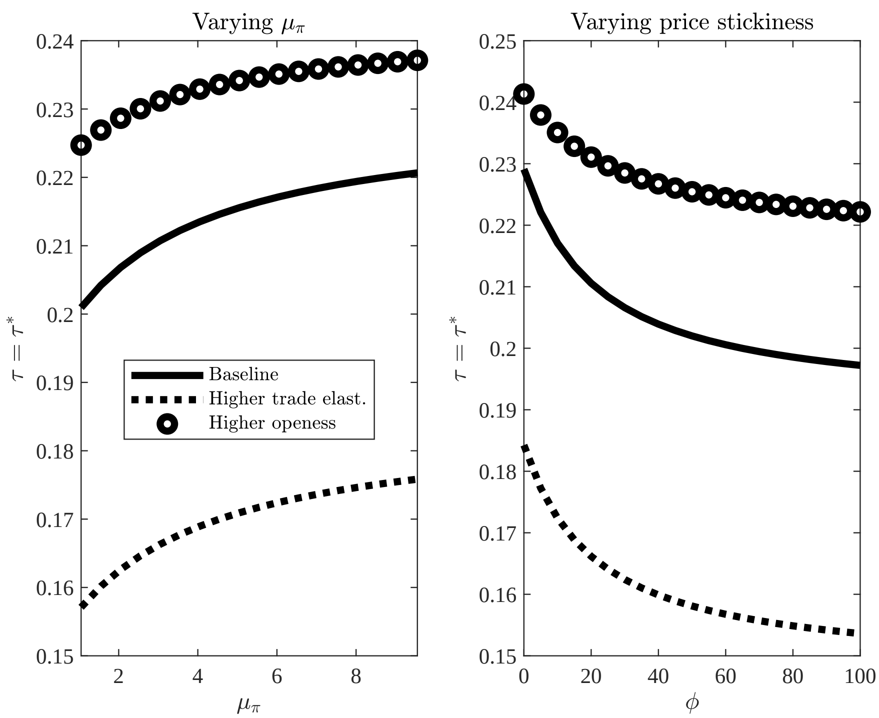
In the above bilateral trade war experiments, tariffs range from roughly 18 percent to 27 percent depending on the set-up. As such they are larger than those observed in the last years under WTO rules, between 5 and 10 percent (see UNCTAD (2013)). But they are not implausibly high in light of the recent U.S.-China trade tariffs, averaging 26 percent.[22] They are however lower than the 60 percent tariffs reported by Ossa (2014) in a calibrated multi-country trade model. In any case, given the key importance of the elasticity of trade flows in shaping the level of non-cooperative tariffs and the lack of consensus about this parameter in the literature, almost any level of tariffs could be achieved in our two-country model with an appropriate choice of the trade elasticity. We choose an elasticity of $\lambda = 5$ in the baseline case, which aligns well with the estimates of Imbs and Mejean (2017), which we use more extensively in Section 6 when computing tariffs arising from a global trade war in multi-country set-up.

[22] According to Bown (2019), average US tariff rates rose from 8 percent in early 2018 to 26 percent at the end of 2019.

4.4 Monetary Policy and Price Stickiness

The results of Section 3 highlight the prominent role of nominal rigidities and of the stance of monetary policy in the determination of Nash tariffs. Here, we want to check whether the insights gained from Result 3 in the example model extend to the more general model. Figure 1 reports the steady-state tariff rates resulting from the Nash game for different values of price stickiness (ϕ) and for different values of the stance of monetary policy (μ_π).

Figure 1: Trade wars: sensitivity to nominal rigidities and monetary policy.



Baseline: $\lambda = 5$ and $\gamma = \gamma_x = 0.7$. Higher trade elasticity: $\lambda = 6$.
Higher trade openness: $\gamma = \gamma_x = 0.6$

As predicted by the example model, equilibrium tariffs of the Nash game are increasing in the stance of monetary policy and decreasing in price stickiness. Tighter monetary policies or more flexible prices reduce the mitigating effects of price stickiness on the optimal tariff decision, and increase tariff rates in the trade war equilibrium.[23] These effects are qualitatively unchanged when considering a greater level of trade openness or a higher trade elasticity, although, all else equal, the former raises the absolute level of tariffs while the latter lowers it. As expected, as the stance of monetary policy increases or price stickiness fades away, tariffs converge to their flexible price levels. In the baseline case, tariffs can be 3 percentage point lower with very sticky prices ($\phi = 100$) than when prices are flexible ($\phi = 0$).

[23]See Appendix A for proofs in the example model.

5 Initial Trade Imbalances

Up to now, we have assumed a zero level of initial net foreign assets (NFA), *i.e.* $b_{-1} = 0$, but this assumption is not consistent with the situation of most countries. In addition, trade imbalances – deficits in particular – have often been cited in practice as a justification for imposing import tariffs. We now relax the assumption of zero NFA and explore the effects of non-zero international asset positions on the trade war equilibrium. Again, we focus on the baseline case, *i.e.* with markup distortions ($\theta < 1$) and sticky prices ($\phi > 0$).

A country with positive net foreign assets can sustain a permanent trade deficit and thus higher levels of private consumption for a given labor effort and production. Even before considering the effects of trade policies, the presence of home bias in final and intermediate goods therefore means that a creditor country has more favorable terms of trade, as well as a higher level of utility. A debtor country finds itself in the opposite situation. In the model, this initial advantage interacts with the strategic motives already highlighted in the previous sections and with a new *wealth* motive, which relates to the interest payments on past net foreign assets. In the presence of this new incentive for tariff setting, a key determinant of non-cooperative tariffs is the currency denomination of internationally traded bonds.

To see this, we incorporate the dynamics of net foreign assets into the determination of tariffs in the trade war. We do this separately, first when bonds are denominated in the Foreign currency, as in the baseline model, and then when bonds are denominated in the Home currency. This introduces net foreign assets as additional state variable in the solution of the trade war. Policymakers must take account of the initial value of net foreign assets in determining the optimal tariff, but also recognize the impact of the tariff on NFA brought into the next period. Although optimal tariffs are chosen under discretion, policymakers must still internalize the effect of their choices on the future value of net foreign assets. Again, we focus on a steady state analysis. In a steady state, the values of net foreign assets will be the same in every period, so that the optimal tariffs chosen in the trade war, conditional on initial net foreign assets, will reproduce that value of net foreign assets for the period following.

When bonds are denominated in the Foreign currency, Home net foreign asset dynamics are given by:

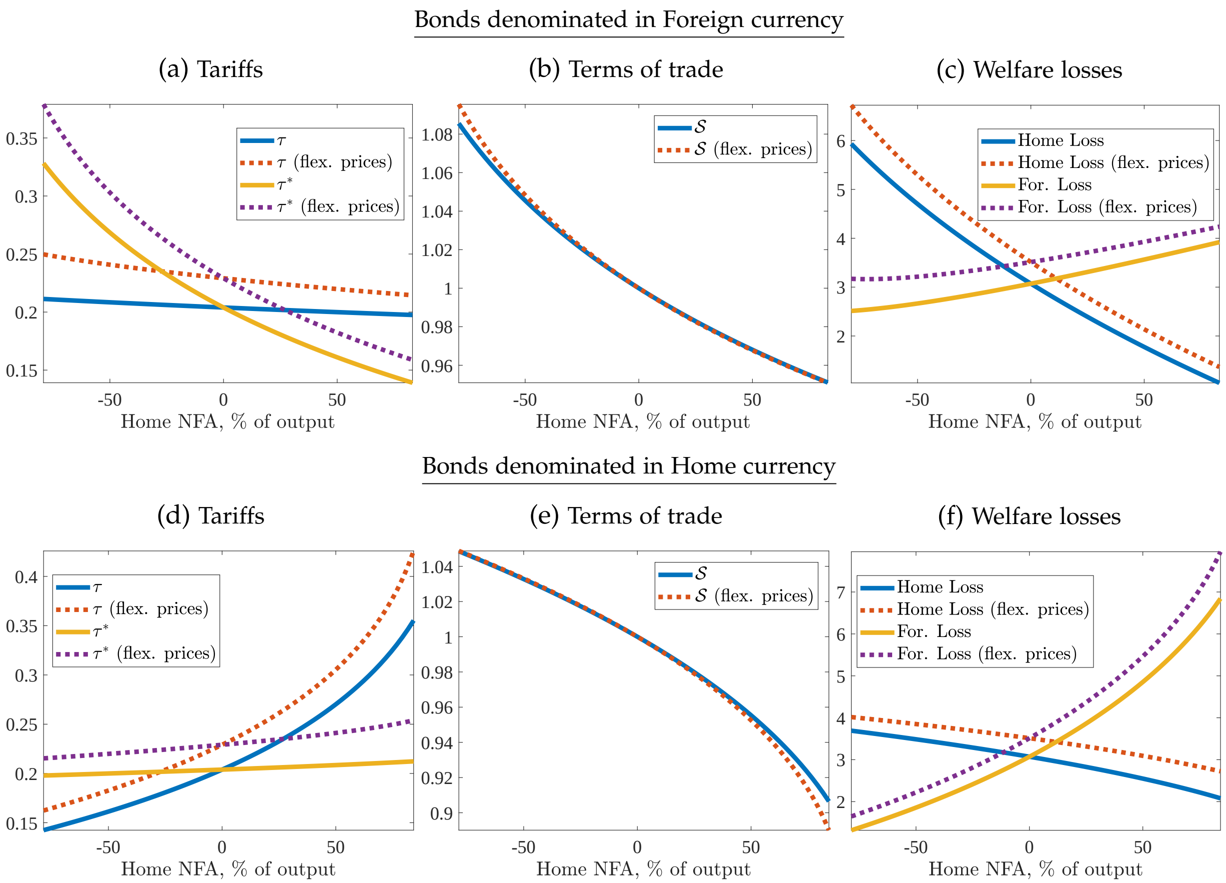
$$b_t = \frac{\mathcal{S}_t \mathcal{P}_{t-1} R^*_{t-1}}{\mathcal{S}_{t-1} \mathcal{P}_t \pi^*_{ft}} b_{t-1} + \mathcal{P}_t^{-1} \left(D^*_{xt} - \frac{1-n}{n} \mathcal{S}_t D_{xt} \right). \tag{41}$$

As shown in Appendix E, when bonds are denominated in the Home currency, most equilibrium conditions of the model are unchanged, except the equation describing the dynamics of Home net foreign assets, which becomes:

$$b_t = \frac{\mathcal{P}_{t-1} R_{t-1}}{\mathcal{P}_t \pi_{ht}} b_{t-1} + \mathcal{P}_t^{-1} \left(D^*_{xt} - \frac{1-n}{n} \mathcal{S}_t D_{xt} \right). \tag{42}$$

Comparing both equations, the main difference lies in the way terms of trade impact the returns on past net foreign asset positions. This gives rise to an additional wealth motive in the determination of tariffs, as explained below. Figure 2 reports the Nash tariffs with trade imbalances when bonds are denominated in the Foreign (top panel) or Home (bottom panel) currency and helps to understand the nature of the wealth motive.

Figure 2: Trade wars with non-zero initial net foreign assets.



Note: Welfare losses are Hicksian percentage consumption equivalent, computed against the free trade equilibrium with zero tariffs and varying net foreign assets.

The top panel of Figure 2 illustrates the results of the trade war for varying levels of steady-state NFA when bonds are denominated in Foreign currency. In this case, the outcome critically depends on the NFA position of the Foreign country. The horizontal axis in each sub-panel illustrates the Home country's net foreign assets, moving from negative (left) to positive (right).[24] If the Home country is a debtor (Foreign is a creditor), then the Foreign tariff setter has an incentive to set a higher tariff, since it will raise its terms of trade and increase the *ex-post* returns on NFA interest payments by the Home country. The Figure shows that the Home country does retaliate, with a higher tariff of its own, but the net effect is an improved Foreign terms of trade

[24]Since we focus on a steady state, there will be a one to one relationship between the NFA position and the steady-state trade balance. Thus, if the Home country is a net creditor (debtor), the Foreign country must be a debtor (creditor) and generate a steady-state trade surplus (deficit) to sustain interest payments (income).

and higher Foreign welfare (lower Foreign losses). By contrast, if the Home country is a creditor, the situation is reversed, the Foreign country faces an incentive to reduce its tariff relative to the zero NFA case, since the wealth motive is negative, and the Foreign country wants to reduce the *ex-post* returns paid to the Home country on its external debt. Again, the Home country retaliates by lessening its tariff rate, but by a smaller amount.

The bottom panel of Figure 2 reports the alternative situation where internationally-traded bonds are denominated in Home currency. The situation is exactly the converse of the top panel. If the Home country is a net creditor, it raises its tariff relative to the benchmark case with zero NFA. In the trade war equilibrium, the Foreign country retaliates and sets a higher tariff as well, but less than the Home country. The Home country thus enjoys improved terms of trade and higher welfare. If it is a debtor, in equilibrium, it will be less protectionist than the Foreign country.

Overall, what can be learned from these results? First, being a net creditor gives a natural terms-of-trade advantage, whether or not NFA is denominated in a country's own currency. Second, considering external debt imbalances introduces an additional wealth motive in the determination of tariffs in a trade war, and leads to differential tariff outcomes. Third, trade and external debt imbalances bring strong asymmetries in trade wars, and cross-country differences in tariffs increase with the size of trade imbalances. Fourth, while there are welfare gains to being an external creditor, being a creditor in its own currency brings an additional advantage in a trade, that leads to further incentive to raise tariffs. For instance, imagine that the U.S. is an external debtor. If internationally-traded bonds are denominated in U.S. dollars, then the U.S. would face an incentive to set lower tariffs in an attempt reduce its terms of trade and lower net interest payments on its external debt.

6 A Global Trade War

We now bring our model closer to the data. We consider a sample of major countries in international trade and use country-specific parameter values to quantify the implications of a full-scale trade war. This approach is a more aggregative exercise than some of the previous trade-oriented studies on the effects of trade wars, which focus on more detailed sectoral measurements. But our objective is to highlight the aggregate impact of alternative monetary policies on the outcome of a full-scale trade war, disciplined by the relevant data on a country-by-country basis. Our sample includes Australia, Canada, China, India, Indonesia, Japan, Korea, Great Britain, the U.S. and a large sub-set of European Union (EU) countries.[25] We consider the EU as a single country, given that the EU has a common trade policy, and construct statistics and calibration targets for the EU using weighted averages. Overall, our sample covers more than 60% of world GDP.

[25] The EU countries of our sample are Austria, France, Finland, Germany, Greece, Hungary, Italy, Portugal, Slovakia, Spain and Sweden.

6.1 Set-up and Data

We compute optimal tariffs in a global trade war in the following way. For each country of the sample, we use a country-specific version of our two-country model. Each country is in turn considered the Home economy against the (Foreign) rest of the world, composed of all the remaining countries of the sample. Bonds are assumed to be denominated in U.S. dollars. The determination of tariffs proceeds in two steps. In step one, for each country in turn, we compute the unilaterally optimal Home tariff assuming a zero tariff in the (Foreign) rest of the world made of the remaining countries of our sample. Take Canada as an example. We consider the optimal unilateral tariff set by Canada in a two-country model (Canada vs. rest of the world), given a zero tariff in the rest of the world. Of course it seems unlikely that all other countries of our sample would coordinate their trade policy to set a common zero tariff rate. So as a result of the first step, we compute a global tariff rate that faces Canada based on a weighted-average of unilateral country-specific tariffs. In step two, we compute the unilaterally optimal tariff for each country in turn, given the positive global tariff found in step one for the (Foreign) rest of the world. We iterate this procedure until the weighted-average global tariff, and thus country-specific tariffs, converge to stable values.

We use the following data for the year 2015 to pin down key parameter values, being understood that the remaining parameters are those reported in the previous Section. First, we use relative GDP per capita to infer steady-state relative productivity levels A/A^*.[26] Second, we use the relative population size of the Home country to fix n. Third, we use trade-to-GDP ratios to infer Home bias parameters, given the imposed share of intermediate goods in production ($\alpha = 0.4$) and the assumption of identical home bias in final and intermediate goods sectors.[27] Fourth, we make use of the estimates provided by Imbs and Mejean (2017) regarding the aggregate elasticity of imports. These are constructed using sectoral data and properly aggregated, resulting in elasticities that are larger than usually considered in macroeconomic frameworks, but consistent with consensual values found in the trade literature. Fifth, we use Lane and Milesi-Ferretti (2018)'s data regarding net foreign asset positions expressed as a percentage of annual GDP, again in 2015. These parameter values are summarized for each country in left-hand panel of Table 2.

We have no clear way to measure the degree of price stickiness or the parameters of monetary policy in each of the countries in our sample. In light of this, we follow the analysis in Section 4. In the baseline case, we assume that prices are sticky and monetary policy follows a Taylor rule, with a similar price adjustment cost parameter and the monetary rule for all countries. We also investigate an alternative set-up where monetary policy is determined under discretion.

[26]Let Y/Y^* the ratio of GDP per capita. Suppose countries supply the same amount of labor per capita. Then our model implies $A/A^* = (Y/Y^*)^{1-\alpha}$ where α is the (common) share of intermediate goods in production. We use relative GDP per capita from the data and the above transformation to obtain a measure of relative productivity.

[27]GDP per capita, population and trade-to-GDP ratios are taken from the World Bank Indicators database in 2015.

6.2 Baseline Case

The tariff rates, consumption and labor efforts stemming from a global trade war in the baseline model are reported in Table 2. The Table also reports the equilibrium consumption and labor efforts under free trade (zero tariffs) and price stability, and computes the welfare losses from a global trade war against this equilibrium. Recall that external debts are all denominated in U.S. dollars.

Table 2: A Global Trade War.

	Calibration (2015 data)					Trade war				Free trade	
Country	A/A^*	λ	$\gamma = \gamma_x$	n	b_{-1}/y	τ	C	L	Loss	C	L
Australia	1.757	6.466	0.861	0.006	−0.577	0.089	0.862	0.916	7.034	0.938	0.929
Canada	1.756	8.018	0.779	0.009	0.225	0.056	0.888	0.906	9.217	0.987	0.918
China	0.792	6.920	0.868	0.355	0.147	0.107	0.203	0.914	3.077	0.209	0.913
India	0.480	4.755	0.860	0.337	−0.288	0.167	0.090	0.915	2.495	0.093	0.925
Indonesia	0.695	4.401	0.860	0.066	−0.453	0.172	0.185	0.910	4.059	0.196	0.928
Japan	1.593	5.420	0.882	0.033	0.610	0.133	0.709	0.902	2.528	0.733	0.912
Korea	1.555	5.113	0.736	0.013	0.141	0.114	0.847	0.895	9.104	0.950	0.919
Great Britain	1.700	5.389	0.814	0.017	−0.254	0.123	0.831	0.908	5.948	0.896	0.925
USA	1.987	4.907	0.907	0.083	−0.432	0.112	0.921	0.924	1.323	0.936	0.928
European Union	1.648	5.096	0.705	0.081	−0.142	0.157	0.736	0.904	4.649	0.783	0.923
Weighted av.	1.000	5.618	0.852	–	−0.106	0.137	0.312	0.913	3.119	0.326	0.920

Welfare losses are computed against the zero-tariff equilibrium with price stability.

With inflation targeting monetary policies, the weighted-average implied tariff across all countries is 13.7 percent. Country-specific tariffs are quite dispersed around this average value, and range from 5.6 percent for Canada to 17.2 percent for Indonesia. Consistent with the theoretical results in Table 1, tariff rates are larger for larger countries, but are also significantly affected by the levels of trade elasticities. The predicted U.S. tariff is 11.2 percent, significantly below the implied tariff for the EU and Japan, but as we see below, this depends a lot on the NFA position of the U.S. Welfare losses relative to free trade are affected by relative tariffs but also by the degree of openness of the country. The distribution of losses varies considerably. The highest losses are for Canada and Korea, while U.S. losses are the lowest among all countries, given that it is relatively less open to trade.

6.3 The Effects of Monetary policy and Price Stickiness

Table 3 shows how these estimates would differ if prices were perfectly flexible. Tariff rates would be almost 4 percentage point higher than those of Table 2, with a global tariff rate equal to 17.9 percent, and country-specific tariffs ranging from 10.2 percent to 23.1 percent. The implied U.S. tariff rate would rise from 11.2 to 14.7 percent. With much higher average non-cooperative tariffs, the welfare losses relative to free trade would also be significantly higher – averaging 4 percent of consumption equivalent relative to 3.1 percent in the case of sticky prices and inflation targeting. All countries lose relative to the baseline case, but again, the U.S. losses are lowest.

Table 3: Global Trade War – Sensitivity

Country	Trade war - sticky prices				Trade war - flex. prices				Trade - discretionary MP				
	τ	C	L	*Loss*	τ	C	L	*Loss*	τ	π	C	L	*Loss*
Australia	0.089	0.862	0.916	7.034	0.136	0.842	0.913	8.932	0.145	1.032	0.825	0.925	11.68
Canada	0.056	0.888	0.906	9.217	0.102	0.864	0.901	11.35	0.110	1.032	0.846	0.912	13.88
China	0.107	0.203	0.914	3.077	0.138	0.202	0.914	3.564	0.148	1.033	0.198	0.927	5.060
India	0.167	0.090	0.915	2.495	0.217	0.089	0.914	3.421	0.231	1.033	0.088	0.927	6.148
Indonesia	0.172	0.185	0.910	4.059	0.231	0.182	0.907	5.615	0.244	1.032	0.179	0.919	7.949
Japan	0.133	0.709	0.902	2.528	0.180	0.702	0.900	3.379	0.190	1.033	0.690	0.912	5.936
Korea	0.114	0.847	0.895	9.104	0.173	0.819	0.888	11.64	0.188	1.029	0.803	0.896	13.90
Great Britain	0.123	0.831	0.908	5.948	0.174	0.813	0.905	7.750	0.186	1.031	0.797	0.916	10.38
USA	0.112	0.921	0.924	1.323	0.147	0.916	0.924	1.785	0.155	1.032	0.900	0.936	4.474
European Union	0.157	0.736	0.904	4.649	0.202	0.722	0.902	6.286	0.223	1.032	0.708	0.913	8.925
Weighted av.	0.137	0.312	0.913	3.119	0.179	0.308	0.913	4.001	0.191	1.033	0.303	0.925	6.239

Welfare losses are computed against the zero-tariff equilibrium with price stability.

The right-hand panel of Table 3 shows the case of the global trade war with discretionary monetary policy. Consistent with our theoretical model, this case predicts a substantial further increase in tariffs and positive inflation rates among all countries. The global tariff rate is now 19.1 percent, with the lowest tariff 11 percent, and the highest 24.4 percent. Welfare losses relative to free trade and zero inflation are now also much higher, given that the losses include the costs of positive inflation rates.

This quantitative empirical application therefore supports the theoretical results from above. The stance of monetary policy may have major consequences for the degree of protection in a global environment where tariffs are chosen non-cooperatively. In the absence of formal or informal trade agreements, the incentive to levy tariffs in distorted economies depends critically on the institutional structure of inflation determination. Comparing a regime of inflation targeting to one of discretionary monetary policy, global tariff rates may be significantly higher and world trade much lower in the latter regime. Indirectly, this represents a significant welfare cost of monetary policy discretion.

6.4 The Effects of External Imbalances

Finally, Table 4 looks at the counterfactual case where each country's NFA position is set to zero. Except for the U.S., this has minimal effects on tariff rates. But the implied U.S. tariff rate rises significantly, from 11.2 percent to 16.3 percent. The intuition is closely tied to the wealth effects affecting tariff incentives as discussed in Section 5 above. As a large net debtor with NFA denominated in its own currency, the U.S. would have an incentive to limit its tariff, since a higher terms of trade would increase the absolute value of its negative NFA. Thus, the model predicts that eliminating the US as a large global debtor would lead to a large increase in US tariffs. Given the large weight of the U.S. in world GDP and in the weighted-average tariff, the latter jumps by 0.4 percentage points, which results in larger welfare losses for all countries but the U.S., which in fact experiences gains in terms of welfare.

Table 4: Global Trade War - zero steady-state net foreign assets.

	Baseline				With $b_{-1}/y = 0$			
Country	τ	C	L	*Loss*	τ	C	L	*Loss*
Australia	0.089	0.862	0.916	7.034	0.089	0.885	0.907	6.956
Canada	0.056	0.888	0.906	9.217	0.057	0.877	0.910	9.484
China	0.107	0.203	0.914	3.077	0.108	0.201	0.917	3.192
India	0.167	0.090	0.915	2.495	0.166	0.092	0.910	2.424
Indonesia	0.172	0.185	0.910	4.059	0.171	0.190	0.903	3.912
Japan	0.133	0.709	0.902	2.528	0.134	0.686	0.911	2.882
Korea	0.114	0.847	0.895	9.104	0.114	0.840	0.897	9.375
Great Britain	0.123	0.831	0.908	5.948	0.122	0.840	0.904	5.983
USA	0.112	0.921	0.924	1.323	0.163	0.942	0.916	1.116
European Union	0.157	0.736	0.904	4.649	0.157	0.740	0.902	4.709
Weighted av.	0.137	0.312	0.913	3.119	0.141	0.314	0.912	3.132

Welfare losses are computed against the zero-tariff equilibrium with price stability.

7 Extensions: constraints on monetary policy

We now extend the baseline model to one where monetary policy is subject to constraints. We first consider a situation of fixed exchange rates, in which the Foreign economy gives up its monetary policy independence by pegging its currency to that of the Home economy. Following this, we consider a world economy with dominant currency pricing (DCP), where all internationally traded goods are invoiced in the Home currency. Both extensions have substantial implications for trade policy and the equilibrium degree of protection, although as we show, the distinction between different monetary policy environments remains the same.

7.1 Fixed Exchange Rate

Assume that the Foreign economy has an exchange-rate target. This implies that it cedes control over its domestic inflation rate, leaving the Home country to run an independent monetary policy. In this case, the Foreign monetary policy rule is replaced by the following condition:

$$\pi_{ht} = \pi^*_{ft} \frac{\mathcal{S}_{t-1}}{\mathcal{S}_t}. \tag{43}$$

Because the nominal exchange rate is fixed, the terms of trade can change only due to changes in nominal price levels, implying that the terms of trade follows the dynamics of relative inflation rates. This rule adds an additional state variable to the model – in addition to net foreign assets – in the form of the lagged terms of trade. Since the nominal exchange rate is pegged, the terms of trade can adjust *only* via differences in inflation rates. Under a fixed exchange rate regime, the problem can be stated as:

$$\max_{\{C_t, C^*_t, Y_t, Y^*_t, b_t, \mathcal{S}_t, \pi_{ht}, \pi^*_{ft}, \tau_t\}} V(\mathcal{S}_{t-1}, b_{t-1}) = U(C_t, H_t) + \beta \mathbb{E}_t \left\{ V(\mathcal{S}_t, b_t) \right\}, \tag{44}$$

subject to (28)-(36) and monetary policy rules (37)-(43) for the Home policymaker and similarly:

$$\max_{\{C_t, C_t^*, Y_t, Y_t^*, b_t, \mathcal{S}_t, \pi_{ht}, \pi_{ft}^*, \tau_t^*\}} V^*(\mathcal{S}_{t-1}, b_{t-1}) = U(C_t^*, H_t^*) + \beta \mathbb{E}_t \left\{ V^*(\mathcal{S}_t, b_t) \right\}, \tag{45}$$

for the Foreign. Assuming $\mathcal{S}_{-1} = 1$ on top of $b_{-1} = 0$ selects only symmetric equilibria in the Nash tariff game, and Table 5 reports the corresponding outcome.

Table 5: Trade wars under fixed exchange rate.

	Inflation targeting rule		Discretionary MP	
	Flexible XR	Fixed XR	Flexible XR	Fixed XR
$\tau = \tau^*$	0.204	0.170	0.264	0.217
$\pi_h = \pi_f^*$	1.000	1.000	1.033	1.038
$\mathcal{S}$	1.000	1.000	1.000	1.000
$C = C^*$	0.284	0.286	0.276	0.276
$L = L^*$	0.910	0.910	0.922	0.926
Welfare loss (%)	3.071	2.464	6.651	6.809

Outcome conditional on $\mathcal{S}_{-1} = 1$ and $b_{-1} = 0$. Welfare losses are computed against the zero-tariff equilibrium with price stability.

With a fixed exchange rate, Table 5 shows that equilibrium tariff rates fall to 17 percent, compared with 20.4 under a flexible exchange rate. With a fixed exchange rate, raising the tariff rate puts upward pressure on the national inflation rate as seen from Equation (43), which conflicts with the monetary rule targeting price stability in the Home country, and by the commitment to the exchange-rate peg in the Foreign economy. This reduces the incentive to engage in a tariff war. When countries follow an inflation targeting monetary rule, both countries equilibrium inflation rates are zero ($\pi_h = \pi_f^* = 1$). Then, since tariffs are lower than those in the baseline case, welfare for both countries is higher under the fixed exchange rate than in the baseline case of flexible exchange rates.

We thus arrive at the novel conclusion that, in a symmetric equilibrium, pegging the exchange rate actually increases welfare of both countries by reducing the severity of the trade war.[28] However, just as in the baseline model, we find that the results change substantially in the case of discretionary monetary policy. With an exchange-rate peg followed by the Foreign country, a discretionary monetary policy involves the Home country choosing both inflation and tariff rates freely, while the Foreign country just chooses a tariff rate, and the Foreign inflation rate adjusts endogenously to satisfy condition (43). Table 5 shows that the implied tariff rates are much higher than under inflation targeting, although still below the tariff rates implied by discretionary monetary policy with flexible exchange rates in Table 1. Also, in the case of discretionary monetary policy, a fixed exchange rate implies higher equilibrium inflation rates, and

[28] Appendix G shows that this result also holds when the steady state is not distorted.

thus higher welfare losses relative to the flexible exchange rate case.[29]

7.2 Dominant Currency Pricing

Recent evidence has pointed to the role of the U.S. dollar as an invoice currency for pricing exports for a large share of the world economy (see Gopinath et al. (2020) and Mukhin (2022)). In terms of our model, this would imply that one country (Home) sets the price of both its exports and domestic sales in its own currency, while the Foreign country sets its domestic sales price in its own currency, but sets its export price in the currency of the Home country. Gopinath et al. (2020) characterize this situation as one of Dominant Currency Pricing (DCP). We explore the implications of DCP for the trade war equilibrium.

The model under DCP differs in only a few features, as explained in details in Appendix F. The nominal exchange rate is still flexible, but the impact of exchange rate changes on the terms of trade is muted, in particular for the Home country, since both the price of its exports and imports are set in its own currency.

The true price index for the Home consumers under DCP now becomes:

$$P_t = \left(\gamma P_{ht}^{1-\lambda} + (1-\gamma)((1+\tau_t)P_{ft})^{1-\lambda}\right)^{1/(1-\lambda)}, \tag{46}$$

where P_{ft} is the price of the Foreign good set in Home currency. By contrast, the price index for the Foreign economy is unchanged compared to the baseline Producer Currency Pricing (PCP) model, since the Home country firm sets all prices in Home currency.

The optimal pricing condition of Home firms is as before, but Foreign firms charge separate prices to the domestic (in Foreign currency) and Home (in Home currency) firms and households respectively buying intermediate and final goods. The profits of the Foreign firm i are then represented as:

$$\Pi_t^*(i) = (1+s)\left(P_{ft}^*(i)\,Y_{ft}^*(i) + S_t^{-1}P_{ft}(i)\,Y_{ft}(i)\right) - MC_t^*\left(Y_{ft}^*(i) + Y_{ft}(i)\right), \tag{47}$$

where MC_t^* is the Foreign nominal marginal production cost and $Y_{ft}(i) = D_{xt}(i)$ and $Y_{ft}^*(i) = D_t^*(i)$ in equilibrium. Foreign firm i maximizes:

$$\mathbb{E}_t\left\{\sum_{j=0}^{\infty}\omega_{t+j}^*\left(\begin{array}{c}\Pi_{t+j}^*(i) - \frac{\phi}{2}\left(\frac{P_{ft+j}^*(i)}{P_{ft+j-1}^*(i)} - 1\right)^2 P_{ft+j}^*(i)\,Y_{ft+j}^*(i) \\ -\frac{\phi}{2}\left(\frac{P_{ft+j}(i)}{P_{ft+j-1}(i)} - 1\right)^2 S_{t+j}^{-1}P_{ft+j}(i)\,Y_{ft+j}(i)\end{array}\right)\right\}. \tag{48}$$

[29]There is an important caveat to these results. There may be many asymmetric equilibria. Indeed, as shown in Auray, Devereux, and Eyquem (2020), there exists a continuum of equilibrium Nash tariff rates conditioned on different values of $\mathcal{S}_{-1}$. If the initial value of terms of trade is $\mathcal{S}_{-1} < 1$, then the Home country will choose a tariff rate higher than that of the Foreign country, so that in equilibrium $\mathcal{S}_t = \mathcal{S}_{t-1} < 1$. Likewise for $\mathcal{S}_{-1} > 1$, then the Home country will choose a lower tariff rate than the Foreign country, and again $\mathcal{S}_t = \mathcal{S}_{t-1} > 1$. Thus, there is a continuum of Nash equilibrium tariff rates in which the Home country is more or less protectionist than the Foreign country, and each delivers a more or less appreciated terms of trade for the Home country.

Note that the Foreign firm incurs costs of price adjustment for sales to the Home country that are separate from those pertaining to local (Foreign) sales. Profit maximization now yields two (not one) inflation equations for Foreign goods depending on whether they are consumed locally or exported. The dynamics of Foreign prices for goods sold in the Home market are:

$$\theta + \phi\epsilon^{-1}\left(\pi_{ft}\left(\pi_{ft}-1\right) - \mathbb{E}_t\left\{\omega^*_{t+1}\frac{D_{xt+1}}{D_{xt}}\frac{\mathcal{S}^*_t}{\mathcal{S}^*_{t+1}}\frac{\mathcal{S}_{t+1}}{\mathcal{S}_t}\pi_{ft+1}\left(\pi_{ft+1}-1\right)\right\}\right) = \left(\frac{\mathcal{S}^*_t}{\mathcal{S}_t}\right)\mathcal{MC}^*_t, \quad (49)$$

while the condition for domestic sales of Foreign goods is:

$$\theta + \phi\epsilon^{-1}\left(\pi^*_{ft}\left(\pi^*_{ft}-1\right) - \mathbb{E}_t\left\{\omega^*_{t+1}\frac{D^*_{t+1}}{D^*_t}\pi^*_{ft+1}\left(\pi^*_{ft+1}-1\right)\right\}\right) = \mathcal{MC}^*_t, \quad (50)$$

where $\mathcal{MC}^*_t$ is the Foreign real marginal production cost. The essential new element that DCP brings to the analysis relates to the terms of trade. In fact, we now have two separate terms of trade. For the Home country, the relative price of imports to exports is now $\mathcal{S}_t = P_{ft}/P_{ht}$, where both prices are set in Home currency. The terms of trade for the Foreign country are expressed as before, $\mathcal{S}^*_t = S_t P^*_{ft}/P_{ht}$. The two measures may differ due to deviations of the law of one price for the Foreign good, since in general with price adjustment costs, P_{ft} will not always equal $S_t P^*_{ft}$. More critically, $\mathcal{S}_t$ can be adjusted only through nominal price adjustment, while $\mathcal{S}^*_t$ adjusts to nominal exchange rate changes for given nominal prices. This effectively means that the Home country terms of trade $\mathcal{S}_t$ displays the same type of persistence as in the case of fixed exchange rates. Since $\mathcal{S}_t = P_{ft}/P_{ht}$, we have:

$$\mathcal{S}_t = \mathcal{S}_{t-1}\frac{\pi_{ft}}{\pi_{ht}}. \quad (51)$$

Thus, the Home terms of trade adjusts according to the differential between the Foreign export price inflation and the Home inflation rate. The terms of trade can be manipulated only indirectly through the impact of tariffs on relative inflation rates. The policy game under DCP is defined in the same way as before, where in the trade war game the Home and Foreign policymakers choose τ_t and τ^*_t respectively conditional on monetary policy rules and equilibrium conditions.

Table 6 describes the equilibrium of the trade war under DCP. This shows substantial asymmetry. The Home country sets tariffs close to zero because it has limited ability to manipulate its own terms of trade. It thus focuses on the domestic markup distortion and sets a low tariff – only 3.1 percent – to limit the negative effects of tariffs on output, which is already inefficiently low. The Foreign country sets a tariff rate slightly above the tariff of the baseline case with producer currency pricing (21.1 percent against 20.4 percent), which results in an appreciation of its terms of trade, higher Foreign consumption and lower Foreign labor effort. In this situation, the Foreign economy gains in terms of welfare and the Home economy loses. Thus, issuing the dominant currency is actually detrimental for a country, when we take into account the endogenous responses of tariffs in a trade war. But on average, tariff rates and welfare losses are lower under

Table 6: Trade wars under Dominant Currency Pricing (DCP)

	Inflation targeting rule		Discretionary MP	
	Baseline	DCP	Baseline	DCP
π_h	1.000	1.000	1.033	1.054
π_f^*	1.000	1.000	1.033	1.054
π_f	–	1.000	–	1.022
τ	0.204	0.031	0.264	0.173
τ^*	0.204	0.211	0.264	0.117
$\mathcal{S}^*$	–	1.093	–	0.978
$\mathcal{S}$	1.000	1.093	1.000	0.972
C	0.284	0.283	0.276	0.274
C^*	0.284	0.294	0.276	0.266
L	0.910	0.919	0.922	0.940
L^*	0.910	0.906	0.922	0.944
Home welfare loss (%)	3.071	4.061	6.651	8.730
Foreign welfare loss (%)	3.071	-0.949	6.651	11.59

Welfare losses are computed against the zero-tariff equilibrium with price stability.

DCP than under PCP, since the Home country sets much lower tariffs.[30]

However, these results are again dependent on the monetary policy setting. Table 6 shows that in the case of discretionary monetary policy, we obtain opposite results. The Home country sets higher tariffs than those of the Foreign country. The logic behind this follows from the fact that, for the Foreign country to improve its terms of trade *via* a tariff, it must engage in costly inflation in the price of its exported good. But in the Nash equilibrium with discretionary monetary policy, inflation is already high. Increasing exported goods inflation even further would be self-defeating. In fact, it is optimal to moderate inflation through a very small tariff. This leads to a terms-of-trade benefit for the Home country.

8 Conclusions

Our paper shows that monetary policy matters for trade wars. Nominal rigidities interact with pre-existing distortions to shape the incentives to apply non-cooperative trade policies. With flexible inflation targeting, trade wars are likely to be much less damaging than if monetary policy is chosen under discretion. There is thus a 'trade dividend' to inflation targeting, in the sense that these policies carry more than just the gains from commitment, as they result in milder trade policies, larger trade flows and higher levels of output and consumption. A quantitative application to a large set of countries representing the bulk of global trade suggests that, in a fully-scale trade war, average tariff rates under inflation targeting may be several percentage points below those under discretion, resulting in much lower welfare losses from trade wars.

[30] Appendix G shows that a similar pattern characterizes the trade war equilibrium without markup distortions, except that the Home economy set positive tariffs, given that output is at a less inefficiently low level.

Our analysis paves the way for additional interesting questions. While we allow for non-zero net foreign assets, our results are confined to steady states, so we do not characterize the dynamics of trade and monetary policy over the business cycle, nor the dynamics of global imbalances. These questions could be of importance in the event of large economic swings, especially in light of the dynamic terms of trade motive. Further, our results could incorporate political economy concerns, such as costs from breaking existing trade agreements or implicit reputational concerns. Finally, our model assumes that countries are collections of homogeneous households and firms. In reality, trade policies may be driven by the balance of gains and losses to different groups of individuals (households, sectors) within each country. We leave these questions for future research.

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A The Example Model

A.1 Optimal Tariff with an Inflation Targeting Monetary Policy Rule

The optimal policy problem for the choice of the Home tariff, given the monetary policy rule writes:

$$V(\mathcal{Z}_t) = \text{Max}_{\{C_{ht}, C_{ft}, H_t, \mathcal{S}_t, \pi_{ht}, \tau_t\}} \; u\left(C_{ht}, C_{ft}\right) - \ell\left(H_t\right) + \beta \mathbb{E}_t\left\{V\left(\mathcal{Z}_{t+1}\right)\right\}, \tag{A.1}$$

subject to Equation (14)-(18). Equation (17) can be omitted, since τ_t is a free variable and so this constraint will always hold. Assuming additive separability between C_{ht} and C_{ft} to reduce the presence of cross terms in the first-order conditions, the latter are:

$$C_{ht} \; : \; u_{c_{ht}} = \xi_{2t} + \xi_{3t} A_t u_{c_{hht}} \mathbb{E}_t\left\{\Omega_{t,t+1}\right\} - \xi_{4t}\mathbb{E}_t\left\{\frac{\pi_{ht}^{\mu_\pi}}{\pi_{ht+1}} \frac{u_{c_{hht}} u_{c_{ht+1}}}{u_{c_{ht}}^2}\right\}, \tag{A.2}$$

$$C_{ft} \; : \; u_{c_{ft}} = \xi_{1t}\mathcal{S}_t, \tag{A.3}$$

$$H_t \; : \; \ell'\left(H_t\right) = \xi_{2t} A_t \Upsilon_t\left(\pi_{ht}\right) + \xi_{3t}\ell''\left(H_t\right), \tag{A.4}$$

$$\mathcal{S}_t \; : \; \xi_{1t}\left(\Lambda\eta\mathcal{S}_t^{\eta-1} - C_{ft}\right) - \xi_{2t}\Lambda\eta\mathcal{S}_t^{\eta-1} = 0, \tag{A.5}$$

$$\pi_{ht} \; : \; -\xi_{2t}\phi\left(\pi_{ht}-1\right)A_t H_t - \xi_{3t}\phi\epsilon^{-1}\left(2\pi_{ht}-1\right)A_t u_{c_{ht}} - \xi_{4t}\mu_\pi\mathbb{E}_t\left\{\frac{\pi_{ht}^{\mu_\pi-1}}{\pi_{ht+1}}\frac{u_{c_{ht+1}}}{u_{c_{ht}}}\right\} = 0. \tag{A.6}$$

Consider the steady state of the above equilibrium, and assume $\pi_h = 1$. Equation (A.2) implies:

$$\xi_4 = -\frac{\xi_3\phi A u_{c_h}}{\mu_\pi\epsilon}, \tag{A.7}$$

which, plugged in Equation (A.6), gives:

$$u_{c_h} = \xi_2 + \xi_3 A u_{c_{hh}}\left(\theta + \frac{\phi}{\mu_\pi\epsilon}\right). \tag{A.8}$$

Further, under our assumption, Equation (A.4) implies:

$$\xi_3 = \frac{\ell'\left(H\right) - \xi_2 A}{\ell''\left(H\right)}. \tag{A.9}$$

Combining the two last equations and using $\ell'(H) = Au_{c_h}\theta$ when $\pi_h = 1$ implies:

$$u_{c_h}\left(1 - \frac{A^2 u_{c_{hh}}}{\ell''\left(H\right)}\theta\left(\theta + \frac{\phi}{\mu_\pi\epsilon}\right)\right) = \xi_2\left(1 - \frac{A^2 u_{c_{hh}}}{\ell''\left(H\right)}\left(\theta + \frac{\phi}{\mu_\pi\epsilon}\right)\right). \tag{A.10}$$

Finally, from Equation (A.3) and (A.3) and using $C_f = \Lambda\mathcal{S}^{\eta-1}$, in a steady state:

$$\xi_2 = \frac{\eta-1}{\eta}\xi_1 = \frac{\eta-1}{\eta}\frac{u_{c_f}}{\mathcal{S}}. \tag{A.11}$$

Combining we get:

$$u_{c_h}\left(1-\frac{A^2u_{c_{hh}}}{\ell''(H)}\theta\left(\theta+\frac{\phi}{\mu_\pi\epsilon}\right)\right)=\frac{\eta-1}{\eta}\frac{u_{c_f}}{\mathcal{S}}\left(1-\frac{A^2u_{c_{hh}}}{\ell''(H)}\left(\theta+\frac{\phi}{\mu_\pi\epsilon}\right)\right), \tag{A.12}$$

and using the optimal spending condition given by Equation (17):

$$1+\tau=\frac{u_{c_f}}{\mathcal{S}u_{c_h}}=\frac{\eta}{\eta-1}\frac{1-\theta\Delta_1}{1-\Delta_1}, \tag{A.13}$$

where $\Delta_1=\frac{A^2u_{c_{hh}}}{\ell''(H)}\left(\theta+\frac{\phi}{\mu_\pi\epsilon}\right)<0$ since $u_{c_{hh}}<0$. If $\theta=1$ (no markup distortion), then the formula implies the following tariff rate:

$$1+\tau=\frac{\eta}{\eta-1}\rightarrow\tau=\frac{1}{\eta-1}. \tag{A.14}$$

Further, since $\xi_2=u_{c_h}$ then $\xi_3=\xi_4=0$ and the assumed zero inflation rate ($\pi_h=1$) is optimal. Is the optimal tariff with markup distortions ($\theta<1$) larger or smaller than the tariff under $\theta=1$?

$$\frac{\eta}{\eta-1}-(1+\tau)=\underbrace{\frac{\eta}{\eta-1}}_{>1}\frac{\overbrace{(\theta-1)}^{<0\text{ since }\theta<1}\overbrace{\Delta_1}^{<0}}{\underbrace{1-\Delta_1}_{>1\text{ since }\Delta_1<0}}>0 \tag{A.15}$$

The tariff rate is smaller with monopolistic distortions. Further, the tariff with markup distortion depends on price stickiness and monetary policy, as shown below:

$$\begin{aligned}\frac{\partial\tau}{\partial\phi}&=\frac{\eta}{\eta-1}\frac{\partial\Delta_1}{\partial\phi}\frac{1-\theta}{(1-\Delta_1)^2}\\&=\frac{\eta}{\eta-1}\underbrace{\frac{A^2u_{c_{hh}}}{\ell''(H)\mu_\pi\epsilon}}_{<0}\underbrace{\frac{1-\theta}{(1-\Delta_1)^2}}_{>0}<0,\end{aligned} \tag{A.16}$$

$$\begin{aligned}\frac{\partial\tau}{\partial\mu_\pi}&=\frac{\eta}{\eta-1}\frac{\partial\Delta_1}{\partial\mu_\pi}\frac{1-\theta}{(1-\Delta_1)^2}\\&=\frac{\eta}{\eta-1}\underbrace{\frac{-A^2u_{c_{hh}}\phi}{\ell''(H)\mu_\pi^2\epsilon}}_{>0}\underbrace{\frac{1-\theta}{(1-\Delta_1)^2}}_{>0}>0.\end{aligned} \tag{A.17}$$

These equations also confirm that the absence of markup distortions ($\theta=1$) implies $\frac{\partial\tau}{\partial\phi}=\frac{\partial\tau}{\partial\mu_\pi}=0$, *i.e.* price stickiness and monetary policy do not affect the tariff rate.

A.2 Optimal Tariff with an Optimal Inflation Rate

Characterizing the optimal policy problem for the joint choice of the Home tariff and Home inflation:

$$V\left(\mathcal{Z}_t\right)=\text{Max}_{\left\{C_{ht},C_{ft},H_t,\mathcal{S}_t,\pi_{ht},\tau_t\right\}}\ u\left(C_{ht},C_{ft}\right)-\ell\left(H_t\right)+\beta\mathbb{E}_t\left\{V\left(\mathcal{Z}_{t+1}\right)\right\}, \tag{A.18}$$

subject to:

$$\begin{aligned}\text{Balanced trade} &: \Lambda\mathcal{S}_t^{\eta}=\mathcal{S}_tC_{ft}, && \text{(A.19)}\\ \text{Home Market clearing} &: A_tH_t\Phi_t=C_{ht}+\Lambda\mathcal{S}_t^{\eta}, && \text{(A.20)}\\ \text{Home labor market} &: \ell'\left(H_t\right)=A_tu_{c_{ht}}\mathbb{E}_t\left\{\Omega_{t,t+1}\right\}. && \text{(A.21)}\end{aligned}$$

Let ξ_{1t}, ξ_{2t}, ξ_{3t} represent the Lagrange multipliers on Equations (A.19), (A.20), and (A.21), respectively. First order conditions imply:

$$\begin{aligned}C_{ht} &: u_{c_{ht}}=\xi_{2t}+\xi_{3t}A_tu_{c_{hht}}\mathbb{E}_t\left\{\Omega_{t,t+1}\right\}, && \text{(A.22)}\\ C_{ft} &: u_{c_{ft}}=\xi_{1t}\mathcal{S}_t, && \text{(A.23)}\\ H_t &: \ell'\left(H_t\right)=\Phi_t\xi_{2t}A_t+\xi_{3t}\ell''\left(H_t\right), && \text{(A.24)}\\ \mathcal{S}_t &: \xi_{1t}(\eta-1)=\xi_{2t}\eta, && \text{(A.25)}\\ \pi_{ht} &: \xi_{2t}\left(\pi_{ht}-1\right)H_t=-\xi_{3t}\frac{u_{c_{ht}}}{\epsilon}\left(2\pi_{ht}-1\right). && \text{(A.26)}\end{aligned}$$

Consider the steady state of the above equations. From Equation (A.24), we get $\xi_3=\frac{\ell'(H)-\Phi\xi_2A}{\ell''(H)}$. Substitute in Equation (A.22) to get:

$$u_{c_h}=\xi_2+\left(\frac{\ell'\left(H\right)-\Phi\xi_2A}{\ell''\left(H\right)}\right)Au_{c_{hh}}\Omega. \tag{A.27}$$

Then using Equation (A.21) to substitute for $\ell'\left(H\right)$ gives:

$$\begin{aligned}u_{c_h} &= \xi_2+\left(\frac{Au_{c_h}\Omega-\Phi\xi_2A}{\ell''\left(H\right)}\right)Au_{c_{hh}}\Omega, && \text{(A.28)}\\ &= \xi_2\frac{\left(1-\frac{A^2u_{c_{hh}}}{\ell''(H)}\Omega\Phi\right)}{\left(1-\frac{u_{c_{hh}}A^2}{\ell''(H)}\Omega^2\right)}. && \text{(A.29)}\end{aligned}$$

From Equation (A.26), we have $\xi_3=-\xi_2\frac{\epsilon(\pi_h-1)H}{u_{c_h}(2\pi_h-1)}$. Substitute in Equation (A.22) to get:

$$u_{c_h}=\xi_2\left(1-\frac{\epsilon\left(\pi_h-1\right)}{\left(2\pi_h-1\right)}\frac{AHu_{c_{hh}}\Omega}{u_{c_h}}\right). \tag{A.30}$$

Substitute this also in Equation (A.24) to get:

$$\ell'(H) = u_{c_h} A\Omega = \xi_2 \left(\Phi A - \frac{\epsilon (\pi_h - 1) H \ell''(H)}{u_{c_h} (2\pi_h - 1)} \right). \tag{A.31}$$

Combine to get:

$$\Omega = \frac{\Phi - \Delta_3 \frac{(\pi_h - 1)}{(2\pi_h - 1)}}{1 - \Delta_4 \frac{(\pi_h - 1)}{(2\pi_h - 1)} \Omega}, \tag{A.32}$$

where $\Delta_3 = \frac{\epsilon \ell''(H) H}{A u_{c_h}}$, and $\Delta_4 = \frac{\epsilon A H u_{c_{hh}}}{u_{c_h}} < 0$. For the tariff rate, combine Equation (A.23) and (A.25) with Equation (17) and replace ξ_2 to get:

$$1 + \tau = \frac{\eta}{\eta - 1} \frac{1 - \Omega^2 \Delta_2}{1 - \Delta_2 \Omega \Phi} < \frac{\eta}{\eta - 1}, \tag{A.33}$$

with $\Delta_2 = \frac{A^2 u_{c_{hh}}}{\ell''(H)} < 0$.

A.3 Optimal inflation with a zero tariff: Proof of Result 4.2

Now we abstract from tariffs altogether, and assume that there is free trade, so that $\tau_t = 0$. The policy problem for the small open economy is defined as:

$$V(\mathcal{Z}_t) = \text{Max}_{\{C_{ht}, C_{ft},\, H_t, \mathcal{S}_t, \pi_t\}} \; u(C_{ht}, C_{ft}) - \ell(H_t) + \mathbb{E}_t \beta \left\{ V(\mathcal{Z}_{t+1}) \right\}, \tag{A.34}$$

subject to (14)-(17). Let ξ_{1t}, ...,ξ_{4t} denote the Lagrange multipliers on the constraints (14)-(17) respectively. Now, in contrast to Result 4.1, the constraint (17) will bind for the policymaker, since she doesn't have the option of choosing a tariff. The first-order conditions for the discretionary policymaker are then listed as:

$$C_{ht} \; : \; u_{c_{ht}} = \xi_{2t} + \xi_{3t} A_t u_{c_{hht}} \mathbb{E}_t \Omega_{t,t+1} - \xi_{4t} \mathcal{S}_t u_{c_{hht}}, \tag{A.35}$$

$$C_{ft} \; : \; u_{c_{ft}} = \xi_{1t} \mathcal{S}_t + \xi_{4t} u_{c_{fft}}, \tag{A.36}$$

$$H_t \; : \; \ell'(H_t) = \xi_{2t} A_t \Phi_t + \xi_{3t} \ell''(H_t), \tag{A.37}$$

$$\mathcal{S}_t \; : \; \xi_{1t}(\eta - 1) C_{ht} + \xi_{4t} u_{c_{ht}} = \xi_{2t} \eta C_{ft}, \tag{A.38}$$

$$\pi_{ht} \; : \; -\xi_{2t} A_t H_t (\pi_{ht} - 1) = \xi_{3t} \frac{u_{c_{ht}}}{\epsilon} (2\pi_{ht} - 1). \tag{A.39}$$

(A.40)

Using Equations (A.35) and (A.37) along with Equation (16), and assuming a steady state, we can obtain:

$$\Omega = \frac{1 - \frac{\phi}{2}(\pi_h - 1)^2 - \Delta_3 \frac{(\pi_h - 1)}{(2\pi_h - 1)}}{1 - \Delta_4 \frac{(\pi_{ht} - 1)}{(2\pi_{ht} - 1)} \Omega - \frac{\xi_4}{\xi_2} \mathcal{S} u_{c_{hh}}} \tag{A.41}$$

where as before, $\Delta_3 = \frac{\ell''(H)H\epsilon}{Au_{c_h}}$, $\Delta_4 = \frac{u_{c_{hh}}AH\epsilon}{u_{c_h}} < 0$, and $\Omega = \theta + \frac{\phi}{\epsilon}(1-\beta)\pi_h(\pi_h - 1)$, $\Phi = 1 - \frac{\phi}{2}(\pi_h - 1)^2$. Note that in contrast to Equation (A.32), Equation (A.41) contains the term $-\frac{\xi_4}{\xi_2}\mathcal{S}u_{c_{hh}}$ in the denominator. This is positive when $\frac{\xi_4}{\xi_2}$ is positive. ξ_2 is the multiplier on the resource constraint, which is positive so long as a rise in TFP increases welfare (no immiserizing growth). ξ_4 is positive, since when the spending condition (17) binds, the planner wishes to reduce the expression $u_{ch}\mathcal{S} - u_{cf}$, given that the optimal tariff condition would be $u_{c_h}\mathcal{S} - u_{c_f}\frac{\eta}{\eta-1}$. Since the denominator in Equation (A.41) is greater than that in Equation (A.32), the equilibrium discretionary inflation rate without optimal tariff must be lower than that of Equation (A.32).

B Example Model with Sticky Wages

What if we consider sticky wages instead of sticky prices in the example model? In this case, the representative household supplies differentiated types of labor $H_t(i)$ at differentiated wage rates $W_t(i)$. There is monopolistic competition in labor types but firms operate under perfect competition. The aggregate labor bundle is

$$H_t = \left(\int_0^1 H_t(i)^{\frac{\epsilon_w - 1}{\epsilon_w}} di \right)^{\frac{\epsilon_w}{\epsilon_w - 1}}, \tag{B.1}$$

and optimal labor demands from firms give

$$H_t^d(i) = \left(\frac{W_t(i)}{W_t} \right)^{-\epsilon_w} H_t. \tag{B.2}$$

Preferences over consumption and hours are now given by:

$$\mathbb{E}_t \sum_{j=0}^{\infty} \beta^s \left[u\left(C_{ht+j}, C_{ft+j}\right) - \ell\left(\int_0^1 H_{t+j}(i)\, di \right) \right], \tag{B.3}$$

and the households budget constraint becomes:

$$\begin{aligned} B_t + P_{ht}C_{ht} \quad &+ \quad (1+\tau_t) S_t P_{ft}^* C_{ft} = R_{t-1}B_{t-1} + \Pi_t + TR_t \\ &+ \quad \int_0^1 \left[(1+s) W_t(i) H_t(i) - \frac{\phi_w}{2} \left(\frac{W_t(i)}{W_{t-1}(i)} - 1 \right)^2 \Xi_t(i) \right] di, \end{aligned} \tag{B.4}$$

where ϕ_w is a wage adjustment cost, s a wage subsidy and $\Xi_t(i) = W_t(i) H_t(i)$. Households maximize welfare subject to the budget constraint, taking into account the individual demands for labor types. The optimal intratemporal and intertemporal choices over consumption are unaffected. The optimal choices of individual wages after substituting individual labor demands and ignoring terms pertaining to $\Xi_t(i)$ are derived. After assuming symmetry among labor suppliers, we get:

$$\frac{P_{ht}\ell'(H_t)}{u_{c_{ht}} W_t} + \frac{(1+s)(1-\epsilon_w)}{\epsilon_w} = \mathbb{E}_t \left\{ \phi_w \epsilon_w^{-1} \left(\pi_t^w (\pi_t^w - 1) - \beta \pi_{t+1}^w (\pi_{t+1}^w - 1) H_{t+1}/H_t \right) \right\}, \tag{B.5}$$

where $\pi_t^w = W_t / W_{t-1}$ denotes wage inflation. Note that, for simplicity, we assumed that the discount factor was $\omega_{t+1} = \beta$, which makes little difference to the example. Under flexible wages ($\phi^w = 0$), Equation (B.5) becomes:

$$\ell'(H_t) = \theta^w u_{c_{ht}} \frac{W_t}{P_{ht}}, \tag{B.6}$$

where $\theta^w = (1+s)(\epsilon^w - 1)/\epsilon^w$ is a subsidy-adjusted measure of wage markup. Up to the markup distortion, Equation (B.6) is very similar to the labor supply Equation (6). Since firms now operate under perfect competition the optimal pricing condition writes:

$$P_{ht} = W_t A_t^{-1} \tag{B.7}$$

which implies:

$$\pi_{ht} = \pi_t^w \frac{A_{t-1}}{A_t} \tag{B.8}$$

In a steady state, wage and price inflation are thus identical, *i.e.* $\pi_h = \pi^w$. Finally, plugging the pricing condition in Equation (B.5) gives:

$$\ell'(H_t) = A_t u_{c_{ht}} \mathbb{E}_t \left\{ \Omega_{t,t+1}^w \right\}, \tag{B.9}$$

where

$$\mathbb{E}_t \left\{ \Omega_{t,t+1}^w \right\} = \mathbb{E}_t \left\{ \theta^w + \phi_w \epsilon_w^{-1} \left(\pi_t^w \left(\pi_t^w - 1 \right) - \beta \pi_{t+1}^w \left(\pi_{t+1}^w - 1 \right) H_{t+1} / H_t \right) \right\}. \tag{B.10}$$

Equation (B.9) is exactly identical to Equation (16), except that the two distortions now relate to the inverse of the subsidy-adjusted wage markup θ^w – instead of inverse of the subsidy-adjusted price markup θ – and to wage inflation π_t^w instead of price inflation π_{ht} – which are equal in a steady state. Further, since wage adjustment costs are paid in units of goods by households, the goods market clearing condition is

$$A_t H_t = C_{ht} + C_{ht}^* + \frac{\phi_w}{2} \left(\pi_t^w - 1 \right)^2 \frac{\Xi_t}{P_{ht}}, \tag{B.11}$$

$$= C_{ht} + C_{ht}^* + \frac{\phi_w}{2} \left(\pi_t^w - 1 \right)^2 \frac{W_t H_t}{P_{ht}}. \tag{B.12}$$

Using the optimal pricing condition from firms $P_{ht} = W_t A_t^{-1}$ we get

$$A_t H_t \Phi_t^w = C_{ht} + C_{ht}^*, \tag{B.13}$$

where

$$\Phi_t^w = 1 - \frac{\phi_w}{2} \left(\pi_t^w - 1 \right)^2 \tag{B.14}$$

Again, Equation (B.13) is isomorphic to the goods market clearing condition obtained under sticky prices, Equation (15). We thus conclude that our results would be unaffected by assuming sticky wages instead of sticky prices in the example model.

C The Extended Model

We describe a model with two countries denoted Home and Foreign. Households supply labor and consume goods from both countries. The world is populated with a unit mass of agents and Home has share n of these, with Foreign share $1-n$. We assume that firms set prices in the currency of producers (PCP), and adjust prices constrained by Rotemberg price adjustment costs. Households in the Home country have preferences over consumption and hours given by:

$$U\left(C_t, H_t\right) = \frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{H_t^{1+\psi}}{1+\psi}. \tag{C.1}$$

and trade bonds across countries.

C.1 Households

The representative Home household maximizes its welfare index:

$$\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left(\frac{C_{t+j}^{1-\sigma}}{1-\sigma} - \chi \frac{H_{t+j}^{1+\psi}}{1+\psi} \right), \tag{C.2}$$

subject to the following budget constraint:

$$S_t B_t^* + B_t + P_{ht} C_{ht} + (1+\tau_t) S_t P_{ft}^* C_{ft} + P_t \Lambda_t = S_t B_{t-1}^* R_{t-1}^* + B_{t-1} R_{t-1} + W_t H_t + \Pi_t + TR_t, \tag{C.3}$$

where B_t^* and B_t are the amounts of Foreign-currency and Home-currency denominated bonds bought by Home households, paying returns R_t^* and R_t between t and $t+1$. Buying Foreign-currency bonds incurs the payment of a small adjustment cost $\Lambda_t = \frac{\nu}{2}\left(\frac{S_t B_t^*}{P_t} - \frac{SB^*}{P}\right)^2$, proportional to the deviation of real Foreign bonds to their steady-state value. The bundle structure of adjustment costs mimics that of final goods. The representative household in the Home economy consumes local goods in quantity C_{ht} at the price P_{ht} and foreign goods in quantity C_{ft} at the price $(1+\tau_t) S_t P_{ft}^*$. The consumption bundle is:

$$C_t = \left(\gamma^{1/\lambda} C_{ht}^{1-1/\lambda} + (1-\gamma)^{1/\lambda} C_{ft}^{1-1/\lambda}\right)^{\frac{1}{1-1/\lambda}}, \tag{C.4}$$

where $\gamma = n + x(1-n)$, and x denotes Home bias. The aggregate consumption price index is:

$$P_t = \left(\gamma P_{ht}^{1-\lambda} + (1-\gamma)\left((1+\tau_t) S_t P_{ft}^*\right)^{1-\lambda}\right)^{\frac{1}{1-\lambda}}, \tag{C.5}$$

so that $P_{ht}C_{ht} + (1+\tau_t)\, S_t P^*_{ft} C_{ft} = P_t C_t$. The demand functions of Home and Foreign goods by Home households are respectively:

$$
\begin{aligned}
C_{ht} &= \gamma \left(\frac{P_{ht}}{P_t}\right)^{-\lambda} C_t = \gamma \mathcal{P}_t^{\lambda} C_t, && \text{(C.6)}\\
C_{ft} &= (1-\gamma) \left(\frac{(1+\tau_t)\, S_t P^*_{ft}}{P_t}\right)^{-\lambda} C_t = (1-\gamma) \left(\frac{\mathcal{P}_t}{(1+\tau_t)\, \mathcal{S}_t}\right)^{\lambda} C_t, && \text{(C.7)}
\end{aligned}
$$

where $\mathcal{P}_t = P_t/P_{ht} = \left(\gamma + (1-\gamma)\left((1+\tau_t)\,\mathcal{S}_t\right)^{1-\lambda}\right)^{\frac{1}{1-\lambda}}$ represents the relative price of the Home consumption good and $\mathcal{S}_t = S_t P^*_{ft}/P_{ht}$ denotes Home terms of trade. The first-order conditions of the Home household imply:

$$
\begin{aligned}
\beta \mathbb{E}_t \left\{ \frac{\mathcal{S}_{t+1} R^*_t \mathcal{P}_t C_t^{\sigma}}{\mathcal{S}_t \pi^*_{ft+1} \mathcal{P}_{t+1} C^{\sigma}_{t+1} \left(1 + \nu \left(\frac{S_t B^*_t}{P_t} - \frac{SB^*}{P}\right)\right)} \right\} &= 1, && \text{(C.8)}\\
\beta \mathbb{E}_t \left\{ \frac{R_t \mathcal{P}_t C_t^{\sigma}}{\pi_{ht+1} \mathcal{P}_{t+1} C^{\sigma}_{t+1}} \right\} &= 1, && \text{(C.9)}\\
\chi H_t^{\psi} C_t^{\sigma} &= \frac{\mathcal{W}_t}{\mathcal{P}_t}, && \text{(C.10)}
\end{aligned}
$$

where $\pi_{ht} = P_{ht}/P_{ht-1}$ and $\pi^*_{ft} = P^*_{ft}/P^*_{ft-1}$ are the gross rates of PPI inflation in the Home and Foreign country respectively, and $\mathcal{W}_t = W_t/P_{ht}$.

The Foreign representative household has a similar utility function, and its consumption bundle and price index are respectively:

$$
\begin{aligned}
C^*_t &= \left(\gamma^{*1/\lambda} C_{ft}^{*1-1/\lambda} + (1-\gamma^*)^{1/\lambda} C_{ht}^{*1-1/\lambda}\right)^{\frac{1}{1-1/\lambda}}, && \text{(C.11)}\\
P^*_t &= \left(\gamma^* P_{ft}^{*1-\lambda} + (1-\gamma^*) \left((1+\tau^*_t)\,\frac{P_{ht}}{S_t}\right)^{1-\lambda}\right)^{\frac{1}{1-\lambda}}, && \text{(C.12)}
\end{aligned}
$$

and the corresponding demand functions are:

$$
\begin{aligned}
C^*_{ft} &= \gamma^* \left(\frac{P_{ft}}{P^*_t}\right)^{-\lambda} = \gamma^* \mathcal{P}_t^{*\lambda} C^*_t, && \text{(C.13)}\\
C^*_{ht} &= (1-\gamma^*) \left(\frac{(1+\tau^*_t)\, P_{ht}}{S_t P^*_t}\right)^{-\lambda} = (1-\gamma^*) \left(\frac{\mathcal{S}_t \mathcal{P}^*_t}{(1+\tau^*_t)}\right)^{\lambda} C^*_t, && \text{(C.14)}
\end{aligned}
$$

where $\mathcal{P}^*_t = P^*_t/P^*_{ft} = \left(\gamma^* + (1-\gamma^*)\left((1+\tau^*_t)/\mathcal{S}_t\right)^{1-\lambda}\right)^{\frac{1}{1-\lambda}}$. The Foreign household faces a different budget constraint, as it only has access to local bonds without paying adjustment costs. Its labor supply equation is:

$$
\chi C_t^{*\sigma} H_t^{*\psi} = \frac{W^*_t}{P^*_t} = \frac{\mathcal{W}^*_t}{\mathcal{P}^*_t}, \qquad \text{(C.15)}
$$

where $\mathcal{W}_t^* = W_t^* / P_{ft}^*$ and the Euler equation associated with Foreign bonds gives:

$$\beta \mathbb{E}_t \left\{ \frac{R_t^* \mathcal{P}_t^* C_t^{*\sigma}}{\pi_{ft+1}^* \mathcal{P}_{t+1}^* C_{t+1}^{*\sigma}} \right\} = 1. \tag{C.16}$$

C.2 Firms

A measure n of firms in the Home economy produce differentiated goods. The aggregate Home good is a composite of these differentiated goods, where the elasticity of substitution between individual goods is denoted $\epsilon > 1$. The production function for firm i in the Home country is

$$Y_t(i) = A_t H_t(i)^{1-\alpha} X_t(i)^{\alpha} \tag{C.17}$$

where A_t is an exogenous aggregate productivity term. Here, $X_t(i)$ represents the use of intermediate goods by the Home firm i and $H_t(i)$ the use of labor. Intermediate good inputs are composed of Home and Foreign goods in a different composition than that of the consumption aggregator. Namely,

$$X_t(i) = \left(\gamma_x^{\frac{1}{\lambda}} X_{ht}(i)^{\frac{\lambda-1}{\lambda}} + (1-\gamma_x)^{\frac{1}{\lambda}} X_{ft}(i)^{\frac{\lambda-1}{\lambda}} \right)^{\frac{\lambda}{\lambda-1}}, \tag{C.18}$$

where $X_{jt}(i)$ is the Home firm's use of inputs from country $j = \{h, f\}$. The profits of Home firm i are then:

$$\Pi_t(i) = ((1+s)P_{ht}(i) - MC_t) Y_t(i), \tag{C.19}$$

where $MC_t = A_t^{-1}(1-\alpha)^{\alpha-1}\alpha^{-\alpha} W_t^{1-\alpha} P_{xt}^{\alpha}$ denotes the firm's nominal marginal cost, and where

$$P_{xt} = \left(\gamma_x P_{ht}^{1-\lambda} + (1-\gamma_x)((1+\tau_t)S_t P_{ft}^*)^{1-\lambda} \right)^{\frac{1}{1-\lambda}}, \tag{C.20}$$

is the price index relevant for the firm's use of intermediate inputs, s represents a subsidy to offset the monopoly distortion in pricing, and where τ_t is the tariff rate on imports. Cost minimization by the firm implies:

$$(1-\alpha)\frac{Y_t(i)}{H_t(i)} = \frac{W_t}{MC_t} \text{ and } \alpha \frac{Y_t(i)}{X_t(i)} = \frac{P_{xt}}{MC_t}, \tag{C.21}$$

with

$$X_{ht}(i) = \gamma_x \left(\frac{P_{ht}}{P_{xt}} \right)^{-\lambda} X_t(i) = \gamma_x \mathcal{P}_{xt}^{\lambda} X_t(i), \tag{C.22}$$

$$X_{ft}(i) = (1-\gamma_x) \left(\frac{(1+\tau_t)S_t P_{ft}^*}{P_{xt}} \right)^{-\lambda} X_t(i) = (1-\gamma_x) \left(\frac{\mathcal{P}_{xt}}{(1+\tau_t)\mathcal{S}_t} \right)^{\lambda} X_t(i), \tag{C.23}$$

where $\mathcal{P}_{xt}$ is the equivalent of $\mathcal{P}_t$ for intermediate goods.[31] The firm chooses its price to maximize the present value of expected profits, net of price adjustment costs:

$$\mathbb{E}_t \sum_{j=0}^{\infty} \omega_{t+j} \left(\Pi_{t+j}(i) - \frac{\phi}{2} \left(\frac{P_{ht+j}(i)}{P_{ht+j-1}(i)} - 1 \right)^2 P_{ht+j}(i)\, Y_{t+j}(i) \right), \tag{C.24}$$

where ω_t is the firm's nominal stochastic discount factor, and ϕ represents a price adjustment cost for the firm. Price adjustment costs are proportional to the nominal value of Home output, to be consistent with the nominal profit objective function of the firm. The first-order condition for profit maximization for the Home firm i takes into account the individual demand of good i, *i.e* $Y_t^d(i) = (P_{ht}(i) / P_{ht})^{-\epsilon} Y_t$ and is the same for all producers so that $P_{ht}(i) = P_{ht}$ and $Y_t(i) = Y_t$ and that the i index can be dropped. It implies:

$$\theta + \phi\epsilon^{-1} \left(\pi_{ht}(\pi_{ht} - 1) - \mathbb{E}_t \{ \omega_{t+1} \pi_{ht+1} (\pi_{ht+1} - 1) Y_{t+1} / Y_t \} \right) = \mathcal{MC}_t, \tag{C.25}$$

where $\theta = \frac{(1+s)(\epsilon-1)}{\epsilon}$, and:

$$\mathcal{MC}_t = MC_t / P_{ht} = \mathcal{MC}_t = \frac{\mathcal{W}_t^{1-\alpha} \mathcal{P}_{xt}^{\alpha}}{A_t \alpha^{\alpha} (1-\alpha)^{1-\alpha}} \text{ as well as } \omega_t = \beta \frac{C_{t-1}^{\sigma} \mathcal{P}_{t-1}}{C_t^{\sigma} \mathcal{P}_t}. \tag{C.26}$$

Using symmetry among producers, the factor demands can be rewritten as:

$$(1-\alpha)\mathcal{MC}_t Y_t = \mathcal{W}_t H_t, \text{ and } \alpha \mathcal{MC}_t Y_t = \mathcal{P}_{xt} X_t, \tag{C.27}$$

where $\mathcal{P}_{xt} = P_{xt} / P_{ht}$.

C.3 Economic Policy

There are three separate levers of policy in the model. Fiscal policy may be used to subsidize monopoly firms. Trade policy may be used to levy tariffs on imports, and monetary policy. In the case where firms are subsidized, we follow the literature in assuming that a fiscal authority chooses a subsidy to offset the steady-state monopoly markup. But we also allow for the possibility that the monopoly markup remains as a pre-existing distortion in the economy. As we see, this may have an important implication for trade policy.

C.4 The Competitive Equilibrium

We assume that governments rebate the proceeds from tariffs – net from the subsidy s – to the household using lump-sum transfers. Given that Rotemberg costs are paid in units of local goods and using the demand functions for intermediate and final goods, the goods market

[31] $\mathcal{P}_t$ and $\mathcal{P}_{xt}$ only differ by the presence of potentially different degrees of home bias.

clearing conditions are given by:

$$Y_t\left(1-\frac{\phi}{2}\left(\pi_{ht}-1\right)^2\right) = D_t + D^*_{xt}, \tag{C.28}$$

$$Y^*_t\left(1-\frac{\phi}{2}\left(\pi^*_{ft}-1\right)^2\right) = D^*_t + D_{xt}, \tag{C.29}$$

where

$$D_t = \gamma\mathcal{P}_t^{\lambda}\left(C_t+\Lambda_t\right)+\gamma_x\mathcal{P}_{xt}^{\lambda}X_t, \tag{C.30}$$

$$D_{xt} = \frac{n}{1-n}\left(\frac{\mathcal{S}_t^{-1}}{1+\tau_t}\right)^{\lambda}\left(\left(1-\gamma\right)\mathcal{P}_t^{\lambda}\left(C_t+\Lambda_t\right)+\left(1-\gamma_x\right)\mathcal{P}_{xt}^{\lambda}X_t\right), \tag{C.31}$$

$$D^*_t = \gamma^*\mathcal{P}_t^{*\lambda}C^*_t+\gamma^*_x\mathcal{P}_{xt}^{*\lambda}X^*_t, \tag{C.32}$$

$$D^*_{xt} = \frac{1-n}{n}\left(\frac{\mathcal{S}_t}{1+\tau^*_t}\right)^{\lambda}\left(\left(1-\gamma^*\right)\mathcal{P}_t^{*\lambda}C^*_t+\left(1-\gamma^*_x\right)\mathcal{P}_{xt}^{*\lambda}X^*_t\right). \tag{C.33}$$

The labor-market clearing conditions are:

$$(1-\alpha)\mathcal{MC}_tA_tH_t^{-\alpha}X_t^{\alpha} = \chi\mathcal{P}_tC_t^{\sigma}H_t^{\psi}, \tag{C.34}$$

$$(1-\alpha)\mathcal{MC}^*_tA^*_tH_t^{*-\alpha}X_t^{*\alpha} = \chi\mathcal{P}^*_tC_t^{*\sigma}H_t^{*\psi}. \tag{C.35}$$

Finally, Home bonds are in zero-net supply so that $B_t = 0$ and the clearing condition on the market for Foreign bonds writes:

$$nB^*_t + (1-n)\,B^{**}_t = 0. \tag{C.36}$$

Defining $b_t = \frac{S_tB^*_t}{P_t}$ and $b^*_t = \frac{B^{**}_t}{P^*_t}$ as the real per-capita net foreign asset positions, Equation (C.36) implies:

$$nb_t + (1-n)\,\frac{\mathcal{S}_t\mathcal{P}^*_t}{\mathcal{P}_t}b^*_t = 0. \tag{C.37}$$

Further, the modified uncovered interest rate parity condition stemming from the combination of Home and Foreign Euler Equations writes:

$$\mathbb{E}_t\left\{\frac{\mathcal{S}_{t+1}\omega_{t+1}}{\mathcal{S}_t\omega^*_{t+1}\left(1+\nu\left(b_t-b\right)\right)}-1\right\}=0, \tag{C.38}$$

where, remember, $\omega_t = \beta\frac{C^{\sigma}_{t-1}\mathcal{P}_{t-1}}{C^{\sigma}_t\mathcal{P}_t}$ and $\omega^*_t = \beta\frac{C^{*\sigma}_{t-1}\mathcal{P}^*_{t-1}}{C^{*\sigma}_t\mathcal{P}^*_t}$. Last, the consolidation of the Home household budget constraint with other equilibrium and market clearing conditions gives:

$$b_t = \frac{\mathcal{S}_t\mathcal{P}_{t-1}}{\mathcal{S}_{t-1}\mathcal{P}_t\omega^*_t}b_{t-1}+\mathcal{P}_t^{-1}\left(D^*_{xt}-\frac{1-n}{n}\mathcal{S}_tD_{xt}\right). \tag{C.39}$$

Using appropriate substitutions, the above equations can be reduced to a system of two Phillips Curves (Equations (C.40) and (C.41) below), two good market clearing conditions (Equations (C.42) and (C.43) below), two Euler equations (Equations (C.45)-(C.46) below), and Equations (C.44), (C.47) and (C.48) that describe the external equilibrium – the terms of trade (Equation (C.47) below) and two net foreign asset positions (Equation (C.44 and Equation (C.48) below). Conditional on a given set of tariffs $\{\tau_t, \tau_t^*\}$ and monetary policies $\{R_t, R_t^*\}$, these equations determine $\left\{\pi_{ht}, \pi_{ft}^*, C_t, C_t^*, Y_t, Y_t^*, b_t, b_t^*, \mathcal{S}_t\right\}$.

$$\theta + \phi\epsilon^{-1}\left(\pi_{ht}\left(\pi_{ht}-1\right) - \mathbb{E}_t\left\{\omega_{t+1}\pi_{ht+1}\left(\pi_{ht+1}-1\right)\frac{Y_{t+1}}{Y_t}\right\}\right) = \mathcal{MC}_t, \quad \text{(C.40)}$$

$$\theta + \phi\epsilon^{-1}\left(\pi_{ft}^*\left(\pi_{ft}^*-1\right) - \mathbb{E}_t\left\{\omega_{t+1}^*\pi_{ft+1}^*\left(\pi_{ft+1}^*-1\right)\frac{Y_{t+1}^*}{Y_t^*}\right\}\right) = \epsilon\mathcal{MC}_t^*, \quad \text{(C.41)}$$

$$Y_t\left(1-\frac{\phi}{2}\left(\pi_{ht}-1\right)^2\right) - D_t - D_{xt}^* = 0, \quad \text{(C.42)}$$

$$Y_t^*\left(1-\frac{\phi}{2}\left(\pi_{ft}^*-1\right)^2\right) - D_t^* - D_{xt} = 0, \quad \text{(C.43)}$$

$$nb_t + (1-n)\frac{\mathcal{S}_t\mathcal{P}_t^*}{\mathcal{P}_t}b_t^* = 0, \quad \text{(C.44)}$$

$$\mathbb{E}_t\left\{\frac{R_t\omega_{t+1}}{\pi_{ht+1}}\right\} = 1, \quad \text{(C.45)}$$

$$\mathbb{E}_t\left\{\frac{R_t^*\omega_{t+1}^*}{\pi_{ft+1}^*}\right\} = 1, \quad \text{(C.46)}$$

$$\mathbb{E}_t\left\{\frac{\mathcal{S}_{t+1}\omega_{t+1}}{\mathcal{S}_t\omega_{t+1}^*\left(1+\nu\left(b_t-b\right)\right)} - 1\right\} = 0, \quad \text{(C.47)}$$

$$b_t - \frac{\mathcal{S}_t\mathcal{P}_{t-1}}{\mathcal{S}_{t-1}\mathcal{P}_t\omega_t^*}b_{t-1} - \mathcal{P}_t^{-1}\left(D_{xt}^* - \frac{1-n}{n}\mathcal{S}_tD_{xt}\right) = 0. \quad \text{(C.48)}$$

where $\omega_t = \beta\frac{C_{t-1}^{\sigma}\mathcal{P}_{t-1}}{C_t^{\sigma}\mathcal{P}_t}$; $\omega_t^* = \beta\frac{C_{t-1}^{*\sigma}\mathcal{P}_{t-1}^*}{C_t^{*\sigma}\mathcal{P}_t^*}$, $\mathcal{MC}_t = \frac{\left(\mathcal{P}_t\chi H_t^{\psi}C_t^{\sigma}\right)^{1-\alpha}\mathcal{P}_{xt}^{\alpha}}{A_t\alpha^{\alpha}(1-\alpha)^{1-\alpha}}$ and $\mathcal{MC}_t^* = \frac{\left(\mathcal{P}_t^*\chi H_t^{*\psi}C_t^{*\sigma}\right)^{1-\alpha}\mathcal{P}_{xt}^{*\alpha}}{A_t\alpha^{\alpha}(1-\alpha)^{1-\alpha}}$ with:

$$H_t = \left(\frac{(1-\alpha)\left(\mathcal{P}_t\chi C_t^{\sigma}\right)^{-\alpha}\mathcal{P}_{xt}^{\alpha}Y_t}{A_t\alpha^{\alpha}(1-\alpha)^{1-\alpha}}\right)^{\frac{1}{1+\alpha\psi}},\ H_t^* = \left(\frac{(1-\alpha)\left(\mathcal{P}_t^*\chi C_t^{*\sigma}\right)^{-\alpha}\mathcal{P}_{xt}^{*\alpha}Y_t^*}{A_t^*\alpha^{\alpha}(1-\alpha)^{1-\alpha}}\right)^{\frac{1}{1+\psi\alpha}}, \quad \text{(C.49)}$$

$$X_t = \frac{\alpha\left(\mathcal{P}_t\chi H_t^{\psi}C_t^{\sigma}\right)^{1-\alpha}\mathcal{P}_{xt}^{\alpha-1}Y_t}{A_t\alpha^{\alpha}(1-\alpha)^{1-\alpha}},\ X_t^* = \frac{\alpha\left(\mathcal{P}_t^*\chi H_t^{*\psi}C_t^{*\sigma}\right)^{1-\alpha}\mathcal{P}_{xt}^{*\alpha-1}Y_t^*}{A_t^*\alpha^{\alpha}(1-\alpha)^{1-\alpha}}. \quad \text{(C.50)}$$

With a flexible exchange rate, the model is closed by the two following monetary policy rules:

$$R_t = \beta^{-1}\pi_{ht}^{\mu_\pi},, \quad \text{(C.51)}$$

$$R_t^* = \beta^{-1}\pi_{ft}^{*\mu_\pi^*}. \quad \text{(C.52)}$$

D Alternative Parameter Values and Calvo Pricing

D.1 Alternative Parameter Values

Table 7 describes the results of a trade war with flexible inflation targeting under alternative parameter values. For a larger trade elasticity, assuming $\lambda = 8$, equilibrium tariffs in the trade war are substantially lower. Tariffs are higher than the baseline when the monopoly markup is lower ($\epsilon = 11$, implying a 10 percent markup), and higher in the case of lesser home bias in preferences and production. In addition, a smaller weight of intermediate goods, and a lower elasticity of intertemporal substitution also leads to higher equilibrium tariff rates. An infinitely elastic labor supply ($\psi = 0$) also leads to increased tariffs.

Table 7: Trade wars under alternative parameter values.

	Trade war - no subsidy ($\theta < 1$)						
	Baseline	$\lambda = 8$	$\epsilon = 11$	$\gamma = \gamma_x = 0.5$	$\alpha = 0.2$	$\sigma = 2$	$\psi = 0$
$\tau = \tau^*$	0.204	0.111	0.252	0.255	0.229	0.246	0.219
$C = C^*$	0.284	0.289	0.296	0.271	0.484	0.373	0.223
$L = L^*$	0.910	0.914	0.944	0.898	0.929	1.205	0.718
Welfare loss (%)	3.071	1.669	3.591	6.386	2.374	60.584	4.150

Welfare losses are computed against the zero-tariff equilibrium with price stability.

D.2 Calvo Pricing

In this variant of the extended model, the firms' pricing conditions, inflation dynamics and market clearing conditions are affected. The profits of Home firm i are represented as:

$$\Pi_t(i) = ((1+s)P_{ht}(i) - MC_t)\, Y_t(i)\,, \tag{D.1}$$

where MC_t denotes the firm's marginal cost. Calvo prices are set optimally subject to the constraint that only a fraction $(1-\varsigma) \in [0,1]$ of producers is randomly selected to reset prices optimally, and the remaining fraction ς keeps prices unchanged. The corresponding optimal pricing condition for re-setters solves:

$$\underset{\overline{P}_{ht}}{\text{argmax}}\mathbb{E}_t \sum_{j=0}^{\infty} (\beta\varsigma)^j \left((1+s)\overline{P}_{ht}(i) - MC_{t+j}\right) \frac{Y_{t+j}(i)}{C^{\sigma}_{t+j}\mathcal{P}_{t+j}}, \tag{D.2}$$

subject to $Y_t(i) = Y_t^d(i) = (P_{ht}(i)/P_{ht})^{-\epsilon}\, Y_t$ and the first-order condition writes:

$$\overline{P}_{ht}(i) = \frac{\epsilon\mathbb{E}_t \sum_{j=0}^{\infty} (\beta\varsigma)^j MC_{t+j} Y_{t+j}(i) \,/\, \left(C^{\sigma}_{t+j}\mathcal{P}_{t+j}\right)}{(1+s)(\epsilon-1)\mathbb{E}_t \sum_{j=0}^{\infty} (\beta\varsigma)^j Y_{t+j}(i)\,/\,\left(C^{\sigma}_{t+j}\mathcal{P}_{t+j}\right)} = \frac{\epsilon\Xi_{1t}}{(1+s)(\epsilon-1)\,\Xi_{2t}}, \tag{D.3}$$

with

$$\begin{aligned}\Xi_{1t} &= \varsigma\beta\mathbb{E}_t\left\{\Xi_{1t+1}\right\} + MC_t Y_t\left(i\right) / \left(C_t^{\sigma}\mathcal{P}_t\right), && \text{(D.4)}\\ \Xi_{2t} &= \varsigma\beta\mathbb{E}_t\left\{\Xi_{2t+1}\right\} + Y_t\left(i\right) / \left(C_t^{\sigma}\mathcal{P}_t\right). && \text{(D.5)}\end{aligned}$$

The general price level evolves as:

$$P_{ht}^{1-\epsilon} = (1-\varsigma)\,\overline{P}_{ht}^{1-\epsilon} + \varsigma P_{ht-1}^{1-\epsilon}, \tag{D.6}$$

which implies that the inflation rate is given by:

$$\varsigma\pi_{ht-1}^{\epsilon-1} + (1-\varsigma)\left(\frac{\epsilon\Xi_{1t}'}{(1+s)\,(\epsilon-1)\,\Xi_{2t}}\right)^{1-\epsilon} = 1, \tag{D.7}$$

with

$$\Xi_{1t}' = \frac{\Xi_{1t}}{P_{ht}} = \varsigma\beta\mathbb{E}_t\left\{\Xi_{1t+1}'\right\}\pi_{ht+1} + \mathcal{MC}_t Y_t\left(i\right) / \left(C_t^{\sigma}\mathcal{P}_t\right). \tag{D.8}$$

Finally, Calvo prices imply price dispersion, that is reflected in a dispersed use of inputs and modifies the goods market clearing conditions, which becomes:

$$Y_t\Phi_t = D_t + D_{xt}^*, \tag{D.9}$$

where

$$\Phi_t = \int_0^1\left(\frac{P_{ht}\left(i\right)}{P_{ht}}\right)^{-\epsilon} di = \varsigma\Phi_{t-1}\pi_{ht}^{\epsilon} + (1-\varsigma)\left(\frac{\epsilon\Xi_{1t}'}{(1+s)\,(\epsilon-1)\,\Xi_{2t}}\right)^{-\epsilon} \geq 1. \tag{D.10}$$

Symmetric conditions holds for the Foreign economy and other equilibrium conditions are unchanged. Table 8 compares the results of Table 1 regarding the trade war equilibrium under Calvo pricing (right panel) against those implied by Rotemberg pricing (left panel). The case with flexible prices is not reported since it delivers identical results by definition. The calibration remains the same except for the Calvo probability of price adjustment $1-\varsigma$, which is adjusted to deliver an identical slope of the Phillips Curves, implying $\varsigma = 0.7$. The results are virtually identical. However, Calvo prices involve a model with more equations and more state variables, while Rotemberg pricing allows to derive a non-linear Phillips Curve with only one equation and hence a more tractable model. This helps build intuition, especially in the example model.

Table 8: Trade wars – Rotemberg *vs.* Calvo pricing.

	Rotemberg pricing				Calvo pricing			
	No subsidy ($\theta < 1$)			$\theta = 1$	No subsidy ($\theta < 1$)			$\theta = 1$
	Base.	Large H.	Unilat.	Base.	Base.	Large H.	Unilat.	Base.
τ	0.204	0.239	0.212	0.304	0.205	0.239	0.213	0.304
τ^*	0.204	0.174	0.000	0.304	0.205	0.175	0.000	0.304
$\mathcal{S}$	1.000	0.954	0.900	1.000	1.000	0.954	0.899	1.000
C	0.284	0.290	0.297	0.308	0.284	0.290	0.297	0.308
C^*	0.284	0.274	0.282	0.308	0.284	0.274	0.282	0.308
L	0.910	0.913	0.905	0.983	0.910	0.913	0.905	0.986
L^*	0.910	0.907	0.921	0.983	0.910	0.907	0.921	0.986
H. loss (%)	3.071	1.088	−1.880	4.006	3.090	1.104	−1.882	4.014
F. loss (%)	3.071	6.120	4.449	4.006	3.090	6.132	4.470	4.014

"Base.": Nash equilibrium with sticky prices. "Large H.": Home country size is $n = 0.7$ (against $n = 0.5$ in the baseline case). "Unilat.": Home tariff setter unilaterally chooses its tariff while the Foreign tariff is $\tau_t^* = 0$. Welfare losses are computed against the zero-tariff equilibrium with price stability.

E Model with Bonds Denominated in Home Currency

When internationally traded bonds are denominated in the Home currency, the Home budget constraint writes:

$$B_t + P_{ht}C_{ht} + (1+\tau_t)\, S_t P_{ft}^* C_{ft} = B_{t-1}R_{t-1} + W_t H_t + \Pi_t + TR_t, \tag{E.1}$$

where B_t is the amount of Home currency-denominated bonds bought by Home households, paying return R_t between t and $t+1$, which implies the following first-order conditions:

$$\beta \mathbb{E}_t \left\{ \frac{R_t \mathcal{P}_t C_t^\sigma}{\pi_{ht+1} \mathcal{P}_{t+1} C_{t+1}^\sigma} \right\} = 1, \tag{E.2}$$

$$\chi H_t^\psi C_t^\sigma = \frac{\mathcal{W}_t}{\mathcal{P}_t}, \tag{E.3}$$

where $\pi_{ht} = P_{ht}/P_{ht-1}$. The representative Foreign household faces a modified constraint, accessing both Foreign and Home bonds:

$$S_t^{-1}B_t^* + B_t^{**} + P_{ht}C_{ht} + (1+\tau_t)\, S_t P_{ft}^* C_{ft} + P_t^* \Lambda_t^* = S_t^{-1} B_{t-1}^* R_{t-1} + B_{t-1}^{**} R_{t-1}^* + W_t H_t + \Pi_t + TR_t, \tag{E.4}$$

where B_t^* and B_t^{**} are respectively the amounts of Home and Foreign currency-denominated bonds bought by Foreign households, paying returns R_t and R_t^* between t and $t+1$. Buying Home bonds incurs the payment of a small adjustment cost $\Lambda_t^* = \frac{\nu}{2}\left(\frac{B_t^*}{S_t P_t^*} - \frac{B^*}{SP^*}\right)^2$, proportional to the deviation of real Foreign bonds from their steady-state value. The bundle structure of adjustment costs mimics that of final goods. The Euler equations associated with Home and

Foreign bonds give:

$$\beta \mathbb{E}_t \left\{ \frac{R_t S_t P_t^* C_t^{*\sigma}}{S_{t+1} P_{t+1}^* C_{t+1}^{*\sigma} \left(1 + \nu \left(\frac{B_t^*}{S_t P_t^*} - \frac{B^*}{SP^*}\right)\right)} \right\} = 1, \tag{E.5}$$

$$\beta \mathbb{E}_t \left\{ \frac{R_t^* \mathcal{P}_t^* C_t^{*\sigma}}{\pi_{ft+1}^* \mathcal{P}_{t+1}^* C_{t+1}^{*\sigma}} \right\} = 1. \tag{E.6}$$

The other equilibrium conditions are unchanged. Foreign bonds are in zero net supply so that $B_t^{**} = 0$ and the clearing condition on the market for Home bonds writes:

$$nB_t + (1-n) B_t^* = 0. \tag{E.7}$$

Defining $b_t = \frac{B_t}{P_t}$ and $b_t^* = \frac{B_t^*}{S_t P_t^*}$ as the real per-capita net foreign asset positions, the latter condition implies:

$$nb_t + (1-n) \frac{\mathcal{S}_t \mathcal{P}_t^*}{\mathcal{P}_t} b_t^* = 0. \tag{E.8}$$

Further, the modified uncovered interest rate parity condition stemming from the combination of Home and Foreign Euler Equations writes:[32]

$$\mathbb{E}_t \left\{ \frac{\mathcal{S}_{t+1} \omega_{t+1} \left(1 + \nu \left(b_t^* - b^*\right)\right)}{\mathcal{S}_t \omega_{t+1}^*} - 1 \right\} = 0, \tag{E.9}$$

where $\omega_t = \beta \frac{C_{t-1}^{\sigma} \mathcal{P}_{t-1}}{C_t^{\sigma} \mathcal{P}_t}$ and $\omega_t^* = \beta \frac{C_{t-1}^{*\sigma} \mathcal{P}_{t-1}^*}{C_t^{*\sigma} \mathcal{P}_t^*}$. Last, the consolidation of the Home household budget constraint with other equilibrium and market clearing conditions gives:

$$b_t = \frac{\mathcal{P}_{t-1}}{\mathcal{P}_t \omega_t} b_{t-1} + \mathcal{P}_t^{-1} \left(D_{xt}^* - \frac{1-n}{n} \mathcal{S}_t D_{xt} \right). \tag{E.10}$$

[32]With bonds denominated in Foreign currency, the modified condition was:

$$\mathbb{E}_t \left\{ \frac{\mathcal{S}_{t+1} \omega_{t+1}}{\mathcal{S}_t \omega_{t+1}^* \left(1 + \nu \left(b_t - b\right)\right)} - 1 \right\} = 0.$$

F Model with Dominant Currency Pricing

The model under DCP differs in only a few features. The nominal exchange rate is still flexible, but the impact of exchange rate changes on the Home terms of trade is muted since both its exports and imports are priced in its own currency. This has significant implications for the equilibrium of the policy game. The true price index for the Home consumer under DCP now becomes:

$$P_t = \left(\gamma P_{ht}^{1-\lambda} + (1-\gamma)((1+\tau_t)P_{ft})^{1-\lambda}\right)^{\frac{1}{1-\lambda}}, \tag{F.1}$$

where P_{ft} (instead of P_{ft}^* previously) is the price of the Foreign good set in Home currency, which implies:

$$\mathcal{P}_t = \frac{P_t}{P_{ht}} = \left(\gamma + (1-\gamma)((1+\tau_t)\mathcal{S}_t)^{1-\lambda}\right)^{\frac{1}{1-\lambda}}, \tag{F.2}$$

where $\mathcal{S}_t = P_{ft}/P_{ht}$. By contrast, the price index for the Foreign economy is unchanged since the Home country firm sets all prices in Home currency, which implies:

$$\mathcal{P}_t^* = P_t^*/P_{ft}^* = \left(\gamma^* + (1-\gamma^*)\left((1+\tau_t)/\mathcal{S}_t^*\right)^{1-\lambda}\right)^{\frac{1}{1-\lambda}}, \tag{F.3}$$

where $\mathcal{S}_t^* = S_t P_{ft}^*/P_{ht}$ is the equivalent of the terms of trade in the baseline (PCP) model. Relative price indices for intermediate goods $\mathcal{P}_t$ and $\mathcal{P}_{xt}$ are modified in the exact same way. The optimal pricing condition of the Home firm is as before, the firm chooses one price which is then converted to the Foreign currency when exported. But the Foreign firm charges separate prices to the local firms and households (in Foreign currency) and to the Home firms and household (in Home currency). The profits of the Foreign firm i are then represented as:

$$\Pi_t^*(i) = (1+s)\left(P_{ft}^*(i)\,Y_{ft}^*(i) + S_t^{-1}P_{ft}(i)\,Y_{ft}(i)\right) - MC_t^*\left(Y_{ft}^*(i) + Y_{ft}(i)\right), \tag{F.4}$$

where $MC_t^* = A_t^{*-1}(1-\alpha)^{\alpha-1}\alpha^{-\alpha}W_t^{*1-\alpha}P_{xt}^{*\alpha}$ and where $Y_{ft}(i) = D_{xt}(i)$ and $Y_{ft}^*(i) = D_t^*(i)$ in equilibrium. Foreign firm i maximizes:

$$\mathbb{E}_t \sum_{j=0}^{\infty} \omega_{t+j}^* \left(\begin{array}{c} \Pi_{t+j}^*(i) - \frac{\phi}{2}\left(\frac{P_{ft+j}^*(i)}{P_{ft+j-1}^*(i)} - 1\right)^2 P_{ft+j}^*(i)\,Y_{ft+j}^*(i) \\ -\frac{\phi}{2}\left(\frac{P_{ft+j}(i)}{P_{ft+j-1}(i)} - 1\right)^2 S_{t+j}^{-1}P_{ft+j}(i)\,Y_{ft+j}(i) \end{array} \right). \tag{F.5}$$

Note that the Foreign firm incurs costs of price adjustment for sales to the Home country that are separate from those pertaining to local sales. The first-order condition for profit maximization for the Foreign firm i selling to the Home country implies:

$$\theta + \phi\epsilon^{-1}\left(\pi_{ft}\left(\pi_{ft}-1\right) - \mathbb{E}_t\left\{\omega_{t+1}^*\frac{D_{xt+1}}{D_{xt}}\frac{\mathcal{S}_t^*/\mathcal{S}_t}{\mathcal{S}_{t+1}^*/\mathcal{S}_{t+1}}\pi_{ft+1}\left(\pi_{ft+1}-1\right)\right\}\right) = \left(\mathcal{S}_t^*/\mathcal{S}_t\right)\mathcal{MC}_t^*, \tag{F.6}$$

where $\mathcal{MC}_t^* = MC_t^*/P_{ft}^*$ while the condition for the Foreign firm i selling to the Foreign country is:

$$\theta + \phi\epsilon^{-1}\left(\pi^*_{ft}\left(\pi^*_{ft}-1\right) - \mathbb{E}_t\left\{\omega^*_{t+1}\frac{D^*_{t+1}}{D^*_t}\pi^*_{ft+1}\left(\pi^*_{ft+1}-1\right)\right\}\right) = \mathcal{MC}^*_t. \tag{F.7}$$

Finally, Home exports and imports become:

$$D_{xt} = \frac{n}{1-n}\mathcal{S}_t^{-\lambda}\left(1+\tau_t\right)^{-\lambda}\left(\left(1-\gamma\right)\mathcal{P}_t^{\lambda}C_t + \left(1-\gamma_x\right)\mathcal{P}_{xt}^{\lambda}X_t\right), \tag{F.8}$$

$$D^*_{xt} = \frac{1-n}{n}\mathcal{S}_t^{*\lambda}\left(1+\tau^*_t\right)^{-\lambda}\left(\left(1-\gamma^*\right)\mathcal{P}_t^{*\lambda}C^*_t + \left(1-\gamma^*_x\right)\mathcal{P}_{xt}^{*\lambda}X^*_t\right), \tag{F.9}$$

and the market clearing conditions:

$$Y_t\left(1-\frac{\phi}{2}\left(\pi_{ht}-1\right)^2\right) = D_t + D^*_{xt}, \tag{F.10}$$

$$Y^*_t\left(1-\frac{\phi}{2}\left(\pi_{ft}-1\right)^2 - \frac{\phi}{2}\left(\pi^*_{ft}-1\right)^2\right) = D^*_t + D_{xt}. \tag{F.11}$$

The modified uncovered interest rate parity condition becomes:

$$\beta\mathbb{E}_t\left\{\frac{\mathcal{S}^*_{t+1}\omega_{t+1}}{\mathcal{S}^*_t\omega^*_{t+1}\left(1+\nu\left(b_t-b\right)\right)} - 1\right\} = 0, \tag{F.12}$$

and net foreign assets:

$$b_t - b_{t-1}\frac{\mathcal{S}^*_t\mathcal{P}_{t-1}}{\mathcal{S}^*_{t-1}\mathcal{P}_t\omega^*_t} - \mathcal{P}_t^{-1}\left(D^*_{xt} - \frac{1-n}{n}\mathcal{S}_tD_{xt}\right) = 0, \tag{F.13}$$

$$nb_t + \left(1-n\right)\frac{\mathcal{S}^*_t\mathcal{P}^*_t}{\mathcal{P}_t}b^*_t = 0. \tag{F.14}$$

G Results with an Undistorted Steady State

The main results are discussed for the case of a distorted steady state where $\theta < 1$. In this Appendix we report the results when an optimal subsidy is in place, implying $\theta = 1$.

G.1 Example Model

Under flexible prices, Result 2 implies:

$$\tau = \frac{1}{\eta - 1} \tag{G.1}$$

where η is the elasticity of Foreign demand. This is the classic monopoly tariff formula according to which tariff setters should exploit the market power of local producers over Foreign demand. As under flexible prices, when $\theta = 1$ with sticky prices under an inflation targeting rule, Result 3 implies $\tau = \frac{1}{\eta-1}$. Finally, under a joint discretionary monetary and trade policy, Result 4.1 implies $\pi_h = 1$ and $\tau = \frac{1}{\eta-1}$. Clearly, $\theta = 1$ insulates the choice of an optimal discretionary tariff from nominal rigidities and from the monetary policy framework.

G.2 Extended Model

Table 9 below reports the results of the baseline experiment when $\theta = 1$ and compares them to the results when the steady state is distorted ($\theta < 1$).

Table 9: Trade Wars under inflation targeting or discretionary monetary policy – $\theta < 1$ *vs* $\theta = 1$.

	No subsidy ($\theta < 1$)		Subsidy ($\theta = 1$)	
	Baseline	Discr. MP	Baseline	Discr. MP
τ	0.204	0.264	0.304	0.304
π	1.000	1.033	1.000	1.000
$\mathcal{S}$	1.000	1.000	1.000	1.000
C	0.284	0.276	0.308	0.308
L	0.910	0.922	0.986	0.986
Welfare loss (%)	3.071	6.651	4.006	4.006

Welfare losses are computed against the zero-tariff equilibrium with price stability.

Table 9 confirms the results from the example model according to which price stability is the optimal outcome under a joint discretionary and trade policy, and tariff rates are similar to those arising under an inflation targeting rule.

G.3 Fixed Exchange Rate

Table 10 compares the outcomes of a trade war under flexible and fixed exchange rates, whether the steady state is distorted (left panel) or not (right panel).

Table 10: Trade wars – Flexible *vs* fixed exchange rate.

	No subsidy ($\theta < 1$)		Subsidy ($\theta = 1$)	
	Flex. ER	Fixed ER	Flex. ER	Fixed ER
$\tau = \tau^*$	0.204	0.170	0.304	0.271
$\mathcal{S}$	1.000	1.000	1.000	1.000
$C = C^*$	0.284	0.286	0.308	0.310
$L = L^*$	0.910	0.910	0.986	0.986
Welfare loss (%)	3.071	2.464	4.006	3.463

Welfare losses are computed against the zero-tariff equilibrium.

With a fixed exchange rate, tariff rates in each country are only 17 percent with markup distortion and 27.1 percent without, against 20.4 and 30.4 percent with a flexible exchange rate. With identical inflation rates ($\pi_h = \pi_f^* = 1$) and lower tariff rates, trade wars under fixed exchange rate produce lower welfare losses than with a flexible exchange rate. The logic underlying the results when the steady state is distorted applies to the case of an undistorted steady state.

G.4 Dominant Currency Pricing

Table 11 compares the outcomes of a trade war in the baseline case against the DCP case, whether the steady state is distorted (left panel) or not (right panel).

Table 11: Trade wars – Baseline *vs* DCP.

	No subsidy ($\theta < 1$)		Subsidy ($\theta = 1$)	
	Baseline	DCP	Baseline	DCP
$\pi_h = \pi_f^* = \pi_f$	1.000	1.000	1.000	1.000
τ	0.204	0.031	0.304	0.136
τ^*	0.204	0.211	0.304	0.311
$\mathcal{S}^*$	–	1.093	1.000	1.082
$\mathcal{S}$	1.000	1.093	1.000	1.082
C	0.284	0.283	0.308	0.309
C^*	0.284	0.294	0.308	0.317
L	0.910	0.919	0.986	0.991
L^*	0.910	0.906	0.986	0.981
Home welfare loss (%)	3.071	4.061	4.006	4.447
Foreign welfare loss (%)	3.071	−0.949	4.006	0.747

Welfare losses are computed against the zero-tariff equilibrium.

The main logic behind the results when the steady state is distorted applies to the case of an undistorted steady state, except that the Home economy sets positive tariffs, given that output is at a less inefficiently low level.