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MAXIMUM IMPACT INTERGENERATIONAL ASSOCIATIONS

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ABSTRACT

This paper presents a new approach to measuring the intergenerational transmission of well-being and a novel perspective on which measures and what age ranges to use to estimate intergenerational social mobility. We select the measures and the age ranges that best predict important human capital outcomes of children. The predictive power of parental resources varies among measures of parental resources as well as the age ranges used to measure them. Lifetime measures outperform traditional snapshot proxies for lifetime incomes based on income flows at certain age windows in predicting child outcomes, regardless of the ages when child outcomes are measured. The sensitivity of IGE estimates to the ages at which parental resources are measured is far smaller than their sensitivity to whether lifetime measures are used or whether snapshot measures are used. We also find that the financial resources of parents compensate in part for nonmonetary inputs to child human capital such as the stability of the family and education of parents. We interpret our estimates using the technology of skill formation modified to account for the emergence of new skills in adolescence.

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A web appendix is available at http://www.nber.org/data-appendix/w31093

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1 Introduction

The pioneering research of Becker and Tomes (1979, 1986) linked the study of intergenerational mobility and the study of child development. In their overlapping generations model under perfect certainty, persons live three periods: (a) as a child; (b) as an altruistic adult who invests in children and (c) as a retiree. Resources available in adulthood shape the potential for investing in children. Welfare of each generation and the entire dynasty is well defined including the return to altruistic investment.

Recent research extends their original framework to allow for multiple stages within each period, and to allow for critical and sensitive stages for effectiveness of investment in childhood. Credit constraints and work demands in the early years of adulthood impact child investment. Hai and Heckman (2017) show that more able and educated people with rising but not easily verifiable wage profiles face evolving credit constraints that vanish as parental income and information are realized. Caucutt and Lochner (2020) develop a model in which parents face evolving credit constraints and its consequences for child investment. Carneiro et al. (2021) present evidence that income received in later years has a substantial impact on child investment. Early childhood is emphasized in many papers despite evidence of multiple stages of developmental sensitivity associated with adolescence (Belsky et al., 2020; Crone, 2016; Steinberg, 2014).

Eshaghnia et al. (2022) analyze the dynamics of the life cycle in terms of the timing of the realization of income and the evolution of information. Intergenerational associations of age specific value functions are shown to vary by the age at which they are measured. There is a continuum of intergenerational parameters linking parental and child incomes. Given what is known about the technology of skill function (Cunha et al., 2010, Heckman and Mosso, 2014; Caucutt and Lochner, 2020) and the presence of credit constraints, the timing of receipt of resources matters for shaping the skills of the next generation.

This paper contributes to this literature. It has three main goals. First, we examine which measures of family resources at what ages are most predictive of child outcomes — such as education — that shape the welfare of the next generation. This analysis explores the empirical consequences of the interplay between constraints and technology of skill formation.

Second, we compute measures of intergenerational associations for the measures of income and its timing that have maximal impact on life-relevant outcomes. Snapshot measures of income designed to proxy lifetime resources based on income in narrow age intervals are usually in age ranges in the 30s. Nybom and Stuhler (2017) discuss this practice.

Third, we interpret our analysis using the technology of skill formation modified to account for the emergence of new skills in adolescence. A large body of literature on adolescence documents that new skills emerge post-puberty and the brain forms and consolidates new pathways that affect decision making and behavior (see Steinberg, 2014).

The focus in much of the received literature is on measurement error and alignment of ages and children, and their impact on estimated social mobility (see e.g. Lee and Solon, 2009; Mazumder, 2014). In contrast, we compute social mobility using maximally predictive income measures. This gives a principled basis for computing dynamic intergenerational measures of social mobility. Assuming that the constraints and technologies facing the offspring resemble those of the parents under the circumstances, it estimates dynastic intergenerational mobility.

Long run measures of expected income are far more predictive of important child outcomes than conventional measures of income traditionally used in the study of intergenerational mobility. In addition, using long run measures, maximal predictive ages for parental measures occur long after their children have outgrown early childhood. This is especially true when we predict adult participation in crime. This finding is consistent with the work of Steinberg (2014) and Moffitt (2018) that shows the importance of the adolescent years in the emergence of skills that affect criminality.

Eshaghnia et al. (2022), following Carneiro et al. (2021), study the impacts of parental resources when children are, on average, around 6 years old. That paper, in turn, builds on a large literature on early childhood skill formation (Cunha and Heckman, 2007). Carneiro et al. (2021) analyze the relationship between measures of realized parental resources and child outcomes measured over a range of childhood ages from birth to 18. This paper extends the analysis by relating parental expected present value of resources by age to child outcomes.

The predictive power of parental resources on child outcomes varies across different measures and also depends on the age ranges used to measure them. The traditional measures of parental resources, i.e., snapshots of lifetime income that are used to proxy lifetime values, are only weakly correlated with important child human capital outcomes. This holds true regardless of the specific age ranges used to measure parental resources. This is not primarily due to measurement error. The explanatory power of traditional measures of parental resources over child outcomes improves only marginally when the age range of the children over which parental resources are measured is varied.

Comparing conventional snapshot measures of parental income flows to lifetime measures of parental resources, we show that the sensitivity of IGE estimates to the age of parents (when their resources are measured) is far smaller than its dependence on whether we use lifetime measures or snapshot measures. We conclude that the type of measure used to study welfare transmission is far more important than the age of measurement. Also, there is no single "overtaking age" at which conventional measures closely proxy individual lifetime measures contrary to practice in the literature (Nybom and Stuhler (2017)).

We also contribute to the literature by studying the heterogeneity in transmission of intergenerational mobility by gender, education and cohabitional stability of parents. This heterogeneity is important for interpreting IGE estimates. Compared to male children, we find a stronger association between parental income and academic achievement for female children. For participation in crime, on the other hand, we find a much stronger association between parental income and child participation in crime for male children. For some outcome measures, parental resources in early childhood best predict the performance of boys whereas parental resources in early adolescence best predict the performance of girls. Moreover, we find that the financial resources of parents compensate in part for non-monetary inputs to children's human capital such as the stability of the family and education of parents.

The rest of this paper is organized in the following way. Section 2 describes our data and our measures of parent and child lifetimes resources. Section 3 presents estimates of the best ages and measures of parental resources for predicting child outcomes. Section 4 presents estimates of the intergenerational elasticity (IGE) based on the most predictive measures. Section 5 examines the heterogeneity of our estimates with respect to parental background. Section 6 links our analysis to that of Carneiro et al. (2021) and amends the standard model of the technology of skill formation to account for the emergence of new skills in adolescence. We present approximations to the technology and search for critical and sensitive periods and for dynamic complementarity. Section 7 presents estimates of the approximating model. Our evidence on sensitive periods is not consistent with that Carneiro et al. (2021). Our evidence on dynamic complementarity is mixed. Section 8 concludes.

2 Data and Measures of Outcomes and Resources

This paper uses full population administrative register data from Denmark in the years 1980 through 2019. The data contain unique identifiers of individuals, which enable us to combine information on a wide range of data across all ages. The data include unique identifiers of parents and spouses, allowing us to link families throughout the entire period. In addition to information on income measures of children and their parents, we also add information on completed education, household structure and demographic characteristics, and crime.[1]

[1]Using the individual identifiers, we link data from registers containing educational attainment (UDDA register), income, assets, transfers, marital status, and fertility (BEF register) for each individual and his or her spouse and parents. We also include information on criminal convictions from the sentencing register (KRAF).

2.1 Main Samples and Definitions

We base our analysis on the sample of children born in 1981 and 1982 for whom we can establish a link to parents, whose parents did not migrate, and who did not themselves migrate. We observe the birth cohorts of 1981 and 1982 from birth to age 38 and 37, respectively (in 2019). We also have information on their parents in all years between 1980 and 2019. For our IGE analysis, we measure children's resources at ages 30–35. In addition, we focus on individuals born between 1995 and 1997 in Denmark when we study the 9th-grade national leaving exam as our measure of child human capital. Precise definitions of the sample used are given in Web Appendix A.

2.2 Measures of Parental Resources

Table 1 summarizes the three measures of resources we analyze. The main component of *wage income* is taxable wage earnings, while *income with transfers* includes self-employment income, capital income, and income from public transfers such as social assistance and unemployment benefits. Finally, the *expected present discounted value* (PDV) of future income includes the expected disposable income stream of parents across their entire life evaluated at each age of the child.

Eshaghnia et al. (2022) consider a wider range of measures including a lifetime wealth approximation to lifetime value functions (the subjective present value of expected lifetime income discounted by an individual discount factor) and disposable income. Eshaghnia et al. (2022) also explicitly analyzes expected vs. realized outcomes and how the former provides a closer link to parents' investment in children.

Table 1: Definitions of Our Measures of Parental Resources

	Variable	Definition
(1)	Wage Income	Taxable family wage earnings and fringes, labor portion of business income, non-taxable earnings, severance pay, and stock options.
(2)	Income with Transfers	Total family personal income (excluding rental value of own home). Total personal income is equal to the sum of wage income, business and self-employment income, capital income, public transfer income, property income, and other non-classifiable income that can be attributed directly to the individual person.
(3)	Expected Present Discounted Value	The expected present discounted value of future total family income, using a deterministic discount factor (β): $$\text{PDV}_{i,t} = \mathbb{E}_{i,t}\left[\sum_{\tau=1}^{T-1} \beta^{\tau} y_{i,t+\tau} \mid \mathcal{I}_{i,t}\right],$$ where $y_{i,t}$ is the total income including interest on assets, public transfers, the estimated rental value of own home for owner-occupied individuals, and unrealized capital gains from housing stock for individuals who are homeowners, minus taxes and interest expenses at age t. β is a common discount factor, and $\mathcal{I}_{i,t}$ is agent i's information set. We set β to 0.96 following Ogaki and Reinhart (1998). The information set is being updated over ages and includes gender, education level (primary school, high school, college, and university), employment status, cohabitation, number of children, quartiles for mean income level, quartiles for mean consumption level, quartiles for mean consumption growth, quartiles for standard deviation of consumption, and homeownership status.

3 Child Outcomes and Parental Resources over the Life Cycle

A large literature focuses on the importance of aligning child and parental ages when estimating intergenerational persistence in income (Solon (1992); Grawe (2006); Mazumder (2008); Nybom and Stuhler (2017)). Different ages of measurement not only potentially lead to life cycle and attenuation bias—it also changes the channels studied for the transmission from parental income to child's income, and therefore leads to different interpretations of the IGE. We find that for many outcomes *there is no unique range of child ages at which parental income is most predictive*.

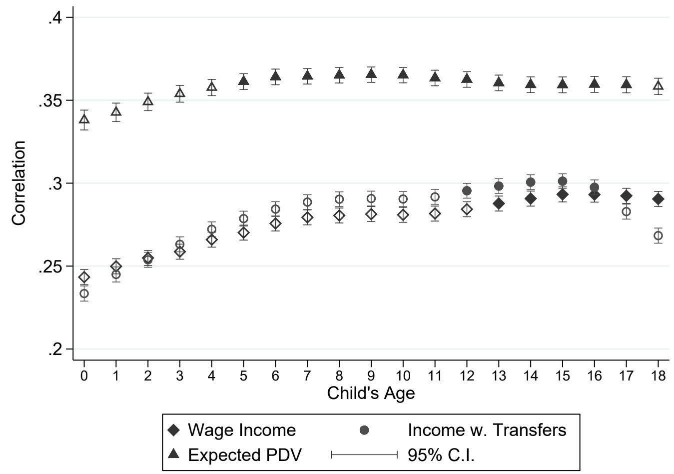
There are documented differences in sensitive periods in child development (see, e.g., Belsky et al., 2020; Knudsen et al., 2006; Steinberg, 2014). A high level of family income during early childhood may support the development of language, while during adolescence, higher levels of family income may prevent children from committing crimes given the development of the prefrontal cortex in adolescence (see, e.g., Crone, 2016; Steinberg, 2014). This finding is supported by long-standing evidence from the child development literature, showing that children develop different faculties at different stages of childhood (Murasko, 2007; Nelson et al., 2014; Belsky et al., 2020).

We provide correlational evidence that the channel through which parental income is transmitted to children depends on the age of the child at which parental income is measured. To make our argument, we proceed by studying the relationship between each child's outcome measure and various parental resources by child's age at measurement. Figure 1 presents correlations between parental income measures at different child ages from zero to 18 and child outcomes. Each panel of Figure 1 focuses on a specific outcome measure and plots the by-age correlation between parental resources (wage income, income with transfers, and the expected PDV) and the child's outcome of interest.[2] For each measure of resources, we use a t-test to evaluate whether the correlation at each age is significantly different from the maximum correlation. We represent the estimate with solid (filled) symbols if it is not significantly different from the maximum value (given the income measure).

[2]To reduce the impact of measurement errors when measuring parental resources, we use a 5-year rolling average of parental resources centered around the corresponding child's age.

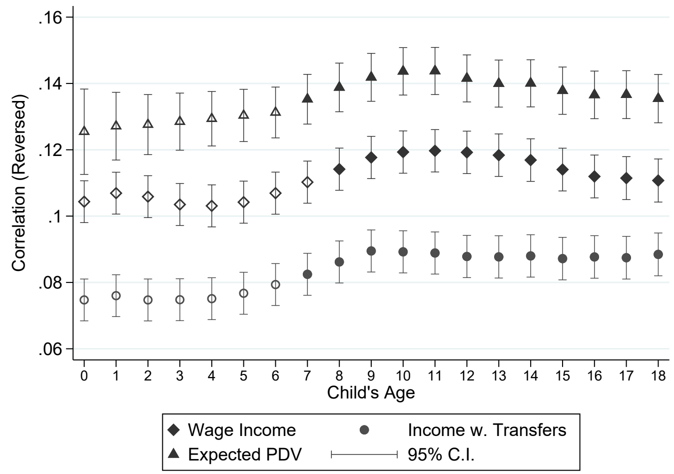
Figure 1: Correlations between Parental Measures of Resources and Various Child Human Capital Outcomes- by Child's Age

(a) Child's Language Test Score at age 11



(b) Math. National Leaving Exam Grade at age 16-17

(c) Years of Education by age 30



(d) Criminality (Ever Been Incarcerated) by age 35

Notes: Panels (a), (b), and (c) plot the correlations between the different parental income measures measured at the different child ages and the child's language test score at age 11, the child's national math test score at age 15, and the child's years of completed schooling at age 35, respectively. Figure (d) plots the inverted correlation between the child's receiving a prison sentence by age 35 and their parental resources. For each measure of resources, we use a t-test to evaluate whether the correlation at each age is significantly different from the maximum correlation (at the 5% level). We depict the estimate with solid (filled) symbols if it is not significantly different from the max.

Panel (a) shows the correlation between the child's language test score at age 11 and their parental measures of resources at the different child's ages between zero and 18. Panels (b), (c), and (d) present the relationship between parental resources in childhood and other child outcome measures, i.e., the child's national math test score at age 15 (panel (b)), the child's years of completed formal schooling by age 35 (panel (c)), and child's incarceration by age 35 (panel (d)). The patterns are in general similar when we decompose the sample by family structure but the levels change. Section 5 below presents these results.[3]

Our main findings are as follows. First, across all ages of childhood, our lifetime measure of parental resources (the expected PDV) outperforms the traditional snapshot measures of parental resources in predicting child outcomes.

Second, traditional snapshot measures of parental resources reach their highest predictive power in middle childhood, and sometimes even after the realization of the child's outcome of interest. For example, while the correlation between the child's language test score at age 11 and the expected PDV of parents peaks in early childhood (at around ages 3–8), the correlation with parental wage income or income with transfers peaks much later in childhood (around ages 13–16); several years after the realization of the outcome in question. The only exception is the child's participation in crime (incarceration) where all measures of parental resources manifest very similar patterns over the child's age where the correlations peak at around ages 9-12 for all measures of parental resources. The measures of parental resources differ significantly in their predictive power, and the lifetime measure outperforms other measures in predicting a child's incarceration by age 35.[4] Additional results for crime for outcomes measured by ages 30, 35, 25 show the same pattern (see Appendix B).

Third, higher levels of parental lifetime income during early childhood (ages 3–8) are associated with higher academic achievement and the development of language, while parental income during adolescent years (ages 9–14) is more tightly linked to children's participation

[3]Appendix F shows that the child's human capital outcome measures such as education and participation in crime are important in predicting child's earnings in adulthood.

[4]Results are similar when we use different ages to define a child's criminality status. Appendix B shows these.

in crimes. Again, the patterns are generally similar when we break down the samples by family structure or by the education of parents. Section 5 develops these results further.

Table 2 summarizes the main results from Figure 1. We list the combination of the child's age at measurement and measure of parental resources with the highest correlation for each of the child outcomes presented in Figure 1. We refer to this combination of child's age at measurement and measure of parental resources as the "best predictor." Column (1) lists the child outcome. We report the corresponding "best predictor" in columns (2) and (3). The "best predictor" is the combination of parental lifetime well-being (column 2) and child age at measurement (column 3), with the highest correlation with the child outcome studied. Column (4) presents the corresponding R^2 resulting from a linear regression of the child outcome in column (1) on the measure of parental resources in column (2), i.e., its best predictor, where parental resources are measured at the child's ages listed in column (3). We discuss the results for IGEs the last two columns (5–6) of Table 2 later in Section 4.

Panel (a) of Table 2 presents the results when we compare all measures of parental resources. The lifetime measure of parental resources (expected PDV) outperforms all other income measures. Panel (b) of Table 2 lists the "best predictor" among the traditional income measures, i.e., where we exclude our lifetime measure of parental resources from the analysis. The only measure of child outcome for which the most predictive ranges of the traditional income measures overlap with those of the lifetime measure is participation in crime (incarceration).

To distinguish those ages with significantly different estimates from other ages, we run pairwise t-tests where we test if each estimate is significantly different from the age with the max correlation. Thus, we benchmark estimate at each age against the age with the max correlation, separately for each income measure and child outcome measure.

The peak age ranges are very similar for crime, but they are different for test scores. The peak ages for years of schooling overlap for some ages. So, it depends on the outcome measure of interest.

Appendix C presents the robustness of the patterns in this section when we control for parental education and family structure. Our empirical results suggest that once we take into account heterogeneity in parental education level and marital status, correlations between parental incomes and children's outcomes drop sharply at all ages for all types of income measures. The results also show less sensitivity with respect to the child's age when parental resources are measured. This may suggest that parental income is only a proxy for the impact of other factors at the family level, such as the education of parents and the family structure, on children's outcomes. The next section focuses on the implications of these results for the age of measurement when estimating the IGEs.

Table 2: Summary: 'Best Predictor' across Child Outcomes and Corresponding Explanatory Power and IGEs

Child's Outcome Measure (1)	Best Predictor: Measure (2)	Best Predictor: Child's Ages (3)	Best Predictor: R^2 (4)	IGE Estimates: Corresponding IGE (5)	IGE Estimates: IGE-R^2 (6)
Panel a: Among All Measures of Parental Resources					
Child's Language Test Score (age 11)	Expected PDV	3-8	0.058	0.45 [0.438,462]	0.113
Math. Leaving Exam (Age 15-16)	Expected PDV	5-17	0.137	0.44 [0.432,0.455]	0.115
Years of Education (by Age 35)	Expected PDV	8-16	0.137	0.43 [0.420,0.444]	0.115
Never Been Incarcerated (by Age 35)	Expected PDV	8-18	0.023	0.42 [0.408,432]	0.115
Panel b: Among Traditional Measures of Parental Resources					
Child's Language Test Score (age 11)	Wage Income	13-16	0.036	0.15 [0.142,0.161]	0.017
Math. Leaving Exam (Age 15-16)	Income w. Transfers	13-18	0.090	0.31 [0.297,0.323]	0.042
Years of Education (by Age 35)	Income w. Transfers	10-18	0.102	0.31 [0.297,0.323]	0.042
Never Been Incarcerated (by Age 35)	Wage Income	7-18	0.014	0.15 [0.142,0.161]	0.017

Notes: This table presents the parental measure (column 2) and age of measurement (column 3), resulting in the highest correlation ('best predictor') with each child outcome (column 1). Column (4) reports the R-squared of the linear regression of the child outcome in column (1) on the measure of parental resources in column (2). Panel (a) includes all measures of parental resources. Panel (b) restricts the analysis to the traditional snapshot measures of parental resources by excluding the expected PDV from the analysis. Column (5) presents the corresponding IGE estimate for each of the different 'best predictors', where individuals are measured over ages 30–35. The 95% confidence intervals are shown in brackets. To compute the IGE, we regress the child's lifetime well-being measure, listed in column (2) and measured at ages 30-35, on the parental lifetime well-being measured by the 'best predictor.' Column (6) reports the R-squared of the IGE regression. To compute the IGE in Panel (B), we regress the child's traditional well-being measure, listed in column (2) and measured at ages 30-35, on the parental lifetime well-being measured by the 'best traditional predictor.'

4 Estimates of Intergenerational Mobility

The literature on intergenerational mobility focuses on intergenerational elasticity (IGE) of income to measure the dependence of well-being across generations. Estimates of the IGE, β, can be obtained by running least square of the following regression:

$$\log(\bar{y}_i^c) = \alpha + \beta_k \log(\bar{y}_{i,k}^h), \tag{1}$$

where $\bar{y}_i^c$ denotes the average of child's resources in adulthood for family i, and $\bar{y}_{i,k}^h$ denotes the average resources of the family (the father and mother) when the child was k years old.[5] Running least squares of Equation (1) separately for different values of k, i.e., children's ages when their parental resources are measured, we obtain a range of IGE estimates, β_k (where $k \in \{0, \ldots, 18\}$), for a given measure of individual's resources.[6]

Using this framework, we discuss the sensitivity of the IGE estimates to the choice of the child's age for measuring parental resources. We find large differences between the IGE estimated using traditional measures and the IGE estimated using lifetime measures. At any given age, the IGE in the expected lifetime measure is around 3 times the IGE for wage income and about 1.5 times the IGE for income with transfers.

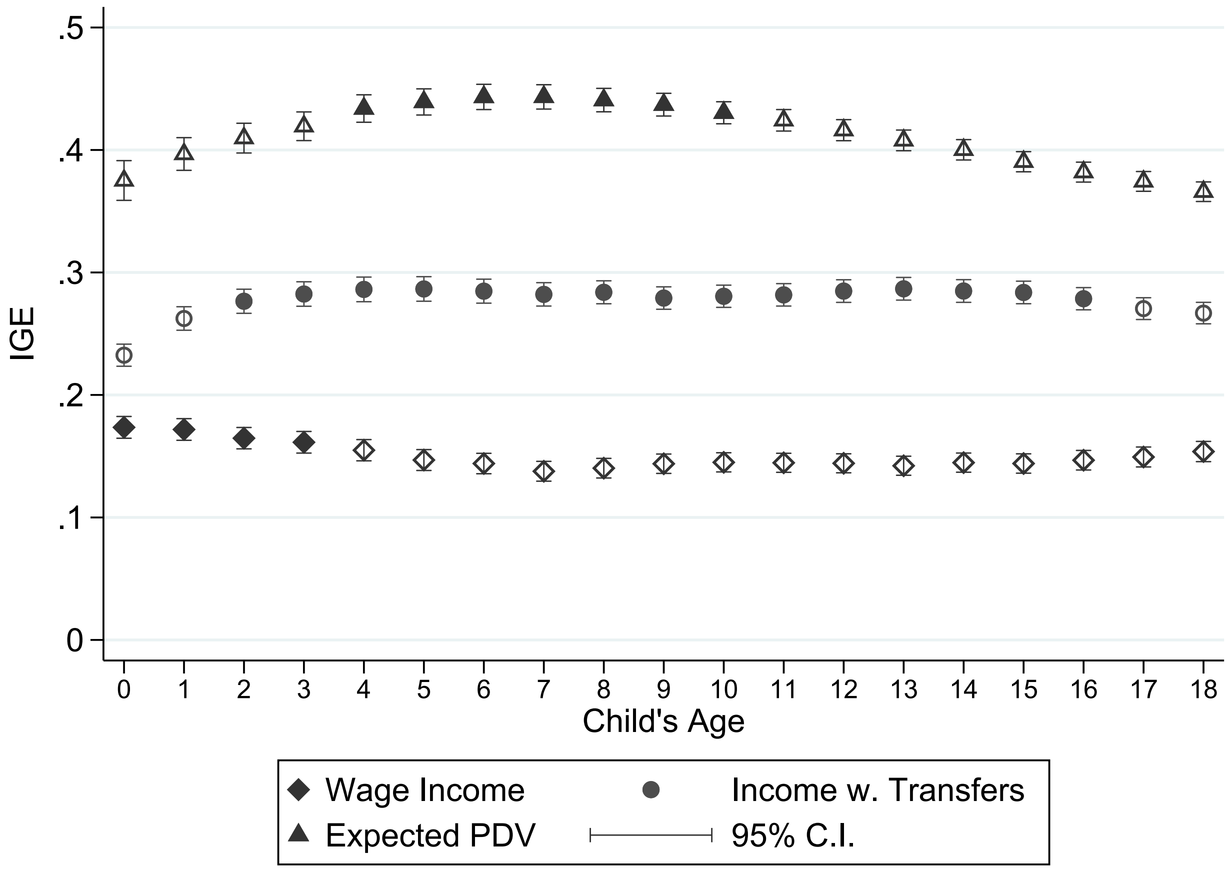
We report the IGE estimates for our traditional measures and lifetime measure in Figure 2. Notably, the IGE for expected PDV is at least 0.37, while the IGE in wage income does not exceed 0.18 and the IGE using income with transfers peaks at around 0.31.[7] The figure shows that the IGE estimate at a given age increases by at least 0.21 when using expected PDV to measure the IGE, rather than wage income.

[5]To mitigate concerns about measurement errors, we use five-year moving average centered around k, i.e., we use the average of family resources over the child's ages $k-2, k-1, k, k+1, k+2$.

[6]To measure child's resources in adulthood, we always use the average of child's resources over ages 30–35. Appendix Section G presents the robustness of our results with respect to different age ranges for measuring children's outcomes.

[7]The corresponding IGE estimates in Eshaghnia et al. (2022), when both parents and children are measured at ages 30-35, are 0.52, 0.28, and 0.34, for expected PDV, wage income, and income with transfers, respectively.

Figure 2: IGE Estimates over Age of Child When Parental Resources Measured: Estimates with Solid (Filled) Symbol Are Not Significantly Different from the Maximum IGE Estimate



Notes: This figure plots the IGE for selected income variables. The child's income is measured at ages 30-35 and the parental income is measured at the household level at different child's ages. We include children from the 1981-1982 cohorts in Denmark and their parents. The vertical lines around the point estimates represent the 95% confidence intervals. For each income measure, we use a t-test to evaluate whether the estimate at each age is significantly different from the maximum IGE estimate (at the 5% level).

Table 3: IGE ESTIMATES WHEN MEASURING PARENTAL RESOURCES OVER DIFFERENT CHILDHOOD AGES

Measure of Resources (1)	Maximum IGE (2)	Max-IGE Age (3)	Minimum IGE (4)	Min-IGE Age (5)
Income w. Transfers	0.31 [0.30,0.32]	4-10	0.25 [0.24,0.26]	15-18
Expected PDV	0.45 [0.44,0.46]	2-16	0.37 [0.36,0.38]	0
Wage Income	0.18 [0.17,0.19]	0–3	0.14 [0.13,0.15]	5-17

Notes: For each measure of resources (Column 1), this table presents the maximum (column 2) and minimum IGE (column 4) when we measure parental resources at different ages of children between 0 and 18 (five-year moving average), along ages of children corresponding to the maximum (column 3) and minimum (column 5) IGEs. We measure the child's resources between ages 30-35. The 95% confidence intervals for the IGE estimates are shown in brackets.

We summarize our findings in Table 3. When comparing IGEs for our lifetime measures with those for income with transfers, we find that the choice of measure has much larger effects on the IGE than the choice of age-interval. The choice of income variable is much more important than the choice of age at which to measure the IGE.

The results of Section 3 established that our lifetime measure, especially during early childhood and adolescent years, has a stronger connection to outcomes of children. In this section, to compute the IGEs, we use these measures of parental resources that are relevant to the transmission of family influence, and we determine those ages that manifest the tightest link with the child human capital outcomes. Columns (5)-(6) of Table 2 in the previous section the estimated IGEs using the age ranges where we find the strongest association between children's human capital outcomes.

We reach the following conclusions: First, the IGE measured at the most predictive ages is high, ranging from 0.42—0.45 (see Panel (a) of Table 2). Second, there is no clear, consistent "correct age" or "correct measure." The measures and ages leading to the maximum correlation vary across child outcomes. That said, among measure of family resources, the "best predictor" is always the lifetime measure of expected parental income, as discussed earlier in Section 3.

Panel (b) of Table 2 showed the "best predictor" for the traditional income measures. The IGEs in this table range from 0.15 to 0.31, the large difference being driven by the choice of income measure, rather than the age range used to measure parental resources.

We interpret these results in the following way. Choosing the correct age range to measure the IGE based on its predictive power for a specific child outcome helps us interpret the channel of transmission of income from parents to children. In the presence of imperfect capital markets and the revelation of life cycle information, the timing of parental income might matter. Which age ranges matter most depends on both the child's outcome of interest and the measure of parental resources. Therefore, for each measure of child outcomes, we estimate the IGE by choosing the most appropriate combination of income measure and age ranges that maximize the prediction power over the child's outcome of interest. We also demonstrate that lifetime measures of income provide us with a stronger correlational link to child outcomes, which is fairly stable across different age ranges during childhood.

We try other ages and present the results in Appendix G. Looking at the correlation and IGEs age by age in the current setting (where we estimate the relationship at each age in isolation from other ages) may not speak to the causal impact of the timing of income presented by Carneiro et al. (2021) since we don't hold income at other ages constant. We develop this point in Section 6 below where we generalize the Carneiro et al. (2021) procedure.

5 Heterogeneity of Estimates

This section studies how our estimates (for both intergenerational correlations and intergenerational elasticises) vary with respect to parental education levels, intactness of the family, marital status of parents, and gender. We also estimate heterogeneity in our results in Section 3 with respect to different types of crimes.

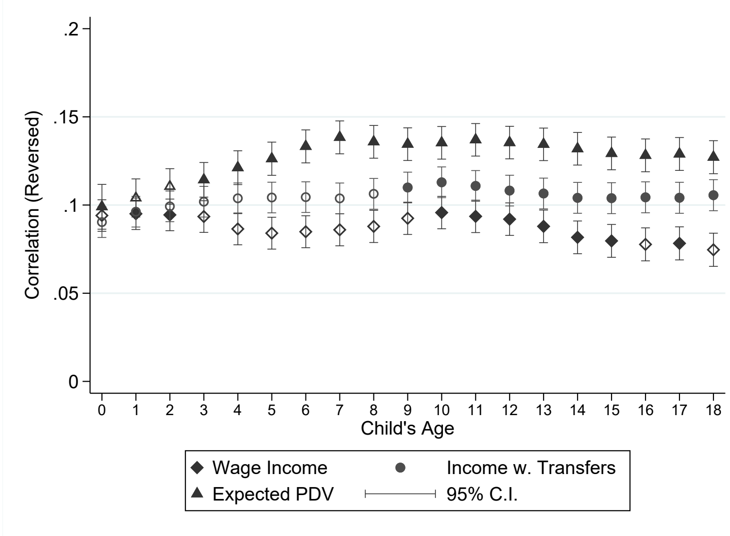
5.1 Crime Types

Figure 3 presents the correlations between parental income measures at different children's ages from zero to 18 and the child's participation in various types of crime, i.e., violent crimes (Panel (a)), property crimes[8] (Panel (b)), and other crimes, i.e., the residual (Panel (c)). "Other crimes" include all crimes other than violent crimes, property crimes, and traffic crimes. It includes, for example, drug crimes, sex crimes, and all other types of crimes from practicing as doctor without license to treason. For this analysis, to get higher precision, we focus on the sample of male children and their paternal resources. Appendix D presents the results for the sample of female children and also for the resources of both parents (the father's resources plus the mother's resources).

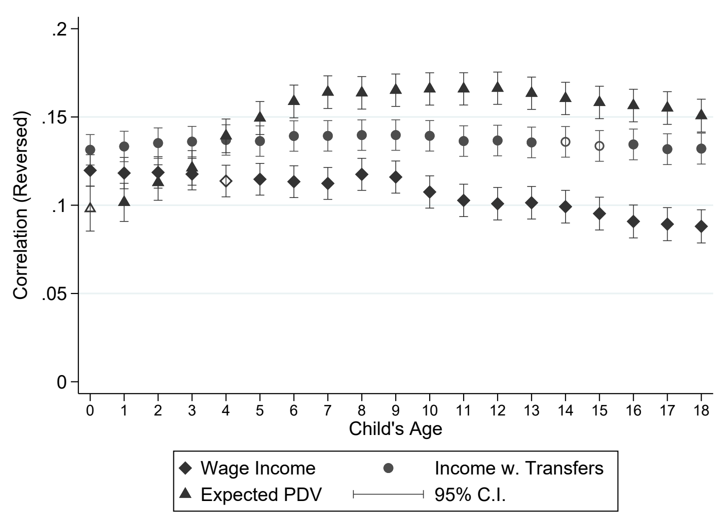
A consistent pattern emerges from Panel (a), (b), and (c) of Figure 3: the expected lifetime resources of parents around ages 7–13 best predict the child's participation in violent crimes, property crimes, and other crimes. Appendix Section B.1 presents the results where we define the criminality of the child for each crime type by ages 20, 25, and 30 of the child, as opposed to age 35 considered in Figure 3. The patterns are similar to Figure 3, but the differences across measures of parental resources are less pronounced when we measure the criminality status of children at earlier ages.

[8] Property crimes include crimes such as document forgery, arson, burglary in banking, business, residences, and uninhabited buildings, theft from car, boat etc, shoplifting, and other thefts, illegal handling of lost property, embezzlement, fraud, extortion, debtor fraud, robbery, tax fraud, vandalism, and property damage.

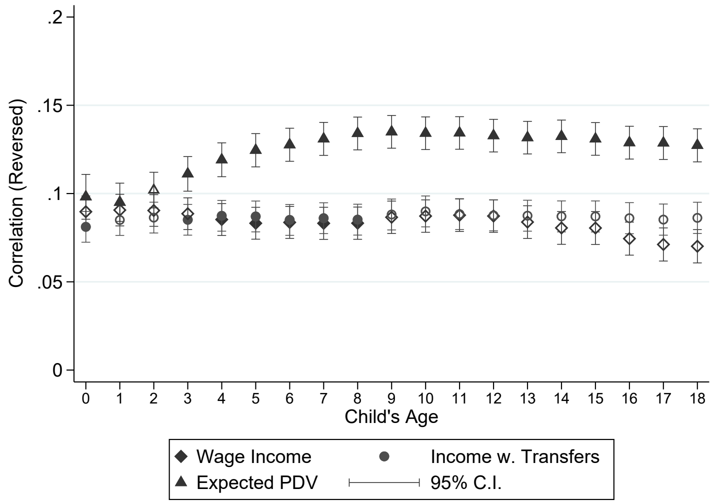
Figure 3: Paternal Resources and Child's Criminality By Age 35 for Different Crime Types- Sample of Male Children



(a) Violent Crimes



(b) Property Crimes



(c) Other Crimes

Notes: This figure plots the correlations between the different parental income measures measured at the different child ages and the child's participation in violent crimes (Panel (a)), property crimes, (Panel (b)), and all other types of crime (Panel (c)). We invert correlations in all figures.

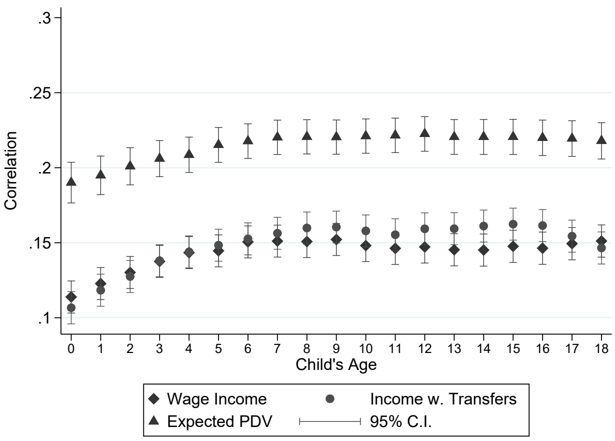
5.2 Parental Education Level

Figure 7 plots correlations between the child's mathematics test scores at age 16 and parental income variables measured at the household level at different ages of children for four different groups of families: college parents where both parents are college or university graduates (Panel a), parents where none of the parents are college or university graduates (Panel b), parents where only the father is a college or university graduate (Panel c). parents where only the mother is a college or university graduate (Panel d).

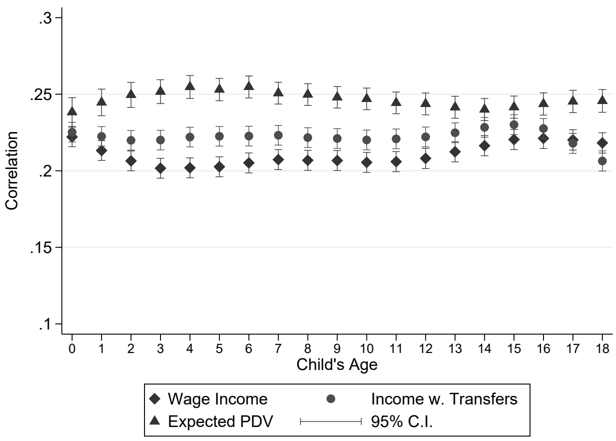
Figure 5 plots the correlation between the child's years of formal schooling by age 35 and parental income variables measured at the household level at different child's ages, separately for the four different groups of parental educational background as described above. Finally, Figure 6 plots the correlation between the child's participation in crime (defined as the absence of any incarceration) by age 35 and parental income variables measured at the household level at different child's ages, separately for the four different groups of parental educational background as in Figures 5 and 6.

The results suggest that the intergenerational correlation patterns are, in general, similar across different groups of parental educational backgrounds. However, intergenerational correlations tend to be significantly lower for the sample of highly educated parents compared to the sample of no-college parents. For example, the correlations between children's math test scores and parental wage income and the expected PDV are about 0.15 and 0.22, respectively, for the sample of college parents in Panel (a), significantly lower than 0.23 and 0.26 for the sample of lower educated parents shown in Panel (b). This suggests that parental resources play a more important role in predicting the child's outcomes for less-educated parents, compared to educated parents where other channels of transmission of human capital (e.g. information about acquiring higher education in college/university) may play a more crucial role.

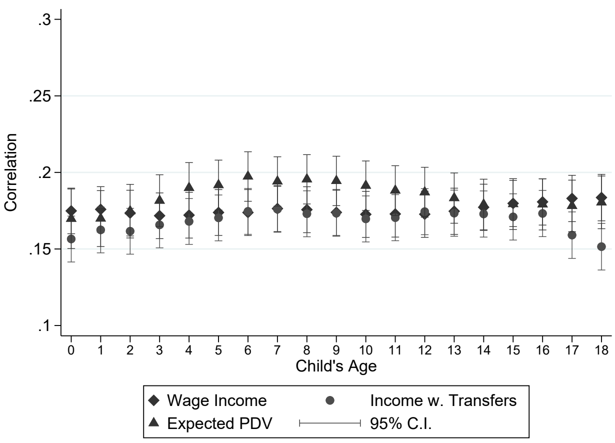
Figure 4: Correlations with Child's Mathematics Test Scores by Parental Education Level



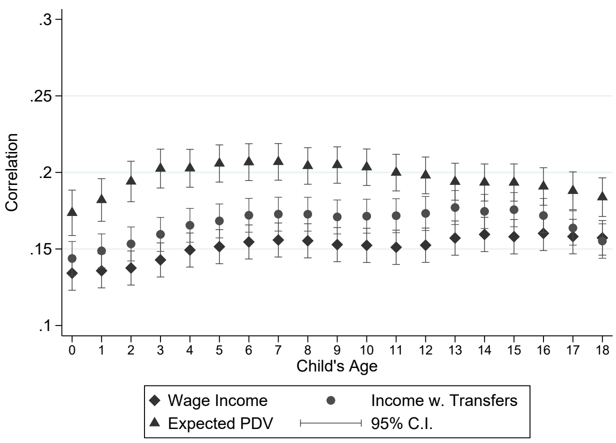
(a) College Parents



(b) Parents without College



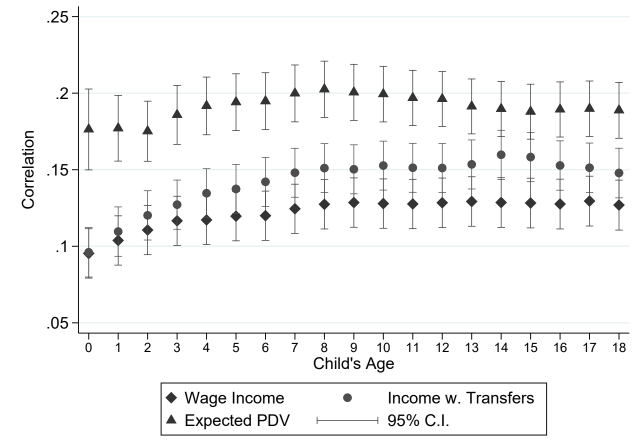
(c) College Fathers



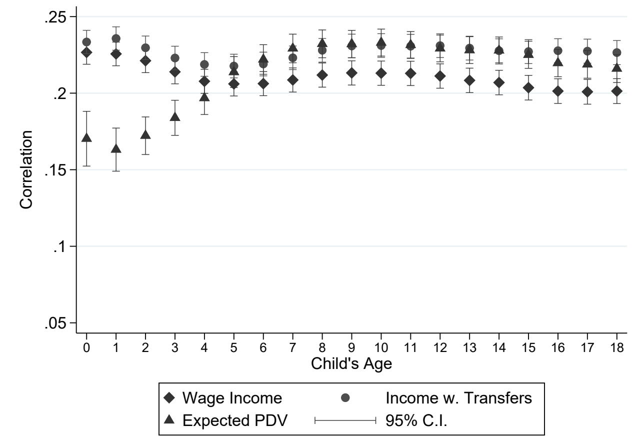
(d) College Mothers

Notes: This figure plots the correlation between Mathematics test scores at age 16 measured and parental income variables measured at the household level at different child's ages. These test scores were measured as part of national leaving examinations for cohorts born in 1995-1997. Panel (a) presents by-age correlations for the sample of college parents where both parents are college (or university) graduates. Panel (b) presents correlations for the sample of parents where non of the parents are college (or university) graduates. Panel (c) restricts the sample to those families where only the father is a college (or university) graduate. Panel (d) restricts the sample to those families where only the mother is a college (or university) graduate.

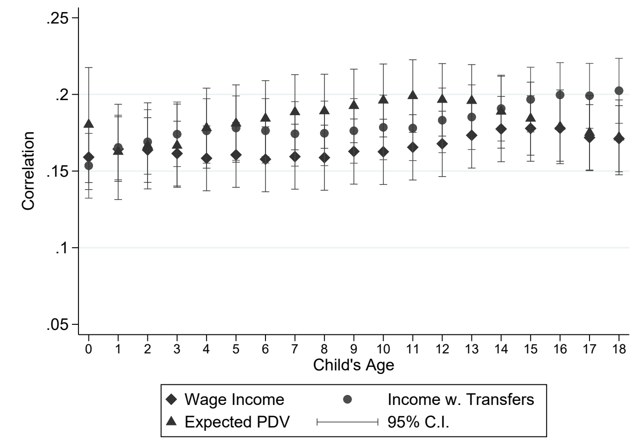
Figure 5: Correlations with Child's Years of Education by Parental Education Level



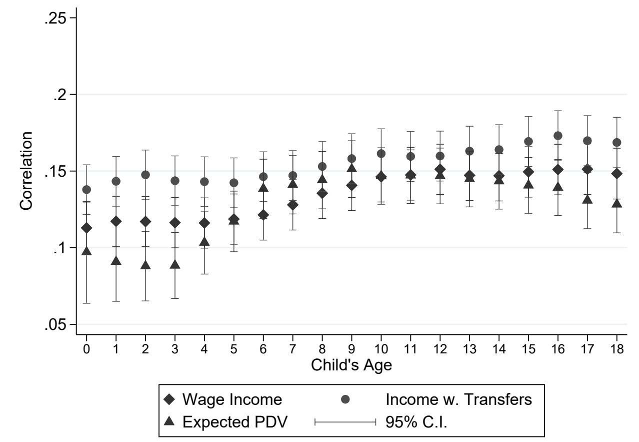
(a) College Parents



(b) Parents without College



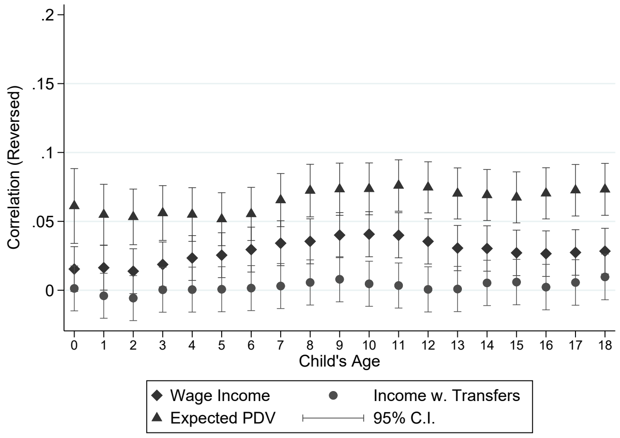
(c) College Fathers



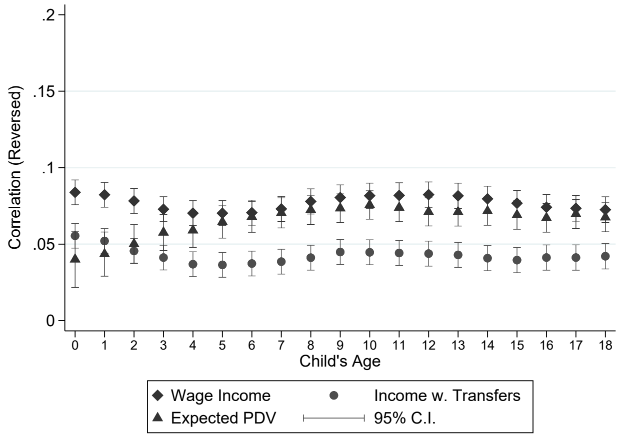
(d) College Mothers

Notes: This figure plots the correlation between years of education and parental income variables measured at the household level at different child's ages. We include children from the 1981-1982 cohorts in these exercises. Panel (a) presents by-age correlations for the sample of college parents where both parents are college (or university) graduates. Panel (b) presents correlations for the sample of parents where non of the parents are college (or university) graduates. Panel (c) restricts the sample to those families where only the father is a college (or university) graduate. Panel (d) restricts the sample to those families where only the mother is a college (or university) graduate.

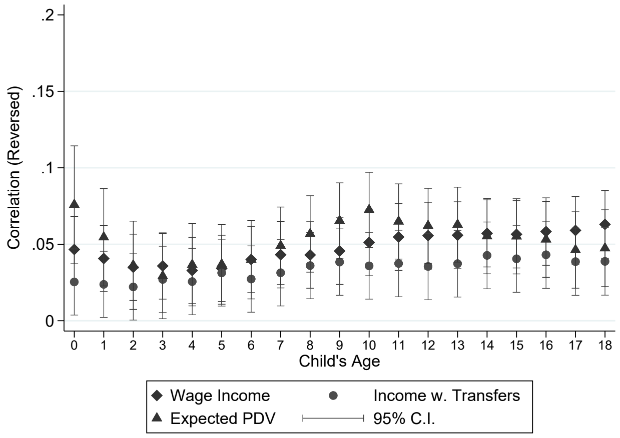
Figure 6: Correlations with Child's Criminality (Never Been Incarcerated) by Parental Education Level



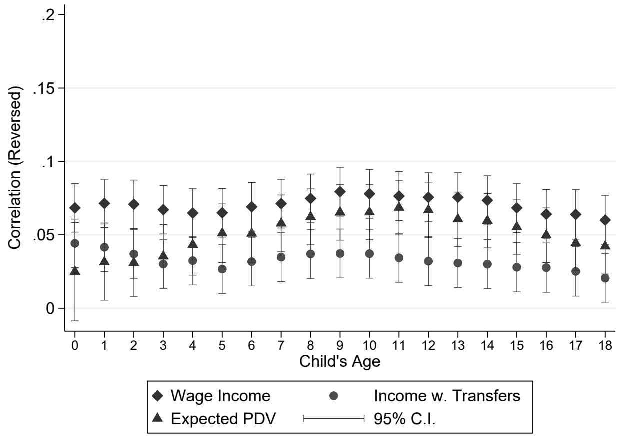
(a) College Parents



(b) Parents without College



(c) College Fathers



(d) College Mothers

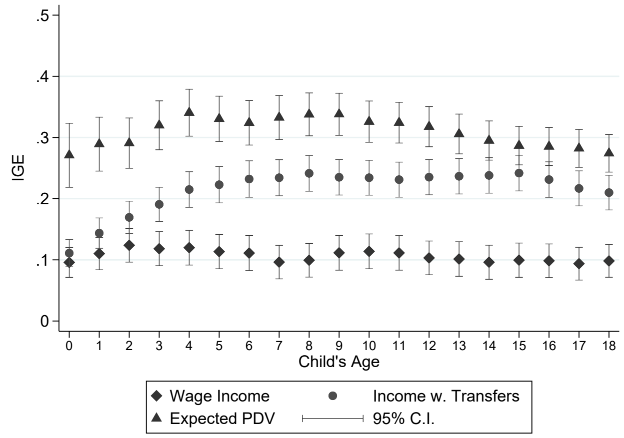
Notes: This figure plots the correlation between criminality and parental income variables measured at the household level at different child's ages. We rely on conviction information from criminal records for children from the 1981-1982 cohorts. Panel (a) presents by-age correlations for the sample of college parents where both parents are college (or university) graduates. Panel (b) presents correlations for the sample of parents where non of the parents are college (or university) graduates. Panel (c) restricts the sample to those families where only the father is a college (or university) graduate. Panel (d) restricts the sample to those families where only the mother is a college (or university) graduate.

We next report the IGE estimate by parental education background when we measure parental resources at different child's ages from zero to 18. The child's income is measured at ages 30-35 and the parental income is measured at the household level at different child's ages shown in the X-axis.[9] We present the results separately for college and non-college parents.

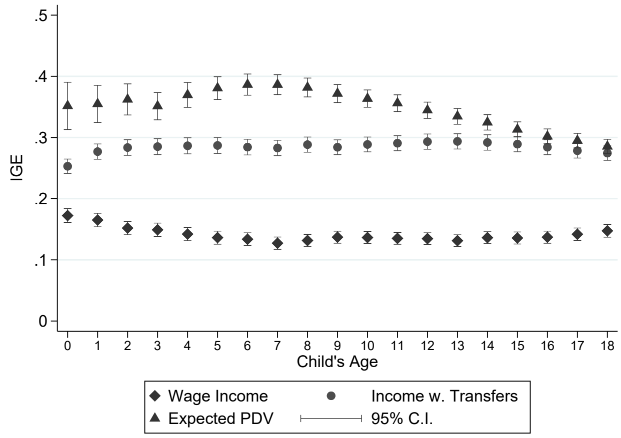
Figure 7 shows that, when comparing measures of resources, the IGE patterns are similar across different groups of parental educational backgrounds. However, social mobility tends to be higher for the sample of children with highly educated parents compared to those coming from families with no college education (expected PDV IGE of 0.40 vs 0.35 and total income IGE of 0.30 vs 0.25 for the no-college parents and college parents, respectively). Appendix G shows comparable results for other ranges of children's ages.

[9] Appendix G.1 presents the resulting IGE estimates when we use different age ranges to measure children's income. The overall pattern is similar to the baseline here.

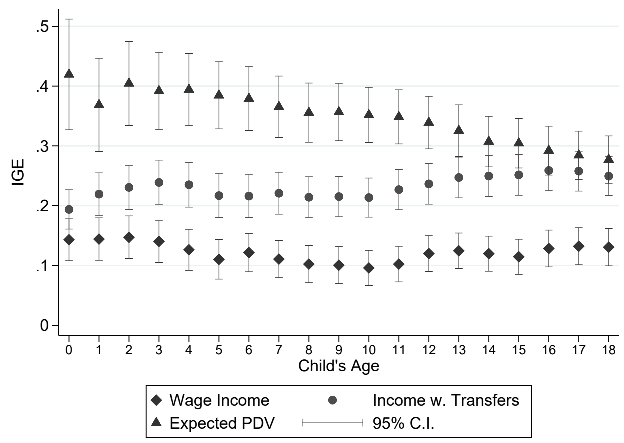
Figure 7: Parents-Child IGE Estimates



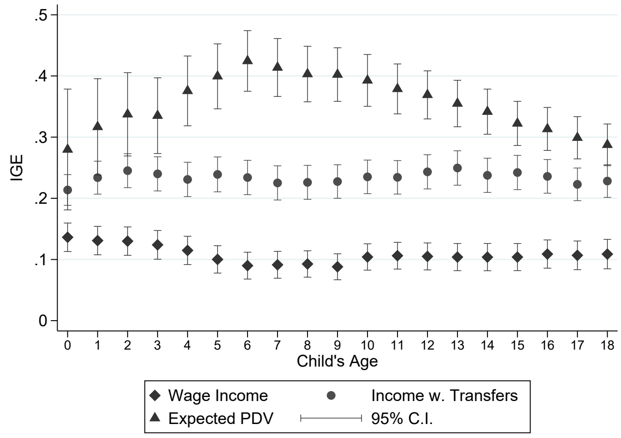
(a) College Parents



(b) No College Parents



(c) College Fathers



(d) College Mothers

Notes: This figure plots the IGE for selected income variables. The child's income is measured at ages 30-35 and the parental income is measured at the household level at different child's ages. We include children from the 1981-1982 cohorts in these exercises. Panel (a) presents by-age correlations for the sample of college parents where both parents are college (or university) graduates. Panel (b) presents correlations for the sample of parents where none of the parents are college (or university) graduates. Panel (c) restricts the sample to those families where only the father is a college (or university) graduate. Panel (d) restricts the sample to those families where only the mother is a college (or university) graduate.

5.3 Intactness of the Family

Figure 8 plots the correlation between the child's mathematics test scores at age 16 and parental income variables measured at the household level at different ages of children, separately for intact and non-intact families and by gender of the child.[10] Panels (a) and (b) present the results for boys and girls growing up in intact families, respectively. Panel (c) and (d) present the results for boys and girls growing up in non-intact families, respectively. For each child, we consider a family as intact if the mother and the father were living together over the whole childhood stage (from age zero to 18 of the child). The patterns reported here are very similar for those based on marital status of parents at the birth of children (see Section 5.4).

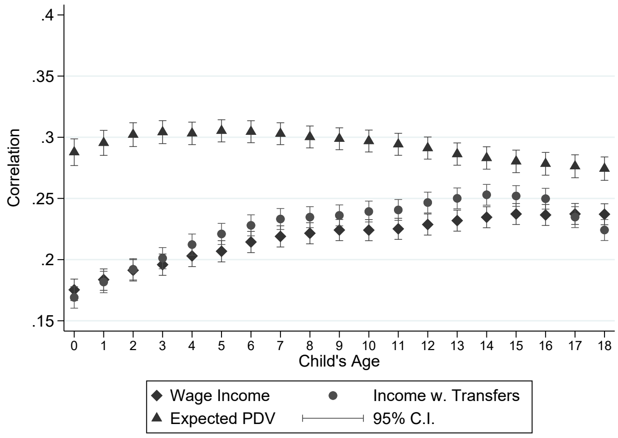
Figure 9 plots the correlation between the child's years of formal schooling by age 35 and parental income variables measured at the household level at different child's ages, separately for the four different groups of families as described above. Finally, Figure 10 plots the correlation between the child's participation in crime (defined as the absence of any incarceration) by age 35 and parental income variables measured at the household level at different child's ages, separately for the four different groups of families as in Figures 8 and 9.

The results suggest that the intergenerational correlation patterns are, in general, similar across different groups of family structures. An important exception is for boys living in intact families. Family structure plays an important role in determining IGEs. Intergenerational correlations tend to be lower for the sample of intact families compared to the sample of non-intact parents. This suggests that monetary resources can compensate for children of disadvantaged less stable families.[11]

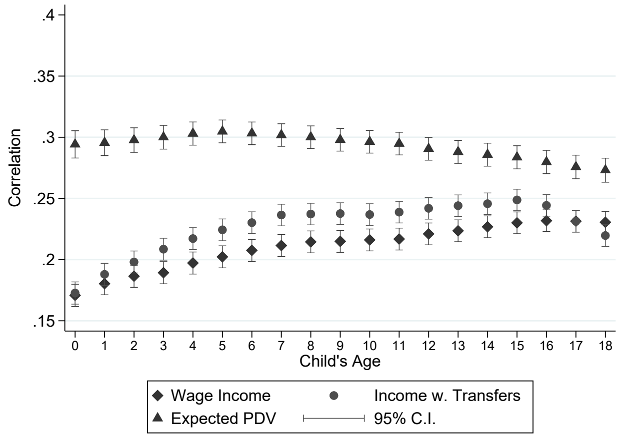
[10] We measure household resources as the sum of the father's and mother's resources, regardless of whether they live together or not.

[11] Figures G.4-G.6 of Appendix Section G.1.2 show similar patterns in IGE estimates by the intactness of the family.

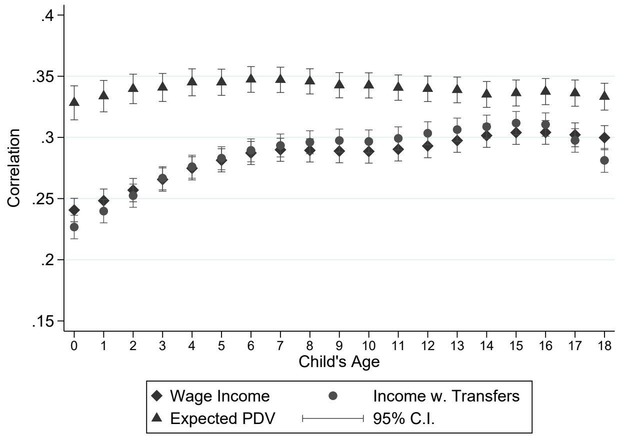
Figure 8: Correlations with Child's Mathematics Test Scores by Family Structure and Gender



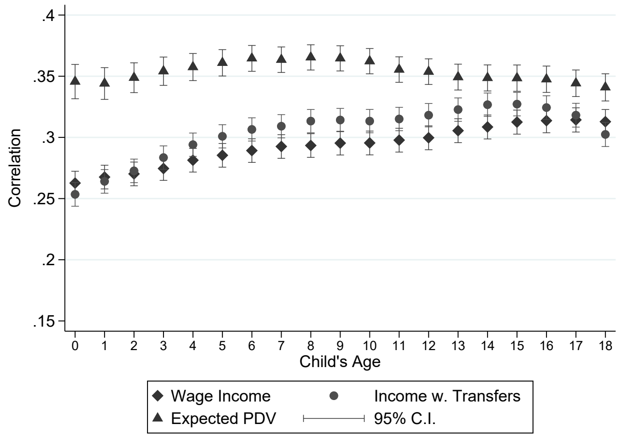
(a) Intact Family- Boys



(b) Intact Family- Girls



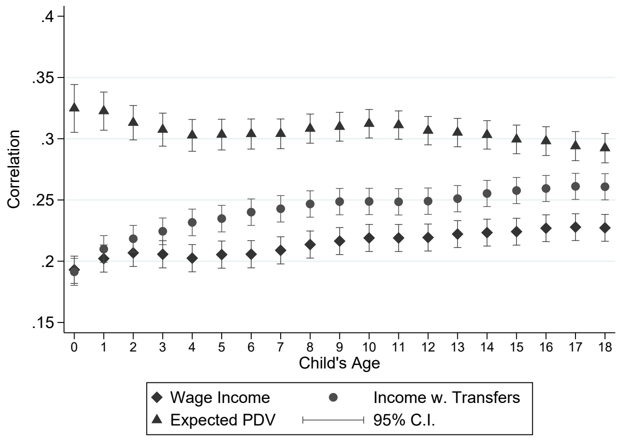
(c) Non-Intact Family- Boys



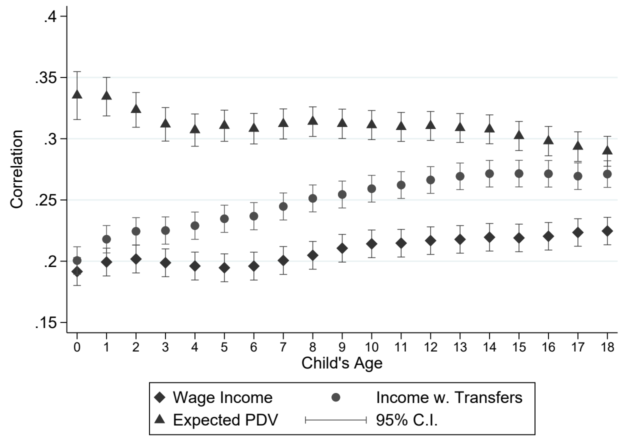
(d) Non-Intact Family- Girls

Notes: This figure plots the correlation between Mathematics test scores at age 16 measured and parental income variables measured at the household level at different child's ages. These test scores were measured as part of national leaving examinations for cohorts born in 1995-1997. Panels (a) and (b) present the results for boys and girls growing up in intact families, respectively. Panel (c) and (d) present the results for boys and girls growing up in non-intact families, respectively. For each child, we consider a family as intact if the mother and the father were living together over the whole childhood stage (from age zero to 18 of the child).

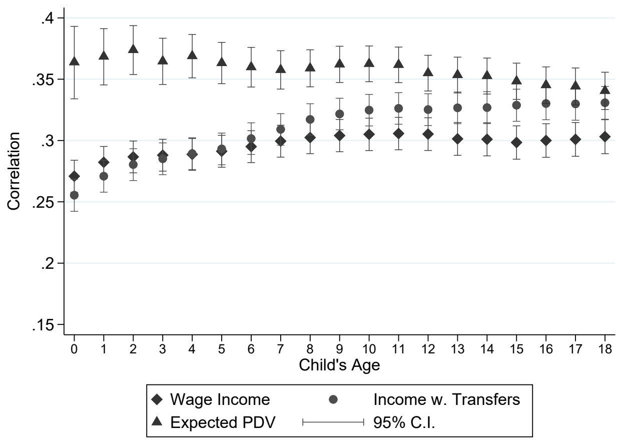
Figure 9: Correlations with Child's Years of Education by Family Structure and Gender



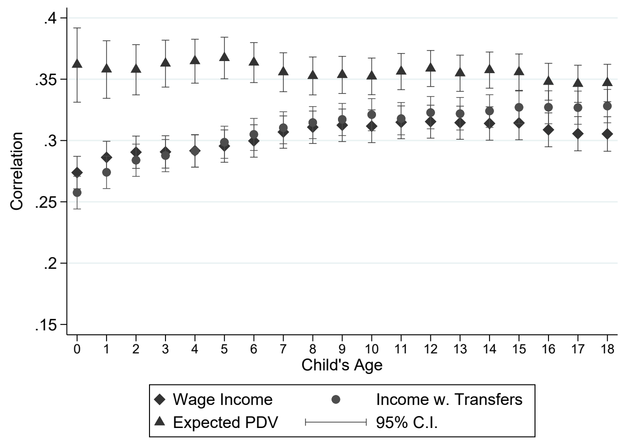
(a) Intact Family- Boys



(b) Intact Family- Girls



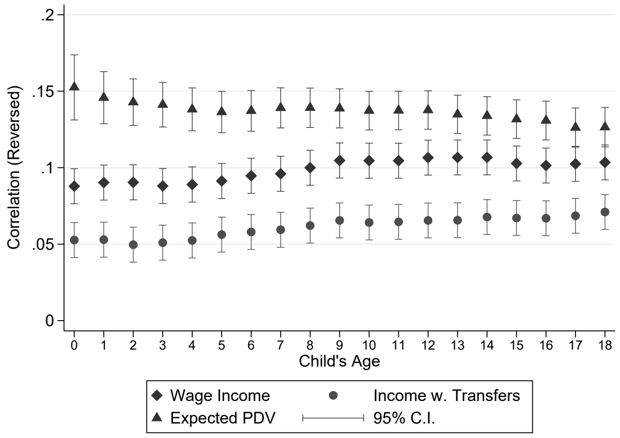
(c) Non-Intact Family- Boys



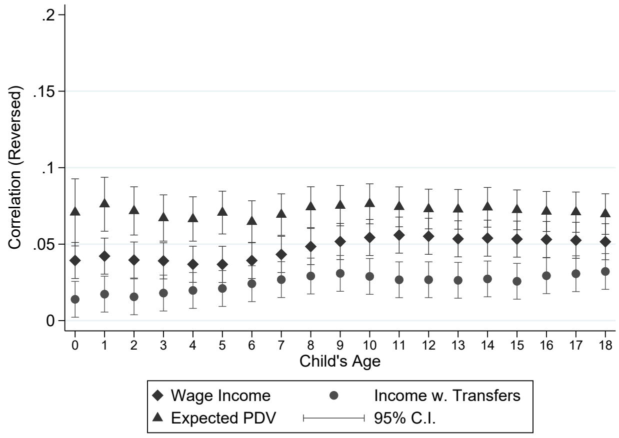
(d) Non-Intact Family- Girls

Notes: This figure plots the correlation between years of education and parental income variables measured at the household level at different child's ages. We include children from the 1981-1982 cohorts in these exercises. Panels (a) and (b) present the results for boys and girls growing up in intact families, respectively. Panel (c) and (d) present the results for boys and girls growing up in non-intact families, respectively. For each child, we consider a family as intact if the mother and the father were living together over the whole childhood stage (from age zero to 18 of the child).

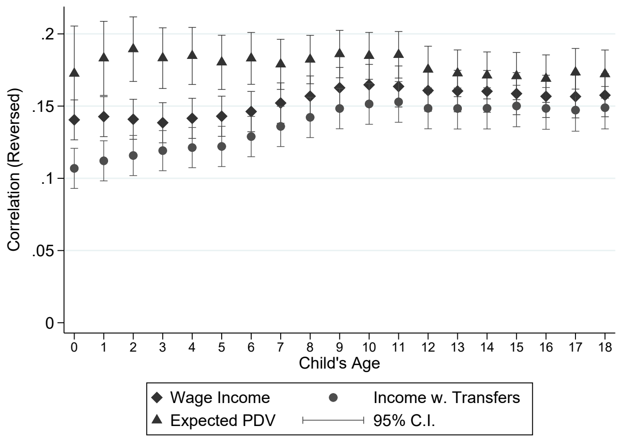
Figure 10: Correlations with Child's Criminality (Never Been Incarcerated) by Family Structure and Gender



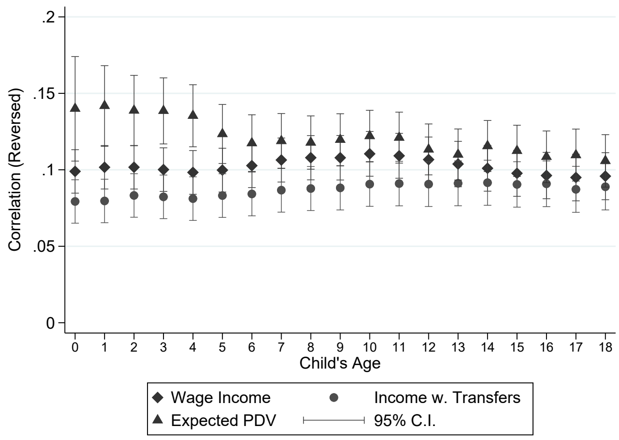
(a) Intact Family- Boys



(b) Intact Family- Girls



(c) Non-Intact Family- Boys



(d) Non-Intact Family- Girls

Notes: This figure plots the correlation between criminality and parental income variables measured at the household level at different child's ages. We rely on conviction information from criminal records for children from the 1981-1982 cohorts. Panels (a) and (b) present the results for boys and girls growing up in intact families, respectively. Panel (c) and (d) present the results for boys and girls growing up in non-intact families, respectively. For each child, we consider a family as intact if the mother and the father were living together over the whole childhood stage (from age zero to 18 of the child).

5.4 Marital Status of Parents When Children Born

Figure 11 plots the correlation between the child's mathematics test scores at age 16 and parental income variables measured at the household level at different child's ages, separately for married and non-married parents and by gender of the child. Panels (a) and (b) present the results for boys and girls growing up in married families, respectively. Panel (c) and (d) present the results for boys and girls growing up in non-married families, respectively. For each child, we consider the family as married if the mother and the father were registered as married at the arrival of the child.

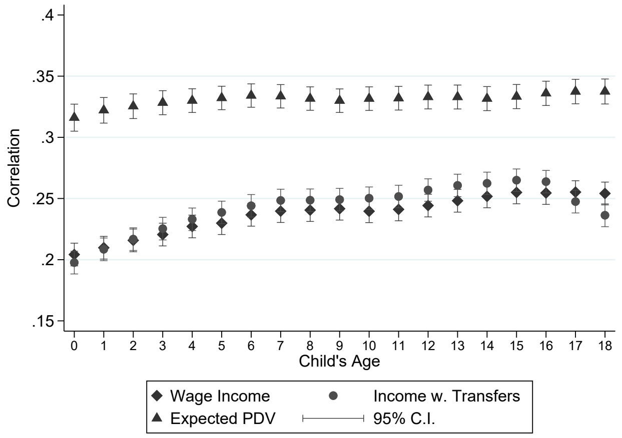
Figure 12 plots the correlation between the child's years of formal schooling by age 35 and parental income variables measured at the household level at different child's ages, separately for the four different groups of families as described above. Finally, Figure 13 plots the correlation between the child's participation in crime (defined as the absence of any incarceration) by age 35 and parental income variables measured at the household level at different child's ages, separately for the four different groups of families.

The results suggest that intergenerational correlations tend to be lower for the sample of married families compared to the sample of non-married parents. Consistent with the previous results in this section, this may suggest that monetary resources can act as compensation mechanisms for nonpecuniary factors.[12]

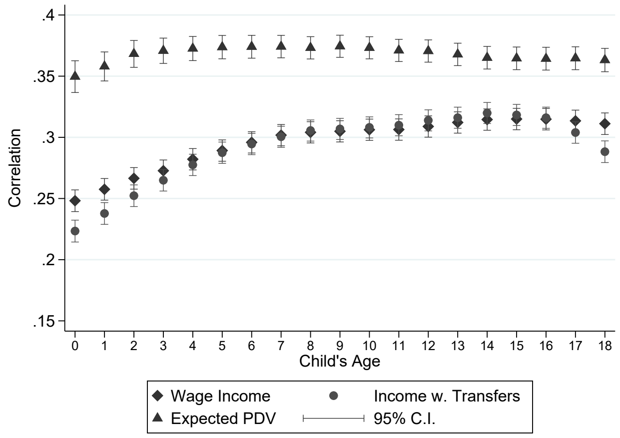
The patterns, in general, are very similar when we break down the sample by whether the father and the mother were cohabitants (regardless of their legal by marital status) at the time of the arrival of the child. Appendix E presents the results. The only difference is that when we break down the sample by cohabitational status (rather than by marital status) of parents, the predictive power of parental resources over child outcomes is slightly higher at most ages.

[12]Figures G.7-G.9 of Appendix Section G.1.3 show similar patterns in IGE estimates by the intactness of the family.

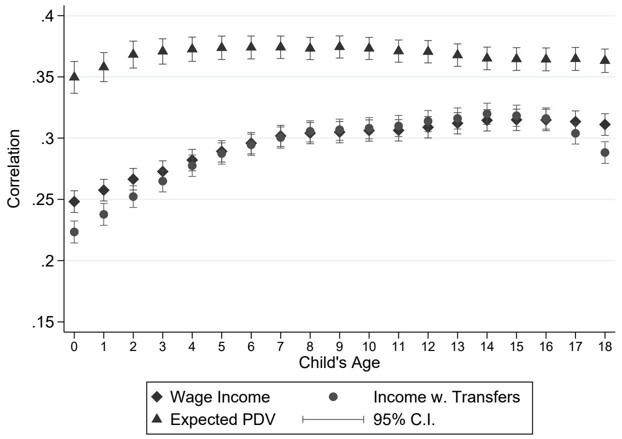
Figure 11: Correlations with Child's Mathematics Test Scores by Parental Marital Status and Gender



(a) Married- Boys



(b) Married- Girls

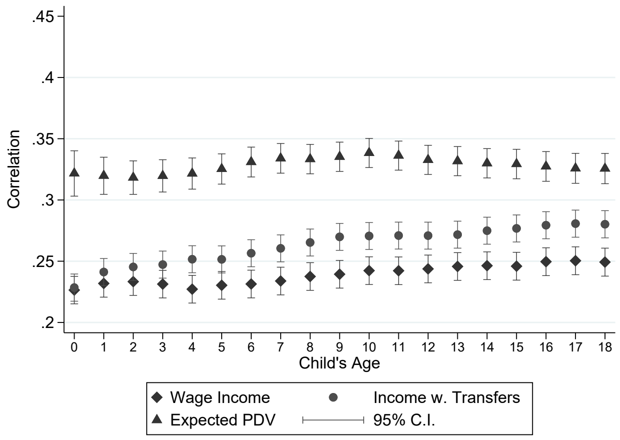


(c) Non-Married- Boys

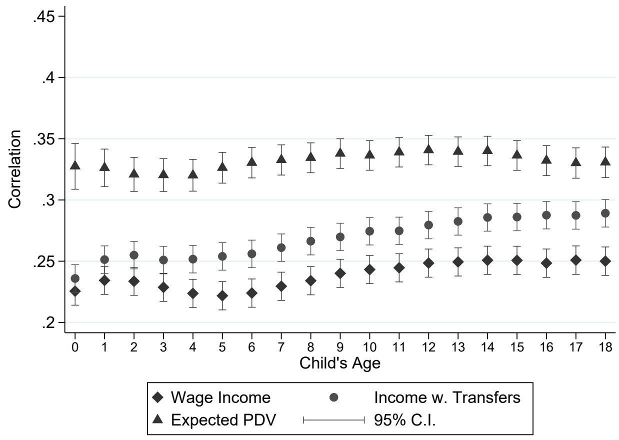
(d) Non-Married- Girls

Notes: This figure plots the correlation between Mathematics test scores at age 16 measured and parental income variables measured at the household level at different child's ages. These test scores were measured as part of national leaving examinations for cohorts born in 1995-1997. Panels (a) and (b) present the results for boys and girls growing up in married families, respectively. Panel (c) and (d) present the results for boys and girls growing up in non-married families, respectively. For each child, we consider the family as married if the mother and the father were registered as married at the arrival of the child.

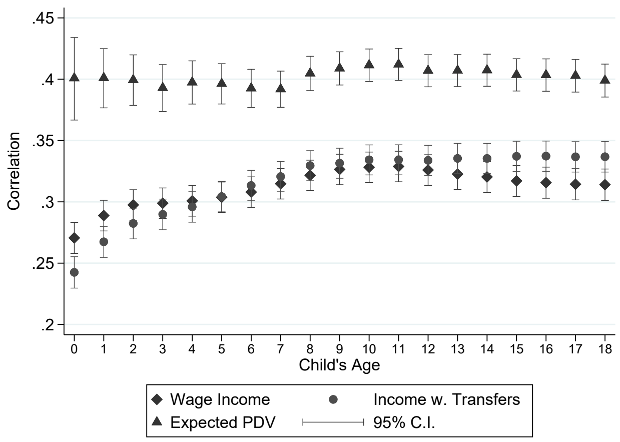
Figure 12: Correlations with Child's Years of Education by Parental Marital Status and Gender



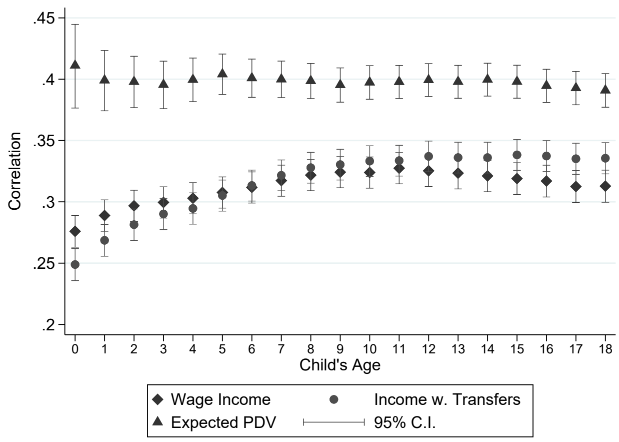
(a) Married- Boys



(b) Married- Girls



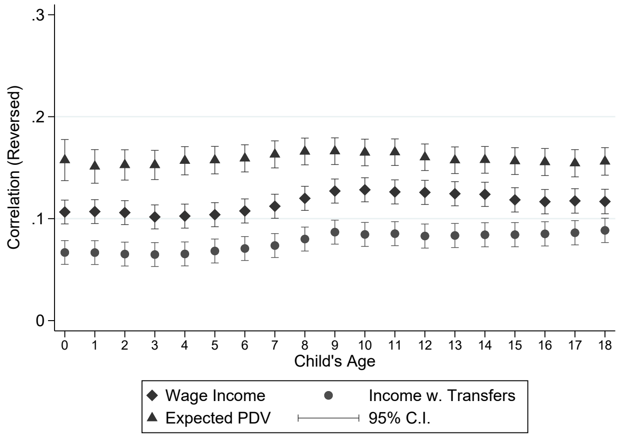
(c) Non-Married- Boys



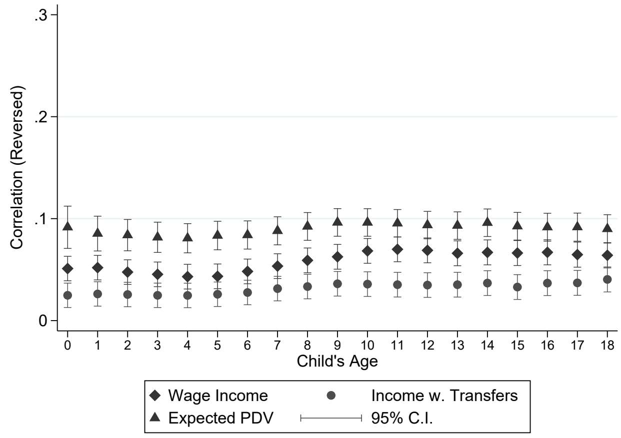
(d) Non-Married- Girls

Notes: This figure plots the correlation between years of education and parental income variables measured at the household level at different child's ages. We include children from the 1981-1982 cohorts in these exercises. Panels (a) and (b) present the results for boys and girls growing up in married families, respectively. Panel (c) and (d) present the results for boys and girls growing up in non-married families, respectively. For each child, we consider the family as married if the mother and the father were registered as married at the arrival of the child.

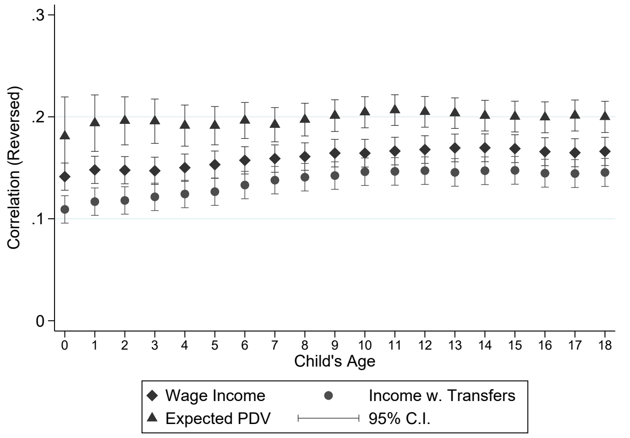
Figure 13: Correlations with Child's Criminality (Never Been Incarcerated) by Parental Marital Status and Gender



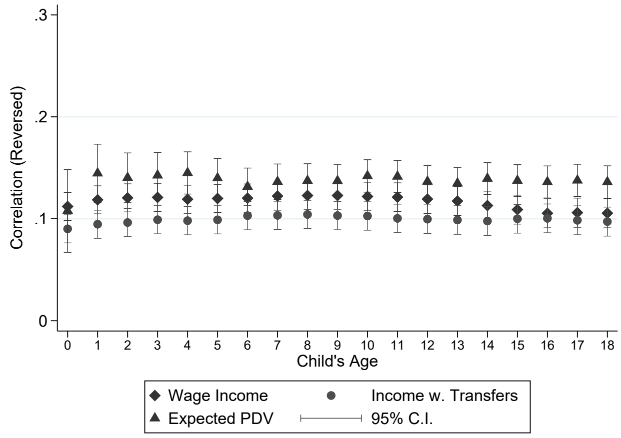
(a) Married- Boys



(b) Married- Girls



(c) Non-Married- Boys



(d) Non-Married- Girls

Notes: This figure plots the correlation between criminality and parental income variables measured at the household level at different child's ages. We rely on conviction information from criminal records for children from the 1981-1982 cohorts. Panels (a) and (b) present the results for boys and girls growing up in married families, respectively. Panel (c) and (d) present the results for boys and girls growing up in non-married families, respectively. For each child, we consider the family as married if the mother and the father were registered as married at the arrival of the child.

5.5 Estimates by Gender

Figure 14(a-b) plots the correlation between the child's language test score at age 11 and parental income variables measured at the household level at different child's ages, separately for male children (Panel a) and female children (Panel b). Figures 14(c-d) and 15(a-d) depict the results for the child's math test score, years of formal schooling, and incarceration, respectively. We evaluate whether the male and female values are statistically significantly different, in which case we show the estimates using solid (as opposed to hollow) symbols.[13]

This heterogeneity is important for interpreting estimates of the IGE. The results suggest that compared to male children, there exists a greater association between parental income and academic achievement for female children. For participation in crime, on the other hand, we find a much stronger association between parental income and children's likelihood of participation in crime for male children. Moreover, for some measures of child human capital, parental resources in early childhood best predict the performance of boys whereas parental resources in early adolescence best predict the performance of girls (See Figure 14).

In all figures, we reject the null hypothesis that all correlations (across different ages of children) are equal. The only exception is wage income in predicting math test scores for the sample of college fathers (Panel (a) of Figure 7) and income with transfers in predicting criminality (Panels (a) and (d) of Figure 6) and wage income in panel (d) of Figure 6.[14]

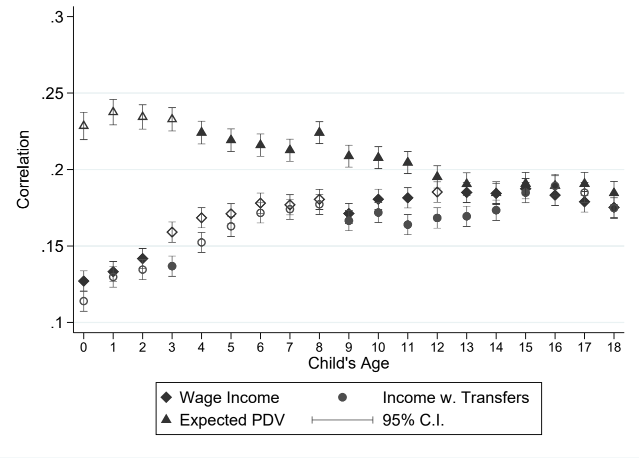
Figure 16 depicts the IGE estimates separately for the sample of male children (Panel a) and female children (Panel b).[15] These results point to substantially higher mobility among female children compared to the sample of male children. This is not due to traditional reasons – that daughters are compared to mothers who tend to work less. The benchmark for both groups is family income. Also, the difference in IGEs across the two genders is more pronounced using expected PDV. The expected PDV IGE estimates are around 0.5 and 0.4,

[13]Note that this differs from the coloring scheme used in sections 3 and 4.

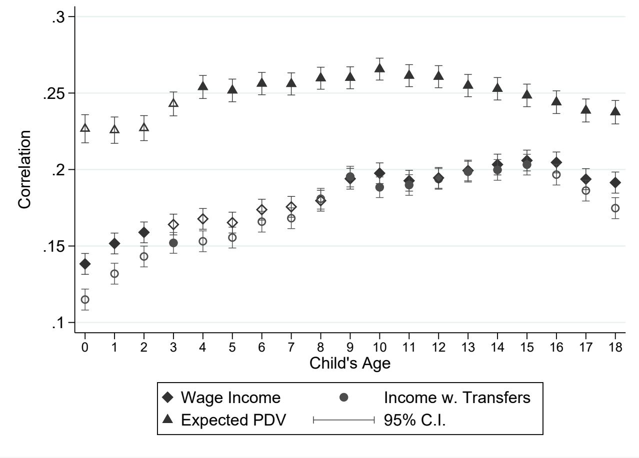
[14]See Appendix H for details of the hypotheses regarding the equality of the correlations across different ages of children, pairwise and jointly.

[15]We evaluate whether the male and female values are statistically significantly different, in which case we color the data points in the graph red. Note that this differs from the coloring scheme used in sections 3 and 4.

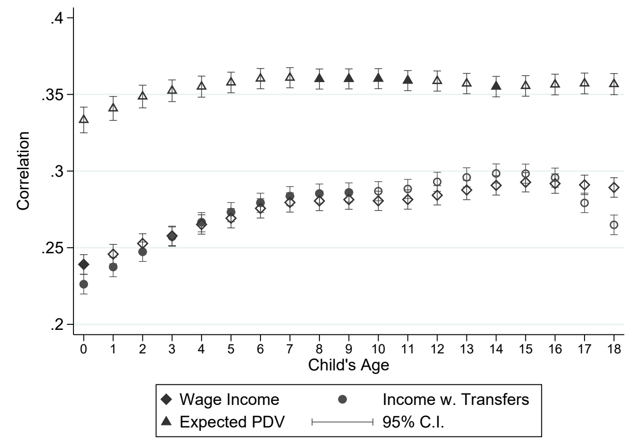
Figure 14: Correlations with Child's Test Scores by Gender of the Child



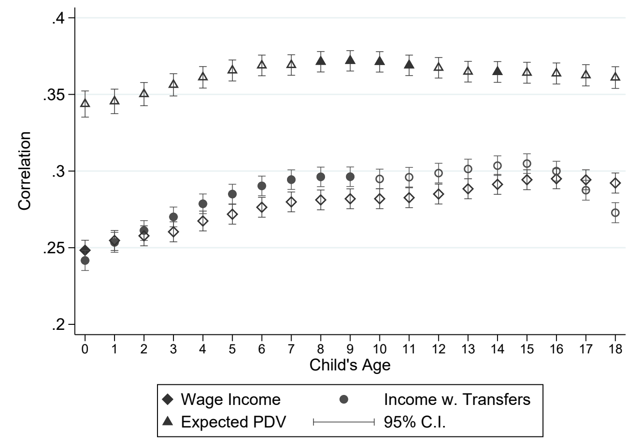
(a) Language Test Score, age 11, Male Children



(b) Language Test Score, age 11, Female Children



(c) Mathematics Test Score, age 16, Male Children



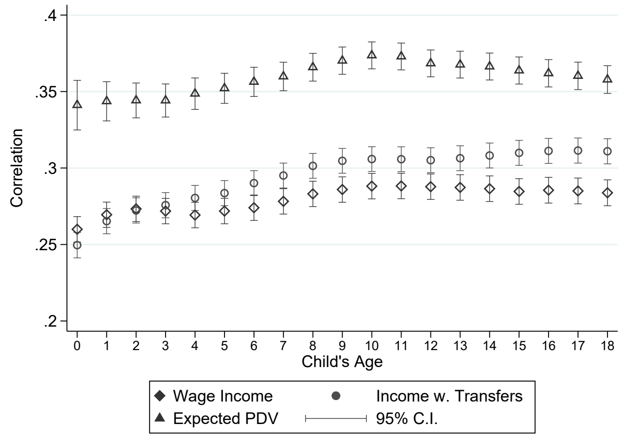
(d) Mathematics Test Score, age 16, Female Children

Notes: This figure plots the correlation between reading scores at age 11 measured for DALSC participants and Mathematics test scores at age 16, and selected parental income variables measured at the household level at different child's ages. We present the correlation for ages 0-18 for consistency, even though income measures after age 11 should have no direct effect on this outcome. DALSC participants were born in September and October of 1995. Panel (a,c) restricts the sample to male children. Panel (b,d) restricts the sample to female children. For each measure of resources, use a t-test to evaluate whether the correlation at each age is significantly different across male and female children (at the 5% level). At any given age, we depict the estimates with solid (filled) symbols if they are significantly different across males and females.

respectively for male and female children. The income with transfers IGEs are about 0.33 and 0.28, respectively for male and female children.

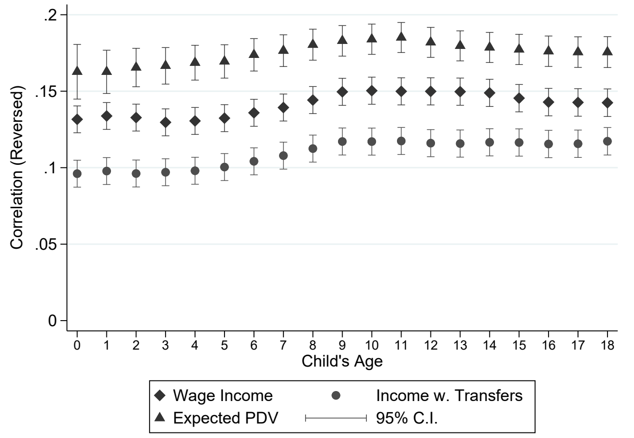
These findings are consistent with the intergenerational changes in educational attain-

Figure 15: Correlations with Child's Education and Criminality by Gender of the Child

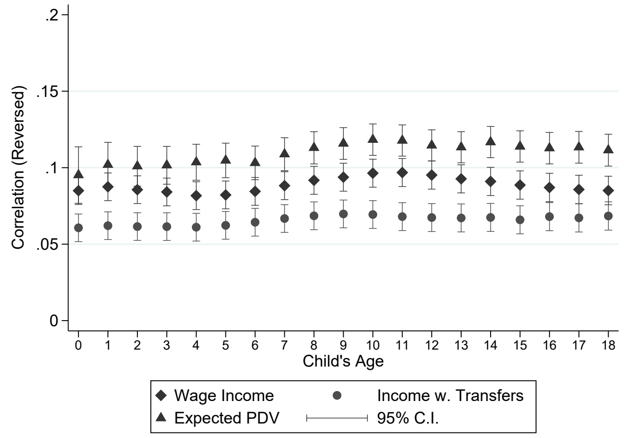


(a) Years of Schooling, Male Children

(b) Years of Schooling, Female Children



(c) Never Been Incarcerated, Male Children

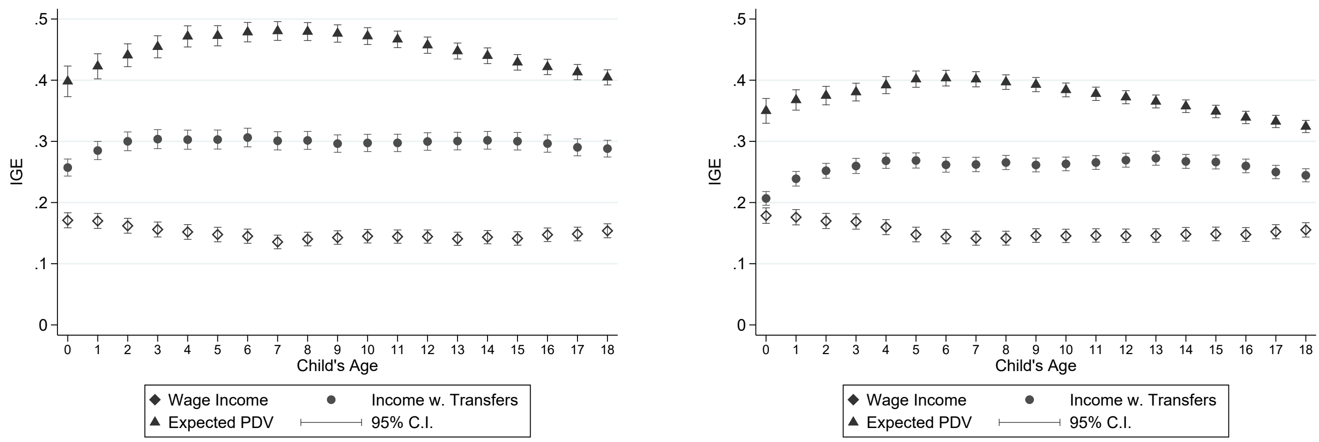


(d) Never Been Incarcerated, Female Children

Notes: This figure plots the correlation between years of education and child has received a prison sentence by age 35, and selected parental income variables measured at the household level at different child's ages. We include children from the 1981-1982 cohorts in these exercises. Panels (a,c) restricts the sample to male children. Panels (b,d) restricts the sample to female children. For each measure of resources, we use a t-test to evaluate whether the correlation at each age is significantly different across male and female children (at the 5% level). At any given age, we depict the estimates with solid (filled) symbols if they are significantly different across males and females.

ment and labor force participation, which varies greatly across genders as documented by Eshaghnia et al. (2022). The sample of female children, on average, outperforms the sample of male children with respect to formal education measured by years of completed schooling. We next turn to relating our results to those in the preceding literature – primarily the work of Carneiro et al. (2021).

Figure 16: Parents-Child IGE Estimates



(a) Male Children

(b) Female Children

Notes: This figure plots the IGE for selected income variables. The child's income is measured at ages 30-35 and the parental income is measured at the household level at different child's ages. We include children from the 1981-1982 cohorts in these exercises. Panel (a) restricts the sample to male children. Panel (b) restricts the sample to female children. The vertical lines around the point estimates represent the 95% confidence intervals. For each measure of resources, we use a t-test to evaluate whether the IGEs at each age are significantly different across male and female children (at the 5% level). At any given age, we depict the estimates with solid (filled) symbols if they are significantly different across males and females.

6 Interpreting our Results Through the Lens of the Technology of Skill Formation

This section follows in the footsteps of Carneiro et al. (2021), who use the technology of skill formation introduced in Cunha and Heckman (2007) and Cunha et al. (2010) to interpret the impact of family income at different stages on child outcomes. We first review the literature on the technology of skill formation and the approach of Carneiro et al. (2021). We then introduce our approach, including an important amendment to the technology of skill formation that recognizes the emergence of new skills in adolescence as documented in Steinberg (2014).

6.1 The Technology of Skill Formation

We write child outcomes ($\boldsymbol{Y}(t)$) at age t as a function of $\boldsymbol{K}(t)$ and other determinants, such as effort, perseverance, and preferences:

$$\boldsymbol{Y(t)} = \Phi_t(\boldsymbol{K(t)}, \boldsymbol{X(t)}). \tag{2}$$

Skills evolve through the technology of skill formation:

$$\boldsymbol{K}(t+1) = \boldsymbol{F}_t(\boldsymbol{K}(t), \boldsymbol{I}(t)). \tag{3}$$

Lagging one period we obtain: $\boldsymbol{K}(t) = \boldsymbol{F}_{t-1}(\boldsymbol{K}(t-1), \boldsymbol{I}(t-1))$.

Recursing we then obtain:

$$\boldsymbol{K}(t+1) = \boldsymbol{F}_t(\boldsymbol{F}_{t-1}(\boldsymbol{K}(t-1), \boldsymbol{I}(t-1)), \boldsymbol{I(t)}). \tag{4}$$

So

$$\boldsymbol{K}(t+1) = \boldsymbol{Q}_t(\boldsymbol{I}(t), \boldsymbol{I}(t-1), \ldots, \boldsymbol{I}(0); \boldsymbol{K}(0)). \tag{5}$$

Agents maximize life cycle programs as in Cunha et al. (2010), Del Boca et al. (2014), Agostinelli and Wiswall (2022) and others. Letting $V(t)$ be the value function at age t,

$$\boldsymbol{I}(t) = \boldsymbol{G}^{(t)}(\Omega_t, \boldsymbol{X}(t)) \tag{6}$$

where $\boldsymbol{X}(t)$ are other determinants of investment and $\Omega(t)$ is shorthand for $\Omega_t \underbrace{(E(V_t)|\boldsymbol{S}_t)}_{\substack{\text{value function} \\ \text{at } t \\ \text{given } \boldsymbol{S_t}}}$.

Thus,

$$\boldsymbol{Y}(t) = \boldsymbol{\Phi}^{(t)}(\boldsymbol{G}^{(t)}, \boldsymbol{G}^{(t-1)}, \boldsymbol{G}^{(0)}, \boldsymbol{X}(t)). \tag{7}$$

6.2 The Carneiro et al. (2021) Approach and Our Approach

Carneiro et al. (2021) approximate Equation (7) by using measures of family income in each period t to approximate Ω_t. They ignore other factors $\boldsymbol{X}(t)$ and approximate initial conditions $\boldsymbol{K}_0$ by the present value of discounted income over the life of the child evaluated at birth. To avoid serious collinearity problems, they use average incomes over three broad intervals corresponding to early childhood, middle childhood, and adolescence.

Instead of their measure of income, we use age specific expected present values to approximate value functions and obtain the approximate state equation:

$$\boldsymbol{K}(t+1) \doteq \boldsymbol{Q}_t(E(PV_t|\boldsymbol{S}_t), E(PV_{t-1}|\boldsymbol{S}_{t-1}), \dots).$$

Define $\boldsymbol{Z_t} = [E(PV_t|\boldsymbol{S}_t), E(PV_{t-1}|\boldsymbol{S}_{t-1}), \dots, |E(PV_0|\boldsymbol{S}_0)]$. Investment in each age produces the stock of skills that govern behavioral Equation (7). Using a Taylor series expansion to second order, the investment and outcome equation obtain:

$$\boldsymbol{Y}(t+1) \doteq \boldsymbol{\alpha_0'(t)} + \boldsymbol{\alpha_1'}(t)\boldsymbol{Z_t} + \boldsymbol{Z_t'}\Psi(t)\boldsymbol{Z_t} \tag{8}$$

where $\boldsymbol{\alpha}_1(t)$ is $t \times 1$ vector. $\boldsymbol{\Psi}(t)$ is $(t+1) \times (t+1)$ matrix. Like Carneiro et al. (2021), we use broad age intervals for our income measures to avoid collinearity. Unlike them, we form the intervals on the basis of the correlation patterns of the $\boldsymbol{Z}_t$.

The coefficients $\boldsymbol{\alpha}_1(t)$ and $\boldsymbol{\Psi(t)}$ give information about critical and sensitive periods. Namely, we can form $\frac{\partial \boldsymbol{Y}(t+1)}{\partial \boldsymbol{Z}_t} = \boldsymbol{\alpha}_1' + \boldsymbol{\Psi}(t)\boldsymbol{Z}_t$. Expected PDVs are updated using the algorithm described in Eshaghnia et al. (2022), which update information sets. They capture period-by-period information updating. Age-specific expectations of PV_t proxy investment (see Equation (6)). Before turning to our empirical results, we first amend the Cunha et al. (2010) model to account for the emergence of skills as documented in Steinberg (2014) and Belsky et al. (2020).

The preceding framework assumes that the dimension and skill categories of $\boldsymbol{K}(t)$ remain the same over the life cycle. This ignores an emerging literature on the flourishing of lifetime skills. As children mature, new preferences and behaviors emerge. Steinberg (2014) documents the emergence of a dual system of adolescent behavior. The centers of the brain that respond to stimulation and pleasure emerge early after the onset of puberty with its hormonal rush. Centers of the brain associated with self control and executive functioning become active later, creating patterns of behavior (and evolution of new skills) unique to the adolescent years.

There are at least two periods of rapid skill and preference development in the life of a child: early childhood and adolescence. Resources available to families in each period of life may play important roles in child development. If markets are incomplete so full insurance against all contingencies is ruled out and borrowing constraints apply for some, as documented in Hai and Heckman (2019), even for able and highly educated people, the timing of income in these sensitive periods may matter.

This helps to explain the U-shaped relationship of child outcomes and parental income through adolescence documented by Carneiro et al. (2021) for which we show partial support. Sensitive periods with binding credit constraints can rationalize the evidence.

We amend the model of Equation (2) to allow the dimension of $\boldsymbol{K}(t)$ to change as new skills emerge and possibly old skills decay. At age t_e, new skills and possibly new investment strategies, emerge. In a simplified model,

$$\begin{bmatrix} \boldsymbol{K}(t+1) \\ \boldsymbol{K}^*(t+1) \end{bmatrix} = \boldsymbol{J}^{(t)}(\boldsymbol{K}(t), \boldsymbol{K}^*(t), \boldsymbol{I}(t), \boldsymbol{I}^*(t))$$

where "$*$" denotes the new skills stocks and investment emerging at and after t_e. We define $\boldsymbol{K}^*(t) = 0$ for $t < t_e$.

The distinctions previously made apply here. New forms of complementarity emerge: $\frac{\partial^2 \boldsymbol{J}^{(t)}(\boldsymbol{K}(t),\boldsymbol{K}^*(t),\boldsymbol{I}(t),\boldsymbol{I}^*(t))}{\partial \boldsymbol{K}(t) \partial (\boldsymbol{K}^* t)'} > 0; t \geq t_e$, and investments of different types may cross fertilize. Outcomes may depend on $\boldsymbol{K}(t)$ (e.g., IQ) and on "soft skills" $\boldsymbol{K}^*(t)$.

Thus, for outcome j (e.g., management skills may depend on both cognitive ability and personality traits: $\boldsymbol{Y_j}(\boldsymbol{K}(t), \boldsymbol{K}^*(t)))$. For example, Mandelbrot (1962) characterizes occupations by bundles of traits. Some skills may have negative marginal product in some occupations, e.g., gregariousness may reduce productivity for an abstract mathematician. Sensitive periods can arise when skills emerge (i.e., after t_e for $\boldsymbol{K}^*(t)$). Early investment may or may not enhance the productivity of later investment. Some skills may interfere with other skills. We extend the definition of $\boldsymbol{I}(t)$ to also account for investment in emergent skills.

7 Empirical Results

This subsection reports estimates of the approximating model, Equation (8), for four different child outcomes realized at different ages. Table 4 documents how we define the periods used for each outcome, and when each child outcome is realized. Effectively, $\boldsymbol{Z}_t$ values are very similar within the selected intervals. Table I.1 of Appendix I shows the correlations between parents' expected PDV(t) across different intervals. Table I.2 shows that correlations within these intervals are very high.[16,17]

We regress child outcomes on parental expected PDV in each interval as well as the interaction between the expected PDVs across different periods. We use two different specifications of the ranges of periods studied:

1. Birth until Realization of the Outcome: All periods starting at birth up to and including the period during which the outcome studied is realized.
2. Birth until age 17: All periods starting at birth up to and including the period during which childhood ends (at age 17)

Tables I.4–I.7 of Appendix I report the coefficients obtained from these two specifications of the model. In what follows, we focus on the first specification. The results for the second specification is presented in Appendix I.

[16] Appendix Table I.2 presents the correlations across all children's ages from zero to 18.

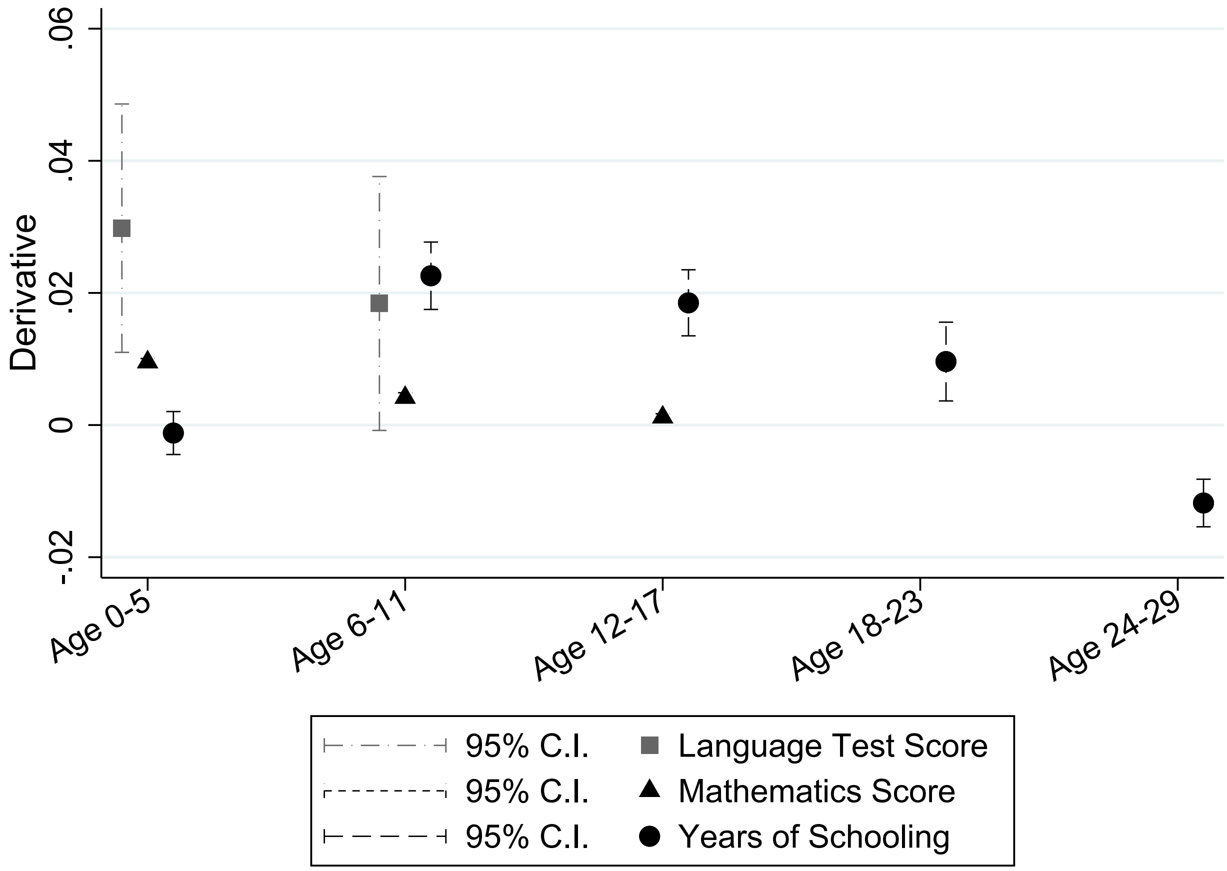
[17] Also, Appendix Figure I.1 presents the eigenvalues from principal components analyses of parents' expected PDV over the child's age intervals of $[0, 5]$, $[6, 11]$, $[12, 17]$, $[18, 23]$, and $[24, 29]$. Except for the first eigenvalue, other eigenvalues are close to zero.

Table 4: Age Ranges Studied for Different Child Outcomes

Outcome	Child's age intervals	Num. of periods
Language Test Score at Age 11	[0,5], [6,11]	2
Math Problem Solving at Age 16-17	[0,5], [6,11], [12,17]	3
Incarcerated by 35	[0,5], [6,11], [12,17],[18,23], [24,29]	5
Total Years of Education	[0,5], [6,11], [12,17],[18,23], [24,29]	5

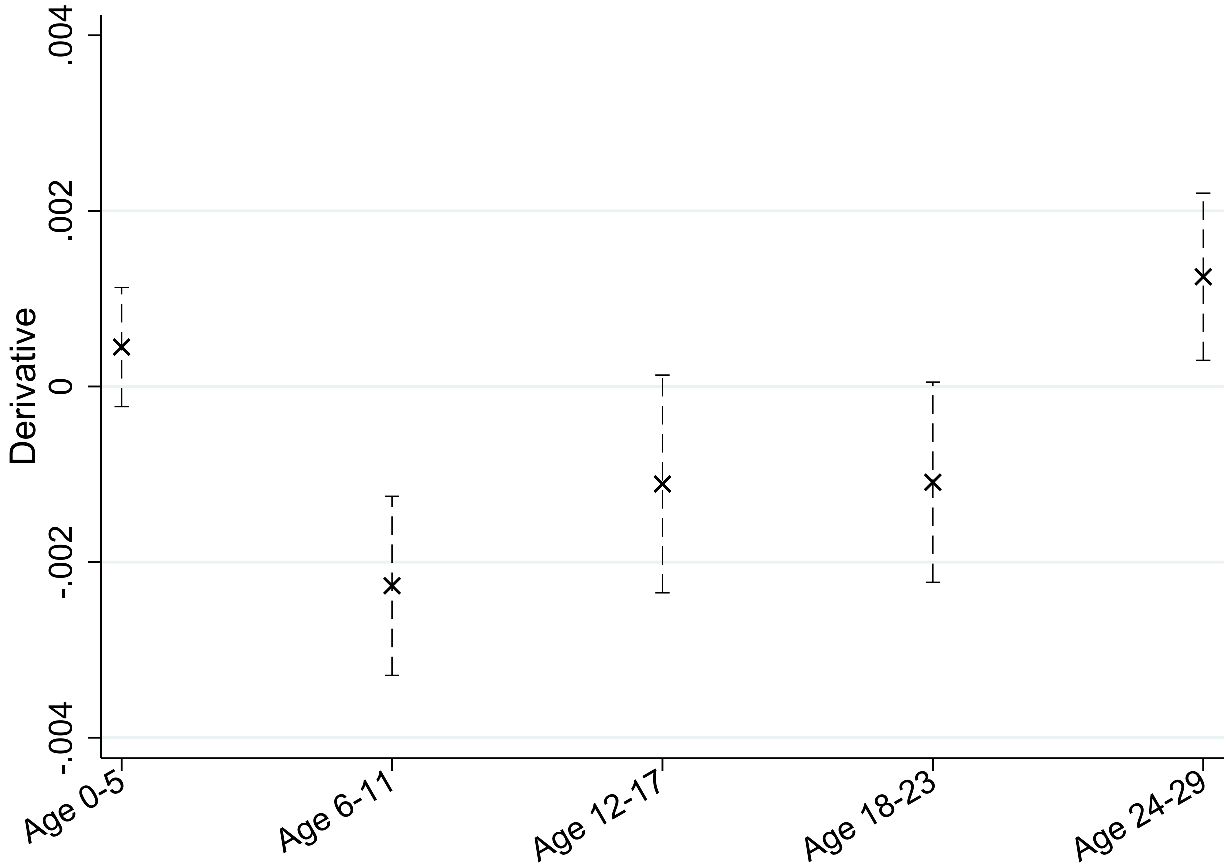
Notes: This table presents the specific periods used for each outcome as well as the number of periods (until the realization of the outcome) to run the model in Equation (8).

Figure 17: Derivative of Child Outcomes with respect to Parental Expected PDV at Different Childhood Age Intervals, Evaluated at the Mean (First Specification)



Notes: This figure depicts the value of the derivative of each of the child outcomes with respect to the parental expected PDV in each interval (in 10,000 USD in 2010 values), evaluated at the mean. Table 5 reports the values. We estimate Equation (8), taking coefficients $\boldsymbol{\alpha}'_0(t)$, $\boldsymbol{\alpha}'_{1,i}(t)$, and $\boldsymbol{\Psi}_{i,j}(t)$ to be constant within the selected intervals. The full set of estimation results including the constant and the coefficients on the interaction terms are reported in Tables I.4-I.7 of Appendix I.

Figure 18: Derivative of Child Outcomes with respect to Parental Expected PDV at Different Childhood Age Intervals, Evaluated at the Mean- **Child Outcome: Ever Been Incarcerated by Age 35** (First Specification)



Notes: This figure depicts the value of the derivative of the child outcome (i.e., ever been incarcerated by age 35) with respect to parental expected PDV in each interval (in 10,000 USD in 2010 values), evaluated at the mean. The dashed lines show the 95% confidence intervals. Table 5 reports the values. We estimate equation (8), taking coefficients $\boldsymbol{\alpha}'_0(t)$, $\boldsymbol{\alpha}'_{1,i}(t)$, and $\boldsymbol{\Psi}_{i,j}(t)$ to be constant within intervals. The full set of estimation results including the constant and the coefficients on the interaction terms are reported in Tables I.6 of Appendix I.

Table 5: DERIVATIVE OF CHILD OUTCOMES WITH RESPECT TO PARENTAL EXPECTED PDV AT DIFFERENT CHILDHOOD INTERVALS, EVALUATED AT THE MEAN (FIRST SPECIFICATION)

(1) Child's Outcome	(2) Child's age $[0, 5]$	(3) Child's age $[6, 11]$	(4) Child's age $[12, 17]$	(5) Child's age $[18, 23]$	(6) Child's age $[24, 29]$
Language Test Score (Age 11)	0.0299** [0.011, 0.05]	0.0185 [-0.00082, 0.0377]			
Mathematics Score (Age 16)	0.00953*** [0.00899, 0.0101]	0.00416*** [0.00343, 0.00504]	0.00119*** [0.000646, 0.00174]		
Incarcerated by Age 35	0.000451 [-0.00023, 0.00115]	-0.00226*** [-0.00329, -0.0013]	-0.0011* [-0.00235, 0.000179]	-0.00108* [-0.00223, 0.0000422]	0.00125*** [0.000298, 0.00207]
Years of Schooling by Age 35	-0.0012 [-0.0045, 0.00203]	0.0226*** [0.0175, 0.0272]	0.0185*** [0.0135, 0.0244]	0.00965*** [0.00365, 0.0152]	-0.0117*** [-0.0154, -0.00756]

95% confidence intervals in brackets.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$
Note: This table shows the value of the derivative of each of the child outcomes with respect to parental expected PDV in each interval, evaluated at the mean. We estimate equation (8), taking coefficients $\boldsymbol{\alpha}_0'(t)$, $\boldsymbol{\alpha}_{1,i}'(t)$, and $\boldsymbol{\Psi}_{i,j}(t)$ to be constant within intervals. The full set of estimation results including the constant and the coefficients on the interaction terms are reported in Tables I.4-I.7 of Appendix I. Column (1) lists the child's outcome of interest. Columns (2)-(6) present the value of the derivative with respect to each age interval, evaluated at the mean parental expected PDV in each interval. The 95% confidence intervals are computed using a bootstrapped method with 200 iterations.

Figures 17-18 and Table 5 plot the derivatives of child outcomes with respect to our proxies for child investments in different childhood intervals. A consistent regularity of these tables is that we do not reproduce the U-shaped profile reported in Carneiro et al. (2021).

Our measures of investment show stronger effects on education in the elementary school years than in the early years, tapering off ever after. For language skills and mathematics skills, the early years are sensitive periods. For participation in crime, the adolescent-young adult years are more potent–consistent with the research of Steinberg (2014) on the maturation of the prefrontal cortex and emergence of self control.

From Equation (6), $\frac{\partial^2 \boldsymbol{G}^{(t)}}{\partial \boldsymbol{Z}_t \partial \boldsymbol{Z}'_{t+j}}$ is a measure of complementarity or substitutability because $\boldsymbol{Z}_t$ proxies $\boldsymbol{I}_t$. The concept of dynamic complementarity has to be extended for skills not in play at early ages. It is meaningful only for ages where the emergent skills are relevant (have non-zero partials). Table 6 shows that dynamic complementarity operates across many ages of investment in producing adult education. Yet all cross partials are not statistically significant and some are negative. Cross effects may be negative. Thus, extraversion may well be a negative aspect of the productivity of a professional mathematician but not for a salesperson. The evidence for dynamic complementarity for other outcomes is more mixed. See Tables I.4-I.7 in the appendix.

Table 6: Dynamic Complementary for Education Attainment at Age 30 (First Specification)

Child's Age	$[0,5]$	$[6,11]$	$[12,17]$	$[18,23]$
$[6,11]$	0.00063*			
	(0.000373)			
$[12,17]$	-0.00162***	0.00170***		
	(0.000428)	(0.000511)		
$[18,23]$	0.00079***	-0.00088*	0.00167**	
	(0.000416)	(0.000514)	(0.00051)	
$[24,29]$	0.00018	-0.00007	-0.00011	0.00179***
	(0.00032)	(0.00041)	(0.00044)	(0.00036)

Standard errors in parentheses.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: This table presents the dynamic complementary for education attainment at age 30. We estimate Equation (8), taking coefficients $\boldsymbol{\alpha}'_0(t)$, $\boldsymbol{\alpha}'_{1,i}(t)$, and $\boldsymbol{\Psi}_{i,j}(t)$ to be constant within the selected intervals. The full set of estimation results are reported in Table I.7 of Appendix I.

8 Conclusion

This paper connects the literature on intergenerational mobility to the literature on child development and suggests a procedure for choosing the appropriate measure of income to estimate the intergenerational transmission of welfare. We present estimates of impacts of age-specific intergenerational expected life cycle income instead of measures of current income over narrow intervals as used in most studies of social mobility.

We introduce a new approach for selecting the appropriate measures of expected lifetime income and the age ranges for measuring it when computing IGEs. Instead of choosing income measures to align ages across generations or to focus on measurement errors, we estimate IGEs on the basis of how well they predict child academic achievement, criminal behavior, and educational attainment. The predictive power of parental income on measures of child outcomes guides our choice of ages of measurement. We select the combination of measures and age ranges that best predict important human capital outcomes of children such as academic performance, educational attainment, and participation in crime.

The predictive power of parental income on child human capital measures crucially depends not only on when parental resources are measured (over the life cycle of children), but also on which measure of parental resources we consider. The choice of income measure makes more difference to estimated IGEs than the choice of measurement intervals. The traditional approach, based on income flows in narrow intervals, substantially overestimates intergenerational mobility and underestimates the persistence of advantage across generations.

Our analyses suggest that parental resources are significantly more important in predicting child outcomes for disadvantaged families, compared to more educated and stable families. We analyze and extend Carneiro et al. (2021). We do not reproduce their U-shaped impact of income timing pattern by age. For math and language skills, early year investment has the biggest impact. For years of schooling, impacts are greatest in the middle school years of children. For crime, investment in the adolescent and young adult years is more impactful. This is consistent with the analysis of Cunha et al. (2010) who show that noncognitive skills emerge later in life and the studies of Steinberg (2014) and Crone (2016). There is evidence of both dynamic complementarity and dynamic substitution of investment over the life cycle, especially for education. We amend the Cunha et al. (2010) model to account for new skills emerging in adolescence. This enables us to account for the appearance of sensitive periods in adolescence and helps to rationalize our results.

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