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NATURAL RESOURCES AND SOVEREIGN RISK IN EMERGING ECONOMIES: A CURSE AND A BLESSING

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Natural Resources and Sovereign Risk in Emerging Economies: A Curse and a Blessing
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ABSTRACT

Emerging economies that are large oil producers have sizable external debt, their sovereign risk rises when oil prices fall, and many of them have defaulted in the past. Interestingly, oil output reduces country risk on impact and in the long-run, but oil reserves increase it in the long-run and reduce it only marginally on impact. We propose a model of sovereign default and oil extraction and derive analytic and quantitative findings consistent with these observations. The sovereign manages oil reserves strategically to make default less painful, and hence its sustainable debt falls. Reserves rise in the run-up to a default and the co-movement of reserves and country risk in response to oil-price shocks switches from negative initially to positive afterwards. These results extend to a setup with rare, large and uncertain oil discoveries. Defaults occur with less severe drops in GDP and oil prices but after long dry spells in discoveries.

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A data appendix is available at http://www.nber.org/data-appendix/w31058

1 Introduction

Fluctuations in the prices and production of commodities are key determinants of macroeconomic performance worldwide. This is the case for advanced, emerging, and developing economies and for both net importers and exporters of commodities.[1] One of the key transmission mechanisms driving this relationship operates via the impact of commodities on access to global financial markets, specifically through changes in sovereign risk. Hence, understanding the link between country risk and commodities is a central question in international economics.

Several important stylized facts reflect the relevance of this question. In a comprehensive study of 42 commodities and 41 countries over 150 years, Domínguez-Cardoza et al. (2022) found a strong, persistent negative association between commodity prices and sovereign risk that became increasingly dominated by mineral commodities (oil and non-oil) over time.[2] They also note that 50 percent of all countries are heavily dependent on commodity exports and that drops in commodity prices have been associated with many debt crises, including the sub-Saharan African crisis of 2014, the defaults in Russia and Ecuador in 1998, and the 1930s and 1980s crises in Latin America. Going forward, we should also expect the transition to renewable energy to affect the access of fossil-fuel producers to financial markets as they face hurdles in extracting and/or selling oil and gas. Hence, a case could be made for providing affected countries with reparation payments or access to special borrowing facilities from international organizations, but their design would hinge on models that capture accurately the link between commodities and country risk.

In this paper, we conduct an empirical, theoretical and quantitative study of the connection between the price and production of commodities and sovereign risk and default. We focus on oil because of the quality and availability of data on reserves, extraction, and discoveries, but our analysis applies to all commodities. Figure 1 summarizes the relationship between oil prices, sovereign risk, and default events in the 1979-2014 period for the 30 largest oil-producing emerging economies as of 2010.[3] The bars show the number of countries in default or financial exclusion each year, the red curve shows sovereign risk measured by the mean of the Institutional Investor Index (III) for all 30 countries (right axis), and the black curve is the real price of Brent crude oil (left axis).[4] Clearly, country risk and the number of countries in default rise sharply as oil prices fall. These co-movements are evident over the medium term (e.g, in the 1980-2005 period) and also in the short-run (with high oil prices in 2005-2007 there were no defaults and country risk fell).

We start with an empirical analysis that formalizes the suggestive evidence shown in Figure 1 and also establishes important new empirical regularities. We document in particular three facts: (1) the 30 largest oil producers have sizable external debt (22.5% on average, weighted by the

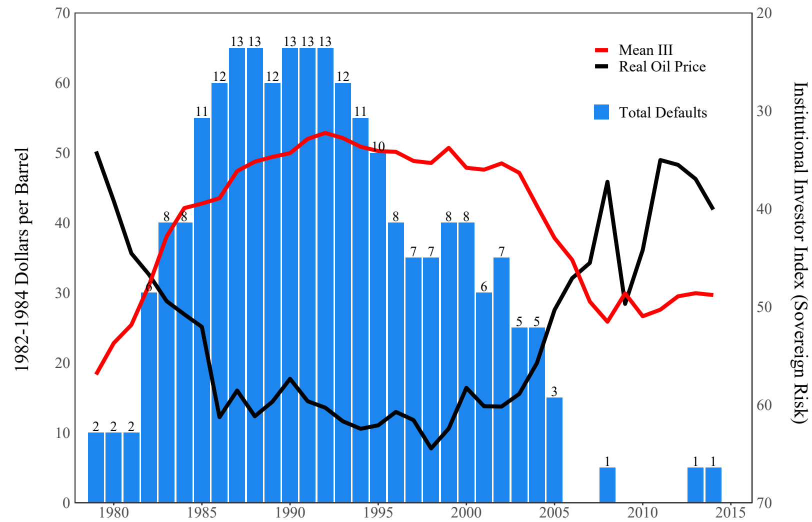
[1] See, for example, Mendoza (1995), Kilian & Vigfusson (2017), Bornstein et al. (2023) and Fernández et al. (2017).

[2] Reinhart et al. (2016), Boehm et al. (2021) and Zhang et al. (2022) also found a negative relationship between commodity prices and country risk in different panel datasets, and Bouri et al. (2017) found large spillovers from the conditional variance of commodity prices to that of sovereign credit default swaps.

[3] See Appendix A for the list of countries, data sources and transformations.

[4] The III goes from zero to 100, with a higher index indicating lower risk. See Section 2 for a detailed description.

Figure 1: Oil Price, Country Risk, and Number of Countries in Default



share of combined oil output), about half of them defaulted at least once, and their debt-GDP ratio and country risk are positively correlated; (2) at the cyclical frequency, country risk is negatively correlated with the real price of oil, with an unconditional (weighted) correlation of -0.69; and (3) in dynamic error-correction panel regressions, higher oil production or non-oil GDP reduce country risk on impact and in the long-run, but the conditional relationship between oil reserves and sovereign risk changes direction over time, with a marginally *positive* effect on the III that reduces risk on impact but a *negative* effect that increases it in the long run.[5]

We then propose a model that extends the standard sovereign default models in the vein of Eaton & Gersovitz (1981) to include oil extraction and reserves. The economy has an endowment of a tradable, non-storable good (non-oil GDP), and produces oil that is sold in world markets at an exogenous relative price in units of the tradable good. The innovation relative to Eaton-Gersovitz (EG) models is in that the sovereign makes oil extraction and reserves decisions, in addition to borrowing from foreign creditors without the ability to commit to repay as in EG models. Oil prices and non-oil GDP are subject to stochastic shocks. When the sovereign defaults, oil exports incur a piece-wise linear cost analogous to the exogenous default cost introduced by Arellano (2008). This cost is modeled as a price penalty akin to a progressive tariff on oil exports, but part of this cost becomes endogenous as oil output responds to the penalty. Importantly, the default payoff of the sovereign features another endogenous response determined by its choice of oil reserves.

This endogenous response is present because, unlike in the canonical EG model where the default payoff is determined by the exogenous realization of endowment income, in the model we propose the value of default rises with oil reserves. Default causes the sovereign to be excluded

[5]The negative correlation between commodity prices and country risk holds also for other mineral and agricultural commodities in the same sample period. For example, correlations between country risk and commodity prices (for the ten largest producers of each commodity) are: -0.6 for barley, -0.5 for green coffee, -0.3 for cotton, -0.5 for soybeans, -0.6 for rice, -0.4 for wheat, -0.7 for coal, and -0.5 for natural gas.

from the credit market but not from the world oil market, and thus oil production and exports continue even after the sovereign defaults. Hence, the value of default depends on oil reserves and this incentivizes the sovereign to strategize over both debt and reserves. When oil prices are high, the standard incentive to extract more oil and reduce reserves competes with the incentive to build reserves to prop up consumption in the event of a default. We derive new analytic results showing conditions under which, keeping other variables constant, default incentives are stronger when oil reserves or oil prices are lower.

We calibrate the model using the same cross-country dataset used in the empirical analysis, and solve it numerically to derive its quantitative predictions and compare them with the stylized facts. The model's oil extraction costs are calibrated separately from the sovereign's problem, to match data targets on oil reserves, the volatility of extraction and the share of oil output in GDP of the subset of countries in our sample that did not default. The resulting parameters are then passed on to the sovereign's problem, which is calibrated to match the observed mean debt ratio and frequency of default. The analytic results hold in this calibrated solution even though some of the assumptions used to derive them do not hold (e.g., shocks are not i.i.d., there is random re-entry to debt markets instead of permanent exclusion).

The quantitative results show that default costs are *lower* than in a variant of the model with constant oil extraction (i.e., with oil as an endowment). This illustrates the sovereign's ability to strategize over oil extraction and reserves to make default less costly. For the same reason, however, the sovereign's borrowing capacity weakens and the debt it can sustain in the long-run falls.

Comparing the model's predictions with the data, we find that the model does well at approximating the second stylized fact we noted: Country risk is negatively correlated with oil prices. The model also does a good job at replicating the observed income correlations and a reasonable job at matching the rest of the oil-price correlations. The model also does well at approximating the variability of gross oil output, and reasonably well at approximating that of total GDP, disposable income, and the trade balance-GDP ratio, but it overestimates the variability of consumption and spreads and underestimates the ones for reserves and extraction. Specifically, consumption is more variable in the model than in a variant of it where the government is committed to repay, but in both models consumption is significantly more volatile than in the data.

The model also reproduces the changing conditional dynamic relationship of oil reserves and sovereign risk over time (country risk falls as reserves rise in the short run but after that they both rise together). We show this by studying the model's local-projection impulse responses to a negative oil-price shock. Reserves rise and country risk falls in the first five years after the shock but the next 15 years both reserves and country risk rise.

Finally, we study the role the oil sector plays in our findings by conducting two sets of experiments. First, we compare long-run properties of debt and spreads and default event dynamics of the proposed model vis-a-vis the variants with constant extraction and without default risk. Allowing default incentives to influence extraction and reserves decisions has large quantitative implications. In the long-run, the lack of commitment to repay reduces the mean debt ratio to 23 percent from 52 percent in the model without default risk. With constant extraction (and default risk), however, the mean debt ratio rises to 28 percent. Hence, strategic use of oil reserves responding to default incentives reduces the sustainable debt by 500 basis points. Around default events, the sovereign reduces oil output and increases reserves sharply above what the risk-free and constant-extraction models predict, and the extra reserves prop up consumption post-default. On the other hand, the sovereign borrow less and at higher spreads before defaults.

The second experiment introduces rare, large and uncertain oil discoveries. Quantitatively, we found that this makes little difference for an oil industry managed by risk-neutral global investors. For the risk-averse sovereign unable to commit to repay, however, the exposure to dry spells without discoveries results in sharply higher oil-sector volatility that strengthens self-insurance incentives, inducing the accumulation of a larger stock of oil reserves in the long-run. Default incentives are stronger, as default are triggered by smaller declines in oil prices and non-oil GDP, after unusually long spells without large discoveries. The changing co-movement of country risk and oil reserves over time and the strategic build-up of reserves prior to a default continue to be important features of the results.

Our work contributes to the sovereign default literature based on the EG framework, particularly the quantitative branch that followed Aguiar & Gopinath (2006) and Arellano (2008) (see the survey by Aguiar et al. (2016)). Three strands of this literature examine models related to ours in that the sovereign's decisions alter the value of default. First, Sosa-Padilla (2018) introduces banks that hold sovereign debt and make working capital loans. The sovereign internalizes that the net worth of the banks alters the value of default because it reduces working capital financing. Second, Hur & Kondo (2016), Bianchi et al. (2018), Bianchi & Sosa-Padilla (2022), and Suarez (2022) examine the optimal choice of foreign reserves and sovereign debt. In the presence of long-term debt, foreign reserves alter the cost of borrowing and the resources available after a default. Third, Hamann (2004), Gordon & Guerron-Quintana (2018), Asonuma & Joo (2021), and Esquivel (2022) introduce capital accumulation managed by the sovereign so that the value of default changes with the capital choice. The effect of oil reserves on the value of default in our model differs from this literature in that reserves prop up consumption directly by supporting oil extraction during periods of exclusion, even with one-period debt, and in that the effect is limited to oil reserves, an asset often controlled by the government in emerging economies, excluding privately-owned capital.

We also contribute to the literature on the business cycle implications of commodity-price and terms-of-trade shocks, as in Mendoza (1995), Bornstein et al. (2023), Fernández et al. (2017), and Schmitt-Grohé & Uribe (2018).[6] These studies typically abstract from modeling sovereign debt explicitly and assume that external debt is risk-free. Hence, our analysis adds to the transmission mechanism of terms-of-trade shocks effects that operate via the sovereign's default incentives. As a result, default and default risk affect fluctuations in consumption, oil reserves (i.e., investment), and net exports. Moreover, because of the lack of commitment to repay sovereign debt, the econ-

[6] See Uribe & Schmitt-Grohé (2017) for a review of this literature.

omy sustains less debt in the long run and is less able to use external borrowing to smooth consumption, relative to what RBC-like models with terms-of-trade shocks predict.

Our work is also related to the empirical studies of the link between commodity prices and country risk cited earlier and to the literature on the open-economy implications of large oil discoveries. Arezki et al. (2017) study giant discoveries as news shocks and find a significant anticipated wealth effect that leads to a current account deficit. Esquivel (2022) examines a model with default and shows that country risk rises after large oil discoveries, in line with our finding that higher oil reserves increase sovereign risk in the long run. In addition, the variant of our model with large, infrequent discoveries predicts that defaults follow unusually long dry spells of discoveries.

Finally, our work contributes to the literature on the "curse of natural resources," initiated by the finding from Sachs & Warner (1995) showing that GDP growth is lower in developing countries with higher resource exports-GDP ratios (see the survey by van der Ploeg (2011)). Our focus is not on growth, but we contribute to this literature by examining how debt, default risk, oil extraction and reserves, and macro-aggregates respond to oil-price shocks. Empirically, we show that while higher extraction reduces country risk on impact and in the long run, higher reserves marginally reduce country risk in the short run and increase it in the long run. The model is consistent with these empirical regularities and also predicts that default risk makes consumption and net exports more volatile and sharply reduces sustainable debt. Hence, our findings suggest that natural resources are both a curse *and* a blessing.

The paper proceeds as follows. Section 2 presents the empirical evidence, Section 3 presents the model, Section 4 presents the model's quantitative analysis, and Section 5 concludes.

2 Stylized Facts

This section documents key empirical regularities linking oil prices, production, and reserves with aggregate economic activity and sovereign risk. We study data for the 1979-2014 period for the subset of emerging economies (as classified by the IMF) that were the 30 largest oil producers as of 2010. The data include oil production, reserves, and net exports; national accounts data on GDP, consumption, trade balance, and oil rents as a percent of GDP; and data on total and external public debt, net foreign assets (NFA), default episodes, and a country risk indicator.[7]

Since international databases do not report oil value added, we used oil rents as a share of GDP from the World Bank's *World Development Indicators* (WDI) to construct a measure of oil GDP by multiplying this series times total GDP at constant local-currency prices from the same source.[8] We then constructed non-oil GDP by subtracting this oil GDP measure from total GDP. Consumption and trade balance data are also from WDI at constant local-currency prices.

[7] See Appendix A for details on data availability, sources and construction of all variables.

[8] Oil rents can differ across countries depending on off- v. on-shore extraction. Using 1962-2009 data for 27 countries in our panel from Andersen et al. (2022), we found that 70% of extraction is on-shore (averaging mean shares across countries) and defaulters and non-defaulters have similar shares of off-shore extraction (28% and 25%, respectively).

Oil-sector data on proven reserves, production and net exports in barrels and oil prices (Brent crude oil spot price, FOB, U.S. dollars per barrel) are from the US Energy Information Administration (EIA). We constructed the real price of oil by deflating the Brent oil price with the US CPI index for all urban consumers, all items (U.S. City average, seasonally adjusted, 1982-1984=100) and gross oil output by multiplying the real price of oil times oil production.

Our measure of country risk is the Institutional Investor's Index (III) of country credit ratings. The III is published biannually in the March and September issues of the *Institutional Investor* and is based on credit ratings constructed with the Institutional Investor's Country Credit Survey, which collects information from senior economists and sovereign-risk analysts at global banks and securities firms, weighting their responses according to their institutions' global exposure. Respondents grade each country in a scale of zero (worst) to 100 (best). Hence, higher scores represent a lower probability of default, although the III captures also other risks related to investing in a particular country (e.g., political risk, exchange rate risk, economic policy risks and macroeconomic risks).[9]

Total public external debt is from the World Bank's *Global Development Finance* database, and NFA from the updated version of the "External Wealth of Nations" dataset constructed by Lane & Milesi-Ferretti (2007). Default data is from Borensztein & Panizza (2009) for 1979-2004 and from Reinhart & Rogoff (2010) for 2005-2014. A sovereign default is defined as failure to meet principal or interest payments on the originally contracted due date (or within the specified grace period), or as an exchange offer of new debt that contains terms less favorable than the original issue.

The data we collected yields three key stylized facts:

1. Large oil-producing emerging economies have an average external public debt ratio of 22.5%, about half of them defaulted at least once, and country risk worsens as debt rises (the average correlation between the debt ratio and the III is -0.6).

Figure 2a shows the mean external public debt-GDP ratio in each country in our dataset. The lowest is around 4% for Iran and the largest is 72% for Vietnam. Across all countries, the mean debt ratio is 22.5%.[10] Figure 2b shows the number of default episodes for the same set of countries. Sixteen of the thirty countries have defaulted at least once since 1979, and several have defaulted more than once (Argentina, Ecuador, Gabon, Indonesia, Nigeria, Russia, and Venezuela).[11] External debt and country risk move together (i.e., debt and III are negatively correlated) in all but two countries for which debt data are available, with correlations ranging from -0.96 to -0.43, and the average correlation across all countries is roughly -0.6.[12]

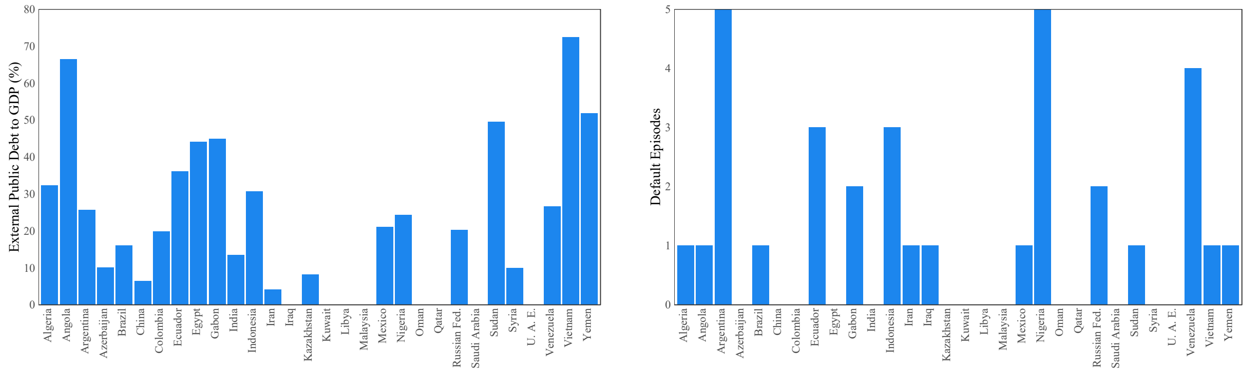
[9] Country risk is often measured using spreads based on the Emerging Markets Bond Index (EMBI) but they are only available since 1994 and for a limited number of countries in our sample. In Appendix B, we show that III is negatively correlated with EMBI spreads in the countries in our sample for which both are available (i.e., both tend to move together to show higher risk), with a median correlation of -0.629. III is also positively correlated with Moody's and Fitch ratings.

[10] This is a weighted average of country-specific averages using as weights shares in oil production for the group of countries in our sample, as described later in this Section.

[11] A zero in Figure 2a denotes that data are not available, but in Figure 2b it denotes a country that has not defaulted.

[12] Appendix I reports country-by-country statistical moments. The average of the country-specific correlations is weighted using oil production shares, as explained in Fact 2.

Figure 2: Debt and Default Episodes per Country



(a) External Public Debt (average 1971-2015)

(b) Default Episodes per Country (1979-2014)

2. Real oil prices and country risk are negatively correlated over the business cycle.

Table 1 reports cross-country weighted averages of business cycle moments, including standard deviations, correlations with GDP, the real price of oil, and oil reserves, and first-order autocorrelations. Weighted averages are computed as follows: The 30 countries are indexed by i. The cross-country weighted average of a moment x is $x = \sum_{i=1}^{30} w_i x_i$, where x_i is the moment for country i and w_i is the country's weight. The weights are time-invariant and they were set by first computing the average of each country's share of oil production in the combined oil production of the 30 countries over the 1979-2014 period, and then normalizing the country averages so that they add to 1. To compute business cycle moments, the data were logged and detrended using the Hodrick-Prescott filter with the smoothing parameter set at 100, except for variables measured as ratios of GDP and the III, which were not detrended (the former because they were stationary and the latter because it is bounded between 0 and 100). For variables where data are missing for some countries, we re-calculate the weights to exclude them from total oil production for all countries.

Table 1 shows that the (weighted) average correlation of real oil prices with the III is 0.693. Thus, when oil prices fall over the business cycle, country risk rises (III falls). This fact also holds country by country, since nearly all country-specific oil price-III correlations are positive and concentrated around the weighted average (see Appendix I). Moreover, this result is validated by several other studies that examine other commodity prices and country risk measures (e.g., Domínguez-Cardoza et al. (2022), Reinhart et al. (2016), Boehm et al. (2021), Zhang et al. (2022))

The average correlations of oil prices with most of the rest of the variables are not very high, as Table 1 shows. In stark contrast with the oil price-III correlations, there is significant heterogeneity across countries. Appendix I shows that the correlations of real oil prices with non-oil GDP, oil production, the trade balance, and GDP vary widely. The correlation between oil prices and the trade balance is the most heterogeneous. It ranges from -0.25 for Indonesia to 0.78 for Kazakhstan. Out of the 30 countries in the sample, eight have correlations above 0.45 and nine have negative correlations. The correlation between oil prices and GDP is between -0.15 and 0.27. Seven countries

Table 1: Oil Prices and Business Cycle Moments

	Mean	Standard Dev.	Corr(i,GDP)	Corr(i,Oil Price)	Corr(i,Reserves)	Autocorr.
Oil price	0	0.182	0.111	1	0.131	0.847
Non-oil GDP	0	0.093	0.631	-0.043	-0.05	0.385
GDP	0	0.069	1	0.111	0.074	0.523
Oil production	0	0.123	0.624	0.041	0.149	0.502
Consumption	0	0.049	0.523	0.120	0.048	0.523
Gross oil output	0	0.235	0.492	0.342	0.110	0.276
Trade balance to GDP	0.073	0.090	0.106	0.190	0.133	0.630
Institutional Investor Index	47.49	11.49	0.208	0.693	0.084	0.869
Debt to GDP	0.224	0.144	-0.284	-0.612	-0.179	0.836
Reserves (billion barrels)	76.97	20.09	0.074	0.131	1	0.833

* Note: Business cycle moments are weighted averages across the thirty countries included in the dataset. The weights were set by first computing the average of each country's share of oil production in the combined oil production of the thirty countries over the 1979-2014 period, and then normalizing the country averages so that they add to 1.

have small but negative correlations, and eleven have correlations exceeding 0.1. The correlation between oil prices and non-oil GDP ranges between -0.24 for Algeria and 0.24 for Russia, although 24 countries have negative correlations that are close to zero. The correlation between oil prices and oil production ranges from -0.23 for Colombia to 0.19 for Algeria, with 16 countries showing negative correlations and 14 positive ones. The mean correlation between real oil prices and gross oil output is relatively high, at 0.342, but this is mostly a valuation effect because oil production is nearly uncorrelated with the price (recall that gross oil output equals production times price).

3. The conditional relationship between oil reserves and sovereign risk changes direction over time. Controlling for the effects of other variables, higher reserves reduce sovereign risk in the short run, albeit the effect is not statistically significant, but in the long run higher reserves have a significant effect that *increases* sovereign risk.

Facts 1 and 2 focused on unconditional moments but Fact 3 focuses on conditional co-movements incorporating all the information in the cross-section of countries and separating the effects of different variables. To handle non-stationarity and separate low-frequency co-movements, we estimate dynamic error-correction panel regressions. We study three specifications (see Appendix C for details). In Model (1), we regress the III on oil production, real non-oil GDP in local currency, oil reserves, external public debt as a share of GDP, oil discoveries, and a default dummy. In Model (2), we control for NFA and exclude the default dummy. Model (3) includes all control variables. The three regressions include country fixed effects (to capture country-specific political situations, for example), and time fixed effects (TFE) to capture variables common to all countries, including oil-prices. The coefficients can be interpreted as elasticities with the exception of the one for discoveries. We estimated balanced and unbalanced panels. Table 2 shows the latter, but the results are qualitatively the same in both (Table C3 in Appendix C shows the balanced panel results).[13]

[13] Due to data limitations, Azerbaijan, Kazakhstan, Kuwait, Iraq, Libya, Oman, Qatar, Saudi Arabia, Syria, United Arab Emirates, and Yemen are dropped from these regressions. Consequently, the estimation is performed taking into account 512, 509 and 509 observations in the regression models 1, 2, and 3 respectively.

In these regressions, the convergence coefficient measures the speed at which the III converges to its long-run trend. In each model, it has the expected sign and is statistically significant at the 1% level. Convergence in the III runs at an annual rate between 0.156% and 0.183%, which means that each year the III covers about 0.17% (depending on the model) of the distance to its trend. Convergence is slightly slower in Model (2), where NFA is included and default is excluded.

The estimated short-run coefficients imply that an increase of 1% in oil production decreases country risk around 5 basis points, and this effect is significant at the 5% confidence level in the three regressions. A one-percent increase in non-oil GDP reduces country risk by 20 to 23 basis points (significant at the 1% level). An increase in oil reserves decreases country risk, but the coefficient is not significant, in line with the mixed results of the country-specific correlation coefficients in Appendix I. Discoveries move country risk in the opposite direction than reserves, but the effect is also not significant. As expected, higher external public debt increases country risk (by about 10 basis points per 100-points of debt), and this result is statistically significant. Finally, the effect of NFA on country risk (keeping external public debt constant) is not significant.

The long-run (co-integration) coefficients yield a key result: All three regressions yield a statistically significant *negative* long-run elasticity of III with respect to oil reserves (at the 5% level for Model (2) and at the 1% level for the other two models). A rise in reserves of 1% worsens country risk by about 0.15%. Thus, the data indicate that oil-producing emerging economies are perceived as more risky in the future when they boost their reserves today. In contrast, the contemporaneous effect operates in the opposite direction but is not statistically significant, as indicated by the short-run coefficient for reserves. Hence, the co-movement between reserves and country risk changes direction from neutral in the short-run to negative as the variables converge to their trends.

The long-run elasticity of III with respect to oil production is positive, in the $0.04 - 0.05$ range, but not statistically significant. With respect to non-oil GDP and discoveries, long-run elasticities are again not statistically significant. External public debt has a negative effect on country risk in the long-run and is statistically significant at the 1% level in the three regressions. Similar to the short-run, in the long-run the *level* of net-foreign assets increases country risk, but is not statistically significant. Finally, as expected, being in default increases country risk. When a country is in default, the III drops about 37%. This last result is statistically significant at the 1% level. As for oil discoveries, an increase in discoveries reduces country risk but the effect is not significant.

Since the inclusion of TFE prevents us from identifying the effect of oil prices on sovereign risk in isolation, we also estimated regressions replacing TFE with the oil price. The results are reported in the column labeled Model (4). The short-run coefficient of the oil-price is positive and significant and its long-run coefficient is negative but not significant. The coefficients of the other variables are similar to those of the TFE regressions. Importantly, oil reserves are still negative and significant in the long run and non-significant in the short run.[14]

[14] These results justified our choice to use the specification with TFE instead of oil prices as the baseline, since it allows us to capture the effects of other important common variables like the world interest rate. It is also important to note that our baseline specification still includes a proxy for country-specific effects of oil prices captured by oil reserves, because

Table 2: Dynamic Fixed Effects Regressions

	Δ Inst. Investor Index				Δ Oil Production
	Model (1)	Model (2)	Model (3)	Model (4)	Model (5)
Convergence coefficient					
Inst. Investor Index (-1)	-0.175***	-0.156***	-0.183***	-0.205***	
	(0.019)	(0.020)	(0.020)	(0.020)	
Oil Production (-1)					-0.097***
					(0.013)
Short-run coefficients					
Δ Oil Production	0.052**	0.047**	0.055**	0.037*	
	(0.021)	(0.022)	(0.022)	(0.022)	
Δ Non-Oil GDP	0.199***	0.231***	0.198***	0.246***	0.241**
	(0.058)	(0.059)	(0.057)	(0.058)	(0.106)
Δ Oil Reserves	0.006	0.014	0.010	0.017	0.107***
	(0.020)	(0.020)	(0.020)	(0.020)	(0.037)
Δ Ext. pub. debt to GDP	-0.104***	-0.094*	-0.107**	-0.052	0.235***
	(0.038)	(0.052)	(0.051)	(0.052)	(0.091)
Δ Oil Discoveries	-0.003	-0.003	-0.003	-0.001	-0.004
	(0.003)	(0.004)	(0.003)	(0.004)	(0.006)
Δ NFA		-0.040	-0.046	-0.004	0.082
		(0.035)	(0.034)	(0.035)	(0.064)
Δ Real Oil Price				0.059***	0.106***
				(0.019)	(0.033)
Long-run coefficients					
Oil Production	0.048	0.048	0.038	0.048	
	(0.041)	(0.049)	(0.041)	(0.038)	
Non-oil GDP	0.095	-0.027	0.101	0.135**	0.133
	(0.106)	(0.120)	(0.100)	(0.057)	(0.211)
Oil Reserves	-0.162***	-0.141**	-0.141***	-0.142***	-0.368*
	(0.051)	(0.060)	(0.050)	(0.044)	(0.192)
Ext. pub. debt to GDP	-0.810***	-1.226***	-1.001***	-0.922***	1.118*
	(0.140)	(0.219)	(0.178)	(0.164)	(0.631)
Default	-0.369***		-0.379***	-0.363***	0.167
	(0.072)		(0.068)	(0.060)	(0.217)
Oil Discoveries	0.045	0.048	0.039	0.030	0.075
	(0.028)	(0.033)	(0.027)	(0.025)	(0.097)
NFA		-0.003	-0.119	-0.106	1.409***
		(0.141)	(0.116)	(0.110)	(0.423)
Constant	0.236	0.781	0.209	0.056	-0.276
	(0.565)	(0.569)	(0.559)	(0.339)	(0.612)
Real Oil Price				-0.036	-0.405***
				(0.052)	(0.179)
TFE	√	√	√	×	×

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Finally, we estimated an additional DFE regression to study the conditional elasticity of oil production to oil prices by setting production as dependent variable in Model (5) of Table 2. The resulting estimate of about 0.11 is very similar to the cross-country panel estimates that Caldara et

proven reserves are defined as the amount of oil that can be extracted profitably given current technology and prices.

al. (2019) obtained using large production drops as instruments (see Appendix J.3 for details).

Summing up, Fact 1, showing that large oil producers display non-trivial debt ratios and default rates and positively-correlated debt and country risk, highlights important empirical regularities consistent with what the sovereign default literature has documented for emerging markets in general. Facts 2 and 3 provide key facts that illustrate the relevance of the oil sector in the sovereign debt and risk dynamics of large oil producers. Fact 2 shows that, over the business cycle, country risk rises when oil prices fall. Fact 3 sheds light on the short- and long-run conditional dynamic relationship of country risk with oil production, reserves, and discoveries, and with macro variables like non-oil GDP and NFA. The main finding is that, while higher oil production or non-oil GDP reduce country risk, the relationship between oil reserves and country-risk changes direction over time. Higher reserves improve country risk marginally in the short-run, but in the long-run they have a statistically significant effect that *worsens* country risk.

3 A Model of Sovereign Default and Oil Extraction

The model we propose introduces oil extraction and reserves into a sovereign default setup. A benevolent social planner cannot commit to repay external debt and chooses optimally whether to default or not. The planner operates the oil industry, and thus chooses also extraction and reserves.[15] Hence, the planner has two vehicles for reallocating resources intertemporally (debt and oil reserves) and can affect the value of default by altering reserves. In addition, the planner's income, repayment capacity and default incentives are exposed to oil-price shocks.

3.1 Model structure

There are two types of goods, oil and a tradable consumption good. The economy has an exogenous stochastic endowment of the latter (non-oil GDP), y, which has an exogenous world price set to 1 without loss of generality. The price of oil relative to the consumption good, p, is stochastic and determined in world markets.[16] A stationary Markov process with transition probability matrix $\pi(p', y'|p, y)$ governs the evolution of p and y, using primes to denote next-period's values.

Extracting x units of oil out of an existing stock of oil reserves s incurs a cost $e(x, s)$ in units of the consumption good, with $e_s(\cdot) < 0$, $e_x(\cdot) > 0$ and $e_s(0, s) = 0$. Oil GDP is thus equal to oil profits $y^O \equiv px - e(x, s)$. The functional form of the cost function is:

$$e(x, s) = \psi \left(\frac{x}{s}\right)^{\gamma} x. \tag{1}$$

Hence, the per-unit extraction cost ($\psi \left(\frac{x}{s}\right)^{\gamma}$) is homogeneous of degree zero in x and s. This for-

[15] This assumption is in line with the dominant role of state-owned enterprises in commodity extraction and/or exports in many emerging and developing economies.

[16] In Appendix E we show that this is a reasonable assumption because most countries in our sample are price takers.

mulation is standard in the resource economics literature following the seminal work of Hotelling (1931). He originally assumed linear extraction costs but subsequent research noted that actual extraction costs are increasing in extraction and decreasing in reserves and are also increasing and convex in the ratio x/s (see Ch. 5 of the textbook by Conrad (1999)).

Reserves follow the law of motion $s' = s - x + \kappa$, where κ denotes a constant amount of oil discoveries each period and s' denotes reserves carried over to the next period.[17] Extraction cannot be negative ($x \geq 0$) and cannot exceed the sum of reserves plus discoveries ($x \leq s + \kappa$). Since oil is a form of capital with an endogenous return, it has an asset valuation that we label the "asset price of oil" defined as $q^O \equiv p - e_x(x, s) + \Delta\tilde{\psi}$, where $\Delta\tilde{\psi} \equiv [\psi^l - \psi^h]/u'(c)$ and ψ^l and ψ^h are the multipliers on the lower and upper bounds of x, respectively.[18]

The world credit market is the same as in EG models. The government maximizes private-sector utility, defined by a standard time-separable expected utility function with constant-relative-risk-aversion period utility $u(c) = \frac{c^{1-\mu}}{1-\mu}$ and subjective discount factor β. The sovereign sells one-period, non-state contingent discount bonds, denominated in units of the consumption good, to risk-neutral foreign investors. The outstanding bond position is denoted b and newly issued bonds are denoted b' (the sovereign is indebted when $b < 0$). The set of feasible bond positions is given by a discrete grid defined over the interval $B = [b_{min}, b_{max}]$where $b_{min} \leq b_{max} = 0$. If the sovereign defaults, it does not repay b in the current period and is excluded from the credit market, so no b' can be issued. Next period, the sovereign re-enters the credit market with probability λ.

We assume that the country continues to participate in the world oil market during the financial exclusion period (i.e., the sovereign can still export oil when it defaults). This is important because it implies that the sovereign's plans for the accumulation of oil reserves affect the value of default, since those reserves can be extracted and exported when foreign borrowing is barred.

The timing of decisions within a period is as follows: First, s and b are known and the shocks p and y are realized. Second, the sovereign decides whether to repay or default by choosing the option that yields the highest value, as explained below. If the sovereign defaults, since the country is not excluded from the oil market, it makes oil extraction and reserves decisions and pays extraction costs. If the sovereign repays, it sells new bonds b' to foreign investors at the price q, makes extraction and reserves decisions, and pays extraction costs. Third, all resources generated from debt and profits from oil exports are transferred to households and used for consumption.

[17] We model κ as a per-period constant for simplicity. To scale it accurately in the quantitative analysis, we calibrate it to long-run averages of oil reserves and output, but we acknowledge that discoveries are lumpy and uncertain in practice. Hence, in the quantitative analysis and in Appendix K we explore the implications of introducing rare and large stochastic discoveries. The baseline model's key results on the changing direction of the reserves-country risk co-movement and the strategic use of oil reserves to make default less painful still hold.

[18] Appendix F shows that, taking as given a bond pricing function, q^O equals the expected present discounted value (discounted with the sovereign's stochastic discount factor) of the income stream composed of oil "dividends", $d^O \equiv -e_s(t) + \psi^h_{t+1}/u'(c_t)$, and the marginal revenue from the effect of accumulating oil reserves on the price of debt.

The planner's payoff at the beginning of the period is:

$$V(b,s,y,p) = \max\left\{v^{nd}(b,s,y,p), v^{d}(s,y,p)\right\},$$

where $v^{nd}(b,s,y,p)$ is the value of repayment and $v^{d}(s,y,p)$ is the value of default.

The value of repayment is characterized by the following maximization problem:

$$v^{nd}(b,s,y,p) = \max_{\{c,x,b',s'\}} \left\{u(c) + \beta E\left[V\left(b',s',y',p'\right)\right]\right\} \tag{2}$$

s.t.

$$c = y - A + px - e\left(x,s\right) + b - q\left(b',s',y,p\right)b', \tag{3}$$

$$s' = s - x + \kappa, \tag{4}$$

$$0 \leq x \leq s + \kappa. \tag{5}$$

Constraint (3) is the resource constraint, (4) is the law of motion of reserves, and (5) states the feasibility constraints on extraction.[19] In the resource constraint, $q\left(b',s',y,p\right)$ is the pricing function for the sovereign bond, which varies with the choices (b',s') and the realizations of (p,y), and A represents autonomous (exogenous) spending allocated to investment expenditures so that the consumption-GDP ratio can be calibrated later to match the data (consumption will include private and public consumption).

The value of default is characterized by the following problem:

$$v^{d}(s,y,p) = \max_{\{c,x,s'\}} \left\{u(c) + \beta\left(1-\lambda\right)Ev^{d}\left(s',y',p'\right) + \beta\lambda EV\left(0,s',y',p'\right)\right\} \tag{6}$$

subject to the same law of motion of reserves and feasibility constraint as in the repayment case and the following resource constraint:

$$c = y - A + h(p)x - e\left(x,s\right). \tag{7}$$

In the right-hand-side of the value of default (6), the sovereign re-enters credit markets with probability λ and a clean slate of debt ($b' = 0$), and it retains its oil reserves s'. It remains in default with probability $(1-\lambda)$ but again it retains its oil reserves s'. The resource constraint (7) includes a piece-wise default cost akin to the one proposed by Arellano (2008) for income but in terms of the price of oil: $h(p) = \hat{p}$ if $p > \hat{p}$ and $h(p) = p$ if $p \leq \hat{p}$. Intuitively, this is similar to a foreign ad-valorem tariff on the country's oil exports that rises with p above the threshold $\hat{p}$. This trade penalty is in line with the empirical observation that international trade is negatively affected by sovereign default. Alternatively, we can focus on the income default cost that $h(p)$ implies in terms of oil output or aggregate GDP. Both are affected not only by the exogenous adjustment in p but by

[19] Note that extraction costs are modeled as factor payments abroad. Alternatively, we could assume that a fraction ϕ of the costs are domestic factor income. In which case $e(x,s)$ is replaced with $(1-\phi)e(x,s)$ in the resource constraint.

the endogenous response of oil production (and hence of total GDP) induced by that adjustment. Hence, unlike in standard EG models, this model's default cost in terms of income includes an endogenous component. We examine this issue in more detail in Section 4.

For a given (b, s), default is optimal for the pairs $\{y, p\}$ for which $v^d(s, y, p) \geq v^{nd}(b, s, y, p)$. Hence, the default set is given by:

$$D(b, s) = \Big\{ \{y, p\} : v^d(s, y, p) \geq v^{nd}(b, s, y, p) \Big\} . \tag{8}$$

The associated default decision rule is given by the function $d(b, s, y, p)$, which takes the value of 1 for $(y, p) \in D(b, s)$ and 0 otherwise (i.e. it equals 1 if the government defaults).

The probability of default next period conditional on current-period information, $P^d(b', s', y, p)$, can then be induced from the default decision rule and the Markov process of the shocks as follows:

$$P^d\left(b', s', y, p\right) = \sum_{y'} \sum_{p'} d(b', s', y', p')\pi(y', p'|y, p). \tag{9}$$

Since foreign investors are risk neutral, bond prices satisfy the standard no-arbitrage condition:

$$q\left(b', s', y, p\right) = q^* \left(1 - P^d\left(b', s', y, p\right)\right),$$

where q^* is the price of a risk-free bond such that $q^* \equiv 1/R^*$ where R^* is the world's risk-free gross real interest rate that represents the opportunity cost of funds for foreign investors.

3.2 Model properties

Appendix G proves six propositions that establish important properties of the model. Two of them show that key features of standard EG models still hold (default sets shrink in b and default incentives strengthen as y falls). The other four provide new results related to the asset price of oil, oil profits, and the effects of oil reserves and oil-price shocks. We describe here these four new results and leave the other two for the appendix. Note that obtaining these results is not a straightforward extension of standard EG models because of the endogeneity of the default payoff on s.

As in the literature (e.g. Arellano (2008)), this analysis establishes sufficiency conditions only and is done assuming i.i.d shocks, permanent exclusion after default ($\lambda = 0$), and no default income costs (i.e., $h(p) = p$ always). We also adopt three conjectures: 1) Asset prices of oil are nonnegative under repayment and default; 2) optimal consumption under repayment is nondecreasing in s; and 3) for (y, p) pairs in the default set, the available contracts for new debt and choices of oil reserves under repayment yield a trade balance at least as large as the difference in oil profits across repayment and default. We evaluated numerically both the conjectures and the propositions for the calibration presented in the next Section (where shocks are not i.i.d., $\lambda > 0$ and there are exogenous default costs). Most of them hold in 100 percent of the possible model evaluations, with three of them holding 97 percent (see Appendix G for details).

Proposition 1. If asset prices of oil are positive, oil profits are increasing in s, for given s', and decreasing in s', for given s.

Given Conjecture 1, oil profits under repayment and default ($M^{nd}(s',s,p), M^{d}(s',s,p)$) are increasing in $s \in [\underline{s}, \overline{s}]$ (the derivatives $M_s^{nd}(\cdot), M_s^{d}(\cdot)$ are positive), and decreasing in $s' \in [s+\kappa - s(p/\psi)^{(1/\gamma)}, s+\kappa]$, namely the derivatives $M_{s'}^{nd}(\cdot), M_{s'}^{d}(\cdot)$ are negative.[20]

This proposition shows that, if the asset prices of oil are positive under repayment and default, the corresponding profits from oil extraction are higher if reserves carried over from the previous period are higher, for a given value of s', and lower if new reserves are higher (i.e. extraction falls) for a given value of s. We show in Appendix F that positive asset prices of oil are equilibrium outcomes in three variants of the model without default risk, one under financial autarky and two with access to world credit markets and an exogenous bond pricing function (one set equal to q^* and one with the same properties as that of the model with default). The result under financial autarky also implies that $q^{Od}(\cdot) > 0$ in the model with default and $\lambda = 0$.

Proposition 2. The default and repayment payoffs are non-decreasing in s.

For all $s^1, s^2 \in [\underline{s}, \overline{s}]$ and $s^1 \leq s^2$, $v^{nd}(b, s^2, y, p) \geq v^{nd}(b, s^1, y, p)$ and $v^{d}(s^2, y, p) \geq v^{d}(s^1, y, p)$.

This result follows from Proposition 1, and demonstrates that one of the conditions needed for the default sets to shrink in b (namely that the default and repayment payoffs are non-decreasing in b) also applies with respect to s. This is not sufficient, however, to yield the result that default sets shrink in s, as the next proposition shows.

Proposition 3. Default sets shrink as s rises (i.e. grow as reserves fall).

Assume $\hat{p} = p$ and $\lambda = 0$ for simplicity. For all $s^1, s^2 \in [\underline{s}, \overline{s}]$ and $s^1 \leq s^2$, if default is optimal for s^2 (i.e., $d(b, s^2, y, p) = 1$) for some states (b, y, p), then default is optimal for s^1 for the same states (b, y, p) (i.e., $D(b, s^2) \subseteq D(b, s^1)$ and $d(b, s^1, y, p) = 1$).

This proposition establishes sufficiency conditions under which the result that default sets shrink as b rises (see Appendix G) also applies to oil reserves. It relies on the three conjectures and Propositions 1 and 2 and establishes that the country risk premium is non-decreasing in the choice of s' (i.e., $q(\cdot)$ is non-decreasing in s'). This result does not follow just from analogy to the results that hold for b, because both the repayment and default payoffs vary with s, whereas the default payoff does not vary with b. The key to this Proposition is Conjecture 3. Intuitively, the net resources that all available debt contracts and reserves choices can generate for consumption under repayment are at most the same as those obtained with the optimal reserves chosen under default.

Proposition 4. If the trade balance is sufficiently large and reserves chosen under default at high p exceed those chosen under repayment at low p, default incentives strengthen as p falls.

Assuming i.i.d shocks, $\lambda = 0$ and $\hat{p} = p$, for all $p_1 < p_2$ and $p_2 \in D(b, s)$, if $tb(b^1, s^1, b) \geq M(s^1, s, p_2) - M(\tilde{s}^2, s, p_2)$ and $s^1 \leq \tilde{s}^2$ (where b^1, s^1 are the optimal bonds and reserves choices under repayment in state (b, s, y, p_1) and $\tilde{s}^2$ is the optimal reserves choice under default in state (s, y, p_2), then $p_1 \in D(b, s)$.

[20] The lower bound of s' follows from assuming oil profits are non-negative. The upper bound is at the point where extraction is set to zero. See Appendix G for details.

This proposition shows sufficiency conditions under which the standard result from EG models showing that the default incentives are stronger when y is low also holds when p is low. This Proposition assumes not only a sufficiently large trade balance but also that the oil reserves chosen under default at a high p are larger than those chosen under repayment at a low p. This last property holds in all of the state space of the calibrated model. Moreover, in the calibrated model, the trade balance condition holds in all of the relevant evaluations, and even ignoring it, default incentives strengthen as oil prices fall in all of the state space (i.e., for all $p_1 < p_2$ and $p_2 \in D(b,s), p_1 \in D(b,s)$).

Next, we use these results and those from Appendix F to study how debt and oil reserves compare in terms of resources disposable for consumption under default and repayment, and to examine how they interact with bond prices and oil prices in determining optimal extraction plans.

Consider first the effects of the choices for new debt b' and reserves s' on resources available for consumption. The constraints of the repayment optimization problem imply:

$$c = y - A + p(s+\kappa) - ps' - e\left(s', s\right) + b - q\left(b', s', y, p\right) b', \tag{10}$$

where we replaced x with s' as an argument of $e(.)$. Note that, since $e(.)_x > 0$ and x decreases with s', $e(.)_{s'} < 0$. This expression shows key differences faced by the sovereign in the choice of b' v. s' for reallocating consumption intertemporally. By borrowing more (reducing b'), resources for current consumption change according to the familiar debt Laffer curve of EG models.[21] Reducing s' is akin to borrowing in that it increases resources for consumption, by the amount by which $-(ps'+e(s',s))$ rises. But there is no Laffer curve when "borrowing with reserves." Conditional on not hitting the upper bound of extraction, lower s' always increases resources available for consumption.[22] Borrowing with s' also differs from b' in that it increases resources even in the default state, by the amount $-(h(p)s' + e(s',s))$ as s' falls (i.e., at a lower rate than under repayment).[23]

Debt and reserves also differ in terms of how outstanding debt b and existing reserves s affect resources for current consumption. They are similar in that arriving at the repayment state with more debt (lower b) reduces resources by the amount b, while arriving with fewer reserves reduces resources by the amount ps. But they differ in that the debt repayment is non-state-contingent while the resources provided by s vary with p. As noted often in the sovereign default literature, debt has poor hedging properties because it does not reduce the burden of repayment in "bad" states (i.e., the repayment is uncorrelated with total income), but oil reserves are *worst* in this regard because the resources they provide correlate positively with oil prices (i.e., they provide fewer resources at lower p). Hence, viewing b and s as assets for hedging income fluctuations, reserves are inferior to

[21] When b' is low so that default risk is low or zero, additional debt always gains resources for consumption, because bond prices fall little or stay at q^*, but as debt rises enough for default risk to reduce $q(\cdot)$ sufficiently, additional debt results in fewer resources for consumption.

[22] Note that $\partial c/\partial s' = -p - e_{s'}\left(s', s\right) = -(p - e_x\left(x, s\right)) < 0$ because $q^{Ond} > 0$ implies that $p - e_x\left(x, s\right) > 0$ for an interior solution of x (see Appendices F and G). Hence, borrowing with reserves always increases resources for consumption because the asset price of oil is positive.

[23] In the default state, $\partial c/\partial s' = -h(p) - e_{s'}\left(s', s\right) = -(h(p)p - e_x\left(x, s\right)) < 0$ because $q^{Od} > 0$ implies that $h(p) - e_x\left(x, s\right) > 0$ for an interior solution of x (see Appendices F and G).

debt. Moreover, the sovereign can default on b to reduce the debt burden ex-post.

Qualitatively, debt and oil reserves have similar effects on conditional default probabilities and default risk. With regard to debt, since the standard result from EG models showing that default sets shrink with b still holds (see Proposition 5 in Appendix G), the conditional probability of default and default risk are non-decreasing in debt. Thus, $q(.)$ is non-decreasing in b'. On the side of oil reserves, Proposition 3 showed that default sets shrink with s and thus the conditional probability of default and default risk are non-decreasing in reserves. Thus, $q(.)$ is non-decreasing in s'. The rationale is that, although in the case of oil reserves the default payoff is increasing in s instead of constant, the repayment payoff grows more than the default payoff as s rises. Notice that these are contemporaneous effects that refer to how country risk at date t responds to the sovereign choosing to increase debt or reduce reserves at t and that, as the propositions assumed, they consider only changes in b' or s' keeping everything else constant.

Next we examine the interaction between sovereign risk and the sovereign's optimal oil extraction plans. For simplicity, so that we can obtain familiar no-arbitrage conditions in sequential form, assume that we give to a sovereign who is committed to repay the model's equilibrium bond pricing function, $q\left(s_{t+1}, b_{t+1}, y_t, p_t\right)$, and that this function is differentiable and satisfies other regularity properties.[24] The optimality conditions of the sovereign's problem yield the following no-arbitrage condition between the expected returns on oil and sovereign bonds (see Appendix F):

$$E_t\left[\tilde{R}^o_{t+1}\right] = R^b_{t+1}\left(s_{t+1}, b_{t+1}\right) - \frac{\text{cov}_t\left(u'\left(c_{t+1}\right), \tilde{R}^o_{t+1}\right)}{E_t\left[u'\left(c_{t+1}\right)\right]}. \tag{11}$$

In this expression, $R^b\left(s_{t+1}, b_{t+1}\right) \equiv \frac{1}{q(t+1)+q_b(t+1)b_{t+1}}$ is the sovereign's gross return on bonds. Since we are assuming commitment, there is no default risk, but because $q(\cdot)$ is the equilibrium pricing function of the model with default, the planner internalizes that higher debt carries a higher interest rate than R^* (since $q_b(\cdot) > 0$). Also, since debt is non-state-contingent, the Euler equation for bonds implies that at equilibrium $R^b_{t+1}\left(s_{t+1}, b_{t+1}\right) = \frac{u'(c_t)}{\beta E[u'(c_{t+1})]}$. The term $\tilde{R}^o_{t+1} \equiv \frac{q^O_{t+1}+d^O_{t+1}}{\left[q^O_t+q_s(t+1)b_{t+1}\right]}$ is the sovereign's gross return on oil inclusive of the financial benefit of higher reserves increasing resources available for consumption by rising the price of newly-issued debt. This rate of return can be rewritten as $\tilde{R}^o_{t+1} \equiv R^o_{t+1}\left[\frac{1}{1+q_s(s_{t+1},b_{t+1})b_{t+1}/q^O_t}\right]$, where $R^o_{t+1} \equiv \frac{q^O_{t+1}+d^O_{t+1}}{q^O_t}$ is the "physical" return on oil and $\left[\frac{1}{1+q_s(s_{t+1},b_{t+1})b_{t+1}/q^O_t}\right]$ is the financial return from higher reserves increasing $q(\cdot)$.

Condition (11) implies that the optimal extraction and reserves plans are set so that the total marginal gross return on oil exceeds the full marginal cost of its liabilities by a premium equal to $-\frac{\text{cov}_t\left(u'(c_{t+1}),\tilde{R}^o_{t+1}\right)}{E_t[u'(c_{t+1})]}$. This is akin to an equity premium, with the caveat that both the return on oil

[24] We assume that $q(\cdot)$ is strictly concave and increasing in b_{t+1} for $b_{t+1} \in [-\bar{b}(s_{t+1}), 0]$, where $-\bar{b}(s_{t+1})$ is the threshold debt above which default is certain for a given s_{t+1} (i.e., $D(\bar{b}(s_{t+1}), s_{t+1})$ includes all (y_{t+1}, p_{t+1}) pairs, which exists because of Prop. 5), with $q(\cdot) = q^*$ for $b_{t+1} \geq 0$ and $q(\cdot) = 0$ for $b_{t+1} \leq \bar{b}(s_{t+1})$. $q(\cdot)$ is also increasing and concave in s_{t+1} for $s_{t+1} \in [\tilde{s}(b_{t+1}), s_t + \kappa]$, where $\tilde{s}(b_{t+1}) = max[s_t + \kappa - s_t(p_t/\psi)^{(1/\gamma)}, \bar{s}(b_{t+1})]$ and $\bar{s}(b_{t+1})$ is the threshold oil reserves below which default is certain for a given b_{t+1} (i.e., $D(b_{t+1}, \bar{s}(b_{t+1}))$ includes all (y_{t+1}, p_{t+1}) pairs, which exists because of Prop. 3). We also assume that $\bar{b}(s_{t+1})$ is increasing in s_{t+1} and $\bar{s}(b_{t+1})$ is increasing in b_{t+1}.

and the return on bonds include financial components. The former (latter) because of the effect of lower oil reserves (higher debt) reducing the price of sovereign debt.

Appendix F examines the implications of condition (11) in two other scenarios: (i) permanent financial autarky (which is the same as the solution of the default payoff if $\lambda = 0$) and (ii) a constant bond price set at $q = q^*$ (which renders the model akin to a small-open-economy RBC model).

Under financial autarky, the model resembles a canonical *closed-economy* RBC model, in which condition (11) reduces to $E_t[u'(c_{t+1}) R_t^O] = u'(c_t)$. Hence, the planner uses oil reserves in a manner similar to capital accumulation in the closed-economy RBC model. Markets are incomplete because there are no assets to insure away the risk of the shocks to p and y. Thus, the planner self-insures with reserves so as to facilitate consumption smoothing. There is also an implicit endogenous domestic real interest rate represented by the stochastic marginal rate of substitution in consumption. In the model with default, the planner has a similar incentive in the default state: being excluded from credit markets, it will use reserves to facilitate consumption smoothing, except that, because $\lambda > 0$ it assigns some probability to being able to re-enter the credit market.

In the case with $q = q^*$, condition (11) reduces to $E_t\left[R_{t+1}^o\right] = R^* - \frac{\text{cov}_t\left(u'(c_{t+1}), R_{t+1}^o\right)}{E_t[u'(c_{t+1})]}$ which is analogous to the one obtained in small-open-economy RBC models for the excess return on physical capital. Markets are again incomplete, but here the sovereign has access to no-state-contingent bonds for self-insurance and consumption smoothing. Oil is a risky asset and carries a risk premium, but the returns on oil and bonds and the risk premium do not include the effects of debt and reserves on the price of bonds. Moreover, since the risk premium is small (as is typical in RBC models), the model is close to yielding Fisherian separation of extraction and reserves plans from savings and consumption plans. This separation holds strictly without uncertainty. As shown in Appendix F, the no-arbitrage condition becomes $R_{t+1}^o = R^*$ and yields a second-order difference equation in s that determines the extraction and reserves decision rules independently of the bonds and consumption choices. Fisherian separation also holds if risk-neutral foreign investors (FI) can invest in the domestic oil industry, in which case the decision rules solve the stochastic second-order difference equation $E_t\left[R_{t+1}^o\right] = R^*$. We refer to this case as the SOE-FI model, which will be used in part of the model's calibration.

In the model with default, since default is infrequent quantitatively, when debt and/or reserves (and the history of oil-price and non-oil GDP shocks) are such that the probability of default is positive only in the distant future, the dynamics of oil extraction and reserves will display similar features (i.e., the model behaves again like a canonical small-open-economy RBC model). One important prediction of this model is that, when oil prices are low, and therefore expected to rise due to mean-reversion, the planner has the incentive to cut extraction and increase reserves. To see this, use the definitions of the asset price of oil and oil dividends to rewrite the no-arbitrage condition $R_{t+1}^o = R^*$ as follows (assuming an internal solution for x_t for simplicity):

$$\frac{p_{t+1} - e_x\left(x_{t+1}, s_{t+1}\right) - e_s\left(x_{t+1}, s_{t+1}\right)}{p_t - e_x\left(x_t, s_t\right)} = R^*. \tag{12}$$

Since $e(\cdot)$ is increasing in x_t and decreasing in s_t, when p_t falls relative to p_{t+1}, the planner reallocates extraction from t to $t+1$ by increasing s_{t+1}.[25] This is a key incentive that is also a work in the model with default, but there it interacts with the planner's incentives to default and to affect the price of issuing new debt by adjusting reserves. As Propositions 3 and 4 show, the incentives to default at date t are stronger when p_t is low but, if the sovereign chooses not to default, the incentive to increase s_{t+1} in response to lower p_t *reduces* the default risk premium paid on bonds sold at t (i.e., increases the price of newly issued bonds) because default sets shrink with s.

4 Quantitative analysis

4.1 Baseline calibration

We calibrate the model using the panel of 30 largest oil producers in emerging economies described in Section 2. We calibrate first the Markov process of the shocks and then the model's parameters.

4.1.1 Exogenous shocks

To construct the y and p stochastic processes, we first estimate this standard VAR for each country:

$$\begin{bmatrix} p_t \\ y_t \end{bmatrix} = \begin{bmatrix} \rho_p & \rho_{yp} \\ \rho_{py} & \rho_y \end{bmatrix} \begin{bmatrix} p_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} \sigma_{\epsilon p} & \sigma_{\epsilon yp} \\ \sigma_{\epsilon py} & \sigma_{\epsilon y} \end{bmatrix} \begin{bmatrix} \epsilon_{pt} \\ \epsilon_{yt} \end{bmatrix},$$

where ϵ_{pt} and ϵ_{yt} are mean-zero, i.i.d. innovations. The data for p (real price of oil) and y (non-oil GDP) are as in Section 2. Since p turned out to be stationary but y did not, we take the logarithm of p and demean it, and for y, we log the data and extract the cyclical component using the Hodrick-Prescott filter. Hence, p and y are in percent deviations from mean and trend, respectively.

Next, we compute the weighted average of the VAR coefficients that are statistically significant across the 30 countries. As explained in Section 2, the weights are the normalized 1979-2014 average shares of each country's oil production in the total oil output of the countries in the panel. The VAR estimation produced statistically-significant coefficients mainly for the autocorrelation coefficients ρ_{pp} and ρ_{yy}, the former in all 30 countries and the latter in 21 countries. Their weighted averages are $\rho_{pp} = 0.901$ and $\rho_{yy} = 0.371$, respectively. In contrast, ρ_{py} is only significant in four countries and ρ_{yp} in two, and their weighted averages are 0.054 and 0.04, respectively. The covariance matrix is aggregated in the same way, by computing weighted averages of its element for all 30 countries.

Table 3 lists the aggregated VAR estimates. Oil-price shocks are more persistent than non-oil-GDP shocks. Because of this, the standard deviation of p is nearly twice as large as that of y (18.2 v. 9.2 percent), even though non-oil-GDP innovations have higher variance (0.007 v. 0.006). The interactions between the two shocks are weak, however, because ρ_{py}, ρ_{yp} and $\sigma_{p,y}$ are low.

[25] Condition (12) also embodies Hotelling's law: the growth of the oil price net of extraction costs equals R^*.

Table 3: VAR Process for Non-Oil Output and Oil Prices

Parameter	Description	Value
ρ_p	oil price auto-correlation	0.90
ρ_y	non-oil output auto-correlation	0.37
ρ_{py}	oil price non-oil output correlation	0.05
ρ_{yp}	non-oil output oil price correlation	0.04
σ_p^2	variance oil price innovations	0.006
σ_y^2	variance non-oil output innovations	0.007
σ_{py}, σ_{yp}	covariance non-oil output, oil price	-0.002

The model is solved using a standard value function iteration algorithm for sovereign default models over a discrete state space. For the VAR process of p and y, we construct a discrete approximation with the coefficients shown in Table 3 using the approach proposed by Tauchen (1986) with a spanning factor of 2.15. This was chosen so as to match the standard deviations of p and y. The realization vectors have seven and five values for p and y, respectively. For b and s, we use discrete grids with 61 and 70 nodes, covering the interval $[-0.6, 0]$ for b and $[16.66, 19.00]$ for s.[26]

4.1.2 Structural parameters

The model has nine parameters: β (discount factor), μ (coefficient of relative risk aversion), κ (oil discoveries), γ and ψ (curvature and scale parameters of extraction costs), r^* (risk-free rate), $\hat{p}$ (oil-price default penalty), λ (credit-market re-entry probability) and A (autonomous spending). Some of these parameters are set to standard values in the literature and others are targeted to match moments from the data.

We set $\mu = 2$, a standard value in the literature. The risk-free rate is set to $r^* = 0.00775$, which corresponds to the average ex-post, US-CPI deflated yield on 3-month U.S. Treasury bills for the 1955-2014 period (see Bianchi et al. (2016)). Total GDP is normalized so that $y + y^o = 1$.

Calibration of extraction costs parameters

In order to separate the calibration of the extraction costs parameters from the analysis of sovereign default, we calibrate the cost parameters to data for the countries in our panel that did not default in the sample period (non-defaulters), assuming that defaulters and non-defaulters operate the same technology and that risk-neutral foreign investors can invest in the oil sector of the latter group. This allows us to calibrate the cost parameters using the SOE-FI model mentioned in Section 3, which only requires solving the Euler equation $E_t\left[R^o_{t+1}\right] = R^*$.

The calibration of extraction costs proceeds as follows:

1. The law of motion of reserves evaluated at steady state yields $x = \kappa$.

[26] Appendix N shows that all our main results are robust to solving the model with larger, finer grids for s and p, and local non-monotonicities in the reserves decision rule are smoothed out.

2. As shown in Appendix F, the deterministic Euler equation for s' (eq. (F.19)) yields this steady-state condition:[27]

$$\left(\tilde{\psi}\frac{\kappa}{s}\right)^{\gamma}\left[\gamma\left(\frac{\kappa}{s}\right)+r^{*}(1+\gamma)\right]=r^{*}, \tag{13}$$

using the assumption that $1/\beta = R^*$ and normalizing the steady-state oil price to 1. Note that $\left(\frac{s}{\kappa}\right)$ defines the years of oil reserves remaining before they are exhausted and that at the steady state the share of Gross Oil Output (px) in GDP is $\frac{\kappa}{y+\kappa-\left(\tilde{\psi}\frac{\kappa}{s}\right)^{\gamma}\kappa}$.

3. We calibrate the extraction costs parameters to match these targets from the non-defaulters' data: (a) γ is set so that the variability of oil extraction in the stochastic SOE-FI solution matches the standard deviation of the cyclical component of x in the data (13.3%); (b) given γ, we impose on the above steady-state condition the expected years of reserves estimated from the data (70.07) and solve for $\tilde{\psi}$; and (c) imposing γ, $\tilde{\psi}$ and the data target of the share of Gross Oil Output in GDP (33.5%) on the definition of this share, we solve for κ.

To perform the calibration, we apply an iterative procedure that starts with a guess for γ and solves for the associated values of $\tilde{\psi}$ and κ as indicated in (b) and (c). Then, we solve the stochastic SOE-FI model and compute its predicted standard deviation of oil extraction, the mean of $(\frac{s}{\kappa})$, and the mean of the ratio of gross oil output to total GDP and iterate until these three model moments get as close as possible (up to a convergence criterion) to their data counterparts. With $\kappa = 0.335$, $\gamma = 3.1$ and $\tilde{\psi} = 32.77$, the standard deviation of x is 13.2% as percentage of the mean, the years of reserves $\frac{s}{\kappa} = 70.07$, and the ratio of gross oil output to GDP is 33%.The implied elasticity of extraction costs with respect to x is $1+\gamma = 4.1$, which is in the range of the estimates in the 2.2-6.8 interval obtained by Bornstein et al. (2023) using the same extraction costs function.[28]

Calibration of remaining parameters

Next we calibrate the remaining parameters. The annual probability of reentry is $\lambda = 0.332$, based on Richmond & Dias (2007) who found a median period of financial exclusion of three years after default across 128 defaults in the 1980-2005 period. The mean interest rate is $r = r^* + spd$, where spd is the weighted average of the sovereign spreads across all 30 countries over the period 1979-2014 and is equal to 707 bpts.[29] Since we normalized GDP to 1, the mean of non-oil GDP is $E[y] = 1 - y^o$, which using $y^o = 0.2069$ (the weighted average of the ratio of oil rents to GDP across all 30 countries) yields $E[y] = 0.793$. Similarly, to calibrate A, we set the steady-state shares of debt and consumption to the 1979-2014 weighted means of the GDP ratios of external debt and private

[27] For simplicity, we use here an equivalent formulation of the $e(\cdot)$ function with the scale parameter expressed as a proportion $\tilde{\psi}$ of the ratio of extraction to reserves, instead of a proportion of the total cost (i.e., $\psi \equiv \tilde{\psi}^{\gamma}$).

[28] Since on- and off-shore extraction technologies differ, if defaulters and non-defaulters differed in this dimension it would be difficult to assume that they operate the same technology. As noted earlier, however, the majority of extraction is on-shore in our dataset (about 70%) and defaulters and non-defaulters have similar off-shore shares (25%-28%). Still, calibrating to defaulters instead, the results are similar with a slightly higher elasticity of 5.75.

[29] This calculation uses JP Morgan's EMBI+GSS spreads data for the period 1998-2016 and then extrapolates the spreads by regressing the EMBI data on the III in the common sample (1998-2016) and using this regression and observed pre-1998 III values to estimate EMBI spreads for 1979-1997 (see Appendix B for details).

plus public consumption also for all countries ($b = -22.45\%$ and $c = 58.88\%$) and then solve from the deterministic steady-state resource constraint $A = 1 + (r/R)b - c = 0.3936$.

The remaining two parameter values (β and $\hat{p}$) are jointly determined so that the stochastic baseline model solution matches the weighted averages of the debt-GDP ratio (22.45%) and the default rate (1.14%) in the complete dataset.[30] This is done using an iterative procedure similar to the one used for the extraction cost parameters: Start with a guess for $(\beta, \hat{p})$, then solve the model and simulate it to compute the model's mean debt-GDP ratio and default rate, and iterate until the model and data moments differ by a convergence criterion. This procedure yields $\beta = 0.831$ and $\hat{p} = 0.643$. The fully calibrated model then yields a mean debt ratio of 22.5% and default rate of 1.19%, both very close to the data. Note, however, that since oil GDP is endogenous and κ was calibrated separately, the model yields $E[GDP] = 1.06$, a little above its normalized steady state and $E[y^o]/E[GDP] = 0.26$, close to the value in the data (0.245).

Table 4 lists the baseline calibration parameters and Table 5 compares key data moments with their baseline model counterparts and variations with constant extraction and without default risk (to be discussed later). The data moments are cyclical components as described in Section 2. The model-generated data are not detrended, since the model is stationary by construction, and thus we measure dispersion using coefficients of variation (standard deviations in percent of means) so that dispersion measures from model and data are both in percent of long-run values.

The baseline model nearly matches the mean debt-GDP ratio and default rate in the data because the data moments were calibration targets. In contrast, the variability of gross oil output and mean reserves were not calibration targets, and the model is close to matching the former but underestimates the latter (the calibration of extractions costs targeted these moments but only for non-defaulters and using the SOE-FI model in which there is no default risk, and also we targeted the standard deviation of x not of gross oil output px). On the other hand, the variability of extraction in the model is 6.3%, about half the data moment (12.3%). Hence, although the SOE-FI model matches the variability of x because of the procedure followed to calibrate extractions costs, using the resulting parameters in the model with default yields a much less volatile extraction process.

The low calibrated discount factor (0.83) is common in sovereign default models as it helps the models to sustain higher debt levels by incentivizing the sovereign to borrow. In our model, however, this incentive (and the lack of commitment that weakens borrowing capacity) also induces the sovereign to maintain lower oil reserves on average in the long run.

[30] In the weighted sum that yields this average default rate, non-defaulters enter with a zero default rate and their corresponding weight in total oil production. Including only defaulters, the default rate would be 2.2%.

Table 4: Parameter Values in the Baseline Calibration

Parameter	Description	Value
β	discount factor	0.83
μ	risk aversion coefficient	2.00
q^*	risk-free debt price	0.99
$\hat{p}$	oil-price default cost threshold	0.64
k	discovery rate	0.33
λ	re-entry probability	0.33
γ	extraction costs curvature	3.1
ψ	extraction costs scale	32.77
A	autonomous spending	0.39

Table 5: Data and Model Moments

Description	Data	Model		
		Baseline BSL	Constant Extraction CE	Risk Free RF
Average External Debt to GDP	0.225	0.225	0.244	0.568
Default Rate	1.14%	1.19%	0.82%	0%
Variability of Gross Oil Output[1]	23.99%	24.73%	19.21%	23.57%
Average Reserves (in years)[2]	62	53	53	53

[1] The variability of gross oil output is the standard deviation of the cyclical component of px in the data and the corresponding coefficient of variation in the model.
[2] Oil reserves data are proven reserves from the US Energy Information Administration.

4.2 Default-repayment sets & default costs

We start the quantitative analysis by examining two aspects that highlight some of the analytic findings from Section 3 and the planner's strategic use of oil reserves. First, we examine the default and repayment sets and then we study default costs and the role that oil extraction plays in them.

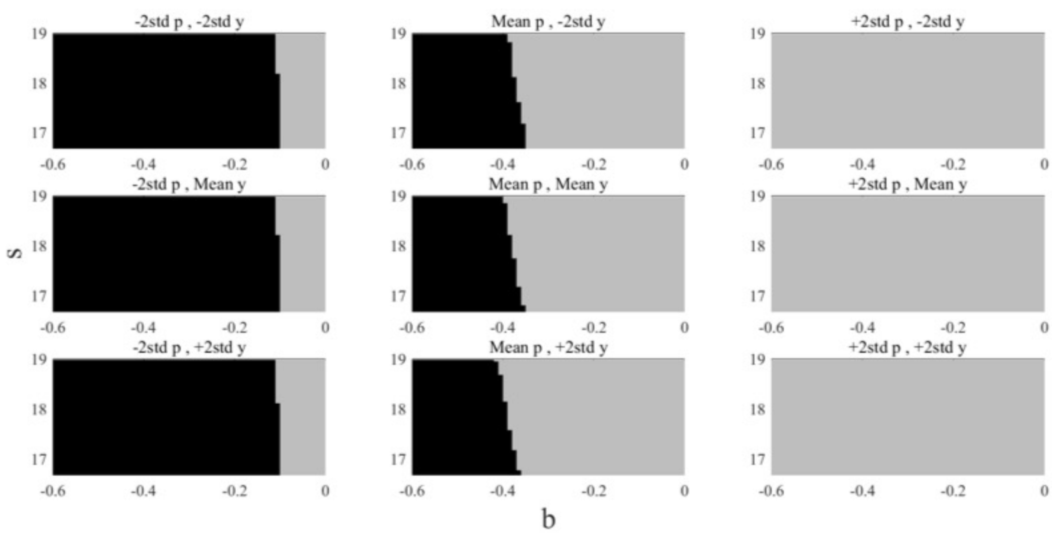
4.2.1 Default and repayment sets

Figure 3 shows nine plots displaying the default and repayment sets across all values of (b, s) in the state space for nine (p, y) pairs that correspond to the means and +/-2 standard deviations of their means. In each plot, s (b) is shown in the vertical (horizontal) axis and the area in black (grey) is the default (repayment) set.

These plots are consistent with the results reported earlier indicating that Proposition 4 (default incentives strengthen as p falls) holds in the calibrated model even though the assumptions used to prove it do not hold, since shocks are not i.i.d, the probability of re-entry is positive and there are default costs in terms of oil prices. Specifically, for a given (b, s, y), if a higher p is in the default set, a lower p is also in the default set. This is evident in the plots. For the three levels of y shown in each row, as we move from the highest to the lowest p, namely from right to left, the repayment sets (in grey) shrink. At the highest p (and at the second-highest, not shown), the default sets are empty, so the sovereign never defaults, but at the lowest p there are still some repayment outcomes (in less than 1/4 of the state space of (b, s) for all y values). Thus, at very high p the sovereign always repays, but at very low p it *does not* always default– it still repays if (b, s) are not "too low."

As noted earlier, the standard result from EG models that default incentives strengthen as y falls also holds in our model: if for a given (b, s, p), a higher y is in the default set, a lower y is also in the default set. This is harder to see in the plots. Careful examination shows that it does hold, but the default sets are nearly invariant in y. For each value of p in the columns, the default and repayment sets are about the same as we increase y, namely from top to bottom. This result is an implication of the fact that the default income cost is modeled via oil prices. Default sets respond more to lower p because the default costs reduce oil revenues by cutting the price to $\hat{p}$ regardless of how high the realized p is. Allowing default costs to affect y too would alter this result.

Figure 3: Default Sets



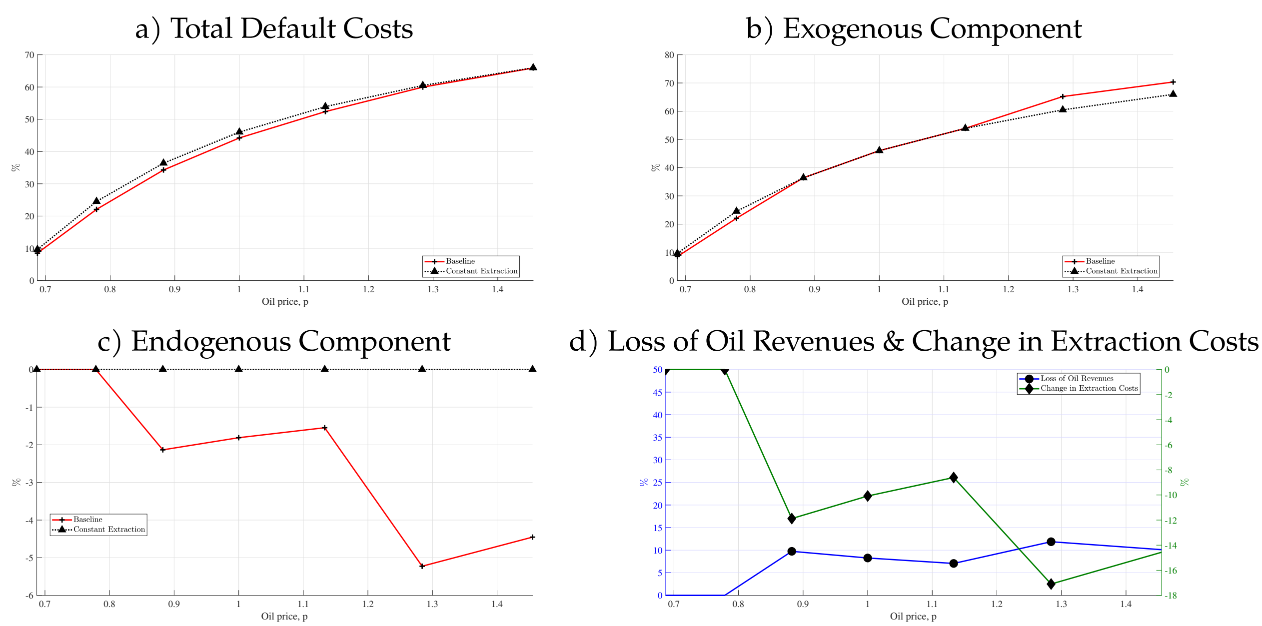
4.2.2 Default income costs

Next, we examine default income costs. As explained before, these costs are not completely exogenous in this model (as they are in standard EG models) due to the endogenous response of x to the default price penalty $\hat{p}$, which is akin to a progressive tariff on oil exports.

We measure default income costs in percent of oil GDP under repayment. Oil GDP under repayment and default are $y^{O,nd}(b, s, y, p) = px^{nd}(b, s, y, p) - e(x^{nd}(b, s, y, p), s)$ and $y^{O,d}(s, y, \hat{p}) = \hat{p}x^{d}(s, y, \hat{p}) - e(x^{d}(s, y, \hat{p}), s)$, respectively. Hence, the total cost of default is $-[y^{O,d}(\cdot)/y^{O,nd}(\cdot) - 1]$. As noted in Section 3, the lower export revenues under default, $\hat{p}x^{d}(s, y, \hat{p})$, can be viewed as a trade penalty imposed when the sovereign defaults. The endogeneity of the default cost follows from the fact that $x^{d}(s, y, \hat{p})$ is affected by $\hat{p}$.

Plot a) of Figure 4 shows default costs as p varies in the baseline model (red-solid line) and in the variant of the model with constant extraction (dotted-black line). Plots b) and c) decompose total costs into exogenous and endogenous components, respectively. The exogenous component captures the loss of oil revenues due to the price penalty keeping extraction as under repayment, $-(\hat{p} - p)x^{nd}(\cdot)/y^{O,nd}(\cdot)$. The endogenous component captures the response in oil profits as extraction and extraction costs change, $-\left[\hat{p}\left(x^{d}(\cdot) - x^{nd}(\cdot)\right) - \left(e(x^{d}(\cdot), s) - e(x^{nd}(\cdot), s)\right)\right]/y^{O,nd}(\cdot))$. Plot d) breaks down the two terms of the endogenous component: the loss in oil revenues if extraction falls under default and the change in extraction costs due to that fall in extraction. The sum of the two curves in Plot d) yields Plot c), and the sum of the curves in Plots b) and c) yields Plot a). In all cases, we evaluate extraction and extraction costs with (b, s, y) set at their long-run averages in the baseline model (and at the same p values) so that the state of nature is the same.

Figure 4: Default Costs and Components
(in percent of Oil GDP under repayment)



Plot a) shows two important results. First, default costs are increasing in p, ranging from 10% to 70%. Hence, default is costlier in better states of nature, as Arellano (2008) showed was needed for EG models to sustain more debt and induce default in bad times. The costs are large because they are in percent of oil GDP. In percent of *total* GDP, they range between 1.42% and 23%, in line with those obtained by Arellano, ranging between 0 and 30% of GDP. Second, default costs are *lower* in the baseline model than with constant extraction. Hence, the sovereign takes advantage of the possibility of strategizing over oil extraction and reserves so as to make default less costly. The gap narrows as p rises, and at the highest p the difference is negligible.

Plot b) shows that the exogenous components display a similar pattern as the total costs. Jointly with plots a) and c), this plot yields the finding that, at the lowest p values, the lower default cost in the baseline model than under constant extraction is only due to the exogenous component. The smaller loss in oil revenues as p falls to $\hat{p}$ is because the sovereign in the baseline chooses lower x under *repayment*. At very low prices, p is expected to rise and hence, under repayment, the planner cuts extraction and builds reserves for future sale at better prices. This is important because, as shown later, all the defaults along the model's equilibrium path occur when p is very low.

At mid-levels of p, the exogenous components are the same in the baseline and with constant extraction. At high p, however, we see the opposite of what we saw at low p. Now the planner in the baseline model expects future prices to fall and increases extraction under repayment to take advantage of the higher current price. But the endogenous component is not zero as it was with low p, and as we show below, oil profits rise to nearly offset the larger exogenous cost.

Plot c) shows that, except at the lowest values of p, the endogenous component is negative (i.e., it *reduces* default costs), and it tends to be more negative at higher p, albeit it does not change monotonically.[31] Oil profits under default are higher than under repayment by up to 5.22% of oil GDP. Plot d) shows that profits are higher under default because, although extraction is lower, the loss of oil revenues due to lower extraction is more than offset by lower extraction costs (which fall with extraction). At the second-highest p, for instance, the revenue loss due to lower extraction is about 12% of oil GDP but extraction costs fall by 17%, resulting in the higher profits of about 5% shown in Plot c). These higher profits under default are about as large as the loss in oil revenues due to higher extraction under repayment (see Plot b)), so that the total default cost is about the same as with constant extraction (see Plot a)).

Jointly, the four plots illustrate the mechanism by which the sovereign chooses extraction to make default less painful. This is particularly the case when oil prices are not too high. As noted above, for the lowest p values, the mechanism is driven solely by the lower exogenous cost due to reduced extraction under repayment. Closer to the mean of p, however, the mechanism is driven by higher oil profits under default, which are attained by reducing extraction so that, upon a default, extraction costs fall by more than the fall in revenue due to a smaller volume of oil exports. The resulting higher reserves are also important, because they prop up the value of default by sustaining consumption while the economy remains in financial autarky. While defaults occur only at the lowest p in the equilibrium path, these responses at higher p levels affect the sovereign's strategic incentives and the expectations of lenders, and thus affect spreads and borrowing capacity.

4.3 Comparing model and data

We compare the model-generated data with the actual data in two key dimensions: business cycle moments and the oil reserves-sovereign risk co-movements.

[31] This non-monotonocity, however, is due to the coarseness of the grid of s. The cost grows monotonically using a finer grid (see Appendix N).

4.3.1 Business Cycle Moments

Columns (1)-(2), (5)-(6), (9)-(10), and (13)-(14) of Table 6 compare variability ratios (relative to the variability of p), income correlations, oil-price correlations, and auto-correlations from the data with those produced by the baseline model. The Table includes other columns showing moments from variants of the model with default risk but constant oil extraction (to examine the relevance of endogenizing oil production in the model with default) and with a sovereign committed to repay at the risk-free rate (to assess the implications of endogenizing default risk). These results are discussed later in this Section. Below each of the data moments, which are weighted averages of country-specific moments, we show in parenthesis the coefficient of variation of each moment across the 30 countries in the panel, so we can tell which ones have higher or lower dispersion.

We show results for both disposable income (DI) and GDP because the autonomous spending A mechanically reduces the models' income variability measured as the coefficient of variation of GDP.[32] For the data, we constructed the comparable DI measure and detrended it with the HP filter, as we did with the other macro time-series.

Columns (1) and (2) show that the model does well at approximating the variability ratio of gross oil output, and reasonably well at approximating those of GDP, disposable income, and the trade balance-GDP ratio, but it overestimates the variability ratios of consumption and spreads and underestimates the ones for reserves and extraction.[33] For consumption, in particular, the model's variability ratio is 0.88 v. 0.27 in the data. We discuss further this result later in this Section.

In terms of income correlations, the model approximates some of them well, except it overestimates those for consumption and GDP (GDP and DI are perfectly correlated in the model because $GDP = A + DI$ and A is a constant). Importantly, the income correlations of x and px are close to those in the data and spreads are negatively correlated with income, as in the data. The model's correlation is more negative than in the data (-0.29 v. -0.09), but in the data the correlation differs significantly across countries, with a coefficient of variation of 163%. The weakest result in these income correlations is that the correlation of DI with consumption is nearly three times higher in the model than in the data (0.91 v. 0.34).

Consider next oil-price correlations. Importantly, the model does well at approximating Fact 2 from Section 2: Country risk (i.e., spreads) and oil prices are negatively correlated, although the model correlation is somewhat higher.[34] The model is also in line with the data in predicting that the trade balance is nearly uncorrelated with oil prices. The correlation is slightly positive (negative) in the data (model), but in the data it differs sharply across countries, spanning the interval from -0.75 to 0.78 which includes the model's correlation of -0.17. The rest of the model's oil-price correlations have the same sign as in the data (albeit the model's correlations are markedly higher), with the exception of reserves. However, the cross-country dispersion in the correlation

[32] Since $GDP = DI + A$ and A is a constant, GDP and DI have the same standard deviation, but the coefficient of variation of GDP is smaller because its mean is larger.

[33] Spreads in the data correspond to EMBI spreads, extended back to 1970 using the III as explained in Appendix B.2.

[34] Regarding Fact 1, the model's calibration uses the mean debt ratio and default frequency as targets.

Table 6: Business Cycle moments - Data v. Model

	Variability relative to Oil Price				Correlation with DI				Correlation with Oil Price				Autocorrelation			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
	Data	BSL	CE	RF	Data	BSL	CE	RF	Data	BSL	CE	RF	Data	BSL	CE	RF
Gross Oil Output	1.32	1.36	1.05	1.30	0.51	0.72	0.72	0.70	0.34	0.96	0.97	0.99	0.29	0.86	0.88	0.87
	(0.30)				(0.45)				(0.22)				(0.51)			
Total GDP	0.38	0.55	0.52	0.54	0.62	1.00	1.00	1.00	0.11	0.71	0.72	0.71	0.52	0.66	0.65	0.65
	(0.62)				(0.52)				(0.90)				(0.39)			
Disposable Income (DI)	0.56	0.87	0.86	0.85	1.00	1.00	1.00	1.00	0.12	0.71	0.71	0.71	0.42	0.66	0.65	0.65
	(0.59)				(0.00)				(0.84)				(0.41)			
Extraction	0.67	0.35	*na*	0.34	0.52	0.58	*na*	0.55	0.04	0.80	*na*	0.82	0.50	0.74	1	0.73
	(0.86)				(0.55)				(2.63)				(0.42)			
Consumption	0.27	0.94	0.93	0.84	0.34	0.91	0.91	0.84	0.12	0.74	0.73	0.73	0.52	0.60	0.59	0.68
	(0.65)				(0.94)				(0.80)				(0.27)			
Trade Balance/GDP	0.49	0.25	0.25	0.04	0.39	0.03	0.02	0.04	0.19	−0.18	−0.19	−0.05	0.63	0.11	0.11	−0.28
	(0.69)				(0.64)				(1.49)				(0.30)			
Reserves	1.78	0.12	*na*	0.11	0.04	−0.20	*na*	−0.20	0.13	−0.31	*na*	−0.32	0.83	0.99	1	0.99
	(0.68)				(3.10)				(3.09)				(0.11)			
Spread	3.53	15.21	14.37	*na*	−0.09	−0.29	−0.22	*na*	−0.46	−0.15	−0.09	*na*	0.66	0.28	0.26	*na*
	(0.54)				(1.63)				(0.75)				(0.27)			

* Data moments are computed using cyclical components of data for the 1979-2014 period, logged and HP-detrended, except for Trade Balance/GDP and Spread which are untransformed. Variability ratios are computed using the standard deviation in the former, and coefficient of variation for the latter measured in basis points. BSL- Baseline, CE- Constant Extraction, RF- Risk Free model. Model moments are from simulated data without detrending, because the models are stationary by construction. Model variability ratios are ratios of coefficients of variation divided by the standard deviation of oil prices (which is the same as its coefficient of variation since $E[p] = 1$).

between reserves and oil prices is very high, with values ranging between -0.60 and 0.74 (which includes the model's correlation of -0.30).

The model's weakly negative correlation of the trade balance with oil prices reflects opposing forces affecting the sovereign's borrowing choice. As shown in Section 3, keeping everything else constant (particularly b, s and y), sovereign risk falls as p rises because the probability of default goes down. As a result, the sovereign has more borrowing capacity and can finance a larger trade deficit. This mechanism pushes for a negative correlation between oil prices and net exports and is active in states of nature at date t in which default is possible at some realizations of p, y at $t+1$. In contrast, when the probability of default is zero and debt is low enough and oil reserves high enough so that the economy is far from the default set, the model behaves as an RBC model, as explained earlier. In this case, higher p incentivizes NFA accumulation (a reduction in debt) so as to save some of the higher income of date t for future consumption. This mechanism pushes for a positive correlation between oil prices and net exports, although the effect is relatively weak because of the low β. Quantitatively, the two forces nearly offset each other to produce a trade balance-oil price correlation of only -0.17. A similar argument explains why the oil-price correlation with the spread is negative but not -1, in the sense that the effect of lower oil prices increasing country risk at date t operates also when default is possible in some states at $t+1$.

In terms of autocorrelations, the baseline model matches the data in that all the autocorrelation coefficients are positive. The magnitudes of those for total GDP, consumption, and reserves are also similar in the model and the data, but those for the rest of the variables show sizable differences.

As noted earlier, an important shortcoming of the model is the high consumption volatility. Comparing the baseline (BSL) model with the variants with constant extraction (CE) and risk-free debt (RF) sheds some light on its cause. While consumption in the three models is much more volatile than in the data, the variability of consumption relative to that of DI is higher in the two models where default is possible at 1.08 in both, compared with 0.99 in the RF model (i.e., consumption is less variable than DI in the RF case but more variable than DI in the other two). This is because in the RF model the sovereign uses both debt and oil reserves to smooth consumption without being hampered by the lack of commitment to repay debt, although consumption variability is still high because of the presence of an ad-hoc debt limit.[35] This limit hampers consumption smoothing but less so than the lack of commitment in other models.

It is important to note that the planner uses oil reserves to smooth consumption in the BSL model without ever hitting the constraints at which there is either zero extraction in a given period or full extraction of the period's existing reserves and new discoveries. The planner adjusts s' taking into account the interaction between debt, debt prices, and reserves when aiming for the consumption path that maximizes private utility. In particular, the lack of commitment limits

[35] The RF model is solved with identical parameters and an ad-hoc debt limit (the lower bound of the b grid) set to the largest debt in the ergodic distribution of the BSL model. Since $\beta R^* = 0.806$ is well below 1, the RF model hits this debt limit often (88% of the time). Hence, the RF model is best viewed as a risk-free model with an exogenous borrowing constraint, whereas the BSL model has an endogenous borrowing constraint driven by the lack of commitment.

borrowing capacity and adds strategic incentives to the choice of reserves, but does not lead the planner to reduce s' to its lowest feasible level. Similarly, in the RF model, even when the sovereign hits the ad-hoc debt limit, it still does not choose to hit the lower bound of s'.

The CE model, in which reserves cannot be used to smooth consumption but default risk remains, is akin to a standard EG model, and its high consumption variability is in line with a common feature of EG models identified by Chatterjee & Eyigungor (2012): When debt is large relative to output, a change in the bond price implies large changes in consumption given that the sovereign must refinance all of the debt at the new price in one period. Because of the ladder-like shape of the equilibrium price of bonds, changes in bond prices can be large. This argument does not necessarily extend to our model, because oil reserves provide an alternative for consumption smoothing, but to the extent that extraction costs hamper consumption smoothing with reserves, the mechanism causing high consumption variability in standard EG models is still at work.

4.3.2 Co-movement of Oil Reserves & Sovereign Risk

Fact 3 of Section 2 showed that the conditional relationship between oil reserves and sovereign risk changes direction from negligibly negative on impact to positive and significant as the variables converge to their trends. Since the model is stationary, however, it does not have predictions comparable to those of the long-run (co-integration) coefficients of the dynamic panel regressions. Still, we can examine whether the model can explain the changing direction of the co-movement between reserves and country risk over time. To this end, we use model-simulated data to construct local-projection impulse responses of (q, b', s', x) to a negative shock to p. [36]

The equations that we estimate to compute the impulse response functions are the following:

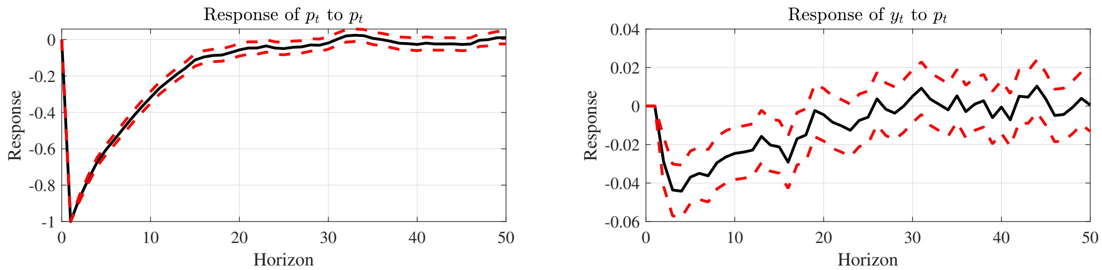
$$m_{t+h} = \beta_0 + \beta_1 s_t + \beta_2 b_t + \beta_3^h p_t + \beta_4 y_t + \beta_5 history_{t+h} + \beta_6 transition_{t+h} + \epsilon_{t+h},$$

where $m_t = \{b_{t+1}, s_{t+1}, q_t, x_t\}$, and the impulse responses are built from β_3^h in $h = 0, ..., T$, with $T = 50$. The sequence of regression coefficients directly maps out the impulse responses. We control for both endogenous and exogenous state variables in the current period and include also two dummies contemporaneous to the dependent variables that capture non-linearities associated with the run-ups to defaults and with the defaults themselves. First, $history_t$ controls for default and exclusion: $history_t = 1$ when either the sovereign defaults or remains in exclusion from financial markets, and is zero otherwise, which are periods where $q_t = b_t = b_{t+1} = 0$ in the model solution. Second, $transition_t$ controls for the fact that during normal times q_t fluctuates slightly around the risk-free price of 0.9923 but as a default approaches it falls rapidly to an interval between 0.8 y 0.9. Hence, $transition_t = 1$ when q_t is in the 0.8-0.9 interval and is zero otherwise.

[36] As noted by Jordà (2005), impulse response functions from local projections have advantages over those from standard VARs for multivariate, nonlinear specifications. Appendix J.2 shows that impulse responses from a standard VAR are quite similar. Importantly, the changing dynamic pattern of co-movement in q and s reported here still holds.

Figure 5: Local-Projection Impulse Responses to Negative Oil Price Shock in the Baseline Model

a) Response of exogenous variables



b) Response of endogenous variables

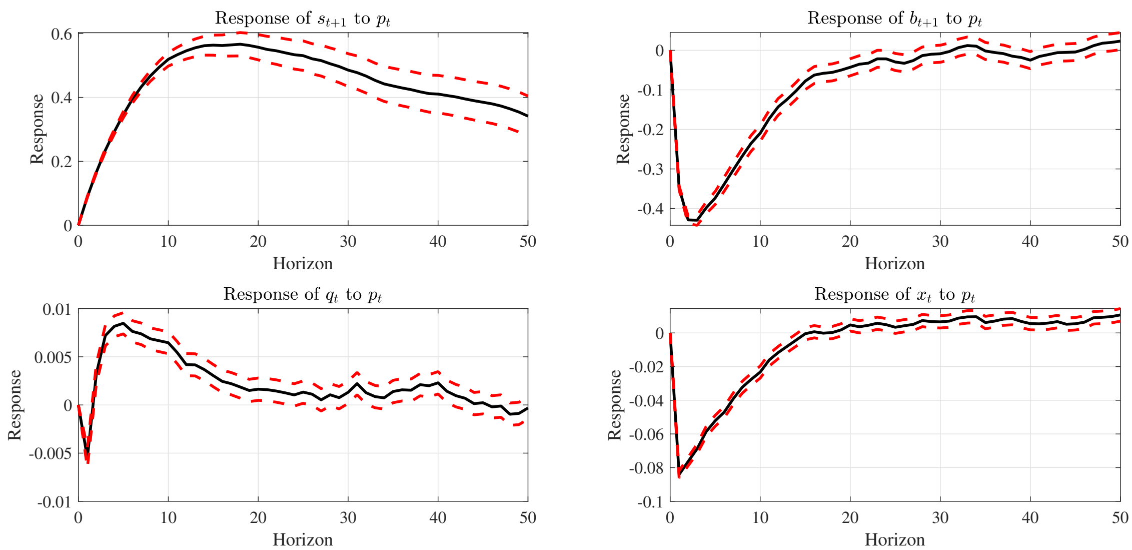


Figure 5 presents plots of the impulse response functions to a negative oil-price shock (including confidence bands at $+/-1.96$ times the standard error of β_3^h). Reserves and bond prices move together (both rising) between periods one and five, but then from $t = 5$ to $t = 19$, reserves rise as bond prices fall. After that, reserves and bond prices again move together. Hence, the model is qualitatively in line with Fact 3 regarding the changing direction of the co-movement between q and s, from a negligibly negative effect of reserves on country risk (positive on q) in the short-run to a positive effect (negative on q) over a longer horizon.

The oil-price shock induces a temporary drop in p that takes about 30 periods to wash out. Because of its covariance structure with the y process, there is also a small, temporary decline in non-oil GDP. In response to these shocks, the sovereign increases reserves (reduces extraction) up to $t = 19$, and after that reserves fall slowly. The sovereign also borrows less initially (b rises), but after the third period debt starts to rise as it reverts to its mean. The initially lower debt and higher reserves cause the early rise in q (after its drop on impact), but then bond prices fall as the mean-reversion of reserves and bonds takes over. The time interval in which reserves are still rising while bond prices are already falling is consistent with the fact that debt starts rising sooner and faster than the reversal in reserves. This accounts for the changing direction in the co-movement between country risk and reserves as time passes (see Section 2).

The response of x is also in line with the empirical evidence from Model (5) in Table 2 showing that the price elasticity of oil production in the data is positive (and significant) in the short run, and negative in the long run. In addition, the model also approximates well the low empirical elasticity estimates of oil production with respect to oil prices. Using the model-simulated data, regressions of x on p, both conditional and unconditional, yield elasticities in the 0.06-0.09 range–recall our empirical elasticity estimate from Table 2 was 0.11 and the estimates from Caldara et al. (2019) are in the 0.02-0.13 range. In the model, these low elasticities are compatible with the high correlation between x and p in Table 6 because the variability of x is much lower than that of p (see Appendix J.3 for details).

4.4 The role of the oil sector

We close the quantitative analysis with an assessment of the role of the oil sector in the model. We first compare the results from the BSL, CE and RF models, and then we introduce large but rare and uncertain oil discoveries, which are an important feature of the oil industry.

4.4.1 Baseline, constant-extraction and risk-free models

The BSL-CE comparison illustrates the role of the optimal extraction choice in the sovereign's choices of debt and default. In the BSL model, the sovereign can prop up consumption when GDP is low by increasing debt, defaulting and/or reducing oil reserves (i.e., exporting oil), and it can do the latter even under financial autarky. In the CE case, oil output cannot be adjusted to assist in consumption smoothing or respond to default incentives. The BSL-RF comparison shows how much default risk matters for extraction and reserves decisions by studying how the choices made by a sovereign committed to repay differ. As mentioned earlier, the RF model is akin to an RBC model (albeit one where βR^* is well below 1 and an ad-hoc debt limit binds) and thus the government can sustain more debt, whereas in the BSL model borrowing capacity is hampered by the lack of commitment and the choice of reserves is affected by strategic incentives.

Table 5 compares the moments used to evaluate the baseline calibration and Table 6 compares the rest of the business cycle moments. Comparing first the BSL and RF models, Table 5 shows that the mean debt ratio rises from 22.5% to 56.8%. Hence, full commitment to repay enhances borrowing capacity by about 34 percentage points of GDP. On the other hand, the debt ratio and net exports fluctuate very little, because again borrowing incentives with $\beta = 0.83$ and $R^* = 1.00775$ are very strong and make the RF model hit the ad-hoc debt limit (-0.60) often. This should incentivize the planner to substitute debt for reserves as a vehicle for consumption smoothing and build-up precautionary reserves for self-insurance. The variability of consumption relative to that of disposable incomes does fall in the RF vis-a-vis the BSL model, as noted earlier, but mean reserves are the same. This occurs because oil is not risk-free and oil reserves are a poor hedge against oil-price shocks since ps falls with p. Thus, the rate of return on oil changes as x, s' and $e(x, s')$ respond to the shocks in (p, y) . While the RF model is similar to a small-open-economy RBC model in theory, the low β that makes the ad-hoc debt limit bind frequently makes it more similar to a *closed-economy* RBC model with oil reserves taking the place of the capital stock. The ability to smooth with reserves is hampered by their endogenous return (effectively, the real interest rate of this economy is endogenous, see Section 3 and Appendix F).

Compare now the BSL and CE models. The mean debt ratio is about 2 percentage points higher in the latter, indicating that removing the ability to use reserves to support consumption during periods of exclusion (i.e. lacking the strategic incentives on reserves) enhances the sovereign's borrowing capacity, and it also reduces slightly the frequency of defaults (about 37 basis points). Moreover, as shown earlier, the ability to strategize over both reserves and debt makes default less costly in the BSL model, and this reduces the debt that the sovereign can sustain.

As noted before, the variability of consumption relative to that of DI is smaller in the RF model than in the other two models, although it is just below DI volatility because the debt limit binds often and the extraction costs hamper the ability to smooth consumption with oil reserves. For the same reason, debt and the trade balance fluctuate much less in the RF model than in the other two, and the trade balance is less negatively correlated with oil prices. Several of the other moments are similar between the BSL and RF models, particularly those for gross oil output and total GDP, suggesting that, even though Fisherian separation of the consumption/borrowing decisions from the choices of reserves does not hold strictly in both models (in the former because of default risk and in the latter because of the binding debt limit), the resulting distortions on the Euler equation of oil reserves do not result in significant differences in the DI and oil-price correlations of gross oil output and total GDP in the long-run. The same is true for the CE model, which suggests that Fisherian separation also holds approximately in terms of the long-run cyclical moments of total GDP and disposable income when default risk is introduced vis-à-vis once it is removed.

The similarity of some of the long-run cyclical moments across models does not imply that endogenizing default risk and oil extraction is irrelevant. Endogenizing extraction does lower the mean debt ratio and the average spread, and higher oil reserves increases default risk. More importantly, the three models yield very different dynamics around default events, as we show next.

We compare dynamics across models when defaults occur in the BSL model. We simulate the model for 10,000 periods, identify default events and construct 19-period windows centered on them, computing for each period the cross-sectional average of the 10,000 observations across all default events. There are 107 defaults–a frequency of 1.1 percent, in line with the model's calibration. Then, we construct comparable time paths for the CE and RF models as follows: First, for each of the baseline model's 107 defaults, we extract the values of (b, s) in the 9th period before the default and the 19 realizations of (p, y) from the 9th period before to the 9th period after the default. Then we feed them into the decision rules of the CE and RF models to construct 107 comparable 19-period time paths for each model, and calculate the averages for each of the 19 periods across the 107 values. Finally, we subtract the RF or CE averages from the BSL model's averages and plot them in Figure 6. To keep the Figure readable, we show 13-period windows, instead of the 19 periods used to generate the data. The red-dashed and dotted-blue lines show the results for the CE and RF models, respectively. For all variables, except those that are GDP ratios or already in percent (i.e., debt, trade balance and interest rate), we plot the difference in percent of the mean in the BSL model, so that the base of the percentages is common for the CE and RF cases. For p and y there is only one time-path, because the same sequences of shocks are used in the three models.

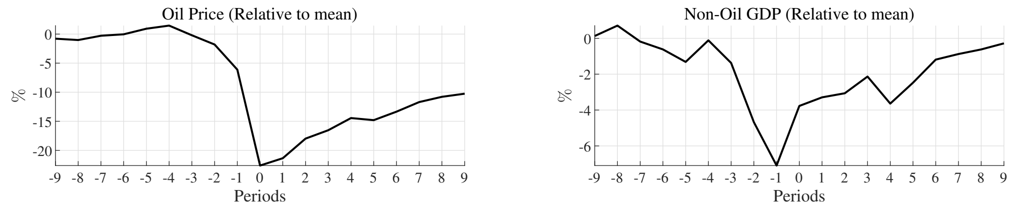
The first result to note in Figure 6 is that defaults in the baseline model occur with a large drop in p, following a gradual decline that starts at $t = -3$, and a large drop in non-oil GDP one period earlier, also following a gradual decline. When defaults occur $(t = 0)$, p hits its lowest realization in the Markov process of (p, y) and y hits a trough roughly 7 percent below its long-run average a period earlier. Post-default, p and y follow mean-reverting, increasing paths, but both are still below their long-run averages. It is also worth noting that at $t = 0$, for the 107 events in which the sovereign in the BSL model defaults, the sovereign in the CE model still repays in 22.6% of the cases. Thus, there is already a meaningful difference across the CE and BSL models in that, faced with the same initial (b, s) pair and identical sequences of (p, y) shocks, the CE sovereign does not default in some instances in which the BSL sovereign does. This is in line with the earlier observation that defaults are less frequent in the long-run (by 37 basis points) when the sovereign cannot strategize over both debt and reserves (see Table 5).

The Figure also shows that the CE and BSL models display very different dynamics. At $t = 0$, when defaults occur in the BSL model, oil extraction and gross oil output (px) are 11% and 7% smaller in the BSL model than the CE model, respectively, and the BSL model builds up an extra 7 months of oil reserves. Since defaults occur when p is very low and thus expected future prices are higher, the sovereign in the BSL model cuts oil extraction and output and increases reserves. Oil GDP (i.e., oil profits) and total GDP, however, are *higher* than in the CE case, because cutting extraction reduces extraction costs by more than the decline in gross oil output. Note also that the cut in extraction and the reserves buildup are beyond what is just the response to expected higher prices, they also reflect the strategic incentives to build reserves to finance future consumption while the economy remains in financial autarky, since the BSL sovereign does default at $t = 0$.

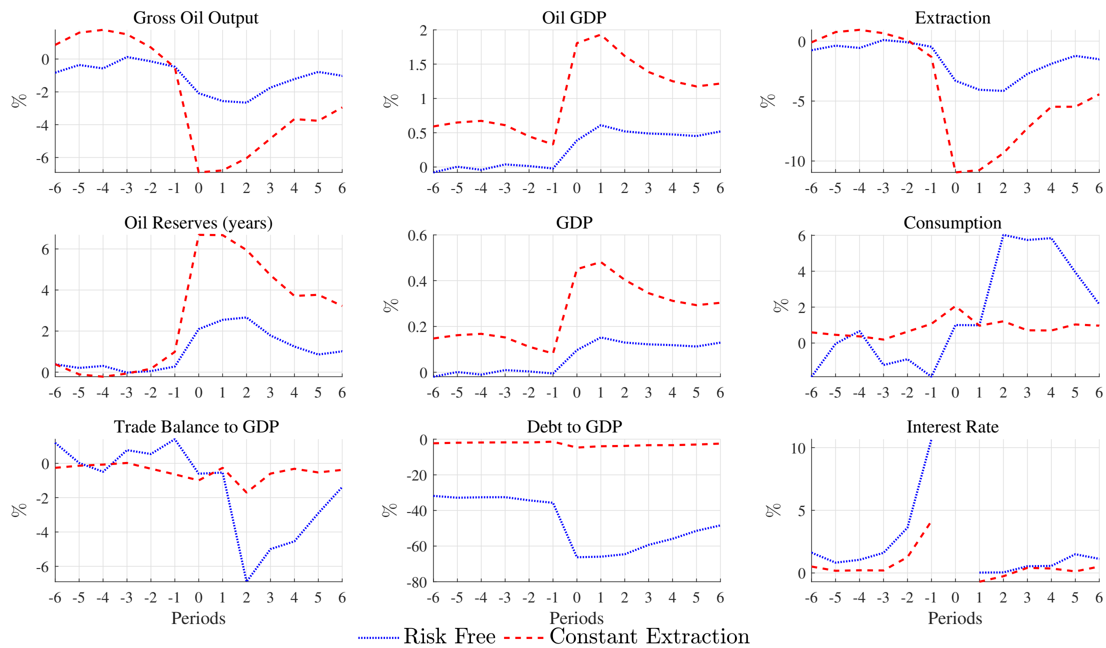
After the default, extraction remains lower and reserves higher in the BSL than the CE model, but the differences narrow to about 5% in extraction and 4 years in reserves. Similarly, oil GDP and total GDP remain higher in the BSL case, but the differences narrow to roughly 1.5% and 0.3%, respectively. These dynamics reflect the sovereign's management of reserves to sustain consumption during the exclusion period. Accordingly, the trade balance is 1 to 2 percentage points of GDP smaller in the BSL than the CE case when default occurs and in the early periods afterwards, and consumption is 1% to 2% higher. Moreover, prior to the default, spreads widen about 400 basis point more in the BSL model, because again strategic incentives are stronger than in the CE case.

Figure 6: Default Event Windows: Baseline, Constant-Extraction and Risk-free Models

a) Exogenous sequences of p and y that trigger defaults at date 0



b) Default dynamics of endogenous variables relative to BSL model



Note: All variables are plotted relative to the BSL model, except for p and y which are plotted relative to their long-run means.

Compare next the BSL and RF models. We observe again that at $t = 0$ extraction and gross oil output are lower in the BSL economy, and oil reserves, oil GDP and total GDP are higher, but the differences are much smaller than in the comparison with the CE model because extraction is endogenous in the RF model. In the RF model, these changes are in response to the sharp drop in p and expected higher future prices affecting the no-arbitrage condition of oil returns vis-a-vis the risk-free rate, but without the effects from strategic default incentives. In fact, the choices of (x, s') are nearly independent of the b' choice, because Fisherian separation nearly holds. Hence, the extra 7 percentage points drop in extraction and extra 5 years of reserves that the BSL sovereign builds when compared to the CE case instead of the RF case (i.e., the gap betwen the CE and RF curves) are due to those strategic incentives. After $t = 0$, we observe the same qualitative differences in extraction, reserves, oil GDP and total GDP as in the BSL-CE comparison, but the differences are smaller. The gap between the CE and RF curves is again reflecting the relevance of the strategic incentives absent in the RF model. Six years after the default, these incentives are equivalent to about an extra three percentage points in extraction and nearly an extra three years of reserves.

Explaining the differences in debt, spreads and consumption requires recalling first that the RF model has a 56.8% mean debt-GDP ratio and deviates little from it, while the BSL model has a much lower mean (22.5%) and is much more variable. This event analysis simulates the RF economy starting from the same b as the BSL model, but that b is therefore too high in the former, and since βR^* is well below 1, the RF economy starts to move gradually back toward its larger debt levels. At $t = 0$, however, the large drop in p causes a sharp increase in debt in the RF economy, so that the debt ratio in the BSL model is more than 60 percentage points lower than in the RF case (in fact, the RF economy hits its ad-hoc debt limit). The trade balance, however, is still 1 percentage point lower in the BSL economy and hence consumption is about 1 percentage point higher. Post-default, the difference in debt ratios narrows and by $t = 6$ debt is about 50 percentage points larger in the RF economy, but the trade balance is still lower and consumption higher in the BSL case.

Intuitively, this pattern reflects how when default occurs and early after that, the BSL sovereign attains higher consumption by defaulting than repaying. In the RF case, the sovereign borrows a lot at $t = 0$ but then reduces its debt (relative to the BSL case) afterwards, while in the BSL the sovereign defaults and then uses its accumulated reserves (and the gamble with the random chance of credit-market re-entry) to attain higher consumption for several periods. On the other hand, the anticipation of this causes spreads to rise sharply before $t = 0$ in the BSL case. At $t = -1$, the BSL sovereign pays nearly 11 percentage points above R^* and spreads start to widen since $t = -4$.

Summing up, this event-analysis comparison across the BSL, CE and RF models shows that endogenizing oil extraction and allowing strategic default incentives to influence extraction and reserves decisions has large quantitative implications. The sovereign accumulates much larger reserves in the run-up to a default and uses them to prop up consumption afterwards. On the other hand, this extra margin for strategic incentives to operate reduces even more the borrowing capacity of the sovereign and pushes up spreads significantly more.[37]

One additional feature of the default event analysis in the BSL model relates to the output drop when defaults occur. This is the ex-post (realized) cost of default, not the strategic default income costs related to the price penalty $\hat{p}$. To shed light on this issue, we show in Appendix M the BSL default event analysis in levels instead of differences relative to CE and RF. At $t = 0$ the sovereign reduces x by 10 percentage points below its mean, which together with the drop in p induces a large drop in gross oil output. Combined with the fact that y is also low at $t = 0$, total GDP falls about 14 percentage points below average. This is somewhat larger than the 9.6% drop reported in Arellano (2008) but is not far from some actual defaults, like the 11% drop in Argentina in 2002. More generally, comparing the variation in oil GDP in the year of default events (relative to the previous year), we obtained average drops of 50.5 and 40 percentage points in the cyclical components of

[37] Pieschacón (2012) obtains a related result showing that responding to oil price shocks with countercyclical fiscal policies instead of procyclical ones improves consumption smoothing and welfare. In our model, the debt, default and oil reserves decisions also represent fiscal policy actions, and the lack of commitment induces the sovereign to use them as an imperfect self-insurance mechanism and results in inefficient use of the trade balance as a smoothing vehicle.

the defaulters in the data and in the deviation from the long-run mean in the model, respectively.

4.4.2 Large, rare & uncertain discoveries

We now introduce large but infrequent and uncertain oil discoveries. Discoveries are modeled as a regime-switching process with drift: $\kappa_t = \bar{\kappa} + \hat{\kappa}_t$, where $\bar{\kappa}$ is a constant and $\hat{\kappa}_t$ is a stochastic regime-switching component with two properties: First, $\hat{\kappa}_t$ takes the value of zero or a large positive value κ^H: $\hat{\kappa}_t = \{0, \kappa^H\}$. Second, the probability of $\hat{\kappa}_{t+1} = 0$ conditional on $\hat{\kappa}_t = 0$ (denoted π^0) is high while the probability of $\hat{\kappa}_{t+1} = \kappa^H$ given $\hat{\kappa}_t = \kappa^H$ (denoted π^H) is low. The term $\bar{\kappa}$ captures smaller, frequent discoveries, and we use it also to address a limitation of the large discoveries data.[38]

There are four new parameters to calibrate $(\kappa^H, \pi^0, \pi^H, \bar{\kappa})$. These are set to match (weighted-average) targets from the non-defaulters' data as follows: The value of π^0 is set to match the frequency of oil discoveries (14.4%), π^H is set to match the average of consecutive annual discoveries (1.14), κ^H targets the ratio of mean discoveries (conditional on having a discovery) to the mean of extraction (2.62), and $\bar{\kappa}$ is set to match the share of gross oil production in GDP (0.335).[39] These four parameters affect mean reserves and the variability of extraction, and hence they are calibrated jointly with new values for the extraction costs parameters, following a similar strategy to the one described earlier (using again the SOE-FI model). The calibration yields $\kappa^H = 0.878, \pi^0 = 0.852, \pi^H = 0.121, \bar{\kappa} = 0.208, \gamma = 4, \tilde{\psi} = 36.7$. We then pass on these parameters to the model with default and re-calibrate the discount factor and default penalty to the same data targets as before, which yields $\beta = 0.71$ and $\hat{p} = 0.604$.

The above parameter values yield unconditional means of $E[\hat{\kappa}] = 0.127$ for large discoveries and $E[\kappa] = 0.127 + .208 = 0.335$ for all discoveries. Large discoveries occur only 14% of the time and they are 4.2 times bigger than $\bar{\kappa}$. The mean duration of a period without large discoveries is about seven years. Notice that the baseline model with constant discoveries is obtained by setting $\hat{\kappa}_t = 0$ and $\bar{\kappa} = 0.335$, because it was calibrated to the same data ratio of gross oil output to GDP.

We compared the baseline (BSL) and stochastic discoveries (SD) models in terms of their long-run moments, default-repayment sets, default costs, local-projection impulse responses to an oil price shock, and default event analysis. We summarize here the main results and provide full de-

[38] Average discoveries implied by the law of motion of reserves (which yields $E[\kappa] = E[x]$) and extraction data exceed average discoveries from the data on giant oil discoveries constructed by Cust et al. (2021). This is in part because only giant discoveries are included, defined as those with Estimated Ultimate Recoverable (EUR) reserves of at least 500 million barrels, where EUR is the sum of proven reserves at a given date plus cumulative extraction up to that date. In addition, as noted earlier, proven reserves are those that can be extracted profitably with current technology and prices, but these change over time. Calibrating π^0, π^H and κ^H to the discoveries data without including $\bar{\kappa}$ yields $E[\kappa] = 0.127$, less than half the mean of the gross oil output-GDP ratio in the data (0.335). Thus, $\bar{\kappa} = 0.208$ is a proxy for "missing discoveries" that ensures that the calibration satisfies the law of motion of reserves and yields $E[\kappa] = 0.335$.

[39] Frequency of discoveries is the weighted average across non-defaulters of the number of years with large discoveries (using the data from Cust et al. (2021)) relative to the total number of years for each country. For the average consecutive annual discoveries, we identify all the periods of large discoveries and determine how many of them have consecutive years with large discoveries, then calculate the mean for each country and the weighted average of these means across non-defaulters. The ratio of mean discoveries to mean extraction is the ratio of the average of the size of large discoveries to the average of extraction. Then we compute the sample average across the non-defaulters.

tails in Appendix K. Overall, we obtained two main findings: First, the sovereign's management of reserves and extraction interacts with the possibility of rare, large discoveries altering the moments and dynamics of oil-sector variables significantly. Second, the main findings from the baseline with constant discoveries (the changing direction of the co-movement of country risk and reserves over time and the build-up of oil reserves to manage default costs) are preserved.

Table 7 compares the values of β, γ, and $\hat{p}$ and lists the long-run moments that changed the most. β and $\hat{p}$ are lower and γ is higher in the SD model. Mean reserves increase sharply from 53 to 83 years, and the variability ratios (relative to the variability of oil prices) of extraction, gross oil output and extractions costs also increase sharply, from 0.35 to 3.28, 1.36 to 3.97, and 1.45 to 9.71, respectively. The variability of oil GDP (i.e., oil profits) also rises but not as much, from 1.37 to 1.97, because the covariance between gross oil output and extractions costs rises sharply too, from 0.002 to 0.046. This moderates also the rise in the relative variability of total GDP, which increases from 0.55 to 0.63 (roughly 14%).

It is important to note that oil-sector volatility increases not because of the rare, large discoveries per se, but because of how they interact with the sovereign's self-insurance and strategic default incentives. With either constant or stochastic discoveries, the oil-sector parameters were calibrated using the SOE-FI setup to match the same data targets (volatility of x, mean of s/x and px/GDP ratio). But using the resulting parameters from each to solve the model with a sovereign that has CRRA preferences and cannot commit to repay changes oil-sector volatility significantly and in different directions (it *falls* in the BSL case but *rises* sharply in the SD case). For instance, the variability ratio of x falls to 0.35 in BSL but increases to 3.28 in SD, relative to the data target of 0.67 that both match in their corresponding SOE-FI calibration.

Table 7: Calibration & Main Long-run Moments: Baseline v. Stochastic Discoveries

	Baseline	Stochastic Discoveries
Extraction costs curvature (γ)	3.10	4.00
Discount factor (β)	0.831	0.71
Default penalty ($\hat{p}$)	0.645	0.604
Mean oil reserves (in years)	53	83
Corr (p, x)	0.80	0.35
Autocorr. x	0.74	0.32
Variability ratios (*coefficients of variation relative to oil price*):		
- Extraction	0.35	3.28
- Gross oil output	1.36	3.97
- GDP	0.55	0.63
- Oil GDP	1.37	1.97
- Extraction costs	1.45	9.71
- Reserves	0.12	0.22

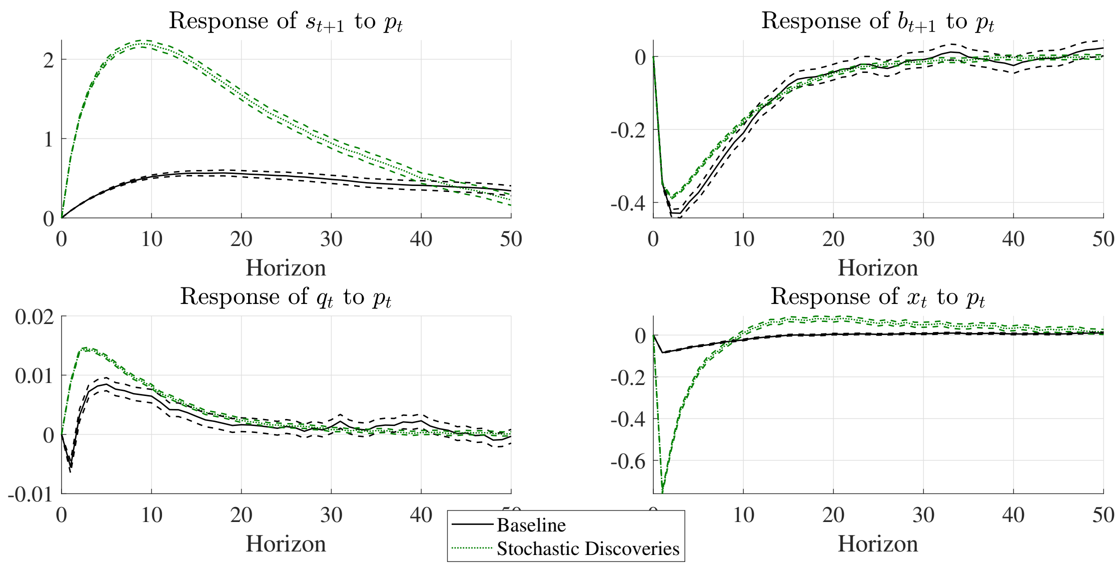
The surge in mean reserves and the higher oil-sector and aggregate GDP volatility in the SD case are related. Higher volatility strengthens the sovereign's self-insurance incentive, particularly since sequences with at least seven-year spells without large discoveries are typical. The calibration then requires lowering β in order to match the actual mean debt ratio, which is also the target in the BSL case. But at that debt ratio, default incentives are too strong and this requires a lower $\hat{p}$ (larger default penalty) to match the observed default frequency. With mean debt and default frequency unchanged relative to the BSL case, the extra precautionary savings are allocated to higher oil reserves. In contrast, introducing rare, large discoveries in the SOE-FI makes little difference. For example, solving the SOE-FI model with the large, rare discoveries but the extraction-cost parameters of the BSL case, mean reserves and the coefficient of variation of x change from 70.25 to 70.21 and 13.3% to 15.5%, respectively. Appendix L provides analytic solutions showing that this is due to the homogeneity of the unitary extraction costs, which makes the derivatives $e_x(\cdot)$ and $e_s(\cdot)$ and the oil-reserves Euler equation depend on x_t/s_t but not on κ_t. With the unconditional mean of x/s pinned down by the reserves data target, optimal extraction depends on future p, and s_{t+1} increases one-to-one with κ_t.

Figure 7 compares impulse responses to a negative one-standard-deviation shock to p, obtained again using local projections from model-simulated data. Reserves rise at first in both models and then decline, but the expansion is larger in the SD model in line with the higher variability ratio of reserves (0.22 v. 0.12). Debt displays similar dynamics in both models, falling about the same in the initial periods and monotonically rising afterwards, with slightly larger debt in the SD case during the transition. Bond prices differ in the impact response (falling in BSL, rising in SD). Afterwards, they show similar inverted J-curve patterns albeit with quantitative differences (rising more in the SD case). Still, the SD model preserves the key result that there is a phase during which reserves and bond prices move in opposite directions, although its duration is shorter (from the second to the ninth period). As before, bond prices rise and then fall because initially both reserves are rising and debt is falling but then debt starts rising and later on reserves fall.

Figure 8 compares default event dynamics.[40] For discoveries, the window shows the fraction of events with κ^H in each period. The event windows of several variables do not change much, with some important exceptions. The higher volatility of the oil sector is reflected in the larger drops in px and x when defaults occur. There are also two interesting new findings. First, defaults in the SD case follow dry spells in large discoveries, in addition to drops in p and y. Large discoveries hover around their long-run 14% frequency at first, but start dropping at t=-3 and fall to just below 2% when defaults occur. Second, defaults occur with smaller drops in p and y and larger increases in reserves. This is indicative of stronger default incentives: smaller declines in p, y make default optimal at the same long-run default frequency and at about the same debt ratio at t=-1. In addition, these results confirm that the result from the BSL case showing a build-up in oil reserves prior to a default continues to hold, and is actually larger (22% above the mean v. 11%).

[40] The default event windows are constructed with the BSL and SD simulations separately, instead of feeding the initial conditions and sequence of shocks of the BSL to the SD model as before, because SD has an additional shock ($\hat{\kappa}$) and its state space of s spans a much larger interval.

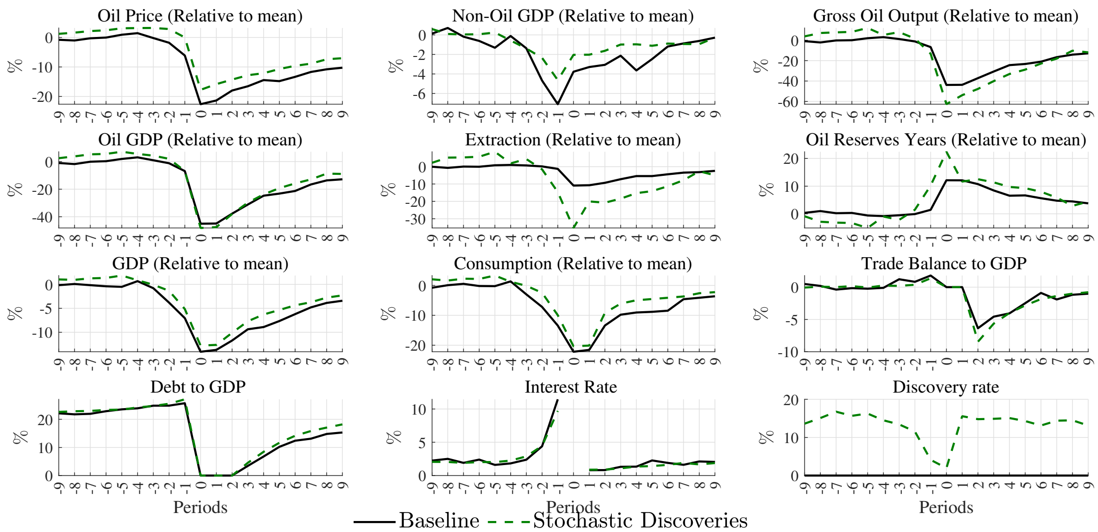
Figure 7: Impulse Responses to an Oil Price Shock: Baseline & Stochastic Discoveries Models



To generate the above SD results, we re-calibrated the model to the same data targets as the BSL case. We also studied rare, large discoveries using counterfactual experiments. Appendix K compares the BSL and SD solutions with two counterfactuals denoted CA and CB. In CA, we introduce rare, large discoveries keeping the BSL parameter values. Comparing SD with CA, oil-sector volatility remains high and mean reserves are slightly lower (the variability ratio of x remains unchanged at 3.28, while mean reserves decrease from 83 to 78 years) but the higher β and $\hat{p}$ result in a lower mean debt ratio (16%) and a lower default frequency (0.79%). In CB, we use the recalibrated parameters of the SD case in the BSL setup but with constant discoveries. Comparing CB with BSL, the volatility of the oil sector is slightly lower, and mean reserves slightly higher (the variability ratio of x drops to 0.25, mean reserves increase from 53 to 55 years) and the lower β and $\hat{p}$ result in a higher mean debt ratio (31%) but again a lower default frequency (0.79%).

Local-projection impulse responses also yield qualitatively similar results across specifications: reserves rise on impact after an oil-price drop in all models, with the peak in reserves occurring sooner in the two specifications with large, infrequent discoveries (about 6–8 periods) than in the two with constant discoveries (about 20–25). The falling (rising) impact response of bond prices is also similar in the two models with constant (stochastic) discoveries, but after the impact effect the four models show similar dynamics. The default event analysis also yields qualitatively similar results across models. In particular, with stochastic discoveries and baseline parameter values, defaults occur with a dry spell in discoveries and are accompanied by larger reserve buffers and a larger cut in extraction (see Fig. K10 Appendix K).

Figure 8: Dynamics around defaults: Baseline v. Stochastic Discoveries



5 Conclusions

This paper examined the implications of commodity production and commodity-price fluctuations for the analysis of sovereign default. We proposed a model in which the sovereign responds to strategic incentives in managing commodity extraction. In particular, the sovereign alters the value of default by adjusting commodity reserves, which are used to prop up consumption while the economy is in financial autarky.

We focus on oil because of the availability and quality of data on oil extraction, reserves and discoveries. Data for the 30 largest oil-producing emerging economies show that these countries have sizable external debt, their country risk is positively correlated with real oil prices, and several of them have defaulted at least once. We also obtained an important new finding: Dynamic error-correction panel regressions show that the conditional co-movement of oil reserves and country risk changes direction over time. Higher reserves increase country risk in the long run and have a negligible effect in the short run.

In the model, the ability to increase the value of default by increasing oil reserves gives the sovereign the incentive to strategize over debt and reserves as it makes its borrowing, default and extraction decisions. We derive analytic results showing that, keeping all other variables constant, default incentives strengthen at lower oil prices or lower oil reserves. Moreover, when oil prices rise, the intertemporal incentive to increase extraction and reduce reserves to increase oil sales competes with the incentive to build up reserves to prop up consumption after a future default.

The model's quantitative predictions were examined using a calibration based on the data for the 30 largest oil producers. We found that the incentive to strategize over oil reserves is quantitatively important. It reduces the cost of default and hence the debt the sovereign can sustain in the long run, relative to both a sovereign committed to repay and one with a fixed oil endowment.

As in the data, country risk and oil prices are negatively correlated in the model. The model also does a good job at approximating several of the second and third moments observed in the data. Some of the second moments deviate from the data, particularly the volatility of consumption. Relative to the case where the government is committed to repay, the model with default yields higher consumption variability because of the reduced ability to smooth consumption without commitment but in both cases consumption is more volatile than in the data. Importantly, the model matches the time-varying dynamic relationship between oil reserves and sovereign risk. In particular, local-projection impulse responses to a negative oil-price shock show that reserves and bond prices rise (country risk falls) together initially but then they go through a lengthy phase in which reserves rise and bond prices fall (country risk rises).

The relevance of endogenous oil extraction for sovereign default was evaluated with two sets of experiments. First, we compared the baseline model results with the variants without default risk and with constant extraction. Second, we introduced rare, large and uncertain oil discoveries. The results of the first comparisons show that allowing default incentives to affect extraction and reserves decisions has large quantitative implications. The sovereign accumulates much larger reserves in the run-up to a default and uses them to prop up consumption afterwards. Introducing large, rare and uncertain discoveries interacts with the sovereign's risk-averse preferences and its lack of commitment to repay, increasing significantly oil-sector volatility and strengthening self-insurance incentives. Defaults are triggered by smaller declines in oil prices and non-oil GDP, indicating stronger default incentives, and they occur after unusually long dry spells in discoveries. The main results regarding the surge in oil reserves in the run-up to a default and the change of direction in the co-movement of reserves and country risk over time are preserved.

Our analysis applies more broadly to developing and emerging economies with sizable production and exports of all commodities. Our findings suggests that issuing debt indexed to commodity prices may enhance borrowing capacity by improving the hedging properties of the debt. Moreover, our model may also be helpful for evaluating the effects of climate change policies on country risk and debt sustainability. In particular, the model can be used to examine how caps on fossil fuel extraction may affect sovereign debt access, country risk and macroeconomic dynamics.

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