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ABSTRACT

During a liquidity trap, increases in the money supply have no real effects, as the nominal interest rate has reached its lower bound. We propose a theory of how helicopter drops of money can be effective during a liquidity trap. We develop a New Keynesian monetary model where the fiscal and monetary authorities are separated, and the latter faces balance sheet constraints. If the monetary authority can commit, helicopter drops are unnecessary in a liquidity trap, even under balance sheet constraints. However, we show that helicopter drops can help stabilize the economy when the monetary authority lacks commitment.

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1 Introduction

During the 2008 global recession and the COVID-19 pandemic, nominal interest rates reached the zero lower bound in many advanced economies, prompting central banks to adopt unconventional policies such as quantitative easing and forward guidance to stabilize their economies. Yet, the persistence and severity of downturns at the zero lower bound have led to the consideration of more unorthodox proposals. One that has garnered significant attention is "helicopter drops," a metaphor introduced by Friedman (1969) to describe a scenario in which the central bank prints money and distributes it directly to the public.[1]

Despite the growing attention on helicopter drops as a potential stabilization tool, standard economic theory predicts that they are irrelevant when nominal interest rates are at zero. In fact, the concept of a liquidity trap, as described by Keynes (1936) and Hicks (1937), refers to a situation where households are satiated with money balances, causing interest rates to reach the lower bound and rendering additional increases in the money supply ineffective.[2]

In this paper, we propose a theory of how helicopter drops can be effective during a liquidity trap. We develop a framework in which the monetary authority is required to remit its profits to the fiscal authority and cannot receive capital injections. Under these balance sheet constraints, we show that while open market operations are irrelevant, helicopter drops can be effective: an increase in unbacked monetary liabilities reduces the monetary authority's net worth, constraining its ability to tighten monetary policy in the future and generating expectations of higher inflation and lower real interest rates. In the absence of commitment, helicopter drops provide a credible promise of higher future inflation and output, helping to stabilize the economy during a liquidity trap.

Our model is a canonical New Keynesian monetary framework with a zero lower bound constraint, extended to include an explicit separation between the fiscal and monetary authorities. We assume that the monetary authority is required to remit its operating profits to the fiscal authority and is restricted in its ability to issue interest-bearing securities. We define a helicopter drop as a transfer to the fiscal authority in excess of the operating profits (or equivalently, as a direct transfer to households).

In the absence of balance sheet constraints, the implications of the zero lower bound follow the canonical results in the literature (Krugman, 1998; Eggertsson and Woodford, 2003; Jung, Teranishi

[1] For proposals, see Bernanke's speech "Deflation: Making Sure It Doesn't Happen Here" (2002), and his Brookings blog post "What tools does the Fed have left? Part 3: Helicopter money" (2016), Gali's VoxEU article on "Helicopter Money: The Time is Now" (2020), and Bartsch, Boivin, Fischer, Hildebrand and Wang SUERF Policy Note "Dealing with the Next Downturn" (2019).

[2] The ineffectiveness of helicopter drops mirrors the irrelevance of open market operations at the zero lower bound (Wallace, 1981; Eggertsson and Woodford, 2003).

and Watanabe, 2005; Werning, 2011). When a temporary discount rate shock reduces aggregate demand triggering a zero lower bound, the monetary authority would like to promise a low real rate in the future to mitigate the recession. However, this promise is time inconsistent, as the monetary authority has incentives to implement the flexible-price outcome with zero inflation and zero output gap once the shock subsides. Without balance sheet constraints, there is in effect a consolidated government budget constraint, and a helicopter drop at the zero lower bound simply leads households to anticipate a future reduction in the money supply—leaving expected real rates and consumption unchanged. As in the canonical Keynesian-Hicksian liquidity trap, an increase in the money supply is ineffective.

We show that helicopter drops are no longer irrelevant when the monetary authority faces balance sheet constraints. Balance sheet constraints limit the monetary authority's ability to reduce the money supply, as the absence of equity injections from the fiscal authority may leave it with insufficient assets to exchange for money. In this economy, a helicopter drop at the zero lower bound can alter expectations of future monetary policy and thus be effective. However, we also show that the ability to implement helicopter drops does not expand the set of private sector equilibria: any equilibrium achieved through a policy involving a helicopter drop remains an equilibrium under an alternative policy that excludes the helicopter drop. Hence, when the monetary authority can commit to future policies, helicopter drops are unnecessary, as the monetary authority can credibly promise future expansionary measures without them. A helicopter drop, on the other hand, may unnecessarily restricts its ability to stabilize output and inflation in the future.

We then turn to our analysis without commitment and show that there is scope for helicopter drops. We study a Markov perfect equilibrium in which the monetary authority, at each point in time, chooses the best available policy, taking as given its inherited states—including its balance sheet—and private sector's expectations about future policies. We show that the resulting fixed point admits an analytical characterization. We construct a log-linear equilibrium and show that it exists and that it is unique.

In the stationary Markov equilibrium, when balance sheet constraints bind, lower net worth induces the monetary authority to pursue a more *expansionary* policy, which results in higher levels of output and inflation. We also show that the nominal rate could increase or decrease with net worth depending on the income elasticity of money demand. Over time, the monetary authority's net worth increases until balance sheet constraints cease to bind and the monetary authority is eventually able to implement the outcome with zero inflation and a zero output gap. The key takeaway is that balance sheet constraints lead a monetary authority with low net worth to adopt a more expansionary policy path.

We then show that, in the absence of commitment, helicopter drops deliver substantial stabilization gains during a liquidity trap. The optimal policy balances the benefits of higher expected inflation at the zero lower bound against the costs associated with tighter balance sheet constraints once the economy exits the liquidity trap. Our numerical simulations indicate that, under the optimal helicopter drop, the level of output and inflation during the liquidity trap closely resembles the forward-guidance policy that would prevail under commitment. However, helicopter drops lead to a significantly more prolonged period of inflation above target once the economy exits the trap.

We conclude the introduction by highlighting that open market operations remain irrelevant in a liquidity trap within our framework. The key distinction is that an open market operation exchanges one asset for another, leaving the monetary authority's net worth unchanged, whereas a helicopter drop increases the money supply without a corresponding increase in assets. The resulting decline in net worth following a helicopter drop generates expectations of lower future real interest rates, leading to higher expected output and inflation that help mitigate the recession today.

Related literature. The seminal contribution on the interactions between monetary and fiscal policy is Sargent and Wallace (1981). They show that fiscal policy constrains the central bank's ability to control the price level. In particular, fiscal deficits force the central bank to eventually raise seigniorage revenue—and thus inflation—to satisfy the consolidated government budget constraint. Despite the central role of fiscal-monetary interactions, much of the literature assumes a consolidated authority that jointly determines fiscal and monetary policy. A more recent literature has explicitly accounted for the fact that the fiscal and monetary authorities have separate budget constraints, focusing on the degree of fiscal support and balance sheet policies that enable the monetary authority to achieve price stability (Sims, 2004; Bassetto and Messer, 2013; Del Negro and Sims, 2015; Hall and Reis, 2015; Benigno and Nisticò, 2020; Barthélemy, Mengus and Plantin, 2024).[3] In contrast, our focus is normative, studying optimal policy when the monetary authority lacks commitment and exploring the role of helicopter drops.

Our paper is also related to a large literature on central bank policies during a liquidity trap. A key insight from this literature (Krugman, 1998; Eggertsson and Woodford, 2003; Jung et al., 2005; Werning, 2011) is that the central bank should commit to an expansionary monetary policy after the liquidity trap is over to help mitigate the contraction in output when the zero lower bound constraint binds. Our contribution to this literature is to show that, in the absence of commitment, a helicopter drop can be effective during a liquidity trap by making future monetary easing credible.

Our paper is closely related to studies examining the role of helicopter drops during liquidity

[3]See Bassetto and Sargent (2020) for a review of the research on monetary and fiscal policy interactions.

traps. Auerbach and Obstfeld (2005) and Galí (2019) show that money-financed stimulus can be expansionary at the zero lower bound.[4] However, these studies assume that money supply remains high beyond the period of the liquidity trap—a policy that may not be time consistent. Benigno and Nisticò (2025) and Michau (2024) also consider environments with commitment, focusing respectively on coordination between fiscal and monetary authorities under the fiscal theory of the price level, and on a situation characterized by secular stagnation and households with a utility preference for wealth. In contrast to these contributions, our paper focuses on the time-consistency problem faced by the central bank in a liquidity trap. We show that helicopter drops of money, by constraining the central bank's future behavior, can credibly raise expectations of future monetary easing.

In the context of models with heterogeneous agents, helicopter drops can have stimulative effects at the zero lower bound by relaxing households' borrowing constraints (see e.g., Guerrieri and Lorenzoni, 2017; Buera and Nicolini, 2020; Bilbiie and Ragot, 2021). However, the same effects can be achieved if the government issues debt instead of increasing the money supply, as open market operations remain irrelevant at the zero lower bound.[5] In contrast, our theory highlights the distinct role of helicopter drops implemented by the monetary authority when it faces balance sheet constraints and lacks commitment.

Our analysis of optimal monetary policy without commitment connects, in turn, with the literature on monetary policy credibility, following the seminal work of Barro and Gordon (1983). Armenter (2018) and Nakata and Schmidt (2019) show that the lack of commitment can induce a deflationary trap equilibrium (see also Benhabib, Schmitt-Grohé and Uribe, 2001b). Afrouzi, Halac, Rogoff and Yared (2024) provide an analytical characterization of the optimal monetary policy in a non-linear framework where the price dispersion is a state variable. While we adopt the canonical linearized model (Clarida, Gali and Gertler, 1999; Woodford, 2011; Galí, 2015), balance sheet constraints introduce a non-linearity linked to the central bank's net worth, yet we show that the Markov equilibria can be solved in closed form.

Finally, our paper is related to the literature exploring the connection between the maturity structure of the government debt and the time consistency of optimal fiscal and monetary policy. Lucas and Stokey (1983) argue that a carefully chosen maturity structure for government debt can make optimal fiscal and monetary policies time consistent when taxation is costly (see also Debortoli, Nunes and Yared, 2021). Alvarez, Kehoe and Neumeyer (2004) study an economy with nominal debt and show that a portfolio can be chosen to deliver time-consistent policies if the Friedman rule is optimal. Calvo and Guidotti (1992) show that a lower level of nominal government

[4]For a survey of the literature on helicopter drops, see Reis and Tenreyro (2022).

[5]Other studies of "fiscal helicopter drops" include Aguiar, Amador and Arellano (2024), Angeletos, Lian and Wolf (2024), and Kaplan, Nikolakoudis and Violante (2023).

debt can help reduce the temptation to inflate ex post. Closer to our paper is the work of Bhattarai, Eggertsson and Gafarov (2022), who present a model of quantitative easing at the zero lower bound. They show that shortening the maturity of consolidated government debt creates incentives to keep interest rates low, due to an aversion to balance sheet losses. By contrast, in our model, the central bank does not suffer balance sheet losses but deliberately reduces its net worth through a helicopter drop.[6]

Outline. The paper is organized as follows. Section 2 presents our model with balance sheet constraints. Section 3 examines the optimal policy under commitment, while Section 4 investigates the optimal policy without commitment. Section 5 concludes. The main proofs are included in the body of the paper and in Appendix A. Additional proofs are in an online appendix.

2 Model

We present a New Keynesian model in which the monetary and fiscal authorities operate under separate budget constraints.

2.1 Main Elements

Monetary Authority. The monetary authority issues monetary liabilities M_t, accumulates a nominal risk-free asset A_t, which pays a nominal interest rate ι_t, and makes nominal transfers τ_t to the fiscal authority.[7] The budget constraint of the monetary authority is given by

$$\frac{A_t}{1+\iota_t} + M_{t-1} + \tau_t \ = M_t + A_{t-1}. \tag{1}$$

Define the beginning-of-period nominal net worth of the monetary authority as $N_t \equiv A_t - M_t$. Using this definition, we can re-express the budget constraint as

$$N_t = N_{t-1} + \frac{\iota_t}{1+\iota_t} A_t - \tau_t.$$

Let $i_t = \log(1+\iota_t)$ be the corresponding instantaneous rate of interest. We define

$$\tau^\star(A, i) \equiv A(1 - e^{-i}).$$

[6]Also related is Jeanne and Svensson (2007), which shows how purchases of foreign currency assets can help avoid a liquidity trap when the central bank faces a lower bound on the level of capital. Amador, Bianchi, Bocola and Perri (2016) show how a large currency mismatch can provoke an early abandonment of a floor on the exchange rate due to the risk of incurring losses.

[7]Alternatively, we could allow the monetary authority to make transfers directly to households. Because the model features Ricardian properties in terms of debt by the fiscal authority, this alternative is equivalent to the current one.

Then, $N_t < N_{t-1}$ if and only if $\tau_t > \tau^\star(A_t, i_t)$. The value of $\tau^\star$ denotes the nominal net gains from holding financial assets ($\imath \times A$) relative to the nominal cost of liabilities ($0 \times M$). When the monetary authority remits more than $\tau^\star$ to the fiscal authority, the net worth of the monetary authority falls in nominal terms. Conversely, if remittances are lower than $\tau^\star$, its net worth increases in nominal terms.

We assume that the monetary authority is required to remit at least $\tau^\star$ every period to the fiscal authority:

$$\tau_t \geq \tau^\star(A_t, i_t), \text{ for all } t, \tag{2}$$

which implies that nominal net worth cannot increase. In effect, this also means that the monetary authority cannot count on equity injections by the fiscal authority. In the terminology of Benigno and Nisticò (2020), our remittance constraint (2) corresponds to a situation where the monetary authority is *financially independent*.

We also assume that the monetary authority cannot issue bonds:

$$A_t \geq 0, \text{ for all } t. \tag{3}$$

These two constraints, (2) and (3), imply that the monetary authority faces a lower bound on monetary liabilities given by

$$M_t \geq -N_{t-1}. \tag{4}$$

According to (4), lower net worth forces the monetary authority to keep a higher level of monetary liabilities. The reason is as follows. If the monetary authority would like to reduce M_t, it has to either sell assets or reduce the transfers it makes to the fiscal authority. Because the monetary authority is constrained to make the minimum transfer and maintain a minimum amount of assets, it faces a limit on how much it can reduce its monetary liabilities.[8] In particular, by selling all its assets and paying the minimum transfer $\tau^\star$, the monetary authority can reduce its monetary liabilities to a maximum level of $-N_{t-1}$.

We refer to constraints (2) and (3) as the "balance sheet constraints", and they constitute our addition to the New Keynesian model. Absent either of these two constraints on the monetary

[8]Notice that one could generalize the borrowing limit in (3) as $A_t/P_t \geq \bar{a}$ for given $\bar{a}$, in which case the constraint (4) would become $M_t \geq -N_{t-1} + \bar{a}P_t$. The results would be similar although the reduction in net worth necessary for the constraint to bind would be larger. Our analysis can similarly be extended to include interest on monetary liabilities (e.g., for commercial bank reserves). To the extent that the interest rate on these liabilities is below the interest rate on securities, or that there is a limit or cost associated with interest-bearing monetary liabilities, balance sheet constraints remain generally binding. While central banks often pay interest on reserves, these payments entail costs—for instance, fiscal costs, as highlighted by Hall and Reis (2015), or a reduction in financial intermediation, as banks allocate relatively less capital to loans, as highlighted by Bianchi and Bigio (2022).

authority, the model would feature a consolidated budget constraint for the government, rendering the net worth of the monetary authority irrelevant. Following early work by Stella (1997, 2005), there is extensive evidence that balance sheet constraints on central banks have influence on how they conduct monetary policy. As these studies note, countries generally have different accounting rules governing the required level of central bank capital, as well as rules for dividends and remittances. These factors determine a central bank's ability to operate with low levels of capital or manage operating losses. For example, the Federal Reserve Act requires the Reserve Banks to remit every year excess earnings after expenses to the U.S Treasury.[9] Other central banks, such as Bank of England and Netherlands are able to retain a fraction of the net earnings (see Chaboud and Leahy, 2013 for a summary of central bank practices). Our assumptions attempt to capture in a simple way the institutional features that characterize the interaction between fiscal and monetary authorities. We also note that recent literature has examined the effects of central bank losses (e.g., Hall and Reis, 2015; Bhattarai et al., 2022), but in our analysis, such losses or concerns about them do not play a role, as we abstract from long-term or other risky assets.

Fiscal Authority. There is a fiscal authority that issues bonds, provides lump-sum transfers to households (or collect taxes) and receives revenue from the monetary authority. The fiscal authority's budget constraint is

$$\frac{B_t}{1+\iota_t} + \tau_t = T_t + B_{t-1}, \tag{5}$$

where B_t denotes the level of debt of the fiscal authority and T_t represents the lump-sum transfers to households.

Households. There is a representative household that has the following preferences over consumption and real money balances:

$$\sum_{t=0}^{\infty} \beta(t) \left[u(C_t) + v\left(\frac{M_t}{P_t}\right) \right]$$

with $\beta(t) = \Pi_{s=0}^{t} e^{-(\rho+\xi_{s-1})} \in (0,1)$ the discount factor from period 0 to t. The value of ξ_t is a time-varying exogenous disturbance to the discount rate (and, as in Werning, 2011, this is the only

[9]Note that when these excess earnings turn negative the Federal Reserve does not receive a transfer from the Treasury. Instead, it accumulates a so-called "deferred asset." This means that future excess earnings are credited against the deferred asset and the Federal Reserve resumes remitting to the Treasury once the deferred asset is extinguished. As emphasized by Greenlaw, Hamilton, Hooper and Mishkin (2013), the accumulation of a deferred asset for a significant amount of time can subject monetary policy decisions to increased public scrutiny and potentially threaten the Federal Reserve's independence. For some discussion in the financial press, see Explainer: Why huge European Central Bank losses matter in Reuters.

non-stationary parameter and the reason for a liquidity trap). The function u is assumed to be strictly increasing, strictly concave, and differentiable in $\mathbb{R}^{+}$. The function v is differentiable in $\mathbb{R}^{+}$; strictly increasing, and strictly concave in $[0, \overline{m}]$; reaching a satiation point at $\overline{m}$: $v(m) = v(\overline{m})$ for $m > \overline{m}$.

The household faces the following sequential budget constraint:

$$P_t C_t + M_t + \frac{W_t}{1+\iota_t} \leq M_{t-1} + W_{t-1} + T_t + P_t Y_t, \tag{6}$$

where P_t is the price level, C_t is the households' real consumption, Y_t is the households' real income (composed of real labor income and profits), and W_t is the financial wealth net of money holdings.

The household is subject to the following No-Ponzi condition:[10]

$$\lim_{t\to\infty} \frac{W_t + M_t}{\prod_{s=0}^{t}(1+\iota_s)} \geq 0. \tag{7}$$

The household's problem is to maximize utility subject to the budget constraint, (6), and the No-Ponzi condition, (7), taking prices and transfers as given. Letting $\pi_t \equiv \log(P_t/P_{t-1})$, we have the following sufficiency result.[11,12]

Lemma 1 (Sufficiency)**.** *Take a sequence of prices and transfers $\{P_t, \iota_t, T_t\}$ and initial conditions $P_{-1}, W_{-1}, \iota_{-1}$. Let $\{C_t, M_t, W_t\}$ be such that (i) $C_t > 0$ for all $t \geq 0$, (ii) the household budget constraint (6) holds, (iii) the No-Ponzi condition (7) holds, (iv) the following first-order conditions (an Euler equation, and a money demand condition) hold*

$$u'(C_t) = e^{i_t - \rho - \xi_t - \pi_{t+1}} u'(C_{t+1}),$$
$$v'\left(\frac{M_t}{P_t}\right) = u'(C_t)\frac{\iota_t}{1+\iota_t},$$

and (v) the following transversality condition holds

$$\lim_{t\to\infty} \frac{W_t + M_t}{\prod_{s=0}^{t}(1+\iota_s)} \leq 0.$$

[10] Buiter and Sibert (2007) propose an alternative No-Ponzi condition that does not include money holdings. Part of their argument is a potential difficulty with obtaining sufficiency results for the household problem. We show in Lemma 1 that this is not an issue.

[11] We will impose the sufficient conditions of this lemma as requirements for household optimality. For the necessity of the transversality condition, we refer the reader to Kamihigashi (2002).

[12] In what follows, we use the notation $\{a_t, b_t, \ldots\}$ to refer to the sequences $\{a_t\}_{t=0}^{\infty}$, $\{b_t\}_{t=0}^{\infty}$, etc.

Then the sequence $\{C_t, M_t, W_t\}$ solves the household's problem.

Proof. In Appendix A.1. □

Note that we can collapse the No-Ponzi condition and the transversality condition into just one condition at an optimal solution:

$$\lim_{t\to\infty} \frac{W_t + M_t}{\prod_{s=0}^{t}(1+\iota_s)} = 0, \tag{8}$$

Firms. The firms' side of the model follows the New Keynesian tradition. The model features monopolistic competition and Calvo-style sticky prices. Because these features are standard, we work directly with the log-linearized Phillips curve, which describes the optimal price-setting behavior of firms:

$$\pi_t = \beta\pi_{t+1} + \kappa y_t, \tag{9}$$

where $\beta \equiv e^{-\rho} \in (0,1)$, $y_t \equiv \log(Y_t/\bar{Y})$ and $\bar{Y} > 0, \kappa \geq 0$.

Market Clearing. In an equilibrium, the goods market clears $Y_t = C_t$. Let the utility function be

$$u(C) = \frac{C^{1-1/\sigma}}{1-1/\sigma},$$

with $\sigma > 0$. The household's Euler equation can be written as:

$$y_t = y_{t+1} - \sigma(i_t - \pi_{t+1} - \rho - \xi_t). \tag{10}$$

The first-order condition for money holdings requires that $\iota_t \geq 0$. Let h denote the inverse of the first derivative of v, we let

$$L(c,i) \equiv h\left(u'(e^c)(1-e^{-i})\right),$$

which is defined for $i \geq 0$, and is increasing in c, decreasing in i, and where $L(c,0) = \overline{m}$.

Thus, the money market clearing condition, together with $C_t = Y_t$, is equivalent to

$$\frac{M_t}{P_t} \geq L(y_t, i_t), \text{with equality if } i_t > 0. \tag{11}$$

2.2 Private Sector Equilibrium

Let us define a private sector equilibrium. The economy starts at $t = 0$ with the monetary authority's balance sheet given by A_{-1} and M_{-1}, an initial fiscal authority debt level B_{-1}, and an initial price level P_{-1}.

Definition 1. A *private sector equilibrium* is a sequence $\{y_t, \pi_t, i_t, P_t, M_t, A_t, B_t, \tau_t, T_t\}$ such that for all $t \geq 0$ and

(i) households optimize and markets clear; that is, (6), (8), (10), and (11) hold for $W_t = B_t - A_t$ for all $t \geq -1$.

(ii) firms optimize; that is, (9) holds.

(iii) the zero lower bound constraint holds, $i_t \geq 0$.

(iv) the budget constraint of the monetary authority, (1), holds.

(v) the budget constraint of the fiscal authority, (5), holds.

Equations (9) and (10), together with the zero lower bound, $i_t \geq 0$, constitute the basic equations of the New Keynesian model. Suppose that we take a sequence $\{y_t, \pi_t, i_t\}$ that satisfies (9) and (10), and the zero lower bound. Based on this sequence, we can construct a private sector equilibrium as follows. We can use the initial price level to solve for the corresponding $\{P_t\}$ and use the condition (11) with equality to recover a sequence for $\{M_t\}$. With this, we can then obtain $\{A_t, B_t\}$ by setting any combination such that $(A_t - B_t) = M_t$ for all t large enough, which guarantees that the transversality condition, (8), holds. Finally, the implied transfers and taxes from (1) and (5) make sure that all the budget constraints hold. This justifies the common approach to focus on just the interest rate policy (satisfying the zero bound) and two equations, the Phillips curve (9) and the Euler equation (10), when studying optimal policy.

Introducing Balance Sheet Constraints and Helicopter Drops. In our definition of a private sector equilibrium we did not yet introduce the restriction that the balance sheet constraints of the monetary policy must hold. We proceed to do this now:

Definition 2. A private sector equilibrium $\{y_t, \pi_t, i_t, P_t, M_t, A_t, B_t, \tau_t, T_t\}$ *is consistent with balance sheet constraints* if equations (2) and (3) hold for all t.

To see the effects of the balance sheet constraints, let us first rewrite the budget constraint of the monetary authority, (1), as follows:

$$M_t - M_{t-1} = (A_t - A_{t-1}) + (\tau_t - \tau^\star(A_t, i_t)).$$

This equation shows that an increase in the monetary base arises as a result of an open market operation where the monetary authority purchases bonds (that is, $A_t - A_{t-1} > 0$) or a remittance to the fiscal authority τ in excess of $\tau^\star$. We refer to the latter as a *helicopter drop*:

Definition 3 (Helicopter Drops)**.** A private sector equilibrium features *a helicopter drop* at time t if $\tau_t > \tau^\star(A_t, i_t)$.

Recall that $N_t = A_t - M_t$; thus, helicopter drops necessarily reduce the nominal net worth of the monetary authority as they increase M_t without a corresponding increase in A_t.

A first result is that the presence of helicopter drops does not expand the set of allocations that constitute a private sector equilibrium consistent with balance sheet constraints. Specifically, if there is a helicopter drop at some t (i.e., $\tau_t > \tau^\star$), then there exists an alternative policy without helicopter drops (i.e., $\tau_t = \tau^\star$) that is also consistent with all equilibrium conditions and where the balance sheet constraints are satisfied. The following lemma formalizes this result.

Lemma 2 (Irrelevance of Helicopter Drops)**.** *Consider a private sector equilibrium with helicopter drops* $\{y_t, \pi_t, i_t, P_t, M_t, A_t, B_t, \tau_t, T_t\}$ *that is consistent with balance sheet constraints. Let* $\{\hat{\tau}_t, \hat{A}_t, \hat{B}_t\}$ *be such that* $\hat{\tau}_t = \tau^\star(\hat{A}_t, i_t)$, $\hat{A}_t = M_t - M_{-1} + A_{-1}$, *and* $\hat{B}_t = B_t + \hat{A}_t - A_t$. *Then,* $\{y_t, \pi_t, i_t, P_t, M_t, \hat{A}_t, \hat{B}_t, \hat{\tau}_t, T_t\}$ *is also a private sector equilibrium consistent with balance sheet constraints.*

Proof. Given $\hat{\tau}_t$ and $\hat{A}_t$, the budget constraint of the monetary authority is

$$\hat{A}_t = M_t - M_{t-1} + \hat{A}_{t-1} = M_t - M_{-1} + A_{-1},$$

where the last follows from repeated substitution of the value of $\hat{A}$.

For the balance sheet constraints, letting $\hat{A}_{-1} = A_{-1}$, we have

$$\hat{A}_t - \hat{A}_{t-1} = M_t - M_{t-1} \geq M_t - M_{t-1} - (\tau_t - \tau^\star(A_t, i_t)) = A_t - A_{t-1},$$

where we use that the original sequence satisfies $\tau_t \geq \tau^\star(A_t, i_t)$. Thus $\hat{A}_t - \hat{A}_{t-1} \geq A_t - A_{t-1}$. Given that $\hat{A}_{-1} = A_{-1}$, it follows that $\hat{A}_t \geq A_t$ and thus condition (3) holds at the new sequence given that it holds at the old. For the new sequence, condition (2) holds with equality. Thus, the balance sheet constraints are satisfied.

The budget constraint of the fiscal authority holds, given the construction of $\hat{B}_t$. Given that $B_t - A_t = \hat{B}_t - \hat{A}_t$. the transversality condition (8) holds given that it was holding at the initial equilibrium.

The new sequence then satisfies all of the conditions for a private sector equilibrium. □

The argument for why helicopter drops do not expand the set of attainable allocations for the monetary authority is as follows: Suppose that starting from an original equilibrium, the monetary authority has increased M_t at some time t while simultaneously setting $\tau_t > \tau^\star$. Such a policy reduces the monetary authority's net worth at time t (this is a "helicopter drop"). Consider instead an alternative in which the monetary authority conducts an open market operation at time t; that is, it increases M_t by the same amount but uses the proceeds to purchase assets, A_t, while leaving $\tau_t = \tau^\star$. The lemma above establishes that this new policy is also consistent with the same original equilibrium outcome for $\{y_t, \pi_t, i_t, P_t, M_t\}$ and consistent with balance sheet constraints.[13]

We highlight that *the reverse of Lemma 2 does not hold.* That is, the claim that a helicopter drop can be introduced without affecting the equilibrium allocation is not generally valid. A helicopter drop reduces net worth, which may lead to a violation of the balance sheet constraint (4), implying that the original allocation can no longer be supported as an equilibrium.[14]

We have established that the ability to implement helicopter drops does not expand the set of private sector equilibria, even in the presence of balance sheet constraints. But do balance sheet constraints matter at all (even in the absence of helicopter drops) for the set of private sector equilibria?[15] The answer to this question depends on the initial networth of the monetary authority. To see this, ignore the balance sheet constraints. Take any allocation $\{y_t, \pi_t, i_t\}$ that satisfies the Euler equation, the Phillips curve, and the zero lower bound constraint. Then, as long as the monetary authority is well capitalized, we can find a policy under which this allocation constitutes a private sector equilibrium consistent with balance sheet constraints.

[13] In the Lemma, the fiscal authority changes B_t to adjust the budget constraint under the alternative policy. In this case, it could well be the case that the new sequences of $\hat{B}_t$ and $\hat{A}_t$ grow without bounds, but $M_t + \hat{B}_t - \hat{A}_t$, which is the crucial object for the household's transversality condition remains unchanged for all t. Note that we could have had the fiscal authority arbitrarily change taxes T_t in response to the removal of the helicopter drop. This would also work as long as the overall fiscal policy guarantees that the transversality condition of the households remains satisfied after the change.

[14] On the other hand, in the absence of balance sheet constraints, introducing a helicopter drop does not require a change to the rest of the equilibrium allocation, as in the standard New Keynesian model. .

[15] Although related, our analysis is different from Benigno and Nisticò (2020) analysis of the neutrality of the monetary authority's balance sheet. They consider an environment where the transfer policy is fixed and the monetary authority chooses an interest on reserves and an asset portfolio. In this environment, they analyze whether a sequence $\{\pi_t, i_t\}$ is a private sector equilibrium for a given transfer/balance sheet policy. In our case, we examine an endogenous transfer, and we ask instead whether the sequence $\{y_t, \pi_t, i_t\}$ constitutes a private sector equilibrium for at least one such policy.

Lemma 3 (Sufficient net worth and balance constraints). *Suppose that $N_{-1} = A_{-1} - M_{-1} \geq 0$. Consider a sequence $\{y_t, \pi_t, i_t\}$ that satisfies the Euler equation, (10), the price setting equation, (9), and the zero lower bound constraint, $i_t \geq 0$. Let $\{P_t\}$ be the corresponding price level given $\{\pi_t\}$ and P_{-1}. Then, there exists a policy $\{M_t, A_t, B_t, \tau_t, T_t\}$ such that $\{y_t, \pi_t, i_t, P_t, M_t, A_t, B_t, \tau_t, T_t\}$ is a private sector equilibrium consistent with balance sheet constraints.*

Proof. The proof is constructive. For $t \geq 0$, let

$$\begin{aligned}
M_t &= P_t L(e^{y_t}, i_t) \\
A_t &= M_t + A_{-1} - M_{-1} \\
\tau_t &= \tau^\star(A_t, i_t) \\
B_t &= A_t - M_t \\
T_t &= \frac{B_t}{1 + \imath_t} + \tau_t - B_{t-1}
\end{aligned}$$

Condition (i) of the private sector equilibrium definition holds, as the household budget constraint, (6), holds with $W_t = B_t - A_t$, the Euler equation (10) holds by assumption, by construction the money demand condition, (11), holds as well, and the private sector total net financial wealth (inclusive of money holdings) $M_t + B_t - A_t = 0$ for all $t \geq 0$, implying that the transversality condition (8) holds. Conditions (ii) and (iii) hold. And by construction, the budget constraints of the monetary and the fiscal authority hold as well. The sequence constitutes a private sector equilibrium. Given that $A_{-1} - M_{-1} \geq 0$, it follows that $A_t \geq 0$, as $M_t \geq 0$ and (3) holds. Finally, by construction condition (2) holds with equality. Thus, the balance sheet constraints are satisfied. □

Lemma 3 thus tells us that, as long as the monetary authority's initial net worth is non-negative and the monetary authority does not introduce helicopter drops, any sequence that satisfies the Euler equation, the Phillips curve, and the zero lower bound constitutes a private sector equilibrium for some policy.

2.3 Policy Objectives

In the previous section, we analyzed the extent to which helicopter drops may affect the set of competitive equilibrium allocations. However, we have not yet addressed how policies are chosen by the monetary and fiscal authorities. Our next goal is to analyze how these policies are determined. To that end, we begin by describing the policy objectives.

Monetary Policy Objective. We assume that the monetary authority seeks to minimize departures from zero for both inflation and the output gap. Specifically, the monetary authority evaluates welfare according to the following objective function:

$$\sum_{t=0}^{\infty} e^{-\rho t}\mathscr{U}(\pi_t, y_t) \tag{12}$$

where $\mathscr{U}(\pi_t, y_t)$ encapsulates the objective of keeping inflation and output close to the target and is strictly maximized at $\mathscr{U}(0, 0)$.

Fiscal Policy Objective. The fiscal authority is assumed to select a path of $\{B_t, T_t\}$ such that its sequential budget constraint holds at all times, and the consolidated debt of the government satisfies the following condition:

$$\lim_{t\to\infty} \frac{B_t + M_t - A_t}{\prod_{s=0}^{t}(1+\iota_s)} = 0, \tag{13}$$

Condition (13) ensures that the household transversality condition is satisfied and that the sequence of taxes chosen by the fiscal authority is neutral, leaving the rest of the equilibrium unaffected.[16]

3 Commitment Solution

Having specified the policy objectives, we start by analyzing the optimal monetary policy under commitment. The monetary authority's problem is as follows: the monetary authority maximizes its objective function by choosing a sequence $\{y_t, \pi_t, i_t, M_t, A_t, \tau_t\}$ while the fiscal authority selects a sequence $\{B_t, T_t\}$ such that $\{y_t, \pi_t, i_t, M_t, A_t, \tau_t, B_t, T_t\}$ constitutes a private sector equilibrium consistent with balance sheet constraints. In this case under commitment, we are assuming that the monetary authority chooses the equilibrium outcome as long as it satisfies the required equilibrium conditions.

Lemma 3 implies that if $A_{-1} \geq M_{-1}$, we can ignore balance sheets *at all times*, allowing us to focus solely on y_t, π_t, i_t when solving for the optimal policy under commitment. Henceforth, we will narrow attention to the case where the initial balance sheet constraint satisfies this condition, as it starkly showcases the difference between the commitment and the no-commitment solutions.

[16]Under this assumption, *the fiscal theory of the price level* does not play a role in our analysis, as the consolidated government sector follows a "Ricardian" fiscal policy in the terminology of Benhabib, Schmitt-Grohé and Uribe (2001a). Woodford (2001) and Kocherlakota and Phelan (1999) provide summaries of the fiscal theory of the price level, Bassetto (2002) performs a game-theoretical analysis of equilibrium selection issues, and Benigno and Nisticò (2025) studies helicopter drops operating through the fiscal theory of the price level. See Appendix C for further discussion.

Liquidity Trap under Commitment

We consider a situation where a liquidity trap may arise in period $t = 0$. Towards this, we let the discount factor disturbance, ξ_t, be

$$\xi_t = \begin{cases} \tilde{\xi} & \text{if } t = 0, \\ 0 & \text{otherwise.} \end{cases} \tag{14}$$

The monetary authority's objective is

$$\mathscr{U}(\pi, y) = -\left[\pi^2 + \varphi y^2\right],$$

for $\varphi \in [0, \infty)$.[17]

Assuming that the initial net worth is positive (so that the balance sheet constraints do not bind), the problem under commitment is in effect, a discrete-time version of Werning (2011), with the liquidity trap lasting for one period. (See also Eggertsson and Woodford (2003) and Jung et al. (2005) for related setups).

Numerical Results. Each period represents a quarter. We set the discount rate to $\rho = 0.01$ and the intertemporal elasticity of substitution to $\sigma = 0.5$. The value for the slope of the Phillips curve is set to $\kappa = 0.35$, which is in the middle range of those typically used in calibrations of New Keynesian models. The coefficient on the output gap in the loss function is set to $\varphi = 0.05$.[18] In addition, we set the shock to the natural rate to $\tilde{\xi} = -0.12$. In the solution without helicopter drops, this shock will generate a fall in output of 6%. Finally, we let money demand be approximated by

$$L(y, i) = \theta e^{\alpha y - \eta i}, \tag{15}$$

(although, as we discussed above in Lemma 3, this does not affect the commitment solution for y_t, π_t, i_t). We set the interest rate elasticity of money demand to $\eta = 0.5$, a standard value (see, e.g., Benati, Lucas, Nicolini and Weber, 2021), the income elasticity to $\alpha = 1$, and the intercept θ to match a currency-to-GDP ratio of 7%, resulting in $\theta = 0.27$.

Figure 1 presents the path for output, inflation, and the nominal interest rate. In line with existing results in the literature, the economy experiences deflation and a negative output gap

[17] This objective can be derived from a second-order approximation to welfare around an efficient steady-state with zero inflation under a cashless limit, following Woodford (2011).

[18] The small weight on output is standard in the literature. A second-order approximation of the welfare function in the canonical New Keynesian model delivers a relative weight of κ divided by the elasticity of substitution between differentiated inputs. Given the value of κ, our choice of φ would be consistent with an elasticity of substitution of 7, which is within the range of the values used in the literature.

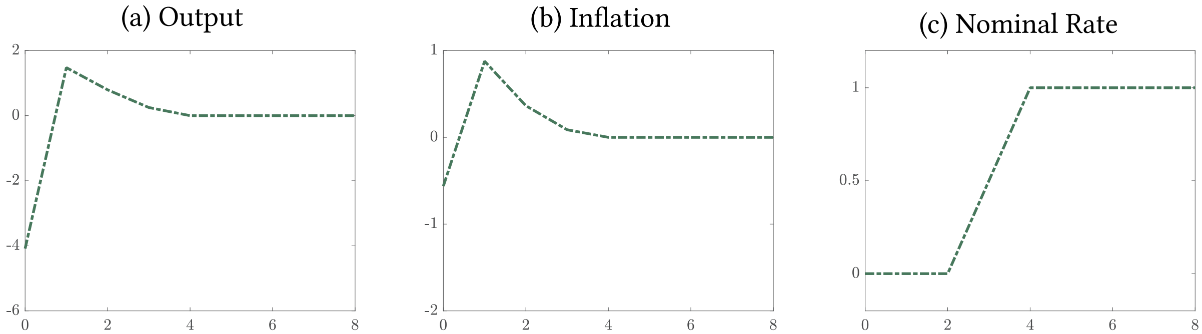

Figure 1: Simulation under Commitment

Note: Output gap, inflation, and the nominal interest rate are expressed as percentages. The x-axes represent time.

at $t = 0$. To help mitigate the effects of the liquidity trap, the monetary authority keeps nominal interest rates low beyond the duration of the liquidity trap. In the example presented, lift-off occurs after four periods, while the discount rate shock ξ_t returns to zero at $t = 1$.[19]

4 Policy without Commitment

We now proceed to the main analysis of the paper: the study of equilibria in which the monetary authority lacks commitment. We focus on Markov equilibria. This is the equilibrium concept used in several previous papers, such as Albanesi, Chari and Christiano (2003); Eggertsson (2003); Adam and Billi (2007); Armenter (2018); Nakata and Schmidt (2019). In what follows, we also narrow attention to the monetary authority's problem and assume that the fiscal authority selects a path of debt and taxes consistent with its budget constraint and the household's transversality condition.

4.1 The Stationary Case: Markov Equilibria

First, we consider equilibria after the discount rate shock has passed, that is $\xi_t = 0$ at all times. Given that all exogenous variables are constant, we refer to this as the "stationary case". But as we will see below, even in this stationary case, there are dynamics induced through the policy choices.

The state variables at time t are the inherited balance sheet positions, M_-, A_-, and the previous period price level, P_-. As usual in monetary models, it suffices to carry $M_-/P_- = m_-$ and $A_-/P_- = a_-$ as state variables.

[19] To complete the characterization of an equilibrium, we need to specify balance sheets and transfers. By Ricardian equivalence, these are not pinned down, but it suffices to use the sequence described in the proof of Lemma 3.

In a Markov equilibrium, the monetary authority takes as given the private sector expectation functions of next-period's output and inflation. At the beginning of the period, the monetary authority inherits its balance sheet, m_- and a_-, and chooses y, π, i, m, a, and $\tilde{\tau} = \tau/P_-$ to maximize its objective function, subject to the Euler equation, the Phillips curve, the zero lower bound constraint, the money market clearing condition, its budget constraint, and the two balance sheet constraints. The budget constraint and balance sheet constraints (1)-(3) can be written as

$$e^{\pi-i}a + m_- + \tilde{\tau} = e^{\pi}m + a_-; \quad a \geq 0; \quad \text{and } \tilde{\tau} \geq e^{\pi}(1-e^{-i})a.$$

Defining $n \equiv a_- - m_-$, and $n' \equiv a - m$, the two balance sheet constraints are equivalent to

$$m \geq -n'; \quad n \geq e^{\pi}n'.$$

The next step is to notice that the inherited values of m_- and a_- do not affect any other constraint, and thus we can simply use real net worth, n, as the only state variable.[20]

We let $V(n)$ denote the value function of the monetary authority as a function of its net worth and Ω its domain set. The functions $\mathcal{Y}(n)$ and $\Pi(n)$ denote the private sector expectations of future output and inflation.

The monetary authority's problem can be formulated as follows.

Problem 1 (The Stationary Case)**.**

$$V(n) = \max_{(y,\pi,i,n'\in\Omega)} \mathcal{U}(\pi, y) + \beta V(n')$$

subject to:

$$y = \mathcal{Y}(n') - \sigma(i - \Pi(n') - \rho) \tag{16}$$

$$\pi = \beta\Pi(n') + \kappa y \tag{17}$$

$$i \geq 0, \tag{18}$$

$$L(e^{y}, i) \geq -n' \textit{ if } i > 0 \tag{19}$$

$$n' \leq e^{-\pi}n. \tag{20}$$

The first three constraints are standard ones: (16) and (17) are the Euler and Phillips curve, respectively, while (18) is the zero lower bound on the nominal interest rate. Constraint (19) is the balance sheet constraint ($a \geq 0$) with the money market equilibrium, that is, $m \geq -n'$. Finally,

[20] Notice that an implication of these two constraints is that when $i = 0$, an open market operation by which the monetary authority increases a and m does not affect net worth, and thus will be irrelevant. As we will see, this irrelevance does not apply to helicopter drops.

constraint (20) is the bound on net worth implied by the lack of fiscal support ($\tau \geq \tau^\star$).

The last two constraints of Problem 1 capture the key innovation of our model. Together they imply $e^{\pi} L(e^y, i) \geq -n$. Because L is decreasing in i and increasing y, this means that a monetary authority that has not enough net worth may need to maintain low nominal rates or generate a large output expansion or high inflation to satisfy its balance sheet constraints.

Further inspection of the monetary authority's problem shows that the inherited net worth, n, only appears in the last constraint. It follows then that increases in n relax the monetary authority's problem, which guarantees that the value function $V(n)$ must be (weakly) increasing in net worth. The result that the value function is monotonic in n raises the question of why would the monetary authority ever conduct a helicopter drop that would deplete its net worth and *thus lower its continuation value*. The answer has to do with how, during a liquidity trap, a low net worth induces higher inflation and output in the future, affecting private sector expectations, as captured by the $\mathcal{Y}$ and Π functions. We will return to this point below once we consider the liquidity trap episode.

Let $\mathcal{N}(n)$ denote the policy function for the evolution of net worth. With this, we are ready to define a Markov equilibria.

Definition 4. A *Markov equilibrium* (in the stationary case) is given by a closed interval $\Omega \subset \mathbb{R}$ and functions $V : \Omega \to \mathbb{R}, \mathcal{Y} : \Omega \to \mathbb{R}, \Pi : \Omega \to \mathbb{R}, \mathcal{N} : \Omega \to \Omega$ such that V solves the monetary authority's problem given $\mathcal{Y}$ and Π for all $n \in \Omega$, and $\mathcal{Y}$, Π, $\mathcal{N}$ are optimal policy functions for output, inflation, and real net worth.

We can now show that given any initial $n_{-1} \in \Omega$ and some initial B_{-1}, the resulting sequence of aggregates constitutes a private sector equilibrium. To see this, iterate the policy function for net worth, $\mathcal{N}$, to obtain a sequence of $\{n_t\}$ starting from n_{-1}. Let $\{y_t, \pi_t\}$ denote the associated sequence of output and inflation given the policies $\mathcal{Y}, \Pi$. We let $\{i_t\}$ be uniquely defined by $\mathcal{Y}(n_t) = \mathcal{Y}(n_{t+1}) - \sigma(i_t - \Pi(n_{t+1}) - \rho)$. For $i_t > 0$, we let $M_t = P_t L(e^{y_t}, i_t)$. For $i_t = 0$, we let $M_t = P_t \max\{L(e^{y_t}, i_t), -n_{t+1}\}$. We let $A_t = P_t n_t - M_t$. Finally $\tau_t = M_t - P_{t-1} n_{t-1} - \frac{1}{1+\iota_t} A_t$. We can then set $B_t = A_t - M_t$, and $T_t = \frac{B_t}{1+\iota_t} + \tau_t - B_{t-1}$, with B_{-1} given. The sequence $\{y_t, \pi_t, i_t, P_t, M_t, A_t, B_t, \tau_t, T_t\}$ we have just constructed *constitutes a private sector equilibrium consistent with balance sheet constraints.*

Let us define $n^\star = -L(1, \rho) < 0$ and assume that $n^\star \in \text{interior}(\Omega)$. We can see that for $n \geq n^\star$, the best allocation ($\pi_t = 0$ and $y_t = 0$ at all t) constitutes a Markov equilibrium. Indeed, the monetary authority's net worth remains constant and choosing $\pi = 0, y = 0$ is optimal given that this allocation achieves the maximum welfare possible and there are no incentives for the monetary authority to deviate from it at any date if future monetary authorities follow this strategy. Note that in this case, $i_t = \rho > 0$, and thus the zero lower bound constraint is automatically satisfied.

The environment we are analyzing, however, has the potential for multiple Markov equilibria, a characteristic that has been highlighted by Albanesi, Chari and Christiano (2003); Armenter (2018); Nakata and Schmidt (2019). In particular, Armenter (2018) and Nakata and Schmidt (2019) show that there exists a "deflationary trap": in our case $\pi = -\rho, y = -\rho(1-\beta)/\kappa$. If the private sector expects these levels of inflation and output tomorrow, then setting $i = 0$ delivers them as outcomes today per equations (16) and (17). And with these expectations, there is not much the monetary authority can do, as raising the nominal interest rate above zero further lowers inflation and reduces output (and thus welfare).[21]

We restrict attention to Markov equilibria where the monetary authority implements the good equilibrium when $n > n^\star$.

Definition 5. An equilibrium is *good* if for $n \geq n^\star$, $\mathcal{Y}(n) = \Pi(n) = 0$ and $\mathcal{N}(n) \geq n$.

In our model, however, when $n < n^\star$, the first-best allocation $\pi = 0, y = 0$ is not a feasible choice because the monetary authority lacks sufficient net worth to implement it. One feasible policy is to generate a sufficiently high level of inflation to ensure that $n' = n^\star$, achieving the first-best allocation from the next period onward. This policy, however, imposes a large inflation cost in the initial period. The optimal policy must therefore balance the tradeoff between inducing higher inflation to improve the continuation value and the costs of a larger departure from the $\pi = 0, y = 0$ allocation, which in turn depend on households' expectations about future policies.

In what follows, we characterize the monetary authority's tradeoff and the Markov equilibrium.

Log-linear Equilibria in the Stationary Case

We show next that when money demand takes the form in (15), the model allows for an analytical characterization of Markov equilibria. We restrict attention to good equilibria that are log-linear for $n < n^\star$:

Definition 6. A good equilibrium is a *log-linear equilibrium* in a closed interval Ω if

[21] The existence of this deflationary trap is anticipated by the self-fulfilling liquidity traps of Benhabib et al. (2001b; 2002) in their analysis of a monetary authority that adheres to a (non-optimizing) Taylor rule but faces a zero lower bound constraint on nominal rates.

$n^\star \in \text{interior}(\Omega)$ and for $n \in \Omega$ and $n < n^\star$,

$$\mathcal{Y}(n) = a(\log(-n) - \log(-n^\star)),$$
$$\Pi(n) = b(\log(-n) - \log(-n^\star)), \text{ and}$$
$$\log(-\mathcal{N}(n)) - \log(-n^\star) = \phi_n(\log(-n) - \log(-n^\star)),$$

for some scalars a, b, and $\phi_n \in [0, 1]$.

The coefficients a, b, and ϕ_n describe respectively the effect of the monetary authority's net worth on output, inflation and the evolution of net worth.[22] Before discussing existence and uniqueness of log-linear equilibria, let us define the following values:

Definition 7. Let $x^\star$ be the value $x \in (0, 1)$ such that

$$1 = x\left(\frac{\eta(1+\sigma\kappa)+\sigma\kappa}{\eta+\alpha\sigma} + 1 + \beta\right) - x^2\left(\frac{\eta(1+\sigma\kappa+\beta)}{\eta+\alpha\sigma} + \beta\right) + x^3\left(\frac{\beta\eta}{\eta+\alpha\sigma}\right). \tag{21}$$

And define $\underline{n}$ to be

$$\underline{n} \equiv \begin{cases} n^\star \exp\left(-\frac{\phi_i}{\rho}\right) & \text{if } \phi_i < 0, \\ -\infty & \text{otherwise} \end{cases}$$

where

$$\phi_i \equiv -\frac{(1-x^\star)[1-(1+\beta+\kappa\sigma)x^\star + \beta(x^\star)^2]}{\kappa\sigma}.$$

There exists a unique value of $x^\star \in (0, 1)$ that satisfies equation (21).[23] Given that $n^\star < 0$, it follows that in general, $\underline{n} < n^\star$. We can use these values to uniquely characterize the log-linear equilibrium: the value of $x^\star$ maps into ϕ_n and $\underline{n}$ represents the value of net worth at which $i = 0$ in the log-linear equilibrium.

Proposition 1 (Existence and uniqueness of a log-linear equilibrium)**.** *Let* $\Omega = [n_0, n_1]$ *with* $\underline{n} < n_0 < n^\star < n_1$*, and money demand is given by* (15) *with* $\alpha \geq 0$ *and* $\eta > 0$*. Then, there*

[22] Notice that the definition restricts attention to $\phi_n \in [0, 1]$, which rules out equilibria where the monetary authority's net worth converges to $-\infty$. In Appendix C, we discuss how this equilibria can be ruled out by a fiscal condition such that the fiscal authority does not accumulate unbounded assets on the private sector

[23] To see this, note that the right-hand side takes a value of zero at $x = 0$ and a value strictly larger than 1 for $x = 1$, and thus by continuity, a root exists. Uniqueness follows from showing that the right-hand side is concave in x for $x \in (0, 1)$.

exists a unique log-linear equilibrium in Ω*, where*

$$\phi_n = x^\star, a = (1 - x^\star)(1 - \beta x^\star)/\kappa, b = (1 - x^\star).$$

The equilibrium interest policy associated with net worth n *is*

$$\mathcal{I}(n) = \phi_i[\log(-n) - \log(-n^\star)] + \rho,$$

and there are no helicopter drops. That is, $\mathcal{N}(n) = e^{-\Pi(n)}n$ *for* $n < n^\star$*.*

Proof. In Appendix A.2. □

Proposition 1 shows that the value of $x^\star$ determines the rate at which net worth converges to $n^\star$. A crucial result here is that both the coefficients a and b in the unique log-linear equilibrium are strictly positive, which implies that higher net worth leads to lower output and lower inflation. This result shows that a sufficiently large helicopter drop (a reduction in n) leads to higher inflation and output in the future. The behavior of nominal rates with respect to net worth is in general ambiguous and depends on parameters. That is, the nominal interest rate could increase or decrease with n according to the strength of the income elasticity of money demand, α. A particularly illustrative example arises when $\alpha = 0$ (no income elasticity). In this case, $\phi_i = -x^\star/\eta < 0$, and the nominal interest rate decreases when n falls.[24]

The proposition also establishes that helicopter drops are not used in the stationary case. The intuition for this result is as follows: Given that $n < n^\star$, in equilibrium, the monetary authority necessarily chooses positive inflation. If the monetary authority were to engineer a helicopter drop, it could achieve the same level of output with a smaller helicopter drop (increasing its continuation value) and a lower interest rate. This satisfies the balance sheet constraints, generates less inflation, and is feasible as long as $i > 0$. Thus, the alternative policy constitutes an improvement relative to the solution with a helicopter drop.

We can use the closed-form solution obtained in Proposition 1 to characterize how the log-linear equilibrium responds to changes to parameter values:

Lemma 4. *The value of* $x^\star$ *is decreasing in* β *and* κ*, and increasing in* α*. When* $\alpha = 0$*, the value of* $x^\star$ *is increasing in* η *and decreasing in* σ*.*

[24] There is another interesting example. When $\alpha = 1$ and $\sigma = 1$, then $\phi_i = 0$. In this case, the nominal rate is independent of net worth. When $\alpha = 1$, the nominal rate is increasing with net worth for values of $\sigma < 1$, and decreasing for $\sigma > 1$. This result, together with the case for $\alpha = 0$, is shown in Appendix B.

Proof. In Appendix A.3. □

Extending the domain below $\underline{n}$ (when $\underline{n}$ is finite) requires us to move beyond the class of log-linear equilibria. Although we are not able to analytically characterize the equilibrium for this larger domain in this case, it is indeed possible to characterize it numerically. Figure 2 presents the results that arise in such a simulation. (We use the same parameter values as in the previous section.) The x-axis in each panel is a transformation of net worth, $-\log(-n)$. In each panel, there are two vertical lines. The first one represents the value of $\underline{n}$, and the second one represents $n^{\star}$. Panels (a), (b), and (c) display output, inflation, and the nominal interest rate as a function of net worth, respectively. Panels (e), and (f) display the policy function for net worth and the value function, respectively.

The simulation is solved without imposing log-linearity (solid blue line), but the figure also displays the log-linear equilibrium prediction (dashed red line). For values of $n \geq n^{\star}$, the good equilibrium is attained, as expected, and all policies are constant. For values of n between $\underline{n} < n < n^{\star}$, the equilibrium is indeed the log-linear one (the linearity can be visually observed in the plots and the solid lines coincide with the dashed ones). For values $n < \underline{n}$, the equilibrium is no longer log-linear (the solid lines deviate from the dashed lines), but it inherits the same monotonic properties of the latter.

For $n < \underline{n}$, welfare is *lower* under the projected log-linear equilibrium, compared with welfare under the actual solution. This occurs because the log-linear equilibrium is solved under the binding balance sheet constraint (19). That is, for $i > 0$, the monetary authority must ensure that monetary liabilities do not fall short of negative net worth. However, once the monetary authority sets $i = 0$, which occurs at $n = \underline{n}$, the constraint (19) no longer binds because households are now indifferent between any combination of bonds and money. The monetary authority is still subject to the remittance constraint (20), but it faces a more relaxed problem. As a result, it reduces inflation and achieves a level of output closer to the efficient level. In other words, in the stationary environment, $i_t \geq 0$ is not a binding constraint.

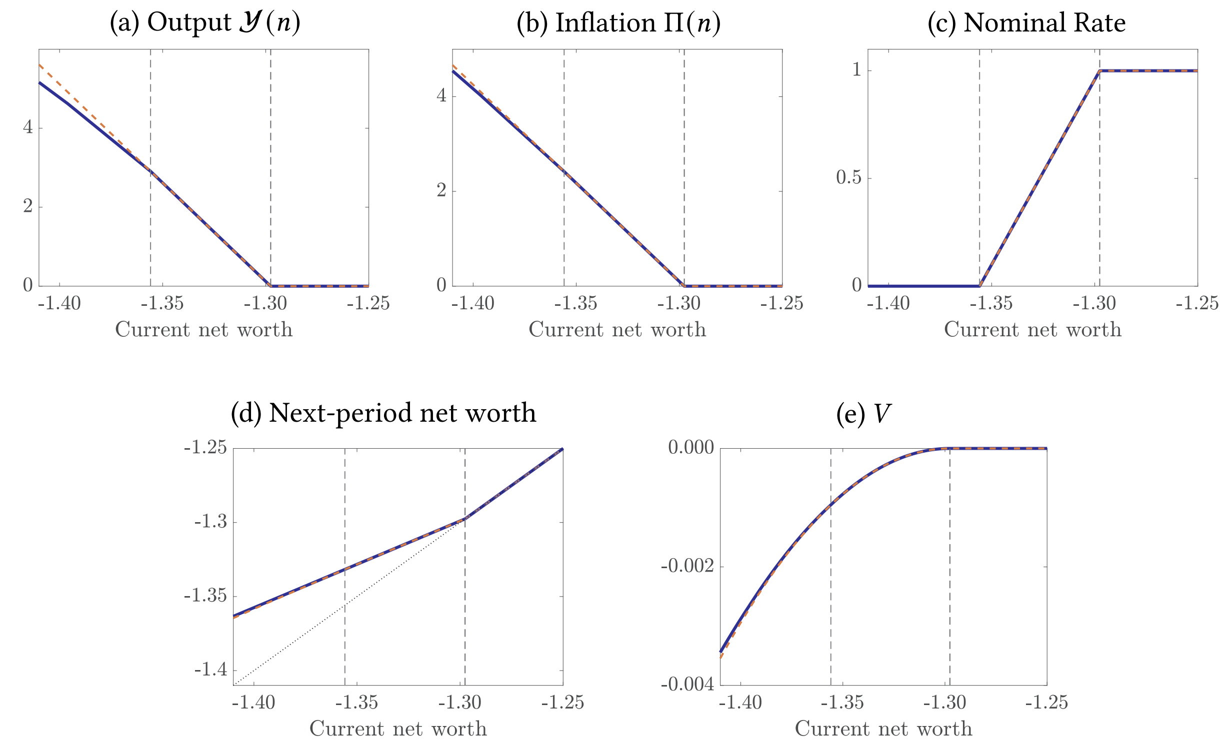

Figure 2: Stationary Environment and Markov Equilibrium

Note: Net worth in this figure is defined to be $-\log(-n)$. The blue solid line in the figure shows the equilibrium policies y, π, i, n' and value V as a function of net worth, n. The red-broken line denotes the values from the log-linear equilibrium. The two vertical lines denote $\underline{n}$ and $n^{\star}$. Output, inflation, and the nominal rate are expressed as percentages.

4.2 The Liquidity Trap

We now let the path of ξ_t be as in (14): temporarily low at $t = 0$ and zero afterwards. Note that from period $t = 1$ onward, the equilibrium is as described in the stationary environment section, and thus we can use the previously computed $\mathcal{Y}(n)$, $\Pi(n)$, and $V(n)$ as the continuation policies and value function.

We are interested in the Markov equilibrium starting from period $t = 0$, the period of the liquidity trap. In this period, the problem of the monetary authority is given by:

Problem 2 (The Liquidity Trap Problem)**.**

$$V_0(n) = \max_{(y,\pi,i,n'\in\Omega)} \mathcal{U}(\pi, y) + \beta V(n')$$

subject to:

$$y = \mathcal{Y}(n') - \sigma(i - \Pi(n') - \rho - \tilde{\xi}),$$

$$\pi = \beta\Pi(n') + \kappa y,$$

$$i \geq 0,$$

$$L(e^y, i) \geq -n' \text{ if } i > 0,$$

$$n \geq e^{\pi} n'$$

Notice that the only difference with Problem 1 from the stationary case is the presence of the discount factor shock $\tilde{\xi}$ in the Euler equation.

We will assume, as in the commitment analysis, that the initial net worth is sufficiently high so that the balance sheet constraints do not bind at time $t = 0$. (For simplicity, we set net worth to zero.) Given that the last two constraints in Problem 2 do not bind, we obtain the following optimality condition with respect to n_1

$$\mathcal{U}_\pi \left[\beta\Pi'(n_1) + \kappa(\mathcal{Y}'(n_1) + \sigma\Pi'(n_1))\right] + \mathcal{U}_y \left[\mathcal{Y}'(n_1) + \sigma\Pi'(n_1)\right] = -\beta V'(n_1), \tag{22}$$

This condition outlines the key trade-off the monetary authority faces during a liquidity trap. A helicopter drop (i.e., a lower n_1) raises expectations of inflation and output, as discussed in the previous section, which in turn boosts current inflation and output. To the extent that in a liquidity trap $\pi < 0$ and $y < 0$, a helicopter drop generates positive benefits today. However, as shown in the previous section, the continuation value $V(n_1)$ increases with net worth, indicating that there are costs associated with conducting a helicopter drop. Condition (22) illustrates how the monetary authority, at the optimum, balances the marginal benefits of improving macroeconomic outcomes today with the marginal costs of worse future outcomes.

Optimal Helicopter Drop. Using the same numerical parameters as in the previous sections, we compute the optimal helicopter drop during a liquidity trap. Figure 3 illustrates the potential equilibrium outcomes across a range of possible net worth choices. The solid dot indicates the optimal policy selected by the monetary authority.

The top panels of Figure 3 illustrate the effects of a helicopter drop on output and inflation at $t = 0$. The vertical line in the figure represents the value of next-period net worth that allows the monetary authority to achieve zero inflation and a zero output gap in the stationary case. As shown in the figure, to the left of the vertical line, a lower choice of net worth (corresponding to a larger helicopter drop) generates higher inflation and output at $t = 0$. As highlighted in Proposition 1, a lower net worth implies higher inflation and output in the stationary case. Therefore, a larger helicopter drop at $t = 0$ stimulates aggregate demand, enabling the monetary authority to increase output and inflation during the liquidity trap.

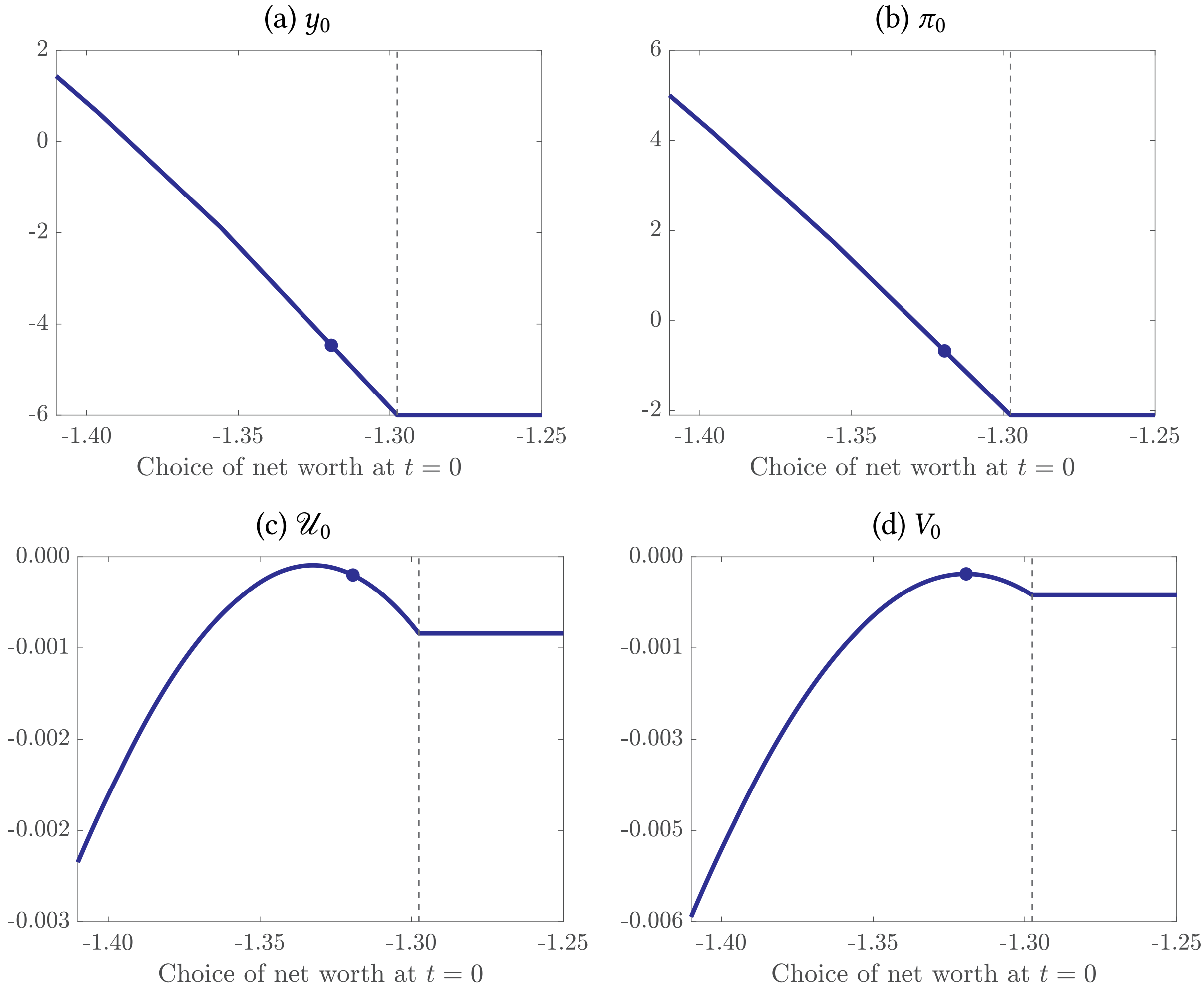

Figure 3: Liquidity Trap and the Markov Equilibrium

Note: This figure shows allocations and welfare in the liquidity trap as a function of possible choices of net worth chosen at $t = 0$. The solid dot represents the optimal choice of the helicopter drop. Net worth in this figure is defined to be $-\log(-n)$. Output and inflation are expressed as percentages.

The bottom panels of Figure 3 illustrate the trade-off faced by the monetary authority. Panel (d) shows how the optimal helicopter drop maximizes the value function during the liquidity trap, while panel (c) presents the monetary authority's objective in the liquidity trap, $\mathscr{U}_0$. As the figure illustrates, maximizing this objective requires a helicopter drop larger than the optimal one. This occurs because, while a larger helicopter drop improves the allocation in the liquidity trap by increasing y_0 and π_0, it also raises future losses from $t = 1$ onward.

Commitment versus Markov Equilibrium. We now compare the simulation outcomes over time of the economy under commitment with the simulations for the Markov perfect equilibrium with and without helicopter drops. In all cases, we initialize the economy at $t = 0$ with a high enough net worth so that balance sheet constraints do not initially bind, and we feed the economy with the same shock $\tilde{\xi}$ used above.

Figure 4 presents the results. The red dashed line represents the Markov perfect equilibrium

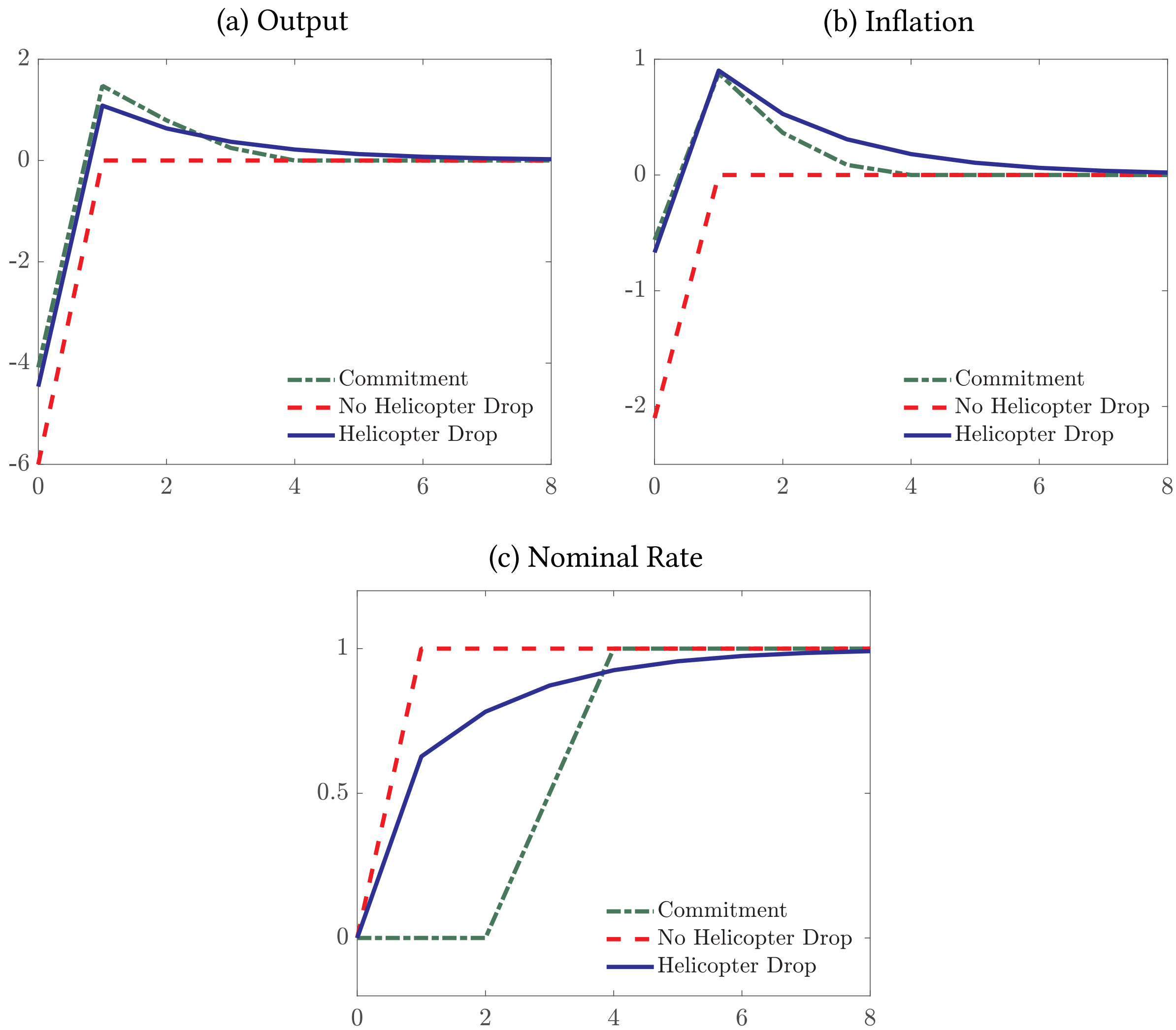

Figure 4: Simulation Comparison

Note: This figure shows the simulations for the three economies starting from a liquidity trap at $t = 0$. All variables in the y-axes are expressed as percentages. The x-axis represents time.

without helicopter drops. This economy features the standard outcome of $\pi_t = 0, y_t = 0$ from $t = 1$ onward. One can see that this economy experiences a much larger recession and deflation at $t = 0$ compared to the commitment solution. This is because when the government can commit, it promises inflation and an output boom from $t = 1$ onward, which mitigates the recession at $t = 0$.

The Markov perfect equilibrium with helicopter drops is represented by the solid blue line. As the figure illustrates, helicopter drops are very effective in alleviating the liquidity trap. Compared to the equilibrium without helicopter drops, the monetary authority reduces the output gap from -6% to -4% and deflation from -2.2% to -0.5%. As explained above, this effectiveness arises because the use of helicopter drops leads households to anticipate higher output and inflation in the future, thereby stimulating aggregate demand during the liquidity trap. Notably, the optimal helicopter drop (approximately 2.5% of GDP) achieves almost the same degree of output and inflation stabilization in the liquidity trap as the commitment solution. However, helicopter drops result in a prolonged period with output and inflation significantly above target.

4.3 Discussion and Policy Implications

The recurrence of ZLB episodes in advanced economies has led to proposals to incorporate helicopter drops of money into the central bank's policy toolkit.[25] These proposals implicitly assume that once the central bank increases the money supply, it will remain elevated, thereby stimulating inflation and alleviating the liquidity trap. Our paper provides a theory of how helicopter drops can serve as a stabilization tool by articulating how the presence of balance sheet constraints can make permanent increases in money supply credible.

Before we conclude, we would like to highlight a few caveats and observations. In terms of implementation, the use of helicopter drops raises of course several challenges. One challenge is that central banks do not currently have the legal authority to provide direct transfers to households. An implication of our analysis, however, is that a transfer from the monetary authority to the fiscal authority has identical effects to a transfer from the monetary authority to households. To the extent that central banks have some discretion in remitting funds beyond their operating profits, this would effectively make helicopter drops easier to implement.

One caveat from our analysis is that we have taken as given the institutional features that give rise to the balance sheet constraints. In particular, we have assumed that such institutional features prevent the fiscal authority from recapitalizing the monetary authority, leading to a strict limit on central banks' net worth. Although our theoretical insights are likely to extend in the case where there are costly recapitalizations, a quantitative analysis on implementation of helicopter

[25] While our analysis centers on a time inconsistency problem arising from the ZLB, the results can be extended to other sources of deflationary bias such as adverse cost-push shocks.

drops—and an integration to standard monetary policy frameworks—would require a more precise measure of balance sheet constraints.[26]

Finally, like any commitment device, helicopter drops may involve a tradeoff between commitment and flexibility. In particular, while helicopter drops can help to credibly raise inflation expectations, it does so by reducing the flexibility of future monetary policy. This may become costly for the central bank if it later needs to reverse the monetary expansion in a response to a shock.

5 Conclusion

We develop an analytically tractable New Keynesian monetary model in which the monetary authority operates under balance sheet constraints. We show that while conventional open market operations are irrelevant in a liquidity trap, helicopter drops are not. When the monetary authority lacks commitment, helicopter drops become a desirable tool for stabilization at the zero lower bound. By reducing the monetary authority's net worth, helicopter drops constrain its ability to reverse the initial increase in the money supply, enabling it to credibly commit to a more expansionary policy and to mitigate the severity of liquidity traps.

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[26]In discussing the implementation challenges from helicopter drops, Bernanke (2016) argues that "However, under certain extreme circumstances—sharply deficient aggregate demand, exhausted monetary policy, and unwillingness of the legislature to use debt-financed fiscal policies—such programs may be the best available alternative. It would be premature to rule them out."

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A Proofs

A.1 Proof of Lemma 1

The argument here follows Buiter and Sibert (2007), except for the use of a different No-Ponzi condition. Let $\{C_t^\star.M_t^\star, W_t^\star\}$ be a sequence that satisfies the conditions in the Lemma. Let $\{C_t, M_t, W_t\}$ be any alternative sequence.

Define D and E to be

$$D \equiv \lim_{T\to\infty} \inf \sum_{t=0}^{T} \beta(t) \left[u(C_t) + v(M_t/P_t) - u(C_t^\star) - v(M_t^\star/P_t^\star) \right],$$

$$E \equiv \lim_{T\to\infty} \inf \sum_{t=0}^{T} \beta(t) \left[u'(C_t^\star)(C_t - C_t^\star) + v'(M_t^\star/P_t^\star) \frac{M_t - M_t^\star}{P_t} \right],$$

where by concavity, it follows that $D \le E$.

Using the budget constraint and rearranging, we have that

$$E = \lim_{T\to\infty} \inf \sum_{t=0}^{T} \beta(t) \left[u'(C_t^\star) \left(\frac{W_{t-1} + M_{t-1} - \frac{W_t}{1+\iota_t} - M_t}{P_t} - \frac{W_{t-1}^\star + M_{t-1}^\star - \frac{W_t^\star}{1+\iota_t} - M_t^\star}{P_t} \right) \right.$$
$$\left. + v'(M_t^\star/P_t^\star) \frac{M_t - M_t^\star}{P_t} \right],$$

$$= \liminf_{T\to\infty} \left\{ \sum_{t=-1}^{T-1} \beta(t+1) u'(C^\star_{t+1}) \left(\frac{W_t + M_t}{P_{t+1}} - \frac{W^\star_t + M^\star_t}{P_{t+1}} \right) \right.$$
$$\left. + \sum_{t=0}^{T} \beta(t) \left[u'(C^\star_t) \left(\frac{\frac{W^\star_t}{1+\iota_t} + M^\star_t}{P_t} - \frac{\frac{W_t}{1+\iota_t} + M_t}{P_t} \right) + v'(M^\star_t / P^\star_t) \frac{M_t - M^\star_t}{P_t} \right] \right\}.$$

Using the Euler equation and the money demand condition

$$E = \liminf_{T\to\infty} \left\{ \sum_{t=0}^{T-1} \beta(t) u'(C^\star_t) \left(\frac{W_t + M_t - W^\star_t - M^\star_t}{(1+\iota_t) P_t} \right) \right.$$
$$\left. + \sum_{t=0}^{T} \beta(t) u'(C^\star_t) \left(\frac{W^\star_t + M^\star_t - W_t - M_t}{(1+\iota_t) P_t} \right) + \beta(0) u'(C^\star_0) \left(\frac{W_{-1} + M_{-1}}{P_0} - \frac{W^\star_{-1} + M^\star_{-1}}{P_0} \right) \right\}.$$

Simplifying this expression and using again the Euler equation

$$E = \frac{\beta(0) u'(C^\star_0)}{P_0} \liminf_{T\to\infty} \frac{W^\star_T + M^\star_T - W_T - M_T}{\prod_{s=0}^{T} (1+\iota_s)}.$$

Using the No-Ponzi condition for the alternative allocation $\{C_t, M_t, W_t\}$, we obtain

$$E \leq \frac{\beta(0) u'(C^\star_0)}{P_0} \liminf_{T\to\infty} \frac{W^\star_T + M^\star_T}{\prod_{s=0}^{T} (1+\iota_s)},$$

Finally, using $D \leq E$ and the transversality condition stated in the Lemma, this implies that $D \leq 0$. □

A.2 Proof of Proposition 1

First note that all the parameters: $\alpha, \kappa, \varphi, \eta, \rho, \sigma$ are non-negative numbers (with the last four strictly positive), and that $\beta \in (0, 1)$.

We are looking within the class of good equilibria. For simplicity, let us renormalize the state to be $k = \log(-n) - \log(-n^\star)$. The value function can then be written as a function of k, and using the log-linear policies we get that

$$V(k) = \begin{cases} -(\varphi a^2 + b^2) k^2 + \beta V(\phi_n k) & k > 0, \\ 0 & \text{otherwise.} \end{cases}$$

Note that the $\varphi a^2 + b^2 > 0$, as for $k > 0$ the $(0, 0)$ solution is not feasible.

If $\beta \phi_n^2 < 1$, the value function must be quadratic, and satisfies $V(k) = -v k^2$ for a value of v

that solves

$$v = \frac{\varphi a^2 + b^2}{1 - \beta\phi_n^2} > 0. \tag{A.1}$$

For $\beta\phi_n^2 \geq 1$, the value function must be minus infinity for $k > 0$.

Using the Euler equation and the Phillips curve, in a log-linear equilibrium, we have that

$$ak = (a + \beta\sigma)\phi_n k - \sigma(i - \rho), \tag{A.2}$$

$$bk = (\beta b + \kappa(a + b\sigma))\phi_n k - \kappa\sigma(i - \rho). \tag{A.3}$$

The first of these implies that

$$i = \rho + \frac{(a + \beta\sigma)\phi_n - a}{\sigma} k.$$

This implies that in the domain Ω, the zero lower bound can only be binding at most at one k, as otherwise ϕ_n could not be a constant.

We can rewrite the government problem using k as the state:

$$V(k) = \max_{k' \geq 0, \pi, y, i} -\pi^2 - \varphi y^2 - \beta v(k')^2$$

subject to:

$$y = (a + b\sigma)k' - \sigma(i - \rho)$$

$$\pi = \left[\beta b + \kappa(a + b\sigma)\right]k' - \kappa\sigma(i - \rho)$$

$$i \geq 0$$

$$\alpha y - \eta(i - \rho) \geq k' \text{ if } i > 0$$

$$k' \geq -\pi + k.$$

The constraint $k' \geq 0$ incorporates that that it could a choice for the government choose the $(0, 0)$ outcome from tomorrow on by setting $k' = 0$ and having the equilibrium outcome played.

Let us now argue that the value function cannot be minus infinite (and thus $\beta\phi_n^2 < 1$). For this it suffices to find a $k > 0$ such that the value to the government is not $-\infty$. The argument is to show that $k' = 0$ is a feasible choice for small enough (but positive k). That is, we need to find i such that

$$i \geq 0, -(\alpha\sigma + \eta)(i - \rho) \geq 0, 0 \geq \kappa\sigma(i - \rho) + k.$$

Setting $i = \rho - k/(\kappa\sigma)$ satisfies all of the constraints if k is positive but small enough. Hence $k' = 0$ is feasible and provides a finite lower bound to the government's payoff.

Define $\phi_k = k'/k$ and $\phi_i = (i - \rho)/k$. Then, by substituting out output and inflation, we can rewrite the government's problem as follows:

$$V(k) = \max_{\phi_i, \phi_k \geq 0} -\Big[\left((\beta b + \kappa(a + b\sigma))\phi_k - \kappa\sigma\phi_i\right)^2 + \varphi\left((a + b\sigma)\phi_k - \sigma\phi_i\right)^2 + \beta v\phi_k^2\Big]k^2 \tag{A.4}$$

subject to:

$$\phi_i k \geq -\rho \tag{A.5}$$

$$[\alpha(a + b\sigma) - 1]\phi_k - (\alpha\sigma + \eta)\phi_i \geq 0 \text{ if } \phi_i k > -\rho \tag{A.6}$$

$$\big[1 + \beta b + \kappa(a + b\sigma)\big]\phi_k - \kappa\sigma\phi_i \geq 1. \tag{A.7}$$

where constraint (A.5) is the zero lower bound constraint, and the last two constraints, (A.6) and (A.7), are the balance sheet constraints.

We have already argued that for almost all values in the domain Ω, the nominal interest rate is above zero. So, we restrict attention to the problem assuming that $\phi_i k > -\rho$. In that case, problem (A.12) becomes:

$$V(k) = \max_{\phi_i, \phi_k \geq 0} -\Big[\left((\beta b + \kappa(a + b\sigma))\phi_k - \kappa\sigma\phi_i\right)^2 + \varphi\left((a + b\sigma)\phi_k - \sigma\phi_i\right)^2 + \beta v\phi_k^2\Big]k^2 \tag{A.8}$$

subject to:

$$[\alpha(a + b\sigma) - 1]\phi_k - (\alpha\sigma + \eta)\phi_i \geq 0 \tag{A.9}$$

$$\big[1 + \beta b + \kappa(a + b\sigma)\big]\phi_k - \kappa\sigma\phi_i \geq 1. \tag{A.10}$$

Suppose that (A.10) is slack. Consider an alternative policy $\hat{\phi}_k = \lambda\phi_k$ and $\hat{\phi}_i = \lambda\phi_i$ for some $\lambda > 0$. This alternative satisfies (A.9), and keeps (A.10) holding for λ close to one. For $0 < \lambda < 1$, this alternative strictly increases welfare, as the objective scales with λ^2 (recall objective cannot be zero for $k > 0$). Hence, (A.10) must bind, and

$$\phi_i = \frac{\big[1 + \beta b + \kappa(a + b\sigma)\big]\phi_k - 1}{\kappa\sigma}. \tag{A.11}$$

Now, plugging this into the objective and the last constraint of Problem (A.8), we have:

$$V(k) = \max_{\phi_k \geq 0} -\Big[(1 - \phi_k)^2 + \frac{\varphi}{\kappa^2}\left(1 - (1 + \beta b)\phi_k\right)^2 + \beta v\phi_k^2\Big]k^2 \tag{A.12}$$

$$\text{subject to:} \quad 1 \geq \big[1 + \beta b + \kappa\frac{\sigma + \eta(a + b\sigma)}{\alpha\sigma + \eta}\big]\phi_k. \tag{A.13}$$

Let us defined the unconstrained optimum, ϕ_k^{int}, to be the optimizer of the objective. It satisfies the following first order condition:

$$-(1-\phi_k) - \frac{\varphi}{\kappa^2}(1-(1+\beta b)\phi_k)(1+\beta b) + \beta v \phi_k = 0,$$

which delivers:

$$\phi_k^{int} = \frac{1+\frac{\varphi}{\kappa^2}(1+\beta b)}{1+\frac{\varphi}{\kappa^2}(1+\beta b)^2+\beta v}. \tag{A.14}$$

Given the simple nature of the government problem (a quadratic objective subject to a linear constraint) we can solve for the solution, $\phi_k^\star$. Towards this, let us define the value of h to be:

$$h \equiv 1+\beta b + \kappa\frac{\sigma+\eta(a+b\sigma)}{\alpha\sigma+\eta}.$$

The solution to the government's problem. If $h \leq 0$, then any non-negative value of ϕ_k satisfies the constraint. Hence the solution is:

$$\phi_k^\star = \max\left\{0, \phi_k^{int}\right\}.$$

If $h > 0$, then ϕ_k is restricted to be in the $[0, 1/h]$ interval. In this case, the solution is

$$\phi_k^\star = \min\left\{\max\left\{0, \phi_k^{int}\right\}, 1/h\right\}. \tag{A.15}$$

Whenever $h > 0$, we define

$$\phi_k^{bound} = \frac{1}{h}. \tag{A.16}$$

An Equilibrium Fixed Point. An equilibrium requires that ϕ_n equals the optimal value of ϕ_k in the problem above. Using (A.11) with $\phi_k = \phi_n$ and (A.2)-(A.3), we can obtain the values of the coefficients a and b in equilibrium:

$$a = \frac{1-(1+\beta b)\phi_n}{\kappa}, \tag{A.17}$$

$$b = 1-\phi_n. \tag{A.18}$$

And given a and b, we recover v using (A.1).

With these values, we can compute ϕ_k^{int} as a function of ϕ_n by using (A.14). Let us denote that map by $\phi_k^{int}(\phi_n)$:

$$\phi_k^{int}(\phi_n) = \frac{(1-\beta\phi_n^2)(\kappa^2+\varphi(1+\beta(1-\phi_n)))}{\varphi+\kappa^2(1+\beta(1-2\phi_n))+\varphi\beta(3+\beta-4(1+\beta)\phi_n+3\beta\phi_n^2)}.$$

We can also compute ϕ_k^{bound} as a function of ϕ_n using (A.16). Let us denote that map by $\phi_k^{bound}(\phi_n)$:

$$\phi_k^{bound}(\phi_n) = \frac{\eta+\alpha\sigma}{\eta(1+\beta(1-\phi_n)^2+(1+\kappa\sigma)(1-\phi_n))+\sigma(\kappa+\alpha(1+\beta(1-\phi_n)))}.$$

In an equilibrium $\phi_n = \phi_k^\star$ and it has to coincide with either ϕ_k^{int}, 0, or ϕ_k^{bound}. Let's go over each case.

Case 1: $\phi_n = \phi_k^{int}$. First,

$$\phi_k^{int}(0) = \frac{1+\frac{\varphi}{\kappa^2}(1+\beta)}{1+\frac{\varphi}{\kappa^2}(1+\beta)^2+\beta(\frac{\varphi}{\kappa^2}+1)} > 0.$$

The corresponding fixed point equation for ϕ_n, $\phi_k^{int}(\phi_n) = \phi_n$, becomes:

$$\frac{(1-\phi_n)(1-\beta\phi_n)(\kappa^2+\varphi(1+\beta(1-2\phi_n))}{\varphi+\kappa^2(1+\beta-2\beta\phi_n)+\varphi\beta(3+\beta-4(1+\beta)\phi_n+3\beta\phi_n^2)} = 0. \tag{A.19}$$

This equation has three solutions:

$$\phi_n \in \left\{1, \frac{1}{\beta}, 1+\frac{\kappa^2+\varphi(1-\beta)}{2\varphi\beta}\right\}.$$

The last two roots are such that $\beta\phi_n^2 > 1$, so are not valid equilibria.

Let us consider then $\phi_n = 1$. In this case, from (A.17) and (A.18) we get that $a = b = 0$, and thus the $(0,0)$ outcome attains in equilibrium, which we already know is not feasible for $n < n^\star$.

To summarize, there is no log-linear equilibrium where $\phi_k^{int} = \phi_n$. In addition, given that $\phi_k^{int}(0) > 0$ and that the denominator of the (A.19) is strictly positive in $[0,1]$, it follows that $\phi_k^{int}(\phi_n) > \phi_n$ for $\phi_n \in [0,1)$. (That the denominator is positive follows by noticing that the denominator is a convex quadratic function of ϕ_n, and it is strictly positive and decreasing and $\phi_n = 1$.)

Case 2: $\phi_n = 0$. In this case, we already know that $\phi_k^{int}(0) > 0$. We can also obtain that $\phi_k^{bound}(0) > 0$. Hence $\phi_k = 0$ cannot be optimal, as it does not satisfy (A.15).

Case 3: $\phi_n = \phi_k^{bound}$**.** So let us now look at the solutions of $\phi_k^{bound}(\phi_n) = \phi_n$. In this case, we obtain there is a unique fixed point $\phi_k^{bound}(\phi_n) = \phi_n$ which corresponds to the cubic equation (21) with $\phi_n = x^\star \in [0, 1]$.

For this to be a valid solution to the government's problem, we need to check that $\phi_k^{int}(\phi_n) \geq \phi_k^{bound}(\phi_n)$ when evaluated at the equilibrium $\phi_n = x^\star$. But this is immediate as we have already argued that $\phi_k^{int}(\phi_n) > \phi_n$ for all $\phi_n \in [0, 1)$ and $\phi_k^{bound}(x^\star) = x^\star$.

One last thing to check is that the nominal interest rate is above the zero lower bound, as assumed. That requires that

$$\phi_i k > -\rho.$$

Using the value of ϕ_i in (A.11), we require that

$$-\frac{(1-\phi_n)(1-(1+\beta+\kappa\sigma)\phi_n+\beta\phi_n^2)}{\kappa\sigma}k > -\rho.$$

Using that $n = \exp(k)n^\star$, this inequality delivers the lower bound for the net worth of the monetary authority, $\underline{n}$, such that interest remains above zero. Given this value of $x^\star$, the values of a and b in the Proposition follow from substitution in (A.17) and (A.18). The value of ϕ_i follows from substituting a, b and $\phi_k = x^\star$ in (A.11).

Resolving a loose thread. There is one final thing. The log-linear equilibrium requires that the interest rate remains strictly positive for almost all k in the domain Ω. We have solved for the unique log-linear equilibrium under this restriction on the policies and move from Problem (A.4) to Problem (A.8).

Now, we need to check that given the prescribed equilibrium behavior, the monetary authority indeed does not want to set $i = 0$ when the equilibrium calls for $i > 0$. One potential advantage of this deviation is that by setting $i = 0$, the monetary authority eliminates the (A.6) constraint from its problem. We show below that this is not an issue.

Consider the alternative policy that sets the nominal interest rate to zero: $(\hat{\phi}_k, \hat{\phi}_i)$ with $\hat{\phi}_i = -\rho/k$. The derivative of the objective function in (A.4) with respect to $\hat{\phi}_k$, maintaining $\hat{\phi}_i$ fixed, is

$$-2\sigma\Big(\beta\kappa b + (\kappa^2+\varphi)(a+\sigma b)\Big)\frac{\rho}{k} - 2\Big(\beta(v+\beta b^2) + (a+\sigma b)(2\beta\kappa b + (\kappa^2+\varphi)(a+\sigma b))\Big)\hat{\phi}_k \leq 0$$

where the inequality follows from $\hat{\phi}_k \geq 0$ and a, b, and v, all strictly positive in the equilibrium. Hence, the monetary policy should choose the lowest possible $\hat{\phi}_k \geq 0$ consistent with its remaining constraint (A.7). So either (A.7) binds or $\hat{\phi}_k = 0$.

If (A.7) binds with $\hat{\phi}_k > 0$, then

$$\hat{\phi}_k = \frac{1 + \kappa\sigma\hat{\phi}_i}{1 + \beta b + \kappa(a + b\sigma)}.$$

In this case, $\hat{\phi}_k \leq \phi_k$ as $\hat{\phi}_i \leq \phi_i$. Using this choice in Problem (A.8), we have that constraint (A.9) can be written as

$$1 \geq \left[1 + \beta b + \kappa \frac{\sigma + \eta(a + b\sigma)}{\alpha\sigma + \eta}\right]\hat{\phi}_k,$$

which must hold as $\hat{\phi}_k < \phi_k$ and (A.13) holds in the equilibrium.

If (A.7) is slack with $\hat{\phi}_k = 0$, then note that (A.9) also holds at $\hat{\phi}_k, \hat{\phi}_i$.

This implies that the best choice of $\hat{\phi}_k$ for $\phi_i = \hat{\phi}_i$ in Problem (A.4) is also in the choice set of Problem (A.8), and thus this alternative policy cannot increase the value above the constructed log-linear equilibrium value.

Helicopter drops. Finally, in the log-linear equilibrium, constraint (A.7) holds with equality, which implies that there are no helicopter drops. □

A.3 Proof of Lemma 4

Let $g(x)$ denote the right-hand side of equation (21). The first thing to note is that $g''(x) < 0$, and thus, g is concave in $[0, 1]$. To see this:

$$g''(x) = -\frac{2(\eta + 2\beta\eta + \alpha\beta\sigma + \eta\kappa\sigma)}{\eta + \alpha\sigma} + 6\frac{\beta\eta}{\eta + \alpha\sigma}x \leq -\frac{2((1-\beta)\eta + \alpha\beta\sigma + \eta\kappa\sigma)}{\eta + \alpha\sigma} < 0$$

where the inequality follows from $x \leq 1$. Now, $g(0) = 0$ and $g(1) = (\eta + \alpha\sigma + \alpha\kappa)/(\eta + \alpha\sigma) > 1$. So $g(x)$ crosses 1, and only once given that it is a concave function, and $g'(x^\star) > 0$.

We have the following, using that $x \in (0, 1)$:

$$\begin{aligned}
\frac{dg(x)}{d\beta} &= \frac{(1-x)x(\eta(1-x) + \alpha\sigma)}{\eta + \alpha\sigma} > 0 \\
\frac{dg(x)}{d\kappa} &= \frac{\sigma x(1 + \eta(1-x))}{\eta + \alpha\sigma} > 0 \\
\frac{dg(x)}{d\alpha} &= -\frac{\sigma(\kappa\sigma + \eta(1-x)(1 - \beta x + \kappa\sigma))}{(\eta + \alpha\sigma)^2} < 0 \\
\left.\frac{dg(x)}{d\eta}\right|_{\alpha=0} &= -\frac{\kappa\sigma}{\eta^2}x < 0
\end{aligned}$$

$$\left.\frac{dg(x)}{d\sigma}\right|_{\alpha=0} = \frac{\kappa x(1+\eta(1-x))}{\eta} > 0$$

The implicit function theorem, using $g'(x^\star) > 0$, completes the proof. □

B Examples

The case of $\alpha = 0$. We want to show that $\phi_i < 0$. In this case, the equation that determines $x^\star$ is

$$1 = (2 + \beta + \kappa\sigma + \kappa\sigma/\eta)\, x - (1 + \kappa\sigma + 2\beta)\, x^2 + \beta x^3.$$

The value of ϕ_i is

$$\kappa\sigma\phi_i = -1 + (2 + \beta + \kappa\sigma)x - (1 + 2\beta + \kappa\sigma)x^2 + \beta x^3.$$

Subtracting these two equations, we get $\kappa\sigma\phi_i = -(\kappa\sigma/\eta)x^\star < 0$, as $x^\star > 0$.

The case of $\alpha = 1$. The equation for $x^\star$ can be rewritten as:

$$w \equiv \left(\frac{\sigma}{\eta} + (1-x)\right)(1 - (1 + \beta + \kappa)x + \beta x^2) - (1-x)(\sigma - 1)\kappa x = 0.$$

Using the equation for ϕ_i and the above, we have that

$$\kappa\eta\phi_i = \kappa\eta\phi_i + \frac{\eta}{\sigma}w = 1 - (1 + \beta + \kappa)x + \beta x^2.$$

Now, rewriting the $w = 0$ equation:

$$\left(\frac{\sigma}{\eta} + (1-x)\right)(1 - (1 + \beta + \kappa)x + \beta x^2) = (1-x)(\sigma - 1)\kappa x.$$

And we can use this to rewrite $\kappa\eta\phi_i$ equation above as:

$$\eta\phi_i = (\sigma - 1)\frac{(1-x)x}{\sigma/\eta + 1 - x}.$$

Given that $x \in (0,1)$, it follows that the sign of ϕ_i equals the sign of $(\sigma - 1)$.

Online Appendix to "Helicopter Drops and Liquidity Traps"

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C The Deflationary Trap and the Monetary Authority's Net Worth

Our analysis of log-linear Markov equilibrium restricts attention to good equilibria with $\phi_n \in [0,1]$, which implies that net worth does not diverge to $-\infty$ and justifies attention to Markov equilibria where Ω is bounded. Here we highlight how equilibria that do not have this property can be ruled out by imposing the following condition on fiscal policy.

> **Condition 1.** *In a private sector equilibrium, the fiscal authority does not accumulate unbounded assets on the private sector:*
>
> $$\lim_{t\to\infty} \frac{B_t}{\prod_{s=0}^{t}(1+\iota_s)} \geq 0.$$

To see the role of this constraint, let us iterate forward on the budget constraint of the monetary authority. Using Condition 1, together with the household's transversality condition, we get that in a private sector equilibrium, we must have that

$$N_{t-1} + \sum_{s=t}^{\infty} \frac{\frac{\iota_s}{1+\iota_s} M_s}{\prod_{\tau=t}^{s-1}(1+\iota_\tau)} \geq \sum_{s=t}^{\infty} \frac{\tau_s}{\prod_{\tau=t}^{s-1}(1+\iota_\tau)} \tag{C.1}$$

The left-hand side can be seen as the "equity value" of the monetary authority: its net worth plus the discounted future value of its "profits" (i.e. seigniorage revenue). This value cannot be lower than the present discounted value of the "dividends," (i.e., the transfers made by the monetary authority).

Note that without Condition 1, the monetary authority's net worth could explode to minus infinity in a private sector equilibrium, even in the presence of balance sheet constraints. In that case, the fiscal authority must accumulate assets to compensate for the ever increasing monetary authority's net liabilities. That is, exploding negative net worth can be part of a private sector equilibrium if the fiscal authority effectively *provides the demand for the monetary authority liabilities*, which is intermediated, in equilibrium, through the household sector. Condition 1 restricts this fiscal behavior, and as a result, inequality (C.1) generates a lower bound on the monetary authority net worth given that $\tau_s \geq 0$ (and assuming that the present value of seigniorage revenue has an upper-bound).

If transfers τ_t were equal to or larger than the seigniorage revenue at all times, then (C.1) would imply that net worth could not be negative. However, the minimum remittance constraint (2) operates on a nominal mark-to-market rule, as in practice.[27] This leaves space for $\tau_s < \iota_s M_s/(1+\iota_s)$, and thus a monetary authority that operates with negative net worth can, in principle, be part of a private sector equilibrium.[28]

Ruling out traps

As we discussed in Section 4.1, there are many sequences of $\{B_t, T_t\}$ that could also constitute an equilibrium, as long as (13) holds. But do any of them satisfy Condition 1? The answer relies on the long-run behavior of the sequence $\{n_t\}$.

To see this, suppose that the sequence $\{n_t \prod_{s=0}^{t} e^{\pi_s - i_s}\}$ converges to a negative number. Then, we have that

$$0 < -\lim_{t\to\infty} n_t \prod_{s=0}^{t} e^{\pi_s - i_s} = \lim_{t\to\infty} \frac{M_t - A_t}{P_t} \prod_{s=0}^{t} \frac{P_s/P_{s-1}}{1+\iota_s} = \lim_{t\to\infty} \frac{M_t - A_t}{P_{-1}\prod_{s=0}^{t}(1+\iota_s)}.$$

For the household transversality condition to hold, it would be necessary that

$$\lim_{t\to\infty} \frac{B_t}{P_{-1}\prod_{s=0}^{t}(1+\iota_s)} < 0,$$

a contradiction of Condition 1. If however, the sequence $\{n_t \prod_{s=0}^{t} e^{\pi_s - i_s}\}$ converges to a non-negative number, then it is possible to construct a fiscal policy that generates a private sector equilibrium where Condition 1 holds.

The next question is whether the deflationary trap equilibrium is consistent with Condition 1. Using equation (20), we can see that in the expectational trap (as $i = 0$),

$$n_t \leq e^{-\pi} n_{t-1} = e^{i-\pi} n_{t-1} \leq \ldots \leq e^{(i-\pi)(t+1)} n_{-1}$$
$$e^{(\pi-i)(t+1)} n_t \leq n_{-1} \Rightarrow \lim_{t\to\infty} e^{(\pi-i)(t+1)} n_t \leq n_{-1}$$

And thus, if $n_{-1} < 0$, the deflationary trap violates Condition 1. We can use the above to also rule

[27] See Hall and Reis (2015) for more details on this.

[28] It is interesting to note that the irrelevance of the helicopter drops, stated in Lemma 2, holds even in the presence of Condition 1. Inspection of the proof of the lemma shows that the alternative allocation without helicopter drops features a fiscal debt that is weakly higher than the original allocation ($\hat{B}_t \geq B_t$ for all t), and thus if Condition 1 held at the original, it remains to hold as well in the alternative.

out transitional paths that converge to the deflationary trap.[29]

In the main body of the paper, we have restricted attention to equilibria where n lies in a closed interval Ω (hence a bounded set). Thus, n_t cannot diverge to minus infinity (if the real rate is asymptotically bounded away from zero), implicitly ruling out the deflationary trap outcome. Given this, it is natural to look for log-linear equilibria where $\phi_n \in [0, 1]$, and thus net worth does not leave the set Ω.

[29] Benigno (2020) shows that a policy commitment by the monetary authority to remit its profits, as well as Condition 1 holding with equality, can lead to price level determinacy in a flexible price model where the monetary policy follows a Taylor rule. In his analysis, it is an exploding positive net worth by the monetary authority that is ruled out by the policy, while in ours it is an exploding negative one. In our model, the monetary authority is operating without commitment and is not following a Taylor rule, but the basic insight that eliminates the deflationary trap is the same as in his model.