NBER WORKING PAPER SERIES

THE RACE BETWEEN EDUCATION, TECHNOLOGY, AND THE MINIMUM WAGE

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Working Paper 31028
http://www.nber.org/papers/w31028

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
March 2023

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NBER Working Paper No. 31028
March 2023
JEL No. E24,J0,J2,J3,J42

ABSTRACT

What is the impact of the minimum wage on the college wage premium? I show that job-ladder models imply that the effect should be small on impact---raising only the wages of workers bound by the minimum wage---and grow over time as workers slowly move up the job ladder. Guided by my theory, I present evidence that these dynamic effects are present and powerful. Estimated at the national level, I show that minimum wages---together with supply and demand---play a central role in shaping the evolution of the U.S. college premium. Estimated at the state level, I show that the elasticity of the college premium to the minimum wage is small on impact and grows dramatically over time. To verify my theory's mechanisms, I additionally document the dynamic impact of the minimum wage over the full wage distribution: on impact, wages rise only for the lowest percentiles (consistent with the literature) but over time this effect spills over up the wage distribution (consistent with my theory and my empirical results on the skill premium). On the basis of these theoretical and empirical results, I conclude that the minimum wage plays a central role in shaping the U.S. college premium and its variation across states.

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1 Introduction

Overview. What is the impact of the minimum wage on the *college premium* (the relative wage of college to non-college workers)? The *canonical model*—introduced by Tinbergen (1974), operationalized by Katz and Murphy (1992), and named by Acemoglu and Autor (2011)—provides the central organizing framework for studying the evolution of the college premium. It straightforwardly relates relative wages of more and less educated workers to their relative supply and demand. I generalize this model to incorporate labor-market institutions—monopsony power and minimum wages—labor-market dynamics, a job ladder, and unemployment while maintaining the original model's tractability. I demonstrate that the extended model generates an estimating equation that is almost identical to the typical empirical implementation of the canonical model except for the presence of one additional term: the real minimum wage. The model predicts that at the moment the minimum wage increases, the elasticity of the skill premium with respect to the minimum wage equals its *bite* (the share of wage earnings of high-education workers at the minimum wage minus the same share among low-education workers), as wages rise only for those earning below the new minimum wage (the *direct effect*). Over time, however, the model predicts that the increase in the minimum wage spills over to wages higher up the distribution (an *indirect effect*) as workers slowly climb the job ladder starting from a higher entry wage. This implies that the long-run elasticity of the skill premium is strictly greater than the short-run elasticity.

The purpose of the framework is both to guide my empirical analysis and interpret its findings. I make the following empirical contributions. First, to motivate the analysis, I incorporate the real minimum wage into the otherwise traditional empirical implementation of the canonical model and document empirically the importance of minimum wages for shaping the national college premium. For example, the decline of the real minimum wage in the 1980s and its subsequent rise explains a substantial share of the rapid rise in the college premium in the 1980s and its slower growth thereafter. Second, and relatedly, whereas the traditional canonical model implies a dramatic slowdown in the rate of skill-biased technical change starting in the late 1980s or early 1990s, the model with the real minimum wage implies a much smaller slowdown (or none at all), since the rise in the minimum wage is sufficient to slow the college premium's growth.

Third, I use the theory to guide a regional estimation of the extended canonical model. I find short-run effects of changes in state-level minimum wages that are quantitatively consistent with both past empirical work finding small wage spillover effects and my model's predictions. I find that the longer-run effects (which have not been the focus of past studies) are about three times larger, implying sizable spillover effects up the wage distribution, consistent with my theoretical predictions and my national estimates. In the longer run, minimum wages—together with supply and demand—play a first-order role in shaping the U.S. college premium and its variation across states.

Finally, while my theory predicts that the mechanism through which the skill premium rises more in the long run than in the short run is wage spillovers up the distribution, the evidence on the skill premium is not itself definitive on this point. To document the mechanism more directly, I study empirically the dynamic impact of the minimum wage over the full wage distribution. On impact, wages rise only for the lowest centiles, consistent with the literature and my theory. However, I show that this effect spills over up the wage distribution slowly, consistent with my theory's mechanism.

Details. In Section 2, I motivate the subsequent analysis by estimating an extended version of the empirical canonical model incorporating the real minimum wage. I find that the elasticity of the college premium with respect to the real minimum wage is between -0.14 and -0.20. This range of point estimates implies (for example) that the 27% decline in the real minimum wage between 1979 and 1989 caused between a 3.7% and a 5.3% increase in the college premium over this time period, which is about a third of the observed increase. I find that the elasticity of the college premium with respect to the relative supply of college is between -0.51 and -0.6. This implies that the slowdown in the growth rate of the relative supply of college between 1979 and 1989, relative to its growth rate between 1969 and 1979, raised the college premium by between 6.2% to 7.2% in the latter period, which is about half of the observed increase. More generally, supply, demand, and the minimum wage each play important roles in the evolution of the U.S. college premium over the period 1963-2017.

Introducing the real minimum wage additionally fundamentally alters a standard conclusion in the literature, that the rate of skill-biased technical change has declined dramatically since the late 1980s or early 1990s. The canonical model estimated on data spanning 1963 - 1987 (the set of years used in the seminal work of Katz and Murphy, 1992) predicts substantially more rapid increases in the college wage premium than actually occur in the data thereafter. Through the lens of the model, this problem with the out-of-sample fit implies a substantial decline in the rate of skill-biased technical change. This issue is mitigated by incorporating the real minimum wage; its rise in the later period reduces the predicted growth rate of the college premium in the absence of any substantial changes in the rate of skill-biased technical change.

In Section 3, I present a generalized version of the canonical model that microfounds the national regression analysis of Section 2, introduces novel predictions on the dynamic implications of changes in the minimum wage, and guides my state-level analysis in Section 4. In addition to supply and demand, the model incorporates monopsony power, minimum wages, a job-ladder, dynamics, and unemployment. My objective is to maintain the simplicity and tractability of the original framework yet facilitate the study of the dynamic implications of minimum wages on inequality. As in the canonical model, an aggregate constant returns to scale production function combines the output of different skills. Unlike the canonical model, the labor market is frictional. In the baseline model, a worker who meets a potential new employer bargains over her wage with her current wage (or unemployment benefits) serving as her outside option.[1]

In equilibrium, a worker exits unemployment at the minimum wage and slowly moves up the job ladder to higher wages as she matches with new employers over time. Each successive wage in the job ladder depends less on the minimum wage and more on the worker's value marginal product. In the steady state, the average wage of skill s workers is a weighted average of the minimum wage and their value marginal product of labor. I characterize how changes in supply, demand, and the minimum wage affect the skill premium across steady states. The minimum wage has no effect on value marginal products, since it does not affect unemployment, but does affect wage markdowns. I show that the elasticity of the skill premium with respect to the real minimum wage is strictly greater than the minimum wage's bite (for any number of worker skills or aggregate production function). Supply and demand affect the skill premium by affecting workers' value marginal products, as in the canonical model; however, changes in value marginal products have muted effects on wages because of wage markdowns. Under the assumptions of the traditional canonical model—two skills, a CES aggregate production function, constant factor-biased growth rates—I micro-found the extended canonical model estimating equation.

I also characterize the transition—solving in closed form for the distribution of wages for each skill group at each date—in response to a one-time increase in the real minimum wage.[2] On impact, an increase in the minimum wage raises wages only for workers whose initial wage is bound by the new minimum wage. Hence, on impact the elasticity of the skill premium equals the minimum wage's bite. Over time, however, workers initially at the new minimum wage slowly rise up the new job ladder and the elasticity of the skill premium rises. Hence, the distributional impact of changes in the minimum

[1]In the baseline wage-bargaining model, I assume that workers and firms are myopic to facilitate the analysis of the transition to changes in the minimum wage. I show that the steady-state results broadly apply in the canonical wage-posting model of Burdett and Mortensen (1998) (with forward-looking agents).

[2]Given that supply and demand are fixed in this exercise, I do not impose the restrictions of the traditional canonical model: there are arbitrarily many skills and an arbitrary aggregate production function.

wage grow over time.

In Section 4 I estimate a regional extended canonical model leveraging variation across U.S. states and time. Motivated by the theoretical results of Section 3, I regress changes over time in state-level college premia on (among other covariates) the interaction between the state-level bite of the minimum wage and the change in the state's minimum wage (instrumenting for this minimum wage interaction using two distinct approaches). Estimated in one-year differences, the coefficient on the minimum wage interaction is very close to one, quantitatively consistent with the model's predictions: in the first year, the direct effect determines the response of skill premium. However, as I increase the length of the time difference in the regression, this elasticity rises, as predicted by the theory, by a factor of approximately three. Lending support to both the national and state-level empirical analysis, I show that the long-run state-level estimates are quantitatively consistent with the national estimates. Hence, the minimum wage plays a central role both in shaping the national skill premium, but also variation across states in state-level skill premia.

My theory predicts that the long-run impact of changes in the minimum wage are larger than in the short run because of dynamic wage spillovers up the wage distribution. To document these dynamic wage spillovers more directly, I study the dynamic impact of the minimum wage over the full wage distribution, as in, e.g. Lee (1999) and Autor et al. (2016). To do so, I replicate the analysis of Autor et al. (2016); however, instead of considering changes in inequality and the minimum wage of one year only, I vary the length of time differences. On impact, wages rise only for the lowest centiles, consistent with the literature and my theory. However, I show that this effect spills over up the wage distribution slowly, confirming the underlying mechanism in my theory.

In summary, in both my theory and empirics, whereas on impact the direct effect explains almost all of the distributional consequences of changes in the minimum wage, over time the indirect effect becomes the dominant mechanism and the effect of the minimum wage on the college premium become larger.

Additional Literature. In terms of static economic questions, my paper is perhaps most related to Autor et al. (2008), who estimate a regression very similar to my motivating empirics in Section 2 leveraging national-time series variation. They contend that the real minimum wage "does not much alter the central role for relative supply growth fluctuations and trend demand growth in explaining the evolution of the college wage premium" and that "institutional factors are insufficient to resolve the puzzle posed by slowing trend relative demand for college workers in the 1990s." They reach their conclusions because they find that the coefficient on the real minimum wage is negative, significant, and of a similar magnitude to my estimates in one of their two specifications (with a linear time trend) but smaller and insignificantly different from zero in their other specification (with a cubic time trend). I conclude differently. My national time-series empirical results are robust across a wide range of degrees of the polynomial of time in my data. I additionally show that these national time-series results also hold using Autor et al.'s data across a wide range of specifications varying in the polynomial of time (up to the sixth degree), excluding only the cubic one. More importantly, my empirical results additionally hold using state-level variation. Based on these empirical findings, I conclude that relative supply growth fluctuations and trend demand growth remain crucial drivers of the college premium, consistent with Autor et al. (2008), but so too are changes in the real minimum wage. I also conclude that changes in the minimum wage are sufficient to resolve the puzzle posed by slowing trend relative demand for college workers in the 1990s. Finally, I also micro-found the regression they and I estimate.

A vast body of work studies the roles of labor-market institutions for shaping inequality. Much of the focus is on minimum wages (e.g., DiNardo et al., 1996; Lee, 1999; Card and DiNardo, 2002; Teulings, 2003; Autor et al., 2016; Cengiz et al., 2019; Dube, 2019; Chen and Teulings, 2022) and monopsony power (see Manning, 2013 for a summary). I build on this literature to identify the impact of minimum wage changes across the full wage distribution not only over short horizons, but also over longer horizons. My short-run theoretical and empirical results are consistent with empirical results that the direct effect is the dominant mechanism through which the minimum wage affects inequality (see, e.g., Autor et al., 2016; Cengiz et al., 2019). My theory, however, implies that the indirect effect strengthens over time. In my empirics, the indirect effect dominates in the longer run. Moreover, I show analytically that a job-ladder model can rationalize these empirical findings both across steady states and in the full transition.[3]

My paper is also related to a recent and fast growing macro-labor literature conducting counterfactual analyses to study the implications of minimum wage policies for inequality, efficiency, and welfare (e.g., Haanwinckel, 2020; Ahlfeldt et al., 2022; Berger et al., 2022; Hurst et al., 2022; Engbom and Moser, 2022; Trottner, 2022). In constrast, I use my qualitative theory to guide and interpret my empirical analyses of the impact of minimum wages on wage inequality. In one respect, my analysis is closest to Engbom and Moser (2022), who use a job-ladder model to study the implications of minimum wages in Brazil. But I am particularly interested in dynamic effects, from which Engbom and

[3]While I show that two types of job ladder models can rationalize my empirical findings, I do not directly test that the wage spillovers I find in the data are caused by movements up the job ladder. Such an exercise would require worker-level wage panel data.

Moser (2022) abstract. In this respect, my analysis is most related to Hurst et al. (2022). Their dynamics are driven primarily by putty-clay capital as opposed to job ladders.

2 Motivation: national time series evidence

I consider regressions of the form

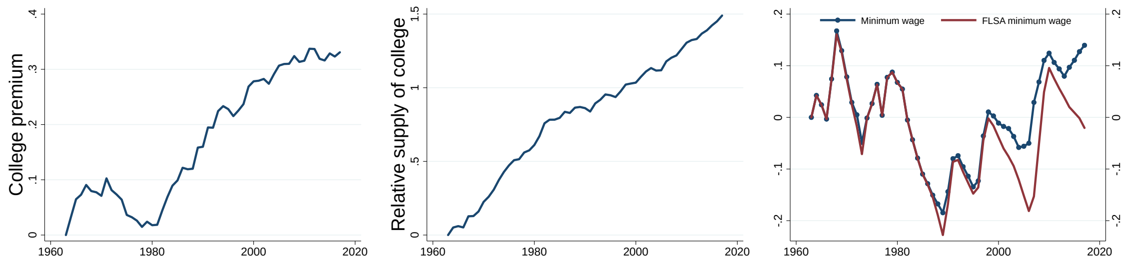
$$\log\left(\frac{w_{ht}}{w_{\ell t}}\right) = \alpha + \beta_m \log m_t + \beta_L \log\left(\frac{Supply_{ht}}{Supply_{\ell t}}\right) + \gamma t + [...] + \iota_t \qquad (1)$$

Here, $\log w_{ht}$ and $\log w_{\ell t}$ are measures of high- and low-education average log wages; $\log Supply_{ht}$ and $\log Supply_{\ell t}$ are measures of high- and low-education labor supply; and m_t is a measure of the real minimum wage, all measured at the national level. The regression includes a linear time trend, γt, and is identical to the traditional empirical implementation of the canonical model (e.g., Katz and Murphy, 1992) except for the inclusion of the real minimum wage.

Measurement. Here I briefly describe how I measure each variable; the Empirical Appendix contains details. I restrict my sample to the working-age population of those between 16 and 64 years old and define high-education workers as those with 16 or more years of education and low-education workers as those with fewer than 16 years of education. I construct the composition-adjusted college premium by first measuring the average log hourly wage within each of 180 groups (defined by the intersection of 9 age bins, 2 genders, 2 races, and 5 education levels) in year t and then averaging across those groups with and those groups without college education using time-invariant weights. I measure the supply of college and non-college workers as the dual of these wages, such that the product of composition-adjusted supply and wages equals the observed total income of college and non-college workers in each year. I measure these variables using the March Annual Demographic Files of the Current Population Survey from 1964 to 2018, which report earnings from 1963 to 2017.

I measure the minimum wage in year t two ways. In one approach, I use the federal (FLSA) minimum wage. In my baseline, I use the average minimum wage across states (the maximum of the legislated state and federal minimum wages), using time-invariant weights. I deflate each series using the GDP deflator. I refer to my baseline measure as the real minimum wage and to my alternative measure as the real FLSA minimum wage.

Panels (a) and (b) of Figure 1 display the college premium and relative supply of college workers, both normalized to zero in 1963. The particularly steep rise in the college



(a) College Premium **(b)** Relative College Supply **(c)** Real Minimum Wages

Figure 1: Relative Wages, Relative Supplies, and the Minimum Wage

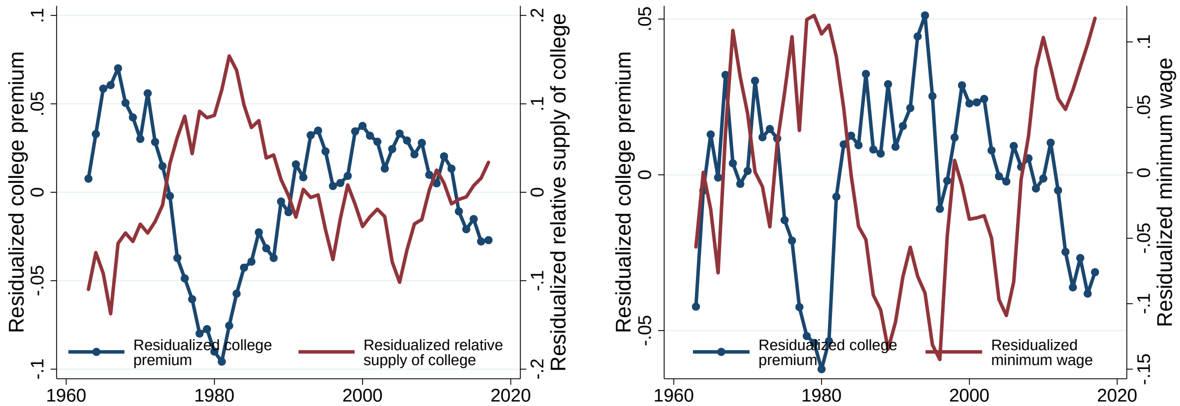
Notes: Panels (a) and (b) display the composition-adjusted college premium and relative supply of college. Panel (c) displays the real FLSA minimum wage and the real minimum wage series, which averages minimum wages across states; both series are deflated by the GDP deflator. All series are normalized to zero in 1963.

premium starting in the early 1980s coincides with a decline in the growth rate of the relative supply of college workers, a fact first emphasized in Katz and Murphy (1992). Panel (c) of Figure 1 displays my two real minimum wage series, each normalized to zero in 1963. The two series move in lockstep until the late 1980s and diverge thereafter (especially in the 2000s) as more states set minimum wages above the federal level. Both real minimum wage series decline dramatically in the 1980s, as fixed nominal minimum wages are eroded by inflation. Finally, there is substantial time variation in both real minimum wage series, which remain even after residualizing on a high-dimensional polynomial of time.

Results. Before providing estimation results, I display the variation in the data that identifies the parameters of interest. Panel (a) of Figure 2 displays the college premium and relative supply of college, each residualized of the real minimum wage and a linear time trend. Panel (b) displays the college premium and real minimum wage, each residualized of the relative supply of college and a linear time trend. The variation in Panel (a) identifies β_L and the variation in Panel (b) identifies β_m when estimating regression (1) using OLS. There is a striking negative relationship in each panel. These qualitative patterns are robust to residualizing on high-dimensional polynomials of time.

Column (a) of Table 1 displays results of estimating regression (1) using OLS including a linear time trend and omitting the real minimum wage on the sample 1963 - 1987.[4] This is the specification and sample years included in the seminal work of Katz and Murphy (1992). Column (b) replicates this analysis, but using the full sample of 1963 - 2017. Consistent with past work, estimates are unstable across samples. The growth rate of

[4] I report robust standard errors in all regressions.



(a) Residualized Relative Supply, College Premium **(b)** Residualized Minimum Wage, College Premium

Figure 2: Residualized Independent and Dependent Variables

Notes: Panel (a) displays the composition-adjusted college premium and relative supply, both residualized of the real minimum wage and a linear time trend. Panel (b) displays the composition-adjusted college premium and real minimum wage, both residualized of composition-adjusted relative supply and a linear time trend.

skill-biased technical change γ falls from 2.4% per year to 1.8%; and the coefficient on relative supply β_L rises from -0.596 to -0.441 when changing the sample from the Katz and Murphy years to the full sample; in spite of differences in data cleaning and measurement, these results are very similar to Acemoglu and Autor (2011). The model estimated on the 1963 - 1987 sample systematically deviates from the data thereafter, predicting a sharper rise in the college premium than actually occurs.

Introducing the real minimum wage goes some way towards fixing this well-known issue. Columns (c) and (d) replicate Columns (a) and (b), but additionally include the value of the real minimum wage. Including the real minimum wage leads to broadly stable parameter estimates across samples, a result that is robust to varying the sample years. The growth rate of skill-biased technical change γ and the coefficient on relative supply β_L are 2.22% and -0.594 when estimated on the 1963 - 1987 sample and are 2.11% and -0.556 when estimated on the full sample. The remaining columns (e), (f), and (g) are estimated on the full sample of years. Columns (e) and (f) include higher-dimensional polynomials of time. Column (g) replaces the real minimum wage (which averages minimum wages across states) with the real FLSA minimum wage.

The estimates in Table 1 highlight the importance of supply (education), demand (technology), and the minimum wage (institutions) in shaping the evolution of the U.S. college premium. The elasticity of the college premium with respect to the real minimum wage over the full sample ranges between -0.136 and -0.195. This elasticity implies that

	1963-1987	1963-2017	1963-1987	1963-2017			
	(a)	(b)	(c)	(d)	(e)	(f)	(g)
Relative supply	-0.596	-0.441	-0.594	-0.556	-0.600	-0.531	-0.510
	(0.116)	(0.044)	(0.137)	(0.054)	(0.052)	(0.102)	(0.049)
Real m.w.			-0.237	-0.195	-0.159	-0.136	
			(0.091)	(0.040)	(0.059)	(0.055)	
Real FLSA m.w.							-0.162
							(0.039)
Time	0.024	0.018	0.022	0.021			0.020
	(0.005)	(0.001)	(0.006)	(0.001)			(0.001)
Constant	0.013	0.024	0.035	0.032	0.021	0.034	0.032
	(0.013)	(0.007)	(0.021)	(0.009)	(0.014)	(0.029)	(0.009)
Time Polynom.	1	1	1	1	2	3	1
Observations	25	55	25	55	55	55	55
R-squared	0.324	0.950	0.517	0.968	0.969	0.969	0.963

Table 1: Regression Models for the College Wage Premium

Notes: Results of estimating (1) using OLS. The dependent variable and "Relative supply" are the logs of the composition-adjusted college premium and relative supply of hours worked. "Real m.w." and "Real FLSA m.w." are the logs of the real minimum wage and real FLSA minimum wage. The sample is 1963-1987 in columns (a) and (b) and 1963-2017 elsewhere. "Time Polynom." refers to the degree of the polynomial of time; the coefficient on the linear trend on "Time" is omitted from the table whenever this polynomial is of degree 2 or greater.

the 27% decline in the real minimum wage between 1979 and 1989 caused between a 3.7% and a 5.3% increase in the college premium over this time period, which is between three to four tenths of the observed 13.4% increase. The elasticity of the college premium with respect to relative supply ranges between -0.51 and -0.6. This elasticity implies that the slowdown in the growth rate of the relative supply of college between 1979 and 1989, relative to the growth rate between 1969 and 1979, raised the college premium by approximately 6.2% to 7.2% in the latter period, which is around one half of the observed 13.4% increase. All three forces—supply, demand, and the minimum wage—play important roles not only in this decade, but also throughout the sample.

Summary. In summary, introducing the real minimum wage into the canonical model's estimating equation demonstrates that supply, demand, and the minimum wage each play central roles in shaping the evolution of the U.S. college premium. Moreover, incorporating the minimum wage improves the out-of-sample fit of the model and, in contrast to the model without the real minimum wage, implies a much smaller slowdown (or none at all) in the rate of skill-biased technical change.

Robustness. Here, I briefly describe results of three types of robustness exercises. I consider the robustness of results to higher-dimensional polynomials of time, measuring relative supply differently, and using data from Autor et al. (2008)'s replication package. Details are provided in the Empirical Appendix.

Appendix Table A.1 extends the baseline analysis by including progressively higher-dimensional polynomials of time. Results on the impact of the minimum wage are largely robust up to an eighth-degree polynomial, although the coefficient on supply becomes insignificant by the fifth degree polynomial.

My measure of supply is the dual of wages, exactly matching wage income for college and non-college workers in each year. This measure omits a portion of labor supply (in the presence of wage markdowns) and depends not only on the supply of labor, but also its demand (in the presence of endogenous unemployment and labor force participation). In the Empirical Appendix, I replicate Table 1 using two alternative measures of labor supply: a composition-adjusted measure of efficiency-unit hours worked (weighing changes in hours worked of each labor group by a fixed-over-time measure of efficiency units per hour worked) in Table A.2 and, alternatively, a composition-adjusted measure of efficiency-unit populations in Table A.3 (weighing changes in the population of each labor group by a fixed-over-time measure of efficiency units). Both alternatives address the issue of changing wage markdowns affecting the measure of changing labor supply. And the population-based measure additionally addresses the issue of changing labor demand affecting the measure of changing labor supply. Results are broadly similar to those displayed in Table 1.

Appendix Table A.4 shows that my conclusions on the importance of the minimum wage for the U.S. college premium holds using data from Autor et al. (2008)'s replication package. The only exception is the case of a third-degree polynomial of time, where the coefficient on the minimum wage is insignificantly different from zero.[5] Autor et al. (2008) focus on this specification and, therefore, reach different conclusions.

3 Theory

My objectives are fourfold: (*i*) to provide a simple extension of the theoretical framework referred to as the canonical model, incorporating monopsony power, minimum wages, unemployment, a job ladder, and dynamics; (*ii*) to derive equation (1) estimated above; (*iii*) to derive a version of equation (1) that can be estimated using regional variation; and, finally, (*iv*) to make predictions on how the resulting regression coefficients vary

[5] The coefficient on supply is insignificant with a fifth or sixth degree polynomial of time.

with the length of the time difference in consideration. In Section 3.1 I describe the simplest model to make these points: a model of wage bargaining with myopic workers and firms. In Section 3.2 I characterize the steady state of this model and provide comparative static results on changes in the minimum wage and labor supply and demand across steady states. In Section 3.3 I describe the transition to a one-time change in the economic environment, focusing on changes in the minimum wage. My job-ladder model is the simplest one to make my points analytically. However, these points apply more broadly: in Section 3.4 I briefly describe related steady-state results in the canonical wage-posting model of Burdett and Mortensen (1998). Additional details and proofs are contained in the Theoretical Appendix.

3.1 Setup

Time is discrete and indexed by t. The economy features S labor skills and types of firm, each indexed by s, with a type s firm hiring only skill s labor to produce type s output. The mass of skill s workers is L_{st}. There are many firms of each type, output across type s firms is perfectly substitutable, and each employed skill s worker produces one unit of type s output. Final output, Y_t, is a constant returns to scale function of the output, Y_{st}, of each type s, $Y_t = Y\left(\{A_{st}Y_{st}\}_{s=1}^{S}\right)$, where A_{st} is time-varying factor-biased productivity. The price of the final good is the numeraire and P_{st} is the endogenous real price of each unit of type s output.

A worker can be employed or unemployed. Each period, a skill s worker-firm match faces an exogenous separation probability $\delta_s \in (0,1)$. Each period, an unemployed skill s worker receives real income b_s and matches with a firm with probability $\lambda_{su} \in (0,1]$. Each period an employed skill s worker meets a new firm with probability $\lambda_{se} \in (0,1)$. If a worker meets a new firm, the worker and firm engage in generalized Nash bargaining to determine if the worker moves from her current employer to the new firm and, if so, the worker's fixed-wage contract. The worker's bargaining weight is $\beta_s \in (0,1)$. If an unemployed worker meets a firm, her outside option is unemployment. If a worker employed at wage w meets a new firm, her outside option is to continue employment in her existing match at wage w.[6]

[6] I treat the matching probabilities as exogenous, as in, e.g., Postel-Vinay and Robin (2002) and Cahuc et al. (2006); this rules out the possibility that an increase in the minimum wage (which reduces employer profit) might reduce matching probabilities disproportionately more for worker skills that are more likely to be bound by the minimum wage. I rule out the possibility that a worker can exploit a new job offer to raise her wage with her current employer, consistent with counteroffers being uncommon empirically (see, e.g., Mortensen, 2003). Finally, unlike Shimer (2006), a worker's outside option is her current wage rather than unemployment; wages are, therefore, not renegotiation proof.

While wages are fixed and not renegotiable, they are also subject to a minimum wage, m_t. If the minimum wage rises above an existing wage contract, the wage in that match rises to the minimum wage if the firm and worker find it optimal to maintain the match at this new wage; otherwise, the firm can endogenously fire the worker and the worker can quit to unemployment. I assume throughout that the minimum wage satisfies $m_t \in ((1-\beta_s) b_s + \beta_s P_{st}, P_{st})$ for each s. The upper bound on m_t implies that firms always find it profitable to hire workers at the minimum wage. The lower bound on m_t implies that firms are constrained by the minimum wage when hiring workers out of unemployment.

Within period t, I assume exogenous separation shocks occur first and then new matches are realized for those workers who did not separate in t. Workers and firms are risk neutral and infinitely lived. In what follows, I focus on the case in which the discount factor is zero, so that each worker maximizes her current wage and each firm maximizes its current profit. This assumption facilitates the analysis of the transition to aggregate shocks. I derive related steady-state results in a canonical (forward-looking) wage-posting model in Section 3.4.

3.2 Steady state

In steady state, the time-varying parameters L_{st}, A_{st}, and m_t are all fixed across time, as is the price of type s output, P_{st}, and the density of wages across workers within each skill s, which I denote by $g_{st}(w)$. Hence, in what follows, I omit time subscripts and re-introduce them in Section 3.3. I first solve for the steady-state distribution of wages across workers within skill s for a given P_s (and, therefore, for any aggregate production function and any number of skills) and then determine P_s for a specific aggregate production function and two skills; see Appendix B.1 for details.

Wages along the job ladder. Suppose that a skill s worker at wage w (where I refer to an unemployed worker's income of b_s as a wage) matches with a new firm. Her new wage is given by the maximum of the unconstrained bargaining problem, $(1-\beta_s) w + \beta_s P_s$, and the minimum wage, m. Hence, in steady state there is a wage ladder $\{w_{j,s}\}_{j=0}^{\infty}$ with a discrete set of wages for each skill s. The unemployment benefit is the wage at the bottom of the ladder: $w_{0,s} = b_s$. The minimum wage is the first rung on the job ladder, since $(1-\beta_s) b_s + \beta_s P_s < m$ implies that workers hired out of unemployment are paid the minimum wage: $w_{1,s} = m$. And the wage of each successive rung $j+1 > 1$ on the job ladder is a simple function of the wage on the previous rung, the worker's value marginal product, and her bargaining power: $w_{j+1,s} = (1-\beta_s) w_{j,s} + \beta_s P_s$. Solving this recursive system yields[7]

$$w_{j,s} = (1-\beta_s)^{j-1} m + \beta_s P_s \sum_{k=0}^{j-2} (1-\beta_s)^k \quad \text{for all } j \geq 1 \tag{2}$$

A new match increases a worker's wage, $w_{j+1,s} - w_{j,s} = \beta_s \left(P_s - w_{j,s}\right) > 0$, but proportionally less for higher initial wages: $d\left(w_{j+1,s}/w_{j,s}\right) \Big/ dw_{j,s} < 0$.

Distribution of workers across the job ladder and the average wage. The probability that a skill s worker at any wage $w_{j+1,s}$ in period t does not work at this wage in period $t+1$ is $\delta_s + (1-\delta_s)\gamma_{se}$, where δ_s is the probability the worker exogenously separates from her firm and, if she does not, γ_{se} is the probability that she matches with a new firm. The probability that a skill s worker begins earning wage $w_{j+1,s}$ at date t is $(1-\delta_s)\gamma_{se} g_s(w_{j,s})$, if $j > 0$ and $\gamma_{su} g_s(w_{j,s})$ if $j = 0$, which is the probability that she was one rung below $j+1$ times the probability that she does not separate (if $j > 0$) and the probability that she matches. Hence, the density of wages can be defined recursively as

$$\begin{aligned} \left[\delta_s + (1-\delta_s)\gamma_{se}\right] g_s(w_{1,s}) &= \gamma_{su} u_s \\ \left[\delta_s + (1-\delta_s)\gamma_{se}\right] g_s(w_{j+1,s}) &= (1-\delta_s)\gamma_{se} g_s(w_{j,s}) \quad \text{for} \quad j \geq 1 \end{aligned}$$

where $u_s = g_s(w_{0,s})$ is the unemployment rate. Solving this recursive system and using the fact that these densities must sum to one across all $j = 0, ..., \infty$, yields both

$$u_s = \frac{\delta_s}{\delta_s + \gamma_{su}} \tag{3}$$

and

$$g_s(w_j) = \left(\frac{(1-\delta_s)\gamma_{se}}{\delta_s + (1-\delta_s)\gamma_{se}}\right)^{j-1} \frac{\gamma_{su}}{\delta_s + (1-\delta_s)\gamma_{se}} \frac{\delta_s}{\delta_s + \gamma_{su}} \quad \text{for} \quad j \geq 1 \tag{4}$$

The average wage among the employed, $\overline{w}_s \equiv \frac{1}{1-u_s} \sum_{j \geq 1} w_{j,s} g_s(w_{j,s})$, is then

$$\overline{w}_s = \frac{\delta_s}{\delta_s + \beta_s(1-\delta_s)\gamma_{se}} m + \left(1 - \frac{\delta_s}{\delta_s + \beta_s(1-\delta_s)\gamma_{se}}\right) P_s \tag{5}$$

which is simply a weighted average of the minimum wage, m, and the value marginal product of labor, P_s.

Comparative statics across steady states. According to equation (5), the minimum wage, m, directly affects the average wage whereas factor supply and demand, L_s and A_s, only

[7] For compactness, here I define $\sum_{k=0}^{0}(1-\beta)^k = 1$ and $\sum_{k=0}^{-1}(1-\beta)^k = 0$.

affect the average wage through skill prices, P_s. The total derivative of $\overline{w}_s$ can be expressed in terms of the share of income earned by skill s at the minimum wage, denoted by

$$b_s \equiv \frac{m g_s(m)}{(1-u_s)\overline{w}_s}$$

as follows

$$d \log \overline{w}_s = \beta_{ms} b_s d \log m + (1 - \beta_{ms} b_s)\, d \log P_s \tag{6}$$

where

$$\beta_{ms} \equiv \frac{\delta_s + (1-\delta_s)\gamma_{se}}{\delta_s + \beta_s(1-\delta_s)\gamma_{se}} > 1 \tag{7}$$

The impact of changes in the price of skill s output on the average skill s wage is less than one-for-one, as opposed to in competitive models, because workers do not receive their value marginal product. Instead, at every step of the job ladder a worker's wage depends on the minimum wage, m, and the value marginal product, P_s, so the average wage does as well, as shown in equation (5).

The result that $\beta_{ms} > 1$ is an important one. The elasticity of the average wage with respect to the minimum wage (across steady states) is strictly greater than what is often referred to in the literature as its direct effect. The direct effect is the impact on workers initially earning below the new minimum, whose wages rise to the new minimum wage, holding fixed wages above this level. If only the direct effect were active, then β_{ms} would equal exactly one. In this job-ladder model, however, there is a direct effect and, at least in the long run, there are wage spillovers up the distribution above the minimum wage.

In particular, let $W_s(c)$ denote the steady-state wage at percentile c of employed skill s workers. In response to an increase in the minimum wage, $W_s(c)$ rises for all percentiles, $dW_s(c)/dm > 0$, but does so disproportionately for lower percentiles,

$$\frac{d\left[W_s(c)/W_s(c')\right]}{dm} > 0 \quad \text{for all} \quad W_s(c) < W_s(c')$$

The intuition for this result and its proof are straightforward. The steady-state share of workers on each rung of the job ladder is invariant to the value of the minimum wage, as shown in equation (4). Moreover, in response to an increase in the minimum wage, wages at lower rungs (and, therefore, at lower centiles of the wage distribution) increase disproportionately more, conditional on employment, as shown in equation (2).

For the steady-state analysis, it remains only to determine how changes across steady states in labor supply, labor demand, and the minimum wage affect skill prices, $d \log P_s$ in equation (6). According to equation (4), the share of workers who are unemployed is fixed across steady states, which implies that changes in skill prices $d \log P_s$ depend only on changes in factor supply and demand. Hence, for any aggregate production function, steady-state skill prices are determined exactly as in a competitive static model (without a job ladder, dynamics, or monopsony power) replacing the level of supply, L_{st}, with a fixed-across-steady-state constant times the level of supply, $(1 - u_s)L_{st}$. These issues have been considered theoretically and empirically in a wide class of aggregate production functions, from the canonical model, which features a CES aggregate production function combining only two skill groups, to nested CES models featuring many skill groups as in, e.g., Card and Lemieux (2001), and beyond. Of course, it should be noted that while the determination of steady-state values of P_s are not affected by the job-ladder model, the relationship between average wages and P_s is, as shown in equation (6).

Given my objective of studying the skill premium and extending the canonical model, in what follows I impose the same restrictions on the aggregate production function as in the canonical model: There are two worker skills—high $s = h$ and low-skill $s = \ell$ workers—and the aggregate production function is CES with elasticity η,

$$Y = \left[(A_h Y_h)^{\frac{\eta-1}{\eta}} + (A_\ell Y_\ell)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{8}$$

where $Y_s = (1 - u_s)L_s$. Additionally, and again only for the sake of obtaining a simple extension of the canonical model, suppose that $\beta_m = \beta_{ms}$ for both skills. Then equation (6) implies

$$d \log \left(\frac{\overline{w}_h}{\overline{w}_\ell} \right) = \beta_m b d \log m - \beta_L d \log \left(\frac{L_h}{L_\ell} \right) + \sum_s \beta_{A_s} d \log A_s \tag{9}$$

where

$$b \equiv b_h - b_\ell$$

is the *minimum wage bite*, which is simply the fraction of wage income earned by minimum wage workers among high-skill, b_h, minus the same fraction among low-skill workers, b_ℓ, and where

$$\beta_L \equiv \frac{1}{\eta} (1 - \beta_m b_\ell) \frac{P_h(1 - u_h)L_h}{Y} + \frac{1}{\eta} (1 - \beta_m b_h) \frac{P_\ell(1 - u_\ell)L_\ell}{Y} \tag{10}$$

Note that if $b_s = 0$ for both skills (i.e. no workers are at the minimum wage), then $\beta_L = 1/\eta$, exactly as in the canonical model.[8] Finally, linearize around a particular year, t,

[8] In general, the model can be consistent with the long-run (steady-state) elasticity of labor demand

assume that each year represents a new steady state (as would arise, for instance, if there were infinitely many periods within a year as in a continuous-time model with discrete changes to supply, demand, and minimum wages each year), and impose the assumption of the canonical model that A_{ht} and $A_{\ell t}$ each grow at constant rates across years with mean zero deviations. These assumptions yield a strict extension of the canonical model,

$$d\log\left(\frac{\overline{w}_{ht}}{\overline{w}_{\ell t}}\right) = \beta_m b d\log m_t - \beta_L d\log\left(\frac{L_{ht}}{L_{\ell t}}\right) + \beta_A t + \iota_t \tag{11}$$

The following proposition summarizes steady-state results.

Proposition 1.

Part 1. *In steady state, the wages on the job ladder, $\{w_{j,s}\}$, are given by equation (2), the unemployment rate is given by equation (3), the share of workers at each wage, $g_s(w_{j,s})$, is given by equation (4), and the average wage, $\overline{w}_s$, is given by equation (5).*

Part 2. *Across steady states, the wage at each centile of the skill s wage distribution, $W_s(c)$, increases disproportionately more with the minimum wage for lower centiles: $W_s(c) < W_s(c') \Rightarrow d\left[W_s(c)/W_s(c')\right]/dm > 0$.*

Part 3. *If the production function is given by (8), $\beta_m = \beta_{ms}$ for $s \in \{h, \ell\}$, A_{st} grows at a constant rate across years t, and each year represents a new steady state, then $d\log\left(\overline{w}_{ht}/\overline{w}_{\ell t}\right)$ is given by equation (11), where $\beta_m > 1$ and β_L are given by equations (7) and (10).*

3.3 Transition dynamics

Consider an economy in steady state at date $t = 0$ that experiences a one-time increase in the minimum wage from m to m' after all matches and bargaining in the period, but before production occurs and wages are paid. In Proposition 2 in Appendix B.2 I solve in closed form for the full transition (the distribution of wages at every date $t \geq 0$). Here, I describe the implications of these results.

On impact, the increase in the minimum wage raises the wage to m' of all workers employed at a wage below m', but has no effect above m'. That is, on impact only the direct effect is active. This implies that the impact elasticity of the average wage of skill s with respect to the minimum wage equals the share of labor income among the employed earned at the minimum wage before the shock $d\log\overline{w}_{s0}/d\log m = b_s$. This elasticity is strictly lower than the steady-state impact on the average wage, which is larger by a

across skills, as in Hurst et al. (2022), since there is a free parameter η in equation (10), as in the traditional canonical model. The same would apply given a generalized aggregate production function.

multiple of $\beta_{ms} > 1$, as shown in equation (6). For any number of skills and any aggregate production function, this implies that the impact elasticity of the skill premium (or, more generally, the relative average wage of any two skill groups) with respect to the minimum wage is the minimum wage bite b (the share of labor income earned at the minimum wage for high- minus the same share for low-skilled workers) just before the shock, $d\log(w_{h0}/w_{\ell 0})/d\log m = b$, which is again strictly lower than the steady-state elasticity in equation (11) by a multiple of $\beta_m > 1$, again assuming that $\beta_m = \beta_{ms}$ for $s \in \{h, \ell\}$.

How do wages adjust over time? For each skill s, two wage ladders coexist at all finite dates following the shock, the steady-state wage ladders associated with m and with m', which I denote by $w_{j,s}$ and $w'_{j,s}$. As described above, in period 0 the increase in the minimum wage only increases the wages of workers earning below m', creating the first rung on the new job ladder, $w'_{1,s} = m'$. One period after the minimum wage increase, $t = 1$, some workers who were employed on the first rung of the new job ladder at date $t = 0$ move to the second rung of the new job ladder, $w'_{2,s}$, and workers hired out of unemployment are employed at wage $w'_{1,s} = m'$. Period-by-period, workers enter successive rungs of the new job ladder and the number of workers on the new job ladder rises as the number of workers on the original job ladder falls by an equal amount; the unemployment rate is constant across time. As the share of workers on the original job ladder falls, it does so at first only for the lower wage rungs. Over time, the share of workers at higher and higher rungs of the original job ladder also begins to decline.[9]

Figure 3 displays the magnification coefficient $\beta_{m,t}$ associated with the t-period change in the skill premium following a small increase in the minimum wage at date 0 from m to $m' > m$, defined as

$$\beta_{m,t} \equiv \log\left(\frac{\overline{w}_{ht}}{\overline{w}_{\ell t}} \Big/ \frac{\overline{w}_{h}}{\overline{w}_{\ell}}\right) \Big/ \left[b\log\left(\frac{m'}{m}\right)\right] \tag{12}$$

Given parameters satisfying $\beta_{mh} = \beta_{m\ell}$ and a small increase in the minimum wage ($m' \to^{+} m$), the analytic results above provide the limits for $\beta_{m,t}$ in equation (12) as time converges to zero and infinity: $\lim_{t\to^{+}0}\beta_{m,t} = 1$ and $\lim_{t\to\infty}\beta_{m,t} = \beta_m$ defined in equation (7). In Figure 3, $\beta_{m,t}$ rises monotonically over time from 1 to β_m.

The figure is constructed under parameter values such that the initial steady-state minimum wage bite $b = b_h - b_\ell$ is approximately = -0.045, consistent with my measure at the national level described in Section 4.2; the log of the college premium is approximately 0.54, consistent with the average value of the national time series data across years in

[9] A decline in the minimum wage is different only because there is no immediate decrease in the wage of workers at the original minimum wage. But any worker who exogenously separates from her employer into unemployment will eventually enter employment at the new minimum wage and will slowly move along the rungs of the new job ladder, which is now associated with a lower minimum wage.

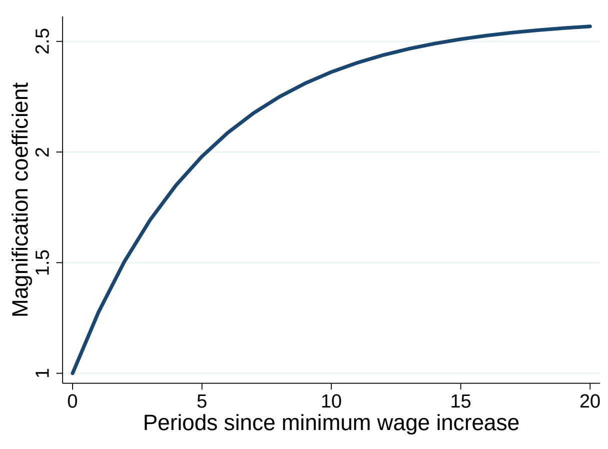


Figure 3: Magnification coefficient $\beta_{m,t}$

Notes: Magnification coefficient $\beta_{m,t}$ from equation (12), calculated changing the minimum wage from $m = 0.45$ to $m = 0.45001$ for $\beta_s = 0.25$, $\gamma_{se} = 0.4$, $\gamma_{su} = 0.9$, and $\delta_s = 0.08$ for both s, and for $P_\ell = 1$ and $P_h = 2$.

my sample; the average wage is approximately 64% of the value marginal product for high- and 74% for low-skill workers, in the range of Berger et al. (2022); and the long-run magnification coefficient is approximately 2.6, in the range of my longer-run estimates in Section 4.2.

The speed of adjustment in Figure 3 depends crucially on the parameters δ_s and γ_{se}. A lower value of δ_s implies that it takes longer for workers to move from the original job ladder to the new job one. A lower value of γ_{se} implies that it takes longer for a non-negligible share of the workers on the new job ladder to ascend up its rungs. See Section B.2 for details.

3.4 Robustness

In the baseline model, workers and firms bargain over wages. I assume that agents are not forward looking to facilitate solving the transition to an aggregate shock; and the model counterfactually predicts that expected job duration is independent of the worker's wage. Here, I show that steady-state results are broadly similar in the canonical wage-posting model of Burdett and Mortensen (1998), extended to include a minimum wage, as in van den Berg and Ridder (1998). In this framework, agents are forward looking and job duration is increasing in the worker's wage. I consider the case of a single skill s (omitting s subscripts), since this is sufficient to show which results are robust, and I focus on steady states, since solving for the transition to an aggregate shock is not straightforward. All results are derived in Appendix B.3.

Assume that parameters are such that the minimum wage is binding: the lowest wage offered by firms if they were not constrained by the minimum wage is less than the minimum wage. In this setting, I obtain a result equivalent to Part 1 of Proposition 1 by characterizing the distribution of wages across employed workers (a result contained in van den Berg and Ridder, 1998) and the average wage. I replicate Part 2 of Proposition 1 by solving explicitly for the wage at centile c of the wage distribution, $W(c)$, and showing that $W(c) < W(c')$ implies $d\left[W(c)/W(c')\right]/dm > 0$. I do not replicate Part 3 of Proposition 1, which requires incorporating two skill groups and an aggregate production function.

From the perspective of my analysis in this paper, there are two key distinctions between this model and my baseline model. First, studying the transition analytically is straightforward in my baseline model whereas solving even quantitatively for the transition in response to an aggregate shock (changing the minimum wage) in Burdett and Mortensen (1998) is more difficult. Second, in my baseline model—as in the data—a mass of workers earn exactly the minimum wage; in the Burdett and Mortensen (1998) model there are no mass points in the wage distribution. Hence, in response to a marginal increase in the minimum wage, the direct effect in my baseline model is positive whereas it is zero here.[10]

4 Empirics leveraging state variation

Section 2 shows that the minimum wage plays a central role—together with supply and demand—in shaping the evolution of the U.S. college premium in the national time series. Nevertheless, however robust are these results to variations in specification and measurement, there are still obvious limitations: the results are derived from a dataset with 55 observations and leverage only national time-series variation. In this section, I instead estimate versions of the extended canonical model of equation (11) leveraging variation across U.S. states and time. In Section 4.1 I describe the baseline empirical specification, data, and measurement. In Section 4.2 I display results—varying the length of time differences—and discuss the extent to which the short- and long-run empirical results are consistent with the theoretical results in Sections 3.2 and 3.3 and the motivating empirical results in Section 2. I additionally describe robustness exercises. Finally, to document the mechanism of dynamic wage spillovers more directly, in Section 4.3 I extend the analysis of Autor et al. (2016)—identifying the impact of changes in the minimum wage on the full wage distribution—to consider not only one-year differences, but also

[10] Related to both points: Engbom and Moser (2022) studies the impact of changes in minimum wages in the Burdett and Mortensen model, focusing exclusively on the steady state, and also extends the model to incorporate a mass point at the minimum wage.

longer ones.

4.1 Specification and measurement

I estimate a regional version of the extended canonical model based on equation (11). To bring this equation to the data, I must take a stand on the extent to which state labor markets face common changes in skill prices or not. States face common skill prices if skill s output is freely traded across space whereas each state faces skill prices determined by local supply and demand if the trade cost for skill s output is infinite. In either case, changes in the local skill premium depend on changes in the minimum wage only locally. In my baseline, I assume that skill prices are set nationally, in which case local supply is omitted from equation (11), yielding

$$\Delta \log \left(\frac{\overline{w}_{hrt}}{\overline{w}_{\ell rt}} \right) = \gamma_t + \gamma_r + \beta_m b_{rt} \Delta \log m_{rt} + [...] + \iota_{rt} \tag{13}$$

In sensitivity, I include measures of changes in local skill supplies and show that results for β_m are largely robust. Here, r indexes region, which correspond to the fifty states, and t indexes time, which correspond to the years 1979 - 2018 (excluding 1994 and 1995 for reasons described below). The term m_{rt} is the real minimum wage in state r in year t (the maximum of the legislated state and federal minimum wages) and $b_{rt} \equiv b_{rht} - b_{r\ell t}$ is the relevant bite of the minimum wage in state r in year t, defined as the share of high-education minus the share of low-education wage income earned by minimum wage workers in state r in year t.[11]

The national time fixed effect, γ_t, absorbs year-specific changes in the relative price of high-to-low-skilled output, $P_{ht}/P_{\ell t}$, which depend on changes in national supply and demand for worker skills. I additionally incorporate a state fixed effect, γ_r, which (together with the fact that the specification is in differences) controls for linear state-specific trends in local skill price deviations from national changes (which are relevant if skill s output is not freely traded). In some specifications I additionally include state-specific linear trends, which absorb quadratic state-specific deviations from national changes in relative skill prices.

Following the theory, I estimate (13) in differences, defining $\Delta x_t \equiv x_{t+T} - x_t$ as the T-period difference in any outcome x starting in year t. I report results using values of T ranging from one-year differences to nine-year differences.

[11] In different empirical contexts, Bailey et al. (2021), Derenoncourt and Montialoux (2021), and Chen and Teulings (2022) use similar measures of the minimum wage bite, which they define as the share of workers (rather than the share of income earned by workers) at or below the minimum wage.

To allow for correlation across time in the error terms ι_{rt}, in all specifications I cluster standard errors by state. In all regressions, I weigh each state by the average across sample years of its share of national population.

Measurement and instrument. I switch from using the March CPS at the national level to the CPS Merged Outgoing Rotation Groups (MORG) at the state level both because the MORG CPS includes a larger sample, which is particularly useful when dividing the data across fifty states, and because individual wages can be measured with less error (see Lemieux, 2006), which is especially important for measuring the bite of the minimum wage. These benefits come at the cost of a shorter time frame, starting with 1979 instead of 1963. I additionally drop the years 1994 and 1995 given missing imputation flags for all of the year (1994) or much of it (1995), leaving 38 years of data across fifty states. I clean the MORG CPS data following closely the approach in Lemieux (2006). See the Empirical Appendix for details.

I measure the state-year bite, b_{rt}, defining a worker as earning the minimum wage if her wage is no higher than 1.15 times her state's minimum wage. I choose 1.15 as the cutoff in order to make my results as comparable as possible to Cengiz et al. (2019), a comparison I describe in detail below.[12] This cutoff is also used in Derenoncourt and Montialoux (2021). I measure composition-adjusted state-year wages as in the national specification, but using state-specific rather than national data both in measuring wages of labor groups and constructing time-invariant weights across them.

Finally, the minimum wage bite, b_{rt}, likely suffers from measurement error. And this error may be correlated with the dependent variable, since the dependent variable (the change in the college premium between year t and $t + T$) itself depends on measures of wages in state r in year t; see Autor et al. (2016). To address this endogeneity concern, I instrument for $b_{rt}\Delta \log m_{rt}$ with $b_{rt-1}\Delta \log m_{rt}$. This instrument addresses the endogeneity concern if measurement error is uncorrelated across consecutive years. I describe an alternative instrument in robustness.

4.2 Results

Table 2 displays baseline estimation results. Each cell of the table displays the coefficient on $b_{rt}\Delta \log m_{rt}$ from a distinct regression. Columns 1 and 2 report results estimating regression (13) using OLS whereas columns 3 and 4 report results estimated using 2SLS.

[12] Cengiz et al. (2019) consider the minimum wage bin to be all wages between the minimum wage and one dollar (deflated to 2016) above. Given a minimum wage of $7 (which is approximately the median across states and sample years), this implies that workers earning up to approximately 1.15 times the minimum wage are considered to be at the minimum wage, as in my measure of b_{rt}.

	OLS		IV	
Time difference (years)	(1)	(2)	(3)	(4)
1	0.73	0.80	1.09*	1.26**
	(0.691)	(0.739)	(0.603)	(0.624)
			[2712]	[2509]
2	0.81	1.02	1.35*	1.63**
	(0.716)	(0.765)	(0.692)	(0.717)
			[7534]	[8870]
3	0.91	1.10	1.48**	1.65**
	(0.737)	(0.783)	(0.728)	(0.726)
			[7703]	[9135]
4	1.29*	1.46*	1.76**	1.93**
	(0.744)	(0.742)	(0.849)	(0.846)
			[1283]	[1419]
5	1.78**	1.98**	2.10**	2.34***
	(0.763)	(0.790)	(0.846)	(0.860)
			[2443]	[2207]
6	1.88**	2.12***	2.06**	2.32**
	(0.704)	(0.735)	(0.868)	(0.907)
			[1514]	[1448]
7	2.06***	2.33***	2.23***	2.52***
	(0.746)	(0.771)	(0.823)	(0.855)
			[1416]	[1335]
8	1.88**	2.20***	2.23***	2.61***
	(0.704)	(0.718)	(0.757)	(0.779)
			[2480]	[2522]
9	2.22***	2.62***	2.69***	2.97***
	(0.709)	(0.747)	(0.691)	(0.752)
			[3611]	[3421]
Year FE	Y	Y	Y	Y
State FE	Y	Y	Y	Y
Linear state trend	N	Y	N	Y

Note: Results of estimating equation (13). Each cell reports the coefficient of $b_{rt}\Delta \log m_{rt}$, and its standard error in parentheses, from a separate regression. Columns 1 and 2 estimate (13) using OLS whereas columns 3 and 4 use 2SLS (and additionally report the first stage F-statistic in brackets). Row T uses T-year differences, $\Delta x_t = x_{t+T} - x_t$. In columns 1 and 2 there are 1,800 observations in row 1 and $50 \times (36 - T)$ in row $T \geq 2$. In columns 3 and 4 there are 100 fewer observations in each row than column 1 since the instrument uses the one-year lagged value of b_{rt}, which is missing in 1979 and 1996.

Table 2: Impact of $b_{rt}\Delta \log m_{rt}$ on State-Level College Wage Premia

Each row $T = 1, ..., 9$ presents results from estimating regression (13) using the corresponding time difference $\Delta x_t \equiv x_{t+T} - x_t$, with one-year changes in row one and nine-year changes (in both skill premia and minimum wages) in row nine. I include time and state fixed effects, γ_t and γ_r, in all columns. I additionally incorporate a linear state trend in columns 2 and 4.

In first differences (row 1), the 2SLS coefficients on the minimum wage interaction range between approximately 1.1 and 1.25. These results are strongly consistent with the theoretical prediction in Section 3.3 that the impact elasticity of the skill premium with respect to the minimum wage equals the minimum wage bite, which is equivalent to a row-one estimate of one.[13] These first-difference results imply that in the first year after the change in the minimum wage, the change in the skill premium is consistent with the direct effect of a change in the minimum wage—raising the wages of workers initially earning below the new minimum wage—with at most very small indirect effects (i.e., very small wage spillovers up the wage distribution above the minimum wage). In particular, I cannot reject that any of the first-row coefficients equal one and, therefore, cannot reject that wage spillovers have no effect on the skill premium on impact.[14] These first-difference results are then broadly consistent with empirical work identifying the impact of changes in a state's minimum wage on its wage distribution. Using one-year differences, Autor et al. (2016) find that the direct effect of the minimum wage is large while wage spillovers are minimal. Cengiz et al. (2019) find that approximately 60% of their total wage effect of a change in the minimum wage is caused by the direct effect, with 40% caused by wage spillover effects above the minimum wage. My estimates imply that between approximately 80% ($\approx 1/1.26$, using the higher 2SLS estimate in column 4) and 92% ($\approx 1/1.09$, using the lower 2SLS estimate in column 3) of the total change in the skill premium is caused by the direct effect in the first year.

The theory in Section 3 not only predicts that one-year difference estimates should be close to one, but also that these elasticities should increase with the length of the time difference.[15] This prediction also fares well in the data. Empirically, coefficient point estimates tend to rise with the length of the time difference in each column. The coefficients in the last row, using nine-year differences, are statistically different from one in all

[13] If a period in the model is shorter than a year, as is likely the case in reality, then the row-one estimates should be slightly greater than one.

[14] Of course, this conclusion depends on the definition of the cutoff wage below which the minimum wage is assumed to bind, which is 1.15 times the minimum wage to facilitate comparison to Cengiz et al. (2019).

[15] Technically, the theory makes this prediction in response to a single change in the minimum wage. In the presence of many changes in the minimum wage, the empirics are guided by the theory but do not identify $\beta_{m,t}$ in equation (12).

columns. In this row, the 2SLS coefficients range between approximately 2.6 and 3. These coefficients imply that approximately 35% of the total impact of the minimum wage on the skill premium is caused by the direct effect, with approximately 65% caused by the indirect effect of wage spillovers above the minimum wage.

How do the coefficients in Table 2, which leverage variation across time and states, relate to the estimates in Table 1, which leverage variation from the national time series alone? These specifications differ in two additional respects: in Section 2, the specification is in levels and the minimum wage is not interacted with its bite whereas here the specification is in time differences and the change in the minimum wage is interacted with its bite. To make the coefficients more comparable, I could estimate the national specification interacting the real minimum wage with its average bite across all years, which is -0.046 when defining a worker in state r to be at the minimum wage if she earns no higher than 1.15 times her state minimum wage, consistent with the construction of b_{rt} in the state-level regressions. This, however, is mathematically identical to simply dividing the minimum wage coefficients in Table 1 by -0.046. This yields coefficients in the range of 3 - 4.2, depending on the specification, making the longer-run state-level estimates broadly consistent with, although at the lower end of the range of values obtained in the national time series in Table 1. This implies that the longer-run elasticity of a state's skill premium with respect to its real minimum wage is similar to the sizable elasticity identified in the national time series.

Summary. Consistent with results from the national time series, changes in the minimum wage play a central role in shaping the differential evolution of college premia across U.S. states between 1979 and 2018. Moreover, consistent with the theory, the impact of changes in the minimum wage is initially almost exclusively driven by its direct effect and, over time, magnified by wage spillovers up the wage distribution.

Robustness. Here, I briefly describe results of three types of robustness exercises: incorporating measures of relative state-specific supply changes, using a different instrument, and incorporating an additional control. Details are provided in the Empirical Appendix.

In my baseline I assumed that skill prices are set nationally, in which case local supply is omitted from equation (13) and the national change in skill prices is subsumed by the time fixed effect. Here, I incorporate changes in relative supply across states. I do so measuring changes in supply three ways: as the dual of wages (as in the national time-series approach), as a composition-adjusted measure of efficiency-unit hours worked (weighing changes in hours worked of each labor group in each state by a fixed-over-time measure of efficiency units per hour worked), and as a composition-adjusted measure of efficiency-unit populations (weighing changes in the population of each labor group in each state by a fixed-over-time measure of efficiency units). Tables A.5, A.6, and A.7 in the Empirical Appendix display results on the impact of changes in the minimum wage on the skill premium. These results are very similar to those displayed in Table 2.

My baseline instrument for $b_{rt}\Delta \log m_{rt}$ is $b_{rt-1}\Delta \log m_{rt}$. This instrument addresses the endogeneity concern if measurement error is uncorrelated across consecutive years. I also consider a different instrument: $\overline{b}_{rt}^{t}\overline{b}_{rt}^{r}\Delta \log m_{rt}$, where $\overline{b}_{rt}^{t}$ is the average bite of the minimum wage in state r across all years in the sample before year t and $\overline{b}_{rt}^{r}$ is the leave-out average bite of the minimum wage in year t across all states other than state r. In a sufficiently long sample, the alternative instrument allows measurement error to be correlated across years and would instead require that measurement error be mean zero on average within each state across years and across states within each year. Table A.8 displays results that are broadly similar to Table 2.

One potential concern is that the bite measure, b_{rt}, rather than the actual change in the minimum wage, $\Delta \log m_{rt}$, is doing the work in these state-by-time regressions. To investigate this issue, I estimate a version of regression (13) including an additional control: b_{rt-1}. Table A.9 displays results, which are again very similar to Table 2.

4.3 Dynamic effects of the minimum wage across the distribution

My results in Table 2—of a growing impact of the minimum wage on the skill premium over time—are consistent with dynamic spillovers up the wage distribution that decline for higher wage centiles, as predicted by my theory in Part 2 of Proposition 1. Here, I provide direct evidence. I leverage the approach and replication package of Autor et al. (2016) (henceforth, AMS). Following Lee (1999), AMS identify the impact of changes in the minimum wage on the wage of centile c relative to the median wage by estimating the following regression in first differences across years (using the MORG CPS between 1979 and 2012)

$$\Delta \log \left(\frac{W_{rt}(c)}{W_{rt}(50)} \right) = \beta_1(c) \Delta \log \left(\frac{m_{rt}}{W_{rt}(50)} \right) + \beta_2(c) \left[\Delta \log \left(\frac{m_{rt}}{W_{rt}(50)} \right) \right]^2 + \alpha_r + \alpha_c + \varepsilon_{rt}(c) \tag{14}$$

using state, r, and time, t, variation. Here, $W_{rt}(c)$ is the wage at centile c in region r in year t, so that $W_{rt}(50)$ is the median wage. Because both the dependent and independent variables depend on the median wage, which is measured with error, they instrument using the first difference of the log of the minimum wage, the first difference of the square of the log minimum, and the first difference of the log minimum interacted with the av-

	Time difference in years								
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
p(5)	0.295	0.341	0.320	0.310	0.292	0.303	0.329	0.352	0.370
	(0.063)	(0.062)	(0.053)	(0.046)	(0.045)	(0.043)	(0.036)	(0.035)	(0.047)
p(10)	0.168	0.199	0.191	0.179	0.159	0.179	0.196	0.196	0.225
	(0.044)	(0.045)	(0.039)	(0.029)	(0.031)	(0.034)	(0.032)	(0.031)	(0.033)
p(20)	0.036	0.036	0.046	0.060	0.065	0.067	0.058	0.070	0.097
	(0.028)	(0.025)	(0.024)	(0.024)	(0.021)	(0.018)	(0.022)	(0.026)	(0.029)
p(30)	-0.003	0.009	0.024	0.037	0.036	0.035	0.025	0.026	0.049
	(0.023)	(0.022)	(0.019)	(0.019)	(0.019)	(0.021)	(0.028)	(0.029)	(0.025)
p(40)	0.016	0.031	0.032	0.025	0.015	0.009	0.019	0.015	0.026
	(0.032)	(0.027)	(0.014)	(0.011)	(0.015)	(0.015)	(0.018)	(0.017)	(0.020)
p(75)	0.014	0.021	0.018	0.018	0.010	-0.004	-0.014	-0.011	0.004
	(0.023)	(0.019)	(0.020)	(0.017)	(0.017)	(0.017)	(0.021)	(0.021)	(0.022)
p(90)	0.021	0.030	0.037	0.030	0.003	-0.013	-0.024	-0.034	-0.007
	(0.034)	(0.033)	(0.034)	(0.034)	(0.034)	(0.034)	(0.038)	(0.040)	(0.040)

Table 3: Distributional effect of minimum wage changes for select percentiles (relative to the median) using one-year to nine-year differences, including females and males

Notes: Estimates of equation (14). Each column $j \in \{1, ...9\}$ replicates column 4 of Table 2B in AMS, which uses one-year differences, but using j-year differences instead.

erage real log median for the state over the sample. AMS report the marginal effects for selected percentiles in column 4 of their table 2. I follow their approach exactly, but varying the length of the time differences (both in the depenedent and independent variables in regression (14), in the regression weights, and in the instruments).

Table 3 displays results for various centiles, with column $j \in \{1, ..., 9\}$ showing results associated with the j-year time difference in regression (14). Since column 1 is a one-year difference, it exactly replicates column 4 of Table 2B in AMS. As noted in AMS, effects are small and insignificant by the 20th centile of the wage distribution. Columns 2 - 9 display new results. Over time, the point estimates associated with the 10th, 20th, and 30th percentiles rise while the point estimates associated with the 75th and 90th percentiles fall (with some noise, of course). While on impact wages rise only at the bottom of the distribution (relative to the median), in the longer run lower centiles of the wage distribution experience disproportionate wage gains than higher wage centiles. Each of these results is broadly consistent with the prediction in Part 2 of Proposition 1. Hence, it appears that the dynamic response of the skill premium to changes in the minimum wage documented in Table 2 and predicted by Part 3 of Proposition 1 are driven by changes throughout the wage distribution, consistent with the theory. Table A.10 in the Empirical

Appendix displays results estimated separately on the subsamples of females and males; similar patterns are evident.

5 Conclusions

What is the impact of the minimum wage on the college premium? In this paper I present a simple theoretical argument and both national and a range of state-level empirical evidence showing that the minimum wage plays a central role in shaping the U.S. college wage premium and its variation across states and showing that these impacts are slow developing.

Theoretically, I have provided a generalization of the canonical model featuring labor-market institutions—monopsony power and minimum wages—labor-market dynamics, a job ladder, and unemployment that maintains the simplicity and tractability of the purely neoclassical model. The model generates a simple estimating equation similar to the canonical model, additionally incorporating the real minimum wage. The model predicts that at the moment the minimum wage increases, the elasticity of the skill premium with respect to the minimum wage equals its bite, as wages rise only for those earning below the new minimum wage. Over time, however, the model predicts that the increase in the minimum wage spills over to wages higher up the distribution as workers slowly climb the job ladder starting from a higher entry wage. This implies that the long-run elasticity of the skill premium is strictly greater than the short-run elasticity.

Empirically, at the national level I have shown that changes in the real minimum wage—together with changes in supply and demand—play a substantial role in generating the observed evolution of the U.S. college premium between 1963 and 2017. I have also documented that incorporating the real minimum wage into the empirical implementation of the canonical model improves its out-of-sample fit and, in contrast to the model without the real minimum wage, implies a much smaller slowdown (or none at all) in the rate of skill-biased technical change. These conclusions differ from the past literature and I show why.

Empirically, at the state level I have shown that the impact elasticity of the college premium roughly equals the minimum wage bite, consistent with my theory and past empirical work finding small wage spillover effects. I find that the longer-run effects are almost three times larger, implying sizable spillover effects up the wage distribution, consistent with my theoretical predictions and national estimates. In the longer run, minimum wages—together with supply and demand—play a first-order role in shaping national and regional college premia (in spite of small wage spillovers in the short run).

Finally, I additionally document that increases in the minimum wage have substantially larger longer-run effects throughout the wage distribution (raising wages at lower percentiles disproportionately more than at higher percentiles) than previously documented in the short run, again confirming the predictions of my theory.

My model is not aimed at conducting a quantitative policy analysis or welfare counterfactual. My theoretical goal was instead qualitative: to guide my empirical analysis and interpret its results. Of course, the model abstracts from many issues, both related to the minimum wage (including the employment effects at the heart of the macro-labor literature) and other institutions (including unions). In addition, while my theory is consistent with a job-ladder model, without worker-level wage panel data I do not provide direct evidence of the minimum wage affecting wages along the job ladder, which I consider an important avenue for future work.

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A Empirical Appendix

Constructing real minimum wages. I construct real minimum wages as follows. In 1974-2012, I use data from Autor et al. (2016) to measure state-year minimum wages. For each month and state, I define the relevant minimum wage as the maximum of the legislated state and federal (FLSA) minimum wages. For each state and year I then average minimum wages across months to construct the state-year minimum wage. For the remaining years, I measure annual federal minimum wages using data from the Department of Labor (averaging across months) and I measure annual legislated state minimum wages using data from FRED. For each state-year, I take the maximum of these annual numbers to obtain the minimum wage. In measuring state real minimum wages, I deflate using the GDP deflator from FRED. This approach yields the state-year series of annual real minimum wages.

To construct the national time series of real minimum wages, I additionally take the following steps. After constructing the state-year series of annual real minimum wages, I average across states in each year using fixed weights and refer to this as the real minimum wage. When using the FLSA minimum wages instead, I also deflate using the GDP deflator.

A.1 National time-series analysis

Basic processing of the March CPS data. I use the March Annual Demographic Files of the Current Population Survey from 1964 to 2018, which report earnings from 1963 to 2017, for workers age 16 to 64 during the earnings year. Thus, throughout when I refer to any year, I am using the following year's March CPS.

I drop respondents with missing schooling, missing or negative earnings, or with missing weeks worked. I additionally drop those who are self employed or engage in unpaid family work and anyone with allocated earnings. Finally, I drop respondents who are part of the 3/8 redesign in the 2014 ASEC sample. Following Autor et al. (2008) I multiply top-coded earnings by 1.5.

In composition adjusting, I bin workers into one of 180 groups, denoted by g, defined by the intersection of 9 age bins, 2 genders, 2 races (white and all other self-reported races), and 5 education levels (high school dropout, high school graduate, some college, college complete, and graduate training).[16] The lowest three educations—high school

[16] Up to and including 1991, I use the highest grade of school completed; I define college complete as having finished the fourth year of college and graduate training as having more than four years of college. Starting in 1992 I use degree completion, assigning associate's degrees to some college.

dropouts, those with a high school degree, and those with some college—are allocated to non-college; the highest two educations—college graduates and graduate training—are allocated to college.

Constructing Wages and Supplies. In each year t and for each of the 180 groups g I construct total hours worked and total wage and salary income (using sample weights) and, from this, the average wage of each group-year pair, w_{gt} for group g. Within each year I average across the log wages of all groups with at least a college degree (denoting the set of these groups by $\mathcal{G}_h$) and, separately, across all groups without (denoting the set of these groups by $\mathcal{G}_\ell$) using time-invariant weights. For instance, for college-educated workers, I have

$$\log w_{ht} = \sum_{g \in \mathcal{G}_h} \omega_g \log w_{gt}$$

where ω_g is the time-invariant weight applied to group g. These weights are constructed using the average across years of the share of hours worked of each group g within the set of college groups $\mathcal{G}_h$ and, separately, within the set of non-college groups $\mathcal{G}_\ell$. The resulting averages are the composition-adjusted wages $\log w_{ht}$ and $\log w_{\ell t}$ used in the analysis.

In my baseline, I construct composition-adjusted supplies of college and non-college workers as the dual of these wages. In particular, I set $\log Supply_{ht} = \log Inc_{ht} - \log w_{ht}$ where Inc_{ht} is the total income of college-educated workers in raw (weighted) data in year t. I similarly construct $\log Supply_{\ell t}$.[17] In robustness exercises I measure supply differently. I describe these alternative approaches below.

Robustness of Table 1. Table A.1 displays results of estimating equation (1) including progressively higher polynomials of time. The minimum wage is negative and significant in all specifications, up to and including an eighth-degree polynomial of time.

Tables A.2 and A.3 replicate Table 1 using two distinct measures of relative skill supply. These results are broadly similar to Table 1.

In Table A.2 I measure changes in relative supply using a composition-adjusted measure of changes in efficiency-unit hours worked. The hours-based measure is constructed as follows. In the first step, for each year and college group (each labor bin is either in the set with completed college education or without), I construct a composition-adjusted weighted average wage. The fixed-over-time weights are identical to those used in the construction of composition-adjusted wages. In the second step, I then divide the av-

[17] When constructing wages and supplies for each state r, I follow exactly the same procedure as above, but using data within state r alone.

	1963-2017							
	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
Relative supply	-0.556	-0.600	-0.531	-0.477	-0.080	0.046	0.050	0.088
	(0.054)	(0.052)	(0.102)	(0.089)	(0.126)	(0.092)	(0.094)	(0.078)
Real m.w.	-0.195	-0.159	-0.136	-0.177	-0.161	-0.117	-0.117	-0.075
	(0.040)	(0.059)	(0.055)	(0.062)	(0.041)	(0.030)	(0.030)	(0.029)
Constant	0.032	0.021	0.034	0.025	-0.015	-0.075	-0.073	-0.027
	(0.009)	(0.014)	(0.029)	(0.028)	(0.017)	(0.012)	(0.013)	(0.009)
Time Polynom.	1	2	3	4	5	6	7	8
Observations	55	55	55	55	55	55	55	55
R-squared	0.968	0.969	0.969	0.971	0.981	0.990	0.990	0.992

Table A.1: Regression Models for the College Wage Premium (Higher Time Polynomials)
Notes: The estimating equation is (1) and the sample is 1963-2017 in all columns. The dependent variable and "Relative supply" are the logs of the composition-adjusted college premium and relative supply of hours worked. "Real min. wage" is the log of the real minimum wage. "Time Polynom." refers to the degree of the polynomial of time. Robust standard errors are reported.

erage wage of each labor group in that year-college pair by the average across all labor groups (in the corresponding year-college pair) created in the first step; this provides a year-specific measure of the relative wage of each group within the college-educated and within the non-college-educated. In the third step, I take an average across years of this relative wage within each labor group. This average across years is a measure of the average efficiency units supplied by each hour of labor of this labor group relative to the average labor group in the same college group. Of course, this average does depend on the average amount of monopsony power confronted by this labor group across time; but *changes* in monopsony power across time do not generate changes across time in this measure of efficiency units. In the fourth step, I take a weighted average of hours worked across all labor groups in the year-college pair, weighting by the average efficiency units supplied by each of these labor groups across time. Finally, I measure supply as the logarithm of this composition-adjusted weighted average of efficiency-unit hours worked.

	1963-1987	1963-2017	1963-1987	1963-2017			
	(a)	(b)	(c)	(d)	(e)	(f)	(g)
Relative supply	-0.519	-0.439	-0.574	-0.575	-0.620	-0.506	-0.519
	(0.109)	(0.047)	(0.111)	(0.057)	(0.055)	(0.092)	(0.051)
Real m.w.			-0.269	-0.211	-0.177	-0.133	
			(0.090)	(0.042)	(0.060)	(0.055)	
Real FLSA m.w.							-0.172
							(0.041)
Time	0.020	0.018	0.021	0.021			
	(0.004)	(0.001)	(0.005)	(0.001)			
Constant	0.018	0.023	0.039	0.032	0.022	0.044	0.032
	(0.014)	(0.007)	(0.020)	(0.009)	(0.015)	(0.029)	(0.009)
Time Polynom.	1	1	1	1	2	3	1
Observations	25	55	25	55	55	55	55
R-squared	0.282	0.946	0.527	0.965	0.966	0.969	0.960

Table A.2: Replicating Table 1 using composition-adjusted changes in efficiency-unit hours worked

Notes: Replicating Table 1 replacing the baseline measure of relative supply using a measure of composition-adjusted changes in hours worked.

	1963-1987	1963-2017	1963-1987	1963-2017			
	(a)	(b)	(c)	(d)	(e)	(f)	(g)
Relative supply	-0.786	-0.317	-0.800	-0.598	-0.804	-0.574	-0.491
	(0.167)	(0.066)	(0.151)	(0.082)	(0.096)	(0.139)	(0.078)
Real m.w.			-0.246	-0.302	-0.271	-0.135	
			(0.078)	(0.057)	(0.065)	(0.055)	
Real FLSA m.w.							-0.235
							(0.054)
Time	0.026	0.014	0.025	0.020			
	(0.006)	(0.002)	(0.005)	(0.002)			
Constant	0.001	0.006	0.022	0.022	-0.006	0.052	0.021
	(0.019)	(0.009)	(0.023)	(0.009)	(0.018)	(0.032)	(0.009)
Time Polynom.	1	1	1	1	2	3	1
Observations	25	55	25	55	55	55	55
R-squared	0.423	0.909	0.630	0.936	0.941	0.964	0.930

Table A.3: Replicating Table 1 using composition-adjusted changes in efficiency-unit populations

Notes: Replicating Table 1 replacing the baseline measure of relative supply using a measure of composition-adjusted changes in population.

In Table A.3 I measure changes in relative supply using a composition-adjusted measure of efficiency-unit populations (in which populations that earn more on average receive higher weight). This population-based measure is constructed very similarly to the composition-adjusted measure of efficiency-unit hours worked. In the first step, for each year and college group, I construct a composition-adjusted weighted average income (again, using weights that are identical to those used in the construction of composition-adjusted wages). In the second step, I divide total income of each labor group by the average across all labor groups (in the corresponding year-college pair) created in the first step. In the third step, I take an average across years of this relative income within each labor group. This average across years is a measure of the total efficiency units supplied across all agents in this labor group relative to the average labor group in the same college group that is invariant across years. In the fourth step, I take a weighted average of populations across all labor groups in the year-college pair, weighting by the share of total efficiency units supplied by each of these labor groups across time. Finally, I measure supply as the logarithm of this composition-adjusted weighted average of populations. As with the composition-adjusted measure of hours worked, *changes* in monopsony power across time do not generate changes across time in this measure of efficiency units. In addition, because I take a weighted average of populations rather than hours worked, changes in unemployment and non-employment more generally do not affect this measure of changes in supply.

Comparison of results with Autor et al. (2008). Autor et al. (2008) estimate a related reduced-form regression including the real minimum wage. They report two specifications including the real minimum wage and relative supply, in columns 6 and 7 of their Table 2. In column 7, they include only a linear time trend and find a significant and negative coefficient on the real minimum wage that is similar to my estimates. In column 6, they include a cubic polynomial of time and find that the coefficient on the real minimum wage is negative, relatively small, and insignificantly different from zero. They conclude that the real minimum wage "does not much alter the central role for relative supply growth fluctuations and trend demand growth in explaining the evolution of the college wage premium" and that "institutional factors are insufficient to resolve the puzzle posed by slowing trend relative demand for college workers in the 1990s."

I find relative supply growth fluctuations and trend demand growth remain crucial drivers of the college premium, consistent with Autor et al. (2008), but so too are changes in the real minimum wage. I also find that changes in the minimum wage are sufficient to resolve the puzzle posed by slowing trend relative demand for college workers in the

	1963-2005					
	(a)	(b)	(c)	(d)	(e)	(f)
Relative supply	-0.431	-0.606	-0.612	-0.216	0.013	0.028
	(0.051)	(0.077)	(0.091)	(0.113)	(0.104)	(0.093)
Minimum wage	-0.113	-0.109	-0.064	-0.174	-0.123	-0.119
	(0.049)	(0.049)	(0.048)	(0.052)	(0.040)	(0.043)
Constant	0.253	0.071	-0.001	0.523	0.581	0.591
	(0.083)	(0.112)	(0.119)	(0.179)	(0.134)	(0.130)
Time Polynom.	1	2	3	4	5	6
Observations	43	43	43	43	43	43
R-squared	0.942	0.947	0.954	0.972	0.979	0.979

Table A.4: Regression Analysis for the College Wage Premium Using Data From the Replication Package for Autor et al. (2008)

Notes: This table estimates (1) using relative wages, relative supply, and the nominal minimum wage from Autor et al. (2008). I deflate their nominal minimum wage using the GDP deflator from FRED rather than the GDP deflator in their replication package. Robust standard errors are reported.

1990s; that is, the extended canonical model fits well out of sample.

Here, I show that these conclusions can be reached using data from Autor et al.'s replication package. Table A.4 displays results using their data moving from a linear time trend in column (a) to a sextic polynomial of time in column (f).[18] Columns (a) and (c) correspond to those reported in Autor et al. (2008). The coefficient on the real minimum wage is negative, significant, and similar to my baseline estimate across all specifications up to and including a sextic polynomial of time except for the case of a third-degree polynomial of time in column (c), in which case the coefficient on the real minimum wage is insignificant and relatively small.

A.2 State-level analysis

Basic Processing of the Merged Outgoing Rotation Groups CPS Data. I use statistics from the Current Population Survey Merged Outgoing Rotation Groups (MORG CPS) in the state-level estimation. I use the MORG CPS from 1979 to 2018, which reflects current wages. Because of missing imputation flags—the CPS did not flag workers with missing wages—in all of 1994, I do not include this year. As above, I restrict attention to worker

[18]Whereas they include the male unemployment rate, here I do not. The pattern of coefficient significance is identical including the male unemployment rate and the estimated minimum wage coefficients are almost identical. Given the shorter time period (1963-2005), I stop at a sextic time trend since both the minimum wage and relative supply coefficients are insignificant thereafter.

ages 16 to 64.

In processing the files, I broadly follow the approaches of Lemieux (2006) and Autor et al. (2008), using hourly wages for workers paid by the hour and using usual weekly earnings divided by hours worked last week for non-hourly workers. I multiply top-coded constructed hourly wages by 1.5. I drop respondents with allocated earnings flags. I identify and drop non-flagged allocated observations between 1989 and 1993 using the unedited earnings values.

In constructing the distribution of wages, I use the product of earnings weights and hours worked. I am therefore constructing the distribution of hourly wages across hours worked rather than across workers.

Robustness of Table 2 to alternative measures of supply. Tables A.5, A.6, and A.7 display results of replicating Table 2 incorporating a measure of the change in state-specific relative skill supply in regression (13), with each table using a distinct measure of relative supply.

In Table A.5 I measure changes in relative supply using the dual of relative wages, exactly as in the national time-series specification, but using data only from state r when constructing supply in state r. In particular, I measure supply of college hours in state r at time t so that the product of the supply of college hours in state r at time t and the hourly wage of college labor in state r at time t exactly equals college income in state r and time t in the data; and the same is true of non-college hours and wages. Unfortunately, this measure of relative supply is directly impacted by changes in the minimum wage, which affect wage income in the state-year pair. The following two measures of relative supply help address this issue.

In Table A.6 I measure changes in relative supply using a composition-adjusted measure of efficiency-unit hours worked. The hours-based measure is constructed as follows. In the first step, for each state, year, and college group (each labor bin is either in the set with completed college education or without), I construct a composition-adjusted weighted average wage. The fixed-over-time weights are identical to those used in the construction of composition-adjusted wages at the state level. In the second step, I then divide the average wage of each labor group in that state-year-college triplet by the average across all labor groups (in the corresponding state-year-college triplet) created in the first step; this provides a state-and-year-specific measure of the relative wage of each group within the college-educated and within the non-college-educated. In the third step, I take an average across years of this relative wage within each labor group-state pair. This average across years is a measure of the average efficiency units supplied by each hour

	OLS		IV	
Time difference (years)	(1)	(2)	(3)	(4)
1	0.56	0.65	1.05*	1.26**
	(0.646)	(0.717)	(0.560)	(0.619)
			[2707]	[2502]
2	0.71	0.94	1.26*	1.57**
	(0.699)	(0.778)	(0.665)	(0.725)
			[7496]	[8827]
3	0.77	0.97	1.34*	1.53**
	(0.713)	(0.793)	(0.680)	(0.703)
			[7638]	[9134]
4	1.22*	1.38*	1.76**	1.92**
	(0.702)	(0.724)	(0.785)	(0.786)
			[1297]	[1426]
5	1.75**	1.93**	2.16**	2.39***
	(0.731)	(0.766)	(0.805)	(0.814)
			[2482]	[2225]
6	1.93***	2.13***	2.15**	2.38***
	(0.692)	(0.717)	(0.851)	(0.869)
			[1519]	[1452]
7	2.07***	2.28***	2.32***	2.53***
	(0.712)	(0.722)	(0.790)	(0.793)
			[1421]	[1338]
8	1.89***	2.13***	2.30***	2.58***
	(0.667)	(0.645)	(0.722)	(0.698)
			[2534]	[2539]
9	2.23***	2.51***	2.71***	2.91***
	(0.660)	(0.623)	(0.664)	(0.652)
			[3642]	[3425]
Year FE	Y	Y	Y	Y
State FE	Y	Y	Y	Y
Linear state trend	N	Y	N	Y

Note: Replicating Table 2 incorporating changes in state-specific relative skill supply, measured as the dual of relative wages.

Table A.5: Replicating Table 2: the dual of wages measures relative supply

Time difference (years)	OLS (1)	OLS (2)	IV (3)	IV (4)
1	0.75	0.82	1.05	1.21*
	(0.716)	(0.757)	(0.647)	(0.657)
			[2714]	[2513]
2	0.87	1.07	1.36*	1.62**
	(0.738)	(0.766)	(0.731)	(0.734)
			[7547]	[8883]
3	0.99	1.19	1.55*	1.72**
	(0.777)	(0.801)	(0.797)	(0.786)
			[7629]	[9146]
4	1.32	1.50*	1.70*	1.89*
	(0.821)	(0.812)	(0.944)	(0.950)
			[1299]	[1429]
5	1.74**	1.97**	1.98**	2.25**
	(0.823)	(0.859)	(0.903)	(0.936)
			[2496]	[2230]
6	1.76**	2.05**	1.89**	2.22**
	(0.740)	(0.792)	(0.905)	(0.975)
			[1520]	[1455]
7	1.97**	2.31***	2.10**	2.46***
	(0.789)	(0.829)	(0.861)	(0.916)
			[1423]	[1339]
8	1.84**	2.23***	2.17***	2.60***
	(0.737)	(0.771)	(0.781)	(0.831)
			[2532]	[2542]
9	2.18***	2.65***	2.64***	2.97***
	(0.748)	(0.816)	(0.724)	(0.821)
			[3690]	[3485]
Year FE	Y	Y	Y	Y
State FE	Y	Y	Y	Y
Linear state trend	N	Y	N	Y

Note: Replicating Table 2 incorporating changes in state-specific relative skill supply, measured as composition-adjusted relative hours worked.

Table A.6: Replicating Table 2: composition-adjusted hours worked (weighted by efficiency units per hour) measures relative supply

	OLS		IV	
Time difference (years)	(1)	(2)	(3)	(4)
1	0.80	0.87	1.15*	1.31**
	(0.710)	(0.756)	(0.632)	(0.651)
			[2730]	[2522]
2	0.89	1.11	1.43**	1.71**
	(0.735)	(0.777)	(0.710)	(0.728)
			[6973]	[8403]
3	0.94	1.14	1.54**	1.72**
	(0.763)	(0.809)	(0.760)	(0.758)
			[7142]	[8634]
4	1.40*	1.60**	1.88**	2.08**
	(0.774)	(0.773)	(0.887)	(0.887)
			[1314]	[1423]
5	1.93**	2.16**	2.25**	2.53***
	(0.786)	(0.821)	(0.872)	(0.896)
			[2469]	[2222]
6	1.99***	2.26***	2.16**	2.49**
	(0.724)	(0.770)	(0.884)	(0.943)
			[1610]	[1489]
7	2.10***	2.41***	2.24***	2.59***
	(0.759)	(0.803)	(0.835)	(0.890)
			[1636]	[1472]
8	1.88**	2.25***	2.21***	2.63***
	(0.705)	(0.741)	(0.759)	(0.799)
			[2594]	[2537]
9	2.25***	2.69***	2.72***	3.06***
	(0.703)	(0.766)	(0.686)	(0.774)
			[3689]	[3391]
Year FE	Y	Y	Y	Y
State FE	Y	Y	Y	Y
Linear state trend	N	Y	N	Y

Note: Replicating Table 2 incorporating changes in state-specific relative skill supply, measured as relative population.

Table A.7: Replicating Table 2: composition-adjusted populations (weighted by efficiency units) measures relative supply of labor of this labor group relative to the average labor group in the same state-college pair. Of course, this average does depend on the average amount of monopsony power confronted by this labor group across time; but *changes* in monopsony power across time do not generate changes across time in this measure of efficiency units. In the fourth step, I take a weighted average of hours worked across all labor groups in the state-year-college triplet, weighting by the average efficiency units supplied by each of these labor groups across time within that state. Finally, I measure supply as the logarithm of this composition-adjusted weighted average of hours worked.

In Table A.7 I measure changes in relative supply using a composition-adjusted measure of efficiency-unit populations. This population-based measure is constructed very similarly to the composition-adjusted measure of hours worked. In the first step, for each state, year and college group, I construct a composition-adjusted weighted average income (again, using weights that are identical to those used in the construction of composition-adjusted wages at the state level). In the second step, I divide total income of each labor group in that state-year-college triplet by the average across all labor groups (in the corresponding state-year-college triplet) created in the first step. In the third step, I take an average across years of this relative income within each labor group-state pair. This average across years is a measure of the total efficiency units supplied across all agents in this labor group relative to the average labor group in the same state-college pair that is invariant across years. In the fourth step, I take a weighted average of populations across all labor groups in the state-year-college triplet, weighting by the share of total efficiency units supplied by each of these labor groups across time within that state. Finally, I measure supply as the logarithm of this composition-adjusted weighted average of populations. As with the composition-adjusted measure of hours worked, *changes* in monopsony power across time do not generate changes across time in this measure of efficiency units. In addition, because I take a weighted average of populations rather than hours worked, changes in unemployment and non-employment more generally do not affect this measure of changes in supply.

Robustness of Table 2 to an alternative instrument. In my baseline, I instrument for $b_{rt}\Delta \log m_{rt}$ with $b_{rt-1}\Delta \log m_{rt}$. This instrument addresses the endogeneity concern if measurement error is uncorrelated across consecutive years. Here, I consider a different instrument: $\overline{b}^t_{rt}\overline{b}^r_{rt}\Delta \log m_{rt}$. This instrument replaces b_{rt} with the product of two terms. The first term, $\overline{b}^t_{rt} \equiv \frac{1}{|t-1979|}\sum_{j<t} b_{rj}$, is the average bite of the minimum wage in state r across all years in the sample before year t. By focusing on years before t, my instrument for the bite is not itself a function of measurement error in year t or $t+T$ and is not a

	IV	
Time difference (years)	(1)	(2)
1	0.81	0.90
	(0.779)	(0.828)
	[91]	[94]
2	0.88	1.00
	(0.868)	(0.906)
	[100]	[105]
3	1.16	1.22
	(0.910)	(0.917)
	[101]	[109]
4	1.77*	1.81*
	(0.895)	(0.912)
	[129]	[132]
5	2.28**	2.50**
	(0.911)	(0.974)
	[210]	[219]
6	2.43***	2.73***
	(0.853)	(0.913)
	[281]	[256]
7	2.30***	2.70***
	(0.825)	(0.850)
	[355]	[274]
8	2.06**	2.48***
	(0.771)	(0.763)
	[577]	[403]
9	2.46***	2.90***
	(0.649)	(0.648)
	[850]	[530]
State FE	Y	Y
Year FE	Y	Y
Linear state trend	N	Y

Note: Replication of columns 3 and 4 of Table 2, but using the alternative instrument.

Table A.8: Replicating Table 2 using the alternative instrument $\bar{b}_{rt}^{t}\bar{b}_{rt}^{r}\Delta \log m_{rt}$

function of current changes in the minimum wage. The second term, $\overline{b}^{r}_{rt} \equiv \frac{1}{49}\sum_{j\neq r} b_{jt}$, is the leave-out average bite of the minimum wage in year t across all states other than state r. In a sufficiently long sample, the alternative instrument allows measurement error to be correlated across years and would instead require that measurement error be mean zero on average within each state across years and across states within each year. Table A.8 displays results. Results are broadly similar to Table 2.

Robustness of Table 2 to controlling for b_{rt-1}. One worry is that the bite measure, b_{rt}, rather than the actual change in the minimum wage, $\Delta \log m_{rt}$, is doing the work in these state-by-time regressions. To investigate this issue, I estimate a version of regression (13) including an additional control: b_{rt-1}. Table A.9 displays results, which are very similar to Table 2. Results are again similar if I instead control for b_{rt}; but given endogeneity concerns, I use b_{rt-1}.

B Theoretical Appendix

B.1 Steady state details

Equation (6) implies

$$\begin{aligned} d\log\left(\frac{\overline{w}_h}{\overline{w}_\ell}\right) &= \chi_m\left(\frac{mg_h(m)}{\overline{w}_h} - \frac{mg_\ell(m)}{\overline{w}_\ell}\right) d\log m \\ &\quad + \left(1-\chi_m\frac{mg_h(m)}{\overline{w}_h}\right) d\log P_h - \left(1-\chi_m\frac{mg_\ell(m)}{\overline{w}_\ell}\right) d\log P_\ell \end{aligned}$$

Equation (8), $P_s = \partial Y/\partial Y_s$, and $Y_s = (1-u_s)L_s$ imply

$$P_s = Y^{\frac{1}{\eta}} A_s^{\frac{\eta-1}{\eta}} (1-u_s)^{\frac{-1}{\eta}} L_s^{\frac{-1}{\eta}}$$

and

$$d\log P_s = \frac{1}{\eta} d\log Y + d\log A_s - \frac{1}{\eta}\left(d\log A_s + d\log L_s\right)$$

From the production function, we have

$$d\log Y = \frac{1}{Y}\sum_{s\in\{h,\ell\}} Y^{\frac{1}{\eta}} \left(A_s(1-u_s)L_s\right)^{\frac{\eta-1}{\eta}} \left(d\log A_s + d\log L_s\right)$$

	OLS		IV	
Time difference (years)	(1)	(2)	(3)	(4)
1	0.95	1.02	1.06*	1.16*
	(0.727)	(0.757)	(0.625)	(0.637)
			[2724]	[2405]
2	1.00	1.10	1.24*	1.42*
	(0.723)	(0.752)	(0.735)	(0.751)
			[7510]	[7920]
3	1.19*	1.37*	1.52**	1.82***
	(0.701)	(0.735)	(0.679)	(0.647)
			[9088]	[6669]
4	1.56*	1.86**	1.93**	2.35***
	(0.780)	(0.821)	(0.850)	(0.853)
			[1748]	[2924]
5	2.02**	2.42***	2.47***	3.00***
	(0.843)	(0.809)	(0.849)	(0.783)
			[4226]	[3386]
6	2.15**	2.47***	2.54***	2.95***
	(0.833)	(0.853)	(0.904)	(0.958)
			[1812]	[2202]
7	2.67***	2.96***	2.98***	3.33***
	(0.894)	(0.939)	(0.889)	(0.946)
			[1428]	[2564]
8	2.69***	3.03***	3.07***	3.47***
	(0.806)	(0.815)	(0.823)	(0.849)
			[3355]	[4151]
9	3.21***	3.41***	3.42***	3.59***
	(0.813)	(0.802)	(0.789)	(0.835)
			[4266]	[4838]
Year FE	Y	Y	Y	Y
State FE	Y	Y	Y	Y
Linear state trend	N	Y	N	Y

Note: Replication of Table 2 but including an additional control: b_{rt-1}.

Table A.9: Replicating Table 2 controlling for b_{rt-1}

	Time difference in years								
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Panel A. Females									
p(5)	0.390	0.423	0.402	0.378	0.347	0.375	0.396	0.418	0.426
	(0.050)	(0.058)	(0.050)	(0.043)	(0.039)	(0.042)	(0.040)	(0.042)	(0.042)
p(10)	0.169	0.217	0.213	0.217	0.235	0.299	0.325	0.321	0.303
	(0.032)	(0.033)	(0.029)	(0.029)	(0.034)	(0.042)	(0.047)	(0.045)	(0.046)
p(20)	0.067	0.087	0.081	0.087	0.109	0.151	0.170	0.158	0.147
	(0.027)	(0.027)	(0.027)	(0.032)	(0.038)	(0.038)	(0.042)	(0.035)	(0.035)
p(30)	0.040	0.053	0.009	0.007	0.018	0.064	0.069	0.064	0.070
	(0.030)	(0.024)	(0.020)	(0.021)	(0.022)	(0.023)	(0.019)	(0.018)	(0.019)
p(40)	0.029	0.017	-0.004	0.004	0.012	0.023	0.002	-0.001	0.001
	(0.035)	(0.022)	(0.021)	(0.019)	(0.022)	(0.030)	(0.036)	(0.029)	(0.028)
p(75)	0.005	0.011	-0.013	-0.033	-0.037	-0.030	-0.038	-0.033	-0.030
	(0.034)	(0.026)	(0.022)	(0.022)	(0.025)	(0.022)	(0.018)	(0.020)	(0.023)
p(90)	0.035	0.052	0.030	0.006	-0.009	-0.015	-0.017	-0.027	-0.024
	(0.042)	(0.041)	(0.043)	(0.039)	(0.035)	(0.034)	(0.033)	(0.036)	(0.039)
Panel B. Males									
p(5)	0.164	0.190	0.197	0.194	0.187	0.199	0.227	0.223	0.246
	(0.041)	(0.037)	(0.030)	(0.026)	(0.030)	(0.034)	(0.031)	(0.034)	(0.042)
p(10)	0.054	0.084	0.091	0.062	0.042	0.053	0.065	0.055	0.095
	(0.031)	(0.026)	(0.029)	(0.027)	(0.033)	(0.035)	(0.042)	(0.051)	(0.046)
p(20)	0.022	0.041	0.029	-0.006	-0.009	0.027	0.052	0.049	0.038
	(0.030)	(0.023)	(0.024)	(0.025)	(0.026)	(0.028)	(0.026)	(0.029)	(0.035)
p(30)	-0.001	0.031	0.042	0.024	0.028	0.054	0.065	0.050	0.046
	(0.029)	(0.024)	(0.020)	(0.018)	(0.022)	(0.023)	(0.018)	(0.018)	(0.019)
p(40)	0.020	0.031	0.035	0.038	0.059	0.095	0.091	0.082	0.078
	(0.041)	(0.032)	(0.027)	(0.026)	(0.028)	(0.036)	(0.036)	(0.033)	(0.028)
p(75)	0.017	0.022	0.028	0.015	0.002	0.008	-0.006	-0.019	-0.012
	(0.022)	(0.022)	(0.024)	(0.021)	(0.023)	(0.023)	(0.023)	(0.030)	(0.023)
p(90)	0.036	0.068	0.061	0.024	-0.009	-0.005	-0.018	-0.026	-0.000
	(0.038)	(0.034)	(0.031)	(0.029)	(0.032)	(0.030)	(0.032)	(0.039)	(0.039)

Table A.10: Distributional effect of minimum wage changes for select percentiles (relative to the median) using one-year to nine-year differences, separately for females and males

Notes: Each column $j \in \{1, ...9\}$ replicates column 4 of Panels A and B of Table 2A in AMS, which uses one-year differences, but using j-year differences instead.

Combining this with the expressions for P_s above, which implies $P_s Y_s = Y^{\frac{1}{\eta}} \left(A_s(1-u_s)L_s\right)^{\frac{\eta-1}{\eta}}$, yields

$$d\log Y = \left[\frac{P_h Y_h}{Y}\left(d\log A_h + d\log L_h\right) + \frac{P_\ell Y_\ell}{Y}\left(d\log A_\ell + d\log L_\ell\right)\right]$$

Substituting into the $d\log P_s$ equation and using $\frac{P_h Y_h}{Y} + \frac{P_\ell Y_\ell}{Y} = 1$ yields

$$d\log P_s = \frac{1}{\eta}\frac{P_{s'}Y_{s'}}{Y}\left[\left(d\log A_{s'} + d\log L_{s'}\right) - \left(d\log A_s + d\log L_s\right)\right] + d\log A_s \quad \text{for } s' \neq s$$

Combining this with the expression for $d\log\left(\frac{\overline{w}_h}{\overline{w}_\ell}\right)$ above and defining β_L as in equation (10) yields equation (9) where

$$\beta_{A_s} \equiv \left(1 - 2\mathbb{I}_{s=\ell}\right)\left[\left(1 - \chi_m \frac{m g_s(m)}{\overline{w}_s}\right)\left(1 - \frac{1}{\eta}\frac{P_{s'}Y_{s'}}{Y}\right) - \frac{1}{\eta}\left(1 - \chi_m \frac{m g_{s'}(m)}{\overline{w}_{s'}}\right)\frac{P_s Y_s}{Y}\right]$$

for $s' \neq s$ and where $\mathbb{I}_{s=\ell}$ is an indicator function that equals one if $s = \ell$ and zero otherwise.

B.2 Transition details

I focus on a single skill s and omit skill subscripts. Let g'_{tj} and g_{tj} denote the shares of all workers at time $t \geq 0$ employed in rung j of the wage ladders associated with m' and m. To simplify notation, focus on the case in which the new minimum wage satisfies $m' \in (m, w_2)$. This determines which rungs of the initial job ladder disappear on impact: in this case, only the first rung.

B.2.1 Results and intuition

Proposition 2. *The share of all workers on any rung $j \geq 1$ of the original and the new job ladders at any date $t \geq 0$ are given by*

$$g_{tj} = \begin{cases} 0 & \text{if } j = 1 \\ \widetilde{g}_{tj} & \text{if } 2 \leq j \leq t+1 \\ g(w_j) & \text{if } j > t+1 \end{cases} \quad \text{and} \quad g'_{tj} = \begin{cases} \widetilde{g'}_{tj} & \text{if } j \leq t+1 \\ 0 & \text{if } j > t+1 \end{cases} \tag{B.1}$$

where

$$\widetilde{g}_{tj} \equiv \sum_{k=2}^{j} \binom{t+1-k}{j-k} g(w_k)(1-\delta)^{2+t-k}(1-\gamma_e)^{2+t-j}\gamma_e^{j-k} \tag{B.2}$$

and

$$\widetilde{g}'_{tj} = \sum_{t'>0}^{t+1-j} \left(\begin{array}{c} t-t' \\ j-1 \end{array} \right) u\gamma_u (1-\delta)^{t-t'} (1-\gamma_e)^{t-t'-(j-1)} \gamma_e^{j-1} + \left(\begin{array}{c} t \\ j-1 \end{array} \right) g(m)(1-\delta)^t (1-\gamma_e)^{t-(j-1)} \gamma_e^{j-1} \tag{B.3}$$

and where $g(w)$ denotes the initial steady-state distribution before date 0.

Consider first the original job ladder, g_{tj}. No one is on the original job ladder at the original minimum wage for any $t \geq 0$, since workers who were employed there were all moved up instantly to the new and higher minimum wage and thereafter this rung is never replenished from unemployment. For any date-rung pair tj satisfying $j > t+1$, the share of workers remains at the initial steady state value, as inflows of workers from the previous rung have yet to be affected by the higher minimum wage. Inflows are reduced into rung j relative to in the original steady state starting on date $t = j-1$. Starting from this date, equation (B.2) tracks in- and outflows as a function of the initial steady state values of $g(w_k)$ for all $k \leq j$, since those in rung j of the original job ladder at date t must have been at a weakly lower rung of this ladder previously. To move from rung k to $j \geq k$ over time requires that the worker *(i)* does not separate in any of the $2+t-k$ periods since we start tracking her (we only start tracking workers when they are on a rung of the job ladder that is not at its original steady-state share), *(ii)* moves up the ladder $j-k$ times, and *(iii)* does not move up in all remaining periods. The binomial coefficient in equation (B.2) tracks the number of routes through which a worker on rung k when we first start tracking her can move up to rung j in period t.

Next, consider the new job ladder. At $t = 0$, only the first rung of this job ladder has any workers. Each successive rung is first reached in each successive period. Hence, $g'_{tj} = 0$ for all $j > t+1$. In period 0, the first rung of the new job ladder has $g(m)$ workers. Over time, these workers either separate or slowly move up the job ladder, reaching rung j at date t through a particular path of wage increases over time with probability $(1-\delta)^t(1-\gamma_e)^{t-(j-1)}\gamma_e^{j-1}$. The binomial coefficient in the second line of equation (B.3) tracks the number of routes that such a worker could have taken. The first line of equation (B.3) counts the workers who are on rung j of the new job ladder at date t, having risen from unemployment to the minimum wage at each possible date $0 < t' \leq t$. The number of workers who are hired from unemployment each period is $u\gamma_u$. The share of these workers who were hired at date $t' \leq t$ who reach exactly rung j of the new job ladder at date t through a particular path is given by $(1-\delta)^{t-t'}(1-\gamma_e)^{t-t'-(j-1)}\gamma_e^{j-1}$. The binomial coefficient in the first line of equation (B.3) tracks the number of routes that such a worker could have taken. And the summation over all t' sums across all possible dates that a worker could have been hired out of unemployment and reached rung j by date t, which requires that $t' \leq t + 1 - j$.

B.2.2 Derivation of g_{tj}

I begin by writing the recursive system. At date $t = 0$, the share of workers at each rung of the original wage ladder is

$$g_{0j} = \begin{cases} 0 & \text{if } j = 1 \\ g(w_j) & \text{otherwise} \end{cases}$$

The share on the first rung remains zero in all dates. At any date $t \geq 1$, the share of workers at each remaining rung of the original wage ladder is given by the recursive system

$$g_{tj} = g_{t-1,j-1}(1-\delta)\gamma_e + g_{t-1,j}(1-\delta)\,(1-\gamma_e) \quad \text{for } j > 1$$

A share $(1-\delta)\gamma_e$ of the workers on the previous rung $j-1$ in the previous period $t-1$ move up to rung j in period t and a share $(1-\delta)(1-\gamma_e)$ of the workers on rung j in period $t-1$ remain there in period t. By mathematical induction, this implies that $g_{tj} = g(w_j)$ for all $j > t+1$, since if $j > t+1$ then $g_{t-1,j-1} = g_{j-1}$ and $g_{t-1,j} = g_j$. It then remains only to solve for g_{tj} for $2 \leq j \leq t+1$, which I refer to as $\widetilde{g}_{tj}$.

To solve the recursive system, note that in period $t = j-1$, the first period in which $g_{tj} \neq g(w_j)$, the share $\widetilde{g}_{j-1,j}$ satisfies

$$\widetilde{g}_{j-1,j} = (1-\gamma_e)\sum_{k=2}^{j} g(w_k)(1-\delta)^{2+t-k}\gamma_e^{j-k}$$

Note that there is a single path (through time and rungs) through which a worker earning w_k in period $k-2$ can have reached exactly rung j in period $t = j-1$. In this path, the worker must have not separated in each of the $2+t-k$ periods, must have not matched with a new firm in the first period, and must have matched with a new firm in each subsequent period. For subsequent periods, there are two differences. First, in each subsequent period, the number of periods in which an agent must not have separated or matched with another firm increases by one. Second, in later periods the number of paths through which a worker earning w_k in period $k-2$ can have reached exactly rung j in period t rises. Hence, $\widetilde{g}_{tj}$ can be expressed as

$$\widetilde{g}_{tj} = \sum_{k=2}^{j} x_{tjk} g(w_k)(1-\delta)^{2+t-k}(1-\gamma_e)^{2+t-j}\gamma_e^{j-k} \text{ for all } 2 \leq j \leq t+1$$

for some x_{tjk}. Here, x_{tjk} is the number of routes a worker earning w_k at date $t = k-2$ could have taken to earn exactly w_j at date t. Each date the worker must move forward in time, and the worker can move one step up in the job ladder or remain on her previous rung. The solution is

$$x_{tjk} = \begin{pmatrix} t+1-k \\ j-k \end{pmatrix} = \frac{(t+1-k)!}{(t+1-j)!\,(j-k)!}$$

The previous two displayed equations yield equation (B.2).

B.2.3 Derivation of g'_{tj}

I again begin by writing the recursive system. At date $t = 0$, the share of workers at each rung of the new wage ladder is

$$g'_{0j} = \begin{cases} g(m) & \text{if } j = 1 \\ 0 & \text{otherwise} \end{cases}$$

and at any date $t \geq 1$, the share of workers at each rung of the new wage ladder is given by the recursive system

$$g'_{tj} = \begin{cases} u\gamma_u + g'_{t-1,1}(1-\delta)(1-\gamma_e) & \text{if } j = 1 \\ g'_{t-1,j-1}(1-\delta)\gamma_e + g'_{t-1,j}(1-\delta)(1-\gamma_e) & \text{if } 1 < j \leq t+1 \\ 0 & \text{if } j > t+1 \end{cases}$$

Given the recursive system and the initial condition, it is straightforward to show that $g'_{tj} = 0$ for all $j > t+1$. It remains to solve for $\widetilde{g'}_{tj}$.

For any tj pair, $\widetilde{g'}_{tj}$ will include a $g(m)$ term and a $u\gamma_u$ term. It is straightforward to solve for the $g(m)$ term. Each term must include $(1-\delta)^t$, since the worker must have not been separated for all t periods. And similarly, each term must include $(1-\gamma_e)^{t-(j-1)}\gamma_e^{j-1}$, since to get to rung j the worker must have moved up exactly $j-1$ times and, therefore, not moved up exactly $t-(j-1)$ times. All that remains then is solving for the coefficient on this term. As in the previous problem this is just a matter of determining how many paths the worker can take from the date 0 and rung 1 cell to the arbitrary tj cell, where to move up a rung requires an increase in time. This coefficient is given by $\begin{pmatrix} t \\ j-1 \end{pmatrix} = \frac{t!}{(t+1-j)!(j-1)!}$.

Each period, $u\gamma_u$ unemployed enter the new wage ladder at the first rung. What is the impact of the inflow to rung 1 of the job ladder in period $t' > 0$ on workers in rung $j \geq 1$ in period $t \geq t'$. This problem is identical to how the $g(m)$ workers in rung 1 at date $t = 0$ transition across rungs in the new job ladder. If $j - 1 > t - t'$, then these workers cannot yet have risen to rung j. If $j - 1 \leq t - t'$, then the share of all skill s workers who *(i)* moved from unemployment to rung 1 at date $t' > 0$ and *(ii)* work in rung j at date t is simply $\begin{pmatrix} t-t' \\ j-1 \end{pmatrix} u\gamma_u(1-s)^{t-t'}(1-\gamma_e)^{t-t'-(j-1)}\gamma_e^{j-1}$. Of course, at rung j and date t, we must sum this across all possible t' satisfying $j - 1 \leq t - t'$, which is equivalent to summing across all $t' \leq t + 1 - j$. Hence, we obtain equation (B.3).

B.3 Wage-posting model

As described in Section 3.4, I use the Burdett and Mortensen (1998) model with homogeneous workers and firms, extended to include a minimum wage, as in van den Berg and Ridder (1998). I consider the case of a single skill s (omitting s subscripts), since this is sufficient to show which results are robust, and I focus on steady states, since solving for the transition to an aggregate shock is not straightforward.

According to equation (2.10) in van den Berg and Ridder (1998), the equilibrium earnings density is

$$g(w) = \frac{\delta \left(P-m\right)^{1/2}}{2\lambda_e} \left(P-w\right)^{-3/2} \text{ for all } w \in [m, w_{\max}]$$

under the assumption that the minimum wage m is binding (an assumption I make throughout). The maximum wage is then

$$w_{\max} \equiv \left(\frac{\delta}{\delta+\lambda_e}\right)^2 m + \left(1 - \frac{\delta}{\delta+\lambda_e}\right)^2 P$$

In these expressions, I have used my notation: P is the value marginal product of labor, δ is the exogenous separation rate, λ_e is the rate at which workers receive offers when employed, and $w_{\max}$ is the (endogenous) supremum of offered wages. Integrating this yields the probability a worker earns less than w,

$$G(w) = \frac{\delta}{\lambda_e}\left(\frac{P-m}{P-w}\right)^{1/2} - \frac{\delta}{\lambda_e} \text{ for all } w \in [m, w_{\max}]$$

which is the earnings distribution (conditional on employment). Given the average wage, solved below, we therefore have Part 1 of Proposition 1 in the wage-posting model.

Impact of a change in the minimum wage on the distribution of wages. Define $W_c(m)$ to be the wage at centile $c \in [0, 100]$. Then from the distribution of wages, I obtain

$$\frac{c}{100} = \frac{\delta}{\lambda_e}\left(\frac{P-m}{P-W_c(m)}\right)^{1/2} - \frac{\delta}{\lambda_e}$$

which yields an explicit solution for the wage at centile c as a function of the minimum wage m,

$$W_c(m) = P - (P-m)\left(\frac{100\delta}{c\lambda_e + 100\delta}\right)^2$$

This implies

$$\frac{W_{c'}(m)}{W_c(m)} = \frac{P - (P-m)\left(\frac{100\delta}{c'\lambda_e+100\delta}\right)^2}{p - (P-m)\left(\frac{100\delta}{c\lambda_e+100\delta}\right)^2}$$

Differentiating with respect to m yields

$$\frac{d\left[W_{c'}(m)/W_c(m)\right]}{dm} < 0 \iff c' > c$$

Hence, as in my baseline model, an increase in the minimum wage increases the relative wage of centile c to centile c' for any $c' > c$. That is, $W_c(m)$ is log sub-modular in c, m.

The average wage. The average wage is given by $\overline{w} = \int_m^{w_{\max}} wg(w)dw$. This can be expressing using the $W_c(m)$ function (the wage at centile c) and noting that the distribution across centiles is uniform (by definition) with density $\widetilde{g}(c) = (w_{\max} - m)^{-1}$ which can be expressed as

$$\widetilde{g}(c) = \left(1 - \frac{\delta}{\delta + \lambda_e}\right)^{-2} (P-m)^{-1}$$

Hence,

$$\overline{w} = \frac{(100\delta)^2}{(w_{\max}\lambda_e + 100\delta)(m\lambda_e + 100\delta)} m + \left(1 - \frac{(100\delta)^2}{(w_{\max}\lambda_e + 100\delta)(m\lambda_e + 100\delta)}\right) P$$

The previous expression gives the average wage. As in the baseline model, equation (5), this is a weighted average of the minimum wage m and the value marginal product of labor, P. Unlike the baseline model, the weights themselves depend on the minimum wage.