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QUALITY LADDERS IN THE THEORY OF GROWTH

Gene M. Grossman

Elhanan Helpman

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ABSTRACT

We develop a model of repeated product improvements in a continuum of sectors. Each product follows a stochastic progression up a quality ladder. Progress is not uniform across sectors, so an equilibrium distribution of qualities evolves over time. But the rate of aggregate growth is constant. The growth rate responds to profit incentives in the R&D sector. We explore the welfare properties of our model. Then we relate our approach to an alternative one that views product innovation as a process of generating an ever expanding range of horizontally differentiated products. Finally, we apply the model to issues of resource accumulation and international trade.

Gene M. Grossman
Woodrow Wilson School
Princeton University
Princeton, NJ 08544
(609)452-4823

Elhanan Helpman
Department of Economics
Tel Aviv University
Ramat Aviv, Tel Aviv 69978
ISRAEL
(03)545-9209

I. Introduction

Casual observation suggests the central role that quality upgrading plays in raising our standard of living. For example, the ease and comfort of transportation have increased enormously since the horse-and-buggy gave way to a succession of automobiles of ever higher quality. New generations of televisions provide finer detail and sharper colors. And the enjoyment of music in the home has been enhanced by the replacement of the gramophone by the phonograph, and the phonograph by the compact disk player.

Intermediate goods too have been subject to product improvement. Witness the recent revolution in desk-top computing. Or consider the advances that have taken place in integrated circuitry. These innovations have raised total factor productivity in the manufacturing of consumer goods and capital equipment, and made possible the production of entirely new final products.

The economics of quality improvement have been studied by industrial organization economists in their work on patent races. Beginning with Loury (1979), Dasgupta and Stiglitz (1980), and Lee and Wilde (1980), much effort has been devoted to understanding the incentive that firms have to bring out new and improved products. This literature typically views the R&D competition as a once-and-for-all race for technological supremacy. While contributing many useful insights, the one-shot framework fails to capture an essential aspect of quality competition. This is the continual and cyclical nature of the process whereby each new product enjoys a limited run at the technological frontier, only to fade when still better products come along. Almost every product exists on a quality ladder, with variants below that may already have become obsolete and others above that have yet to be discovered.

Recent work by Segerstrom et.al.(1987) and Aghion and Howitt (1989) provides the beginnings of a theory of repeated quality innovations. These

authors cast the patent-race paradigm in a dynamic, general-equilibrium setting. The result in each case is a model of long-run growth based on endogenous technical change.¹ Their models enable us to study the structural and institutional determinants of ongoing technological progress.

Each of these interesting efforts contains at least one unappealing element, however. In the work by Segerstrom et.al.(1987), patent races take place in a multitude of industries in sequence. That is, all research effort in the economy first is devoted to improving a single product, then another, and so on, until all products have been improved exactly once. Then the cycle repeats. In Aghion and Howitt (1989), by contrast, the patent race takes place at an economy-wide level. A successful research project improves all products simultaneously. The sole innovator thereby gains monopoly power across all industries. Evidently unhappy with this implication, Aghion and Howitt make reference to antitrust laws to justify their imposition of a requirement that the monopolist must license the portfolio of patents to a continuum of arms-length competitors.

In what follows, we propose an approach that resolves these difficulties. This approach draws on the building blocks provided by Segerstrom et.al.(1987) and Aghion and Howitt (1989). We envision a continuum of products, each with its own quality ladder. Entrepreneurs target individual products and race to bring out the next generation. These races take place simultaneously. In any given time interval some of the efforts succeed while others fail. Successful ventures call forth efforts aimed at still further improvement, with each

¹ Related work by Schleifer (1986) and Krugman (1988) deals with a continual process of cost innovation. Reductions in cost and improvements in quality are two different ways for firms to supply more services at a given price.

innovation building upon the last. This specification accords well with the description of the research process in, for example, Freeman (1982) and Dosi (1988).

Our model generates an equilibrium distribution of product qualities that evolves over time. Each individual product follows a stochastic progression up the quality ladder. But, in the aggregate, the innovation process we describe is a smooth one. An index of consumption grows at a constant and determinate rate in the steady state. This rate is readily calculable.

We believe that our approach will prove useful in many applications. In this paper, for example, we explore the relationship between the accumulation of resources and the long-run rate of growth, and describe the long-run pattern of specialization in a two-country world economy with innovation and trade. In Grossman and Helpman (1989e), we use a similar framework to study the product life cycle with concurrent innovation by high-wage producers and imitation by low-wage producers.

Our approach is related to an alternative one that views product innovation as a process of generating an ever expanding range of horizontally differentiated products.² This latter framework has been applied to positive and normative topics in economic growth in recent work by Judd (1985), Romer (1988) and ourselves (Grossman and Helpman, 1989a,b,c,d). At first glance, the economics of the development of horizontally differentiated products seems quite distinct from that of product improvement. Yet, as we shall demonstrate below, the two approaches yield quite similar answers to many questions.

² Scherer (1980, p.409) cites survey evidence that firms devote 59% of their research outlays to product improvement, 28% to developing new products and 13% to developing new processes. These data may understate the importance of vertical relative to horizontal differentiation in the innovation process, since many "new products" replace old products that perform similar functions.

Indeed, it is possible to construct comparable variants from each class of model that share identical reduced forms (at least in simple cases). The alternative analyses do diverge, however, when it comes to normative issues.

The remainder of this paper is organized as follows. We develop our simplest one-factor model of quality-based growth in the next section and explore its positive and normative properties in Section III. In Section IV we pursue the analogy between quality-based growth and variety-based growth. Then, in Section V, we extend the model to include an outside good and a second primary input, and investigate the implications of factor accumulation for growth. In Section VI we introduce a second country and international trade, and describe the long-run pattern of trade. Section VII concludes.

II. The Basic Model

We consider an economy with a continuum of goods indexed by ω . The set of goods consists of the interval $[0,1]$ whose measure is one. We refer to the measure of a subset of goods as the "number" of goods in the subset. Hence, the total number of goods, which we take to be fixed, is equal to one.

Each product ω can be supplied in a countable number of qualities. We choose units so that the lowest quality equals one; i.e., $q_0(\omega)=1$. Quality j of product ω is given by $q_j(\omega)=\lambda^j$, where $\lambda>1$ is the same for every ω . In order to attain quality j , a product must be improved j times. Each step up the quality ladder requires R&D. We describe this activity below.

Consumers allocate spending according to a common intertemporal utility function

$$(1) \quad U = \int_0^{\infty} e^{-\rho t} \log u(t) dt ,$$

where ρ is a subjective discount rate, t indexes time, and $\log u(t)$ represents the flow of utility at time t . Instantaneous utility is given by

$$(2) \quad \log u(t) = \int_0^1 \log[\sum_j q_{jt}(\omega) d_{jt}(\omega)] d\omega ,$$

where $d_{jt}(\omega)$ denotes consumption of quality j of product ω at time t . Every consumer maximizes utility subject to an intertemporal budget constraint

$$(3) \quad \int_0^{\infty} e^{-R(t)} E(t) dt \leq A(0) ,$$

where $E(t)$ represents the flow of spending at time t , $R(t)$ is the cumulative interest factor up to time t , and $A(0)$ denotes the present value of the stream of factor incomes plus the value of initial asset holdings at $t=0$. Naturally,

$$(4) \quad E(t) = \int_0^1 \sum_j p_{jt}(\omega) d_{jt}(\omega) d\omega ,$$

where $p_{jt}(\omega)$ is the price of quality j of product ω at time t .

The consumer maximizes utility in two stages. First, he allocates $E(t)$ to maximize $u(t)$ given prices at time t . Then he chooses the time pattern of spending to maximize U . To solve the static problem, the consumer selects for each product the single quality $j=J_t(\omega)$ that carries the lowest quality-adjusted price $p_{jt}(\omega)/q_{jt}(\omega)$.³ Then he allots identical expenditure shares to all products. This gives static demand functions

³ In what follows, $J_t(\omega)$ is unique.

$$(5) \quad d_{j_t(\omega)} = \begin{cases} E(t)/p_{j_t(\omega)} & \text{for } j=J_t(\omega) , \\ 0 & \text{otherwise.} \end{cases}$$

Substituting these demand functions into (2), and the result into (1), we can reformulate the intertemporal maximization problem as one of choosing the time pattern of spending to maximize

$$(6) \quad U = \int_0^{\infty} e^{-\rho t} (\log E(t) + \int_0^1 [\log q_t(\omega) - \log p_t(\omega)] d\omega) dt$$

subject to (3), where $q_t(\omega)$ and $p_t(\omega)$ represent the quality level and price, respectively, of $J_t(\omega)$. The solution to this problem is characterized by

$$(7) \quad \dot{E}/E = \dot{R} - \rho$$

and either (3) or the transversality condition. Since preferences are homothetic, aggregate demands are those of the representative consumer. In what follows, we use E to denote aggregate spending and (5) to represent economy-wide demand functions.

Turning to the production side, we endow the economy with a single primary factor called "labor" (but see Sections 5 and 6). One unit of labor is needed to manufacture one unit of any product, regardless of quality. Of course, better quality products cannot be produced until they have been invented. Patent restrictions may apply as well, as we shall discuss further below.

At any point in time producers in any "industry" ω compete as price-setting oligopolists with common costs equal to the wage rate $w(t)$. Then, if

several firms in an industry are able to produce goods of the same quality, each sets a price equal to unit cost and each earns zero profit. Alternatively, one producer may enjoy a quality lead over her industry rivals. If she charges a quality-adjusted price below that of other producers of the same good, she faces unitary-elastic demand. Demand falls to zero for quality-adjusted prices above the minimum among those charged by competitors. As a consequence, the leader maximizes profits by "limit-pricing"; i.e., by setting a price that, adjusted for quality, falls epsilon below the production cost of the nearest competitor. We shall show below that the nearest competitor is always one step behind. Then all products with a unique quality leader sell for

$$(8) \quad p = \lambda w .$$

This price yields demand per product of $E/\lambda w$. The leader earns a flow of profits $\pi = (1 - \delta)E$, where $\delta = 1/\lambda$.

A blueprint is needed to produce any commodity. These blueprints are costly to develop. Research efforts might, in principle, be devoted either towards imitating existing products or towards developing the next generation. However, we can rule out the first activity, since a successful imitator will earn zero profit in the resulting Bertrand competition and so be unable to recoup any R&D costs. Even if imitation were costless, it might be precluded by patent protection afforded to the leader. For these reasons, we concentrate here on product improvements.⁴

⁴ In Grossman and Helpman (1989e), we develop a two-country model in which factor prices differ across countries. Then imitation may be profitable in the low wage country, because success in creating a clone yields strictly

An entrepreneur may attempt to improve any product. If she undertakes R&D at intensity λ and targets her research efforts at product ω for a time interval of length dt , then she will succeed in taking the next step up the quality ladder for that product with probability λdt . This formulation mimics the one-shot, partial equilibrium patent-race models of Lee and Wilde (1980) and others. It implies that R&D success bears a Poisson probability distribution with an arrival rate that depends upon the level of R&D activity.

A unit of R&D activity requires a_i units of labor per unit of time. An entrepreneur who is not currently an industry leader may target any product in her R&D efforts. If she succeeds, she will take over industry leadership and begin to earn a flow of profits π . Since these profits are independent of ω , the individual entrepreneur is indifferent as to which product she targets. We shall focus on the symmetric equilibrium in which all products are targeted to the same extent. Several entrepreneurs may be involved in the race to improve the same product. However, we need not distinguish among them, and so let λ represent aggregate R&D intensity per product.

Without any cost advantage, an industry leader will not engage in R&D.⁵ To see this, note that a research success would leave the leader with a two-step lead over her nearest competitor, and thus allow her to increase her price to $\lambda^2 w$. This would yield incremental profits of $(1-1/\lambda^2)E - (1-1/\lambda)E - (1-1/\lambda)E/\lambda$, which is strictly less than the incremental profits $(1-1/\lambda)E$ that accrue to a non-leader who achieves a research success.

positive profits to the imitator in the resulting duopoly equilibrium.

⁵ In Grossman and Helpman (1989e) we allow for the possibility that leaders, by dint of their past R&D successes, might be able to improve upon their products at lower (expected) cost than outsiders. This modification of the model enriches the set of possible outcomes at some cost in terms of simplicity.

We derive now an equation for the equilibrium level of R&D activity per product. Let v represent the stock-market value of an industry leader. An entrepreneur who devotes resources $a_I \iota$ to R&D for interval dt at cost $wa_I \iota dt$ attains an expected reward of $v \iota dt$. Since the risks involved in any particular research effort are idiosyncratic, free entry ensures that costs are equal to expected reward when $\iota > 0$. Thus $v = wa_I$.

The ownership shares in industry leaders pay dividends πdt over a time interval of length dt , and appreciate by $\dot{v} dt$ if no entrepreneur succeeds in supplanting the firm's leadership position. However, with probability ιdt the leader's product is improved upon during the interval dt , in which case the shareholders suffer a capital loss of amount v . All told, the shares bear an expected rate of return of $(\pi + \dot{v})/v - \iota$. This return is risky. But once again the risk is idiosyncratic, so arbitrage in the capital markets ensures equality between the expected rate of return and the safe interest rate \dot{R} . Using $v = wa_I$, we may write the no-arbitrage condition as⁶

$$\frac{\pi}{wa_I} + \frac{\dot{w}}{w} = \dot{R} + \iota .$$

We choose labor as numeraire; i.e., $w(t) = 1$ for all t . Recall that $\pi = (1 - \delta)E$. This, together with (7) and the no-arbitrage condition, implies

$$(9) \quad \frac{\dot{E}}{E} = \frac{(1 - \delta)E}{a_I} - \rho - \iota .$$

This provides a differential equation for spending. The rate of growth in

⁶ We derive a similar condition in Grossman and Helpman (1989b) by equating R&D costs to expected present discounted profits, and then differentiating with respect to time. The same method could be used here.

spending increases with the level of spending and decreases with aggregate R&D intensity.

We close the model with a market-clearing condition. Let L be the total supply of labor. Total manufacturing employment equals⁷

$$\int_0^1 [E(\tau)/p(\tau)] d\omega - \delta E(\tau) ,$$

while the R&D sector employs $a_I \iota$ workers. Therefore, equilibrium in the labor market implies

$$(10) \quad a_I \iota + \delta E = L .$$

The differential equation (9) together with the side condition (10) describe the evolution of our economy for every initial value of spending E . We depict the dynamics in Figure 1, where LL represents the resource constraint (10), and IIII describes combinations of E and ι such that $\dot{E}=0$; i.e.,

$$(11) \quad \frac{(1-\delta)E}{a_I} = \rho + \iota .$$

The economy must always lie along LL, with spending rising above IIII and falling below this line. For any initial value of E below that labelled \bar{E} , spending eventually falls to zero, a violation of the transversality

⁷ We assume here that even at time 0 each industry has a unique leader plus a competitor one rung down the quality ladder. This implies that $p=1/\delta$ for all products. We will show that, with this assumption, the economy jumps immediately to the steady state. Alternatively, we might start the economy with a universally known backstop technology for quality q_0 of each good. Then the steady state that we describe is approached in the limit, after all goods have been improved for the first time.

condition. For initial values of E above \bar{E} , ι approaches zero at a point where the level of spending implies expected profits in excess of R&D costs. This event would contradict profit maximization by entrepreneurs. We conclude that the economy must jump immediately to the steady state at point A. The equilibrium values of E and ι solve (10) and (11).

In the dynamic equilibrium, each product is improved with probability ιdt in a time interval of length dt . By the law of large numbers, a fraction ι of the products are continually being improved. These will not be the same products in every "period", nor will technological progress be uniform across sectors. In fact, our model predicts an evolving distribution of product qualities, with individual products constantly swapping relative positions within that distribution.

Before we proceed to investigate the market determinants of R&D and the welfare properties of the equilibrium, it is worthwhile to point out an alternative interpretation of our model. This interpretation treats each good ω as an intermediate input into the production of a single final consumer good. Then (2) gives the constant-returns-to-scale (Cobb-Douglas) production function for the final good, and u represents the flow of final output. The consumers' utility function remains as in (1), but now u there stands for the consumption of the single final good. This good is priced at marginal cost in a competitive equilibrium; i.e.,

$$\log p_u = \int_0^1 \log[p_t(\omega)/q_t(\omega)]d\omega .$$

This provides a natural interpretation of (6) as the discounted value of the logarithm of real spending E/p_u .

From this point on the analysis proceeds as before to arrive at the differential equation (9) and the resource constraint (10). With this new interpretation, technological progress entails improvement in intermediate inputs, which serves to raise total factor productivity in the manufacture of consumer goods. It would be a simple matter to augment the model by including direct use of primary factors in the production of final goods, in which case we would have a specification similar in many respects to Romer (1988). We shall pursue this analogy further in Section IV below.

III. Equilibrium and Optimal Growth

We define the growth rate g to be the rate of increase of u . With the interpretation of the ω 's as intermediate goods, g represents the rate of growth of final output. With our original interpretation of the ω 's as consumer goods, g corresponds to the rate of increase in a quality-adjusted consumption index.

To calculate g , we substitute (5) and (8) into (2) to derive

$$\log u(t) = \log E - \log \lambda + \int_0^1 \log q_t(\omega) d\omega .$$

The last term depends upon t and τ . For any given ω , the probability of exactly m improvements in a time interval of length τ is (see Feller, 1968, p.159)

$$f(m, \tau) = \frac{(\lambda \tau)^m e^{-\lambda \tau}}{m!}$$

Since in equilibrium the same intensity of R&D applies to all products, $f(m, \tau)$

represents the measure of products that are improved exactly m times over an interval of length τ . Therefore,

$$\int_0^1 \log q_t(\omega) d\omega = \sum_{m=0}^{\infty} f(m, \tau) \log \lambda^m .$$

The right-hand side equals $\log \lambda$ times the expected number of improvements in an interval of length τ , or $\iota \tau \log \lambda$. Hence,

$$(12) \quad \log u(t) = \log E - \log \lambda + \iota \tau \log \lambda,$$

and the growth rate $g = \iota \log \lambda$.

We are now prepared to study the determinants of the growth rate. We solve (10) and (11) to derive the following reduced-form expression for ι :

$$(13) \quad \iota = (1-\delta)L/a_1 - \delta \rho .$$

This equation or Figure 1 can be used to examine the comparative statics. In terms of the figure, an increase in L shifts the LL curve out and so increases equilibrium ι . Thus, as in Romer (1988), Aghion and Howitt (1989) and elsewhere, a larger resource base implies faster growth (but see Section V below). Similarly, a decline in a_1 shifts LL out and also shifts III down. R&D effort expands due both to a resources effect and an incentive effect. The same is true about an increase in λ (decrease in δ), which not only raises ι but also spurs growth directly, because the technology jumps become larger. In sum, R&D responds to profitability incentives and the economy exhibits dynamic increasing returns to scale.

We turn now to issues of welfare. Using (1), (12), and the knowledge that E and ι are constant in equilibrium, we can calculate the following exact expression for U :

$$(14) \quad \rho U = \log E - \log \lambda + (\iota/\rho) \log \lambda .$$

This representation of lifetime utility induces a preference ordering on E and ι that can be depicted by well behaved indifference curves such as UU in Figure 1. Greater spending means higher utility early on, which may compensate for fewer quality improvements and hence lower utility later.

We find the optimal growth rate by maximizing (14) subject to (10).⁸

This gives the optimal intensity of innovation

$$(15) \quad \iota^* = \frac{L}{a_1} - \frac{\rho}{\log \lambda} ,$$

which we find in Figure 1 at the tangency of an indifference curve and the resource constraint LL .

We discuss the possible discrepancies between the optimal and equilibrium

⁸ In a model with more than two activities, such as that of Section V below, we would need to distinguish between the second-best growth rate which takes the oligopoly pricing of innovative products as given and the first-best rate that sets all prices equal to marginal cost (see Grossman and Helpman, 1989c). To achieve the first best we would generally require two policy instruments, one to correct for externalities generated in R&D and the other to ensure optimal output of the innovative products. However, with all manufactured goods priced similarly and with labor supplied inelastically, the problem of optimal resource allocation becomes simply one of determining whether the resources devoted to R&D in the market equilibrium are too many or too few.

growth rates with the aid of Figure 2. In the figure, we have plotted the curve for $\lambda/(\lambda-1)$. This must not exceed $L/\rho a_I + 1$ if we are to have positive growth in equilibrium (compare the vertical intercepts of LL and III in Figure 1). Hence, $\iota = 0$ for $\lambda \leq \lambda_0$ and $\iota > 0$ for $\lambda > \lambda_0$. Next, we compute from (13) and (15)

$$(16) \quad \iota^* - \iota = \frac{\rho}{\lambda} \left(\frac{L}{\rho a_I} + 1 - \frac{\lambda}{\log \lambda} \right).$$

Figure 2 also shows the curve for $\lambda/\log \lambda$. As is clear from the figure, $\iota > \iota^*$ for $\lambda \in (\lambda_0, \lambda_1)$ and $\lambda > \lambda_2$, whereas $\iota < \iota^*$ for $\lambda \in (\lambda_1, \lambda_2)$. In other words, the market incentives for R&D are excessive in our economy when the steps of the quality ladder are quite small or quite large, but are insufficient for steps of intermediate size.

This finding can be understood with reference to the market distortions identified by Aghion and Howitt (1989). First, successful innovators generate a positive externality for consumers. Consumers pay the same price as before the innovation but receive a product of higher quality. This externality certainly lasts as long as the innovator maintains her monopoly position. Actually, it lasts indefinitely into the future, since all later innovations improve upon a product that is one step higher up the quality ladder than otherwise. This externality, which combines what we shall call a consumer-surplus effect (during the life of the new product) and what Aghion and Howitt term an intertemporal spillover effect (extending over the lives of later products), is proportional to $\log \lambda / \rho$.

Second, a successful innovator generates a negative externality for the extant industry leader. The entrepreneur is motivated by the profits she can earn, whereas total industry profits are bound to be unaffected by her success

or failure. In effect, the innovator "destroys" the producer surplus of the firm it displaces (hence the title of the paper by Aghion and Howitt). Aghion and Howitt term this the business-stealing effect. Here it is proportional to $(\lambda-1)/(\iota^*+\rho)$, since the flow of profits is proportional to $(\lambda-1)$, and $\iota^*+\rho$ is the effective rate of discount of profits, taking into account the optimal expected rate of arrival of the next innovation.⁹ For low or high values of λ this effect dominates (because $\lambda/\log\lambda$ is then large), while the combined consumer-surplus and intertemporal-spillover effects dominate when λ takes on an intermediate value.

The optimal growth rate is more likely to exceed the equilibrium rate when L/a_1 is large; i.e., in large economies as measured in units of (R&D) efficiency labor. Then the optimal rate of innovation is great. Larger values of ι^* reduce the size of the business-stealing effect per unit output without changing the size of the consumer-surplus effect per unit output.

The optimum can be decentralized here by means of a tax or subsidy on R&D outlays. Let T be the multiple (or fraction) of R&D costs borne by the firm, with $T>1$ for a tax and $T<1$ for a subsidy. With such a policy in effect the no-arbitrage condition (11) is replaced by one with the left-hand side divided by T . An increase in T shifts the III curve upward in Figure 1 and so generates an equilibrium with greater spending and less innovation. A decrease in T has the opposite effects on resource allocation. By varying T , the government can achieve any point along LL , including of course the optimum.

⁹ For both external effects, the factor of proportionality is the optimal level of sales per product. Using the expression for ι^* in (15), it is easy to show that the difference between the combined consumer-surplus plus intertemporal spillover effect and the business-stealing effect has the same sign as the right-hand side of (16).

IV. Quality versus Variety

We have followed Segerstrom et.al. (1988) and Aghion and Howitt (1989) in treating endogenous product improvements as the engine of growth. As we noted in the introduction, this approach to technological progress differs from the one pursued in recent papers by Romer (1988) and ourselves (1989a,b,c).¹⁰ This work treats technology-based growth as a process of generating an ever-expanding variety of horizontally differentiated products. In this section, we shall develop a simple variant of a variety-based growth model to demonstrate a remarkable similarity between the alternative approaches. In particular, we show below that the variety-based growth model and the quality-based growth models have identical reduced forms and we discuss the analogous roles that different parameters play in each formulation.

We continue to represent consumer preferences by (1). Now, however, the consumption index $u(t)$ exhibits love of variety over an infinite set of horizontally-differentiated products, as in Dixit and Stiglitz (1977). We replace (2) by

$$(17) \quad u(t) = \left[\int_0^{n(t)} d_t(\omega)^\alpha d\omega \right]^{1/\alpha}, \quad 0 < \alpha < 1,$$

where $n(t)$ denotes the measure (number) of differentiated products available at time t and $d_t(\omega)$ represents consumption of brand ω at time t . In this case too the differentiated products can be interpreted either as final consumer goods or as intermediate inputs. Under the latter interpretation, (17) represents a production function and u is output of a homogeneous consumer

¹⁰ See also Judd (1985), who studies the introduction of new products via R&D, but in a dynamic framework in which growth ceases in the long run.

good (see Ethier, 1982). We shall not dwell on the intermediate goods interpretation in order to save space.

This preference structure implies an intertemporal allocation of spending given by (7) and well known static demand functions that exhibit a constant price elasticity of demand of $1/(1-\alpha) > 1$ and a unitary expenditure elasticity of demand for each variety (see, for example, Grossman and Helpman, 1989d). We assume that a unit of any brand can be produced with one unit of labor, with labor again being the sole primary input. In this case, marginal production costs equal the wage rate w and oligopolistic price competition among suppliers of available varieties implies mark-up pricing, or

$$(18) \quad p = (1/\alpha)w ,$$

for every product at each point in time. This equation is analogous to (8). It implies that profits per brand are $\pi = (1-\alpha)E/n$, where E is total spending.

Entrepreneurs must devote resources to R&D in order to bring out new products.¹¹ Product development requires a_D/K units of labor per increment to the set of varieties, where K is the stock of knowledge capital at a point in time. Knowledge capital is a public good; it is freely available to all potential innovators. Moreover, knowledge capital is generated as a by-product of product development.¹² For simplicity, let $K=n$. Then product development costs are $c_n = wa_D/n$. We assume that the developer of a new variety maintains indefinite monopoly power in the sub-market for her specific brand.

¹¹ Or, more precisely, resources are spent to expand the measure of the set of available products by dn .

¹² For discussion of this assumption, see Romer (1988).

Free entry by entrepreneurs ensures that, whenever innovation takes place, the present value of the infinite stream of future profits exactly matches the cost of product development. The time derivative of this zero-profit condition gives the following no-arbitrage condition:

$$\frac{(1-\alpha)E}{wa_D} + \frac{\dot{w}}{w} - \frac{\dot{n}}{n} = \dot{R} .$$

This condition equates the interest rate to the instantaneous profit rate $\pi/c_n = (1-\alpha)E/wa_D$ plus the capital gain $\dot{c}_n/c_n = \dot{w}/w - \dot{n}/n$.

We again choose labor as numeraire; i.e., $w(t)=1$ for all t . Let $\gamma = \dot{n}/n$ be the growth rate of the number of varieties. Then, using (7), we can re-express the no-arbitrage condition as

$$(19) \quad \frac{\dot{E}}{E} = \frac{(1-\alpha)E}{a_D} - \rho - \gamma ,$$

which is analogous to (9) above. Here a_D plays the role of a_I and γ plays the role of ι ; the latter analogy will become clearer from what follows.

Total demand for labor is the sum of employment in R&D, $(a_D/n)\dot{n}$, and that in manufacturing, $E/p = \alpha E$. Thus, labor-market clearing implies

$$(20) \quad a_D \gamma + \alpha E = L ,$$

which is analogous to (10). The differential equation (19) together with the side condition (20) determine the evolution of the economy over time, given an initial value of spending. Clearly, the system can be described by means of a figure that is analogous to Figure 1, with γ replacing ι on the horizontal

axis. Hence, the economy jumps to a steady state that satisfies (20) and a steady-state no-arbitrage condition,

$$(21) \quad (1-\alpha)E/a_D = \rho + \gamma ,$$

that is analogous to (11).

Comparing (10)-(11) with (20)-(21) establishes the equivalence of the two reduced-form systems. In the latter, γ replaces ι , a_D replaces a_I , and α replaces δ . Clearly, all comparative static calculations that apply to one system must apply to both.

The following observations may clarify the similarities between the two approaches and the economic structures that they generate. In both models, agents invest resources to acquire the exclusive ability to manufacture a new product. Moreover, the R&D activity generates unappropriable spillovers in both cases. In the variety-based growth model, the R&D externality is quite explicit. Each completed product development project lowers the cost of later R&D efforts. In the quality-based model, the externality is implicit. When one improvement project succeeds, other researchers can quit their efforts to achieve that same innovation and begin to work on the next improvement. In both instances we assume that by observing the results from one innovative success, researchers can learn scientific and engineering facts that are useful in their own research endeavors. This seems a natural and important characteristic of research and reflects the public good nature of technology as information.

The two approaches do diverge, however, in their welfare implications. Proceeding with the analysis of the variety-based growth model, we note that

(17) implies¹³

$$(22) \quad \log u(t) = \log E + \log \alpha + \gamma t(1/\alpha - 1) ,$$

which is analogous to (12). Equation (22) gives the growth rate in the variety-based model as $g = \gamma(1/\alpha - 1)$. Substituting (22) into (1), we obtain the welfare function

$$(23) \quad \rho U = \log E + \log \alpha + (\gamma/\rho)(1/\alpha - 1) ,$$

which is comparable to (14). Now we can maximize (23) subject to (20), and compare the resulting optimal γ^* to the market equilibrium γ . We find in contrast to our earlier result that whenever the optimal rate of innovation is positive this rate exceeds the market determined rate (see Romer, 1988 and Grossman and Helpman, 1989c for similar results). The reason is as follows. Each new product initially contributes $(1-\alpha)En^{1/\alpha-2}$ to consumer surplus. The marginal entrant inflicts an aggregate loss of profits of $(1-\alpha)E/\alpha n$ on the n existing firms. The marginal utility of income is $\alpha n^{1/\alpha-1}$. Thus, the static consumer-surplus effect and the static business-stealing effect just offset one another. Moreover, both of these effects compound similarly over time. What remains then is the intertemporal spillover effect whereby current technological advance reduces the cost of later R&D. Therefore, the marginal innovation conveys a net positive externality in the variety-based growth model and equilibrium growth is always too slow.

¹³ In writing (22) we assume without loss of generality that the initial number of differentiated products $n(0)=1$.

V. Resources and Growth

We endeavor now to extend our model of quality-based innovation in order to bring out certain features of the growth process that are not evident from a one-factor, one-manufacturing-sector formulation. Specifically, we study in this section the relationship between the growth rate and the accumulation of particular factors of production. In the next section we examine the long-run pattern of trade in a two-country world economy. It should be apparent as we proceed, in view of our findings in the previous section, that similar results apply to an appropriately extended variant of the variety-based growth model.

We add to the model of Section II a sector that produces a homogeneous product of fixed quality. This might be a service sector, for example. We replace (1) with the augmented preferences,

$$(24) \quad U = \int_0^{\infty} e^{-\rho t} [s \log u(t) + (1-s) \log y(t)] dt ,$$

where $y(t)$ represents consumption of the homogenous product and $u(t)$ is as before (see (2)). Then consumers allocate at every point in time a share s of their spending to the vertically differentiated products and a share $(1-s)$ to the outside good y . The time pattern of spending follows (7).

We add as well a second primary factor of production. We shall refer to the two factors as unskilled labor (L) and skilled labor (H). Let unit manufacturing costs of the vertically differentiated products be $c_x(w_L, w_H)$ and those of the outside good be $c_y(w_L, w_H)$, where w_i denotes the reward to input i , $i=L, H$. The cost of a unit of innovative activity is given by $c_i(w_L, w_H)$.

We assume perfect competition in the market for the outside good and Bertrand competition as before in the markets for the vertically differ-

entiated products. Let the former good serve now as numeraire. Then we have in place of (8) the following pricing equations:

$$(25) \quad p_X = \lambda c_X(w_L, w_H);$$

$$(26) \quad 1 = c_Y(w_L, w_H) .$$

Factor market clearing requires

$$(27) \quad a_I(w_L, w_H)\iota + a_X(w_L, w_H)X + a_Y(w_L, w_H)Y = \begin{bmatrix} H \\ L \end{bmatrix} ,$$

where $a_i(\cdot)$ represents the cost minimizing input vector per unit of output for $i=I, X, Y$, and X and Y denote output quantities. The input vectors are given by the gradients of the respective unit-cost functions. Static equilibrium in the commodity market entails

$$(28) \quad s/(1-s) = p_X X / p_Y Y .$$

Finally, the steady-state no-arbitrage condition reads (in place of (11)):

$$(29) \quad (1-\delta)p_X X / c_I(w_L, w_H) = \rho + \iota .$$

As before, convergence to the steady state is immediate.

We wish to explore the relationship between the growth rate and the size of the resource base. A common feature of recent models of endogenous technological progress has been that larger economies grow faster (see Aghion

and Howitt, 1989, Romer, 1988, and Grossman and Helpman, 1989a). We shall show that this result depends critically on the general equilibrium structure of these models. In general, only accumulation of factors that are used intensively in the growth-generating activities guarantees faster growth.

To make this point, we pursue the special case in which R&D and manufacturing of X use only skilled labor while production of the outside good uses both primary factors. With this restriction, we can combine (25)-(28) to obtain

$$(30) \quad a_{HI} \iota + \frac{s}{1-s} \frac{Y(w_H, L)}{\lambda w_H} + H_Y(w_H, L) = H,$$

where $Y(\cdot)$ represents the profit-maximizing supply of the outside good and $H_Y(\cdot)$ represents that sector's demand for skilled labor when it employs the entire unskilled labor force. Similarly, we combine (25)-(26) and (28)-(29) to obtain

$$(31) \quad (1-\delta) \frac{s}{1-s} \frac{Y(w_H, L)}{w_H a_{HI}} = \rho + \iota.$$

We plot these two equations as HH and III, respectively, in Figure 3.

Now consider an increase in H . This shifts the HH curve up and to the left, while leaving III unaffected. The new equilibrium involves more innovation and faster growth.¹⁴ Here we see that accumulation of the factor used intensively in R&D indeed is conducive to growth.

When L increases, both curves shift to the right. The rate of innovation grows if and only if the rightward shift of III from A exceeds the rightward

¹⁴ The growth rate of a consumption index now is $s \cdot \log \lambda$.

shift of HH. This occurs when the elasticity of substitution between skilled and unskilled labor in the production of the outside good is larger than one, but not when it is smaller than one.¹⁵ Intuitively, the outside good draws skilled labor away from R&D and X due to an output effect, but releases skilled labor as w_H rises due to a substitution effect. When the elasticity of substitution is large, the latter effect dominates and so more skilled labor finds itself employed in R&D in the new general equilibrium.

A more striking result emerges from a different special case. Suppose R&D uses only skilled labor, the outside good uses only unskilled labor, and the vertically differentiated products require both factors in fixed proportions. It is straightforward to show that, in these circumstances, an increase in the supply of unskilled labor must slow growth. The general point is that a positive monotonic relationship between resource supplies and employment of those resources in the growth-generating activities exists only for certain general equilibrium structures.

VI. International Trade

In this section we show how quality ladders can be embedded in a model of international trade. We use the two-factor, two-sector structure of Section V to explore the nature of a trading equilibrium and to examine the long-run

¹⁵ The rightward shift in III is given by the change in w_H that leaves $Y(w_H, L)/w_H$ constant as L increases. The rightward shift in HH is larger or smaller than this as H_Y increases or decreases with L when $Y(w_H, L)/w_H$ is kept constant. Using the condition for optimal H_Y (marginal product equals wage), we find that H_Y increases with L when $Y(w_H, L)/w_H$ is constant if and only if $\theta_H + \epsilon_H > 1$, where θ_H is the share of H in the cost of producing Y and ϵ_H is the absolute value of the elasticity of the marginal product of skilled labor in the Y sector. Since $\theta_H + \epsilon_H = 1 + \epsilon_L(1 - \sigma)$, where ϵ_L is defined analogously to ϵ_H and σ is the elasticity of substitution between skilled and unskilled labor in the production of the outside good, our claim is established.

pattern of trade when resources in each country are devoted to improving the quality of some products.

To begin with, suppose that each vertically differentiated product must be manufactured in the country in which the most recent product improvement has taken place. That is, we rule out international licensing and multinational corporations for the moment. In Figure 4 we draw a rectangle whose dimensions represent the aggregate world endowments of H and L. If there were no country borders, then the equilibrium would be that described in Section V above. We ask now whether an equilibrium with free trade can reproduce the essential features of that integrated equilibrium, despite the fact that factors now are restricted to stay within their countries of origin.

The vector OQ^* in the figure represents the total employment of H and L in the activities of product improvement and manufacturing of X in the equilibrium of Section V (henceforth, the "integrated equilibrium"). This vector is given by $a_{1I} + a_{1X}$. The vector Q^*O^* similarly represents employment in the production of the outside good in the integrated equilibrium. We depict the endowments of the two countries by a point in the rectangle, with the vector of factors measured from the origin at O representing the endowment of the home country and that measured from O^* representing the endowment of the foreign country. We claim that if the endowment point, marked E, falls within the parallelogram OQO^*Q^* , then there exists a trading equilibrium with all aggregate variables identical to those of the integrated equilibrium.

At E, the home country is relatively well endowed with skilled labor. Suppose that factor prices and interest rates in the two countries were equalized nonetheless and that their levels were the same as in the integrated

equilibrium.¹⁶ Then techniques of production would be the same. The home country achieves full employment if it employs OP in the combined activity of product improvement and manufacturing of X and OP_Y in the production of outside goods. Full employment obtains abroad when that country employs O^*P^* in the combined activity and $O^*P_Y^*$ in manufacturing of Y . These employment vectors give the same aggregate levels of activity as in the integrated equilibrium. The ratio of the line segments $\overline{OP}/\overline{OQ}$ gives the number n of vertically differentiated products that is produced at home. The home country also performs a fraction n of world R&D activity, and thereby maintains leadership in n products in all periods.¹⁷ The foreign country produces the remaining n^*-1-n vertically differentiated products and undertakes the fraction n^* of R&D effort.

It remains to be shown only that, with these proposed allocations, the product markets clear and all profitability conditions are satisfied. Since we have provisionally assumed that factor prices are the same in each country as they were in the integrated equilibrium, all activities break even in each country, as they all did in the integrated equilibrium. Also, with interest rates as before, the no-arbitrage condition continues to be satisfied in each country. With the same costs of production, commodity prices are the same as in the integrated equilibrium. Aggregate income is the same as well. Since

¹⁶ International trade in financial assets would of course guarantee equalization of interest rates. However, as we shall see, a steady-state equilibrium exists with identical interest rates in the two countries even when financial assets are not traded.

¹⁷ Entrepreneurs in the home country might, for example, be the only ones who attempt to improve a fraction n of the products, each at intensity ι . Or home entrepreneurs may attempt to improve the entire spectrum of vertically differentiated products at intensity $n\iota$. Other allocations are possible as well, so long as the aggregate innovation effort devoted to each product ω is the same and equal to ι from the integrated equilibrium.

preferences are homothetic, the distribution of income does not matter for aggregate demand, and so product markets clear. This completes our demonstration that commodity trade (with or without trade in financial assets) suffices to reproduce the essential features of the integrated steady-state equilibrium.

What then is the pattern of world trade? Suppose, to begin with, that financial assets are not traded; i.e., each country must finance all R&D that takes place within its borders from domestic savings. Then the trade account must balance. The homotheticity of preferences implies an identical composition of aggregate demand in the two countries. But from Figure 4 we see that the unskilled labor abundant (foreign) country specializes relatively in the production of the unskilled labor-intensive (outside) good. Hence the home country imports the outside good. With trade balanced, this country must be a net exporter of vertically differentiated products. This pattern of intersectoral trade corresponds of course to the predictions of the Heckscher-Ohlin model. Here it applies to the steady state of a dynamic world economy with continual quality upgrading.

The two-country world economy does not converge immediately to a steady state, unless the initial ownership shares in blueprints for frontier products happen to coincide with the n and n^* of the steady-state equilibrium. In general, these shares are attained during a phase of dynamic adjustment. If international trade in financial assets takes place along the adjustment path, then typically the steady state will not be characterized by balanced trade. Although the production patterns of the steady state remain as described above, it may happen that one country will import both the outside good and (on net) vertically differentiated products. It can do so if its steady-state

surplus on service account is large enough. Trade imbalance can never reverse the pattern of trade from that predicted by the Heckscher-Ohlin theorem, however.

Finally, we relax the assumption that product improvement and manufacturing of the improved product must take place in the same country. In the region of factor price equalization (OQO^*Q^*) there is of course no incentive to separate these activities, because the same costs prevail everywhere. Suppose, however, that the endowment point lies in triangle ODQ of Figure 4. Then factor price equalization does not obtain when products are improved and produced in the same country. If these two activities differ in their factor proportions, then each country enjoys a comparative cost advantage in one of them. Assume for concreteness that at common factor prices R&D employs more skilled labor relative to unskilled labor than does manufacturing of the vertically differentiated products. Let OD and DQ be the employment vectors in these two activities, respectively, in the integrated equilibrium. Then, following Grossman and Helpman (1989d), it is easy to show that a suitable number of multinational corporations will form. These firms undertake their R&D at home, but perform their subsequent manufacturing activities in the foreign country. For endowments in ODQ , the formation of multinationals restores factor price equalization and enables the world economy to once again reproduce the integrated equilibrium. For still larger differences in factor composition across countries, multinational corporations still emerge but the trade equilibrium differs from that of the integrated economy.

VII. Concluding Remarks

We have developed a model of ongoing product improvements. This model draws several building blocks from earlier work by Segerstrom et.al (1987) and Aghion and Howitt (1989). Entrepreneurs race to bring out the next generation of a continuum of goods. In each industry success occurs with a probability per unit time that is proportional to the total R&D resources targeted to improving that product. Each product follows a stochastic progression up the quality ladder. But the equilibrium is characterized by an aggregate rate of innovation that is determinate, and constant in the steady state.

The model captures many realistic aspects of the innovation process. Individual products become obsolete after a time. Progress is not uniform across sectors. Research responds to profit incentives. And innovators are able to benefit from observing and analyzing the research successes of their rivals. These features fit the detailed historical descriptions of industrial R&D provided by Freeman (1982) and others.

We related our approach to an alternative one that treats industrial R&D as a process of creating an ever expanding range of horizontally differentiated products. The latter framework has been applied to issues of long-run technological progress and growth by Romer (1988) and ourselves (1989a,b,c). We showed that the two approaches though seemingly quite distinct actually share identical reduced forms for their simplest variants. Thus, both approaches yield the same answers to many positive questions about the determinants of the long-run growth rate. However, as Aghion and Howitt (1989) pointed out, the normative analyses of the variety-based and quality-based growth models differ. In the former, the equilibrium rate of innovation always is too slow. But when growth derives from product improvements, it may

be slower or faster than is optimal. We have been able to derive and contrast these results using very comparable versions of the alternative models.

In one section of this paper, we cast the innovation process in a two-country setting. We described a long-run equilibrium with product improvements taking place in each country, with intra-industry trade in vertically differentiated products, and with inter-industry trade of technologically progressive goods for homogeneous, unchanging goods. The Heckscher-Ohlin theorem predicts the long-run pattern of intersectoral trade despite the diversion of resources to R&D and the continual technological advance that takes place.

Building on the framework developed here, we provide a richer story of international trade in a companion paper (Grossman and Helpman, 1989e). There, all product improvements take place in a high-wage region with comparative advantage in R&D. But entrepreneurs in the low-wage region are able to produce clones of state-of-the-art products if they succeed in reverse engineering. Imitation, like innovation, requires resources and entails uncertain prospects. Unlike earlier models of the product life cycle based on horizontally differentiated products (e.g., Krugman, 1979, or Grossman and Helpman, 1989b), this one predicts that the locus of output of a particular type of good will move back and forth between the North and the South as the former region captures market share when quality improvements take place and the latter then begins the process of imitating the new, improved product. This description of the product cycle would seem apt for many industries; e.g., personal computers and many consumer electronics.

We believe that our model is rich in its predictions, yet technically quite manageable. It might gainfully be extended to include endogenous

accumulation of primary factors. We hope that it will prove useful in application to additional issues concerning innovation and long-run growth.

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Figure 1

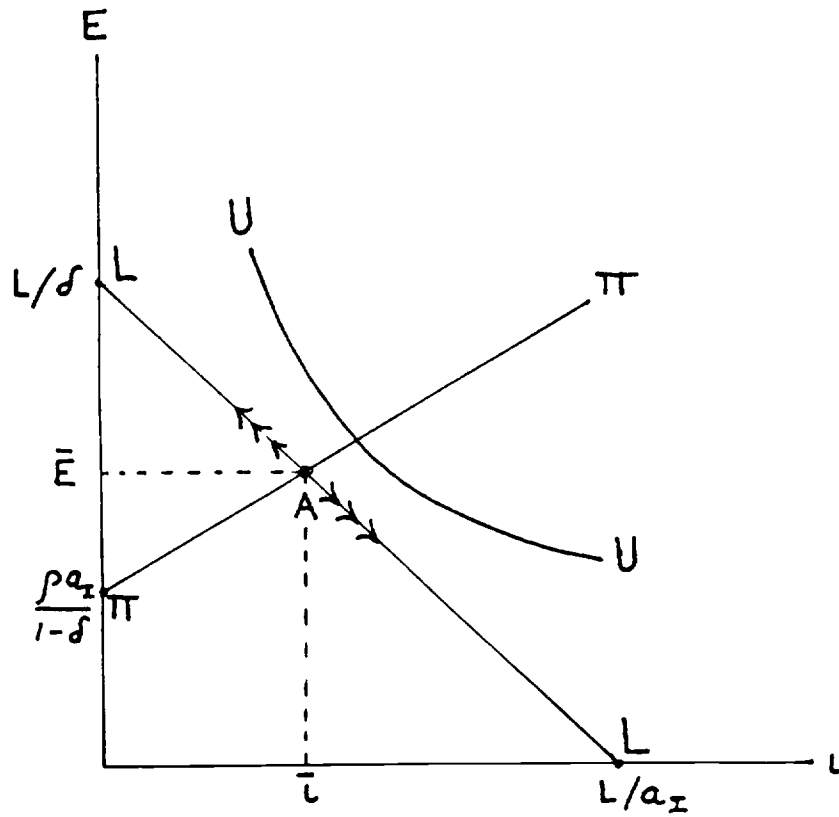


Figure 2

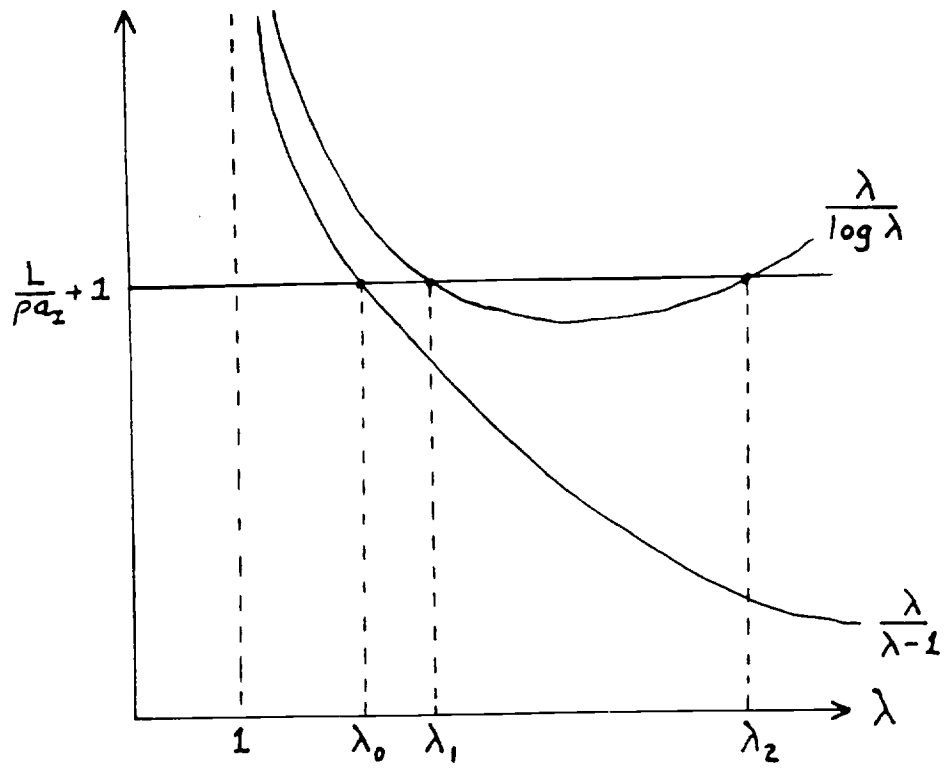


Figure 3

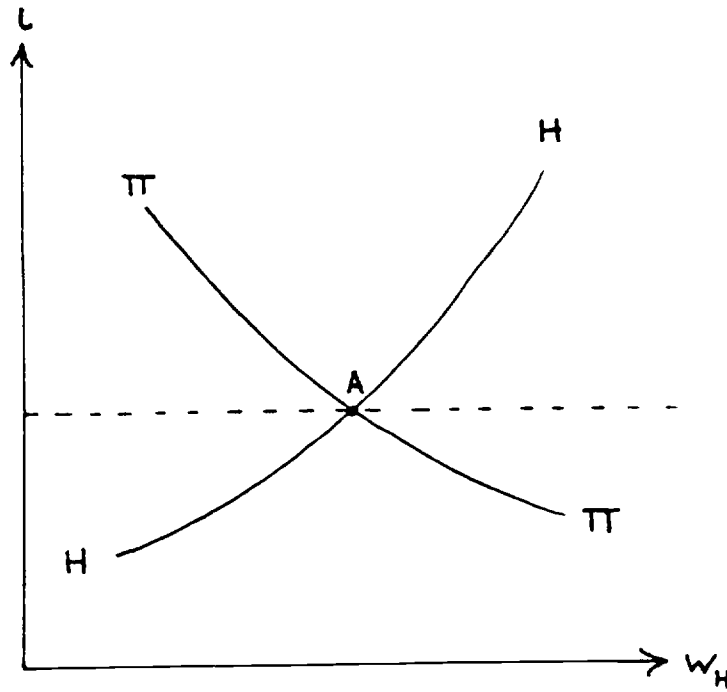


Figure 4

