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PRODUCTION NETWORK FORMATION, TRADE, AND WELFARE

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ABSTRACT

We study the aggregate implications of production network formation in a quantitative multi-location general equilibrium trade model. Firms search for suppliers and buyers across locations subject to matching frictions, generating a gravity structure of production networks. We develop sufficient statistics for global and regional welfare and characterize the deviations from the fixed network environment, including the role of inefficiency and amplification effects of search and matching. We calibrate our multi-sector model to Chilean domestic and international firm-to-firm trade data and show that our model can rationalize the observed increase in domestic supplier linkages after Chile's recent trade agreements. Abstracting from endogenous networks reduces Chile's aggregate welfare losses by 20 percent when import costs are raised to their pre-agreement levels, consistent with inefficiently low equilibrium levels of search. Fixing the trade elasticity, the welfare gains from trade relative to municipality autarky drop by 40 percent due to amplification effects of search.

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1 Introduction

The trade of intermediate inputs constitutes the majority of international trade. This trade is enabled by the formation of production networks between firms, which plays a crucial role in the transmission of shocks. The recent rounds of tariff increases across the globe raise questions about how supply chains may reorganize and how it impacts the aggregate economy. The standard quantitative trade model, the mainstay of applied analysis for global value chains, treats these production linkages as fixed, overlooking the fact that firms actively shape these networks by establishing relationships with their suppliers and buyers. While a growing body of research provides evidence on firms' decisions in forming production linkages, a gap remains in understanding how these decisions collectively shape the aggregate structure of production networks, economic activity, and regional welfare — both theoretically and quantitatively.

We address this gap by developing and analyzing a tractable, quantitative multi-location general equilibrium trade model that incorporates firms' production network formation. Our model embeds firms' production network formation decisions in an otherwise standard trade model with input-output linkages, as reviewed by Costinot and Rodríguez-Clare (2014) and Antrás and Chor (2022). Firms search for both suppliers and buyers across space while facing ad-valorem trade costs and search costs. Relationships materialize depending on the suppliers' and buyers' bilateral search efforts and the matching technology. We show that our model implies a gravity structure of production networks and bilateral trade. Production networks, in turn, shape aggregate economic activity across locations in general equilibrium. We use this framework to develop sufficient statistics for global and regional welfare and characterize the deviations from the fixed network environment.

A key feature of our framework is the search and matching externalities arising from firms' production network formation decisions. They play a critical role in the aggregate effects of trade shocks and the departures of our setup from the standard trade model with fixed production networks. To clarify the nature of these externalities, we solve a planning problem and derive a simple optimal policy formula for sales and search cost taxes. We show that the search taxes are generally non-zero unless the elasticities of matching technology take particular values so that the social marginal benefit of search coincides with the firms' equilibrium spending for search. Under these parameter values, the non-appropriability and business stealing effects of relationship formation exactly cancel out. This result extends the classical insight of search externality by Hosios (1990) to the setting with multiple layers of production networks spanning across multiple locations.

Equipped with this result, we develop a series of sufficient statistics for the aggregate welfare effects of trade cost shocks. First, up to first order, global welfare changes are proportional to the product of observed nominal trade flows times the trade cost changes, scaled by a specific coefficient. This observation that nominal trade flows are key summary statistics relates to the *ex-ante* welfare sufficient statistics analyzed by Atkeson and Burstein (2010) and Baqaee and Farhi (2024) with fixed production networks, who in turn build on the insight of Hulten (1978) in the closed economy. The specific coefficient may not be equal to one, precisely because of the presence of equilibrium inefficiencies. In the fixed network environment, this coefficient is greater than one due to a classic double marginalization inefficiency that arises from monopolistic competition. With endogenous production networks, this coefficient depends on the value of the social marginal benefit relative to the private spending for search and how it interacts with the inefficiency from the double marginalization.

Second, for any magnitude of the shock, welfare changes of a region are proportional to the changes in intra-regional sourcing shares, scaled by a specific coefficient. This result extends the familiar *ex-post* welfare sufficient statistics of welfare gains from trade with fixed production networks (Arkolakis, Costinot and Rodríguez-Clare 2012; Blaum, Lelarge and Peters 2018). This coefficient deviates from what is implied by a fixed production network environment for two reasons. First, endogenous network formation increases the aggregate trade elasticity – the responsiveness of aggregate trade flows to variable trade costs – by enabling adjustments in supplier and buyer linkages. Second, trade cost shocks can directly change the search cost payment, which amplifies these shocks.

We also demonstrate that our welfare sufficient statistics results extend beyond the particular microfoundation of the network formation based on search and matching. Following the approach of Arkolakis et al. (2012), we derive reduced-form macro restrictions that lead to the same *ex-ante* and *ex-post* welfare sufficient statistics. We show that these restrictions hold for alternative microfoundations under appropriate parametric assumptions, such as relationship-specific fixed costs (as in Lim (2018); Huneeus (2018); Bernard, Dhyne, Magerman, Manova and Moxnes (2022); Dhyne, Kikkawa, Kong, Mogstad and

Tintelnot (2023)) or discrete choices of suppliers (as in Oberfield (2018), Acemoglu and Azar (2020), Antràs and De Gortari (2020) and Eaton, Kortum and Kramarz (2024)), providing a coherent account of how endogenous production network formation affect the aggregate economy. We also show that our framework can be extended to a multi-sector environment following the specification of Caliendo and Parro (2015).

How much does endogenous production network formation affect the aggregate effects of trade cost shocks in practice? We answer this question using Chilean domestic and international firm-to-firm trade data. We first provide reduced-form evidence that a permanent trade shock –tariff changes from Chile's recent bilateral trade agreements with the U.S. and China– leads to the reorganization of production networks surrounding Chilean firms. Using a difference-in-differences design with firms' pre-agreement import mix to construct an exposure measure, we show that firms with higher exposure to import tariff reduction differentially increase both international and domestic supplier linkages. Our finding of the *increase* in domestic supplier linkages is in contrast to existing research studying the impacts of *temporary* international demand shocks (Demir, Fieler, Xu and Yang, 2024a) and supply shocks (Huneeus, 2018) on domestic production networks, and suggests that domestic and international supplier linkages are gross complements.

We next calibrate our model to data. We estimate the key structural parameters – matching function elasticities and labor coefficients on search costs– using an indirect inference approach, targeting the reduced-form effects of Chilean tariff reforms on domestic production network reorganization. We show that our estimates of large matching function elasticities and small labor coefficients on search costs can indeed rationalize the differential *increase* in domestic supplier linkages when import tariffs go down.

Using the calibrated model, we first assess the impacts of moderate international trade cost shocks on the Chilean economy. When we increase the iceberg trade costs from U.S. and China to Chile by a magnitude similar to the aforementioned trade agreements (about a 6.5 percentage point increase in tariffs for all imported goods from U.S. and China), the aggregate welfare in Chile decreases by 0.35 percent. When we shut down endogenous networks, this number decreases to 0.28 percent. Therefore, shutting down endogenous network formation reduces the aggregate welfare losses by approximately 20%. This result is consistent with the interpretation that the equilibrium level of search, and hence the trade flow, is inefficiently low. Given observed trade flows, the aggregate welfare effects are larger if we account for the fact that observed production networks are formed endogenously.

Finally, we assess the welfare gains from trade (GFT) relative to municipality autarky (i.e., no trade across Chilean municipalities and with international countries) and those relative to international autarky (i.e., allowing for trade across Chilean municipalities but no trade with international countries). Fixing the trade elasticity, abstracting from endogenous networks decreases the average GFT relative to *municipality* autarky from 169 to 97 percentage points, which is a 42% reduction. This reduction reflects the amplification of trade shocks through search costs. We also find that abstracting from endogenous networks decreases the average GFT relative to *international* autarky from 6.2 to 5.6 percentage points, which is a 10% reduction. The smaller gap for this counterfactual reflects the fact that the reorganization of domestic production networks across Chilean municipalities has an additional mitigation effect. Overall, our findings suggest that endogenous network formation has a large quantitative implication for the aggregate welfare effects of various trade shocks.

This paper contributes to several strands of literature on international trade and macroeconomics. First, we contribute to the literature on production networks by developing a tractable quantitative multi-region general equilibrium trade model featuring endogenous production network formation and by characterizing its aggregate implications. Existing work has established that shocks propagate through fixed production networks. Existing work has also established that firms endogenously form production networks depending on the economic environment (see Johnson (2018), Bernard and Moxnes (2018), Carvalho and Tahbaz-Salehi (2019), Antrás and Chor (2022), and Baqaee and Rubbo (2023) for recent reviews). Some authors, such as Antràs and De Gortari (2020) and Eaton et al. (2024), incorporate discrete choices of suppliers in multi-location general equilibrium trade models with input-output linkages. However, this literature offers a limited theoretical characterization of how endogenous network formation under various frictions shapes the aggregate effects of trade shocks. We contribute by extending familiar *ex-ante* and *ex-post* welfare sufficient statistics in trade models with fixed production networks to the endogenous network environment.

Our emphasis on sufficient statistics relates to Baqaee, Burstein, Duprez and Farhi (2024), who provide a nonparametric accounting framework to evaluate the firm-level and aggregate effects of observed changes in production networks. A key distinction of our work is that we endogenize production network formation. We show that this feature crucially influences the aggregate effects of trade shocks, both theoretically and quantitatively.

Several existing papers study the equilibrium inefficiency of production network formation. Grossman, Helpman and Lhuillier (2023) and Grossman, Helpman and Sabal (2024) examine the equilibrium inefficiency from markup distortions in a stylized small open economy model. Boehm and Oberfield (2020) analyze a model with a discrete choice of suppliers under wedges and quantify the aggregate distortion. Acemoglu and Tahbaz-Salehi (2024) study a model where a discrete number of firms form linkages and bargain over surplus and highlight equilibrium inefficiency due to a hold-up problem. In contrast, our work highlights the equilibrium inefficiency from search and matching externalities. We do so in the multi-location general equilibrium environment with flexible geographic frictions, which allows us to map our model to data for quantification.

Finally, we contribute to the literature on search and matching frictions in trade. This literature has provided empirical evidence for the search and matching frictions in inter- and intra-national trade and developed various models to capture these frictions.[1] Our model extends the firm-level supplier and buyer search decisions of Demir et al. (2024a) to multi-location general equilibrium environment, and use this model to characterize equilibrium inefficiency and welfare sufficient statistics.

2 Model

The economy is segmented by a finite number of locations (such as regions or countries) denoted by $u, i, d \in \mathcal{N}$. In each location, there is an L_i measure of households. Each household supplies one unit of labor inelastically and earns a competitive wage w_i. There is a fixed mass of intermediate goods producers in each location, which we call "firms" in short. We denote each firm in location i by $\omega \in \Omega_i$ and the measure of firms in location i

[1] Examples include Chaney (2014); Allen (2014); Bernard, Moxnes and Saito (2019); Brancaccio, Kalouptsidi and Papageorgiou (2020); Krolikowski and McCallum (2021); Eaton, Jinkins, Tybout and Xu (2022); Lenoir, Martin and Mejean (2023); Startz (2024); Miyauchi (2024); Huang, Manova, Perello and Pisch (2024); Demir, Javorcik and Panigrahi (2024b).

by N_i. We denote the distribution function of total factor productivity (TFP) by firms in location i by $G_i(\cdot)$, which can flexibly depend on the location.

Firms produce differentiated intermediate goods, combining labor and intermediate goods. Intermediate goods can be traded across firms and locations connected by production linkages subject to iceberg trade costs. These connections, in turn, are determined by firms' search and matching decisions. Local competitive retailers source intermediate goods from local firms and create retail goods within each location. The retail goods are used for final consumption and for firms' search activity. We take the global nominal GDP as the numéraire unless explicitly stated otherwise.

We denote $S_{ui}(\omega) \subseteq \Omega_u$ to indicate the set of suppliers producing in u that a firm ω in location i can purchase from. Therefore, $\{S_{ui}(\cdot)\}_{u,i}$ summarize the structure of production networks in this economy. We first describe how production occurs given networks $\{S_{ui}(\cdot)\}_{u,i}$. We then describe how these networks are endogenously formed through a search and matching process.

2.1 Production given Networks

Firms. The production function of firm $\omega \in \Omega_i$ is given by

$$q_i(\omega) = z_i(\omega) \left(\frac{l_i(\omega)}{\beta}\right)^{\beta} \left(\frac{\tilde{q}_i(\omega)}{1-\beta}\right)^{1-\beta}, \tag{1}$$

where $z_i(\omega)$ is the TFP of firm ω, $l_i(\omega)$ is labor inputs, and $\tilde{q}_i(\omega)$ is the composite of intermediate inputs, β is the parameter proxying the input share for labor. The composite of intermediate inputs is a constant elasticity of substitution (CES) aggregator of the input varieties sourced from their connected suppliers, given by

$$\tilde{q}_i(\omega) = \left(\sum_{u\in\mathcal{N}} \int_{\upsilon\in S_{ui}(\omega)} q_{ui}(\upsilon,\omega)^{\frac{\sigma-1}{\sigma}} d\upsilon\right)^{\frac{\sigma}{\sigma-1}}, \tag{2}$$

where $q_{ui}(\upsilon,\omega)$ is the quantity of input for the variety from connected supplier $\upsilon \in S_{ui}(\omega)$, and σ is the elasticity of substitution. From cost minimization, the marginal cost of production of firm ω is given by

$$c_i(\omega) = \frac{1}{z_i(\omega)} w_i^{\beta} \left(\sum_{u \in \mathcal{N}} \int_{\upsilon \in S_{ui}(\omega)} p_{ui}(\upsilon, \omega)^{1-\sigma} \, d\upsilon \right)^{\frac{1-\beta}{1-\sigma}}, \tag{3}$$

where $p_{ui}(\upsilon, \omega)$ is the intermediate goods price that supplier υ in location u charges to firm ω in location i. On top of these production costs, when a firm sells their intermediate goods to location d, they incur an iceberg trade cost of $\tau_{id} \geq 1$.

We assume that all firms are matched with a continuum of suppliers and, therefore, suppliers are under monopolistic competition to supply to each buyer. Thus, given the isoelastic intermediate goods demand, suppliers charge a constant markup to their marginal cost net of the iceberg trade cost;

$$p_{id}(\upsilon, \omega) = \tilde{\sigma} c_i(\upsilon) \tau_{id}, \tag{4}$$

where $\tilde{\sigma} = \sigma / (\sigma - 1)$ is the markup ratio.

Retailers. Perfectly competitive retailers in each location i combine intermediate inputs from all firms in location i and produce standardized nontradable retail goods. Their production function is given by

$$Q_i = g_i \left(\left\{ q_i^R(\omega) \right\}_{\omega \in \Omega_i} \right), \tag{5}$$

where $g_i(\cdot)$ is a function that satisfies homogeneous of degree one, and $q_i^R(\omega)$ is the quantity of intermediate inputs from firm ω. The retail goods are used for final consumption and for firms' search activity to form production linkages, as we describe further below.

We also assume that retailers have the entire bargaining power when purchasing intermediate inputs from each firm, and therefore, purchase goods at the marginal cost $c_i(\upsilon)$. From cost minimization, retail goods prices are given by

$$P_i = \tilde{g}_i \left(\left\{ c_i(\omega) \right\}_{\omega \in \Omega_i} \right), \tag{6}$$

where $\tilde{g}_i(\cdot)$ is a solution to the cost minimization problem by retailers.

Final Consumers. Measure L_i households supply labor inelastically at wage w_i. They also own an equal share of local firms and earn their profits. Therefore, their budget constraint is given by

$$P_i Q_i^F = w_i + \frac{\Pi_i}{L_i}, \tag{7}$$

where Q_i^F is the amount of final consumption of retail goods per capita, and Π_i is aggregate profit by firms producing in location i.

2.2 Production Network Formation

Next, we describe how the production network structure, $\{\mathcal{S}_{ui}(\cdot)\}$, is endogenously determined through a search and matching process.

2.2.1 Firms' Search Decision

Each firm ω from a given location i decides the buyer search effort in different destinations, $\{n_{id}^B\}_{d\in\mathcal{N}}$, and the supplier search effort across supplier origins, $\{n_{ui}^S\}_{u\in\mathcal{N}}$.[2] Following Arkolakis (2010); Boehm and Oberfield (2023); Demir et al. (2024a), we assume that these search efforts are associated with iso-elastic upward-sloping search costs. The total search cost is given by

$$f_i\left(\{n_{id}^B\}_d, \{n_{ui}^S\}_u\right) = e_i \left\{ \sum_{d\in\mathcal{N}} f_{id}^B \frac{\left(n_{id}^B\right)^{\gamma^B}}{\gamma^B} + \sum_{u\in\mathcal{N}} f_{ui}^S \frac{\left(n_{ui}^S\right)^{\gamma^S}}{\gamma^S} \right\}, \tag{8}$$

where e_i is the unit cost of supplier and buyer search in location i (we describe how this is determined in the end of this section), and $\gamma^B > 1$ and $\gamma^S > 1$ are parameters capturing the decreasing returns in search effort. $\{f_{id}^B\}$ and $\{f_{ui}^S\}$ are location-pair-specific search cost shifters, capturing the possibility that the cost of searching for suppliers and buyers may depend on geographic frictions.[3]

[2] Whenever the equilibrium variables involve two locations with an upstream and downstream relationship (e.g., n_{id}^B, n_{ui}^S), we adopt the convention of denoting the subscripts in the order of upstream and then downstream locations.

[3] In Appendix B.2, we consider an alternative specification where either the supplier or buyer search is undirected to a specific location. The only difference in the aggregate welfare implications arises from the differences in trade elasticities, as we discuss in Section 4.3.

Supplier search efforts, $\{n_{ui}^S\}_u$, turn into successful supplier relationships at location-pair specific match rate, $\{m_{ui}^S\}_u$. Similarly, buyer search efforts, $\{n_{id}^B\}_d$, turn into successful buyer relationships at rate $\{m_{id}^B\}_d$. Each firm is atomistic and hence takes these matching rates as given. In the equilibrium, these matching rates are endogenously determined by the aggregate search efforts and matching technology, as we describe in the next section.

Firms in the same location i with the same productivity z face the same equilibrium revenue and cost functions, and thus, they will make the same supplier and buyer decisions. Therefore, without loss of generality, we denote equilibrium search efforts by $\{n_{id}^B(\omega), n_{ui}^S(\omega)\}$ and $\{n_{id}^B(z), n_{ui}^S(z)\}$ interchangeably.

Applying the law of large numbers, the cost function (3) can be written as a function of firm efficiency z and the set of successful supplier matches, $n_{ui}^S \times m_{ui}^S$, such that

$$c_i\left(z, \{n_{ui}^S\}_u\right) = \frac{1}{z_i} w_i^{\beta} \left(\sum_{u \in \mathcal{N}} n_{ui}^S m_{ui}^S {C_{ui}}^{1-\sigma} \right)^{\frac{1-\beta}{1-\sigma}}, \tag{9}$$

where $C_{ui} \equiv \int_z \left(\tilde{\sigma} c_u(z)\, \tau_{ui}\right)^{1-\sigma} dG_{ui}^B(z)$ is the CES aggregator of the prices of suppliers producing in location u to supply to location i, and $G_{ui}^B(z)$ is probability density function of productivities weighted by the equilibrium buyer search efforts, i.e. $dG_{ui}^B(z) = n_{ui}^B(z)\, dG_u(z) / \int_{z'} n_{ui}^B(z')\, dG_u(z')$. Given this cost, a firm in location i with productivity z has expected revenue from each matched buyer from location d, $r_{id}(z, \{n_{ui}^S\}_u) = \left(\tilde{\sigma} \tau_{id} c_i\left(z, \{n_{ui}^S\}_u\right)\right)^{1-\sigma} D_d$, where D_d is the average demand per buyer in location d net of the buyer-specific price index.

The firm's total revenue from a destination d is the revenue per match times the expected number of buyers, $n_{id}^B \times m_{id}^B$. The firm profit is then determined by the optimal search decisions given by the difference of the variable profit and search costs,

$$\tilde{\pi}_i(z) \equiv \max_{\{n_{id}^B\}_d, \{n_{ui}^S\}_u} \frac{1}{\sigma} \sum_{d \in \mathcal{N}} n_{id}^B m_{id}^B r_{id}(z, \{n_{ui}^S\}_u) - f_i\left(\{n_{id}^B\}_d, \{n_{ui}^S\}_u\right), \tag{10}$$

subject to (8) and (9). We impose a parameter restriction that $\delta \equiv (\sigma - 1) / \left(1 - \frac{1}{\gamma^B} - \frac{1-\beta}{\gamma^S}\right) > 0$, which guarantees that firms make positive sales and profit. The following lemma characterizes the solution to this problem.

Lemma 1. *Consider the optimization problem* (10) *and let* $\delta > 0$. *Then,*
a) The solution to the supplier and buyer search problem (10) is given by

$$n_{id}^{B}\left(z\right)=a_{id}^{B}z^{\frac{\delta}{\gamma^{B}}},\qquad n_{ui}^{S}\left(z\right)=a_{ui}^{S}z^{\frac{\delta}{\gamma^{S}}},\tag{11}$$

where $a_{id}^{B}\equiv\left(\Gamma_{i}^{B}\frac{X_{id}}{e_{i}f_{id}^{B}}\right)^{\frac{1}{\gamma^{B}}}$ *and* $a_{ui}^{S}\equiv\left(\Gamma_{i}^{S}\frac{X_{ui}}{e_{i}f_{ui}^{S}}\right)^{\frac{1}{\gamma^{S}}}$ *with* $\Gamma_{i}^{S},\Gamma_{i}^{B}>0$ *defined in Appendix A.1, and* X_{id} *is aggregate nominal sales of intermediate goods from* i *to* d.
b) Furthermore, its unit cost, $c_i\left(z\right)$, *can be expressed as*

$$c_{i}\left(z\right)=C_{i}z^{-\frac{\delta}{\gamma^{S}}\frac{1-\beta}{\sigma-1}-1},\qquad C_{i}^{1-\sigma}\equiv w_{i}^{\beta(1-\sigma)}\left(\sum_{u\in\mathcal{N}}a_{ui}^{S}m_{ui}^{S}C_{ui}^{1-\sigma}\right)^{1-\beta},\tag{12}$$

The proof of this lemma and all other propositions are in Appendix A. The Lemma extends the characterization in the single location environment in Demir et al. (2024a) to many locations. It implies that optimal search decisions $\{n_{id}^{B}\left(z\right), n_{ui}^{S}\left(z\right)\}$ are multiplicatively separable between the firm-specific component that is iso-elastic in productivity z and the location-pair-specific components $\{a_{id}^{B}, a_{ui}^{S}\}$.

It is worth pointing out that the unit cost, $c_i\left(z\right)$, decays at a faster rate than z^{-1}. This is because more productive firms search for suppliers more intensively (Equation 11), which leads to disproportionately lower production costs.

The firm's total revenue (excluding sales to retailers) is

$$r_{i}^{*}\left(z\right)=\tilde{\sigma}^{1-\sigma}D_{i}^{*}C_{i}^{1-\sigma}z^{\delta},\qquad D_{i}^{*}=\sum_{d\in\mathcal{N}}a_{id}^{B}m_{id}^{B}D_{d}\tau_{id}^{1-\sigma}.\tag{13}$$

In addition, we can substitute these results in Equation (10) to obtain firm profit as a constant fraction of revenue,

$$\tilde{\pi}_{i}\left(z\right)=\frac{1}{\sigma}\left(1-\left(\frac{1}{\gamma^{B}}+\frac{1-\beta}{\gamma^{S}}\right)\right)r_{i}^{*}\left(z\right),\tag{14}$$

where $\frac{1}{\gamma^{B}}$ and $\frac{1-\beta}{\gamma^{S}}$ corresponds to the fraction of firms' variable profit that goes to buyer and supplier search efforts, respectively.

Finally, we describe how the unit cost of search, e_i, is determined. In order to engage in search activity, firms use both labor and retail goods. These search costs capture the broad notion of costs associated with finding and establishing supplier and buyer connections, including the cost of advertisement, sales promotion, product customization, quality assurance, and investment for relationship building. Assuming a Cobb-Douglas technology and by cost minimization, the unit cost of search effort is given by

$$e_i = (w_i)^{\mu} (P_i)^{1-\mu}, \tag{15}$$

where μ is the labor share in search activity.

2.2.2 Matching Technology and Aggregation

We now describe how the equilibrium matching rates between suppliers and buyers, m_{ud}^S and m_{ud}^B, are determined. Following the literature of labor search and matching (Diamond 1982; Mortensen 1986; Pissarides 1985), we assume that the aggregate measure of successful matches between a pair of locations, M_{ud}, is determined by the Cobb-Douglas matching technology, given by:

$$M_{ud} = \kappa_{ud} \left(\overline{M}_{ud}^B\right)^{\lambda^B} \left(\overline{M}_{ud}^S\right)^{\lambda^S}, \tag{16}$$

where λ^B, $\lambda^S \geq 0$ denote the elasticities of total matches created for the pair of regions with respect to the aggregate supplier and buyer search intensity, respectively;[4] κ_{ud} is the parameter governing the efficiency of matching technology that can flexibly depend on the location pairs; and $\overline{M}_{ud}^B = N_u \int_z n_{ud}^B(z) dG_u(z)$ and $\overline{M}_{ud}^S = N_d \int_z n_{ud}^S(z) dG_d(z)$ are aggregate buyer and supplier search intensity. Given M_{ud}, the matching rates m_{ud}^B and m_{ud}^S are determined as:

$$m_{ud}^B = \frac{M_{ud}}{\overline{M}_{ud}^B}, \qquad m_{ud}^S = \frac{M_{ud}}{\overline{M}_{ud}^S}. \tag{17}$$

The following lemma provides an analytical expression of the aggregate production networks and trade flows.

[4]Notice that λ^B represents the elasticity with respect to *buyer* search (by the suppliers), and λ^S represents the elasticity with respect to *supplier* search (by the buyers).

Lemma 2. *The measure of supplier-to-buyer relationships from supplier location u to buyer location d (extensive margin), M_{ud}, and the average transaction volume per relationship (intensive margin), $\overline{r}_{ud}$, are given by the following gravity equations:*

$$M_{ud} = \varrho^{E}\chi_{ud}^{E}\zeta_{u}^{E}\xi_{d}^{E}, \quad \overline{r}_{ud} = \varrho^{I}\chi_{ud}^{I}\zeta_{u}^{I}\xi_{d}^{I}, \tag{18}$$

with bilateral resistance shifters

$$\chi_{ud}^{E} = \left[\kappa_{ud}\left(f_{ud}^{B}\right)^{-\tilde{\lambda}^{B}}\left(f_{ud}^{S}\right)^{-\tilde{\lambda}^{S}}\left(\tau_{ud}^{1-\sigma}\right)^{\tilde{\lambda}^{B}+\tilde{\lambda}^{S}}\right]^{\frac{1}{1-\tilde{\lambda}^{S}-\tilde{\lambda}^{B}}}, \quad \chi_{ud}^{I} = \left(\tau_{ud}\right)^{1-\sigma},$$

where we define $\tilde{\lambda}^{S} \equiv \lambda^{S}/\gamma^{S}$, $\tilde{\lambda}^{B} \equiv \lambda^{B}/\gamma^{B}$; $\{\varrho^{E}, \varrho^{I}\}$ are constants invariant across locations, and the origin and destination shifters $\{\zeta_{u}^{E}, \xi_{d}^{E}, \zeta_{u}^{I}, \xi_{d}^{I}\}$ are given in Appendix A.2.

This lemma shows that aggregate trade flows follow a gravity structure, where the extensive and intensive margins have distinct geographic structures. The intensive margin is only affected by the iceberg trade cost, $\left(\tau_{ud}\right)^{1-\sigma}$, as the search and matching frictions do not affect trade flows once a link is formed. The extensive margin is, in addition, affected by the matching technology, κ_{ud}, and the bilateral search costs, f_{ud}^{B} and f_{ud}^{S}.

For later analysis, we define trade elasticity as the partial derivative of X_{ud} with respect to iceberg cost, τ_{ud}, holding factor and intermediate goods prices as given:

$$\varepsilon \equiv \frac{\sigma - 1}{1 - \tilde{\lambda}^{S} - \tilde{\lambda}^{B}}. \tag{19}$$

2.3 General Equilibrium

The general equilibrium is defined by the set of prices $\{p_{id}(\upsilon, \omega), P_i, w_i, e_i\}$ and quantities $\{q_{id}(\omega, \psi), q_{i}^{R}(\omega), Q_i, n_{id}^{B}(\omega), n_{ui}^{S}(\omega), l_i(\omega)\}$ with which (i) households maximize consumption given the budget constraint (7) with income from firm profit given by $\Pi_i = \int_z \tilde{\pi}_i(z)\, dG_i(\omega)$; (ii) firms make optimal pricing and production decisions for intermediate goods (3), (4) and search decisions (10); (ii) retailers make optimal optimal production decisions for retail goods (6); and (iv) intermediate goods, retail goods, and labor markets clear (see Appendix A.3 for precise market clearing conditions).

The following proposition shows that our model yields a tractable mathematical structure, one that resembles standard multi-location general equilibrium trade models.

Proposition 1. *The equilibrium satisfies the following properties:*
a) Equilibrium wages $\{w_i\}$ and cost shifters $\{C_i\}$ solve the following set of equations

$$w_i^{1+\tilde{\lambda}^B\mu\frac{\varepsilon}{\sigma-1}} C_i^{\varepsilon\left(1+\tilde{\lambda}^B\frac{1-\mu}{\sigma-1}\right)} = \frac{1}{L_i}\sum_d K_i^U K_d^D \chi_{id}^I \chi_{id}^E w_d^{\left(\frac{1-\beta\sigma}{1-\beta}-\tilde{\lambda}^S\mu\right)\frac{\varepsilon}{\sigma-1}} C_d^{\varepsilon\left(\frac{1}{1-\beta}-\tilde{\lambda}^S\frac{1-\mu}{\sigma-1}\right)}, \tag{20}$$

$$w_i^{1-\left(\frac{1-\beta\sigma}{1-\beta}-\tilde{\lambda}^S\mu\right)\frac{\varepsilon}{\sigma-1}} C_i^{-\varepsilon\left(\frac{1}{1-\beta}-\tilde{\lambda}^S\frac{1-\mu}{\sigma-1}\right)} = \frac{1}{L_i}\sum_u K_u^U K_i^D \chi_{ui}^I \chi_{ui}^E w_u^{-\tilde{\lambda}^B\mu\frac{\varepsilon}{\sigma-1}} C_u^{-\varepsilon\left(1+\tilde{\lambda}^B\frac{1-\mu}{\sigma-1}\right)}, \tag{21}$$

where $\{K_i^U, K_i^D\}$ are exogenous constants defined in Appendix A.4.
b) If $\frac{\beta(\sigma-1)}{1-\beta} \geq (1-\mu)\left(\tilde{\lambda}^B+\tilde{\lambda}^S\right)$ and $\left(\frac{1-\beta\sigma}{1-\beta}-\tilde{\lambda}^S\mu\right)\frac{\varepsilon}{\sigma-1} \leq 1$, the equilibrium exists and is unique up-to-scale.

Part (a) of this proposition shows that the key region-level economic variables, $\{w_i, C_i\}$, can be solved using only two sets of fixed-point equations. This proposition drastically simplifies the equilibrium conditions to two sets of equations. Therefore, we can apply existing techniques to establish positive properties of the equilibrium system, such as equilibrium uniqueness. Furthermore, as an immediate consequence of Proposition 1, we can apply the exact-hat algebra approach of Dekle, Eaton and Kortum (2008) to solve for counterfactual equilibrium changes given external shocks as long as we set the values of the structural parameters $\{\sigma, \beta, \mu, \tilde{\lambda}^B, \tilde{\lambda}^S\}$ and the observed aggregate trade flows, $\{X_{id}\}$. We use this approach for our quantitative analysis in Section 5, extended to a multi-sector environment.

A special case with $\lambda^S = \lambda^B = 0$ corresponds to a scenario where production networks are fixed. In our analysis below, we refer to this case as "fixed production networks" and contrast it to our baseline with endogenous production networks.[5]

[5]To eliminate spending for search, we also take the limit $\gamma^S, \gamma^B \to \infty$ when we implement the "fixed production network" case.

3 Planning Problem and Equilibrium Inefficiency

An important feature of our model is the search and matching externalities resulting from firm network formation decisions. To shed light on these externalities and their role in determining the aggregate welfare, we solve a planning problem. We consider a global planner that controls all locations and has a set of policy tools that differ across origin-destination pairs but are the same across firms within that pair.[6] First, the planner can introduce ad-valorem subsidies for intermediate goods sales specific to origin i and destination d, s^I_{id}. Under these subsidies, the intermediate goods prices change from Equation (4) to

$$p_{id}\left(\upsilon,\omega\right)=\left(1-s^I_{id}\right)\tilde{\sigma}c_i\left(\upsilon\right)\tau_{id}. \tag{22}$$

Second, the planner can introduce ad-valorem taxes for supplier and buyer search, t^S_{id} and t^B_{id}, for each pair of supplier and buyer locations. Therefore, total search costs by firms in location i is modified from Equation (8) as

$$f_i\left(\{n^B_{id}\}_d,\{n^S_{ui}\}_u\right)=e_i\left\{\sum_{d\in\mathcal{N}}\left(1+t^B_{id}\right)f^B_{id}\frac{\left(n^B_{id}\right)^{\gamma^B}}{\gamma^B}+\sum_{u\in\mathcal{N}}\left(1+t^S_{ui}\right)f^S_{ui}\frac{\left(n^S_{ui}\right)^{\gamma^S}}{\gamma^S}\right\}. \tag{23}$$

Finally, we introduce lump-sum transfers for households in location i, T^F_i, so that households' budget constraint is modified from Equation (7) to $P_iQ^F_i=w_i+\frac{\Pi_i}{L_i}+T^F_i$.

The planner chooses the optimal policy to maximize the global welfare

$$\max_{\left\{\left\{s^I_{id},t^B_{id},t^S_{id}\right\},T^F_i\right\}}\mathcal{W}\equiv\sum_i\psi_iL_iQ^F_i \tag{24}$$

subject to equilibrium constraints and the government budget constraint (see Appendix A.6 for the formal definition), where $\psi_i\geq 0$ corresponds to the welfare weights attached to the households in each location. The following proposition provides a simple formula for the optimal policy and illustrates how each set of taxes/subsidies corrects for each source of market failure.

[6]Specifically, we rule out firm-level search taxes within a location pair. With those taxes, the planner can address additional externalities arising from the firm heterogeneity in the matching market, where lower productivity firms create congestion externality to more productive firms (Acemoglu, 2001; Bilal, 2023). We do not focus on this inefficiency as it does not interact with aggregate trade cost shocks in Section 4.

Proposition 2. *[Optimal Taxes and Subsidies] For any weakly positive welfare weights* $\{\psi_i\}$*, the optimal set of taxes and subsidies* $\{s_{id}^I, t_{id}^B, t_{id}^S\}$ *must satisfy*

$$s_{id}^I = \frac{1}{\sigma}, \qquad t_{id}^B = \frac{1}{\lambda^B} - 1, \qquad t_{id}^S = \frac{1}{\lambda^S}(1-\beta)\frac{E_d}{R_d} - 1, \tag{25}$$

for all i, d*, where* $R_d \equiv \sum_\ell X_{d\ell}$ *and* $E_d \equiv \sum_\ell X_{\ell d}$*.*

The proposition illustrates how each set of taxes/subsidies corrects for each source of market failure. First, the intermediate goods subsidy s_{id}^I is set at a constant rate across location pairs to exactly offset the markups. These subsidies address the double marginalization distortions that commonly arise in a model with input-output linkages and imperfect competition (e.g., Baqaee and Farhi, 2020).

Second, buyer search taxes, t_{id}^B , target the search and matching externality by balancing two potential inefficiencies: the non-appropriability effect and the business stealing effects. The non-appropriability effects arise because suppliers capture only a fraction of social surplus as revenue. Business stealing effects arise because an additional connection leads to the loss of profit of other suppliers. When $\lambda^B = 1$, there is no externality in the matching rates (Equation 16), and the business stealing effect only arises through input substitution by buyers given connections. Serendipitously, the two effects exactly cancel each other, similarly to the entry models with CES demand (Dixit and Stiglitz, 1977; Mankiw and Whinston, 1986). With $\lambda^B < 1$, instead, optimal buyer search taxes are positive in order to discourage search and offset the congestion externality.

The optimal taxes for supplier search, t_{id}^S, similarly balances the non-appropriability effect and the business stealing effects. However, the expression is slightly different from t_{id}^B. This difference stems from the fact that, while firms' *buyer* search incentives increase proportionally with revenue, firms' *supplier* search incentives proportionally increase in intermediate input expenditure, and $(1-\beta)\,E_d/R_d$ capture this ratio.

Proposition 2 can be rewritten in the form of a necessary condition for equilibrium efficiency in the tradition of labor search and matching literature.

Corollary 1. *[Constrained Efficiency] Suppose the subsidy for intermediate goods sales is set at the optimal level,* $s_{id}^I = 1/\sigma$*. Then, there exists welfare weights* $\{\psi_i\}$ *that maximize* *the global welfare with $t^B_{id} = t^S_{id} = 0$ for all i, d if and only if*

$$\lambda^B = 1, \qquad \lambda^S = 1 - \beta. \tag{26}$$

Furthermore, the supporting welfare weights ψ_i are proportional to equilibrium retail prices P_i.

This condition resonates Hosios (1990), which establishes a necessary condition of the efficiency of two-sided search and matching models between firms and workers. He shows that, when this knife-edge condition is satisfied, the equilibrium is (constrained) efficient since the hold-up problem that arises due to Nash bargaining – because firms cannot exploit the entire surplus from the match – exactly cancels out the congestion externalities. The intuition behind Corollary 1 is similar, with the non-appropriability effects taking the place of the hold-up problem, and the congestion externality arising from search and matching and input substitution from other connected suppliers.[7]

4 Welfare Effects of Trade Shocks

We now turn to the analysis of the impacts of shocks on aggregate welfare. For expositional purposes, we focus on a shock in iceberg trade costs $\{d \ln \tau_{ij}\}$, while it is straightforward to extend our analysis to other shocks on technology, such as those on search costs or firm productivity. Unless explicitly stated, we focus on the laissez-faire equilibrium without taxes and transfers.

4.1 Ex-Ante Sufficient Statistics on Global Welfare

We first analyze the first-order effect of shocks on global welfare. Let us define the global welfare according to Equation (24). To isolate our focus from the redistribution effects, we set welfare weights ψ_i equal to each region's retail goods price, P_i. The following proposition provides a sharp characterization of the shock's first-order effect.

[7]Interestingly, constrained efficiency requires the increasing returns to scale (IRS) matching function instead of the constant returns to scale (CRS) in the Hosios' environment. This difference reflects the presence of additional business stealing effects through input substitution given links. See Miyauchi (2024) and Eaton et al. (2024) for empirical evidence that matching function in firm-to-firm trade exhibits IRS.

Proposition 3. *[Ex-ante sufficient statistics] The first-order effect of a shock in iceberg trade costs $\{d \ln \tau_{ij}\}$ on global welfare with welfare weights $\psi_i = P_i$ is given by:*

$$d \ln \mathcal{W} = -\frac{\varsigma}{\frac{\beta}{1-\beta} - \frac{1-\mu}{\sigma-1}\left(\tilde{\lambda}^B + \tilde{\lambda}^S\right)} \sum_{u,d} X_{ud} d \ln \tau_{ud}, \tag{27}$$

where $\varsigma \equiv \left(\sum_i w_i L_i + \sum_i \Pi_i\right) / \sum_{u,i} X_{ui}$ is the ratio of nominal world GDP to nominal world intermediate goods expenditure by firms, given by:

$$\varsigma = \frac{\beta}{1-\beta} + \frac{1}{\sigma} - \frac{1-\mu}{\sigma}\left(\frac{1}{\gamma^B} + \frac{1-\beta}{\gamma^S}\right). \tag{28}$$

This proposition shows that, up to first order, global welfare changes are proportional to nominal trade flows, scaled by a specific coefficient. Notice that the expression depends only on the baseline trade flows and a set of structural parameters. In particular, it does not require solving for a counterfactual equilibrium, thereby providing a powerful *ex-ante* sufficient statistic. The observation that changes in global welfare are proportional to nominal trade flows relates to the analysis of Atkeson and Burstein (2010) and Baqaee and Farhi (2024) with fixed production networks, who in turn build on Hulten (1978) for a closed economy. However, because of the presence of the equilibrium inefficiency discussed in Section 3, the coefficient of proportionality may not be equal to one.

We build intuition behind Proposition 3 focusing on several special cases and alternative versions. First, consider the case with fixed production networks ($\lambda^S = \lambda^B = 0$) and firms do not use any resources for search activity ($\gamma^S, \gamma^B \to 0$) . In this case, the only equilibrium inefficiency is the double marginalization of intermediate inputs discussed in the previous section. Therefore, Proposition 3 comes down to

$$d \ln \mathcal{W}^{\text{Fixed}} = -\frac{\varsigma^{\text{Fixed}}}{\frac{\beta}{1-\beta}} \sum_{u,d} X_{ud} d \ln \tau_{ud}, \qquad \varsigma^{\text{Fixed}} = \frac{\beta}{1-\beta} + \frac{1}{\sigma}. \tag{29}$$

In the denominator of the coefficient, $\beta/(1-\beta)$ is the standard input-output multiplier, which captures the propagation of production cost shocks toward downstream firms. The numerator ς^{Fixed} is the GDP-to-input-expenditure ratio. The coefficient $\varsigma^{\text{Fixed}} / \frac{\beta}{1-\beta}$ is greater than one, which reflects the fact that the equilibrium intermediate goods expenditure is inefficiently small due to double marginalization.

Next, consider the case where we allow for endogenous networks, while the optimal sales subsidy $s^I_{id} = 1/\sigma$ is in place to eliminate the inefficiency from double marginalization. Then, Proposition 3 becomes (see Appendix A.7)

$$d \ln \mathcal{W}^{\text{SalesSubsidy}} = -\frac{\frac{\beta}{1-\beta} - \frac{1-\mu}{\sigma-1}\left(\frac{1}{\gamma^B} + \frac{1-\beta}{\gamma^S}\right)}{\frac{\beta}{1-\beta} - \frac{1-\mu}{\sigma-1}\left(\tilde{\lambda}^B + \tilde{\lambda}^S\right)} \sum_{u,d} X_{ud} d \ln \tau_{ud}. \tag{30}$$

Under a fixed network environment ($\lambda^S = \lambda^B = 0$; $\gamma^S, \gamma^B \to 0$), the coefficient in Equation (30) is one, as the double marginalization is the only source of inefficiency. With endogenous networks, search and matching externality influences the coefficient through both its denominator and the numerator.

In the denominator, the additional term $\frac{1-\mu}{\sigma-1}\left(\tilde{\lambda}^B + \tilde{\lambda}^S\right)$ reflects the multiplier effects through search costs.[8] In particular, negative iceberg trade cost shocks decrease search costs depending on the intensity of retail goods used for search, $1-\mu$. This effect increases buyer and supplier search effort with elasticity, $1/\gamma^B$ and $1/\gamma^S$, respectively. In turn, these responses increase the number of equilibrium links with elasticity, λ^B and λ^S, through the matching technology. Finally, the increase in production linkages reduces the production costs through the love-of-variety effect with elasticity, $1/(\sigma - 1)$. In the numerator, the additional term $\frac{1-\mu}{\sigma-1}\left(\frac{1}{\gamma^B} + \frac{1-\beta}{\gamma^S}\right)$ corresponds to the aggregate spending for retail goods for firms' search activity as a share of aggregate firm revenue, if optimal sales subsidy $s^I_{id} = 1/\sigma$ is implemented.

The role of inefficiency is clear. To see this, if we set $\lambda^B = 1$ and $\lambda^S = 1 - \beta$ such that the equilibrium is constrained efficient with sales subsidy $s^I_{id} = 1/\sigma$ (Corollary 1), the coefficient in Equation (30) becomes one. However, interestingly, the converse is not true. One such case arises when $\mu = 1$, i.e., labor is the only resource used for search activity. In this case, the coefficient in Equation (30) becomes one (because there is no distortion in the intermediate goods market other than double marginalization), even though labor is misallocated between production and search activity (and hence equilibrium is constrained inefficient). Hence, there is no deviation from Hulten's characterization. Another case

[8] See Buera, Hopenhayn, Shin and Trachter (2023) for a similar multiplier effect in a growth model, where a part of fixed cost for technology adoption originates from produced goods.

arises when $\tilde{\lambda}^B + \tilde{\lambda}^S = \frac{1}{\gamma^B} + \frac{1-\beta}{\gamma^S}$. In this case, even though resources for search activity is misallocated between supplier and buyer search, its total use is not, and hence aggregate trade flows are not distorted.[9]

Finally, we come back to the laissez-faire equilibrium ($s^I_{id} = 0$) as considered in Proposition 3 and discuss the differences between the fixed and endogenous networks. With fixed networks, the welfare effect is influenced by double marginalization, and with endogenous networks, it is additionally influenced by the search and matching inefficiency. These two inefficiencies interact: Due to double marginalization, firm revenue is too low, which implies that the private incentive for search is also too low relative to the social optimum. Therefore, even under $\lambda^B = 1$ and $\lambda^S = 1 - \beta$, the coefficient on Equation (27) is larger with endogenous than fixed network environment.

4.2 Ex-Post Sufficient Statistics on Each Location's Welfare

We now turn to the welfare of each region. There is no convenient *ex-ante* sufficient statistics expression for each region's welfare, unlike the global welfare with price weights as in the previous section. However, the following proposition presents a simple *ex-post* sufficient statistics for each location's welfare.

Proposition 4. *[Ex-post sufficient statistics] For any magnitude of trade cost shocks that satisfies $d \ln \tau_{ii} = 0$ for location i, changes in location i's real GDP is given by:*

$$d \ln Q^F_i = -\frac{1}{\varepsilon} \frac{1}{\frac{\beta}{1-\beta} - \frac{1-\mu}{\sigma-1}\left(\tilde{\lambda}^S + \tilde{\lambda}^B\right)} d \ln \Lambda_{ii}, \tag{31}$$

where ε is the trade elasticity defined by Equation (19), and $\Lambda_{ii} = X_{ii}/\sum_u X_{ui}$ is the aggregate share of intermediate inputs by firms in location i sourced from location i.

Therefore, for any magnitude of the shocks (notice that the coefficient in front of $d \ln \Lambda_{ii}$ is constant), welfare changes are solely summarized by the changes in intra-region

[9]Due to the Cobb-Douglas production and search and matching technology, aggregate trade cost shocks do not induce any reallocation of labor between search and production activity. Therefore, there are no changes in "allocative efficiency" in our framework (as highlighted for example by Baqaee and Farhi, 2020), and the only deviation from Hulten's characterization arises due to the distortion in pre-shock trade flows.

expenditure share. This result resonates with the familiar *ex-post* welfare sufficient statistics for welfare gains from trade with fixed production networks (Arkolakis et al. 2012; Blaum et al. 2018).

Notice that under the fixed network specification, this expression comes down to

$$d\ln Q_i^{F,\text{Fixed}} = \frac{1}{\sigma-1}\frac{1}{\frac{\beta}{1-\beta}} d\ln \Lambda_{ii}^{\text{Fixed}}, \tag{32}$$

where $\sigma - 1$ corresponds to the trade elasticity with the fixed network environment, and $(1-\beta)/\beta$ is the input-output multiplier. Comparing Equations (31) and (32) reveals two key differences between the endogenous and fixed network environment. First, an additional amplification term, $\frac{1-\mu}{\sigma-1}\left(\tilde{\lambda}^S + \tilde{\lambda}^B\right)$, appears in the endogenous network case. This term coincides with the one that appears in the denominator of Proposition 3, and it reflects the multiplier effect of trade shocks through search costs. Second, trade elasticity is higher with endogenous networks than fixed networks, i.e., $\varepsilon > \sigma - 1$. This is because the adjustment of supplier and buyer networks act as an additional substitution margin. This force decreases the welfare changes conditional on the same change in intra-region expenditure share.

Proposition 4 is inherently related to Proposition 3. To see the connection, we can rewrite Equation (31) as

$$d\ln Q_i^F = \frac{\varsigma}{\frac{\beta}{1-\beta} - \frac{1-\mu}{\sigma-1}\left(\tilde{\lambda}^S + \tilde{\lambda}^B\right)} \times \frac{1}{\varsigma}\left(-\frac{1}{\varepsilon} d\ln \Lambda_{ii}\right), \tag{33}$$

where the first term in the right-hand-side is the same coefficient as in Proposition 3. Notice that $-\frac{1}{\varepsilon} d\ln \Lambda_{ii}$ is the changes in terms-of-trade ($d\ln P_i - \sum_u \Lambda_{ui} d\ln P_u$) and $1/\varsigma$ is the Domar weight of intermediate goods sector. The term, $\frac{1}{\varsigma}\left(-\frac{1}{\varepsilon} d\ln \Lambda_{ii}\right)$ is analogous to Hulten's expression from terms-of-trade shocks to location i. The constant in front of this term reflects the equilibrium inefficiency, and is identical to the coefficient in Proposition 3.

4.3 Isomorphisms with Other Network Formation Models

So far, we have derived our theoretical results under a particular microfoundation for production network formation through search and matching. In this section, we argue that these results extend to a broader class of production network formation models. To do so, we follow the approach of Arkolakis et al. (2012) and derive common macro restrictions that lead to the same theoretical results as Propositions 3 and 4. Here we develop our main argument, and delegate the formal results in Appendix B.

Suppose that the economy satisfies the same three macro restrictions considered by Arkolakis et al. (2012) (i.e., trade balance, constant profit and labor share to intermediate goods sales, and constant aggregate trade elasticity).[10] We introduce two additional macro restrictions pertaining to endogenous production network formation. First, in response to trade cost shocks $\{d\ln\tau_{ui}\}$, the changes in aggregate bilateral linkages follow

$$\begin{aligned} d\ln M_{ud} = &\left(\delta_{L,U}+\delta_{Q,U}+\delta_{L,D}+\delta_{Q,D}\right) d\ln X_{ud} \\ &- \delta_{L,U} d\ln w_u - \delta_{Q,U} d\ln P_u - \delta_{L,D} d\ln w_d - \delta_{Q,D} d\ln P_d, \end{aligned} \tag{34}$$

where $\delta_{L,U}$, $\delta_{Q,U}$, $\delta_{L,D}$, $\delta_{Q,D}$ are constant parameters capturing the reliance of link formation in labor and retail goods in upstream and downstream locations. In our baseline model, iso-elastic search decisions (8) and Cobb-Douglas matching technology (16) jointly imply that $\delta_{L,U}=\mu\tilde{\lambda}^B$, $\delta_{Q,U}=(1-\mu)\,\tilde{\lambda}^B$, $\delta_{L,D}=\mu\tilde{\lambda}^S$, and $\delta_{Q,D}=(1-\mu)\,\tilde{\lambda}^S$. Alternatively, M_{ui} can be determined by other mechanisms, such as relationship-specific fixed cost or discrete supplier choices, as we further discuss below.

Second, the changes in retail goods prices are given by

$$d\ln P_i = \beta d\ln w_i + (1-\beta)\sum_u \Lambda_{ui}\left(d\ln P_u + d\ln\tau_{ui} - \nu d\ln M_{ui}\right). \tag{35}$$

Equation (35) is a version of Shephard's lemma, relating the retail price index of location i to the average input costs weighted by the expenditure share. ν is a constant parameter capturing the elasticity of input bundle price with respect to supplier linkages. In our baseline model, $\nu = 1/(\sigma-1)$ captures the love-of-variety in intermediate inputs. Al-

[10] If there are no intermediate goods, as originally considered by Arkolakis et al. (2012), the second condition comes down to the constant profit to GDP ratio.

ternatively, ν can also be affected by the selection of suppliers to serve the buyer (under relationship-specific fixed cost) or buyers' selection of suppliers (under a discrete choice of suppliers), as we further discuss below.

Under these conditions, Proposition 3 generalizes to

$$d \ln \mathcal{W} = \frac{\varsigma}{\frac{\beta}{1-\beta} - \nu \left(\delta_{Q,U} + \delta_{Q,D}\right)} \sum_{u,d} X_{ud} d \ln \tau_{ud}, \tag{36}$$

where ς is the GDP-to-intermediate-goods-expenditure ratio, restricted to be constant as a part of our macro restrictions. Furthermore, Proposition 4 generalizes to

$$d \ln Q_i^F = -\frac{1}{\varepsilon} \frac{1}{\frac{\beta}{1-\beta} - \nu \left(\delta_{Q,U} + \delta_{Q,D}\right)} d \ln \Lambda_{ii}. \tag{37}$$

The intuition behind Equations (36) and (37) is similar to those of Propositions 3 and 4. $\nu \left(\delta_{Q,U} + \delta_{Q,D}\right)$ in the denominators captures the multiplier effects through retail goods costs used for production network formation. The deviation of Equation (36) from Hulten (1978) arises because of the potential equilibrium inefficiency in the size of the intermediate goods sector, ς.

It is easy to verify that our model in Section 2 satisfies our macro restrictions. In Appendix B, we also argue that they hold in an alternative specification where either the supplier or buyer search is undirected toward a specific location. In those cases, the only difference from our main specification is the differences in the trade elasticity, which is smaller as it only depends on the search and matching elasticities of only one side.

In the same appendix, we discuss two alternative microfoundations of production network formation that give rise to these macro restrictions. The first one is with models based on relationship-specific fixed costs incurred by suppliers. In these models, heterogeneous monopolistically competitive suppliers pay a relationship-specific fixed cost to form a relationship with buyers. Similar to our framework, this model exhibits the non-appropriability and business stealing effects of relationship formation. To establish the isomorphism, we extend the two-sided partial equilibrium model of Bernard, Moxnes and Ulltveit-Moe (2018) to a multi-location general equilibrium environment with production networks and roundabout inputs and show that this model satisfies the proposed macro restrictions if the productivity distribution follows a power law.[11] In particular, trade elasticity is given by $\varepsilon = \gamma_C \theta$, where θ is the shape parameter of firms' productivity distribution, and $\gamma_C > 1$ is a composite parameter that translates the productivity distribution to the marginal cost distribution. The elasticity of input bundle price with respect to supplier linkages is given by $\nu = 1/(\sigma - 1) - 1/(\gamma_C \theta)$, which is affected by the love-of-variety in inputs, $1/(\sigma - 1)$, and the supplier selection, $-1/(\gamma_C \theta)$.

The second alternative microfoundation is with models based on discrete supplier choice. This approach is taken by Oberfield (2018) and Acemoglu and Azar (2020) without geographic dimensions, and Antràs and De Gortari (2020) and Eaton et al. (2024) with geographic dimensions. In these models, competitive firms produce homogeneous intermediate goods and the buyer chooses the least cost supplier. We show that a version of these models satisfies the proposed macro restrictions if the productivity distribution follows a power law. In particular, the aggregate trade elasticity coincides with the productivity shape parameter, i.e., $\varepsilon = \theta$, similarly to Ricardian trade models (Eaton and Kortum, 2002). Furthermore, firms do not incur any resource costs for network formation and simply choose the optimal suppliers under perfect competition and constant returns to scale, i.e., $\delta_{L,U} = \delta_{Q,U} = \delta_{L,D} = \delta_{Q,D} = 0$. Therefore, $\nu (\delta_{Q,U} + \delta_{Q,D}) = 0$; and the coefficient of Equation (36) becomes one (notice that $\varsigma = \beta / (1 - \beta)$ due to perfect competition); and Equation (37) reflects the formula derived by Arkolakis et al. (2012) with the standard input-output multiplier ($\beta / (1 - \beta)$).

4.4 Extension to Multiple Sectors

So far, we have focused on the environment where the only source of firm heterogeneity is the TFP. However, in reality, firms may produce different types of intermediate goods using different production technologies and may face different degrees of search and matching frictions. To accommodate such heterogeneity, we extend our model to incorporate multiple sectors $k, m \in K$ connected through input-output linkages following the specification of Caliendo and Parro (2015), as detailed in Appendix C. In this extended environment, production technology takes a Cobb-Douglas form with labor and

[11] Lim (2018); Huneeus (2018); Bernard et al. (2022); Dhyne et al. (2023) consider this microfoundation without multi-location dimension.

intermediate inputs with expenditure shares, $\{\beta_{m,L}, \{\beta_{km}\}_k\}$, and intermediate inputs aggregate with elasticity of substitution, σ_k. Final consumption is Cobb-Douglas with sectoral share, α_k. Iceberg costs depends on location-and-sector pairs, $\tau_{ij,km}$. Matching occurs for each location-and-sector pairs with search cost elasticities, $\{\gamma_k^B, \gamma_k^S\}$, matching elasticities, λ_{km}, and the labor share in search costs, μ_k.

We show in Appendix C that this version of the model predicts gravity equations of trade flows for each sector pair. Consequently, we can use the exact-hat algebra approach to undertake counterfactual simulations. We also extend the optimal policy formula in Section 3 and the *ex-post* sufficient statistics in Section 4.2, where the Leontief inverse of the input-output matrix, adjusted by the search cost multiplier, become the key statistics. Due to the potential misallocation across sectors, there is no convenient *ex-ante* sufficient statistics as in Section 4.1, and obtaining the welfare effects from an arbitrary shock generally requires solving the full equilibrium system.

5 Data and Reduced-Form Evidence

In this section, we describe our data from Chile we use to quantify our theoretical predictions. We also use this data to provide reduced-form evidence that production networks reorganize in response to tariff changes from Chile's recent trade agreements.

5.1 Data Sources

Our key data source is a firm-to-firm transaction-level data set that covers the universe of domestic and international trade by Chilean firms. For domestic firm-to-firm transaction data, we draw on the electronic receipts reported to the fiscal authority for the purpose of value-added tax (VAT) collections. Since 2018, all corporate entities in Chile are mandated to submit electronic receipts of all the transactions that occur across firms to the Chilean Internal Revenue Service, SII (for its acronym in Spanish).[12] Each receipt includes information on the supplier's and buyer's unique tax-ID, transaction dates and values, and the municipalities of the establishments where the transaction occurs. For our model calibration in Section 6, we use data from 2019. For our reduced-form analysis of bilateral

[12] Informal firms in Chile, which do not appear in our data sets, represent only 2% of Chile's GDP.

trade agreements in Section 5.2, we use data from 2003 to 2007, which also come from the VAT records, but aggregated at the level of supplier's and buyer's unique tax-ID at the biannual frequency.[13]

For international firm-to-firm transaction data, we draw on customs data. This data set reports the export and import activity of each tax-ID, with the information about the products traded, country of origin or destination, transaction value. Importantly, the data also reports the identity of the foreign firm involved in the transaction, which allows us to construct the firm-to-firm international transaction information.

We merge these two data sets using the unique tax-ID that is common across sources. We also merge these data sets with balance sheet information (SII tax form 29) and labor information (SII tax form 1887). We drop tax-IDs that report no value-added or employment and samples that report negative values of value-added, sales, or material inputs. [14] We also use the Inter-Country Input-Output (ICIO) sectoral tables from the Organization for Economic Cooperation and Development (OECD) to capture the international trade surrounding Chile for our model calibration.

5.2 Reduced-Form Evidence for Endogenous Production Networks

Before proceeding with the model quantification, we provide causal evidence that production networks endogenously reorganize in response to time-varying trade shocks. Specifically, we study the impacts of Chile's bilateral trade agreements with the United States

[13]For the data in 2003 to 2007, only firms that have total expenditures on intermediates in a given year above US$390000 have to report this information, which account for around 80 percent of value added in the Chilean economy. See Huneeus (2018) for further details.

[14]To secure privacy, the Central Bank of Chile (CBC) mandates that the development, extraction, and publication of the results should not allow the identification, directly or indirectly, of natural or legal persons. Officials of the CBC processed the disaggregated data and merged them across sources. The authors implemented all the analysis and did neither involve nor compromise the CBC nor the Chilean tax authority. This study was developed within the scope of the research agenda conducted by the CBC in economic and financial affairs of its competence. The CBC has access to anonymized information from various public and private entities, by collaboration agreements signed with these institutions. The information contained in the databases of the Chilean tax authority is of a tax nature originating in self-declarations of taxpayers presented to the authority; therefore, the veracity of the data is not its responsibility.

(US) and China. Chile signed a Preferential Trade Agreement (PTA) with the US in 2004 and a Free Trade Agreement (FTA) with China in 2006. Both of these agreements reduced the average tariffs from 6.9 percentage points to nearly zero percentage points, depending on the products traded (see Appendix E.2 for details). Previous studies have analyzed how these episodes affected Chile's aggregate product-level international imports (Fontagné, Guimbard and Orefice, 2022). Here, we instead focus on how these trade shocks have affected the architecture of international and domestic production networks.

A key challenge in identifying the impacts of these trade policy changes is to isolate them from general macroeconomic trends. A simple comparison of Chile's overall production network architecture before and after these trade agreements does not allow us to identify the impacts of trade policy shocks. Therefore, we adopt the differences-in-differences design using firm-level exposure on the import tariff changes as an additional cross-sectional variation (e.g., Goldberg, Khandelwal, Pavcnik and Topalova, 2010). Specifically, we estimate the following firm-level regression specification:

$$\Delta \log y_\omega = \alpha \text{ImportTariffShock}_\omega + \zeta_{h(\omega)} + \beta' X_\omega + \epsilon_\omega, \tag{38}$$

where ω is the firm, Δ indicates that we take differences of variables between 2003 (pre-agreements) and 2007 (post-agreements), y_ω is the outcome variable of firm ω (e.g., number of import and domestic production linkages), $\text{ImportTariffShock}_\omega$ is the proxy for firm-level import tariff shocks as we further discuss below, $\zeta_{h(\omega)}$ is the 6-digit sector fixed effect for firm ω's sector $h(\omega)$, X_ω is a vector of firm-level control variables, including the shares of imports in firms' total material inputs and a proxy for the firm-level export tariff shocks (import tariffs charged by the counterparty countries).[15]

We use two different proxies for firm-level import tariff shocks depending on the outcome variables. First, when we study the impacts on the international import linkages, we use the following proxy:

[15] We control for pre-period import share in firms' total material purchases to deal with the concern that firms with a higher import penetration may have differential trends in outcome variables (Borusyak, Hull and Jaravel, 2022). We focus on import tariff changes, instead of export tariff changes, because of the significantly fewer number of firms that engage in exports than imports in Chile.

$$\text{ImportTariffShock}_{\omega} \equiv \sum_{n}\sum_{g} \frac{\text{Import}_{\omega ng,t_0}}{\text{ImportSum}_{\omega,t_0}} \times \Delta \log(1+\mathcal{T}_{ng}), \tag{39}$$

where n is the origin country, g is the HS-6 product, $\text{Import}_{\omega ng,t_0}$ is the value of imports by firm ω from country n and product g in the baseline year $t_0 = 2003$, $\text{ImportSum}_{\omega,t_0} \equiv \sum_n \sum_g \text{Import}_{\omega ng,t_0}$ is firm ω's total import values in the baseline year, $\mathcal{T}_{ng}$ is the rate of applied import tariff for product g from country n. Intuitively, this proxy captures the weighted average of import tariff changes in the basket of imported goods by firm ω, where the weights are based on the import shares in the pre-agreement period.

When we study the impacts on the domestic production linkages (i.e., the number of domestic suppliers and buyers), we use the following proxy:

$$\text{ImportTariffShock}_{\omega} \equiv \sum_{n}\sum_{g} \frac{\text{Import}_{\omega ng,t_0}}{\text{ImportSum}_{\omega,t_0} + \text{DomPurchase}_{\omega,t_0}} \times \Delta \log(1+\mathcal{T}_{ng}), \tag{40}$$

where the difference from Equation (39) arises from the inclusion of total domestic material purchases by firm ω in the baseline period, $\text{DomPurchase}_{\omega,t_0}$, in the denominator of the weight. Intuitively, this proxy captures the weighted average of import tariff changes in the basket of *all purchased materials* by firm ω. We take this definition for the domestic network outcome variables because, even if firms face large import tariff reduction within their import portfolio (hence Equation 39 takes a large negative value), it may not affect firms' behavior regarding domestic production networks if overall imports are a relatively small share of their intermediate inputs.

Table 1 presents the estimation results of Equation (38). Columns (1) and (2) present the impacts on international production linkages of import tariff shocks defined by Equation (39). Column (1) shows that a one percentage point increase in international import tariffs is associated with a 3.2 percent reduction in the value of international imports. Column (2) shows that more than half of this response is driven by the extensive margin, i.e., the increase in the number of foreign suppliers by Chilean firms. Therefore, the reorganization of international supplier linkages substantially contributes to the total import responses, consistent with our theoretical model.

Columns (3) and (4) present the impacts of import tariff shocks on domestic production linkages defined by Equation (40). We find the coefficient of -1.52 for the number of domestic suppliers and -1.30 for the number of domestic buyers. Therefore, the reduction in import tariffs not only increased the international supply linkages but also increased the domestic supplier and buyer linkages within Chile.

The positive effects on the number of domestic suppliers are notable, as it implies that the import tariff reduction complemented the formation of domestic supplier linkages, instead of substituting them. In a recent literature, researchers have studied how the temporary international trade shocks affect domestic sales (Dhyne, Kikkawa, Mogstad and Tintelnot 2021; Dhyne, Kikkawa, Komatsu, Mogstad and Tintelnot 2024) and the formation of production linkages (Demir et al. 2024a; Huneeus 2018), following the design of Autor, Dorn and Hanson (2013). Interestingly, this line of research has not found significant impacts on the number of domestic supplier linkages from international demand shocks (Demir et al., 2024a) and from supply shocks (Huneeus, 2018). These differences potentially stem from the permanent feature of the import tariff reduction that we study. We show below that our model can successfully replicate these domestic production network responses through endogenous search decisions, and that these responses crucially determine the aggregate welfare effects of trade shocks.

6 Quantitative Analysis

In this final section, we explore the quantitative implications of endogenous production network formation for the effects of trade cost shocks on aggregate welfare.

6.1 Calibration

We calibrate our multi-sector model in Section 4.4 to the Chilean economy for 2019. We define locations in our model as 345 within Chile and three international locations: United States, China, and the Rest of the World. To avoid the sparseness of the sector-region trade flows, we broadly divide sectors into "goods" and "services" sectors, where "goods" sector includes agriculture and fishing, mining and quarrying, and manufacturing, and "services" sector includes all other sectors.

To undertake counterfactuals, we need to calibrate the baseline trade flows across locations and sectors $\{X_{ud,hk}\}$ and a subset of structural parameters $\{\alpha_k$, $\beta_{k,L}$, β_{hk}, γ_k^B, γ_k^S,

Table 1: Impact of Import Tariff Shocks on International and Domestic Production Links

	Total Imports (1)	Number Int. Suppliers (2)	Number Dom. Suppliers (3)	Number Dom. Buyers (4)
Import Tariff Shock	-3.20	-1.88	-1.52	-1.30
	(1.26)	(0.61)	(0.71)	(1.10)
Number Observations	33260			
Sector FE (6 digit)	Yes			
Prior Import Share	Yes			
Export Shock Residualized	Yes			
Period	2003-2007			

Notes: This table reports the estimates of regression equation (38). Import shocks are defined by Equation (39) for Columns (1)-(2) and by Equation (40) for Columns (3)-(4). All outcome variables are log changes between 2003 (pre-agreements) and 2007 (post-agreements). The samples include all Chilean firms that exist in both 2003 and 2007. Export shocks are constructed similarly to import shocks and controlled for. Standard errors are computed following Borusyak et al. (2022). Appendix E.2 presents summary statistics with the magnitude of the tariff changes. Appendix E.3 shows a set of standard tests to assess the validity of the shift-share design, including placebo regressions as suggested by Borusyak et al. (2022).

$\varepsilon_k, \lambda_{kl}^B, \lambda_{kl}^S, \mu_k, \sigma_k\}$. We construct baseline trade flows $\{X_{ud,hk}\}$ using various data sources described in Section 5. For trade between municipalities within Chile, we aggregate our domestic firm-to-firm trade data, and for trade between Chilean municipalities and international countries, we aggregate our customs data. For trade across international countries that do not involve Chile, we obtain the values using the Inter-Country Input-Output (ICIO) table. The trade flows constructed in this way may not satisfy our model's equilibrium conditions. To enable well-defined counterfactuals, we adjust the trade flows so that they are consistent with the equilibrium conditions by interpreting that the observed trade flows involve measurement errors (see Appendix F.1 for details).

We now turn to the calibration of structural parameters, summarized in Table 2. We calibrate the final expenditure shares, $\{\alpha_k\}$, labor coefficient in production, $\{\beta_{k,L}\}$, and intermediate input coefficient in production, $\{\beta_{hk}\}$, by aggregating our domestic firm-to-firm trade data and firm-level labor compensation at the sectoral level across all firms and municipalities, analogously to Caliendo and Parro (2015).

Table 2: Calibrated Parameters

Parameters	Goods	Services	Description
α_k	0.25	0.75	Final consumption share
$\{\beta_{k,L}\}$	0.14	0.33	Labor coefficient in production
$\{\beta_{lk}\}$			Intermediate input coefficient in production
l: Goods Sector	0.45	0.10	
l: Services Sector	0.41	0.57	
γ_k^S	2.6	2.7	Search cost curvature w.r.t. suppliers
γ_k^B	4.5	2.8	Search cost curvature w.r.t. buyers
ε_k	5.0	4.6	Trade elasticity
μ_k	0.05	0.05	Labor coefficient in search
$\lambda_{kl}^S = \lambda_{kl}^B \ (\forall k, l)$	0.64	0.64	Matching function elasticity
σ_k	4.1	3.5	Elasticity of substitution

To calibrate the search cost elasticities, $\{\gamma_k^B, \gamma_k^S\}$, we use the model-predicted log-linear relationship between the number of suppliers and buyers and aggregate sales at the firm-level, given by

$$\log \sum_{d \in \tilde{\mathcal{N}}} \sum_{l \in K} n_{id,kl}^{\mathcal{X}}(z) = \frac{1}{\gamma_k^{\mathcal{X}}} \log r_{i,k}(z) + \phi_{i,k}^{\mathcal{X}}, \qquad \mathcal{X} \in \{S, B\} \tag{41}$$

for any subset of locations $\tilde{\mathcal{N}} \subset \mathcal{N}$, where $\{\phi_{i,k}^B, \phi_{i,k}^S\}$ is a composite variable that depend on location and sector but not by firm productivity z.[16] Specifically, we estimate $\{1/\gamma_k^B, 1/\gamma_k^S\}$ from the regression of the log number of domestic suppliers and buyers within Chile (by taking $\tilde{\mathcal{N}}$ as all municipalities within Chile) on the log of the aggregate intermediate goods sales, conditional on the location and sector fixed effects (capturing $\{\phi_{i,k}^B, \phi_{i,k}^S\}$). We find the values of $\{\gamma_k^B, \gamma_k^S\}$ ranging from 2.6 to 4.5, which satisfy our equilibrium assumptions $\gamma_k^B > 1$ and $\gamma_k^S > 1$.

We calibrate the trade elasticities $\{\varepsilon_k\}$ from the existing literature. For the goods sector, we set it to 5.0 following Fontagné et al. (2022), who estimate this parameter using Chile's aggregate import responses to import tariff changes. For the services sector, we set it to 4.6 from Gervais and Jensen (2019), who estimate this parameter using service trade

[16]This relationship is obtained by reformulating Lemma 3 and Equation (C.15) in Appendix C of our multi-sector model, which corresponds to Lemma 1 and Equation (13) for the single sector case.

flows within the United States. Notice that we can also obtain the elasticity of substitution $\sigma_k = \varepsilon_k \left(1 - \lambda_{kl}^B/\gamma_k^B - \lambda_{kl}^S/\gamma_l^S\right)$ once we additionally choose the values for $\{\lambda_{kl}^B, \lambda_{kl}^S, \mu_k\}$ as we do below.

We estimate the matching function elasticities, $\{\lambda_{kl}^B, \lambda_{kl}^S\}$, and the labor share in search costs, $\{\mu_k\}$, through the indirect inference approach. Specifically, we calibrate these parameters so that the model replicates the impacts of import tariffs on the reorganization of international and domestic production linkages as documented in Table 1. The procedure is summarized below (see Appendix F.2 for details).

First, for each sector k and origin country u, we construct the average import tariff changes $\tilde{\mathcal{T}}_{uk}$ as the value-weighted average across HS-6 products. Next, for each candidate value of $\{\lambda_{kl}^B, \lambda_{kl}^S, \mu_k\}$, we undertake the counterfactual simulations of the changes in the iceberg trade costs $\hat{\tau}_{ud,kl} = 1 + \tilde{\mathcal{T}}_{uk}$ for all municipalities d and sector l within Chile. Notice that, even though $\hat{\tau}_{ud,kl}$ are common for all d, l, they have different effects across municipalities and sectors depending on the baseline import exposure to the US and China. We then run the analogous regressions as in Table 1 using the model-predicted counterfactual changes as the location and sector within Chile as a sample. We look for the values of $\{\lambda_{kl}^B, \lambda_{kl}^S, \mu_k\}$ that minimize the squared distance between the regression coefficients on domestic production linkages in the data (Columns 3-4) and in the model prediction. We use the regression coefficients on international production linkages (Columns 1-2) as untargeted moments to assess the external validity.

Due to the limited variations in tariff changes outside tradable sectors, we assume that these parameters are common across all sectors $k, l \in K$. We also assume that the matching function elasticities are symmetric $\lambda^B = \lambda^S$ as these two parameters tend to jointly affect the equilibrium system and it is difficult to identify each of them separately. Alternative values for λ^B, λ^S while keeping the sum $\lambda^S + \lambda^B$ unchanged yields virtually identical implications for the aggregate welfare changes (Appendix Table G.1).

Following this procedure, we obtain the estimates of $\lambda^S = \lambda^B = 0.64$ and $\mu = 0.05$. Table 3 shows that the model-predicted regression coefficients under these parameter values align with the targeted reduced-form regression coefficients (Columns 3 and 4). Furthermore, our model predicts similar regression coefficients for the total imports and the number of international suppliers as we find in the data, even though we do not directly target these regression coefficients (Columns 1 and 2).

Table 3: Impact of Import Tariff Shocks: Model vs Data

	Untargeted		Targeted	
	Total Imports (1)	Number Int. Suppliers (2)	Number Dom. Suppliers (3)	Number Dom. Buyers (4)
Data	-3.20	-1.88	-1.52	-1.30
	(1.26)	(0.61)	(0.71)	(1.10)
Model	-3.93	-1.44	-1.40	-1.54

Notes: "Data" reports the firm-level impacts of import tariff changes from Equation (38), replicating Table 1. "Model" reports the corresponding regression using model-predicted counterfactual changes using our calibrated parameters targeting the last two columns, as discussed in Section 6.1 and Appendix F.2.

Our estimates of the matching function elasticity at $\lambda^S = \lambda^B = 0.64$ imply that equilibrium production networks elastically respond to firms' search decisions. Interestingly, $\lambda^S = \lambda^B > 0.5$ implies that the matching technology exhibits increasing returns to scale. This finding is consistent with existing work that estimates the matching technology in firm-to-firm trade (Miyauchi, 2024; Eaton et al., 2024), although these papers consider different models, and hence the estimates are not directly comparable.

Our estimate of the labor coefficient in search costs at $\mu = 0.05$ means that search costs are substantially influenced by intermediate inputs costs. This low value of μ is consistent with the negative regression coefficients of import tariff shocks on the domestic supplier linkages, indicating that international import tariff reduction has complemented the formation of domestic supplier linkages. Our model can replicate this complementarity through the changes in search costs. A decrease in international input costs reduces not only the marginal production costs but also the search costs (Equation 15). This effect encourages firms to search reduces not only international suppliers but also domestic suppliers. In Appendix F.2, we show that the regression coefficient for Column (3) indeed turns positive if we set higher values of μ.

6.2 Impacts of Trade Cost Shocks

We first undertake counterfactuals of increasing the iceberg trade costs from US and China to all Chilean municipalities by the same magnitude of the import tariff changes under the trade agreements used in Section 5.2. We then discuss how the effects vary by the sign and magnitudes of the shock.

Table 4 reports the results. To highlight the role of the endogenous networks, we undertake this simulation under four scenarios: (a) using our baseline parameters (Table 2), (b) shut down endogenous networks ($\lambda^S = \lambda^B = 0$) while keeping the trade elasticity ε_k at our baseline scenario, (c) shut down endogenous networks while keeping the elasticity of substitution σ_k at our baseline scenario, and (d) allow for endogenous networks but alternatively set the labor shares in search costs to one ($\mu_k = 1$). Column (1) reports the changes in aggregate welfare across all Chilean municipalities (weighted average of GDP changes across Chilean municipalities with pre-shock GDP weights), along with the ratio to the values in our baseline specification; Column (2) reports the average percent changes in imports from US and China; Column (3) reports the average changes in the number of supplier linkages from US and China; Column (4) reports the average changes in the number of supplier linkages within Chile.

Table 4: Aggregate Effects From Import Cost Increase From China and the US (%)

	1) $\widehat{\text{Welfare}}$ (%)	2) Rel. to Baseline	3) $\hat{X}_{ui,u\in\text{US,CN}}$	4) $\hat{M}_{ui,u\in\text{US,CN}}$	5) $\hat{M}_{ui,u\in\text{CL}}$
a) Baseline	-0.35	100	-20.5	-7.6	-0.01
b) Fixed Network, fix ε_k	-0.27	77	-20.2	0	0
c) Fixed Network, fix σ_k	-0.28	81	-12.7	0	0
d) Baseline, $\mu_k = 1$	-0.23	68	-20.3	-7.5	0.1

Notes: The results of counterfactual simulations to increase the iceberg trade costs from US and China to all Chilean municipalities by the same magnitude of the import tariff changes under the trade agreements used in Section 5.2 under four scenarios: (a) using our baseline parameters (Table 2), (b) shut down endogenous networks, while keeping the trade elasticity ε_k at our baseline scenario, (c) shut down endogenous networks ($\lambda^S = \lambda^B = 0$), while keeping the elasticity of substitution σ_k at our baseline scenario, and (d) allow for endogenous networks but alternatively set the labor coefficients in search costs to one ($\mu_k = 1$). Column (1) reports the changes in aggregate welfare across all Chilean municipalities (weighted average of GDP changes across Chilean municipalities with pre-shock GDP weights); Column (2) reports the ratio of the values in Column (1) to our baseline specification; Column (3) reports the average percent changes in imports from US and China by Chilean municipalities; Column (4) reports the average changes in the number of supplier linkages from US and China; Column (5) reports the average changes in the number of supplier linkages within Chile.

Using our baseline specification, we find a 0.35 percent decline in Chile's aggregate welfare from the import cost increase (Column 1, Row a), indicating a modest but non-negligible aggregate welfare effect. These effects are associated with a decrease in aggregate imports from US and China by 20.5 percent (Column 3). Nearly a third of this decrease is attributed to the decrease in the extensive margin of Chilean import relationships from US and China (Column 4). We also find a reduction in supplier linkages within Chile (Column 5). The signs of Columns (3)-(5) are consistent with our differences-in-differences findings in Table 1. The magnitudes of the decrease in domestic supplier linkages are smaller, suggesting that aggregate effects on domestic production network formation are smaller than the difference-in-differences effects as documents in Table 1.

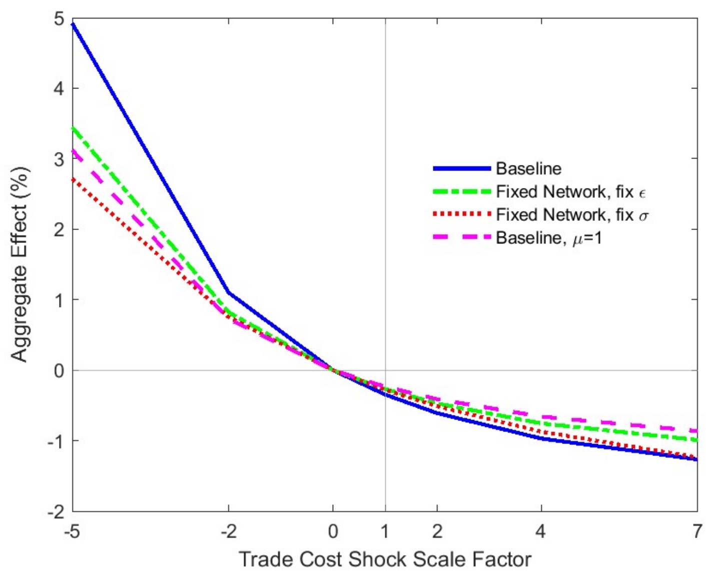
We find that these aggregate welfare effects become smaller when we shut down endogenous production network formation. When we do so while fixing the trade elasticity, ϵ_k (Row b), we find that the welfare effect decreases to 0.27 percent, which is 77 percent of our baseline specification. Interestingly, this reduction is similar by alternatively fixing the elasticity of substitution, σ_k, when we shut down endogenous networks (Row c), despite that this specification predicts a smaller aggregate import response (Column 3, Rows c vs b).

These findings are consistent with our theoretical observation in Proposition 2. Up to first order, whether allowing for endogenous networks increases the aggregate welfare effects depends on the search externality and how they interact with the inefficiency from the double marginalization. In particular, it increases if the equilibrium trade flows with endogenous networks (subject to both search externality and double marginalization) are inefficiently lower than the case with fixed networks (only subject to double marginalization). Our results suggest that this is indeed the case under our calibrated parameters.

In Row (d), we allow for endogenous networks but instead set the labor coefficient in search costs at $\mu = 1$. We find that the welfare effect under this specification is smaller than our baseline specification (68 percent), and the predicted welfare changes are more similar to those with fixed networks (Rows b-c). This finding is also consistent with Proposition 2, where endogenous networks do not affect the first-order aggregate welfare changes if $\mu = 1$. As discussed before, in this case, even though labor is misallocated between production and search activity, equilibrium trade flows are not distorted. Therefore, endogenous networks do not influence the aggregate welfare changes to a first order. Notice also that, in contrast to our baseline specification, the number of supplier linkages within Chile increases in response to the import cost increase (Row d, Column 4). In other words, with $\mu = 1$, the model predicts the substitution of production linkages toward domestic suppliers, instead of complementarity in our baseline specification (Row a). Therefore, the failure to capture these complementary patterns of network reorganization is consequential for assessing aggregate welfare effects.

In Figure 1, we present how these patterns depend on the sign and the magnitude of the shock. Specifically, we report the aggregate welfare effects of the iceberg trade cost changes from US and China to all Chilean municipalities by the magnitude of reverting the trade agreements, multiplied by the value in the horizontal axis. A value of zero in the horizontal axis indicates no trade cost shock; a value of one indicates the same increase in the iceberg trade cost as in Table 4; a negative value indicates a decrease in the iceberg trade costs.

Figure 1: Aggregate Welfare Effects From Import Cost Change: Nonlinearity



Notes: The figure shows the aggregate welfare effects of the iceberg trade cost changes from US and China to all Chilean municipalities by the magnitude of reverting the trade agreements, multiplied by the value in the horizontal axis. A value of zero in the horizontal axis indicates no trade cost shock; a value of one indicates the same increase in the iceberg trade cost as in Table 4; a negative value indicates a decrease in the iceberg trade costs. The four lines correspond to the same set of alternative model specifications as used in Table 2.

When we abstract from endogenous networks while keeping trade elasticity ϵ_k fixed, we find that the welfare changes are always attenuated regardless of the sign and the magnitude of the shock. Furthermore, these patterns are similar to the alternative scenario in which we allow for endogenous networks but instead set $\mu = 1$. This result is consistent with the interpretation that, fixing trade elasticities, amplification of trade shocks through search costs is the key margin that shapes the aggregate welfare effects, as highlighted in Proposition 4.

When we abstract from endogenous networks while keeping the elasticity of substitution σ_k fixed, thereby using smaller values of trade elasticities, we find somewhat different patterns. When we increase the trade costs (positive and larger values in the horizontal axis), the differences in welfare gains from our baseline specification remain small and even narrow for a large increase in trade costs. On the other hand, when we decrease the trade costs (negative values in the horizontal axis), the gap instead widens. This pattern is consistent with the interpretation that endogenous network formation facilitates the reallocation of trade flows toward regions with large positive shocks and away from regions with large negative shocks.

6.3 Welfare Gains from Trade Relative to Autarky

We next study how endogenous production networks affect the welfare gains from trade (GFT) relative to autarky. While this is the central question in the literature on international trade, previous literature has not quantified how endogenous production network formation affects those numbers. In Table 5, we report the estimates of two types of GFT. In Panel (a), we report the GFT relative to municipality autarky, i.e., average welfare changes by shutting down all trade with other Chilean municipalities and international countries. In Panel (b), we report the GFT from international autarky, i.e., the same values by shutting down all trade with international countries but keeping the trade within Chile across municipalities.

Starting from the GFT relative to municipality autarky (Panel a), we find an estimate of 169 percentage points using our baseline specification (Row 1). This number is substantially larger compared to GFT relative to international autarky (Panel b), reflecting the significantly larger trade flows within a country than across countries.

Table 5: Welfare Gains from Trade

(a) Relative to Municipality Autarky

	$\widehat{\text{Welfare}}$ (%)	% of Baseline
1) Baseline	169	100
2) Fixed Networks, fix ε_k	97	58
3) Fixed Networks, fix σ_k	172	102
4) Baseline, $\mu_k = 1$	99	59

(b) Relative to International Autarky

	$\widehat{\text{Welfare}}$ (%)	% of Baseline
1) Baseline	6.2	100
2) Fixed Networks, fix ε_k	5.6	90
3) Fixed Networks, fix σ_k	8.8	143
4) Baseline, $\mu_k = 1$	4.6	88

Notes: Panel (a) reports the welfare gains from trade relative to regional autarky by Chilean municipalities, i.e., average welfare changes by shutting down all trade with other Chilean municipalities and international countries. Panel (b) reports the same values by shutting down all trade with international countries but keeping the trade within Chile across municipalities. Rows (1)-(4) correspond to the same set of alternative model specifications as used in Table 2.

When we shut down endogenous networks while keeping the trade elasticity ϵ_k fixed, GFT decreases to 97 percentage points, which is 42% smaller than that of our baseline specification. This result is consistent with Proposition 4: Fixing trade elasticities, GFT is larger through the search cost multiplier. Consistent with this interpretation, we find a similar value of GFT if we allow for endogenous networks but instead shut down search cost multiplier by setting $\mu_k = 1$ (Rows 4 vs 2).[17]

When we abstract from endogenous networks while keeping the elasticity of substitution σ_k fixed (Row 3), thereby using larger values of trade elasticities, we find that the average GFT relative to municipality autarky is 172 percentage points, which is similar to our baseline specification. Therefore, the mitigation effects of the larger trade elasticity

[17]Proposition 4 shows that, in the single-sector environment, Rows (4) and (2) should exactly equal to each other given observed trade flows. This may not be the case in the multi-sector environment due to differences in sectoral reallocation in the two models (see Proposition 8 for the multi-sector version of Proposition 4), which generates the small difference between Row (4) and (2) in Panel (a) of Table 5.

roughly cancel out with the amplification effects through search cost.

We next discuss the GFT relative to international autarky (Panel b). We find that the average GFT relative to international autarky by Chilean municipalities is 6.2 percentage points. As mentioned above, this value is by an order of magnitude smaller than GFT relative to municipality autarky (Panel a) and more similar to typical estimates of GFT at the country level (Costinot and Rodríguez-Clare, 2014).

We find somewhat different patterns regarding endogenous networks from Panel (a). When we shut down endogenous networks while keeping the trade elasticity ϵ_k fixed, GFT decreases to 5.6 percentage points, which is 90% of that in our baseline specification (Row 2). Therefore, compared to Row (2) of Panel (a), the gap from the baseline specification becomes smaller. Similarly, when we shut down endogenous networks while keeping the elasticity of substitution σ_k fixed, thereby using larger trade elasticities, GFT increases to 8.8 percentage points, which is 143% of that in our baseline specification (Row 3). Therefore, compared to Row (3) of Panel (a), we find a larger attenuation of GFT by allowing for endogenous networks.

The different patterns of endogenous networks between the municipality and international autarky counterfactuals indicate that the reorganization of production networks across Chilean municipalities plays an important role. For the latter counterfactual, we allow for the reorganization of domestic production networks within Chilean municipalities. This reorganization attenuates the welfare loss from the shock of shutting down international trade. This result is consistent with Korovkin, Makarin and Miyauchi (2024), who study the role of reorganization of production networks in the aggregate effects of localized conflict shocks in Ukraine. Using an extension of our model with richer firm heterogeneity, they find that allowing for endogenous networks mitigates the aggregate output loss from the spillover effects of conflict shocks, fixing the elasticity of substitution σ_k. This occurs through the reorganization of production networks within non-conflict areas.

7 Conclusion

We study the aggregate implications of endogenous production network formation in a quantitative multi-location general equilibrium trade model. We develop sufficient statistics formulas for global and each region's welfare and characterize the precise deviation from the fixed network environment. The deviation occurs due to inefficiency for the *ex-ante* welfare sufficient statistics, and due to the differences in trade elasticities and the multiplier effects from search costs for the *ex-post* welfare sufficient statistics. We also provide macro restrictions under which these sufficient statistics hold for any microfoundation of production network formation, providing a coherent account of how and why endogenous production network formation matters for aggregate welfare.

We then calibrate our model to the Chilean economy. Our calibrated model is able to replicate the new empirical finding that import tariff reductions generate increases in both foreign and domestic suppliers, thereby generating gross complementarity in production network formation. We show that the deviation from the fixed network environment is quantitatively large (20-40%), depending on the signs and magnitudes of the shock.

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Appendix for "Production Network Formation, Trade, and Welfare"

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April 15, 2025

A Proofs and Mathematical Details

For notational convenience, we define $\mathbb{M}_i(\delta) \equiv \int z^{\delta} dG_i(z)$.

A.1 Proof of Lemma 1

We first note that Problem (10) is a strictly convex optimization problem when $\gamma^B > 1$ and $\gamma^S > 1$. Therefore, there is a unique solution to the problem, and the first order conditions (FOCs) are necessary and sufficient for the solution.

Conjecture that solution entails $n_{id}^B(z) = a_{id}^B z^{\frac{\delta}{\gamma^B}}$, $n_{ui}^S(z) = a_{ui}^S z^{\frac{\delta}{\gamma^S}}$, and $\delta \equiv (\sigma - 1) / \left(1 - \frac{1}{\gamma^B} - \frac{1-\beta}{\gamma^S}\right)$ as in the proposition. By plugging these solutions to (9), we obtain the expression for marginal cost $c_i(z)$ as in Equation (12). By plugging these solutions to $r_i^*(z) = \sum_d r_{id}(z) m_{id}^B n_{id}^B(z)$, where $r_{id}(z) \equiv (\tilde{\sigma}\tau_{id} c_i(z))^{1-\sigma} D_d$ is the revenue per buyer link, we obtain the expression for revenue $r_i^*(z)$ as in Equation (13).

The FOC with respect to n_{id}^B is given by

$$e_i f_{id}^B \left(n_{id}^B\right)^{\gamma^B - 1} = \frac{\tilde{\sigma}^{1-\sigma}}{\sigma} m_{id}^B D_d (\tau_{id})^{1-\sigma} \underbrace{\frac{w_i^{\beta(1-\sigma)} \left(\sum_{u \in \mathcal{N}} n_{ui}^S m_{ui}^S (C_{ui})^{1-\sigma}\right)^{1-\beta}}{z^{1-\sigma}}}_{= c_i(z)^{1-\sigma}} \tag{A.1}$$

The FOC with respect to n_{ui}^S is given by

$$e_i f_{ui}^S \left(n_{ui}^S\right)^{\gamma^S - 1} = \frac{\tilde{\sigma}^{1-\sigma}}{\sigma} \left\{ \sum_d n_{id}^B m_{id}^B D_d (\tau_{id})^{1-\sigma} \right\} \times (1-\beta) \frac{w_i^{\beta(1-\sigma)} \left(\sum_{u'} n_{u'i}^S m_{u'i}^S (C_{u'i})^{1-\sigma}\right)^{-\beta}}{z^{1-\sigma}} m_{ui}^S (C_{ui})^{1-\sigma} \tag{A.2}$$

Note that all terms related to z cancel out under the conjectures of $\{n_{id}^{B}(z), n_{ui}^{S}(z)\}$ in the proposition. Furthermore, Equation (A.1) is rewritten as

$$
\begin{aligned}
e_i f_{id}^{B}\left(a_{id}^{B}\right)^{\gamma^B} &= a_{id}^{B}\frac{\tilde{\sigma}^{1-\sigma}}{\sigma} m_{id}^{B} D_d \left(\tau_{id}\right)^{1-\sigma} w_i^{\beta(1-\sigma)}\left(\sum_{u\in\mathcal{N}} a_{ui}^{S} m_{ui}^{S}\left(C_{ui}\right)^{1-\sigma}\right)^{1-\beta} \\
&= a_{id}^{B}\frac{\tilde{\sigma}^{1-\sigma}}{\sigma} m_{id}^{B} D_d \left(\tau_{id}\right)^{1-\sigma} C_i^{1-\sigma} \\
&= a_{id}^{B}\frac{\tilde{\sigma}^{1-\sigma}}{\sigma} m_{id}^{B} D_d \left(\tau_{id}\right)^{1-\sigma} z^{-\frac{\delta}{\gamma^S}(1-\beta)-(\sigma-1)} c_i(z)^{1-\sigma} \\
&= \frac{1}{\sigma} r_{id}(z)\, m_{id}^{B} a_{id}^{B} z^{-\frac{\delta}{\gamma^S}(1-\beta)-(\sigma-1)} \\
&= \frac{1}{\sigma} r_{id}(z)\, m_{id}^{B} n_{id}^{B}(z)\, z^{-\frac{\delta}{\gamma^B}-\frac{\delta}{\gamma^S}(1-\beta)-(\sigma-1)} \\
\iff \quad e_i f_{id}^{B}\left(a_{id}^{B}\right)^{\gamma^B} z^{\delta} &= \frac{1}{\sigma} r_{id}(z)\, m_{id}^{B} n_{id}^{B}(z).
\end{aligned} \tag{A.3}
$$

By integrating wrt z using density function $G_i(\cdot)$ and multiplying by N_i,

$$
e_i f_{id}^{B}\left(a_{id}^{B}\right)^{\gamma^B} N_i \mathbb{M}_i(\delta) = \frac{1}{\sigma} X_{id} \iff a_{id}^{B} = \left(\Gamma_i^{B}\frac{X_{id}}{e_i f_{id}^{B}}\right)^{\frac{1}{\gamma^B}}, \tag{A.4}
$$

where we define $\Gamma_i^{B} = \frac{1}{\sigma \mathbb{M}_i(\delta) N_i}$, and $X_{id} = N_i \int r_{id}(z)\, m_{id}^{B} n_{id}^{B}(z)\, dG_i(z)$ is the total intermediate goods revenue sold by firms in i to firms in d. This expression corresponds to the one in Lemma 1.

Equation (A.2) is rewritten as

$$
\begin{aligned}
e_i f_{ui}^{S}\left(a_{ui}^{S}\right)^{\gamma^S-1} &= \frac{\tilde{\sigma}^{1-\sigma}}{\sigma}\underbrace{\left\{\sum_d a_{id}^{B} m_{id}^{B} D_d \tau_{id}^{1-\sigma}\right\}}_{=D_i^*=z^{-\delta} r_i^*(z)(\tilde{\sigma}C_i)^{\sigma-1}}(1-\beta) \\
&\quad \times \underbrace{w_i^{\beta(1-\sigma)}\left(\sum_{u'} a_{u'i}^{S} m_{u'i}^{S}\left(C_{u'i}\right)^{1-\sigma}\right)^{-\beta} m_{ui}^{S}\left(C_{ui}\right)^{1-\sigma}}_{=C_i^{1-\sigma}\left(a_{ui}^{S}\right)^{-1}\Lambda_{ui}} \\
\iff \quad e_i f_{ui}^{S}\left(a_{ui}^{S}\right)^{\gamma^S} z^{\delta} &= \frac{1}{\sigma}(1-\beta)\,\Lambda_{ui} r_i^*(z)
\end{aligned} \tag{A.5}
$$

where $\Lambda_{ui} = X_{ui}/\sum_{u'} X_{u'i}$. By integrating wrt z using density function $G_i(\cdot)$ and multiplying by N_i,

$$e_i f_{ui}^S \left(a_{ui}^S\right)^{\gamma^S} N_i \mathbb{M}_i(\delta) = \frac{1}{\sigma}(1-\beta)\Lambda_{ui} R_i$$

$$\iff a_{ui}^S = \left(\frac{1}{\sigma \mathbb{M}_i(\delta) N_i} \frac{(1-\beta) R_i}{e_i f_{ui}^S} \frac{X_{ui}}{E_i}\right)^{\frac{1}{\gamma^S}} = \left(\Gamma_i^S \frac{X_{ui}}{e_i f_{ui}^S}\right)^{\frac{1}{\gamma^S}}, \tag{A.6}$$

where $R_i = \sum_d X_{id}$ is intermediate goods sales by firms in i (excluding sales to retailers), $E_i = \sum_u X_{ui}$ is intermediate goods expenditure by firms in i, and $\Gamma_i^S \equiv \frac{1-\beta}{\sigma \mathbb{M}_i(\delta) N_i} \frac{R_i}{E_i}$. Note that, under trade balance, $R_i = E_i$. This expression corresponds to the one in Lemma 1.

Finally, firms' profit subtracting the expenditure for search activity is given by

$$\tilde{\pi}_i(z) = \frac{1}{\sigma} r_i^*(z) - e_i \left\{\sum_{d \in \mathcal{N}} f_{id}^B \frac{\left(n_{id}^B\right)^{\gamma^B}}{\gamma^B} + \sum_{u \in \mathcal{N}} f_{ui}^S \frac{\left(n_{ui}^S\right)^{\gamma^S}}{\gamma^S}\right\} = \frac{1}{\sigma}\left\{1 - \frac{1}{\gamma^B} - \frac{1-\beta}{\gamma^S}\right\} r_i^*(z),$$

where we used Equations (A.3) and (A.5). This equation coincides with Equation (14).

A.2 Proof of Lemma 2

We first obtain the intensive margin of trade flows, i.e., the average transaction volume per relationship (intensive margin). Noting that the average sales per buyer by firms in u with productivity z selling in d is given by $r_{ud}(z) = (\tilde{\sigma}\tau_{ud} c_u(z))^{1-\sigma} D_d$,

$$\overline{r}_{ud} = \frac{\int r_{ud}(z)\, n_{ud}^B(z)\, dG_u(z)}{\int n_{ud}^B(z)\, dG_u(z)} = (\tilde{\sigma}\tau_{ud})^{1-\sigma} D_d \frac{\int c_u(z)^{1-\sigma} n_{ud}^B(z)\, dG_u(z)}{\int n_{ud}^B(z)\, dG_u(z)}.$$

Using the expression for the marginal costs and buyer search from Lemma 1,

$$\overline{r}_{ud} = \varrho^I (\tau_{ud})^{1-\sigma} \left(\frac{\mathbb{M}_u(\delta)}{\mathbb{M}_u\left(\frac{\delta}{\gamma^B}\right)} C_u^{1-\sigma}\right) D_d, \tag{A.7}$$

with $\varrho^I = \tilde{\sigma}^{1-\sigma}$, which corresponds to $\overline{r}_{ud}$ in Equation (18) with $\zeta_u^I = \frac{\mathbb{M}_u(\delta)}{\mathbb{M}_u\left(\frac{\delta}{\gamma^B}\right)} C_u^{1-\sigma}$ and $\xi_d^I = D_d$.

We next obtain the extensive margin, i.e., the measure of supplier-to-buyer relationships from supplier location u to buyer location d, M_{ud}. From Equation (16),

$$M_{ud} = \kappa_{ud} \left(N_d a^S_{ud} \mathbb{M}_d \left(\frac{\delta}{\gamma^S} \right) \right)^{\lambda^S} \left(N_u a^B_{ud} \mathbb{M}_u \left(\frac{\delta}{\gamma^B} \right) \right)^{\lambda^B} = \kappa^*_{ud} \frac{X_{ud}^{\tilde{\lambda}^S + \tilde{\lambda}^B}}{e_u^{\tilde{\lambda}^B} e_d^{\tilde{\lambda}^S}}$$

where $\kappa^*_{ud} = \kappa_{ud} \left(f^S_{ud} \right)^{-\tilde{\lambda}^S} \left(f^B_{ud} \right)^{-\tilde{\lambda}^B} \left(N_d \mathbb{M}_d \left(\frac{\delta}{\gamma^S} \right) \right)^{\lambda^S} \left(N_u \mathbb{M}_u \left(\frac{\delta}{\gamma^B} \right) \right)^{\lambda^B} \left(\Gamma^S_d \right)^{\tilde{\lambda}^S} \left(\Gamma^B_u \right)^{\tilde{\lambda}^B}$. Plugging $X_{ud} = \overline{r}_{ud} M_{ud}$ into this equation yields

$$\begin{aligned} M_{ud} &= \kappa^*_{ud} \left(M_{ud} \overline{r}_{ud} \right)^{\tilde{\lambda}^S + \tilde{\lambda}^B} e_u^{-\tilde{\lambda}^B} e_d^{-\tilde{\lambda}^S} \\ &= \left(\kappa^*_{ud} \right)^{\frac{1}{1-\tilde{\lambda}^S-\tilde{\lambda}^B}} \overline{r}_{ud}^{\frac{\tilde{\lambda}^S+\tilde{\lambda}^B}{1-\tilde{\lambda}^S-\tilde{\lambda}^B}} e_u^{-\frac{\tilde{\lambda}^B}{1-\tilde{\lambda}^S-\tilde{\lambda}^B}} e_d^{-\frac{\tilde{\lambda}^S}{1-\tilde{\lambda}^S-\tilde{\lambda}^B}} \\ &= \left(\kappa^*_{ud} \right)^{\frac{1}{1-\tilde{\lambda}^S-\tilde{\lambda}^B}} \left[\tau_{ud}^{1-\sigma} \varrho^I C_u^{1-\sigma} \frac{\mathbb{M}_u (\delta)}{\mathbb{M}_u \left(\frac{\delta}{\gamma^B} \right)} D_d \right]^{\frac{\tilde{\lambda}^S+\tilde{\lambda}^B}{1-\tilde{\lambda}^S-\tilde{\lambda}^B}} e_u^{-\frac{\tilde{\lambda}^B}{1-\tilde{\lambda}^S-\tilde{\lambda}^B}} e_d^{-\frac{\tilde{\lambda}^S}{1-\tilde{\lambda}^S-\tilde{\lambda}^B}}, \end{aligned} \tag{A.8}$$

which corresponds to M_{ud} in Equation (18) with $\rho^E = \left(\varrho^I \right)^{\frac{\tilde{\lambda}^S+\tilde{\lambda}^B}{1-\tilde{\lambda}^S-\tilde{\lambda}^B}}$, and

$$\chi^E_{ud} = \left[\kappa_{ud} \left(f^B_{ud} \right)^{-\tilde{\lambda}^B} \left(f^S_{ud} \right)^{-\tilde{\lambda}^S} \left(\tau_{ud}^{1-\sigma} \right)^{\tilde{\lambda}^B + \tilde{\lambda}^S} \right]^{\frac{1}{1-\tilde{\lambda}^S-\tilde{\lambda}^B}}, \tag{A.9}$$

$$\zeta^E_u = \left(\frac{\mathbb{M}_u (\delta)}{\mathbb{M}_u \left(\frac{\delta}{\gamma^B} \right)} \right)^{\frac{\tilde{\lambda}^S+\tilde{\lambda}^B}{1-\tilde{\lambda}^S-\tilde{\lambda}^B}} C_u^{\frac{\tilde{\lambda}^S+\tilde{\lambda}^B}{1-\tilde{\lambda}^S-\tilde{\lambda}^B}(1-\sigma)} e_u^{-\frac{\tilde{\lambda}^B}{1-\tilde{\lambda}^S-\tilde{\lambda}^B}} \left(N_u \mathbb{M}_u \left(\frac{\delta}{\gamma^B} \right) \right)^{\lambda^B} \left(\Gamma^B_u \right)^{\tilde{\lambda}^B}, \tag{A.10}$$

$$\xi^E_d = D_d^{\frac{\tilde{\lambda}^S+\tilde{\lambda}^B}{1-\tilde{\lambda}^S-\tilde{\lambda}^B}} e_d^{-\frac{\tilde{\lambda}^S}{1-\tilde{\lambda}^S-\tilde{\lambda}^B}} \left(N_d \mathbb{M}_d \left(\frac{\delta}{\gamma^S} \right) \right)^{\lambda^S} \left(\Gamma^S_d \right)^{\tilde{\lambda}^S} \tag{A.11}$$

A.3 Market Clearing Conditions

Intermediate goods for each firm ω are used as inputs for production by other firms and local retailers:

$$q^R_i (\omega) + \sum_{d \in \mathcal{N}} \int_{\psi:\, \omega \in S_{id}(\psi)} q_{id} (\omega, \psi)\, \tau_{id} d\psi = q_i (\omega). \tag{A.12}$$

Search activity uses local labor and retail goods:

$$\int_{\omega\in\Omega_i(\omega)}\left(\sum_d n^B_{id}\left(z\right)+\sum_u n^S_{ui}\left(\omega\right)\right)d\omega=\left(\frac{L^A_i}{\mu}\right)^{\mu}\left(\frac{Q^A_i}{1-\mu}\right)^{1-\mu}, \tag{A.13}$$

where L^A_i and Q^A_i corresponds to the aggregate amount of labor and retail goods used for search activity.

Retail goods are consumed by final consumers and used for search activity:

$$L_iQ^F_i+Q^A_i=Q_i. \tag{A.14}$$

Labor is used for intermediate goods production and for search activity:

$$\int_{\omega\in\Omega_i(\omega)} l_i\left(\omega\right)d\omega+L^A_i=L_i. \tag{A.15}$$

A.4 Proof of Proposition 1

Preliminaries We first obtain several useful expressions. First, we derive $C^{1-\sigma}_{ui}\equiv\int\left(\tilde{\sigma}c_u\left(z\right)\tau_{ui}\right)^{1-\sigma}dG^B_{ui}\left(z\right)$, where $G^B_{ui}\left(\cdot\right)$ denotes the weighted distribution of z by buyer search intensity, given by

$$dG^B_{ui}\left(z\right)=\frac{n^B_{ui}\left(z\right)dG_u(z)}{\int n^B_{ui}\left(z'\right)dG_u(z')}=\frac{z^{\frac{\delta}{\gamma^B}}dG_u(z)}{\int z'^{\frac{\delta}{\gamma^B}}dG_u(z')}=\frac{z^{\frac{\delta}{\gamma^B}}dG_u(z)}{\mathbb{M}_u\left(\frac{\delta}{\gamma^B}\right)}.$$

Therefore we have

$$C^{1-\sigma}_{ui}=\int\left(\tilde{\sigma}C_uz^{-\frac{\delta}{\gamma^S}\frac{1-\beta}{\sigma-1}-1}\tau_{ui}\right)^{1-\sigma}dG^B_{ui}\left(z\right)=\left(\tilde{\sigma}C_u\tau_{ui}\right)^{1-\sigma}\frac{\mathbb{M}_u\left(\delta\right)}{\mathbb{M}_u\left(\frac{\delta}{\gamma^B}\right)}, \tag{A.16}$$

where we used the fact that $\delta=\left(-\frac{\delta}{\gamma^S}\frac{1-\beta}{\sigma-1}-1\right)\left(1-\sigma\right)+\frac{\delta}{\gamma^B}$.

Next, we derive D_d. Notice the aggregate intermediate input expenditure E_d satisfies

$$\begin{aligned}
E_d &= \sum_u N_u \int (\tau_{ud})^{1-\sigma} D_d \left(\tilde{\sigma} c_u (z)\right)^{1-\sigma} n^B_{ud}(z)\, m^B_{ud} dG_u(z) \\
&= D_d \sum_u N_u (\tau_{ud})^{1-\sigma} (\tilde{\sigma})^{1-\sigma} \int (C_u)^{1-\sigma} a^B_{ud} m^B_{ud} z^{\frac{\delta}{\gamma^B} + \frac{\delta}{\gamma^S}(1-\beta)+(\sigma-1)} dG_u(z) \\
&= D_d \sum_u (\tau_{ud})^{1-\sigma} (\tilde{\sigma})^{1-\sigma} (C_u)^{1-\sigma} N_u a^B_{ud} m^B_{ud} \mathbb{M}_u(\delta) \\
&= D_d \sum_u (\tau_{ud})^{1-\sigma} (\tilde{\sigma})^{1-\sigma} (C_u)^{1-\sigma} N_d a^S_{ud} m^S_{ud} \frac{\mathbb{M}_d\left(\frac{\delta}{\gamma^S}\right)}{\mathbb{M}_u\left(\frac{\delta}{\gamma^B}\right)} \mathbb{M}_u(\delta) \quad \text{(from 17 and } \overline{M}^S_{ud}, \overline{M}^B_{ud}\text{)} \\
&= D_d N_d \mathbb{M}_d\left(\frac{\delta}{\gamma^S}\right) \sum_u C^{1-\sigma}_{ud} a^S_{ud} m^S_{ud} \quad \text{(from Equation A.16)} \\
&= D_d N_d \mathbb{M}_d\left(\frac{\delta}{\gamma^S}\right) \left[w_d^{-\beta} C_d\right]^{\frac{1-\sigma}{1-\beta}} \quad \text{(from Equation 12)}
\end{aligned}$$

Denote nominal labor compensation to intermediate input expenditure ratio:

$$\varsigma^L \equiv \frac{w_d L_d}{E_d} = \frac{\frac{\sigma-1}{\sigma}\beta + \frac{1}{\sigma}\mu\left(\frac{1}{\gamma^B} + \frac{1-\beta}{\gamma^S}\right)}{\frac{\sigma-1}{\sigma}(1-\beta)}. \tag{A.17}$$

Then,

$$D_d = \frac{1}{\varsigma^L} \frac{L_d}{N_d \mathbb{M}_d\left(\frac{\delta}{\gamma^S}\right)} (w_d)^{\frac{1-\beta\sigma}{1-\beta}} (C_d)^{\frac{\sigma-1}{1-\beta}}. \tag{A.18}$$

Finally, from Equation (18) and Appendix A.2, total margin trade flows are given by

$$X_{ud} = \varrho \chi_{ud} \zeta_u \xi_d \tag{A.19}$$

where $\rho = \varrho^I \varrho^E$, $\chi_{ud} = \chi_{ud}^I \chi_{ud}^E$, and

$$\zeta_u = C_u^{-\varepsilon} e_u^{-\frac{\tilde{\lambda}^B}{1-\tilde{\lambda}^S-\tilde{\lambda}^B}} \tilde{K}_u^U, \qquad \tilde{K}_u^U \equiv \left(\frac{\mathbb{M}_u(\delta)}{\mathbb{M}_u\left(\frac{\delta}{\gamma^B}\right)}\right)^{\frac{1}{1-\tilde{\lambda}^S-\tilde{\lambda}^B}} \left(N_u \mathbb{M}_u\left(\frac{\delta}{\gamma^B}\right)\right)^{\lambda^B} \left(\Gamma_u^B\right)^{\tilde{\lambda}^B}$$

$$\xi_d = w_d^{\frac{1-\beta\sigma}{1-\beta}\frac{\varepsilon}{\sigma-1}} C_d^{\frac{\varepsilon}{1-\beta}} e_d^{-\frac{\tilde{\lambda}^S}{1-\tilde{\lambda}^S-\tilde{\lambda}^B}} K_d^D, \qquad K_d^D \equiv \left(N_d \mathbb{M}_d\left(\frac{\delta}{\gamma^S}\right)\right)^{\lambda^S} \left(\Gamma_d^S\right)^{\tilde{\lambda}^S}$$

Part (a) From the accounting identity,

$$w_i L_i = \varsigma^L \sum_d X_{id}, \tag{A.20}$$

where ς^L is the nominal labor compensation to intermediate input expenditure ratio as defined in Equation (A.17). Plugging Equation (A.19) into this equation, and after some algebra, we obtain Equation (20) with $K_i^U \equiv \varrho \varsigma^L \tilde{K}_i^U$.

Next, by combining Equation (A.20) and the trade balancing condition (implied by the market clearing conditions,[1] we obtain:

$$w_i L_i = \varsigma^L \sum_u X_{ui}. \tag{A.21}$$

Plugging Equation (A.19) into this equation, we obtain Equation (21).

Part (b) We apply Allen, Arkolakis and Li (2024). Define the matrices

$$\Gamma \equiv \begin{bmatrix} 1+c_1 & c_2 \\ 1-\delta_G & -c_3 \end{bmatrix}, \qquad B \equiv \begin{bmatrix} \delta_G & c_3 \\ -c_1 & -c_2 \end{bmatrix}$$

where we define

$$\delta_G = \left(\frac{1-\beta\sigma}{1-\beta} - \tilde{\lambda}^S \mu\right) \frac{\varepsilon}{\sigma - 1}$$

[1]By integrating intermediate goods clearing condition (A.12) across firms, we have $P_i Q_i + \sum_d X_{id} = \tilde{R}_i$, where $\tilde{R}_i$ is firms' aggregate revenue. From the accounting identity, $\tilde{R}_i = \sum_u X_{ui} + \Pi_i + w_i L_i + P_i Q_i^A$. The retail goods market clearing (A.14) implies that $\Pi_i + w_i L_i + P_i Q_i^A = P_i Q_i$. Putting together, we have $\sum_u X_{ui} = \sum_d X_{id}$.

$$c_1 = \tilde{\lambda}^B \frac{\varepsilon}{\sigma-1}\mu, \quad c_2 = \varepsilon\left(1+\tilde{\lambda}^B \frac{1-\mu}{\sigma-1}\right), \quad c_3 = \varepsilon\left(\frac{1}{1-\beta} - \tilde{\lambda}^S \frac{1-\mu}{\sigma-1}\right)$$

where $c_1 > 0$ and $c_2 > 0$ under our model parameter assumptions. A sufficient condition for the equilibrium uniqueness is that the spectral radius of $A = |B\Gamma^{-1}|$ is equal to 1, where

$$\begin{aligned} B\Gamma^{-1} &= \frac{1}{-c_3(1+c_1) - c_2(1-\delta_G)} \begin{bmatrix} \delta_G & c_3 \\ -c_1 & -c_2 \end{bmatrix} \begin{bmatrix} -c_3 & -c_2 \\ -(1-\delta_G) & 1+c_1 \end{bmatrix} \\ &= \frac{1}{-c_3(1+c_1) - c_2(1-\delta_G)} \begin{bmatrix} -c_3 & -\delta_G c_2 + c_3(1+c_1) \\ c_1 c_3 + (1-\delta_G) c_2 & -c_2 \end{bmatrix} \end{aligned}$$

We now show that, when $\delta_G \leq 1$ and $\frac{\beta(\sigma-1)}{1-\beta} \geq (1-\mu)\left(\tilde{\lambda}^B + \tilde{\lambda}^S\right)$ as assumed in the proposition, the largest eigenvalue of $|B\Gamma^{-1}|$ is indeed less than one. From the second condition, we have $c_3 > c_2 > 0$. Furthermore, together with $\delta_G \leq 1$, $-\delta_G c_2 + c_3(1+c_1) \geq c_1 c_3 + (1-\delta_G) c_2 > 0$. Therefore,

$$|B\Gamma^{-1}| = \frac{1}{c_3(1+c_1) + c_2(1-\delta_G)} \begin{bmatrix} c_3 & -\delta_G c_2 + c_3(1+c_1) \\ c_1 c_3 + (1-\delta_G) c_2 & c_2 \end{bmatrix}$$

Note that the sum of the rows for the first column and second column are both one. Therefore, from Collatz–Wielandt Formula (see Allen et al. (2024)), the largest eigenvalue of $|B\Gamma^{-1}|$ is one under this condition. Therefore, when $\delta_G \leq 1$ and $\frac{\beta(\sigma-1)}{1-\beta} \geq (1-\mu)\left(\tilde{\lambda}^B + \tilde{\lambda}^S\right)$, the equilibrium exists and it is unique up to scale.

A.5 Exact-hat Algebra

Consider the changes in fundamentals $\{\hat{\tau}_{ud}, \hat{\kappa}_{ud}, \hat{f}^B_{ud}, \hat{f}^S_{ud}\}$, where $\hat{x} = x'/x$ denote the proportional change in variable x and x' denotes the value of x after the change in fundamentals. The changes in equilibrium wages $\{\hat{w}_i\}$ and cost shifters $\{\hat{C}_i\}$ are given by the solution to the following set of equations:

$$\hat{w}_i^{1+\tilde{\lambda}^B \mu \frac{\varepsilon}{\sigma-1}} \hat{C}_i^{\varepsilon\left(1+\tilde{\lambda}^B \frac{1-\mu}{\sigma-1}\right)} = \sum_d \Psi_{id} \hat{\chi}_{id} \hat{w}_d^{\left(\frac{1-\beta\sigma}{1-\beta} - \tilde{\lambda}^S \mu\right)\frac{\varepsilon}{\sigma-1}} \hat{C}_d^{\varepsilon\left(\frac{1}{1-\beta} - \tilde{\lambda}^S \frac{1-\mu}{\sigma-1}\right)}, \tag{A.22}$$

$$\hat{w}_i^{1-\left(\frac{1-\beta\sigma}{1-\beta}-\tilde{\lambda}^S\mu\right)\frac{\varepsilon}{\sigma-1}}\hat{C}_i^{-\varepsilon\left(\frac{1}{1-\beta}-\tilde{\lambda}^S\frac{1-\mu}{\sigma-1}\right)} = \sum_u \Lambda_{id}\hat{\chi}_{ui}\hat{w}_u^{-\tilde{\lambda}^B\mu\frac{\varepsilon}{\sigma-1}}\hat{C}_u^{-\varepsilon\left(1+\tilde{\lambda}^B\frac{1-\mu}{\sigma-1}\right)}, \tag{A.23}$$

where $\Psi_{id} = X_{id}/\sum_n X_{in}$ and $\Lambda_{ui} = X_{ui}/\sum_n X_{ni}$ are import and export shares in the baseline equilibrium, and

$$\hat{\chi}_{ud} = \hat{\tau}_{ud}^{\varepsilon}\left[\hat{\kappa}_{ud}\left(\hat{f}_{ud}^B\right)^{-\tilde{\lambda}^B}\left(\hat{f}_{ud}^S\right)^{-\tilde{\lambda}^S}\right]^{\frac{1}{1-\tilde{\lambda}^S-\tilde{\lambda}^B}}. \tag{A.24}$$

A.6 Planning Problem and Proof of Proposition 2 and Corollary 1

Formal Definition of Planning Problem Given the set of subsidies/taxes described in Section 3, the government budget constraint is given by

$$\sum_i T_i^F L_i + \sum_{i,d}\frac{s_{id}^I}{1-s_{id}^I}X_{id} + \sum_{i,d}\frac{t_{id}^B}{1+t_{id}^B}\frac{1}{\sigma}\frac{1}{\gamma^B}X_{id} + \sum_{i,d}\frac{t_{id}^S}{1+t_{id}^S}\frac{1}{\sigma}\frac{1-\beta}{\gamma^S}X_{id} = 0, \tag{A.25}$$

where $\frac{1}{\sigma}\frac{1}{\gamma^B}X_{id}$ and $\frac{1}{\sigma}\frac{1-\beta}{\gamma^S}X_{id}$ correspond to the aggregate expenditure for buyer and supplier search, respectively. Then, defining the equilibrium prices and quantities by $\mathcal{X} \equiv \left\{X_{id}, p_{id}(v,\omega), P_i, e_i, w_i, C_i, q_{id}(\omega,\psi), q_i^R(\omega), Q_i^F, Q_i^A, n_{id}^B(\omega), n_{ui}^S(\omega), l_i(\omega), L_i^A\right\}$, the planning problem is defined by

$$\begin{aligned} &\max_{\left\{\left\{s_{id}^I, t_{id}^B, t_{id}^S\right\}, T_i^F\right\}, \mathcal{X}} \sum_i \psi_i L_i Q_i^F && \text{(A.26)}\\ &\text{s.t.} \quad P_i Q_i^F = w_i + \frac{\Pi_i}{L_i} + T_i^F \\ &\qquad X_{id} = \int_{\omega\in\Omega_i,\psi\in\Omega_d} q_{id}(\omega,\psi)\, dG_i(\omega) dG_d(\psi) \\ &\qquad (1)\text{-}(3), (5)\text{-}(6), (9)\text{-}(17), (22), (23), (A.12)\text{-}(A.15), (A.25) \end{aligned}$$

Proof of Proposition 2 We proceed in two steps. First, we solve for a relaxed planning problem (A.26) where the planner directly specifies the aggregate allocation. Next, we show that the planner can implement this allocation using the subsidies/taxes available to the planner.

Consider the following relaxed planning problem:

$$\max_{\{Q_i^F, Q_i^A, L_i^A, L_i^P, A_{ui}^S, A_{ui}^B, q_i^R, q_{id}, M_{ui}\}} \sum_i \psi_i L_i Q_i^F \tag{A.27}$$

$$\text{s.t.} \quad Q_i^A + L_i Q_i^F = g_i\left(q_i^R\right) \qquad [P_i]$$

$$L_i^A + L_i^P = L_i \qquad [w_i]$$

$$\sum_u A_{ui}^S + \sum_d A_{id}^B = \left(\frac{L_i^A}{\mu}\right)^{\mu} \left(\frac{Q_i^A}{1-\mu}\right)^{1-\mu} \qquad [e_i]$$

$$q_i^R + \sum_d q_{id}\tau_{id} = Z_i \left(\frac{L_i^P}{\beta}\right)^{\beta} \left(\frac{\left(\sum_u M_{ui}^{\frac{1}{\sigma}} q_{ui}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}}{1-\beta}\right)^{1-\beta} \qquad [C_i]$$

$$M_{ui} = \kappa_{ui} \left(\frac{\gamma^B}{f_{ui}^B} A_{ui}^B\right)^{\frac{\lambda^B}{\gamma^B}} \left(\frac{\gamma^S}{f_{ui}^S} A_{ui}^S\right)^{\frac{\lambda^S}{\gamma^S}} \qquad [\eta_{ui}]$$

where A_{ui}^B and A_{ui}^S correspond to the aggregate buyer and supplier search effort between suppliers in u and buyers in i, and L_i^P is the aggregate labor used for production. The brackets in each constraint represent the Lagrange multipliers. In the final constraint, we combined the matching technology (16) and search effort (8). In the second to last constraint, we define q_i^R and q_{ui} as aggregate intermediate good quantify (not per firm or link).

Notice that we dropped the firm identifiers ω within each location, because the planner has no policy tool to discriminate firms within a location. We later verify that these allocations are achieved given tax instruments $\left\{\left\{s_{id}^I, t_{id}^B, t_{id}^S\right\}, T_i\right\}$. The FOCs of this problem with respect to Q_i^F, Q_i^A, q_i^R, L_i^A, L_i^P, A_{ui}^B, A_{ui}^S, M_{ui}, q_{ui} are given by:

$$P_i = \psi_i \tag{A.28}$$

$$P_i = e_i A_i (1-\mu) \frac{1}{Q_i^A} \tag{A.29}$$

$$C_i = P_i g_i'\left(q_i^R\right) \tag{A.30}$$

$$w_i = e_i A_i \mu \frac{1}{L_i^A} \tag{A.31}$$

$$w_i = \beta C_i q_i \frac{1}{L_i^P} \tag{A.32}$$

$$e_u = \eta_{ui} \frac{\lambda^B}{\gamma^B} M_{ui} \frac{1}{A_{ui}^B} \tag{A.33}$$

$$e_i = \eta_{ui} \frac{\lambda^S}{\gamma^S} M_{ui} \frac{1}{A_{ui}^S} \tag{A.34}$$

$$\eta_{ui} = \frac{1}{\sigma - 1} (1 - \beta) C_i q_i \Lambda_{ui} \frac{1}{M_{ui}} \tag{A.35}$$

$$\tau_{ui} C_u = (1 - \beta) C_i q_i \Lambda_{ui} \frac{1}{q_{ui}} \tag{A.36}$$

where $A_i \equiv \sum_u A_{ui}^S + \sum_d A_{id}^B$, $q_i \equiv q_i^R + \sum_d q_{id}\tau_{id}$, and $\Lambda_{ui} \equiv \frac{M_{ui}^{\frac{1}{\sigma}} q_{ui}^{\frac{\sigma-1}{\sigma}}}{\sum_\ell M_{\ell i}^{\frac{1}{\sigma}} q_{\ell i}^{\frac{\sigma-1}{\sigma}}}$. From (A.33), (A.34), (A.35),

$$\frac{\lambda^B}{\gamma^B} \frac{1}{\sigma - 1} (1 - \beta) C_i q_i \Lambda_{ui} = e_u A_{ui}^B \tag{A.37}$$

$$\frac{\lambda^S}{\gamma^S} \frac{1}{\sigma - 1} (1 - \beta) C_i q_i \Lambda_{ui} = e_i A_{ui}^S \tag{A.38}$$

We now consider the equilibrium subsidies/taxes that implement this allocation. Suppose that the Lagrange multipliers $\{P_i, w_i, e_i, C_i\}$ correspond to equilibrium prices. Then, from firms' profit maximization given sales subsidy,

$$\left(1 - s_{ui}^I\right) \frac{\sigma}{\sigma - 1} \tau_{ui} C_u q_{ui} = (1 - \beta) C_i q_i \Lambda_{ui} \tag{A.39}$$

By comparing this condition with (A.36), the optimal s_{ui}^I satisfies $1 - s_{ui}^I = 1 - s^I = \frac{\sigma-1}{\sigma} \iff s^I = 1/\sigma$.

Next, following Lemma 1 applied to the economy with taxes, equilibrium allocation for buyer and supplier search is given by

$$\left(1 + t_{ui}^B\right) e_u A_{ui}^B = \frac{1}{\sigma} \frac{1}{\gamma^B} X_{ui} \frac{1}{1 - s^I} \tag{A.40}$$

$$\left(1 + t_{ui}^S\right) e_i A_{ui}^S = \frac{1}{\sigma} \frac{1 - \beta}{\gamma^S} X_{ui} \frac{R_i}{E_i} \frac{1}{1 - s^I} \tag{A.41}$$

where we accommodate the possibility that $R_i \neq E_i$ due to the potential trade imbalance from taxes and transfers. By comparing these equations with (A.37) and (A.38), and by noting that $X_{ui} = (1-\beta)\, C_i q_i \Lambda_{ui}$, we obtain the expressions in Proposition 2.

Finally, we confirm that these allocations are indeed consistent with the equilibrium conditions. The constraints in (A.27) coincide with resource constraints, production of search effort, and matching technology; (A.32) and (A.36) coincide with intermediate goods producers' optimal production decisions; (A.30) coincides with retailers' optimal production decisions; (A.29), (A.31), (A.40) and (A.41) coincide with firms' optimal search decisions. Finally, assume that lump-sum transfers are set such that

$$T_i^F = P_i Q_i^F - \frac{\Pi_i^F}{L_i} - w_i$$

By inserting these equilibrium allocations to Equation (A.25), we can verify that these allocations satisfy the government budget constraints from market clearing conditions.

Proof of Corollary 1 We first prove that Condition (26) is a necessary condition for the constrained efficiency (i.e., "if" statement). Suppose Condition (26) holds. Suppose that the taxes/subsidies are set according to Corollary 1, i.e., $s_{id}^I = 1/\sigma$ and $t_{id}^B = t_{id}^S = 0$ for all i, d. Suppose also that all supplier subsidies are financed by the local lump-sum taxes, i.e.,

$$T_i^F = -\sum_d \frac{s_{id}^I}{1-s_{id}^I} X_{id} = -\frac{1}{\sigma-1}\sum_d X_{id}$$

Clearly these taxes and transfers satisfy the government budget constraint (A.25) and all equilibrium conditions. Together with Proposition 2, these taxes/subsidies are indeed optimal. Furthermore, from Equation (A.28), ψ_i is proportional to equilibrium retail prices P_i. Finally, since the taxes are financed within each location, trade balance holds from market clearing conditions, i.e., $E_i = R_i$ for all i. This completes the proof of the "if" statement.

Next, we prove that Condition (26) is a sufficient condition for the constrained efficiency (i.e., "only if" statement). From Proposition 2, $\lambda^B \neq 1$ cannot be optimal under the taxes/subsidies in according to Corollary 1. Furthermore, $\lambda^S \neq (1-\beta)\frac{E_d}{R_d}$ cannot be optimal under these taxes/subsidies. At the same time, the only case where $\lambda^S = (1-\beta)\frac{E_d}{R_d}$ holds for all d is when trade balance holds, i.e., $E_d = R_d$, and $\lambda^S = 1-\beta$. Hence,

$\lambda^S \neq 1-\beta$ cannot be optimal under the taxes/subsidies in Corollary 1. Therefore, $\lambda^B \neq 1$ or $\lambda^S \neq 1-\beta$ cannot attain constrained efficiency. This completes the proof of the "only if" statement.

A.7 Proof of Proposition 3

Assume that the welfare weights coincide with the retail goods prices, $\psi_i = P_i$. Denote the nominal GDP by Y_i^F. Then, by noting that we define global nominal GDP as the numeraire ($\sum_i Y_i^F = 1$), and that $d\ln C_i = d\ln P_i$ from cost minimization of retailers (Equation 6),

$$d\ln \mathcal{W} = \sum_i Y_i^F \left(d\ln Y_i^F - d\ln C_i\right) = -\sum_i Y_i^F d\ln C_i. \tag{A.42}$$

We first characterize $d\ln C_i$ through forward (cost) linkages. By applying Shephard's Lemma to firm's cost minimization problem (12),

$$d\ln C_i = (1-\beta)\sum_u \Lambda_{ui}\left(d\ln C_u + d\ln \tau_{ui} - \frac{1}{\sigma-1}d\ln M_{ui}\right). \tag{A.43}$$

Furthermore, from Lemmas 1 and 2,

$$\begin{aligned} d\ln M_{ui} = &\left(\tilde{\lambda}^S + \tilde{\lambda}^B\right)\left(d\ln \Lambda_{ui} + d\ln w_i\right) \\ &- \tilde{\lambda}^S\left(\mu d\ln w_u + (1-\mu)\, d\ln C_u\right) - \tilde{\lambda}^B\left(\mu d\ln w_i + (1-\mu)\, d\ln C_i\right), \end{aligned} \tag{A.44}$$

where we used the fact that $d\ln X_{ui} = d\ln \Lambda_{ui} + d\ln E_i = d\ln \Lambda_{ui} + d\ln w_i$ and $d\ln e_i = \mu d\ln w_i + (1-\mu)\, d\ln C_i$. Reformulating,

$$\left(1-(1-\beta)\frac{1-\mu}{\sigma-1}\tilde{\lambda}^B\right)d\ln C_i = (1-\beta)\sum_u \Lambda_{ui}\left(\left(1+\frac{1-\mu}{\sigma-1}\tilde{\lambda}^S\right)d\ln C_u + d\ln \tau_{ui} + d\ln Z_{ui}\right),$$

where

$$d\ln Z_{ui} = \frac{1}{\sigma-1}\left(\tilde{\lambda}^S \mu d\ln w_u - \left(\tilde{\lambda}^S + \tilde{\lambda}^B (1-\mu)\right)d\ln w_i\right)$$

In matrix notation,

$$d\ln\mathbf{C} = \Phi^{-1}(1-\beta)\left[\mathbf{\Lambda}\cdot(\tau+\mathbf{Z})\right]'\mathbf{1}, \tag{A.45}$$

where $d\ln\mathbf{C}$ is a $1\times|\mathcal{N}|$ vector of $\{d\ln C_i\}$, $\mathbf{\Lambda}$, τ, $\mathbf{Z}$ are matrices of $\{\Lambda_{ui}\}$, $\{d\ln\tau_{ui}\}$, and $\{d\ln Z_{ui}\}$, $\mathbf{1}$ is a $1\times|\mathcal{N}|$ vector of ones, and

$$\Phi = \left[\left(1-\underbrace{(1-\beta)\frac{1-\mu}{\sigma-1}\tilde{\lambda}^B}_{\equiv c^B}\right)\mathbf{I}-(1-\beta)\left(1+\underbrace{\frac{1-\mu}{\sigma-1}\tilde{\lambda}^S}_{\equiv c^S}\right)\mathbf{\Lambda}\right],$$

where $\mathbf{I}$ is the identity matrix. Together,

$$d\ln\mathcal{W} = -\mathbf{Y}^{\mathbf{F}'}\Phi^{-1}(1-\beta)\left[\mathbf{\Lambda}\cdot(\tau+\mathbf{Z})\right]'\mathbf{1}. \tag{A.46}$$

We next characterize nominal GDP Y_i^F through backward (demand) linkages. Let us define $\tilde{R}_i$ as location i's final consumption expenditure (Y_i^F) plus the intermediate goods sales to other firms. Then, $\tilde{R}_i = \frac{1}{\varphi}Y_i^F$, where

$$\varphi = \frac{w_iL_i+\Pi_i}{w_iL_i+\Pi_i+R_i} = \frac{\frac{\beta}{1-\beta}+\frac{\mu}{\sigma}\left(\frac{1}{\gamma^B}+\frac{1-\beta}{\gamma^S}\right)+\frac{1}{\sigma}\left(1-\frac{1}{\gamma^B}-\frac{1-\beta}{\gamma^S}\right)}{\underbrace{\frac{\beta}{1-\beta}+\frac{\mu}{\sigma}\left(\frac{1}{\gamma^B}+\frac{1-\beta}{\gamma^S}\right)}_{=w_iL_i/R_i}+\underbrace{\frac{1}{\sigma}\left(1-\frac{1}{\gamma^B}-\frac{1-\beta}{\gamma^S}\right)}_{=\Pi_i/R_i}+1}. \tag{A.47}$$

Then,

$$\tilde{R}_i = \frac{1}{\varphi}Y_i^F + \underbrace{(1-\beta)\left(1+c^S\right)\sum_d\Lambda_{id}\tilde{R}_d + c^B\tilde{R}_i + \left(-(1-\beta)\left(1+c^S\right)-c^B\right)\tilde{R}_i}_{=0}$$

$$\iff \left(1-c^B\right)\tilde{R}_i = (1-\beta)\left(1+c^S\right)\sum_d\Lambda_{id}\tilde{R}_d + \frac{\left(1-(1-\beta)\left(1+c^S\right)-c^B\right)}{\varphi}Y_i^F,$$

where we used the trade balance $\tilde{R}_i = \sum_d \Lambda_{id}\tilde{R}_d$. In matrix notation,

$$\tilde{\mathbf{R}} = \frac{\left(1-(1-\beta)\left(1+c^S\right)-c^B\right)}{\varphi}\left[\Phi^{-1}\right]'\mathbf{Y^F} = \frac{\left(\beta-(1-\beta)\frac{1-\mu}{\sigma-1}\left(\tilde{\lambda}^B+\tilde{\lambda}^S\right)\right)}{\varphi}\left[\Phi^{-1}\right]'\mathbf{Y^F}.$$

Plugging this expression into (A.46),

$$d\ln\mathcal{W} = -\frac{\varphi}{\left(\beta-(1-\beta)\frac{1-\mu}{\sigma-1}\left(\tilde{\lambda}^B+\tilde{\lambda}^S\right)\right)}\tilde{\mathbf{R}}'(1-\beta)\left[\mathbf{\Lambda}\cdot(\tau+\mathbf{Z})\right]'\mathbf{1}.$$

From the definition of φ in Equation (A.47), $\tilde{R}_i\Lambda_{ui} = \frac{1}{1-\varphi}X_{ui}$. Furthermore, $\tilde{R}_i$ is proportional to Y_i^F and w_iL_i. Therefore, under the normalization that $\sum_i Y_i^F = 1$, $\tilde{\mathbf{R}}'\left[\mathbf{\Lambda}\cdot\mathbf{Z}\right]' = \mathbf{0}$. Hence we have

$$d\ln\mathcal{W} = -\frac{\varsigma}{\left(\frac{\beta}{1-\beta}-\frac{1-\mu}{\sigma-1}\left(\tilde{\lambda}^B+\tilde{\lambda}^S\right)\right)}\sum_{u,d}X_{ud}d\ln\tau_{ud}, \tag{A.48}$$

where $\varsigma = \frac{w_iL_i+\Pi_i}{E_i} = \frac{\varphi}{1-\varphi}$ is the ratio of nominal world GDP to nominal world intermediate goods expenditure, as defined in the main text. Hence we obtain the expression in Proposition 3.

Proposition 3 under optimal sales subsidy (Derivation of Equation 30) Suppose that optimal sales subsidy $s_{id}^I = s^I = 1/\sigma$ are in place as considered in Corollary 1. The only difference from laissez-faire equilibrium above is the expression for ς, which becomes nominal final consumption expenditure (instead of GDP) such that

$$\begin{aligned}\varsigma^{\text{SalesSubsidy}} &\equiv \frac{w_iL_i+\Pi_i+T_i^FL_i}{E_i}\\ &= \frac{\beta}{1-\beta}+\frac{1}{1-s^I}\frac{1}{\sigma}\mu\left(\frac{1}{\gamma^B}+\frac{1-\beta}{\gamma^S}\right)+\frac{1}{1-s^I}\frac{1}{\sigma}\left(1-\frac{1}{\gamma^B}-\frac{1-\beta}{\gamma^S}\right)-\frac{1}{1-s^I}\frac{1}{\sigma}\end{aligned}$$

Reformulating the expression and plugging it into Equation (A.48) yields Equation (30).

A.8 Proof of Proposition 4

Applying Shephard's Lemma to firm's cost minimization problem (12), and by taking an alternative numeraire such that$w_i = 1$,

$$\begin{aligned} d\ln C_i &= (1-\beta)\sum_u \Lambda_{ui}(d\ln C_u + d\ln\tau_{ui} - \frac{1}{\sigma-1}d\ln M_{ui}) \\ &= (1-\beta)(d\ln C_i - \frac{1}{\sigma-1}d\ln M_{ii} + \frac{1}{\sigma-1}d\ln\Lambda_{ii}) \\ &= \frac{1}{\sigma-1}\frac{1-\beta}{\beta}d\ln\Lambda_{ii} - \frac{1}{\sigma-1}\frac{1-\beta}{\beta}d\ln M_{ii}, \end{aligned} \tag{A.49}$$

where the second transformation used the fact that $d\ln\Lambda_{ii} = -d\ln C_i + \frac{1}{\sigma-1}d\ln M_{ii} - \sum_u \Lambda_{ui}(-d\ln C_u - d\ln\tau_{ui} + \frac{1}{\sigma-1}d\ln M_{ui})$.

Notice also that, from cost minimization of retailers (Equation 6), $d\ln P_i = d\ln C_i$. Furthermore, firm profit is a constant fraction of labor compensation. Hence $d\ln Q_i^F = -d\ln C_i$.

Furthermore, from Lemmas 1 and 2,

$$\begin{aligned} d\ln M_{ii} &= \left(\tilde{\lambda}^S + \tilde{\lambda}^B\right)d\ln X_{ii} - \left(\tilde{\lambda}^S + \tilde{\lambda}^B\right)d\ln e_i \\ &= \left(\tilde{\lambda}^S + \tilde{\lambda}^B\right)d\ln\Lambda_{ii} - (1-\mu)\left(\tilde{\lambda}^S + \tilde{\lambda}^B\right)d\ln P_i, \end{aligned} \tag{A.50}$$

where the last formulation uses the fact that $d\ln X_{ii} = d\ln\Lambda_{ii} + d\ln E_i = d\ln\Lambda_{ii}$ under our normalization of $w_i = 1$. Combining these together, we have

$$d\ln Q_i^F = -\frac{\frac{1-\beta}{\beta}\frac{1}{\sigma-1}\left(1-\left(\tilde{\lambda}^S + \tilde{\lambda}^B\right)\right)}{1-\frac{1}{\sigma-1}\frac{1-\beta}{\beta}(1-\mu)\left(\tilde{\lambda}^S + \tilde{\lambda}^B\right)}d\ln\Lambda_{ii}.$$

Manipulating this equation, we obtain the expression in Proposition 4.

B Details on Isomorphisms

B.1 General Results

We first establish the general results. Consider a generalized economy of Section 2 by allowing for an arbitrary aggregation of intermediate input bundles (instead of CES in Equation 2) and arbitrary microfoundation of the network formation $\{\mathcal{S}_{ui}(\cdot)\}$ (instead of search and matching in Section 2.2); see Appendix D for the formal set-up. We first introduce three macro restrictions extending Arkolakis et al. (2012) to an environment with endogenous production networks.

Condition 1. Aggregate trade balance holds, $\sum_i X_{ui} = \sum_d X_{id}$ for all i.

Condition 2. Aggregate profit Π_i and labor compensation $w_i L_i$ are constant fraction of aggregate intermediate goods sales (excluding those to retailers) R_i for all i. We denote GDP-to-intermediate-goods-expenditure ratio $\varsigma \equiv (w_i L_i + \Pi_i) / R_i$.

Condition 3. Aggregate trade elasticity is CES. That is, $\frac{\partial \ln X_{ui}/X_{ii}}{\partial \ln \tau_{ui}} = \varepsilon$ for all u, i.

The three conditions are the same as Arkolakis et al. (2012) except that Condition 2 is extended to an environment with intermediates. In addition, we introduce two more macro restrictions that are relevant for the endogenous production networks.

Condition 4. Denote the aggregate measure of links by $M_{ui} = \int_{\omega \in \Omega_i} S_{ui}(\omega)\, d\omega$. Then, it follows the following equilibrium relationships:

$$\begin{aligned} d \ln M_{ui} = {} & (\delta_{L,U} + \delta_{Q,U} + \delta_{L,D} + \delta_{Q,D})\, d \ln X_{ui} \\ & - \delta_{L,U} d \ln w_u - \delta_{Q,U} d \ln P_u - \delta_{L,D} d \ln w_d - \delta_{Q,D} d \ln P_d \end{aligned} \tag{B.1}$$

Condition 5. The retail goods cost has the following relationships

$$d \ln P_i = \beta d \ln w_i + (1-\beta) \sum_u \Lambda_{ui} \left(d \ln P_u + d \ln \tau_{ui} - \nu d \ln M_{ui} \right) \tag{B.2}$$

where ν is some constant parameter.

The following proposition shows that versions of Propositions 3 and 4 hold if these macro restrictions are satisfied.

Proposition 5. *Consider global welfare with GDP weights: $\psi_i = w_i L_i + \Pi_i$. Under Conditions 1- 5, the first-order effect of a shock in iceberg trade costs $\{d\ln\tau_{ij}\}$ on global welfare is given by:*

$$d\ln\mathcal{W} = \frac{\varsigma}{\frac{\beta}{1-\beta} - \nu\left(\delta_{Q,U} + \delta_{Q,D}\right)} \sum_{u,d} X_{ud} d\ln\tau_{ud}, \tag{B.3}$$

where ς is the ratio of nominal world GDP to nominal world intermediate goods expenditure by firms defined in Condition 2.

Proposition 6. *If Conditions 1- 5 are satisfied, the welfare changes in location i from external shocks of any magnitude is given by:*

$$d\ln Q_i^F = -\frac{1}{\varepsilon}\frac{1}{\frac{\beta}{1-\beta} - \nu\left(\delta_{Q,U} + \delta_{Q,D}\right)} d\ln\Lambda_{ii}, \tag{B.4}$$

The proof of this proposition, as well as the one below, follows a similar structure as the proof of Proposition 3 (by replacing Equations A.44 and A.43 with Equations B.1 and B.2) and Proposition 4 (by replacing Equations A.50 and A.49 with Equations B.1 and B.2), and hence omitted here.

B.2 Search and Matching

We start by discussing that our baseline model satisfies Conditions 1-5. Condition 1 is trivial from the market clearing conditions (see Footnote 1). Condition 2 holds because our model predicts constant aggregate profit and labor share, as discussed in Section 2.2. Finally, Condition 3 holds as with ε given by Equation (19). Condition 4 is satisfied by the isoelastic search decisions (8) and Cobb-Douglas matching technology (16), with $\delta_{L,U} = \mu\tilde{\lambda}^B$, $\delta_{Q,U} = (1-\mu)\tilde{\lambda}^B$, $\delta_{L,D} = \mu\tilde{\lambda}^S$, and $\delta_{Q,D} = (1-\mu)\tilde{\lambda}^S$. Condition 5 holds from cost minimization (12) with $\nu = 1/(\sigma-1)$, which captures the degree of love of variety in intermediate inputs.

We also argue that the macro restrictions hold in the alternative specifications where either supplier or buyer search is undirected with respect to locations. First, consider the case where buyer search is undirected. The search costs are modified from Equation (8) to

$$f_i\left(n_i^B, \{n_{ui}^S\}_u\right) = e_i \left\{ f_i^B \frac{\left(n_i^B\right)^{\gamma^B}}{\gamma^B} + \sum_{u\in\mathcal{N}} f_{ui}^S \frac{\left(n_{ui}^S\right)^{\gamma^S}}{\gamma^S} \right\},$$

where notice that buyer search n_i^B does not depend on destinations d. The matching technology is modified from Equation (16) to

$$M_{ud} = \kappa_{ud} \left(\overline{M}_u^B\right)^{\lambda^B} \left(\overline{M}_{ud}^S\right)^{\lambda^S},$$

where $\overline{M}_u^B = N_u \int_z n_u^B(z) dG_u(z)$ and $\overline{M}_{ud}^S = N_d \int_z n_{ud}^S(z) dG_d(z)$. Given M_{ud}, the matching rates m_u^S and m_u^B are determined as:

$$m_u^B = \sum_d \frac{M_{ud}}{\overline{M}_u^B}, \qquad m_{ud}^S = \frac{M_{ud}}{\overline{M}_{ud}^S}.$$

Following the same algebra as in Lemmas 1 and 2, we can verify that this model satisfies the macro restrictions with $\varepsilon = \frac{\sigma-1}{1-\tilde{\lambda}^S}$, $\delta_{L,U} = -(1-\mu)\tilde{\lambda}^B$, $\delta_{Q,U} = (1-\mu)\tilde{\lambda}^B$, $\delta_{L,D} = \mu\tilde{\lambda}^S$, $\delta_{Q,D} = (1-\mu)\tilde{\lambda}^S$, and $\nu = 1/(\sigma-1)$. Therefore, the only difference in our welfare sufficient statistics (36) and (37) arises through the differences in trade elasticity ε. Importantly, trade elasticity is smaller than our baseline model, as it only depends on the search and matching elasticities on the supplier search side.

The case where supplier search is undirected can be considered analogously. In this case, this model satisfies the macro restrictions with $\varepsilon = \frac{\sigma-1}{1-\tilde{\lambda}^B}$, $\delta_{L,U} = \mu\tilde{\lambda}^B$, $\delta_{Q,U} = (1-\mu)\tilde{\lambda}^B$, $\delta_{L,D} = -(1-\mu)\tilde{\lambda}^S$, $\delta_{Q,D} = (1-\mu)\tilde{\lambda}^S$, and $\nu = 1/(\sigma-1)$.

B.3 Relationship-Specific Fixed Cost

This section develops a production network formation model with relationship-specific fixed cost incurred by suppliers. The structure of production given networks follow exactly as in Section 2.1. In addition, we assume that the productivity $z_i(\omega)$ follows Pareto distribution with shape parameter θ with lower bound z^*. In terms of network formation, we assume that any pair of a supplier in u and a buyer in d form a relationship as long as the supplier is willing to pay a fixed cost f_{ud}. These fixed cost are paid as a Cobb-Douglas composite of supplier's location's labor and retail goods with labor share μ, similarly to the search costs in our baseline specification.

Let us conjecture that firm's production cost and intermediate goods revenue take the following form:

$$c_i(z) = C_i z^{-\gamma_C}, \qquad r_i(z) = R_i z^{\gamma_R},$$

where γ_C, γ_R are constants, and C_i and R_i are location-level cost and revenue shifter. Furthermore, conjecture that the upper-bound of the marginal cost of firms in location u to supply to a firm in location i with productivity z (by paying a fixed cost) takes the following form:

$$\overline{c}_{ui}(z) = \Omega_{ui} z^{\gamma_P},$$

where γ_P is a constant. We now confirm these conjectures are correct and derive the expressions for $\gamma_C, \gamma_R, \gamma_P$.

First, from cost minimization,

$$c_i(z) = \frac{1}{z} w_i^{\beta} \tilde{p}_i(z)^{1-\beta}, \qquad \tilde{p}_i(z) = \left(\sum_u \left(\frac{\sigma}{\sigma-1} \overline{c}_{ui}(z) \right)^{\gamma_C \theta} \int_{p \geq \frac{\sigma}{\sigma-1} \overline{c}_{ui}(z)} p^{1-\sigma} dG_c(p) \right)^{\frac{1}{1-\sigma}} \tag{B.5}$$

where $G_c(\cdot)$ is an inverse Pareto distribution with lower bound $\frac{\sigma}{\sigma-1}\overline{c}_{ui}(z)$ and shape parameter $\gamma_C \theta$.

Next, we consider the zero-profit condition for a marginal supplier to sell to a buyer. Denote the revenue by a supplier with marginal cost c to a buyer with productivity z, conditional on positive transaction, by

$$r_{ui}(c\tau_{ui}, z) = \frac{\left(\frac{\sigma}{\sigma-1} c\tau_{ui}\right)^{1-\sigma}}{\tilde{p}_i(z)^{1-\sigma}} r_i(z)$$

Similarly as in Bernard et al. (2018), we consider a limit where the lower-bound of productivity $z^* \to 0$. Then, from zero-profit condition of a marginal supplier,

$$r_{ui}(\overline{c}_{ui}(z)\tau_{ui}, z) = \frac{\left(\frac{\sigma}{\sigma-1} \overline{c}_{ui}(z)\tau_{ui}\right)^{1-\sigma}}{\tilde{p}_i(z)^{1-\sigma}} r_i(z) = f_{ui} w_u^{\mu} P_u^{1-\mu} \tag{B.6}$$

Finally, from the definition of intermediate firm revenue,

$$r_u(z) = r_u^F(z) + \sum_d N_d \int_{c_u(z) \geq \bar{c}_{ui}(z')} r_{ud}(c_u(z)\,\tau_{ud}, z') dG_d(z') \tag{B.7}$$

where $r_u^F(z)$ is sales to local retailers, and N_d proxies the measure of firms in location d. To obtain closed form solution, we assume that retail goods production technology (D.5) takes the form of CES with the elasticity of substitution γ_R, such that $r_u^F(z) = R_u^F z^{\gamma_R}$.

By focusing on the exponents on z, Equations (B.5), (B.6), (B.7) are satisfied with

$$\gamma_C = 1 + (1-\beta)\,\gamma_P \frac{\gamma_C\theta - (\sigma - 1)}{\sigma - 1}$$

$$\gamma_R = \gamma_P \gamma_C \theta$$

$$\gamma_R = \gamma_C \left(\gamma_C\theta - \frac{\gamma_R}{\gamma_P}\right)$$

Notice now that aggregate expenditure share is given by

$$\Lambda_{ui} = \frac{\int_{c_u(z)\geq\bar{c}_{ui}(z')} r_{ud}(c_u(z)\,\tau_{ud}, z') dG_d(z')}{\sum_u \int_{c_u(z)\geq\bar{c}_{ui}(z')} r_{ud}(c_u(z)\,\tau_{ud}, z') dG_d(z')} = \frac{\bar{c}_{ui}(z')^{\gamma_C\theta}}{\sum_u \bar{c}_{ui}(z')^{\gamma_C\theta}}$$

Hence Condition 3 holds with

$$\varepsilon = \gamma_C\theta.$$

Furthermore, from Equation B.5,

$$\nu = \frac{1}{\sigma - 1} - \frac{1}{\gamma_C\theta}.$$

It is also straightforward to verify that the other conditions hold.

B.4 Discrete Choice of Suppliers

In this section, we discuss a version of Oberfield (2018) and Eaton et al. (2024). Firms choose the optimal supplier among the randomly drawn potential suppliers. Suppliers provide homogenous products for each task. Therefore, production technology (D.2) can be given by

$$\tilde{q}_i(\omega) = \sum_u \sum_{\upsilon \in S_{ui}(\omega)} q_{ui}(\upsilon, \omega).$$

Hence the marginal cost of production (D.3) is given by

$$c_i(\omega) = \frac{1}{z_i(\omega)} w_i^{\beta} \left(\min_{\upsilon \in S_{ui}(\omega)} c_u(\upsilon) \tau_{ui} \right)^{1-\beta}.$$

We also assume that the set of suppliers $S_{ui}(\omega)$ is determined simply by exogenous random process, i.e., probability of supplier in u to match with i is given by a Poisson process with exogenous location-pair-specific parameter.[2]

As Oberfield (2018) and Eaton et al. (2024), this model yields a tractable solution if we assume the Pareto productivity distribution. In particular, aggregate expenditure share is given by

$$\Lambda_{ui} = \frac{(P_u \tau_{ui})^{\theta}}{\sum_u (P_u \tau_{ui})^{\theta}}$$

Therefore, Condition 3 holds with

$$\varepsilon = \theta$$

Furthermore, given that $S_{ui}(\omega)$ is determined simply by exogenous random process without using resource costs, Condition 3 holds with

$$\delta_{L,U} = \delta_{Q,U} = \delta_{L,D} = \delta_{Q,D} = 0$$

It is also straightforward to verify that the other conditions hold.

[2] Eaton et al. (2024) considers a set up where the matching rate depends on the supplier's production cost. In this case, the aggregate trade elasticity is affected by how the matching rate varies by the supplier's production efficiency. Furthermore, we abstract in-house production for intermediate goods, which leads to a violation of the constant labor share restriction.

Online Supplement for "Production Network Formation, Trade, and Welfare"

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C Multiple Sectors

This section develops a model of endogenous production network formation across space and sectors.

The economy is segmented by a finite number of locations denoted by $u, i, d \in \mathcal{N}$. In each location, there is an L_i measure of households. Each household supplies one unit of labor inelastically and earns a competitive wage w_i. There is a fixed mass of intermediate goods producers in each location, which we call "firms" in short. Firms also belong to a sector, denoted by $h, k, l \in K$. We denote each firm in location i and sector k by $\omega \in \Omega_{i,k}$ and the measure of firms in location i and sector k by $N_{i,k}$. We denote the distribution of total factor productivity (TFP) by firms in location i and sector k by$G_{i,k}(\cdot)$, which can flexibly depend on the location. Firms produce differentiated intermediate goods combining labor and intermediate goods. Intermediate goods can be traded across firms in different locations subject to iceberg trade costs as long as there are production linkages between them. Local competitive retailers, and the retailers sell the combined composites to local consumers.

We model production networks as the presence of connections between supplier firms and buyer firms. Specifically, we denote $S_{ui,hk}(\omega) \subseteq \Omega_{u,h}$ to indicate the set of suppliers producing in u that a firm ω in location i can purchase from. In what follows, we first describe how production occurs given networks $\{S_{ui,hk}(\cdot)\}_{u,i,h,k}$. We then describe how these networks are endogenously formed through a search and matching process.

C.1 Production given Networks

Firms. A continuum of firms produces a distinct variety in each location and sector. Production of intermediate goods requires labor and intermediate inputs. Intermediate inputs are sourced from firms that are directly connected by production networks. The production function of firm $\omega \in \Omega_{i,k}$ is given by

$$q_{i.k}\left(\omega\right) = z_{i,k}\left(\omega\right)\left(\frac{l_{i,k}\left(\omega\right)}{\beta_{k,L}}\right)^{\beta_{k,L}} \prod_{h}\left(\frac{\tilde{q}_{i,hk}\left(\omega\right)}{\beta_{hk}}\right)^{\beta_{hk}}, \tag{C.1}$$

where $z_{i,k}\left(\omega\right)$ is the total factor productivity (TFP) of firm ω which follows distribution $G_{i,k}(\cdot)$, $l_{i,k}\left(\omega\right)$ is labor inputs, and $\tilde{q}_{i,k}\left(\omega\right)$ is the composite of intermediate inputs, $\beta_{k,L}$ is the parameter proxying the input share for labor, and β_{hk} is the parameter proxying the input share for intermediate inputs from sector h. We assume that the production technology is constant returns to scale such that $\beta_{s,L} + \sum_{k\in K}\beta_{ks} = 1$. The composite of intermediate inputs is a constant elasticity of substitution (CES) aggregator of the input varieties sourced from their connected suppliers. Therefore,

$$\tilde{q}_{i,hk}\left(\omega\right) = \left(\sum_{u}\int_{\upsilon\in S_{ui,hk}(\omega)} q_{ui,hk}\left(\upsilon,\omega\right)^{\frac{\sigma_h - 1}{\sigma_h}} d\upsilon\right)^{\frac{\sigma_h}{\sigma_h - 1}}, \tag{C.2}$$

where $q_{ui,hk}(\upsilon,\omega)$ is the quantity of input for each variety, and σ_h is the elasticity of substitution.

Given these production functions, the marginal cost of production by firm ω is given by

$$c_{i,k}\left(\omega\right) = \frac{1}{z_{i,k}\left(\omega\right)} w_i^{\beta_{k,L}} \prod_{h}\left(\sum_{u}\int_{\upsilon\in S_{ui,hk}(\omega)} p_{ui,hk}\left(\upsilon,\omega\right)^{1-\sigma_h} d\upsilon\right)^{\frac{\beta_{hk}}{1-\sigma_h}}, \tag{C.3}$$

where $p_{ui,hk}\left(\upsilon,\omega\right)$ is the intermediate goods price that supplier υ in location u and sector h charges to firm ω in location i and sector k. On top of these production cost, when a firm sells their intermediate goods to location d and sector l, they incur an iceberg trade cost of $\tau_{id,kl} \geq 1$.

We assume that all firms are matched with a continuum of suppliers. Furthermore, suppliers are under monopolistic competition to supply to each buyer. Thus, given the isoelastic intermediate goods demand (Equation C.3), suppliers charge a constant markup of their marginal cost net of the iceberg trade cost;

$$p_{id,kl}\left(\upsilon,\omega\right)=\tilde{\sigma}_{k}c_{i,k}\left(\omega\right)\tau_{id,kl}, \tag{C.4}$$

where $\tilde{\sigma}_k = \sigma_k / \left(\sigma_k - 1\right)$ is the markup ratio.

Notice that firms producing in the same location with the same productivity $z_{i,k}\left(\omega\right)$ charge the same prices and earn the same profit. As we describe below, they also make the same decisions regarding supplier and buyer search. Therefore, without risk of confusion, we sometimes index the cost function using z instead of ω, e.g., $c_{i,k}(z)$ instead of $c_{i,k}(\omega)$ for firm ω whose productivity is $z = z_{i,k}\left(\omega\right)$.

Retailers. Perfectly competitive retailers in each location i and sector k combine intermediate inputs from all firms in location i and sector k and produce a standardized nontradable retail goods. Their production function is given by

$$Q_{i,k}=g_{i,k}\left(\left\{q_{i,k}^{R}\left(\omega\right)\right\}_{\omega\in\Omega_{i,k}}\right), \tag{C.5}$$

where $g_{i,k}\left(\cdot\right)$ is a function that satisfies homogeneous of degree one, and $q_{i,k}^{R}\left(\omega\right)$ is the quantity of intermediate inputs from firm ω. The retail goods are used for final consumption purposes and by search service sector for production network formation, as we describe further below.

We also assume that retailers have the entire bargaining power when purchasing intermediate inputs from each firm at their marginal cost $c_{i,k}\left(\upsilon\right)$. Under cost minimization, retail prices are given by

$$P_{i,k}=\tilde{g}_{i,k}\left(\left\{c_{i,k}\left(\omega\right)\right\}_{\omega\in\Omega_{i,k}}\right), \tag{C.6}$$

where $\tilde{g}_{i,k}\left(\cdot\right)$ is a solution to the cost minimization problem by retailers.

Final Consumers. Measure L_i Households supply labor inelastically at wage w_i. They also own an equal share of local firms. Therefore their budget constraint is given by

$$P_{i}Q_{i}^{F}=w_{i}+\frac{\sum_{k}\Pi_{i,k}}{L_{i}}, \tag{C.7}$$

where $\Pi_{i,k}$ is aggregate profit by firms producing in location i. Q_i^F is a Cobb-Douglas aggregator of retail goods across sector, i.e.,

$$Q_i^F = \prod_k \left(Q_{i,k}^F\right)^{\alpha_k}. \tag{C.8}$$

C.2 Production Network Formation

We now describe how the production network structure, $\{\mathcal{S}_{ui,hk}(\cdot)\}$, is endogenously determined. To capture the notion that establishing supplier and buyer connections are costly and frictions, we model that these connections arise as a consequence of search and matching process, building on the literature in labor search and matching (Diamond 1982; Mortensen 1986; Pissarides 1985).

C.2.1 Search Decision

To search for suppliers and buyers, firms use services from local search intermediaries. In particular, firms in location i determine how much search services to use for supplier search in many different locations and sectors, $\{n_{ui,hk}^S\}_u$, and for buyer search in many different locations and sectors, $\{n_{id,kl}^B\}_{d,l}$. Each unit of supplier search $\{n_{ui,hk}^S\}_u$ will turn into a successful supplier relationship at rate $\{m_{ui,hk}^S\}_{u,h}$; and each unit of buyer search $\{n_{id,kl}^B\}_{d,l}$ will turn into a successful buyer relationship at rate $\{m_{id,kl}^B\}_{d,l}$; where we describe how $\{m_{ui,hk}^S\}_{u,h}$ and $\{m_{id,kl}^B\}_{d,l}$ are endogenously determined through the matching technology in the next section. While our terminology of "search services" may sound specific, one should interpret these search services to include a broader notion of the cost to establish supplier and buyer connections; which not only includes the literal external intermediation cost, but also other costs such as identifying the right suppliers and buyers, customization, quality assurance, or investment for relationship building. The total search costs payed by the firm is given by

$$f_{i,k}\left(\{n_{id,kl}^B\}_{d,l}, \{n_{ui,mk}^S\}_{u,m}\right) = e_{i,k}\left\{\sum_{d\in\mathcal{N}}\sum_{l} f_{id,kl}^B \frac{\left(n_{id,kl}^B\right)^{\gamma_k^B}}{\gamma_k^B} + \sum_{u\in\mathcal{N}}\sum_{h} f_{ui,hk}^S \frac{\left(n_{ui,hk}^S\right)^{\gamma_k^S}}{\gamma_k^S}\right\}, \tag{C.9}$$

where $e_{i,k}$ is the unit cost of search service in location i and sector k, and γ_k^B and γ_k^S are parameters capturing the decreasing returns in search investment. $\{f_{id,kl}^B\}$ and $\{f_{ui,hk}^S\}$ are location-pair-specific search cost shifters, capturing the notion that the cost of searching for suppliers and buyers may depend on spatial frictions.

We now define firms' search decisions. Given the random matching with suppliers, and the cost function (3), the intermediate goods cost is affected by supplier search decisions $\{n_{ui}^S\}_u$ as follows:

$$c_{i,k}\left(z,\{n_{ui,hk}^S\}_{u,h}\right) = \frac{1}{z} w_{i,k}^{\beta_{k,L}} \prod_h \left(\sum_{u\in\mathcal{N}} n_{ui,hk}^S m_{ui,hk}^S C_{ui,hk}^{1-\sigma_h}\right)^{\frac{\beta_{hk}}{1-\sigma_h}}, \tag{C.10}$$

where $C_{ui,hk}^{1-\sigma} \equiv \int \left(\tilde{\sigma}_h c_{u,h}(z)\,\tau_{ui,hk}\right)^{1-\sigma_h} dG_{ui,hk}^B(z)$ is the CES aggregator of the price of a supplier producing in location u in sector h to supply to location i and sector k, and $G_{ui,hk}^B(z)$ is the distribution of productivity weighted by the buyer search intensity. Depending on this cost, the firm in location i with productivity z's expected profit per buyer in location d is given by $\frac{1}{\sigma_k}\left(\tilde{\sigma}_k \tau_{id,kl} c_i\left(z,\{n_{ui,hk}^S\}_{u,h}\right)\right)^{1-\sigma_k} D_{d,kl}$, where $D_{d,kl}$ is the average demand net of the price index averaged across all buyers. Together, the optimal search decisions for $\{n_{ui,hk}^S\}_u$, $\{n_{id,kl}^B\}_{d,l}$ are given by:

$$\tilde{\pi}_{i,k}(\omega) \equiv \max_{\{n_{id,kl}^B\}_{d,l},\{n_{ui,hk}^S\}_{u,h}} \sum_{l,d} m_{id,kl}^B n_{id,kl}^B \frac{1}{\sigma_k}\left(\tilde{\sigma}_k \tau_{id,kl} c_{i,k}\left(z,\{n_{ui,hk}^S\}_{u,h}\right)\right)^{1-\sigma_k} D_{d,kl}$$

(C.11)

$$- e_{i,k}\left\{\sum_{d\in\mathcal{N}}\sum_l f_{id,kl}^B \frac{\left(n_{id,kl}^B\right)^{\gamma_k^B}}{\gamma_k^B} + \sum_{u\in\mathcal{N}}\sum_h f_{ui,hk}^S \frac{\left(n_{ui,hk}^S\right)^{\gamma_k^S}}{\gamma_k^S}\right\},$$

subject to $(C.10)$.

The first term inside the max operator represents the firm profit, and the last term is the search cost, as discussed above. We impose a parameter restriction that $1 - \frac{1}{\gamma_k^B} - \frac{\sum_h \beta_{hk}}{\gamma_k^S} > 0$, which guarantees that firms make positive sales and profit. The following lemma characterizes the solution to this problem.

Lemma 3. *The solution to the supplier and buyer search problem (C.11) is given by*

$$n^B_{id,kl}(z) = a^B_{id,kl} z^{\frac{\delta_k}{\gamma^B_k}};\; n^S_{ui,hk}(z) = a^S_{ui,hk} z^{\frac{\delta_k}{\gamma^S_k}}, \tag{C.12}$$

where $\delta_k \equiv (\sigma_k - 1) / \left(1 - \frac{1}{\gamma^B_k} - \frac{1}{\gamma^S_k}\sum_h \beta_{hk}\frac{\sigma_k-1}{\sigma_h-1}\right)$*, where*

$$a^B_{id,kl} = \left(\Gamma^B_{i,k}\frac{X_{id,kl}}{e_{i,k} f^B_{id,kl}}\right)^{\frac{1}{\gamma^B_k}}, \qquad a^S_{ui,hk} = \left(\Gamma^S_{i,k}\frac{R_{i,k}}{E_{i,k}}\frac{X_{ui,hk}}{e_{i,k} f^S_{ui,hk}}\right)^{\frac{1}{\gamma^S_k}} \tag{C.13}$$

where $\Gamma^S_{i,k} = \frac{1}{\sigma_k \mathbb{M}_{i,k}(\delta_k) N_{i,k}}$ *and* $\Gamma^B_{i,k} = \frac{1}{\sigma_k \mathbb{M}_{i,k}(\delta_k) N_{i,k}}\beta_{hk}\frac{\sigma_k-1}{\sigma_h-1}$*, and* $X_{id,kl}$ *is total nominal trade flows from* i, k *to* d, l*.*

Furthermore, the unit cost of firms, $c_i(z)$, can be expressed as

$$c_{i,k}(z) = C_{i,k} z^{-\frac{\delta_k}{\gamma^S_k}\sum_h \frac{\beta_{hk}}{\sigma_h-1} - 1}, \quad (C_{i,k})^{1-\sigma_k} \equiv w_i^{\beta_{k,L}(1-\sigma_k)} \prod_h \left(\sum_{u\in\mathcal{N}} a^S_{ui,hk} m^S_{ui,hk} (C_{ui,hk})^{1-\sigma_h}\right)^{\frac{\beta_{hk}}{1-\sigma_h}(1-\sigma_k)}, \tag{C.14}$$

Total revenue from sales to other firms (excluding sales to retailers) is

$$r_{i,k}(z) = (\tilde{\sigma}_k)^{1-\sigma_k} D^*_{i,k} (C_{i,k})^{1-\sigma_k} (z)^{\delta_k}, \quad D^*_{i,k} = \sum_{d\in\mathcal{N}}\sum_l m^B_{id,kl} a^B_{id,kl} D_{d,kl} (\tau_{id,kl})^{1-\sigma_k}. \tag{C.15}$$

Search intermediaries. In each location, perfectly competitive search intermediaries provide search services by combining labor and retail goods. Their production function is given by

$$A_{i,k} = \left(\frac{L^A_{i,k}}{\mu_k}\right)^{\mu_k}\left(\frac{Q^A_{i,k}}{1-\mu_k}\right)^{1-\mu_k}, \tag{C.16}$$

where $L^A_{i,k}$ and $Q^A_{i,k}$ corresponds to the amount of labor and retail goods, and μ_k is the labor share for search intermediaries. The profit maximization condition implies that the price of search services is given by

$$e_{i,k} = (w_i)^{\mu_k} (P_{i,k})^{1-\mu_k} . \tag{C.17}$$

C.2.2 Matching Technology

The matching rates between suppliers and buyers, $m^S_{ud,hl}$ and $m^B_{ud,hl}$, are determined for each pair of locations and sectors. We follow a long tradition in the literature of labor search and matching (Diamond 1982; Mortensen 1986; Pissarides 1985) and assume that only a fraction of supplier and buyer search lead to a successful match. The measure of total matches created for each pair of locations is determined by the matching function that takes the aggregate supplier and buyer postings as arguments. The aggregate supplier search by buyers in location d in sector l in sector for suppliers in location u and sector h is given by:

$$\overline{M}^S_{ud,hl} = N_{d,l} \int n^S_{ud,hl}(z) dG_{d,l}(z) = N_{d,l} a^S_{ud,hl} \mathbb{M}_{d,l} \left(\frac{\delta_l}{\gamma^S_l} \right), \tag{C.18}$$

where we define $\mathbb{M}_{d,l}(\chi) \equiv \int z^\chi dG_{d,l}(z)$. Similarly, the aggregate buyer search by suppliers in location u and sector h for buyers in location d and sector l is given by:

$$\overline{M}^B_{ud,hl} = N_{u,h} \int n^B_{ud,hl}(z) dG_{u,h}(z) = N_{u,h} a^B_{ud,hl} \mathbb{M}_{u,h} \left(\frac{\delta_h}{\gamma^B_h} \right). \tag{C.19}$$

The aggregate measure of successful matches between a pair of locations, M_{ud}, is determined by the following Cobb-Douglas matching function:

$$M_{ud,hl} = \kappa_{ud,hl} \left(\overline{M}^S_{ud,hl} \right)^{\lambda^S_{hl}} \left(\overline{M}^B_{ud,hl} \right)^{\lambda^B_{hl}}, \tag{C.20}$$

where λ^S_{hl}, $\lambda^B_{hl} \geq 0$ denote the elasticities of total matches created for the pair of regions with respect to the supplier and buyer search, respectively, and $\kappa_{ud,hl}$ is the parameter governing the efficiency of matching technology. Given $M_{ud,hl}$, the matching rates m^S_{ud}

and m^B_{ud} are defined by:

$$m^S_{ud,hl} = \frac{M_{ud,hl}}{\overline{M}^S_{ud,hl}}, \qquad m^B_{ud,hl} = \frac{M_{ud,hl}}{\overline{M}^B_{ud,hl}}. \tag{C.21}$$

C.2.3 Aggregate Trade Flows and Trade Elasticity

The analytical characterization of the firm search decision combined with the Cobb-Douglas matching technology yields a tractable expression for the aggregate production networks and trade flows.

Lemma 4. *The measure of supplier-to-buyer relationships from supplier location u to buyer location d (extensive margin), $M_{ud,hl}$, and the average transaction volume per relationship (intensive margin), $\overline{r}_{ud,hl}$, are given by the following gravity equations:*

$$M_{ud,hl} = \rho^E_{hl}\chi^E_{ud,hl}\zeta^E_{u,hl}\xi^E_{d,hl}, \quad \overline{r}_{ud,hl} = \varrho^I_{hl}\chi^I_{ud,hl}\zeta^I_{u,h}\xi^I_{d,hl}, \tag{C.22}$$

where we define $\tilde{\lambda}^S_{hl} \equiv \lambda^S_{hl}/\gamma^S_l$, $\tilde{\lambda}^B_{hl} \equiv \lambda^B_{hl}/\gamma^B_h$, and

$$\rho^E_{hl} = \left(\varrho^I_{hl}\right)^{\frac{\tilde{\lambda}^S_{hl}+\tilde{\lambda}^B_{hl}}{1-\tilde{\lambda}^S_{hl}-\tilde{\lambda}^B_{hl}}(1-\sigma_h)}$$

$$\zeta^E_{u,hl} = C_{u,h}^{\frac{\tilde{\lambda}^S_{hl}+\tilde{\lambda}^B_{hl}}{1-\tilde{\lambda}^S_{hl}-\tilde{\lambda}^B_{hl}}(1-\sigma_h)} e_{u,h}^{-\frac{\tilde{\lambda}^S_{hl}}{1-\tilde{\lambda}^S_{hl}-\tilde{\lambda}^B_{hl}}}$$

$$\xi^E_{d,hl} = \left(\frac{R_{d,l}}{E_{d,l}}\right)^{\frac{\tilde{\lambda}^B_{hl}}{1-\tilde{\lambda}^S_{hl}-\tilde{\lambda}^B_{hl}}} D_{d,hl}^{\frac{\tilde{\lambda}^S_{hl}+\tilde{\lambda}^B_{hl}}{1-\tilde{\lambda}^S_{hl}-\tilde{\lambda}^B_{hl}}} e_{d,l}^{-\frac{\tilde{\lambda}^S_{hl}}{1-\tilde{\lambda}^S_{hl}-\tilde{\lambda}^B_{hl}}}$$

$$\chi^E_{ud,hl} = \tau_{ud,hl}^{\frac{\tilde{\lambda}^S_{hl}+\tilde{\lambda}^B_{hl}}{1-\tilde{\lambda}^S_{hl}-\tilde{\lambda}^B_{hl}}(1-\sigma_h)} \times \left[\frac{\kappa_{ud,hl}}{\left(f^S_{ud,hl}\right)^{\tilde{\lambda}^S_{hl}}\left(f^B_{ud,hl}\right)^{\tilde{\lambda}^B}} \left(N_{d,l}\mathbb{M}_{d,l}\left(\frac{\delta_l}{\gamma^S_l}\right)\right)^{\lambda^S_{hl}} \left(N_{u,h}\mathbb{M}_{u,h}\left(\frac{\delta_h}{\gamma^B_h}\right)\right)^{\lambda^B_{hl}} \left(\Gamma^S_{d,h}\right)^{\tilde{\lambda}^S_{hl}}\left(\Gamma^B_{u,l}\right)^{\tilde{\lambda}^B_{hl}}\right]^{\frac{1}{1-\tilde{\lambda}^S_{hl}-\tilde{\lambda}^B_{hl}}}$$

and

$$\varrho^I_{hl} = \tilde{\sigma}_h^{1-\sigma_h}$$

$$\chi^I_{ud,hl} = (\tau_{ud,hl})^{1-\sigma_h}$$

$$\zeta^I_{u,h} = (C_{u,h})^{1-\sigma_h}$$

$$\xi^I_{d,l} = D_{d,hl}$$

We define trade elasticity, defined by the partial derivative of $X_{ud,hl}$ with respect to iceberg cost $\tau_{ud,hl}$ fixing factor and intermediate goods prices as given:

$$\varepsilon_{hl} \equiv \frac{\sigma_h - 1}{1 - \tilde{\lambda}^S_{hl} - \tilde{\lambda}^B_{hl}} \tag{C.23}$$

C.3 Market Clearing and Equilibrium

To close the model, we introduce the various market clearing conditions. Intermediate goods for each firm ω are used as inputs for production by other firms and local retailers:

$$q^R_{i,k}(\omega) + \sum_d \sum_l \int_{\psi:\ \omega \in S_{id}(\psi)} q_{id,kl}(\omega, \psi)\, \tau_{id,kl} d\psi = q_{i,k}(\omega)\,. \tag{C.24}$$

Retail goods are used by final consumers and by search service intermediaries:

$$L_i Q^F_{i,k} + Q^A_{i,k} = Q_{i,k}. \tag{C.25}$$

Search services in location i and sector k are used for supplier and buyer search toward all supplier and buyer locations:

$$\int_{\omega \in \Omega_{i,k}(\omega)} \left(\sum_{d,l} n^B_{id,kl}(\omega) + \sum_{u,h} n^S_{ui,hk}(\omega) \right) d\omega = A_{i,k}. \tag{C.26}$$

Labor is used for intermediate goods production and for search services:

$$\sum_k \left(\int_{\omega \in \Omega_{i,k}(\omega)} l_{i,k}(\omega)\, d\omega + L^A_{i,k} \right) = L_i. \tag{C.27}$$

The equilibrium is defined as follows:

Definition 1. The equilibrium is defined by the set of prices $\{p_{id,kl}(\upsilon,\omega), P_{i,k}, e_{i,k}, w_i, C_{i,k}\}$ and quantities $\{q_{id,kl}(\omega,\psi), q^R_{i,k}(\omega), Q^F_{i,k}, Q^A_{i,k}, n^B_{id,kl}(\omega), n^S_{ui,hk}(\omega), A_{i,k}, l_{i,k}(\omega), L^A_{i,k}, \tilde{\pi}_{i,k}(\omega)\}$ that satisfy the following set of conditions:

1. Households maximize consumption given the budget constraint (C.7) with firm profit given by $\Pi_{i,k} = \int_{\omega\in\Omega_{i,k}(\omega)} \tilde{\pi}_{i,k}(\omega)\, d\omega$
2. Firms make optimal pricing and production decisions for intermediate goods (C.3), (C.4) and search decisions (C.11); retailers make optimal optimal production decisions for retail goods (C.6); search service intermediaries make optimal production decisions (C.17)
3. All markets clear (C.24), (C.25), (C.26), and (C.27)

C.4 System of Equilibrium Equations

C.4.1 Baseline Equilibrium

- Trade flows

$$X_{ud,hl} = \varrho_{hl}\chi_{ud,hl}\zeta_{u,hl}\xi_{d,hl}$$

where

$$\rho_{hl} = \left(\varrho^I_{hl}\right)^{-\varepsilon_{hl}}$$

$$\chi_{ud,hl} = \left(\kappa^*_{ud,hl}\right)^{\frac{1}{1-\tilde{\lambda}^S_{hl}-\tilde{\lambda}^B_{hl}}} \left(\tau_{ud,hl}\right)^{-\varepsilon_{hl}}$$

$$\zeta_{u,hl} = \frac{\mathbb{M}_{u,h}\left(\delta_h\right)}{\mathbb{M}_{u,h}\left(\frac{\delta_h}{\gamma^B_h}\right)} C^{-\varepsilon_h}_{u,h} e_{u,h}^{-\frac{\tilde{\lambda}^S_{hl}}{1-\tilde{\lambda}^S_{hl}-\tilde{\lambda}^B_{hl}}}$$

$$\xi_{d,hl} = \left(\frac{R_{d,l}}{E_{d,l}}\right)^{\frac{\tilde{\lambda}^S_{hl}}{1-\tilde{\lambda}^S_{hl}-\tilde{\lambda}^B_{hl}}} D_{d,hl}^{\frac{1}{1-\tilde{\lambda}^S_{hl}-\tilde{\lambda}^B_{hl}}} e_{d,l}^{-\frac{\tilde{\lambda}^B_{hl}}{1-\tilde{\lambda}^S_{hl}-\tilde{\lambda}^B_{hl}}}$$

- Intensive and extensive margin

$$\overline{r}_{ud,kl} = \varrho^I_k \left(\tau_{ud,kl}\right)^{1-\sigma_k} \left(\frac{\mathbb{M}_{u,k}\left(\delta_k\right)}{\mathbb{M}_{u,k}\left(\frac{\delta_k}{\gamma^B_k}\right)} C^{1-\sigma_k}_{u,k}\right) D_{d,kl}$$

$$M_{ud,kl} = \frac{X_{ud,hl}}{\overline{r}_{ud,kl}}$$

- Revenue and expenditure

$$R_{i,k} = \sum_d \sum_l X_{id,kl} \tag{C.28}$$

$$E_{i,k} = \sum_u \sum_h X_{ui,hk} \tag{C.29}$$

- Cost shifter

$$(C_{i,k})^{1-\sigma_k} = \left(N_{i,k}\mathbb{M}_{i,k}\left(\frac{\delta_k}{\gamma_k^S}\right)\right)^{-1} w_i^{\beta_{k,L}(1-\sigma_k)} \prod_h \left(\sum_{u\in\mathcal{N}} M_{ui,hk}\,(C_{u,h})^{1-\sigma_h}\,(\tau_{ui,hk})^{1-\sigma_h}\right)^{\frac{\beta_{hk}}{1-\sigma_h}(1-\sigma_k)}$$

- Labor compensation

$$w_i L_i = \sum_k \underbrace{\frac{\beta_{k,L}}{\sum \beta_{hk}} E_{i,k}}_{\text{firms}} + \underbrace{\frac{1}{\sigma_k}\mu_k\left(\frac{1}{\gamma_k^B} + \frac{\sum_h \beta_{hk}\frac{\sigma_k-1}{\sigma_h-1}}{\gamma_k^S}\right) R_{i,k}}_{\text{search intermediaries}} \tag{C.30}$$

- Profit

$$\Pi_i = \sum_k \frac{1}{\sigma_k}\left(1 - \left(\frac{1}{\gamma_k^B} + \frac{\sum_h \beta_{hk}\frac{\sigma_k-1}{\sigma_h-1}}{\gamma_k^S}\right)\right) R_{i,k} \tag{C.31}$$

- Search cost

$$e_{i,k} = (w_i)^{\mu_k}\,(C_{i,k})^{1-\mu_k}$$

- Aggregate expenditure for sector k: by noting that there is no profit from sales to retailers,

$$\begin{aligned} E_{i,k} &= \left(\sum_h \beta_{hk}\right)\left[P_{i,k}Q_{i,k}^F + P_{i,k}Q_{i,k}^A + \frac{\sigma_k - 1}{\sigma_k} R_{i,k}\right] \\ &= \left(\sum_h \beta_{hk}\right)\left[\alpha_k\,(w_i L_i + \Pi_i) + \left[\frac{1}{\sigma_k}\,(1-\mu_k)\left(\frac{1}{\gamma_k^B} + \frac{\sum_h \beta_{hk}\frac{\sigma_k-1}{\sigma_h-1}}{\gamma_k^S}\right) + \frac{\sigma_k - 1}{\sigma_k}\right] R_{i,k}\right] \end{aligned} \tag{C.32}$$

C.4.2 Counterfactual Equilibrium

- Trade flows

$$\hat{X}_{ud,hl} = \hat{\chi}_{ud,hl}\hat{\zeta}_{u,h}\hat{\xi}_{d,l}, \quad \hat{\zeta}_{u,h} = \hat{C}_{u,h}^{-\varepsilon_{hl}}\hat{e}_{u,h}^{-\frac{\tilde{\lambda}^S_{hl}}{1-\tilde{\lambda}^S_{hl}-\tilde{\lambda}^B_{hl}}}, \quad \hat{\xi}_{d,l} = \left(\frac{\hat{R}_{d,l}}{\hat{E}_{d,l}}\right)^{\frac{\tilde{\lambda}^S_{hl}}{1-\tilde{\lambda}^S_{hl}-\tilde{\lambda}^B_{hl}}} \hat{D}_{d,l}^{\frac{1}{1-\tilde{\lambda}^S_{hl}-\tilde{\lambda}^B_{hl}}}\hat{e}_{d,l}^{-\frac{\tilde{\lambda}^B_{hl}}{1-\tilde{\lambda}^S_{hl}-\tilde{\lambda}^B_{hl}}}$$

- Intensive and extensive margin

$$\hat{M}_{ui,hk} = \frac{\hat{X}_{ud,hl}}{\hat{\bar{r}}_{ud,kl}}, \quad \hat{\bar{r}}_{ud,kl} = (\hat{\tau}_{ud,kl})^{1-\sigma_k}\hat{C}_{u,k}^{1-\sigma_k}\hat{D}_{d,l}$$

- Revenue

$$\hat{R}_{i,k} = \sum_d \sum_l \Psi_{id,kl}\hat{X}_{id,kl}$$

- Cost shifter

$$\left(\hat{C}_{i,k}\right)^{1-\sigma_k} = \hat{w}_i^{\beta_{k,L}(1-\sigma_k)} \prod_h \left(\sum_{u\in\mathcal{N}} \hat{M}_{ui,hk}\left(\hat{C}_{u,h}\right)^{1-\sigma_h}(\hat{\tau}_{ui,hk})^{1-\sigma_h}\Lambda_{ui,hk}\right)^{\frac{\beta_{hk}}{1-\sigma_h}(1-\sigma_k)}$$

- Labor compensation

$$\hat{w}_i = \sum_k S^{L,F}_{i,k}\hat{E}_{i,k} + S^{L,A}_{i,k}\hat{R}_{i,k}$$

- Profit

$$\hat{\Pi}_i = \sum_k S^{\Pi}_{i,k}\hat{R}_{i,k}$$

- Search cost

$$\hat{e}_{i,k} = (\hat{w}_i)^{\mu_k}\left(\hat{C}_{i,k}\right)^{1-\mu_k}$$

- Expenditure

$$\hat{E}_{i,k} = S^{E,L}_{i,k}\hat{w}_i + S^{E,\Pi}_{i,k}\hat{\Pi}_i + S^{E,R}_{i,k}\hat{R}_{i,k}$$

- Demand shifter

$$\hat{D}_{d,hl} = \hat{e}_{d,l}^{\tilde{\lambda}_{hl}^{B}} \left(\frac{\hat{R}_{d,l}}{\hat{E}_{d,l}}\right)^{-\tilde{\lambda}_{hl}^{S}} \left(\widehat{\sum_{u} X_{ud,hl}}\right)^{1-\tilde{\lambda}_{hl}^{S}-\tilde{\lambda}_{hl}^{B}} \left(\sum_{u,h} \Lambda_{ui,hk} \hat{\chi}_{ud,hl} \hat{C}_{u,h}^{-\varepsilon_{hl}} \hat{e}_{u,h}^{-\frac{\tilde{\lambda}_{hl}^{S}}{1-\tilde{\lambda}_{hl}^{S}-\tilde{\lambda}_{hl}^{B}}}\right)^{-\left(1-\tilde{\lambda}_{hl}^{S}-\tilde{\lambda}_{hl}^{B}\right)}$$

where the baseline shares are given by

- Revenue share

$$\Psi_{id,kl} = \frac{X_{id,kl}}{\sum_{d'} \sum_{l'} X_{id',kl'}}$$

- Expenditure share

$$\Lambda_{ui,hk} = \frac{X_{ui,hk}}{\sum_{u'} X_{u'i,h'k}}$$

- Labor compensation shares across sectors, by firms and by search intermediaries

$$S_{i,k}^{L,F} = \frac{\frac{\beta_{k,L}}{\sum \beta_{hk}} E_{i,k}}{\sum_{k} \frac{\beta_{k,L}}{\sum \beta_{hk}} E_{i,k} + \frac{1}{\sigma_k} \mu_k \left(\frac{1}{\gamma_k^B} + \frac{\sum_h \beta_{hk} \frac{\sigma_k - 1}{\sigma_h - 1}}{\gamma_k^S}\right) R_{i,k}}$$

$$S_{i,k}^{L,A} = \frac{\frac{1}{\sigma_k} \mu_k \left(\frac{1}{\gamma_k^B} + \frac{\sum_h \beta_{hk} \frac{\sigma_k - 1}{\sigma_h - 1}}{\gamma_k^S}\right) R_{i,k}}{\sum_{k} \frac{\beta_{k,L}}{\sum \beta_{hk}} E_{i,k} + \frac{1}{\sigma_k} \mu_k \left(\frac{1}{\gamma_k^B} + \frac{\sum_h \beta_{hk} \frac{\sigma_k - 1}{\sigma_h - 1}}{\gamma_k^S}\right) R_{i,k}}$$

- Profit share across sectors

$$S_{i,k}^{\Pi} = \frac{\frac{1}{\sigma_k} \left(1 - \mu_k \left(\frac{1}{\gamma_k^B} + \frac{\sum_h \beta_{hk} \frac{\sigma_k - 1}{\sigma_h - 1}}{\gamma_k^S}\right)\right) R_{i,k}}{\sum_k \frac{1}{\sigma_k} \left(1 - \mu_k \left(\frac{1}{\gamma_k^B} + \frac{\sum_h \beta_{hk} \frac{\sigma_k - 1}{\sigma_h - 1}}{\gamma_k^S}\right)\right) R_{i,k}}$$

- Demand shares

$$S_{i,k}^{E,L} = \frac{\alpha_k w_i L_i}{\alpha_k \left(w_i L_i + \Pi_i\right) + \left[\frac{1}{\sigma_k} \left(1 - \mu_k\right) \left(\frac{1}{\gamma_k^B} + \frac{\sum_h \beta_{hk} \frac{\sigma_k - 1}{\sigma_h - 1}}{\gamma_k^S}\right) + \frac{\sigma_k - 1}{\sigma_k}\right] R_{i,k}}$$

$$S_{i,k}^{E,\Pi} = \frac{\alpha_k \Pi_i}{\alpha_k \left(w_i L_i + \Pi_i\right) + \left[\frac{1}{\sigma_k}\left(1-\mu_k\right)\left(\frac{1}{\gamma_k^B} + \frac{\sum_h \beta_{hk} \frac{\sigma_k - 1}{\sigma_h - 1}}{\gamma_k^S}\right) + \frac{\sigma_k - 1}{\sigma_k}\right] R_{i,k}}$$

$$S_{i,k}^{E,R} = \frac{\left[\frac{1}{\sigma_k}\left(1-\mu_k\right)\left(\frac{1}{\gamma_k^B} + \frac{\sum_h \beta_{hk} \frac{\sigma_k - 1}{\sigma_h - 1}}{\gamma_k^S}\right) + \frac{\sigma_k - 1}{\sigma_k}\right] R_{i,k}}{\alpha_k \left(w_i L_i + \Pi_i\right) + \left[\frac{1}{\sigma_k}\left(1-\mu_k\right)\left(\frac{1}{\gamma_k^B} + \frac{\sum_h \beta_{hk} \frac{\sigma_k - 1}{\sigma_h - 1}}{\gamma_k^S}\right) + \frac{\sigma_k - 1}{\sigma_k}\right] R_{i,k}}$$

where

$$w_i L_i = \underbrace{\sum_k \frac{\beta_{k,L}}{\sum \beta_{hk}} E_{i,k}}_{\text{firms}} + \underbrace{\frac{1}{\sigma_k}\mu_k \left(\frac{1}{\gamma_k^B} + \frac{\sum_h \beta_{hk} \frac{\sigma_k - 1}{\sigma_h - 1}}{\gamma_k^S}\right) R_{i,k}}_{\text{search intermediaries}}$$

$$\Pi_i = \sum_k \frac{1}{\sigma_k}\left(1 - \left(\frac{1}{\gamma_k^B} + \frac{\sum_h \beta_{hk} \frac{\sigma_k - 1}{\sigma_h - 1}}{\gamma_k^S}\right)\right) R_{i,k}$$

C.5 Planning Problem

We consider a planner with a restrictive set of policy tools. First, the planner can introduce linear (ad-valorem) subsidies for intermediate goods sales specific to origin i and destination d, $s_{id,kl}^I$. Under these subsidies, the intermediate goods prices change from Equation (4) to

$$p_{id,kl}\left(\upsilon, \omega\right) = \left(1 - s_{id,kl}^I\right) \tilde{\sigma}_k c_{i,k}\left(\upsilon\right) \tau_{id,kl}. \tag{C.33}$$

Second, the planner can introduce linear taxes for supplier and buyer search, $t_{id,kl}^S$ and $t_{id,kl}^B$, for each pair of supplier and buyer locations. Therefore, total search costs by firms in location i is modified as

$$\begin{aligned} & f_{i,k}\left(\{n_{id,kl}^B\}_{d,l}, \{n_{ui,mk}^S\}_{u,m}\right) \\ & = e_{i,k}\left\{\sum_{d\in\mathcal{N}}\sum_l \left(1 + t_{id,kl}^B\right) f_{id,kl}^B \frac{\left(n_{id,kl}^B\right)^{\gamma_k^B}}{\gamma_k^B} + \sum_{u\in\mathcal{N}}\sum_h \left(1 + t_{ui,hk}^S\right) f_{ui,hk}^S \frac{\left(n_{ui,hk}^S\right)^{\gamma_k^S}}{\gamma_k^S}\right\}, \end{aligned} \tag{C.34}$$

Finally, we introduce lump-sum transfers for households in location i, T_i^F, so that households' budget constraint is modified from Equation (7) to $\sum_k P_{i,k} Q_{i,k}^F = w_i + \frac{\Pi_i}{L_i} + T_i^F$.

The planner chooses the optimal policy to maximize the global welfare

$$\max_{\left\{\left\{s^I_{id,ml},t^B_{id,ml},t^S_{id,ml}\right\},T^F_i\right\}} \mathcal{W} \equiv \sum_i \psi_i L_i \prod_k \left(Q^F_{i,k}\right)^{\alpha_k} \tag{C.35}$$

subject to equilibrium constraints and the government budget constraint, where $\psi_i \geq 0$ corresponds to the welfare weights attached to each location. The following proposition provides a simple formula for the optimal policy.

Proposition 7. *The optimal set of taxes and subsidies* $\left\{s^I_{id}, t^B_{id}, t^S_{id}\right\}$ *must satisfy*

$$s^I_{id,ml} = \frac{1}{\sigma_m}, \qquad t^B_{id,ml} = \frac{1}{\lambda^B_{ml}} - 1, \qquad t^S_{id,ml} = \frac{1}{\lambda^S_{ml}} \frac{\sigma_m - 1}{\sigma_m} \frac{\sigma_l}{\sigma_l - 1} \beta_{ml} \frac{E_{d,l}}{R_{d,l}} - 1, \tag{C.36}$$

for all i, d, *where* $R_{i,k} = \sum_{d,l} X_{id,kl}$ *corresponds to aggregate intermediate goods sales (excluding those to retailers) in location* d *and* $E_{i,k} = \sum_{u,h} X_{ui,hk}$ *corresponds to the aggregate intermediate goods expenditure in location* d. *Lump-sum transfers* $\{T^F_i\}$ *is set depending on the welfare weights* ψ_i.

Proof. Similarly as Proposition 2, we solve for a relaxed planning problem where the planner directly specifies the allocation. The relaxed problem is given by

$$\max_{\{Q^F_{i,k}, Q^A_{i,k}, L^A_{i,k}, L^P_{i.k}, A^S_{ui,hk}, A^B_{ui,hk}, q^R_{i,k}, q_{id,hk}, M_{ui,hk}\}} \sum_i \psi_i L_i \prod_k \left(Q^F_{i,k}\right)^{\alpha_k} \tag{C.37}$$

$$\text{s.t.} Q^A_{i,k} + L_i Q^F_{i,k} = g_{i,k}\left(q^R_{i,k}\right) \qquad [P_{i,k}]$$

$$\sum_k \left(L^A_{i,k} + L^P_{i,k}\right) = L_i \qquad [w_i]$$

$$\sum_h \sum_u A^S_{ui,hk} + \sum_l \sum_d A^B_{id,kl} = \left(\frac{L^A_{i,k}}{\mu_k}\right)^{\mu_k} \left(\frac{Q^A_{i,k}}{1-\mu_k}\right)^{1-\mu_k} \qquad [e_{i,k}]$$

$$q^R_{i,k} + \sum_m \sum_d q_{id,km}\tau_{id,km} = Z_{i.k} \left(\frac{L^P_{i,k}}{\beta_{k,L}}\right)^{\beta_{k,L}} \prod_h \left(\frac{\left(\sum_u M_{ui,hk}^{\frac{1}{\sigma_h}} q_{ui,hk}^{\frac{\sigma_h - 1}{\sigma_h}}\right)^{\frac{\sigma_h}{\sigma_h - 1}}}{\beta_{hk}}\right)^{\beta_{hk}} \qquad [C_{i,k}]$$

$$M_{ui,hk} = \kappa_{ui,hk} \left(\frac{\gamma^B_h}{f^B_{ui,hk}} A^B_{ui,hk}\right)^{\frac{\lambda^B_{hk}}{\gamma^B_h}} \left(\frac{\gamma^S_k}{f^S_{ui,hk}} A^S_{ui,hk}\right)^{\frac{\lambda^S_{hk}}{\gamma^S_k}} \qquad [\eta_{ui,hk}]$$

The FOCs of this problem with respect to $Q^F_{i,k}$, $Q^A_{i,k}$, $q^R_{i,k}$, $L^A_{i,k}$, $L^P_{i,k}$, $A^B_{ui,hk}$, $A^S_{ui,hk}$, $M_{ui,hk}$, $q_{ui,hk}$ is given by:

$$P_i = \psi_i L_i \alpha_k \frac{W_i}{Q^F_{i,k}} \tag{C.38}$$

$$P_{i,k} = e_{i,k} A_{i,k} \left(1 - \mu_k\right) \frac{1}{Q^A_{i,k}} \tag{C.39}$$

$$C_{i,k} = P_{i,k} g'_{i,k} \left(q^R_{i,k}\right) \tag{C.40}$$

$$w_i = e_{i,k} A_{i,k} \mu_k \frac{1}{L^A_{i,k}} \tag{C.41}$$

$$w_i = \beta_k C_{i,k} q_{i,k} \frac{1}{L^P_{i,k}} \tag{C.42}$$

$$e_{u,h} = \eta_{ui,hk} \frac{\lambda^B_{hk}}{\gamma^B_h} M_{ui,hk} \frac{1}{A^B_{ui,hk}} \tag{C.43}$$

$$e_{i,k} = \eta_{ui,hk} \frac{\lambda^S_{hk}}{\gamma^S_k} M_{ui,hk} \frac{1}{A^S_{ui,hk}} \tag{C.44}$$

$$\eta_{ui,hk} = \frac{1}{\sigma_h - 1} \beta_{hk} C_{i,k} q_{i,k} \Lambda_{ui,hk} \frac{1}{M_{ui,hk}} \tag{C.45}$$

$$\tau_{ui,hk} C_{u,h} = \beta_{hk} C_{i,k} q_{i,k} \Lambda_{ui,hk} \frac{1}{q_{ui,hk}} \tag{C.46}$$

where $W_i = \prod_k \left(Q^F_{i,k}\right)^{\alpha_k}$, $A_{i,k} \equiv \sum_h \sum_u A^S_{ui,hk} + \sum_l \sum_d A^B_{id,kl}$, $q_{i,k} \equiv q^R_{i,k} + \sum_m \sum_d q_{id,km} \tau_{id,km}$, and $\Lambda_{ui,hk} \equiv \frac{M^{\frac{1}{\sigma_h}}_{ui,hk} q^{\frac{\sigma_h - 1}{\sigma_h}}_{ui,hk}}{\sum_\ell M^{\frac{1}{\sigma_h}}_{\ell i,hk} q^{\frac{\sigma_h - 1}{\sigma_h}}_{\ell i,hk}}$. From (C.43), (C.44), (C.45),

$$\frac{\lambda^B_{hk}}{\gamma^B_h} \frac{1}{\sigma_h - 1} \beta_{hk} C_{i,k} q_{i,k} \Lambda_{ui,hk} = e_{u,h} A^B_{ui,hk} \tag{C.47}$$

$$\frac{\lambda^S_{hk}}{\gamma^S_k} \frac{1}{\sigma_k - 1} \beta_{hk} C_{i,k} q_{i,k} \Lambda_{ui,hk} = e_{i,k} A^S_{ui,hk} \tag{C.48}$$

We now consider the equilibrium subsidies/taxes that implement this allocation. Suppose that the Lagrange multipliers $\{P_{i,k}, w_i, e_{i,k}, C_{i,k}\}$ correspond to equilibrium prices. Then, from the intermediate goods market clearing condition,

$$\left(1-s^I_{ui,hk}\right)\frac{\sigma_h}{\sigma_h-1}\tau_{ui,hk}C_{u,h}q_{ui,hk}=\beta_{hk}C_{i,k}q_{i,k}\Lambda_{ui,hk} \tag{C.49}$$

By comparing this condition with (C.46), the optimal $s^I_{ui,hk}$ satisfies $1-s^I_{ui,hk}=1-s^I_h=\frac{\sigma_h-1}{\sigma_h} \iff s^I_h=1/\sigma_h$.

Next, following Lemma 3 applied to the economy with taxes, equilibrium allocation for buyer and supplier search is given by

$$\left(1+t^B_{ui,hk}\right)e_{u,h}A^B_{ui,hk}=\frac{1}{\sigma_h}\frac{1}{\gamma^B_h}X_{ui,hk}\frac{1}{1-s^I_h} \tag{C.50}$$

$$\left(1+t^S_{ui,hk}\right)e_{i,k}A^S_{ui,hk}=\frac{1}{\sigma_k}\frac{\beta_{hk}}{\gamma^S_k}X_{ui,hk}\frac{R_{i,k}}{E_{i.k}}\frac{1}{1-s^I_h} \tag{C.51}$$

By comparing these equations with (C.47) and (C.48), and by noting that $X_{ui,hk}=\beta_{hk}C_{i,k}q_{i,k}\Lambda_{ui,hk}$, we obtain the expressions in Proposition 7.

Finally, we confirm that these taxes can be implemented in the equilibrium. Notice that the constraints in (C.37) coincide with resource constraints, production of search effort, and matching technology; (C.42) and (C.46) coincide with intermediate goods producers' optimal production decisions; (C.40) coincides with retailers' optimal production decisions; (C.39), (A.31), (C.50) and (C.51) coincide with intermediate producers' optimal search decisions. Finally, lump-sum transfers can be set such that

$$T^F_i=\sum_k P_{i,k}Q^F_{i,k}-\frac{\sum_k \Pi^F_{i,k}}{L_i}-w_i$$

We can verify that these allocations satisfy the government budget constraints from market clearing conditions. □

C.6 Ex-Post Sufficient Statistics on Each Location's Welfare

The welfare gains from trade from an external shock to location i is characterized by the following proposition:

Proposition 8. *The welfare changes in location i from external shocks is given by $d\ln W_i = -\sum_h \alpha_h d\ln C_{i,h}$, where*

$$d\ln C_{i,h} = \sum_k \phi_{hk} \sum_l \beta_{lh} \left(\frac{1}{\varepsilon_{lh}} d\ln \Lambda_{ii,lh} + \frac{1}{\sigma_l - 1} \left(\tilde{\lambda}^S_{lh} d\ln R_{i,h} + \tilde{\lambda}^B_{lh} d\ln E_{i,h} \right) \right) \tag{C.52}$$

where $\Lambda_{ii,kl} = X_{ii,kl} / \sum_u X_{ui,kl}$, and ϕ_{kh} is an (k,h)-th element of a matrix Φ given by

$$\Phi = \left[\mathbf{c}^S \mathbf{I} - \mathbf{C}^B \right]^{-1} \tag{C.53}$$

where $\mathbf{I}$ is the $|K| \times |K|$ identity matrix, $\mathbf{c}^S$ is the $|K| \times 1$ vector with k-th element corresponds to $1 - \sum_h \beta_{hk} \frac{1-\mu_k}{\sigma_h - 1} \tilde{\lambda}^B_{hk}$, and $\mathbf{C}^B$ is the $|K| \times |K|$ matrix with (k,h)-th element corresponds to $\beta_{hk} \left(1 + \frac{1-\mu_h}{\sigma_h - 1} \tilde{\lambda}^S_{hk} \right)$.

Note that Proposition 8 collapses to Proposition 4 in the case of single sector. Note that, in the case of single sector, $d\ln R_i = d\ln E_i = d\ln w_i$ because of the trade balance and constant share of intermediate goods sales to labor, and hence $d\ln \Lambda_{ii}$ is the only endogenous variable summarizing the welfare gains from trade.

Proof. We start by applying the Shephard's Lemma to firm's cost minimization problem (C.14). For expositional simplicity, we only consider shocks to external trade costs $\{d\ln \tau_{ui}\}$, while final equation (C.54) is unchanged for any other external shocks considered in Proposition 8.

Applying the Shephard's Lemma to firm's cost minimization problem (C.14),

$$\begin{aligned} d\ln C_{i,k} &= \sum_h \beta_{hk} \sum_u \Lambda_{ui,hk} (d\ln C_{u,h} + d\ln \tau_{ui,hk} - \frac{1}{\sigma_h - 1} d\ln M_{ui,hk}) \\ &= \sum_h \beta_{hk} (d\ln C_{i,h} - \frac{1}{\sigma_h - 1} d\ln M_{ii,hk} + \frac{1}{\sigma_h - 1} d\ln \Lambda_{ii,hk}) \end{aligned} \tag{C.54}$$

At the same time, from Lemmas 3 and 4, and from the cost minimization of retailers,

$$
\begin{aligned}
d\ln M_{ii,hk} = & \left(\tilde{\lambda}^S_{hk} + \tilde{\lambda}^B_{hk}\right)\big(\underbrace{d\ln X_{ii,hk}}_{= d\ln \Lambda_{ii,hk} + d\ln E_{i,k}} \big) + \tilde{\lambda}^S_{hk}\left(d\ln R_{i,k} - d\ln E_{i,k}\right) \\
& - \left(\left(1-\mu_h\right)\tilde{\lambda}^S_{hk} d\ln C_{i,h} + \left(1-\mu_k\right)\tilde{\lambda}^B_{hk} d\ln C_{i,k}\right) \\
= & \left(\tilde{\lambda}^S_{hk} + \tilde{\lambda}^B_{hk}\right) d\ln \Lambda_{ii,hk} + \tilde{\lambda}^S_{hk} d\ln R_{i,k} + \tilde{\lambda}^B_{hk} d\ln E_{i,k} \\
& - \left(\left(1-\mu_h\right)\tilde{\lambda}^S_{hk} d\ln C_{i,h} + \left(1-\mu_k\right)\tilde{\lambda}^B_{hk} d\ln C_{i,k}\right)
\end{aligned}
$$

where $R_{i,k}$ and $E_{i,k}$ denote location i and sector k's intermediate goods sales and expenditure. Note that, unlike the case for a single sector, these two objects may not take the same value. Combining this expression with Equation (C.54),

$$
\begin{aligned}
& \left(1 - \sum_h \beta_{hk} \frac{1-\mu_k}{\sigma_h - 1} \tilde{\lambda}^B_{hk}\right) d\ln C_{i,k} \\
& = \sum_h \beta_{hk} \left(1 + \frac{1-\mu_h}{\sigma_h - 1}\tilde{\lambda}^S_{hk}\right) d\ln C_{i,h} + \frac{\beta_{hk}}{\varepsilon_{hk}} d\ln \Lambda_{ii,hk} + \frac{\beta_{hk}}{\sigma_h - 1}\left(\tilde{\lambda}^S_{hk} d\ln R_{i,k} + \tilde{\lambda}^B_{hk} d\ln E_{i,k}\right)
\end{aligned}
$$

Therefore, denoting $\mathbf{c}^S$ as $|K| \times 1$ vector with k-th element corresponds to $1 - \sum_h \beta_{hk} \frac{1-\mu_k}{\sigma_h - 1}\tilde{\lambda}^B_{hk}$, and $\mathbf{C}^B$ as $|K| \times |K|$ matrix with (k,h)-th element corresponds to $\beta_{hk}\left(1 + \frac{1-\mu_h}{\sigma_h - 1}\tilde{\lambda}^S_{hk}\right)$. Then,

$$
d\ln C_{i,k} = \sum_h \phi_{kh} \sum_l \beta_{lh} \left(\frac{1}{\varepsilon_{lh}} d\ln \Lambda_{ii,lh} + \frac{1}{\sigma_l - 1}\left(\tilde{\lambda}^S_{lh} d\ln R_{i,h} + \tilde{\lambda}^B_{lh} d\ln E_{i,h}\right)\right)
$$

where ϕ_{kh} is an (k,h)-th element of a matrix Φ given by

$$
\Phi = \left[\mathbf{c}^S \mathbf{I} - \mathbf{C}^B\right]^{-1}
$$

where $\mathbf{I}$ is the $|K| \times |K|$ identity matrix. Finally, welfare gains from trade in location i is given by

$$
d\ln W_i = -\sum_k \alpha_k d\ln C_{i,k}.
$$

□

Now we derive the expression for aggregate intermediate goods sales (excluding those to retailers), $R_{i,h}$, and aggregate intermediate goods expenditure, $E_{i,h}$, under regional autarky, similarly as in Costinot and Rodríguez-Clare (2014). Note the normalization of region i's GDP to one such that $w_i L_i + \Pi_i = 1$. First, note that

$$R_{i,k} = \sum_h \frac{\beta_{kh}}{\sum_l \beta_{lh}} E_{i,h}$$

Furthermore,

$$\begin{aligned}
E_{i,k} &= \left(\sum_l \beta_{lk}\right) \left[P_{i,k} Q_{i,k}^F + P_{i,k} Q_{i,k}^A + \frac{\sigma_k - 1}{\sigma_k} R_{i,k}\right] \\
&= \left(\sum_l \beta_{lk}\right) \left[\alpha_k \underbrace{(w_i L_i + \Pi_i)}_{=1} + \left[\frac{1}{\sigma_k}(1-\mu_k)\left(\frac{1}{\gamma_k^B} + \frac{\sum_h \beta_{hk} \frac{\sigma_k - 1}{\sigma_h - 1}}{\gamma_k^S}\right) + \frac{\sigma_k - 1}{\sigma_k}\right] R_{i,k}\right] \\
&= \left(\sum_l \beta_{lk}\right) \left[\alpha_k + \left[\frac{1}{\sigma_k}(1-\mu_k)\left(\frac{1}{\gamma_k^B} + \frac{\sum_h \beta_{hk} \frac{\sigma_k - 1}{\sigma_h - 1}}{\gamma_k^S}\right) + \frac{\sigma_k - 1}{\sigma_k}\right] \sum_h \frac{\beta_{kh}}{\sum_l \beta_{lh}} E_{i,h}\right]
\end{aligned}$$

$$\tilde{E}_{i,k} = \left[\alpha_k + \left[\frac{1}{\sigma_k}(1-\mu_k)\left(\frac{1}{\gamma_k^B} + \frac{\sum_h \beta_{hk} \frac{\sigma_k - 1}{\sigma_h - 1}}{\gamma_k^S}\right) + \frac{\sigma_k - 1}{\sigma_k}\right] \sum_h \beta_{kh} \tilde{E}_{i,h}\right]$$

where we define $\tilde{E}_{i,k} \equiv E_{i,k} / \left(\sum_l \beta_{lk}\right)$. Then,

$$\tilde{\mathbf{E}}_i = \tilde{\mathbf{E}} = [\mathbf{I} - \mathbf{D}]^{-1} \mathbf{a}$$

where $\tilde{\mathbf{E}}$ is $|K| \times 1$ vector with k'th element correspond to $\tilde{E}_{i,k}$, $\mathbf{D}$ is $|K| \times |K|$ matrix with (k,h)-th element corresponds to $\beta_{kh} \left[\frac{1}{\sigma_k}(1-\mu_k)\left(\frac{1}{\gamma_k^B} + \frac{\sum_h \beta_{hk} \frac{\sigma_k - 1}{\sigma_h - 1}}{\gamma_k^S}\right) + \frac{\sigma_k - 1}{\sigma_k}\right]$, and $\mathbf{a}$ is $|K| \times 1$ vector of α_k.

Once we obtain $E_{i,k}$, we can further back out $R_{i,k} = \sum_h \frac{\beta_{kh}}{\sum_l \beta_{lh}} E_{i,h}$.

D Details on Isomorphisms with Alternative Network Formation Models

We first establish a general framework that encompass different microfoundation of production network formation models. The key generalization of the model in Section 2 is that (1) we allow for arbitrary aggregation of intermediate input bundles (Equation D.2 instead of 2) and (2) we do not impose a structure of the network formation $\{\mathcal{S}_{ui}(\cdot)\}$ as specified in Section 2.2.

Firms. A continuum of firms produces a distinct variety in each location. Production of intermediate goods requires labor and intermediate inputs. Intermediate inputs are sourced from firms that are directly connected by production networks. The production function of firm $\omega \in \Omega_i$ is given by

$$q_i(\omega) = z_i(\omega) \left(\frac{l_i(\omega)}{\beta} \right)^{\beta} \left(\frac{\tilde{q}_i(\omega)}{1-\beta} \right)^{1-\beta}, \tag{D.1}$$

where $z_i(\omega)$ is the total factor productivity (TFP) of firm ω which follows distribution $G_i(\cdot)$, $l_i(\omega)$ is labor inputs, and $\tilde{q}_i(\omega)$ is the composite of intermediate inputs, β is the parameter proxying the input share for labor, given by

$$\tilde{q}_i(\omega) = F_i \left(\{q_{ui}(\upsilon, \omega)\}_{\upsilon \in S_{ui}(\omega), u \in \mathcal{N}} \right). \tag{D.2}$$

Given these production functions, the marginal cost of production by firm ω is given by

$$c_i(\omega) = \frac{1}{z_i(\omega)} w_i^{\beta} \tilde{F}_i \left(\{p_{ui}(\upsilon, \omega)\}_{\upsilon \in S_{ui}(\omega), u} \right), \tag{D.3}$$

where $p_{ui}(\upsilon, \omega)$ is the intermediate goods price that supplier υ in location u charges to firm ω in location i. On top of these production cost, when a firm sells their intermediate goods to location d, they incur an iceberg trade cost of $\tau_{id} \geq 1$.

We assume that suppliers charge a constant markup of their marginal cost net of the iceberg trade cost;

$$p_{id}\left(\upsilon,\omega\right)=\varrho_{id}\left(\upsilon,\omega\right)c_{i}\left(\upsilon\right)\tau_{id}, \tag{D.4}$$

where $\varrho_{id}\left(\upsilon,\omega\right)$ is the markup ratio.

Retailers. Perfectly competitive retailers in each location i combine intermediate inputs from all firms in location i and produce a standardized nontradable retail goods. Their production function is given by

$$Q_{i}=g_{i}\left(\left\{q_{i}^{R}\left(\omega\right)\right\}_{\omega\in\Omega_{i}}\right), \tag{D.5}$$

where $g_i\left(\cdot\right)$ is a function that satisfies homogeneous of degree one, and $q_i^R\left(\omega\right)$ is the quantity of intermediate inputs from firm ω. The retail goods are used for final consumption purposes and by search service sector for production network formation, as we describe further below.

We also assume that retailers have the entire bargaining power when purchasing intermediate inputs from each firm at their marginal cost $c_i\left(\upsilon\right)$. Under cost minimization, retail prices are given by

$$P_{i}=\tilde{g}_{i}\left(\left\{c_{i}\left(\omega\right)\right\}_{\omega\in\Omega_{i}}\right), \tag{D.6}$$

where $\tilde{g}_i\left(\cdot\right)$ is a solution to the cost minimization problem by retailers.

Final Consumers. Measure L_i Households supply labor inelastically at wage w_i. They also own an equal share of local firms. Therefore their budget constraint is given by

$$P_{i}Q_{i}^{F}=w_{i}+\frac{\Pi_{i}}{L_{i}}, \tag{D.7}$$

where Q_i^F is the amount of consumption of retail goods per capita, and Π_i is aggregate profit by firms producing in location i.

Production Network Formation. Next we describe how the production network structure, $\left\{\mathcal{S}_{ui}\left(\cdot\right)\right\}$, is endogenously determined. Existing papers have taken different approaches to these networks, such as through the entry decisions into a markets or forming a relationship, search and matching decisions, or a choice of suppliers. Here, we simply assume that they are produced by combining the labor and retail goods by firms in upstream locations and downstream locations. We denote that there are aggregate relationships between the profiles of labor and retail goods by firms in the upstream and downstream locations, such that

$$\{\mathcal{S}_{ui}(\cdot)\} = G_{ui}\left(\left\{l_u^U(\omega)\right\}_{\omega\in\Omega_u}, \left\{Q_u^U(\omega)\right\}_{\omega\in\Omega_u}, \left\{l_d^D(\omega)\right\}_{\omega\in\Omega_d}, \left\{Q_d^D(\omega)\right\}_{\omega\in\Omega_d}\right) \quad \text{(D.8)}$$

Market clearing. We introduce various market clearing conditions to close the model. Intermediate goods for each firm ω are used as inputs for production by other firms and local retailers:

$$q_i^R(\omega) + \sum_d \int_{\psi:\ \omega\in S_{id}(\psi)} q_{id}(\omega,\psi)\,\tau_{id} = q_i(\omega). \quad \text{(D.9)}$$

Retail goods are consumed by final consumers and used for search activity:

$$L_i Q_i^F + \int_{\omega\in\Omega_i} Q_i^U(\omega)\,d\omega + \int_{\omega\in\Omega_i} Q_i^D(\omega)\,d\omega = Q_i. \quad \text{(D.10)}$$

Labor is used for intermediate goods production and for search activity:

$$\int_{\omega\in\Omega_i(\omega)} l_i(\omega)\,d\omega + \int_{\omega\in\Omega_i} l_i^U(\omega)\,d\omega + \int_{\omega\in\Omega_i} l_i^D(\omega)\,d\omega = L_i. \quad \text{(D.11)}$$

The general equilibrium is defined as follows:

Definition 2. The equilibrium is defined by the set of prices $\{p_{id}(\upsilon,\omega), P_i, e_i, w_i, C_i\}$ and quantities $\{q_{id}(\omega,\psi), q_i^R(\omega), Q_i^F, Q_i^A, n_{id}^B(\omega), n_{ui}^S(\omega), l_i(\omega), L_i^A, \tilde{\pi}_i(\omega)\}$ that satisfy the following set of conditions:

1. Households maximize consumption given the budget constraint (7) with income from firm profit given by Π_i

2. Firms make optimal pricing and production decisions for intermediate goods (3), (4); retailers make optimal optimal production decisions for retail goods (6)

3. The structure of the network follows (D.8), with some rules to determine $\left\{l_u^U(\omega)\right\}_{\omega\in\Omega_u}$, $\left\{Q_u^U(\omega)\right\}_{\omega\in\Omega_u}$, $\left\{l_d^D(\omega)\right\}_{\omega\in\Omega_d}$, $\left\{Q_d^D(\omega)\right\}_{\omega\in\Omega_d}$.

4. All markets clear (D.9), (D.10), and (D.11)

E Appendix for Data and Reduced-Form Facts

E.1 Empirical Estimates of Gravity Equations

This appendix presents the estimates of gravity equations of domestic firm-to-firm trade flows across Chilean municipalities. In Table E.1, we document that geographic frictions shape the architecture of production networks. Specifically, we construct the total trade flows, average transaction per relationship (intensive margin), and the number of supplier-to-buyer relationships (extensive margin), across 345 municipalities within Chile, by aggregating our domestic firm-to-firm trade data in 2019. We then regress the log of these values on the log of travel distance between the municipalities, controlling for the origin and destination municipality fixed effects. These regressions correspond to the structural gravity equations implied by our model (Lemma 2), where we parameterize iceberg trade costs and search and matching frictions as an isoelastic function of travel distance.

We find that the overall trade flows sharply decline in geographic distance, with the estimated elasticity of -1.43. This estimate within the range but on the higher end of the typical estimates for international trade flows (0.28-1.55; Disdier and Head, 2008). Interestingly, this decay is driven both by the transaction volume per relationship (intensive margin), with the elasticity of -0.52, and the number of the number of supplier-to-buyer relationships (extensive margin), with the elasticity of -0.91. This pattern is consistent with Lemma 2, which predicts that iceberg trade costs and search and matching frictions drive distinct spatial structures for extensive and intensive margins.

E.2 Tariff Changes from Bilateral Trade Agreements

The U.S. and Chile implemented a Preferential Trade Agreement (PTA) in 2004.[3] This PTA reduced Chile's (average) preferential import tariff towards US products by 93% (from an

[3]In 2018, imports from China and the US constituted about 24% and 19% of overall imports, which corresponds to 6% and 4% of Chile's GDP, respectively. Exports to China and the US were about 33% and 14% of overall exports, which amounts to 8% and 4% of Chile's GDP, respectively.

Table E.1: Estimates of Empirical Gravity Regression within Chile

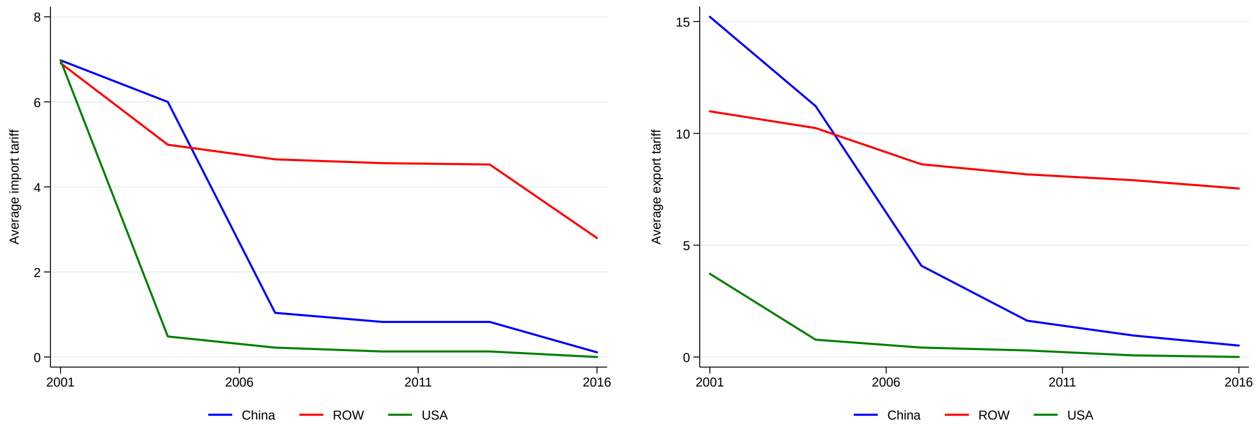
	Total	Intensive	Extensive
	(1)	(2)	(3)
Log Distance	-1.43	-0.52	-0.91
	(0.01)	(0.01)	(0.00)
R^2	0.63	0.30	0.83
Origin Municipality FE	✓	✓	✓
Destination Municipality FE	✓	✓	✓
Same Municipality FE	✓	✓	✓
Number of Observations	72668	72668	72668

Notes: This table presents the results of estimating the gravity regressions, where we regress the logarithm of the total transaction volume between a pair of municipalities on the logarithm of travel distance, controlling for origin and destination fixed effects and the dummies for the same municipalities, across 345 municipalities within Chile, using our domestic firm-to-firm trade data in 2019. The dependent variable corresponds to log total trade flow, log average trade flow (intensive margin), and the log number of links between municipalities (extensive margin).

average applied tariff of 6.9 percentage point to 0.5 percentage point (Fontagné et al., 2022)), with a peak of a 100% tariff cut (i.e. the complete removal of import tariffs) for many organic and inorganic chemical products as well as for many plastic and rubber products (Fontagné et al., 2022). It had similar effects on export tariffs of Chile to the U.S.In addition, Chile has implemented a trade liberalization agenda that in particular reduced tariffs from and to China. Average import tariffs with China were reduced from 6.9 percentage points in 2001 to 0.1 percentage points in 2016. Figure E.1 shows a significant tariff decline from and to China and the US, while there is only a moderate decline to the Rest of the World (ROW). These tariff cuts were particularly relevant for intermediate imports, as one of the products that are most imported from China are computers and engines whereas from the US are energy-related inputs such as gas and also chemical products. Table E.2 summarizes the tariff changes between 2001 to 2016 from and to the US and China for three main sectors where the majority of trade liberalization occurred: Agricultural and Fishing, Mining, and Manufacturing. While import tariff reduction is relatively homogenous across sectors, export tariff reduction is heterogeneous across sectors.

Figure E.1: Import and Export Tariffs of Chile during 2001-2016

(a) Average Import Tariffs (b) Average Export Tariffs



Notes: These figures present the average import and export tariffs of Chile (averaged across sectors) with iChina, the US, and the rest of the world (ROW), computed using the dataset built by Fontagné et al. (2022). Panel (A) presents average import tariffs and Panel (B) presents average export tariffs (imposed by the counterpart countries).

Table E.2: Import and Export Tariff Change of Chile with Main Trade Partners: Across Sectors (%)

	Imports		Exports	
	China	US	China	US
a) Agriculture and Fishing	-6.54	-6.54	-12.84	-1.86
b) Mining	-6.45	-6.45	-2.63	-0.20
c) Manufacturing	-6.45	-6.45	-13.06	-3.85

Notes: This table presents the average percentage point changes in tariffs from and to China and the US, between 2001 and 2016, across different sectors and for import and export tariffs, computed using the dataset built by Fontagné et al. (2022)

E.3 Shift-Share Design

To assess the validity of the shift-share design of our trade shocks we check whether the assumptions in Borusyak et al. (2022) (henceforth BHJ) hold in our context. Identification in BHJ leverages quasi-experimental shock variation of the shifts allowing exposure shares to be potentially endogenous. Concretely, the consistency of the shift-share instrumental variable (SSIV) estimator requires the shifts to be i) randomly assigned, ii) numerous and mutually uncorrelated, and iii) relevant. We focus on this appendix in discussing conditions (ii) and (iii). In out context, shifts are defined by tariff changes between 2003 and 2007 at the country–product level. Shares of exposure are defined as import shares relative to total imports (Equation 39) and relative to total expenses (Equation 40).

Table E.3: Shift-Share Import Shock: Summary Statistics

Share Denominator	Total Imports (1)	Total Expenses (2)
Mean	-0.05	-0.05
Standard Deviation	0.03	0.03
Interquartile range	0.04	0.04
Effective sample size		
Across country-products	2453	2324
Across region-HS3	187	187
Across HS3	61	61
Largest weight		
Across country-products	0.00	0.00
Across region-HS3	0.02	0.02
Across HS3	0.04	0.04
Number of countries	113	113
Number of products	4322	4322
Number of region-HS3	869	869
Number of HS3	217	217
Number of shocks (country-products)	33260	33260

Notes: This table presents summary statistics of the shift-share import shock used for the empirical and quantitative analysis. Column 1 uses the definition of the shock where the denominator of the share is total imports as described in Equation (39). Column 2 uses the definition of the shock where the denominator of the share is total expenses as described in Equation (40).

We start with condition ii. Table (E.3) presents summary statistics of importance weights $s_{cp} = \sum_f s_{fcp}$ as in BHJ, where s_{fcp} is the share associated to the triple of firm f, seller country c, and HS-6 product p. For our main analysis we use two shocks which in which is the denominator of the share used: (1) total imports as in Equation (39), (2) total expenses as in Equation (40). Both shocks have similar means, standard deviations, and interquartile ranges. We rely on tariff variation across 113 countries, 4322 products, and 33260 country-product combinations. To assess whether shocks are not too concentrated, we compute their inverse HHI $1/\sum_f s_{fcp}^2$ across all country-product pairs which is coined as the effective sample size. Following BHJ, asymptotics of this design relies on this measure being large. The effective sample size is indeed high, suggesting that condition ii) holds. We also see that the largest weight in our sample is below 0.01, so no single country-product pair is particularly important for firms in the sample. Following the discussion in BHJ, this suggests that a shock-level law of large numbers applies for these shocks.[4]

Table E.4: Shift-Share Import Shock: Dispersion

Shock Denominator	Controls	Mean	SD	IQR
(1) Total Imports	Raw	-0.052	0.019	0.027
	Net of FE	0.000	0.019	0.024
	Net of FE and weights	0.000	0.019	0.024
(2) Total Expenses	Raw	-0.021	0.020	0.032
	Net of FE	0.000	0.019	0.029
	Net of FE and weights	0.000	0.010	0.007

Notes: This table presents summary statistics of the mean and dispersion of the shift-share import shock used for the empirical and quantitative analysis. Row 1 uses the definition of the shock where the denominator of the share is total imports as described in Equation (39). Row 2 uses the definition of the shock where the denominator of the share is total expenses as described in Equation (40). For each of the two shocks, the table displays the standard deviation and IQR for i) the raw shock ii) the shock net of sectoral fixed effects iii) the shock net of sectoral fixed effects, and controls for incomplete shares, as measured by the sum of weights used in the construction of a shock.

Table (E.4) shows further statistics of the shock at the firm-level. We conclude three features of the shock. First, shocks have similar standard deviations. Second, adding sectoral fixed effects does not remove much of the variation in both shocks. Third, controlling for "incomplete shares" does play a role for the shock that uses total expenses.[5]

Finally, we check whether our shocks are in fact relevant (condition iii). We follow Demir et al. (2024a) in constructing placebo shift-share shocks and run the same shock-level regressions as before with the placebo shocks instead. Specifically, we keep exposure shares fixed and draw placebo shifts. Contrary to Demir et al. (2024a), we draw shifts from the empirical distribution of tariff changes of Chilean imports in our dataset rather than imposing any parametric assumption on the distribution of placebo shifts. We impose no structure in the randomization, except that country-products with zero changes remain unaltered. Table (E.5) shows that results are insignificant when implementing the placebo shift-share shock.

To compute standard errors when using the shift-share shocks we proceed as follows.

[4]Table (E.3) also shows that once we compute importance weights at either the country region-product group or product-group levels, the effective sample size decreases substantially. This implies that there is much less variation available if we allow shocks to be correlated or clustered at this level.

[5]It does not play a role for the shock that uses total imports in the denominator as shares are complete in this case.

Table E.5: Impact of Import Tariff Shocks on International and Domestic Production Links: Placebo

	Total Imports (1)	Number Int. Suppliers (2)	Number Dom. Suppliers (3)	Number Dom. Buyers (4)
Import Tariff Shock	-0.93	-1.27	0.53	0.12
	(1.63)	(0.77)	(0.97)	(1.51)
Number Observations	33260			
Sector FE (6 digit)	Yes			
Prior Import Share	Yes			
Period	2003-2007			

Notes: This table reports the estimates of regression equation (38) using a placebo shift-share shock where the tariff changes are drawn randomly from the empirical distribution of tariff changes. Import shocks are defined by Equation (39) for Columns (1)-(2) and by Equation (40) for Columns (3)-(4). All outcome variables are log changes between 2003 (pre-agreements) and 2007 (post-agreements). The samples include all Chilean firms that exist in both 2003 and 2007. Standard errors are computed following Borusyak et al. (2022).

As in Adão, Kolesár and Morales (2019), BHJ shows that inference with shift-share shocks is complicated by the fact that observations with similar exposure shares probably have correlated unobservables. That is, two firms with similar exposure to changes in a product's tariffs are more likely to have correlated unobservable shocks. To address this issue, BHJ propose a shares-weighted second stage IV regression at the shock-level, i.e. at the country-product level:

$$y_{cp} = \alpha + \beta x_{cp} + \epsilon_{cp}$$

where y_{cp} and x_{cp} are the same variables as the regression at the firm level, but appropriately weighted so that they are measured at the country-product level. In this regression, tariff changes serve as instruments for the shift-share shock x_{cp}. This approach yields correct, exposure-robust standard errors and numerically equivalent point estimates of the SSIV coefficient in a firm-level regression.

To assess the appropriate level of clustering of the shock-level regressions from above we perform the following test. Shocks must be mutually uncorrelated, or at least across clusters. In the same spirit as in BHJ, we compute intra-class correlation coefficients of the shocks within different country-product groups. We obtain the intra-class correlations (ICC) from a random effects model hierarchically decomposing residual variation in the shifts:

$$\Delta\tau_{cp} = a_{region-1digit(cp)} + b_{region-2dig(cp)} + c_{region-3digit(cp)} + e_{cp}$$

where $a_{region-1digit(cp)}$ are country region-1 digit product code random effects, $b_{region-2digit(cp)}$ are country region-2 digit product code random effects, and $c_{region-3digit(cp)}$ are country region-3 digit product code random effects. Table (E.6) presents the estimated ICCs suggesting that clustering is present only at the finest level of aggregation - region by 3-digit product groups. Furthermore, we cluster our standard errors of the shock-level regressions at this level as there is not much correlation across shocks at higher aggregation levels.

Table E.6: Shift-Share Import Shock: Clustering

	(1) Log Tariff Change
Country region by 1 dig product group	0.285 (0.248)
Country region by 2 dig product group	0.0645 (0.0384)
Country region by 3 dig product group	0.123 (0.0509)
Number of country-products	33260

Notes: This table presents summary statistics of the tariff changes. Estimates come from a maximum likelihood procedure with an exchangeable covariance structure for each country region-product group. Standard errors in parentheses.

F Appendix for Calibration

F.1 Adjustment of Trade Flows

As mentioned in Section 6.1, we calibrate aggregate trade flows $\{X_{ud,hk}\}$ using various data sources. However, the observed trade flows constructed in this way do not necessarily satisfy our model's equilibrium conditions. To enable well-defined counterfactuals, we adjust the trade flows consistent with the equilibrium conditions under the calibrated structural parameters $\{\alpha_k,\ \beta_{k,L},\ \beta_{hk},\ \gamma_k^B,\ \gamma_k^S,\ \varepsilon_k,\ \lambda_{kl}^B,\ \lambda_{kl}^S,\ \mu_k,\ \sigma_k\}$ below by interpreting that the observed trade flows involve measurement errors.

Denote that the observed aggregate expenditure in location d and sector l by $\check{E}_{d,l} = \sum_{u,h} \check{X}_{ud,hl}$, where $\check{X}_{ud,hl}$ is observed trade flows from (u,h) to (d,l). We assume that the actual trade flow from (u,h) to (d,l) is given by

$$X_{ud,hl} = \check{\Lambda}_{ud,hl}\beta_{hl}\check{E}_{d,l}\epsilon_{d,l}, \tag{F.1}$$

where $\check{\Lambda}_{ud,hl} \equiv \frac{\check{X}_{ud,hl}}{\sum_{u'} \check{X}_{u'd,hl}}$, and $\{\epsilon_{d,l}\}$ is the measurement errors of the aggregate expenditure. We recover $\{\epsilon_{d,l}\}$ that satisfy all the market clearing conditions. Specifically, we iterative over $\{\epsilon_{d,l}\}$ starting from initial guess until convergence:Obtain $\{X_{ud,hl}\}$ from Equation (F.1)

1. Compute $\{R_{i,k},\ w_i L_i,\ \Pi_i\}$ from Equations (C.28), (C.29), (C.30), and (C.31),

2. Recompute $\{E_{i,k}\}$ using $\{R_{i,k},\ w_i L_i,\ \Pi_i\}$ and Equation (C.32)

3. Update $\{\epsilon_{i,k}\}$ by

$$\epsilon_{i,k} = \frac{E_{i,k}}{\tilde{E}_{i,k}},$$

where $\tilde{E}_{i,k} = \sum_{u,h} \check{\Lambda}_{ui,hk}\beta_{hk}\check{E}_{i,k}$, and we normalize one element of $\{\epsilon_{i,k}\}$ to one.

Notice that this procedure requires the knowledge of the structural parameters $\{\alpha_k,\ \beta_{k,L},\ \beta_{hk},\ \gamma_k^B,\ \gamma_k^S,\ \varepsilon_k,\ \lambda_{kl}^B,\ \lambda_{kl}^S,\ \mu_k,\ \sigma_k\}$. Therefore, when we estimate parameters $\{\lambda_{kl}^B, \lambda_{kl}^S, \mu_k\}$ for indirect inference (Appendix F.2), we repeat this procedure for each candidate value of $\{\lambda_{kl}^B, \lambda_{kl}^S, \mu_k\}$.

F.2 Estimation of $\{\lambda, \mu\}$

In this appendix, we describe the detailed estimation procedure for the matching function elasticities, $\{\lambda_{kl}^B, \lambda_{kl}^S\}$, and the labor share in search costs, $\{\mu_k\}$, through the indirect inference approach in Section 6.1. Recall that, due to the limited variations in tariff changes outside "goods" sectors, we assume that these parameters are common across all sectors $k, l \in K$. We also assume that the matching function elasticities are symmetric

$\lambda^B = \lambda^S \equiv \lambda$ as these two parameters tend to jointly affect the equilibrium system and it is difficult to identify each of them separately. We show in the the appendix that alternatively setting $\lambda^B = 0$ or $\lambda^S = 0$ while keeping the sum $\lambda^S + \lambda^B$ unchanged yields virtually identical implications for the aggregate welfare changes. The procedure of estimating $\{\lambda, \mu\}$ is as follows.

1. For each sector k and origin country u, we construct the average import tariff changes $\tilde{\mathcal{T}}_{uk}$ as the weighted average across HS-6 products within each category.

2. For each candidate value of $\{\lambda, \mu\}$, we undertake the counterfactual simulations of the changes in the iceberg trade costs $\hat{\tau}_{ud,kl} = 1 + \tilde{\mathcal{T}}_{uk}$ for all municipalities d and sector l within Chile. That is, we set

$$\hat{\tau}_{ud,kl} = \begin{cases} 1 & d \notin Chile \\ \Delta \log(1 + \tilde{\mathcal{T}}_{uk}) & d \in Chile \end{cases} \tag{F.2}$$

3. We run the following regression using the model-predicted outcome variables, analogous to Equation (38) using actual data, but at the location and sector within Chile as a sample:

$$\log \hat{y}_{d,l} = \beta \text{ImportTariffShock}_{d,l} + \epsilon_{d,l}. \tag{F.3}$$

Similarly for the regression (38) using actual data, we define $\text{ImportTariffShock}_{d,l}$ as the weighted averages of $\hat{\tau}_{ud,kl}$ using baseline import shares. For the outcome variable of the international import linkages, it is defined by

$$\text{ImportTariffShock}_{d,l} \equiv \sum_n \sum_k \frac{\text{Import}_{nd,kl}}{\text{ImportSum}_{d,l}} \times \log \hat{\tau}_{ud,kl}. \tag{F.4}$$

For the outcome variable of the domestic production linkages, it is defined by

$$\text{ImportTariffShock}_{d,l} \equiv \sum_n \sum_k \frac{\text{Import}_{nd,kl}}{\text{ImportSum}_{d,l} + \text{DomPurchase}_{d,l}} \times \log \hat{\tau}_{ud,kl}. \tag{F.5}$$

These definitions align with those of the data regressions (Equations 39 and 40), correspondingly.

4. We search for $\{\lambda, \mu\}$ that minimizes the squared distance between the regression coefficients on domestic production linkages in the data (Columns 3-4 of Table 1) and in the model prediction. That is,

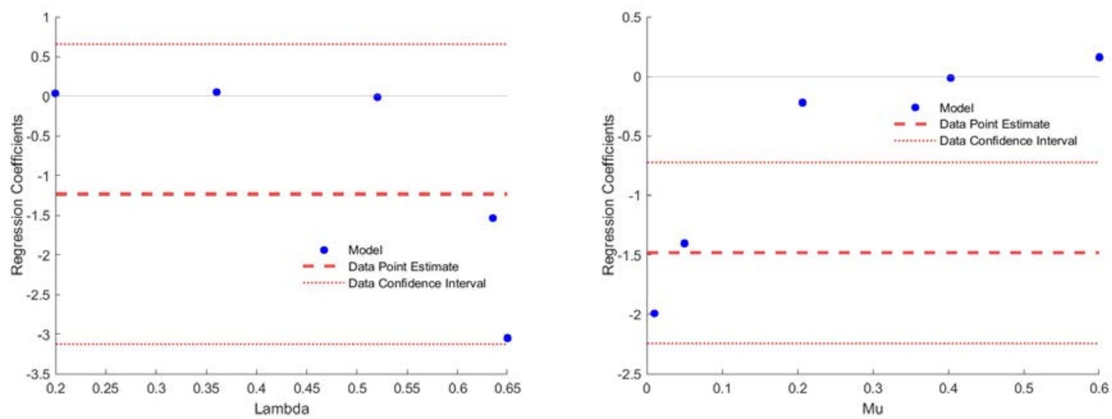
$$\{\hat{\lambda}, \hat{\mu}\} \equiv \min \sum_c \frac{\left(\hat{\alpha}_c^{\text{Model}} - \hat{\alpha}_c^{\text{Data}}\right)^2}{\text{Avar}\left(\hat{\alpha}_c^{\text{Data}}\right)},$$

where c corresponds to the regression coefficients of each of Columns 1-4, and Avar $(\hat{\alpha}_c^{\text{Data}})$ corresponds to the asymptotic variance of the regression coefficient for $\hat{\alpha}_c^{\text{Data}}$.

Weighting the regressions using the model counterfactuals using the number of firms in each municipality and sector, aligning the firm-level regression in Table 1, has virtually no effects on the regression coefficients. We cannot run regression Table 1 at the municipality and location level using our data as it does not report the location of transactions for 2003-2007.

In Figure F.1, we show the sensitivity of $\{\lambda, \mu\}$ on the targeted regression coefficients. In Panel (a), given our estimates of $\hat{\mu}$, we vary the value of λ, and show how the regression coefficients on the number of domestic buyers in the model regression change. In Panel (b), given our estimates of $\hat{\lambda}$, we vary the value of λ, and show how the regression coefficients on the number of domestic suppliers in the model regression change. It is clear that the targeted model coefficients are responsive to these parameter values.

Figure F.1: Sensitivity of Parameters to Targeted Moments



(a) Sensitivity of λ on the impacts on number of domestic buyers

(b) Sensitivity of μ on the impacts on number of domestic suppliers

Notes: These figures present average import and export tariffs of Chile (averaged across sectors) with iChina, the US, and the rest of the world (ROW), computed using the dataset built by Fontagné et al. (2022). Panel (A) presents average import tariffs and Panel (B) presents average export tariffs (imposed by the counterpart countries).

G Additional Results for Counterfactual Simulations

Table G.1: Aggregate Effects From Import Cost Increase (%): Different Values of λ

	1) $\widehat{\text{Welfare}}$ (%)	2) Rel. to Baseline	3) $\hat{X}_{ui,u\in\text{US,CN}}$	4) $\hat{M}_{ui,u\in\text{US,CN}}$	5) $\hat{M}_{ui,u\in\text{CL}}$
a) Baseline	-0.35	100	-20.5	-7.6	-0.01
b) Baseline, $\lambda^S = 0.32$, fix $\lambda^S + \lambda^B$	-0.35	100	-16.2	-3.3	-0.04
c) Baseline, $\lambda^B = 0.32$, fix $\lambda^S + \lambda^B$	-0.33	95	-19.7	-6.9	0.05
d) Fixed Networks, fix ε_k	-0.27	77	-20.2	0	0

Notes: The results of counterfactual simulations to increase the iceberg trade costs from US and China to all Chilean municipalities by the same magnitude of the import tariff changes under the trade agreements used in Section 5.2, under four scenarios: (a) using our baseline parameters (Table 2), (b) allow for endogenous networks but alternatively set $\lambda^S = 0.3$ while keeping $\lambda^s + \lambda^B$ fixed, (c) allow for endogenous networks but alternatively set $\lambda^B = 0.3$ while keeping $\lambda^s + \lambda^B$ fixed and (d) shut down endogenous networks ($\lambda^S = \lambda^B = 0$), while keeping the trade elasticity ϵ_k at our baseline scenario. Column (1) reports the changes in aggregate welfare across all Chilean municipalities (weighted average of GDP changes across Chilean municipalities with pre-shock GDP weights); Column (2) reports the ratio of the values in Column (1) to our baseline specification; Column (3) reports the average percent changes in imports from US and China by Chilean municipalities; Column (4) reports the average changes in the number of supplier linkages from US and China; Column (5) reports the average changes in the number of supplier linkages within Chile.