

NBER WORKING PAPER SERIES

BITCOIN MINING MEETS WALL STREET:
A STUDY OF PUBLICLY TRADED CRYPTO MINING COMPANIES

Hanna Halaburda
David Yermack

Working Paper 30923
<http://www.nber.org/papers/w30923>

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
February 2023, Revised September 2023

We appreciate comments from Andreas Park, Fahad Saleh, and seminar participants at Deakin University, the Deutsche Bundesbank, the University of Missouri, and the University of Western Australia. Part of this research was completed while Yermack was a visiting professor at Erasmus University Rotterdam. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

NBER working papers are circulated for discussion and comment purposes. They have not been peer-reviewed or been subject to the review by the NBER Board of Directors that accompanies official NBER publications.

© 2023 by Hanna Halaburda and David Yermack. All rights reserved. Short sections of text, not to exceed two paragraphs, may be quoted without explicit permission provided that full credit, including © notice, is given to the source.

Bitcoin Mining Meets Wall Street: A Study of Publicly Traded Crypto Mining Companies
Hanna Halaburda and David Yermack
NBER Working Paper No. 30923
February 2023, Revised September 2023
JEL No. G23,L23,L94

ABSTRACT

This paper studies the operations and financial valuations of 13 cryptocurrency mining companies that are listed on the NASDAQ stock exchange and have facilities in North America. We find that miners using Texas wind power are offline more than other miners, in a more erratic pattern. Yet, despite having relatively low activity levels, these Texas miners are more profitable than those using more stable sources of energy such as hydro power or solar power, as reflected in significantly higher enterprise values. Our model shows that miners using sustainable energy may be more profitable than those using conventional sources, despite the shutdowns, as they benefit from extremely low prices when there is oversupply of sustainable energy (e.g., strong winds). The model also shows that it may be beneficial for the electric utility to offer miners compensation for curtailment of their activity when there is undersupply of energy (e.g. lack of wind), which we also observe in our sample. This compensation further increases profits of the miners. We find a negative and significant beta between crypto mining stocks and an index of electric utilities, suggesting that ownership of a crypto mining company might provide a useful channel for risk management in the electric power industry.

Hanna Halaburda
Stern School of Business
New York University
44 W 4th St.
New York, NY 10012
hhalaburda@gmail.com

David Yermack
Stern School of Business
New York University
44 West Fourth Street, Suite 9-160
New York, NY 10012
and NBER
dyermack@stern.nyu.edu

Bitcoin mining meets Wall Street: A study of publicly traded crypto mining companies

1. Introduction

In the spring of 2021, China’s government unexpectedly banned proof-of-work cryptocurrency mining. Up to that point, China had hosted a majority of the global bitcoin network hashrate, and on short notice miners had to shut their Chinese operations and seek new venues. North America emerged as a prime destination, and by the end of 2021 the United States had emerged as the largest site for proof-of-work data centers, with a large concentration in Texas.¹

Many U.S. bitcoin miners have elected to organize themselves as listed companies on the NASDAQ stock market, and by August 2023 more than a dozen publicly traded crypto mining companies, representing about 16.4% of the global hashrate, had floated their shares alongside those of more traditional miners of gold, copper, aluminum, and other minerals. Operating as public corporations represented a sharp break with the most common patterns of organization in the industry, which had previously been dominated by private partnerships and lone-wolf entrepreneurs. For

¹ The news media has closely chronicled the migration of the crypto mining industry to the U.S. and in particular Texas. For example, see Dalvin Brown, “Bitcoin miners break new ground in Texas, a state hailed as the new cryptocurrency capital,” *The Washington Post*, July 8, 2021. The evolving global footprint of proof-of-work mining is tracked by the Cambridge University Centre for Alternative Finance at https://ccaf.io/cbeci/mining_map, which shows the rearrangement of hashrate shares among countries since May 2021 in slow-motion animation.

stockholders and bondholders, agreeing to become risk-sharing investors in crypto mines might have seemed unusual, since the essential task in proof-of-work mining involves little more than guessing random integers in an attempt to solve puzzles by trial-and-error. The comparative advantage of crypto miners lies in their ability to guess random integers rapidly, akin to somebody skilled at approaching a lottery kiosk and buying many tickets very quickly.

Disclosures by the publicly traded crypto miners have provided new transparency into the operating risks, leverage, cost structure and supply chain relationships in the mining industry. Our paper studies how outside shareholders have valued bitcoin miners, and how the publicly traded mining companies have adapted their strategies in an environment that requires regular shareholder reporting and interaction with Wall Street analysts. Along with daily stock prices and standard disclosures such as Forms 10-K, 10-Q, and 8-K, we rely heavily on monthly reports of mining success that all of the crypto miners now publish via press releases shortly after the end of each month. Our study covers a particularly difficult period for the mining industry, as the plunge in cryptocurrency prices in 2022 badly hurt miners' revenue, such that by late 2022 one of the mining companies was operating in Chapter 11 bankruptcy and several others underwent debt restructurings.

Proof-of-work mining, as used in the bitcoin network, involves trial-and-error computations in which a miner appends a positive integer to a string that represents a "block" of unconfirmed bitcoin transactions. The augmented string becomes the input to a hash function, and the miner tests whether that the output hashcode falls below a critical value set by the network. Most of the time the miner's guess will not yield a low enough

hashcode, and the process is then repeated many more trillions of times until a miner somewhere in the world finds a valid solution and collects a reward that is currently set at 6.25 bitcoins, or about \$170,000 at recent prices.² The network algorithmically adjusts the critical value so that on average ten minutes of trial-and-error “work” is expected to be necessary across the entire network until some miner wins the next block. No creativity or strategy is involved in the sequence of trial-and-error guessing; the process simply requires brute-force repetition of uninteresting work at the highest possible velocity.

Due to the absence of skill required in mining, we examine other possible sources for a company’s comparative advantage that might create investor demand for its shares. Two explanations are related to procurement: companies may have priority access to scarce mining equipment, or they may secure relationships with cheap and reliable energy providers. A third possibility is that a miner may have superior engineering skill that keeps its machinery consistently online. A fourth hypothesis is that miners may accumulate bitcoin over time in such a way that they begin to resemble bitcoin closed-end funds, thereby attracting investors.

We can quickly rule out the importance of access to mining hardware, at least during the crypto bear market that has characterized most of our sample period. When improved models become available, miners’ specialized hardware can be highly sought-after, particularly those models manufactured by market leader Bitmain Technologies Ltd. in China. Among our sample companies, we observe many shareholder

² The system of mining incentives, which also involves customer user fees set continuously by auction, is detailed by Easley, O’Hara and Basu (2019). Lehar and Parlour (2021) study the possibility of collusion among miners to induce bitcoin customers to pay higher user fees.

communications during the 2021 crypto bull market that give great attention to the status of their Bitmain orders, including the quantity of mining rigs ordered, the expected delivery schedule, the actual shipment and delivery events, and the schedules for planned installation and activation of newly received machines. However, by the Fall of 2022 the mining market had become saturated with Bitmain's newer models,³ and some mining companies were canceling orders or re-selling units at a discount.⁴

We also see little evidence that mining companies build large inventories of bitcoin in order to become surrogate closed-end funds. Only one company in our sample, Hut 8 Mining Corp. of Canada, followed this strategy on a sustained basis, and by the end of 2022 it held \$150 million worth of bitcoin in inventory and had an enterprise value (debt plus equity) of about \$200 million, implying that the bitcoin represented three-quarters of the value of the firm. However, Hut 8 halted the accumulation strategy in early 2023. The alignment between Hut 8's daily stock returns and the bitcoin price index, as measured by a linear regression, is estimated at 1.03, but this value is quite close to the estimates for most other companies in the sample, all of which have much smaller bitcoin inventories relative to their sizes. A few other firms did purport to follow accumulation or "hodl" strategies of retaining their mined bitcoin during part of the sample period, but all of them except Hut 8 abandoned this strategy during 2022, often selling their bitcoin at depressed market prices in order to raise cash to forestall financial distress.

³ See, e.g., Eliza Gkritsi, "A Huge Glut of Bitcoin Mining Rigs Is Sitting Unused in Boxes," *Coindesk*, October 14, 2022.

⁴ For example, see the January 12, 2023 announcement by Iris Energy, stating that it had purchased and immediately re-sold miners previously ordered from Bitmain, with the transaction reducing prepaid expenses by \$8 million on its balance sheet while improving its cash position by \$6 million; this appears to represent the liquidation of its Bitmain deposits at 75 cents on the dollar.

We therefore focus on miners' relationships with electric utilities as sources of comparative advantage. The electricity consumption of crypto mining has received considerable public scrutiny. The Congressional Research Service (2019) provides a detailed overview of the mining process along with a chronology of regulatory interventions by the U.S. and other countries due to the high levels of energy use. Some North American jurisdictions such as New York state and the province of British Columbia have passed or are considering restrictions on proof-of-work mining due to its impact on consumer utility prices (Benetton, Compiani, and Morse, 2021). Partly in response to the controversy over their high energy consumption, mining companies have sought out sources of sustainable or renewable energy. Along with its public relations benefit, using renewable energy may be attractive to miners because wind or hydropower are often generated in remote geographical areas where a mining company may be less likely to compete with nearby households for a share of the energy supply and will also avoid criticism for nuisance externalities such as the emission of continuous loud noise.

Most of the miners in our sample claim to be engaging in "green," "sustainable," or "renewable" energy consumption, but researchers have been skeptical (Solomon 2022), and many of the companies' claims often apply to plans for future electrical installations not yet operating. One environmentally friendly strategy adopted by several companies in our sample involves "load balancing," whereby they agree to become customers of utilities that produce erratic sources of renewable energy, especially wind power. Under a load balancing strategy, a miner provides a stable source of demand for electricity but agrees to shut down operations when supply of electricity drops, essentially becoming a buffer between the utility and highly variable supply of renewable

electricity. In this setting the crypto miner may receive a rebate or subsidy when it curtails its consumption at the utility's request, or a discounted retail electricity price with the proviso that the utility can interrupt the flow of power at its own discretion.⁵

The presence of a crypto miner willing to operate on this basis can encourage the construction of more wind power capacity, leading to greater generation of renewable energy across the entire grid (Cassauwers, 2021). The state of Texas, which already obtains about one-fourth of its electrical power from wind, has taken a special interest in crypto mining for this reason and is actively encouraging mining companies to locate there, even if Texas' hot weather is less than ideal for keeping mining rigs cool.

Our paper presents a basic model of a miner's choice between sustainable energy and conventional sources of electric power, and we identify market conditions under which a sustainable miner may be more profitable despite shutdowns induced by erratic energy supply resulting in price surges. The model generates several predictions that we test using data from the 13 publicly traded mining companies, three of which currently rely heavily upon wind power in Texas. As predicted by the model, we find that these miners are less productive due to frequent shutdowns of their operations, but they benefit from extremely low prices when there is oversupply of sustainable energy (e.g., strong winds), and also from compensation for curtailment of their activity when there is undersupply of energy (e.g. lack of wind). As shown in analysis below, these companies trade in the stock market at higher valuations than their competitors.

We also analyze the structure of daily stock returns for all mining companies. We find positive market reactions to monthly mining reports that indicate high efficiency.

⁵ An executive of one crypto miner told us that his company is required to power down its rigs on 15 seconds' notice from its utility provider under the terms of their contract.

Miners' stock returns exhibit strong positive alignment with the NASDAQ market index, which is heavily weighted with technology stocks, and also with the daily returns on bitcoin. After controlling for moves in the NASDAQ market and bitcoin, mining stock returns exhibit significantly negative associations with an index of electric utility stocks. This result suggests that crypto mining offers a natural hedge for the risks of operating an electric utility, implying that direct ownership of proof-of-work miners by utilities may be optimal for risk-sharing purposes (see the analysis of the Brazilian market in Bastian-Pinto *et al.*, 2021). We observe “behind-the-meter” installations of crypto mines at some electric utilities that bypass the consumer transmission grid and appear to represent risk-sharing joint ventures.

By studying the demand side of the market for renewable energy that is subject to irregular fluctuations in supply, our paper complements an emerging literature, often published in operations research or engineering journals, that shows the potential of load balancing for stimulating the supply side of the renewables market. These papers include Shan and Sun (2019); Bruno, Weber and Yates (2022); and Niaz, Liu, and You (2022).

The remaining sections of the paper are organized as follows. Section 2 presents our model. Section 3 describes our dataset. Section 4 contains our analysis. Section 5 includes a discussion and conclusions.

2. Model

A. Environment

Consider a market environment which includes three types of miners that differ by the electricity prices they face and barriers to entry: *individual* miners, who use

electricity at retail price, p_I ; *conventional energy* powered mining facilities, who get a commercial rate $p_C < p_I$; and *sustainable energy* powered mining facilities, whose electricity cost, p_S depends upon the energy state.

Individual miners face no barriers to entry (Prat and Walter 2021, Budish 2022, Halaburda et al 2022), hence more will enter if they find mining profitable. We will consider them in aggregate and denote this aggregate as I . Mining facilities, however, face some barriers to entry, related for example to contracting with a limited number of electricity providers, so there is an upper limit on how many can be present in the market. We assume there is one conventional-energy-powered mining facility (C), and one sustainable-energy-powered mining facility (S). All miners use the same equipment which is available without scarcity, so relative usage of electricity corresponds to relative hashing power employed.

Unlike conventional electricity, sustainable electricity has a high variability of supply depending on the weather or season — more water for hydro in the spring, more sunshine during the day, wind maybe present or not. The sustainable energy state may be good (G) or bad (B). In the good state, which occurs with frequency α , the supply of energy is high and the price of sustainable electricity, $p_S(G)$, is lower than the price of conventional energy.

Given the level of hashing power H_{-i} applied by the rest of the market, the profit miner i obtains from applying h_i hashing power is

$$\pi(h_i, H_{-i}) = \frac{h_i}{h_i + H_{-i}} R - p_i h_i$$

where R is the value of the mining reward. Note that h_i and H_{-i} are at the same time measures of electricity consumption and hashing power.

B. Participation and Profitability of a Sustainable-Energy Mining Facility

Within this environment, we ask: (1) When is it worthwhile for the sustainable miner to participate in the market, and (2) When is mining with sustainable energy more profitable than mining with conventional energy?

Case i: Participation with Spot Market Adjustments

B.1. Baseline without S.

Consider first the baseline case without the sustainable mining facility, i.e., when only individual miners and the conventional mining facility participate in the mining market. For any level of $H_{-I} = h_C$ that is less than R/p_I , individual miners find it profitable to enter, and therefore increase I 's electricity use until approximately $h_I + H_{-I} = \frac{R}{p_I}$. That determines the overall use of energy for mining in this baseline equilibrium to be $H_0 = \frac{R}{p_I}$. In equilibrium, the conventional mining facility maximizes its profits by using $h_C^*(H_0) = \left(1 - \frac{p_C}{p_I}\right) \frac{R}{p_I}$ energy, obtaining $\pi_C^*(H_0) = \left(1 - \frac{p_C}{p_I}\right)^2 R$. While each individual miner uses infinitesimal amount of energy and breaks even, in aggregate, they use $h_I^* = \frac{p_C}{p_I} \frac{R}{p_I}$.

Lemma 1. *The effect of changes in electricity prices (p_I , p_C) and Bitcoin price (R) on electricity consumption without S :*

- i. *Increase in R directly increases H_0 as well as both $h_C^*(H_0)$ and h_I^* .*
- ii. *Increase in p_I directly decreases H_0 and h_I^* . Its effect on $h_C^*(H_0)$ is ambiguous: $h_C^*(H_0)$ is decreasing in p_I when $2p_C < p_I$, but increasing otherwise.*
- iii. *Increase in p_C has no effect on the overall electricity consumption H_0 , and it moves the electricity consumption from C to I .*

B.2. Participation of the sustainable miner (S).

Whenever $p_S < p_I$, the sustainable mining facility finds it profitable to mine rather than not mine. S 's entry has a differential impact on the equilibrium and participation of other miners depending on whether $p_S < p_I - p_C$ (which we will call *very low* p_S) or the reverse (which we call *higher* p_S). Whether S is more profitable in mining than C depends on the relative costs p_S and p_C as well as the proportion of the good weather state, α .

If there is already H_0 electricity consumed for mining in the market, S finds it profitable to add more electricity if $\pi(h_S, H_0) = \frac{h_S}{h_S + H_0} R - p_S h_S > 0$ which is equivalent to $\frac{R}{p_S} - H_0 > h_S$. Therefore, for any $p_S < p_I$, a sustainable mining facility finds it profitable to participate. And the lower the p_S , the more additional hashing power that S adds. Other miners adjust their electricity use in response. The overall electricity consumption exceeding H_0 makes individual miners unprofitable, and some or all will exit, depending how low p_S is.

For *higher* p_S , i.e., $p_S > p_I - p_C$, if sufficiently many individual miners exit, the remaining ones become borderline profitable (or indifferent). That implies that the overall use of the energy in the market is the same as before S 's entry, $H_0 = \frac{R}{p_I}$.⁶ In this equilibrium, the sustainable mining facility obtains the highest profits $\pi_S^*(H_0) = \left(1 - \frac{p_S}{p_I}\right)^2 R$ at the electricity consumption level $h_S^*(H_0) = \left(1 - \frac{p_S}{p_I}\right) \frac{R}{p_I}$. The electricity consumption and profitability of C does not change: $h_C^*(H_0) = \left(1 - \frac{p_C}{p_I}\right) \frac{R}{p_I}$ and $\pi_C^*(H_0) =$

⁶ This value may be higher than H_0 if individual miners face different costs of retail energy, or if there were entry frictions for individual miners in the baseline case before S 's entry.

$\left(1 - \frac{p_C}{p_I}\right)^2 R$. Now the individual miners in aggregate consume $h_I^* = \frac{p_C + p_S - p_I}{p_I} \frac{R}{p_I}$, each one breaking even.

When p_S is *very low*, i.e., $p_S < p_I - p_C$, a sustainable mining facility finds it optimal to use so much energy that even when all but one individual miners exit, the last one will not be profitable. Therefore, all individual miners leave, and only S and C remain in the market. In this equilibrium the remaining miners use more overall energy than in the presence of individual miners: $H_1 = \frac{R}{p_C + p_S} > H_0$. Within that, S uses

$$h_S^*(H_1) = \left(1 - \frac{p_S}{p_C + p_S}\right) \frac{R}{p_C + p_S} \text{ and } h_C^*(H_1) = \left(1 - \frac{p_C}{p_C + p_S}\right) \frac{R}{p_C + p_S}, \text{ yielding } \pi_S^*(H_1) = \left(1 - \frac{p_S}{p_C + p_S}\right)^2 R \text{ and } \pi_C^*(H_1) = \left(1 - \frac{p_C}{p_C + p_S}\right)^2 R.$$

To compare the energy consumption and profits as p_S varies, recognize that p_S may represent different values in these formulas. Therefore, we use p_S^H to denote *higher* values of p_S and p_S^L to denote *very low* values of p_S . By these definitions, $p_S^H > p_I - p_C$ and $p_S^L < p_I - p_C$.

If we hold p_I and p_C constant while varying p_S , then for $R > 0$, $\pi_S^*(H_0) = \left(1 - \frac{p_S^H}{p_I}\right)^2 R < \left(1 - \frac{p_I - p_C}{p_I}\right)^2 R = \left(\frac{p_C}{p_I}\right)^2 R$. At the same time, $\pi_S^*(H_1) = \left(1 - \frac{p_S^L}{p_C + p_S^L}\right)^2 R = \left(\frac{p_C}{p_C + p_S^L}\right)^2 R > \left(\frac{p_C}{p_I}\right)^2 R$. Therefore, $\pi_S^*(H_1) > \pi_S^*(H_0)$, i.e., a sustainable mining facility's profit always increases as p_S decreases. At the same time, the profit of the conventional mining facility does not change with small changes of p_S so long as p_S is *higher*. Once p_S drops to *very low*, C profit decreases ($\pi_C^*(H_1) < \pi_C^*(H_0)$) and keeps decreasing as p_S decreases within the *very low* range.

Lemma 2. *Participation decisions of miners solely depend on the electricity prices, p_S , p_C and p_I , and are independent of the value of block reward (via Bitcoin price), R :*

- i. *I participates when $p_I < p_C + p_S$,*
- ii. *S participates when $p_C < p_I$.*
- iii. *C always participates, as $p_C < p_I$.*

Lemma 3. *The level of electricity consumption, and profitability of miners depends both on the electricity prices and the value of block reward (via price of Bitcoin), R .*

- i. *electricity consumption (and profit) of each miner increases with R increasing,*
- ii. *decreasing p_i increases h_i^* (and π_i^*).*

The model predicts that electricity consumption will drop as the bitcoin price falls.

This occurs not so much due to exit as the decreased intensity of the same miners. When R decreases, each of the miners decreases their electricity use proportionally. If all miners respond optimally, then each miner's proportion of the electricity use remains the same and only depends on the relative electricity prices, while overall electricity use declines.⁷

Proposition 1. *S who entered the market finds it optimal to shut down or decrease their mining operations in state B , i.e., $0 \leq h_S^*(B) < h_S^*(G)$.*

This result follows directly from the variable supply of sustainable electricity, resulting in a variable electricity price.

B.3. Relative profitability of the sustainable miner (S):

Per analysis above, a sustainable mining facility is profitable whenever $p_S < p_I$. Moreover, it is more profitable than the conventional mining facility every period it operates and $p_S < p_C$. If p_S would always be lower than p_C , then we would get a straightforward result that S is more profitable than C . However, p_S depends on the sustainable energy state, and it may be higher than p_C , or even not available at all in bad

⁷ This is consistent with the main result in Arnosti and Weinberg (2022).

states. Thus, whether S is more profitable than C overall depends on $p_S(G)$ and $p_S(B)$ relative to p_C , as well as α .

Suppose now that $p_S(B) > p_I$. Then the sustainable miner S is active in the good state but shuts down and gets 0 profit in the bad state. Then, using simply $p_S(G) = p_S$, we find that S is more profitable than C when

$$\alpha > \begin{cases} \left(\frac{p_I - p_C}{p_I - p_S}\right)^2 & \text{for } p_S > p_I - p_C, \\ \frac{(p_I - p_C)^2(p_C + p_S)}{(p_C - p_S)p_I^2 + (p_I - p_C)^2(p_C + p_S)} & \text{for } p_S < p_I - p_C. \end{cases}$$

In both cases the threshold for α is less than 1. That means that

Proposition 2. *S may be more profitable than C , even though it is mining fewer bitcoins than its mining power capacity.*

Case ii: Load Balancing Agreements with Curtailment Payments

In the previous section, we showed that a mining facility connected to a sustainable energy utility can be more profitable than the conventional one, and it will mine fewer blocks than its share of the mining power suggests. This may occur purely in response to the variance in the market price of electricity resulting from the variance in supply of sustainable energy. That is, the mining facility finds it optimal to shut off when there is a shortage of energy and the price is high.

Similar results hold when $p_S(B) < p_I$, even though in this case S is limiting the use of mining power in state B , instead of shutting down. More precisely, $h_S^*(G) =$

$$\left(1 - \frac{p_S(G)}{p_I}\right) \frac{R}{p_I}, h_S^*(B) = \left(1 - \frac{p_S(B)}{p_I}\right) \frac{R}{p_I}, \pi_S^* = \alpha \left(1 - \frac{p_S(G)}{p_I}\right)^2 R + (1 - \alpha) \left(1 - \frac{p_S(B)}{p_I}\right)^2 R$$

$$\text{and } \pi_S^* > \pi_C^* \Leftrightarrow \alpha > \frac{(p_I - p_C)^2 - (p_I - p_S(B))^2}{(p_I - p_S(G))^2 - (p_I - p_S(B))^2}$$

The energy utility, however, may face strict capacity constraints in *the B* state. Exceeding this limit is costly for the utility, with cost L . The cost may arise from possible damage to the infrastructure or from loss of reputation and future customers. At the same time, the utility may be limited in how much it increases the price. This is because some essential businesses (e.g. hospitals) may need to keep using the electricity no matter how costly it becomes, and increasing it based on the spot market may be considered price gouging.

In such a case, the utility may be willing to pay non-essential businesses, such as a mining facility, for curtailing their usage when the supply state is B , in order to avoid overwhelming the grid at the cost X .

Shutting down upon request

The mining facility loses $\left(1 - \frac{p_S(B)}{p_I}\right)^2 R$ when it shuts down in the B state. If the cost of overwhelming the grid is higher, the utility is willing to compensate the miner for this loss in return for curtailment. In state B , the utility receives $p_S(B)h_S(B) - X = p_S(B) \left(1 - \frac{p_S(B)}{p_I}\right) \frac{R}{p_I} - X$ without curtailment. It finds it beneficial to pay $\left(1 - \frac{p_S(B)}{p_I}\right)^2 R$ for curtailment when $X > \left(1 - \frac{p_S(B)}{p_I}\right) R$.

For comparison, some other types of producers may have higher costs of shutdown than just the current production. Therefore, mining facilities are the cheapest to compensate.

Curtailment schemes may take different forms. First, the utility may offer $\left(1 - \frac{p_S(B)}{p_I}\right)^2 R$ for stopping mining when the state is B . Second, the utility may set up a

long-term contract where it pays up front $(1 - \alpha) \left(1 - \frac{p_S(B)}{p_I}\right)^2 R$ and the mining facility stops mining upon request. The utility will only request stopping when the state is B , so in expectations both the spot payment and the up-front long term payment lead to the same payoffs for the utility and the mining facility.

Another alternative is a long-term contract where instead of up-front payment, the utility offers a lower rate in G state, $p'_S < p_S(G)$ in return for shutting down mining upon request in the B state. A lower rate will encourage the mining facility to use more energy in the G state, but since in that state there is an excess supply of energy, higher usage does not overwhelm the grid nor put pressure on price for other users. In this scheme, the utility offers p'_S such that it gets the same expected payment

$$\begin{aligned} & \alpha p'_S (1 - (p'_S)/p_I) R / p_I \\ &= \alpha p_S(G) (1 - (p_S(G))/p_I) R / p_I - (1 - \alpha) \underbrace{\left(\left(1 - \frac{p_S(B)}{p_I}\right)^2 R \right)}_{\text{lump-sum curtailment payment}}. \end{aligned}$$

Under this scheme with lower rate p'_S , the mining facility is strictly better off than under the lump sum payment, because

$$\begin{aligned} \alpha \left(1 - \frac{p'_S}{p_I}\right)^2 R &> \alpha \left(1 - \frac{p_S(G)}{p_I}\right)^2 R + (1 - \alpha) \left(1 - \frac{p_S(B)}{p_I}\right)^2 R \Leftrightarrow \\ \alpha \left(1 - \frac{p'_S}{p_I}\right)^2 R &> \alpha \left(1 - \frac{p_S(G)}{p_I}\right)^2 R + \alpha p_S(G) \left(1 - \frac{p_S(G)}{p_I}\right) \frac{R}{p_I} - \alpha p'_S \left(1 - \frac{p'_S}{p_I}\right) \frac{R}{p_I} \Leftrightarrow \\ \left(1 - \frac{p'_S}{p_I}\right)^2 + \frac{p'_S}{p_I} \left(1 - \frac{p'_S}{p_I}\right) &> \left(1 - \frac{p_S(G)}{p_I}\right)^2 + \frac{p_S(G)}{p_I} \left(1 - \frac{p_S(G)}{p_I}\right) \Leftrightarrow \\ 1 - \frac{p'_S}{p_I} &> 1 - \frac{p_S(G)}{p_I} \\ p_S(G) &> p'_S. \end{aligned}$$

B.4. Fixed price contracts with resale option

Suppose that the mining facility and the utility sign a long-term contract with fixed price, independent of the state. The parties may want to sign such a contract to encourage building facilities in closer proximity and prevent future hold-up. (Signing such a contract may attract a mining facility away from locating close to a conventional electrical utility. For that, however, the new fixed p_S needs to be lower than p_C .)

The mining facility would agree to p_S such that

$$\left(1 - \frac{p_S}{p_I}\right) p_S \geq \alpha \left(1 - \frac{p_S(G)}{p_I}\right) p_S(G) + (1 - \alpha) \left(1 - \frac{p_S(B)}{p_I}\right) p_S(B).$$

And the mining facility would agree to p_S when

$$\left(1 - \frac{p_S}{p_I}\right)^2 \geq \alpha \left(1 - \frac{p_S(G)}{p_I}\right)^2 + (1 - \alpha) \left(1 - \frac{p_S(B)}{p_I}\right)^2.$$

There exist values of p_S where both these conditions are met; for some values of $p_S(G)$ and $p_S(B)$, α cannot be too high for such a p_S to exist.

Note that under the fixed price, the mining facility will use more energy than $h_S^*(B)$ in state B , which may put pressure on the grid. A way to avoid increasing pressure on the grid under the fixed price is offering the mining facility the option to resell their electricity back to the grid at the market price. The facility will take this option in state B if $\left(1 - \frac{p_S}{p_I}\right) \frac{R}{p_I} p_S(B) > \left(1 - \frac{p_S}{p_I}\right)^2 R \Leftrightarrow p_S(B) > p_I - p_S$. In such a case, the resale option effectively curtails the energy use in state B , which allows the utility avoid the cost C . At the same time, it makes the mining facility more profitable than operating under the spot market prices.

Overall, mining facilities using sustainable energy not only can be more profitable while mining fewer coins than their share of the mining capacity, but these curtailment payments and similar schemes may also make them even more profitable. This analysis yields the proposition below.

Proposition 3. *If the sustainable electricity utility faces cost X for overwhelming the grid in state B , there exist conditions for which the utility optimally offers curtailment schemes to induce mining shutdowns. Moreover, such curtailment payments increase profitability of the mining facility.*

C. Welfare Impact

C.1. New coins created.

Changing energy prices should have no effect on the number of new coins created if the difficulty adjustment works reasonably well. This is because the network is programmed to issue the same number of bitcoins within a given time interval – 6.25 BTC per 10 minutes on average – independently of the overall or individual electricity consumption.

C.2. Overall energy consumed.

Lower electricity prices and higher Bitcoin prices increase overall consumption of electricity for mining. Consider first the situation with no contracting frictions. Overall electricity consumption is

$$H_0 = \frac{R}{p_I}, \quad \text{when } p_S > p_I - p_C \text{ or } S \text{ inactive}$$

$$H_1 = \frac{R}{p_C + p_S}, \quad \text{when } p_S < p_I - p_C$$

It is straightforward that as p_S decreases from *higher* to *very lower*, overall energy consumption increases, $H_1 > H_0$. By the same logic both H_0 and H_1 increase in R .

In addition, penalties for not using the electricity, $p_P > 0$, may further increase the overall energy consumed ($H = \frac{R}{p_C + p_S - p_P} > H_1$ if \bar{h}_S not binding) when $p_S - p_P < p_I - p_C$.

Corollary 1. *The effect of lowering p_S on the overall consumption of electricity is non-linear: For $p_S > p_I - p_C$, the overall electricity consumption stays constant at H_0 as p_S declines. Once $p_S < p_I - p_C$, the electricity consumption, H_1 , increases as p_S declines. (For a constant R .)*

C.3. Conventional vs sustainable energy consumed.

When p_S declines (holding R constant), less conventional electricity is used for mining, even though more overall electricity is used. At the same time, as R increases both types of electricity use increase.

Earlier results show that when p_S is *higher*, the overall consumption, H_0 does not change as p_S declines, but the market share of S increases at the cost of I . If at least some of I are using conventional energy, decreasing p_S results in decreasing use of conventional energy in the market.

For *very low* p_S , we take an earlier result $h_C^*(H_1) = \frac{p_S}{(p_C + p_S)^2} R$. Note that for $p_S > p_C$, $\frac{p_S}{(p_C + p_S)^2}$ is increasing in p_S . Together with $p_S < p_I - p_C$, it yields $h_C^*(H_1) < \frac{p_I - p_C}{p_I^2} R = h_C^*(H_0)$. So even if all h_I^* is coming from sustainable energy, *very low* p_S results in lower conventional energy use. The effect is exacerbated if some individual miners use conventional energy. Moreover, the result will increase with $p_P > 0$.

Corollary 2. *When p_S declines (holding R constant), less electricity is used for mining by miners other than S , even though more overall electricity is used.*

This implies that reports pointing to the damaging environmental impact of Bitcoin mining may be misleading if only focusing on total electricity used.

Corollary 3. *When p_S declines (allowing R to move arbitrarily), a lower proportion of mining is done by miners other than S .*

D. Predictions of the model

The main conclusions of the model are that the high variance of supply of sustainable energy leads to following results:

1. Miners using sustainable energy sources find it optimal to periodically shut down (or decrease) their mining operations, which results in mining fewer coins than their mining power would suggest (Proposition 1).
2. Miners using sustainable energy sources may be more profitable despite the periodic shut-downs (Proposition 2).
3. A sustainable energy utility may find it profitable to set up curtailment schemes, inducing periodic shut-downs. These curtailment schemes further increase profitability of the mining facilities (Proposition 3).

In the following sections, we show that these results are consistent with empirical evidence. Additionally, looking at the potential environmental impact of mining, the model shows that

1. After the introduction of sustainable energy at a lower price than the retail price for conventional energy, the overall use of energy for mining either stays the same or increases. The latter happens when the price of sustainable energy is so low that it pushes individual miners out of the market.
2. In both cases, lowering the price of sustainable energy (e.g. in the period of the over-supply of the energy) decreases conventional energy used for mining.
3. Increasing prices of cryptocurrencies increase the use of both conventional and sustainable energy for mining.

3. Data description

To test the implications of our model, we collect data on all publicly traded bitcoin mining companies with North American operations that are listed on the

NASDAQ market during a sample period that begins in May 2021 and continues up to mid-August 2023. We include only those companies that achieve market capitalizations of at least \$100 million at some point during the sample period, 13 firms in all, and we ignore a number of penny stocks that also have listed their shares. The roster of companies appears in Table 1. One firm, Riot Platforms,⁸ has been public for more than five years, but the majority of the companies represent recent additions to the public markets, with most having become listed on the NASDAQ during a wave of special purpose acquisition company (SPAC) transactions that occurred in the technology industry in 2021. Table 1 shows that the NASDAQ-listed crypto miners are legally registered in a diverse set of common law jurisdictions, including the U.S., United Kingdom, Canada, Australia, and the Cayman Islands, and their NASDAQ listings require all of them to make standard financial and governance disclosures to the U.S. Securities and Exchange Commission (SEC). Only two of the companies have operations outside North America, and just one represents a relocation of hardware that had previously been installed in China. The most popular location for mining is Texas (nine companies with facilities either in place or under construction), followed by Georgia (four companies), New York (four), and Quebec (three). According to media reports, all of the U.S. mining companies except Marathon Digital Holdings belong to one or more mining pools, which are mutual organizations in which operators reduce risk by sharing resources and rewards (Cong, He, and Li, 2021; Makarov and Schoar, 2021).

⁸ Riot Platforms was known as Riot Blockchain until changing its name at the start of 2023. The company had been called Biopix and was mainly focused on veterinary pharmaceuticals until taking the Riot Blockchain name in October 2017 and entering the bitcoin mining business during a run-up in the crypto markets.

Figure 1 shows the evolution of the market capitalization of the equity of the publicly traded bitcoin miners since April 30, 2021. At the inception of the sample three companies were public, and together they had a market cap of about \$7.0 billion. Companies are added to the chart at the different times that they list their shares, and the high watermark for the industry's market cap occurs in mid-November 2021, when 10 companies were public with an aggregate market cap of about \$23 billion. As the market price of bitcoin declined after November 2021, so too did the value of mining stocks, albeit at a somewhat faster rate. As of August 2023, 13 companies were listed with a collective market cap of about \$10.1 billion, which represented a rebound from a low of \$1.7 billion reached near the end of 2022. The chart does not reflect the value of the entire U.S. mining industry since it only includes public companies and ignores some penny stocks.

Table 2 provides information about the assets of the 13 public mining companies, including the total reported hashrates of their mining hardware and their holdings of bitcoins, which were either mined by the companies or, in some cases, purchased on the open market to augment their inventories. All values are measured as of June 30, 2023. Not all companies report their bitcoin inventories, but in most of these cases other information suggests these undisclosed inventories are negligible.

While bitcoin prices had significantly dropped during the “crypto winter” of 2022, reducing the value of rewards earned by crypto miners, the industry nevertheless continued to expand its capacity with the global hashrate growing almost continuously during the year. As of January 2023, four companies in our sample – CleanSpark, Marathon, Riot, and TeraWulf – were actively building new facilities and/or installing

new mining units, and their announced plans would add about 24.5 Eh/second of hashing power to the network by mid-2023, representing nearly a 10% increase of the global hashrate. However, companies had experienced difficulty in achieving their announced growth targets, for various reasons related to local permitting delays, supply chain bottlenecks, adverse weather events, and the like.

Companies' bitcoin retention policies vary considerably, and the time series of monthly bitcoins mined and sold is shown for three companies in Figure 2. The top company, CleanSpark Inc., tends to sell its mining output in real time, as its monthly sales track fairly closely the number of coins mined. For the most part, this policy resembles those of commodity mining companies that extract ores and minerals from the crust of the earth. The middle company, in Figure 2, Hut 8 Mining Corp., simply accumulates all the bitcoins it mines through the end of 2022, following a policy known in the crypto community as "hodling." As noted above, Hut 8 is really in two businesses, (i) mining bitcoin, and (ii) speculating on the future appreciation of bitcoin. To the extent its bitcoin inventories grow over time, it might begin to resemble a closed-end fund focused on cryptocurrency, and it might attract an investor clientele seeking exposure to crypto assets via a surrogate for a bitcoin ETF, a product that the SEC has repeatedly refused to approve despite dozens of applications and evident investor demand. The company on the bottom of Figure 2, Argo Blockchain, had been partially following an accumulation policy similar to Hut 8's until it abruptly reversed course in mid-2022 and began to liquidate its inventories of cryptocurrency in order to create liquidity for addressing financial problems. By late 2022 it was following a strategy similar to CleanSpark's, selling its bitcoin almost immediately after they were mined.

The 13 companies in our sample have very different capital structure policies. Six out of 13 have low to moderate debt on their balance sheets and would appear to have low bankruptcy risk. Among the other seven miners, borrowing is sometimes aggressive, often with bitcoin held in inventory being pledged as collateral for loans that in turn are used to put down deposits on new orders of mining rigs or to finance construction of additional data centers. These strategies could provide considerable leverage that would amplify profits if bitcoin prices were rising, but they could accelerate a descent into financial distress if bitcoin prices were dropping, as was the case with a number of firms by the end of 2022. Many companies that had moderate leverage in mid-2022 had backed into highly leveraged capital structures by year-end due to the declines in the market values of their equity, even as new borrowing had largely been cut off by the debt markets.⁹

4. Analysis

We begin our analysis by studying the relations between daily company stock price returns and relevant market indexes. Table 3 contains regression analysis for a pooled sample of all 13 companies during a period beginning May 3, 2021, and continuing to August 11, 2023 (most companies were not publicly traded for the entire

⁹ A detailed analysis of mining companies' capital raises and indebtedness, including commitments for equipment financing, is provided by Luxor Technology Corp. in its 2022 year-end report, available at hashrateindex.com/blog/content/files/2023/01/Hashrate-Index-2022-Year-In-Review--FINAL-.pdf. In one of the more unusual capital raises by mining firms, Marathon announced a \$500 million convertible debt offering in November 2021 that it twice expanded, reaching a final total of \$747.5 million for five-year bonds with a 1% coupon and the right to convert into Marathon stock at a price of \$76.17 per share. Shareholders registered sharp displeasure with the terms of the issue, sending Marathon's stock into a nosedive from \$75.92 to \$55.40 per share. See <https://www.fool.com/investing/2021/11/15/why-marathon-digital-plummeted-more-than-20-today/>. The company nevertheless pushed ahead with the bond sale and used the proceeds to place a large order for more bitcoin mining equipment.

sample period). Standard errors are clustered by company. In the first three columns we regress daily stock returns against the daily returns on the NASDAQ market index, bitcoin, and the SP500 electric utilities index, respectively (for bitcoin, returns on weekends and market holidays are folded into the next Wall Street market day's observations). In the fourth column, all three indexes are included together. Focusing on the fourth column, we see a very strong association between mining stocks and movements in the NASDAQ index, with an estimated market beta of 1.57, and not surprisingly a strong association also exists between the bitcoin return and the bitcoin mining stocks. For the electric utility industry, we obtain a significantly negative beta of -0.65 after controlling for movements in bitcoin and the overall market. This result indicates that cryptocurrency mining has the economic properties of a hedge against returns in the utility industry, a pattern that makes sense since electricity represents the main variable cost in crypto mining.

In the right column of Table 3, we augment the model with an indicator variable for those days (usually one per month) on which companies make public announcements about their mining results for the prior calendar month. We define a statistic of mining "success" equal to the ratio between a company's actual coins mined and the expected amount, with the expectation based upon the ratio between the company's own hashrate and the network hashrate, multiplied by the global bitcoin quantity (including user fees) earned by miners worldwide during that month. To calculate the hashrate ratio, we take the starting and ending ratios for a company each month and average them together.

Many companies' mining success statistics cluster around 1.00, but some are lower, a pattern we discuss further below.¹⁰

The regression estimates in the right column of Table 3 show positive and significant market reactions to announcements of successful mining months. While positive investor reactions to good operating performance are not surprising, we note that our result suggests that investors do not monitor the bitcoin blockchain with great attention. In principle, interested researchers should be able to identify the public keys of individual mining companies on the blockchain and take note of their mining progress in real time, so that month-end announcements of mining success shouldn't move the market.

We next move to test some of the predictions from our model introduced in Section 2 above. Based upon Proposition 1, we conjecture that miners that are located in jurisdictions with large usage of wind power will have more variable success rates and will spend more time offline, because the generation of wind power is much more erratic than other sources such as fossil fuels, nuclear, or renewables including solar and hydro. Prices of energy should be much higher during periods in which the wind is not blowing, and when demand for energy is otherwise high due to the weather. Figure 3 presents information about the electricity market in Texas, showing the variability of the monthly demand for energy (the red series) and the availability of wind power (the blue series). Data are based upon daily observations in North Central Texas for the Load Mix and

¹⁰ While many companies have mining success near 1.00, none of them exceed 1.00 which seems surprising, since by construction the statistic should equal 1.00 for the mining industry in aggregate. The simplest explanation is probably that some (or most) companies exaggerate their true hashrates. Consistent with this possibility, in its recent *2022 Annual Crypto Review* (slide 14), Coindesk shows a disagreement between the calculated bitcoin network hashrate and the somewhat higher hashrates reported by major mining pools in the aggregate.

Native Load reports that are available on the website of the Electric Reliability Council of Texas (ERCOT). The red series in Figure 3 shows that demand for electricity is highest during the summer period of June-August, with demand also rising by a lesser amount in January-February during the winter. All else equal, the price of energy should be highest during these months. From the supply side of the market, the blue series in Figure 3 shows that the most abundant period of the year for generating wind power is the springtime, March through May, as well as the fall, October through December. During these periods, which could be interpreted as the “good” state $S(G)$ in our model, the price of renewable energy should be lowest. Crypto miners engaging in load balancing strategies in Texas should therefore expect to be offline especially during the summer months, when relatively little wind power is available and demand for energy is high. These months would correspond to the “bad” state, $S(B)$, in our model. These miners should also exhibit a higher overall fraction of days off-line, which would translate into lower mining success rates.

We infer companies’ offline behavior from their data for mining success, a variable that equals 1.00 if a company mines exactly the number of coins that would be expected given the size of its hashrate relative to the overall network hashrate. We have three companies mining the majority of their coins in Texas: Cipher Mining, Marathon Digital Holdings and Riot Platforms. Argo Blockchain and Bit Digital also have operations in Texas as well as other U.S. states and Canada. (Table 1 indicates that 9 of our 13 companies mine crypto in Texas, and in the cases of the other five companies, their Texas sites are under construction or under option for future development.) Table 2 shows that Cipher, Riot and Marathon have three of the lowest success rates in our

sample, indicating that they are often offline. In Figure 4, we plot the monthly success values for Riot against those for BitFarms, a miner that mostly uses hydro power in Quebec. It is evident by inspection of the plot that Riot Blockchain is off-line far more frequently than BitFarms (in 23 out of 24 months) while exhibiting much greater variation in success; the standard deviations of the bi-monthly series in Figure 4 are 0.11 for Riot and 0.05 for BitFarms.

Though we are relying upon information from a small sample of companies, we interpret the lower and more variable success rates for Texas miners as being broadly consistent with Proposition 1 in our model, which implies that these miners will disconnect their machines on days with low availability of renewable wind power and/or high external energy demand. For example, Riot has been very open about this behavior in its monthly reports.

Moreover, our Proposition 3 suggests that curtailment schemes may be profitable both for the electric utilities and the mining facilities. Consistent with this prediction, Riot generally discloses the value of rebates that it receives from local utilities due to periodic “curtailments” of mining. For example, in its July 2022 monthly mining update, Riot reported obtaining \$9.5 million worth of power credits as compensation for not mining on certain days, an amount that significantly exceed its revenues from mining bitcoin during the same month.¹¹ Its balance sheet for September 30, 2022, at the end of the summer, showed a current asset of \$40 million worth of future power credits which

¹¹ See Shawn Tully, “How Riot Blockchain capitalized on a hot Texas summer to make more money selling power than mining crypto,” *Fortune*, August 13, 2022. For December 2022, Argo Blockchain (Texas), ClearSpark (Georgia), Core Scientific (various), Hive Blockchain (Quebec), and Riot Blockchain (Texas) all reported curtailments of mining during a severe winter storm that disrupted the electric utility industry especially in the central U.S. and upstate New York.

could be used to offset future electricity bills; the accounting is handled in a similar way to prepaid future revenue. This asset had been reduced to near zero according to the balance sheet dated June 30, 2023, implying that the credits had been used to offset energy bills during the winter and spring. Although other companies in the sample make occasional reports of earning these credits or similar subsidies, none of them appears to track them on its balance sheet as Riot does.

Another implication of our model, stated in Proposition 2, is that miners relying on sustainable electricity may be more profitable than ordinary miners, even if they end up mining far less. This implication is further reinforced by access to subsidies for curtailment periods, as predicted by Proposition 3. While it is difficult for us to obtain data about the monthly profits of miners, we can observe the stock prices of these firms and examine whether they are valued more highly by the market.

Table 4 and Figure 5 investigate the recent value of the 13 companies in our sample as a function of their installed hashrates. We use the Merton-KMV model for this purpose. We observe the market value of equity for each company as well as the equity volatility over the past year. We assume that each firm has a default point equal to the value of short-term liabilities and one-half of long-term liabilities. We treat equity as having a one-year option to purchase the assets of the firm at an exercise price equal to the book value of total liabilities. The one-year U.S. Treasury bill rate serves as the risk-free rate.

Table 4 tabulates the market value of equity, book value of debt, and equity volatility (which exceeds 100% annually for nearly every firm), along with the solution for the estimated market value of the entire firm. We subtract the equity market value

from the firm's total value to obtain an estimate of the market value of debt. The table shows that some firms have debt with market values deeply discounted from book values, including Core Scientific, which has already filed for bankruptcy and negotiated some debt forgiveness, and several others. Many of the firms' shares therefore trade in ways similar to out-of-the-money call options, a property that would appeal to risk-seeking investors of the type that also gravitate toward the cryptocurrency markets.

In Figure 5, we compare each firm's enterprise value with the hashrate of its mining equipment. Using the data from Table 4, we define enterprise value as the market value of equity plus the market value of debt, minus the market value of bitcoins, in order to capture the value of the underlying mining business independent of the value of any bitcoin inventory. August 11, 2023 serves as the measurement date. Note that not all companies disclose bitcoin inventories, and debt values are recorded on June 30, 2023, the most recent public disclosure date for most of the companies.¹² We fit a regression line to the enterprise values of the different companies based upon their hashrates, and this line also appears in Figure 5. The regression has a good r^2 of 0.68. We label the data points for three firms, Marathon Digital Holdings, Riot Platforms, and Cipher Mining Technologies, which all rely heavily on the Texas power grid for their supplies of electricity.¹³ As shown in Figure 5, all three firms' values lie above the regression line,

¹² We use the most recent balance sheets for each firm to determine their book values of debt and the division of debt between short-term and long-term. For seven companies the June 30, 2023 balance sheet appears in a recent quarterly filing, but for four companies the March 31 balance sheet is the most recent available, and for two foreign-based companies that do not file quarterly, the December 31, 2022 balance sheet is used.

¹³ The companies' facilities are described in detail in their March 2023 Form 10-K filings. Marathon, currently the largest company in the industry, has nine mining facilities in five states, with a majority of its miners at four sites in Texas. The largest facility in McCamey, TX, uses wind power. Riot has two mining facilities, both in Texas, and contracts with a utility company that has diverse energy sources. Cipher has

indicating that their shares are valued at a premium, even though data in Table 2 shows that all three have low rates of mining success due to relatively large amounts of downtime. Another way to evaluate the data shown in Figure 5 is to consider each company's enterprise value per unit of hashrate. The 13 companies have a mean enterprise value of \$120 million per eh/s, and a median of \$117 million. The three companies highlighted rank among the top five in our sample of 13 firms, with Riot Platforms (\$249 million per eh/s) far and away the most valuable, and Marathon and Cipher at \$150 million and \$144 million per eh/s, respectively.

5. Discussion and conclusions

This paper studies the operations and financial valuations of 13 publicly traded cryptocurrency mining companies. We find that miners using Texas wind power are offline more than other miners, in a more erratic pattern. Nevertheless, these Texas miners are more highly valued than those using more stable sources of energy such as hydro power or solar power, as reflected in significantly higher enterprise values.

One of the most intriguing results in the paper appears in Table 3, where regression estimates in the fourth column imply a strongly negative beta for crypto mining stocks with respect to an index of electric utilities. The estimate implies that owning a cryptocurrency mining unit would provide an effective hedge, or risk management tool, for utilities. In absence of full integration, the second-best are the load-balancing schemes attracting mining facilities while offering significant revenue augmentations from "curtailment" payments, which we observe in our data.

its entire mining operation spread across four centers in Texas, three of which are operated as part of a joint venture with WindHQ LLC, a wind power producer.

Our data also indicate that a subset of crypto miners actively pursue risky strategies involving high degrees of leverage and the pledging of bitcoins held in inventory.

Bibliography

Arnosti, Nick S. and Matthew Weinberg, 2022, Bitcoin: A natural oligopoly. *Management Science* 68, 4755-4771

Bastian-Pinto, Carlos L., Felipe V. de San Araujo, Luiz E. Brandão, and Leonardo L. Gomes, 2021, Hedging renewable energy investments with bitcoin mining, *Renewable and Sustainable Energy Reviews* 138, 110520.

Benetton, Matteo, Giovanni Compiani, and Adair Morse, 2021, When cryptomining comes to town: High electricity-use spillovers to the local economy, unpublished manuscript, University of California at Berkeley.

Bruno, August, Paige Weber, and Andrew J. Yates, 2022, Can bitcoin mining increase renewable electricity capacity? Unpublished manuscript, University of North Carolina at Chapel Hill.

Budish, Eric, 2022, The economic limits of Bitcoin and anonymous, decentralized trust on the blockchain, University of Chicago, Becker Friedman Institute for Economics Working Paper, (83).

Cassauwers, Tom, 2021, An environmental upside to bitcoin? *Sierra*, March 30.

Cong, Lin William, Zhiguo He, and Jiasun, Li, 2021, Decentralized mining in centralized pools, *Review of Financial Studies* 34, 1191-1235.

Congressional Research Service, 2019, *Bitcoin, blockchain, and the energy sector*, report no. 45,863, August 9.

Easley, David, Maureen O'Hara, and Soumya Basu, 2019, From mining to markets: The evolution of bitcoin transaction fees, *Journal of Financial Economics* 134, 91-109.

Halaburda, Hanna, Guillaume Haeringer, Joshua Gans, and Neil Gandal, 2022, The microeconomics of cryptocurrencies, *Journal of Economic Literature*, 60(3), pp.971-1013.

Lehar, Alfred, and Christine A. Parlour, 2021, Miner collusion and the bitcoin protocol, unpublished manuscript, University of California at Berkeley.

Makarov, Igor, and Antoinette Schoar, 2021, Blockchain analysis of the bitcoin market, unpublished manuscript, Massachusetts Institute of Technology.

Niaz, Haider, Jay J. Liu, and Fengqi You, 2022, Can Texas mitigate wind and solar curtailments by leveraging bitcoin mining? *Journal of Cleaner Production* 364, 132700.

Prat, Julien, and Benjamin Walter, 2021, An equilibrium model of the market for bitcoin mining, *Journal of Political Economy* 129, 2415-2452.

Shan, Rui, and Yaojin Sun, 2019, Bitcoin mining to reduce the renewable curtailment: A case study of CAISO, unpublished manuscript, University of North Carolina.

Solomon, Marissa, 2022, ICYMI: Three new reports paint damning picture of climate-killing & community impacts of cryptocurrency mining, *Earthjustice*, October 18.

Table 1**Publicly traded bitcoin mining companies**

The table lists 13 publicly traded bitcoin mining companies with the year of their NASDAQ stock market listings, countries of corporate registration, and principal geographic locations of mining operations. Some mining sites are still under construction.

Company	Listed	Registered	North America sites	Other sites
Riot Platforms Inc.	2017	USA	Texas	(formerly China)
CleanSpark Inc.	2020	USA	Georgia, New York, Texas	
Bit Digital Inc.	2020	Caymans	Georgia, New York, Texas, Nebraska	
Marathon Digital Holdings Inc.	2021	USA	Texas, North Dakota	Argentina, Paraguay Iceland, Sweden
Hut 8 Mining Corp.	2021	Canada	Alberta, Ontario	
BitFarms Ltd.	2021	Canada	Quebec, Washington	
Hive Blockchain Technologies Inc.	2021	Canada	Quebec, Texas	
Cipher Mining Technologies Inc.	2021	USA	Texas	
Greenidge Generation Holdings Inc.	2021	USA	New York, Texas, South Carolina	
Argo Blockchain PLC	2021	UK	Georgia, Kentucky, Texas, North Carolina, Quebec	
Iris Energy Ltd.	2021	Australia	British Columbia, Texas	
TeraWulf Inc.	2021	USA	New York, Pennsylvania	
Core Scientific Inc.	2022	USA	Georgia, Kentucky, North Carolina, North Dakota	

Table 2**Mining data for publicly traded bitcoin mining companies**

The table provides statistics about 13 publicly traded bitcoin mining as sourced from companies' monthly mining update press announcements. Each company's hashrate and BTC holdings are reported as of June 30, 2023, at which point these companies collectively accounted for 16.4% of the worldwide bitcoin mining hashrate. For each company, mining success equals the actual bitcoin mined divided by the expected amount, based upon the company's hashrate relative to the overall network hashrate. Mining success reported in the table is the average based upon all available monthly mining reports released by each company. Equity market value is measured as of August 11, 2023. Debt market value is calculated based on the KMV method using the company's most recent balance sheet (June 30, 2023 for most firms).

Company	Hashrate eh / sec.	Equity MV millions	Debt MV millions	BTC held	BTC mined 2022	Mining Success
Riot Platforms Inc.	10.7	\$2,276	\$102	7,250	5,536	0.73
CleanSpark Inc.	6.7	859	\$46	529	4,621	0.94
Bit Digital Inc.	1.8	286	4	485	1,247	0.97
Marathon Digital Holdings Inc.	17.7	2,592	423	12,232	4,144	0.73
Hut 8 Mining Corp.	2.6	657	42	9,136	3,568	0.77
BitFarms Ltd.	5.3	408	83	549	5,168	0.96
Hive Blockchain Technologies Inc.	3.5	389	38	n.a.	3,270	0.97
Cipher Mining Technologies Inc.	6.7	919	60	417	> 516	0.83
Greenidge Generation Holdings Inc.	2.4	41	94	n.a.	> 2,318	0.89
Argo Blockchain PLC	2.5	68	53	512	2,157	0.68
Iris Energy Ltd.	5.6	286	118	n.a.	2,295	0.98
TeraWulf Inc.	5.5	475	122	n.a.	495	0.78
Core Scientific Inc. (<i>in Chapter 11</i>)	15.0	320	703	n.a.	14,420	0.89

Table 3**Analysis of mining company stock returns**

The presents ordinary least squares estimates of daily stock returns of 13 publicly traded bitcoin mining companies between May 3, 2021, and August 11, 2023. Explanatory variables include the daily returns on the NASDAQ market index, bitcoin, and the SP500 Electric Utilities index. The indicator for monthly updates equals one on days in which companies report their mining results for the previous month. The success variable equals the company's monthly mining output divided by its expected output, based on the company's reported hashrate relative to the overall bitcoin blockchain hashrate. The growth variable represents the percentage increase in the company's hashrate over the prior month. Standard errors are clustered at the company level.

Dependent variable: Daily stock return

Intercept	-0.0028 *** (0.0006)	-0.0022 *** (0.0006)	-0.0031 *** (0.0006)	-0.0023 *** (0.0006)	-0.0024 *** (0.0007)
NASDAQ return	2.1598 *** (0.1073)			1.5734 *** (0.1075)	1.5749 *** (0.1065)
Bitcoin return		0.9712 *** (0.0370)		0.6866 *** (0.0345)	0.6862 *** (0.0344)
Utilities return			0.6374 *** (0.0436)	-0.6513 *** (0.0672)	-0.6512 *** (0.0685)
Update indicator					-0.0360 (0.0226)
Update indicator x Success					0.0466 * (0.0262)
Update indicator x Growth					-0.0117 (0.0213)
Observations	6,513	6,513	6,513	6,513	6,513
R ²	0.189	0.215	0.009	0.281	0.282

*** Significant at 1% (**), 5% (*), and 10% (·) levels

Table 4**Estimated market values of crypto mining companies**

The table shows the market value of equity and estimated market values for debt and the overall firm for a sample of 13 publicly traded bitcoin mining companies.

August 11, 2023. Equity volatility is measured over one year prior to that date. Debt book value is obtained from the company's most recent balance sheet, in most cases dated June 30, 2023. The firm value estimate is derived using the KMV model, using a one-year U.S. Treasury rate and other assumptions described in the next. Implied debt value equals the difference between the firm value estimate and the equity market value. All dollar values are in millions. Core Scientific filed for Chapter 11 bankruptcy protection in late 2022.

Company	Equity market value	Equity volatility	Debt book value	Firm value estimate	Debt value implied
Riot Platforms Inc.	\$2,775.7	1.01	\$118.0	\$2,877.2	\$101.5
CleanSpark Inc.	859.2	0.99	49.2	904.9	45.7
Bit Digital Inc.	295.8	1.07	5.2	289.7	3.9
Marathon Digital Holdings Inc.	2,592.3	1.18	763.0	3015.7	423.4
Hut 8 Mining Corp.	657.2	1.09	51.1	698.9	41.7
BitFarms Ltd.	408.1	1.07	87.5	490.8	82.7
Hive Blockchain Technologies Inc.	388.6	0.96	46.8	426.1	37.5
Cipher Mining Technologies Inc.	919.0	1.37	73.5	978.6	59.6
Greenidge Generation Holdings Inc.	40.8	1.62	140.6	134.5	93.7
Argo Blockchain PLC	68.3	1.62	72.2	120.8	52.5
Iris Energy Ltd.	311.2	1.27	133.1	429.6	118.4
TeraWulf Inc.	475.0	1.46	170.4	597.2	122.2
Core Scientific Inc.	320.0	2.92	1,143.3	1,022.8	702.8

Figure 1

Equity market capitalizations of crypto mining companies

The figure shows the equity market capitalizations of 13 publicly traded bitcoin mining companies between April 30, 2021 and August 11, 2023. Each company is tracked from the date of its initial listing on the NASDAQ market; three companies were publicly traded prior to the start of the sample period.

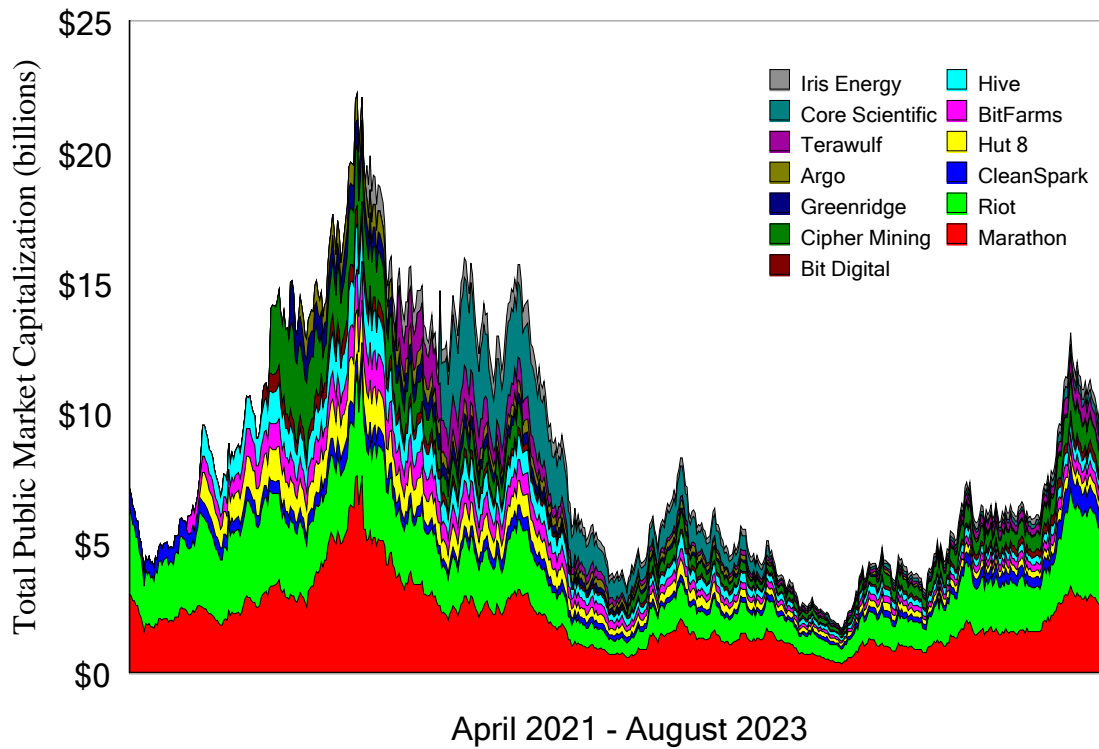


Figure 2

Examples of mining retention policies

The figures show monthly data for three companies during the 22 month period October 2021 through July 2023. In each graph the red bars represent new bitcoins mined during the month, while the green bars show bitcoins sold. The first company, CleanSpark Inc., follows a steady-state strategy of selling an amount each month that approximately equals its month mining output. The second company, Hut 8 Mining Corp., retains its entire mining output and accumulates an increasing inventory of bitcoins until reversing course in early 2023. The third company, Argo Blockchain, also follows an accumulation policy until May 2022, when it abruptly liquidates most of its bitcoin inventory and switches to a steady-state strategy similar to CleanSpark's.

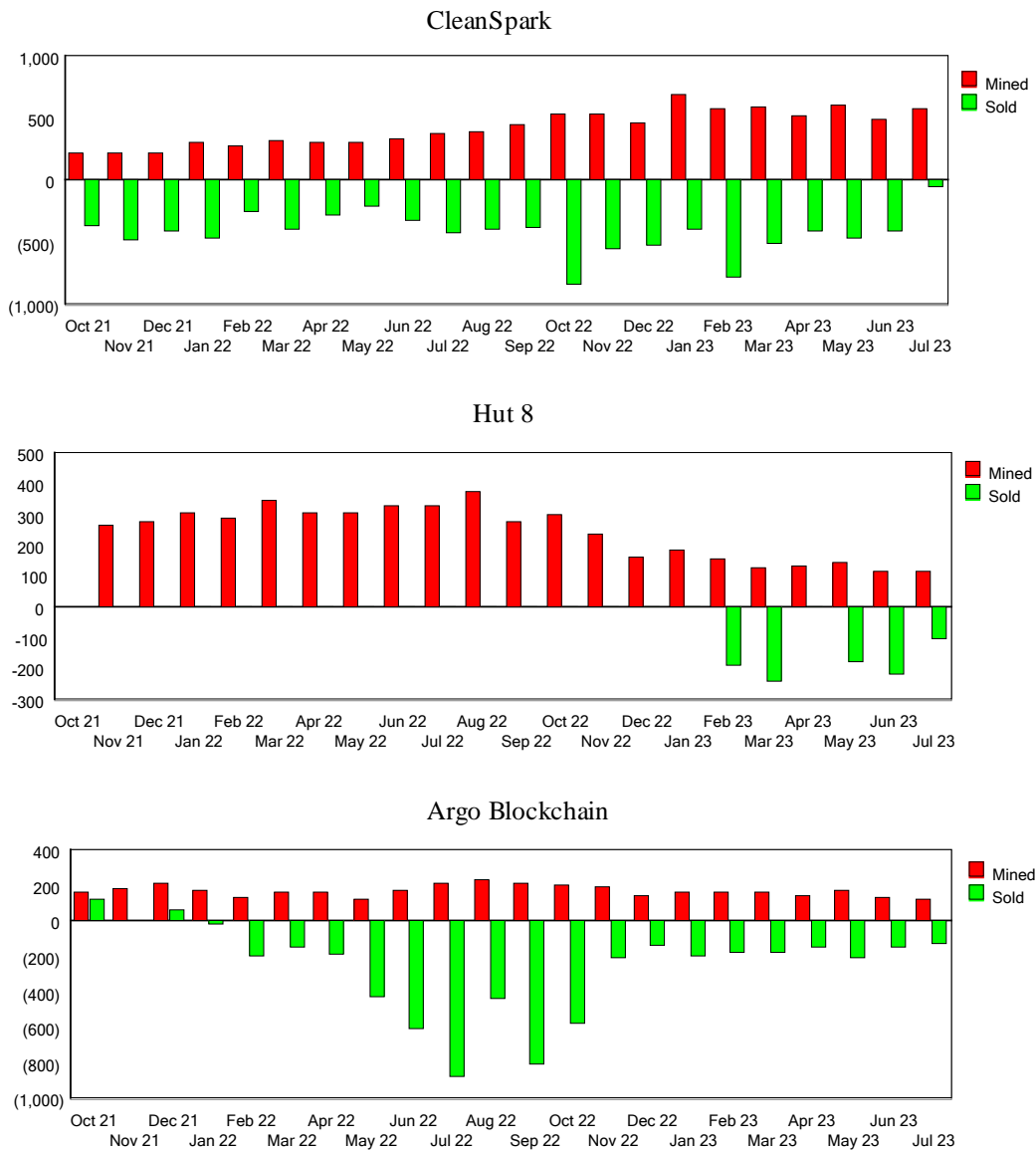


Figure 3
Monthly data from Texas electricity market

The graph shows monthly information about the availability of wind power and the demand for electricity in the state of Texas between January 2021 and July 2022. For each time series, the plotted value equals the percentage of the series' monthly average during the sample period. The supply of wind power is based on daily statewide data from the Fuel Mix report posted on the website of the Electric Reliability Council of Texas (ERCOT). The demand for electricity is based on the average of daily peak values within each month as reported in ERCOT's Native Load report using observations for the North Central market area.

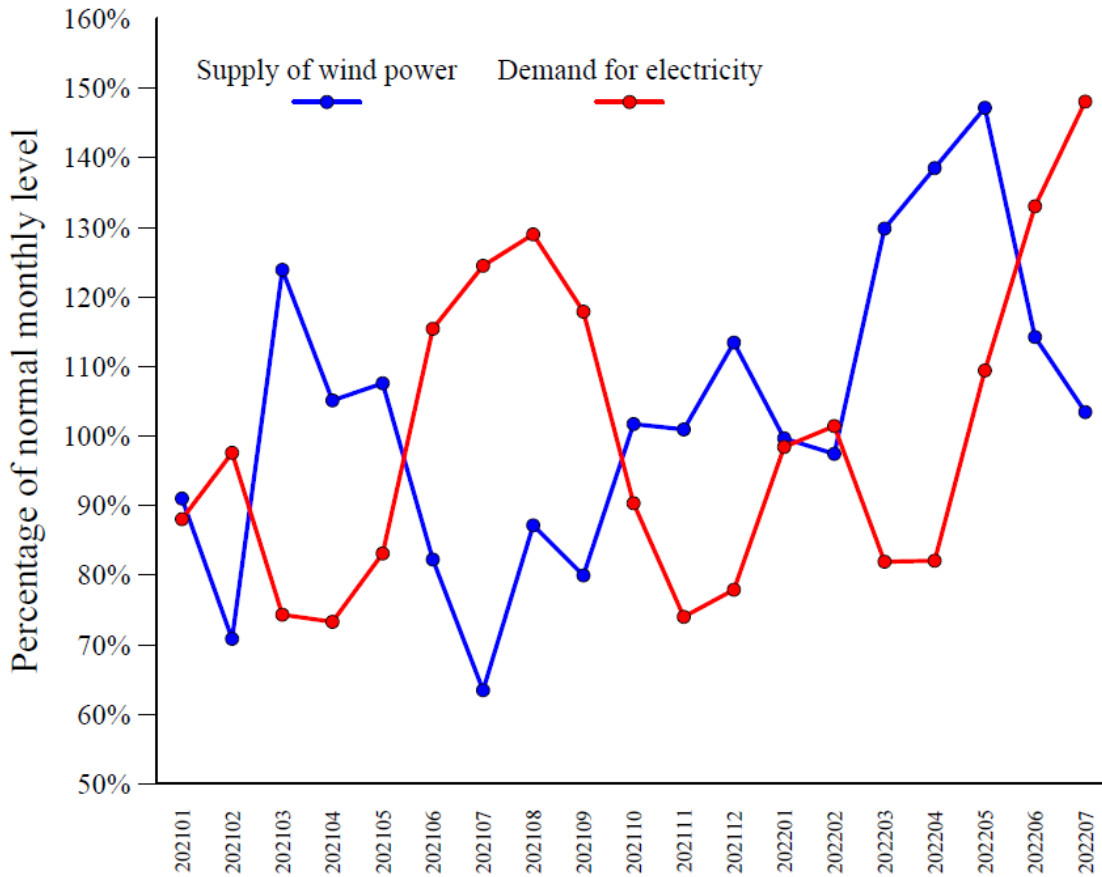


Figure 4
Monthly mining “success” for two companies

The graph shows monthly data for two companies’ mining “success,” which equals the company’s monthly mining output divided by its expected output, based on the company’s reported hashrate relative to the overall bitcoin blockchain hashrate. BitFarms uses mainly hydropower from facilities in Quebec and Washington State. Riot Platforms uses a mix of energy, including wind power, mainly at its mining site in Texas.

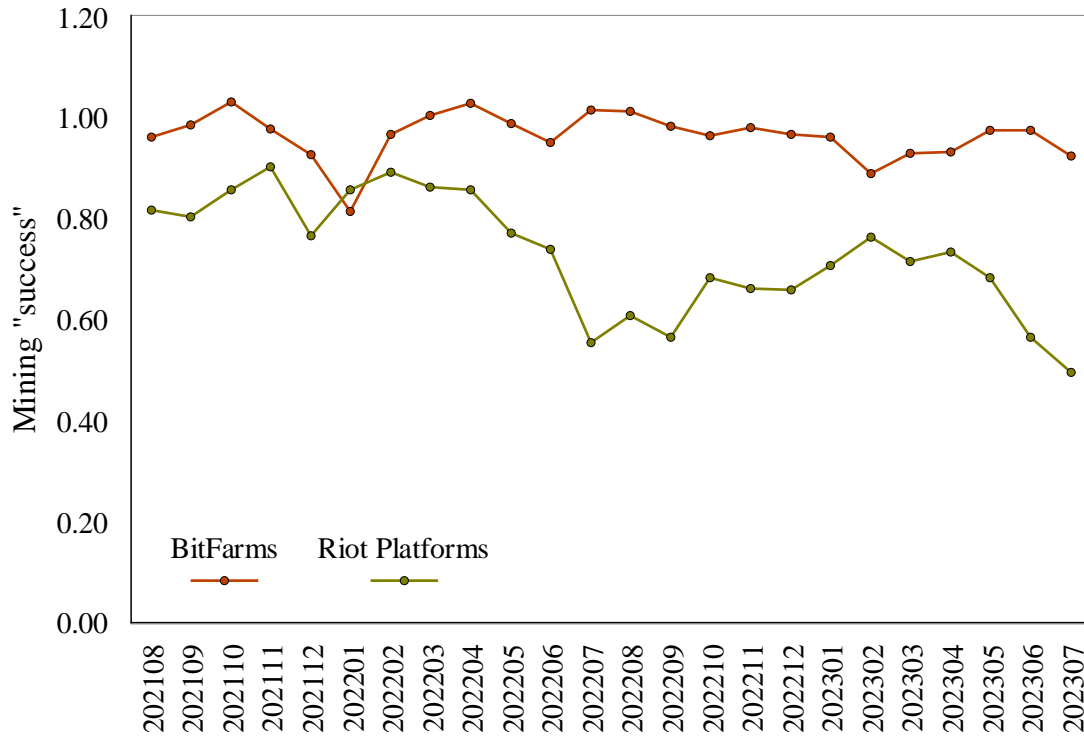
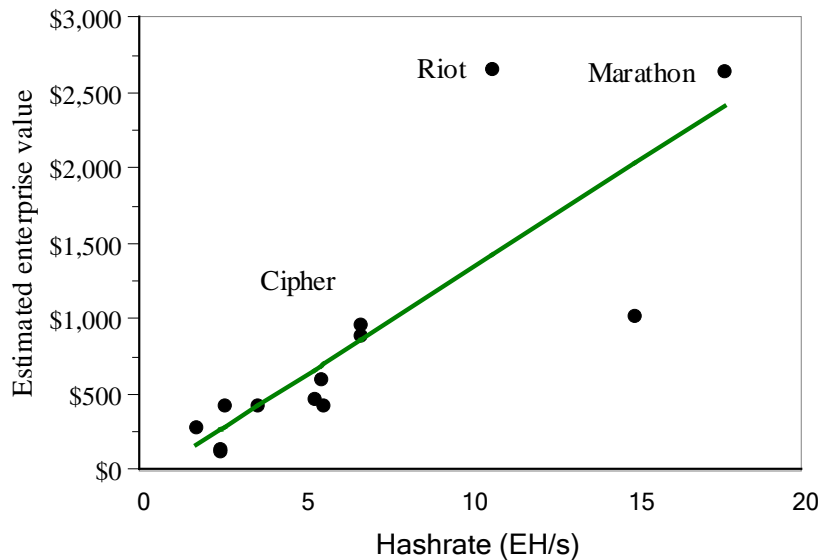


Figure 5
Enterprise value as a function of hashing capacity

The figure shows the estimated enterprise values (in millions) of 13 publicly traded bitcoin mining companies on August 11, 2023 as a function of their mining hash rates. Enterprise value equals equity value plus debt value minus the market value of bitcoin held in inventory. Debt value is estimated using the KMV model in Table 4, and bitcoin inventories are based on company reports and are assumed to equal zero for five companies that make no disclosures. The dark green line is a least-squares regression based upon the 13 company observations. The labels identify three observations for companies with mining operations predominantly in Texas. The regression has an r^2 of 0.68.

W



Appendix with Proofs

Proof of Lemma 1

With only I, C maximizes $\pi(h_C, H_I) = \frac{h_C}{h_C + H_I} R - p_C h_C$. The FOC:

$R \left(\frac{H_I + h_C^* - h_C^*}{(H_I + h_C^*)^2} \right) - p_C = 0$. Knowing that in equilibrium $H_I^* + h_C^* = H_0 = \frac{R}{p_I}$, the condition

for equilibrium h_C^* becomes $R \left(\frac{H_0 - h_C^*}{H_0^2} \right) - p_C = 0 \Leftrightarrow \frac{R}{H_0} \left(1 - \frac{h_C^*}{H_0} \right) = 0 \Leftrightarrow h_C^* =$

$\left(1 - \frac{p_C}{p_I} \right) \frac{R}{p_I}$. Moreover, $h_I^* = H_0 - h_C^* = \frac{p_C R}{p_I p_I}$. Then:

- $\frac{\partial H_0}{\partial R} > 0$, as well as both $\frac{\partial h_C^*(H_0)}{\partial R} > 0$ and $\frac{\partial h_I^*}{\partial R} > 0$ since $p_I > p_C$.
- $\frac{\partial H_0}{\partial p_I} < 0$, $\frac{\partial h_I^*}{\partial p_I} < 0$;

$$\begin{aligned} \frac{\partial h_C^*(H_0)}{\partial p_I} &= \frac{\partial \frac{(p_I - p_C)^R}{p_I^2}}{\partial p_I} = \frac{R p_I^2 - 2 p_I (p_I - p_C) R}{p_I^4} = \frac{R p_I}{p_I^4} (p_I - 2(p_I - p_C)) \\ &= \frac{R p_I}{p_I^4} (2 p_C - p_I) \end{aligned}$$

$$\text{so } \frac{\partial h_C^*(H_0)}{\partial p_I} < 0 \Leftrightarrow 2 p_C < p_I.$$

- $\frac{\partial H_0}{\partial p_C} = 0$, $\frac{\partial h_I^*}{\partial p_C} > 0$ and $\frac{\partial h_C^*(H_0)}{\partial p_C} < 0$

Proof of Lemma 2

Each i participates when for $h_i > 0$, $\pi(h_i, H_{-i}) = \frac{h_i}{h_i + H_{-i}} R - p_i h_i \geq 0 \Leftrightarrow h_i \leq \frac{R}{p_i} - H_{-i}$. An individual infinitesimal I miner enters as long as $H_{-i} < \frac{R}{p_i} = H_0$. Thus, there will be some I miners participating (i.e., in aggregate $h_I > 0$) only if $H_{-I} = h_C + h_S < \frac{R}{p_I}$, and then $h_I + H_{-I} = H_0$.

In the case that individual miners, I , participate, C maximizes $\pi(h_C, H_{-C}) = \frac{h_C}{h_C + H_{-C}} R - p_C h_C$. The FOC: $R \left(\frac{H_{-C} + h_C^* - h_C^*}{(H_{-C} + h_C^*)^2} \right) - p_C = 0 \Leftrightarrow R \left(\frac{H_0 - h_C^*}{H_0^2} \right) - p_C = 0 \Leftrightarrow h_C^* = \left(1 - \frac{p_C}{p_I} \right) \frac{R}{p_I}$.

Similarly, in the case that individual miners, I , participate, S maximizes

$\pi(h_S, H_{-S}) = \frac{h_S}{h_S + H_{-S}} R - p_S h_S$. The FOC: $R \left(\frac{H_{-S} + h_S^* - h_S^*}{(H_{-S} + h_S^*)^2} \right) - p_S = 0 \Leftrightarrow R \left(\frac{H_0 - h_S^*}{H_0^2} \right) - p_S = 0 \Leftrightarrow h_S^* = \left(1 - \frac{p_S}{p_I} \right) \frac{R}{p_I}$.

Then $h_C^* + h_S^* < \frac{R}{p_I} \Leftrightarrow \left(2 - \frac{p_C + p_S}{p_I} \right) \frac{R}{p_I} < \frac{R}{p_I} \Leftrightarrow p_I < p_C + p_S$. Thus, when this condition holds, some I will participate. This proves the first part of Lemma 2.

Notice that in the case that I participates, $h_C^* > 0 \Leftrightarrow p_C < p_I$ and $h_S^* > 0 \Leftrightarrow p_S < p_I$. When no I participates, the total hash power used is $H_1 = h_C + h_S$. Then by FOCs

$\frac{H_1 - h_C^*}{H_1^2} = \frac{p_S}{R}$ and $\frac{H_1 - h_S^*}{H_1^2} = \frac{p_C}{R}$, which in equilibrium yields $\frac{h_C^*}{H_1^2} = \frac{p_S}{R}$ and $\frac{h_S^*}{H_1^2} = \frac{p_C}{R}$, so that

$H_1 = h_C^* + h_S^* = \frac{R}{p_C + p_S}$ and $h_C^* = \left(1 - \frac{p_C}{p_C + p_S} \right) \frac{R}{p_C + p_S}$, $h_S^* = \left(1 - \frac{p_S}{p_C + p_S} \right) \frac{R}{p_C + p_S}$, which are

strictly positive whenever $p_S > 0$ and $p_C > 0$. Note that I does not participate when $p_I >$

$p_C + p_S$, so that directly $p_I > p_S$ and $p_I > p_C$. This proves the remainder of Lemma 2.

Proof of Lemma 3

When $p_I < p_S + p_C$, then for $i = C, S$, $h_i^* = \left(1 - \frac{p_i}{p_I}\right) \frac{R}{p_I}$, and $\pi_i^* = \left(1 - \frac{p_i}{p_I}\right)^2 R$.

Moreover, individual miners in aggregate consume $h_i^* = \left(\frac{p_C + p_S}{p_I} - 1\right) \frac{R}{p_I}$. When $p_I >$

$p_S + p_C$, then $h_i^* = \left(1 - \frac{p_i}{p_C + p_S}\right) \frac{R}{p_C + p_S}$ and $\pi_i^* = \left(1 - \frac{p_i}{p_C + p_S}\right)^2 R$.

All these values strictly increase with R . Moreover, for $i = C, S, I$, $\frac{\partial h_i^*}{\partial p_i} < 0$ and for $i = C, S$, $\frac{\partial \pi_i^*}{\partial p_i} < 0$. Since individual miners always break even, their profit stays constant.

Proof of Proposition 1

Because of the variability of the supply of sustainable energy, $p_S(B) > p_S(G)$.

So S enters the market when $p_S(G) < p_I$ (by Lemma 2), with

$$h_S^*(G) = \begin{cases} \left(1 - \frac{p_S(G)}{p_I}\right) \frac{R}{p_I}, & \text{when } p_S(G) > p_I - p_C \\ \left(1 - \frac{p_S(G)}{p_C + p_S(G)}\right) \frac{R}{p_C + p_S(G)}, & \text{when } p_S(G) \leq p_I - p_C. \end{cases}$$

When $p_S(B) > p_I$, the miner shuts down in state B (also by Lemma 2), i.e.,

$$h_S^*(B) = 0.$$

When $p_S(B) < p_I$,

$$h_S^*(B) = \begin{cases} \left(1 - \frac{p_S(B)}{p_I}\right) \frac{R}{p_I}, & \text{when } p_S(B) > p_I - p_C \\ \left(1 - \frac{p_S(B)}{p_C + p_S(B)}\right) \frac{R}{p_C + p_S(B)}, & \text{when } p_S(B) \leq p_I - p_C. \end{cases}$$

Any pair of prices $p_S(G)$, $p_S(B)$ falls into one of three cases:

- (i) $p_S(G) < p_S(B) < p_I - p_C$
- (ii) $p_S(G) < p_I - p_C < p_S(B)$
- (iii) $p_I - p_C < p_S(G) < p_S(B)$

Now, we show that if $h_S^*(B) > 0$, then $h_S^*(B) < h_S^*(G)$.

For case (i), $h_S^*(G) = \left(1 - \frac{p_S(G)}{p_I}\right) \frac{R}{p_I} > \left(1 - \frac{p_S(B)}{p_I}\right) \frac{R}{p_I} = h_S^*(B)$.

For case (ii),

$$\begin{aligned} h_S^*(G) > h_S^*(B) &\Leftrightarrow \\ \left(1 - \frac{p_S(G)}{p_I}\right) \frac{R}{p_I} &> \left(1 - \frac{p_S(B)}{p_C + p_S(B)}\right) \frac{R}{p_C + p_S(B)} \Leftrightarrow \\ \left(\underset{>\check{p}_I}{p_C + p_S(B)}\right)^2 \left(\underset{>\check{p}_C}{p_I - p_S(G)}\right) &> p_C p_I^2. \end{aligned}$$

So, it always holds in this case.

For case (iii),

$$\begin{aligned} h_S^*(G) > h_S^*(B) &\Leftrightarrow \\ \left(1 - \frac{p_S(G)}{p_C + p_S(G)}\right) \frac{R}{p_C + p_S(G)} &> \left(1 - \frac{p_S(B)}{p_C + p_S(B)}\right) \frac{R}{p_C + p_S(B)} \Leftrightarrow \\ (p_C + p_S(B))^2 p_C &> (p_C + p_S(G))^2 p_C, \end{aligned}$$

which always holds.

Therefore, $0 \leq h_S^*(B) < h_S^*(G)$, which completes the proof of the proposition.

Proof of Proposition 2

First, suppose that prices are such that individual miners participate, $p_I < p_C + p_S \Leftrightarrow p_S > p_I - p_C$. In this case, C profit over a given period of time is $\pi_C^* =$

$\left(1 - \frac{p_C}{p_I}\right)^2 R$. In the same period of time, given α , S 's profit is $\pi_S^*(\alpha) = \alpha \left(1 - \frac{p_S}{p_I}\right)^2 R$.

$\pi_S^*(\alpha) > \pi_C^* \Leftrightarrow \alpha \left(1 - \frac{p_S}{p_I}\right)^2 R > \left(1 - \frac{p_C}{p_I}\right)^2 R \Leftrightarrow \alpha > \left(\frac{p_I - p_C}{p_I - p_S}\right)^2$.

Notice that in such a case, while S 's share of the mining power is $\frac{h_S^*}{H_0}$, it mines only $\alpha \frac{h_S^*}{H_0}$ blocks.

Next, consider the case where individual miners do not participate when S is active, i.e., $p_I > p_C + p_S \Leftrightarrow p_S < p_I - p_C$. S is active with frequency α . When S is active, its profit is $\pi_S^*(H_1) = \left(1 - \frac{p_S}{p_C + p_S}\right)^2 R$ and C 's profit is $\pi_C^*(H_1) = \left(1 - \frac{p_C}{p_C + p_S}\right)^2 R$.

When S is not active, C 's profit is $\pi_C^*(H_0) = \left(1 - \frac{p_C}{p_I}\right)^2 R$.

For any period of time, S 's overall profit is $\pi_S^*(\alpha) = \alpha \left(1 - \frac{p_S}{p_C + p_S}\right)^2 R$, and C 's overall profit is $\pi_C^*(\alpha) = \alpha \left(1 - \frac{p_C}{p_C + p_S}\right)^2 R + (1 - \alpha) \left(1 - \frac{p_C}{p_I}\right)^2 R$. S 's profit is higher

than C 's when $\alpha > \frac{(p_I - p_C)^2 (p_C + p_S)}{(p_C - p_S) p_I^2 + (p_I - p_C)^2 (p_C + p_S)}$.

On average, the network has $\alpha H_1 + (1 - \alpha)H_0$ hashing power. S 's share of it is $\frac{h_S^*}{\alpha H_1 + (1 - \alpha)H_0}$. However, it only wins the share $\alpha \frac{h_S^*}{\alpha H_1 + (1 - \alpha)H_0}$ of the blocks. Alternatively, for comparison, we can consider that if S kept its h_S active at all times, it would have $\frac{h_S}{H_1}$ share of the total hashing power. This is higher than the share of the blocks it wins given α , i.e., $\alpha \frac{h_S^*}{\alpha H_1 + (1 - \alpha)H_0} < \frac{h_S}{H_1}$.