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ABSTRACT

I develop a stylized model of multi-stage production in which the time length of each stage is endogenously determined. Letting the production process mature for a longer period of time increases labor productivity, but it comes at the cost of higher working capital needs for firms. Under autarky, countries with lower interest rates feature longer production processes, higher labor productivity, and higher wages. In a free trade equilibrium, countries with lower interest rates specialize in relatively 'time intensive' stages in global value chains (GVCs). Yet, if free trade brings about interest rate equalization, wages are also equalized and the pattern of trade is instead shaped by capital intensity and capital abundance, regardless of the time intensity of the various stages. Reductions in trade costs lead to patterns of specialization associated with higher amounts of vertical specialization in world trade. A worldwide decline in interest rates similarly fosters an increase in the share of GVC trade in world trade. The framework also sheds light on the role of trade credit and trade finance in shaping international specialization.

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> "The disadvantage connected with the capitalist method of production is its sacrifice of time. The roundabout ways of capital are fruitful but long; they procure us more or better consumption goods, but only at a later period of time. This proposition [...] is one of the ground pillars of the theory of capital." *The Positive Theory of Capital*, Eugen Böhm-Bawerk (1889, p. 82).

1 Introduction

In the 1870s, when Carl Menger was pioneering the 'Marginal Revolution' in economic theory and laying the foundations for the Austrian school of economics, the world economy was witnessing the initial phases of the so-called 'first globalization'. Largely fueled by decreases in transportation costs (most notably, the advent of steamships), merchandise exports as percentage of GDP grew from 5.9% in 1870 to 8.2% in 1913 (O'Rourke and Williamson, 2001). As noteworthy as this 'first globalization' was, it was dwarfed about one hundred years later by the remarkable growth in world trade recorded in the 1980s, 1990s, and early 2000s. This so-called 'hyperglobalization period' also changed the landscape of international trade in many industries by reshaping world production into global value chains (GVC). As in the case of the 'first globalization', the emergence of GVCs was fostered by technological factors (especially the ICT revolution), but reductions in man-made trade barriers and political developments that greatly increased the world labor force under capitalism also played a key role in the deepening of trade integration (Antràs, 2015).

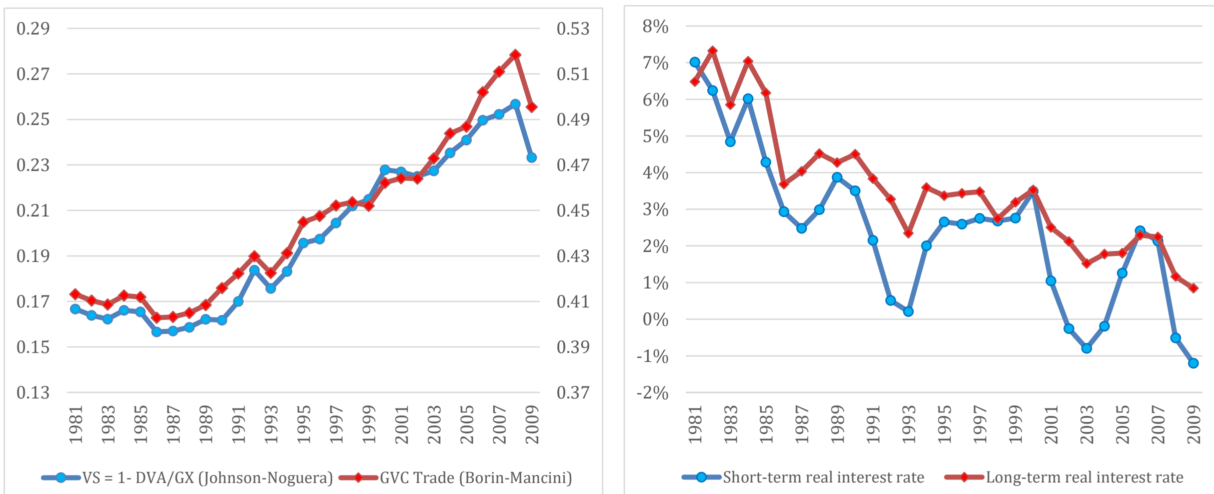
The 'unbundling' of production associated with the growth of GVCs is illustrated in the left panel of Figure 1, which depicts the ratio of foreign value added to gross exports ($1-VAX/GX$), as proposed by Johnson and Noguera (2012), and the share of *GVC trade* (i.e., trade that involves more than one border crossing) in gross exports, from Borin and Mancini (2019). Both series point at a clear increase in the importance of GVCs in the 1980s, 1990s, and early 2000s (see Antràs and Chor, 2022, for a discussion of alternative measures of GVC integration).

As the right panel of Figure 1 indicates, this same period was characterized by a substantial decline in real interest rates. More specifically, the figure includes proxies from Farhi and Gourio (2018) for the one-year and ten-year US real interest rates by subtracting inflation expectations from nominal Treasury yields. A similar decline is observed in many countries (Jordà et al., 2019), and not only for relatively risk-free rates, but also for broader measures of the cost of capital faced by firms (Barkai, 2020). The causes of this secular decline in real interest rates are still being debated by macroeconomists, but candidate explanations include a slowdown in trend real output growth, demographic forces leading to an aging world population, a global saving glut, a shortage of safe assets, and increased wealth inequality.

Independent of the causes of the observed fall in real interest rates, could this reduction in the cost of capital be partly responsible for the 'hyperglobalization' of the 1980s, 1990s, and

Figure 1: GVC Participation and Real Interest Rates

(a) Rising GVC Participation **(b)** Declining Interest Rates



Sources: Johnson and Noguera (2012) and Borin and Mancini (2019) for GVC participation and Farhi and Gourio (2018) for real interest rates.

early 2000s and for the associated growth in GVC? A natural and intuitive mechanism quickly comes to mind: although GVCs were set up to arbitrage away production cost differences across countries, carrying out such 'arbitrage' increases the extent of time it takes to complete a product, thereby raising the working-capital and trade-finance needs of firms. Firms in GVCs may try to attenuate these higher capital demands via faster shipping or just-in-time inventory management, but the profitability of GVCs remains importantly shaped by the cost of capital faced by firms in these production chains. For these reasons, a period of low interest rates facilitates the sustainability of longer production processes and delivery times that can better allocate worldwide resources to their more efficient use.[1]

The goal of this paper is to develop a framework to study the interplay between the emergence and functioning of GVCs and real interest rates. Although the main hypothesis that motivates this paper is relatively easy to understand, formalizing this hypothesis is less straightforward because it requires developing a framework with an explicit notion of production length and of delivery time, concepts that standard frameworks of GVCs (and of international trade, more broadly) are typically silent about. To accomplish this goal, this paper develops a model in the 'Austrian' tradition of Böhm-Bawerk (1889) and Wicksell (1934), a tradition that associates the use of capital and capital intensity with the degree of 'roundaboutness' of production processes, i.e., the time structure of the period of production.[2] Although, this theoretical framework can

[1]Interestingly, the 'first globalization' period of 1870-1913 also witnessed a significant decline in real interest rates (Jordà et al., 2019).

[2]Jevons (1871) independently developed a theory of capital that bears some resemblance to Böhm-Bawerk's.

be reduced to a competitive trade model with two factors of production ('capital' and labor), its equilibrium differs in important ways from the Heckscher-Ohlin model, which instead builds on John Bates Clark's neoclassical notion of capital as a stock of *physical* capital (Clark, 1888), a tradition furthered developed and popularized by Paul Samuelson and Robert Solow, among many others.

I begin in section 2 by developing a stylized model of sequential value chains in which production takes time, and firms need capital to be able to pay their workers before the production process has been completed. Initially, I consider a closed-economy environment in which a single final good is produced by completing a pre-determined order of N stages, with each stage combining labor with the good produced up to the prior stage. This baseline framework builds closely on the GVC framework developed in Antràs and de Gortari (2020). In modeling the temporal dimension of production and the associated working capital needs for firms, I build most closely on the 'Austrian' model in Findlay (1978), which in turn is based on Wicksell (1934) and Metzler (1950). More specifically, although production could in principle be carried out instantaneously, by letting the production process 'mature' for a period of time, more output can be obtained per unit of labor.

Metzler (1950) and Findlay (1978) offer the following metaphor to motivate their frameworks. They envision an environment in which firms initially hire workers to plant the seeds of trees, which deliver wood in the future. The longer the trees are allowed to grow, the more 'capital' is accumulated, and the more wood the firm will be able to sell in the future. The optimal time to chop down the tree is shaped by the rate of growth of trees as well as by the interest rate faced by firms. The lower the interest rate, the longer production processes will be, and the higher will be the productivity of labor. Despite the powerful visual aspects of this metaphor, I take a more general but also more reduced-form approach and simply posit that by letting production of a given process take longer (or become more 'roundabout') the productivity of labor in that production process grows over time. The main closed-economy results of my model link lower interest rates with longer production times at each stage, larger wages, and higher levels of final-good output. I also briefly analyze the possibility that firms may adjust the number of stages depending on the interest rate they face, but the effect of interest on that notion of production length turns out to be ambiguous, and, in fact, null in a benchmark symmetric case. To close the model I introduce a supply side to the capital market, building on Antràs and Caballero (2009, 2010), and show that differences in patience across countries generate

The fact that the theory is as British as it is Austrian, and that it was turned mainstream by a Swedish economist (Wicksell), led Findlay (1978) to write Austrian between quotation marks in the title of his paper. I also adopt quotation marks in my title, but in doing so, I am frankly more motivated by the fact that the term 'Austrian economics' is now commonly associated with the latest phases of the Austrian school of economics, when Hayek and especially Mises turned this school into a heterodox branch of economics largely focused on advocating libertarian ideals (Wasserman, 2019).

differences in autarkic interest rates across these countries as well as differences in production lengths, labor productivity, wages, and output.

In section 3, I consider a world economy in which two economies (Home and Foreign) of the type described in section 2 engage in international trade. To simplify matters and to isolate mechanisms, I assume that countries are symmetric in all respects except for the fact that agents at Home are more patient than those in Foreign, and I also (initially) abstract from trade costs and from international borrowing or lending. I show that in the presence of interest rate differences across countries, labor productivity will also vary across countries and stages. Furthermore, the resulting wage differences across countries confer Home comparative advantage in the most *'time intensive'* stages, that is, in the stages for which a longer maturation of the production process translates into disproportionately larger labor productivity increases. The equilibrium of the model in the presence of factor price differences is thus akin to that of a multi-product Ricardian model, except for the fact that labor productivity differences are endogenous and shaped by the differential cost of capital across countries. If a worldwide increase in patience (e.g., a 'savings glut') reduces interest rates proportionately in all countries, the pattern of comparative advantage is unaltered, but production lengths and therefore labor efficiency and real wages grow worldwide.

I next study the conditions under which the free trade equilibrium indeed features differences in interest rates and in wage levels. I show that if (i) differences in patience across countries are small, (ii) there is a strong correlation between capital intensity and time intensity across stages, and (iii) there is enough variation in capital and time intensity across stages, then the equilibrium will instead feature factor price equalization (FPE). Furthermore, the equalized wages and interest rates worldwide correspond to those that would apply in a fictional *integrated equilibrium* (cf., Dixit and Norman, 1980) of a world economy in which capital and labor are freely mobile across countries. Because with common interest rates, labor productivity differences across countries vanish, the resulting pattern of specialization is purely determined by the interaction of patience and capital intensity, regardless of variation across stages in time intensity. In sum, specialization is governed by distinct primitives with and without factor price equalization.

In section 4, I extend the analysis beyond the special case of zero trade costs. I first model trade costs as 'instantaneous' iceberg costs, following the bulk of the trade literature. Somewhat surprisingly, I demonstrate that the optimal production length of all stages is *independent* of the level of trade costs. Furthermore, and analogously to the results in Harms et al. (2012) and Baldwin and Venables (2013), I show that instantaneous trade costs affect the pattern of specialization in an intricate manner. Despite these complexities, by adopting the dynamic programming approach in Antràs and de Gortari (2020), I formally show that higher trade costs increase the benefits of bunching contiguous production stages in the same location. This of course implies that processes of trade liberalization lead to an 'unbundling' of production processes and to increases in GVC participation.

I next leverage the explicit treatment of time in my model to study the effect of trade costs associated with the time it takes to ship goods across borders, and show how interest rates shape the pattern of specialization. A first implication of temporal trade costs is that interest rate differences across countries generate asymmetric bilateral trade costs which are lower when exporting from relatively capital-abundant countries, in line with the results in Waugh (2010). Second, and consistently with the patterns described in Figure 1, a reduction in interest rates worldwide tends to reduce trade costs and leads to an increase in the extent to which goods cross borders, hence generating a higher amount of GVC participation.

The results above are obtained under frictionless capital markets within countries but zero capital mobility across countries. I relax these assumptions in section 5. To do so, I introduce frictions in borrowing and lending in the form of monitoring costs. These monitoring costs vary across countries and are assumed to be disproportionately large in international financial transactions, a feature that naturally limits financial integration. I also allow cross-border monitoring costs to be lower in the presence of *trade credit*, namely in situations in which exporters provide trade credit to their foreign buyers. I demonstrate that trade credit naturally expands the region of the parameter space for which FPE attains with free trade. More interestingly, outside the FPE set, the introduction of trade credit deepens vertical specialization relative to the baseline model, in the sense that the extent to which intermediate goods cross borders along the production process is higher with trade credit than without it. I finally show that trade finance (borrowing and lending associated with international commerce) generates similar effects and also leads to an increase in production fragmentation and GVC participation.

As is evident from my earlier discussion, my work is closely related to a sequence of 'Austrian' models that begins with the descriptive work of Jevons (1871) and Böhm-Bawerk (1889), continues with the early influential formalization in Wicksell (1934), and culminates with the modern treatments in Metzler (1950) and Findlay (1978). My work is particularly related to Findlay (1978) who also consider a multi-stage model of international trade. There are, however, important differences between my work and his. First, Findlay (1978) considered a two-stage (upstream-downstream) model in which capital was only required in the upstream stage of production. My framework clarifies that his result that patient countries should specialize in relatively upstream stages of production has nothing to do with the fact that those stages are far from final demand, and is instead driven by the assumption that upstream stages are more time intensive, which may or may not be an empirically plausible scenario. Second, Findlay (1978) focused throughout on a trade equilibrium with factor price equalization, and thus most of the results I derive in section 3 are novel. Similarly, the results in sections 4 and 5 regarding costly trade and trade credit are also original to my work.

Another closely related paper to mine is Kim and Shin (2023), who also develop a model of sequential production with 'Austrian' features. Their emphasis, however, is on how the optimal number of symmetric stages of production is shaped by interest rates, and how the longer production times associated with cross-border fragmentation shape the optimal extent of offshoring. Furthermore, they show that, in equilibrium, offshoring, inventories, trade and productivity are all decreasing in interest rates.[3] In my framework, the number of stages N is fixed, and I instead emphasize cross-stage heterogeneity in time intensity and in production lengths. These features are crucial for some of the key results in the paper, including the determinants of comparative advantage or the role of trade costs in shaping GVC participation.

Naturally, this paper also builds extensively on the economics literature on GVCs (see Antràs and Chor, 2022, for a recent survey), and more specifically on contributions that emphasize the sequential nature of these production chains. A non-exhaustive list of contributions featuring sequential production includes (in chronological order): Dixit and Grossman (1982), Sanyal and Jones (1982), Yi (2003, 2010), Harms et al. (2012), Antràs and Chor (2013), Baldwin and Venables (2013), Costinot et al. (2013), Fally and Hillberry (2018), Kikuchi et al. (2018) Alfaro et al. (2019), Johnson and Moxnes (2019), Antràs and de Gortari (2020), and Tyazhelnikov (2022). None of these frameworks studies the role of interest rates in shaping GVCs. In fact, most recent models of GVCs are Ricardian in nature and ignore capital altogether. Apart from the work of Kim and Shin (2023) described above, two noteworthy exceptions are the recent work by Sposi et al. (2021) and Ding (2022), though the focus of those papers is very distinct from that of this paper. The work by Kim and Shin (2012) on the role of financial linkages between producers to sustain production chains also connects with my results in section 5.1 on credit market imperfections, but I do not study financial contracting, which is the key contribution of their paper.

Although an explicit treatment of production and delivery time is rare in international trade, a few pioneering papers, other than Findlay (1978) and Kim and Shin (2023), deserve discussion. Deardorff (2003) explores the importance of time in international trade by developing a model in which products depreciate over time, perhaps due to demand changes, and producers can choose to produce and trade faster using production techniques that are more physical capital intensive. Evans and Harrigan (2005) and Hummels and Schaur (2013) also develop frameworks in which consumers value speedier delivery times, and they both show that the pattern of specialization and mode of transport appear to be consistent with the notion that relatively *time sensitive* goods are more likely to be shipped from nearby locations – in Evans and Harrigan (2005) – or via faster shipping methods, such as air shipping – in Hummels and Schaur (2013). Relatedly,

[3] Bruno et al. (2018) use balance sheet proxies for the length of production chains (specifically, inventories, accounts payable and accounts receivable) to unveil a negative impact of a strong dollar (a proxy for a tightening of dollar credit) on production length.

Djankov et al. (2010) show that shipping delays in exporting countries (due, for instance, to inefficient trade facilitation practices) significantly depress exports and particularly so for time-sensitive goods, such as perishable agricultural products. In my framework, production and delivery times are important, but rather than emphasizing delay costs motivated by changes in demand, I focus on financial capital costs associated with letting production processes mature to improve their productivity. Furthermore, unlike in Deardorff (2003), I do not assume that faster production involves more capital intensive techniques; instead, I show that longer production processes are in fact associated with a relatively higher demand for working capital. Still, in the presence of interest rate differences across countries, the pattern of international specialization is driven by time intensity rather than by capital intensity. A related literature has studied the role of inventories and of 'just-in-time' techniques in shaping specialization and the reactions of firms to trade shocks. The work of Alessandria et al. (2011), and the more recent contributions by Ferrari (2022), Pisch (2020) and Carreras-Valle (2022) exemplify some of the insightful work in this area. Although I abstract from inventory choices, I believe this would be a fruitful extension to the theoretical framework developed in this paper.

2 Closed-Economy Model

I begin my theoretical analysis by presenting the model in its closed-economy version. There are three main building blocks to the model. First, I specify a sequential production process – or value 'chain' – according to which firms hire workers to a produce a final good following a pre-determined order of N stages, with each stage combining labor with the good produced up to the prior stage. The framework is closest to the one developed in Antràs and de Gortari (2020). Second, I incorporate a temporal dimension to production and the associated working capital needs following closely the 'Austrian' model in Findlay (1978). Finally, I close the model by incorporating a supply side to the capital market, building on Antràs and Caballero (2009, 2010).

Environment Time evolves continuously. Infinitesimal agents are born at a rate ρ per unit of time and die at the same rate; the population mass is constant and equal to L. All agents are endowed with one unit of labor services which they supply inelastically to the market. Consumers value a single final good, which is taken to be the numéraire.

Multi-Stage Production To be demanded by consumers, production of the single final good needs to undergo N stages in a pre-determined, sequential order. The last stage of production can be interpreted as final-good assembly and is indexed by N. I mostly focus on the case in which the number of stages is discrete (so N is an integer number), but the analysis can be extended to the case of a continuum of stages without difficulty. At each stage $n > 1$, production combines labor with the good finished up to the previous stage $n - 1$. Production in the initial stage $n = 1$ only uses labor, and production technologies in all stages are homogeneous of degree one. Although this should not be crucial for most qualitative results (see below), I will further restrict the analysis to Cobb-Douglas production technologies at all stages $n > 1$. More precisely, and following Antràs and de Gortari (2020), I assume

$$y_n = (z_n L_n)^{\alpha_n} (y_{n-1})^{1-\alpha_n}, \tag{1}$$

where y_n is output after stage n is performed. In this expression, z_n denotes labor productivity at stage n, and $\alpha_n \in [0, 1]$ is labor- or value-added-intensity at stage n. From the above assumption that the first stage only uses labor, it follows that $\alpha_1 = 1$.

The technologies in equation (1) are freely available to all agents in the economy, and the various stage-specific goods are produced under perfect competition. I will first describe a *decentralized* equilibrium in which different agents optimize at different stages of the chain, but will later demonstrate that the equilibrium is isomorphic to the one that would result from a *lead firm* organizing the whole chain.

Optimal Production Length The main innovation relative to most prior work on sequential value chains is in acknowledging that production *takes time*, and that the more time is devoted to production, the more output will be obtained. I formalize this in an 'Austrian' manner following the approach in Findlay (1978).

To build intuition, let us begin by describing the first stage ($n = 1$). Firms initially hire an amount L_1 of labor and could instantaneously produce an amount $z_1(0) L_1$ of stage-1 output. But by 'waiting' and letting the production process 'mature', labor efficiency increases as a function of time, though at a diminishing rate. This assumption can be interpreted literally as workers improving their skills over time, or it could reflect the accumulation of some additional factor of production that enhances the productivity of labor. As mentioned in the Introduction, Metzler (1950) and Findlay (1978) refer to this additional factor as wood: in their metaphor, the initial labor hired is used to plants trees, and these trees grow over time producing more and more wood as time goes by. As I will discuss below, the assumption that labor is only paid at the very beginning of a stage's production is not important for the results below, but it simplifies the algebra.

Although lengthening the time taken to produce stage-1 output enhances its efficiency, delaying production and sale comes at the cost of increasing the working capital needs of the firm, as firms need to borrow in order to cover the initial wage bill wL_1. Given our continuous time environment, and letting the interest rate be given by r, the firm faces costs (inclusive of capital costs) equal to wL_1e^{rt} after t periods.

Denoting by t_1 the optimal length of the first stage – or what Böhm-Bawerk (1889) labelled the 'roundaboutness' of production – we can write the net present discounted value of profits for firms producing the first stage (at the time they pay workers) as

$$\pi_1 = p_1 z_1 (t_1) L_1 e^{-rt_1} - wL_1. \tag{2}$$

This profit function appears to implicitly impose that stage-1 producers receive payment at the end of their production process, rather than when the final good is sold. I will demonstrate below, however, that the timing of payments is actually irrelevant in this closed-economy version of the model. Firms treat good and factor prices as given, so t_1 will be set to maximize $z_1(t_1) e^{-rt_1}$. The optimal length of stage 1 is thus implicitly given by:

$$\frac{z_1'(t_1^*)}{z_1(t_1^*)} = r. \tag{3}$$

I will introduce below explicit conditions on $z_1(t_1)$ to ensure the existence of a unique optimal length t_1^*. Given equation (2), the zero-profit condition at stage 1 further imposes that

$$p_1 = \frac{w}{z_1(t_1^*)} e^{rt_1^*}. \tag{4}$$

In words, the price of stage-1 output is equal to its unit cost, which consists of the unit labor cost magnified by the capital costs of letting labor productivity mature (or 'wood' grow) for t_1 periods.

Consider now any stage $n > 1$. Producers at that stage must initially hire L_n workers and purchase the prior stage output y_{n-1}. These inputs could be instantaneously combined, but as in the case of stage 1, 'taking time' to produce is beneficial because the efficiency of labor at that stage grows over time though at a diminishing rate. More precisely, I assume that $z_n'(t) > 0$ and $z_n''(t)/z_n'(t) < 0 < z_n'(t)/z_n(t)$, and also that $z_n'(t_1)/z_n(t_n) \to \infty$ when $t_n \to 0$ and $z_n'(t_1)/z_n(t_n) \to 0$ when $t_n \to \infty$.

In light of equation (1), if stage-n producers let efficiency mature for t_n periods, the net present discounted value of profits of these firms at the time they pay wages and purchase the good finished up to the prior stage $n-1$ is

$$\pi_n = p_n (z_n(t_n) L_n)^{\alpha_n} (y_{n-1})^{1-\alpha_n} e^{-rt_n} - wL_n - p_{n-1}y_{n-1}. \tag{5}$$

Because firms take all prices as given (and, anyway, p_{n-1} is independent of t_n), it is straightforward to see that the optimal stopping time t_n^* will satisfy

$$\alpha_n \frac{z_n'(t_n^*)}{z_n(t_n^*)} = r. \tag{6}$$

In fact, this equation holds for all $n \geqslant 1$ because remember that $\alpha_1 = 1$, and thus equation (3) is a special case of equation (6). Given my assumptions above on the function $z_n(t_n)$, there exists a unique t_n^* satisfying (6).

From equation (6), it is clear that the length and labor productivity of all production processes is decreasing in the interest rate r, while other things equal, more labor intensive stages achieve higher labor productivity. It is also interesting to note that the optimal production length is independent of the price p_n or the amount of labor hired L_n, and more generally, it would be independent of any firm-level shifter of revenue that is independent of t_n. The reason for this is that the marginal benefit of letting production mature is associated with a reduction in labor costs, while the marginal cost of letting production mature is associated with delaying the collection of revenue. Because wage payments and revenue are proportional to each other, the choice is independent of scale. Relatedly, in section 4, I will show that the optimal production length is also independent of the level of trade costs faced by firms.[4]

Although many of my results hold for a general function $z_n(t_n)$ satisfying weak assumptions, as an illustrative example, consider the log-linear case in which $z_n(t_n) = (t_n)^{\zeta_n}$ for a vector of constants $\zeta = \{\zeta_1, \zeta_2, ..., \zeta_N\}$ with $\zeta_n \in (0,1)$ for all n. These constants ζ_n govern the importance of 'maturation' for productivity, or the *time intensity* of production of the various stages n. In such a case, equation (3) delivers $t_n^* = \alpha_n \zeta_n / r$. The optimal length of a given stage is thus increasing in the labor and time intensity of stage n, and is decreasing in the interest rate r. A convenient feature of this log-linear case is that the cumulative interest $e^{rt_n^*}$ turns out to be independent of the interest rate r, which will make this case particularly transparent in illustrating some of the main results below.

For alternative functional forms for $z_n(t_n)$, t_n^* continues to necessarily fall with r, but whether rt_n^* increases or decreases in r depends on whether the growth rate of $z_n(t_n)$ rises faster or slower than under the log-linear case. For instance, if $z_n(t_n) = \exp\left((t_n)^{\omega_n}\right)$ for $\omega_n \in (0,1)$, $z_n(t_n)$ features a declining growth rate as a function of time, but with a lower decline than the log-linear case considered above. Under this specification, we obtain $rt_n^* = \alpha_n \omega_n (t_n^*)^{\omega_n}$ and rt_n^* is higher, the lower is the interest rate (due to the higher t_n^*). I will sometimes refer to this as the 'elastic' case. Conversely, when $z_n(t_n) = 1/\exp\left((t_n)^{-\psi_n}\right)$ for $\psi_n > 0$, the growth rate of $z_n(t_n)$ declines faster than in the log-linear case, and we have $rt_n^* = \alpha_n \psi_n (t_n^*)^{-\psi_n}$, implying that rt_n^* is lower, the lower is the interest rate r. I will at times refer to this latter case as the

[4] In Antràs (2023) I show that the same result applies when the firm has market power but charges a constant markup over marginal cost.

'inelastic' case.

Discussion of Assumptions Some readers might be put off by some of the strong assumptions I have made above. I next describe three generalizations that might help me regain the sympathy of these readers. First, how important is the assumption of Cobb-Douglas technologies for the result that production lengths are a decreasing function of interest rates? To answer this, consider the case in which the production technology at stage n is given by $y_n = F_n\left(z_n\left(t_n\right), L_n, y_{n-1}\right)$, where F_n is a twice-continuously differentiable function assumed to be increasing in all arguments and homogeneous of degree one in L_n and y_{n-1}. Following the same steps as above, the optimal choice of t_n^* now satisfies

$$\varepsilon_{F,z}\frac{z_n'\left(t_n^*\right)}{z_n\left(t_n^*\right)} = r, \tag{7}$$

where $\varepsilon_{F,z} \equiv \left(\partial F_n/\partial z_n\right)\left(z_n/F_n\right)$ is the elasticity of F_n with respect to $z_n\left(t_n\right)$. As long as $\varepsilon_{F,z}$ does not increase in $z_n\left(t_n\right)$ too fast, it is clear that the left-hand-side of equation (7) will continue to be decreasing in t_n, and thus t_n^* will be decreasing in r.

Second, is it material for my results that producers are paid at the end of their production process, rather than when the final good is finished and sold to consumers? To answer this question, consider the case in which producers at stage n only receive payment at the end of stage $n+s$, with $n < n+s \leqslant N$. In such a case, producers are effectively lending their sale revenue to downstream producers from the end of stage n up to stage $n+s$, and in our perfectly frictionless capital-market environment, they will charge an interest rate r on those loans. As a result, the net present discounted value of their profits can be expressed as

$$\pi_n = p_n\left(z_n\left(t_n\right)L_n\right)^{\alpha_n}\left(y_{n-1}\right)^{1-\alpha_n} e^{-r\sum\limits_{m=n}^{n+s} t_m^*} \times e^{r\sum\limits_{m=n+1}^{n+s} t_m^*} - wL_n - p_{n-1}y_{n-1},$$

which ends up coinciding with the expression for profits in equation (5). The timing of payments is thus irrelevant in a closed economy in which all agents can borrow or lend at a common interest rate r. Matters will be different in the open-economy version of the model whenever interest rates differ across countries, so I will return to this issue later in the paper.

Third, how important is it that workers are only paid at the beginning of stage-n production? If instead workers were paid at every instant (while maintaining their employment level at L_n), the total wage build would accumulate to

$$wL_n\int_0^{t_n^*} e^{rt}dt = wL_n\frac{e^{rt_n^*}-1}{r},$$

Note that the right-hand-side of this expression is necessarily an increasing function of the interest rate (for a given t_n^*). In that case, and reverting back to the Cobb-Douglas technologies in (1), the choice of t_n^* is governed by

$$\alpha_n \frac{z_n'\left(t_n^*\right)}{z_n\left(t_n^*\right)} = r\left(1-\alpha_n+\alpha_n \frac{e^{rt_n^*}}{e^{rt_n^*}-1}\right), \tag{8}$$

where simple differentiation shows that the right-hand-side is increasing in r and decreasing in t_n^*. Thus, as long as the growth in $z_n\left(t_n\right)$ falls sufficiently fast in t_n, equation (8) continues to deliver a negative relationship between the interest rate and optimal production lengths.

Equilibrium Allocation of Labor Across Stages Having solved for all optimal production lengths, we can next solve for the allocation of labor across stages. From the profit function (5) at stage n, it is clear that the share of costs (including capital costs) associated with labor and the prior stage $n-1$ are given by

$$w L_n e^{rt_n} = \alpha_n p_n y_n \tag{9}$$

$$p_{n-1} y_{n-1} e^{rt_n} = \left(1-\alpha_n\right) p_n y_n. \tag{10}$$

Iterating the second equation forward and plugging the result into the first one, delivers

$$w L_n e^{r \sum_{m=n}^{N} t_m} = \alpha_n \beta_n y_N, \tag{11}$$

where

$$\beta_n \equiv \prod_{m=n+1}^{N} \left(1-\alpha_m\right), \tag{12}$$

(with the convention $\prod_{m=N+1}^{N}\left(1-\alpha_m\right) = 1$), and where y_N is final output and final-good revenue (given $p_N = 1$). The latter can in turn be expressed as

$$y_N = \prod_{n=1}^{N} \left(z_n\left(t_n\right) L_n\right)^{\alpha_n \beta_n}. \tag{13}$$

Notice that for any two stages $n' > n$, we have

$$\frac{L_{n'}}{L_n} = \frac{\alpha_{n'} \beta_{n'}}{\alpha_n \beta_n} e^{r \sum_{m=n}^{n'-1} t_m}. \tag{14}$$

In words, more labor is allocated to relatively more important stages of production (i.e., those featuring a higher value of $\alpha_n \beta_n$), and more labor is also allocated to relatively downstream stages of production, since the financial cost (i.e., the cumulative interest up to the point at which all stages are completed) associated with labor payments in those stages is lower relative to the capital cost of wages paid in upstream stages.

***Lead-Firm* Problem** So far, I have focused on characterizing a decentralized equilibrium in which different producers make decisions at different stages in the chain. Following Antràs and de Gortari (2020), we can alternatively characterize the equilibrium by considering the problem of a *lead firm* seeking to maximize the overall profits obtained along the chain. It is convenient to write these profits as evaluated at the end of the production process:

$$\max_{L_n,t_n} \prod_{n=1}^{N} \left(z_n\left(t_n\right)L_n\right)^{\alpha_n\beta_n} - \sum_{n=1}^{N} wL_n e^{r\left(\sum\limits_{m=n}^{N} t_m\right)}. \tag{15}$$

The lead firm will then satisfy the following first-order-condition:

$$wL_n e^{r\left(\sum\limits_{m=n}^{N} t_m\right)} = \alpha_n\beta_n \prod_{n=1}^{N} \left(z_n\left(t_n\right)L_n\right)^{\alpha_n\beta_n}, \tag{16}$$

which given (13), is identical to (11). Naturally, the relative use of labor at different stages continues to be given by equation (14).

Perhaps less trivially, the optimal length of each stage of production also continues to be given by (6). To see this, note that the lead firm's first-order condition for the choice of t_n is given by:

$$\alpha_n\beta_n \frac{z_n'\left(t_n^*\right)}{z_n\left(t_n^*\right)} \prod_{n=1}^{N} \left(z_n\left(t_n^*\right)L_n\right)^{\alpha_n\beta_n} = r\sum_{m=1}^{n} wL_m e^{r\sum\limits_{m'=m}^{N} t_{m'}^*}.$$

or, plugging in (16),

$$\alpha_n\beta_n \frac{z_n'\left(t_n^*\right)}{z_n\left(t_n^*\right)} = r\sum_{m=1}^{n} \alpha_m\beta_m. \tag{17}$$

But from the definition of β_n in equation (12), one can verify that $\beta_n = 1 - \sum_{m=n+1}^{N} \alpha_m\beta_m$, so (17) reduces to

$$\alpha_n \frac{z_n'\left(t_n^*\right)}{z_n\left(t_n^*\right)} = r,$$

which is of course identical to (6).

The Optimal Number of Stages The above formulation of the *lead firm* problem also allows us to briefly consider the possibility that firms do not take the number of stages N as fixed, but instead choose it to maximize their profits. As I show in Online Appendix A.1, it is straightforward to work with the cost minimization problem dual to (15) to obtain the following expression for the overall marginal cost of production of the final good:

$$c_Y = \prod_{n=1}^{N} \left(\frac{w}{\alpha_n \beta_n z_n \left(t_n^*\right)} e^{r \sum_{m=n}^{N} t_m^*} \right)^{\alpha_n \beta_n}. \tag{18}$$

How is this marginal cost affected by a change in N? The answer to this question is complex because the choice of N affects the marginal cost in equation (18) in a number of different ways. Starting from a given N, adding an additional stage $N+1$ will imply a direct increase in marginal costs associated with the costs of paying the factors used in that new stage. But the addition of this stage will also affect the cost of the prior stages both by delaying the collection of final revenue and by affecting the terms $\alpha_n \beta_n$ in equation (18), given that β_n is a 'forward-looking' term (see equation (12)). A more formal discussion of these effects is relegated to Online Appendix A.1. Here in the main text, I focus on a special symmetric case which highlights that the model does not generate a clean link between interest rates and the length of production as captured by the number of stages N.

With this goal in mind, consider a symmetric case in which the labor intensity parameters satisfy $\alpha_n = 1/n$ while stage-specific labor productivity is governed by $z_n(t_n) = \chi_n (t_n)^{\zeta n}$ with $\chi_n = \bar{z}/(\zeta/r)^{\zeta n}$. I refer to this case as *symmetric* because it has the implication that the contribution to value added ($\alpha_n \beta_n = 1/N$), the time length of all stages ($t_n^* = t^* = \zeta/r$), and labor productivity ($z_n(t_n) = \bar{z}$) are identical across all stages . Given these symmetry assumptions, equation (18) can be shown to simplify (see Online Appendix A.1) to

$$c_Y = \frac{w}{\bar{z}} N e^{\frac{\zeta}{2}(N+1)}, \tag{19}$$

which is clearly increasing in N. Thus, the framework cannot justify why firms would ever set $N > 1$.

As I show in Online Appendix A.1, it is straightforward to append a 'love-for-variety' term to the marginal cost function to engineer a nontrivial choice of the number of stages N. Nevertheless, equation (19) indicates that the marginal cost of increasing N is independent of the interest rate r, so unless the 'love-for-variety' term is a function of the interest rate, the choice of N will remain independent of r. For example, if the 'love-for-variety' takes the simple form $N^{-(1+\gamma)}$, as in Benassy (1996), marginal costs are minimized whenever $N^* = 2\gamma/\zeta$, independent of the interest rate r.

Although the above discussion has focused on a very special case, it highlights that whether the 'optimal' N increases or decreases in r will be a function of subtle features of the environment. For this reason, for the remainder of the paper I will take N as a fixed parameter of the model.

Labor Market Clearing From the above discussion, production processes feature a length equal to $\sum_{n=1}^{N} t_n^*$ broken up into $n = 1, ..., N$ intervals of length t_n^* each. At any point in time, the economy features various production processes at various times of completion. Following Findlay (1978), I focus on stationary equilibria in which the share of the economy's labor allocated to each stage is constant (more on this below). Labor-market clearing further imposes

$$\sum_{n=1}^{N} L_n = L.$$

Since all chains hire labor at different stages according to (14), we have

$$L_n = \frac{\alpha_n \beta_n e^{-r \sum_{m=n}^{N} t_m^*}}{\sum_{n'=1}^{N} \alpha_{n'} \beta_{n'} e^{-r \sum_{m=n'}^{N} t_m^*}} L. \tag{20}$$

As anticipated above, more labor is allocated to stages with a higher final-output elasticity – see equation (14) – and also to more downstream stages. Given our Cobb-Douglas assumption, the allocation of labor across stages is independent of the wage rate w. Whether a lower interest rate r moves labor towards downstream or towards upstream stages depends on how the terms $e^{-rt_n^*}$ in (20) change with r. When all functions $z_n(t_n)$ are log-linear (i.e., $z_n(t_n) = (t_n)^{\zeta_n}$), we have seen above that rt_n^* is independent of r, and thus the allocation of labor across sectors is also independent of the interest rate r. Whenever rt_n^* is affected by r, changes in the interest rate will shift labor from upstream stages to downstream stages or vice versa, depending on the properties of the $z_n(t_n)$ functions.

To solve for the equilibrium wage w, we can invoke the zero-profit condition at the last stage N and invoke equation (18) to obtain

$$w = \prod_{n=1}^{N} \left(\alpha_n \beta_n z_n(t_n^*) e^{-r \sum_{m=n}^{N} t_m^*} \right)^{\alpha_n \beta_n}. \tag{21}$$

Although it is not obvious from inspection, one can establish that the lower is the interest rate r, the higher is the wage rate w (see Online Appendix A.2 for a formal proof). Intuitively, lower interest rates lead to longer production processes that accumulate more 'capital' (or 'wood'), and this enhances the efficiency of labor. It is worth stressing that this result is independent of whether the various terms rt_n^* increase or decrease in r.

Final-Good Output Given that all labor productivities $z_n(t_n)$ decrease with the interest rate r, it is intuitive that final-good output y_N should also be decreasing in the interest rate r. For a fixed allocation of labor across sectors, this is obvious from inspection of equation (13). This result is useful because, as argued above, when all functions $z_n(t_n)$ are log-linear (i.e., $z_n(t_n) = (t_n)^{\zeta_n}$), the allocation of labor across sectors is indeed independent of the interest rate r. Taking into account the potential effect of the interest rate on the allocation of labor complicates the evaluation of how changes in r affect final-good output. But in Online Appendix A.3, I show that final output y_N is decreasing in the interest rate r regardless of whether rt_n^* increases in, decreases in, or is independent of the interest rate r.

Aggregate Demand for Capital I finally turn to discussing equilibrium in the capital market, beginning with the demand side of that market. Notice that firms pay wages at the beginning of production, and while they can obtain revenue and pay workers at the end of their stage, downstream producers need to borrow to pay their inputs, and these embody those upstream wage costs. As a result, the effective interval of time for which wages paid at stage n need to be financed is equal to $\sum_{m=n}^{N} t_m$. In other words, the cumulated working capital needs associated with stage-n production are given by

$$e^{r \sum\limits_{m=n}^{N} t_m^*} wL_n.$$

In a stationary equilibrium with a uniform time-invariant distribution of production processes of different ages, aggregate demand for capital equal to

$$K^d = \sum_{n=1}^{N} wL_n \int_0^{\sum_{m=n}^{N} t_m^*} e^{rt} dt. \tag{22}$$

Evaluating the last integral in equation (22), we obtain:

$$K^d = \sum_{n=1}^{N} wL_n \frac{e^{r \sum\limits_{m=n}^{N} t_m^*} - 1}{r}. \tag{23}$$

This equation can alternatively be derived by computing capital demand stage by stage, and then integrating over a uniform distribution of 'ages' at each stage, or

$$K^d = \sum_{n=1}^{N} \left(wL_n + p_{n-1} y_{n-1}\right) \int_0^{t_n^*} e^{rt} dt.$$

Using $\alpha_n p_{n-1} y_{n-1} = (1 - \alpha_n) wL_n$, equation (23) can thus alternatively be written as

$$K^d = \sum_{n=1}^{N} \frac{1}{\alpha_n} wL_n \frac{e^{rt_n^*} - 1}{r}. \tag{24}$$

How does aggregate capital demand depend on the interest rate r? It is clear from inspection of (23) or (24) that in the log-linear case $z_n(t_n) = (t_n)^{\zeta_n}$, aggregate capital demand is decreasing in the interest rate r. This result follows from the fact that (i) a lower interest rate r increases the wage rate w; (ii) the denominator in equation (23) includes a direct negative impact of r on K^d; and (iii) whenever the functions $z_n(t_n)$ are log-linear, all terms rt_n^* are independent of r, and the allocation of labor L_n is also independent of r. In Online Appendix A.4, I show that the negative impact of interest rates on aggregate capital demand persists in the 'elastic' case in which the functions $z_n(t_n)$ are such that the terms rt_n^* are weakly increasing in r.

It is interesting to next invoke the zero-profit condition at all stages (recoverable from equation (15) for the lead firm) to obtain

$$\sum_{n=1}^{N} wL_n e^{r\left(\sum_{m=n}^{N} t_m\right)} = y_N.$$

Combining this expression with (23) delivers

$$y_N = wL + rK^d,$$

which of course constitutes the standard national identity equating the value of final-good production to national income, the latter being the sum of labor and capital income.

Consumer Preferences and the Supply of Capital So far, I have treated the interest rate r as exogenous, but I now 'close' the model by introducing a supply for capital. I do so following the approach in Antràs and Caballero (2009, 2010), which in turn build on the work of Caballero et al. (2008). Remember that I have assumed that agents are born at a rate ρ per unit of time and die at the same rate. I now further assume that agents save all their income and consume only when they (are about to) die.[5] For simplicity, I assume that the final good is the only store of value, so agents save in terms of units of (or claims on) this final good and lend it to firms seeking to pay workers before selling their goods. If K_t^s denotes aggregate savings (or capital) accumulated up to date t, then aggregate consumption at time t is ρK_t. The parameter ρ is thus inversely related to the aggregate propensity to save of this economy, and thus can also

[5]This can be interpreted as agents saving to provide for their long retirement. Caballero et al. (2008) show that the crucial features of the equilibrium described below survive to more general overlapping generation structures, such as that in Blanchard (1985) and Weil (1989)

be interpreted as reflecting the 'impatience' of agents. For now, I abstract from frictions in the capital market, and assume that all agents take the interest rate r_t as given and that lenders have no trouble collecting interest from debtors. I will relax this assumption in section 5.

For an exogenously given initial capital stock K_0, aggregate capital evolves according to

$$\frac{dK_t^s}{dt} = w_t L + r_t K_t^s - \rho K_t^s.$$

Therefore, in a steady state in which all variables are time invariant, we must have

$$K^s = \frac{wL}{\rho - r} = wL \times \sigma(r, \rho). \tag{25}$$

The supply of capital is thus proportional to labor income, with the factor of proportionality $\sigma(r, \rho)$ being positively affected by the interest rate r and negatively affected by a parameter ρ governing the impatience of agents.

With this capital supply at hand, it is straightforward to characterize the equilibrium interest rate à la Metzler (1951). Notice, in particular, that combining (23) and (25), we have

$$\frac{K^d}{wL} = \sum_{n=1}^{N} \frac{L_n}{L} \frac{e^{r\left(\sum\limits_{m=n}^{N} t_m^*\right)} - 1}{r} = \sigma(r, \rho) = \frac{K^s}{wL}. \tag{26}$$

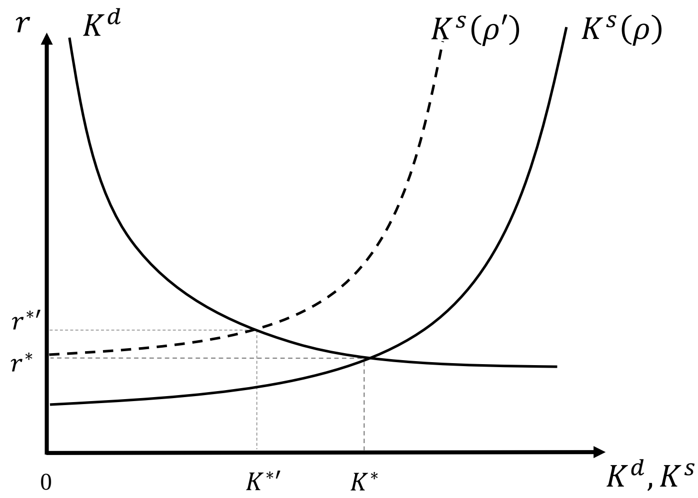
As I mentioned above and established in Online Appendix A.4, as long as rt_n^* is independent of or decreasing in r, the ratio K^d/wL is a monotonically decreasing function of the interest rate r, while the ratio $K^s/wL = \sigma(r, \rho)$ is monotonically increasing in r. Furthermore, K^d/wL goes to $+\infty$ when $r \to 0$, while it goes to 0 when $r \to \infty$. As a result, equation (26) uniquely determines the economy's interest rate r, which is a monotonically increasing function of the impatience ρ of agents. Figure 2 depicts the equilibrium in the capital market as well as the effects of an increase in impatience ρ on that equilibrium.

To sum up, the results above indicate that the more impatient agents are, the higher is the interest rate and the lower are production lengths, wages, and final-good output.

3 Open-Economy Model

I now consider a world economy in which two economies (Home and Foreign) of the type described in the previous section engage in international trade. I will make a number of simplifying assumptions to streamline the discussion of the model's key results. More specifically, I assume that countries are symmetric in all respects except for the 'impatience' of their agents. In particular, agents at Home are assumed to be more patient than those in Foreign ($\rho^H < \rho^F$),

Figure 2: Equilibrium in the Capital Market



Notes: The figure plots the demand and supply for capital in equations (23) and (25), respectively. An increase in impatience ρ shifts capital supply to the left and leads to a higher equilibrium interest rate.

so that at Home the autarky interest rate is lower ($r^H < r^F$) and the wage is higher ($w^H > w^F$) than in Foreign. Conversely, all production technologies and all schedules $z_n(t_n)$ are common in both countries, though obviously labor efficiencies may vary across countries if the equilibrium interest rate differs between the two economies. Indeed, I will initially impose that factor prices differ across countries in the trade equilibrium, but will later consider the possibility of trade bringing about factor price equalization (or FPE). For the time being, I also abstract from any trade costs and rule out the possibility of international borrowing or lending, or of international trade finance. I will tackle these important aspects in later sections.

I also focus attention to the log-linear case in which $z_n(t_n) = (t_n)^{\zeta_n}$. I have explored the equilibrium of the model under alternative functional forms, but these efforts delivered a very low ratio of additional (robust) insights to algebra (see Online Appendix A.5).

3.1 The Pattern of Specialization

A Misconception In the closed-economy version of the model, firms producing at stage n generate working capital needs along the whole chain that accumulate to an amount

$$e^{r\sum_{m=n}^{N} t_m^*} wL_n.$$

It may thus seem intuitive that the more patient Home would have a comparative advantage in the most upstream stages of production because the resulting cumulative interest would be disproportionately lower if those stages are performed in a low interest rate country. This is reflected in the fact that the exponent $r\sum_{m=n}^{N} t_m^*$ increases in r and disproportionately so for the most upstream stages. Consistent with this intuition, one of the main results in the two-stage model in Findlay (1978) is that the more patient country is indeed a net exporter of the output of the upstream stage of production, a result that the current author confidently expected would generalize to the N-stage model in this paper.

As is often the case, however, this author's initial intuition proved to be wrong. As I will show below, the pattern of specialization has nothing to do *per se* with the upstreamness or downstreamness of different stages.

Pattern of Specialization: The Two-Stage Case To build intuition, consider a two-stage environment ($N = 2$) along the lines of the one considered by Findlay (1978). From equation (4), the ratio of the Home to the Foreign price for the first stage output when sold to stage-2 producers is given by

$$\frac{p_1^H}{p_1^F} = \frac{w^H}{z_1\left(t_1^H\right)} \frac{z_1\left(t_1^F\right)}{w^F} \frac{e^{r^H t_1^H}}{e^{r^F t_1^F}},$$

which in the log-linear case $z_n\left(t_n\right) = \left(t_n\right)^{\zeta_n}$ reduces to

$$\frac{p_1^H}{p_1^F} = \frac{w^H}{w^F}\left(\frac{r^H}{r^F}\right)^{\zeta_1}. \tag{27}$$

Now consider the same ratio of Home to Foreign prices at stage 2:

$$\frac{p_2^H}{p_2^F} = \left(\frac{w^H}{w^F}\frac{z_2\left(t_1^F\right)}{z_2\left(t_1^H\right)}\frac{e^{r^H t_2^H}}{e^{r^F t_2^F}}\right)^{\alpha_2}\left(\frac{p_1}{p_1}\frac{e^{r^H t_2^H}}{e^{r^F t_2^F}}\right)^{1-\alpha_2},$$

which is a function of the labor costs (inclusive of working capital needs) at stage 2 as well as of the working capital needs associated with purchasing the stage-1 output. Note that I impose that Home and Foreign firms have access to the same price for stage-1 output at the beginning of stage-2 production, reflecting the absence of trade costs in the model. In the log-linear case $z_n\left(t_n\right) = \left(t_n\right)^{\zeta_n}$, this price ratio can be simplified to

$$\frac{p_2^H}{p_2^F} = \left(\frac{w^H}{w^F}\left(\frac{r^H}{r^F}\right)^{\zeta_2}\right)^{\alpha_2}. \tag{28}$$

Comparing equations (27) and (28), we see that as long $w^H > w^F$ and $r^H < r^F$, Home is a net exporter of the upstream stage if and only if $\zeta_1 > \zeta_2$. More specifically, if $\zeta_1 > \zeta_2$, $p_1^H/p_1^F > 1$ implies $p_2^H/p_2^F > 1$, and $p_2^H/p_2^F < 1$ implies $p_1^H/p_1^F < 1$. Both of these cases would be inconsistent with positive labor demand in both countries. Hence, the only possible configuration when

$\zeta_1 > \zeta_2$ is $p_1^H/p_1^F \leqslant 1$ and $p_2^H/p_2^F \geqslant 1$, with at least one of these inequalities being a strict inequality. Under those circumstances, Home is a net exporter of the upstream stage and Foreign is a net exporter of the downstream stage. When $\zeta_2 > \zeta_1$, the arguments above are reversed, and so is the pattern of trade, while when $\zeta_1 = \zeta_2$, there is no trade. In sum, $\zeta_1 > \zeta_2$ is both a necessary and a sufficient condition for Home to be a net exporter of the upstream stage.

In more plain words, Home has comparative advantage upstream if and only if labor productivity increases faster with time in the upstream sector than in the downstream sector, which reflects a higher benefit of using working capital to lengthen production in the upstream sector than in the downstream sector. Indeed, this is the case Findlay (1978) implicitly restricted attention to. In his framework, production in the downstream stage occurs instantaneously without any working capital needs. So, this amounts to assuming $\zeta_2 = 0$ in terms of the notation in the current paper. It is then intuitive why Findlay (1978) found that his framework delivered comparative advantage in the upstream stage for the patient country. Yet, the derivations above demonstrate that this result has nothing to do *per se* with the relative upstreamness of the two stages. Instead, what is relevant is the relative importance of *time* in enhancing productivity in the various stages of production.

Pattern of Specialization: The N-Stage Case To further demonstrate the above claim, consider the N-stage case. The ratio of Home to Foreign prices at stage n when selling to producers at stage $n+1$ is given by

$$\frac{p_n^H}{p_n^F} = \left(\frac{w^H}{w^F} \frac{z_n\left(t_n^F\right)}{z_n\left(t_n^H\right)} \frac{e^{r^H t_n^H}}{e^{r^F t_n^F}} \right)^{\alpha_n} \left(\frac{p_{n-1}}{p_{n-1}} \frac{e^{r^H t_n^H}}{e^{r^F t_n^F}} \right)^{1-\alpha_n},$$

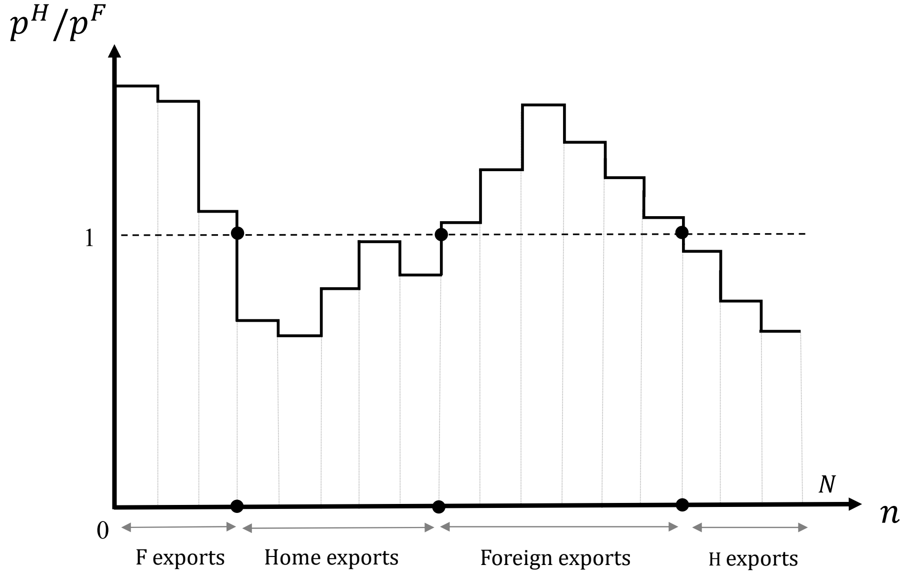
which in the log-linear case reduces to

$$\frac{p_n^H}{p_n^F} = \left(\frac{w^H}{w^F} \left(\frac{r^H}{r^F} \right)^{\zeta_n} \right)^{\alpha_n}. \tag{29}$$

This equation makes it clear again that what matters for the pattern of trade is the relative size of the parameter ζ_n in the different sectors – or their relative *time intensity* – rather than their relative positioning in the value chain. More specifically, if Home is a net exporter of a stage n, so $p_n^H/p_n^F \leqslant 1$, then it will also be a net exporter of any stage n' such that $\zeta_{n'} > \zeta_n$ (since $r^H < r^F$ in equation (29)).

In Online Appendix A.5, I show that this result is *not* an artifact of the log-linear functional form assumption for the function $z_n(t_n)$. The intuition is much more general. Upstream stages are indeed further away from final consumption than relatively downstream stages, but firms

Figure 3: Comparative Advantage and Downstreamness



Notes: The figure plots the relative price at which Home and Foreign can provide the various stages of production. With no trade costs, Home exports all goods for which $p_n^H/p_n^F < 1$ and Foreign exports all goods for which $p_n^H/p_n^F > 1$.

obtain sale revenue when they sell their own stage output and *not* when the final good is sold to consumers. In international trade transactions, this is implied by the assumption that there are *no* international capital flows: for instance, if Home exporters at stage n were not paid by downstream Foreign importers immediately after completing the production at stage n, then this would amount to Home exporters effectively extending trade credit to Foreign importers. Although I have rule out trade credit for the time being, I will allow for these forms of capital flows in section 5 below.[6]

In sum, the time interval for which working capital is needed by a specific producer is a function of the length of the stage's production process and not of the overall length of production or how far from final-good assembly a given stage is. As a result, as long as there are factor price differences across countries, the pattern of specialization is governed by the relative impatience of countries and by the relative time intensity of the various stages of production. I illustrate this result in Figure 3 by plotting the ratio p_n^H/p_n^F for a case with many stages. In the figure, the time intensity of the various stages is not perfectly correlated with their positioning in the chain, and as a result production moves back and forth across countries.

[6]As in the closed-economy model, the timing of payments continues to be irrelevant for domestic transactions involving agents facing the same interest rate.

3.2 Trade Equilibrium with Factor Price Differences

In light of the above preliminary results, I am now ready to describe a trade equilibrium in which factor price differences persist even with free trade (see the next section for an analysis of equilibria with FPE). It should be clear that as long as wages and interest rates differ across countries and time intensity ζ_n is distinct in all stages, the relative price in equation (29) can only be equal to 1 for at most one stage n. To avoid unproductive discussions, I further assume that the equilibrium is one with complete specialization, that is, in which we can partition the set of stages into two disjoint sets $\mathcal{N}^H$ and $\mathcal{N}^F$ with $\mathcal{N}^j$ being the set of stages in which country $j = H, F$ specializes. Naturally, we have $\mathcal{N}^H \cup \mathcal{N}^F = \{1, 2, 3, ..., N\}$.

Let us next reindex stages in decreasing order of their time intensity ζ_n. More precisely, when indexing stages by $v = 1, ..., N$, we have ζ_v decreasing in v. Next, as in Dornbusch et al. (1977), define the relative labor efficiency schedule

$$A(v) \equiv \frac{z_n\left(t_n^H\right)}{z_n\left(t_n^F\right)} = \left(\frac{r^F}{r^H}\right)^{\zeta_v}. \tag{30}$$

As long as factor prices vary across countries, with $r^H < r^F$, the function $A(v)$ is decreasing in v, and there exists a unique $v^* \in \mathbb{R}$, such that $A(v) > w^H/w^F$ for $v < v^*$ and $A(v) < w^H/w^F$ for $v > v^*$. In words, Home has a cost advantage in stages $v \in \mathbb{N}$ with $v < v^*$, while Foreign has a cost advantage in all stages with $v > v^*$. As long as v^* is not an integer, such an equilibrium will indeed feature complete specialization.

Next, consider labor demand and labor-market clearing. Following the same derivations as in the closed-economy model, it can be shown that equation (11) continues to apply with factor price differences in a slightly modified way. More precisely, for each country $j = H, F$, we have

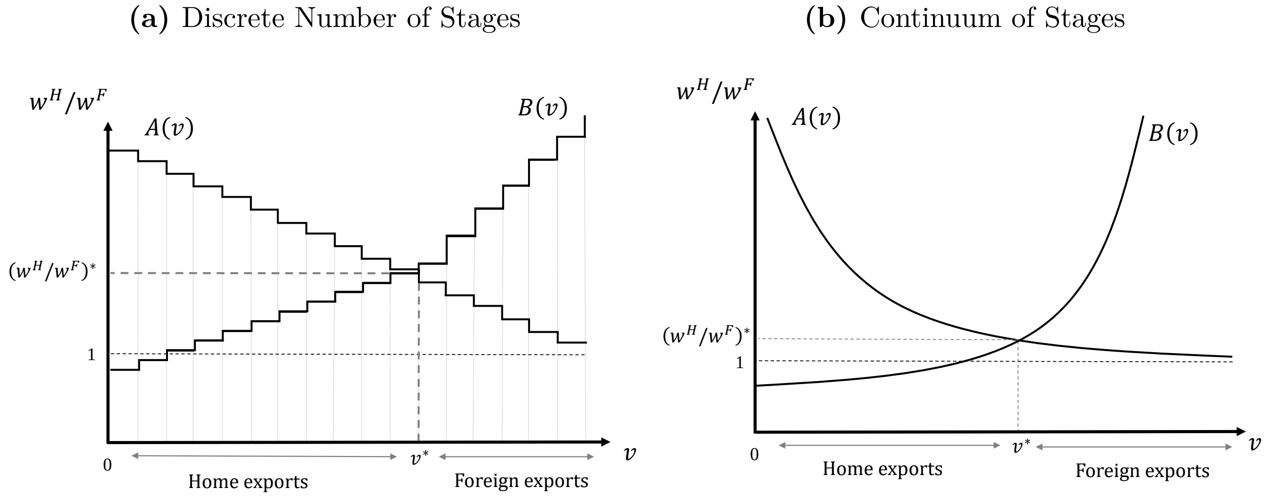
$$L_n^j = \frac{\alpha_n \beta_n}{w^j} e^{-\sum_{m=n}^{N} \alpha_m \zeta_m} y_N, \quad n \in \mathcal{N}^j,$$

and $L_n^j = 0$ if $n \notin \mathcal{N}^j$. Note that, relative to equation (11), I already imposed here that $r_n^j t_n^j = \alpha_n \zeta_n$ for both $j = H$ and $j = F$, as obtained when the functions $z_n(t_n)$ are log-linear. From this equation, we then obtain

$$\frac{w^H}{w^F} = \frac{\sum_{n \in \mathcal{N}^H} \alpha_n \beta_n e^{-\sum_{m=n}^{N} \alpha_m \zeta_m}}{\sum_{n \in \mathcal{N}^F} \alpha_n \beta_n e^{-\sum_{m=n}^{N} \alpha_m \zeta_m}} \equiv B(v^*). \tag{31}$$

The schedule $B(v^*)$ is an increasing function of the threshold stage v^* because as more stages

Figure 4: Trade Equilibrium with Factor Price Differences



Notes: The figure plots the trade equilibrium with no factor price equalization. Stages are reordered in decreasing order of their time intensity ζ_n. The schedules $A(v^*)$ and $B(v^*)$ are defined in equations (30) and (31), respectively.

are performed at Home, the numerator in (31) increases while the denominator decreases.

For given interest rates $r^H < r^F$, equations (30) and (31) determine the equilibrium threshold v^* and the relative wage w^H/w^F in a manner completely analogous to the analysis in Dornbusch et al. (1977). Figure 4 depicts such an equilibrium. In the left panel, I plot the equilibrium when the number of stages is large but discrete, and in the right panel, I plot the same equilibrium for the case in which the number of stages is so large that it can be well-approximated by a continuum, as in Dornbusch et al. (1977).[7]

A key distinction relative to Dornbusch et al. (1977) is that, in my model, labor efficiency is not exogenous but is instead shaped by the length of production of the various stages, which is in turn shaped by the interest rate faced by producers, as is evident from equation (30). To complete the description of the equilibrium, we thus need to determine the relative interest rate in the two countries.

Equilibrium interest rates can be obtained by deriving capital demand and invoking capital-market clearing. Remember that funding labor costs at stage n in country j requires an amount of capital equal to $w^j L_n^j e^{r^j t_n^j}$, and to fund intermediate input costs requires an amount of capital equal to $p_{n-1} y_{n-1} e^{r^j t_n^j}$. Given our Cobb-Douglas assumptions, the optimal mix of labor and intermediate inputs at stage n is such that $\alpha_n p_{n-1} y_{n-1} = (1 - \alpha_n) w L_n$ (see equations (9) and (10)). Using this equality, we can express capital intensity at stage n in country j in a stationary

[7]The equilibrium conditions for the case with a continuum of stages are analogous to those derived in this section, with summations being replaced by integrals. Whenever the multiplication operator $\prod$ appears in the equations, one can first take logs and then replace summations with integrals in the relevant equations.

equilibrium with a uniform time-invariant distribution of production processes as

$$\frac{K_n^j}{L_n^j} = \frac{w^j}{r^j}\frac{1}{\alpha_n}\left(e^{\alpha_n\zeta_n} - 1\right), \tag{32}$$

where I have again invoked $r^j t_n^j = \alpha_n \zeta_n$.

The aggregate demand for capital in country j is therefore equal to:

$$\left(K^d\right)^j = \frac{w^j}{r^j}\sum_{n\in\mathcal{N}^j}\frac{1}{\alpha_n}L_n^j\left(e^{\alpha_n\zeta_n} - 1\right). \tag{33}$$

Imposing capital-market clearing and the definition of the function $\sigma(r,\rho)$ in equation (25), we can express the ratio of interest rates in the two countries as

$$\frac{r^F}{r^H} = \frac{\sigma\left(r^H,\rho^H\right)}{\sigma\left(r^F,\rho^F\right)} \times \frac{\sum\limits_{n\in\mathcal{N}^F} L_n^F \frac{1}{\alpha_n}\left(e^{\alpha_n\zeta_n} - 1\right)}{\sum\limits_{n\in\mathcal{N}^H} L_n^H \frac{1}{\alpha_n}\left(e^{\alpha_n\zeta_n} - 1\right)}. \tag{34}$$

This equation equates the ratio of interest rates to the product of the ratio of each country's aggregate capital supply and a labor-weighted average capital intensity of the stages in which each country specializes. As long as the right-hand side of equation (34) is higher than one, the equilibrium will indeed entail a lower interest rate at Home and a pattern of specialization as described above.

When is such an equilibrium more likely to attain? Because Home specializes in high ζ_n sectors, if there is little variation in labor intensity α_n across sectors, Home also specializes in relatively capital-intensive sectors, and this constitutes a force towards factor price convergence, by making the second ratio in the right-hand-side of equation (34) lower than one. Nevertheless, if variation in α_n is such that the positive correlation between time-intensity ζ_n and capital intensity in equation (32) is low or if differences in patience across countries are sufficiently large – so $\sigma\left(r^H,\rho^H\right)/\sigma\left(r^F,\rho^F\right)$ is large–, then the equilibrium will necessarily feature factor price differences across countries and a pattern of specialization as described in Figure 4.

Comparative Statics Equation (34) can also be used to discuss some comparative statics. Suppose first that Home becomes even more patient relative to Foreign, which corresponds to an increase in ρ^H holding ρ^F fixed. This leads to an increase in $\sigma\left(r^H,\rho^H\right)$, and from equation (34), r^F/r^H goes up on impact. This in turn shifts the $A(v)$ curve up in Figure 4, increasing the Home relative wage w^H/w^F and pushing v^* up. As a result $\mathcal{N}^H$ grows and $\mathcal{N}^F$ shrinks, partly undoing (but not overturning) the effect of the higher ρ^H in equation (34). As a result, factor price differences are magnified by this increase in Home impatience.

Next, consider a worldwide increase in patience ρ (e.g., a global savings glut) that decreases interest rates proportionately in both countries. From equation (34), this corresponds to a proportional shift in $\sigma\left(r^H,\rho^H\right)$ and $\sigma\left(r^F,\rho^F\right)$, and from equations (30) and (31), this shift has no effect on the relative wage w^H/w^F or the threshold stage v^*. Does this imply that a savings glut has no impact on the equilibrium of the model? Not quite. Because interest rates will be lower in both countries, production lengths will grow worldwide and so will labor efficiency and the level of the real wage rate in both countries.

Although I have assumed that both countries share the same population level $L^H = L^F = L$, it is interesting to briefly study the implications of an increase in the relative population in Foreign (i.e., an increase in L^F/L^H). As in Dornbusch et al. (1977), the impact effect of such a shock is to shift the $B(v)$ up and to the left, thereby increasing the relative wage w^H/w^F and reducing the threshold sector v^*.[8] In the current model, however, this is not the end of the story. From equation (34), the implied shift of production from Home to Foreign tends to bid up the ratio of interest rates r^F/r^H, which will lead to an upward shift in the $A(v)$ curve and an implied further increase in the relative wage w^H/w^F. Thus, population growth abroad leads to a magnified terms-of-trade improvement at Home.

3.3 Trade Equilibrium with Factor Price Equalization

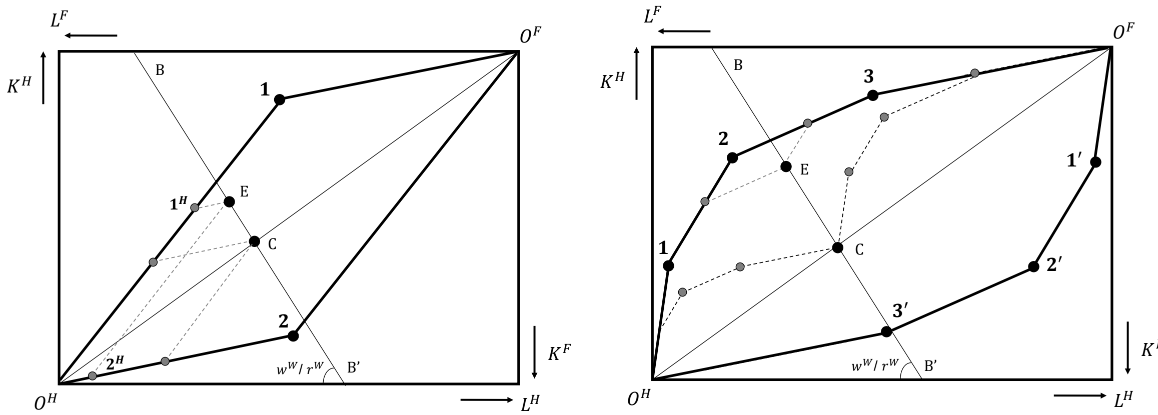
So far, I have focused on equilibria in which factor price differences across countries persist even in a free trade equilibrium. In this section, I consider a situation in which, perhaps due to small differences in patience across countries or perhaps due to a strong correlation between capital and time intensity across sectors, equation (34) would result in a ratio r^F/r^H lower than one. Rather than implying that Home ends up with a lower wage and a higher interest rate than Foreign, the equilibrium in those situations is one in which both wages and interest rates are equalized across countries, and these factor prices correspond to those that would apply in a fictional *integrated equilibrium* of a world economy in which capital and labor are freely mobile across countries (cf., Dixit and Norman, 1980). As I discuss below, the resulting pattern of production and commodity trade is generally indeterminate, except for the special case in which there are only two stages of production. Yet, the pattern of trade is such that the patient country ends up specializing disproportionately in capital-intensive stages, in a sense to be formalized, regardless of the time intensity of the various stages. The pattern of specialization

[8] More precisely, when population levels differ across countries, equation (31) becomes

$$\frac{w^H}{w^F} = \frac{L_F \sum\limits_{n\in\mathcal{N}^H} \alpha_n\beta_n e^{-\sum\limits_{m=n}^{N}\alpha_m\zeta_m}}{L_H \sum\limits_{n\in\mathcal{N}^F} \alpha_n\beta_n e^{-\sum\limits_{m=n}^{N}\alpha_m\zeta_m}} \equiv B\left(v^*\right).$$

Figure 5: Equilibrium with Factor Price Equalization

(a) Two-Stage ($N = 2$) Case **(b)** Four-Stage ($N = 4$) case



Notes: The figure plots the trade equilibrium with factor price equalization. Details are provided in the main text.

can thus be quite distinct with and without factor price equalization.

Integrated Equilibrium Let us begin by describing the integrated equilibrium. In that equilibrium, all workers in the world command the same wage w^W and the interest rate is also equalized worldwide at a level r^W. The integrated equilibrium wage rate is pinned down by equation (21), with t_n^W determined by equation (6) and the interest rate r^W being implicitly defined by the following variant of equation (26):

$$\sum_{n=1}^{N} \frac{L_n^W}{L^H + L^F} \frac{e^{\sum_{m=n}^{N} \alpha_n \zeta_n} - 1}{r} = \sigma\left(r^W, \rho^H\right) \frac{L^H}{L^H + L^F} + \sigma\left(r^W, \rho^F\right) \frac{L^F}{L^H + L^F},$$

with the share of world labor allocated to sector n being given by

$$\frac{L_n^W}{L^H + L^F} = \frac{\alpha_n \beta_n e^{-\sum_{m=n}^{N} \alpha_m \zeta_m}}{\sum_{n'=1}^{N} \alpha_{n'} \beta_{n'} e^{-\sum_{m=n'}^{N} \alpha_m \zeta_m}}.$$

With the implied solutions for w^W, r^W, and t_n^W in hand, it is straightforward to use equation (33) to compute the amount of capital the world allocates to the various stages of production or

$$\left(K_n^d\right)^W = \frac{w^W}{r^W} L_n^W \frac{1}{\alpha_n} \left(e^{\alpha_n \zeta_n} - 1\right).$$

Figure 5 provides a graphical representation of the integrated equilibrium for the case of two stages (left panel) and for the case of four stages (right panel). These graphs should be familiar to readers of Dixit and Norman (1980) or Helpman and Krugman (1985). O^H and O^F represent the origins for Home and Foreign, respectively. In the left panel, the vectors $O^H\mathbf{1}$ and $O^H\mathbf{2}$ represent world employment of capital and labor at stages $n = 1$ and $n = 2$, respectively, in the integrated economy equilibrium (that stage 1 is more capital intensive than stage 2 is immaterial). The set of factor endowments for which factor price equalization (FPE) is attained corresponds to the set of points inside the parallelogram $O^H\mathbf{1}O^F\mathbf{2}$. Point E defines a potential distribution of capital between Home and Foreign. In the graph, Home is capital abundant relative to Foreign, reflecting its higher patience. Line BB' goes through point E and has a slope of w^W/r^W. The relative income of each country is thus held fixed for all points on line BB' and inside the FPE set. Following Helpman and Krugman (1985), one can always choose units of measurement such that (i) $\|O^H\mathbf{1}\|$ and $\|O^H\mathbf{2}\|$ correspond to the levels of production of stages 1 and 2, and such that (ii) $\|O^H O^F\| = r^W K^W + w^W L^W$. With the first two normalizations, we can graphically determine the level of production of each stage carried out in each country (e.g., $\|O^H\mathbf{1}^H\|$ and $\|O^H\mathbf{2}^H\|$, for the case of Home). Moreover, with the last normalization, we can write $s^H = \|O^H C\| / \|O^H O^F\|$, where s^H is the share of final-good consumption accruing to Home.

The case with four stages in the right-panel of Figure 5 is analogous. The vectors $O^H\mathbf{1}$, $\mathbf{12}$, $\mathbf{23}$, and $\mathbf{3}O^F$ represent the allocation of capital and labor to the four stages (sorted by capital intensity) in the integrated equilibrium. The FPE set now consists of the set of relative factor endowments inside the parallelogram $O^H\mathbf{123}O^F\mathbf{1'2'3'}$, regardless of the positioning of the four stages in value chains. Again, if countries feature sufficiently similar levels of impatience, the equilibrium will indeed be one with factor price equalization.[9] Given factor price equalization, point C in the right panel identifies the share $s^H = \|O^H C\| / \|O^H O^F\|$ of final-good spending corresponding to Home consumers.

Pattern of Production and Trade The pattern of production in an equilibrium with factor price equalization can be studied by invoking the factor-market clearing conditions. In particular, the capital-market clearing condition in each country imposes

$$\frac{w^W}{r^W} \sum_{n \in \mathcal{N}^j} L_n^j \frac{1}{\alpha_n} \left(e^{\alpha_n \zeta_n} - 1 \right) = (K^s)^j , \quad \text{for } j = H, F,$$

[9]The fact that impatience pins down differences in aggregate capital-labor ratios across countries for common factor prices is evident from the last equality in equation (26).

where w^W, r^W, and $(K^s)^j$ are the integrated equilibrium wage rate, interest rate, and country j's capital supply. Meanwhile, labor-market clearing imposes

$$\sum_{n \in \mathcal{N}^j} L_n^j = L^j, \quad \text{for } j = H, F.$$

This defines a system of 2×2 equations in $2 \times N$ unknowns, which can deliver a unique solution only when $N = 2$. This is illustrated in the left-panel of Figure 5, where $\|O^H \mathbf{1}^H\|$ and $\|O^H \mathbf{2}^H\|$ are indeed the unique volumes of production of stage 1 and stage 2 that are consistent with the integrated equilibrium factor prices and with factor markets clearing at Home. It is graphically clear that in the case $N = 2$, the pattern of specialization is such that the relatively capital-abundant country (Home) is a net exporter of the capital-intensive stage 1.[10]

In sum, for $N = 2$, the pattern of trade is determinate, and it is governed by the interaction of relative factor intensity and relative factor abundance, as in the Heckscher-Ohlin model. If the vector of value-added intensity parameters α_n is negatively correlated with the vector of time intensity parameters ζ_n, the pattern of comparative advantage and trade can thus be quite distinct from the case with no FPE, where time intensity is the *only* aspect that mattered for the pattern of trade. The reason for this is that, in a world of FPE, all countries will face the same interest rate and will thus choose the same production length for all stages. As a result, differences in time intensity will not generate endogenous labor productivity differences across countries, and thus time intensity differences across stages do not generate cross-stage differences in relative production costs across countries (see the next subsection for more on this).

As the right panel of Figure 5 indicates, the case in which the number of stages is higher than the number of countries is more intricate because the pattern of production, and thus the pattern of trade, is indeterminate. In the figure, Home exports goods 1, 2, and 3, while Foreign only exports good 4, but there are alternative ways to clear factor markets that involve quite distinct trade patterns. As in the canonical Heckscher-Ohlin model, however, the pattern of trade is still shaped by differences in relative capital endowments in the sense that, no matter what the production and trade vectors are, the relatively capital-abundant country is necessarily a net exporter of capital services embodied in the traded goods, while the relatively capital-scarce country is a net exporter of labor services embodied in the traded goods. These net factor flows are represented by the difference between points E and C in the two panels of Figure 5. In sum, and as in the case $N = 2$, variation in time intensity has no effect *per se* on the pattern of commodity trade or on the net factor content of trade whenever FPE is attained.

[10]Note that at the integrated equilibrium factor prices, s^H corresponds to the share of any stage's production (not just final-good production) that is ultimately consumed at Home.

3.4 Link to the Heckscher-Ohlin Model

As I have mentioned in the last section, whenever trade brings about factor price equalization, my model seemingly collapses to a standard two-factor Heckscher-Ohlin model. More specifically, note that we can write capital intensity in sector n using equation (32) as

$$\frac{K_n^j}{L_n^j} = \frac{\lambda_n}{1-\lambda_n}\frac{w^j}{r^j}, \tag{35}$$

where

$$\lambda_n = \frac{e^{\alpha_n \zeta_n} - 1}{\alpha_n + e^{\alpha_n \zeta_n} - 1}. \tag{36}$$

It would thus appear that the FPE equilibrium of the model is isomorphic to the one obtained in a two-sector Heckscher-Ohlin model with Cobb-Douglas technologies in both sectors. Indeed, the allocation of labor and capital to the two stages is identical in both models. The two models are however *not* isomorphic because labor productivity is endogenous in my model and shaped by the integrated economy's interest rate. Having said this, because productivity levels are common across countries when facing a common interest rate, these productivity levels have no bearing for the pattern of comparative advantage or for the allocation of factors of production to countries whenever trade brings about FPE.

Whenever FPE does not attain, differences in interest rates generate endogenous differences in labor productivity across countries, and it is intuitive that this might have an effect on the set of determinants of comparative advantage. It is however much less obvious why the time intensity parameters ζ_n become the *unique* determinants of the pattern of specialization outside the FPE set. In other words, a general Heckscher-Ohlin model of international trade with labor productivity differences across countries and sectors would still generate a role for the interaction of factor intensity and factor abundance in shaping comparative advantage. Why is this not a feature of the current model?

The key is that capital intensity is shaped by different forces in my model relative to a neoclassical model. To see this, consider the zero-profit condition associated with stage n, which we can write as

$$p_n^j = a_{Kn}^j r^j + a_{Ln}^j w^j + a_{n-1,n}^j p_{n-1},$$

where a_{Kn}^j is the unit capital requirement at stage n in country j, and a_{Ln}^j and $a_{n-1,n}^j$ are the analogous unit labor and unit prior-stage input requirements. Using either equation (32) or equation (35), together with $\alpha_n p_{n-1} y_{n-1} = (1-\alpha_n)\, wL_n$, we can write the relative price at Home at stage n in both the Heckscher-Ohlin model and my model as

$$\frac{p_n^H}{p_n^F} = \frac{a_{Ln}^H}{a_{Ln}^F}\frac{w^H}{w^F}.$$

In a FPE equilibrium, countries share the same factor prices (w^j, r^j) and thus the same technologies (a^j_{Ln}), and thus all these prices are common across countries. But outside the FPE set, these prices will differ across countries. More precisely, in a Heckscher-Ohlin model with Cobb-Douglas technologies as implied by equation (35) and with $\alpha_n p_{n-1} y_{n-1} = (1-\alpha_n) wL_n$, we would have

$$a^j_{Ln} = \frac{\kappa_n}{w^j} \left(r^j\right)^{\frac{\alpha_n \lambda_n}{1-\lambda_n(1-\alpha_n)}} \left(w^j\right)^{\frac{\alpha_n(1-\lambda_n)}{1-\lambda_n(1-\alpha_n)}} \left(p_{n-1}\right)^{\frac{(1-\alpha_n)(1-\lambda_n)}{1-\lambda_n(1-\alpha_n)}}$$

for some constant κ_n.[11] Thus, the ratio of Home and Foreign prices at stage n would be

$$\frac{p^H_n}{p^F_n} = \frac{a^H_{Ln}}{a^F_{Ln}} \frac{w^H}{w^F} = \left(\left(\frac{w^H}{w^F}\right)^{1-\lambda_n} \left(\frac{r^H}{r^F}\right)^{\lambda_n} \right)^{\frac{\alpha_n}{1-\lambda_n(1-\alpha_n)}}.$$

This equation indicates that whether p^H_n/p^F_n is higher or lower than one, i.e., whether Home is a net importer or a net exporter of stage n output in this Heckscher-Ohlin model, is shaped by capital intensity $\lambda_n/(1-\lambda_n)$ even outside the FPE set.[12]

In my model, unit labor requirements are *not* shaped by wages and interest rates in a manner governed by the smooth isoquants associated with a Cobb-Douglas cost function. The demand for capital is instead tightly linked to wage payments and to cumulative interest, and the resulting unit labor requirements are given by

$$a^H_{Ln} = \frac{\iota_n}{w^j} \left(w^j \left(r^j\right)^{\zeta_n} \right)^{\alpha_n} \left(p_{n-1}\right)^{1-\alpha_n}$$

for some constant ι_n, and the ratio of Home and Foreign prices at stage n is

$$\frac{p^H_n}{p^F_n} = \frac{a^H_{Ln}}{a^F_{Ln}} \frac{w^H}{w^F} = \left(\frac{w^H \left(r^H\right)^{\zeta_n}}{w^F \left(r^F\right)^{\zeta_n}} \right)^{\alpha_n},$$

as already pointed out in equation (29). Whether p^H_n/p^F_n is higher or lower than one in the different stages is thus shaped *solely* by factor price differences and by the vector of time intensities ζ_n, independently of capital intensity λ_n (which shapes capital intensity together with ζ_n). This results illustrates a key difference between Böhm-Bawerk's notion of capital as a factor that allows production processes to become more roundabout while still being able to compensate labor before production is completed, and John Bates Clark's notion of capital as a physical stock that can smoothly substitute for labor in production.

Naturally, the vectors ζ_n and λ_n are not orthogonal to each other as the parameter λ_n in

[11] To derive this equation, note that (35) and $\alpha_n p_{n-1} y_{n-1} = (1-\alpha_n) wL_n$ lead to a Cobb-Douglas cost function with exponents ϕ_K, ϕ_L, and $\phi_{y_{n-1}}$ satisfying $\phi_K/\phi_L = \lambda_n/(1-\lambda_n)$ and $\phi_{y_{n-1}}/\phi_L = (1-\alpha_n)/\alpha_n$.

[12] More specifically, if $w^H > w^F$ and $r^H < r^F$, then Home $p^H_n/p^F_n < 1$ in stages with a high enough λ_n and $p^H_n/p^F_n > 1$ in stages with a low enough λ_n.

equation (36) is increasing in ζ_n. But depending on how value-added intensity varies along the chain, these two vectors could be far from being perfectly correlated.

4 Costly Trade

In this section, I relax the assumption that international trade does not entail trade costs. I consider first the typical formulation of trade costs as being 'instantaneous' iceberg costs proportional to the value of the good being transacted. I show that these instantaneous trade costs generate nontrivial effects on the pattern of comparative advantage, in line with insights from the work of Harms et al. (2012), Baldwin and Venables (2013), and Antràs and de Gortari (2020). I next leverage the explicit treatment of time in my model to study the effect of trade costs associated with the time it takes to ship goods across borders, and show how world interest rates shape the pattern of specialization.

4.1 Standard Iceberg Trade Costs

To introduce costly trade in the simplest possible way, I begin by assuming that in shipping between Home and Foreign a fraction of goods melts in transit, so τ units of a good need to be shipped for 1 unit to make it to the other country. For the time being, I ignore the temporal dimension of shipping and assume goods are received in the same instant they are shipped, as with digital trade.

If one were to ignore the sequentiality of production, treating each stage as an independent production process, these iceberg trade costs would create a range of nontraded goods, as in the classical model of Dornbusch et al. (1977). More specifically, if we denote by c_n^j the cost of production of 'good' n in country j, Home would export good n to Foreign only if $\tau c_n^H < c_n^F$, while Foreign would export that good to Home only if $c_n^H > \tau c_n^F$. As a result, a range of goods featuring small production cost differences across countries would become nontraded. More precisely, goods n satisfying

$$\tau \geqslant \max\left\{\frac{c_n^H}{c_n^F}, \frac{c_n^F}{c_n^H}\right\}$$

would not be traded. Naturally, the lower are trade costs, the larger would be the set of goods traded and the larger the volume of international trade.

In the current model with sequential production, the effects of introducing trade costs are much more complex. First of all, and as formally shown in Antràs and de Gortari (2020), with trade costs it is no longer possible to determine comparative advantage stage by stage. More specifically, the choice at stage n between purchasing stage $n-1$ from Home or Foreign is not independent of where producers at stage $n-1$ source the stage $n-2$ input. Determining the cost minimization production path to sell final goods in a given country (Home or Foreign) involves solving an optimal *path* problem. More formally, iterating the cost function dual to the stage production functions in equation (1), we can write the problem of a *lead firm* choosing the location of production of all stages $n \in N$ to minimize the overall cost of serving consumers in a given country $i \in \{H, F\}$ as

$$\boldsymbol{\ell}^i = \arg\min_{\boldsymbol{\ell} \in 2^N} \left\{ \prod_{n=1}^{N} \left(c_n^{\ell(n)}\right)^{\alpha_n \beta_n} \times \prod_{n=1}^{N-1} \left(\tau_{\ell(n)\ell(n+1)}\right)^{\beta_n} \times \tau_{\ell(N)i} \right\}, \tag{37}$$

where $c_n^{\ell(n)}$ is the cost of production (inclusive of interest payments) of stage n in location $\ell(n)$, where β_n was defined in equation (12), and where $\tau_{\ell(n)\ell(n+1)} = \tau$ when $\ell(n) \neq \ell(n+1)$ and $\tau_{\ell(n)\ell(n+1)} = 1$ when $\ell(n) = \ell(n+1)$.

In the absence of trade costs, the problem above reduces to minimizing production costs stage by stage, or choosing $\ell^i(n) = \arg\min \left\{c_n^{\ell(n)}\right\}$ regardless of the destination i of final consumption. With trade costs, however, the location $\ell^i(n)$ minimizing $c_n^{\ell(n)}$ might not be part of a firm's optimal path. More specifically, and as emphasized by Harms et al. (2012) and Baldwin and Venables (2013), the presence of trade costs may lead to the *bunching* (or co-location) of two or more contiguous production stages in the same location, even if some of those stages could be produced more cheaply in the other country. Intuitively, facing a trade-off between selecting the cheapest location for an individual step and incurring additional transport costs, firms may tend to lump together several parts of the production chain in one country, even if this implies that some steps are not performed at the cheapest location. Harms et al. (2012) and Baldwin and Venables (2013) develop very stylized examples that illustrate the rich comparative advantage patterns that emerge depending on different levels of trade costs.[13]

I next offer an alternative formal derivation of the benefits of bunching contiguous production stages in the presence of trade costs. To do so, I build on a dynamic programming formulation of the path problem (37). As Antràs and de Gortari (2020) and Tyazhelnikov (2022) show, as long as technologies feature constant returns to scale, the *lead firm* can break the problem into a series of stage- and country-specific optimal sourcing problems, and then solve the problem via forward induction (starting in the most upstream stage).

More specifically, consider the (sub)problem of a producer at any stage $n+1$ choosing the optimal source of stage n inputs. In our baseline model with free trade, this producer would simply choose a supplier at Home if $w^F \left(r^F\right)^{\zeta_n} > w^H \left(r^H\right)^{\zeta_n}$ and a supplier in Foreign

[13] A second noteworthy aspect of the minimand in equation (37), also pointed out by Antràs and de Gortari (2020), is that the trade-cost elasticity of the unit cost of serving consumers in country j increases along the value chain due to the compounding effect of trade costs along the chain. An implication of this compounding effect, in a multi-country environment, is that relatively central countries gain comparative advantage in relatively downstream stages of production.

if $w^F\left(r^F\right)^{\zeta_n} < w^H\left(r^H\right)^{\zeta_n}$, as implied by equation (29). In the presence of iceberg trade costs, this choice will obviously be a function of trade barriers, but one also needs to take into account whether these trade barriers might affect the cost of intermediate inputs as well as the optimal length of production, and thus labor productivity and production costs.

My first result is that, perhaps surprisingly, the level of (iceberg) trade costs has *no* bearing on the length of the production process at any stage. To see this, consider the problem of producers at stage n in country $j \in \{H, F\}$, who set the time length t_n^j to maximize their profits,

$$\pi_n^j = \sum_{j' \in \{H,F\}} \left[p_n^{jj'} \left(z_n\left(t_n^j\right) L_n^{jj'} \right)^{\alpha_n} \left(y_{n-1}^{jj'} \right)^{1-\alpha_n} e^{-r^j t_n^j} - w^j L_n^{jj'} - p_{n-1}^j y_{n-1}^{jj'} \right], \tag{38}$$

which are derived from equation (5) and are the sum of revenue minus costs associated with sales in each market. Regardless of trade costs, which shape the wedge between domestic (p_n^{jj}) and export prices $(p_n^{jj'})$, the optimal production length will satisfy

$$\alpha_n \frac{z_n'\left(t_n^j\right)}{z_n\left(t_n^j\right)} = r^j,$$

just as in the case without trade costs. The intuition for this result was anticipated in our closed-economy analysis in section 2. Although the marginal benefit of letting production mature increases proportionately with the scale of production, the marginal cost of delaying the collection of revenue also grows proportionately with that scale, as wage payments are also proportional to scale.

When the function $z_n(t)$ is log-linear, it follows from the previous discussion that the cost of production for a producer from country j at stage n is given by

$$c_n^j = \kappa_n \left(\frac{w^j \left(r^j\right)^{\zeta_n}}{\left(\alpha_n \zeta_n\right)^{\zeta_n}} \right)^{\alpha_n} \left(p_{n-1}^j\right)^{1-\alpha_n} e^{\alpha_n \zeta_n}, \tag{39}$$

for some constant κ_n. Thus, a producer in Foreign at stage $n+1$ would choose a Foreign supplier at n whenever

$$\tau \left(w^H \left(r^H\right)^{\zeta_n} \right)^{\alpha_n} \left(p_{n-1}^H\right)^{1-\alpha_n} > \left(w^F \left(r^F\right)^{\zeta_n} \right)^{\alpha_n} \left(p_{n-1}^F\right)^{1-\alpha_n}. \tag{40}$$

This expression differs from the free-trade comparison of $w^H\left(r^H\right)^{\zeta_n}$ and $w^F\left(r^F\right)^{\zeta_n}$ in two respects. First, the left-hand-side includes the iceberg trade costs associated with the Foreign buyer at $n+1$ importing inputs from Home. Naturally, this force produces a direct benefit of co-locating stages $n+1$ and n in the same country (Foreign in this case). A second new aspect of the above inequality (40) is that the price paid for inputs at $n-1$ may also differ depending on the choice of location of stage n inputs, due again to trade costs. As a result, the producer at $n+1$ in Foreign might favor sourcing from Home at n if the price of $n-1$ inputs is significantly lower for Home producers at n than for Foreign producers at n, perhaps because Home has strong cost advantage at stage $n-1$. Notice that this possibility would also lead to some 'bunching' at Home of stages n and $n-1$. But leaving this aside, I next show that a producer at $n+1$ in Foreign will *necessarily* be more likely to choose a supplier from Foreign at n than as implied by a simple comparison of $w^H\left(r^H\right)^{\zeta_n}$ and $w^F\left(r^F\right)^{\zeta_n}$ in the absence of trade costs.

To show this formally, consider the prices p_{n-1}^H and p_{n-1}^F that partly shape the choice at n, as indicated by inequality (40). Given our discussion above, it should be clear that these prices satisfy:

$$\begin{aligned} p_{n-1}^H &= \min\left\{\left(w^H\left(r^H\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}}\left(p_{n-2}^H\right)^{1-\alpha_{n-1}}, \tau\left(w^F\left(r^F\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}}\left(p_{n-2}^F\right)^{1-\alpha_{n-1}}\right\}; \\ p_{n-1}^F &= \min\left\{\tau\left(w^H\left(r^H\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}}\left(p_{n-2}^H\right)^{1-\alpha_{n-1}}, \left(w^F\left(r^F\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}}\left(p_{n-2}^F\right)^{1-\alpha_{n-1}}\right\}, \end{aligned}$$

where p_{n-2}^H is the price of stage $n-2$ faced by producers in H of stage $n-1$, and p_{n-2}^F is the analogous price for producers in F of stage $n-1$ output. It is then relatively straightforward to show (see Online Appendix A.6) that the ratio p_{n-1}^H/p_{n-1}^F satisfies

$$\frac{p_{n-1}^H}{p_{n-1}^F} \geqslant \frac{1}{\tau},$$

which can be interpreted as a no-arbitrage condition in the sourcing of inputs at stage $n-1$. Furthermore, this inequality is strict unless both H and F producers source from H at $n-2$. This inequality then implies that a sufficient but *not necessary* condition for (40) to hold is

$$w^H\left(r^H\right)^{\zeta_n} > w^F\left(r^F\right)^{\zeta_n}.$$

In other words, Foreign producers at stage $n+1$ may choose F as a source of stage n inputs even when $w^F\left(r^F\right)^{\zeta_n} > w^H\left(r^H\right)^{\zeta_n}$, which indicates a disproportionate desire to bunch contiguous stages in the same location.

This disproportionate desire to source inputs from a local supplier applies to all stages $n > 1$. But it also applies to the case $n = 1$, in which case, the condition in (40) reduces to $\tau w^H\left(r^H\right)^{\zeta_1} > w^F\left(r^F\right)^{\zeta_1}$, which clearly also favors local sourcing. Following a completely analogous set of steps (exploiting $\tau \geqslant p_{n-1}^H/p_{n-1}^F$), one can similarly demonstrate that Home suppliers at $n+1$ are also disproportionately more likely to choose Home suppliers at n than as implied by the simple condition $w^H\left(r^H\right)^{\zeta_n} < w^F\left(r^F\right)^{\zeta_n}$ emanating from our baseline model with free trade.

The above discussion shows that, for given factor prices, in a costly-trade equilibrium, producers have a disproportionate incentive to use local versus foreign intermediate inputs in their production, which translates into lower levels of foreign value added in exports. An important caveat, however, is that this result is partial equilibrium in nature. In general equilibrium, changes in trade costs will not only lead to more 'home bias' in sourcing, but it will also affect factor prices. The manner in which factor prices change when trade costs increase depends in subtle ways on the pairwise correlations between upstreamness, time intensity, and capital intensity, as hinted by equations (31) and (34) in the free-trade equilibrium. I relegate a description of the equations characterizing this general equilibrium with costly trade to Online Appendix A.6, where I also discuss under which circumstances these general equilibrium effects are likely to be modest.

4.2 Time as a Trade Cost

An advantage of having developed a framework with an explicit time value of money is that we can apply the same machinery to introduce a temporal dimension of trade costs. Specifically, I now assume that, relative to domestic transactions, shipping goods across borders involves an additional interval of time d between the time at which production is completed and the time at which the shipment is received and payment is made. In terms of standard terminology in trade finance, I thus assume that all international transactions are carried out on 'open account' terms (i.e., involving post-shipment payments), but I will explore alternative trade finance arrangements in the next section. I also continue to assume that shipping involves iceberg trade costs τ (shipping fees, insurance, etc.) incurred after production but before the shipment arrives at the importer's country.

As in the case of iceberg trade costs, the presence of temporal trade costs turns out to be immaterial for the length of production processes.[14] As a result, the cost of production c_n^j for producers at stage n in country j is still given by (39), while the minimum price at which these

[14]In particular, going back to equation (38), the production length t_n^j that maximizes

$$\pi_n^j = \sum_{j'=\{H,F\}} \left[p_n^{jj'} \left(z_n \left(t_n^j \right) L_n^{jj'} \right)^{\alpha_n} \left(y_{n-1}^{jj'} \right)^{1-\alpha_n} e^{-r^j \left(t_n^j + d^{jj'} \right)} - w^j L_n^{jj'} - p_{n-1}^j y_{n-1}^{jj'} \right]$$

is still given by

$$\alpha_n \frac{z_n' \left(t_n^j \right)}{z_n \left(t_n^j \right)} = r^j,$$

regardless of the shipping time $d^{jj'} = d$ if $j \neq j'$ (and $d^{jj'} = 0$ if $j = j'$).

goods can be sold in country j' is

$$p_n^{jj'} = \tau^{jj'} c_n^j = \begin{cases} \tau e^{r^j \delta} c_n^j & \text{if } j' \neq j \\ c_n^j & \text{if } j' = j \end{cases} .$$

Some of the implications of these temporal trade costs are analogous to those resulting from iceberg trade costs. Naturally, shipping delays are relatively more costly for downstream stages than for upstream stages (as in Antràs and de Gortari, 2020), and high shipping times lead to more bunching (or co-location of contiguous stages) than implied by the model without these international trade costs. Nevertheless, the above modeling of trade costs as an interval of time between production and delivery generates at least two additional implications.

First, as long as the equilibrium is one without factor price equalization, we necessarily have that $\tau^{HF} < \tau^{FH}$, and thus trade costs are asymmetric and relatively lower in relatively capital-abundant, low interest rate countries. This implication is in line with the results in Waugh (2010), who shows that to reconcile bilateral trade volumes and price data within a standard gravity model, trade frictions between rich and poor countries must be systematically asymmetric, with poor countries facing higher costs to export relative to rich countries. Furthermore, Antràs (2015) shows that these asymmetries are not only correlated with income per capita differences, but also with measures of the quality of contracting institutions, which tend to be negatively correlated with interest rates across countries (see Djankov et al., 2007).

Second, because trade costs are now partly shaped by the time value of money, reductions in interest rates worldwide tend to reduce trade costs. From our results above on the link between iceberg trade costs and GVC participation, we can thus conclude that proportional reductions in interest rates that leave relative wages unchanged (see the discussion in Online Appendix A.6) will necessarily lead to an increase in the extent to which goods cross borders, hence generating a higher volume of GVC participation. As a result, and as anticipated in the Introduction, one potential contributor to the growth in GVC activity in the late 1980s, 1990s and 2000s is the secular decline in interest rates during that period.

5 Imperfect Capital Markets and Capital Mobility

The analysis in the previous sections assumed that capital markets are perfectly integrated and efficient within countries (where producers in country j can borrow at a competitive interest rate r^j), while they are completely segmented across borders (since I have not allowed for international borrowing and lending). In this section, I relax these two assumptions. First, I introduce frictions in lending in the form of monitoring costs, which may vary across countries, perhaps reflecting variation in financial development. Next, I allow for cross-border borrowing and lending, though I assume that this entails additional monitoring costs, which naturally limits financial integration. Finally, I consider cross-border capital flows in the form of trade credit, which I associate with borrowing and lending between producers of contiguous stages, and which I assume are subject to disproportionately low monitoring costs. For the most part, I revert to my baseline model without trade costs, but I end this section with a brief discussion of the implications of trade finance (i.e., the financing associated with the time lag between shipment and receipt in international trade transactions) in my framework.

5.1 Imperfect Capital Markets without Capital Mobility

I first relax the assumption that credit markets are frictionless within borders. Suppose collecting interest demands that lenders in country j incur monitoring costs m^j per unit of capital lent. In analogy to iceberg trade costs, I model these monitoring costs as 'melted' resources, as in Ramos-Menchelli and Van Doornik (2022). For the time being, I assume that this monitoring technology is widely available in the population, so whether these lenders are individuals, firms, or banks is immaterial. In the presence of monitoring costs, aggregate capital then evolves according to

$$\frac{d\left(K_t^s\right)^j}{dt} = w_t^j L^j + \left(r_t^j - m^j\right)\left(K_t^s\right)^j - \rho^j \left(K_t^s\right)^j ,$$

with an exogenously given initial capital stock $\left(K_0\right)^j$. In a steady state in which all variables are time invariant, we must then have

$$\left(K^s\right)^j = \frac{w^j L^j}{\rho^j + m^j - r^j} = w^j L^j \times \sigma\left(r^j, \rho^j, m^j\right).$$

As in our baseline model, the supply of capital is increasing in the interest rate r^j and is decreasing in impatience ρ^j. The main novel feature is that the supply of capital is now also decreasing in the level of monitoring costs because these monitoring costs depress the return to capital for savers.

The existence of monitoring costs does not affect the demand for capital *for a given* equilibrium interest rate r^j paid by borrowers. As a result, under autarky, interest rates are determined by the following equilibrium condition, analogous to equation (26) in the baseline model:

$$\frac{\left(K^d\right)^j}{w^j L^j} = \sum_{n=1}^{N} \frac{L_n^j}{L^j} \frac{e^{r^j\left(\sum\limits_{m=n}^{N} t_m^j\right)} - 1}{r^j} = \sigma\left(r^j, \rho^j, m^j\right) = \frac{\left(K^s\right)^j}{w^j L^j}.$$

As long as $r^j t_n^j$ is independent of or decreasing in r^j, the ratio of capital demand $\left(K^d\right)^j$ to the wage bill $w^j L^j$ is necessarily a decreasing function of the interest rate r^j, and equilibrium in the capital market is as depicted in Figure 2 with the autarkic equilibrium interest rate increasing in the impatience ρ^j of agents and now also in the monitoring costs m^j associated with country j.

Consider now the case of free trade with no capital integration across countries, which amounts to assuming that lenders face infinite monitoring costs when lending to agents abroad (I will relax this assumption below). Under these circumstances, and as long as trade integration does not equalize factor prices across countries, the country with the lower autarky interest rate will again specialize in the most time intensive stages of production, just as in the baseline model without monitoring costs. The main difference here is that the pattern of specialization is no longer shaped *only* by variation in impatience across countries, but also by cross-country variation in the level of monitoring costs. To the extent that a high level of m^j is associated with a lower degree of financial development, the novel implication that arises from this extension with monitoring costs is that, other things equal, countries that are more financially developed should specialize in relatively time intensive stages of production.

As in the baseline model, however, if differences in impatience and monitoring costs are small across countries, and if time intensity and capital intensity are sufficiently highly correlated, the free trade equilibrium will feature factor price equilibrium, and the pattern of trade in that case turns out to be governed by capital intensity rather than by time intensity.[15]

5.2 Capital Market Integration

I now consider the possibility of borrowing and lending across countries. In the absence of monitoring costs within or across countries, it is clear that arbitrage will ensure that all borrowers face the same interest rate r^W worldwide, even in the absence of integration in goods markets. Intuitively, producers in the high interest rate Foreign will borrow from savers at Home, and the supply of capital in Foreign $(K^s)^F$ and at Home $(K^s)^H$ will adjust to ensure interest rate equalization (see equation (34)). As I have shown above, this equalization of interest rates in turn implies the equalization of wage rates, and the equilibrium is akin to one with factor price equalization (given some distribution of available capital in each country). Naturally, and as in Mundell (1957), if trade integration already brings about factor price equalization, there is no incentive for capital to flow across countries.

Next consider the case in which lending is associated with monitoring costs. As before, when financing producers in j, lenders in j incur monitoring costs m^j. I assume that Home is both more patient ($\rho^H > \rho^F$) and more financially developed ($m^H < m^F$), so under autarky, we necessarily have $r^H < r^F$. I now further assume that when lending to producers in $j' \neq j$,

[15] Of course, the pattern of trade is indeterminate with $N > 2$ stages, but relatively capital-abundant countries necessarily are net exporters of capital services embodied in the traded goods.

lenders incur a monitoring cost $m^{jj'}$ in addition to the domestic cost m^j. I assume that this monitoring cost is bilaterally symmetric, so we can simply define $\mu = m^{HF} = m^{FH}$. If r^H and r^F are the prevailing interest rates without capital flows, then if $r^F < r^H + \mu$, the equilibrium will feature *no* capital flows and will continue to be as described in section 5.1.

The case in which (with trade but without capital mobility) we have $r^F > r^H + \mu$ is more interesting because in such a scenario, Foreign producers can reduce their financial costs by borrowing from Home lenders. Indeed, in that case, capital market integration will lead to $r^F = r^H + \mu$. As a result, interest rates faced by producers at Home and in Foreign will converge but will *not* equalize even with capital mobility, and the trade equilibrium will be characterized by the analysis in section 3.2 rather than by the analysis in section 3.3. The only subtle difference in equations (30)–(34) characterizing that equilibrium is that the supply of capital in Foreign $(K^s)^F$ and at Home $(K^s)^H$ are now shaped by capital mobility. Because capital flows from Home to Foreign, this will increase $(K^s)^F$ and will reduce $(K^s)^H$, and some production stages previously produced at Home may move to Foreign. The implications of these capital movements for the volume of trade or for the share of GVC trade in world trade are however ambiguous.

5.3 Trade Credit

I finally consider environments in which cross-border monitoring costs may be shaped by the identity of lenders and borrowers. More specifically, notice that in a trade equilibrium with no factor price equalization and high cross-border monitoring costs μ, there are situations along the chain in which importers in Foreign face higher interest rates than their suppliers at Home. Would these Home suppliers not have an incentive to provide credit to their Foreign buyers to reduce the latter's production costs? More specifically, Home exporters at stage n could provide the good finished up to that stage but allow the buyer to pay for this good only after stage $n+1$ has been completed and has been sold to downstream producers at stages $n+2$. This would allow Foreign producers to limit their borrowing to the wage costs incurred during production.[16] The quantitative relevance of trade credit has been well established in the literature (see, among many others, Burkart and Ellingsen, 2004, Klapper et al., 2011, or Jacobson and Von Schedvin, 2015). In fact, the literature on trade credit has emphasized that buyers and sellers may be in a better position than 'outside' lenders in monitoring each other (see Petersen and Rajan, 2015, and references therein).

With this in mind, I next consider a situation in which cross-border monitoring costs are lower (and given by $\underline{\mu}$) whenever the agent extending credit also sells goods to the producer

[16]In principle, suppliers could provide financing to also cover these wage costs, but this is rarely seen in practice and would go beyond the limits of what is typically referred to as trade credit. In any case, and for completeness, the analysis in Online Appendix A.7 also covers this case.

obtaining credit. To simplify the exposition, I consider the case in which $\underline{\mu} \to 0$, and I also focus on cases in which domestic borrowing and lending entails no monitoring costs ($m^H = m^F = 0$). These assumptions are immaterial for the main results to be derived below. Similarly, the less frequent case (see Burkart and Ellingsen, 2004) in which buyers extend credit to sellers could be studied in an analogous manner.

What are the implications of allowing for trade credit in the model? It should be clear that as long as the trade equilibrium is one with factor price equalization, there will be *no* incentive for firms to engage in trade credit, as importers and exporters will face the exact same cost of capital. For this reason, I hereafter focus on situations in which, without trade credit, differences in wages and interest rates persist even with free trade, as in section 3.2 above.

Given that I focus on situations in which $w^H > w^F$ and $r^H < r^F$, trade credit will only emerge in situations in which Foreign firms import from Home firms, who may thus be in a particularly favorable position to offer them finance at a rate $r^H < r^F$. Trade credit thereby increases the demand for Home capital relative to Foreign capital, and this in turn generates an increase in the demand of Foreign labor and a decrease in the demand of Home labor. Both of these forces foster the reduction of factor price differences across countries and expand the range of parameter values for which FPE is attained. These effects are very much in line with those generated by the broader type of capital market integration studied above, but they are smaller in magnitude in the sense that the arbitrage of interest rate differentials requires that borrowers and lenders produce in subsequent stages and are located in different countries. It is thus not hard to construct examples in which trade credit reduces but does not eliminate interest rate differences across countries, even in the absence of monitoring costs.[17]

Leaving aside these general equilibrium effects, the possibility of trade credit may have significant effects on trade patterns. More specifically, and as I demonstrate formally in Online Appendix A.7, the introduction of trade credit tends to deepen vertical specialization and results in an increase in the extent to which intermediate goods cross borders along the production process. Demonstrating this result is not straightforward because, even in the absence of trade costs, with trade credit, the pattern of specialization can no longer be elucidated stage by stage. Intuitively, whether Home or Foreign have a cost advantage in a stage may well be a function of the location of production in the prior stage whenever Home producers extend credit to their Foreign buyers. In Online Appendix A.7 I show, however, that the dynamic programming approach already developed in section 4.1 to study the implications of trade costs can be applied to the study of the implications of trade credit. There, I formally show that,

[17] For instance, suppose that there are a large number of stages N and there is a perfect correlation between upstreamness and time intensity, so Home specializes in all stages $n \leqslant \hat{n}$, and Foreign specializes in all stages $n > \hat{n}$. In such a case, trade credit is only available for Foreign producers at stage $\hat{n}+1$, who are allowed to borrow from Home producers. As a result, if there are a large number of stages, and the contribution of stage $\hat{n}$ to overall value added is small, the implied effect on equilibrium interest rates will be very modest.

relative to the baseline model, the introduction of trade credit typically decreases the probability of consecutive stages being performed in the same country (at least when holding factor prices constant). Intuitively, Foreign producers at stage n now find Home suppliers at $n-1$ relatively more appealing than without trade credit because these Home producers can now provide them with cheaper finance than they would have access to otherwise. In addition, Home producers at stage n may now find Foreign suppliers at stage $n-1$ a more appealing source of inputs whenever these foreign suppliers gain access to finance from $n-2$ Home producers via trade credit. As a result of these forces, with trade credit, firms in more stages are engaged in GVC participation, embodying foreign value added in their exports, and exporting goods value added that is in turn re-exported embodied in the receiving country's exports.[18]

5.4 Trade Finance

When introducing temporal trade costs in section 4.2, I assumed that goods were shipped on an open account basis, that is, the exporter bore the financial costs of the delayed payment associated with shipping times. Whenever the exporter faces a lower interest rate than the importer, this is indeed the efficient way to finance the shipment transaction. In situations in which the importer faces a lower cost of capital, however, it is instead more efficient for goods to be transacted on a 'cash in advance' basis (see Antràs and Foley, 2015). When exporters and importers are able to finance the transaction efficiently, it thus follows that trade costs cease to be asymmetric and are instead given by $\tau^{jj'} = \tau^{j'j} = \tau e^{\min\{r^j, r^{j'}\}\delta}$.

Holding constant wage rates and interest rates, the possibility of using trade finance to minimize the financing costs of trade transactions generates implications that constitute a combination of the effects of reductions in temporal trade costs studied in section 4.2 and those of trade credit studied in the last subsection. More specifically, in the presence of trade costs, Home producers at stage n favor Home suppliers at stage $n-1$ (relative to a baseline without trade costs), but with trade finance, this preference is diminished when Home producers at n can reduce the financing costs faced by Foreign suppliers at $n-1$ by paying for goods *before* they are shipped. In other words, the possibility of extending trade finance reduces temporal trade costs in a manner analogous to interest rate reductions, and thus makes Home more likely to source inputs from Foreign (though still less so than in the baseline with no trade costs). As a result, trade finance enhances GVC participation. In general equilibrium, trade finance tends

[18] As I show in the Online Appendix A.7, it is *always* the case that, with trade credit, two consecutive stages are less likely to be bunched together at Home than in the baseline model without trade credit. The same result applies to Foreign whenever labor intensity α_n and time intensity ζ_n do not vary significantly along the chain. When labor intensity or time intensity are particularly high at some stage n, Foreign producers at stage $n+1$ may be inclined to source from Foreign rather than Home even when $w^F \left(r^F\right)^{\zeta_n} > w^H \left(r^H\right)^{\zeta_n}$, but this can only happen when Foreign producers at n source inputs from Home producers at $n-1$. Therefore, these instances of persisting 'local bunching' are in fact ultimately associated with GVC activity upstream.

to shift labor demand toward Foreign and capital demand toward Home, thereby constituting a force toward factor price equalization, similar to the effect of trade credit studied above.

6 Conclusions

In this paper, I have developed a stylized model of sequential production with a pre-determined number stages in which the time length of each stage is endogenously determined. Lengthening production processes, or making them more 'roundabout' (Böhm-Bawerk, 1889), increases labor productivity, but it also increases the working capital needs of firms. I have shown that this 'Austrian' notion of capital generates different implications for the pattern of specialization than the Clark-Samuelson notion of capital as a physical stock. Even when countries differ in their supply of capital and sectors differ in their capital intensity, comparative advantage is shaped by time intensity (i.e., the extent to which time increases productivity) rather than by capital intensity, except for the extreme case in which trade integration leads to factor price equalization.

Building a model with an explicit notion of time featuring interest rates and working capital needs has allowed me to draw implications for the role of temporal trade costs and of trade credit in shaping specialization. I have shown that the model formalizes the intuitive idea that a period of low interest rates – such as the one witnessed in recent decades – facilitates the sustainability of longer production processes and also allows for a better allocate worldwide resources to their more efficient use, with less regard to the time it takes to combine these worldwide resources. As a result, a worldwide decline in interest rates fosters an increase in the share of GVC trade in world trade.

Although I have intentionally kept the model stylized, I am confident that the model could be extended in a variety of ways, perhaps making it more amenable to empirical investigations of the role of *time* in international trade and in other fields in Economics. Within international trade, a particularly natural extension of the model would be to incorporate scale economies and imperfect competition. These are standard features of state-of-the-art models of international trade, and they seem particularly relevant in this setting because the expansion of GVCs in the 1980s, 1990s, and early 2000s was associated with significant investments by firms, with these investments often being fixed in nature and incurred before any production abroad could occur. In Antràs (2023) I have sketched some of the implications of introducing scale economies into an 'Austrian' model, but I have done so in a framework without GVCs.

Reflecting my own comparative advantage, this paper has unapologetically focused on abstract or conceptual matters. It is my hope, however, that my work will inspire empirical work in this area. Such an endeavor will be complicated by the fact that precise measures of the notion of production length are undeniable hard to come by. As mentioned in the

Introduction, some authors have employed balance sheet data to proxy for the length of production chains (see Bruno et al., 2018), but there may be scope for more accurate measures. For some production processes, such as the making of wine or cheese, firms face nontrivial choices regarding the optimal time of letting their output mature or ferment. Similarly, in many modern manufacturing processes, firms decide on how long to conduct R&D before beginning to manufacture their goods. These decisions would naturally be shaped by interest rates in ways captured by my model, and information on those decisions could thus constitute a valuable laboratory to study the empirical relevance of the 'Austrian' approach in this paper.

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A Online Appendix

A.1 The Choice of N

In this section, I provide the formal details on the determination of the optimal number of stages in the closed-economy version of the model.

Consider first the cost minimization problem dual to (15):

$$\min_{L_n, t_n} \quad \sum_{n=1}^{N} w L_n e^{r\left(\sum_{m=n}^{N} t_m\right)}$$
$$s.t. \quad \prod_{n=1}^{N} \left(z_n\left(t_n\right) L_n\right)^{\alpha_n \beta_n} \geqslant Q.$$

We have shown in the main text that the optimal choice of t_n is the same as in the decentralized equilibrium without *lead* firms. Let us denote this solution by t_n^*. The choice of L_n is in turn associated with the following first-order condition (FOC):

$$we^{r\left(\sum_{m=n}^{N} t_m^*\right)} = \lambda \frac{\alpha_n \beta_n Q}{L_n},$$

where λ is the Lagrange multiplier associated with the constraint, which is the marginal cost c_Y we are trying to solve for. Solving out for L_n from the FOC and plugging it into the constraint, delivers after some straightforward manipulations:

$$c_Y = \lambda = \prod_{n=1}^{N} \left(\frac{we^{r\left(\sum_{m=n}^{N} t_n^*\right)}}{\alpha_n \beta_n z_n\left(t_n^*\right)} \right)^{\alpha_n \beta_n},$$

as claimed in the main text.

How is this marginal cost affected by a change in N? The answer is complex because the choice of N affects the overall chain marginal cost in a number of different ways.

First, starting from a given N, moving to $N+1$ stages will imply a direct increase in marginal costs associate with the costs of paying the factors used in that stage. In terms of the above expression, it would imply the inclusion of a term

$$\left(\frac{we^{rt_{N+1}^*}}{\alpha_{N+1} \beta_{N+1} z_{N+1}\left(t_{N+1}^*\right)} \right)^{\alpha_{N+1} \beta_{N+1}},$$

which would naturally increase marginal costs.

Second, an increase from N to $N+1$ will also affect the cost of the prior stages by not only delaying the collection of final revenue but also by affecting the exponents $\alpha_n \beta_n$ if one wants to maintain the assumption of constant returns to scale. More specifically, the term in the marginal cost function c_Y associated with stage n now becomes

$$\left(\frac{w}{\alpha_n \beta_n' z_n\left(t_n^*\right)} \right)^{\alpha_n \beta_n'},$$

where $\beta_n' = \left(1 - \alpha_{N+1}\right) \beta_n$. Overall, we can write

$$c_Y' = \left(c_Y \frac{e^{rt_{N+1}^*}}{1 - \alpha_{N+1}} \right)^{1 - \alpha_{N+1}} \left(\frac{we^{rt_{N+1}^*}}{\alpha_{N+1} z_{N+1}\left(t_{N+1}^*\right)} \right)^{\alpha_{N+1}}$$

Overall, the balance of these effects is ambiguous. Furthermore, even if we were to be able to establish that $c_Y' > c_Y$, and thus that adding stage $N+1$ does not reduce costs, this still leaves the possibility that adding stage $N+2$ reduces costs by so much that adding both $N+1$ and $N+2$ is beneficial.

To further elucidate these effects, and to see whether our framework is likely to generate a clean link between interest rates and the length of production as captured by the number of stages N, I next develop a special symmetric case. Consider then a case in which the labor intensity parameters satisfy $\alpha_n = 1/n$, while stage-specific labor productivity is governed by $z_n(t_n) = \chi_n (t_n)^{\zeta n}$ with $\chi_n = \bar{z}/(\zeta/r)^{\zeta n}$. I refer to this case as symmetric because it has the implication that $\alpha_n \beta_n = 1/N$, $t_n^* = t^* = \zeta/r$, and $z_n(t_n) = \bar{z}$ for all n. In that case, equation (18) simplifies as follows:

$$\begin{aligned} c_Y &= \prod_{n=1}^{N} \left(\frac{w e^{r\left(\sum_{m=n}^{N} t_n^*\right)}}{\alpha_n \beta_n z_n(t_n^*)} \right)^{\alpha_n \beta_n} \\ &= \prod_{n=1}^{N} \left(\frac{w}{\bar{z}} N \right)^{1/N} \prod_{n=1}^{N} \left(e^{r\left(\sum_{m=n}^{N} \zeta/r\right)} \right)^{1/N} \\ &= \frac{w}{\bar{z}} N \prod_{n=1}^{N} \left(e^{\zeta(N-n+1)} \right)^{1/N} \\ &= \frac{w}{\bar{z}} N e^{\zeta \frac{(N+1)}{2}}. \end{aligned}$$

It is then clear that the marginal cost c_Y is increasing in N. Thus, the framework cannot justify why firms would ever set $N > 1$.

To engineer a nontrivial choice of N, consider a simple modification of the marginal cost function (or of the production technology) to incorporate a 'love-for-variety' term $N^{-(1+\gamma)}$, with $\gamma > 0$, as in Benassy (1996). We then have

$$c_Y = \frac{w}{\bar{z}} N^{-\gamma} e^{\zeta \frac{(N+1)}{2}},$$

which is minimized (the second-order condition is satisfied for $\gamma > 0$) whenever the number of stages is given by $N^* = 2\gamma/\zeta$, as stated in the main text. More generally, as long as any (unmodeled) love-for-variety benefits from a larger number of stages are independent of the interest rate, in this symmetric example, the optimal choice of N will also be independent of the interest rate.

A.2 The Effect of the Interest Rate on the Wage Rate

In this Appendix, I show that the closed-economy wage w is decreasing in the interest rate r. Take equation (21) determining the wage, which I reproduce below for convenience:

$$w = \prod_{n=1}^{N} \left(z_n(t_n^*) e^{-r \sum_{m=n}^{N} t_m^*} \right)^{\alpha_n \beta_n}.$$

Consider first the effect of an increase in the interest rate holding the vector of production lengths t_n^* constant. Clearly, this effect is negative. I next show that the remaining effect of r on wages working through its effect on production lengths is negligible due to the envelope theorem.

To see this, consider the effect of r on a given time length t_n^*. From the wage equation, this will only impact the wage rate via changes in $z_n(t_n^*)$ and via changes in capital costs $e^{-r\sum\limits_{m=n}^{N} t_m^*}$ that are a function of t_n^*, which can only be capital costs related to stages n and lower. The relevant terms affected by t_n^* can be written as

$$\left(z_n(t_n^*)\, e^{-r\sum\limits_{m=n}^{N} t_m^*}\right)^{\alpha_n\beta_n} \times \left(e^{-r\sum\limits_{m=n-1}^{N} t_m^*}\right)^{\alpha_{n-1}\beta_{n-1}} \times \ldots \times \left(e^{-r\sum\limits_{m=1}^{N} t_m^*}\right)^{\alpha_1\beta_1}.$$

Differentiating the above expression with respect to t_n^*, we obtain

$$\left[\alpha_n\beta_n \frac{z_n'(t_n^*)}{z_n(t_n^*)} - r\sum\nolimits_{m=1}^{n} \alpha_n\beta_n\right]$$
$$\times \left(z_n(t_n^*)\, e^{-r\sum\limits_{m=n}^{N} t_m^*}\right)^{\alpha_n\beta_n} \left(e^{-r\sum\limits_{m=n-1}^{N} t_m^*}\right)^{\alpha_{n-1}\beta_{n-1}} \ldots^{\alpha_n\beta_n} \left(e^{-r\sum\limits_{m=1}^{N} t_m^*}\right)^{\alpha_1\beta_1}.$$

But from the definition of β_n in (12), we can derive $\beta_n = \sum_{m=1}^{n} \alpha_n\beta_n$, and thus the formula for the optimal production length t_n^* in equation (6) implies that this whole indirect effect is negligible.

A.3 The Effect of the Interest Rate on Final-Good Output

As mentioned in the main text, given that

$$y_N = \prod_{n=1}^{N} \left(z_n(t_n)\, L_n\right)^{\alpha_n\beta_n},$$

it is clear that as long as the allocation of labor across sectors is independent of the interest rate, y_N is necessarily decreasing in r on account of the positive effect of a lower interest rate on the production lengths t_n^*. Furthermore, from equation (20), L_n will indeed be independent of r whenever $e^{rt_n^*}$ is independent of r for all n, which in turn is implied by a specification of isoleastic functions $z_n(t_n) = (t_n)^{\zeta_n}$.

I will now consider deviations from this log-linear case. Consider first the 'elastic' case in which all functions $z_n(t_n)$ are such that rt_n^* (and thus $e^{rt_n^*}$) decreases in r for all n. Remember that this 'elastic' case corresponds to the scenario in which a reduction in the interest rate shifts labor from upstream to downstream stages (see equation (14)). Thus, a reduction in r must necessarily increase the allocation of labor to (at least) the most downstream stage N. Now recall from equation (11) that for $n = N$, we have

$$wL_N e^{rt_N^*} = \alpha_N\beta_N y_N. \tag{A.1}$$

As shown above in Appendix A.2, w increases when r goes down, and I have just argued that L_N goes up as well when r falls. Finally, the last term $e^{rt_N^*}$ on the left-hand-side of (A.1) also goes up when r goes down because $e^{rt_N^*}$ decreases in r for all n. It then follows from this expression that y_N must increase too when the interest rate r falls.

Now consider the 'inelastic' case in which rt_n^* (and thus $e^{rt_n^*}$) increases in r for all n. Building again on equation (A.1), we can plug in (21) and (20) to obtain

$$wL_N e^{rt_N^*} = \prod_{n=1}^{N} \left(z_n \left(t_n^*\right) e^{-r \sum\limits_{m=n}^{N} t_m^*} \right)^{\alpha_n \beta_n} \frac{\alpha_N \beta_N}{\sum\limits_{n'=1}^{N} \alpha_{n'} \beta_{n'} e^{-r \sum\limits_{m=n'}^{N} t_m^*}} L.$$

Differentiating with respect to r, we obtain

$$\frac{\partial \left(wL_N e^{rt_N^*} \right)}{\partial r} = \frac{\alpha_N \beta_N}{\left(\sum\limits_{n'=1}^{N} \alpha_{n'} \beta_{n'} e^{-r \sum\limits_{m=n'}^{N} t_m^*} \right)^2} L \times \Gamma,$$

where

$$\Gamma = \frac{\partial \left(\prod\limits_{n=1}^{N} \left(z_n \left(t_n^*\right) e^{-r \sum\limits_{m=n}^{N} t_m^*} \right)^{\alpha_n \beta_n} \right)}{\partial r} \times \sum_{n'=1}^{N} \alpha_{n'} \beta_{n'} e^{-r \sum\limits_{m=n'}^{N} t_m^*} - \prod_{n=1}^{N} \left(z_n \left(t_n^*\right) e^{-r \sum\limits_{m=n}^{N} t_m^*} \right)^{\alpha_n \beta_n} \times \frac{\partial \left(\sum\limits_{n=1}^{N} \alpha_n \beta_n e^{-r \sum\limits_{m=n}^{N} t_m^*} \right)}{\partial r}$$

Evaluating the two derivatives in the above expression, while invoking the envelope theorem as in the proof developed in Appendix A.2, we obtain

$$\frac{\partial \left(\prod\limits_{n=1}^{N} \left(z_n \left(t_n^*\right) e^{-r \sum\limits_{m=n}^{N} t_m^*} \right)^{\alpha_n \beta_n} \right)}{\partial r} = - \prod_{n=1}^{N} \left(z_n \left(t_n^*\right) e^{-r \sum\limits_{m=n}^{N} t_m^*} \right)^{\alpha_n \beta_n} \sum_{n=1}^{N} \frac{\alpha_n \beta_n}{z_n \left(t_n^*\right) e^{-r \sum\limits_{m=n}^{N} t_m^*}} z_n \left(t_n^*\right) e^{-r \sum\limits_{m=n}^{N} t_m^*} \sum_{m=n}^{N} t_m^*$$

and

$$\frac{\partial \left(\sum\limits_{n=1}^{N} \alpha_n \beta_n e^{-r \sum\limits_{m=n}^{N} t_m^*} \right)}{\partial r} = - \sum_{n=1}^{N} \alpha_n \beta_n e^{-r \sum\limits_{m=n}^{N} t_m^*} \left(\sum_{m=n}^{N} t_m^* + r \frac{\partial t_m^*}{\partial r} \right).$$

Plugging back into the definition of Γ delivers

$$\frac{\Gamma}{\prod\limits_{n=1}^{N} \left(z_n \left(t_n^*\right) e^{-r \sum\limits_{m=n}^{N} t_m^*} \right)^{\alpha_n \beta_n}} = - \sum_{n=1}^{N} \alpha_n \beta_n \sum_{m=n}^{N} t_m^* \times \sum_{n'=1}^{N} \alpha_n \beta_n e^{-r \sum\limits_{m=n'}^{N} t_m^*} + \sum_{n=1}^{N} \alpha_n \beta_n e^{-r \sum\limits_{m=n}^{N} t_m^*} \left(\sum_{m=n}^{N} t_m^* + r \frac{\partial t_m^*}{\partial r} \right).$$

To show that $wL_N e^{rt_N^*}$ decreases in r thus amounts to showing that $\Gamma < 0$, or

$$\sum_{n=1}^{N} \alpha_n \beta_n \sum_{m=n}^{N} t_m^* \times \sum_{n'=1}^{N} \alpha_n \beta_n e^{-r \sum\limits_{m=n'}^{N} t_m^*} > \sum_{n=1}^{N} \alpha_n \beta_n e^{-r \sum\limits_{m=n}^{N} t_m^*} \left(\sum_{m=n}^{N} t_m^* + r \frac{\partial t_m^*}{\partial r} \right).$$

Because $\partial t_m^*/\partial r < 0$, it will thus suffice to show that

$$\sum_{n=1}^{N} \alpha_n \beta_n \sum_{m=n}^{N} t_m^* \times \sum_{n'=1}^{N} \alpha_n \beta_n e^{-r \sum_{m=n'}^{N} t_m^*} > \sum_{n=1}^{N} \alpha_n \beta_n \sum_{m=n}^{N} t_m^* e^{-r \sum_{m=n}^{N} t_m^*}$$

or

$$\sum_{n=1}^{N} \alpha_n \beta_n \sum_{m=n}^{N} t_m^* > \sum_{n=1}^{N} \frac{\alpha_n \beta_n e^{-r \sum_{m=n}^{N} t_m^*}}{\sum_{n'=1}^{N} \alpha_n \beta_n e^{-r \sum_{m=n'}^{N} t_m^*}} \sum_{m=n}^{N} t_m^* = \sum_{n=1}^{N} \frac{L_n}{L} \sum_{m=n}^{N} t_m^*,$$

where I have again invoked (20) in equality in the right-hand-side. Now because $L_{n'}/L_n > \alpha_{n'}\beta_{n'}/\alpha_n\beta_n$ for $n' > n$ (see equation (14)), and because $\sum_{m=n'}^{N} t_m^* < \sum_{m=n}^{N} t_m^*$ for $n' > n$, the above inequality necessarily holds because the right-hand-side is a weighted sum (with weights L_n/L) that puts higher weight on relatively smaller terms, relative to the weighted sum on the left-hand-side.

In sum, when r goes down, $wL_N e^{rt_N^*}$ must increase, and thus equation (A.1) implies that y_N must increase as well.

A.4 The Effect of the Interest Rate on Aggregate Capital Demand

Plugging equation (20) into (23) delivers

$$K^d = \sum_{n=1}^{N} w \frac{\alpha_n \beta_n e^{-r \sum_{m=n}^{N} t_m^*}}{\sum_{n'=1}^{N} \alpha_{n'} \beta_{n'} e^{-r \sum_{m=n'}^{N} t_m^*}} L \frac{e^{r \sum_{m=n}^{N} t_m^*} - 1}{r},$$

which can be written as

$$K^d = \frac{wL}{r} \sum_{n=1}^{N} \frac{\alpha_n \beta_n - \alpha_n \beta_n e^{-r \sum_{m=n}^{N} t_m^*}}{\sum_{n'=1}^{N} \alpha_{n'} \beta_{n'} e^{-r \sum_{m=n'}^{N} t_m^*}} = \frac{wL}{r} \left(\frac{1}{\sum_{n'=1}^{N} \alpha_{n'} \beta_{n'} e^{-r \sum_{m=n'}^{N} t_m^*}} - 1 \right).$$

As long as rt_n^* weakly increases when the interest rate r falls, the last term in parenthesis will also be higher, the lower the interest rate (since all terms $e^{-r \sum_{m=n'}^{N} t_m^*}$ will be lower). Because wages are also higher the lower is r, the overall effect of r on K^d is necessarily negative in the 'elastic' case when rt_n^* weakly decreases in r.

A.5 Pattern of Specialization

In the main text, I showed that whenever the function $z_n(t_n)$ is log-linear, i.e., $z_n(t_n) = (t_n)^{\zeta_n}$, the pattern of specialization is shaped solely by the relative size of the parameter ζ_n in the different stages. Neither the relative positioning of the various stages, nor their relative labor intensity α_n mattered for comparative advantage. In this Appendix, I explore the robustness of these results for alternative labor productivity functions $z_n(t_n)$.

The Elastic Case Consider first the 'elastic case' introduced in the main text, in which $z_n(t_n) = \exp\left((t_n)^{\omega_n}\right)$. It is straightforward to show that this case delivers $rt_n^* = \alpha_n \omega_n (t_n^*)^{\omega_n}$ and

$$z_n(t_n) = \exp\left(\left(\frac{\alpha_n \omega_n}{r}\right)^{\omega_n/(1-\omega_n)}\right).$$

For the second-order conditions to be satisfied, we need $z_n''(t)/z_n'(t) < 0 < z_n'(t)/z_n(t)$, which imposes $\omega_n < 1$.

Consider next the ratio of Home and Foreign prices at stage n, which generally is given by:

$$\frac{p_n^H}{p_n^F} = \left(\frac{w^H}{w^F}\frac{z_n(t_n^F)}{z_n(t_n^H)}\frac{e^{r^H t_n^H}}{e^{r^F t_n^F}}\right)^{\alpha_n}\left(\frac{p_{n-1}}{p_{n-1}}\frac{e^{r^H t_n^H}}{e^{r^F t_n^F}}\right)^{1-\alpha_n}.$$

In this particular case, we have

$$\begin{aligned}\frac{p_n^H}{p_n^F} &= \left(\frac{w^H}{w^F}\frac{e^{(t_n^F)^{\omega_n}}}{e^{(t_n^H)^{\omega_n}}}\right)^{\alpha_n}\frac{e^{\alpha_n\omega_n(t_n^H)^{\omega_n}}}{e^{\alpha_n\omega_n(t_n^F)^{\omega_n}}}\\ &= \left(\frac{w^H}{w^F}\exp\left((1-\omega_n)\left((t_n^F)^{\omega_n}-(t_n^H)^{\omega_n}\right)\right)\right)^{\alpha_n}\\ &= \left(\frac{w^H}{w^F}\exp\left((1-\omega_n)(\alpha_n\omega_n)^{\frac{\omega_n}{1-\omega_n}}\left((r^F)^{-\frac{\omega_n}{1-\omega_n}}-(r^H)^{-\frac{\omega_n}{1-\omega_n}}\right)\right)\right)^{\alpha_n}.\end{aligned}$$

This expression is a bit messy, but it is straightforward to derive some results from it. First, note that whenever interest rates are equalized, prices are equalized too only if wages are equalized. And if wages differ across countries (while interest rates remain equalized), one country would always feature a lower price than the other one, which would violate labor-market clearing. Second, it is clear that the relative upstreamness or downstreamness of stages does not matter per se for comparative advantage. Third, it is also clear from inspection of the relative price p_n^H/p_n^F that $\left(p_n^H/p_n^F\right)^{1/\alpha_n}$ is decreasing in α_n whenever $r^F > r^H$, which of course implies that Home has a cost advantage in high α_n sectors. Conversely, the effect of ω_n on the price ratio is now more complicated than in our baseline log-linear case, since it is the balance of two effects. On the one hand, $(r^F)^{-\frac{\omega_n}{1-\omega_n}} - (r^H)^{-\frac{\omega_n}{1-\omega_n}}$ decreases in ω_n whenever $r^H < r^F < 1$, but it is also the case that $(1-\omega_n)(\omega_n)^{\frac{\omega_n}{1-\omega_n}}$ – which pre-multiplies a negative term – is also decreasing function of ω_n.

Inelastic Case Consider next the 'inelastic case' introduced in the main text, in which $z_n(t_n) = 1/\exp\left((t_n)^{-\psi_n}\right)$. It is straightforward to show that this case delivers $rt_n^* = \alpha_n\psi_n(t_n^*)^{-\psi_n}$ and

$$z_n(t_n) = 1/\exp\left(\left(\frac{\alpha_n\psi_n}{r}\right)^{-\psi_n/(1+\psi_n)}\right).$$

The ratio of Home and Foreign prices at stage n is in turn given by

$$\begin{aligned}\frac{p_n^H}{p_n^F} &= \left(\frac{w^H}{w^F}\frac{z_n\left(t_n^F\right)}{z_n\left(t_n^H\right)}\frac{e^{r^H t_n^H}}{e^{r^F t_n^F}}\right)^{\alpha_n}\left(\frac{p_{n-1}}{p_{n-1}}\frac{e^{r^H t_n^H}}{e^{r^F t_n^F}}\right)^{1-\alpha_n} \\ &= \left(\frac{w^H}{w^F}\frac{\exp\left((1+\psi_n)\left(t_n^H\right)^{-\psi_n}\right)}{\exp\left((1+\psi_n)\left(t_n^F\right)^{-\psi_n}\right)}\right)^{\alpha_n} \\ &= \left(\frac{w^H}{w^F}\exp\left((1+\psi_n)\left(\alpha_n\psi_n\right)^{-\psi_n/(1+\psi_n)}\left(\left(r^H\right)^{\psi_n/(1+\psi_n)}-\left(r^F\right)^{\psi_n/(1+\psi_n)}\right)\right)\right)^{\alpha_n}\end{aligned}$$

As in the 'elastic case', we can derive a series of results from this expression. First, whenever interest rates are equalized, prices are equalized too only if wages are equalized. And if wages differ across countries (while interest rates remain equalized), one country would always feature a lower price than the other one in all stages, which would violate labor-market clearing. Second, it is clear that the relative upstreamness or downstreamness of stages does not matter per se for comparative advantage. Third, it is also clear from inspection of the relative price p_n^H/p_n^F that $\left(p_n^H/p_n^F\right)^{1/\alpha_n}$ is increasing in α_n whenever $r^H < r^F$, which of course implies that Home has a cost advantage in low α_n sectors. This is exactly the opposite result to the one we obtained in the 'elastic case'. Finally, the effect of ψ_n on the price ratio appears to be ambiguous, as it is the balance of two effects that are themselves ambiguous.

A.6 Costly Trade

This Appendix fills in some of the details of the version of the model with costly trade developed in section 4.

Partial Equilibrium Remember that in the main text we established that the prices p_{n-1}^H and p_{n-1}^F that partly shape the choice at n, as indicated by inequality (40) satisfy:

$$\begin{aligned}p_{n-1}^H &= \min\left\{\left(w^H\left(r^H\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}}\left(p_{n-2}^H\right)^{1-\alpha_{n-1}}, \tau\left(w^F\left(r^F\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}}\left(p_{n-2}^F\right)^{1-\alpha_{n-1}}\right\}; \\ p_{n-1}^F &= \min\left\{\tau\left(w^H\left(r^H\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}}\left(p_{n-2}^H\right)^{1-\alpha_{n-1}}, \left(w^F\left(r^F\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}}\left(p_{n-2}^F\right)^{1-\alpha_{n-1}}\right\},\end{aligned}$$

where p_{n-2}^H is the price of stage $n-2$ faced by producers in H of stage $n-1$, and p_{n-2}^F is the analogous price for producers in F of stage $n-1$ output. The ratio p_{n-1}^H/p_{n-1}^F can thus be bounded below as follows:

$$\frac{p_{n-1}^H}{p_{n-1}^F} = \begin{cases} \dfrac{\left(w^H\left(r^H\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}}\left(p_{n-2}^H\right)^{1-\alpha_{n-1}}}{\tau\left(w^H\left(r^H\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}}\left(p_{n-2}^H\right)^{1-\alpha_{n-1}}} = \frac{1}{\tau} \\ \dfrac{\tau\left(w^F\left(r^F\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}}\left(p_{n-2}^F\right)^{1-\alpha_{n-1}}}{\left(w^F\left(r^F\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}}\left(p_{n-2}^F\right)^{1-\alpha_{n-1}}} = \tau \\ \dfrac{\left(w^H\left(r^H\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}}\left(p_{n-2}^H\right)^{1-\alpha_{n-1}}}{\left(w^F\left(r^F\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}}\left(p_{n-2}^F\right)^{1-\alpha_{n-1}}} > \dfrac{\left(w^H\left(r^H\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}}\left(p_{n-2}^H\right)^{1-\alpha_{n-1}}}{\tau\left(w^H\left(r^H\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}}\left(p_{n-2}^H\right)^{1-\alpha_{n-1}}} = \frac{1}{\tau} \\ \dfrac{\tau\left(w^F\left(r^F\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}}\left(p_{n-2}^F\right)^{1-\alpha_{n-1}}}{\tau\left(w^H\left(r^H\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}}\left(p_{n-2}^H\right)^{1-\alpha_{n-1}}} > \dfrac{\tau\left(w^F\left(r^F\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}}\left(p_{n-2}^F\right)^{1-\alpha_{n-1}}}{\left(w^F\left(r^F\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}}\left(p_{n-2}^F\right)^{1-\alpha_{n-1}}} = \tau \end{cases},$$

where I have appealed to the *min* operator to insert the inequality in the third and fourth values.

As claimed in the main text, we thus have that the ratio p^H_{n-1}/p^F_{n-1} satisfies

$$\frac{p^H_{n-1}}{p^F_{n-1}} \geqslant \frac{1}{\tau},$$

which can be interpreted as a no-arbitrage condition in the sourcing of inputs at stage $n-1$. Notice from the expression above that this inequality is strict unless both H and F producers source from H at $n-2$. As demonstrated in the main text, the fact that $p^H_{n-1}/p^F_{n-1} \geqslant 1/\tau$ suffices to show that Foreign producers at stage $n+1$ may choose F as a source of inputs at stage n even when $w^F \left(r^F\right)^{\zeta_n} > w^H \left(r^H\right)^{\zeta_n}$, which indicates a disproportionate desire to bunch contiguous stages in the same location.

General Equilibrium Trade costs not only affect the location of production, but by doing so, they also affect factor markets and thus factor prices. I next outline the equations that determine the general equilibrium of the model with costly trade. Notice first that for given wages and interest rates, the dynamic programming approach in the main text can be used to solve for the path of production $\ell^i = \left\{\ell^i(1), \ell^i(2), ..., \ell^i(N)\right\}$ that ultimately delivers consumption to country i. Notice that there are two such chains, one for $j = H$ and one for $j = F$. Let us now define an indicator function $I^i(j,n)$ which takes a value of 1 if country j produces stage n for the chain ℓ^i and takes a value of 0 otherwise. Analogously to our derivations in the version of the model without trade costs, we can express demand for country j's labor at stage n as

$$L^j_n = \sum_{i \in \{H,F\}} \mathbb{I}^i(j,n) \frac{\alpha_n \beta_n}{w^j} e^{-\sum_{m=n}^{N} \alpha_m \zeta_m} \left(w^i L^i + r^i K^i\right), \tag{A.2}$$

where I have already replaced final-good consumption y^i_N in country i with the sum of factor income. Rearranging this expression and aggregating across stages, we can express equilibrium in the labor market as a system of two equations:

$$w^j L^j = \sum_{n=1}^{N} \sum_{i \in \{H,F\}} \mathbb{I}^i(j,n) \alpha_n \beta_n e^{-\sum_{m=n}^{N} \alpha_m \zeta_m} \left(w^i L^i + r^i K^i\right). \tag{A.3}$$

To close the model, we similarly need to invoke capital-market clearing. The derivations are analogous to those in the model without trade costs. Funding labor costs at stage n in country j requires an amount of capital equal to $w^j L^j_n e^{r^j t^j_n}$, and to fund intermediate input costs requires an amount of capital equal to $p^j_{n-1} y^j_{n-1} e^{r^j t^j_n}$. But given our Cobb-Douglas assumptions, we have that $\alpha_n p^j_{n-1} y^j_{n-1} = (1-\alpha_n) w^j L^j_n$, and thus capital intensity at stage n in country j in a stationary equilibrium with a uniform time-invariant distribution of production processes can be expressed as

$$\frac{K^j_n}{L^j_n} = \frac{w^j}{r^j} \frac{1}{\alpha_n} e^{\alpha_n \zeta_n}.$$

The aggregate demand for capital in country j is in turn equal to:

$$\left(K^d\right)^j = \frac{w^j}{r^j} \sum_{n=1}^{N} L^j_n \frac{1}{\alpha_n} \left(e^{\alpha_n \zeta_n} - 1\right),$$

where L^j_n is defined in equation (A.2), and thus already takes into account that L^j_n (and thus K^j_n) will take a value of 0 for some stages.

Imposing capital-market clearing and the definition of the function $\sigma\left(r^j,\rho^j\right)$ in equation (25), we can finally express equilibrium in the capital market as a system of two equations:

$$r^j \times \sigma\left(r^j,\rho^j\right) = \sum_{n=1}^{N} \frac{L_n^j}{L^j}\frac{1}{\alpha_n}\left(e^{\alpha_n \zeta_n} - 1\right). \tag{A.4}$$

The two systems of equations (A.3) and (A.4) jointly determine the wage rates w^H and w^F and interest rates r^H and r^F as a function of specialization, as captured by the indicator functions $I^i(j,n)$. Remember, in turn, that these indicator functions are determined via forward induction and are naturally a function of wages and interest rates, implying that solving the model requires finding a fixed point for wages that is consistent with factor market clearing given the equilibrium pattern of specialization.

I conclude this section by briefly discussing how changes in trade costs qualitatively affect these equilibrium factor prices. From our partial equilibrium results, we know that the larger trade costs are, the more bunching there will be, and this will lead to relocation of production stages from some countries to others. How does that impact the key equations pinning down wages and interest rates. Consider first the system in (A.3). This equation makes it clear that the wage in country j could be negatively impacted by trade costs if such trade barriers lead to a shift of production (and labor) (i) from stages with large value added $(\alpha_n\beta_n)$ to stages with low value added, (ii) from relatively downstream stages to relatively upstream stages, and (iii) from stages belonging to chains serving the high-income country to stages belonging to chains serving a low-income country. But if the changes in specialization are largely unrelated to the importance of these stages in value added, to their relative position in value chains, or to the ultimate destination of consumption, then the relative wage implications of changes in trade costs will typically be modest.

Turning to the system in (A.4), it is clear from that expression that the impact of changes in specialization on country j's interest rate will be shaped by the relative capital intensity of the stages that release labor and those that absorb it. If differences in capital intensity across stages are modest, changes in specialization will have little impact on interest rates.

A.7 Trade Credit

In this Appendix, I formalize the implications of trade credit for comparative advantage and for the importance of GVC trade in world trade. To build intuition, I begin with the simpler case in which sellers are allowed to finance both the wage and intermediate input expenditures of buyers, and then I turn to the more realistic case in which only the intermediate input expenditures are financed. I assume throughout that wage and interest rate differences persist even after trade integration, and I also abstract from trade costs to isolate the role of trade credit on specialization.

A.7.1 Trade Credit Cum Wages Financing

Given that the interest rate at Home is lower than that in Foreign, trade credit will only arise whenever a Home producer sells goods to Foreign buyers. Facing that lower interest rate, Foreign buyers of Home inputs will set a production length identical to that chosen by Home buyers, and thus their marginal cost of production for a

Foreign producer at n buying Home inputs at $n-1$ will be

$$c_n\left(\ell\left(n\right)=F\mid\ell\left(n-1\right)=H\right)=\kappa_n\left(\frac{w^F\left(r^H\right)^{\zeta_n}}{\left(\alpha_n\zeta_n\right)^{\zeta_n}}\right)^{\alpha_n}\left(p_{n-1}^H\right)^{1-\alpha_n}e^{\alpha_n\zeta_n}, \tag{A.5}$$

where p_{n-1}^H is the price of $n-1$ inputs when purchased from a Home producer at $n-1$. If instead the Foreign buyer at n where to purchase inputs from a Foreign producer at $n-1$, its marginal cost would be

$$c\left(\ell\left(n\right)=F\mid\ell\left(n-1\right)=F\right)=\kappa_n\left(\frac{w^F\left(r^F\right)^{\zeta_n}}{\left(\alpha_n\zeta_n\right)^{\zeta_n}}\right)^{\alpha_n}\left(p_{n-1}^F\right)^{1-\alpha_n}e^{\alpha_n\zeta_n}, \tag{A.6}$$

where p_{n-1}^F is the price of $n-1$ inputs when purchased from a Foreign producer at $n-1$.

Consider next the choices faced by a Home producer at n deciding where to buy the stage $n-1$ from. This producer will not make use of trade credit, so its marginal cost would be

$$c\left(\ell\left(n\right)=H\mid\ell\left(n-1\right)=H\right)=\kappa_n\left(\frac{w^H\left(r^H\right)^{\zeta_n}}{\left(\alpha_n\zeta_n\right)^{\zeta_n}}\right)^{\alpha_n}\left(p_{n-1}^H\right)^{1-\alpha_n}e^{\alpha_n\zeta_n}, \tag{A.7}$$

when purchasing from a Home supplier at $n-1$, and

$$c\left(\ell\left(n\right)=H\mid\ell\left(n-1\right)=F\right)=\kappa_n\left(\frac{w^H\left(r^H\right)^{\zeta_n}}{\left(\alpha_n\zeta_n\right)^{\zeta_n}}\right)^{\alpha_n}\left(p_{n-1}^F\right)^{1-\alpha_n}e^{\alpha_n\zeta_n}, \tag{A.8}$$

when purchasing from a Foreign supplier at $n-1$.

Foreign Bias for Home Producers Given our results above for the choice of Foreign producers at n (and applying them to $n-1$), it should be clear from equations (A.5)–(A.8) that we can then express the choice of a Home producer at n as solving:

$$\begin{aligned}\ell &= \arg\min_{j\in\{H,F\}}\left\{p_{n-1}^j\right\}\\ &= \min\left\{\begin{array}{c}\left(w^H\left(r^H\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}}\left(\min\left\{p_{n-2}^H,p_{n-2}^F\right\}\right)^{1-\alpha_{n-1}},\\ \min\left\{\left(w^F\left(r^H\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}}\left(p_{n-2}^H\right)^{1-\alpha_{n-1}},\left(w^F\left(r^F\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}}\left(p_{n-2}^F\right)^{1-\alpha_{n-1}}\right\}\end{array}\right\}\end{aligned} \tag{A.9}$$

There are two cases to consider: (i) $p_{n-2}^H<p_{n-2}^F$ and (ii) $p_{n-2}^H>p_{n-2}^F$. In the first case, and because $r^H<r^F$, it is evident that regardless of Home producers' choice for inputs at $n-1$, the optimal source of inputs at $n-2$ is necessarily Home. In such a case, Home producers at n will procure inputs from Foreign at $n-1$ whenever

$$\left(w^H\left(r^H\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}}>\left(w^F\left(r^F\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}}\left(\frac{r^H}{r^F}\right)^{\alpha_{n-1}\zeta_{n-1}},$$

which is more likely to hold than under the baseline condition $w^H\left(r^H\right)^{\zeta_{n-1}}>w^F\left(r^F\right)^{\zeta_{n-1}}$. In the second case $p_{n-2}^H>p_{n-2}^F$, then Home producers at $n-1$ choose Foreign as a source of $n-2$ inputs, while the choice of Foreign producers at $n-1$ is ambiguous. If both Home and Foreign producers at $n-1$ choose Foreign as a source of inputs at $n-2$, Home producers at n will procure inputs from Foreign at $n-1$ whenever

$$\left(w^H\left(r^H\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}}>\left(w^F\left(r^F\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}},$$

which is identical to the condition under our baseline model. If Home producers $n-1$ source from Foreign (at $n-2$), while Foreign producers source from Home (at $n-2$), then Home producers at n will procure inputs from Foreign at $n-1$ whenever

$$\left(w^H\left(r^H\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}} > \left(w^F\left(r^F\right)^{\zeta_{n-1}}\right)\left(\frac{r^H}{r^F}\right)^{\alpha_{n-1}\zeta_{n-1}}\left(\frac{p^H_{n-2}}{p^F_{n-2}}\right)^{1-\alpha_{n-1}}. \tag{A.10}$$

But remember from (A.9) that for Foreign producers at $n-1$ to prefer sourcing from Home at $n-2$, it must be the case that

$$\left(w^F\left(r^H\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}}\left(p^H_{n-2}\right)^{1-\alpha_{n-1}} < \left(w^F\left(r^F\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}}\left(p^F_{n-2}\right)^{1-\alpha_{n-1}},$$

which implies that

$$\left(\frac{r^H}{r^F}\right)^{\zeta_{n-1}\alpha_{n-1}}\left(\frac{p^H_{n-2}}{p^F_{n-2}}\right)^{1-\alpha_{n-1}} < 1.$$

This result then implies that the condition in (A.10) is again more likely to hold than under the baseline condition $w^H\left(r^H\right)^{\zeta_{n-1}} > w^F\left(r^F\right)^{\zeta_{n-1}}$.

In sum, I have shown that Home producers at n are more likely to favor Foreign as a source of inputs at $n-1$ than would be implied by the simple comparison of $w^H\left(r^H\right)^{\zeta_{n-1}}$ and $w^F\left(r^F\right)^{\zeta_{n-1}}$ in the baseline model. As a result, Home producers are more likely to embody foreign value added (or more likely to participate in GVCs) than in the baseline model.

Home Bias for Foreign Producers The choice of Foreign at n is a bit more involved, but following analogous steps as above (see equations (A.5)–(A.8)), we can express it as

$$\begin{aligned}\ell &= \min\left\{\left(\frac{r^H}{r^F}\right)^{\zeta_n\alpha_n}\left(p^H_{n-1}\right)^{1-\alpha_n}, \left(p^F_{n-1}\right)^{1-\alpha_n}\right\}\\ &= \min\left\{\begin{array}{c}\left(\frac{r^H}{r^F}\right)^{\zeta_n\alpha_n}\left(\left(w^H\left(r^H\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}}\left(\min\left\{p^H_{n-2}, p^F_{n-2}\right\}\right)^{1-\alpha_{n-1}}\right)^{1-\alpha_n},\\ \left(\min\left\{\left(w^F\left(r^H\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}}\left(p^H_{n-2}\right)^{1-\alpha_{n-1}}, \left(w^F\left(r^F\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}}\left(p^F_{n-2}\right)^{1-\alpha_{n-1}}\right\}\right)^{1-\alpha_n}\end{array}\right\}\end{aligned} \tag{A.11}$$

As before, there are two cases to consider: (i) $p^H_{n-2} < p^F_{n-2}$ and (ii) $p^H_{n-2} > p^F_{n-2}$.

In the first case, it is clear that regardless of the choice by Foreign producers at $n-1$, Home is the optimal choice as a source of inputs at stage $n-2$. In that case, then Foreign producers at n will procure inputs from Home at $n-1$ whenever

$$\left(\frac{r^H}{r^F}\right)^{\zeta_n\alpha_n}\left(\frac{r^H}{r^F}\right)^{-\zeta_{n-1}\alpha_{n-1}(1-\alpha_n)}\left(w^H\left(r^H\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}(1-\alpha_n)} < \left(w^F\left(r^F\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}(1-\alpha_n)}. \tag{A.12}$$

Whether the term

$$\left(\frac{r^H}{r^F}\right)^{\zeta_n\alpha_n}\left(\frac{r^H}{r^F}\right)^{-\zeta_{n-1}\alpha_{n-1}(1-\alpha_n)}$$

is higher or lower than one depends on the relative labor and time intensity of stages n and $n-1$. Intuitively, by choosing Home inputs, Foreign producers at n are able to extend the production length at stage n, and this will be more appealing the larger the labor intensity or time intensity at stage n. By choosing Foreign inputs at $n-1$, Foreign producers at n give up these gains, but now gain the ability to source inputs at stage $n-1$ benefiting from a lower cost of capital obtained by Foreign producers at $n-1$ whenever they source stage $n-2$

inputs from Home. The latter effect however happens up the chain and is discounted by the exponent $1-\alpha_n$. As a result, note that if $\zeta_n \simeq \zeta_{n-1}$ and $\alpha_n \simeq \alpha_{n-1}$, we necessarily have that

$$\left(\frac{r^H}{r^F}\right)^{\zeta_n\alpha_n}\left(\frac{r^H}{r^F}\right)^{-\zeta_{n-1}\alpha_{n-1}(1-\alpha_n)} < 1, \tag{A.13}$$

and in light of equation (A.12), Foreign producers at n are more likely to source from Home suppliers at $n-1$ than under the baseline condition $w^H\left(r^H\right)^{\zeta_{n-1}} < w^F\left(r^F\right)^{\zeta_{n-1}}$. It is also worth stressing that whenever condition (A.13) fails to hold, perhaps because labor intensity or time intensity are particularly high at stage n, such a 'local bunching' in Foreign (in stages n and $n-1$) can only happen when Foreign producers at $n-1$ source inputs from Home producers at $n-2$, and thus these instances of 'local bunching' are still associated with GVC activity.

Consider next the case in which $p^H_{n-2} > p^F_{n-2}$. In such a case, Home producers at $n-1$, necessarily choose Foreign as a source of inputs at $n-2$. Conversely, Foreign producers at $n-1$ may still choose Home producers at $n-2$, since those producers can offer them finance. If both Home and Foreign producers at $n-1$ choose Foreign as a source of stage $n-2$ inputs, then in light of the problem in (A.11), Foreign producers at n will procure inputs from Home at $n-1$ whenever

$$\left(\frac{r^H}{r^F}\right)^{\zeta_n\alpha_n}\left(\left(w^H\left(r^H\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}}\right)^{1-\alpha_n} < \left(\left(w^F\left(r^F\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}}\right)^{1-\alpha_n},$$

which is clearly more likely to hold than under the baseline condition $w^H\left(r^H\right)^{\zeta_{n-1}} < w^F\left(r^F\right)^{\zeta_{n-1}}$.

Finally if Home producers at $n-1$ choose Foreign as a source of stage $n-2$ inputs, while Foreign producers at $n-1$ choose Home as a source of stage $n-2$ inputs, then Foreign producers at n will procure inputs from Home at $n-1$ whenever

$$\left(\frac{r^H}{r^F}\right)^{\zeta_n\alpha_n}\left(\frac{r^H}{r^F}\right)^{-\zeta_{n-1}\alpha_{n-1}(1-\alpha_n)}\left(\frac{p^F_{n-2}}{p^H_{n-2}}\right)^{(1-\alpha_{n-1})(1-\alpha_n)}\left(\left(w^H\left(r^H\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}}\right)^{1-\alpha_n} < \left(\left(w^F\left(r^F\right)^{\zeta_{n-1}}\right)^{\alpha_{n-1}}\right)^{1-\alpha_n}.$$

Whether the term

$$\left(\frac{r^H}{r^F}\right)^{\zeta_n\alpha_n}\left(\frac{r^H}{r^F}\right)^{-\zeta_{n-1}\alpha_{n-1}(1-\alpha_n)}\left(\frac{p^F_{n-2}}{p^H_{n-2}}\right)^{(1-\alpha_{n-1})(1-\alpha_n)}$$

is larger or smaller than one is ambiguous, but because under this case, $p^H_{n-2} > p^F_{n-2}$, it is clear that this term is more likely to be lower than one than the simpler condition in equation (A.13). Thus, Foreign producers at n are more likely to choose Home suppliers at $n-1$ than in the baseline model without trade credit provided that labor intensity α_n and time intensity ζ_n do not vary significantly along the chain. And whenever this condition is violated and Foreign producers prefer a Foreign supplier to a Home one even when $w^F\left(r^F\right)^{\zeta_n} > w^H\left(r^H\right)^{\zeta_n}$, it must be the case that Foreign producers at n source inputs from Home producers at $n-1$; and thus these instances of 'local bunching' are still associated with GVC activity.

A.7.2 Pure Trade Credit

When Home producers only provide credit to Foreign buyers by allowing them to delay their payment for the intermediate inputs (which is closer in spirit to the notion of trade credit), then Foreign producers will face a different interest rate for different types of spending. When receiving trade credit from Home producers at $n-1$, the profits of Foreign producers at n can be expressed as

$$\pi_n^F = p_n \left(z_n^F \left(t_n^F \right) L_n^F \right)^{\alpha_n} \left(y_{n-1}^F \right)^{1-\alpha_n} - w^F L_n^F e^{r^F t_n^F} - p_{n-1} y_{n-1}^F e^{r^H t_n^F}.$$

The production time length that maximizes these profits satisfies:

$$\alpha_n p_n \left(z_n^F \left(t_n^F \right) L_n^F \right)^{\alpha_n} \left(y_{n-1}^F \right)^{1-\alpha_n} \frac{\partial z_n^F \left(t_n^F \right) / \partial t_n^F}{z_n^F \left(t_n^F \right)} = r^F w^F L_n^F e^{r^F t_n^F} + r^H p_{n-1} y_{n-1}^F e^{r^H t_n^F},$$

which reduces to

$$\alpha_n \frac{\partial z_n^F \left(t_n^F \right) / \partial t_n^F}{z_n^F \left(t_n^F \right)} = r^F \alpha_n + r^H \left(1 - \alpha_n \right).$$

Whenever the function $z_n^F \left(t_n^F \right)$ is log-linear, i.e. $z_n^F \left(t_n^F \right) = \left(t_n^F \right)^{\zeta_n}$, this delivers

$$t_n^F = \frac{\alpha_n \zeta_n}{\widetilde{r_n^F}},$$

where

$$\widetilde{r_n^F} = r^F \alpha_n + r^H \left(1 - \alpha_n \right) \in \left[r^H, r^F \right].$$

The marginal cost of production for a Foreign producer at n buying Home inputs at $n-1$ will thus be given by:

$$\begin{aligned} c_n \left(\ell \left(n \right) = F \mid \ell \left(n-1 \right) = H \right) &= \kappa_n \left(\frac{w^F \left(\widetilde{r_n^F} \right)^{\zeta_n}}{\left(\alpha_n \zeta_n \right)^{\zeta_n}} e^{r_n^F t_n^F} \right)^{\alpha_n} \left(p_{n-1} e^{r_n^H t_n^F} \right)^{1-\alpha_n} \\ &= \kappa_n \left(\frac{w^F \left(\widetilde{r_n^F} \right)^{\zeta_n}}{\left(\alpha_n \zeta_n \right)^{\zeta_n}} \right)^{\alpha_n} \left(p_{n-1} \right)^{1-\alpha_n} e^{\widetilde{r_n^F} t_n^F}. \end{aligned}$$

Notice that this expression is analogous to that in equation (A.5) with $\widetilde{r_n^F}$ replacing r^H. Furthermore, $\widetilde{r_n^F} t_n^F = r_n^H t_n^H = \alpha_n \zeta_n$, so a comparison of marginal costs will again boil down to a comparison of the terms studied in the case with both trade credit and wage financing.

Next, note that because $r^H < \widetilde{r_n^F} < r^F$, all the main results derived for the case of trade credit cum wage financing will continue to hold when sellers only provide trade credit. In particular, for Home producers at n to have a higher preference for Foreign supplier at $n-1$, what was key was that $\left(r^H / r^F \right)^{\alpha_{n-1} \zeta_{n-1}} < 1$. But because $\left(\widetilde{r_{n-1}^F} / r^F \right)^{\alpha_{n-1} \zeta_{n-1}} < 1$, we can now similarly conclude that pure trade credit will also lead Home producers to be more likely to buy from Foreign producers (and thus more likely to engage in GVC activity) than in the baseline model without trade credit. Similarly, the discussion of the effect of trade credit on the sourcing decision of Foreign firms at stage n boil down to a discussion of the term

$$\left(\frac{r^H}{r^F} \right)^{\zeta_n \alpha_n} \left(\frac{r^H}{r^F} \right)^{-\zeta_{n-1} \alpha_{n-1} (1-\alpha_n)},$$

while it will now be shaped by the term

$$\left(\frac{\widetilde{r_n^F}}{r^F} \right)^{\zeta_n \alpha_n} \left(\frac{\widetilde{r_{n-1}^F}}{r^F} \right)^{-\zeta_{n-1} \alpha_{n-1} (1-\alpha_n)};$$

but since $\widetilde{r_n^F} \simeq \widetilde{r_{n-1}^F}$ whenever $\alpha_n \simeq \alpha_{n-1}$, the same conclusions drawn above will continue to apply to this case.