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FEEDBACK AND CONTAGION THROUGH DISTRESSED COMPETITION

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ABSTRACT

Firms tend to compete more aggressively in financial distress; this intensified competition, in turn, reduces profit margins, pushing themselves further into distress and adversely affecting their industry peers. To study such feedback and contagion effects, we incorporate strategic competition into a dynamic model with long-term defaultable debt, exploring various peer interactions like predation and price war. The feedback effect represents a novel source of financial distress costs associated with leverage, which helps explain the negative profitability-leverage relation across industries. Owing to the contagion effect, firms' optimal leverage is often excessively high from an industry perspective, undermining the industry's financial stability.

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A data appendix is available at http://www.nber.org/data-appendix/w30841
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1 Introduction

Over the past three decades, there has been a significant surge in product market concentration (e.g., Gutiérrez and Philippon, 2017; Autor et al., 2020; Loecker and Eeckhout, 2019). Industries are increasingly characterized as "winner takes most," with a small number of firms controlling a large share of the market.[1] Notably, the positions of market leaders are often highly persistent (e.g., Sutton, 2007; Bronnenberg, Dhar and Dubé, 2009), affording them the ability to engage in strategic competition and sustain elevated profit margins through tacit collusion.[2] The increasingly ubiquitous adoption of pricing algorithms powered by artificial intelligence has further facilitated such collusive behavior.[3] Considering the pivotal roles these dominant firms play in shaping aggregate earnings and output (Gabaix, 2011), coupled with the important impact of price markups on aggregate economic fluctuations (Bils, Klenow and Malin, 2012), a deeper understanding of market leaders' strategic competition is vital not just for grasping industry dynamics but also for understanding macroeconomic fluctuations.

In this paper, we study the dynamic strategic competition among firms facing threats of financial distress. We demonstrate that the interplay between tactical competition and the threat of financial distress engenders a distressed competition mechanism, leading to a competition-distress feedback loop and a novel form of financial contagion. These feedback and contagion effects will influence a firm's credit risk and the overall financial stability of the industry. Furthermore, by introducing strategic market competition into the tradeoff theory of capital structure, our model helps resolve the longstanding "profitability-leverage puzzle."

Figure 1 illustrates the intuition behind the competition-distress feedback loop. Consider a firm-specific shock that raises the distress risk for one of the major firms in an industry. The increased probability of default effectively makes the firm more impatient, leading to a greater focus on the short term gains. In the context of tacit collusion within a repeated game, firms retaliate against any rival's deviation from the collusive arrangement by reverting to non-collusive competition. When a firm becomes more impatient, the value it places on future cooperation profits diminishes, and its concern about future retaliation weakens. This

[1] According to the U.S. Census data, on average, the top four firms within each four-digit SIC industry account for about 48% of each industry's total revenue (see Dou, Ji and Wu, 2021*a*).

[2] See, e.g., Gutiérrez, Jones and Philippon (2019), Grullon, Larkin and Michaely (2019), and Corhay, Kung and Schmid (2020*b*) for evidence of high markups and profit margins, and Anderson, Rebelo and Wong (2018) and Dou, Ji and Wu (2021*a*,*b*) for evidence of strong fluctuations in aggregate profit margins. There is extensive granular and direct evidence showing that tacit collusion, as a form of strategic competition, is prevalent (see Online Appendix 2).

[3] See, e.g., Calvano et al. (2020), Asker, Fershtman and Pakes (2022), Brown and MacKay (2022), Sanchez-Cartas and Katsamakas (2022), Clark et al. (2023), and Dou, Goldstein and Ji (2023).

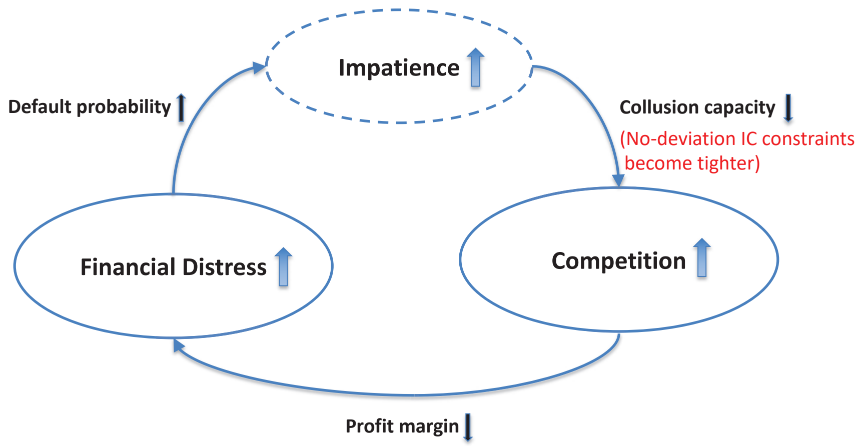

Figure 1. Interplay between financial distress and product-market competition.

tightening of the no-deviation incentive compatibility (IC) constraints reduces the firms' capacity for collusion and intensifies competition in the product market, resulting in reduced profit margins. Lower profit margins, in turn, push the firms further into financial distress, bringing the feedback loop full circle.

Drawing parallels, our feedback mechanism resonates with Fudenberg and Maskin (1986)'s folk theorem; yet, our model adds a unique dimension by endogenizing firms' discount rates through the impact of product market competition on default probabilities. Importantly, the firms do not actively choose to compete more aggressively; rather, it is the tightening of the no-deviation IC constraints that obliges them to reduce their profit margins. This feedback loop constitutes a key aspect of our analysis and highlights the intricate relationship between market competition and financial distress.

The competition-distress feedback loop yields two important implications about capital structure. First, it diminishes firms' incentives to increase leverage, as higher leverage curtails their ability to collude and reduces profitability. This effect introduces a novel source of financial distress costs. Second, the feedback loop provides an explanation for the profitability-leverage puzzle at the industry level. Profitability is negatively associated with leverage across both firms and industries, a robust empirical pattern that is at odds with the traditional tradeoff theory of capital structure (e.g., Myers, 1993; Rajan and Zingales, 1995; Fama and French, 2002). Our model can account for this negative relationship by endogenously connecting profitability with financial leverage. Specifically, industries with higher profitability tend to be those in which firms enjoy higher capacity for tacit collusion. This capacity is particularly vulnerable to the financial distress risk. As a result, to preserve their collusion capacity, firms in these more profitable industries opt for lower leverage.[4]

[4]The same idea can also offer a new perspective on the financial distress anomaly, differing from recent studies (e.g., Garlappi and Yan, 2011; Boualam, Gomes and Ward, 2020; Chen, Hackbarth and Strebulaev, 2022).

Strategic competition can also give rise to financial contagion within an industry. As illustrated in Figure 1, if a firm experiences an adverse idiosyncratic shock and becomes more financially distressed, competition intensifies within the industry, reducing the profits of other firms in the industry. As a result, the financial conditions of the firm's industry peers will also deteriorate, especially in the short run, such as one or two years. Moreover, in the case of multi-industry firms, financial contagion can spread across industries, amplifying the propagation of distress risk.[5]

This contagion effect has several implications. First, it expands the channels through which idiosyncratic shocks are transmitted across firms and industries. An adverse idiosyncratic shock to one market leader has ripple effects, negatively impacting the profit margins of its competitors due to endogenous competition responses. Such ripple effects imply that firms' cash flow risks and their financial distress levels become interdependent, justifying a fundamental assumption supporting information-based theories of credit market freezes (e.g., Bebchuk and Goldstein, 2011). Second, the strength of the contagion effect hinges on the industry structure. It is more pronounced in industries characterized by higher concentration, higher leverage, or a lower entry threat. This observation underscores the importance of considering the industry structure and the financial conditions of major competitors when analyzing firm-level credit risk, which differs from standard credit risk models that typically explain firm-level credit risk solely based on firm-specific and systematic (industry- or market-wide) economic conditions. Third, the contagion effect has implications for firms' optimal leverage ratios. Firms may choose leverage ratios that appear excessively high from the industry's perspective, compromising the industry's financial stability. The contagion effect delineated in our model sheds light on how common ownership can impact the financial stability of industries.

We present two models that formally explore the interactions between financial distress risk and strategic market competition. We start with a simple model in Section 2, which embeds the Merton model of credit risk (Merton, 1974) into a repeated-game model of Bertrand competition with potential tacit collusion (e.g., Fudenberg and Maskin, 1986; Rotemberg and Saloner, 1986). This simple model, which can be solved in closed form, provides a transparent and analytical demonstration of the feedback and contagion effects. Specifically, in the simple model, we focus on the strategic interactions between two firms that act as market leaders in a duopoly industry.

These firms derive their revenue from selling goods to consumers. They also need to service their debt, which are in the form of a consol, and pay taxes. In the event that their operating cash flows fall below a predetermined threshold, the firms default on their debt

[5]See Dou, Johnson and Wu (2023) for causal evidence of the cross-industry financial contagion effects.

and then exit the industry at the end of the period. To maintain stationarity and the duopoly market structure, a new firm promptly enters the industry to replace any firm that defaults. This turnover resets the game of market competition, which resumes with fresh interactions between the surviving incumbent firm and the new entrant.

We now describe how product prices and demands are determined under product market competition. Because goods are perishable and their prices are set above the costs, firms produce goods to exactly meet the demand in equilibrium. Demand for a firm's goods depends on the expected demand, which is endogenously determined according to a demand system, and an independent and identically distributed (i.i.d.) idiosyncratic demand shock. Under the demand system, if the two firms set the same product price, each firm obtains unity expected demand. However, if their prices differ, the firm with the higher price will lose all demand, whereas the firm with the lower price will capture additional expected demand that rises with its rival's price. This demand system implies that, in the absence of collusion, firms would undercut each other until they reach the unique non-collusive equilibrium which features zero profits for both firms.

However, as firms interact in a repeated game, they have the incentives to tacitly collude with each other to achieve positive profits. We focus on the collusive equilibrium sustained by grim trigger strategies. Assuming that the rival firm will honor the collusive price, a firm may be tempted to boost its current revenue by undercutting the price; however, this deviation risks reducing future revenue due to retaliation by the rival firm in subsequent periods. Following the literature,[6] we adopt the non-collusive equilibrium as the incentive compatible punishment for deviation. Using the closed-form solutions, we prove the existence of the competition-distress feedback loop and the financial contagion effect arising from the distressed competition mechanism.

To explore the model's implications for firms' capital structure decisions, we also allow firms to strategically set their debt levels. The determination of the optimal capital structure occurs at the inception, hinging on the fixed point of each firm's best response function of leverage choice. This function reflects the firm's optimal reaction to the rival's debt level within the collusive equilibrium. As in traditional tradeoff theories of capital structure, firms in our model weigh tax-shield benefits against financial distress costs when determining their optimal leverage. A distinctive and original aspect of our theory is the introduction of a new source of financial distress costs. This source is endogenous, stemming from the feedback loop between market competition and financial distress. Specifically, higher leverage levels within the industry tighten the no-deviation IC constraints in the collusive equilibrium, escalate industry competition, and ultimately reduce profitability and cash

[6] See, e.g., Green and Porter (1984), Brock and Scheinkman (1985), and Rotemberg and Saloner (1986).

flows. As a result, firms are incentivized to pursue lower financial leverage ex ante, even in the absence of bankruptcy costs. By varying the magnitude of this new source of financial distress costs across industries through heterogeneous levels of idiosyncratic left-tail risk, we show that the model can reproduce the negative correlation between profitability and leverage observed at the industry level.

After examining the simple model, we develop a full-fledged quantitative model to assess the quantitative significance of the distressed competition mechanism and its implications for capital structure. This quantitative model, presented in Section 3, extends the simple model above by incorporating more realistic assumptions and functional form specifications across four dimensions. First, the firm-specific demand dynamics are modeled to follow a sequentially dependent process, akin to Leland (1994), deviating from the i.i.d. process in the simple model. Consequently, the firm's distance-to-default becomes a persistent state variable, introducing another dynamic element to our analysis. Second, we consider a more flexible and realistic demand system. Third, firms make optimal default decisions based on endogenously determined default boundaries as opposed to the exogenously specified ones used in the simple model. Fourth, the quantitative model accommodates both partial debt recoveries and industry structure changes in the event of default. Specifically, by varying the size of entrant firms, we analyze industries with diverse entry barriers to investigate how entry threats influence rivals' predatory pricing behaviors.

The main contributions of this paper are theoretical. In a companion paper, Dou, Johnson and Wu (2023) provide extensive causal evidence of the contagion effects based on granular data. In addition, we provide three sets of empirical results in Online Appendix 3 to support the main theoretical implications in this paper. First, for evidence of the feedback effect, we find that industry-level profit margins load negatively on the discount rate and more so in industries where firms are closer to their default boundaries. This finding is consistent with our model's prediction that higher discount rates reduce firms' collusion capacity, and that the competition-distress feedback effect is stronger when firms' distances to default are lower. Second, we provide evidence of the financial contagion effect. By sorting the top six firms (ranked by sales) within each industry into three groups based on their distances to default, we find that adverse idiosyncratic shocks to the financially distressed group diminish the profit margins of the financially healthy group. Furthermore, we find that the within-industry spillover effect is more pronounced in the industries facing lower entry threats (i.e., higher entry barriers) as predicted by our quantitative model. Third, we provide evidence of the model's implications for the profitability-leverage relationship. Specifically, we show in the data that industries with lower profit margins or higher idiosyncratic left-tail risk are associated with higher leverage ratios. Moreover, the negative profitability-leverage relationship across industries becomes insignificant after controlling for idiosyncratic left-tail risk. Thus, consistent with our model's prediction, varying degrees of idiosyncratic left-tail risk across industries can explain a major part of the industry-level profitability-leverage anomaly.

Related Literature. Our study adds to the literature on structural credit models, emphasizing the complex interplay between imperfections in the primary credit market and assorted other market frictions. Previous studies by Philippon (2009), Kuehn and Schmid (2014), and Gomes, Jermann and Schmid (2016), among others, have delved into the interplay between a firm's long-term debt financing and corporate investment frictions. Additionally, He and Xiong (2012), He and Milbradt (2014), Chen et al. (2018), and Chen, Xu and Yang (2021) have investigated the interaction between a firm's default decisions and the secondary market liquidity for defaultable corporate bonds. Corhay (2017) examines credit risk in an oligopoly model, focusing on non-collusive competition and short-term debts. In contrast, our model centers on long-term debts and tacit collusion. Our paper highlights the dynamic interplay between tactical competition in imperfect product markets and the threat of financial distress.

Our study also enriches the theoretical literature on financial contagion, both across firms and between diverse assets. Previous studies have demonstrated that financial contagion operates through various economic channels, involving common (levered) investors (e.g., Kyle and Xiong, 2001; Gromb and Vayanos, 2002; Kodres and Pritsker, 2002; Kaminsky, Reinhart and Végh, 2003; Goldstein and Pauzner, 2004; Martin, 2013; Gârleanu, Panageas and Yu, 2015), cross-holding relations of firms (e.g., Elliott, Golub and Jackson, 2014), and information-based feedback effects (e.g., Cespa and Foucault, 2014). In contrast, our paper introduces a novel channel of financial contagion that underscores the importance of market competition.

A growing body of theoretical research delves into the complex interaction between market competition and corporate financial decisions, with early contributions dating back to Titman (1984), Brander and Lewis (1986), Maksimovic (1988), and Dumas (1989), among others. Over the past decades, this critical topic has seen significant theoretical advancements (e.g., Chevalier and Scharfstein, 1996; Allen, 2000; Grenadier, 2002; Aguerrevere, 2009; Back and Paulsen, 2009; Hackbarth and Miao, 2012; Bustamante, 2015; He and Matvos, 2016; Bova and Yang, 2017; Dou and Ji, 2021; Dou et al., 2021*b*; Dai, Jiang and Wang, 2022). Our paper makes a unique contribution to this body of theoretical literature by establishing the competition-distress feedback and financial contagion effects. We demonstrate the importance of market competition in distress propagation and revive the trade-off theory's explanatory power concerning the joint pattern of profitability and financial leverage. While Spiegel and Tookes (2013, 2020) also develop analytically tractable oligopoly dynamic models, our paper proposes a different strategic competition mechanism. In contrast to the studies by Spiegel and Tookes (2013, 2020), we determine firm-level profitability endogenously rather than specifying it exogenously. As a result, the competition-distress feedback and financial contagion effects arising from endogenous profitability are unique predictions of our theory.

The rest of the paper is organized as follows. Section 2 presents a simple model with closed-form solutions. Section 3 presents a full-fledged quantitative model that extends the simple model. Section 4 calibrates the full-fledged quantitative model and undertakes quantitative analyses. Section 5 concludes. The appendix contains proofs of the theoretical results of the simple model. The internet appendix provides empirical evidence supporting the theoretical results, as well as additional analyses of the full-fledged quantitative model. Moreover, a note on additional materials and literature discussions can be found in Chen et al. (2023), which is available on the authors' personal websites.

2 Simple Model

In this section, we present a simple model that combines the Merton model of credit risk (Merton, 1974) with a repeated-game model that allows for tacit collusion (e.g., Fudenberg and Maskin, 1986; Rotemberg and Saloner, 1986). This model clearly illustrates the interaction between financial leverage decisions and product market competition, which in turn endogenously determines a firm's profitability and its risk of financial distress. This illustration sheds light on the key insights of both the competition-distress feedback loop and the financial contagion effect.

This model also demonstrates that the competition-distress feedback effect can help reconcile the tradeoff theory of capital structure with the observed negative profitability-leverage relation in the data. This negative relation, often referred to as the profitability–leverage puzzle in the literature, has been widely documented and is generally regarded as a challenge to the tradeoff theory of capital structure (e.g., Myers, 1993; Rajan and Zingales, 1995; Fama and French, 2002).[7]

2.1 Model Formulation

We consider an infinite-horizon economy in discrete time with $t = 0, 1, 2, ...$ The risk-free rate is constant and zero, that is, $r_f = 0$. We consider a duopolistic industry with each firm

[7]A partial list of papers documenting an inverse relation between leverage and profitability include Titman and Wessels (1988), Fischer, Heinkel and Zechner (1989), Booth et al. (2001), Graham and Tucker (2006), Frank and Goyal (2014), Chen, Harford and Kamara (2019), and Eckbo and Kisser (2021).

producing a differentiated good. These firms repeatedly engage in price competition each period. We denote the generic firm by i and its competitor by j, that is, $i = 1, 2$ and $j \neq i$. At $t = 0$, firm i is financed by external equity and long-term consol debt with coupon payment of e^{b_i} each period. Due to the i.i.d. setting, we suppress the subscript t.

Cash Flows Driven by Market Competition. The total demand for firm i's goods is $e^{z_i} C_i$, where z_i captures the i.i.d. idiosyncratic demand shock that satisfies $\mathbb{E}\left[e^{z_i}\right] \equiv 1$; C_i is the expected demand, which is endogenously determined in the Nash equilibrium of the competition games. The marginal cost of production is zero, and thus each firm i would always choose its product price $P_i \geq 0$.

We now explain the demand system that lies at the heart of the determination of (C_1, C_2). These two firms simultaneously name prices $P_1, P_2 \in [0, \overline{P}]$, being aware that the expected demand faced by firm i is governed by the following demand system:

$$C_i = \begin{cases} 1 + \rho P_j^{\eta}, & \text{if } P_i < P_j \\ 1, & \text{if } P_i = P_j \\ 0, & \text{if } P_i > P_j \end{cases}, \quad \text{with } \; i \neq j \in \{1, 2\}, \tag{1}$$

where $\eta > 1$, $\rho > 0$, and $\overline{P} > 1$ satisfy $\rho \overline{P}^{\eta} \leq 1$.

Several important points warrant further discussion. First, when firms do not undercut each other ($P_1 = P_2$), the expected demand is normalized to unity ($C_1 = C_2 = 1$). Second, following the literature (e.g., Rotemberg and Saloner, 1986; Asker, Fershtman and Pakes, 2022), if firm i sets a lower price than its rival ($P_i < P_j$), firm j loses all of its demand, while firm i gains additional expected demand of ρP_j^{η}. The coefficients ρ and η together determine the extent of consumers' willingness to substitute between these two differentiated products. Intuitively, when either ρ or η is larger, setting a higher price P_j by firm j gives its rival, firm i, a stronger incentive to undercut firm j's price. Third, as in Asker, Fershtman and Pakes (2022), an upper bound $\overline{P}$ is imposed on the price. To ensure that the expected demand gain from undercutting never exceeds the expected demand loss experienced by its rival, we assume that $\rho \overline{P}^{\eta} \leq 1$. In the remainder of this section, we consider sufficiently large values of η and sufficiently small values of ρ to guarantee the existence of the collusive equilibrium. By analyzing the demand system with these assumptions, we can gain valuable insights into the important properties of the collusive equilibrium in the model.

Production Profits. Prices P_1 and P_2 are determined prior to the realization of the idiosyncratic demand shocks within the same period. Consequently, these prices remain unresponsive to the transitory idiosyncratic demand shocks, reflecting a mild degree of price rigidity in line with empirical observations (e.g., Bils and Klenow, 2004; Bils, Klenow and Malin, 2012). Specifically, both firms simultaneously name their product prices (P_1, P_2) before observing the idiosyncratic demand shocks, which determine the expected demand for their goods (C_1, C_2) according to the demand system (1). Then, after observing its specific demand shock z_i, firm i makes an optimal production decision. It chooses to produce $Y_i = e^{z_i} C_i$ units of outputs to meet the realized demand for its products. This production decision is well-founded from the firm's perspective, considering the assumption that the marginal cost of production is zero, and the outputs are perishable or nonstorable.[8]

Firm i generates a profit of $P_i e^{z_i} C_i$ from selling its products. It first uses this profit to cover interest expenses and taxes, and then distributes the remaining amount to shareholders as dividends. Let $\theta_i \equiv \ln(P_i C_i)$ denote the log profitability of firm i, which, as an intrinsic firm characteristic, is not influenced by transient idiosyncratic shocks, such as z_i. Firm i's earnings, after accounting for interest expenses and taxes, are $(1-\tau)\mathcal{E}_i$, where $\tau \in (0,1)$ represents the corporate tax rate, and $\mathcal{E}_i$ is given by:

$$\mathcal{E}_i = e^{\theta_i + z_i} - e^{b_i}. \tag{2}$$

Financial Distress and Borrowing Constraints. As in the standard Merton model, firm i defaults on its debts and exits the market when its operating cash flow $e^{\theta_i + z_i}$ is lower than an exogenous default boundary $e^{\varphi(b_i)}$.[9] For tractability, we postulate a functional form for the default boundary, $\varphi(b) \equiv \frac{1}{2}\bar{\varphi} b^2$, with $\bar{\varphi} > 0$. This assumption captures the idea that when a firm is sufficiently underwater, it will not be able to raise external funding to meet its debt obligation. Moreover, the higher the debt level the more likely default becomes.

In the meantime, financial distress can also stem from the possibility of covenant violations. To stay running, firm $i \in \{1, 2\}$ needs to make sure that the following earnings-based borrowing constraint is not violated in any period:

$$\theta_i - b_i \geq \ell, \quad \text{for a constant } \ell > 0.^{10} \tag{3}$$

To ensure stationarity and maintain tractability, we assume that a new firm enters the

[8] The adoption of a demand-driven optimal production, which captures the notion of costly inventories, has been widely embraced in the literature as a simplification assumption (e.g., Gourio and Rudanko, 2014; Corhay, Kung and Schmid, 2020*a*; Dou et al., 2021*b*; Dou, Ji and Wu, 2021*a*,*b*).

[9] We consider endogenous default boundaries in the full-fledged quantitative model in Section 3. As shown in the literature (e.g., Longstaff and Schwartz, 1995; Chen, Collin-Dufresne and Goldstein, 2008; Cremers, Driessen and Maenhout, 2008; Huang and Huang, 2012; Leland, 2012), models with exogenous and endogenous default boundaries can generate quantitatively similar predictions for default probabilities and credit spreads.

[10] Earnings-based borrowing constraints as specified in equation (3) are prevalent in practice (e.g., Roberts and Sufi, 2009; Lian and Ma, 2020).

industry when an incumbent firm defaults and exits.[11] In particular, when incumbent firm i exits, a new firm enters immediately, with coupon payment e^{b_i}. The game of industry competition is then "reset" to a new one between the surviving incumbent firm j and the new entrant. Upon default and exit, the firm is liquidated and its debt holders obtain a fraction δ of the unlevered asset value. We assume that δ equals zero for simplicity.[12]

Left-Tail Risk. We assume that for firm i, the idiosyncratic demand shock z_i follows a logistic distribution, Logistic $(\mu(\nu), \nu)$, with $\nu \in (0, 1/2)$ and

$$\mu(\nu) \equiv \ln(\sin(\nu\pi)) - \ln(\nu\pi) < 0. \tag{4}$$

It holds that $\mathbb{E}[z_i] = \mu(\nu)$, $\text{var}[z_i] = \nu^2\pi^2/3$, and $\mathbb{E}[e^{z_i}] \equiv 1$ for $i \in \{1, 2\}$.

This distributional assumption helps us obtain closed-form solutions. Moreover, under this assumption, the operating cash flow shock e^{z_i} follows the log-logistic distribution, also known as the *Fisk distribution*, which is widely applied in economic research (e.g., in modeling the distribution of wealth or income). This distribution has a heavy tail close to the Pareto tail, which is more realistic compared to the log-normal distribution. The parameter restriction $\nu \in (0, 1/2)$ ensures that the moments of z_i are well defined.[13] The probability of a left-tail event occurring, defined as $z_i \leq \underline{z}$ for a given threshold $\underline{z} < 0$, increases with ν.[14]

It is important to highlight that while the left-tail event is idiosyncratic, the left-tail risk, captured by ν, applies to the entire industry. We consider ν as a critical fundamental characteristic that differentiates industries. Intuitively, ν could represent the risk associated with radical innovations that can potentially displace market leaders. In industries with higher idiosyncratic left-tail risk, firms are more prone to radical and disruptive innovation that could dramatically alter industry landscape within a relatively short span of time (e.g., Christensen, 1997). For instance, the mobile phone industry has witnessed the market leadership shift from Nokia and Motorola to Apple and Samsung within a short span. In Section 2.5, we present theoretical evidence highlighting the substantial impact of heterogeneity in the industry-level tail risk on determining the financial leverage and profitability across industries. In Online Appendix 3.4, we further support these theoretical findings with empirical evidence.

[11] This assumption is similar in spirit to the "return process" of Luttmer (2007) and the "exit and reinjection" assumption in the industry dynamics models of Miao (2005) and Gabaix et al. (2016). The same assumption is commonly adopted in the industrial organization (IO) literature on oligopolistic competition and predation (e.g., Besanko, Doraszelski and Kryukov, 2014) and can be interpreted as the reorganization of the exiting firm.

[12] In the full-fledged quantitative model in Section 3, we calibrate the recovery rate of debt to match the data.

[13] Specifically, $\mathbb{E}[z_i] / \sqrt{\text{var}[z_i]}$ is a monotonically decreasing and strictly concave function of ν over $(0, 1/2)$.

[14] The proof is in Appendix A.

2.2 Nash Equilibrium

Firms play a supergame, in which the stage games of setting prices are infinitely repeated over time. A non-collusive equilibrium, which is the repetition of the one-shot Nash equilibrium, exists and is Markov perfect. Multiple subgame-perfect collusive equilibria can also exist, in which competition strategies are sustained by grim trigger strategies.

Unique Non-Collusive Equilibrium. In the following proposition, we characterize the non-collusive equilibrium. The proof is in Appendix B.

Proposition 2.1 (Non-Collusive Equilibrium)**.** *There exists a unique non-collusive equilibrium, in which firms set $P_1^N = P_2^N = 0$ and obtain $C_1^N = C_2^N = 1$ in each period. Thus, both firms have zero profits and zero equity values ($E_1^N = E_2^N = 0$).*

This result is quite intuitive. If $P_i > 0$, firm j can always choose a price infinitesimally lower than P_i to undercut firm i, given that the marginal cost is zero.

Competition Under Tacit Collusion. Let $\theta_i^C \equiv \ln\left(P_i^C C_i^C\right)$ denote the log profitability of firm i in the collusive equilibrium. As depicted in Figure 2, we consider the collusive equilibrium sustained by grim trigger strategies in which deviations from the tacit collusion scheme results in a punishment known as Bertrand reversion. Under this punishment, the firms shift from tacit coordination to non-coordination, leading to zero profits and zero equity values. The concept of retaliation through terminating coordination and reverting to the zero-profit non-collusive equilibrium is commonly employed in many studies (e.g., Rotemberg and Saloner, 1986; Hatfield et al., 2020) due to its simplicity.[15] Here, the effectiveness of the grim trigger strategy hinges on the ability of both firms to observe their competitor's price without incurring any cost, enabling swift detection and punishment of deviations.

The analytical expression for the default probability in a collusive equilibrium is presented in the following lemma. The proof is in Appendix C.

Lemma 1 (Default Probability)**.** *In a collusive equilibrium, the default probability of firm i over one period, denoted by λ_i^C, is equal to*

$$\lambda_i^C = \lambda^C(\theta_i^C, b_i), \quad \textit{with } \lambda^C(\theta, b) \equiv \frac{1}{1 + e^{[\mu(\nu)+\theta-\varphi(b)]/\nu}}. \tag{5}$$

We now explain the IC constraints that prevent the occurrence of deviation on the equilibrium path. The price-setting scheme at $\left(P_i^C, P_j^C\right)$ can be sustained as an equilibrium

[15] In the full-fledged quantitative model in Section 3, profits are positive in the non-collusive equilibrium, which increases the practical relevance of the same retaliation scheme.

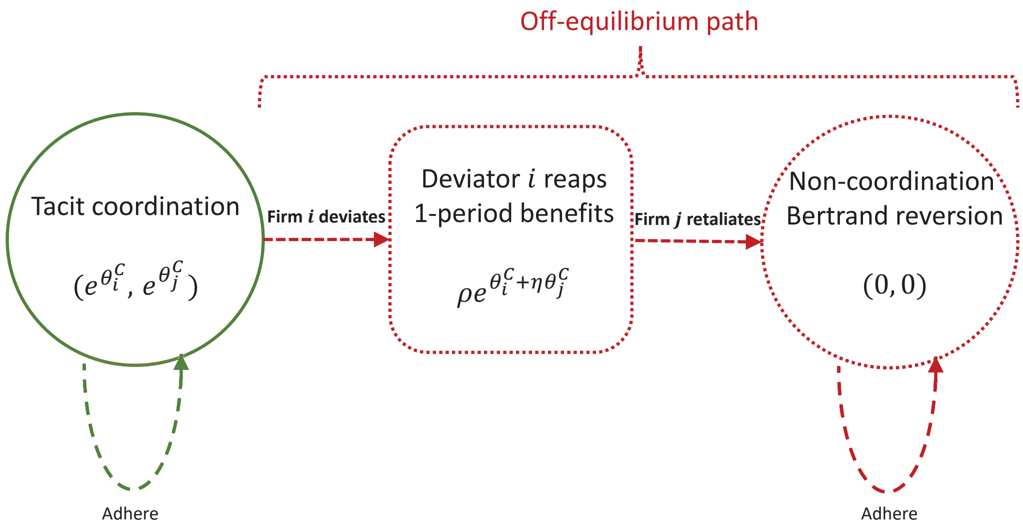

Figure 2. Graphic illustration of the classes of equilibria and the off-equilibrium-path threats. The two circled nodes represent the two phases of different forms of competition, and the squared node represents the transition based on the deviant behavior of the game players i or j with $i \neq j \in \{1,2\}$. The two firms have profits $(e^{\theta_i^C}, e^{\theta_j^C})$ under tacit coordination and $(0,0)$ under non-coordination Bertrand reversion.

outcome only if neither firm has incentive to deviate by setting a lower price, or equivalently, a lower log profitability. Deviation of firm i by setting a slightly lower price would result in an increase in expected demand of firm i by $\Delta C_i = \rho e^{\eta\theta_j^C}$ according to (1). However, such a deviation triggers retaliation from firm j in the next period, shifting the market to the non-collusive equilibrium, where profits and equity values become zero for both firms. Therefore, the gain from deviation, due to gaining additional market shares by undercutting its rival in the current period, and the subsequent loss, due to losing the future profits from collusion, which is firm i's equity value in the collusive equilibrium conditional on survival, can be characterized as follows:

$$\text{Expected benefits of deviation for firm } i = P_i^C \Delta C_i = \rho e^{\theta_i^C+\eta\theta_j^C}, \tag{6a}$$

$$\text{Expected costs of deviation for firm } i = \left[1-\lambda^C(\theta_i^C, b_i)\right] E_i^C(b_i, b_j), \tag{6b}$$

where $E_i^C(b_i, b_j)$ is the equity value of firm i in the collusive equilibrium.

To ensure that firm i has no incentive to deviate from tacit coordination, it must hold that

$$\underbrace{\rho e^{\theta_i^C+\eta\theta_j^C}}_{\text{Benefits of deviation}} \leq \underbrace{\left[1-\lambda^C(\theta_i^C, b_i)\right] E_i^C(b_i, b_j),}_{\text{Costs of deviation}} \quad \text{with } i \neq j \in \{1,2\}. \tag{7}$$

The IC constraints in (7) determine the maximal log profitabilities (θ_i^C, θ_j^C) that the grim trigger strategy can sustain. Additionally, within this equilibrium, both firms must set the same price, denoted as P^C. Indeed, under the demand system as specified by (1), no firm would willingly set a higher price than its rival, knowing that this would lead to zero profits.[16] This ensures equal expected demand for both firms, with $C_1^C = C_2^C = 1$, as well as equal log profitability, denoted as θ^C. These derived profitabilities then dictate the values of equity and debt in the collusive equilibrium. Having discussed the intuition, we formally characterize the collusive equilibrium in Proposition 2.2. The proof is in Appendix D.

Proposition 2.2 (Collusive Equilibrium)**.** *The equity and debt values of firm i in the collusive equilibrium are*

$$E_i^C(b_i, b_j) = (1-\tau)\left(e^{\theta^C} - e^{b_i}\right)\lambda^C(\theta^C, b_i)^{-1}, \tag{8}$$

$$D_i^C(b_i, b_j) = e^{b_i}\lambda^C(\theta^C, b_i)^{-1}, \tag{9}$$

where θ^C is the log profitability for both firms in the collusive equilibrium, and (θ^C, θ^C) is the unique Pareto efficient point of the following incentive compatible region (IC region) in the log-profitability space:

$$\mathcal{C} \equiv \left\{(\theta_1, \theta_2) \ : \ \theta_i \le \Psi(\theta_j, b_j)/\eta \ \text{ and } \ \theta_i = \theta_j, \ \text{ with } i \ne j \in \{1,2\}\right\}, \tag{10}$$

for b_1 and b_2 such that $\mathcal{C}$ is not empty. The function $\Psi(\theta, b)$ in (10) has the expression:

$$\Psi(\theta, b) \equiv \ln[(1-\tau)/\rho] + [\mu(\nu) + \theta - \varphi(b)]/\nu + \ln\left[1 - e^{-(\theta-b)}\right]. \tag{11}$$

Consequently, the log profitability in the collusive equilibrium is characterized by:

$$\theta^C = \Psi(\theta^C, b_1 \vee b_2)/\eta, \tag{12}$$

where $b_1 \vee b_2$ denotes the maximum of b_1 and b_2, and θ^C is a function of $b_1 \vee b_2$.

The characterizations of the equilibrium equity and debt values in (8) and (9) are intuitive. The debt value in (9) is the discounted coupon until the time of default. The equity value in (8) is the asset value minus the debt value, plus the value of tax shield:

$$E_i^C(b_i, b_j) = \underbrace{(1-\tau)e^{\theta^C}\lambda^C(\theta^C, b_i)^{-1}}_{\text{Asset value}} - \underbrace{D_i^C(b_i, b_j)}_{\text{Debt value}} + \underbrace{\tau D_i^C(b_i, b_j),}_{\text{Value of tax shield}} \ \text{with } i \ne j \in \{1,2\}. \tag{13}$$

The total value of firm i in the collusive equilibrium is $V_i^C(b_i, b_j) \equiv E_i^C(b_i, b_j) + D_i^C(b_i, b_j)$.

[16] In the full-fledged quantitative model in Section 3, firms can set different prices in the collusive equilibrium.

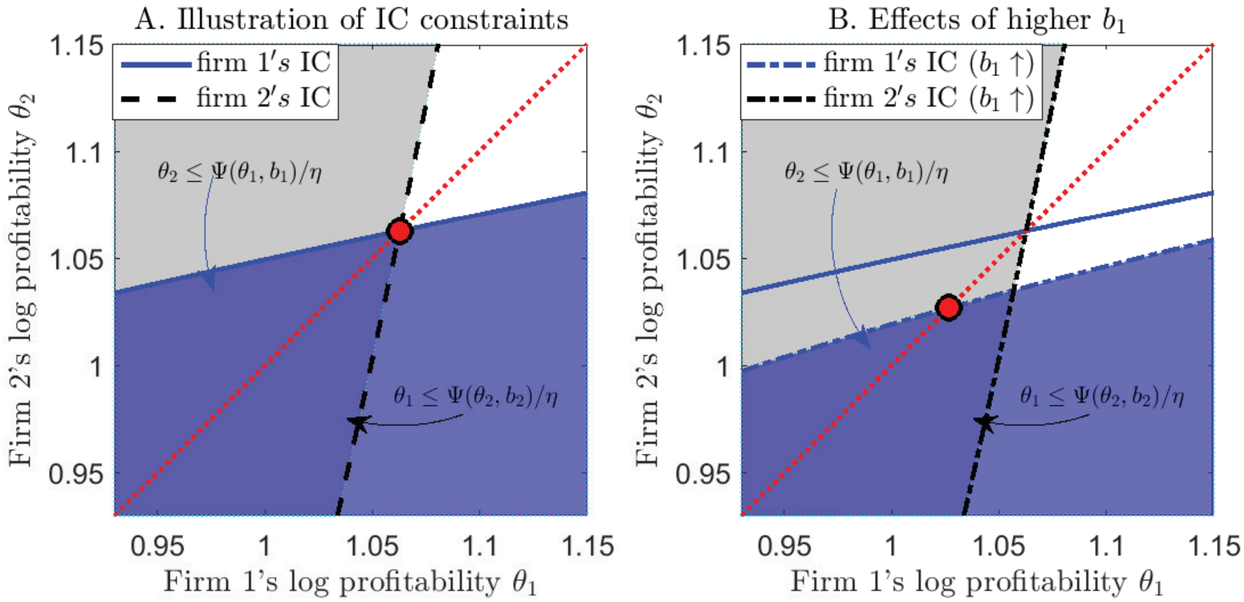

Figure 3. Illustration of the characterization of collusive equilibria and IC constraints. In panel A, we consider the symmetric case with $b_1 = b_2 = \ln(1.5)$. In panel B, we set $b_1 = \ln(2)$ and $b_2 = \ln(1.5)$. Other parameter values are set at $\tau = 0.35$, $\bar{\varphi} = 0.2$, $\rho = \exp(-20.5)$, $\eta = 21$, $\nu = 0.3$, and $\overline{P} = 3.5$.

The market leverage of firm i, denoted by $lev_i^C(b_i, b_j) \equiv D_i^C(b_i, b_j)/V_i^C(b_i, b_j)$, is equal to

$$lev_i^C(b_i, b_j) = \frac{1}{(1-\tau)e^{\theta^C - b_i} + \tau}, \quad \text{with } i \neq j \in \{1, 2\}. \tag{14}$$

Figure 3 illustrates the central idea of Proposition 2.2. First, as discussed above, the firms must agree on the collusive price and log profitability each period. As a result, the equilibrium pair of log profitabilities must lie on the 45-degree dotted lines in panels A and B, the feasible set for any collusive equilibrium. In panel A, we consider the symmetric case with $b_1 = b_2$. Firm 1's IC constraint is satisfied in the region below the solid line. Intuitively, given firm 1's log profitability θ_1, to prevent it from deviating from an agreed scheme, firm 2's log profitability θ_2 cannot be too high. Similarly, firm 2's IC constraint is satisfied in the region to the left of the dashed line. The segment of the 45-degree dotted line within the overlapping area of the two firms' IC regions represents all collusive equilibria sustained by grim trigger strategies. However, we focus on the unique Pareto efficient collusive equilibrium represented by the large dot at the intersection of the three lines, which corresponds to (θ^C, θ^C) as stated in Proposition 2.2.

In panel B, we consider the case where firm 1's leverage becomes higher (i.e., $b_1 > b_2$ while b_2 is fixed). The dash-dotted lines represent the new IC constraints faced by the two firms. Firm 1's IC constraint shifts downward, while firm 2's IC constraint remains unchanged. The increase in b_1 leads to a reduction in $\Psi(\theta, b)$ for all θ, as indicated by equation (11). Intuitively, the higher leverage of firm 1 increases its default probability $\lambda^C(\theta^C, b_1)$, strengthening its incentives to deviate from collusion since the cost of deviation in (7) is now lower. Consequently, firm 2 cannot set its profitability level as high as before. However, firm 2's IC constraint remains unchanged as it depends solely on b_2. As a result, the segment of the 45-degree dotted line that lies within the overlapping area of the two firms' IC regions, representing all collusive equilibria sustained by grim trigger strategies, shrinks compared to that in panel A. The large dot at the intersection of firm 1's new IC constraint and the 45-degree dotted line depicts the new unique Pareto efficient collusive equilibrium with a lower log profitability. In this situation, firm 2's IC constraint is not binding. The example in Figure 3 demonstrates that higher leverage can intensify price competition.

2.3 Financial Contagion Through Endogenous Competition

The following proposition shows how a firm's financial leverage affects its own and its peer's profitability through endogenous changes in competition intensity. The proof can be found in Appendix E.

Proposition 2.3 (Effects of Financial Leverage on Product Pricing Behaviors)**.** *For $i \neq j \in \{1,2\}$, the partial derivative of θ^C with respect to b_i is*

$$\frac{\partial \theta^C}{\partial b_i} = \begin{cases} 0, & \text{if } \ b_i < b_j \\ -\dfrac{\dot{\varphi}(b_i)/\nu + d_i}{\eta - (1/\nu + d_i)}, & \text{if } \ b_i > b_j, \end{cases}$$

where $d_i \equiv 1/(e^{\theta^C - b_i} - 1)$ and $\dot{\varphi}(b)$ is the derivative of $\varphi(b)$. Therefore, the log profitability θ^C is decreasing in both b_1 and b_2 in the collusive equilibrium.

Proposition 2.3 shows that if firm i increases its financial leverage by increasing b_i, both firm i and its rival firm j will have lower profitability (i.e., θ^C decreases) in the collusive equilibrium. As summarized in the following corollary, this reduced profitability will raise firm j's default probability, leading to a spillover effect of financial distress. The proof is in Appendix F.

Corollary 2.1 (Financial Distress Spillovers Through Endogenous Competition)**.** *An increase in leverage for firm i, with b_j kept unchanged, increases the default probability of rival firm j in the collusive equilibrium. That is,*

$$\frac{\partial \lambda_j^C}{\partial b_i} \geq 0, \text{ with } \ i \neq j \in \{1,2\}, \tag{15}$$

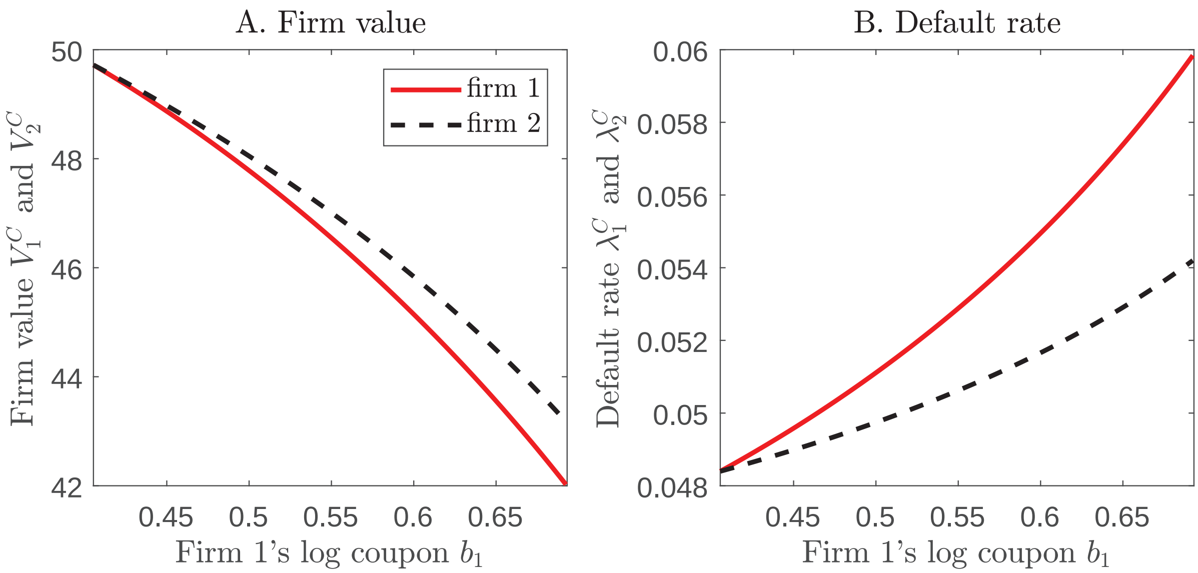

Figure 4. Illustration of contagion effects and negative externality of financial leverage. Parameter values are set at $\tau = 0.35$, $\bar{\varphi} = 0.2$, $\rho = \exp(-20.5)$, $\eta = 21$, $\nu = 0.3$, $\overline{P} = 3.5$, and $b_2 = \ln(1.5)$.

where "$>$" holds if and only if $b_i > b_j$.

The financial contagion effect refers to the phenomenon where an increase in one firm's leverage reduces the market value of its rivals in the same industry due to a combination of lower profitabilities and higher default probabilities. Figure 4 provides a graphical illustration of this negative externality. In panel A, we observe that as b_1, capturing the leverage of firm 1, increases, the market value of both firms declines. Notably, the market value of firm 2, the rival firm in the same industry, may decrease at a rate comparable to that of firm 1. In panel B, we observe that as b_1 increases, the default probability of both firms increases. Both panels of Figure 4 demonstrate that the increase in leverage by one firm has spillover effects on its competitors, resulting in a decline in their market values and a rise in their default probabilities.

The classical tradeoff theory assumes that a firm's capital structure choice is independent of the actions or financial conditions of its product market rivals. However, recent evidence suggests that there is an important spillover effect of product market rivals' actions on a firm's distress risk level, as documented in studies such as Leary and Roberts (2014) and Dou, Johnson and Wu (2023). Proposition 2.3 and its Corollary 2.1 provide a theoretical foundation for these observed contagion patterns. They indicate that, due to the contagion effect, in a decentralized equilibrium, leverage is excessively high from an industry perspective, compromising the industry's financial stability.

In essence, our model shows that, under the collusive form of competition, firms do not fully internalize the impact of their leverage choices on their rivals in the same industry, even though they tacitly collude on profitability. As a result, the optimal levels of leverages, which are chosen separately by each firm to maximize their individual firm values in a decentralized manner, do not collectively maximize the total market value of the whole industry (i.e., the sum of the values of the firms in the industry).

The implications of the theoretical setting of industry-level optimal leverages are relevant in practice. For example, in some industries, firms have common institutional investors as owners. Such common ownership can influence the firms' leverage decisions so that they are jointly chosen to maximize the total market value of the firms. This coordination may occur even though the common owners may not have control over the firms' production or price-setting decisions. The contagion effect identified in our model sheds light on the impact of common ownership on financial stability in real-world industries.

2.4 Competition-Distress Feedback Loop

The following proposition demonstrates that an increase in financial leverage (b_i) by firm i results in a higher default probability, not only due to the direct effect of higher leverage but also due to a positive feedback effect arising from the endogenously intensified product market competition. This feedback mechanism between the financial and product markets is intuitive. When firm i increases b_i, its default probability rises directly. Consequently, both firms experience a reduction in profitability due to the endogenous decrease in coordination capacity, as discussed in Section 2.3. The decreased profitability, in turn, further amplifies the increase in the default probability of firm i. The proof is in Appendix G.

Proposition 2.4 (Feedback Loop Between Financial and Product Markets)**.** *With firm j's coupon b_j remaining unchanged, an increase in firm i's financial leverage via an increase in b_i triggers a feedback loop between the financial and product markets, amplifying the impact of b_i on the default probability λ_i^C:*

$$\begin{bmatrix} d\lambda_i^C \\ d\theta^C \end{bmatrix} = \underbrace{\begin{bmatrix} \partial\lambda^C(\theta, b_i)/\partial b_i\big|_{\theta=\theta^C} \\ 0 \end{bmatrix}}_{\textit{Initial direct effect} \geq 0} db_i + \underbrace{\begin{bmatrix} 0 & \partial\lambda^C(\theta, b_i)/\partial\theta\big|_{\theta=\theta^C} \\ \partial\theta^C/\partial\lambda_i^C\big|_{b_j} & 0 \end{bmatrix}}_{\textit{Higher-order feedback effect} \leq 0} \begin{bmatrix} d\lambda_i^C \\ d\theta^C \end{bmatrix},$$

where the term $\partial\theta^C/\partial\lambda_i^C\big|_{b_j}$ denotes the sensitivity of log profitability θ^C to firm i's default probability λ_i^C, with b_j kept unchanged, in the collusive equilibrium, and the expressions $\partial\lambda^C(\theta, b_i)/\partial b_i$ and $\partial\lambda^C(\theta, b_i)/\partial\theta$ are standard partial derivatives of $\lambda^C(\cdot, \cdot)$. Importantly, the system is stable because the eigenvalues of the matrix that captures the higher-order feedback effect lie on $(-1, 1)$.

As shown in Proposition 2.4, an increase in b_i drives up the default probability λ_i^C not only

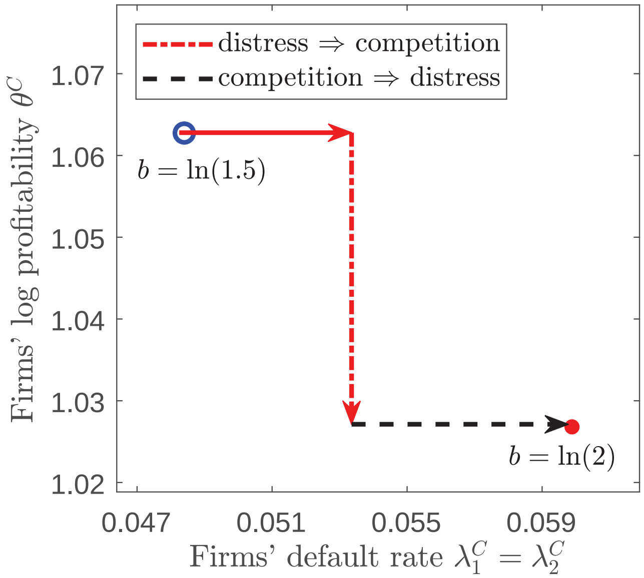

Figure 5. Illustration of the competition-distress feedback loop. Consider the symmetric case with $b_1 = b_2 = b$. The open circle represents the initial collusive equilibrium with $b = \ln(1.5)$. The dot represents the collusive equilibrium with $b = \ln(2)$, which is reached after the shock to financial distress (i.e., after b_1 and b_2 both increase from $\ln(1.5)$ to $\ln(2)$). The arrows represent the process of the firms' responses regarding θ^C and λ^C. The horizontal solid line represents the initial direct effect of an increase in b from $\ln(1.5)$ to $\ln(2)$ on the default probability, while holding firms' log profitability θ^C unchanged. The vertical dash-dotted line represents the decrease in firms' log profitability caused by the increase in their default probability via the endogenously decreased collusion capacity. Finally, the horizontal dashed line represents the further increase in firms' default probability due to the decrease in their profitability. Other parameters are set at $\tau = 0.35$, $\bar{\varphi} = 0.2, \rho = \exp(-20.5), \nu = 0.3, \eta = 21$, and $\overline{P} = 3.5$.

through the initial direct effect, captured by the first term with $\partial\lambda^C(\theta, b_i)/\partial b_i|_{\theta=\theta^C} > 0$, but also through the feedback effect, captured by the second term with $\partial\lambda^C(\theta, b_i)/\partial\theta|_{\theta=\theta^C} < 0$ and $\partial\theta^C/\partial\lambda_i^C|_{b_j} < 0$. Specifically, an increase in b_i directly leads to an increase in the default probability λ_i^C, which will in turn lead to a decline in the log profitability of both firms, θ^C, because a higher default probability suppresses the coordination capacity of rivals (see the IC constraints in (7) and Proposition 2.3). To close the feedback loop, the decline in log profitability θ^C further increases the default probability λ_i^C (see equation (5)).

Figure 5 visually illustrates the competition-distress feedback loop. An increase in both firms' log coupon levels from $\ln(1.5)$ to $\ln(2)$ leads to two distinct effects on default probability. While the direct effect is represented by the horizontal solid line, the indirect effect, even slightly more pronounced, is illustrated by the horizontal dashed line.

Naturally, the competition-distress feedback loop amplifies the effect of changes in firms' coupon on their default probability to a greater extent if firms are more financially distressed. This is formalized in the following corollary. The proof is in Appendix H.

Corollary 2.2 (Amplification Due to Feedback Effects)**.** *Suppose firms are symmetric with*

$b_1 = b_2 = b$. The sensitivity $\partial\theta^C/\partial b$ becomes more negative as b increases.

2.5 Profitability-Leverage Puzzle

We now demonstrate how the competition-distress feedback effect can reconcile the tradeoff theory of capital structure and the observed negative profitability-leverage relation. Following Leland (1994), we assume that firm i optimally chooses its log coupon b_i to maximize its initial firm value, and b_i remains unchanged afterwards.[17] Each firm maximizes its own value by optimally choosing its leverage, taking the rival's leverage choice as given. The firms do not choose their capital structures to jointly maximize the total value of the industry. The following proposition characterizes the optimal capital structure decisions in the equilibrium. The proof can be found in Appendix I.

Proposition 2.5 (Optimal Leverage Decisions)**.** *The best response function $\mathbf{b}(b)$ is defined as follows:*

$$\mathbf{b}(b_j) \equiv \underset{b_i}{\operatorname{argmax}}\, V_i^C(b_i, b_j), \ \ \textit{with } i \neq j \in \{1,2\}. \tag{16}$$

We focus on the Nash equilibrium in which (b^C, b^C) is the unique Pareto efficient point of the following Nash equilibrium set in the log-coupon space:

$$\mathcal{B} \equiv \left\{(b_1, b_2) \ : \ b_i = \mathbf{b}(b_j) \ \textit{ and } \ b_i = b_j, \ \textit{ with } i \neq j \in \{1,2\}\right\}. \tag{17}$$

Consequently, the log coupon in the equilibrium, b^C, is characterized by:

$$b^C = \min\{b \ : \ (b,b) \in \mathcal{B}\}. \tag{18}$$

Similar to characterizing the collusive equilibrium of firms' profitabilities (θ_1, θ_2) given their coupon levels (b_1, b_2) in Proposition 2.2, there are also infinitely many Nash equilibria that satisfy (17). We use Proposition K.1 in Appendix K to formally show that $\mathcal{B}$ contains infinitely many points. To maintain consistency, we adopt the same intuitive equilibrium selection procedure to focus on the unique Pareto efficient Nash equilibrium. A Pareto efficient Nash equilibrium $(b_1, b_2) \in \mathcal{B}$ cannot be Pareto dominated by a different Nash equilibrium $(\tilde{b}_1, \tilde{b}_2) \in \mathcal{B}$ in terms of firm values.

[17] Models with a dynamic capital structure (e.g., Goldstein, Ju and Leland, 2001; Hackbarth, Miao and Morellec, 2006; Bhamra, Kuehn and Strebulaev, 2010; Chen, 2010) allow firms to optimally issue more debt when current cash flows surpass a threshold, which helps generate stationary default rates under a general-equilibrium setup. Adopting the static optimal capital structure makes the model more tractable but does not change the main insights or results of this paper.

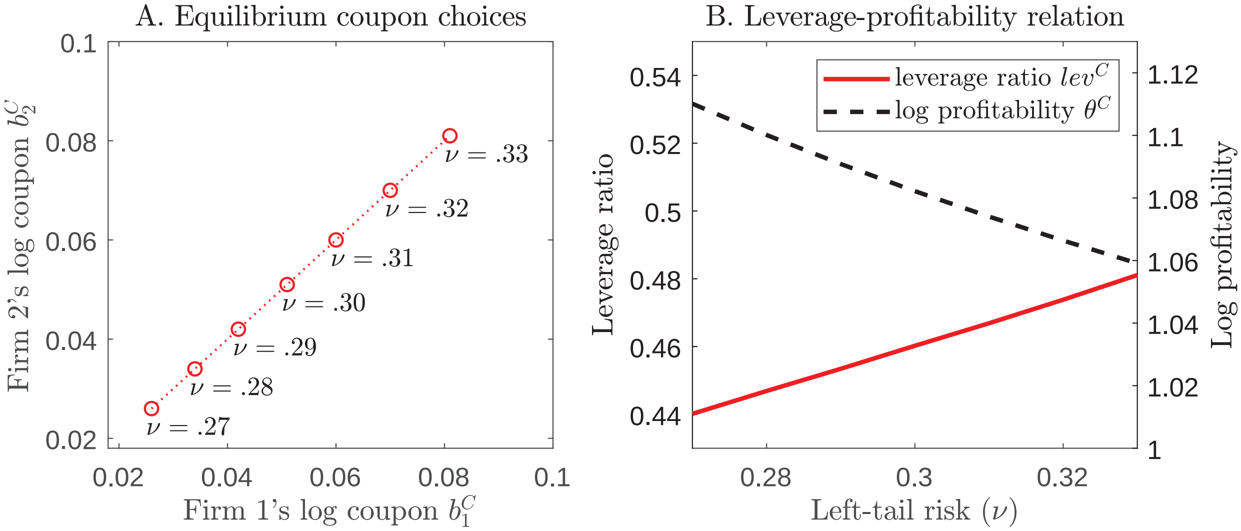

Figure 6. Illustration of the impact of idiosyncratic left-tail risk on financial leverage and product pricing decisions. We set $\tau = 0.35$, $\bar{\varphi} = 0.2$, $\rho = \exp(-20.5)$, $\eta = 21$, and $\overline{P} = 3.5$.

As a key contribution, we show that the competition-distress feedback effect rationalizes the profitability-leverage puzzle. The following proposition formalizes this result and the proof is in Appendix J.

Proposition 2.6 (Negative Profitability-Leverage Relation). *As the idiosyncratic left-tail risk, ν, increases, the equilibrium optimal log coupon b^C increases, the equilibrium market leverage ratio $lev^C(b^C, b^C)$ increases, and the equilibrium log profitability $\theta^C(b^C, b^C)$ decreases.*

Panel A of Figure 6 illustrates the result of Proposition 2.6 that an increase in the left-tail risk ν scales up the optimal log coupon levels (b^C, b^C). Panel B shows that, as ν increases, the leverage ratio $lev^C(b^C, b^C)$ increases, whereas log profitability $\theta^C(b^C, b^C)$ decreases.

Not only does Proposition 2.6 show how the competition-distress feedback effect rationalizes the observed profitability-leverage puzzle, but it also offers a novel insight into the tradeoff theory of optimal capital structure — the competition-distress feedback effect imposes an additional source of "financial distress costs," which are incurred to raise leverage. Under traditional tradeoff theory, when a firm raises its leverage, its default probability increases immediately, and thus its financial distress costs also increase due to the ex-post efficiency losses of bankruptcy. Our theory adds a new source of financial distress costs to traditional tradeoff theory. Specifically, we show that an increase in the default probability endogenously reduces the capacity for tacit collusion, thus suppressing the firms' profitability, which in turn leads to the competition-distress feedback loop as illustrated in Figure 5 and Proposition 2.4. Such feedback effects create an additional source of financial distress costs.

Proposition 2.6 shows that industries with higher idiosyncratic left-tail risk (i.e., a higher

ν) have a higher leverage ratio lev^C, but a lower log profitability θ^C; as a result, profitability and financial leverage are negatively correlated across industries. The reason is that firms in an industry with higher idiosyncratic left-tail risk face a higher likelihood of default and thus find it harder to tacitly collude, which leads to lower profitability. Meanwhile, these industries face a weaker competition-distress feedback effect and thus have lower financial distress costs, which leads to higher optimal financial leverage. In Online Appendix 3.4, we provide direct empirical tests of the proposed economic mechanism, under which the heterogeneous idiosyncratic left-tail risk of industries plays a vital role in generating the negative association between profitability and financial leverage in the cross-section of industries.

3 Full-Fledged Quantitative Model

In Section 2, we present a simple model built upon simplifying assumptions to ensure its analytical tractability. Here, we develop a full-fledged quantitative model that generalizes the simple model across several dimensions. This quantitative model is solved numerically with the following objectives. First, we aim to validate that the primary economic mechanisms and their implications from the simple model remain robust under more general and more realistic specifications. Second, by aligning parameters with data moments, we aim to gauge the quantitative strength of the mechanism and evaluate its empirical relevance within the context of this quantitative model. Finally, this quantitative model enables us to study important theoretical implications that are beyond the capacity of the simple model, like the effect of entry barriers on predatory pricing behaviors. Predatory pricing is a strategy in which a firm sells its products or services at a very low price with the intent of driving rivals out of the market. Once the competition is eliminated or sufficiently reduced, the predatory firm may raise its prices to recoup losses or even to monopolize the market, exploiting its increased market power. The rationale behind this strategy is that the predatory firm bears the short-term losses from the low prices because the long-term gains (once rivals are out of the market) will more than compensate for these losses. In the simple model studied in Section 2, predatory pricing is absent. This is because, upon the event of default and exit, a new firm of comparable size to the incumbent enters the market.

3.1 Model Formulation

We consider an infinite-horizon, discrete-time model where the time periods are indexed by t. Within this model, two firms operate in the same industry. The economy features a constant risk-free rate, denoted by r_f, with $r_f \geq 0$. At the initial time $t = 0$, each firm i, where $i \in \{1, 2\}$, is financed through external equity and long-term consol debt, bearing a coupon payment of e^{b_i}.

Cash Flows Driven by Market Competition. Similar to the simple model, firm i's earnings after interest expenses in period t are

$$\mathcal{E}_{i,t} = (P_{i,t} - \omega)e^{z_{i,t}}C_{i,t} - e^{b_i}, \tag{19}$$

where ω is the marginal cost of production, $P_{i,t}$ is the price, $e^{z_{i,t}}C_{i,t}$ is the demand for firm i's products, and $z_{i,t}$ is a firm-specific demand shock, for firm $i \in \{1, 2\}$. The timeline of a firm's decisions in each period is as follows. The firm first names its price and receives the demand for its goods. Next, the firm uses its operating cash flows to make interest payments, and then decides whether to default. Finally, if it chooses not to default, the firm pays taxes and distributes the remaining cash flows to its shareholders as dividends. Specifically, firm i's earnings after interest expenses and taxes are $(1 - \tau)\mathcal{E}_{i,t}$, where the corporate tax rate is $\tau \in (0, 1)$.

In each period t, the two firms simultaneously name their prices $P_{1,t}$ and $P_{2,t}$, knowing that they face downward-sloping within- and cross-industry demand functions. Once prices are set, the demand $e^{z_{1,t}}C_{1,t}$ and $e^{z_{2,t}}C_{2,t}$ are determined according to the following demand system. To characterize how industry demand depends on the industry-level price index P_t, we follow the literature (e.g., Hopenhayn, 1992; Pindyck, 1993; Caballero and Pindyck, 1996) and postulate a downward-sloping isoelastic industry demand curve. Specifically, the industry demand is $e^{a_t}C_t$, where $e^{a_t} \equiv \sum_{i=1}^{2} e^{z_{i,t}}$ is the industry-level demand shock and C_t is governed by

$$C_t = P_t^{-\epsilon}, \tag{20}$$

where the parameter $\epsilon > 1$ captures the industry-level price elasticity of demand.[18] The industry-level ϵ typically represents the elasticity of substitution across goods produced in different industries.

Next, we introduce the demand system for differentiated goods within an industry. Given C_t and P_t, consumers decide on a basket of differentiated goods based on the prices $P_{1,t}$ and $P_{2,t}$ charged by firms 1 and 2, respectively. Specifically, C_t equals a Dixit-Stiglitz constant

[18] To microfound such an isoelastic industry demand curve, consider a continuum of industries that exist in the economy and produce differentiated industry-level composite goods. The elasticity of substitution across industry-level composite goods is ϵ and the preference weight for an industry-level composite good is equal to e^{a_t} (Dou, Ji and Wu, 2021*b*). The CES utility function that embodies the aggregate preference for diversity over differentiated products can be further microfounded by the characteristics (or address) model and discrete choice theory (e.g., Anderson, Palma and Thisse, 1989).

elasticity of substitution (CES) aggregator that combines $C_{1,t}$ and $C_{2,t}$, specified as follows:

$$C_t = \left[\sum_{i=1}^{2} \left(e^{z_{i,t}-a_t}\right) C_{i,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}, \tag{21}$$

where the parameter $\eta > 1$ captures the elasticity of substitution among goods produced by the two firms in the same industry. Intuitively, the weight $e^{z_{i,t}-a_t}$ captures consumers' relative preference for firm i's goods. We assume that $\eta \geq \epsilon > 1$, meaning that goods within the same industry are more substitutable than those across industries.[19]

From the CES aggregator, the firm-level demand curve immediately follows. Specifically, given the prices $P_{i,t}$ for $i = 1, 2$ and C_t, the demand for firm i's goods $e^{z_{i,t}}C_{i,t}$ can be obtained by solving a standard expenditure minimization problem:

$$e^{z_{i,t}}C_{i,t} = e^{z_{i,t}-a_t}\left(\frac{P_{i,t}}{P_t}\right)^{-\eta} e^{a_t}C_t, \quad \text{with the price index } P_t = \left[\sum_{i=1}^{2} e^{z_{i,t}-a_t} P_{i,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}. \tag{22}$$

All else equal, the demand for firm i's goods, $e^{z_{i,t}}C_{i,t}$, increases with consumers' relative preference $e^{z_{i,t}-a_t}$ for firm i's goods in equilibrium. A larger $e^{z_{i,t}-a_t}$ implies that firm i's price $P_{i,t}$ has a greater influence on the price index P_t. The demand curves at the industry level (20) and the firm level (22) yield:

$$C_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\eta} P_t^{-\epsilon}. \tag{23}$$

The short-run price elasticity of demand for firm i's goods is

$$-\frac{\partial \ln(e^{z_{i,t}}C_{i,t})}{\partial \ln P_{i,t}} = \underbrace{\mu_{i,t}\left[-\frac{\partial \ln(e^{z_{i,t}}C_{i,t})}{\partial \ln P_t}\right]}_{\text{cross-industry}} + \underbrace{(1-\mu_{i,t})\left[-\frac{\partial \ln(e^{z_{i,t}}C_{i,t}/(e^{a_t}C_t))}{\partial \ln(P_{i,t}/P_t)}\right]}_{\text{within-industry}} \tag{24}$$

$$= \mu_{i,t}\epsilon + (1-\mu_{i,t})\eta, \tag{25}$$

where $\mu_{i,t}$ is the (revenue) market share of firm i, which equals $\mu_{i,t} = e^{z_{i,t}-a_t}\left(P_{i,t}/P_t\right)^{1-\eta}$. Equation (24) shows that the short-run price elasticity of demand is given by the average of η and ϵ, weighted by the firm's market share $\mu_{i,t}$. When its market share $\mu_{i,t}$ shrinks (grows), within-industry (cross-industry) competition becomes more relevant for firm i, so its price elasticity of demand depends more on η (ϵ). There are two extreme cases, $\mu_{i,t} = 0$

[19] For example, the elasticity of substitution between the Apple iPhone and the Samsung Galaxy is likely to be much higher than that between a cell phone and a cup of coffee. This assumption is consistent with those in the literature (e.g., Atkeson and Burstein, 2008; Corhay, Kung and Schmid, 2020*a*; Dou, Ji and Wu, 2021*a*).

and $\mu_{i,t} = 1$. In the former case, firm i becomes atomistic and takes the industry price index P_t as given; as a result, firm i's price elasticity of demand is exactly η. In the latter case, firm i monopolizes the industry, and its price elasticity of demand is exactly ϵ.

In every period t, each firm i faces the risk of an idiosyncratic left-tail shock, which occurs with a probability of ν. Should this shock materialize, the affected firm will experience a complete cessation in demand, with $e^{z_{i,t}} = 0$. Consequently, this firm will be compelled to immediately exit the market.[20] If the left-tail shock does not occur, the firm-specific demand shock $z_{i,t}$ evolves according to:

$$z_{i,t} = z_{i,t-1} + g + u_t + u_{i,t}, \tag{26}$$

where the parameter g captures the firm's expected growth rate, the random variables $u_t \sim N(0, \varsigma^2)$ and $u_{i,t} \sim N(0, \sigma^2)$ capture aggregate and idiosyncratic shocks, respectively. The left-tail shocks, u_t, and $u_{i,t}$ are mutually independent.

Several elements in our model merit emphasis. First, the aggregate shock u_t in equation (26) captures economy-wide or industry-wide shocks. It carries a market price of risk $\gamma > 0$. In Section 4.2 and Online Appendix 1.3.2, we study the role of γ in determining the endogenous distressed competition mechanism. Second, the idiosyncratic shocks, $u_{1,t}$ and $u_{2,t}$, are firm-specific shocks. They are needed for the model to quantitatively match the default frequency and generate a nondegenerate cross-sectional distribution of market shares in the stationary equilibrium. Third, the idiosyncratic left-tail shocks play a crucial role in our theory and empirical results. Idiosyncratic left-tail risk has been proven useful in explaining credit spreads and credit default swap index (CDX) spreads (e.g., Delianedis and Geske, 2001; Collin-Dufresne, Goldstein and Yang, 2012; Kelly, Manzo and Palhares, 2018; Seo and Wachter, 2018).

Endogenous Profitabilities and Externalities. Now, we characterize the profitability function. Firm i's operating profits are

$$\left(P_{i,t} - \omega\right) e^{z_{i,t}} C_{i,t} = \Pi_i(\theta_{i,t}, \theta_{j,t}) e^{z_{i,t}}, \text{ with } \Pi_i(\theta_{i,t}, \theta_{j,t}) \equiv \omega^{1-\epsilon} \theta_{i,t} \left(1 - \theta_{i,t}\right)^{\eta-1} \left(1 - \theta_t\right)^{\epsilon-\eta}, \tag{27}$$

where $\theta_{i,t}$ and θ_t represent the firm- and industry-level profit margins,

$$\theta_{i,t} \equiv \frac{P_{i,t} - \omega}{P_{i,t}} \text{ and } \theta_t \equiv \frac{P_t - \omega}{P_t}, \text{ respectively,} \tag{28}$$

[20] Our specification is close to that of Seo and Wachter (2018), who calibrate a disastrous idiosyncratic jump of almost -100%, under which a firm's exit is certain.

and it directly follows from equation (22) that the relation between $\theta_{i,t}$ and θ_t is

$$1-\theta_t = \left[\sum_{i=1}^{2} e^{z_{i,t}-a_t}(1-\theta_{i,t})^{\eta-1}\right]^{\frac{1}{\eta-1}}. \tag{29}$$

Equation (27) shows that firm i's profits depend on its rival j's profit margin $\theta_{j,t}$ through the industry's profit margin θ_t. This reflects the externality of firm j's profit margin decisions. For example, holding firm i's profit margin fixed, if firm j cuts its profit margin $\theta_{j,t}$, the industry's profit margin θ_t will drop, which will reduce the demand for firm i's goods $C_{i,t}$ (see equation (23)), compromising firm i's profits. Below, we explain the Nash equilibrium, which determines the profit margin strategies $(\theta_{1,t}, \theta_{2,t})$.

Financial Distress. Firm i can optimally choose to file for bankruptcy and exit when its equity value drops to zero because of negative demand shocks to $z_{i,t}$. As in the simple model, we maintain tractability by ensuring that industry entry of a new firm is contingent upon the exit of an incumbent. This new entrant is characterized by an initial demand shock $e^{z_{new}} = \kappa e^{z_{j,0}} > 0$ and an optimally set coupon $e^{b_{new}}$. Here, the parameter $\kappa > 0$ offers insight into the relative size of the new firm when juxtaposed against the initial size of the surviving incumbent, firm j. Intuitively, the magnitude of κ serves as a barometer for the entry threat posed by potential new entrants to the incumbent firms. Upon the entry of new firm, the dynamic game of industry competition, which we describe in Section 3.2 below, is "reset" to a new one between the surviving incumbent firm and the new entrant.

3.2 Nash Equilibrium

Non-Collusive Equilibrium. In our model, the two firms within an industry engage in a supergame (Friedman, 1971). This involves an infinite repetition of stage games where profit margins are set, taking into account both exogenous and endogenous state variables that change over time. The strategies employed by the firms depend on "payoff-relevant" states $z_t \equiv \{z_{1,t}, z_{2,t}\}$ in the state space $\mathcal{Z}$, as in Maskin and Tirole (1988*a*,*b*).

Specifically, the non-collusive equilibrium comprises a profit-margin-setting scheme, $\Theta^N(\cdot) = (\theta_1^N(\cdot), \theta_2^N(\cdot))$, alongside a default choice, $\mathcal{D}^N(\cdot) = (d_1^N(\cdot), d_2^N(\cdot))$. Both pairs of functions are defined on the state space $\mathcal{Z}$. For each firm i, it strategically chooses its optimal profit margin $\theta_i^N(z_t)$ and the optimal binary default decision $d_i^N(z_t) \in \{0,1\}$ under the assumption that the rival firm j remains committed to the non-collusive equilibrium profit margin $\theta_j^N(z_t)$ and binary default choice $d_j^N(z_t) \in \{0,1\}$.

We formulate the optimization problems as a pair of recursive equations:

$$
\begin{aligned}
E_i^N(z_t) = \max_{\theta_{i,t}, d_{i,t}} \ & (1-\tau)\left[\Pi_i(\theta_{i,t}, \theta_{j,t}^N)e^{z_{i,t}} - e^{b_i}\right] \\
& + (1-d_{i,t})(1-d_{j,t}^N)(1-\nu)\mathbb{E}_t\left[M_{t,t+1}E_i^N(z_{t+1})\right] \\
& + (1-d_{i,t})d_{j,t}^N(1-\nu)\mathbb{E}_t\left[M_{t,t+1}\widetilde{E}_i^N(\widetilde{z}_{t+1})\right], \text{ with } i \neq j \in \{1,2\},
\end{aligned} \tag{30}
$$

where $\theta_{i,t}^N \equiv \theta_i^N(z_t)$ and $d_{i,t}^N \equiv d_i^N(z_t)$ are strategies in the non-collusive Nash equilibrium, the profitability function $\Pi_i(\cdot,\cdot)$ is defined in (27), $E_i^N(\cdot)$ is the equity value of firm i in the non-collusive Nash equilibrium, $M_{t,t+1}$ is the exogneously-specified intertemporal marginal rate of substitution, $\widetilde{z}_{t+1} = \{z_{i,t+1}, z_{new}\}$ captures the scenario where firm j chooses to default in period t and a new firm enters with initial idiosyncratic demand shock z_{new} in period $t+1$, and $\widetilde{E}_i^N(\cdot)$ represents the equity value of firm i, accounting for the situation in which firm j chooses to default in period t and a new firm enters with an optimal log coupon level of b_{new}.

The coupled recursive equations provide the solutions for profit margins $\theta_i^N(z_t)$ and default decisions $d_i^N(z_t)$ for $i = 1,2$ in the non-collusive equilibrium. Each firm i's optimal default decision $d_i^N(z_t)$ implies an endogenous default boundary $\underline{z}_i^N(z_{j,t})$ such that the firm defaults if $z_{i,t} \leq \underline{z}_i^N(z_{j,t})$ in the non-collusive equilibrium.[21]

Competition Under Tacit Collusion. Our main focus is the collusive equilibrium, which is sustained using the non-collusive equilibrium as a punishment strategy. Firms tacitly collude with each other in setting higher profit margins, with any deviation potentially triggering a switch to the non-collusive equilibrium.

In the collusive equilibrium, strategies not only depend on "payoff-relevant" states z_t, but also on a pair of indicator functions that track whether either firm has previously deviated from the collusive agreement, as in Fershtman and Pakes (2000, p. 212).[22] Consider a generic collusive equilibrium in which the two firms follow a collusive profit-margin-setting scheme. If one firm deviates from the collusive profit-margin-setting scheme in period t, then with probability ξ, the other firm will implement a punishment strategy in the next period $t+1$, under which it will forever set the non-collusive profit margin.[23]

[21] In a continuous-time model, multi-dimensional free boundary problems can be handled by generalized value matching and smooth pasting conditions (e.g., Chen et al., 2022; Kakhbod et al., 2022).

[22] For notational simplicity, we omit the indicator states of historical deviations.

[23] One interpretation is that, with a probability of $1-\xi$, the deviator can persuade its rival not to enter the non-collusive equilibrium. Ex-post renegotiations can occur because the non-collusive equilibrium is not renegotiation-proof or "immune to collective rethinking" (Farrell and Maskin, 1989). The strategy we consider is essentially a probabilistic punishment strategy.

In our quantitative model, each period represents a month. With the span per period fixed, the parameter ξ captures the time frame in which a deviating firm can reap additional profits prior to any retaliatory actions by its rival. Consequently, a higher ξ leads to a more immediate punishment, thereby diminishing the incentives for deviation and bolstering the capacity for collusion. The relationship between ξ and the competition-distress feedback loop is characterized by its non-monotonic nature. When $\xi = 0$, any punitive measures become unenforceable, preventing any form of tacit collusion between firms. Consequently, firms engage in non-collusive competition, effectively nullifying the competition-distress feedback loop. Conversely, when $\xi = 1$, the perfect cartel can almost be maintained irrespective of the financial health of the firms. This results in firms behaving akin to a single monopolistic entity, again causing the competition-distress feedback loop to dissipate.

Similarly, with ξ held constant, the duration of each period in the model influences collusion capacity. A shorter period hastens punitive actions, reducing deviation incentives and enhancing the capacity for collusion. Crucially, to preserve a consistent collusion capacity, adjustments to the value of parameter ξ are necessary when modifying the period length in the model.

In Section 4.1, we calibrate the monthly quantitative model to match the key relevant moments in the data. There, we adjust ξ to ensure the model-implied average gross profit margin aligns with its empirical counterpart in the data. Importantly, around our calibrated value of ξ, the intensity of the competition-distress feedback loop increases as ξ rises, primarily due to a more pronounced decline in profit margin resulting from increased financial distress.

Formally, the set of incentive compatible collusion agreements, denoted by $\mathcal{C}$, consists of all profit-margin-setting schemes $\Theta^C(\cdot) \equiv (\theta_1^C(\cdot), \theta_2^C(\cdot))$ such that the following participation constraint (PC) and IC constraint are satisfied:

$$E_i^N(z) \leq E_i^C(z), \quad \text{for all } z \in \mathcal{Z}, \text{ and} \qquad \text{(PC)} \qquad (31)$$

$$E_i^D(z) \leq E_i^C(z), \quad \text{for all } z \in \mathcal{Z}, \qquad \text{(IC)} \qquad (32)$$

where $i \in \{1, 2\}$, $E_i^D(z)$ is firm i's equity value if it chooses to deviate from collusion, and $E_i^C(z)$ is firm i's equity value in the collusive equilibrium.

Given a specified profit-margin-setting scheme, $(\theta_1^C(\cdot), \theta_2^C(\cdot))$, the value functions $E_i^C(z_t)$ and their corresponding optimal default decisions $d_i^C(z_t)$ are determined by the following coupled recursive equations:

$$
\begin{aligned}
E_i^C(z_t) = \max_{d_{i,t}} \ & (1-\tau)\left[\Pi_i(\theta_{i,t}^C, \theta_{j,t}^C)e^{z_{i,t}} - e^{b_i}\right] \\
& + (1-d_{i,t})(1-d_{j,t}^C)(1-\nu)\mathbb{E}_t\left[M_{t,t+1}E_i^C(z_{t+1})\right] \\
& + (1-d_{i,t})d_{j,t}^C(1-\nu)\mathbb{E}_t\left[M_{t,t+1}\widetilde{E}_i^C(\widetilde{z}_{t+1})\right], \text{ with } i \neq j \in \{1,2\}, \qquad (33) \\
& \text{subject to the PC constraint (31) and the IC constraint (32),}
\end{aligned}
$$

where $\theta_{i,t}^C \equiv \theta_i^C(z_t)$ and $d_{i,t}^C \equiv d_i^C(z_t)$ are strategies in the collusive Nash equilibrium, the profitability function $\Pi_i(\cdot,\cdot)$ is defined in (27), $E_i^C(\cdot)$ represents the equity value of firm i in the collusive Nash equilibrium, $M_{t,t+1}$ is the exogneously-specified intertemporal marginal rate of substitution, $\widetilde{z}_{t+1} = \{z_{i,t+1}, z_{new}\}$ is defined in (30), and $\widetilde{E}_i^C(\cdot)$ represents the equity value of firm i, accounting for the situation in which firm j chooses to default in period t and a new firm enters with an optimal log coupon level of b_{new}.

Equilibrium Deviation Values. When firm i tacitly deviates from collusion, its rival j would continue to follow the collusive profit-margin-setting scheme, $\theta_j^C(\cdot)$, and the optimal default decision, $d_j^C(\cdot)$. As a result, the optimal deviation value can be formulated recursively as follows:

$$
\begin{aligned}
E_i^D(z_t) = \max_{\theta_{i,t}, d_{i,t}} \ & (1-\tau)\left[\Pi_i(\theta_{i,t}, \theta_{j,t}^C)e^{z_{i,t}} - e^{b_i}\right] \\
& + (1-d_{i,t})(1-\nu)(1-\xi)\mathbb{E}_t\left\{M_{t,t+1}\left[(1-d_{j,t}^C)E_i^D(z_{t+1}) + d_{j,t}^C\widetilde{E}_i^D(\widetilde{z}_{t+1})\right]\right\} \\
& + (1-d_{i,t})(1-\nu)\xi\mathbb{E}_t\left\{M_{t,t+1}\left[(1-d_{j,t}^C)E_i^N(z_{t+1}) + d_{j,t}^C\widetilde{E}_i^N(\widetilde{z}_{t+1})\right]\right\}, \qquad (34)
\end{aligned}
$$

where $i \neq j \in \{1,2\}$, $\theta_{i,t}^C \equiv \theta_i^C(z_t)$, $d_{i,t}^C \equiv d_i^C(z_t)$, and $\Pi_i(\cdot,\cdot)$ is defined in (33), $E_i^N(\cdot)$, $\widetilde{E}_i^N(\cdot)$, and $\widetilde{z}_{t+1} = \{z_{i,t+1}, z_{new}\}$ are defined in (30), $E_i^D(\cdot)$ denotes the equity value of firm i if it chooses to deviate from the collusion, $\widetilde{E}_i^D(\cdot)$ denotes the equity value of firm i, accounting for the situation in which firm j chooses to default and a new firm enters with an optimal log coupon level of b_{new}, and $M_{t,t+1}$ is the exogneously-specified intertemporal marginal rate of substitution.

Two aspects of this quantitative model warrant further discussion. First, the PC constraint (31) can become binding in the collusive equilibrium, triggering the two firms to switch to the non-collusive equilibrium. The endogenous switch captures the endogenous outbreak of price wars. We assume that once the two firms switch to the non-collusive equilibrium, they will stay there forever.[24] Endogenous switching from the collusive to the non-collusive equilibrium because of increased financial distress (i.e., higher leverage ratios), coupled with high entry barriers of the market, is one of our model's key differences from that of Dou, Ji and Wu (2021*a*), in which firms are financed wholly by equity, insulating them from financial distress, and they never face the threat of new entrants. In their model, the PC constraint is never binding because higher profit margins always lead to higher equity values in the absence of financial distress costs owing to costly default.

Second, there exist infinitely many elements in the set of incentive compatible collusion agreements, $\mathcal{C}$, and hence infinitely many collusive equilibria. We focus on a subset of $\mathcal{C}$, denoted by $\overline{\mathcal{C}}$, consisting of all profit-margin-setting schemes $\Theta^C(\cdot)$ such that the IC constraints (32) are binding state by state, that is, $E_i^D(z_t) = E_i^C(z_t)$ for all $z_t \in \mathcal{Z}$ and $i \in \{1,2\}$.[25] The subset $\overline{\mathcal{C}}$ is nonempty because it contains the profit-margin-setting scheme in the non-collusive equilibrium. Consistent with the simple model presented in Section 2, we further narrow our focus to the "Pareto efficient frontier" of $\overline{\mathcal{C}}$, denoted by $\overline{\mathcal{C}}_p$, consisting of all pairs of $\Theta^C(\cdot)$ such that there does not exist another pair $\widehat{\Theta}^C(\cdot) = (\widehat{\theta}_1^C(z_t), \widehat{\theta}_2^C(z_t)) \in \overline{\mathcal{C}}$ such that the implied firm values are higher for all $z_t \in \mathcal{Z}$ and $i \in \{1,2\}$, with strict inequality held for some i and z_t.[26] Our numerical algorithm is similar to that of Abreu, Pearce and Stacchetti (1990).[27] Deviation never occurs on the equilibrium path. The one-shot deviation principle (Fudenberg and Tirole, 1991) makes it clear that the collusive equilibrium characterized above is subgame perfect.

Debt Value. The debt value equals the sum of the present value of cash flows that accrue to debtholders until the occurrence of an endogenous default or an idiosyncratic left-tail jump shock (i.e., exogenous displacement), whichever occurs first, plus the recovery value. We follow the literature on dynamic debt models (e.g., Mello and Parsons, 1992; Leland, 1994; Hackbarth, Miao and Morellec, 2006) and set the recovery value of endogenous default to a fraction $\delta \in (0,1)$ of the firm's unlevered asset value, $A_i^C(z_t)$, which is the value of an all-equity firm. In the collusive equilibrium, the unlevered asset value $A_i^C(z_t)$ is similarly

[24] As the firm that proposes switching to the non-collusive equilibrium is essentially deviating, we assume that the two firms will not return to the collusive equilibrium. We make this assumption to be consistent with our specification for the punishment strategy.

[25] This equilibrium refinement is similar in spirit to Abreu (1988), Alvarez and Jermann (2000, 2001), and Opp, Parlour and Walden (2014).

[26] One can show that the "Pareto efficient frontier" is nonempty based on the fundamental theorem of the existence of Pareto efficient allocations (e.g., Mas-Colell, Whinston and Green, 1995), as $\overline{\mathcal{C}}$ is nonempty and compact, and the order that we consider is complete, transitive, and continuous.

[27] Alternative methods include those of Pakes and McGuire (1994) and Judd, Yeltekin and Conklin (2003), who use similar ingredients to this paper in their solution method. Proving the uniqueness of the equilibrium under our selection criterion is beyond the scope of this paper. We use different initial points in our numerical algorithm and find robust convergence to the same equilibrium. We provide proof for the uniqueness in the simple model of Section 2.

determined by equation (30) subject to the IC and PC constraints, except for setting $e^{b_i} = 0$ and $d_{i,t} = 0$.

The value of debt in the collusive equilibrium can be characterized by the following recursive equation equations:

$$D_i^C(z_t) = e^{b_i} + d_{i,t}^C \delta A_i^C(z_t) \tag{35}$$

$$+ (1 - d_{i,t}^C)(1 - \nu)\mathbb{E}_t \left\{ M_{t,t+1} \left[(1 - d_{j,t}^C) D_i^C(z_{t+1}) + d_{j,t}^C \widetilde{D}_i^C(\widetilde{z}_{t+1}) \right] \right\}, \tag{36}$$

where $i \neq j \in \{1,2\}$, $d_{i,t}^C \equiv d_i^C(z_t)$ for $i = 1,2$ are optimal default decisions that solve (33), $\widetilde{z}_{t+1} = (z_{i,t+1}, z_{new})$ is defined in (30), $D_i^C(\cdot)$ represents the debt value of firm i in the collusive Nash equilibrium, $\widetilde{D}_i^C(\cdot)$ represents the debt value of firm i, accounting for the situation in which firm j chooses to default in period t and a new firm enters with an optimal log coupon level of b_{new}, and $M_{t,t+1}$ is the exogneously-specified intertemporal marginal rate of substitution.

Optimal Leverage Decisions. We now illustrate the optimal leverage decisions of firms at $t = 0$. The log coupons b_1 and b_2 are optimally determined in the Nash equilibrium, as in the simple model presented in Section 2. The best response function $\mathbf{b_i}(b_j)$ is defined as follows:

$$\mathbf{b_i}(b_j) \equiv \underset{b_i}{\operatorname{argmax}}\, V_i^C(z_0; b_i, b_j), \quad \text{with } i \neq j \in \{1,2\}. \tag{37}$$

The firm value at $t = 0$ is captured by $V_i^C(z_0; b_i, b_j) \equiv E_i^C(z_0; b_i, b_j) + D_i^C(z_0; b_i, b_j)$. In this representation, we slightly modify the notations of $E_i^C(\cdot)$ and $D_i^C(\cdot)$ to clearly denote their dependence on the leverage terms (b_i, b_j). The equilibrium leverage, captured by (b_1^C, b_2^C), is then determined by the subsequent condition:

$$b_i^C = \mathbf{b_i}(b_j^C), \quad \text{for } i \neq j \in \{1,2\}. \tag{38}$$

In alignment with the simple model, firms determine their optimal coupons strategically at $t = 0$, and these decisions remain unchanged subsequently. Importantly, this choice for a static capital structure ensures model simplicity while preserving the paper's primary insights. This model simplification is underpinned by subsequent theoretical developments. When firms have the flexibility to adjust leverage freely, either by issuing or repurchasing debt at any moment to optimize their present equity value without a credible, pre-committed debt policy, the "leverage ratchet effect" emerges (Admati et al., 2018; DeMarzo and He, 2021). In such scenarios, equity holders are never willing to voluntarily reduce leverage but always have an incentive to borrow more, regardless of whether the current leverage is excessive or new debt must be junior to existing claims. As highlighted by DeMarzo and He (2021), the absence of a reliable commitment to a future debt policy makes it prohibitively difficult for firms to raise extra equity to buy back debt. Consequently, firms only issue debt smoothly and inadequately adjust debt levels in response to fluctuations in their cash flows. Other pertinent research, such as those by Benzoni et al. (2022) and Malenko and Tsoy (2020), explores non-Markov perfect equilibria (non-MPE). They theorize the possibility of sustaining non-MPE using a "grim trigger" strategy against any deviations by equity holders, utilizing the MPE characterized by DeMarzo and He (2021) as a viable, incentive-compatible punitive measure. A central insight from this body of research underscores that the leverage ratchet effect becomes especially acute during periods of financial distress for firms. Taken together, the strong reluctance of firms to repurchase debt and the resulting excessively high leverages due to the leverage ratchet effect, especially in financial distress phases, reinforce both the competition-distress feedback and the financial contagion effects driven by market competition proposed in this paper.

4 Quantitative Results

In this section, we calibrate the full-fledged quantitative model and undertake quantitative analyses.

4.1 Calibration and Parameter Choices

We consider a monthly model where each period corresponds to one month. Panel A of Table 1 presents the externally calibrated parameters. The risk-free rate is set at $r_f = 5\%/12$, following Chen et al. (2018). The within-industry elasticity of substitution is set at $\eta = 16$, and the cross-industry price elasticity of demand at $\epsilon = 2$, which are broadly consistent with the calibration and estimation in the IO and international trade literature (e.g., Harrigan, 1993; Head and Ries, 2001; Atkeson and Burstein, 2008). We set $g = 1.8\%/12$, $\gamma = 0.4/\sqrt{12}$, and $\varsigma = 10\%/\sqrt{12}$ as in He and Milbradt (2014). In our model, the tax rate captures the net tax-shield benefit of debt from the perspective of equity holders. Consistent with the literature, we thus set $\tau = 20\%$.[28] We normalize the two firms in the same industry to have unit demand shocks initially, that is, we set $z_{1,0} = z_{2,0} = 0$. The parameter κ is inversely related to entry barrier because a smaller κ leads to a smaller new entrant relative to the size

[28]Following Graham (2000), Chen (2010) calibrates a corporate tax rate of 35%, a personal dividend income tax rate of 12%, and a personal interest income tax rate of 29.6%. Using these values, the implied effective tax rate for the net tax-shield benefit of debt is calculated as $1 - (1 - 35\%)(1 - 12\%)/(1 - 29.6\%) = 18.75\%$, which is close to our calibration.

Table 1: Calibration and parameter choices.

Panel A: Externally determined parameters.					
Parameter	Symbol	Value	Parameter	Symbol	Value
Risk-free rate	r_f	0.42%	Within-industry elasticity	η	16
Industry price elasticity	ϵ	2	Growth rate of cash flows	g	0.15%
Market price of risk for u_t	γ	0.12	Volatility of aggregate shocks	ς	2.89%
Tax rate	τ	20%	Relative size of new entrants	κ	0.3
Initial demand shocks	$z_{1,0}, z_{2,0}$	0	Idiosyncratic left-tail risk	ν	0

Panel B: Internally calibrated parameters and targeted moments.					
Parameter	Symbol	Value	Moments	Data	Model
Recovery ratio of assets	δ	0.5	leverage ratio (Baa rated)	34%	31%
Volatility of idiosyncratic shocks	σ	7.22%	10-year default rate (Baa rated)	4.9%	4.9%
Marginal cost of production	ω	3.2	net profitability	3.9%	4.0%
Punishment rate	ξ	0.5%	gross profit margin	31.4%	30.9%

of the surviving incumbent (i.e., a smaller κ leads to a weaker entry threat). As a benchmark, we calibrate $\kappa = 0.3$, indicating that the new entrant's size is 30% of the initial size of the surviving incumbent. In Section 4.2, we show that the behavior of predatory pricing, as a strong form of contagion effects, becomes particularly prevalent when κ is sufficiently low (i.e., when the entry barrier is high). We set $\nu = 0$ in our benchmark calibration, and in Section 4.3 we analyze how the cross-industry dispersion in ν explains the observed negative relation between profitability and leverage ratios across industries.

The remaining parameters are calibrated by matching the relevant moments summarized in panel B of Table 1. When constructing the model moments, we simulate one industry for 20 years.[29] We then compute the model counterparts of the data. For each moment, the table reports the average value of 100,000 independent simulations. To ensure the model yields an optimal leverage ratio (i.e., the debt-to-asset ratio) of 31%, close to the estimation in our sample, we calibrate $\delta = 0.5$.[30] We calibrate the volatility of idiosyncratic shocks at $\sigma = 25\%/\sqrt{12}$ to match the 10-year default rate of 4.9% estimated by Chen (2010). The marginal cost of production ω is a scaling parameter, which does not affect the model's quantitative implications. We calibrate its value at $\omega = 3.2$ to match the average net profitability of 3.9% estimated in our sample. We set the punishment rate at $\xi = 0.5\%$

[29]We focus on the initial 20 years of simulation because our model has a static capital structure. We emphasize that although our model allows for a stationary distribution in the long run, it is achieved by the exit and entry assumption rather than by the dynamic leverage adjustment that is adopted in models of dynamic capital structure (e.g., Goldstein, Ju and Leland, 2001).

[30]The implied bond recovery rate (i.e., the defaulted bond price divided by its promised face value) is close to the estimate in the literature (e.g., Chen, 2010; Dou et al., 2021*a*; Dou, Wang and Wang, 2022).

so that the gross profit margin implied by the model is 30.9%, consistent with the average value in our sample. In the data, net profitability and gross profit margins are measured as in Dou, Ji and Wu (2021*a*).

4.2 Impact of Market Structure on Peer Interactions

Through the lens of the calibrated quantitative model, we quantify the competition-distress feedback effect and the financial contagion effect through distressed competition in Online Appendices 1.2 and 1.3. In this subsection, we examine how variations in industry market structures impact these feedback and contagion effects.

Price Elasticity of Demand. The cross-industry price elasticity of demand, ϵ, reflects the extent to which the products of an industry are substitutable with those of other industries. Focusing on the collusive equilibrium, panel A of Figure 7 illustrates how the industry's price elasticity of demand influences the competition-distress feedback effect. One way to measure the magnitude of the feedback effect is to examine the sensitivity of profit margins to changes in the market price of risk γ. Intuitively, a rise in γ would effectively increase leverage as it would suppress the equity value more than the debt value, and according to Proposition 2.4, the magnitude of the resulting decrease in profit margins reflects the strength of the competition-distress feedback effect.

The solid and dashed lines in panel A of Figure 7 plot the profit-margin beta to γ when the industry's price elasticity of demand is set at $\epsilon = 2$ and 2.5, respectively. Relative to the baseline case with $\epsilon = 2$, the profit-margin beta is less negative in the case with $\epsilon = 2.5$, especially when the industry is close to the default boundary (i.e., the dashed line is flatter than the solid line). Intuitively, firms find it harder to tacitly collude on high profit margins when the industry's price elasticity of demand is higher, because they would suffer more when the rival firm deviates due to a larger loss of industry-level demand. The weakened collusion capacity reduces the response of profit margins to changes in the distance-to-default, and thus dampens the competition-distress feedback loop, which in turn makes the profit-margin beta to γ less negative, especially when the industry is close to the default boundary.

To examine the impact of the industry's price elasticity of demand on the financial contagion effect, we conduct impulse-response experiments. In particular, we measure the within-industry contagion effect on profit margins by computing the percentage change in firm j's profit margin, relative to the counterfactual scenario without shocks, in response to an unexpected idiosyncratic shock that increases the coupon of firm i at the beginning of

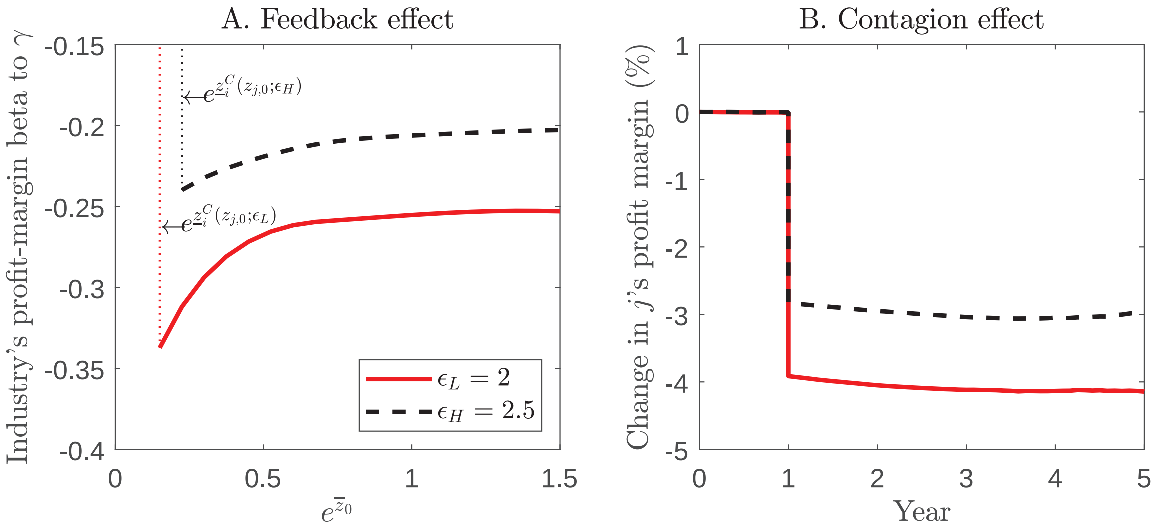

Figure 7. Feedback and contagion effects in industries facing different price elasticities of demand. In panel A, the industry's profit-margin beta to γ at $t = 0$ is $\beta^\theta(\bar{z}_0) \equiv \theta^C(\bar{z}_0; \gamma_H)/\theta^C(\bar{z}_0; \gamma_L) - 1$, where $\gamma_L = 0.12$, $\gamma_H = 0.2$, and $z_{1,0} = z_{2,0} = \bar{z}_0$. The two vertical dotted lines represent firms' default boundaries corresponding to each industry when $\gamma = \gamma_H$. The initial log coupons at $t = 0$ are set at b^C, the optimal log coupon in the collusive equilibrium of each industry when $e^{\bar{z}_0} = 1$ and $\gamma = \gamma_L$. Parameters are calibrated as in Table 1.

year 1, which corresponds to the beginning of period $t = 12$ in our model, as we consider each period to represent one month.[31] Panel B of Figure 7 shows that the contagion effect on profit margins becomes less pronounced as the industry's price elasticity of demand ϵ increases. Intuitively, the financial contagion effect through the distressed competition channel becomes weaker when ϵ is higher because the collusion capacity declines.

Entry Barriers and Predatory Pricing. Higher entry barriers lead to stronger competition-distress feedback and financial contagion effects, because a firm has stronger predatory incentives when its rival firm becomes more financially distressed in an industry with higher entry barriers.[32] Dou, Johnson and Wu (2023) provide causal evidence on the spillover of distress risk through market competition, based on granular data, consistent with predatory pricing behavior.

The central idea of the predatory pricing strategy is the attempt to gain increased market power in the long run at the cost of heightened distress risk in the short run due to narrowed profit margins. Specifically, the firm that engages in predatory pricing behavior forces lower profit margins and higher distress risk on itself in the short run. However, in the long run, once its major competitor is eliminated, the firm will dominate the market and raise its

[31] That is, firm i's coupon e^{b_i} increases unexpectedly from e^{b^C} to $e^{b_i^{shock}}$. The value of $e^{b_i^{shock}}$ is chosen so that firm i's leverage ratio increases by 10% at the beginning of year 1, similar to the experiment described for Figure 3 in Online Appendix 1.2.

[32] Our results are related to Wiseman (2017), who shows that in a market with exits but no entries, the financially strong firm may wage a price war against the weak firms until only one firm survives the industry.

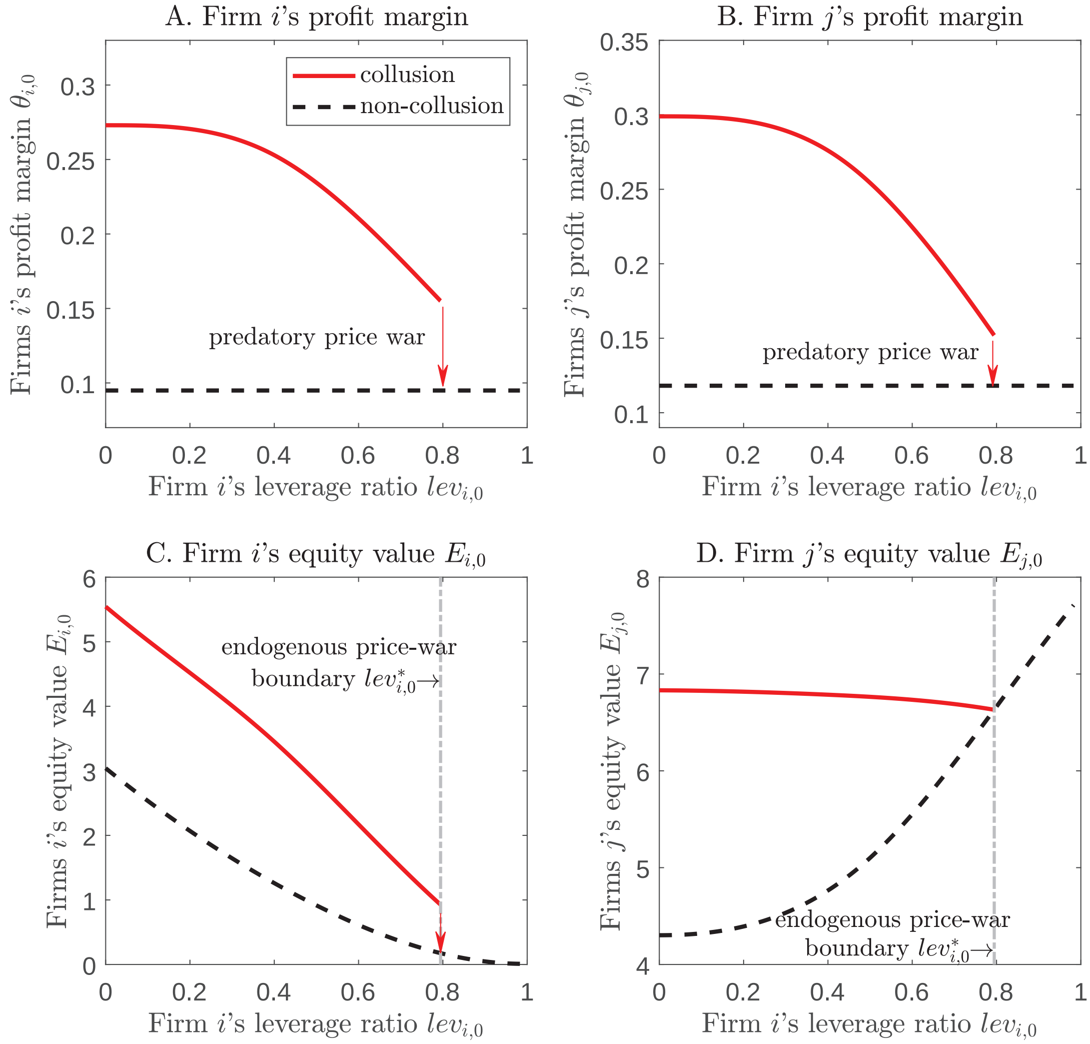

Figure 8. Predatory pricing behavior and full-blown predatory price war. Consider the industry with $\kappa = 0$, in which $e^{z_{i,0}} = 0.5$ and $e^{z_{j,0}} = 1$, so firm i is more distressed. By varying firm i's coupon e^{b_i}, panels A and C plot firm i's profit margin and equity value as a function of its own leverage ratio, and panels B and D plot those of firm j as a function of firm i's leverage ratio. The solid and dashed lines represent the collusive and non-collusive equilibrium, respectively. In panels A to C, the vertical arrows represent the endogenous jump in the two firms' profit margins and firm i's equity value, respectively, as the equilibrium endogenously switches from collusion to non-collusion. In panels C and D, the vertical dash-dotted lines represent the threshold of the endogenous jump, which is determined by the leverage ratio $lev^*_{i,0}$ of firm i above which firm j' PC constraint (31) is violated. The initial log coupons at $t = 0$ are set at b^C, the optimal log coupon in the collusive equilibrium when $e^{z_{i,0}} = e^{z_{j,0}} = 1$. Parameters are calibrated as in Table 1 except for $\kappa = 0$.

profit margin to recoup the profits it lost during the period of predatory pricing. Such predatory pricing incentives are weak in industries with low entry barriers because major new competitors can easily enter the market.[33]

[33]For example, Loualiche (2021) develops a model to emphasize that firm entries can increase competition and reduce incumbents' profit margins.

In Figure 8, we show that our model can generate full-blown predatory price wars when $\kappa = 0$, which represents an industry with sufficiently high entry barrier in the sense that new entrants are negligible in size relative to incumbent firms. Panels A and B show that as firm i's leverage ratio $lev_{i,0}$ increases, both firms' profit margins decrease in the collusive equilibrium (the solid lines),[34] whereas their profit margins remain unchanged in the non-collusive equilibrium (the dashed lines). Notably, the firms' profit margins in the collusive equilibrium fall to the level of profit margins in the non-collusive equilibrium when firm i's leverage ratio $lev_{i,0}$ is greater than $lev^*_{i,0} \approx 0.8$, which is the threshold of endogenous price war.

We now delineate the mechanism through which a full-blown predatory price war endogenously emerges within our model. In panel D of Figure 8, the equity value of firm j in the collusive equilibrium (represented by the solid line) meets its equity value in the non-collusive equilibrium (represented by the dashed line) at a leverage ratio of $lev^*_{i,0} \approx 0.8$ for firm i, as marked by the vertical dash-dotted line. When firm i's leverage ratio $lev_{i,0}$ is lower than $lev^*_{i,0}$, both firms' PC constraints are satisfied for the Pareto efficient collusive profit-margin schemes that make their IC constraints (32) binding state by state. Thus, both firms choose to collude with each other. However, when $lev_{i,0}$ lies above $lev^*_{i,0}$, the PC constraint becomes violated for firm j, and thus the collusion cannot be sustained even though the two firms' IC constraints are still satisfied. Importantly, panel C of Figure 8 illustrates that in the collusive equilibrium, firm i's equity value strictly surpasses its value in the non-collusive equilibrium for $lev_{i,0} \leq lev^*_{i,0}$. At the endogenous price-war boundary, represented by the vertical dash-dotted line at $lev^*_{i,0}$, firm i's equity value experiences a stark drop from its collusive level to the non-collusive level. Upon comparing panels C and D, it becomes evident that the firm in a more robust financial position (specifically firm j in our experiment) is more inclined to break away from collusion and initiate a price war. To further elaborate, when the leverage ratio $lev_{i,0}$ exceeds $lev^*_{i,0}$, firm i is dangerously close to its default boundary. Recognizing this vulnerability, firm j deems it optimal to push firm i out of the market, anticipating the long-term monopoly benefits. This strategy drives firm j to forgo the implicit collusive agreement, launching a predatory price war against firm i and transitioning to the non-collusive equilibrium. As a result, industries with higher barriers to entry witness a more pronounced within-industry contagion effect in the short run due to such predatory behavior from rivals.

We further study the central intertemporal tradeoff behind the predatory pricing strategy,

[34]The narrowed profit margin of firm j caused by the increased leverage of firm i reflects both its self-defensive and predatory incentives. The self-defensive component captures firm j's reduced profit margin as a response to defend its market demand against firm i's aggressive pricing. Through the lens of a structural model, predatory incentives can be isolated from self-defensive incentives (Besanko, Doraszelski and Kryukov, 2014).

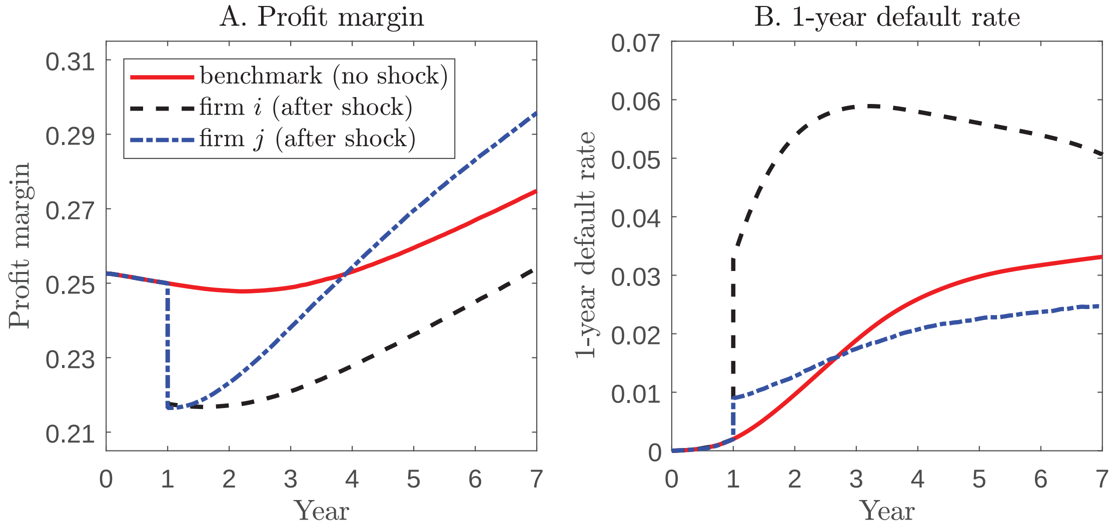

Figure 9. Predatory pricing as an extreme case of financial contagion in the short run. In each period t, the solid lines in panels A and B plot the average dynamics of one firm's profit margin and 1-year default rate, conditional on not defaulting (i.e., not exiting) at t, based on 200,000 independent simulations of the benchmark scenario with no distress shock to firm i. The dashed and dash-dotted lines represent the average dynamics of firms i and j, respectively, in the scenario where firm i's log coupon increases unexpectedly from b^C to b_i^{shock} at the beginning of year 1, whereas firm j's log coupon is held at b^C for $t \geq 0$. The value of b_i^{shock} is chosen so that firm i's leverage ratio increases by 10% at the beginning of year 1. We set $e^{z_{1,0}} = e^{z_{2,0}} = 0.5$, so both firms are already distressed at $t = 0$. The two firms' initial log coupons at $t = 0$ are set at b^C, the optimal log coupon in the collusive equilibrium when $e^{z_{1,0}} = e^{z_{2,0}} = 1$. Parameters are calibrated as in Table 1 except for $\kappa = 0$.

focusing on the balance between long-term monopolistic gains and short-term distress risk. Our exploration of the dynamic effects of financial contagion within an industry is centered on scenarios where $\kappa = 0$. As depicted in Figure 9, we model two firms in financial distress at $t = 0$, both characterized by an identical initial demand shock of $e^{z_{1,0}} = e^{z_{2,0}} = 0.5$.[35] At the beginning of year 1, there is a "large" adverse distress shock to firm i, which increases its leverage ratio by 10%. For each subsequent period t, we compute the average dynamics of each firm's profit margin and 1-year default rate, conditional on that the firm has not defaulted by t.

In panel A of Figure 9, the solid line serves as a benchmark case, depicting the profit margin when no large adverse distress shock is present. When the large adverse distress shock hits firm i at the beginning of year 1, both firms significantly reduce their profit margins. The dash-dotted line in the figure shows that firm j reduces its profit margin by 3.4 percentage points at the beginning of year 1. This reduction compels firm i to lower its prices to retain demand, subsequently depressing its profit margin. Both the feedback and contagion effects amplify the consequences of the initial distress shock on firm i, elevating its default rate. This elevation in turn enhances the prospects of firm j consolidating market

[35]To generate predatory pricing behavior, it is not necessary for both firms to be distressed (for example, in Figure 8, only firm i is distressed, not firm j). We consider initially distressed firms for illustrative purposes.

power in the future. Such power would enable firm j to achieve elevated profit margins. Supporting this, the dash-dotted line in panel A of Figure 9 reveals that, over time, firm j's profit margin consistently rises on average, surpassing the benchmark (represented by the solid line) post year 4.

A key economic insight we have gleaned is that, following an idiosyncratic distress shock to firm i, its rival, firm j, stands to benefit in the long-term. Yet, in the short-term, firm j must strategically and aggressively reduce its profit margin to stifle firm i's recovery from financial distress. The intertemporal tradeoff of firm j's predatory pricing can be clearly illustrated by the evolution of its short-term default risk. Panel B of Figure 9 plots the 1-year default rate, which is the conditional default probability over the next year. The solid line serves as a benchmark case, depicting the 1-year default rate when no large adverse distress shock is present at the beginning of year 1.

Upon comparing the solid and dash-dotted lines, it is evident that when firm j engages in predatory pricing, its 1-year default rate right after the shock to firm i exceeds the rate in the benchmark scenario, where no large distress shock is present at the outset of year 1. The reason for this is that firm j opts for significantly reduced profit margins immediately following the shock, as evidenced by the dash-dotted line in panel A. However, when firm j engages in predatory pricing, it is evident that, on average, its 1-year default rate starts to become lower than the rate in the benchmark scenario (the solid line) after year 3. This reduction in the 1-year default rate of firm j in the long run is due to its predatory behavior, which, in the short run, elevates the default probability for the shocked firm, firm i. As a consequence, firm j positions itself to reap monopoly rents in the long run.

4.3 Capital Structure Under Strategic Competition

Profitability-Leverage Puzzle. In this section, we show that the profitability-leverage puzzle can be rationalized by our full-fledged quantitative model, due to the same intuitions discussed in the simple model in Section 2. As an illustration, consider two industries with low ($\nu_L = 0$) and high ($\nu_H = 0.1$) levels of idiosyncratic left-tail risk. Panel A of Figure 10 shows that when $e^{z_{1,0}} = e^{z_{2,0}} = 1$, firm i's profit margin in the industry with ν_L is higher than that in the industry with ν_H by 14.1% (= 31% − 16.9%) at $t = 0$. In panel B of Figure 10, we compare the optimal leverage ratio in the two industries. The industry with ν_H has an optimal leverage ratio of 39%, which is higher than the industry with ν_L by 8% (= 39% − 31%). Taking the results in panels A and B of Figure 11 together, our model implies that industries with higher profitability are associated with lower leverage ratios.

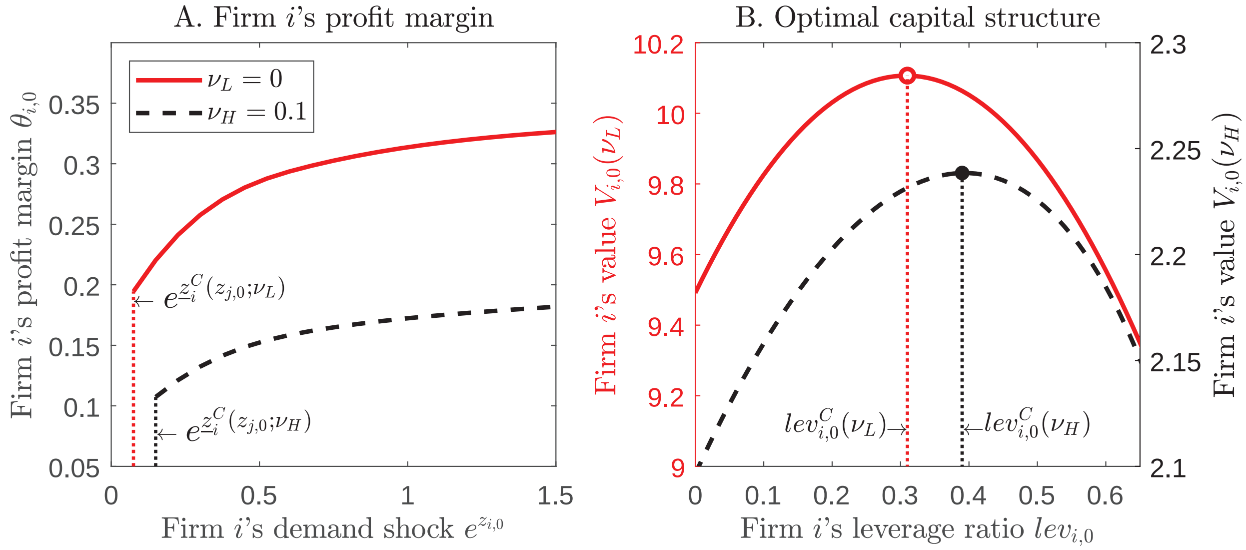

Figure 10. Higher idiosyncratic left-tail risk leads to lower profitability but higher optimal leverage. In both panels, the solid and dashed lines represent industries with low ($\nu_L = 0$) and high ($\nu_H = 0.1$) levels of idiosyncratic left-tail risk, respectively. Panel A plots the profit margin $\theta_{i,0}$ of firm i as a function of $e^{z_{i,0}}$, holding $e^{z_{j,0}} = 1$ unchanged. The two firms' initial log coupons at $t = 0$ are set at b^C, the optimal log coupon in the collusive equilibrium when $e^{z_{1,0}} = e^{z_{2,0}} = 1$. The vertical dotted lines represent the default boundaries of firm i in the corresponding industries. Panel B plots firm i's value $V_{i,0}$ as a function of its leverage ratio $lev_{i,0}$, by varying firm i's log coupon b_i while holding firm j's log coupon at b^C and $e^{z_{1,0}} = e^{z_{2,0}} = 1$ unchanged. The vertical dotted lines represent the optimal leverage ratios of firm i in the corresponding industries. Parameters are set according to Table 1.

Costly Leverage Due to the Feedback Effect. When determining its optimal amount of debt issuance, a firm recognizes the implications of increasing its leverage ratio on its profit margin, stemming from the competition-distress feedback effect. A higher leverage ratio can result in decreased collusive profit margins and cash flows. As a result, the firm behaves more cautiously, adopting a relatively low leverage ratio ex ante. To further understand this, we conduct experiments to measure the quantitative significance of the competition-distress feedback effect on the optimal capital structure. We consider two symmetric firms with initial demand shocks $e^{z_{1,0}} = e^{z_{2,0}} = 1$. These firms have optimal log coupons of $b_1 = b_2 = b^C$ in the collusive equilibrium. Panel A of Figure 11 helps illustrate our findings. The solid and dashed lines plot firm i's value $V_{i,0}$ in relation to its leverage ratio $lev_{i,0}$ at $t = 0$. The solid line corresponds to our baseline model with collusive profit margins, while the dashed line represents a model where firms' profit margins do not adjust according to their coupon levels. The peak values of firm i in both models are indicated at the optimal leverage ratios, which are represented by two vertical dotted lines in the same panel.

In the baseline model, which has firm values represented by the solid line, the optimal leverage ratio stands at 31%. In contrast, in the model depicted by the dashed line, firms' profit margins remain unaffected by their coupon levels. In this case, where profit margins

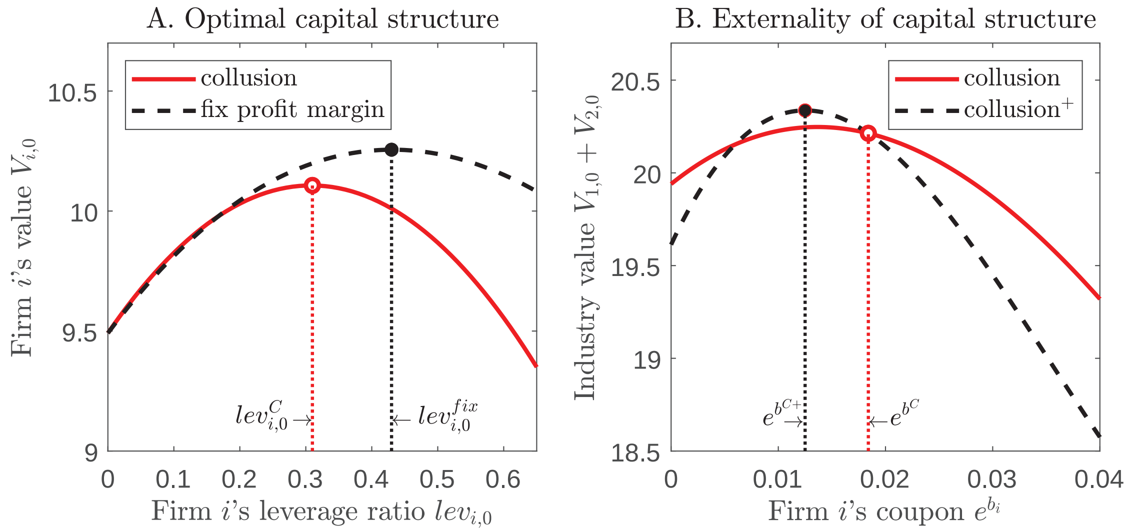

Figure 11. Implications of feedback and contagion on optimal capital structure. We consider two symmetric firms with $e^{z_{1,0}} = e^{z_{2,0}} = 1$. Panel A plots firm i's value $V_{i,0}$ as a function of its leverage ratio $lev_{i,0}$, by varying firm i's log coupon b_i while holding firm j's log coupon at b^C, the optimal log coupon in the collusive equilibrium when $e^{z_{1,0}} = e^{z_{2,0}} = 1$. The solid and dashed lines represent the baseline model with collusive profit margins and the model in which each firm's profit margin does not respond to its coupon level, respectively. In panel B, the solid line plots the industry's value ($V_{1,0} + V_{2,0}$) as a function of firm i's coupon e^{b_i} in the collusive equilibrium, holding $b_j = b^C$ unchanged. The right vertical dotted line represents e^{b^C}. The dashed line represents the industry's value as a function of firm i's (or firm j's) coupon e^{b_i} in the collusion$^+$ equilibrium, holding $b_i = b_j$. The left vertical dotted line represents the coupon $e^{b^{C+}}$ that maximizes the industry's value. Parameters are set according to Table 1.

are fixed, the competition-distress feedback is absent. This is because, by design, firm i's profit margin and cash flows remain independent of its leverage ratio. It is evident that firm i's optimal leverage ratio increases from 31% to 43% in the absence of the competition-distress feedback loop, indicating that the feedback effect reduces the optimal leverage ratio by a sizable amount, approximately 12% (= 43% − 31%). Our quantitative exercise suggests that the competition-distress feedback loop is an important mechanism that makes leverage costly and thus reduces firms' optimal leverage ratios. The large bankruptcy costs emphasized in the traditional tradeoff theory of capital structure (e.g., Leland, 1994) are not a precondition for generating large financial distress costs that result in costly leverage.

Excessive Leverage Relative to the Industry's Optimal Leverage. As previously discussed in the simple model from Section 2, the financial contagion effect suggests that an increase in one firm's debt level can adversely impact the value of its rival. This implies that excessive debt might be present when viewed from an industry-wide perspective. To better understand this, we quantitatively assess this potential inefficiency in firms' capital structure decisions from the standpoint of the entire industry. Consider two symmetric firms with initial demand shocks $e^{z_{1,0}} = e^{z_{2,0}} = 1$. The solid line in panel B of Figure 11 plots the industry's value

$V_{1,0} + V_{2,0}$ in the collusive equilibrium as a function of firm i's coupon e^{b_i}, holding $b_j = b^C$, where b^C is the optimal log coupon in the collusive equilibrium when $e^{z_{1,0}} = e^{z_{2,0}} = 1$. It is evident that the vertical dotted line on the right, which represents e^{b^C}, does not maximize the industry's value even though it is the optimal coupon that maximizes each firm's value in the collusive equilibrium. The solid line is downward sloping in the local region around e^{b^C}, indicating that the industry's value would be higher if firm i chooses a coupon level below e^{b^C}.

We now shift our focus to determining the leverage choice that maximizes the entire industry's value. Instead of allowing each firm to choose its own coupon non-cooperatively in a Nash equilibrium (see equations (37) and (38)), we introduce a cooperative approach for leverage decisions. Here, the industry jointly chooses the log coupons b_i and b_j for both firms. This strategy is aimed at maximizing the combined value of the industry, represented as $V_{1,0} + V_{2,0}$, and in doing so, addresses the externality concerns associated with individual debt choices. We introduce a distinct label for this equilibrium to differentiate it from the traditional collusive equilibrium. We term it the "collusive$^+$ equilibrium". We focus on the symmetric equilibrium, wherein both firms must adopt the same optimal coupon to maximize the industry's overall value. To visualize this, consider panel B of Figure 11. The dashed line in this panel illustrates the industry's value in relation to firm i's coupon e^{b_i}, under the condition that $b_i = b_j$. The vertical dotted line on the left represents the optimal coupon $e^{b^{C+}}$ that maximizes the industry's value, which implies a leverage ratio of 21.5% for the industry, much lower than the leverage ratio of 31% in the collusive equilibrium, corresponding to the choice of the log coupon b^C. Thus, from the perspective of the entire industry, both firms in the collusive equilibrium adopt an excessively high debt level.[36]

5 Conclusion

This paper investigates the dynamic interactions between endogenous strategic competition and financial distress. We develop the first elements of a tractable dynamic framework for distressed competition by incorporating a supergame of strategic rivalry into a dynamic model of long-term defaultable debt. In our model, firms tend to compete more aggressively when they are in financial distress, and the intensified competition, in turn, diminishes the profit margins of all firms in the industry, pushing some further into distress. Thus, the

[36] He and Matvos (2016) argue that firms may be under-leveraged from a social efficiency perspective since debt generates positive externalities for peers by inducing early exit, especially for inefficient firms. Although our model highlights negative externalities of debt, the key mechanism of our model shares similarities with He and Matvos (2016) in that a higher level of debt increases the likelihood of default and exit, which, in turn, alters firms' strategic interactions by increasing the discount rate for equity holders.

endogenous distressed competition mechanism implies novel competition-distress feedback and financial contagion effects.

Our study raises interesting questions for future research. Equity issuance is costless in our model but costly in reality. The costless issuance of equity is a simplification widely adopted in standard credit risk models. Do firms compete more aggressively when they become more liquidity constrained, but not yet more financially distressed? We focus on frictions related to debt financing, but equity financing frictions should also be investigated. Extending the model to incorporate external equity financing costs and allowing firms to hoard cash, as in Bolton, Chen and Wang (2011, 2013), Dou et al. (2021*b*), and Dou and Ji (2021), would be interesting for future research. In addition, our paper highlights an important source of cash flow risk — endogenous competition risk — that depends on industries' market structures. Extending the model to study the joint determination of optimal capital structure and risk management, as in Rampini and Viswanathan (2010, 2013), is another potentially fruitful research area.

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Appendix

In the appendix section, we make the dependence of endogenous variables on ν explicit to ensure clarity. We first introduce the following technical assumptions for the simple model.

Assumption 1. *The coupon level $b_i > \varphi(b) - \ell$, for $\ell > 0$ in the borrowing constraint (3).*

This technical assumption is quite intuitive. It is consistent with the fact that the covenants are typically triggered before firms choose to default.

Assumption 2. *The log coupon level $b_i > \underline{b}$, for some $\underline{b} > \tau\nu/\bar{\varphi}$.*

This technical assumption is a sufficient condition for Proposition 2.6. The assumption is innocuous. We can always adjust the cash flow to be represented as $Me^{z_i}C_i$. By selecting a sufficiently large value for M, the firm's leverage ratio can be brought close to zero, given a predefined lower bound for the coupon level, $\underline{b}$. We choose the lower bound $\underline{b}$ such that $\underline{b} > \tau\nu/\bar{\varphi}$ to fix the scale of the economy and ensure that increasing a firm's coupon payment has a non negligible effect on its default probability relative to the tax shield benefit.

A Proof of the Statement on Left-Tail Risk

According to the cumulative distribution function of a logistic distribution, it suffices to show that both $e^{-\underline{z}/\nu}$ and $[\sin(\nu\pi)/(\nu\pi)]^{1/\nu}$ are decreasing in ν. Because $\underline{z} < 0$, it is obvious that $e^{-\underline{z}/\nu}$ is decreasing in ν. Let $\mathcal{H}(\nu) \equiv \ln[\sin(\nu\pi)/(\nu\pi)]^{1/\nu}$. We only need to show that the following function is decreasing in ν:

$$\mathcal{H}(\nu) \equiv \frac{1}{\nu}\mu(\nu) = \frac{1}{\nu}\left[\ln(\sin(\nu\pi)) - \ln(\nu\pi)\right]. \tag{39}$$

Using the facts that $\sin x < x < \tan x$ for $x \in (0, \pi/2)$, $\lim_{x\to 0} x/\sin x = 1$, and $\lim_{x\to 0} x/\tan x = 1$, it is easy to verify that $\mathcal{H}(\nu)$ is a monotonically decreasing and strictly concave function over $(0, 1/2)$.

B Proof of Proposition 2.1

We first show that $P_1^N = P_2^N = 0$ is a one-shot static Nash equilibrium. No firm would like to increase the price because this deviation would lead to zero demand according to the demand system specification (equation (1) of the main text). Meanwhile, no firm would like to lower the price to some negative value because this deviation would lead to a negative profit (i.e., an operating loss). Next, we show that $P_1^N = P_2^N = 0$ is the unique one-shot static Nash equilibrium. Suppose (P_1, P_2) is a pair of prices with at least one price being positive. Without loss of generality, we assume that $P_1 \leq P_2$ and $P_2 > 0$. There are two cases. If $P_1 = P_2 > 0$, either firm can reduce its price by an infinitesimal amount to obtain a positive gain. If $P_1 < P_2$ and $P_2 > 0$, firm 1 can increase its price to a level that is lower than P_2 by an infinitesimal amount to obtain a positive gain. Therefore, $P_1^N = P_2^N = 0$ is the unique non-collusive equilibrium, in which $C_1^N = C_2^N = 1$ according to the demand system specification and $E_1^N = E_2^N = 0$.

C Proof of Lemma 1

In the collusive equilibrium, it holds that

$$\lambda_i^C = \mathbb{P}\{e^{\theta_i^C + z_i} \leq e^{\varphi(b_i)}\}, \tag{40}$$

where $\theta_i^C \equiv \ln\left(P_i^C\right)$ for $i \in \{1,2\}$. The idiosyncratic demand shock z_i is i.i.d., following a logistic distribution, Logistic $(\mu(\nu), \nu)$. Thus, the default probability is equal to

$$\lambda_i^C = \mathbb{P}\{z_i \leq \varphi(b_i) - \theta_i^C\} = \frac{1}{1 + e^{[\mu(\nu)+\theta_i^C - \varphi(b_i)]/\nu}}, \quad \text{for } i \in \{1,2\}. \tag{41}$$

D Proof of Proposition 2.2

The two firms must set the same price P^C and thus set the same log profitability θ^C in the collusive equilibrium. The equity value of firm i is

$$E_i^C(b_i, b_j) = \mathbb{E}\left\{(1-\tau)\left(e^{\theta^C + z_i} - e^{b_i}\right) + E_i^C(b_i, b_j)\mathbf{1}_{\{\theta^C + z_i > \varphi(b_i)\}}\right\}, \text{ for } j \neq i,$$

where $\mathbf{1}_{\{\theta^C + z_i > \varphi(b_i)\}}$ is the indicator function for the event of $\theta^C + z_i > \varphi(b_i)$. Using the definition of the default probability $\lambda^C(\theta^C, b_i)$ in equation (5) of the main text and the distribution of z_i (i.e., $\mathbb{E}[e^{z_i}] \equiv 1$), the above equation can be rewritten as follows:

$$E_i^C(b_i, b_j) = (1-\tau)\left(e^{\theta^C} - e^{b_i}\right) + \left[1 - \lambda^C(\theta^C, b_i)\right] E_i^C(b_i, b_j), \tag{42}$$

Rearranging terms further leads to

$$E_i^C(b_i, b_j) = (1-\tau)\left(e^{\theta^C} - e^{b_i}\right)\lambda(\theta^C, b_i)^{-1}. \tag{43}$$

Plugging (41) into (43), we obtain that

$$E_i^C(b_i, b_j) = (1-\tau)\left(e^{\theta^C} - e^{b_i}\right)\left\{1 + e^{[\mu(\nu)+\theta^C - \varphi(b_i)]/\nu}\right\}. \tag{44}$$

The debt value can be characterized by the following recursive equation:

$$D_i^C(b_i, b_j) = \mathbb{E}\left\{e^{b_i} + D_i^C(b_i, b_j)\mathbf{1}_{\{\theta^C + z_i > \varphi(b_i)\}}\right\} \tag{45}$$

$$= e^{b_i} + \left[1 - \lambda^C(\theta^C, b_i)\right] D_i^C(b_i, b_j). \tag{46}$$

Rearranging terms in the equation above, we can solve the equilibrium debt value as follows:

$$D_i^C(b_i, b_j) = e^{b_i}\lambda^C(\theta^C, b_i)^{-1}. \tag{47}$$

The incentive compatibility (IC) constraint in equation (7) of the main text can be rewritten as follows:

$$\rho e^{\theta^C + \eta\theta^C} \leq \left[1 - \lambda(\theta^C, b_i)\right] E_i^C(b_i, b_j) \tag{48}$$

$$= (1-\tau)\left[1 - \lambda(\theta^C, b_i)\right]\left(e^{\theta^C} - e^{b_i}\right)\left\{1 + e^{[\mu(\nu)+\theta^C - \varphi(b_i)]/\nu}\right\}. \tag{49}$$

Plugging (41) into (49), we obtain the following inequality:

$$\rho e^{\theta^C + \eta\theta^C} \leq (1-\tau)\left(e^{\theta^C} - e^{b_i}\right) e^{[\mu(\nu)+\theta^C - \varphi(b_i)]/\nu}. \tag{50}$$

Taking logs on both sides, it follows that

$$\ln(\rho) + \eta\theta^C \leq \ln(1-\tau) + [\mu(\nu) + \theta^C - \varphi(b_i)]/\nu + \ln\left[1 - e^{-(\theta^C - b_i)}\right]. \tag{51}$$

Therefore, the IC constraints can be written as follows:

$$\theta^C \leq \left\{\ln[(1-\tau)/\rho] + [\mu(\nu) + \theta^C - \varphi(b_i)]/\nu + \ln\left[1 - e^{-(\theta^C - b_i)}\right]\right\}/\eta, \quad \text{with } i \in \{1,2\}. \tag{52}$$

We define a function $\Psi(\theta, b)$ as follows:

$$\Psi(\theta, b) \equiv \ln[(1-\tau)/\rho] + [\mu(\nu) + \theta - \varphi(b)]/\nu + \ln\left[1 - e^{-(\theta - b)}\right]. \tag{53}$$

Thus, the two IC constraints in (52) for the pairs of log profitability (θ^C, θ^C) can be expressed in the following way:

$$\theta^C \leq \Psi(\theta^C, b_i)/\eta, \quad \text{with } i \in \{1,2\}. \tag{54}$$

Hence, the incentive compatible region (IC region) can be characterized as follows:

$$\mathcal{C} \equiv \left\{(\theta_1, \theta_2) \ : \ \theta_i \leq \Psi(\theta_j, b_j)/\eta \ \text{ and } \ \theta_i = \theta_j, \ \text{ for } i \neq j\right\}, \tag{55}$$

for b_1 and b_2 such that $\mathcal{C}$ is not empty.

The firms try to maximize their log profitability and thus equity values. As a result, in the Pareto efficient collusive equilibrium with grim trigger (punishment) strategies, at least one of the two IC constraints in (54) must be binding. Knowing that the function $\Psi(\theta, b)$ is decreasing in b, we can obtain the following equality:

$$\theta^C = \Psi(\theta^C, b_1 \vee b_2)/\eta, \tag{56}$$

where $b_1 \vee b_2$ denotes the maximum of b_1 and b_2.

Next, we show that the Pareto efficient collusive equilibrium exists if b_1 and b_2 satisfy certain conditions. The function $f(x) \equiv \ln[1 - e^{-x}]$ is a strictly increasing and concave function in x, it must hold that

$$\frac{1}{e^{\overline{\theta}} - 1}(\theta - b_1 \vee b_2 - \ell) \leq \ln\left[1 - e^{-(\theta - b_1 \vee b_2)}\right] - \ln\left[1 - e^{-\ell}\right] \leq \frac{1}{e^{\ell} - 1}(\theta - b_1 \vee b_2 - \ell), \tag{57}$$

for all $\theta \in \left[b_1 \vee b_2 + \ell, \ln(\overline{P})\right]$, where ℓ is the constant defined in the borrowing constraint, specified in equation (3) of the main text.

We now define a function $G(\cdot, \cdot, \cdot)$ as follows:

$$\begin{aligned} G(\theta, b_1, b_2) &\equiv \Psi(\theta, b_1 \vee b_2) - \eta\theta & (58) \\ &= C(b_1 \vee b_2) + \theta/\nu - \eta\theta + \ln\left[1 - e^{-(\theta - b_1 \vee b_2)}\right] - \ln\left[1 - e^{-\ell}\right], & (59) \end{aligned}$$

where

$$C(b) \equiv \ln[(1-\tau)/\rho] + [\mu(\nu) - \varphi(b)]/\nu + \ln\left[1 - e^{-\ell}\right]. \tag{60}$$

Based on the equilibrium characterization in (56), we know that the log profitability in the Pareto efficient collusive equilibrium θ^C is the root of the function $G(\theta, b_1, b_2)$ if it exists. Because of the inequalities in (57), it holds that

$$\underline{G}(\theta, b_1, b_2) \le G(\theta, b_1, b_2) \le \overline{G}(\theta, b_1, b_2), \quad \text{for all } \theta \in \left[b_1 \vee b_2 + \ell, \overline{\theta}\right], \tag{61}$$

where

$$\underline{G}(\theta, b_1, b_2) \equiv C(b_1 \vee b_2) - (b_1 \vee b_2 + \ell)/(e^{\overline{\theta}} - 1) - \left\{\eta - \left[1/\nu + 1/(e^{\overline{\theta}} - 1)\right]\right\}\theta, \tag{62}$$

$$\overline{G}(\theta, b_1, b_2) \equiv C(b_1 \vee b_2) - (b_1 \vee b_2 + \ell)/(e^{\ell} - 1) - \left\{\eta - \left[1/\nu + 1/(e^{\ell} - 1)\right]\right\}\theta, \tag{63}$$

When η is sufficiently large and ρ is sufficiently small, both the lower-bound function $\underline{G}(\theta, b_1, b_2)$ and the upper-bound function $\overline{G}(\theta, b_1, b_2)$ have the following positive roots:

$$\underline{\theta}^*(b_1, b_2) \equiv \frac{C(b_1 \vee b_2) - (e^{\overline{\theta}} - 1)^{-1}(b_1 \vee b_2 + \ell)}{\eta - \left[\nu^{-1} + (e^{\overline{\theta}} - 1)^{-1}\right]}, \quad \text{and} \tag{64}$$

$$\overline{\theta}^*(b_1, b_2) \equiv \frac{C(b_1 \vee b_2) - (e^{\ell} - 1)^{-1}(b_1 \vee b_2 + \ell)}{\eta - \left[\nu^{-1} + (e^{\ell} - 1)^{-1}\right]}, \quad \text{respectively.} \tag{65}$$

Because of the inequalities in (61), it must hold that $\theta^C \in [\underline{\theta}^*(b_1, b_2), \overline{\theta}^*(b_1, b_2)]$.

Therefore, the equilibrium exists (i.e., θ^C exists and lies between $b_1 \vee b_2 + \ell$ and $\overline{\theta}$), as long as the parameters, including b_1 and b_2, make the following conditions hold:

$$b_1 \vee b_2 + \ell \le \underline{\theta}^*(b_1, b_2) \le \overline{\theta}^*(b_1, b_2) \le \overline{\theta}. \tag{66}$$

Lastly, we show that the Pareto efficient collusive equilibrium characterized by θ^C, the root of the function $G(\theta, b_1, b_2)$, is unique if it exists. This is true because $\frac{\partial G(\theta, b_1, b_2)}{\partial \theta} < 0$ if η is sufficiently large. Indeed, $\frac{\partial G(\theta, b_1, b_2)}{\partial \theta}$ has the following analytical expression:

$$\frac{\partial G(\theta, b_1, b_2)}{\partial \theta} = -\eta + 1/\nu + \frac{1}{e^{\theta - b_1 \vee b_2} - 1} \tag{67}$$

Because of the borrowing constraint, it follows that $0 < 1/(e^{\theta - b_1 \vee b_2} - 1) \le 1/(e^{\ell} - 1)$, which further implies that

$$\frac{\partial G(\theta, b_1, b_2)}{\partial \theta} < -\eta + 1/\nu + \frac{1}{e^{\ell} - 1}, \tag{68}$$

where the right hand side is negative if η is sufficiently large.

E Proof of Proposition 2.3

We define a function $G(\cdot, \cdot, \cdot)$ as in (58). According to Proposition 2.2, the collusive equilibrium log profitability θ^C is the solution to $G(\theta, b_1, b_2) = 0$ for given b_1 and b_2. For $i \ne j \in \{1, 2\}$, if $b_i < b_j$, the function $G(\theta, b_1, b_2) \equiv \Psi(\theta, b_j) - \eta\theta$ is independent of b_i. As a result, θ^C is independent of b_i, and thus $\partial\theta^C/\partial b_i = 0$. Alternatively, if $b_i \ge b_j$, the function $G(\theta, b_1, b_2) \equiv \Psi(\theta, b_i) - \eta\theta$ depends on b_i, and thus θ^C depends on b_i. The Implicit Function Theorem implies that

$$\frac{\partial \theta^C}{\partial b_i} = -\left[\left.\frac{\partial G(\theta, b_1, b_2)}{\partial \theta}\right|_{\theta = \theta^C}\right]^{-1} \left.\frac{\partial G(\theta, b_1, b_2)}{\partial b_i}\right|_{\theta = \theta^C}, \tag{69}$$

where the derivatives, evaluated at $\theta = \theta^C$, are

$$\left.\frac{\partial G(\theta, b_1, b_2)}{\partial \theta}\right|_{\theta=\theta^C} = -\eta + (1/\nu + d_i) \quad \text{and} \quad \left.\frac{\partial G(\theta, b_1, b_2)}{\partial b_i}\right|_{\theta=\theta^C} = -\dot{\varphi}(b_i)/\nu - d_i, \tag{70}$$

with $d_i \equiv 1/[e^{\theta^C - b_i} - 1]$. Therefore,

$$\frac{\partial \theta^C}{\partial b_i} = -\frac{\dot{\varphi}(b_i)/\nu + d_i}{\eta - (1/\nu + d_i)}. \tag{71}$$

Because of the borrowing constraint and the fact that $\theta^C = \ln(P^C) \leq \ln(\overline{P})$, it follows that $0 < 1/\left(\overline{P} - 1\right) < d_i \leq 1/(e^{\ell} - 1)$, which further implies that $\dot{\varphi}(b_i)/\nu + d_i > \dot{\varphi}(b_i)/\nu \geq 0$ and $\eta - (1/\nu + d_i) > \eta - \left[1/\nu + 1/(e^{\ell} - 1)\right]$, with the latter being positive if η is sufficiently large. Therefore, if η is sufficiently large, it follows from equation (71) that $\frac{\partial \theta^C}{\partial b_i} < 0$.

F Proof of Corollary 2.1

Note that $\lambda_j^C = \lambda^C(\theta^C, b_j)$. It follows from the chain rule that

$$\frac{\partial \lambda^C(\theta^C, b_j)}{\partial b_i} = \left.\frac{\partial \lambda^C(\theta, b_j)}{\partial \theta}\right|_{\theta=\theta^C} \times \frac{\partial \theta^C}{\partial b_i}. \tag{72}$$

Based on equation (5) of the main text, it follows that $\partial \lambda^C(\theta, b_j)/\partial \theta < 0$. In addition, according to Proposition 2.3, it follows that $\partial \theta^C(b_1, b_2)/\partial b_i \leq 0$. Taken together, the default probability of the rival firm j is increasing in firm i's coupon level b_i, that is, $\partial \lambda^C(\theta^C, b_j)/\partial b_i \geq 0$.

G Proof of Proposition 2.4

According to equation (5) of the main text, the default probability is

$$\lambda_i^C = \lambda^C(\theta^C, b_i), \quad \text{where } \lambda^C(\theta, b) \equiv \frac{1}{1 + e^{[\mu(\nu) + \theta - \varphi(b)]/\nu}}, \tag{73}$$

where θ^C is a function of b_i and b_j in the collusive equilibrium for $i \neq j$. It holds that

$$d\lambda_i^C = \left.\frac{\partial \lambda^C(\theta, b_i)}{\partial b_i}\right|_{\theta=\theta^C} db_i + \left.\frac{\partial \lambda^C(\theta, b_i)}{\partial \theta}\right|_{\theta=\theta^C} d\theta^C. \tag{74}$$

The above total differential representation of $\lambda^C(\theta, b)$ is general in the sense that it does not depend on any specific economic mechanism.

According to our proposed theory, a change in b_i affects λ_i^C, which in turn affects the log profitability θ^C. Holding b_j unchanged, changes in either λ_i^C or θ^C can only be caused by variations in b_i. As a result, it follows that

$$d\theta^C = \frac{\partial \theta^C(b_1, b_2)}{\partial b_i} db_i = \frac{\partial \theta^C(b_1, b_2)}{\partial b_i} \left[\frac{\partial \lambda^C(\theta^C(b_1, b_2), b_i)}{\partial b_i}\right]^{-1} d\lambda_i^C = \left.\frac{\partial \theta^C}{\partial \lambda_i^C}\right|_{b_j} d\lambda_i^C, \tag{75}$$

where $\partial \theta^C / \partial \lambda_i^C|_{b_j}$ is the sensitivity of log profitability θ^C to firm i's default probability λ_i^C, holding b_j

unchanged, in the collusive equilibrium.

Combining (74) and (75), the feedback loop representation immediately follows:

$$\begin{bmatrix} \mathrm{d}\lambda_i^C \\ \mathrm{d}\theta^C \end{bmatrix} = \underbrace{\begin{bmatrix} \partial\lambda^C(\theta,b_i)/\partial b_i\big|_{\theta=\theta^C} \\ 0 \end{bmatrix}}_{\text{Initial direct effect } \geq 0} \mathrm{d}b_i + \underbrace{\begin{bmatrix} 0 & \partial\lambda^C(\theta,b_i)/\partial\theta\big|_{\theta=\theta^C} \\ \partial\theta^C/\partial\lambda_i^C\big|_{b_j} & 0 \end{bmatrix}}_{\text{Higher-order feedback effect } \leq 0} \begin{bmatrix} \mathrm{d}\lambda_i^C \\ \mathrm{d}\theta^C \end{bmatrix}. \tag{76}$$

Define $A \equiv \begin{bmatrix} 0 & \partial\lambda^C(\theta,b_i)/\partial\theta\big|_{\theta=\theta^C} \\ \partial\theta^C/\partial\lambda_i^C\big|_{b_j} & 0 \end{bmatrix}$, which captures the higher-order feedback effect of changes in b_i on λ_i^C and θ^C. The matrix A has two eigenvalues given by $\pm\sqrt{\partial\lambda^C(\theta,b_i)/\partial\theta\big|_{\theta=\theta^C} \times \partial\theta^C/\partial\lambda_i^C\big|_{b_j}}$. It is straightforward to show that these two eigenvalues lie on $(-1,1)$ because $\partial\lambda_i^C/\partial\theta^C\big|_{b_j} < \partial\lambda^C(\theta,b_i)/\partial\theta\big|_{\theta=\theta^C} < 0$. Intuitively, $\partial\lambda^C(\theta,b_i)/\partial\theta\big|_{\theta=\theta^C}$ only captures the initial direct effect of varying log profitability θ on the default probability $\lambda^C(\theta,b_i)$, holding both b_1 and b_2 unchanged. In contrast, $\partial\lambda_i^C/\partial\theta^C\big|_{b_j}$ captures the effect of varying b_i on λ^C through the change in the equilibrium log profitability θ^C, which reflects not only the initial direct effect of varying θ on λ^C but also the feedback effect between the financial and product markets, holding only b_j unchanged. Mathematically, according to (75), it holds that

$$\begin{aligned} \left.\frac{\partial\lambda_i^C}{\partial\theta^C}\right|_{b_j} &= \left(\frac{\partial\theta^C}{\partial b_i}\right)^{-1} \frac{\partial\lambda^C(\theta^C,b_i)}{\partial b_i} && (77) \\ &= \left(\frac{\partial\theta^C}{\partial b_i}\right)^{-1} \left[\left.\frac{\partial\lambda^C(\theta,b_i)}{\partial b_i}\right|_{\theta=\theta^C} + \left.\frac{\partial\lambda^C(\theta,b_i)}{\partial\theta}\right|_{\theta=\theta^C} \frac{\partial\theta^C}{\partial b_i}\right] && (78) \\ &= \left(\frac{\partial\theta^C}{\partial b_i}\right)^{-1} \left.\frac{\partial\lambda^C(\theta,b_i)}{\partial b_i}\right|_{\theta=\theta^C} + \left.\frac{\partial\lambda^C(\theta,b_i)}{\partial\theta}\right|_{\theta=\theta^C}. && (79) \end{aligned}$$

In general, we observe that $\partial\theta^C/\partial b_i \leq 0$. However, for the derivative $\left.\frac{\partial\lambda_i^C}{\partial\theta^C}\right|_{b_j}$ to be well-defined, the condition $\partial\theta^C/\partial b_i \neq 0$ must be met. Consequently, we conclude that $\partial\theta^C/\partial b_i < 0$. Because $\partial\theta^C/\partial b_i < 0$ and $\partial\lambda^C(\theta,b_i)/\partial b_i\big|_{\theta=\theta^C} > 0$, it follows that

$$\left.\frac{\partial\lambda_i^C}{\partial\theta^C}\right|_{b_j} - \left.\frac{\partial\lambda^C(\theta,b_i)}{\partial\theta}\right|_{\theta=\theta^C} = \left(\frac{\partial\theta^C}{\partial b_i}\right)^{-1} \left.\frac{\partial\lambda^C(\theta,b_i)}{\partial b_i}\right|_{\theta=\theta^C} < 0. \tag{80}$$

Thus, $I - A$ is invertible and the inversion is equal to

$$(I-A)^{-1} = I + A + A^2 + A^3 + \cdots. \tag{81}$$

We have shown that the system is stable and invertible above. We now demonstrate the resulting amplification effect that arises from the feedback loop. According to equations (76) and (81), it follows that

$$\begin{bmatrix} \mathrm{d}\lambda_i^C \\ \mathrm{d}\theta^C \end{bmatrix} = [I + \underbrace{A + A^2 + A^3 + \cdots}_{\substack{\text{Amplification due to} \\ \text{feedback loops}}}] \underbrace{\begin{bmatrix} \partial\lambda^C(\theta,b_i)/\partial b_i\big|_{\theta=\theta^C} \\ 0 \end{bmatrix}}_{\text{Initial direct effect } \geq 0} \mathrm{d}b_i. \tag{82}$$

The powers of A, for $k = 1, 2, \cdots$, are

$$A^{2k} = \begin{bmatrix} (\partial\lambda^C(\theta, b_i)/\partial\theta|_{\theta=\theta^C})^k (\partial\theta^C/\partial\lambda_i^C|_{b_j})^k & 0 \\ 0 & (\partial\lambda^C(\theta, b_i)/\partial\theta|_{\theta=\theta^C})^k (\partial\theta^C/\partial\lambda_i^C|_{b_j})^k \end{bmatrix}, \text{ and} \tag{83}$$

$$A^{2k-1} = \begin{bmatrix} 0 & (\partial\lambda^C(\theta, b_i)/\partial\theta|_{\theta=\theta^C})^k (\partial\theta^C/\partial\lambda_i^C|_{b_j})^{k-1} \\ (\partial\lambda^C(\theta, b_i)/\partial\theta|_{\theta=\theta^C})^{k-1} (\partial\theta^C/\partial\lambda_i^C|_{b_j})^k & 0 \end{bmatrix}. \tag{84}$$

H Proof of Corollary 2.2

We now define a function $G(\cdot, \cdot)$ as follows:

$$G(\theta, b) \equiv \Psi(\theta, b) - \eta\theta \tag{85}$$

$$= C(b) + \theta/\nu - \eta\theta + \ln\left[1 - e^{-(\theta-b)}\right] - \ln\left[1 - e^{-\ell}\right], \tag{86}$$

where $C(b)$ is defined in (60). The Implicit Function Theorem implies that

$$\frac{\partial\theta^C}{\partial b} = -\left[\frac{\partial G(\theta, b)}{\partial\theta}\bigg|_{\theta=\theta^C}\right]^{-1} \frac{\partial G(\theta, b)}{\partial b}\bigg|_{\theta=\theta^C}, \tag{87}$$

where the derivatives, evaluated at $\theta = \theta^C$, are

$$\frac{\partial G(\theta, b)}{\partial\theta}\bigg|_{\theta=\theta^C} = -\eta + (1/\nu + d) \quad \text{and} \quad \frac{\partial G(\theta, b)}{\partial b}\bigg|_{\theta=\theta^C} = -\dot{\varphi}(b)/\nu - d, \tag{88}$$

with $d \equiv 1/[e^{\theta^C(b,b)-b} - 1]$. Therefore,

$$\frac{\partial\theta^C(b, b)}{\partial b} = -\frac{\bar{\varphi}b/\nu + 1/[e^{\theta^C(b,b)-b} - 1]}{\eta - \{1/\nu + 1/[e^{\theta^C(b,b)-b} - 1]\}}, \tag{89}$$

which becomes more negative when b increases.

I Proof of Proposition 2.5

The characterization of the set $\mathcal{B}$ is straightforward from the definition of a Nash equilibrium. We focus on the symmetric equilibrium in which $b_1 = b_2$ because firms are ex-ante symmetric in the simple model. In this case, $V_1^C(b, b) = V_2(b, b)$ for any $(b, b) \in \mathcal{B}$.

We prove this proposition by contradiction. Let b^C denote the smallest value of $\mathcal{B}$. Suppose (b^C, b^C) is not a Pareto efficient point of the set $\mathcal{B}$ or it is a Pareto efficient point of the set $\mathcal{B}$ but not the unique one. As a result, there exists $\bar{b}^C$ such that $\bar{b}^C > b^C$ and $(\bar{b}^C, \bar{b}^C) \in \mathcal{B}$, satisfying the following inequality:

$$V_1^C(b^C, b^C) \leq V_1^C(\bar{b}^C, \bar{b}^C). \tag{90}$$

The inequality above implies that

$$
\begin{aligned}
V_1^C(b^C, b^C) &\leq V_1^C(\bar{b}^C, \bar{b}^C) && (91)\\
&\leq V_1^C(\bar{b}^C, b^C) \quad \text{(because } V_i^C(b_i, b_j) \text{ is decreasing in } b_j\text{)} && (92)\\
&< V_1^C(b^C, b^C) \quad \text{(because } (b^C, b^C) \text{ is a Nash equilibrium and Proposition K.1 holds for } b^C\text{)} && (93)
\end{aligned}
$$

Taken together, the inequalities (91) to (93) lead to a contradiction, because $V_1^C(b_1^C, b_2^C)$ cannot be strictly smaller than itself. Therefore, (b^C, b^C) must the unique Pareto efficient pair of log coupons in the set $\mathcal{B}$.

We now elaborate how the inequalities (92) and (93) are derived. The inequality (93) arises because (b^C, b^C) is a Nash equilibrium and according to Proposition K.1, b^C is the unique best response of firm 1 to firm 2's leverage decision b^C. The inequality (92) requires more explanations. According to Proposition 2.2, the firm value in the collusive equilibrium is $V_i^C(b_i, b_j) \equiv E_i^C(b_i, b_j) + D_i^C(b_i, b_j)$, where the equity value $E_i^C(b_i, b_j)$ and the debt value $D_i^C(b_i, b_j)$ are characterized in closed-form expressions. To be specific, the firm value can be expressed as follows:

$$
V_i^C(b_i, b_j) \equiv \Phi(\theta^C(b_i, b_j), b_i), \tag{94}
$$

where $\theta^C(b_i, b_j)$ is characterized in Proposition 2.2, and the function $\Phi(\theta, b)$ is specified as follows:

$$
\begin{aligned}
\Phi(\theta, b) &\equiv \underbrace{(1-\tau)e^{\theta}\lambda^C(\theta, b)^{-1}}_{\text{Asset value}} + \underbrace{\tau e^{b}\lambda^C(\theta, b)^{-1}}_{\text{Value of tax shield}}, && (95)\\
&= (1-\tau)e^{\theta} + (1-\tau)e^{\theta + [\mu(\nu)+\theta-\varphi(b)]/\nu} + \tau e^{[\mu(\nu)+\theta-\varphi(b)]/\nu + b} + \tau e^{b}. && (96)
\end{aligned}
$$

Thus, it follows that

$$
\frac{\partial V_i^C(b_i, b_j)}{\partial b_j} = \Phi_\theta(\theta^C(b_i, b_j), b_i)\frac{\partial \theta^C(b_i, b_j)}{\partial b_j}. \tag{97}
$$

From (109), it follows that $\Phi_\theta(\theta, b_i) > 0$. Moreover, Proposition 2.3 shows that $\dfrac{\partial \theta^C(b_i, b_j)}{\partial b_j} \leq 0$. Thus, $\dfrac{\partial V_i^C(b_i, b_j)}{\partial b_j} \leq 0$, i.e., $V_i^C(b_i, b_j)$ is decreasing in b_j. As a result, because $b^C < \bar{b}^C$, then $V_1^C(\bar{b}^C, \bar{b}^C) \leq V_1^C(\bar{b}^C, b^C)$.

J Proof of Proposition 2.6

We define a function $G(\theta, b_1, b_2) \equiv \Psi(\theta, b_1 \vee b_2) - \eta\theta$ as in (58). According to Proposition 2.2, the collusive equilibrium log profitability θ^C is the solution to $G(\theta, b_1, b_2) = 0$ for given b_1 and b_2. The Implicit Function Theorem implies that

$$
\frac{\partial \theta^C}{\partial \nu} = -\left[\left.\frac{\partial G(\theta, b_1, b_2)}{\partial \theta}\right|_{\theta=\theta^C}\right]^{-1} \left.\frac{\partial G(\theta, b_1, b_2)}{\partial \nu}\right|_{\theta=\theta^C}, \tag{98}
$$

where the derivatives, evaluated at $\theta = \theta^C$, are

$$
\left.\frac{\partial G(\theta, b_1, b_2)}{\partial \theta}\right|_{\theta=\theta^C} = -\eta + (1/\nu + d_i) \quad \text{and} \quad \left.\frac{\partial G(\theta, b_1, b_2)}{\partial \nu}\right|_{\theta=\theta^C} = -\frac{\theta^C - \varphi(b_1 \vee b_2)}{\nu^2} + \dot{\mathcal{H}}(\nu), \tag{99}
$$

with $d_i \equiv 1/[e^{\theta^C - b_i} - 1]$ and $\mathcal{H}(\nu)$ defined in the proof of Appendix A. Therefore,

$$\frac{\partial \theta^C}{\partial \nu} = -\frac{[\theta^C - \varphi(b_1 \vee b_2)]/\nu^2 - \dot{\mathcal{H}}(\nu)}{\eta - (1/\nu + d_i)}. \tag{100}$$

Based on the borrowing constraint specified in equation (3) of the main text and Assumption 1, we can obtain that $\theta^C > b_1 \vee b_2 + \ell > \varphi(b_1 \vee b_2)$. According to the proof in Appendix A, we know that $\dot{\mathcal{H}}(\nu) < 0$. Furthermore, we have $\eta - (1/\nu + d_i) > \eta - \left[1/\nu + 1/(e^{\ell} - 1)\right]$, which is positive if η is sufficiently large. Therefore, if η is sufficiently large, it follows from equation (100) that $\frac{\partial \theta^C}{\partial \nu} < 0$. In other words, $\theta^C(b_1, b_2)$ decreases as the left-tail risk ν increases for any given b_1 and b_2.

Next, we show that the optimal log coupon level b^C increases when ν increases. Based on equations (94) – (96), we define a new function, denoted by $\Upsilon_i^C(b)$, as follows:

$$\Upsilon_i^C(b) \equiv \frac{\partial V_i^C(b,b)}{\partial b} = \Phi_\theta(\theta^C(b,b), b)\frac{\partial \theta^C(b,b)}{\partial b} + \Phi_b(\theta^C(b,b), b), \tag{101}$$

where the first-order partial derivatives of $\Phi(\theta, b)$ are:

$$\Phi_\theta(\theta, b) \equiv (1-\tau)e^{\theta} + (1-\tau)\left(1 + \frac{1}{\nu}\right) e^{\theta + [\mu(\nu) + \theta - \varphi(b)]/\nu} + \frac{\tau}{\nu} e^{[\mu(\nu)+\theta-\varphi(b)]/\nu + b}, \tag{102}$$

$$\Phi_b(\theta, b) \equiv -(1-\tau)\frac{\dot{\varphi}(b)}{\nu} e^{\theta + [\mu(\nu)+\theta-\varphi(b)]/\nu} + \tau\left[1 - \frac{\dot{\varphi}(b)}{\nu}\right] e^{[\mu(\nu)+\theta-\varphi(b)]/\nu + b} + \tau e^{b}. \tag{103}$$

According to Proposition 2.5, the optimal log coupon level b^C maximizes $V_i^C(b,b)$. The first-order condition and the Hessian condition to characterize b^C are

$$\Upsilon_i^C(b^C) \equiv 0 \quad \text{and} \quad \left.\frac{\partial \Upsilon_i^C(b)}{\partial b}\right|_{b=b^C} < 0, \quad \text{respectively.} \tag{104}$$

It follows immediately from (102) that $\Phi_\theta(\theta, b) > 0$. By the definition of $\Upsilon_i^C(b)$ in (101) and the first-order condition in (104), we obtain the following relation:

$$\Phi_b(\theta^C(b^C, b^C), b^C) = -\, \Phi_\theta(\theta^C(b^C, b^C), b^C) \times \left.\frac{\partial \theta^C(b,b)}{\partial b}\right|_{b=b^C}. \tag{105}$$

According to Proposition 2.3, it follows that $\frac{\partial \theta^C(b,b)}{\partial b} < 0$. As a result, it must hold that

$$\Phi_b(\theta^C(b^C, b^C), b^C) > 0. \tag{106}$$

According to the Implicit Function Theorem, the derivative of b^C with respect to ν can be written as

$$\frac{\partial b^C}{\partial \nu} = -\frac{\left.\frac{\partial \Upsilon_i^C(b)}{\partial \nu}\right|_{b=b^C}}{\left.\frac{\partial \Upsilon_i^C(b)}{\partial b}\right|_{b=b^C}}. \tag{107}$$

To show that $\frac{\partial b^C}{\partial \nu} > 0$, it suffices to show that $\left.\frac{\partial \Upsilon_i^C(b)}{\partial \nu}\right|_{b=b^C} > 0$ because the optimality condition (104) ensures that $\left.\frac{\partial Y_i^C(b)}{\partial b}\right|_{b=b^C} < 0$. The partial derivative $\frac{\partial Y_i^C(b)}{\partial \nu}$ has the following expression:

$$\begin{aligned}\frac{\partial Y_i^C(b)}{\partial \nu} &= \Phi_{\theta\theta}(\theta^C, b)\frac{\partial \theta^C}{\partial b}\frac{\partial \theta^C}{\partial \nu} + \Phi_{\theta\nu}(\theta^C, b)\frac{\partial \theta^C}{\partial b} + \Phi_{\theta}(\theta^C, b)\frac{\partial^2 \theta^C}{\partial b \partial \nu} \\ &\quad + \Phi_{b\theta}(\theta^C, b)\frac{\partial \theta^C}{\partial \nu} + \Phi_{b\nu}(\theta^C, b), \end{aligned} \tag{108}$$

where $\theta^C = \theta^C(b, b)$ and the partial derivatives of $\Phi(\theta, b)$ are

$$\Phi_\theta(\theta, b) \equiv (1-\tau)e^\theta + (1-\tau)\left(1+\frac{1}{\nu}\right)e^{\theta+[\mu(\nu)+\theta-\varphi(b)]/\nu} + \frac{\tau}{\nu}e^{[\mu(\nu)+\theta-\varphi(b)]/\nu+b}, \tag{109}$$

$$\Phi_{\theta\theta}(\theta, b) \equiv \Phi_\theta(\theta, b) + \widetilde{\Phi}_{\theta\theta}(\theta, b), \quad \text{with} \tag{110}$$

$$\widetilde{\Phi}_{\theta\theta}(\theta, b) \equiv (1-\tau)\frac{1}{\nu}\left(1+\frac{1}{\nu}\right)e^{\theta+[\mu(\nu)+\theta-\varphi(b)]/\nu} + \frac{\tau}{\nu}\left(\frac{1}{\nu}-1\right)e^{[\mu(\nu)+\theta-\varphi(b)]/\nu+b}, \tag{111}$$

$$\begin{aligned}\Phi_{\theta\nu}(\theta, b) &\equiv -\frac{1-\tau}{\nu^2}\left\{1+\left(1+\frac{1}{\nu}\right)[\mu(\nu)+\theta-\varphi(b)-\nu\dot{\mu}(\nu)]\right\}e^{\theta+[\mu(\nu)+\theta-\varphi(b)]/\nu}, \\ &\quad -\frac{\tau}{\nu^2}\left\{1+\frac{1}{\nu}[\mu(\nu)+\theta-\varphi(b)-\nu\dot{\mu}(\nu)]\right\}e^{[\mu(\nu)+\theta-\varphi(b)]/\nu+b}, \end{aligned} \tag{112}$$

$$\Phi_b(\theta, b) \equiv -(1-\tau)\frac{\dot{\varphi}(b)}{\nu}e^{\theta+[\mu(\nu)+\theta-\varphi(b)]/\nu} + \tau\left[1-\frac{\dot{\varphi}(b)}{\nu}\right]e^{[\mu(\nu)+\theta-\varphi(b)]/\nu+b} + \tau e^b, \tag{113}$$

$$\begin{aligned}\Phi_{bb}(\theta, b) &\equiv (1-\tau)\left\{\left[\frac{\dot{\varphi}(b)}{\nu}\right]^2 - \frac{\ddot{\varphi}(b)}{\nu}\right\}e^{\theta+[\mu(\nu)+\theta-\varphi(b)]/\nu} \\ &\quad + \tau\left\{\left[\frac{\dot{\varphi}(b)}{\nu}-1\right]^2 - \frac{\ddot{\varphi}(b)}{\nu}\right\}e^{[\mu(\nu)+\theta-\varphi(b)]/\nu+b} + \tau e^b, \end{aligned} \tag{114}$$

$$\Phi_{b\theta}(\theta, b) \equiv -(1-\tau)\frac{\dot{\varphi}(b)}{\nu}\left(1+\frac{1}{\nu}\right)e^{\theta+[\mu(\nu)+\theta-\varphi(b)]/\nu} + \frac{\tau}{\nu}\left[1-\frac{\dot{\varphi}(b)}{\nu}\right]e^{[\mu(\nu)+\theta-\varphi(b)]/\nu+b}, \tag{115}$$

$$\begin{aligned}\Phi_{b\nu}(\theta, b) &\equiv \frac{\dot{\varphi}(b)}{\nu^2}\left[(1-\tau)e^{\theta-b}+\tau\right]e^{[\mu(\nu)+\theta-\varphi(b)]/\nu+b} \\ &\quad - \left[\Phi_b(\theta, b) - \tau e^b\right]\left\{[\theta-\varphi(b)]/\nu^2 - \dot{\mathcal{H}}(\nu)\right\}. \end{aligned} \tag{116}$$

Below, we show that $\Phi_{b\theta}(\theta^C, b) < \Phi_b(\theta^C, b)$. Using equations (113) and (115), we can obtain the following inequality:

$$\begin{aligned}\Phi_b(\theta, b) - \Phi_{b\theta}(\theta, b) =& \frac{1}{\nu}(1-\tau)\frac{\dot{\varphi}(b)}{\nu}e^{\theta+[\mu(\nu)+\theta-\varphi(b)]/\nu} + \tau e^b \\ &+ \tau\left(1-\frac{1}{\nu}\right)\left[1-\frac{\dot{\varphi}(b)}{\nu}\right]e^{[\mu(\nu)+\theta-\varphi(b)]/\nu+b}, \end{aligned}$$

where $\dot{\varphi}(b) = \bar{\varphi}b > \tau\nu$ according to Assumption 2. Moreover, because $\nu \in (0, 1/2)$, which implies $1 - 1/\nu < 0$, we can obtain the following relation:

$$\begin{aligned}\Phi_b(\theta, b) - \Phi_{b\theta}(\theta, b) &> \frac{1}{\nu}(1-\tau)\tau e^{\theta+[\mu(\nu)+\theta-\varphi(b)]/\nu} + \tau\left(1-\frac{1}{\nu}\right)(1-\tau)e^{[\mu(\nu)+\theta-\varphi(b)]/\nu+b}, \\ &> \frac{1}{\nu}(1-\tau)\tau\left(e^\theta - e^b\right)e^{[\mu(\nu)+\theta-\varphi(b)]/\nu} \\ &> 0, \end{aligned}$$

where the last inequality is due to the borrowing constraints, which imply that $\theta > b$.

Now, because $\Phi_{b\theta}(\theta^C, b) < \Phi_b(\theta^C, b)$ and $\frac{\partial \theta^C}{\partial \nu} < 0$, equation (108) implies that

$$\frac{\partial Y_i^C(b)}{\partial \nu} > \Phi_{\theta\theta}(\theta^C, b)\frac{\partial \theta^C}{\partial b}\frac{\partial \theta^C}{\partial \nu} + \Phi_{\theta\nu}(\theta^C, b)\frac{\partial \theta^C}{\partial b} + \Phi_\theta(\theta^C, b)\frac{\partial^2 \theta^C}{\partial b \partial \nu}$$
$$+ \Phi_b(\theta^C, b^C)\frac{\partial \theta^C}{\partial \nu} + \Phi_{b\nu}(\theta^C, b), \tag{117}$$

where $\theta^C = \theta^C(b, b)$.

According to (105), it follows that the inequality below must hold:

$$\left.\frac{\partial Y_i^C(b)}{\partial \nu}\right|_{b=b^C} > \Phi_{\theta\theta}(\theta^C, b^C) \left.\frac{\partial \theta^C}{\partial b}\frac{\partial \theta^C}{\partial \nu}\right|_{b=b^C} + \Phi_{\theta\nu}(\theta^C, b^C)\left.\frac{\partial \theta^C}{\partial b}\right|_{b=b^C} + \Phi_\theta(\theta^C, b^C)\left.\frac{\partial^2 \theta^C}{\partial b \partial \nu}\right|_{b=b^C}$$
$$- \Phi_\theta(\theta^C, b^C) \left.\frac{\partial \theta^C}{\partial b}\frac{\partial \theta^C}{\partial \nu}\right|_{b=b^C} + \Phi_{b\nu}(\theta^C, b^C) \tag{118}$$
$$= \widetilde{\Phi}_{\theta\theta}(\theta^C, b^C) \left.\frac{\partial \theta^C}{\partial b}\frac{\partial \theta^C}{\partial \nu}\right|_{b=b^C} + \Phi_{\theta\nu}(\theta^C, b^C) \left.\frac{\partial \theta^C}{\partial b}\right|_{b=b^C} + \Phi_\theta(\theta^C, b^C) \left.\frac{\partial^2 \theta^C}{\partial b \partial \nu}\right|_{b=b^C}$$
$$+ \Phi_{b\nu}(\theta^C, b^C), \quad \text{where } \theta^C = \theta^C(b, b). \tag{119}$$

The summation $\widetilde{\Phi}_{\theta\theta}(\theta^C, b^C)\left.\frac{\partial \theta^C}{\partial b}\frac{\partial \theta^C}{\partial \nu}\right|_{b=b^C} + \Phi_\theta(\theta^C, b^C)\left.\frac{\partial^2 \theta^C}{\partial b \partial \nu}\right|_{b=b^C}$ is positive, because $\widetilde{\Phi}_{\theta\theta}(\theta^C, b^C) > \Phi_\theta(\theta^C, b^C) > 0$, $\frac{\partial \theta^C}{\partial b} < 0$, $\frac{\partial \theta^C}{\partial \nu} < 0$, and $\left.\frac{\partial \theta^C}{\partial b}\frac{\partial \theta^C}{\partial \nu}\right|_{b=b^C} + \left.\frac{\partial^2 \theta^C}{\partial b \partial \nu}\right|_{b=b^C} > 0$ if η is sufficiently large. The term $\Phi_{\theta\nu}(\theta^C, b^C)\left.\frac{\partial \theta^C}{\partial b}\right|_{b=b^C}$ is positive, because $\Phi_{\theta\nu}(\theta^C, b^C) < 0$ and $\frac{\partial \theta^C}{\partial b} < 0$. The term $\Phi_{b\nu}(\theta^C, b^C)$ is positive, because of Assumption 2. Therefore, $\left.\frac{\partial Y_i^C(b)}{\partial \nu}\right|_{b=b^C} > 0$, and consequently, it holds that $\frac{\partial b^C}{\partial \nu} > 0$. In what follows, we will elaborate more on why $\Phi_{b\nu}(\theta^C, b^C) > 0$. In fact, we can obtain that

$$\Phi_{b\nu}(\theta, b) \equiv \frac{\dot{\varphi}(b)}{\nu^2}\left[(1-\tau)e^{\theta - b} + \tau\right] e^{[\mu(\nu)+\theta-\varphi(b)]/\nu+b} - \left[\Phi_b(\theta, b) - \tau e^b\right]\left\{[\theta - \varphi(b)]/\nu^2 - \dot{\mathcal{H}}(\nu)\right\}$$
$$> -\left[\Phi_b(\theta, b) - \tau e^b\right]\left\{[\theta - \varphi(b)]/\nu^2 - \dot{\mathcal{H}}(\nu)\right\}. \tag{120}$$

In the above equation, we have

$$-\left[\Phi_b(\theta, b) - \tau e^b\right] = (1-\tau)\frac{\dot{\varphi}(b)}{\nu}e^{\theta + [\mu(\nu)+\theta-\varphi(b)]/\nu} - \tau\left[1 - \frac{\dot{\varphi}(b)}{\nu}\right]e^{[\mu(\nu)+\theta-\varphi(b)]/\nu+b}$$
$$\geq (1-\tau)\tau e^{\theta + [\mu(\nu)+\theta-\varphi(b)]/\nu} - \tau(1-\tau)e^{[\mu(\nu)+\theta-\varphi(b)]/\nu+b} \tag{121}$$
$$> (1-\tau)\tau e^{[\mu(\nu)+\theta-\varphi(b)]/\nu+b} - \tau(1-\tau)e^{[\mu(\nu)+\theta-\varphi(b)]/\nu+b}$$
$$= 0,$$

where the first inequality is due to Assumption 2, which implies $\dot{\varphi}(b) = \bar{\varphi}b_i > \bar{\varphi}\underline{b} \geq \tau\nu$. The second inequality is due to the borrowing constraint (3) in the main text. Moreover, the borrowing constraint (3) implies that $[\theta - \varphi(b)]/\nu^2 > 0$. According to the proof in Appendix A, we have $\dot{\mathcal{H}}(\nu) < 0$. Therefore, it follows that $\Phi_{b\nu}(\theta, b) > 0$.

Finally, we have shown that $\frac{\partial \theta^C(b, b)}{\partial b} < 0$ for any given b, $\frac{\partial \theta^C(b_1, b_2)}{\partial \nu} < 0$ for any given b_1 and b_2, and

$\frac{\partial b^C}{\partial \nu} > 0$. Taking the three results together, we know that $\theta^C(b^C, b^C)$ decreases with ν. Additionally, as in equation (14) of the main text, firm i's market leverage can be rewritten as

$$lev^C(b^C, b^C) = \frac{1}{(1-\tau)e^{\theta^C(b^C,b^C)-b^C} + \tau}. \tag{122}$$

As ν increases, $\theta^C(b^C, b^C)$ decreases and b^C increases. Therefore, as ν increases, $lev^C(b^C, b^C)$ increases according to equation (122).

K Proposition K.1 and Its Proof

Proposition K.1 (Best Response Function and Nash Equilibrium Set $\mathcal{B}$)**.** *There exists $h > 0$ such that $\mathbf{b}(b) \equiv b$ on the interval $[b^C, b^C + h)$, and thus, every pair of (b, b) with $b \in [b^C, b^C + h)$ belongs to $\mathcal{B}$, the Nash equilibrium set in the log-coupon space characterized in Proposition 2.5. In addition, when $b < b^C$, $\mathbf{b}(b) \equiv b^C$, independent of b.*

In what follows, we provide a proof for Proposition K.1. We first show that $\Upsilon_i^C(b) < 0$ for $b > b^C$, where $\Upsilon_i^C(b)$ is defined in (101). According to (104), there exists $h > 0$ such that $\Upsilon_i^C(b) < 0$ for any $b \in (b^C, b^C + h)$ because of continuity. This means that, as log coupon level b increases, the marginal increase in tax shield benefits of debt is strictly dominated by the marginal decrease in firm value due to lower profits and a higher default probability, when $b \in (b^C, b^C + h)$. The dominance of the marginal decrease in firm value due to lower profits and a higher default probability is more pronounced with a higher log coupon level b (see Corollary 2.2). Thus, $\Upsilon_i^C(b) < 0$ for $b > b^C$.

We next show that, for any $b \in (b^C, b^C + h)$, it holds that $b = \mathbf{b}(b)$. On the one hand, we show that there does not exist $\tilde{b} > b$ such that $V_i^C(\tilde{b}, b) > V_i^C(b, b)$ for $i = 1, 2$. We prove it by contradiction. If such $\tilde{b}$ exists, then it holds that $V_i^C(\tilde{b}, \tilde{b}) = V_i^C(\tilde{b}, b) > V_i^C(b, b)$. However, this contradicts our established fact that $\Upsilon_i^C(b) < 0$ for $b > b^C$. Therefore, such a $\tilde{b}$ cannot exist. On the other hand, we show that, for any $b \in (b^C, b^C + h)$, there does not exist $\tilde{b} < b$ such that $V_i^C(\tilde{b}, b) > V_i^C(b, b)$ for $i = 1, 2$. This is straightforward because reducing the log coupon level from b to $\tilde{b}$ would lead to lower tax-shield benefits for firm i without increasing its profitability θ^C (see Proposition 2.3) or decreasing its default probability λ^C (see Corollary 2.1). Therefore, (b, b) is a Nash equilibrium for any $b \in [b^C, b^C + h)$.

Lastly, we show that, when $b < b^C$, $\mathbf{b}(b) \equiv b^C$, independent of b. For any $b < b^C$, similar to the argument above, we can show that any coupon level $\tilde{b}$ that is lower than b cannot be the best response to the rival's coupon level b. Further, it cannot be the case $b = \mathbf{b}(b)$. This is because such a scenario would contradict the result of Proposition 2.5, which establishes that $b^C = \min\{b \,:\, (b, b) \in \mathcal{B}\}$. As a result, if the best response function is well defined, it must hold that

$$\mathbf{b}(b) \equiv \underset{\tilde{b} > b}{\operatorname{argmax}}\, V_i^C(\tilde{b}, b), \quad \text{for } i \in \{1, 2\}. \tag{123}$$

Importantly, it holds that $V_i^C(\tilde{b}, \tilde{b}) \equiv V_i^C(\tilde{b}, b)$ for any log coupon levels $\tilde{b}$ and b such that $\tilde{b} > b$. Thus, the relation characterized by (123) above implies that

$$\mathbf{b}(b) \equiv \underset{\tilde{b} > b}{\operatorname{argmax}}\, V_i^C(\tilde{b}, \tilde{b}), \quad \text{for } i \in \{1, 2\}, \tag{124}$$

which is independent of b when $b < b^C$. Therefore, based on the characterization of b^C in Proposition 2.5 and equation (104), we can obtain that $\mathbf{b}(b) \equiv b^C$ for all $b < b^C$.