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ABSTRACT

Economic interactions, such as crowdfunding, often involve sequential actions, observational learning, and contingent project implementation. We incorporate all-or-nothing thresholds in a canonical model of information cascades. Early supporters effectively delegate their decisions to a "gatekeeper," resulting in uni-directional cascades without herding on rejections. Project proposers consequently can charge higher prices. Proposal feasibility, project selection, and information aggregation all improve, even when agents can wait. Equilibrium outcomes depend on the crowd size, and project implementation and information aggregation achieve efficiency in the large-crowd limit. Our key insights remain robust under thresholds in dollar amounts, alternative equilibrium selection, among other model extensions.

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A data appendix is available at http://www.nber.org/data-appendix/w30820

1 Introduction

Financing business activities and gathering support often involve sequential contributors, observational learning, and project implementation contingent on achieving certain threshold levels of support. Over the past decade, crowd-based fundraising, which includes equity and reward crowdfunding, peer-to-peer lending, and initial coin offerings, constitutes the most salient example. Such economic interactions among sequential, privately informed agents are prone to information cascades that cause incomplete information aggregation and suboptimal financing. Standard theories (e.g., Banerjee, 1992; Bikhchandani, Hirshleifer, and Welch, 1992) focus on pure informational externalities with each agent's payoff structure independent of others' actions. We incorporate into a model of dynamic contribution games the fact that many projects or proposals in practice are only implemented with a sufficient level of support—an "all-or-nothing" (AoN) threshold. We show that threshold implementation drastically alters informational environments and economic outcomes, with implications for financing projects and aggregating information—arguably the two most important functions of modern financial markets.[1]

Specifically, we introduce threshold implementation in a standard framework of information cascade à la Bikhchandani, Hirshleifer, and Welch (1992). A project proposal is sequentially considered by N agents who choose to support or reject. Each supporter pays a pre-specified contribution price, and gets an eventual payoff normalized to one if the project is good. All agents are risk-neutral with a common prior belief about the project's quality. They each receive a private, informative signal and observe the actions of preceding agents before deciding whether to support. Deviating from the literature, supporters pay the price and receive the payoff if and only if the support level reaches an AoN threshold, which is either exogenously given or endogenously determined jointly with the price by the proposer.

AoN thresholds lead to uni-directional cascades in which agents never rationally ignore positive private signals to reject the project (i.e., there is practically no DOWN cascade, which we define precisely in the model), but may rationally ignore negative private signals to support the project (i.e., UP cascades are possible), making the agents appear to

[1] AoN threshold is predominant on crowdfunding platforms and in venture financing. Moreover, super-majority rule or q-rule is a common practice in many voting procedures; assurance contract or crowdaction in public goods provision is characterized by sequential decisions and implementation thresholds (e.g., Bagnoli and Lipman, 1989). In a similar spirit, charitable projects set target levels of fundraising to proceed (e.g., Andreoni, 1998).

have fears of missing out. Information aggregation also becomes more efficient, especially with a large crowd. With endogenous implementation threshold and price, the proposer no longer underprices the issuance as seen in Welch (1992). Consequently, proposal feasibility (positive probability for implementation), project selection (good projects being more likely implemented than bad projects), and information aggregation (public supporting history revealing project quality) all improve. In particular, when the number of agents approaches infinity, equilibrium project implementation and information aggregation become efficient, which is in stark contrast to findings in the prior literature on information cascades (Banerjee, 1992; Lee, 1993; Bikhchandani, Hirshleifer, and Welch, 1998; Ali and Kartik, 2012).

To derive these results, we first take the AoN threshold and price as given in the subgame of agent contribution and learning. We show that before reaching the threshold, the aggregation of private information only stops upon an UP cascade. The intuition is that the AoN threshold links agents' payoffs to subsequent agents' actions, making them partially internalize the informational externalities of their action. Such forward-looking considerations lead to interesting asymmetries: Even before an UP cascade, agents with positive private signals always support because they essentially delegate decisions to a future "gate-keeping" agent whose supporting decision brings the total support to the threshold. This "delegation" hedges against mistakenly supporting a bad project because the subsequent "gate-keeper" makes the contribution decision with better information by observing a longer sequence of previous actions. DOWN cascades are therefore always interrupted by agents with positive signals before the AoN threshold is reached. In contrast, an agent with a negative signal is reluctant to support a project before it reaches the AoN threshold or an UP cascade, for fear that their supporting actions (which would be indistinguishable from the actions of agents with positive signals now) may mislead subsequent agents to positively update the belief on the project quality in spite of the negative signal the agent privately observes. This agent's supporting action then increases the likelihood of a bad project being funded, reducing her expected payoff. But once the agent's belief on the project quality is sufficiently high, an UP starts, the agent no longer worries about misleading subsequent agents because they do not positively updates from her action anyway.

We next analyze how the entrepreneur or proposer endogenously designs the AoN threshold, as well as the contribution price, to maximize the level of support. A higher AoN threshold, albeit less likely to be reached, delays potential DOWN cascades because as we argued earlier, a DOWN cascade cannot happen before the AoN threshold has been reached. The entrepreneur's optimal AoN is thus set to be just sufficient so that achieving it implies a high valuation relative to the contribution price and essentially excludes DOWN cascades. Meanwhile, the proposer trades off increasing the proceeds from supporters (by charging a higher price) with lowering the AoN (and charging a correspondingly lower price so as to still effectively exclude DOWN cascades) to boost the probability of implementing the project. In general, a larger crowd mitigates the concern about implementation failure and generally permits a higher optimal price, making prices endogenously dependent on the crowd size.

AoN thresholds and uni-directional cascades have three important implications. First, they improve project feasibility by allowing good projects with high production costs to be supported. Standard information cascade theories suggest that for projects with high production costs, the contribution price to at least cover the cost is so high that the first agent will reject it even with a positive private signal, resulting in a DOWN cascade and a guaranteed funding failure (Welch, 1992). AoN thresholds mitigate the concern over DOWN cascades, making it possible to charge a high price to cover the production costs. Second, AoN thresholds improve project implementation efficiency because charging a high price implies implementation only when the posterior belief is sufficiently positive, which is correlated with the project's positive quality. Third, AoN thresholds facilitate information aggregation by mitigating DOWN cascades and delaying the arrival of UP cascades. A proposer facing a large number of potential supporters can utilize threshold implementation to guard against DOWN cascades and to charge a high contribution price (which delays UP cascades) for greater proceeds or support, regardless of whether the threshold is eventually reached.

While outcomes in standard models of information cascades are independent of the size of the agent base, the case with AoN thresholds differs: the errors of mis-supporting or mis-rejecting decrease with the crowd size, and the endogenous price converges to the highest level at which the proposer extracts full surplus. In the limit, projects are implemented if and only if they are of high quality. Public knowledge about the project's true type also becomes perfect. We therefore obtain socially efficient project implementation (under private signals) and full information aggregation with a large crowd, hitherto unachievable in most models of information cascades. These findings are especially relevant in the age of digital platforms and the Internet, which feature outreaches to extremely large crowds.

Finally, we demonstrate that our key insights apply even when agents have the option to postpone their decisions or to delay expending effort to acquire information, and are thus less subject to the usual critiques of exogenous action timing. We also show that our findings are robust to introducing investor heterogeneity and thresholds based on dollar amounts (and to introducing small contribution frictions or learning costs, which is discussed in the appendix). We further analyze other perfect Bayesian Nash equilibria under the same mild tie-breaking convention and to understand the strategic complementarity introduced by AoN thresholds. In terms of project implementation and information aggregation, the equilibrium outcomes converge to the ones characterized in our baseline model.

The theoretical insights we derive apply to many sequential contribution games such as venture financing or syndicated loans. We highlight the application to crowdfunding for several reasons: First, crowdfunding has quickly become a mainstream source of capital for entrepreneurs, with its total volume surpassing the market size for angel funds in 2015 and reaching a whopping 35 billion USD globally in 2017 even before the explosion of crypto-token offerings. Second, it presents a setting where the technology allows an outreach to large crowds, which renders the limiting results for large crowds relevant and important. Third, the sequential nature of contributions and threshold implementation are salient in crowdfunding, making it representative of general dynamic economic interactions with observational learning and threshold implementation, unlike auctions.

Moreover, other forms of entrepreneurial or corporate finance also feature investors frequently inquiring about preceding investments as well as threshold implementation written as clauses in the contingency offering contracts, IPOs, subscription money-back guarantees, or private placement memoranda.[2] Therefore, they can also be analyzed through our conceptual lens, further demonstrating the practical importance of threshold implementation design in a considerable variety of economic interactions and financing situations.

Literature — Our paper adds to the theory of informational cascades, sequential decisions, and observational learning. The insights from prior dynamic informational models primarily concern signal structure and learning bias (Banerjee, 1992; Bikhchandani, Hirsh-

[2]In an angel or A round of financing, investors who are approached later in the fundraising process often learn which other financiers indicated their support for the project and offer additional contributions on the condition that the fundraising reaches certain thresholds (Halac, Kremer, and Winter, 2020). In IPO processes, late investors learn from observing the behavior of early investors, and the issuer may choose to withdraw the offering if the market reaction is lukewarm (e.g., Ritter and Welch, 2002). In fact, in the early 1980s, many tiny firms in the United States conducted an IPO with a best efforts contract that frequently had an AoN feature. We thank Jay Ritter for providing this example and Steve Kaplan for showing us sample proprietary documents of private placement memoranda.

leifer, and Welch, 1992; Welch, 1992; Bikhchandani, Hirshleifer, and Welch, 1998; Chamley, 2004; Callander, 2007). Traditionally, informational cascades can be asymmetric or even uni-directional only when some actions are not observable (Chari and Kehoe, 2004; Guarino, Harmgart, and Huck, 2011; Herrera and Hörner, 2013). Our contributions to this literature are two-fold. First, we obtain asymmetric informational cascades endogenously due to threshold implementation even with observable actions. Second, we show that full learning can be achieved with bounded signals once we allow for payoff interdependence via threshold implementation. Importantly, we obtain perfect information aggregation in large-crowd limits, which is typically unachievable with information cascades (Ali and Kartik, 2012). Our model therefore describes a new set of equilibrium behavior by large crowds and adds to the understanding of how the latest technologies, such as the Internet and blockchain, impact the social efficiency in information aggregation and fundraising in financial markets.

Our paper also adds to an emerging literature on AoN design in the context of crowdfunding and marketplace lending. Strausz (2017) and Ellman and Hurkens (2015) find that AoN is crucial for mitigating moral hazards and price discrimination. Chemla and Tinn (2018) share the concern for moral hazard as in Strausz (2017), but in addition emphasize the real option of learning through crowdfunding. Chang (2016) shows that in simultaneous move games as in Chemla and Tinn (2018), AoN also generates more profit under common-value assumptions by making the expected payments positively correlated with values. Hakenes and Schlegel (2014) argue that endogenous loan rates and AoN thresholds encourage information acquisition by individual households in lending-based crowdfunding. Brown and Davies (2020) focus on a simultaneous-action setting in which threshold implementation, when set by an entrepreneur after observing the total contribution, creates a losers' blessing that discourages investors' information acquisition and reduces financing efficiency.

Instead of introducing moral hazard or financial constraint, we offer the first dynamic model of sequential contribution under threshold implementation. Our emphasis on observational learning, a salient feature of crowdfunding and support-gathering processes in real life, distinguishes our paper from and complements the existing crowdfunding literature and studies such as Kremer (2002) and García and Urošević (2013).[3] While demand is exogenous

[3] An average crowdfunding campaign lasts 9 weeks or longer (*https://blog.fundly.com/crowdfunding-statistics/*). As articulated by Canal (2020), one of the best features of crowdfunding platforms is that "users can see the success of a campaign as it progresses," not to mention the ample empirical evidence for agents' sequential arrivals (e.g., Vismara, 2018).

in Strausz (2017), in our setting the demand during crowdfunding campaign is endogenously determined by both the true underlying quality of the project and the dynamic learning under informational frictions. We also confirm the superiority of AoN designs over "keep-it-all" designs in a dynamic environment and the value of committing to threshold implementation for improving financing efficiency (for which Brown and Davies, 2020, also contains an example in a simultaneous-action setting) and information aggregation.

Models of dynamic learning become complicated very quickly. Regarding the particular application of our theory, we do not claim to cover all aspects of crowdfunding, especially those concerning information acquisition and information design (e.g., Kremer, Mansour, and Perry, 2014; Glazer, Kremer, and Perry, 2015). Our paper should be viewed as a first step in understanding the consequences of introducing threshold implementations in dynamic contribution games with large crowds. Instead of allowing the entrepreneur to possess private information about production cost as in Strausz (2017), we emphasize the aggregation of investors' private signals about project quality. Whereas Brown and Davies (2020) emphasizes investors' information acquisition, we focus on entrepreneurs' ex-ante commitment to implementation thresholds in affecting information aggregation, and we derive the optimal thresholds in a dynamic setting.

The rest of the paper is organized as follows: Section 2 sets up the model; Section 3 characterizes the equilibrium, starting with the subgame of contribution to illustrate the main mechanism before endogenizing contribution prices and implementation thresholds; Section 4 discusses model implications on proposal feasibility, project selection, and information aggregation; Section 5 extends the model to allow options to wait, budget heterogeneity and thresholds in dollar amounts, and characterizations of other equilibria; Section 6 concludes. The internet appendices contain all the proofs and details of various model extensions.

2 A Dynamic Model of Crowd-based Support-gathering

2.1 Model Setup

Consider a project proposal presented to agents $i = 1, 2, \ldots, N$ who sequentially take actions $a_i \in \{-1, 1\}$ to either support ($a_i = 1$) or reject ($a_i = -1$) it.[4] In crowdfunding,

[4]We use "support" and "invest" interchangeably, although our model can be applied to situation where the contribution is non-pecuniary. In practice, crowdfunders typically observe both the total capital raised

supporting means contributing financially; more broadly, supporting can be interpreted as adopting or advocating for certain behaviors by incurring a personal cost. If the proposal is implemented, then the proposer collects from every supporting agent a pre-specified "contribution" p, and each agent in the end receives a project payoff V, which is either 0 or 1.[5] Given that crowdfunding often serves a demand discovery function in many cases (Strausz, 2017), V can be interpreted as a crude transformation of the uncertain aggregate market demand, which could be high ($V = 1$) or low ($V = 0$).

Threshold implementation. We depart from the prior literature by incorporating "all-or-nothing" (AoN) thresholds commonly observed in practice: the proposer receives "all" contributions if the campaign reaches a pre-specified threshold level of support, or "nothing" otherwise.[6] In other words, the project is implemented if and only if at least T agents support it, where the threshold T could be exogenous, e.g., driven by the need to cover a minimum scale of the project that is outside the entrepreneur's control, once the contribution price is specified. In many cases including crowdfunding, however, T is endogenously set by the entrepreneur, which is equivalent to setting a total dollar amount when agents face the same contribution price. We discuss thresholds in dollar amounts under investor heterogeneity in Section 5.2. Note also that supporters pay p only when the project is implemented. Threshold implementations are an salient feature of crowdfunding markets, and our contribution centers around providing insights on their informational effects, especially concerning financing and information aggregation outcomes.

Agents' information and decision. All agents (indexed by i) and the proposer are rational, risk-neutral, and share the common prior that the project pays $V = 0$ and $V = 1$ with equal probability. Our specification describes fittingly equity-based crowdfunding and

and the number of supporters to-date (Vismara, 2018), whose distinction is immaterial in the baseline model. Importantly, our setting differs from that for voting because non-contributors do not bear the risky outcomes of the project whereas non-voters typically face the consequences of a voting outcome.

[5]A separate literature allows price to dynamically change and focuses on asset pricing implications (Avery and Zemsky, 1998; Brunnermeier, 2001; Vives, 2010; Park and Sabourian, 2011). We follow the standard cascade models to fix the price for taking an action ex ante, which closely matches applications in crowdfunding and entrepreneurial finance. In other activities such as political petitions, p can be interpreted as the supporting effort or reputation cost if the petition goes through and becomes public.

[6]The JOBS Act mandates that crowdfunding platforms adopt threshold implementation (Sec. 4A.a.7. See $http://beta.congress.gov/bill/112th-congress/senate-bill/2190/text$). The AoN mechanism, alternatively known as "provision point mechanism," has also been used in Regulation D filings since 1982 (Bagnoli and Lipman, 1989). As in Hakenes and Schlegel (2014); Chang (2016), we assume an entrepreneur can commit ex ante to an implementation threshold.

peer-to-peer lending, which constitute 80% of the entire crowdfunding market as of 2020. Even in reward-based crowdfunding whereby agents have private valuations and idiosyncratic preferences, there is a common value corresponding to the basic quality of the product. While it does not fully capture cases such as sales of art piece or music where private value dominates, the common value assumption allows unambiguous comparisons concerning project implementation and information aggregation with prior studies (e.g., Fey, 1996; Wit, 1997).

Each agent i observes one conditionally independent informative private signal $x_i \in \{1, -1\}$ such that:

$$Pr(x_i = 1|V = 1) = Pr(x_i = -1|V = 0) = q \in \left(\frac{1}{2}, 1\right). \tag{1}$$

We denote the sequence of private signals by $x = (x_1, \ldots, x_N)$ and the set of all such sequences by $X = \{1, -1\}^N$.[7]

The order of agents' decision-making is exogenous and known to all.[8] When agent i makes her decision, she observes x_i and the history of actions $\mathcal{H}_{i-1} \equiv (a_1, a_2 \ldots, a_{i-1}) \in \{-1, 1\}^{i-1}$. Her strategy can thus be represented as $a_i(\cdot, \cdot) : \{1, -1\} \times \{-1, 1\}^{i-1} \to \Delta(\{-1, 1\})$, which includes mixed strategies in terms of probability distributions of the action set $\{-1, 1\}$. To simplify exposition, we define $A_i = \sum_{j=1}^{i} a_j \mathbb{1}_{\{a_j=1\}}$, for $1 \le i \le N$ as the total number of supporters up to agent i. When $1 \le i' < i \le N$ and $\mathcal{H}_{i'}$ has the same first i' elements as $\mathcal{H}_i$ does, we say $\mathcal{H}_i \in \{-1, 1\}^i$ *nests* $\mathcal{H}_{i'} \in \{-1, 1\}^{i'}$, a concept we use for equilibrium definition later. Agent i's optimization is:

$$\max_{a_i \in \{-1,1\}} \mathbb{1}_{\{a_i=1\}} \mathbb{E}\left[(V - p)\mathbb{1}_{\{A_N \ge T\}} \mid x_i, \mathcal{H}_{i-1}, a_i = 1\right], \tag{2}$$

where, A_N is the total number of supporters among all agents, and $\mathbb{1}_{\{A_N \ge T\}}$ is the indicator

[7]The binary information and action structure here are standard in the literature (Bikhchandani, Hirshleifer, and Welch, 1992). We show in the Internet Appendix C) that the main results and intuition are robust with multiple investment amounts and when signals are asymmetrically distributed.

[8]While real world examples such as crowdfunding may involve endogenous orderings of agents, our setup allows a comparison with the large literature on information cascades which typically assumes exogenous orders of agents (Kremer, Mansour, and Perry, 2014). Moreover, because agents in practice update their beliefs based on the passage of campaign time (also seen in Herrera and Hörner, 2013) and use contribution information alone to predict final funding outcomes (Dasgupta, Fan, Li, and Xiao, 2020), our setup can capture the case in which the agents roughly know their position in line by referencing the usual accumulation and rejection with the passage of calendar time. We show in Section 5.1 that our key findings are robust when agents have the option to wait.

function for project implementation. Agent i gets zero payoff from rejecting ($a_i = -1$) and gets $(V - p)\mathbb{1}_{\{A_N \geq T\}}$ from supporting ($a_i = 1$) the proposal.[9] $a_i = 1$ appears in the conditioning term in (2) because given equilibrium strategies of other agents a^*_{-i}, subsequent agents' decisions and project implementation generally depend on agent i's action.

Following the convention in the literature (e.g., Banerjee, 1992; Bose, Orosel, Ottaviani, and Vesterlund, 2008), we now introduce a tie-breaking rule for agents.

Assumption 1 (Tie-breaking)**.** *When the AoN threshold can be reached in equilibrium (if all remaining agents support), agents indifferent between supporting and rejecting always support.*

Threshold implementations introduce strategic complementarity of agents' actions. Naturally, the strategic complementarity creates equilibrium multiplicity, some of which are undesirable in terms of implementation outcomes. For example, one equilibrium entails everyone rejecting and the project failing for sure regardless of agents' private signals.[10] Assumption 1, which is an equilibrium refinement, rules out such trivial equilibria with extreme forms of coordination. This selection natural in that in practice, when implementation is not completely infeasible, the proposer can always provide an infinitesimal subsidy contingent on implementation to break agents' indifference and induce more support.

Moreover, when one cares about information aggregation even when the project is not implemented, one face redundant equilibria that share the same implementation outcomes and payoffs, but with different public information sets when AoN cannot be reached. We therefore need a weak refinement:

Assumption 2. *In equilibria where the crowdfunding fails (AoN not reached), agents supports if and only if she has a positive private signal.*

This assumption allows information aggregation to be strictly monotone in A_N (e.g., Proposition 6) and to achieve social efficiency in the large crowd limit and even when the crowdfunding fails . It is not needed for other results.

[9] We implicitly assume free information acquisition and no contribution cost in the baseline model. We extend our model with (small) contribution/information acquisition cost in the Internet Appendix B.

[10] We also discuss such undesirable equilibria when we extend our model to allow for contribution or information acquisition costs in Internet Appendix B.

Proposer's optimization. Let $\nu \geq 0$ be the cost per supporter incurred by the proposer. ν can be the production cost of each product in reward-based crowdfunding or private valuation (outside option) of issuer's shares when the project is funded without an equity-based crowdfunding. We assume $\nu < \frac{q^N}{q^N+(1-q)^N}$, i.e., the cost is below the expected investment payoff when it is public that all agents have positive signals; otherwise, the project trivially fails to be implemented. Given the campaign length N (the proposer cannot terminate prematurely), a proposer chooses price p and AoN threshold T to solve:

$$\max_{p,T} \ \pi(p,T,N) = E\left[(p-\nu)A_N \mathbb{1}_{\{A_N \geq T\}}\right]. \tag{3}$$

The proposer's expectation depends on investor agents' equilibrium strategies $\{a_i^*\}_{i=1,2...,N}$. In fundraising, the proposer maximizes his expected profit; in non-financial scenarios, the proposer solicits the maximum amount of support, with p interpreted as each agents' additive amount of support. We leave alternative funding objectives to future studies.

When an agent's action does not reflect her private signal, the market fails to aggregate the dispersed information from her. Our notion of informational cascade is then rather conventional (e.g., Bikhchandani, Hirshleifer, and Welch, 1992):

Definition 1 (Information Cascade)**.** *An* UP *cascade occurs following a history of actions* $\mathcal{H}_n$ *(*$1 \leq n < N$*) if along the equilibrium path, all subsequent agents support the proposal, regardless of their private signal, while Agent n herself is not part of any cascade. We denote the set of such histories by* $\mathbb{H}^U$*. A* DOWN *cascade is similarly defined, replacing "support" with "reject," and* $\mathbb{H}^U$ *with* $\mathbb{H}^D$*.*

Standard models feature both UP and DOWN cascades. If a few early agents observe positive signals, their support may push the posterior so high that the project remains attractive even for someone with a private negative signal. Similarly, a series of negative signals may doom the offering. An early preponderance towards supporting or rejecting causes all subsequent individuals to ignore their private signals, which are then never reflected in the public pool of knowledge.

2.2 Equilibrium

In our analyses, we use the concept of perfect Bayesian Nash equilibrium (PBNE). In equilibrium, each agent's action strategy $a_i^*(x_i, \mathcal{H}_{i-1}, p^*, T^*)$ is a function of her own private signal x_i, the history $\mathcal{H}_{i-1}$, the proposer's proposal choice $\{p^*, T^*\}$, and implicitly conditional on other agents' action strategies $a^*_{-i} = \{a^*_j\}_{j=1,2,...,i-1,i+1,...,N}$. The strategic complementarity among agents introduce multiple equilibria surviving Assumption 1. In our baseline analysis, we focus on a subset of equilibria in which all actions are informative outside a cascade, as formalized here:

Definition 2 (Informer Equilibrium)**.** *An equilibrium is called an "informer equilibrium" if for every i and history $\mathcal{H}_{i-1} \in \{-1, 1\}^{i-1}$ that does not nest any history in $\mathbb{H}^U$ or $\mathbb{H}^D$, agent i's action differs for different x_i, i.e. $a_i(1, \mathcal{H}_{i-1}) \neq a_i(-1, \mathcal{H}_{i-1})$.*

In other words, agents' actions are informative before an information cascade, making them "informers." Informer equilibria are intuitive and clearly illustrate our economic insights. In Section 5.3, we analyze all other PBNE that survive Assumption 1 and establish that these variants asymptotically converge ($N \to \infty$) to the "informer equilibrium" in pricing, project implementation, and information aggregation.

2.3 Benchmark without Threshold Implementation

Let us consider a benchmark without threshold implementation (equivalently, $T = 1$), as in Bikhchandani, Hirshleifer, and Welch (1992) and Welch (1992). The project is implemented for sure and an agent's payoff does not depend on subsequent agents' actions. Agent i simply chooses to support if and only if $\mathbb{E}[V|x_i, \mathcal{H}_{i-1}] \geq p$.

With an exogenous p, both UP and DOWN cascades can occur, which halt the information aggregation. With an endogenous p, imprecise signals can cause "underpricing":

Lemma 1. *The proposer always charges $p \leq q$. In particular, when $\nu = 0$ and $q \leq \frac{3}{4} + \frac{1}{4}\left(3^{\frac{1}{3}} - 3^{\frac{2}{3}}\right) \approx 0.59$, the optimal price is $p^* = 1 - q < \frac{1}{2} = \mathbb{E}[V]$.*

The lemma generalizes the underpricing results ($\nu = 0$) in Welch (1992). The pricing upper bound q is not tight but reflects the concern for early DOWN cascades.[11] If $p > q$, then even with a positive signal $x_1 = 1$, the first agent rejects and so does every subsequent agent, yielding zero payoff for the proposer. The second part of the lemma concerns the optimal pricing when agents' signals are imprecise. UP and DOWN cascades affect the proposer's payoff asymmetrically because he benefits from UP cascades by attracting support from

[11] Without parameter constraint on q, the proposer's optimal price is either $1 - q$ or $\frac{1}{2}$.

future agents with negative signals while DOWN cascades mean that a few early rejections may doom his offering. Consequently, he optimally underprices $p = 1 - q < \frac{1}{2}$ to ensure an UP cascade from the first agent, even with a negative signal. Note that the proposer's incentive to create an UP cascade dominates because the signal is imprecise, i.e., she does not have to lower the price too much to trigger such an UP cascade.

3 Equilibrium Characterization

We now solve the equilibrium in several steps. First, we examine agents' decisions taking the price p and AoN threshold T as given. We then derive the proposer's endogenous p and T prior to the crowdfunding and compare the equilibrium outcomes to the benchmark outcomes without implementation thresholds.

3.1 An Illustration with $T = 2$

Before a formal analysis, we illustrate the key economic intuition through a simple example. Consider a funding campaign with $p \in (\frac{1}{2}, q)$ and $N = 3$. As in Bikhchandani, Hirshleifer, and Welch (1992), without any AoN threshold, an UP cascade occurs when both Agents 1 and 2 observe positive signals and choose to support, and a DOWN cascade starts when Agent 1 observes a negative signal and chooses to reject, which leads to Agents 2 and 3 to follow suit regardless of their own signals.

Now suppose we set the AoN threshold to $T = 2$. When both Agents 1 and 2 receive positive signals and support, Agent 3 still finds the project attractive regardless of her private signal, i.e., threshold implementation does not affect the UP cascade. When Agent 1 observes a negative signal and rejects the proposal, however, DOWN cascade no longer ensues. If Agent 2 observes a positive signal, her optimal choice is to support because she only pays the price p when Agent 3 chooses to support as well. If Agent 3's signal is negative, then she finds the project unattractive and chooses to reject, and the proposal fails to implement. Anticipating Agent 3's action, Agent 2 effectively only pays p when Agent 3 observes a positive signal as well, which implies that the project is profitable. In a sense, Agent 2 is hedged from the downside risk of supporting.

However, the argument does not imply that Agent 2 supports regardless of her private signal, because Agent 2's supporting decision is informative to Agent 3. Agent 2 does not want to mislead Agent 3. For example, when Agent 1 supports, Agent 2 has no incentive to support if she observes a negative signal. If she deviates to support, then Agent 3 (incorrectly) infers that Agent 2's signal is positive and (incorrectly) chooses to support and implement the proposal even if she observes a negative signal.

Lastly, when the AoN threshold is about to be reached (when there exist $T-1$ supporting agents), we are back to the standard information cascade settings and both UP and DOWN cascades could happen. To see this, suppose we have $N = 4$ while keeping $T = 2$. If Agent 1 rejects and Agent 2 supports, and Agent 3 observes a negative signal and chooses to reject, then there is a DOWN cascade, and Agent 4 will reject regardless of her private signal. Here, given Agent 2's support,the project is certainly implemented if Agent 4 contributes.

3.2 Observational Learning under Threshold Implementation

As seen in the simple example, no DOWN cascade happens before the AoN threshold is approached. When the AoN threshold is about to be reached, the model reduces to a standard cascade setting, and both UP and DOWN cascades become possible. Define k_i as the difference between the numbers of inferred positive signals and of inferred negative signals up to agent i. Then k_i depends on both the history $\mathcal{H}_i$ and the agents' equilibrium strategies (which affect how informative the action history is). Similar to Bikhchandani, Hirshleifer, and Welch (1992), we show later in our analysis that the public posterior valuation of the project follows:

$$Pr(V = 1|\mathcal{H}_i) = \frac{q^{k_i}}{q^{k_i} + (1-q)^{k_i}} \equiv V_{k_i}. \tag{4}$$

We define $\bar{k}(p) \equiv \min\{k \mid p \leq V_k, k \in \mathbb{Z}\}$ be the minimum number of "excess" inferred positive signals needed to justify a contribution, where $\mathbb{Z}$ denotes the set of integers. We have:

Proposition 1. *For any given pair of (p, T), a unique informer equilibrium exists. In equilibrium, when there is no cascade yet, an agent supports if and only if the private signal is positive. Importantly, an UP cascade occurs following history $\mathcal{H}_i$ when $k_i \geq \bar{k}(p) + 1$; a DOWN cascade occurs following history $\mathcal{H}_i$ only when $A_i \geq T-1$ and $k_i < \bar{k}(p) - 1$.*

As in the earlier example, Proposition 1 shows that there is no DOWN cascade before approaching the AoN threshold ($A_{i-1} < T-1$) and all actions are informative unless a cascade follows immediately. Once $A_{i-1} = T - 1$, Agent i and subsequent ones face exactly the same decision as they do in standard cascade models, and they update beliefs accordingly.

In stark contrast to the benchmark without threshold implementation, DOWN cascades disappear before approaching the AoN threshold ($A_{i-1} < T - 1$) because an agent with a positive signal is protected when supporting in that she does not pay if the project turns out to be bad (and does not achieve enough support to be implemented). The agent observing $T - 1$ preceding supporters is the "gatekeeper" for all preceding supporters because her decision affects whether other supporters incur the contribution and receive the project payoffs. Because she observes a longer history and makes a more informed decision, preceding supporters benefit from "delegating" the implementation decision to her.

Having said that, observing a longer history is helpful only when actions convey private information. To complete the argument above, we argue that when there is no UP cascade yet and before the AoN threshold is approached, agents with negative signals reject the proposal: If an agent with a negative signal deviates to support, then all subsequent agents would misinterpret her action and form wrong posterior beliefs. The over-optimistic beliefs imply that subsequent agents either start an UP cascade too early or reach the AoN threshold when the true posterior is not high enough. Taking that into account, agents with negative signals find deviations unattractive. Here the information breakdown associated with UP cascades interestingly facilitates information aggregation before $\bar{k} + 1$ is reached.

Note that an UP cascade can still occur before reaching the AoN threshold due to the asymmetry between the payoffs of supporting versus rejecting. A rejecting agent does not share the upside if the eventual posterior valuation makes supporting profitable and the project is implemented. Yet a supporting agent is hedged from potential losses, making agents more motivated to support (and potentially creating UP cascades). We illustrate the two scenarios (with and without an UP cascade) with project implementation in Figure 1, which plots the difference between supporting agents and rejecting agents when n agents have arrived. The figure also includes a sample path that leads to an implementation failure because AoN threshold is not reached.

Exogenous price or AoN threshold. Proposition 1 characterizes the subgame-perfect equilibrium from which we build to endogenize p and T. It also provides insights for the

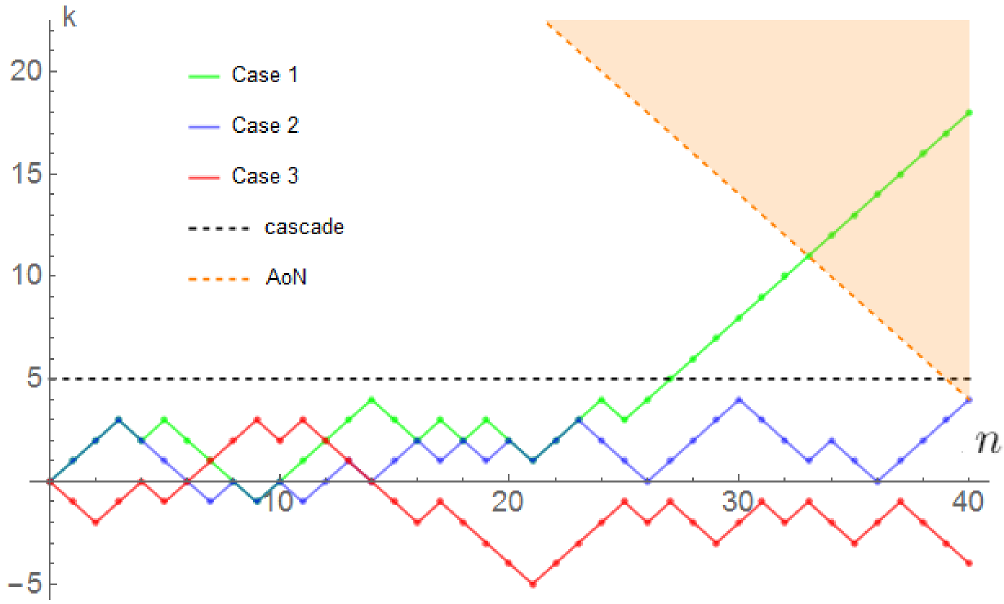


Figure 1: Evolution of support-reject differential

Simulated paths for $N = 40$, $q = 0.7$, $p^* = V_4 = 0.9673$, and AoN threshold $T^*(N) = 22$. Case 1 indicates a path that crosses the cascade trigger $\bar{k}(p) + 1 = 5$ at the 26th agent and all subsequent agents support regardless of their private signal; Case 2 indicates a path with no cascade, but the project is still funded by the end of the fundraising; Case 3 indicates a path where AoN threshold is not reached and the project is not funded. The orange shaded region above the AoN line indicates that the project is funded.

situation in which the entrepreneur has little say in the price and funding target. For example, T could be the minimal capital required to fund a film production that is exogenous to the model. Because the proposition holds for any p and T, the asymmetry in information cascades is robust whenever we have threshold implementation.

3.3 Optimal Price and AoN Threshold

In practice, project proposers and entrepreneurs typically set contribution prices and implementation thresholds. We next endogenize the proposer's choices in the model. We find two main effects. First, they essentially rule out DOWN cascades even after the thresholds are approached. Second, they make the optimal price dependent on the number of agents. The two effects jointly affect proposal feasibility, project selection, and information aggregation, especially with large crowds.

Proposition 2. *An informer equilibrium exists for each N. In equilibrium, agents' action strategies are as characterized in Proposition 1, and the proposer's optimal proposal choice $\{p^*, T^*\}$ satisfies $p^* = V_{k^*}$ and $T^* = \lfloor \frac{N+k^*}{2} \rfloor$, for some integer $k^* \in \{-1, 0, \ldots, N\}$.*

Proposition 2 establishes the equilibrium existence and shows that the set of potential optimal prices and AoN targets is finite. To see this, notice that based on Proposition 1, for a given T, any $p \in (V_{k-1}, V_k]$ induces the same supporting decisions. The proposer always finds $p = V_k$ strictly dominating any $p \in (V_{k-1}, V_k)$. We therefore can focus our analysis on $p \in \{V_{-1}, V_0, \ldots, V_N\}$. We exclude $k < -1$ because $V_{-1} = 1 - q$ is already sufficiently low to induce an UP cascade from the very beginning. Different from Proposition 1, it is possible to have multiple equilibria here because there may exist more than one k^* generating the same maximum expected revenue for the proposer.

In equilibrium, the proposer chooses the optimal level of AoN threshold jointly with price to maximize the expected revenue. The proposer is concerned about DOWN cascades because once they start, subsequent agents all reject. As shown in Proposition 1, there is no DOWN cascade before approaching the threshold. For a given equilibrium price V_{k^*}, a higher AoN target either reduces the possibility of DOWN cascades or delays their arrival, so that the proposer can collect more proceeds conditional on implementation. Yet a higher threshold itself is more difficult to reach. The proposer facing the tradeoff finds the optimal AoN threshold to be the one that is just large enough to exclude DOWN cascades that reduce the total expected proceeds. The following corollary characterizing DOWN cascades proves useful for our discussion of model implications later.

Corollary 1 (Uni-directional Cascades). *With endogenous threshold implementation and contribution price, the sufficient and necessary condition for a DOWN cascade entails only the last agent ($i = N$) herding (i.e., ignoring the private signal and reject) and an implementation failure even when all private signals are aggregated publicly.*

Corollary 1 states that for all practical purposes, all surviving DOWN cascades are of no concern here because they can only start from the last agent and do not affect project implementation. To see this, notice that when the price is $p = V_k$, if the DOWN cascade entails only agent N, then agent $N - 1$ must observes a negative private signal $x_{N-1} = -1$, and the posterior before her action is V_{k-1}. Now the highest possible evaluation of the project is V_{k-1}, smaller than price. This implies that the project will be rejected regardless of the last agent N's private signal x_k. The down cascade, when it exists, does not affect proposer's payoff much and has almost no impact on information aggregation. This uni-directional nature of cascades has important implications for project implementation and information aggregation, as we demonstrate in Section 4.

We next turn to the optimal pricing. Without an AoN threshold, i.e., $T = 1$, the concern about DOWN cascades is so severe that in equilibrium the proposer avoids DOWN cascades by choosing a low price to trigger an UP cascade at the very beginning (Lemma 1). Endogenous AoN thresholds mitigate the concern. The optimal pricing problem now is very similar to the one in the auction literature: a higher price allows the proposer to extract more rent from each supporter, but is associated with a higher optimal AoN target, reducing the probability of implementation. Figure 2 illustrates how the entrepreneur's profit, being dependent on implementation outcomes, varies with the optimal AoN threshold and thus its corresponding price.

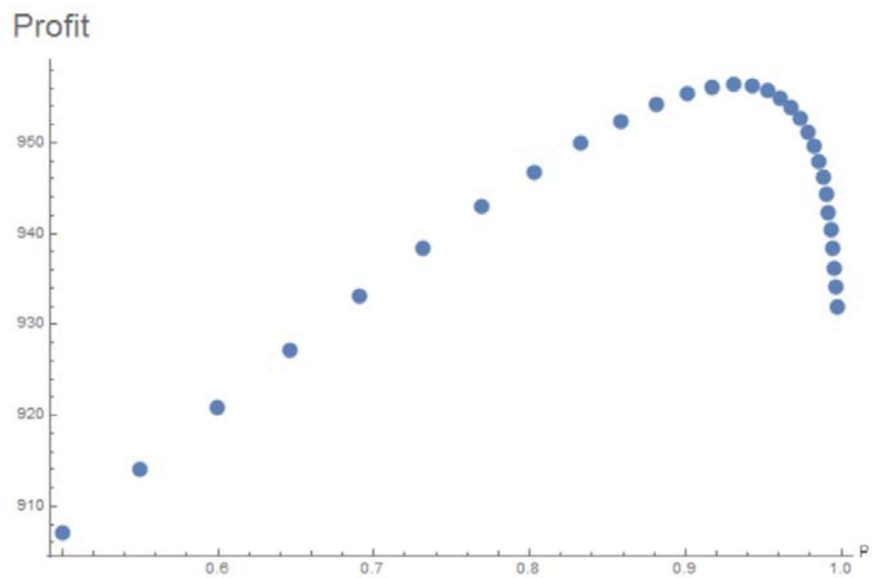


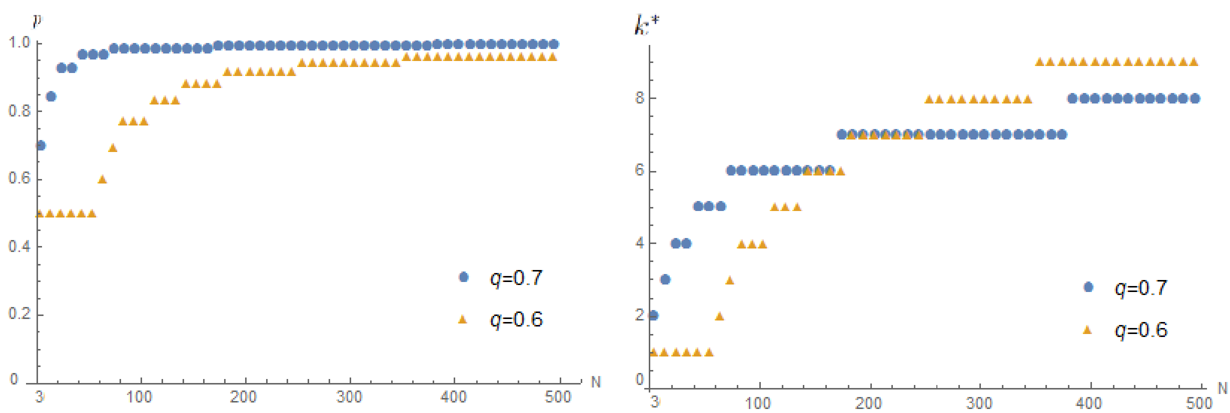
Figure 2: Proposal profit as a function of price

Simulated expected profit as a function of price for $N = 2000$, $\nu = 0$ and $q = 0.55$. For each price V_k, the threshold is set as the optimal one given the price $T = \lfloor \frac{N+k}{2} \rfloor$. The optimal $k^* = 13$, and the profit-maximizing price is around 0.93.

Recall that in Lemma 1 without threshold implementation, the optimal price is *independent* of the number of agents when there is no AoN threshold. With AoN thresholds, the number of remaining agents and thus the total number of agents matter for k^*. Proposition 2 then implies that the optimal price depends on the number of agents N. The following corollary shows that the optimal price p^* in general increases as N increases.

Proposition 3. *The optimal price p^* is dependent on N and has a lower bound weakly increasing in N and approaching 1 as $N \to \infty$. In particular,* $\lim_{N\to\infty} p^*(N) = 1$.

A larger agent base implies a greater chance to learn about the project quality, and because the probability to reach a certain AoN target is higher for good projects, the concern about implementation failure is mitigated, allowing the proposer to set the optimal price higher to increase the expected proceeds. Figure 3 shows the optimal pricing for different values of N, where the left panel plots the absolute price level and the right panel plots the associated k^*. With an endogenous AoN threshold, the proposer can charge a higher price for a larger crowd, which can even appear "overpriced" ex ante, i.e., $p > \mathbb{E}[V]$. Our findings on pricing are important because the underpricing or overpricing of securities or products may affect the success or failure of a project proposal, and thus impact the real economy (Welch, 1992). We discuss these model implications next.



(a) Optimal price for $N = 3$ to 500.

(b) Corresponding k^* for $N = 3$ to 500.

Figure 3: Cascades and Optimal Prices as N Changes

Numerical solution of optimal price as a function of N for $\nu = 0$. The blue plots represent the case for $q = 0.7$, and the orange plots represent the case for $q = 0.6$. Subfigure (a) plots the optimal price V_k, while subfigure (b) illustrates the corresponding k^*.

4 Implications of Thresholds and Large Crowds

Two key functionalities of modern financial markets and digital platforms are funding good projects and aggregating localized/decentralized information to inform investors and economic decisions. Meanwhile, one salient distinguishing feature of crowdfunding platforms from venture capital lies in the large crowds they access. For example, according to Kickstarter's official statistics as of November 2020, the crowdfunding platform has 18.87 million backers in total and the top 10 most popular projects have 74,410 to 219,380 backers; The Crowdfunding Center reports that fully funded projects have on average 300 backers.[12] We

[12]See, for example, https://www.statista.com/statistics/288345/number-of-total-and-repeat-kickstarter-project-backers/ and https://www.statista.com/statistics/378054/most-backed-kickstarter-projects/;

therefore examine the immediate implications of all-or-nothing thresholds for project implementation and information aggregation as well as equilibrium outcomes when the crowd size gets very large.

In particular, we show that threshold implementations generally improve proposal feasibility, project selection, and information aggregation. In the limit of large crowds, we prove that project implementation and information aggregation become fully efficient–results not obtainable in earlier models of dynamic observational learning and crowdfunding.

4.1 Project Implementation

A financial marketplace serves to match capital with worthy projects; it is socially efficient to ensure good projects and good projects only are financed.

Proposal feasibility. Lemma 1 establishes an upper bound for the price in standard cascade models above which the proposal is infeasible, i.e., good projects with production cost $\nu > q$ cannot be supported because even the break-even price triggers DOWN cascades. Threshold implementations allow proposers to charge $p > q$ and still implement the projects.

Proposition 4 (Proposal Feasibility). *Projects with $\nu > q$ cannot be implemented without AoN thresholds; all projects have a positive ex ante probability to be implemented under endogenous pricing and AoN thresholds.*

Obviously, projects with $\nu > V_N$ cannot be financed because agents' posterior valuation can never exceed V_N. Charging $p > \nu$ does not trigger a DOWN cascade if T is set to be sufficiently high. As a result, crowdfunding and the like with either endogenous AoN thresholds or sufficiently high exogenous thresholds enable the financing of projects with high production costs for which funding is otherwise infeasible (Welch, 1992). This result is consistent with Mollick and Nanda (2015), which empirically documents that crowdfunding is more likely to finance projects with costly production that a group of experts would not finance in traditional settings.

Project selection. Without threshold implementation, UP cascades start from the very beginning and all projects are implemented, resulting in a poor project selection (Welch,

https://www.thecrowdfundingcenter.com/data/projects.

1992). With AoN thresholds, DOWN cascades do not occur before reaching the implementation threshold; neither do UP cascades start from the beginning.[13] Good projects thus have a higher chance of reaching the target threshold due to prolonged public information aggregation before any cascade. Project selection therefore improves. Denote the probabilities of missing a good project (Type I error) and financing a bad project (Type II error) by $\mathcal{P}^I = 1 - Pr(A_N \geq T|V = 1)$ and $\mathcal{P}^{II} = Pr(A_N \geq T|V = 0)$ respectively. Then we have:

Proposition 5 (Project Selection)**.** *With endogenous pricing and threshold, good projects are more likely to be implemented than bad projects (i.e.* $Pr(A_N \geq T|V = 1) \geq Pr(A_N \geq T|V = 0)$*). Moreover,* $\lim_{N\to\infty} \mathcal{P}^I_N = \lim_{N\to\infty} \mathcal{P}^{II}_N = 0$*.*

Whereas N does not matter in standard cascade models, threshold implementation links the timing and correctness of cascades to the size of the crowd! Both types of errors approach zero as N becomes large because a larger crowd implies a higher endogenous optimal price, which in turn delays the arrival of UP cascades and improves the correctness of implementation. In general, while UP cascades do lead to some bad projects being financed, such Type II errors are not as frequent as in Welch (1992), in which all bad projects are financed with endogenous pricing and the probability of the cascade being correct is $\frac{1}{2}$. In our setting, the wisdom of the crowd is fully harnessed to distinguish good projects from the bad ones. As for the allocation of surplus, the investors' share vanishes in the limit because the price approaches the true value of a good project, and the proposer eventually gets all the surplus from the project implementation.

4.2 Information Aggregation

Sequential support-gathering processes such as crowdfunding allow the market to aggregate investors' private signals and (partially) reveal the aggregated information to the public. Recent studies provide both theoretical arguments (Strausz, 2017; Chemla and Tinn, 2018; Brown and Davies, 2020) and convincing empirical evidence that entrepreneurs use crowdfunding as an information aggregation mechanism (Mollick and Kuppuswamy, 2014; Viotto da Cruz, 2016; Xu, 2017).[14] In the previous session we analyzes improvement in

[13]Uni-directional cascade and threshold implementation also mean that offerings in our setting can fail whereas offerings never fail in Welch (1992). Our model is thus consistent with the possibility that some offerings fail occasionally and/or are withdrawn, without invoking insider information as Welch (1992) does.

[14]Reduction of search and matching effort, divisibility of funding and low communication costs, greater outreach, decentralized participation, timely disclosure and monitoring, etc. are generally recognized as

project implementation as a result of better information aggregation improvement is partially reflected.[15] The following proposition shows that the information aggregation improves even within projects that fail to reach the AoN threshold.

Proposition 6. *The crowdfunding process is informative of projects' quality and achieves full information aggregation in the large crowd limit. Specifically,* $\mathbb{E}[V|\mathcal{H}_N]$ *is weakly increasing in* A_N *and* $\mathbb{E}[V|\mathcal{H}_N, A_N < T^*]$ *is strictly increasing in* A_N*, with* $\mathbb{E}[V|\mathcal{H}_N] \xrightarrow{prob.} V$ *as* $N \to \infty$.

Different from standard cascade models with DOWN cascades conditional on failing to reach the AoN threshold, the proposer updates the belief more positively with more supporting agents. Our model has a natural implication that the belief updates on V based on incremental support is smaller, conditional on project implementation, because it likely involves an UP cascade and information aggregation is more limited. This is consistent with Xu (2017), who finds that conditional on fundraising success, a 50% increase in pledged amount leads to a 9% increase in the probability of commercialization outside the crowdfunding platform — a small sensitivity of the updating on the prospect of the project to the changes in the level of support.

5 Discussion and Extensions

5.1 Option to Wait

Potential contributors in crowdfunding often cannot or do not wait because of non-trivial attention or monitoring costs. Moreover, the shares or products sold are often in limited supply, and waiting may cause an agent to miss out on the opportunity. That said, in some cases, agents may choose to wait to observe more information. We now extend the model by expanding agents' action space to $a_i^t \in \{-1, 0, 1\}$, where 0 means that agent i delays her decision in period t to the next period. Agents can keep waiting until the last period. In

the key advantages of Internet-based platforms for aggregating information and harnessing wisdom from the crowd. In fact, the SEC also recognizes in its final rule of regulating crowdfunding that "individuals interested in the crowdfunding campaign – members of the 'crowd'...fund the campaign based on the collective 'wisdom of the crowd' " (Li, 2017).

[15] Project selection discussed earlier corresponds to $\Pr(A_N \geq T|V=1) \geq \Pr(A_N \geq T|V=0)$, which is not the same as that good projects are more likely to have a higher A_N. We can show that $V=1$ indeed gives higher A_N in expectation, but it is equivalent to Proposition 6 (a higher A_N informs that the project is more likely to be good) only under the common prior belief on V that $V=0$ and $V=1$ with equal probability. In general, project selection and information aggregation results differ with Bayesian updates.

any period t, after agent t's decision, all agents already waiting from earlier periods make decisions one by one (ordered by their first arrival time). We stick with the Tie-breaking rules in the extension. Internet Appendix A.9 contains a formal description of the extension. The option to wait results in potential equilibrium multiplicity due to the coordination problem on waiting decisions and off-equilibrium beliefs. Nevertheless, the equilibrium characterized in Proposition 1, in a slightly modified form, survives.

Proposition 7. *For any given (p, T), a subgame-perfect equilibrium exists in which those agents who would reject in Proposition 1 now delay their actions as much as possible; those who would support in Proposition 1 support upon their first decision-making opportunity.*

Proposition 7 shows that given (p, T), the option to wait does not affect information aggregation because a decision to wait reveals a negative signal. To see this, if there is already an UP cascade, then no one wants to deviate to wait. Now suppose there is no cascade yet; then it is one equilibrium action for agents with positive signals to support because supporting always dominates rejection and thus there is no need to wait in this equilibrium. For agents with negative signals, waiting till the end weakly dominates rejection and they wait. The next proposition shows that with endogenous (p, T), our finding on project implementation with large crowds is also robust to options to wait.

Proposition 8. *When agents can wait, there exists a sequence of equilibria as $N \to \infty$ in which the optimal price $p_N^* \to 1$. In the limit, all agents support and only support good projects, i.e., the first-best project implementation is obtained.*

The intuition for the results is similar to that for Corollary 3. The absence of DOWN cascades helps us avoid missing good projects and the high price screens out bad projects whose valuation cannot be sufficiently high as information gradually gets aggregated. Notably, because agents with negative signals who wait can still invest or support later so that they do not miss out on good projects, the project scale also reaches full efficiency in the limit, which is the first-best outcome not achievable without agents' options to wait.

5.2 Investor Heterogeneity and Thresholds in Dollar Amounts

According to The Crowdfunding Center, successful campaigns rely on large numbers of comparable, small contributions instead of concentrated, large contributions. Therefore, in the baseline model, the homogeneous contribution amount and threshold specification in the number of supporters reasonably balance tractability and realism. Nevertheless, many crowdfunded projects offer multiple options regarding the level of contribution, and require a minimum dollar amount to be feasible. We next extend the model to illustrate the impact of investor heterogeneity in wealth and of implementation thresholds involving dollar amounts. We demonstrate a novel phenomenon of "prolonged learning" through partial support, which derives from a hitherto absent "funding gap versus belief gap" tradeoff. Internet Appendices A and D further formalize the key insights and provide numerical procedures for designing optimal AoN thresholds under investor heterogeneity and thresholds in dollar amounts.

Example 1. *Suppose there are* 30 *agents. Each agent can choose* $\{H, L, 0\}$*, where* $H = 1$ *and* $L = 0.3$ *are different contribution levels the entrepreneur specifies. Poorer agents can only afford* L*. The project payoff is still* $V = 0$ *or* 1 *per dollar contribution. The AoN target is* $T = 10.1$*. Now consider a history such that* $A_{i-1} = 9, k_{i-1} = \bar{k}(p) - 3$*. Suppose* $x_i = 1$*. Should a rich Agent* i *fully support (*$a_i = H$*), partially support (*$a_i = L$*), or reject (*$a_i = 0$*)?*

Agent i understands that she gets nothing if she rejects. If she chooses to fully support with H, then $A_i = A_{i-1} + 1 = 10$, and $k_i = k_{i-1} + 1 = \bar{k}(p) - 2$. Then in equilibrium, even if next three agents (i.e., $i + 1$, $i + 2$, $i + 3$) all receive positive signals, Agent $i + 1$ (without knowing subsequent agents' signals) still chooses not to support and starts a DOWN cascade. Consequently, Agent i gets zero payoff. However, if Agent i chooses partial support L, then $A_i = A_{i-1} + 0.3 = 9.3$, and $k_i = k_{i-1} + 1 = \bar{k}(p) - 2$. If the next three agents are poor but all receive positive signals and are of type L, then we have $a_{i+1} = a_{i+2} = a_{i+3} = L = 0.3$, $k_{i+3} = \bar{k}(p) + 1$, and $A_{i+3} = 10.2 > 10.1 = T$. The project will still be implemented and Agent i receives a positive expected profit.

In this example, considering all the options, partial support ($L = 0.3$) evidently can be preferred because it generates a positive expected payoff. Hence, it would be naive to conclude that one always contributes to the full extent if one supports, or that the wealthier agent never makes a low contribution. As we show in the internet appendix, in equilibrium, agents with negative signals refrain from contributing. But with positive signals, the example reveals that it could be optimal for a rich agent to switch the contribution from H to L to prolong learning at the expense of slower fundraising.

When the funding gap (the additional support needed to reach implementation) is small but the belief gap (the distance to the break-even valuation) is still big, an agent uses partial support to create "prolonged learning" because a partial support allows for more rounds of trials before triggering implementation or a DOWN cascade. Such episodes of prolonged learning may occur multiple times before eventually the agent returns to full support or a DOWN cascade takes place, depending on whether the break-even belief (captured by $\bar{k}(p)$ in the baseline model) is reached. This tradeoff between reaching the implementation threshold and allowing more observational learning can affect the ex ante information design of the crowdfunding, which constitutes an interesting topic for future research.

Moreover, we demonstrate in the internet appendices that with variable dollar amounts in contribution, the entrepreneur can no longer extract the full surplus even as N goes to infinity. To extract the full surplus, the proposer needs to set prices such that all agents are indifferent between supporting and rejecting. This is problematic now because if the price is high enough to approach $V_{\bar{k}}$, a necessary condition for full surplus extraction, and the gap between H and L is too big, then an agent will switch from high support to low support to prolong the campaign and information aggregation. As such, there exist some signal sequences such that an UP cascade is more likely to happen, which generates a positive payoff to the agents. Note that in an UP cascade the agent with positive signal must be getting some positive payoff, because he is also supporting when his signal is negative which generates a lower but still nonnegative payoff.

Note that similar to adding options to wait, enlarging the action space of agents augments the communication space as well. Consequently, multiple equilibria may exist. For example, with small p and N but a relatively large T, the threshold is sufficiently far away that a contribution L would still allow sufficient subsequent signal aggregation. Yet as long as contributing L would not mislead subsequent agents too much, i.e., the equilibrium belief is that both rich agents with negative signals and poor agents with positive signals would contribute L, and the proportion of rich agents is sufficiently large. This way, rich agents with negative signals can still be protected from below while not missing out from the investment. We leave a more comprehensive characterization of various equilibria to future work.

5.3 Free-riders and Characterization of All Equilibria

We can characterize PBNEs satisfying Assumptions 1 and 2 even beyond informer equilibria—a daunting task most models of observational learning leave out. We first show that all possible equilibria involve a group of "informers" and a group of "free riders" whose actions before a cascade are ignored in equilibrium. Mathematically, Agent i is a "free rider" if $\mathbb{E}[V|\mathcal{H}_{i-1}] = \mathbb{E}[V|\mathcal{H}_i] < V_{\bar{k}(p)+1}$ before any UP cascade. In other words, following sub-history $\mathcal{H}_{i-1}$, it is common knowledge that subsequent rational agents would not update their beliefs based on agent i's action, even though an UP cascade has not started yet.

Free-riding differs from information cascades. Although an agent's action is uninformative as in information cascades, agents still can take informative actions after the free rider's move, and information aggregation resumes until a cascade starts, another free rider appears, or the game ends. We call an equilibrium with a positive number of free riders a "free rider equilibrium." To give an example, suppose $\nu < \frac{1}{2}$, $p = \frac{1}{2} = \frac{q^0}{q^0+(1-q)^0}$ and the target is $T = N$. Then there is a subgame free rider equilibrium in which all but the Nth agent support regardless of their private signal, and the Nth agent supports if and only if $x_N = 1$.

In a free rider equilibrium, who become free riders is generally path-dependent (i.e., specific to sequential realizations of private signals). Whether an agent becomes a free rider depends on subsequent agents' higher order beliefs. As in the informer equilibrium, such a phenomenon is absent in conventional models because absent threshold implementation, the agent's expected payoff at the time of decision-making is independent of subsequent agents' actions.

Lemma 2. *Under Assumption 1 for tie-breaking, a PBNE is either an informer equilibrium or a free rider equilibrium. For $p \in \{V_k, k = -1, 0, \ldots N\}$, all free rider subgame equlibria are weakly Pareto-dominated by the informer subgame-perfect equilibrium described in Proposition 1; free rider subgame equlibria involving at least two free riders are strictly Pareto-dominated.*

To analyze the large crowd limit, we use a standard equilibrium selection based on payoff dominance (Harsanyi and Selten, 1988, which can be motivated in our context by communications before agents receive signals) to focus on *Pareto-undominated* subgame equilibria. This refinement rules out nuisance equilibria such as the example given before the lemma where investors coordinate on Pareto inferior outcomes, but still allows the large class of free rider equilibria for general $p \notin \{V_k, k = -1, 0, \ldots N\}$.[16] With the refinement, whenever $p \in \{V_k, k = -1, 0, \ldots N\}$, we only need to consider informer equilibria and free rider equilibria with one free rider. This allows us to show that the number of informers is un-

[16]Note that nuisance equilibria can also be ruled out by considering agents' option to wait. Obviously, every agent observing signal $x_i = -1$ is better off waiting.

bounded as N increases and that informer and Pareto-undominated free rider equilibria deliver qualitatively the same results as $N \to \infty$.

Proposition 9. *In any sequence of endogenous designs* $\{p(N), T(N)\}_{N=1}^{\infty}$ *and Pareto-undominated subgame equilibria, as* $N \to \infty$*,* $p^*(N) \to 1$*, good projects and only good projects are implemented, and public posterior valuation converges to the true quality* V*.*

The proposition implies that no matter which equilibrium is selected among all the Pareto-undominated ones, in the limit the proposer charges a high enough price, which precludes DOWN cascades and ensures efficient project implementation and full information aggregation. In the proof, we actually show that for large N, the implementation efficiency and information aggregation generally improve relative to that in standard information cascade settings without threshold implementations. Given that financing projects and aggregating information are arguably the most important functions of financial markets, the impact of threshold implementation, especially with large crowds, cannot be overstated.

6 Conclusion

We incorporate All-or-Nothing threshold implementation (AoN) into a standard model of information cascade and find that agents' payoff interdependence results in uni-directional cascades in which agents rationally ignore private signals and imitate preceding agents only if the preceding agents decide to support. Information aggregation, proposal feasibility, and project selection all improve. As the number of agents approaches infinity, equilibrium project implementation and information aggregation achieve socially efficient levels despite information frictions. These findings add to the theories of observational learning and dynamic contribution games, as well as to the emerging literature on entrepreneurial crowdfunding and FinTech platforms.

An important implication of our model is that digital funding platforms can help entrepreneurs reach out to a larger agent base to better harness the wisdom of the crowd than traditional funding channels, as envisioned by regulatory authorities. We highlight that specific features and designs such as endogenous AoN thresholds are crucial in capitalizing these potential benefits, especially for sequential sales in the presence of informational frictions. For parsimony and generality, we have left out some application-specific details. For example, third-party certification can significantly impact equity crowdfunding (Knyazeva and

Ivanov, 2017). A project proposer may also price discriminate or control the information flow to potential investors. Incorporating these institutional features as well as jointly considering the information and mechanism designs constitute promising future research topics.

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