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TARGET RETIREMENT FUND: A VARIANT ON TARGET DATE FUNDS THAT USES DEFERRED LIFE ANNUITIES RATHER THAN BONDS TO REDUCE RISK AS RETIREMENT

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Target Retirement Fund: A Variant on Target Date Funds that uses Deferred Life Annuities rather than Bonds to Reduce Risk as Retirement Approaches
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ABSTRACT

This paper evaluates a new variant of the popular target date funds used in employer-based retirement savings plans. We call this new variant a "target retirement plan." Instead of increasing the allocation to bond funds as retirement approaches, a target retirement fund gradually purchases deferred life annuities beginning at age 50. In the particular straw model target retirement fund examined in the paper, the defined contribution participant makes deferred life annuity purchases at ages 50, 52, 54, 56, 58, 60 and 62. We compare how a target retirement fund participant would fare compared with someone who stays with a traditional TDF until retirement and then buys an immediate life annuity. We examine 1,000 possible 30-year futures for stock returns, bond fund returns and Treasury interest rates. The main result from this paper is that buying a retirement annuity in advance (by accumulating deferred life annuities) is superior to sticking with a Target Date Fund until retirement and then buying an immediate annuity in most scenarios of future stock returns, interest rates and bond returns.

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1. Introduction

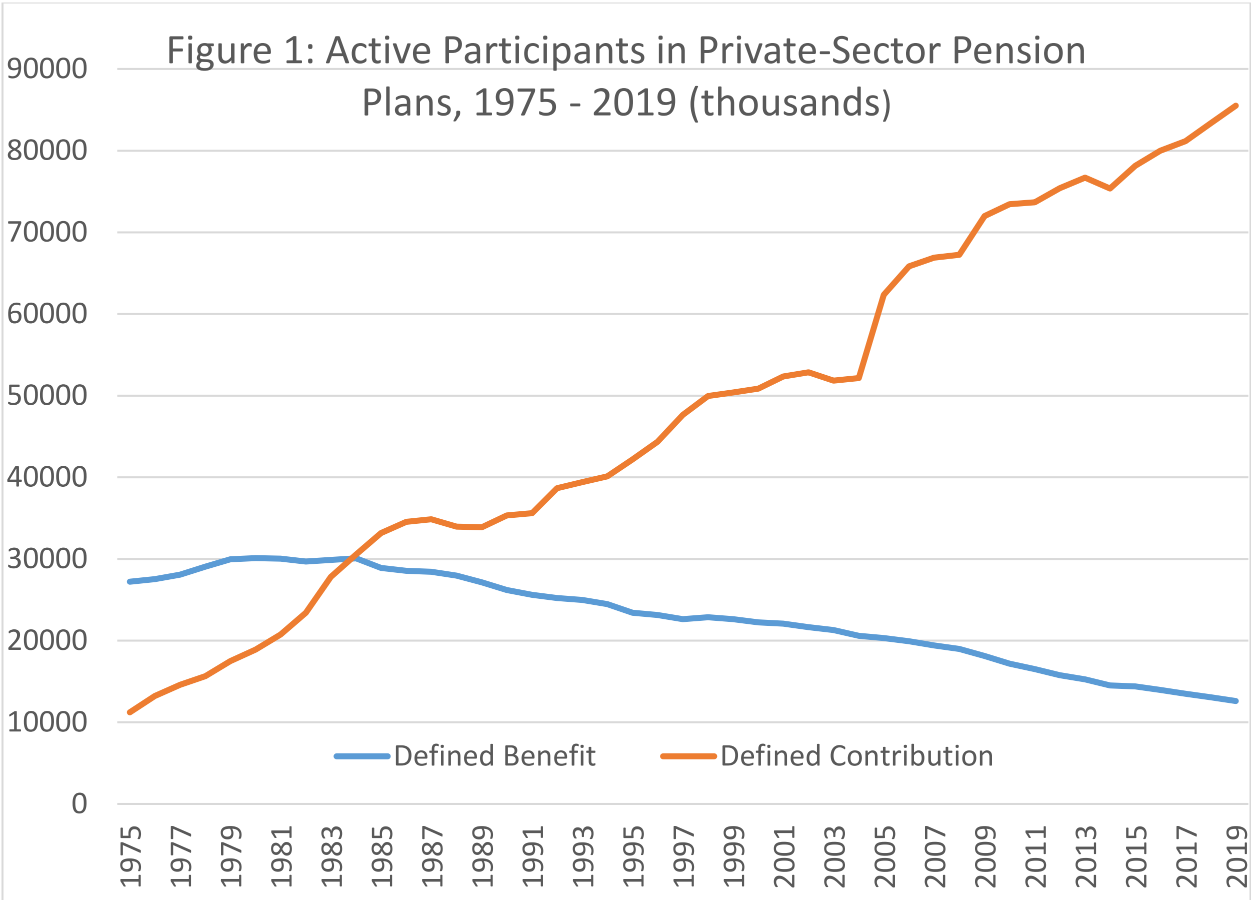
Target Date Funds (TDFs) have been a huge market success in defined contribution retirement plans. Their share of 401(k) assets has grown from 8 percent in 2007 to 31 percent in 2019. In fact, by 2019, 60 percent of 401(k) participants had at least some money in Target Date Funds. 87 percent of 401(k) plans offered TDFs and 87 percent of 401(k) participants had access to them. All of these facts are taken from the 2022 ICI Fact Book (ICI, 2022).

In earlier work (Shoven and Walton (2021)), we evaluated the performance of TDFs in the stock market crash that occurred at the onset of the Covid-19 pandemic and also pointed out the one-size-fits-all nature of TDFs. Target Date Funds offer a dynamic asset allocation that depends on only one thing – the participant's age and thus does not take into account such important matters as education, occupation, marital status, children, home ownership status, health status, other assets and attitudes towards risk. This paper is not going to address these issues. Instead, we will focus on the possibility of introducing a variant of target date funds that we refer to as a "target retirement fund" or TRF.

In addition to the dramatic growth of target date funds, there has been an even larger and longer trend towards defined contribution retirement plans and away from defined benefit ones. The dramatic change in the number of active participants in private companies in the two types of plans is shown in Figure 1. In 1975 there were about 2.5 times as many people in DB plans as in DC plans. By 2019, this had completely reversed to where private sector DC plan participants outnumber DB participants by approximately 7 to 1.

The default payout in a defined benefit plan is typically a single-life annuity. That is, at a specified retirement age (such as 65), the participant is promised a monthly income for as long as they live. The amount of the monthly checks typically depends on years of service with the employer and final salary. Choices are typically offered such as joint and survivor annuities and occasionally a lump-sum cash payout. In contrast, DC plans are primarily retirement asset accumulation vehicles, with no structured payout in retirement. The Target Retirement Fund that we are evaluating as a straw model would combine the simplicity of TDFs with part of the retirement assets devoted to life annuity payouts.

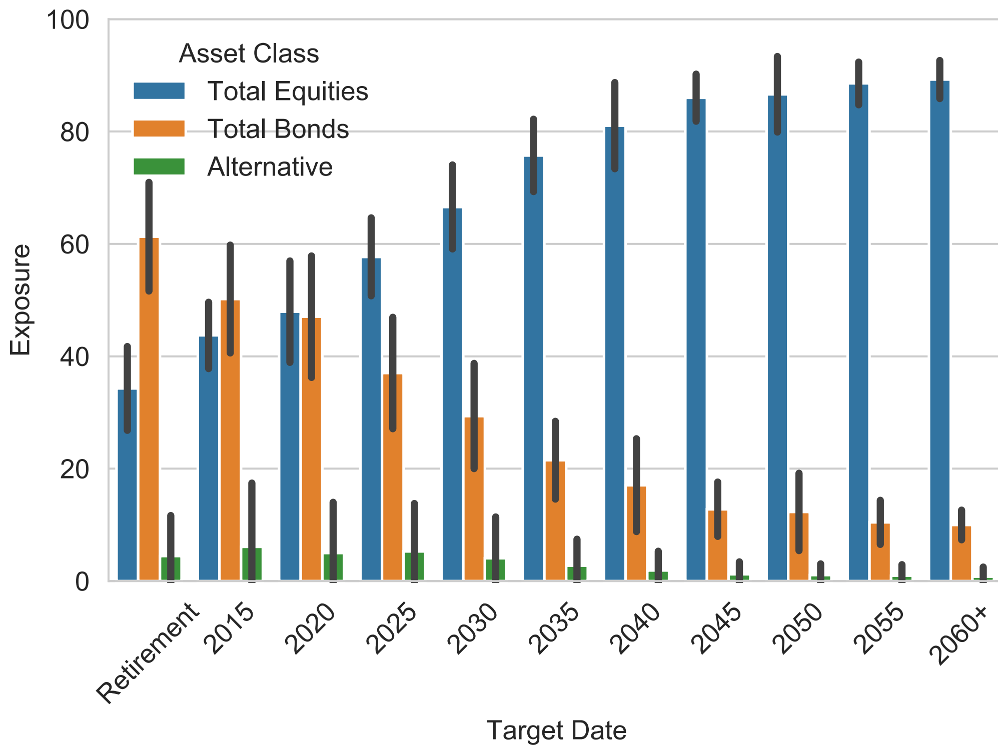
Two key drawbacks to DB plans are that they penalize job changers relative to career employees and that, from the employer's perspective, the necessary funding is unpredictable and depends on the performance of asset markets. Typically, when asset markets perform poorly, required funding increases. In contrast, DC plans are highly portable across employers and highly predictable in terms of employer funding amounts. There is no reason why a DC plan cannot have annuity payouts and still preserve the advantages that they enjoy relative to DB plans.



Target date funds are offered with a range of target retirement dates such as 2020, 2025, 2030, 2035, all the way to 2065. At the more distant target dates, such as 2045 and beyond, the portfolio is roughly 90 percent invested in equities, but for the nearer target dates, equity exposure is reduced and bond exposure is increased. Figure 2 is taken from our previous paper and shows the average asset allocation of target date funds as a function of the target date as of 2020. The dynamic asset allocation is often referred to as the glide path of the TDF.

Our alternative to a TDF, the Target Retirement Fund, would add deferred life annuities instead of bond funds beginning at age 50. As a straw model, not optimized in any respect, we evaluate initially investing all of 401(k) contributions in equities, but then devoting 10 percent of accumulated assets to the purchase of a deferred life annuity at 50, 52, 54, 56, 58, 60 and 62. Each of these seven purchases use 10 percent of the equity balance at the time of purchase. To determine the price of the annuities, we use online quotations from immediateannuities.com. We model someone who starts contributing 9 percent of salary at age 35 (6 percent as an employee contribution and 3 percent as an employer match). The general nature of our results would be the same we believe if retirement contributions started earlier than 35. In our straw model analysis, the employee retires on their 65th birthday and each of the deferred annuities purchased has a monthly payout commencing at 65.

Figure 2: Asset Allocation of Target Date Funds as a Function of Target Date as of 2020



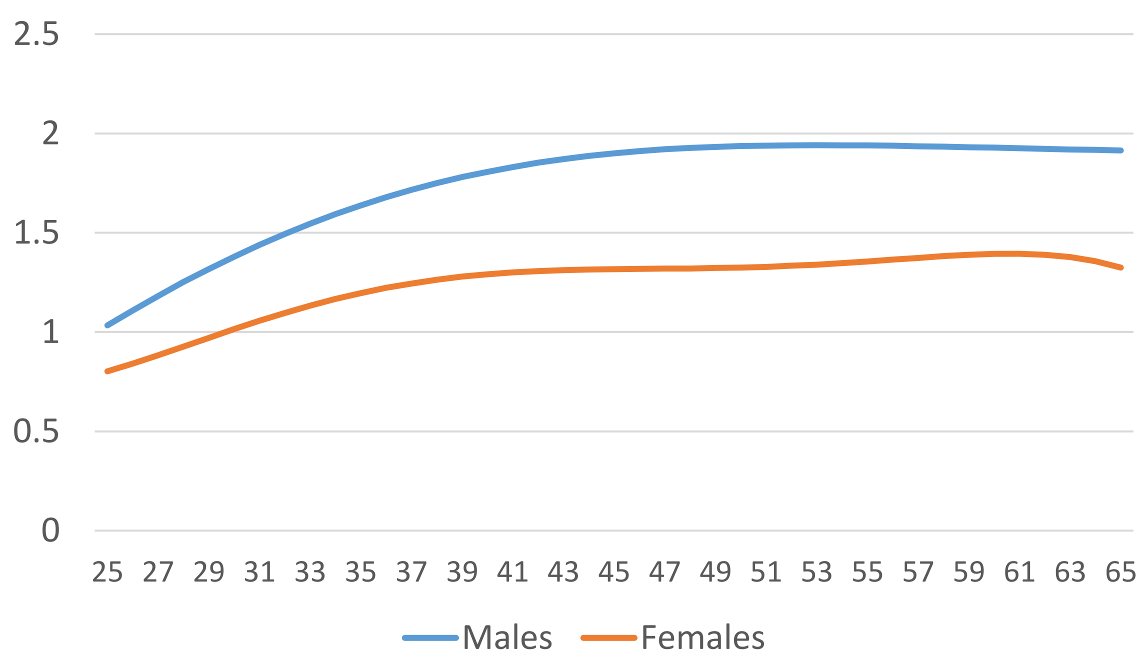
There is a large economics literature suggesting the optimality of using life annuities in retirement. The path-breaking paper was Yaari (1965) which finds that all retirement wealth should be annuitized under a number of assumptions, most noticeably no bequest motive. Davidoff, Brown and Diamond (2005) considerably weaken the assumptions needed to get the full annuitization result and further find that partial annuitization is almost always welfare enhancing if individuals do not have a bequest motive. We take from the literature that the case for at least partial annuitization is quite strong. We also refer to Scott (2008) and Scott, Watson and Hu (2011) who emphasize the advantages of longevity annuities (annuities whose payouts start relatively late in life such as 75, 80 or 85). With their work in mind, we also evaluate a case where the deferred annuities accumulated in the last 15 years of work have initial payouts at 75 rather than 65. The insurance value of annuities is greater in years where survival is more uncertain. This gives an advantage to late-life annuities.

The main result from this paper is that buying a retirement annuity in advance (by accumulating deferred annuities) is superior to sticking with a Target Date Fund until retirement and then buying an immediate annuity in most scenarios of future stock returns, interest rates and bond returns. We get this result by fitting a VAR model to historical returns data and running a Monte Carlo simulation, generating 1,000 sets of future returns and interest rates over a 30-year horizon. For each of the 1,000 scenarios, we calculate the balance of a stylized target date fund for a 401(k) participant with an age-earnings profile typical for college graduates. We calculate the monthly annuity payout accumulated by a TRF participant as well as their liquid asset balance at retirement. The general result is that buying annuities starting at age 50 is superior to waiting until retirement at 65 in the large majority of future return scenarios. It applies in approximately 85 to 90 percent of the scenarios that we evaluated. We examine a second method for generating the 1,000 futures as a robustness check, using a model with parameters not estimated or chosen by the authors. The results are qualitatively the same. This superiority of buying annuities towards the end of one's career rather than waiting until retirement also applies to the purchase of annuities whose payouts start at 75. In fact, the advantage of the TRF approach is larger in the case of annuities whose payout starts at 75.

2. Age-Earnings Profiles

In our simulations of retirement plan accumulations over 1,000 possible returns futures, we use the average earnings of college graduates by age relative to the Average Wage Index (AWI) as measured by Social Security. To obtain the earnings/AWI ratio, we utilize the work of Scott, Shoven, Slavov and Watson (2022). Their earnings data came from the Center for Economic Policy Research's Current Population Survey extract for March 2018. They used the average wage and salary income for individuals with a bachelor's degree. After dividing the results by the Social Security Average Wage Index (AWI) in 2018 ($52,145.80), they smoothed the resulting profiles by fitting a fifth degree polynomial to them. The results are shown in Figure 3.

Figure 3: Average Earnings Relative to AWI by Age for College Graduates



We project the nominal earnings of college graduates in this study by taking these fitted ratios and multiplying them by our projection of the future levels of AWI. We start with the AWI in 2020 ($55,628.60) and assume that it will grow in nominal terms at 3 percent per year. Roughly, this assumes 1 percent growth in real wages and a 2 percent rate of inflation. Most of our analyses have workers starting retirement contributions at age 35.

We recognize that in the real world there is uncertainty about future wages, productivity growth and inflation. For the purposes of this paper, we concentrate on the impact of future real stock and bond returns on the attractiveness of Target Date Funds relative to Target Retirement Funds which utilize deferred life annuities.

3. Quotes for Deferred Annuities

We obtained quotes for single-life deferred annuities with monthly payouts beginning at age 65. We did this for both men and women, with the purchases taking place at ages 50, 52, 54, 56, 58, 60 and 62. We also obtained a quote for an immediate life annuity beginning at 65. We obtained these quotes online from immediateannuities.com on four different occasions: November 2, 2021, April 19, 2022, May 26, 2022 and June 19, 2022. The quotes are for a simple single-life deferred annuity. That is to say, we declined options such as life annuities with a minimum of 10 years of payouts, ones that paid out if you died before the commencement of benefits, and various forms of joint and survivor annuities. The quotes are for fixed nominal monthly payments for life. There certainly were options for graduated payments, such as those that increase 2, 3, or 4 percent per year. We recorded the quote with the highest monthly benefit for a $100,000 purchase from a top rated insurance company (A++). The ratings were from A.M. Best. In all cases, the highest such quotes came from Massachusetts Mutual or Guardian Life.

In Table 1 we show the quotes from June 19, 2022. The quotes for women are less than those for men, primarily because of higher life expectancies for women (i.e. lower age-specific mortality rates). It is also true that the duration of a woman's life annuity contract is greater than one for a man and this may allow the insurance company to hedge its obligation with longer maturity bonds.

Table 1. Annuity Quotes on 6/19/22 with monthly payout beginning at age 65

Purchase Age	Single Male	Single Female
50	1120	1070
52	1010	965
54	928	887
56	838	809
58	769	736
60	704	674
62	637	610
65	567	558

A useful way to transform the data of Table 1 is to divide the purchase price ($100,000) by the quotes in order to get the price of $1/month for life. This can be interpreted as the price of the annuity. The results for a $1/ month annuity starting at 65 as of 6/19/22 are shown in Table 2.

Table 2. Price of $1/month life annuity starting at Age 65 as of 6/19/22

Purchase Age	Single Male	Single Female
50	89.29	93.46
52	99.01	103.63
54	107.76	112.74
56	119.33	123.61
58	130.04	135.87
60	142.05	148.37
62	156.99	163.93
65	176.37	179.21

One can see in Table 2 that the price of $1/month starting at 65 is substantially cheaper for those buying the annuity in advance. This is largely due to the interest that the insurance company can earn before payments commence, but it also factors in the chance that the purchaser will die before 65 and therefore never collect anything and the fact that adverse selection is less of a problem with a substantial deferral period.

Another way to quantify the advantage of buying retirement annuities in advance by purchasing deferred life annuities can be derived from Table 2. If one assumes that the immediate annuity price when you are 65 is the same as the immediate annuity's price on the date of the quotation, then one can calculate the return needed to make passing up the purchase of a deferred annuity and later buying an immediate annuity. For instance, if a 50-year old man chooses to pass on the opportunity to buy a $1 deferred annuity for $89.29 and instead invests the $89.29 and buys an immediate annuity at 65 for $176.37, what rate of return on the investment would make the two strategies equally attractive? The assumption that the immediate annuity's price will remain unchanged is not a bad expectation, although it is extremely unlikely that the realized future price will be exactly unchanged.

Table 3 reports the rate of return on money not invested in deferred annuities required to make postponing the annuity purchase to the time of retirement a breakeven proposition. Again, this is assuming that the price of immediate annuities stays unchanged. The answers are that the investments must earn a rate of return during the potential deferral period of well over the yield on government bonds and even more than investment grade corporate bonds as of 6/19/2022. This is relevant because our Target Retirement Fund strategy involves purchasing deferred annuities as retirement approaches instead of purchasing investment grade corporate bond funds, as is typical of Target Date Funds.

Table 3: Breakeven Return for Postponing Annuity Purchase until Retirement (based on 6/19/22 quotes)

Purchase Age	Single Male	Single Female
50	4.64	4.44
52	4.54	4.30
54	4.58	4.30
56	4.44	4.21
58	4.45	4.03
60	4.42	3.85
62	3.96	3.01

4. Expected Rate of Return on Deferred Annuities

With the four sets of quotes in hand (from November 2021 and April, May and June 2022), we then calculated the expected rate of return offered by these deferred annuities factoring in the probability of being alive to collect the monthly benefits. For the survival probabilities conditional on being alive at the time of purchase, we used Social Security's cohort life tables that were used in the 2018 Social Security Trustees Report. For instance, for the November 2021 quote for a deferred annuity purchased at age 50 with payments starting at 65, we used the cohort life tables for men and women for people born in 1971. The information that we used was the probability of being alive at each age between 65 and 120 conditional on being alive at age 50. These tables involve Social Security's forecasts of future mortality rates. We used the "Alternative 2" or intermediate forecasts, which are the ones that are most commonly used. The survival probabilities are the average for the entire population. In all likelihood the participants in our plan would be in better than average health and have higher than average income, since these features are characteristic of defined contribution participants. On average, DC participants have more education and higher income than non-participants. We did not take this into account in our calculations.

The expected rate of return on these deferred life annuity contracts is given by the internal rate of return that equates the expected value of future payments received to the $100,000 purchase price. That is, it is the r(k) which solves the following equation, where k is the age at the time of purchase, Q(k) is the best quote at that age of purchase, and S(t,k) is the chance of being alive at age t given that the purchaser was alive at age k.

(1) $$100{,}000 = Q(k)\sum_{t=65}^{120} S(t,k)/(1+r(k))^{t-k}$$

The internal rates of return are shown for purchases at age 50, 52, 54, 56, 58, 60 and 62 in Figure 4A, 4B, 4C and 4D. The figures also show a quote for an immediate annuity at 65 and the 10-year Treasury rate at the time of purchase. Next to the figures is a table of Treasury interest rates on the day of the annuity quotation.

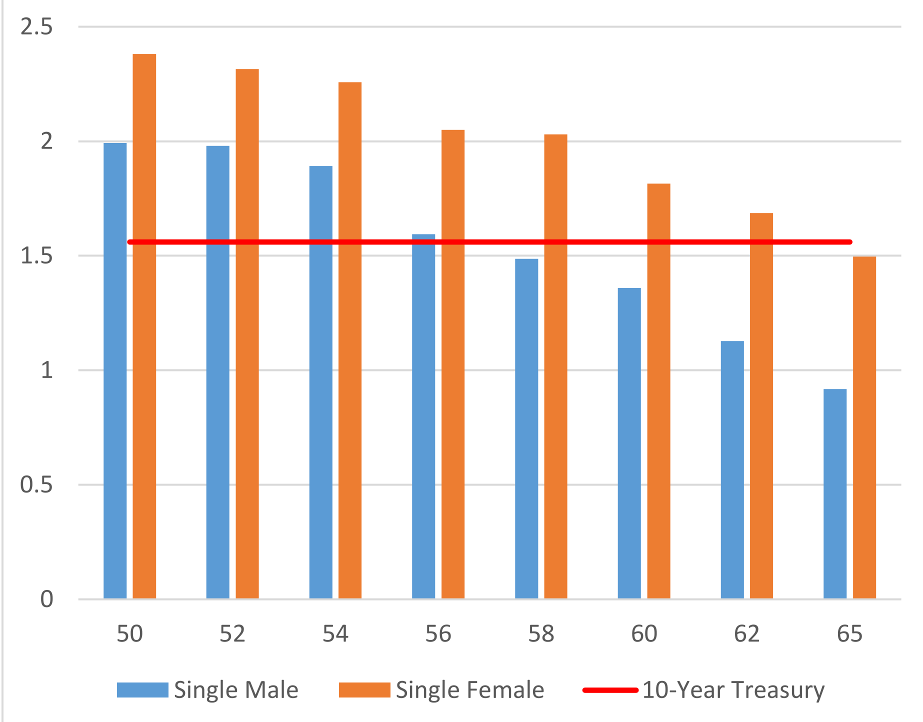


Figure 4A: IRR for Deferred 65 Annuities, November 2, 2021

Treasury Yields	
1-Yr	0.15
2-Yr	0.46
3-Yr	0.73
5-Yr	1.15
7-Yr	1.42
10-Yr	1.56
20-Yr	1.97
30-Yr	1.96

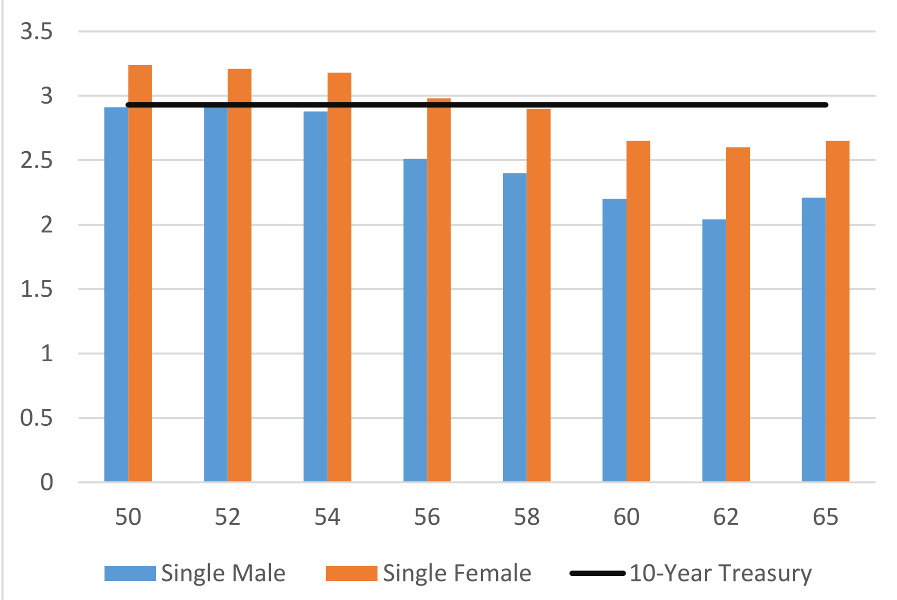


Figure 4B: IRR for Deferred 65 Annuities, April 19,2022

Treasury Yields	
1-Yr	1.94
2-Yr	2.61
3-Yr	2.81
5-Yr	2.91
7-Yr	2.95
10-Yr	2.93
20-Yr	3.19
30-Yr	3.01

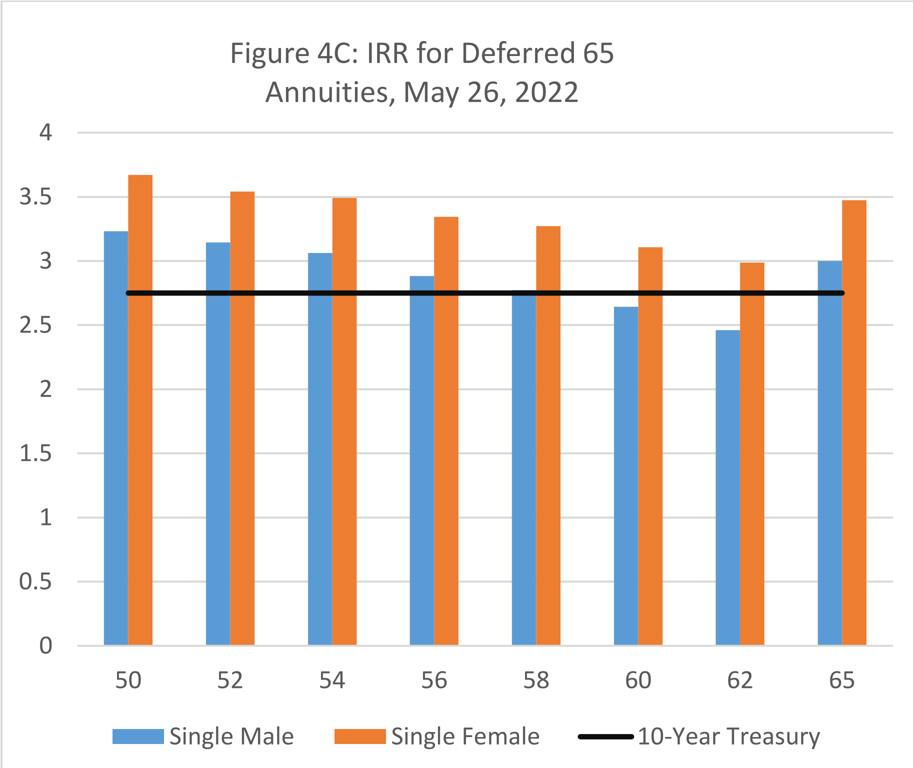


Figure 4C: IRR for Deferred 65 Annuities, May 26, 2022

Treasury Yields	
1-Yr	1.99
2-Yr	2.46
3-Yr	2.63
5-Yr	2.70
7-Yr	2.75
10-Yr	2.75
20-Yr	3.18
30-Yr	2.99

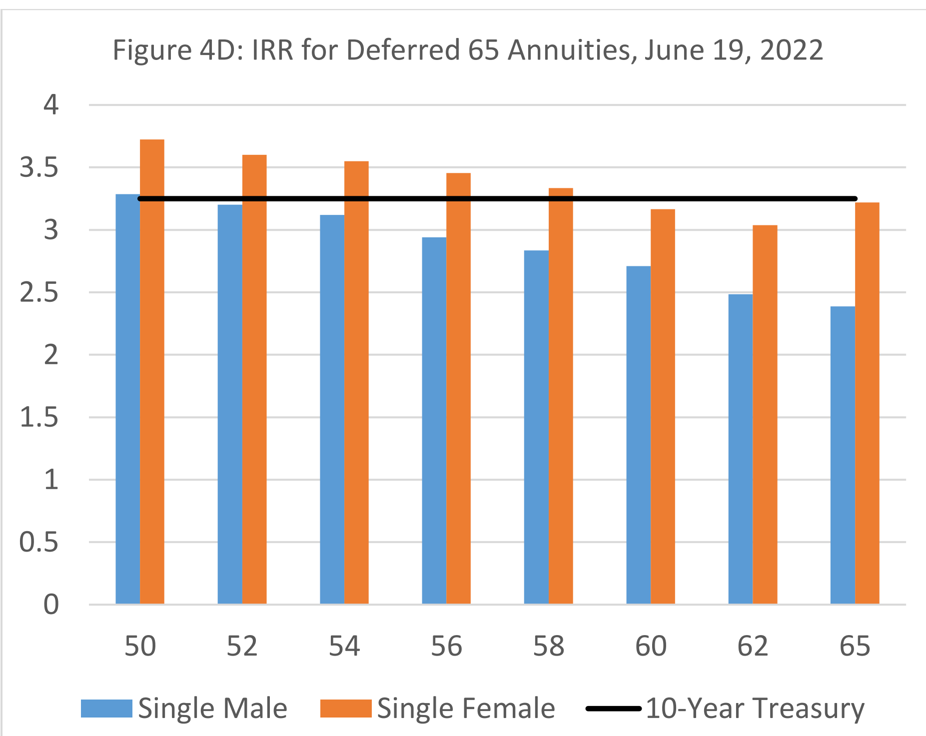


Figure 4D: IRR for Deferred 65 Annuities, June 19, 2022

Treasury Yields	
1-Yr	2.80
2-Yr	3.17
3-Yr	3.35
5-Yr	3.34
7-Yr	3.34
10-Yr	3.25
20-Yr	3.55
30-Yr	3.36

We also collected data for deferred annuities whose payouts begin at age 75. As emphasized by Scott (2008) and Scott, Watson and Hu (2011), the insurance element of annuities is more valuable for ages where survival is more uncertain, so an annuity that begins its payouts at 75 has more valuable insurance per dollar spent than one starting at 65. That simply is due to increasing mortality risk with age. The quotes (obtained on August 23, 2022) for deferred annuities with monthly payouts starting at age 75 are shown in Figure Table 4.

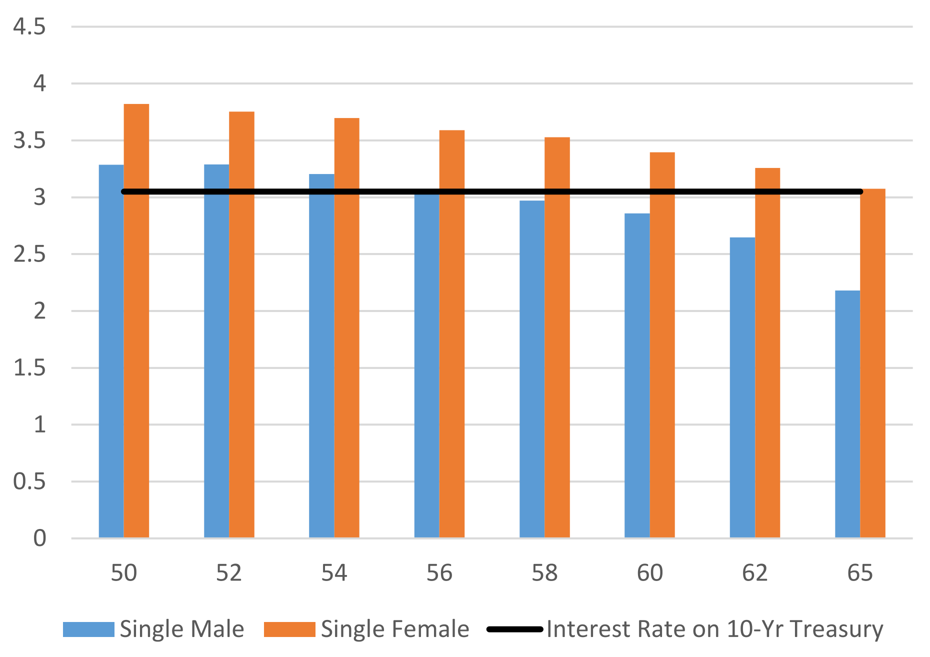
Table 4: Annuity Quotes on 8/23/22 for Monthly Payments Starting at 75

Purchase Age	Single Male	Single Female
50	2,617	2,434
52	2,384	2,216
54	2,184	2,029
56	1,974	1,833
58	1,807	1,678
60	1,649	1,532
62	1,476	1,371
65	1,230	1,192

Not surprisingly, these quotes for monthly payouts are much higher than corresponding annuities whose payout starts at 65. On average, the payouts were about 2.25 times the quotes on 8/23/22 for annuities with payouts commencing at 65.

The expected internal rate of return offered by these annuities whose payouts start at 75 were comparable to the ones starting at 65 using population cohort mortality tables from Social Security. Figure 4E shows the calculated internal rates of return compared with the 10-year Treasury interest rate on the date of purchase. It is interesting that the expected rate of return on these deferred life annuities with payouts starting at 75 still decline with the age of purchase between 50 and 65. Our interpretation of this fact is that people learn useful information about their prospects for a long life between 50 and 65. For instance, they learn whether they experience any episodes of cancer or heart disease in their 50s or early 60s. This implies that adverse selection (at least from the insurance company's point of view) becomes more of a problem with the later purchase ages.

Figure 4E: IRR for Deferred 75 Annuities, As of August 23, 2022

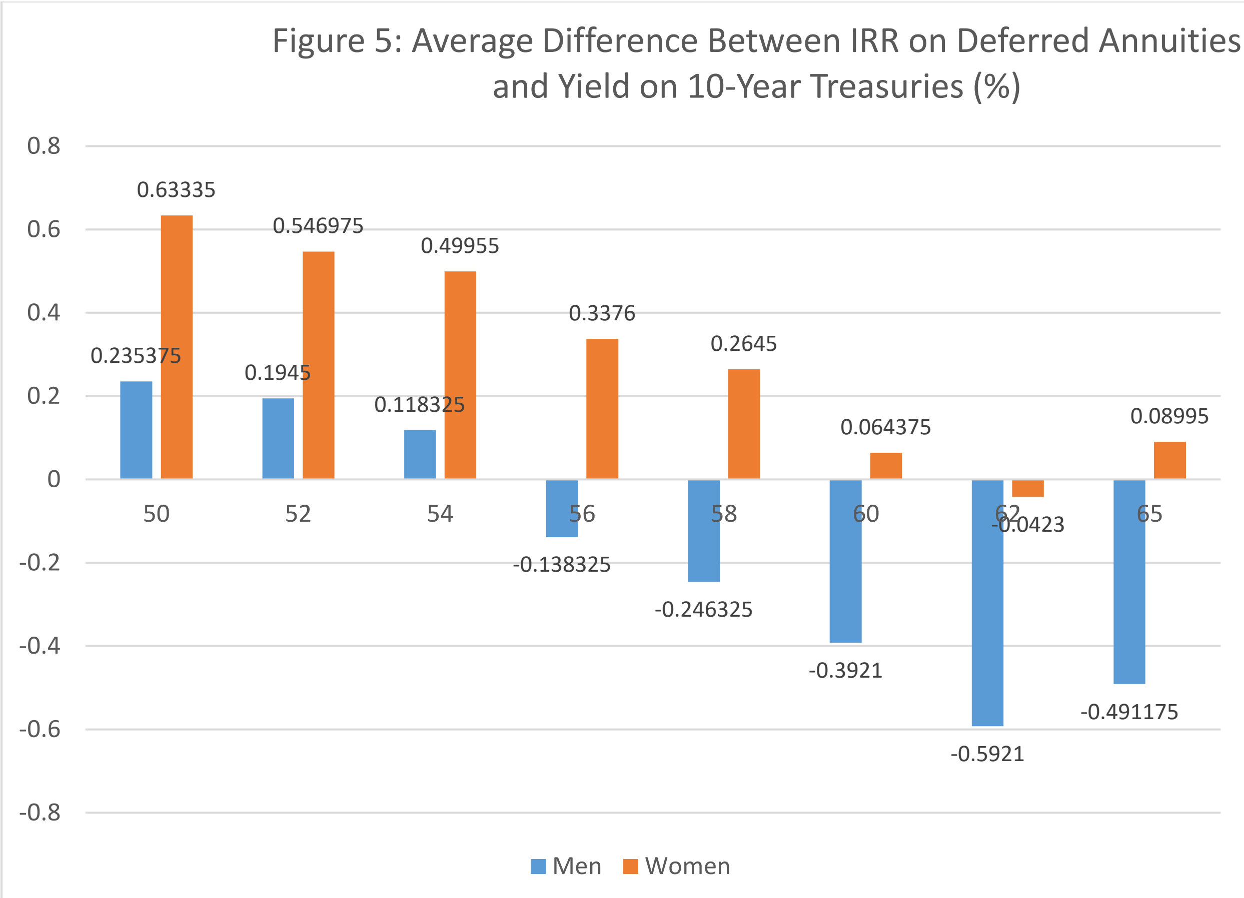


Treasury Yields	
1-Yr	3.29
2-Yr	3.29
3-Yr	3.35
5-Yr	3.18
7-Yr	3.14
10-Yr	3.05
20-Yr	3.49
30-Yr	3.26

At first blush, out results seem at odds with the recent paper by Poterba and Solomon (2021), who calculate money's worth figures for both immediate and deferred annuities. However, upon closer examination the qualitative results are quite consistent. The approaches are somewhat different. We calculate the expected internal rate of return offered by the annuity contracts and compare the results with the 10-year Treasury rate. They calculate the expected present value of annuity payments using the term structure of interest rates and then compare the EPV to the purchase price of the annuity contract ($100,000). They use both the term structure of Treasury interest rates and the term structure of corporate BBB rates and they also use mortality rates for the general population as well as the mortality rates for annuity purchasers. The most comparable cases (from their Table 7 and our Figure 4E) are for deferred annuities with payouts commencing at 75, using general population mortality and Treasury interest rates. First, the similarities: (1) we both find that the annuity deal is better for women than for men and (2) the purchaser at 55 (54 and 56 in our case) gets a better deal than the purchaser at 65. The second of these result is presumably because adverse selection is less severe at 55 than at 65. People know more about their prospects for a long life at 65 than they do at 55. Their numbers indicate that women get an expected PV of $101,155 using Treasury discount rates whereas men have expected receipts of $89,433. Our results are somewhat better than that for the deferred annuity purchaser. A large part of the difference (perhaps all of it) comes from the fact that we use the highest quote from a top rated insurance company, whereas they use the average of 16 quotes. In our case, restricting ourselves to the highest rated companies (there were only two) did not bind in that they always offered the highest quotes as well. This difference, using the highest quote rather than the average quote, largely accounts for the difference in results. Most of our quotes were obtained in 2022 whereas theirs were obtained in June 2020, a month of very low interest safe interest rates with relatively high credit spreads. In the end, we find the most comparable results to be consistent. Finally, we think that the annuity contracts purchased from the highest rated insurance companies (Massachusetts Mutual and Guardian Life) have a riskiness much closer to Treasury bonds than BBB corporates. The companies themselves are very strong financially and the contract holder also has the protection of state-level insurance coverage.

The general result is that deferred life annuities, even using the total population cohort life tables (that is, ignoring the better mortality experience of annuity buyers relative to the general population), offer an expected return that is competitive with Treasury bonds. The expected return is higher for women than for men and, in general, is higher the longer the deferral period. It should be noted that these expected returns are net of fees unlike bond mutual funds which would charge anywhere from 5 to 70 basis points in fees. Of course, bond funds would typically be invested in investment grade corporate bonds which have a higher yield than Treasuries. One question is how the safety of the annuity contract compares to a bond fund or U.S. government Treasuries. It is our opinion that a contract with an A++ insurance company is a very safe investment. It should be noted that deferred life annuities provide valuable long-life insurance, unlike a bond fund. To the extent that their expected return is competitive with bonds, the insurance element seems to be relatively low cost. It also should be mentioned that life annuity contracts are illiquid assets, which certainly could be considered a drawback.

Figure 5 plots the difference between the average IRR offered by deferred life annuities starting at 65 and the yield on ten-year Treasuries as a function of the purchase age.



While the data is somewhat noisy, it is clear that women are offered higher IRRs and that longer deferral periods tend to also have higher IRRs relative to 10-year Treasuries. The average IRR on contracts offered women are almost always greater than the yield on 10-year Treasuries. For men, the contracts offered at 50, 52 and 54 also have IRRs greater than the 10-year Treasury bond. We assume that these differentials relative to the 10-year Treasury bond will persist in the future. This will allow us to compare the typical TDF strategy of adding bonds to the portfolio vs. our alternative strategy of buying deferred annuities starting at age 50.

5. Future Deferred Annuity Quotes as a Function of Future Interest Rates

We seek to predict future annuity quotes as a function of future yields on U.S. Government 10-Year Treasuries. We assume that the differentials between annuity IRRs and the 10-year yields, shown in Figure 5, will persist. With that assumption, we have

(2) $i(k,g) = y + d(k,g)$

where $i(k,g)$ is the internal rate of return on a deferred life annuity purchased by a person of gender g at age k, y is the yield on 10-year Treasuries and $d(k,g)$ is the yield differentials shown in Figure 2 for purchasers of gender g and age k. We also do the comparable calculation for deferred annuities whose payouts begin at 75 with the assumption that the differentials shown in Figure 4E will persist.

With this notation and the constant differentials assumption, future quotes for a $100,000 purchase by someone of gender g and age k for an annuity with payouts starting at 65 would be

(3) $$\mathrm{q(k,g)} = \left(\frac{100{,}000}{12}\right) / \sum_{t=65}^{120} \mathrm{S(t,k,g)}(1/(1+i(k,g))^{t-k}$$

where q(k,g) is the quote of monthly benefits for a purchaser of gender g at age k and S(t,k,g) is the survival rate to age t for someone of gender g who was alive and purchased a deferred life annuity contract at age k. The "12" in the equation is there just to convert annual payouts to monthly payouts.

Deferred annuity contracts are long-duration assets, obviously longer for lengthier deferral periods. For instance, a deferred annuity contract purchased at age 50 with payouts commencing at 65 is a much longer duration contract than an immediate annuity purchased at 65. Because of that, the quotations and pricing of deferred life annuities are more sensitive to interest rates. Table 5 uses equation 3 to show hypothetical monthly quotes for yields on 10-year Treasuries ranging from 2.5 percent to 7.5 percent. One can see that all life annuities benefit levels are sensitive to interest rates, but the effect is much more dramatic for a 15 year deferral period than for a 5 year deferral period or an immediate annuity.

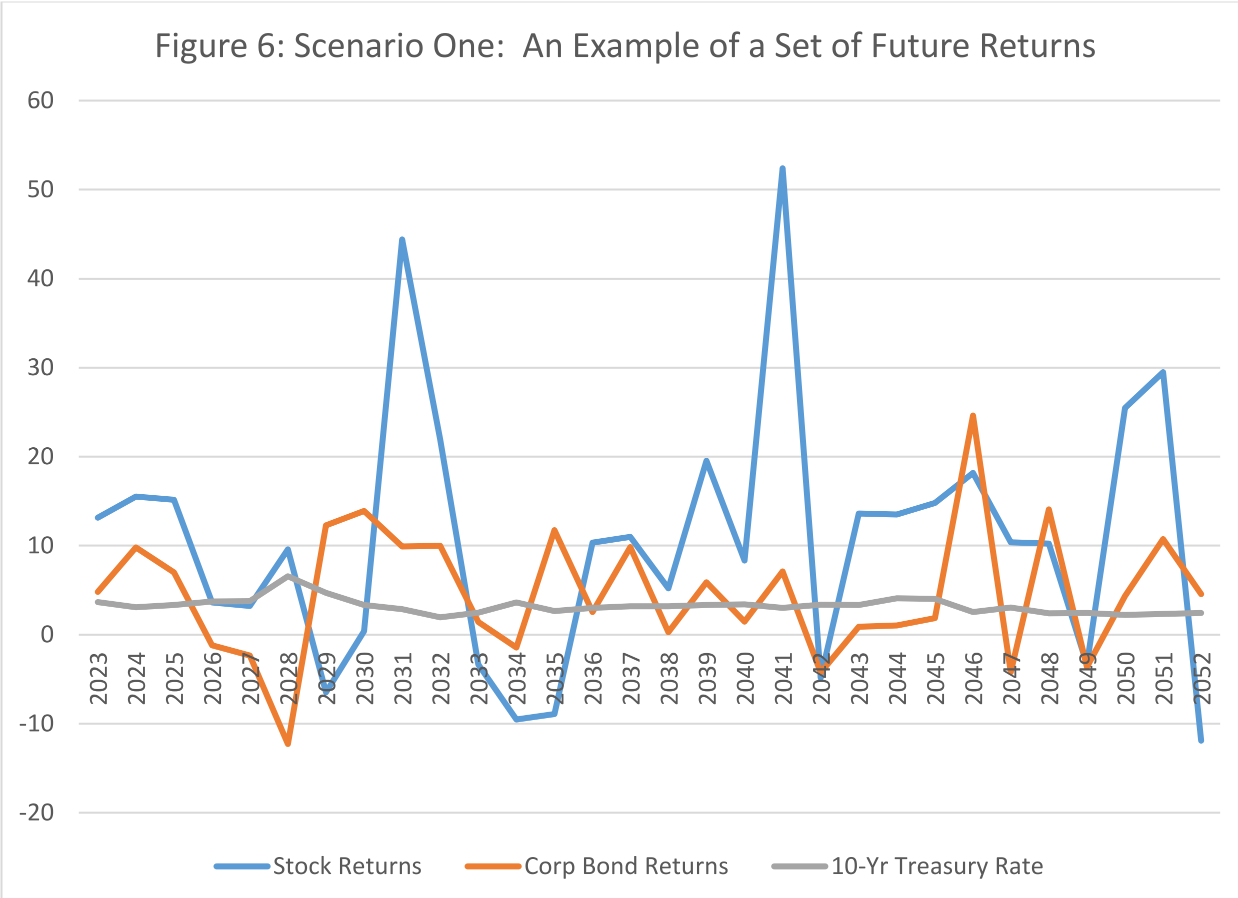
Table 5. Hypothetical Quotes for $100,000 Annuities with Payouts Commencing at 65

10-Yr yield	Man@50	Woman@50	Man@60	Woman@60	Man@65	Woman@65
2.50%	947	894	623	596	528	507
3.50%	1201	1141	720	692	582	560
4.50%	1514	1444	827	798	636	615
5.50%	1894	1814	944	914	692	671
6.50%	2354	2062	1071	1040	748	728
7.50%	2918	2803	1209	1177	805	785

The advantage of buying early (i.e. using annuities with a longer deferral period) can be seen by scanning across the rows of Table 5. At higher interest rates, the advantage is greater. The sensitivity of annuity quotes and prices to interest rates is shown in the columns of Table 5. In this case, the longer the deferral period, the more sensitive quotes are to interest rates.

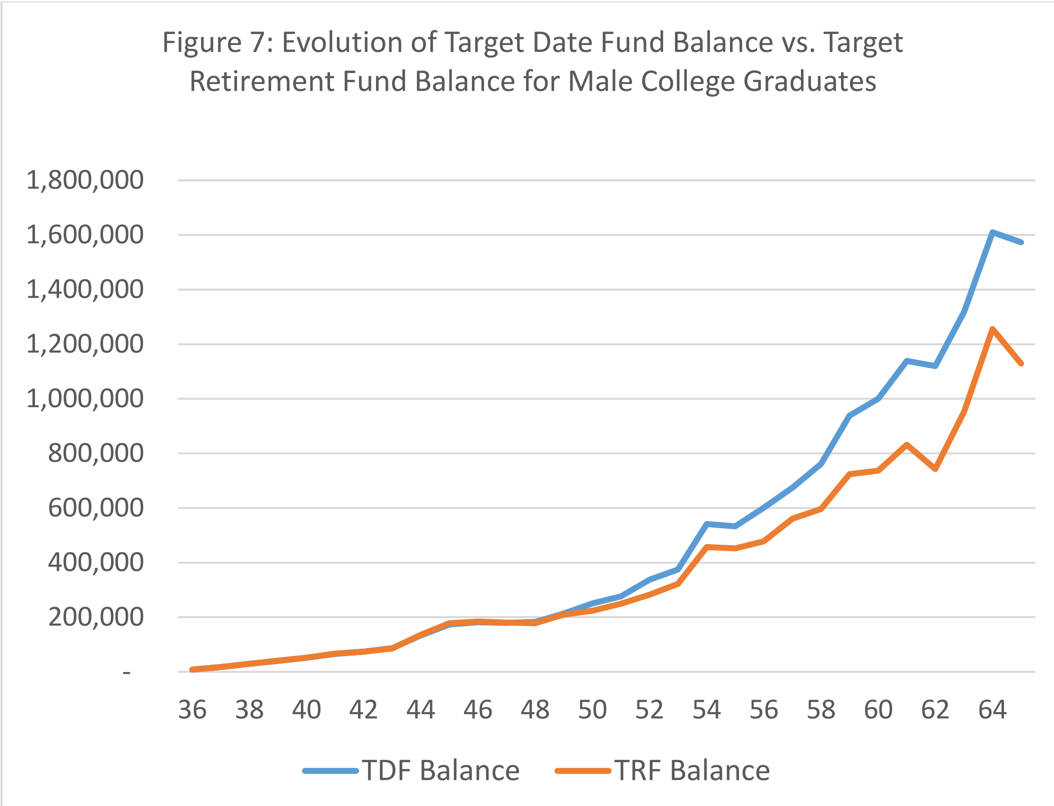
6. An Example Return Scenario

Our plan is to compare how a stylized Target Date Fund compares to our straw model Target Retirement Fund under a large number of possible future return environments. To be clear about our approach, we present a single possible return scenario generated from the Academy of Actuaries Interest Rate Generator which provides a possible future vector of stock market returns, bond returns and Treasury interest rates. We refer to this as Scenario One. In the next section of the paper, we will develop our own scenario generator and evaluate 1,000 possible futures. We take the relative earnings profile of college graduate men and women shown in Figure 3, multiply those ratios by our projected growth of the Social Security Average Wage Index (AWI). We assume that 9 percent contributions are made to a 401(k) account starting at age 35 and ending at the last day as a 64 year old. Contributions are made to the account at the end of each year. The 30 returns in Scenario One are shown in Figure 6 for stock market, a long corporate bond fund and the level of the 10-year Treasury interest rate.



The evolution of the balance of a TDF account compared with the liquid assets of a Target Retirement Account is shown in Figure 7 for the average male college graduate. Recall that in the straw model TRF plan, 10 percent of the balance at 50, 52, 54, 56, 58, 60 and 62 is used to purchase deferred annuities. The difference between the two balances at 65 is $444.611. On the other hand, under the straw model TRF system, at 65 the participant would be entitled to a monthly annuity starting at age 65 of $3.208. If the TDF participant decided to buy an immediate annuity at 65 paying the same monthly benefit, it would cost $577, 796. After such a purchase, the TRF participant, who bought their annuity in advance, would have liquid assets that are $133, 185 greater than the TDF participant. Just to be clear, both individuals would have exactly the same retirement annuity, but the TRF would have additional liquid assets in excess of $133,000. For reference, the final nominal salary of this hypothetical man (at age 64) was $251, 280. So the advantage of the TRF program relative to the TDF one amounts to approximately 6.36 months of final salary earnings.

It is probably worth noting that in Scenario One for average college graduate men, the TRF participant ends up with a liquid asset balance of $1,128,629 and has a life annuity with a replacement value (cost of an immediate purchase) of $577,796. The annuity portion of retirement assets in this case is just over one-third of the total retirement assets. This just points out that the straw model TRF involves partial annuitization rather than full annuitization as in most DB plans.



We did exactly the same analysis with Scenario One returns for a male college graduate accumulating deferred annuities whose payout start at age 75. In the accumulation phase, there is no difference in the returns or the amount spent on deferred annuities, so the difference in the balance of liquid assets is the same, namely the TDF balance exceeds the balance of liquid assets at retirement in the TRF plan by $444,611. But, in this case, the TRF participant would have accumulated a total annuity that paid out $7,007 per month starting at 75. For the TDF participant to duplicate that annuity by buying at age 65 a deferred annuity with payouts of $7,007 per month starting at 75 would cost $663,478. This leaves the TRF participant $218,867 ahead in liquid balances at retirement. Once again, the TRF participant ends up ahead, this time by 10.45 months of his final salary. At least in this scenario, the gain from starting accumulating annuities at 50 rather than waiting until 65 is quite substantial.

It is also worth noting that the TRF participant ends up with liquid assets of $1,128,624 at retirement. This is more than enough to fund consumption between 65 and 75 at the level of the annuity income that they have purchased starting at 75 ($7.007 per month) even under a zero percent return scenario.

We looked at how women would fare with Scenario One returns with our stylized TDF and our straw model TRF. The results were very similar to those for men. If a woman college graduate with average career earnings trajectory accumulated her retirement savings in a TDF and then purchased an annuity at age 65 with the same monthly payout that she would have had as a TRF participant, she would end up with less liquid assets with the TDF approach. The differences in terms of months of final salary amounts to 6.17 months in the case of buying annuities that begin payouts at 65 and 8.23 months in the case with initial payouts at 75. At least in the case of Scenario One, buying annuities in the TRF plan beats waiting until retirement to buy them.

Our next goal is to look at lots of return scenarios to determine whether this result is likely over a wide range of return scenarios. We turn to that now.

7. Simulating Future Scenarios and Modeling Uncertainty

To evaluate the impact on savings and liquidity at retirement, we simulate many future paths of equity returns, bond returns, interest rates, and life-annuities prices. We assume that the saver has access to multiple assets; in particular, a diversified equity fund, a diversified bond fund, and life annuities. As the focus of the paper is to consider simple saving strategies to compare with the generic target date fund. we assume that the saver allocates their entire savings across these three assets and, in particular, does not hold a risk-free liquid asset like a savings account or money-market fund, although this assumption can easily be relaxed.

Any retirement saving strategy should be evaluated not just based on an expected return, but the uncertainty of the outcome as well. We propose a model for the joint evolution of equity returns, bond returns, and long-term interest rates. Rather than rely on a model of term structure of interest rates, as in Konicz et al (2016), which uses the Nelson-Siegel model to determine the returns on bonds through assumptions about the debt structure, we simply take historical data on the returns of a diversified long-term corporate bond fund and directly estimate the impact of long-term risk-free interest rates on corporate bonds, along the lines of Campbell et. al (2003). This has the advantage of not assuming a particular coupon structure or bond duration, and also does not rely on an assumption that the bonds are riskless, i.e., there are (small) default probabilities of assets contained in the bond fund that we use. This is particularly relevant for target date funds, which typically contain risky bonds as part of their portfolios.

We model the time-varying returns and long-term interest rates with a VAR(1)-process, which is typical for models of uncertainty to evaluate asset allocation decisions (see Campbell et al (2003)). The dependent variable in the VAR(1) process is a 3 x 1 vector

$$\xi_t = \begin{pmatrix} s_t \\ b_t \\ i_t \end{pmatrix}$$

The VAR(1) process can be written as

$$\xi_t = c + A\xi_t + \varepsilon_t$$

where c is the 3 x 1 vector of intercepts, A is the 3 x 3 matrix of slope coefficients, and ε_t is the 3 x 1 vector of i.i.d. errors, with the assumption of mean 0 and finite variance σ^2. The covariance of the errors Σ is given by $E[\varepsilon\varepsilon^T]$. The assumption in a typical VAR model and that we adopt here is that the errors are cross-sectionally correlated, but are homoscedastic and independently distributed over time. Stability (a fixed point that is robust to perturbations) is a desired property of a dynamic system such as the VAR, and that stability is achieved in a finite-variance VAR when the eigenvalues of A all have modulus less than 1. In a stable VAR, there exists a long-term steady state with expectation μ and variance Γ (see Lutkepohl (2005)) as follows:

$$\mu = (I - A)^{-1}c$$
$$vec(\Gamma) = (I - A \otimes A)^{-1}vec(\Sigma)$$

where I is the identity matrix, $\otimes$ is the Kronecker product, and $vec(\cdot)$ transforms a K x K matrix to a $K^2 \times 1$ vector by stacking the columns.

For estimation of the VAR model, we use several data sources. For all time series, we take yearly real returns between 1982 and 2020 (38 years of returns). For measuring a diversified portfolio of equity returns, we use the Wilshire 5000 index annual year-end change, retrieved from FRED. To measure the return on corporate bonds, we use a mutual fund, the Vanguard Long-Term Investment-Grade Fund (Investor shares), ticker VWESX. This fund began in 1973 and has a history of returns running through the current date. We retrieved the 10-year Treasury bill yields from FRED as well, from the annual effective yield, semi-yearly compounding time series. All of these series give nominal returns. To convert the series to real returns, we pulled the annual inflation index from FRED for the same period, and transformed nominal returns to real returns following $1 + r = \frac{1+n}{1+i}$, where r is the real return, n is the nominal return, and i is the inflation rate.

Given that this econometric model is very lightweight in terms of complexity and number of parameters, and estimated at an annual frequency, a relatively coarse unit, we consider this VAR as a first pass to produce an interpretable and configurable model that one can use to generate Monte Carlo simulations for computing moments of interest from the distribution of portfolio returns and annuity wealth at the age of retirement. In this view, we considered it appropriate to adjust the estimated VAR parameters in order to better fit several stylized expectations we think are appropriate for a future-looking model. The flexibility of the VAR allows us to address the issues of the estimated model.

First, we observed that high-grade bonds (both long- and short-duration) have experienced a remarkable run over the estimation period in terms of consistently producing high returns to bondholders, mainly due to the trend of declining rates in the same period. We observe that the VWESX mutual fund in the estimation period averaged a real return of 6.33%, while over the same period, the Wilshire 5000 index averaged a real return of 8.55%, generating a spread between stocks and bonds of only 2.22%, whereas the Wilshire 500 index experienced a standard deviation of 15.87% and the VWESX fund experienced nearly half the standard deviation, at 8.09%. We adjust our future expectations of the real yield on high-grade long-term corporate bonds downwards, so that we achieve a real rate of 2.5% on bonds. This is achieved by adjusting the intercept of the bond-equation of the VAR downwards. Further, we adjusted the first-difference intercept of the 10-year Treasury-rate equation of the VAR to be 0 (it was estimated to be nearly -1% initially), which is in line with the assumption that long-term Treasury rates are stationary. We also adjusted the real rate of stock returns downwards from 8.55% to 7%, again by decreasing the intercept of the corresponding VAR equation. This actually makes our simulations more conservative in terms of relative performance of TRFs to TDFs, since the simple TRF strategy outlined here utilizes an all-stock portfolio before annuity purchases.

We report the parameters of the VAR model below in Table 6 and the steady state expectations and covariance matrix in Table 7. We also verify that all the eigenvalues of the process' characteristic polynomial have moduli less than 1, implying that the model is stable.

Table 6: Estimated and Adjusted VAR model parameters

	Real Equity Return	Real Bond Return	Real 10-year Treasury Return
Constant	0.0736 (0.0360)	0.0299 (0.0139)	-0.0001 (0.0028)
Lag Equity Return	-0.0696 (0.1785)	-0.0241 (0.0688)	-0.0103 (0.0137)
Lag Bond Return	-0.1176 (0.3700)	-0.4935 (0.1427)	0.0181 (0.0283)
Lag 10-year Tbill Return	-0.6747 (0.7986)	-1.1592 (0.3080)	-0.0671 (0.0612)

Predicting future returns on equities, corporate bonds, and long-term treasury bills is difficult. There are clearly many relevant factors that these models miss, and that fact is reflected in the low R^2 values from the VAR regressions. We think this predictive shortcoming is not as important as long as the model produces moments of the return distributions that are realistic and representative of what we think may happen over the next 30 years in US financial markets.

In Table 7, we see that the real steady-state expected returns on equities is 9.1%, on bonds is 4%, and long-term Treasury bills is near zero, but slightly negative at -0.3%. We observe that equities and, surprisingly, 10-year interest rates, have a higher variance than that of bonds, and that bonds and 10-year rates covary the most, and all 3 state variables positively covary one with another.

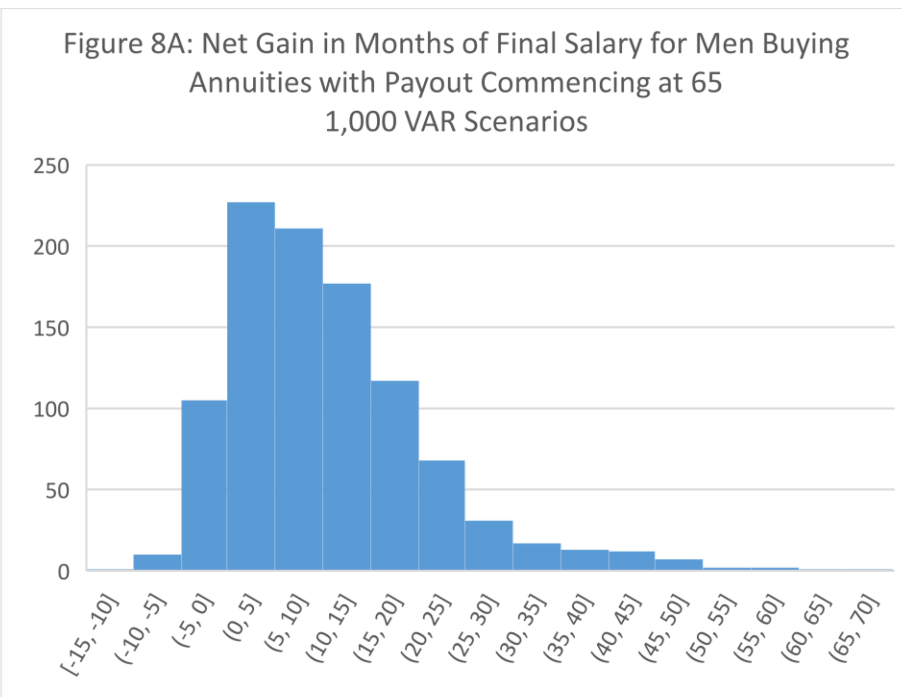
Table 7. Steady state expectations and covariance matrix of state variables

	Equity	Bond	10-year Tbill
Expectation	0.07	0.025	0.019
Equity	0.0276	0.0038	0.0018
Bond	-	0.0063	0.0074
10-year Tbill	-	-	0.0286

We then use this model to produce many simulated future scenarios, and compare the performance of a stylized Target Date Fund with our simple Target Retirement Fund. To further control the simulation, we impose a filter on the draws from the VAR model. This has the effect of impacting the empirical distribution of returns. We chose to do this because of the well-known issue that standard VARs estimated on financial data have a poor fit of error variance, due to assumption of i.i.d. normality of errors, whereas financial time series distributions often exhibit fatter tails. As a result, the normal errors are often simulated with a high variance. Our simulation approach is a low-tech but effective way to correct for this. We truncate the distribution of returns by redrawing from the simulation if any of the following conditions are not met: (1) that the nominal interest rate lies between -1% and +11.5% and (2) that stock returns are capped at 38%. The restriction (2) is the maximum observed historical return of the Wilshire 5000 in our dataset, and the restriction (1) is not as severe as it may seem at first. Our modeling framework assumes a constant 2% inflation in all futures, so capping real interest rates at 9.5% seemed to be reasonable.

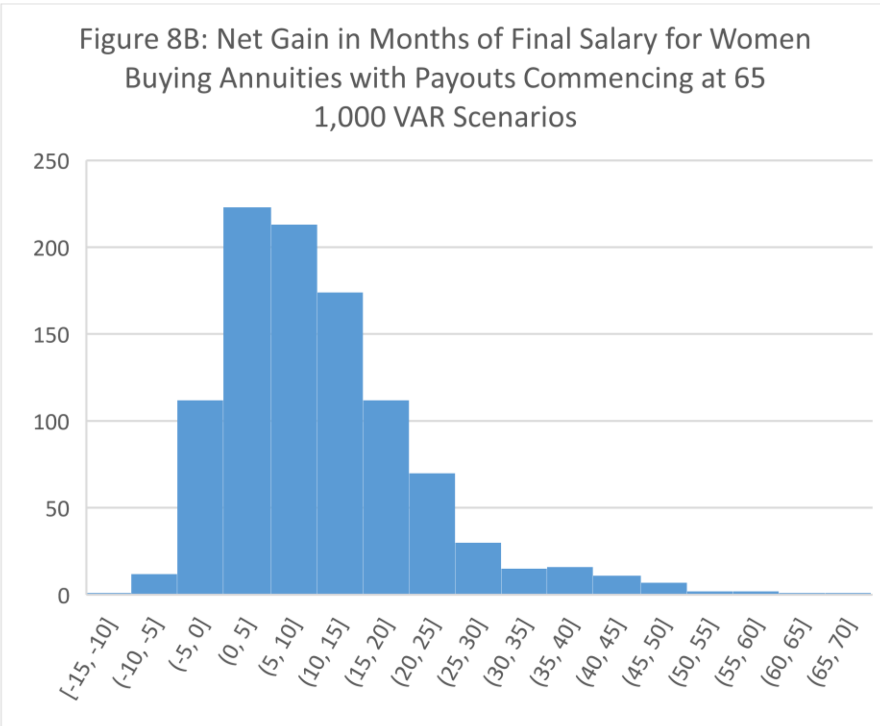
Both investment strategies follow the same glide paths as outlined in Section 5. We draw 30-year future return simulations corresponding to the market returns during years 35 to 65 of the hypothetical retirement-saver's life for 1000 times. To cut off unrealistic tail risk that is a result of our linear model and normally distributed error term assumptions, we redraw a scenario until the simulated 10-year real interest rate never drops below -1%.

To compare outcomes between the TDF and the TRF, we consider, for each of the 1,000 simulated scenarios, the difference in the liquid portions of the TDF and TRF, if, at retirement age, the TDF saver purchased the same amount of annuities as had been purchased in the TRF. This metric of comparison has the advantage that it accounts for equal annuitization, as well as the difference in liquid assets due to the diverging investment paths across the two strategies as retirement nears. Figure 8A shows the number of the 1,000 scenarios that generates different net gains for the TRF participant relative to the TDF participant. In both cases the participant has made identical contributions and has the same monthly income from annuities in retirement. The advantage of the TRF strategy is simply the difference in liquid assets in the 401(k) account at the time of retirement. This difference is expressed in months of final salary and can be positive or negative. What the table next to Figure 8A shows is that the TRF participant has more liquid assets in 88.5 percent of the scenarios. On average the advantage for the TRF participant is equal to 10.59 months of final salary, a considerable difference. Perhaps more importantly, the average advantage in the cases where the TRF participant is ahead is slightly more than one year of salary. In the 11.5 percent of scenarios in which the TRF participant is behind, the average loss is a relatively low 2.25 months of final salary.



%+	88.5
%-	11.5
Average Gain	10.59
Average + Gain	12.26
Average – Gain	-2.25

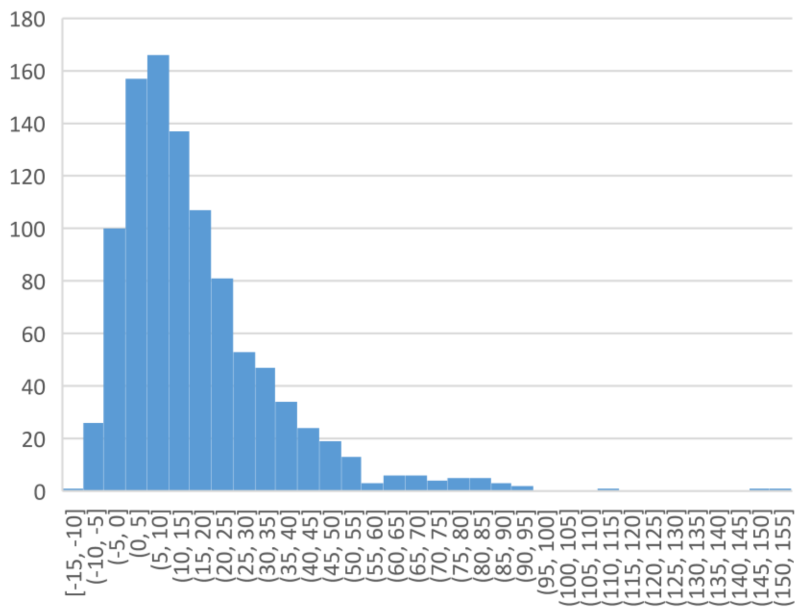
Figure 8B shows the same information for women buying annuities with payouts starting at 65. Once again, the TRF plan which buys annuities in advance beats the TDF participant who buys an equivalent annuity at retirement in roughly 88 percent of the 1,000 future scenarios. The dollar numbers for the gains or losses are less for women, but that is almost entirely due to the lower salary trajectory shown in Figure 3 and therefore the lower dollar contributions. In terms of months of final salary as illustrated in Figures 8A and 8B, there is very little difference between the circumstances offered men and women. In both cases, buying annuities in advance instead of waiting until retirement looks like a very good bet.



%+	87.6
%-	12.4
Average Gain	10.46
Average + Gain	12.26
Average – Gain	-2.27

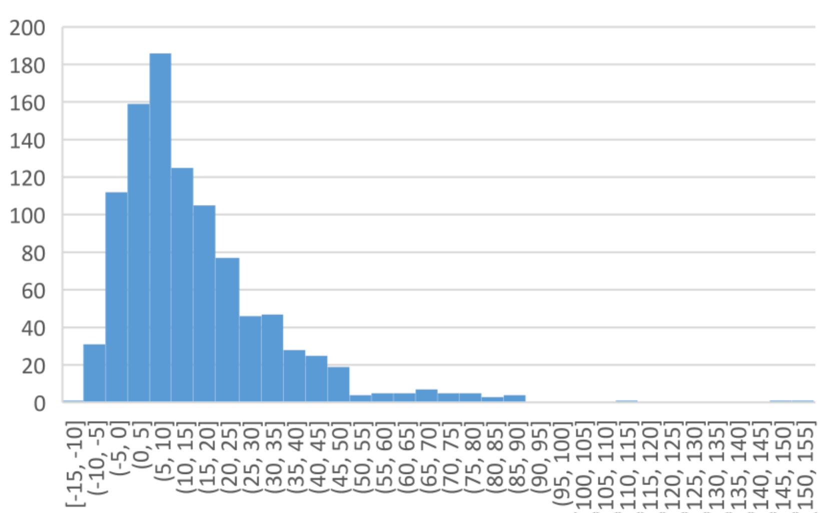
Figures 8C and 8D present the same information from our 1,000 VAR generated scenarios, but this time for the acquisition of annuities whose payouts start at 75. Each of the bins in the histograms is still 5 months of final salary wide. The gains and losses are somewhat bigger in the case of annuities starting at 75, but the odds of the TRF participant having more liquid assets at the time of retirement are about the same and the general shape of the histogram (with the long tail in the positive direction for the TRF participant) is also the same.

Figure 8C: Net Gain in Months of Final Salary for Men Buying Annuities with Payouts Commencing at 75
1,000 VAR Scenarios



%+	87.4
%-	12.6
Average Gain	16.23
Average + Gain	19.01
Average – Gain	-3.11

Figure 8D: Net Gain in Months of Final Salary for Women Buying Annuities with Payouts Commencing at 75, 1,000 VAR Scenarios



%+	85.7
%-	14.3
Average Gain	15.05
Average + Gain	18.12
Average – Gain	-3.27

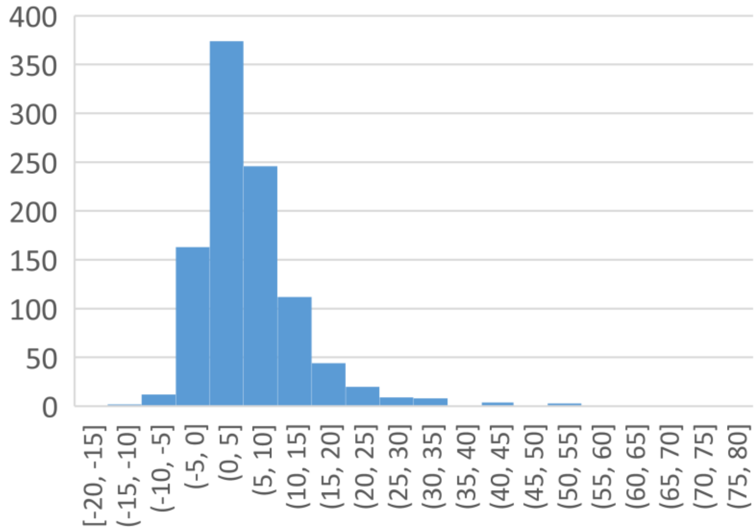
From Figures 8A through 8D, it is clear that if annuitization is desired going into retirement, there is an advantage to annuitizing earlier and over multiple tranches. This takes advantage of the adverse selection premium that early annuity purchasers enjoy, as well as the reduction of “sequence risk” from smaller, more frequent annuity purchases. The result is somewhat dramatic: in that the Target Retirement Fund participant ends up ahead of the TDF participant in more than 85 percent of the 1,000 simulations. Further, the average gain is quite large, ranging from 10 to 16 months of final pre-retirement salary.

8. A Robustness Check: Alternative Asset Simulator

As a robustness check on the results just covered using our own VAR scenario generator, we also evaluated 1,000 scenarios generated by the same Academy of Actuaries’ Interest Rate Generator that produced Scenario One. The evaluation is the same as above. We calculate the total annuitized monthly income that a TRF participant would have accumulated in each scenario. We note the balance of liquid assets that the TRF participant would have had at retirement. Then, we calculate the balance that a TDF participant would have after purchasing an annuity at retirement with the same monthly payouts that the TRF participant will receive. At this point, the TDF and the TRF participant will have had the same earnings history, made the same 401(k) contributions and will enjoy the same monthly income for life. The only thing left to compare is their remaining liquid assets in their 401(k) account. We do this for 1,000 scenarios generated by the Academy of Actuaries scenario generator.

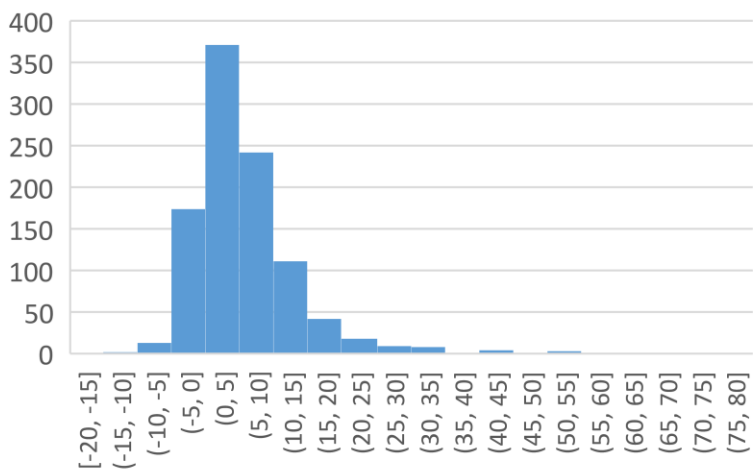
We first present the results for men and women buying annuities with payouts commencing at 65. We express the difference between the liquid assets of the TRF participant and the TDF participant in months of final salary, just like in the previous section. This way of expressing the results is easier to interpret than dollar amounts after 30 years of inflation. The results for men are shown in Figure 9A and for women in Figure 9B. Next to the figures are the same summary statistics that we showed in the previous section. You can see that the difference once again favors the TRF participant in 82.3 percent of the scenarios in the case of men and 81.1 percent of the time in the case of women. To the naked eye, the two histograms for men and women are nearly identical. The average gain when the gain is positive is about 7.5 months of salary, whereas the average loss when the gain is negative is slightly over two months of salary. 7.5 months of salary strikes us as a large gain, although not quite as large as produced by our VAR scenario generator. In these 1,000 scenarios, the TRF seems like the better ex-ante retirement strategy

Figure 9A: Net Gain in Months of Final Salary for the TRF Strategy for Men Buying Deferred Annuities with Payout Commencing at 65



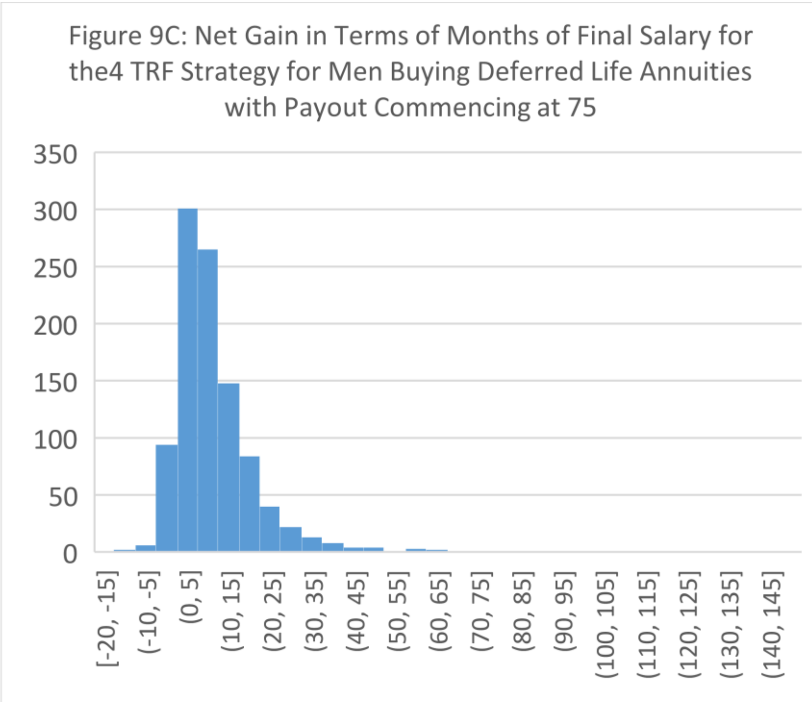
%+	82.3
%-	17.7
Average Gain	5.82
Average + Gain	7.53
Average – Gain	-2.12

Figure 9B: Net Gain in Terms of Months of Final Salary for the TRF Strategy for Women Buying Deferred Annuities with Payout Commencing at 65

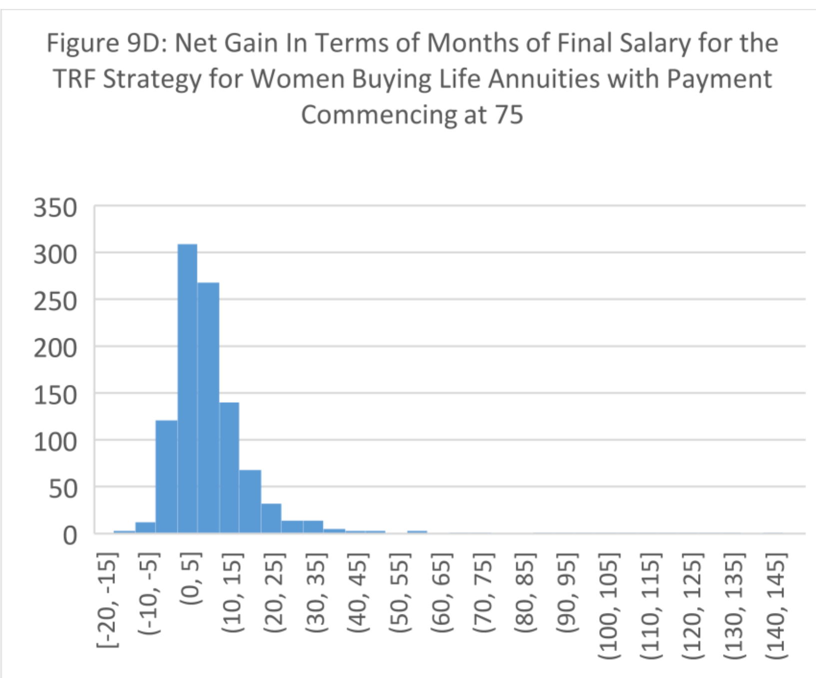


%+	81.1
%-	18.9
Average Gain	5.67
Average + Gain	7.49
Average – Gain	-2.12

Once again, we ran the stylized TDF and the straw model TRF for cases where the annuities purchased begin paying out at 75. The results are shown in Figures 9C for men and 9D for women. In general, buying deferred annuities while working instead of waiting until retirement works even better for annuities that don't payout until 75. In this case, the earlier purchase benefits the retiree between 86 and 90 percent of the time. Also, the average gain for the cases where the benefits are positive is approximately equal to ten months of final salary. The average loss, in the relatively few cases where there is a loss, is still close to two months of final salary.



%+	89.8
%-	10.2
Average Gain	8.93
Average + Gain	10.20
Average – Gain	-2.28



%+	86.4
%-	13.6
Average Gain	7.85
Average + Gain	9.46
Average – Gain	-2.39

We view these results with the Academy of Actuaries scenario generator as strengthening the same conclusions that we reached with our VAR scenario generator. By using an externally estimated model, we remove the objection that the parameters of simulation are cherry-picked by us to generate more favorable scenarios for the Target Retirement Fund. The simulations in this section show that using an alternative model for future returns, we still arrive at qualitatively the same conclusions.

9. Causes of TRF outperformance of TDFs when annuitizing

Given the strong outperformance of the target retirement fund over the target date fund, when it comes to annuitization at retirement, it is worth asking what are the sources of advantage of the TRF. Knowing the driving factors of the good TRF performance sheds light on the intuition behind the strategy (and, in our opinion, making the results more believable) as well as giving the main principles through which one could create a more sophisticated strategy to achieve even more efficient annuitization at retirement. In other words, the TRF is a simple investing and annuitization strategy one could use to take advantage of the 2 key principles that we lay out below.

We claim that the two main principles that drive the efficiency of annuitization in the TRF over the stylized TDF are (1) the benefits of early annuitization from the mitigation of adverse selection, and (2) the mitigation of sequence risk through multiple purchases of annuities at different dates over the span of a decade or more. The first principle, that adverse selection is not as severe at younger deferred purchases, is observed in Tables 1, 2, 3, and 4, and directly encoded into our simulation in Table 5. The second principle, that sequence risk is mitigated through multiple purchases, is based on the simple notion that one can reduce the variance of the overall investment outcome by repeatedly sampling from the distribution of yields (by buying once every 2 years in our case) rather than taking a single sample (buying at the time of retirement).

To demonstrate that our results are driven by these two factors, we run the simulation by "taking away" these advantages for the target retirement fund and once again compare the gain against the target date fund. We take away the adverse selection benefit by setting the yield differential $d(k, g)$ to 0 for every age k and gender g. Thus, given the same interest rate, we force the annuity to "cost the same" to a 50-year-old and a 65-year-old in terms of present value. We take away the mitigation from sequence risk by simply running a single purchase equal to 70% of the saver's total portfolio value in the target retirement fund at age 54. The results for men purchasing deferred annuities with payouts commencing at 65 are in Table 8. The results for the other simulations (women and deferral to 75 years old) are similar, and are available from the authors upon request.

We observe that, as expected, removing the adverse selection advantage and multiple purchase dates results in virtually no advantage of the TRF strategy over the TDF strategy in terms of annuitization. While the combination of the two factors has a nonlinear effect, we do see that the multiple purchases seems to be more impactful than the removal of the adverse selection yield differential. While it is apparent that there is right skewness in all of the distributions, we also observe that by doing a single purchase, the distribution of gains becomes a lot less skewed because there are many more cases in which the single annuity purchase yields a negative gain. These findings suggest that the number and timing of purchases can have a large impact, and the optimization of annuity purchase times could yield additional efficiency.

Table 8. Comparison of simulations with and without the adverse selection advantage and multiple purchase dates in the TRF, for men purchasing annuities with payment commencing at age 65.

	Original Simulation	**No Adverse Selection**	**Single Purchase at 54**	**No Adverse Selection and Single Purchase at 54**
Percentage with a positive gain	86.59%	78.18%	56.76%	47.75%
Average gain (months)	9.68	5.04	2.57	0.59
Median gain (months)	7.80	3.44	1.16	-0.38
25th percentile gain (months)	2.62	0.41	-3.25	-4.83
75th percentile gain (months)	14.58	7.56	7.00	4.99

10. Conclusion

Target Date Funds have been incredibly popular, but that does not mean that innovation regarding their design should be dismissed. In this paper, we examine the possibility of preserving the general idea of moving to a safer portfolio as retirement approaches, but doing so by gradually accumulating deferred life annuities rather than bonds or bond funds. Equity exposure is reduced as retirement approaches just like with TDFs, but at retirement the participant is partially annuitized rather than simply having a large balance of liquid assets. Such annuitization can simplify the participant's problem of planning an achievable consumption plan in retirement.

We obtained quotes for deferred life annuities using the online website immediateannuities.com. We only used quotes from insurance companies that received the highest safety rating from A. M. Best. We used the highest quote from an A++ company. In order to calculate the expected return on a deferred life annuity investment, we used cohort life tables from Social Security. We consistently found that annuities purchased with a considerable deferral period offered a higher internal rate of return than those purchased with a shorter deferral period. We believe that is due to there being less adverse selection in the case of long deferral periods. We also found that the internal rate of return offered for women was consistently higher than for men. Finally, the internal rates of return were comparable to the interest rate on 10-year Treasuries. This suggests that the insurance element of deferred life annuities is relatively inexpensive since they offer relatively competitive returns compared with uninsured products.

For people who want some annuities in retirement, we compare using target date funds and buying annuities at the time of retirement with a straw model target retirement fund that begins buying deferred life annuities at age 50. We model how annuity prices change based on interest rates and the age of the purchaser. We generate 1,000 different futures in terms of stock market returns, bond market returns and Treasury interest rates. For each such "future" we calculate how a typical college graduate who begins participating in a 401(k) retirement plan at age 35 would fare with a generic TDF and TRF.

Our main finding is that if the TDF participant buys an annuity at retirement with the same lifetime monthly payouts as the TRF participant has accumulated in advance, the TDF participant ends up with less liquid assets In the large majority of the return scenarios that we generate. The TRF strategy beats the TDF strategy in this sense more than 80 percent of the time. Many times, the TRF participant is ahead by a very substantial amount such as an amount equal to more than 10 months of final salary. The TRF participant does not always end up better off. However, when they do fall short of the TDF participant, it is not by very much money. The average loss in the cases where the TRF strategy ends up behind is of the order of two months of final salary. If these scenario generators accurately describe the array of possible future returns that might happen, then a target retirement fund which uses deferred life annuities looks like a good way to acquire annuities for retirement.

We repeatedly have referred to our TRF calculations as a straw model. That is because we have made a number of particular assumptions, such as starting retirement saving at 35, contributing a total of 9 percent of earnings, retiring at 65, beginning to purchase deferred annuities at 50, purchasing them every two years until 62, devoting 10 percent of liquid assets in the 401(k) for each purchase, etc. All of these assumptions can be changed and parameters could conceivably be optimized. We have no reason to believe that changing any of these assumptions or even several of them at once would materially alter our main conclusion that purchasing annuities in advance usually beats buying them at the time of retirement.

It is our opinion that Target Retirement Funds are worthy of further study as a possible alternative to Target Date Funds. By incorporating annuities, they recapture one aspect of defined benefit plans for defined contribution participants. They may make retirement planning easier for retirees. And, this work suggests that the cheaper way to obtain annuities is by buying them in advance on a deferred purchases basis.

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