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THE ECONOMICS OF CARBON ACCOUNTING AND CARBON OFFSETS

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ABSTRACT

I provide a rigorous framework for accounting for corporate greenhouse gas emissions, based on the Greenhouse Gas Protocol. I show that only Scope 1 emissions are of interest from a national policy perspective: that emissions in Scopes 2 and 3 are duplicative, and that downstream Scope 3 emissions in total are equal to total Scope 1 emissions. The correct measure of a company's contributions to national GHG emissions is its Scope 1 emissions plus a part of the emissions of the household and government sectors. This is not generally the same as the level of emissions for which a company bears some responsibility, legal or otherwise. I apply some of these ideas to an analysis of carbon offsets, trying to understand the conditions under which offsets are a valid mechanism for reducing emissions by formalizing the ideas of additionality and leakage.

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1 Introduction

The aim here is to provide a rigorous framework for thinking about how to classify and add up emissions of greenhouse gases generated by firms and other participants in an economic system.[1] The conventional approach is to use the Greenhouse Gas Protocol (GHGP: www.ghgprotocol.org. For a discussion of the GHG protocol see Kaplan and Ramanna [2022].) developed by a partnership between the World Resources Institute and the Business Council for Sustainable Development. This classifies emissions into Scopes 1, 2 and 3. Scope 1 emissions come from a company's own operations, Scope 2 are generated by the production of electricity that it purchases on the market, and Scope 3 emissions are those generated in the upstream and downstream supply chains of a company. I show below that the total emissions of an economy are captured by the sum of Scope 1 emissions, plus any emissions associated with uses of products other than as inputs to other companies - that is as exports, or as inputs to the activities of households and governments. In this sense Scopes 2 and 3 are redundant: they are not needed to capture total emissions at the national level. I also show that if there are no sales of goods to households or governments and there are no exports exports[2] then the sum of downstream Scope 3 emissions is the same as the sum of Scope 1 emissions. I use the framework developed to study carbon offsets and the conditions under which these can make a valid contribution to emissions reductions. There is considerable concern that in many cases they are not making a contribution to reducing greenhouse gas emissions, and in fact are doing the opposite (see Calel et al. [2021], Bushnell [2010] and references cited there). I formalize the ideas of additionality and leakage, and also introduce a distinction between offsets that reduce existing emis-

[1]The analysis applies to any kinds of emissions, but GHGs are the ones that are currently of interest.

[2]or if the emissions-intensities of these sectors are the same as those of the productive sectors

sions into the atmosphere, and those that remove CO_2 from the atmosphere, arguing that in the long term only the latter can be valid offsets.

2 The Model

To generate a simple and familiar framework, we work with a standard economic input-output model.[3] This is an easy way of modeling supply chains: it allows us to keep track of how much of each good is used in the production of every other good. In this there model are n goods and n firms and each firm produces one good only, potentially using as inputs all goods. All production processes are linear. A is an nxn non-negative matrix typical element a_{ij} where a_{ij} is the amount of good i needed to produce one unit of good j. $x \in R^n$ is a vector of outputs of the n goods. e_i is the emissions of GHGs associated with using one unit of good i and $e = (e_1, ..., e_n) \in R^n$ and E is an nxn diagonal matrix with diagonal elements $e_{ii} = e_i$. Likewise X is a diagonal matrix with diagonal elements $x_{ii} = x_i$.

2.1 Scope 1

The SCOPE 1 EMISSIONS of firm i are those directly associated with the production of good i: if this good is produced at level x_i then the inputs used are $a_{ji}x_i,\ j = 1, ..., n$. The associated Scope 1 emissions for firm i are $E_i^1 = x_i \sum_j a_{ji}e_j$. The vector E^1 of all Scope 1 emissions is then

$$E^1 = eAX \tag{1}$$

[3] For a similar use of input-output models to analyze emissions from supply chains see Levinson [2010].

2.2 Scope 3

The SCOPE 3 EMISSIONS can also be modeled easily in this framework: they are the emissions produced in making the inputs to an industry or in using its products. The former I call upstream Scope 3 and the latter downstream. $a_{ij}x_j$ is the amount of good i used in producing good j at level x_j and the emissions from this (downstream emissions) are $a_{ij}x_je_i$. So the total emissions from the use of good i are $e_i \sum_j a_{ij}x_j$. The vector of all such emissions is thus

$$E^{3d} = EAx \tag{2}$$

These are emissions from the use of each good, denoted by the superscript d for downstream. Note we are assuming here that all goods are fully used as inputs to the production of other goods, and none are diverted to exports or to use by governments or households (I relax this assumption below).

Scope 3 also includes those associated with the production of the inputs to the goods, i.e. upstream emissions. The Scope 1 emissions per unit of output for the production of good k are given by $\sum_j a_{jk}e_j$ and the inputs of good k and others to the production of good i are $a_{ki}x_i$, $k = 1, ..., n$. So the total Scope 3 upstream emissions from the production of the inputs to good i are

$$E_i^{3u} = \sum_k a_{ki}x_i \sum_j a_{jk}e_j$$

We can write the vector of these emissions as

$$E^{3u} = eAAX \tag{3}$$

noting that $\sum_j a_{jk}e_j$ is the inner product of the row vector e with a column of the nxn matrix A so that the list of these numbers is eA, and then that each element of the vector $\left(\sum_j a_{jk}e_j\right)_{k=1,..,n}$ is multiplied by a_{ki} so we are taking the inner product of the vector $\left(\sum_j a_{jk}e_j\right)_{k=1,...,n}$ with a column of

A. Finally we are multiplying each element of the resulting vector by x_i. It follows that the vector of total Scope 3 emissions is

$$E^3 = eAAX + EAx \tag{4}$$

2.3 Scope 2

A company's Scope 2 emissions are those arising from the production of energy that it purchases. Typically these are emissions from the production of electric power by fossil fuels. Clearly these are included in the upstream Scope 3 emissions given by the expression $eAAX$. We can break them out separately: let the index $p \in [1, .., n]$ denote power, so that a_{pj} is the amount of power used to produce one unit of good j. The emissions associated with producing a unit of power are $\sum_j a_{jp}e_j$ so the Scope 2 emissions of industry m are $a_{pm}x_m \sum_j a_{jp}e_j$, which is one of the terms in the sum E_i^{3u} above. This term is already contained in the total of Scope 1 emissions, and as we will see below is also contained in the sum of downstream Scope 3 emissions.

2.4 Illustration

In the 3x3 case expanding these expressions gives

$$eAX = (a_{11}e_1 + a_{21}e_2 + a_{31}e_3)x_1, (a_{12}e_1 + a_{22}e_2 + a_{32}e_3)x_2, (a_{13}e_1 + a_{23}e_2 + a_{33}e_3)x_3$$

$$EAx = (a_{11}e_1x_1 + a_{12}e_1x_2 + a_{13}e_1x_3), (a_{21}e_2x_1 + a_{22}e_2x_2 + a_{23}e_2x_3), (a_{31}e_3x_1 + a_{32}e_3x_2 + a_{33}e_3x_3)$$

and $eAAX$ is

$$\begin{array}{l}
a_{11}(a_{11}e_1 + a_{21}e_2 + a_{31}e_3) + a_{21}(a_{12}e_1 + a_{22}e_2 + a_{32}e_3) + a_{31}(a_{13}e_1 + a_{23}e_2 + a_{33}e_3))x_1 \\
a_{12}(a_{11}e_1 + a_{21}e_2 + a_{31}e_3) + a_{22}(a_{12}e_1 + a_{22}e_2 + a_{32}e_3) + a_{32}(a_{13}e_1 + a_{23}e_2 + a_{33}e_3))x_2 \\
a_{13}(a_{11}e_1 + a_{21}e_2 + a_{31}e_3) + a_{23}(a_{12}e_1 + a_{22}e_2 + a_{32}e_3) + a_{33}(a_{13}e_1 + a_{23}e_2 + a_{33}e_3))x_3
\end{array}$$

These expressions can be given a relatively straightforward explanation. Consider $E^1 = eAX$, the vector of Scope 1 emissions of the three industries. $a_{11}e_1$ is the emissions from the good 1 used in producing good 1, and it is multiplied by the amount of good 1 produced, x_1. Likewise $a_{21}e_2$ is the level of emissions from the good 1 used to produce 1 unit of good 1, and it is multiplied by x_1. The terms colored in green do the same for the inputs to good 2, and those in red for good 3. So eAX is clearly the total Scope 1 emissions, the emissions generated by the use of each of the inputs to each good.

The same expressions occur in the expansion of $eAAX$, highlighted on the diagonal in blue, green and red, but this time multiplied by a_{ii}.

It is clear that the total Scope 1 emissions are the total emissions from this system, obtained by adding the three components of the vector eAX. The reasoning is that the only source of emissions is the use of goods 1, 2 and 3 in production, and the expression eAX captures all emissions associated with the use of any of the three inputs in the production of all of the three outputs.

If we add the coordinates of $eAX$and EAx to obtain total emissions in each expression we find that they are respectively

$$(a_{11}e_1 + a_{21}e_2 + a_{31}e_3)x_1 + (a_{12}e_1 + a_{22}e_2 + a_{32}e_3)x_2 + (a_{13}e_1 + a_{23}e_2 + a_{33}e_3)x_3 \tag{5}$$

and

$$(a_{11}e_1 + a_{21}e_2 + a_{31}e_3)\,x_1 + (a_{12}e_1 + a_{22}e_2 + a_{32}e_3)\,x_2 + (a_{13}e_1 + a_{23}e_2 + a_{33}e_3)\,x_3 \tag{6}$$

so that the total emissions are the same in the two expressions eAX, EAx. Thus the total of Scope 1 emissions is equal to the total of all downstream Scope 3 emissions (though they are not equal on a firm-by-firm basis). We can prove this analytically: to sum the components of a vector we take its inner product with a vector of ones, $1 = (1, 1, 1, ..., 1)$. We need to show that

$$eAX.1 = 1.EAx \tag{7}$$

as the LHS is $1xn$ and the RHS is $nx1$ and we need conformability for multiplication. See the appendix below for a proof.

It is intuitive that the sum of Scope 1 emissions is equal to the sum of all Scope 3 emissions associated with the use of the products, which is what $E^{3d} = EAx$ is. Scope 1 emissions are generated by the use of the products.

2.5 Household and Government Sectors

Recall from standard input-output models (see e.g. Heal et al. [1974]) that to produce enough to meet a vector f of final demands we need to operate all industries at levels $x = (x_1, x_2, ..., x_n)$ given by

$$\{I - A\} x = f \tag{8}$$

where I is the identity matrix. Hence x is given by $x = \{I - A\}^{-1} f$ and the literature contains many sufficient conditions for $\{I - A\}^{-1}$ to exist and be non-negative. We can now ask how much a small increase in final demand will increase emissions. We can calculate

$$\frac{\partial x}{\partial f_i}$$

and from eAX calculate the resulting change in emissions.

Suppose that use of a product in final demand (households or governments or exports) generates emissions. Then this is a source of emissions additional to those considered above. The emissions considered above come from the use of a good in the production of other goods: if some of it is not used to produce other goods, then no emissions are attributed to this part by the calculations above. Let f be the vector of final demands, clearly satisfying$f \leqq x$. Suppose the use of a unit of good i in final demand generates emissions of ε_i, with $\varepsilon = (\varepsilon_1, ..., \varepsilon_n)$ so that final demands as a whole generate emissions of $f.\varepsilon$. In this case the vector of downstream emissions from the use of goods in production is given by $EA\,(x - f)$ and then the emissions from non-inputs uses are given by Ξf where Ξ is the diagonal matrix with ε_i on the diagonal. So now downstream Scope 3 emissions are

$$EA\,(x - f) + \Xi f = EAx + f\,(\Xi - EA)$$

We know that EAx is the same as Scope 1 emissions so how this total compares with Scope 1 depends on the sign of $\Xi - EA$, which in intuitive terms depends on whether consumption of the produced goods outside the production system (exports, households, governments) is more or less emissions intensive than their consumption as inputs to production. If they are the same then the second term on the RHS here is zero and downstream emissions are again EAx. The issue here is whether the use of for example gasoline by a household or a government agency generates the same level of emissions per unit consumed as the use of gasoline by a corporation. It seems that in many cases this will be true, but it is not inevitable.

The sum of all emissions is now Scope 1 plus the emissions by households etc $(f.\varepsilon\ )$, that is the sum of all emissions is given by $f.\varepsilon + eAX.1$

3 GHG Accounting

In choosing a measure of a firm/sector's emissions, the best choice depends on what exactly we want to achieve. If we want to add up all companies' emissions for an estimate of total corporate emissions then the right number is Scope 1 emissions, plus the emissions of the household and government sectors given by $f.\varepsilon$: these latter are the Scope 3 emissions which are not duplicative of other firms' Scope 1 emissions. If we are only interested in domestic emissions we will exclude exports from the term $f.\varepsilon$. More generally how we treat emissions associated with imports and exports will depend on whether we are aiming to understand global or national emissions. As most countries are setting national targets policy, makers will generally be interested in the latter, in which case we will exclude emissions associated with the production of imports and the use of exports.

Scope 1 emissions are the appropriate measure for corporate contributions to national emissions. In particular if we wish to know how a cut in corporate emissions of x% will change national emissions then it is appropriate to look at the effect of an x% cut in Scope 1 emissions.

Scope 3 emissions give us all the emissions that are directly and indirectly associated with a company's activities, including those associated with the manufacture of its inputs and the use of its outputs. Most of these emissions will be Scope 1 emissions for other companies and so adding emissions across companies raises obvious dangers of double counting. It is also true that most of these emissions will be under the control of other companies and so policy-makers cannot expect company i to be able to reduce its Scope 3 emissions: most of these will be the Scope 1 emissions of other companies and under their control. Scope 3 emissions do not seem to be a very useful concept from a policy-making perspective. Some environmental groups argue that firms are "responsible" for their Scope 2 and Scope 3 emissions, but as these are under the control of other firms and produced by other firms this point seems arguable. For example, the Center for American Progress comments on this issue that[4]

> Notably, because Scope 3 includes emissions up and down the value chain, it is often the largest category of emissions. This is especially true for certain industries. An obvious example is oil and gas companies, whose products are responsible for a wide range of emissions down the value chain—including combustion

[4]https://www.americanprogress.org/article/why-companies-should-be-required-to-disclose-their-scope-3-emissions/

> of fuel in aircraft, trucks, heavy equipment, and cars. In fact, Scope 3 emissions account for about 88 percent of total emissions from the oil and gas sector. A low-carbon economy cannot be achieved without addressing these emissions, which is why shareholders have been pressuring companies to disclose their Scope 3 emissions and demonstrate how they are managing them.

This neglects the point that Scope 3 emissions are Scope 1 for another corporation and if that corporation manages its Scope 1, then it will take care of the problem. The US Environmental Protection Agency, however, seems aware of this: "The scope 3 emissions for one organization are the scope 1 and 2 emissions of another organization."[5]

4 Carbon Offsets

I next apply some of the ideas just discussed to analyzing carbon offsets. Offsets are deals that allow a company to claim as its own a reduction in emissions that was actually implemented by another entity. Emissions here are taken to be Scope 1 emissions only, in view of the arguments of the earlier sections. Offsets are widely used by corporations to claim a reduction in net emissions, and there is growing concern that many of the offsets traded do not represent genuine reductions in emissions (see for example Calel et al. [2021], Bushnell [2010] and literature cited there. For a humorous perspective see John Oliver at https://www.youtube.com/watch?v=6p8zAbFKpW0).

First we need some definitions and notation. An entity is an organization, or a group of them, that emits greenhouse gases. It is typically a corporation, but could also be he household sector of an economy or one of the government sectors - federal, state or municipal in the case of the US, in keeping with the earlier discussions. Let $e_{i,t}$ be the emissions of entity i in period t: i runs from 1 to I and is a list of all legal entities that emit GHGs. The natural

[5]https://www.epa.gov/climateleadership/scope-3-inventory-guidance

unit of time is a year, as we measure emissions annually. The index t runs from 1 to T and is a list of all years that are relevant to the resolution of the climate problem, so it is likely that $T \geq 80$. We let e_t be a list of all emissions of all entities in year t, and $E_t = \sum_i e_{i,t}$ is the total emissions in year t. The list of all emissions in all periods is e with no subscript.

Emissions are generated by actions, a. We let $a_{i,t}$ be a vector of actions taken by entity i in period t : we suppose the $a_{i,t} \in R^N$ so that there are N actions that any entity can take, all being continuously variable. The scale of $i's$ $j-th$ action in period t is $a_{i,j,t}$. a_i is the list of all actions chosen by i in all periods, and a is the list of all actions taken by any entity in any period. $a_i \in R^{NT}$ and $a \in R^{NTI}$.

Finally we assume the emissions of any entity in any period are determined by its actions and those of others, and to be precise are a differentiable function of the actions that all entities take in all periods, so that $e_{i,t}(a)$. We will focus on the sensitivity of $e_{i,t}$ to the actions of entities, which we denote by the partial derivatives $\partial e_{i,t}/\partial a_{j,k,r}$: this gives the rate of change of $i's$ emissions of GHGs in period t with changes in $j's$ scale of action k in period r. We expect that for most combinations of subscripts $\partial e_{i,t}/\partial a_{j,k,r} = 0$, so there is no effect of $j's$ choice of k in r on $i's$ emissions in t.

The system of actions and emissions is assumed to be in an **equilibrium** and the equilibrium values are represented by $*$: hence the equilibrium is $\{a^*, e^*\}$. These equilibrium values are benchmark values, the values we expect to prevail in the absence of changes and the values from which we measure deviations. This could for example be an equilibrium of the input-output system discussed above. More generally we would represent it as the equilibrium of a model of economic activity and GHG emissions.[6]

If $\partial e^*_{i,t}/\partial a^*_{j,k,r} \neq 0$ then we say that action k by entity j in period r is a **potential offset** for $j's$ emissions in any period because a change in the

[6]This could be a competitive equilibrium or a Nash equilibrium of an imperfectly competitive system. This is a multi-period equilibrium so the values of emissions and actions will very from one period to the next.

level of action k (increase or decrease according as the derivative is negative or positive) by j can reduce $i's$ emissions in t, thereby compensating for some of $j's$ own emissions. (This action could be making a payment.)

There are other conditions that $\partial e^*_{i,t}/\partial a^*_{j,k,r}$ must satisfy if action k by j in r is to be an actual rather than a potential offset. These are generally referred to as additionality and non-leakage.

A reduction in emissions by i in period t satisfies the **additionality** condition with respect to $a_{j,k,r}$ if it would not have happened except for the change in action k by entity j in period r. This means that the change in $a_{j,k,r}$ caused the drop in emissions and nothing else caused it: there is nothing else to which it could be attributed and we can be sure that it would not have happened were it not for $j's$ action. For this to be the case it must be true that either the emissions $e_{i,t}$ are insensitive to any actions other than $a_{j,k,r}$ so that formally $\partial e_{i,t}/\partial a_{b,c,d} = 0$ for any $(b, c, d) \neq (j, k, r)$ or alternatively if $e_{i,t}$ is sensitive to actions other than $a_{j,k,r}$, meaning $\partial e_{i,t}/\partial a_{b,c,d} \neq 0$ for some $a_{b,c,d}$, then $a_{b,c,d}$ is constant. Note that additionality requires a comparison of what happened after $a_{j,k,r}$ was changed with what would have happened if there had been no change in $a_{j,k,r}$: it is a counterfactual concept, and in practice these are often hard to implement.

Leakage occurs if entity j takes an action k at time r to reduce emissions at i at time t, and this leads to some other entity b taking an action c at some date d, not necessarily r, which increases emissions at another date. A classic example is a reduction in deforestation in Papua New Guinea leading to an increase in deforestation in Indonesia or Malaysia. My use of the phrases "leads to" and "leading to" here suggests causation, and this is generally seen as an integral part of leakage. We can formalize this as $\partial a_{b,c,d}/\partial a_{j,k,r} \neq 0$: in words the two actions are functionally dependent and a change in one drives a change in the other.[7]

[7] A form of leakage occurs if the regulation of emissions in one country leads industries to move to another - for a review see Levinson [2010]

It is clear that if the additionality and no-leakage conditions are met, then the production of an offset leads to a drop in total emissions relative to the equilibrium or baseline. However failure of additionality or of the no-leakage condition means that the offset is compromised, and that the net reduction in emissions is less than the offset, perhaps even zero.

At various UNFCCC meetings, notably in Paris in 2015, the world community has agreed to aim for zero annual GHG emissions by sometime in the second half of this century. Recall that $E_t = \sum_i e_{i,t}$ is the total emissions in year t. So by that date we need to have $E_t = \sum_i e_{i,t} = 0$. There are two ways in which this can happen. One is that all entities have zero emissions, so that every $e_{i,t}$ is zero. The second is that some components are positive and others negative and the sum is zero. A negative emission corresponds to the withdrawal of GHGs from the atmosphere, by for example reforestation or by direct air capture (or, more speculatively, by the use of bioenergy with carbon capture and storage).

At this point we need to draw a distinction between offsets that reduce the extent of existing emissions, which nevertheless remain positive, and those that remove CO_2 from the atmosphere, thereby reducing the stock of CO_2 in the atmosphere. We will call these two categories **reductions** and **removals**. A reduction occurs when $e_{i,t} > 0$ and $\partial e_{i,t}/\partial a_{b,c,d}.\Delta a_{b,c,d} < 0$. In words, emissions are positive but the action taken reduces them. A removal corresponds to a situation where $e_{i,t} < 0$. Most of the offsets currently used correspond to reductions - reductions in the amount of CO_2 produced by generating power or by transportation, for example. A more complex example is the reduction of emissions from deforestation (see Coutiño et al. [2022] for a discussion).

In the case of a reduction offset, the level of emissions remains non-negative even after the reduction associated with the offset, because even if the seller of the offset has now zero emissions, the purchaser of the offset continues to emit. So in a world where all offsets are reductions it is not possible for $E_t = \sum_i e_{i,t} = 0$ - we cannot achieve zero emissions by offsets based on reductions. If some entities are to continue to produce CO_2, that is there are positive components in $e_{i,t}$, then they cannot offset these emissions by reductions if the world as a whole is to have zero emissions. They must be offset by removals, which correspond to negative components to $e_{i,t}$.

The conclusions to emerge from this analysis are

1. Testing for additionality is hard and requires the analysis of a counterfactual: it requires a full understanding of all factors that might influence the reduction in emissions that is to be claimed as an offset. You cannot determine if additionality holds by just studying the emissions of one entity or even one country.

2. The same is true of testing for leakage: it requires a comprehensive understanding of how the global economic system generates emissions and how the actions of one entity influence the emissions of all others. In principle a forest management policy change in south east Asia could lead to a change in deforestation in west Africa several years later.

3. It is important to distinguish between offsets based on a reduction in existing emissions and those based on removal from the atmosphere of GHGs that are already there. Only the latter are consistent with a net zero world.

5 Carbon Taxes

A widely-discussed possible policy response to climate change is a carbon tax: entities would be taxed on the amount of carbon they emit. But what would be the measure of carbon emissions - Scope 1 alone, or this plus Scope 3? Generally it is suggested that a carbon tax be levied “upstream” on fossil fuel companies as they sell fossil fuels to other participants in the economy.[8] The argument for this is normally administrative simplicity: there are few fossil fuel producers relative to users. This amounts to taxing fossil fuel producers on their downstream Scope 3 emissions, and not on the emissions that they actually produce themselves, their Scope 1 emissions. In this case there is not a problem with double counting: the total amount of fossil fuel burned is clearly equal to the total amount sold, modulo any changes in inventories, and this is the correct basis for estimating greenhouse gas emissions. There are however some incentive issues: it is not obvious that the tax on fossil fuel producers will all be passed on to the consumers. Tax incidence is a complex issue. There is even a possibility that a carbon tax falling on producers will provide an incentive for them to delay production - see for example Heal and Schlenker [2019].

6 Appendix

Here we show that $eAX.1 = 1.EAx$. The proof is just a matter of setting out the typical terms of the two products. Start with eAX. A and X are nxn matrices so their product is an nxn matrix. e is an $1xn$ vector so its product with A or AE is a $1xn$ vector. The typical term of AX is $(a_{ij}x_j)_{i,j=1,..,n}$. The typical term in the $1xn$ product eAX is $\sum_{i=1}^{n} e_i a_{ij} x_j$. The sum of such terms, which results from the inner product with 1, is $\sum_i \sum_j e_i a_{ij} x_j$which is the total emissions from Scope 1. Looking at EAx we see that a typical term in the product EA is $(a_{ij}e_i)_{i,j=1,..,n}$ and in the product EAx the typical term is $\sum_{j=1}^{n} a_{ij} e_i x_j$. Hence the sum of such terms, total downstream Scope 3 emissions, is $\sum_i \sum_j a_{ij} e_i x_j$ and the two sums are clearly the same.

[8] See for example Center for Climate and Energy Solutions at https://www.c2es.org/content/carbon-tax-basics/

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