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THE POLITICS OF INTERGENERATIONAL REDISTRIBUTION

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ABSTRACT

This paper studies the political-economic equilibrium of a two-period model with overlapping generations. In each period the policy is chosen under majority rule by the generations currently alive. The paper identifies a "sustainable set" of values for public debt. Any amount of debt within this set is fully repaid in equilibrium, even in the absence of commitments. By issuing debt within this set, the first generation of voters redistributes revenue in its favor and away from the second generation. The paper characterizes the determinants of the equilibrium intergenerational redistribution carried out in this way, and points to a difference between debt policy and social security legislation as instruments of redistribution. The key features of the model are heterogeneity within each generation and altruism across generations.

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1. Introduction

Issuing government debt shifts the tax burden on future generations of tax payers. Two key features distinguish this policy from other redistributive policies. First, issuing debt involves the promise of some future transfer from yet unborn (or yet non-immigrated) generations. Second, the promise is made without the consent of the future generations who bear the burden of the redistribution. Two natural questions arise: Under what circumstances are these promises kept, and why? And why don't older generations take full advantage of future generations? These questions are addressed in this paper.

Even though both issues have been the focus of some interesting recent research, a satisfactory answer does not yet exist in the literature, particularly with reference to the first question of why debt isn't repudiated. Cukierman and Meltzer (1989) focus exclusively on the second issue. They show that the extent of intergenerational redistribution in a political equilibrium depends on the strength of the voters' bequest motive and on the general equilibrium effects of the policy. However, their results are driven by two central assumptions. First, that only the currently old generation votes on the policy. Second, that the current majority can precommit the future majority to honor its debt obligations.[1] Hansson and Stuart (1989) reach similar conclusions in a model of social security legislation, but they too are unable to deal satisfactorily with the issue of precommitment. They assume that a policy can be changed only if there is unanimity. Since in their model the old are always opposed to changing the policy, the unanimity requirement is equivalent to a commitment technology. Rothemberg (1989) and Kotlikoff, Persson and Svensson (1989) relax the assumption of precommitment. Rothemberg (1989) analyzes a non-cooperative bargaining model between the two generations currently alive, and studies the properties of the steady state. But his bargaining equilibrium is difficult to interpret with reference to a specific political institution. Kotlikoff, Persson and Svensson (1989) study a "social contract" enforced by an implicit reputation mechanism. If the currently young generation reneges on the social contract, it will be unable to write similar contracts with future generations. However, this social contract is not renegotiation proof. More importantly, the equilibrium amount of intergenerational redistribution is indeterminate.

This paper assumes that the policy is chosen under majority rule by rational voters. Precommitment is ruled out: a vote is taken in each period, and all the generations currently alive are eligible to vote. They explicitly vote on how much debt to repay. The model has two central features. First, there is bi-directional altruism between fathers and sons; this altruism moderates the intergenerational conflict. Second, there is heterogeneity within each generation. As a consequence, the debt policy has both inter- and intra-generational redistributive effects. The equilibrium outcome reflects both features.

The paper provides an answer to both previous questions. First, despite the absence of a commitment technology, the currently old generation is able to redistribute income towards itself and away from future, yet unborn, generations. This happens because issuing government debt creates a constituency in favor of repaying it. Hence, issuing government debt "creates facts" that can alter future collective decisions, even in the absence of any commitment technology. As will be shown below, intragenerational heterogeneity is essential to this result.[2]

Second, if too much debt is issued, it is repudiated by the voters who bear the burden of servicing it. Hence, the absence of a commitment technology is not irrelevant: it generally prevents the currently old generation from achieving all the desired intergenerational redistribution.

Finally, the paper characterizes how the equilibrium amount of intergenerational redistribution depends on the features of society and in particular on the rate of growth of output and of the population, and on the initial distribution of wealth.

The paper outline is as follows. The model is described in Section 2. Section 3 characterizes the economic equilibrium. Section 4 analyzes the political equilibrium in which voters choose how much debt to repay. The equilibrium amount of intergenerational redistribution is characterized in Section 5. Finally, Section 6 contains some concluding remarks.

2. The Model

Consider a two-period closed economy. In period 1 only one generation -- called "parents" -- is alive. In period 2 another generation -- called "kids" -- is born. Each parent has $(1+n)$ kids. Thus, $n \gtreqless 0$ is the rate of growth of the population. Parents live two periods and kids live one period. Both generations are altruistic. Thus, the i^{th} parent maximizes

$$W^i = \text{Max}[U(c_1^i) + c_2^i + \delta(1+n)V(x^i)], \quad 1 > \delta > 0, \tag{1}$$

where c_t^i denotes the parent's consumption in period t and x^i denotes the kid's consumption in period 2. And the i^{th} kid maximizes

$$J^i = \left[\frac{\gamma}{1+n} c_2^i + V(x^i)\right], \quad 1 > \gamma > 0 \tag{2}$$

The functions $U(\cdot)$ and $V(\cdot)$ are twice continuously differentiable concave utility functions, and the coefficients δ and γ measure the altruism of parents and kids. Altruism is weighted by the rate of growth of the population. Thus, as the family size increases (as n grows), parents give less weight to their own welfare relative to their kids' welfare, and the opposite is true about the kids' altruism. This specification of preferences is plausible and simplifies the algebra, but is not crucial for results.

Different households have the same preferences but different endowments. At the beginning of his life, the i^{th} parent receives $1 + e^i$ units of non-storable output. The individual-specific variable e^i can be either positive or negative, and is distributed in the population according to a known distribution $G(\cdot)$, with zero mean, non-positive median, and bounded support inside the unit circle. Let s^i denote parent i's savings and let g_1 denote a lump sum transfer received in period 1. Then write the i^{th} parent's budget constraint for period 1 as:

$$c_1^i + s^i \leq 1 + e^i + g_1 \tag{3}$$

The only store of value is government debt, b, that earns a gross of tax gross rate of return R, and is taxed (or repudiated) at the rate θ.

In period 2, parents receive a second individual specific endowment, a^i. This endowment is not publicly observable. Hence, in period 1 individuals cannot borrow against it, and savings is constrained to be non-negative: $s^i \geq 0$ for all i. The only role of this second period endowment is to insure that in equilibrium $c_2^i > 0$ for all i. Since the parents' utility is linear in c_2^i, and since they cannot borrow, all the income effects of a^i are absorbed by their own period 2 consumption, with no effect on the private intergenerational transfers. Thus, heterogeneity of a^i plays no role, except in motivating the no-borrowing constraint. On the other hand, as will be seen below, heterogeneity of the first period endowments, e^i, is crucial, since it generates heterogeneous savings behavior.

Kids of different households are all alike. They receive w units of output at the beginning of period 2, and pay a tax τ_2. Moreover, in period 2 parents can leave non-negative bequests to their kids and kids can give non-negative transfers (gifts) to their parents. Hence, the i^{th} family budget constraints for period 2 can be written as:

$$c_2^i + x^i(1+n) \leq (w-\tau_2)(1+n) + R(1-\theta)s^i + a^i \tag{4a}$$

$$c_2^i \leq R(1-\theta)s^i + f^i(1+n) + a^i \tag{4b}$$

$$x^i \leq w - \tau_2 + t^i/(1+n) \tag{4c}$$

where $f^i \geq 0$ and $t^i \geq 0$ denote gifts and bequests respectively. Thus, (4a) is the family budget constraint; (4b) and (4c) are implied by the non-negativity constraints on bequests and gifts respectively.

There is no government consumption. Hence, if we denote average variables by omitting the i-superscript, the government budget constraints can be written as

$$\begin{aligned} g_1 &\leq b \\ R(1-\theta)b &\leq \tau_2(1+n) \end{aligned} \tag{5}$$

Tax policy is chosen by majority rule, at the beginning of each period and before any private economic decision is made. In period 1 parents vote on how much debt to issue. And in period 2 both parents and kids vote on the tax rate on debt, θ. The government budget constraints determine the lump sum transfers and taxes g_1 and τ_2 residually. Finally, period 1 equilibrium in the asset market requires that average savings equal average government debt:

$$\int_0^\infty s^i dF(s^i) = b \tag{6}$$

where $F(\cdot)$, the cumulative distribution of s^i over the population, is characterized in the next section. By Walras' law, equations (3)-(6) imply that the good markets are also in equilibrium.

There are two features of the model that deserve special attention. First, whereas parents have heterogeneous endowments, all kids have the same income. As will be seen below, this feature of the model plays an important role, since it implies that a tax on accumulated savings redistributes wealth across households whereas a tax on the kids income does not. This assumption is a simple way to capture the well known fact that wealth inequality is much more pronounced than income inequality.[3] This extreme asymmetry of the model can be relaxed, at the price of some complications, provided it remains true that a tax on the parents' wealth is more redistributive than the income tax on kids. Second, since the parents' preferences are linear in their own period 2 consumption, private intergenerational transfers are the same for all households, irrespective of the parents' initial endowments. This feature considerably simplifies the description of the political equilibrium, but it is not crucial for the qualitative results.

3. The Economic Equilibrium

In this section individuals are considered in their role as economic agents. The current and expected future policy is taken as given when the economic decisions are being made. It is straightforward to verify that optimality for all families in period 2 implies:

$$1 \geq \delta V_x(x^i) \geq \delta\gamma, \quad \text{all} \quad i. \tag{7}$$

where a subscript on a function denotes a derivative. If the first inequality is strict parents are bequest constrained, and if the second inequality is strict kids are gift constrained. That is, the non-negativity constraint on private transfers (bequests and gifts respectively) is binding. As noted above, by (7) all households are in the same position with respect to the gift and bequest constraints: If one household is constrained, so are all the others.[4] The paper assumes throughout that

$$1 > \delta V_x(w) > \delta\gamma \tag{7'}$$

Thus, in the absence of any government intervention, the kids do not provide any gifts to their parents, and the parents do not leave bequests to their kids. This assumption guarantees that there is a role for public policy in affecting the intergenerational distribution of income.

The amount saved by each parent in period 1 is determined by combining (3) - (5) and taking the first order condition of (1) with respect to s^i. We obtain:

$$U_c(1+e^i+b-s^i) \geq R(1-\theta^e) \tag{8}$$

with equality if $s^i > 0$, where θ^e denotes the expectation of θ.

We want to describe the equilibrium distribution of savings across parents. To simplify notation, denote the expected net of tax rate of return on public debt by r. Thus, $r = R(1-\theta^e)$. Furthermore, let the variable z be implicitly defined by:

$$U_c(1+b-z) - r = 0 \tag{9}$$

Then the savings of parent i can be expressed as:

$$s^i = \mathrm{Max}(0, z+e^i) \tag{10}$$

Thus, all parents with $e^i \le -z$ save a zero amount. All other parents save an amount $s^i = z + e^i$.

Recalling that e^i is distributed in the population according to the cumulative function $G(\cdot)$, we can express the equilibrium condition in the market for government debt, (6), as:

$$b - z(1-G(-z)) - \int_{-z}^{\infty} e^i \, dG(e^i) = 0 \tag{6'}$$

Together, equations (6′) and (9) implicitly define the equilibrium values of r and z as functions of government debt: $r^* = r(b)$, $z^* = z(b)$. By the implicit function theorem:

$$\frac{dr^*}{db} = -G(-z^*)U_{cc}/[1-G(-z^*)] \ge 0 \tag{11}$$

$$\frac{dz^*}{db} = 1/[1-G(-z^*)] \ge 0$$

Let $[\underline{e}, \bar{e}]$ be the support of the cumulative distribution $G(\cdot)$, with $\underline{e} < 0 < \bar{e}$. Then, according to (6′), if $b \ge -\underline{e}$, we have $z = b$ and $G(-z) = 0$ -- that is, all the parents save a positive amount. In this case, (9) implies $r^* = U_c(1)$ for any $b \ge -\underline{e}$. Conversely, if $b = 0$, then everybody saves a zero amount, and the equilibrium interest rate is $r^* = U_c(1+\bar{e}) < U_c(1)$. This information is summarized in Figure 1 below.

Intuitively, when $b = 0$, the interest rate on government debt must be sufficiently low that even the wealthiest parent, for which $e^i = \bar{e}$, is willing to save a zero amount. As debt is issued, the interest rate must rise in order to induce poorer parents to forego current consumption. Finally, once a sufficiently large amount of debt has been issued (i.e., for

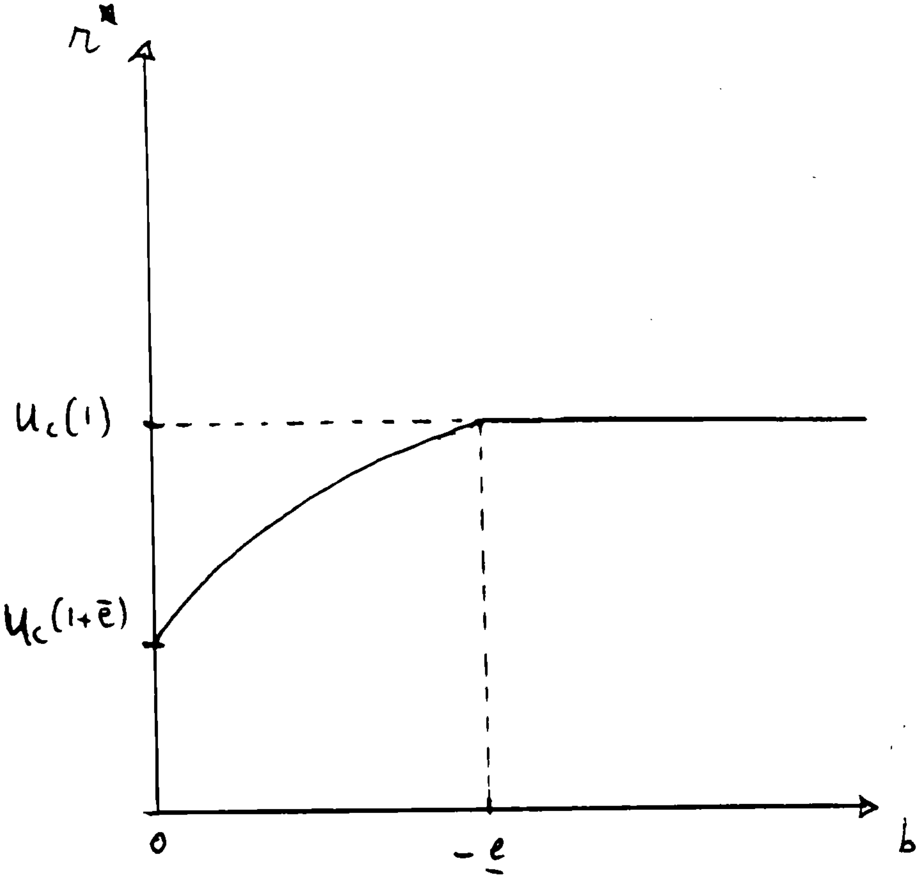

Figure 1

$b \geq -\underline{e}$), and hence once a sufficiently large transfer to the parents has taken place, no household is borrowing constrained. At this point, the constancy of the equilibrium interest rate simply reflects the constant marginal utility of consumption in period 2.

Finally, having determined the equilibrium interest rate net of taxes, it is straightforward to obtain the equilibrium rate gross of taxes:

$$R^* = r^*/(1-\theta^e) \tag{12}$$

We now turn to the analysis of the political equilibrium.

4. Voting on Debt Repudiation

This section describes the political equilibrium in which voters choose how much outstanding debt to repay. The vote occurs in period 2, and the stock of debt outstanding is taken as given. The main result of this section is that, under appropriate conditions, a majority of the voters is in favor of repaying the debt outstanding.

4.1 The Voters Preference

The equilibrium repudiation rate is a value of θ, say θ^*, such that there is no other value of θ preferred to θ^* by a majority of the voters in a pairwise comparison. To compute the equilibrium, we first characterize the voters' preferences about θ. Since θ is chosen once expectations, and hence market prices, have been formed, the voters evaluate θ ex-post, taking R as given. Inserting the government budget constraint, (5), in the private budget constraints, (4), we obtain:

$$c_2^i + (1+n)x^i \leq w(1+n) + R(1-\theta)(s^i-b) + a^i \tag{13a}$$

$$c_2^i \leq R(1-\theta)s^i + f^i(1+n) \tag{13b}$$

$$(1+n)x^i \leq w(1+n) - R(1-\theta)b + t^i \tag{13c}$$

In period 2, both parents and kids are eligible to vote. Consider first the effect of changing θ on the i^{th} parent's welfare, W^i_θ. By the envelope theorem, this effect can be computed by differentiating the Lagrangian of the parents' optimization problem with respect to b, taking the parents' choice variables as given. The Lagrangian is constructed from equation (1), with equations (13) as constraints. After some transformations, we find:[5]

$$W^i_\theta = Rb[\delta V_x - s^i/b] \tag{14}$$

By (14), each dollar of debt repudiated affects the i^{th} parent welfare in two ways. On the one hand, it reduces the tax burden on the kids by one dollar; this gives the parent a marginal utility of δV_x. This effect is the same for all parents. On the other hand, it reduces the parent wealth by $s^i/_b$, which yields a disutility of $-s^i/_b$. The net welfare effect depends on the wealth of the i^{th} parent relative to the average, and is more likely to be negative the wealthier is the parent.

The effect of changing θ on the i^{th} kid's welfare, J^i_θ, depends on whether or not the bequest constraint (13c) is binding. While the constraint is not binding (i.e., if $\delta V_x = 1$), every kid is affected by θ exactly like his parent. Intuitively, debt repudiation here has the only effect of redistributing wealth across families and not across generations. On the other hand, if the bequest constraint (13c) binds, so that $t^i = 0$ and $\delta V_x < 1$, then J^i_θ can be computed as illustrated above for W^i_θ, to obtain:

$$J^i_\theta = \frac{Rb}{1+n} (V_x - \gamma \frac{s^i}{b}) \tag{15}$$

which has the same interpretation as (14).

It is easy to show that:

Lemma 1

In equilibrium, the non-negativity constraint on bequests, (13c), is always binding.

Proof: By assumption, the median e^i does not exceed the average e^i. Hence, by (10) and (6'), the savings of the median parent do not exceed average savings, b.[6] Thus, when the bequest constraint does not bind, so that $\delta V_x = 1$, by (14) at least 50% of the parents always favor more debt repudiation. Next, consider the kid of a parent with average wealth. From his point of view, debt repudiation only redistributes across generations, and not across families. Since he discounts the welfare of his parents, this average kid always prefers repudiation past the point where the bequest constraint just binds. All the kids of poorer parents (at least 50% of the kids) prefer even more repudiation, since for them repudiation also redistributes from other families to their own family. Hence, a value of θ such that the bequest constraint is not binding cannot be supported as a political equilibrium under majority rule. Q.E.D.

This result is important for two reasons. First, it underscores that the absence of commitment matters: there is an upper bound to the amount of intergenerational redistribution that can take place in equilibrium. Second, this result implies that we can restrict our attention to the case in which the bequest constraint binds. In this case, the kids' preferences for the wealth tax are summarized by (15).

Based on (7), (14) and (15), the voters can be classified into four classes, according to the period 2 relative wealth of the parents, s^i/b: the "wealthy", for which $s^i/b > 1/\gamma\delta$; the "upper middle class", for which

$1/\gamma\delta \geq s^i/b \geq 1$; the "lower middle class" for which $1 \geq s^i/b \geq \gamma\delta$; and the "poor", for which $s^i/b < \gamma\delta$.

At the extremes of the wealth distribution, parents and kids vote in the same way: By (7), the wealthy favor $\theta = 0$ over anything else; and the poor favor $\theta = 1$ over anything else. In these two classes, the redistributive effect across families of debt repudiation always dominates the intergenerational effect.

In the middle classes, on the other hand, the intergenerational and the intragenerational effects of repudiation are traded off against one another. Since the intergenerational effect is evaluated differently by parents and kids (because $\delta, \gamma < 1$), parents and kids in the same middle class family vote in different ways. Specifically, the upper-middle class parent always votes for $\theta = 0$, and the lower middle class kid always votes for $\theta = 1$. The lower middle class parent and the upper middle class kid, on the other hand, prefer a value of θ in the $[0,1]$ interval; let θ^i be the value of θ preferred by the i^{th} voter in this group. Then it can be shown that θ^i is a non-increasing function of s^i/b, and strictly decreasing in s^i/b if $1 > \theta^i > 0$. Thus, individual preferences can be ranked in terms of the parents' relative wealth position: more wealthy voters prefer lower repudiation rates. Finally, it can be shown that individual preferences are single peaked. Hence, the equilibrium policy is that preferred by the median voter of period 2.

All this information is summarized in Figure 2.

4.2 The Median Voter

In order to identify the median voter, we have to combine the two groups of voters, parents and kids. Consider a parent with period 2 relative wealth equal to s^i/b. From (14) and (15), the welfare effect of

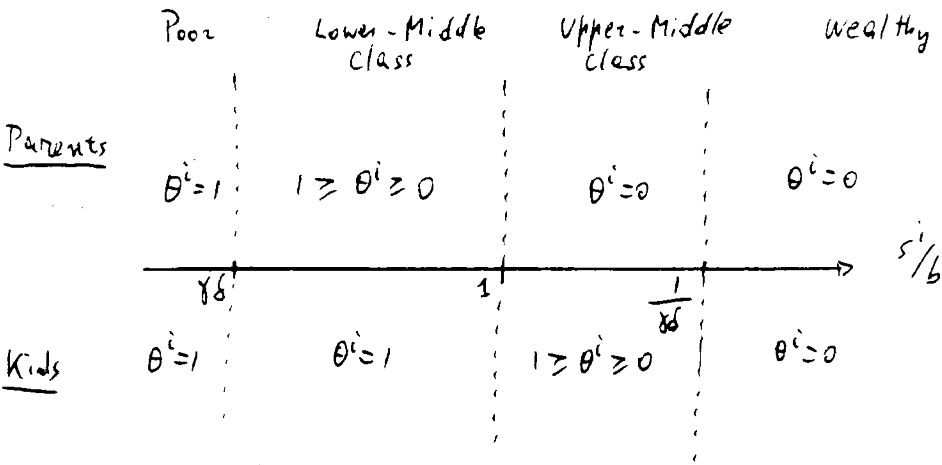
Poor
Lower-Middle Class
Upper-Middle Class
Wealthy
Parents
$\theta^i = 1$
$1 \geq \theta^i \geq 0$
$\theta^i = 0$
$\theta^i = 0$
$\gamma\delta$
1
$\frac{1}{\gamma\delta}$
s^i/b
Kids
$\theta^i = 1$
$\theta^i = 1$
$1 \geq \theta^i \geq 0$
$\theta^i = 0$

Figure 2

θ on this parent is the same as the effect on the kid of a parent whose relative wealth s^j/b is defined by:[7]

$$\frac{s^j}{b} = \frac{1}{\delta\gamma}\frac{s^i}{b} > s^i/b. \tag{16}$$

Equation (16) enables us to match up each parent with a kid (not his own kid) that votes exactly like him and to prove the following important:

Lemma 2

The median voter in period 2 is a parent with initial endowment e_2^m, defined implicitly by:

$$G(e_2^m) + (1+n)G[(e_2^m+(1-\delta\gamma)z)/\delta\gamma] - 1 - \frac{n}{2} = 0 \tag{17}$$

Proof: Let $H(\cdot)$ be the cumulative distribution of the parents' relative wealth in the population at the beginning of period 2. That is, let the variable s^i/b be distributed according to $H(\cdot)$. The median voter in period 2 is the parent with relative wealth s^m/b (or the kid of the parent with relative wealth $s^m/\gamma\delta b$), where s^m/b is defined by:

$$H(s^m/b) + (1+n)H(s^m/\delta\gamma b) = (1-H(s^m/b)) + (1+n)(1-H(s^m/\delta\gamma b)) \tag{18}$$

The left hand side of (18) represents all the parents and kids who prefer a repudiation rate greater than or equal to that preferred by the parent s^m/b. For s^m/b to be the median voter, there must be an equal number of voters on the opposite side of s^m/b.

Equation (18) simplifies to:

$$H(s^m/b) + (1+n)H(s^m/\gamma\delta b) = 1 + \frac{n}{2} \tag{19}$$

which uniquely identifies the relative wealth of the median voter parent, s^m/b.

By (10), s^m/b can be expressed as a function of government debt and of the median voter's initial endowment, e_2^m:

$$\frac{s^m}{b} = \text{Max}(0,(z+e_2^m)/b) \tag{20}$$

To determine e_2^m, consider the random variable $(z+e^i)/b$, which is a known transformation of the random variable e^i. For any $y \geq 0$, $\text{Prob}((z+e^i)/b \leq y) = G(by-z)$. This, together with (19) and (20), implies (17).

Q.E.D.

Note from (17) that e_2^m is smaller than the median initial wealth (otherwise (17) would be violated, since $\gamma\delta < 1$). Thus, the median voter is a parent who is poorer than the median parent. Since by assumption the median value of e^i is non-positive, we obtain that the median voter is also poorer than the average (i.e., $e_2^m < 0$).

Lemma 2 points out an important result: Issuing government debt changes the relative wealth position of the median voter, s^m/b. It does so in two ways. First, it increases the relative wealth of all parents poorer than the average, including whoever happens to be the median voter. Second, it changes the identity of the median voter. In this model, the proceeds of the debt issue are distributed as an equal lump sum to each parent. Thus, on the one hand, issuing debt reduces the inequality of period 2 wealth. In particular, since as noted above the median voter parent is poorer than the average, issuing government debt increases the relative wealth of whichever parent happens to be the median voter.[8] On the other hand, issuing government debt also changes the identity of the median voter. According to (17), e_2^m is a decreasing function of b: A larger debt changes the distribution of voters preferences so as to make the median voter correspond to a poorer parent.[9] The reason is that issuing government debt increases the relative wealth of the median voter kid's family by even more than it increases the relative wealth of the median voter parent. Thus, the interval $[s^m/b, s^m/\gamma\delta b]$ contains more kids as debt increases. These kids all vote in favor of more debt repudiation, since they come from poorer families than the median voter kid. Thus, issuing government debt makes more kids in favor of higher repudiation rates, which in turn makes the median voter correspond to a poorer parent (i.e., e_2^m falls as b rises).

Thus, issuing government matters for two distinct reasons. First, it changes the preferences of whoever happens to be the median voter, by increasing its relative wealth. Second, it changes the identify of the median voter, because it changes the distribution of wealth within society. This finding is crucial, because it implies that issuing government debt changes the political equilibrium in period 2. As a consequence, even in the absence of commitment, debt can be used strategically to influence future policy decisions. This implication is analyzed more thoroughly throughout the remainder of the paper.

4.3 The Political Equilibrium

I now turn to a discussion of the repudiation rate chosen in the political equilibrium, and of how this constrains the redistributive policies that can be implemented in period 1.

Combining (6′), (17) and (20), the relative wealth position of the median voter parent in period 2 can be expressed as a continuous function of government debt:

$$\frac{s^m}{b} = F(b) \tag{21}$$

The slope of this function is generally ambiguous, since it reflects the two opposite effects of issuing debt described above, on the median voter's preferences and on his identity.

By (14), the value of this function determines the repudiation rate chosen under majority rule, θ^*.

$$\begin{aligned}
&\text{if } F(b) < \delta V_x(w), && \text{then } \theta^* = 1 \\
&\text{if } F(b) > \delta V_x(w - \frac{Rb}{1+n}), && \text{then } \theta^* = 0 \\
&\text{otherwise, } 1 > \theta^* > 0 \text{ is defined by:} \\
&\delta V_x(w - \frac{Rb(1-\theta^*)}{1+n}) - F(b) = 0
\end{aligned} \tag{22}$$

These conditions define θ^* as a function of Rb. But with rational expectations and complete information, the equilibrium repudiation rate must be fully anticipated when the debt is issued, in period 1. Hence, $\theta^e = \theta^*$ and, by (12), $R^*(1-\theta^*) = r^*$ for any $\theta^* < 1$. If $\theta^e = 1$, no finite interest rate can protect the investor from the forthcoming repudiation. Hence, it is reasonable to postulate that if $\theta^e = 1$ nobody is willing to buy public debt.

Imposing this additional equilibrium requirement results in the following:

<u>Proposition 1</u>

<u>In equilibrium, government debt can only be issued in amounts that satisfy the inequality</u>:

$$F(b) - \delta V_x(w - \frac{r^*b}{1+n}) \geq 0 \tag{23}$$

<u>Proof</u>: If (23) holds with strict inequality, and if $\theta^e = 0$, then by (12) and (22), $\theta^* = 0$. If (23) holds with equality, and if $\theta^e < 1$, then again $\theta^* = \theta^e < 1$. Hence if (23) is satisfied, there always exists an equilibrium

in which no repudiation is expected and the debt is fully repaid. But if (23) is violated, then by (12) and (22) there exists no equilibrium in which $\theta^e = \theta^* < 1$. Hence, in equilibrium no debt can be issued in amounts that violate (23).[10] Q.E.D.

Thus, Proposition 1 defines a sustainable set. Only amounts of debt within this set can be issued in equilibrium. We already know from Lemma 1 that this sustainable set is bounded from above. It turns out that the sustainable set is also bounded away from zero, from below. This can be seen by noting that at the point $b = 0$, $F(b) = 0$ whereas $\delta V_x(w) > 0$, so that (23) is violated. Thus, under majority rule society will choose not to repudiate only if government debt is large enough. This result may seem surprising, but it has a simple explanation. If debt is too small, it is held by a minority of the parents. Hence there will always be a majority of the voters in favor of debt repudiation. But once enough debt is issued, and hence debt is sufficiently widely held, the constituency of debt holders may be large enough that repudiation is no longer viable.

The sustainable set corresponding to inequality (23) is illustrated in Figure 3, by the interval $[\underline{b}, \bar{b}]$. The second term in (23), $\delta V_x(\cdot)$, is drawn as the upwards sloping curve. This term always has a positive slope, since the function $V(\cdot)$ is concave. The first term, $F(b)$, is drawn as a curve that first increases and then decreases. In fact, the slope of $F(b)$ is unambiguously positive only at point A in the diagram. To the right of A, the function $F(b)$ could be either increasing or decreasing, depending on the value of b and the properties of the cumulative distribution of initial endowments, $G(\cdot)$. Hence, the sustainable set could be non-convex, or it could even be empty. The Appendix provides an example of a non-empty

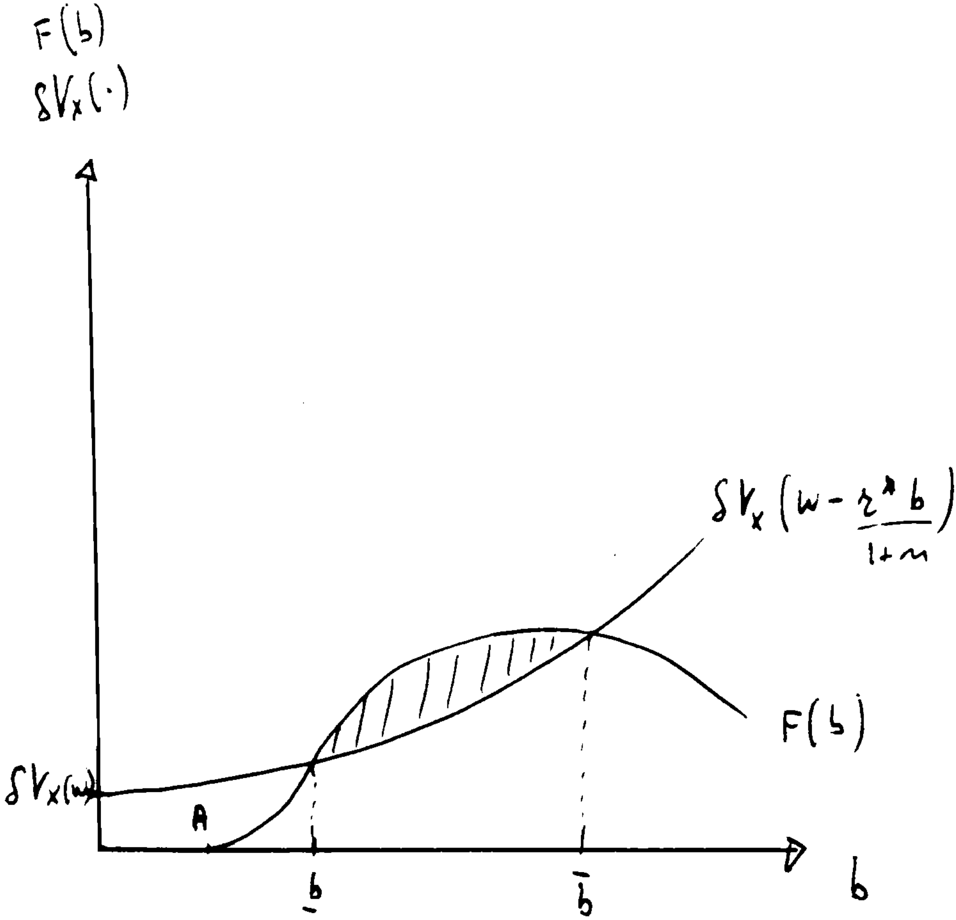

Figure 3

and convex sustainable set, similar to that of Figure 3, for the case of a uniform distribution $G(\cdot)$ and logarithmic utility function $U(\cdot)$.

If the sustainable set is nonempty, some intergenerational redistribution can take place in equilibrium, even in the absence of any commitment. This finding, that the absence of commitment does not preclude some intergenerational redistribution, is particularly striking in light of the following observation. Suppose that no action is taken in period 1, and that in period 2 a vote is taken on a simple social security system that redistributes lump sums from the kids to the parents. If the rate of growth of the population is positive (if $n > 0$), then clearly in this model such a social security system would be opposed by a majority of the voters (by all the kids). Hence, no intergenerational redistribution from the kids to the parents would take place through the social security system. So why does issuing government debt succeed where a simple social security system fails?

The answer is that, by issuing government debt, the parents tie together the intergenerational and the intragenerational effects of the policy. By doing so, they are able to gain the support of a fraction of the kids for a policy that redistributes wealth to the parents. This point is best seen with reference to an example. Suppose that, as in the example of the Appendix, a large enough stock of debt is issued, so that all parents save a positive amount. In this case, a vast majority of the parents is against full repudiation. In addition, all the kids of the wealthy parents, and some of the upper middle class kids, also oppose the repudiation. Their opposition is motivated exclusively by the adverse intragenerational redistributive effects of repudiation, and occurs even though their income would increase by repudiating the debt and even though in equilibrium the kids do not receive any bequests from their parents.

The intragenerational redistributive effects of debt repudiation occur because debt is distributed unequally among the parents, whereas the tax burden of servicing the debt falls equally on each kid. This feature of the model attempts to capture in a simple way the well known fact that wealth inequality is much greater than labor income inequality (see also footnote 3). Since there is no crucial discontinuity in the model, the results are likely to generalize to less extreme assumptions about wealth versus income inequality.

5. Equilibrium Intergenerational Redistributions

I now turn to a description of the political equilibrium of period 1, in which the parents vote on how much debt to issue. The main result of this section is that the absence of commitment generally imposes a binding constraint on the period 1 voters: the equilibrium amount of intergenerational redistribution coincides with the upper bound of the sustainable set of Proposition 1. To emphasize the importance of commitment, I first consider the benchmark case in which period 1 voters can commit future government to honor their debts.

5.1 Equilibrium Debt with Commitment

Suppose that the period 2 government is committed to repay the debt in full. Thus, $\theta = \theta^e = 0$. In this case, period 2 voters have nothing to choose. Issuing debt in period 1 forces the future government to transfer an amount r^*b to every parent in period 2; according to the government budget constraint, this transfer is financed by a tax $\tau_2 = r^*b/(1+n)$ on the kids. Consider the effect of changing b on the i^{th} parent's welfare, W_b^i. Repeating the procedure illustrated in subsection 4.1 and in footnote 5, we find:

$$W_b^i = U_c^i - r^*\delta V_x + b\,\frac{dr^*}{db}\,(s^i/b - \delta V_x) \tag{24}$$

The first two terms on the right hand side of (24) summarize the net direct effect of issuing debt. Namely, to increase the parents' income by one unit (which yields a marginal utility of $U_c^i = U_c(c_1^i)$ to voter i), and to decrease the kids' income by r^* units (which costs a disutility of $-r^*\delta V_x$). Since $U_c^i \geq r^*$ and $1 - \delta V_x \geq 0$, this direct effect is always non-negative, and strictly positive if the parent is bequest-constrained (i.e., if $1 - \delta V_x > 0$). The third term on the right hand side of (24) summarizes the indirect general equilibrium effect of issuing debt, operating through the change in the interest rate. This indirect effect is evaluated differently by different consumers. Issuing debt raises the interest rate, and this redistributes income from poor to wealthy households. Hence this third term is non-negative for households wealthier than the average, but it can be negative for poor households.

To simplify the analysis, and to underscore the importance of the commitment assumption, throughout the rest of the paper it is assumed that for all parents the direct effect always dominates the indirect effect. As shown in the Appendix, this happens if:

$$(1-G(-z))(1-\delta V_x)r^* + G(-z)\delta V_x U_{cc} b > 0 \tag{25}$$

for all $b \leq -\underline{e}$, where U_{cc} is evaluated at the point $(1+b-z)$, V_x is evaluated at the point $(w-r^*b)$, and the equilibrium conditions (9) and (6′) hold. Intuitively, this condition says that for poor parents the non-negativity constraint on bequests binds much more than the non-negativity constraint on savings. The Appendix provides an example where $U(\cdot)$ is logarithmic and $G(\cdot)$ is uniform, in which condition (25) is satisfied for an appropriate $V(\cdot)$ function. With this simplification, we can prove:

Lemma 3

If government debt cannot be repudiated, and if condition (25) holds, in equilibrium debt is issued up to the point where the parents' bequest constraint is not binding.

Proof: As shown in the appendix, under (25) $W_b^i > 0$ for all i and for any $b \leq -\underline{e}$. Moreover, recalling the results of section 2, if $b \geq -\underline{e}$ then $s^i > 0$ for all i; in this case, $U_c^i = U_c(1) = r*$ for any $b \geq -\underline{e}$ and all i, and only the direct effects of issuing debt matter. Hence, for $b \geq -\underline{e}$, equation (24) reduces to:

$$W_b^i = U_c(1) \; [1 - \delta V_x(w - U_c(1)b)] \tag{24'}$$

Clearly then, all parents are unanimous: they want to issue government debt up to the point where $\delta V_x = 1$, and the non-negativity constraint on bequests is not binding. Q.E.D.

This result is analogous to that of Cukierman and Meltzer (1989). If debt repudiation is ruled out, issuing government debt matters for two reasons. It redistributes revenue across generations. And it has a general equilibrium effect that is evaluated differently by different voters. Condition (25) ensures that for all voters, either the two effects go in the same direction, or the direct intergenerational effect of issuing debt dominates the general equilibrium effect. Hence, in equilibrium debt is issued up to the point where the parents achieve their desired intergenerational distribution of income. Government debt is the instrument with which society gets around the non-negativity constraint on bequests. And the equilibrium allocation of resources across generations depends exclusively on the degree of altruism of the parents. Cukierman and Meltzer (1989) do not impose a condition analogous to (25) and allow different parents to have a bequest motive of different intensity. As a result, in their paper the political equilibrium also reflects the general equilibrium effects of fiscal deficits.

5.2 Equilibrium Debt without Commitment

If the period 2 government is not committed to fully repay the debt, then as discussed in section 4, the equilibrium stock of debt must belong to the sustainable set of Proposition 1, since otherwise nobody would buy it. Under condition (25) all parents would like to issue debt up to the point where the non-negativity constraint on bequests is just binding (i.e., up to where $\delta V_x = 1$). But by Lemma 1, this point is outside the sustainable set. The political equilibrium of period 1 is then very simple:

Proposition 2

Under Condition (25), with unanimity the equilibrium level of debt coincides with the upper bound of the sustainable region (point $\bar{b}$ in Figure 3).[11]

This result underscores that incomplete political participation matters. As explained in the previous section, the equilibrium intergenerational redistribution is supported by the wealthier fraction of the kids because it is tied to the intragenerational effects of the policy. But *ex-ante* this tie is much weaker than *ex-post*. *Ex-post*, once the debt is issued and expectations have been formed, repudiating the debt redistributes wealth from rich to poor families. *Ex-ante*, on the other hand, the intragenerational consequences of issuing debt are only due to the interest rate effect. By (15), this effect is not very large, and it disappears altogether for $b \geq -\underline{e}$. Hence, there is a difference between the *ex-ante* and *ex-post* attitude of the kids towards intergenerational transfers through public debt. Incomplete political participation matters because it enables the parents to exploit this difference. If the kids could also vote in period 1, they would anticipate their *ex-post* preferences towards repaying the debt, and they would generally oppose a fiscal deficit.[12]

I now turn to a discussion of how the equilibrium intergenerational redistribution is affected by changes in the underlying parameters. Throughout I assume that condition (25) holds, so that Proposition 2 applies. Consider first an increase in the kids' per capita income, w. Referring to Figure 3, we see that increasing w leaves the $F(b)$ curve unaffected and shifts the δV_x curve downwards. Hence, a higher value of w increases the upper bound of the sustainable region, and thus leads to more intergenerational redistribution. The intuition is simply that when the kids' income increases, the altruistic motive of kids becomes stronger and that of the parents becomes weaker. Hence, all voters shift their preferences in favor of lower repudiation rates in period 2, which in turn enables the parents to issue a larger amount of debt in period 1. This finding is similar to that derived by Cukierman and Meltzer (1989) under the commitment assumption. Even though the extension to a stochastic framework is beyond the scope of this paper, this similarity suggests that the absence of commitments may not prevent the implementation of a policy of risk sharing among generations.

Next, consider an increase in the rate of growth of the population, n. The curve δV_x in Figure 3 shifts down, since the burden of repaying the debt is now shared among a larger kids' population. It can be shown that the curve $F(b)$ is also shifted downwards.[13] Intuitively, as n increases the proportion of voters in favor of more debt repudiation (the kids) rises, so that the political equilibrium of period 2 supports a smaller amount of intergenerational redistribution. Thus, the net effect is ambiguous: a higher rate of population growth can lead to either more or less intergenerational redistribution, depending on the specific properties of the kids' utility function and of the initial wealth distribution.

Finally, the size of the feasible set depends on the distribution of initial endowments, $G(\cdot)$. The more concentrated is the initial wealth distribution, and hence the larger is the fraction of parents in the poor and lower-middle classes, the smaller is the endowment of the median voter parent, e_2^m. As a consequence, the smaller is the upper bound of the feasible region. In the limit, if e_2^m is too low, the feasible region is empty, in which case no domestic government debt can be issued. This feature of the equilibrium may contribute to explain why developing countries rarely issue domestic government debt and often rely on external sources of funds. At low stages of development, wealth is highly concentrated and a political equilibrium in favor of domestic debt repudiation would easily materialize. In the case of external debt, on the other hand, the threat of external sanctions and trade disruptions can provide an enforcement technology not available on domestic capital markets.

6. Concluding Remarks

There is a widespread opinion that domestic government debt is honored because of reputation incentives.[14] Recently Bulow and Rogoff (1989) have cast doubts on this idea, by showing that reputation incentives only work if a repudiating government is shut out from world wide capital markets also as a lender and not just as a borrower. It is hard to believe that domestic debt repudiation would have such dismal consequences.

This paper has explored an alternative line of thought, which emphasizes the redistributive consequences of debt repudiation.[15] The main insight of the paper is that issuing debt creates a constituency in support of repaying it. Thus, issuing debt "creates facts" even in the absence of any commitment technology. This is because, once debt is issued, repudiation has redistributive consequences. Opposition to such a redistribution may create a majority in favor of repaying the debt.

This idea has been applied in the paper to explain why a generation can extract resources from future yet unborn generations. By issuing government debt, the intergenerational redistribution is tied to the intragenerational consequences of choosing how much debt to repay. Young voters motivated by the desire to avoid intragenerational redistributions may accept transferring resources to the older generation, even though they would have opposed such a transfer if it was voted on in isolation. This may explain why alternative methods of intergenerational redistribution, such as social security and government debt, coexist at the same time in the same society. These methods may be equivalent from an economic point of view. But they differ in their political viability, since they tie the intergenerational aspect to other redistributive issues in a different way.

This same idea can be investigated in alternative frameworks, unrelated to the intergenerational issue. Aghion and Bolton (1989) have independently applied it to an economy where there is no intergenerational conflict but individuals differ in their preferences for private versus public consumption. Other possible applications are to more general forms of wealth taxation, besides debt repudiation, or to privatization decisions. Persson and Tabellini (1989) discuss a few related examples.

Footnotes

[1]A third assumption is that voters, even though rational, are "naive" in the sense that they disregard the effect of the current policy on the future voting decisions. In other words, future voting decisions are taken as given by current voters, even though such decisions are affected by a state variable under the control of current voters.

[2]This same insight has been independently derived in a recent very interesting paper by Aghion and Bolton (1989), who disregard intergenerational aspects of debt policy and focus on different issues.

[3]In the U.S., in 1985 the distribution of income and wealth was as follows:

X	% of Income of Top X% Families	% of Wealth Owned by Top X% Families
2%	5.8%	10.4%
10%	15.1%	36.0%
50%	73.3%	90.0%

The data are from the 1986 Survey of Consumers' Finance, by R. Avery and A. Kennickell, sponsored by the Board of Governors of the Federal Reserve System.

[4]In deriving (7) I relied on the fact that $c_2^i > 0$ for all i, and hence that $a^i > 0$ and sufficiently large for all i.

[5]Equation (14) has been derived as follows. Let λ^i and μ^i be the Lagrange multipliers associated with (13b) and (13c). Then the envelope theorem implies:

$$W_\theta^i = -\lambda^i(s^i-b)\,R - \mu^i s^i\,R + \mu^i(1+n)\frac{df^i}{d\theta} \qquad \text{(F.1)}$$

By the parents' first order conditions, $\mu^i = 1 - \delta V_x$ and $\lambda^i = \delta V_x$.

Moreover, by (7′), the kids are always gift constrained for any $b \geq 0$. Hence, $df_i/d\theta = 0$. Using these facts in (F.1) yields (14).

[6]This can be seen by noting that equation (9) in the text and Figure 1 imply $b \geq z$, with strict inequality if $b < -\underline{e}$. Hence, $s^i = \text{Max}(0, z+e^i) \leq \text{Max}(0, b+e^i)$. The average e^i is 0. Hence, for at least 50% of the parents, $s^i \leq \text{Max}(0,b)$.

[7]Equation (16) has been obtained by setting (14) and (15) equal to zero, and by noting that only δV_x in (14) and (15) depends on θ.

[8]This can also be seen by noting from (10) and (11) that if $s^i > 0$, then $\frac{\partial s^i/b}{\partial b} = \frac{1}{b}\,[b-(1-G(-z))s^i]$. Thus, $\frac{\partial s^i/b}{\partial b} > 0$ for relatively poor parents and $\frac{\partial s^i/b}{\partial b} < 0$ for relatively wealthy parents.

[9]Applying the implicit function theorem to (20), one obtains that:

$$\frac{de_2^m}{db} = \frac{-G[(e_2^m+(1-\delta\gamma)z/\delta\gamma]}{(1-G(-z))\{g(e_2^m)\delta\gamma+g[(e_2^m+(1-\delta\gamma)z)/\delta\gamma]\}} < 0$$

where $g(\cdot) = G'(\cdot)$.

[10]Note that when (23) is satisfied with equality, the value of $\theta*$ is indeterminate: any $1 > \theta \geq 0$ can be an equilibrium. This occurs because, since debt is the only asset, a fully anticipated wealth tax is of no consequence whatsoever. This would not be true if there were other taxable forms of wealth with returns technologically fixed, such as land or capital. A previous version of this paper considered this extension, and derived analogous but much more complicated results.

[11]If condition (25) does not hold, then the general equilibrium effects of the fiscal deficit may induce a majority of the parents to oppose issuing

debt up to the upper bound of the sustainable set. In this case, the absence of commitment would not impose a binding constraint on the period 1 voters. Naturally, the results of Proposition 1 concerning the sustainable set itself do not depend on condition (25). Finally, if (25) is violated, the parents' preferences are not necessarily single peaked.

[12]All of the kids would always oppose a deficit larger than $-\underline{e}$. Some of the wealthy kids may vote in favor of a deficit smaller than $-\underline{e}$.

[13]Using (17), and since e_2^m is smaller than the median of e^i, it can be shown that e_2^m is a decreasing function of n. That is, increasing n leads to a poorer median voter parent. By (20), this then implies that $s^m/_b$ is decreasing in n.

[14]The literature on reputation and wealth taxation is surveyed in Persson and Tabellini [1989]. Grossman and Van Huyck [1988] and Chari and Kehoe [1989] study reputational incentives with reference to debt repudiation.

[15]Alesina [1988] provides historical evidence in support of this line of research.

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Appendix

1. Proof of Proposition 2. (i) Consider the case $b \le -\underline{e}$. Using (11), equation (24) can be rewritten as:

$$W_b^i = U_c^i - r^*\delta V_x - \frac{G}{1-G} U_{cc} b(s^i/b - \delta V_x) \tag{A.0}$$

where $G(\cdot)$ is evaluated at the point $-z^*$ and U_{cc} is evaluated at the point $(1+b-z)$. Consider parent j for which $e^j = -z$. This parent is just borrowing constrained. Hence, for him $s^j = 0$ and (8) holds as an equality. Thus, for this parent, (A.0) yields:

$$W_b^j = r^*(1-\delta V_x) + \frac{G}{1-G} U_{cc} b\delta V_x \tag{A.1}$$

which is positive by (25).

All parents with $e^i < e^j$ also have $s^i = 0$. But since they are borrowing constrained, by (8) they also have $U_c^i > r^*$. Hence, by (A.0) and (A.1), for all of these parents $W_b^i > W_b^j > 0$. Finally, all parents with $e^i > e^j$ save a positive amount. Hence, for them $U_c^i = r^*$ and (A.0) becomes:

$$W_b^i = r^*(1-\delta V_x) + \frac{G}{1-G} U_{cc}\, \delta b V_x - \frac{G}{1-G} U_{cc} s^i \tag{A.2}$$

Thus, again, $W_b^i > W_b^j > 0$. Thus, under (25) $W_b^i > 0$ for all voters when b is in the range $[0,-\underline{e}]$. The case $b > -\underline{e}$ is discussed in the text.

Q.E.D.

2. Example. Suppose that the distribution of initial endowments is uniform, with support $[-e,e]$, where $1 > e > 0$. Thus,

$$G(e^i) = \frac{e^i+e}{2e}, \quad g(e^i) = \frac{1}{2e} \tag{A.3}$$

Suppose further that $U(c) = \ell n\ c$. By (6′), after some transformations, we obtain:

$$z = -e + 2\sqrt{eb} \quad \text{for} \quad b \le e$$
$$z = b \qquad\qquad \text{for} \quad b > e \tag{A.4}$$

Combining (A.3) and (A.4), we have:

$$1 - G(-z) = \frac{\sqrt{eb}}{e} \tag{A.5}$$

Moreover, by (9) and (A.4):

$$r^* = \frac{1}{1+b-z} = \frac{1}{1+b+e-2\sqrt{eb}} \tag{A.6}$$

and

$$U_{cc}(1+b-z) = -(\frac{1}{1+b-z})^2 = -(r^*)^2 \tag{A.7}$$

Combining all this information, we can rewrite (25) as:

$$\frac{\sqrt{eb}}{e}\left[1-\delta V_x\right] - \left[1-\frac{\sqrt{eb}}{e}\right]\delta V_x br^* > 0 \tag{A.8}$$

Using (A.6) this expression simplifies to:

$$\frac{1+b+e-2\sqrt{eb}}{1+e-\sqrt{eb}} > \delta V_x\ (w-r^*b) \tag{A.9}$$

which is satisfied for appropriate specifications of the function $V(\cdot)$.

Retaining the same specifications for $G(\cdot)$ and $U(\cdot)$, consider now equation (17) in lemma 2. It can be rewritten as:

$$\frac{e_2^m+e}{2e} + (1+n)\frac{\left[e_2^m + (1-\delta\gamma)z\right]\frac{1}{\delta\gamma}+e}{2e} - 1 - \frac{n}{2} = 0 \tag{A.10}$$

Making use of (A.4) and simplifying yields:

$$e_2^m = \frac{(1+n)(1-\gamma\delta)(e-2\sqrt{eb})}{1+n+\gamma\delta} \tag{A.11}$$

Moreover, by (20) and (A.4):

$$s^m = \mathrm{Max}\left(0, 2\sqrt{eb}-e+\frac{(1+n)(1-\gamma\delta)(e-2\sqrt{eb})}{1+n+\gamma\delta}\right) \quad \text{(A.12)}$$

which in turn yields:

$$F(b) = \frac{s^m}{b} = \mathrm{Max}\left\{0, \frac{(2\sqrt{eb}-e)}{b}\ \phi\right\} \quad \text{(A.13)}$$

where $\phi = \frac{\gamma\delta(2+n)}{1+n+\gamma\delta}$.

Thus, $F(b) = 0$ for $b \le e/_4$ and $F(b) > 0$ for $b > e/_4$. Moreover, for $b \ge e/_4$, we have:

$$F_b(b) = \frac{\phi e}{b^2\sqrt{eb}}\ (\sqrt{eb}-b) \quad \text{(A.14)}$$

$$F_{bb}(b) = -\frac{\phi}{b}\left[\frac{e}{b^2} - \sqrt{eb}\right] \quad \text{(A.15)}$$

By (A.14), $F(b)$ reaches a maximum at the point $b = e$. Incidentally, this is the smallest value of b for which even the poorest parent is not borrowing constrained. To the left of this point, $F_b > 0$. To the right, $F_b < 0$. At the point $b = e$, we have:

$$F(e) = \phi = \frac{\gamma\delta(2+n)}{1+n+\gamma\delta} \quad \text{(A.16)}$$

Thus, F_b can be drawn as in Figure 4. By (23) a sufficient condition for the sustainable set to be non-empty is:

$$V_x(w-e) < \frac{\gamma(2+n)}{1+n+\gamma\delta} \quad \text{(A.17)}$$

For if (A.17) holds, then by (A.6) and (A.15) $\delta V_x < F(b)$ at the point $b = e$.

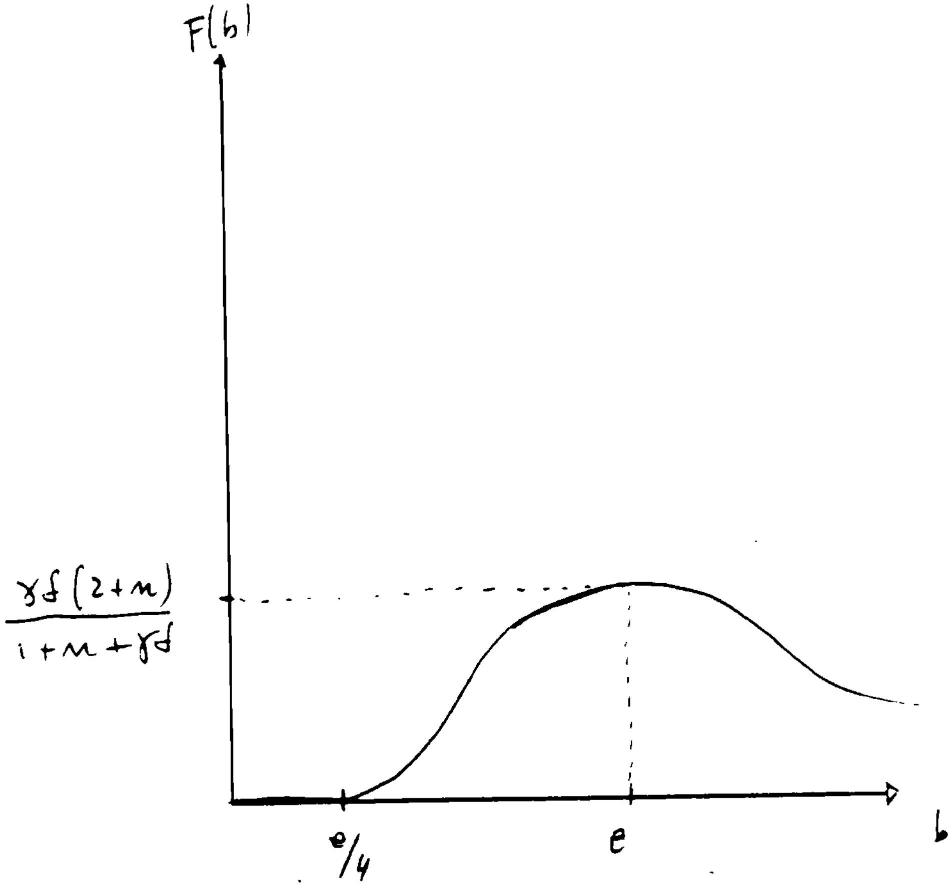

Figure 4