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INFORMATIONALLY EFFICIENT CLIMATE POLICY:
DESIGNING MARKETS TO MEASURE AND PRICE EXTERNALITIES

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ABSTRACT

I study how policymakers can access and act on the information about climate change damages that is dispersed throughout the economy. I analyze a new dynamic deposit-refund instrument (called “carbon shares”). I show that there exists a limit-case rational expectations equilibrium in which it: i) efficiently prices emissions conditional on information, ii) efficiently incentivizes removal of past emissions conditional on information, and iii) efficiently aggregates dispersed information about the social cost of emissions. Conventional emission taxes generally succeed at only the first of these objectives. Rather than projecting damages in all future periods and all possible states of the world in order to calculate the optimal tax, the regulator here estimates damages as they are realized and empowers markets to perform price discovery about future damages.

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Despite the validity in principle of the tax-subsidy approach in the Pigouvian tradition, in practice it suffers from serious difficulties. For we do not know how to estimate the magnitudes of the social costs, the data needed to implement the Pigouvian tax-subsidy proposals.

– Baumol (1972, p. 316)

The practical problem, however, arises precisely because these facts are never so given to a single mind, and because, in consequence, it is necessary that in the solution of the problem knowledge should be used that is dispersed among many people.

– Hayek (1945, p. 530)

1 Introduction

Economists have long emphasized the informational advantages of market-based policies for controlling pollution. Market-based policies require the regulator only to measure the social cost of pollution (i.e., the externality) and aggregate abatement costs, whereas command-and-control policies also require the regulator to measure individual firms' costs of reducing pollution. Because actual firms and agents know more about their own options to reduce pollution than does the regulator, market-based policies increase efficiency by empowering them to employ their most cost-effective options. However, even market-based policies may demand a lot of information from a regulator: it is often quite challenging to measure the social cost of pollution, as Baumol (1972) recognizes in the opening quotation. In particular, economists have struggled to measure the social cost of the carbon emissions that drive global climate change.¹ As a result, economists have still not converged on an emission price to recommend to policymakers interested in market-based solutions.²

To date, measurement of social costs has been centralized among academics and regulators, but much information about the cost of climate change is dispersed throughout society.

¹Nordhaus (2019, p. 1998) acknowledges, “In reality, projecting impacts is the most difficult task and has the greatest uncertainties of all the processes associated with global warming.” Some economists even criticize the social cost estimates underlying the most prominent climate-economy models as “completely made up, with no theoretical or empirical foundation” (Pindyck, 2013, p. 868). Recent work estimates the effects of weather shocks and uses these effects to project the consequences of future climate change (Deschênes and Greenstone, 2007; Carleton and Hsiang, 2016), but despite many advances in this literature, fundamental questions remain about how to map consequences of weather shocks to consequences of climate change (Dell et al., 2014; Lemoine, 2021b).

²In a survey of 289 economists with expertise on climate change (Holladay et al., 2009), estimates of the social cost of carbon had a standard deviation of \$339 per tCO₂, around three times larger than the average estimate. The social costs of carbon calculated in Pindyck (2019) from another survey of 113 economists also shows substantial dispersion, with the standard deviation again of comparable magnitude to the mean.

Every person and firm on the planet is exposed to climate change. Each may have some information about their own particular exposure and about their own particular ability to adapt. Since at least Hayek (1945), a rich tradition in economics views markets as an algorithm that aggregates dispersed information about the costs and benefits of the many goods produced in society. Yet economists have not studied how to design markets to aggregate information about externalities, and thereby to perform price discovery for social costs.

I design and analyze a new market-based policy instrument that simultaneously measures and controls externalities. In my setting, firms in different sectors of the economy trade off the benefit of emitting carbon against the cost of complying with current policies, as is customary. Carbon emissions generate warming that impacts each sector of the economy in an uncertain, stochastic, and potentially correlated fashion. I introduce two novel features to this environment. First, firms can pay to remove old emissions from the atmosphere.³ Second, information about climate change damages is heterogeneous. Agents measure climate impacts in their own sectors, and a regulator measures the aggregate effect of climate change from data on final good production and temperature. Both types of measurements are unbiased but may be arbitrarily noisy. Agents' measurements are private information, whereas the regulator reports its measurement to all actors in the economy and can use it to set policy.⁴

I show that a regulator who uses an emission tax policy is unable to optimally use new information about social costs or to observe all of the information about social costs that is dispersed throughout society.⁵ First, if the regulator did have access to all agents' information about social costs, then it could use an emission tax to efficiently control new emissions (as has long been known). However, an emission tax on its own cannot incentivize the removal of past emissions other than as means to offset ongoing emissions: once an emitter has paid the penalty, subsequent information is irrelevant even if it dramatically alters the estimated social cost of previously emitted carbon and warrants paying for its removal. An emission tax ultimately places the risk of needing to fund large-scale carbon removal on taxpayers, which could impose an impractical fiscal burden (Bednar et al., 2019).⁶ Second, the regulator

³Carbon dioxide removal, or negative emission, strategies include chemically separating carbon dioxide from air ("direct air capture"), capturing emissions from power plants that burn biomass ("bioenergy with carbon capture and storage"), accelerating the weathering of rocks, enhancing uptake of carbon by forests or oceans, and more. See National Research Council (2015), Fuss et al. (2018), and National Academies of Sciences, Engineering, and Medicine (2018) for recent reviews. Recently, Microsoft and Stripe each received bids to undertake carbon removal for around \$150 per tCO₂ on average (Joppa et al., 2021).

⁴The regulator could choose to measure whatever it cares about. Whatever the regulator attempts to measure will determine the market incentives to reduce emissions and to remove old emissions. For instance, the regulator could apply equity weighting, value nonmarket impacts, and/or, as analyzed in Appendix C, impose its own discount rate on the market's damage expectations.

⁵Similar critiques apply to cap-and-trade programs, the quantity version of an emission tax.

⁶A forward-thinking regulator could save the revenue collected by an emission tax for the procurement of future carbon removal, but as analyzed in Appendix D, even that revenue could be insufficient if carbon removal becomes desirable because the regulator learns climate change is more damaging than it had believed

will not actually have access to all agents' information. I show that this regulator's emission tax is informationally inefficient in the plausible case that the regulator measures aggregate consequences only imperfectly and also in the plausible case that sectors have heterogeneous value shares and correlated exposure to climate change. In either case, an all-seeing regulator with access to all of the information dispersed throughout society would choose a different emission tax than would a more realistic, information-constrained regulator.

I design a new instrument that I show can incentivize optimal carbon removal without spending taxpayer funds and can efficiently aggregate dispersed information. The new instrument is a dynamic deposit-refund scheme. The regulator requires that emitters post a deposit at the time of emission and in exchange gives emitters a tradeable security that I call a "carbon share". In each period, the regulator refunds part of the deposit to current shareholders based on whether its measure of aggregate realized damages was as bad as implied by the deposit. The equilibrium value of the carbon share reflects expected refunds, which are by construction smaller than the value of the deposit and which vary inversely with market expectations of the regulator's future damage measurements. Emitters have an incentive to reduce emissions in order to avoid giving up the deposit for the less valuable carbon share. In later periods, a carbon shareholder may decide to remove the underlying unit of carbon from the atmosphere in order to retire the carbon share and recover its deposit. If, for example, traders become more pessimistic about climate change, then expected refunds fall. Emitting becomes less attractive because the value of the carbon share received falls, and carbon removal becomes more attractive because the opportunity cost of retiring a carbon share falls.

I show that as period-to-period stochasticity in damages becomes small, there exists a fully revealing rational expectations equilibrium in which the price of a carbon share perfectly aggregates the information dispersed throughout society and in which the incentives to reduce emissions and remove carbon are the same as in the welfare-maximizing, informationally efficient benchmark.⁷ This optimal outcome depends on the deposit being sufficiently large and on the regulator making a good-faith (albeit potentially imperfect) effort to measure and report aggregate recent damages. A large deposit is critical because the private cost of emitting carbon and the private benefit of removing carbon are both defined by traders' expectation of the difference between the deposit and expected refunds. I call this difference the expected stream of "damage charges" that correspond to the regulator's future measurements of aggregate damages. If the initial deposit is small, then damage charges will be constrained by the deposit in periods with high measured climate change damages. As the

at the time of emission.

⁷Following the convention in finance and information economics, I use "rational expectations" to indicate that Bayesian traders use prices to learn about others' information and trade optimally conditional on their posterior beliefs. The usage common in macroeconomics is slightly different, as that literature does not typically model asymmetric information. See Vives (2008, Chapter 3) and Campbell (2017, Chapter 12) for discussions.

deposit becomes large, traders' expectations of damage charges converge to their expectations of future measurements of damages and thus to their current estimates of the marginal damage from carbon emissions, so that the private cost of emitting converges to the social cost of carbon emissions. A numerical calibration suggests that a deposit around 2–3 times as large as the estimated social cost of emissions approximates optimal emission and removal incentives.

I also study the Bayesian Nash equilibrium of a game in demand functions in order to assess equilibria that are “implementable” via a specific trading mechanism (following Vives, 2014). Each trader submits a demand schedule that accounts for their private information and for what they would infer about other traders' information from any given carbon share price they might observe. Traders' risk aversion, their concern for a winner's curse, and the existence of noise traders prevent the equilibrium carbon share price from being fully revealing. Nonetheless, I show that emission incentives depend on carbon share prices, which traders use to learn about correlated impacts and to reduce the impact of measurement error in aggregate data. Emission incentives therefore respond to dispersed information that the regulator does not possess, bringing emission outcomes closer to the informationally efficient benchmark.

This new policy can be understood by comparison to two other policies. First, conditional on information, emission and removal incentives are identical to incentives under an idealized policy that taxes firms for their past contributions to the current stock of atmospheric carbon (as opposed to taxing their current flow of emissions).⁸ Under such a policy, emission and removal incentives depend on expectations of future stock taxes, which I show in Appendix B should be tied to measurements of realized damages precisely like the damage charges in a carbon share policy. However, in practice emitters will not fully internalize future stock taxes when they recognize that there is a high likelihood of going out of business over the many decades that carbon persists in the atmosphere, and in the absence of a market to coordinate expectations and reveal information, different agents could expect different stock taxes and thus set their marginal cost of emissions to different levels, violating a basic tenet of efficiency. In contrast, carbon shares are valuable securities with positive payoffs funded by an upfront deposit, rendering them immune to problems posed by judgment-proofness, and agents base their marginal cost of emissions on the same observed carbon share price.

Second, a regulator could report measurements of realized climate damages, run a prediction market on those measurements, and set an emission tax based on that market's revealed predictions.⁹ As with carbon shares, such a policy shifts much of the work of projecting possi-

⁸Stock taxes have been proposed in the context of climate change (Lemoine, 2007), mine remediation (White et al., 2012; Yang and Davis, 2018), and space orbits (Rao et al., 2020).

⁹Some prior work focuses on the revelation of beliefs about the magnitude of climate change rather than about climate change damages: Schlenker and Taylor (2021) show that weather derivatives are sensitive to climate model projections, and Hsu (2011) proposes futures on emission taxes set, following McKittrick (2011), according to a predefined function of temperature (see also Aliakbari and McKittrick, 2018). In

ble future climate change damages from the regulator to markets. And as with carbon shares, such a policy could in principle aggregate dispersed information. However, many prediction markets suffer from low liquidity, and the tax-cum-prediction-market scheme lacks incentives to remove carbon beyond the point at which net emissions reach zero. In contrast, liquidity in a carbon share market is assured because each unit of emissions creates a new stake in the market that its holder will need to value, and that market does incentivize negative emissions if damage estimates become sufficiently pessimistic.

A central theme throughout environmental economics is the importance of using market-based instruments to control pollution, whether in the form of emission taxes or cap-and-trade programs (see, among others, Metcalf, 2009; Stavins, 2022).¹⁰ Broadening the typical focus on the role of emission prices in determining the budget sets of firms and households, I consider how to design markets so that emission prices convey information about damages from climate change. Where dispersed information about damages is relatively unimportant and cleanup of past emissions is irrelevant (as may be true of particulate matter or lead pollution), the proposed policy performs like an emission tax or cap-and-trade program. But where information about damages is dispersed, the proposed policy acts like improving the information underlying an emission tax or cap-and-trade program, and where cleanup of past emissions is potentially relevant, the present policy can incentivize such cleanup without requiring the regulator to directly fund it. Climate change clearly demonstrates dispersed information about impacts and the possibility of ex post cleanup, and many other externalities will too.¹¹

My proposed instrument is a dynamic deposit-refund instrument. Static deposit-refund schemes resolve difficulties monitoring—and thus taxing—improper waste disposal (e.g., Bohm, 1981; Russell, 1987; Fullerton and Kinnaman, 1995; Torsello and Vercelli, 1998).¹² I posit no problem monitoring either the act of emission or the act of carbon removal, but the proposed policy does resolve difficulties that arise from the regulator’s imperfect ability to

the latter papers, price discovery provides a signal useful for long-run investment but does not determine emission and carbon removal decisions.

¹⁰The recommendation to address climate change by taxing emissions dates to at least Nordhaus (1977), and attempts to econometrically estimate the consequences of climate change date to at least Mendelsohn et al. (1994). Weitzman (1974) shows that asymmetric information about abatement costs can break the equivalence between an emission tax and cap. I emphasize asymmetric information about the externality, which I show can make an emission tax or cap informationally inefficient.

¹¹For instance, consider the externalities produced by orbital debris in space. Satellite owners could post a bond to fund an “orbital-use share” that would incentivize both optimal debris creation and optimal debris cleanup. Fees for launching satellites are the analogue of an emission tax. They fail to incentivize either active measures to avoid creating debris post-launch or cleanup of debris post-impact. Rao et al. (2020) propose orbital-use fees that are the analogue of taxing the stock of pollution, a policy option discussed above.

¹²Deposit-refund schemes have also been understood as means to avoid the fiscal costs of subsidies and the distributional costs of taxes (Bohm, 1981). Here one of the motivations is to avoid the fiscal costs of using the public purse to directly fund carbon removal.

incentivize optimal carbon removal by taxing past emitters for emerging climate damages. From the perspective of emitters, my policy combines a tax (in the form of the deposit) and a subsidy (in the value of the carbon share received), with emission incentives determined by the difference between the deposit and the value of the carbon share. This type of emission incentive is familiar from the static generalization of deposit-refund schemes in Fullerton and Wolverton (2000). Here, however, the level of the subsidy is not fixed by the regulator but is instead determined in equilibrium (in the form of the carbon share price) by private actors' information about climate change impacts.¹³

The possibility of removing enough carbon from the atmosphere to make aggregate emissions “net negative” has become a prominent part of the climate policy discourse. The Intergovernmental Panel on Climate Change projects that limiting warming to 1.5°C (2°C) would require up to 700 (250) Gt CO₂ of net negative emissions over this century (IPCC, 2022), the 2021 U.S. Infrastructure Investment and Jobs Act provided \$3.5 billion to establish carbon removal hubs, and the 2022 U.S. Inflation Reduction Act increased tax credits for capturing and storing carbon from the air from \$50 to \$180 per ton of CO₂. Despite the increasingly prominent discussion and promotion of carbon removal, I know of no work on market-based approaches to incentivizing optimal use of these technologies.¹⁴ In the absence of alternative policy instruments, many assume that governments would directly subsidize carbon removal, despite concerns about the fiscal burden such subsidies would impose (see Bednar et al., 2019; Edenhofer et al., 2021).¹⁵ I here propose a policy instrument that is revenue-positive for the government and efficiently adapts the scale of carbon removal to new information about the cost of climate change.

Finally, this paper constitutes a novel link between environmental economics and a younger literature on asymmetric information in financial markets. Formalizing the insight of Hayek (1945), much work since Grossman (1976, 1978) studies financial markets' ability to efficiently aggregate dispersed information. I here study an asset tied to an externality. Payoffs are common across traders because they depend on damage charges that are applied uniformly to all shareholders. Common value models are typical of this literature, but my setting differs from nearly all such work by permitting correlation in private information

¹³I show that the ideal deposit would equal the worst-case social cost of carbon. Others have previously proposed that fees on materials or products be set to their most harmful possible environmental fate, with fees refunded in accord with the harmfulness of actual outcomes (e.g., Solow, 1971; Mills, 1972; Bohm and Russell, 1985; Costanza and Perrings, 1990; Boyd, 2002). These informal proposals employ arguments based on ambiguity aversion, difficulties monitoring pollution, or difficulties posed by judgment-proofness.

¹⁴Conventional emission pricing policies could incentivize use of carbon dioxide removal technologies up to the point at which net emissions are zero. However, the European Union's flagship cap-and-trade program historically has not provided the credits for carbon dioxide removal that could sustain even this limited incentive (Scott and Geden, 2018; Rickels et al., 2020). Bednar et al. (2021) propose “carbon removal obligations” that would extend standard cap-and-trade schemes to allow temporarily overshooting longer-run carbon targets.

¹⁵Bednar et al. (2019) calculate that the subsidies required for carbon removal could exceed even the share of output that the U.S. spends on defense.

about the common value (exceptions include Ozsoylev and Walden, 2011; Lambert et al., 2018; Lou et al., 2019). I show that correlation can be critical to informationally efficient updating and also to the informativeness of carbon share prices.

The next section describes the economic and informational environment. Section 3 derives outcomes in the informationally efficient, welfare-maximizing benchmark. Section 4 analyzes informational properties of emission taxes. Section 5 formally defines the carbon share policy and analyzes its informational properties. The final section discusses issues arising. The appendix contains proofs, extensions, and numerical details.

2 Setting

Throughout, I use a hat ($\hat{\cdot}$) to indicate outcomes under the informationally efficient welfare-maximizing benchmark, a tilde ($\tilde{\cdot}$) to indicate outcomes under an emission tax policy, and a breve ($\breve{\cdot}$) to indicate outcomes under a carbon share policy.

2.1 Production, Consumption, and Emissions

Let there be a unit mass of households and $N > 1$ intermediate-good sectors, each of which is perfectly competitive. Output from sector i in period t is

$$Y_{it} = \exp[-\zeta_{it}T_t] L_{it} Y^{it}(e_{it}).$$

$L_{it} \in [0, 1]$ is labor offered by households to sector i in exchange for wage w_{it} . The representative firm in sector i has gross production function $Y^{it}(e_{it})$, with $e_{it} \geq 0$ indicating emissions. $Y^{it}(\cdot)$ is strictly increasing and concave and can evolve with technology. Temperature T_t imposes damages ζ_{it} in sector i at time t .¹⁶ The multiplicative effect of climate damages follows the DICE model (Nordhaus, 1992, 2013), among others, and the exponential form for damages follows Golosov et al. (2014) and Lemoine (2021a), among others.

The representative firm in sector i can fund the removal of quantity $R_{it} \geq 0$ of emissions from the atmosphere. It purchases this emission removal from a competitive industry whose costs $c_t(R_t)$ as a share of total output (see below) depend on aggregate removal $R_t \triangleq \sum_{i=1}^N R_{it}$. The cost function $c_t(\cdot)$ is strictly positive, strictly increasing, and strictly convex and can evolve over time.¹⁷

Cumulative emissions up to time t are $M_t = M_0 + \sum_{s=0}^{t-1} \left[\sum_{i=1}^N e_{is} - R_s \right]$, with pre-policy cumulative emissions $M_0 \geq 0$ given. Time t warming is $T_t = \alpha M_t$, with $\alpha > 0$. This representation recognizes that carbon dioxide is a globally mixed pollutant and follows recent scientific findings that global temperature is approximately a linear function of cumulative

¹⁶The index i could equivalently be interpreted as indicating either regions or sector-region pairs.

¹⁷Convexity in removal costs reflects both the cost of removing carbon from the atmosphere and the potential scarcity of sites for storing carbon after removal.

emissions (see Dietz and Venmans, 2019, among others). Firms are small, so they ignore the effects of their own emissions on temperature.

Total output is Cobb-Douglas:

$$Y_t = \prod_{i=1}^N (Y_{it})^{\kappa_i},$$

with each $\kappa_i > 0$ and $\sum_{i=1}^N \kappa_i = 1$. Substituting from Y_{it} ,

$$Y_t = e^{-\tilde{\zeta}_t T_t} \prod_{i=1}^N (L_{it} Y^{it}(e_{it}))^{\kappa_i},$$

where $\tilde{\zeta}_t \triangleq \sum_{i=1}^N \kappa_i \zeta_{it}$ is the aggregate damage realization. Aggregate consumption C_t is no greater than net output:

$$C_t \leq (1 - c_t(R_t))Y_t.$$

The representative household has logarithmic utility:

$$u(C_t) = \ln(C_t).$$

Time t welfare is the present value of expected utility:

$$\sum_{s=t}^{\infty} \frac{1}{(1+r)^{s-t}} E_t[u(C_s)],$$

with per-period discount rate $r > 0$ and with the information set defined in each application below.

In equilibrium, firms maximize the expected present value of profits subject to prices, households maximize utility subject to budget constraints, and all markets clear. Time 0 consumption is the numeraire.

2.2 Informational Environment

I now describe the informational environment. All agents are Bayesian.

Agents affiliated with sector i observe $\zeta_{it} + \lambda_{it}$, where $\zeta_{it} = \zeta_i + \bar{\zeta}_i + \epsilon_{it}$. The ζ_i are unknown and unobserved. The $\bar{\zeta}_i$ are public knowledge and represent prior expected damages in each sector. Assume only that $\sum_{k=1}^N \kappa_k \bar{\zeta}_k < (r/\alpha) \kappa_i Y^{i0'}(0)/Y^{i0}(0)$ for some $i \in \{1, \dots, N\}$, which will ensure that welfare-maximizing aggregate emissions are strictly positive in the initial period. The ϵ_{it} and λ_{it} are random variables that are each normally distributed, mean-zero, unobserved, and uncorrelated either across sectors or over time. The variance of each ϵ_{it} is $\sigma^2 \geq 0$, which represents random exposure to global temperature. That randomness

could result from randomness in the mapping from global temperature to temperatures in locations relevant to sector i and/or from randomness in sector i 's exposure to its locations' temperatures. The variance of each λ_{it} is $\omega^2 \geq 0$, which represents agents' potentially imperfect ability to measure the effect of temperature on sectoral production.

The regulator and firms have a common jointly normal prior over the ζ_i at time 0. Each ζ_i has a prior mean of zero and has prior variance $\tau_0^2 > 0$. The correlation between any pair ζ_i and ζ_j (for $i \neq j$) is $\Gamma \in [0, 1)$, a known parameter.¹⁸ This correlation determines how signals of damages in one sector provide information about damages in another sector. If $\Gamma = 0$, then the unknown component of damages is independent across sectors. If $\Gamma > 0$, then the unknown component of damages has a common component across sectors, as when impacts in one sector affect other sectors or as when uncertain vulnerability to weather is correlated across sectors.

The regulator does not observe sectoral production or input choices.¹⁹ Instead, at the end of time t , the regulator uses observed total output to measure $\tilde{\zeta}_t + \tilde{\lambda}_t$.²⁰ The $\tilde{\lambda}_t$ are random variables that are normally distributed, mean-zero, and serially uncorrelated. Their variance is $\tilde{\omega}^2 \geq 0$. They reflect the possibility of measurement error in aggregate data and of imprecision due to having to estimate $\tilde{\zeta}_t$ from aggregate data. The regulator shares the measured $\tilde{\zeta}_t + \tilde{\lambda}_t$ with all agents in the economy.

The timing within a period t is that intermediate-good firms make emission decisions, markets clear based on realized production, agents observe $\zeta_{it} + \lambda_{it}$, and finally the regulator observes $\tilde{\zeta}_t + \tilde{\lambda}_t$.

3 Informationally Efficient, Welfare-Maximizing Benchmark

Begin by considering welfare-maximizing emissions and carbon removal. Define \hat{E}_t as the expectation operator based on all information available up to time t , $\hat{\mu}_t$ and $\hat{\Omega}_t$ as the $N \times 1$ vector of posterior means and the $N \times N$ posterior covariance matrix for the ζ_i based on information up to time t , and $\hat{\mu}_t$ and $\hat{\Omega}_t$ as the posterior mean and variance of $\sum_{i=1}^N \kappa_i \zeta_i$ based on information up to time t .

¹⁸ Assuming a prior mean of zero is not restrictive, as nonzero means are absorbed into the $\bar{\zeta}_i$. Assuming that the prior variance τ_0^2 and correlation Γ are constant over sectors is for ease of exposition, and assuming that Γ is nonnegative simplifies the discussion by focusing on an especially plausible case.

¹⁹ The subsequent analysis would be fundamentally similar if we instead permitted the regulator to observe sectoral production and assumed that agents had private information at the subsectoral level.

²⁰ Firms' equilibrium production choices are independent of T_t (see (A-14) through (A-16)), so the regulator can estimate $\tilde{\zeta}_t$ from a time series of Y_t and T_t . For this same reason, the assumption below that firms in sector i do not know ζ_{it} when choosing their time t emissions is not critical.

Welfare-maximizing outcomes solve the following Bellman equation:

$$\hat{W}(T_t, \hat{\boldsymbol{\mu}}_t, \hat{\boldsymbol{\Omega}}_t) = \max_{\mathbf{L}_t, \mathbf{e}_t, R_t \geq 0} \hat{E}_t \left[u(C_t) + \frac{1}{1+r} \hat{W}(T_{t+1}, \hat{\boldsymbol{\mu}}_{t+1}, \hat{\boldsymbol{\Omega}}_{t+1}) \right],$$

where \mathbf{L}_t and \mathbf{e}_t indicate vectors of labor and emissions in each sector. Taking first-order conditions and then recursively substituting from the envelope theorem yields the following conditions that must hold for all i :

$$\frac{\kappa_i Y^{it'}(e_{it})}{Y^{it}(e_{it})} \begin{cases} = \frac{1}{r} \alpha \left[\sum_{k=1}^N \kappa_k \bar{\zeta}_k + \hat{\mu}_t \right] & \text{if } e_{it} > 0 \\ \leq \frac{1}{r} \alpha \left[\sum_{k=1}^N \kappa_k \bar{\zeta}_k + \hat{\mu}_t \right] & \text{if } e_{it} = 0 \end{cases}, \quad (1)$$

$$\frac{c'_t(R_t)}{1 - c_t(R_t)} \begin{cases} = \frac{1}{r} \alpha \left[\sum_{k=1}^N \kappa_k \bar{\zeta}_k + \hat{\mu}_t \right] & \text{if } R_t > 0 \\ \geq \frac{1}{r} \alpha \left[\sum_{k=1}^N \kappa_k \bar{\zeta}_k + \hat{\mu}_t \right] & \text{if } R_t = 0 \end{cases}. \quad (2)$$

On the right-hand side, the terms in brackets yield per-period expected damages per unit of warming, the α converts to units of emissions, and the $1/r$ converts to present value. The first condition equates the marginal benefit of emissions to the marginal social cost of emissions when emissions are strictly positive. If $Y^{it'}(0)$ is sufficiently small, then $e_{it} = 0$. The second condition equates the marginal cost of carbon removal to the marginal social cost of emissions (i.e., the marginal benefit of carbon removal) when carbon removal is strictly positive. If $c'_t(0)$ is sufficiently large, then $R_t = 0$. As $\sum_{k=1}^N \kappa_k \bar{\zeta}_k + \hat{\mu}_t$ increases, e_{it} falls and R_t either increases or remains zero. Negative emissions, in which $R_t > \sum_{i=1}^N e_{it}$, become optimal when $\sum_{k=1}^N \kappa_k \bar{\zeta}_k + \hat{\mu}_t$ is sufficiently large.

Now consider beliefs. Following convention, the benchmark of informational efficiency updates beliefs from the available signals $\{\zeta_{it} + \lambda_{it}\}_{i=1}^N$ and $\tilde{\zeta}_t + \tilde{\lambda}_t$.

Proposition 1 (Informationally Efficient Beliefs). *There exists $\hat{Z}_t \in [0, 1)$ such that $\hat{Z}_t \rightarrow 0$ as $\tilde{\omega}^2/\omega^2 \rightarrow \infty$ and*

$$\begin{aligned} \hat{\mu}_t = & \hat{Z}_t \left[\frac{1}{t} \sum_{j=0}^{t-1} [\tilde{\zeta}_j + \tilde{\lambda}_j] - \sum_{k=1}^N \kappa_k \bar{\zeta}_k \right] \\ & + \frac{(1 - \hat{Z}_t)(1 - \Gamma)\tau_0^2 - \hat{Z}_t\sigma^2/t}{(1 - \Gamma)\tau_0^2 + \sigma^2/t + \omega^2/t} \sum_{k=1}^N \kappa_k \left[\frac{1}{t} \sum_{j=0}^{t-1} [\zeta_{kj} + \lambda_{kj}] - \bar{\zeta}_k \right] \\ & + \frac{\sigma^2/t + (1 - \hat{Z}_t)\omega^2/t}{(1 - \Gamma)\tau_0^2 + \sigma^2/t + \omega^2/t} \frac{N\Gamma\tau_0^2}{(1 - \Gamma)\tau_0^2 + \sigma^2/t + \omega^2/t + N\Gamma\tau_0^2} \frac{1}{N} \sum_{k=1}^N \left[\frac{1}{t} \sum_{j=0}^{t-1} [\zeta_{kj} + \lambda_{kj}] - \bar{\zeta}_k \right]. \end{aligned} \quad (3)$$

Proof. Apply the projection theorem to a random vector formed from $\sum_{k=1}^N \kappa_k \zeta_k$, the N sectoral signals, and the aggregate signal. See Appendix E. \square

Corollary 1 (Special Cases for Informationally Efficient Beliefs).

i If $\omega^2 = 0$, then

$$\hat{\mu}_t = \frac{(1 - \Gamma)\tau_0^2}{(1 - \Gamma)\tau_0^2 + \sigma^2/t} \overbrace{\sum_{k=1}^N \kappa_k \left[\frac{1}{t} \sum_{j=0}^{t-1} \zeta_{kj} - \bar{\zeta}_k \right]}^{\text{=lim}_{\tilde{\omega}^2 \rightarrow 0} \frac{1}{t} \sum_{j=0}^{t-1} [\tilde{\zeta}_j + \tilde{\lambda}_j] - \sum_{k=1}^N \kappa_k \bar{\zeta}_k} + \frac{\sigma^2/t}{(1 - \Gamma)\tau_0^2 + \sigma^2/t} \frac{N\Gamma\tau_0^2}{(1 - \Gamma)\tau_0^2 + \sigma^2/t + N\Gamma\tau_0^2} \frac{1}{N} \sum_{k=1}^N \left[\frac{1}{t} \sum_{j=0}^{t-1} \zeta_{kj} - \bar{\zeta}_k \right].$$

ii If $\tilde{\omega}^2 = 0$ and $\sigma^2 = 0$, then

$$\hat{\mu}_t = \frac{1}{t} \sum_{j=0}^{t-1} \tilde{\zeta}_j - \sum_{k=1}^N \kappa_k \bar{\zeta}_k.$$

iii If $\tilde{\omega}^2 = 0$ and $\Gamma = 0$, then

$$\hat{\mu}_t = \frac{\tau_0^2}{\tau_0^2 + \sigma^2/t} \left[\frac{1}{t} \sum_{j=0}^{t-1} \tilde{\zeta}_j - \sum_{k=1}^N \kappa_k \bar{\zeta}_k \right].$$

iv If $\tilde{\omega}^2 = 0$ and each $\kappa_i = 1/N$, then

$$\hat{\mu}_t = \frac{(1 - \Gamma)\tau_0^2 + N\Gamma\tau_0^2}{(1 - \Gamma)\tau_0^2 + \sigma^2/t + N\Gamma\tau_0^2} \left[\frac{1}{t} \sum_{j=0}^{t-1} \tilde{\zeta}_j - \sum_{k=1}^N \kappa_k \bar{\zeta}_k \right].$$

Proof. See Appendix F. □

The informationally efficient benchmark aggregates information from the private signals and the public signals. The first part of Corollary 1 describes mean beliefs when dispersed agents do not suffer measurement error ($\omega^2 = 0$). In this case, the informationally efficient benchmark has no use for the aggregate signals $\tilde{\zeta}_j + \tilde{\lambda}_j$. Instead, it weights the sectoral signals as if a perfectly measured version of the aggregate signal were available (first line). In the presence of correlation ($\Gamma > 0$), it also uses the unweighted average of the sectoral signals (second line): each sector's signals provide the same information about damages in the other sectors, regardless of whether a given sector has large or small κ_i .

The remaining three parts of Corollary 1 describe conditions under which efficient updating has no use for the sectoral signals $\zeta_{kj} + \lambda_{kj}$. All three conditions require that there be no measurement error in the aggregate signal ($\tilde{\omega}^2 = 0$). When there is also no stochasticity

($\sigma^2 = 0$), the aggregate signal is without noise and the other signals are superfluous (part ii). When sectoral signals are independent ($\Gamma = 0$), there is no scope for learning through correlation and thus no scope for improving on the perfectly measured aggregate signal (part iii). And when sectors have identical value shares of output (each $\kappa_i = 1/N$), the weights in the perfectly measured aggregate signal are identical to the weights that optimally use the sectoral signals' correlation (part iv).

Proposition 1 shows that informationally efficient beliefs in general use both aggregate and dispersed information. The aggregate signal (first line in (3)) provides information that can mitigate the consequences of measurement error in sectoral signals, and the sectoral signals (second line) provide information used to construct an alternate version of the aggregate signal that mitigates the consequences of measurement error in the aggregate signal. Efficient updating also leverages correlation across sectoral effects (third line) to learn from sectors whose small κ_i mean they do not directly matter much for aggregate outcomes.

4 Regulation by Emission Taxes

Now consider a regulator who maximizes welfare by taxing firms' period t net emissions at rate ν_t . Firms can avoid the tax either by reducing emissions or by contracting for removal to offset ongoing emissions. The regulator returns any tax revenue to households as lump-sum transfers. I initially assume that the regulator's tax revenue must be weakly positive in each period and summarize the consequences of weakening that assumption in Section 4.1, drawing on analysis in Appendix D.

The regulator sets the time t tax at the beginning of the period so as to maximize welfare conditional on its time t beliefs and subject to market equilibrium. The regulator's chosen time t tax is therefore a function of the aggregate measurements from times 0 through $t - 1$. Denote the regulator's mean belief about $\sum_{k=1}^N \kappa_k \zeta_k$ at the time t information set as $\tilde{\mu}_t$.

The following proposition gives the optimal emission tax:

Proposition 2 (Emission Tax). *There exists $\bar{\nu}_t > 0$ such that $\sum_{i=1}^N e_{it} - R_t = 0$ if and only if $\nu_t \geq \bar{\nu}_t$. The regulator maximizes welfare with a tax of*

$$\nu_t = \min \left\{ \bar{\nu}_t, C_0 \frac{\alpha}{r} \left[\sum_{k=1}^N \kappa_k \bar{\zeta}_k + \tilde{\mu}_t \right] \right\}.$$

Proof. See Appendix G. □

When $\nu_t < \bar{\nu}_t$, the tax is determined by the present value of expected aggregate damages.²¹

²¹The combination of logarithmic utility and the damage specification means that the optimal tax does not depend on total consumption and uncertainty is not priced directly, both as in Golosov et al. (2014). For a more general constant relative risk aversion utility function, the optimal tax would be sensitive to future consumption and would include a risk premium (see Lemoine, 2021a).

Using that tax, firms' first-order conditions (A-15) and (A-16) become

$$\frac{\kappa_i Y^{it}(e_{it})}{Y^{it}(e_{it})} = \frac{\alpha}{r} \left[\sum_{k=1}^N \kappa_k \bar{\zeta}_k + \tilde{\mu}_t \right],$$

$$\frac{c'_t(R_t)}{1 - c_t(R_t)} = \frac{\alpha}{r} \left[\sum_{k=1}^N \kappa_k \bar{\zeta}_k + \tilde{\mu}_t \right].$$

Once we account for the possibility of corner solutions, these are the conditions for welfare-maximization given in (1) and (2) as long as $\tilde{\mu}_t = \hat{\mu}_t$.

We see two possible reasons why a tax may not attain the informationally efficient welfare-maximizing benchmark. First, it could be that $C_0 \frac{\alpha}{r} \left[\sum_{k=1}^N \kappa_k \bar{\zeta}_k + \hat{\mu}_t \right] > \bar{\nu}_t$. In this case, the benchmark from Section 3 would have negative net emissions whereas the regulator's feasible equilibrium has zero net emissions. I refer to this possibility as inducing a loss due to inefficiency in using information, as it indicates a failure to implement the information collected from the economy. Second, it could be that $\tilde{\mu}_t \neq \hat{\mu}_t$. In this case, the benchmark from Section 3 would be based on beliefs that are different from the regulator's beliefs. I refer to this possibility as inducing a loss from observing information, as it indicates a failure to collect all of the information in the economy. I explore each in turn.

4.1 Loss Due to Inefficiency in Using Information

Temporarily assume that the regulator observes all signals $\zeta_{it} + \lambda_{it}$, as when firms truthfully communicate to the regulator or the regulator collects the same data as firms. In this case, $\tilde{\mu}_t = \hat{\mu}_t$, so the regulator can set the informationally efficient tax and we have no loss from inefficiency in observing information.

The policy described in Proposition 2 imposes losses relative to the welfare-maximizing benchmark when there is some chance that the constraint $\bar{\nu}_t$ binds (i.e., that negative net emissions become optimal). That chance is driven by the possibility of observing information that makes $\tilde{\mu}_t$ large and by the possibility that technological progress in carbon removal makes R_t large for any given ν_t (i.e., makes $\bar{\nu}_t$ small). The constraint itself reflects the inability of an emission tax to procure negative emissions. At $\nu_t = \bar{\nu}_t$, the economy has zero net emissions. It may be that carbon removal is undertaken ($R_t > 0$) for $\nu_t < \bar{\nu}_t$, but that removal acts merely to offset contemporary emissions. What the emission tax cannot do is procure removal that undoes old emissions and thereby makes $\sum_{i=1}^N e_{it} - R_t < 0$. Because no current actor is financially responsible for old emissions, no current actor is incentivized to bear the cost of removing them.

A regulator could attain net negative emissions if it were allowed to subsidize carbon removal. But we ruled this out by assuming revenue neutrality: a subsidy would have to be funded by new revenue because tax revenue in a period with net negative emissions is zero

by construction. One might object that requiring revenue neutrality within each period is too stringent. A forward-thinking regulator could save the revenue collected from emission taxes and dedicate it to funding carbon removal. In effect, such a policy would establish a lockbox for emission tax revenue that allows the regulator to procure some level of negative net emissions without needing to raise money from taxpayers. It would change the revenue constraint from a static one that must hold in each period to a dynamic one that must hold across periods.

Appendix D studies such a constraint. Summarizing the foregoing and these additional results, we learn:

1. A regulator who must obey a period-by-period revenue constraint cannot procure negative emissions using an emission tax. An emission tax therefore cannot attain first-best should negative emissions eventually become optimal, even if the regulator has perfect information.
2. A regulator who stores emission tax revenue in a lockbox for funding future carbon removal can fund the optimal level of carbon removal if the regulator does not learn about damages over time. In the current setting, an emission tax-plus-lockbox therefore can attain first-best in the presence of technological progress in carbon removal.
3. A regulator who stores emission tax revenue in a lockbox for funding future carbon removal might not be able to fund the optimal level of carbon removal if the regulator learns about damages over time. An emission tax-plus-lockbox therefore may not attain first-best in the presence of new information about climate damages. And the optimal use of the lockbox distorts emissions and removal decisions in all other periods so as to increase funds in the lockbox.²²

The third point shows that information is critical to inefficiencies in procuring negative emissions, as it is the possibility of pessimistic information about the social cost of emissions that would justify spending beyond the revenue collected from earlier emission taxes.

4.2 Loss Due to Inefficiency in Observing Information

Now allow asymmetric information so that we can analyze the loss from inefficiency in observing information.

The following result describes the regulator's time t posterior estimate of $\sum_{k=1}^N \kappa_k \zeta_k$ formed from observing the aggregate signals $\tilde{\zeta}_j + \tilde{\lambda}_j$.

²²The distortions in the other periods would also arise if we allowed the regulator to subsidize carbon removal with public funds raised through distortionary taxes.

Proposition 3 (Regulator's Beliefs).

$$\tilde{\mu}_t = \frac{(1 - \Gamma)\tau_0^2 \sum_{i=1}^N \kappa_i^2 + \Gamma\tau_0^2}{(1 - \Gamma)\tau_0^2 \sum_{i=1}^N \kappa_i^2 + \Gamma\tau_0^2 + \frac{1}{t}[\tilde{\omega}^2 + \sigma^2 \sum_{i=1}^N \kappa_i^2]} \frac{1}{t} \sum_{j=0}^{t-1} \left(\tilde{\zeta}_j + \tilde{\lambda}_j - \sum_{k=1}^N \kappa_k \tilde{\zeta}_k \right). \quad (4)$$

Proof. Follows from application of the conventional univariate normal-normal Bayesian updating formula, observing that the prior variance is $\tau_0^2 \sum_{i=1}^N \kappa_i^2 + 2\Gamma\tau_0^2 \sum_{i=1}^N \sum_{k=i+1}^N \kappa_i \kappa_k$ and using $\sum_{i=1}^N \kappa_i = 1$. \square

The weight placed on the aggregate measurement in (4) increases in Γ : positive correlation increases the regulator's prior uncertainty about aggregate damages and thus mechanically increases the weight placed on the aggregate measurement. Posterior beliefs are exactly the same as those formed by a counterfactual regulator in a world with variance $\tilde{\tau}_0^2 = (1 - \Gamma)\tau_0^2 + \Gamma\tau_0^2 / [\sum_{i=1}^N \kappa_i^2]$ and correlation $\tilde{\Gamma} = 0$. In contrast, Proposition 1 showed that positive correlation among the unknown sector-specific effects ζ_i increases the weight that informationally efficient beliefs place on the disentangled sectoral measurements, because such beliefs recognize that each sector's measurement contains information about all other sectors.

The following corollary delineates conditions under which the regulator's beliefs are informationally efficient.

Corollary 2 (Informationally Efficient Regulator). *For $t > 0$, $\tilde{\mu}_t = \hat{\mu}_t$ with probability 1 if (i) $\tilde{\omega}^2 = 0$ and either (iia) $\sigma^2 = 0$, (iib) $\Gamma = 0$, or (iic) each $\kappa_i = 1/N$.*

Proof. Follows straightforwardly from Corollary 1 and Proposition 3. \square

The regulator's beliefs are informationally efficient if there is no measurement error at the aggregate level ($\tilde{\omega}^2 = 0$) and either there is no stochasticity in climate damages ($\sigma^2 = 0$), there is no correlation among sectoral effects ($\Gamma = 0$), or sectors have identical weights in production (each $\kappa_i = 1/N$) that render correlation unimportant for learning. Otherwise Proposition 1 showed that informationally beliefs generally use disaggregated signals unavailable to the regulator. Because it is unlikely that any of the conditions in Corollary 2 hold in reality, an actual regulator's beliefs are likely to be informationally inefficient.²³

Whereas the drawback in Section 4.1 was inefficiency in using the available information should that information warrant negative emissions, the drawback here is the inefficiency in observing all available information. The best emission tax that a regulator can implement will generally differ from the emission tax that the regulator would choose based on all of the information in the economy.

²³Appendix A.1 provides numerical examples.

5 A Policy Framework that Dominates Conventional Emission Pricing

We have seen that conventional emission pricing does not perform ideally at either collecting or using information about the social cost of greenhouse gas emissions. There is space for a policy to do better than conventional market-based instruments. I now describe such a policy, in the form of a novel dynamic deposit-refund instrument.

This new type of policy requires each emitter to post a deposit $D \geq 0$ per unit of emissions. We can express D as

$$D \triangleq \frac{1}{r} C_0 \alpha \left[\sum_{k=1}^N \kappa_k \bar{\zeta}_k + \bar{\mu} \right]. \quad (5)$$

Choosing the deposit is equivalent to choosing a parameter $\bar{\mu}$ that defines implied per-period climate damages.²⁴ The regulator invests the deposit and earns interest at rate r . In exchange for the deposit, the emitter receives a transferable security that is attached to the unit of carbon emitted. I refer to the security as a “carbon share” because it reflects a claim on a part of the carbon in the atmosphere.

At the end of each period, the policymaker applies a damage charge Δ_t to each outstanding carbon share. This charge is set equal to the lesser of the period t measured marginal damage from carbon emissions and the per-period damages implied by the deposit:

$$\Delta_t \triangleq C_0 \alpha \min \left\{ \tilde{\zeta}_t + \tilde{\lambda}_t, \sum_{k=1}^N \kappa_k \bar{\zeta}_k + \bar{\mu} \right\}. \quad (6)$$

Damage charges are returned lump sum to households.²⁵ The policymaker refunds to carbon shareholders the difference between the damage charge and the per-period damages implied by the deposit:²⁶

$$\begin{aligned} d_t &\triangleq r D - \Delta_t \\ &= C_0 \alpha \max \left\{ 0, \sum_{k=1}^N \kappa_k \bar{\zeta}_k + \bar{\mu} - (\tilde{\zeta}_t + \tilde{\lambda}_t) \right\}. \end{aligned} \quad (7)$$

²⁴The deposit would equal the emission tax ν_t from Proposition 2 if $\bar{\mu} = \tilde{\mu}_t$ and $\nu_t < \bar{\nu}_t$.

²⁵In a second-best setting, revenue from damage charges could be used to offset revenue from distortionary taxes. A full analysis of such a setting should consider how to adapt both the damage charges and the deposit (see Fullerton and Wolverton, 2000, 2005).

²⁶Two notes on the refund. First, Appendix C shows how the regulator could adjust the refund in order to impose a different discount rate than the market rate. Second, the optimal refund will be common to all outstanding shares because marginal damage is the same for all units of carbon (see equation (1)). In other applications, shares could be vintaged by date or location of emission.

	Time t	Time $t+1$	Time $t+2$...	Time $t+s$
Emitter:	Pays D , Receives share worth q_t	Sells share for q_{t+1}			
Shareholder:		Buys share for q_{t+1} , Receives d_{t+1}	Receives d_{t+2}	...	Pays p_{t+s}^R , Receives $(1+r)D - \Delta_{t+s}$

Figure 1: Example of cashflows over the life of a carbon share. Here the share is attached to a unit of time t emissions, the emitter decides to sell the share at time $t + 1$, and the new shareholder decides to remove the underlying unit of carbon from the atmosphere at time $t + s$.

The refunds d_t are weakly positive. No refund is paid in the period of emission. The deposit acts like principal, some of which is returned to agents over time in the form of refunds and some of which is reclaimed by the regulator in the form of damage charges. Over the lifetime of a carbon share, the present value of total refunds and damage charges recovers the deposit:

$$\sum_{s=1}^{\infty} \frac{1}{(1+r)^s} [d_{t+s} + \Delta_{t+s}] = \sum_{s=1}^{\infty} \frac{1}{(1+r)^s} rD = D.$$

When deposits are invested at the market interest rate, the carbon share policy is revenue-neutral if climate change damages completely fail to materialize and raises revenue via nonzero damage charges otherwise.

In each period subsequent to emission, a carbon share's owner decides whether to leave its attached unit of carbon in the atmosphere. If the owner removes the carbon from the atmosphere in time t , they receive $(1+r)D - \Delta_t$ and the share is retired; otherwise they receive refund d_t and can keep or sell the share. Leaving carbon in the atmosphere for one more period means losing the damage charge Δ_{t+1} .

Figure 1 provides an example of cashflows over time under the carbon share policy. At time t , an emitter posts the deposit D and in return receives a carbon share whose market value is q_t (to be analyzed below). At time $t + 1$, the emitter in this example decides to sell the share to a third party for the market price q_{t+1} . That third party claims the time $t + 1$ refunds d_{t+1} and continues to do so until either selling the share or removing the underlying

unit of carbon from the atmosphere. At time $t + s$, the third party in this example does decide to remove the underlying unit of carbon from the atmosphere, which costs p_{t+s}^R . At that point, the regulator retires the carbon share and pays the third party $(1 + r)D - \Delta_{t+s}$.

I assume all agents discount at rate r . Use \check{W} to denote welfare along a realized trajectory under the carbon share policy defined above and use \hat{W} to denote welfare along a realized trajectory under the welfare-maximizing, informationally efficient benchmark:

$$\check{W} = \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} u(\check{C}_s),$$

$$\hat{W} = \sum_{s=0}^{\infty} \frac{1}{(1+r)^s} u(\hat{C}_s).$$

The full-information expected loss from using the carbon share policy is:

$$\check{L} = E_0 \left[\hat{W} - \check{W} \mid \boldsymbol{\zeta} \right],$$

where $\boldsymbol{\zeta}$ is a vector of the ζ_i .

5.1 Improved Efficiency in Using Information

Temporarily assume that all actors in the economy see all of the $\zeta_{it} + \lambda_{it}$ at each time t . We saw in Section 4.1 that an emission tax may fail to use information efficiently when information justifies negative emissions. I will show that carbon shares can improve outcomes if the deposit D is sufficiently large.

Define \hat{q}_t as the carbon share's value in period t prior to observing the $\zeta_{it} + \lambda_{it}$, $\tilde{\zeta}_t + \tilde{\lambda}_t$, Δ_t , or d_t , where the hat notation reflects that the carbon share price in this section uses informationally efficient beliefs (as opposed to the share price to be defined in Section 5.2). The following lemma establishes the equilibrium value of the carbon share:

Lemma 1 (Carbon Share Value).

$$\hat{q}_t = \sum_{j=0}^{\infty} \frac{1}{(1+r)^j} \hat{E}_t[d_{t+j}] \geq 0. \quad (8)$$

Proof. See Appendix H. □

The equilibrium value of the carbon share is the expected present value of the refunds that it claims. The value of holding a carbon share derives from the possibility that damages

will not be as bad as implied by $\bar{\mu}$. At the time of emission, a firm's net outlays per unit of non-abated emissions are $D - (\hat{q}_t - \hat{E}[d_t]) \in [0, D]$.²⁷

The following assumption ensures that it would never be optimal to remove enough carbon to bring atmospheric carbon and temperature below their initial levels:

Assumption 1 (Pre-Policy Carbon Should Not Be Removed). $\hat{R}_t \leq M_t - M_0$ for all $t \geq 0$.

Even the highest-removal scenarios for the coming century do not project bringing carbon or temperature below current levels (IPCC, 2022), so this assumption is likely to be met by any carbon share policy begun in the next few years. The following proposition relates the period t loss to the deposits required at earlier times:

Proposition 4 (Efficiency Conditional on Information). *Let Assumption 1 hold. Then $\check{L} \rightarrow 0$ as $D \rightarrow \infty$.*

Proof. See Appendix I. □

The proposition establishes that the carbon share policy achieves the welfare-maximizing benchmark as the deposit becomes large. The proof shows that the time t private values for reducing emissions and removing carbon are each equal to

$$\sum_{j=1}^{\infty} \frac{1}{(1+r)^j} \hat{E}_t[\Delta_{t+j}].$$

Emitters lose the difference between the initial deposit and the initial value of the share they receive, and that difference is the present value of expected damage charges. Carbon removal benefits shareholders by preventing the loss of future damage charges. From (6), damage charges are the current period's realized marginal damage when $\bar{\mu}$ (and thus D) is large. Therefore the present value of expected future damage charges under large D is simply the present value of expected marginal damage from emissions, which is the social cost of carbon familiar from much work on the economics of climate change.²⁸

As the deposit becomes large, the carbon share policy attains the emission reduction incentives of an emission tax and also approaches efficiency in using information even in the presence of carbon removal. Comparing to results in Section 4.1, the carbon share

²⁷If future damages were guaranteed to be zero in every period, then the present value of the stream of refunds at the time of emission would be D , and if future damages were guaranteed to exceed the per-period value implied by D in every period, then the present value of the stream of refunds at the time of emission would be zero. Therefore $\hat{q}_t - \hat{E}[d_t] \in [0, D]$.

²⁸I have used normal distributions for tractability and ease of exposition. If I instead assumed that the distribution of damages had finite support, then the carbon share policy would achieve the welfare-maximizing benchmark as $\bar{\mu}$ (and thus D) approaches some finite value from below. Under this interpretation, the carbon share policy approaches efficiency as the deposit approaches the worst-case social cost of carbon.

policy outperforms an emission tax with a static revenue constraint if net negative emissions might ever become optimal, and the carbon share policy outperforms an emission tax with a dynamic revenue constraint if optimal removal might exhaust the cumulative revenue collected from emission taxes.²⁹

The optimal carbon share policy provides the same incentives as would the optimal tax on the stock of carbon previously emitted by a firm (as opposed to the conventional tax on the flow of carbon emissions studied in Section 4).³⁰ However, whereas firms could avoid a carbon stock tax by declaring bankruptcy, carbon shares are valuable assets that investors want to hold, financed at the time of emission by the deposit. If the owner of a carbon share were to declare bankruptcy or otherwise liquidate, its creditors would want the carbon share so they could receive its refunds and have the option to eventually reclaim the full deposit. Carbon shares therefore avoid judgment-proofness problems that would bedevil stock taxes. (Moreover, a stock tax would fail to coordinate emission incentives across firms because it would lack the ability to aggregate information, as described in Section 5.2 below.)

One might be concerned about whether the deposit would challenge firms' liquidity (see Shogren et al., 1993). Recall that firms receive a carbon share in return for their deposit and can immediately sell this valuable asset on. From (A-22), their net outlays per unit of emissions are the exact same outlays required by the traditional Pigouvian emission tax. This is why an arbitrarily large deposit does not distort firms' emission incentives. If the market for carbon shares is thick, a carbon share policy need not be any more financially challenging than a conventional carbon emission tax.³¹

But one might still wonder about the scale of the deposit. If the deposit is not sufficiently large, then the highest potential damage charges are truncated by the constraints imposed by the deposit, which reduces the expected damage charges that firms use to guide emission and removal decisions. The possibility of hitting a deposit's constraints therefore increases emissions and reduces removal. Ex post, the prices of options on carbon shares would reveal whether traders deemed it likely that the value of a carbon share would approach zero, as when damage charges are constrained by the deposit. Ex ante, a numerical exercise detailed in Appendix A provides some indication of how large a deposit may be necessary. This exercise takes damage estimates from the survey in Pindyck (2019) and considers the probability that any given deposit would be insufficient to cover the implied damage charges

²⁹Compare incentives under the carbon share policy and under an emission tax policy that attaches a distinct lockbox to each unit of emissions, so that the most a regulator can spend to remove some particular old carbon is what the regulator collected at the time of emission. Increasing the emission tax at some time would increase the amount the regulator could later spend on removal, but such a change in the emission tax would overincentivize emission reductions. In contrast, increasing the carbon share's deposit increases both the efficiency of its emission price and the efficiency of its removal incentives.

³⁰Appendix B shows that the optimal time t stock tax would be $\lim_{\bar{\mu} \rightarrow \infty} \Delta_t$.

³¹Gross outlays are also capped because any firm could avoid posting the deposit by reducing its emissions. The growing number of firms making zero emission pledges and recent cost projections for removal technologies both suggest that even the maximum gross outlays are limited to a reasonable scale.

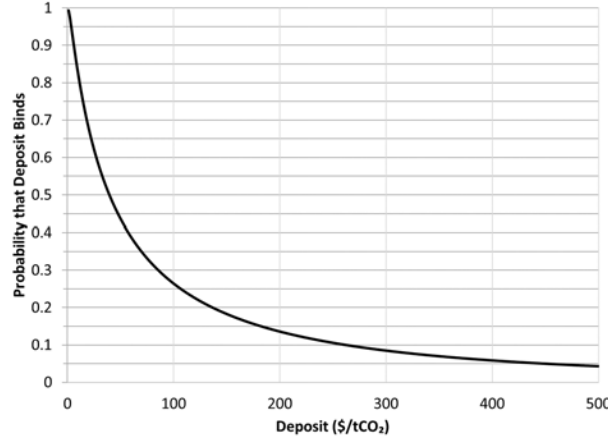


Figure 2: The probability that deposits of various sizes would bind, based on the calibration in Appendix A and using $\tilde{\omega}^2 = \sigma^2 = 0$.

(i.e., that $\Delta_t < C_0 \alpha [\tilde{\zeta}_t + \tilde{\lambda}_t]$ in equation (6)). In this calibration, expected damages imply a tax of \$118 per tCO₂. Figure 2 shows that a deposit roughly twice as large (\$250 per tCO₂) would suffice in all but the worst 10% of cases, and a deposit just over three times as large (\$400 per tCO₂) would suffice in all but the worst 5% of cases. An adequate deposit may therefore be well within an order of magnitude of what the carbon tax would have been.

5.2 Improved Efficiency in Observing Information

I now investigate how the market for carbon shares aggregates dispersed information about climate change damages. I therefore allow asymmetric information, as in Section 4.2. I condition results on arbitrarily large D so as to highlight potential inefficiencies in observing information rather than in using information. From Section 5.1, emissions and carbon removal will be optimal conditional on information.

A continuum of traders of measure $n_i > 0$ is attached to sector i . At the beginning of time t , all agents have a symmetric, common prior over $\sum_{k=1}^N \kappa_k \zeta_k$, based on the regulator's measured aggregate damages in earlier periods and the prices of carbon shares in earlier periods. The time 0 prior is as described in Section 2.2, the posterior at the beginning of time t assigns variance τ_t^2 to each ζ_k , and the posterior mean of $\sum_{k=1}^N \kappa_k \zeta_k$ at the beginning of time t is $\check{\mu}_t$. The price of carbon shares at the beginning of time t is q_t . Firms make emission and removal decisions based on this price and households consume accordingly. Subsequently, traders attached to sector i measure $\zeta_{it} + \lambda_{it}$.³² They trade carbon shares

³²The division of a continuum of traders into a finite number of types avoids the “schizophrenia” problem described by Hellwig (1980). We have a cousin to the large-market setting of Hellwig (1980) either if $N \rightarrow \infty$ or if we let each trader within a sector receive a different signal.

based on this differentiated information. The market clears at price \check{q}_t^* . Between periods t and $t + 1$, the regulator measures $\check{\zeta}_t + \check{\lambda}_t$ from its data on aggregate output, returns refunds d_t to shareholders based on shareholdings at the end of period t , and issues new shares to firms based on period t emissions.³³

At the beginning of time t , each trader of type i has share holdings y_{it} and wealth w_{it} . The y_{it} include shares issued in all previous periods that are still active (i.e., for which the underlying unit of carbon has not yet been removed). Traders have the ability to invest in a riskless asset with return r . After observing their private signals, traders choose their net demand X_{it} to maximize their expected utility of wealth at the beginning of period $t + 1$:

$$\max_{X_{it}} \check{E}_t \left[-\exp \left\{ -A_i \left((1+r)(w_{it} + (X_{it} + y_{it})d_t - X_{it}\check{q}_t) + (y_{it} + X_{it})q_{t+1} \right) \right\} \middle| \zeta_{it} + \lambda_{it}, \check{q}_t \right],$$

with $A_i > 0$ the coefficient of absolute risk aversion among traders attached to sector i and \check{E}_t indicating expectations based on common information available at the beginning of time t . Traders have exponential utility, as opposed to the logarithmic utility function of the representative household. Exponential utility is critical to the analysis in this section because exponential utility yields linear asset demand functions that are independent of wealth and amenable to aggregation. For these reasons, exponential utility (including its implementation as quadratic payoffs) is used in nearly all literature on asymmetric information in asset markets.³⁴

In a rational expectations equilibrium, markets clear with traders inferring from prices whatever information they can and maximizing utility conditional on that information. This equilibrium is defined as fully revealing if the carbon share price \check{q}_t^* reveals the same information about $\check{\zeta}_t$ as would observing the N disentangled signals $\{\zeta_{kt} + \lambda_{kt}\}_{k=1}^N$. The following proposition establishes properties of this equilibrium:

Proposition 5 (Efficient Equilibrium). *Let Assumption 1 hold. A fully revealing rational expectations equilibrium with $\check{\mu}_t = \hat{\mu}_t$ and $\check{L} = 0$ exists as $\sigma^2 \rightarrow 0$ and $D \rightarrow \infty$.*

Proof. See Appendix K.

Sketch: Shows that a fully revealing price both clears the market in period 0 and, as $\sigma^2 \rightarrow 0$, generates informationally efficient beliefs at the beginning of period 1. Shows analogous results for all t by induction. Result then follows from Proposition 4. □

³³Traders do not need to be only in emitting sectors; they could be in any sector with information about damages. Here, that possibility would be reflected by $Y^{it'}(0) = 0$, in which case sector i would have zero time t emissions but could be affected by damages. The creation of new shares can be handled by including time t emitters in the set of time $t + 1$ traders.

³⁴If we give the representative household in Section 2.1 exponential utility over consumption with $A > 0$ the coefficient of absolute risk aversion and also let damages be additive rather than multiplicative, then the learning dynamics are unchanged and, as $A \rightarrow 0$ (so as to eliminate risk premia, as with log utility), the regulator's tax in Section 4 and the damage charge in Section 5.1 are altered only by losing the C_0 normalization.

A fully revealing rational expectations equilibrium exists and recovers informational efficiency as the fundamental shocks ϵ become small and the deposit becomes large. Informationally efficient updating sees the stochastic shocks to each ζ_{it} as noise to be filtered out for predicting $\sum_{i=1}^N \kappa_i \zeta_i$, but it is not noise to time t traders attempting to predict the coming aggregate measurement $\tilde{\zeta}_t + \tilde{\lambda}_t$. These two predictions converge as $\sigma^2 \rightarrow 0$ because the shocks in the observed $\zeta_{it} + \lambda_{it}$ become pure noise in either case. By aggregating traders' private information about damages, the carbon share market improves on the regulator's ability to estimate damages, and by defining the marginal cost of emitting, the carbon share market simultaneously implements that information to control emissions and incentivize carbon removal. We have therefore designed a decentralized policy instrument that attains the welfare-maximizing, informationally efficient benchmark even though the regulator has the exact same information as in Section 4.

However, it is well-known that a fully revealing rational expectations equilibrium is not always implementable: it may be that no trading mechanism can actually deliver this equilibrium.³⁵ In particular, if the carbon share price is a sufficient statistic for all information in the economy, then traders should ignore their private information, in which case it is unclear how their private information ends up being summarized by the equilibrium price. I therefore also study an equilibrium in demand functions. Here traders submit demand functions that account for their observed sectoral signals $\zeta_{it} + \lambda_{it}$ and for the information they would infer from an observed carbon share price. These demand functions yield the market-clearing price \check{q}_t^* . Noise traders make the observed price $\check{q}_t = \check{q}_t^* + \theta_t$, where θ_t is a mean-zero, independently and identically distributed, normal random variable, with variance $\Theta^2 > 0$.³⁶ Traders treat \check{q}_t^* as exogenous (i.e., they are price-takers) but do recognize how their observed signals influence that price through the beliefs of other traders in their sector. Following much previous literature, I associate an implementable equilibrium with a Bayesian Nash equilibrium of this game and study linear equilibria.³⁷

The following proposition characterizes emissions in an implementable equilibrium.

Proposition 6 (Implementable Equilibrium). *As $D \rightarrow \infty$, a linear equilibrium in demand functions sets the marginal cost of emissions and marginal benefit of carbon removal equal to*

$$\frac{1}{r} C_0 \alpha \left[\sum_{k=1}^N \kappa_k \bar{\zeta}_k + \check{\mu}_t \right]$$

³⁵The foregoing analysis postulates a price and tests whether it meets market-clearing and rational expectations criteria. The coming analysis postulates a mechanism that converts strategies into a market-clearing price, which meets rational expectations criteria when strategies are in equilibrium. See Kyle (1989) and Vives (2014).

³⁶Noise in price or, equivalently, supply has long been recognized as critical for the existence of partially revealing equilibria when traders are small and payoffs are pure common values (e.g., Hellwig, 1980; Grossman and Stiglitz, 1980; Diamond and Verrecchia, 1981).

³⁷The equilibrium is symmetric in terms of strategies defined over ζ_{it} and \check{q}_t . Of course, the actual demand schedules will not be symmetric as each will depend on the observed ζ_{it} .

and generates

$$\begin{aligned}
\check{\mu}_t = & \check{\pi}_t \frac{1}{t} \sum_{k=0}^{t-1} \left(\check{\zeta}_k + \check{\lambda}_k - \sum_{j=1}^N \kappa_j \check{\zeta}_j \right) \\
& + \sum_{k=0}^{t-1} \check{\pi}_{kt} \underbrace{\left[\frac{(1-\Gamma)\tau_k^2 + \sigma^2}{\tau_k^2 + \sigma^2 + \omega^2} \sum_{i=1}^N \check{\kappa}_{ik} \kappa_i (\zeta_{ik} + \lambda_{ik} - \bar{\zeta}_i) + \frac{N\Gamma\tau_k^2}{\tau_k^2 + \sigma^2 + \omega^2} \frac{1}{N} \sum_{i=1}^N \check{\kappa}_{ik} (\zeta_{ik} + \lambda_{ik} - \bar{\zeta}_i) \right]}_{\text{from } \check{q}_k} \\
& - \sum_{k=0}^{t-1} \check{\pi}_{kt} \frac{r}{C_0 \alpha(\check{\chi}_k + r)} \theta_k,
\end{aligned} \tag{9}$$

where the $\check{\kappa}_{kt}$ and $\check{\chi}_k$ are each $\in (0, 1)$.

Proof. See Appendix L.

Sketch: Use traders' first-order conditions to determine demand for carbon shares in each sector. The equilibrium price \check{q}_t^* sets aggregate net demand to zero given the beliefs traders form from their private signals, the observed carbon share price, and expectations of q_{t+1} . Because normal-normal Bayesian updating implies that $\check{\mu}_{t+1}$ is a linear function of $\check{\mu}_t$, \check{q}_t , and $\check{\zeta}_t + \check{\lambda}_t$, so too is expected q_{t+1} . The proof then constructs a signal \check{q}_t of aggregate damages implied by \check{q}_t . By normal-normal Bayesian updating, each sector i trader's posterior mean for $\check{\zeta}_t + \check{\lambda}_t$ is a linear function of $\check{\mu}_t$, \check{q}_t , and $\zeta_{it} + \lambda_{it}$. We can thus express the price signal \check{q}_t as an unknown linear function of sectoral signals. The projection theorem yields each type of trader's posterior mean for aggregate damages conditional on observed sectoral information and on the observed price signal. Matching coefficients and applying Brouwer's fixed-point theorem yields posterior beliefs that are self-fulfilling via the price and yields the market-clearing price, both as functions of the unknown coefficients that determine $\check{\mu}_{t+1}$. Beliefs $\check{\mu}_{t+1}$ follow from the projection theorem and the solution for earlier carbon share prices. \square

A carbon share policy acts like imposing an emission tax based on beliefs $\check{\mu}_t$. The first line in (9) determines the weight placed by $\check{\mu}_t$ on previous periods' aggregate measurements of damages. The second line describes how agents learn from the past prices \check{q}_k of carbon shares (use (A-48) in (A-32)). Those past prices embed two types of information: a first piece learns from a version of the aggregate signal constructed from sectoral signals, and a second piece takes advantage of the correlation among sectoral effects to learn from the unweighted average of the sectoral signals. The ability to construct a version of the aggregate signal that is affected by sectoral measurement error but not by aggregate measurement error and the ability to use the correlation across sectors to learn more efficiently were critical to informationally efficient beliefs in Proposition 1 but were missing from the regulator's beliefs in Proposition 3.

The following corollary establishes the conditions under which past carbon share prices provide useful information:

Corollary 3 (Informative Carbon Share Prices). *Consider the $\{\check{\pi}_{0t}, \dots, \check{\pi}_{(t-1)t}\}$ defined in Proposition 6. These are each equal to zero if (i) $\tilde{\omega}^2 = 0$ and either (iia) $\sigma^2 = 0$, (iib) $\Gamma = 0$, or (iic) each $\kappa_i = 1/N$. Otherwise $\check{\pi}_{(t-1)t} > 0$.*

Proof. See Appendix M. □

Traders' posterior beliefs do not rely on the sectoral signals embedded in past share prices ($\check{\pi}_{kt} = 0$) under the same conditions that informationally efficient beliefs rely only on the aggregate signals (see Corollary 1), which are also the conditions under which an emission tax-wielding regulator's beliefs are informationally efficient (see Corollary 2). In other cases, traders and firm do learn from carbon share prices. Because these beliefs determine emission reduction incentives and use information unavailable to the regulator, we can conclude that the carbon share policy is generally more informationally efficient than an emission tax policy.

However, the carbon share policy does not generally achieve full informational efficiency. There are several differences with respect to the informationally efficient beliefs described in Proposition 1. First, noise traders mechanically induce randomness in the share price that hinders learning (the θ_k in the third line of (9)). Second, the $\check{\kappa}_{ik}$ downweight information from the carbon share price (in the second line of (9)). The following corollary examines that downward adjustment in more detail:

Corollary 4 (Traders' Distortions). *Consider the $\{\check{\kappa}_{1t}, \dots, \check{\kappa}_{Nt}\}$ defined in Proposition 6. We can write*

$$\check{\kappa}_{it} = \check{\kappa}_{it}^{BS} \check{\kappa}_{it}^{RA},$$

where $\check{\kappa}_{it}^{BS} \in (0, 1)$, $\check{\kappa}_{it}^{RA} \in (0, 1)$, and

$$i \lim_{\Theta^2 \rightarrow \infty} \check{\kappa}_{it}^{BS} = 1.$$

$$ii \lim_{\Theta^2 \rightarrow 0} \check{\kappa}_{it}^{BS} = 0.$$

$$iii \lim_{A_i \rightarrow 0} \check{\kappa}_{it}^{RA} = 1 \text{ and } \lim_{A_i \rightarrow 0} \check{\kappa}_{jt}^{RA} = 0 \text{ for all } j \neq i.$$

$$iv \lim_{A_i \rightarrow \infty} \check{\kappa}_{it}^{RA} = 0.$$

$$v \text{ If } \kappa_i = 1/N \text{ for all } i \in \{1, \dots, N\}, \text{ then } \check{\kappa}_{it}^{RA} = n_i A_i^{-1} / \sum_{j=1}^N n_j A_j^{-1}.$$

vi Without loss of generality, order sectors by κ_i . If the sequence $\{n_1 A_1^{-1}, \dots, n_N A_N^{-1}\}$ is weak monotone increasing, then as $\Theta^2 \rightarrow \infty$, the sequence $\{\check{\kappa}_{1t}^{RA}, \dots, \check{\kappa}_{Nt}^{RA}\}$ is weak monotone increasing, with $\check{\kappa}_{1t}^{RA} \leq n_1 A_1^{-1} / \sum_{j=1}^N n_j A_j^{-1}$ and $\check{\kappa}_{Nt}^{RA} \geq n_N A_N^{-1} / \sum_{j=1}^N n_j A_j^{-1}$. The two inequalities are strict if, in addition, $\kappa_1 < 1/N$.

Proof. See Appendix N. □

The downward adjustments due to the $\check{\kappa}$ can be decomposed into two terms. The first term, labeled $\check{\kappa}_{it}^{BS}$, reflects how traders in each sector shade their bids to reflect the possibility of a winner's curse in this market with a pure common value: from equations (A-38) and (A-47), they underweight their own signals when forming beliefs. This downward adjustment vanishes as noise traders fully determine the price (part i of the corollary), but carbon share prices fail to aggregate any private information as noise traders become irrelevant (part ii). The latter result is a manifestation of the same force that prevents the fully revealing rational expectations equilibrium from being implementable: traders whose information is fully revealed by the equilibrium price do not trade on that information. Thus there is a tension between minimizing the consequences of the third line in (9) and minimizing the consequences of bid shading for the $\check{\kappa}$.

The second term, labeled $\check{\kappa}_{it}^{RA}$, reflects how risk-averse traders' demand for carbon shares decreases in the variance of the returns they will earn: from equations (A-24) and (A-26), the market price is more sensitive to traders who have more precise beliefs and less aversion to risk. As traders in some sector approach risk-neutrality, their demand becomes perfectly elastic and the carbon share price incorporates only their private information (part iii of the corollary), but as traders in some sector become infinitely risk-averse, the carbon share price incorporates information only from the other sectors (part iv). When sectors are symmetric, perceived risk is the same in each sector and it is the traders with less risk aversion who reveal more information to the market (part v). When sectors are not symmetric and bid-shading is small, the effects of risk aversion are more severe for sectors that have smaller value shares in final-good production, because traders in those sectors observe signals that are less informative about returns to carbon shares and thus perceive additional risk that makes them less willing to trade carbon shares (part vi).³⁸

In sum, noise traders, bid shading, and traders' risk aversion prevent the implementable equilibrium's carbon share price from being fully revealing,³⁹ but even that equilibrium does aggregate information in a fashion that is useful to traders and analogous to the aggregation performed within informationally efficient updating. By using markets to perform price discovery for social cost, a carbon share policy goes some distance towards resolving the difficulties that a regulator wielding an emission tax faces in using all of the information

³⁸Vives (2014) also studies a game in demand functions within a competitive environment, but his traders achieve informational efficiency. The reason is that he permits imperfect correlation among traders' values for the asset, whereas I study a pure common value with correlation instead arising among the signals of that value. Vives (2014) finds informational efficiency in the limit as his correlation approaches a pure common value, but his equilibrium does break down in that limit because he lacks noise traders. The present setting becomes loosely analogous to the perfect correlation case of Vives (2014) in a limit economy with each $A_i \rightarrow 0$, $\Theta^2 \rightarrow 0$, $\Gamma = 0$, and each $\kappa_i = 1/N$.

³⁹Moreover, even if it were somehow the case that each $\check{\kappa}_{it} = 1$ with $\Theta^2 = 0$, the carbon share price would still fail to aggregate information in the most efficient manner possible. In particular, stochasticity governed by σ^2 makes traders' predictions target a slightly different object than would be targeted in an efficient equilibrium, as described following Proposition 5.

available in the economy to inform the private cost of emitting.

6 Discussion

I have advanced a new perspective on environmental policymaking. Extending the traditional emphasis on asymmetric information about firms' costs of eliminating emissions, I have emphasized asymmetric information about the social cost of emissions. I have shown that conventional emission taxes neither aggregate dispersed information nor enable full use of potential information about the severity of externalities. Instead, I have shown that a new market-based instrument I call "carbon shares" aggregates dispersed information and enables full use of new information without losing the desirable properties of emission taxes and similar market-based instruments.

The new policy instrument conceives of a different role for the regulator. Traditionally, the regulator must project the marginal harm from emissions in all possible states of the world and in all future time periods in order to determine an emission price (or cap). Here, however, the regulator need only measure damage as it is realized and determine the deposit based on approximate worst-case outcomes. A carbon share policy shifts the burden of projecting possible future damages from the regulator to market traders. These traders form their own damage estimates based on information produced by the regulator, the observed prices of carbon shares, and their own private information. This type of belief updating is a common task in markets.

This proposal generates five immediate questions. First, how would a regulator actually estimate the realized aggregate impacts of climate change? This task is different from the task economists have traditionally undertaken in projecting future damages from climate change. It is closer to the attribution studies now regularly undertaken in which climate scientists test how climate change affected the likelihood of realized weather events (e.g., Philip et al., 2020). Governments already do regularly produce measures of recent economic outcomes that are noisy yet are of critical importance for policymaking and directly determine monetary payments. As but one example, the U.S. consumer price index determines social security benefits and other transfer payments and is a prominent input to monetary policy, but it is imperfectly estimated and there is disagreement even about what it should be estimating (National Research Council, 2002; Schultze, 2003). The present challenge may be no greater. Economists should begin tackling it.

Second, what if market actors value impacts differently from the regulator or discount the future more than the regulator would like? It is important to recognize that emission and removal incentives are determined by the price of a carbon share, which in turn depends on market actors' expectations of refunds and thus of damage charges. Traders care about forecasting whatever damage charge calculation the regulator will use, which could equity-weight impacts or include nonmarket impacts. Appendix C even shows that the regulator

can induce traders to apply its own favored discount rate to future climate change damages by merely rescaling its calculated damage charges. Traders may have better or worse private information about different types of measurements the regulator will make, but they do not impose their own preferences regarding how measurements should be done.

Third, would the regulator have credibility to estimate realized impacts faithfully? If an econometric framework could be developed that became widely accepted, then the estimation could be institutionalized as with the production of other national statistics—and to the extent this estimation relies on standard data, it may be less vulnerable to political influence than the U.S. government’s estimates of the social cost of carbon have been (see Voosen, 2021). A real-world implementation of the policy might also constrain the change in damage charges from period to period, which would reduce the flexibility to respond to new information but also reduce vulnerability to transient political influence.

Fourth, how would this instrument affect incentives to coordinate policy internationally? I have followed a long tradition in analyzing the benchmark of a global regulator. However, climate policy is in practice fragmented among countries. Future work should compare international dimensions of this policy to carbon taxes, cap-and-trade programs, and other policy options. In particular, the ability to institutionalize the damage charge calculations and to explicitly adopt country weights in the damage charge calculations could each affect incentives to coordinate policy: these calculations may have more credibility than a global carbon tax would enjoy, and countries may be incentivized to join a coalition in order to have their damages counted.

Fifth, would traders have an incentive to collect additional information about climate impacts? I have treated information about climate change in the economy as exogenous. To date, the development of better scientific monitoring and modeling systems has primarily been the task of governments and universities. However, such information should have a market value under a carbon share policy, as I conjecture that implementable equilibria do not suffer the paradox of Grossman and Stiglitz (1976, 1980). Such information might also have a market value under an emission tax (as agents want to understand their own exposure to climate change and to forecast future emission taxes), but this information is plausibly much more valuable under a carbon share policy because it determines the immediate payoffs from trading carbon shares. One might thus expect a carbon share policy to spur private investment in information production, so that a carbon share policy not only aggregates the information already dispersed throughout the economy but also improves that information. I encourage future work on the incentives to collect information in various policy environments.

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