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FAMILY BARGAINING WITH ALTRUISM

Robert A. Pollak

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ABSTRACT

The theoretical literature on bargaining ignores altruism and assumes that everyone is an egoist. Because the importance of altruism in the family is widely recognized, the egoism assumption is especially problematic in the economics of the family. This paper shows that incorporating altruism into cooperative bargaining models shrinks the set potential cooperative bargaining solutions. The analysis depends on the implications of altruism for Pareto efficiency and the implications of Pareto efficiency for potential cooperative bargaining solutions. The analysis here applies not only to Nash bargaining but to all cooperative bargaining models. For noncooperative bargaining, the analysis implies that any solution that lies outside the shrunken set of potential cooperative bargaining solutions is not Pareto efficient.

Robert A. Pollak
Washington University in St. Louis
Arts and Sciences
and the Olin Business School
Campus Box 1133
1 Brookings Drive
St. Louis, MO 63130-4899
and IZA
and also NBER
pollak@wustl.edu

1. Introduction

The theoretical literature on bargaining ignores altruism and assumes that everyone is an egoist. Because the importance of altruism in the family is widely recognized, the egoism assumption is especially problematic in the economics of the family. This paper shows that incorporating altruism into cooperative bargaining models shrinks the set potential cooperative bargaining solutions. The analysis depends on the implications of altruism for Pareto efficiency and the implications of Pareto efficiency for potential cooperative bargaining solutions. The analysis in this paper applies not only to Nash bargaining but to all cooperative bargaining models.

Becker (1981, 1991, Chapter 8) correctly insisted on the importance of altruism in families. He explicitly rejected bargaining in families and proposed instead noncooperative game, although he did not use that term. The centerpiece of his discussion of altruism in families is his "voluntary transfer model." The simplest version of Becker's model assumes that one spouse – the husband, in Becker's telling -- is an altruist and the other spouse is an egoist. The model begins with an initial consumption allocation that is determined by the requirements of equilibrium in the marriage market. If the initial allocation gives the husband "too much" and his wife "too little" according to the husband's preferences, then the husband makes a voluntary transfer to the wife. The resulting allocation is the Pareto efficient allocation that maximizes the utility of the altruistic spouse. (For a discussion of the case in which both spouses are altruistic, see section 6.) We can avoid the need to model strategic interactions by assuming that the rules of the game do not allow the egoistic spouse to reject the altruist's offer to induce him to make a better offer.

This paper investigates the consequences of incorporating altruism into cooperative bargaining

models. Although the analysis has applications well beyond family economics, bargaining between spouses in marriage provides a plausible context for bargaining with altruism. The results apply more generally, for example, to bargaining between parents and children and to bargaining among adult children about long-term care for disabled elderly parents. An advantage of beginning with the marriage context, however, is that it simplifies the exposition by focusing on a two-player game and allowing us to use gendered pronouns.

I show that incorporating altruism into a cooperative bargaining model shrinks the set potential solutions. More specifically, altruism rules out some allocations that were potential bargaining solutions with egoism without simultaneously introducing any new solutions (i.e., no allocations that were not solutions with egoism become solutions with altruism.) With cooperative bargaining, all solutions must lie in the shrunken set of potential cooperative bargaining solutions.

Although Nash bargaining is the workhorse of the family bargaining literature, focusing on Nash bargaining conceals rather than clarifies the implications of introducing altruism into bargaining models. The familiar Nash Product Function

$$N(x) = (V^h(x) - \bar{V}^h) \times (V^w(x) - \bar{V}^w)$$

plays no role in the analysis, nor do the spouses' von Neumann-Morgenstern utility functions on which the Nash bargaining solution is based. The analysis of bargaining with altruism relies entirely on the spouses' ordinal preferences.

The approach in this paper is intuitive and novel. It leads to a simple, nontechnical analysis that is supported by simple figures. The analysis rests on the implications of altruism for Pareto efficiency and the implications of Pareto efficiency for potential cooperative bargaining solutions. Instead of considering separately every possible pair of von Neumann-Morgenstern utility functions corresponding to the spouses' ordinal utility functions, this approach permits a wholesale rather than a retail analysis of

potential cooperative bargaining solutions. Neither the numbering of spouses' indifference curves corresponding to their von Neumann-Morgenstern utility functions nor the location of the threat point play any role in the analysis. The price we pay for this "batch processing" is that instead of identifying *the* solution to a particular cooperative bargaining problem, we identify the set of all solutions to a larger, well-specified class of bargaining problems -- those corresponding to all threat points and all von Neumann-Morgenstern utility functions consistent the spouses' ordinal preferences.

To motivate the analysis of bargaining with altruism, I offer two examples from family economics that are best explained by altruism. First, spouses with weak bargaining power often (usually?) do better than egoistic bargaining models would predict. This is true not only in societies long ago and far away in which women had few legal or political rights, but also in the United States and England. In the 19th century child custody laws in the United States and England and strongly favored fathers. Divorce, which was difficult in the US, was virtually impossible in England. In England and Wales, despite divorce law liberalization in 1857, there were fewer than 1000 divorces per year until 1918; Stone (1990, p. 435). Until the Married Women's Property Acts (Mississippi, 1839; New York, 1848; England and Wales, 1870), married women could not own property and did not have the right to their own earnings. The Nineteenth Amendment to the US Constitution giving women the right to vote was ratified in 1920. In the United Kingdom women over the age of 30 who met certain property qualifications were given the right to vote in 1918 but women were not given the right to vote on the same terms as men until 1928. Although inexplicable in bargaining models with egoistic spouses, altruism allows us to understand why spouses with little bargaining power do better than egoistic bargaining models predict.

Second, as Becker (1981, 1991, Ch. 8) and Weiss (1997) argue, spouses provide informal insurance for each other. The prospect of reciprocity allows egoistic spouses to credibly commit to

insuring each other against shocks that are relatively small and likely to recur such as temporary unemployment and temporary disability. The difficulty arises, however, when we consider how egoistic spouses can credibly commit to providing insurance against shocks that are large and nonrecurring such as permanent unemployment and permanent disability.

The organization of the paper is as follows: In section 2, I briefly review the rich but disjoint literatures on bargaining in families. In section 3, I introduce notation in the context of the standard case in which both spouses are egoists. In section 4, I consider the case in which one spouse is an egoist and the other an altruist. This case features prominently in the literature because Becker (1981, 1991) often assumes that only the "head of the household" is an altruist. I show that in this transparent case altruism shrinks the set of potential cooperative bargaining solutions; although this does not prove the claim that altruism shrinks the set of potential cooperative bargaining solutions when both spouses are altruists, it confirms its plausibility. The assumption that only one spouse is an altruist simplifies the analysis in part because it avoids the need to distinguish between "ordinary altruism" and what Becker calls "excessive altruism," a distinction I explain in section 5. Section 6 establishes the basic result: ordinary altruism shrinks the set of potential cooperative bargaining solutions. Section 7 shows that the basic result also holds with excessive altruism. In section 8, I show that the basic result – that is, altruism shrinks the set of potential cooperative bargaining solutions -- continues to hold when the model is generalized to include household public goods. Section 9 discusses Becker's "voluntary transfer model," a noncooperative game but, as Becker repeatedly asserts, not a bargaining game. Becker's voluntary contribution is closely related to the model of cooperative bargaining wo I analyze in this paper. Section 10 concludes.

2. Bargaining in Economics, Especially in the Economics of the Family

Egoism is the default assumption in economics. A major exception is the altruism of parents

toward their children. Altruism also plays a major role in the analysis of charitable giving (see, for example, Andreoni, 1990), in explaining the motives of individuals who support redistributive policies (see, for example, Fehr, Epper, and Senn, 2022), and in the analysis of experimental games such as ultimatum games and dictator games; see Eckel and Grossman, 1996; Levine, 1998; and van Damme, et al., 2014). When economists turn to formal models of cooperative and noncooperative bargaining, however, altruism disappears and we return to the realm of egoistic preferences.

The remainder of this section illustrates the breadth of the economics literature on bargaining in families. A comprehensive survey of the literature on cooperative bargaining models of marriage would be too lengthy, but the family bargaining literature now extends far beyond cooperative bargaining and far beyond marriage.

Bargaining models of allocation in marriage were introduced by Manser and Brown (1980) and McElroy and Horney (1981). The early bargaining models were cooperative -- Nash and Kalai-Smorodinsky bargaining in Manser and Brown, Nash bargaining in McElroy and Horney. These pathbreaking articles analyzed egoistic spouses bargaining about allocation in marriage with divorce as the threat point. A decade later, Thomas (1990) refocused the bargaining literature by showing that in two-parent families child survival probabilities were much greater if mothers controlled a larger fraction of household nonlabor income. Taken together, these results suggest that, contrary to the traditional assumption in economics, spouses do not "pool" their resources as economists traditionally assumed and that allocation in marriage depends on control over resources.

Lundberg and Pollak (1993, 1996) also considered egoistic spouses bargaining over allocation in marriage, but they proposed a model in which the threat point is internal to the marriage and reflects traditional gender norms. Lundberg, Pollak, and Wales (1997) exploited the natural experiment presented by a change in the British Child Allowance in the late 1970s which

transferred resources "from the wallet to the purse." They found that in two-parent families the increase in the wife's share of household resources changed household expenditure patterns (e.g., in favor of women's clothing and children's clothing). Browning, et al. (1994), an early version of Chiappori's "collective model," did not specify an explicit bargaining model but assumed that allocation in marriage was determined by Pareto efficient agreements that spouses made within marriage.[1]

Asymmetric information complicates bargaining and its analysis. Relaxing the assumption that each spouse can monitor the other's behavior, Ashraf (2009, 2014) uses field experiments to investigate the role of asymmetric information in marriage; the earlier study investigates fertility control and the later savings behavior. Rangel (2006) investigates the effect on the time allocation of cohabiting couples of legal changes in Brazil that extended to cohabiting couples alimony rights similar to those previously available only to married couples. Weiss and Willis (1985) investigate bargaining by divorced couples over child support, arguing that a noncooperative model is appropriate because divorced couples face monitoring and enforcement issues that differ fundamentally from those facing married couples.

Turning from spouses to parents and adolescent children, Hao, Hotz, and Jin (2008) propose and analyze a noncooperative model in which parents signal their seriousness to younger children by "over-punishing" older children for misbehavior. Using a laboratory experiment, Peters et al., (2004) investigate the extent to which parents and children cooperate in public goods games and conclude that they cooperate more than strangers but less than Becker's "Rotten Kid Theorem" predicts. Economists' analyses of the relationship between elderly parents and their adult children

[1] Browning et al. cite Manner and Brown and McElroy and Horney, but mention the marriage market only once and cite neither Becker's argument that the marriage market determines allocation within marriage (Becker, 1981, 1991, Chs. 3,4) nor any of the marriage market literature.

focuses on inter vivos transfers, and end-of-life transfers (e.g., bequests) Mukherjee (2022) investigates whether transfers from elderly parents to adult children in response to the over indexation of social security benefits (the so-called "notch") were motivated by altruism or by exchange. She finds that the substantial increases in transfers to adult children were associated with *decreases* [her italics] in care provided by children and concludes that "These results suggest that altruistic preferences -- ones that are potentially pure-- play a significant role in retirement transfers" (p. 1495). Pezzin and Schone (1999) emphasize the importance of altruism as well as bargaining, but altruism plays no role in their formal analysis. Using survey data, but without an explicit bargaining model, Light and McGarry (2004) evaluate the importance of parents' altruistic, exchange, and "evolutionary" motives (i.e., favoring genetic children over stepchildren and adopted children) for the division of their estates among their children and find that all three motives play substantial roles. Bernheim, Shleifer, and Summers (1986) propose a model in which exchange motives determine bequests; they contrast their analysis with that of Barro (1974), a widely cited paper that builds on Becker's voluntary transfer model and argues that if parents and adult children are "altruistically linked," then government bonds are not net worth to a "dynastic family."[2] Duflo (2003) analyzes the effect of the South African old age pension program on the grandchildren of black pension recipients. She finds that grandfathers' pensions had little effect on the grandchildren but that grandmothers' pensions had a substantial positive effect on the weight for height and the height for age of granddaughters but not grandsons. Both Barro and Duflo avoid the word "altruism."

Family "insurance" extends beyond marriage and beyond parent-child interactions and

[2] Using US data, Altonji, Hayashi, and Kotlikoff (1992) show that parents and children are not "altruistically linked" in the sense required for Barro's result; the phrase "altruistically linked" is theirs, not Barro's.

includes the provision of foster care and adoption by grandparents, uncles, and aunts of their orphaned grandchildren, nieces, and nephews; see Bald, et al., (2022) and Brahm (2021). Some states now use subsidies to incentivize the provision of kin foster care and kin adoptions, but the prevalence of kin foster care and adoption before government subsidization is evidence of altruism.

The current version of Chiappori's collective model assumes that allocation in marriage is determined by Pareto efficient agreements that prospective spouses make in the marriage market. Hence, one might expect bargaining in the marriage market to play a prominent role in the collective model. It does not. The collective model allows no scope for bargaining in the marriage market, just as the familiar textbook analysis of the market for wheat allows no scope for bargaining. The current version of the collective model imposes assumptions that ensure that allocation in marriage is determined by the requirements of equilibrium in a perfectly competitive marriage market in which all participants meet simultaneously and have full information about prospective spouses. The scope of bargaining in the marriage market is further limited in the United States because courts will not enforce terms of premarital agreements specifying allocation in marriage; see Levmore (1995) and Hasday (2014). Pollak (2019) summarizes my reservations about assumptions that eliminate the need for search in the marriage market and I will not repeat that discussion here.

3. Setting the Stage: Bargaining between Egoists

I begin with the simplest version of bargaining in marriage: spouses must allocate a fixed quantity of a private good between them. The set of feasible allocations is given by

(1) $x_h + x_w \leq \hat{x}$

where $\hat{x}$ is the resource constraint. (Fig. 1).

Figure 1 Goes Here

An allocation is a potential cooperative bargaining solution if and only if there exists a threat point for which it is a bargaining solution. Hence, if the threat point coincides with an allocation on the frontier of the feasible set, then the cooperative bargaining solution coincides with that allocation.

The "allocation representation" of ordinal preferences is a pair of utility functions in the quantity space

$$\{U^h = U^h(x_h,x_w),\ U^w = U^w(x_h,x_w)\}.$$

If a spouse is an egoist, we can represent his or her preferences by an ordinal utility function of the form

$$(2) \quad U^i(x_h,x_w) = x_i$$

If the wife (husband) is an egoist, we can represent her (his) preferences by an indifference map consisting of parallel horizontal (vertical) lines. Unless a spouse is an egoist, we assume that his or her utility function is strictly quasi-concave and increasing in both x_h and x_w. This assumption rules out envy.

The "mixed representation" of ordinal preferences is a pair of utility functions defined in a mixed quantity-utility space (i.e., the quantity of his or her own consumption and the utility of his or her spouse)

$$\{U^h = W^h[x_h, U^w],\ U^w = W^w[x_w, U^h]\}.$$

Becker (1981, 1991, Ch. 8) uses both representations. Bergstrom (1999) shows that under very general conditions the mixed representation can be solved for the allocation representation, so basing the analysis

on the allocation representation involves no loss of generality. For our purposes, the allocation representation is more convenient because it allows the standard indifference map analysis.

Let A* denote the allocation that the wife prefers to all other allocations in the feasible set; that is, A* is the allocation the wife would choose if she were a dictator. Similarly, let B* denote the husband's preferred allocation in the feasible set. For an egoistic wife (husband) the preferred allocation is at the intersection of the frontier of the feasible set with the y-axis (x-axis).

This section retains the usual assumption that both spouses are egoists -- that is, each cares only about his or her own consumption. We assume that consumption allocation is determined as the solution to a cooperative bargaining game. Although it is often useful to think of Nash bargaining, for our purposes it is better to focus on the general case to avoid being distracted by the special features of Nash bargaining. We do not need to specify a particular cooperative bargaining game because we know that, by assumption, solutions to cooperative games must be Pareto efficient. Together with our assumptions about preferences, Pareto efficiency implies

Proposition 1: With egoistic preferences, allocations are Pareto efficient if and only if they are on the frontier. Hence, if both spouses are egoists, then the set of potential cooperative bargaining solutions is the entire frontier of the feasible set, AB. Allocations that lie on the frontier are Pareto efficient; feasible allocations that do not lie on the frontier are not Pareto efficient.

Proof of Proposition 1: It is easy to see that interior allocations are not Pareto efficient and that with egoistic preferences A* = A and B* = B and the Pareto efficient set coincides with the frontier.

4a. Both Spouses Altruists: Preview of Where We Are Going

Altruism changes everything. If both spouses are altruists, then the set of potential cooperative

bargaining solutions shrinks from the entire frontier of the feasible set (i.e., the line segment AB), to proper subset of AB that I denote by CD (Fig. 2). The size and location of CD depends on the spouses' ordinal preferences (i.e., their indifference maps).

Figure 2 Goes Here

In sections 6 and 7, I provide interpretations of the allocations C and D, which are the end points of the set of potential cooperative bargaining solutions.

From the standpoint of this paper, Pareto efficiency is the key cooperative bargaining axiom. I will show that two-sided altruism implies that

(a) all potential cooperative bargaining solutions lie on CD which is a proper subset of AB, and

(b) any noncooperative bargaining solution that lies outside the subset CD is not Pareto efficient; more generally, any feasible consumption allocation.

4b. Only One Spouse an Altruist

For definiteness, suppose the wife is an altruist and the husband an egoist. Let A* denote the wife's preferred allocation in the feasible set (Fig. 3). For purposes of the figure, I assume that the wife's preferences over allocations are Cobb-Douglas

(3a) $U^w(x_h,x_w) = x_h^{\alpha_w} \; x_w^{1-\alpha_w}$

The Cobb-Douglas is an especially convenient functional form because the parameter α_w represents the share of total resources allocated to the husband at A*, the allocation the wife prefers to all other allocations in the feasible set. This will be useful in section 6 when we construct measures of the size and location of the set of potential cooperative bargaining solutions.

Figure 3 Goes Here

Proposition 2: If the wife is an altruist and the husband is an egoist, then the set of potential cooperative bargaining solutions is A*B. Allocations on the peripheral line segment AA* are not Pareto efficient.

Figure 4 Goes Here

Proof that any allocation on AB is Pareto efficient: From any allocation on A*B, a move toward A* makes the wife better off and the husband worse off; any move toward B makes the husband better off and the wife worse off.

Proof that, apart from A*, no allocation on AA* is Pareto efficient and, hence, not a potential solution to any cooperative bargaining problem: Select an allocation F on AA* other than A*.

(1) The altruistic wife prefers A* to F. (Indeed, the wife prefers A* to every other allocation in the feasible set.)

(2) The egoistic husband prefers A* to F. (Because the husband is egoistic and his consumption is greater at A* than at F.)

Hence, apart from A*, no allocation on AA* is Pareto efficient. Every allocation on A*B is Pareto efficient and, hence, a potential cooperative bargaining solution.

Now suppose we reverse our assumption about which spouse is the altruist and which the egoist. Recall that B* denotes the husband's preferred allocation in the feasible set.

If the husband is a Cobb-Douglas altruist

(3b) $U^h(x_h,x_w) = x_h^{(1-\alpha_h)} x_w^{\alpha_h}$

Then the Cobb-Douglas parameter α_h is the wife's share at B* -- that is, α_h is the share of $\hat{x}$ the husband would allocate to the wife if he were a dictator.

Proposition 3: If the husband is an altruist and the wife is an egoist, then the set of potential cooperative bargaining solutions is AB* (Fig. 5). Allocations on the peripheral line segment B*B are not Pareto efficient.

Proof: The proof is the same as that of the previous proposition; only the names differ.

Figure 5 Goes Here

Taken together, these two propositions lend plausibility to the claim that, if both spouses are altruists, then the set of potential cooperative bargaining solutions shrinks from AB to CD, where C and D are distinct from A and B. The analysis of two-sided altruism is complicated, however, by the need to distinguish between "ordinary altruism" and "excessive altruism."

5. Both Spouses Altruists

Ordinary altruism and excessive altruism are properties of the preferences of both spouses, not of an individual spouse: it makes no sense to say that the husband is an ordinary altruist or that the wife is an excessive altruist. Ordinary altruism is the case in which the husband's preferred share for himself (i.e., his share at B*) is greater than the wife's preferred share for him (i.e., his share at A*). That is, (x_h |B*) > (x_h |A*) where (x_h |B*) and (x_h |A*) denote the allocations at B* and A*. Equivalently, ordinary altruism is the case in which the wife's preferred share for herself (i.e., her share at A*) is greater than the husband's preferred share for her (i.e., her share at B*). That is, (x_w |A*) > (x_w|B*). Ordinary altruism implies that if spouses disagree, as they do about allocations on the frontier between C and D, then each spouse prefers a greater share of the marital pie for himself or herself: C = A* and D = B*.

"Excessive altruism" is the case in which for some allocations, each spouse would prefer to consume a smaller share of the marital pie so that the other spouse can consume a greater share.[3] In terms of the figures, the relative positions of the spouses' preferred allocations, A* and B*, follow from the definitions of ordinary and excessive altruism. Replacing ordinary altruism with excessive altruism interchanges the positions of A* and B*: instead of C = A* and D = B*, excessive altruism implies C = B* and D = A*.

"Super altruism" – the term is mine – is the razor's edge case in which A* and B* coincide: A* = B*. With super altruism the only Pareto efficient allocation is A* = B*.

[3] The term "excessive altruism" is Becker's; see Becker [1991, p. 286, ftn. 6].) The 1905 O. Henry short story "The Gift of the Magi" provides a much-anthologized literary example of excessive altruism.

With ordinary altruism, the frontier can be partitioned into three regions corresponding to the line segments {AA*, A*B*, B*B} (Fig. 6). At every allocation in the interior of A*B* the wife and the husband prefer to move in opposite directions along the frontier; hence, these allocations, as well and A* and B*, are Pareto efficient.

Figure 6 Goes Here

Excessive altruism reverses the positions of A* and B* on the frontier.. Hence, the frontier can be partitioned into three regions corresponding to the line segments (AB*, B*A*, A*B) (Fig. 9b). At every allocation in the interior of B*A* the wife and the husband prefer to move in opposite directions along the frontier, so these allocations, as well as A* and B*, are Pareto efficient.

6. Ordinary Altruism

Proposition 4: With ordinary altruism, the set of potential cooperative bargaining solutions is the central region, A*B*. Allocations in the two peripheral regions, AA* and B*B, are not Pareto efficient. With ordinary altruism, C corresponds to A* (the wife's preferred allocation) and D corresponds to B* (the husband's preferred allocation).

Proof of Proposition 4: To show that allocations on A*B* are Pareto efficient, consider a threat point F on A*B*. From any interior allocation on A*B*, a small reallocation in either direction makes one spouse better off and the other worse off (Fig. 6).

Figure 6 Goes Here

To prove that allocations on AA* are not Pareto efficient, consider a small reallocation from the altruistic wife to her altruistic husband along the linear frontier of the feasible set, AA*. We consider only reallocations that are sufficiently small that they remain in the region AA*. If a small reallocation is in the direction of the wife's preferred allocation, then she prefers the reallocation to the original allocation. But a small reallocation toward A* also makes the husband better off because it is in the direction of his preferred allocation, B*. Because the reallocation makes both spouses better off, the original allocation was not Pareto efficient. An analogous argument shows that allocations on the peripheral line segments B*B are not Pareto efficient; a small reallocation in the direction of B* makes both altruistic spouses better off. This implies that with ordinary altruism the set of Pareto efficient allocations shrinks from AB to A*B*. The argument also shows that, with ordinary altruism, the end points of the bargaining set (C,D) correspond to the wife's preferred allocation and the husband's preferred allocation: C=A* and D=B*.

Compared with egoism, altruism always shrinks the set of potential cooperative bargaining solutions, but the resulting bargaining set may be relatively large (if A* is close to A and B* is close to B), or relatively small (if A* and B* are close to each other).

The size of the set of possible bargaining solutions is inversely related to the likely role of

altruism or, to borrow Becker's terminology, to whether altruism is likely to be "effective." If the bargaining set is large (and the "altruism set" correspondingly small), then it is more likely that allocation in marriage will be determined by bargaining that by altruism. (See section 9 for a discussion of this in the context of Becker's voluntary transfer model.) If the set of potential bargaining solutions is small, then all bargaining solutions imply similar allocations; super altruism is the limiting case. If bargaining solutions were usually close together, a better title for this paper would be "altruism with bargaining" rather than "bargaining with altruism."

We can describe the size and the location of the set of potential bargaining solutions using two angles, θ and σ, where θ is the angle between the rays corresponding to A* and B* and σ the angle of the ray connecting the origin to the midpoint of the bargaining set, the line segment A*B* (Figs. 7 and 8). In the Cobb-Douglas case, the slope of the wife's preferred ray, OA*, is equal to the ratio

$$x_w / x_h = (1 - a_w) / a_w$$

Hence,

$$\tan \theta_w = (1 - a_w) / a_w$$

and, for those who remember high school trigonometry,

(4a) $$\theta_w = \arctan (x_w / x_h) = \arctan (1 - a_w) / a_w$$

The slope of the husband's preferred ray, OB*, is equal to the ratio

$$x_w / x_h = a_h / (1 - a_h)$$ so

(4b) $$\theta_h = \arctan x_w / x_h = \arctan a_h / (1 - a_h)$$

The difference between the slopes of the preferred rays OA* and OA* determines the length of the

line segment A*B*. Thus, the angle θ, a measure of the size of the Pareto efficient set, is given by

$$(5) \quad \theta = \theta_w - \theta_h$$

and we can calculate θ_w and θ_h using the arctan formulae (4a) and (4b).

Figure 7 Goes Here

Figure 8 Goes Here

Proposition 5: With super altruism, the wife's preferred allocation coincides with the husband's preferred the allocation (A* = B*). Hence, the set of Pareto efficient allocations is a singleton and, hence, the set of potential cooperative bargaining solutions is a singleton.

Proof: It is easy to verify that σ is the mean of θ_w and θ_h:

$$(6) \quad \sigma = \frac{1}{2}[\theta_w + \theta_h]$$

Super altruism is the boundary case separating ordinary altruism and excessive altruism, so that

$$\theta_w = \theta_h$$

and the angle $\theta = 0$.

7. Excessive Altruism

Despite its limited practical importance, excessive altruism requires discussion because establishing the results for ordinary altruism (Proposition 4) depended on assuming it away. With excessive altruism, at Pareto efficient allocations (i.e., at allocations in the bargaining region) the spouses disagree because both spouses want to consume a *smaller* share of the marital pie so that the other spouse

may consume a *larger* share. The defining characteristic of excessive altruism is that the wife's (husband's) preferred share for herself (himself) is *less* than the share the husband (wife) would prefer her (him) to have. As we have shown, excessive altruism reverses the relative positions of A* and B* (Figs. 9a and 9b).

Proposition 6: With excessive altruism, the set of potential cooperative bargaining solutions is the central region, B*A*. The two peripheral regions, AB* and A*B, are not Pareto efficient. That is, with excessive altruism, C = B* and D = A*.

The proof is essentially the same as the proof of Proposition 5. As in the proof of Proposition 5, the restriction to small reallocations is needed to keep the reallocation in the same region as the initial allocation.

8. Household Public Goods

Household public goods play a major role in family economics. Sometimes the household public goods are investments in children's human capital, sometimes they goods consumed by children, sometimes they are home produced (e.g., home-cooked meals or clean houses), and sometimes they are purchased on the market (e.g., private school tuition). Regardless of the interpretation, household public goods complicate the analysis of cooperative bargaining, but it remains true that altruism shrinks the set of potential cooperative bargaining solutions. The argument is straightforward and similar to the argument used when all goods were private.

Suppose there is a single household public good, (x_p), that must be purchased on the market; the set of feasible allocations (x_h, x_w, x_p) lie in the three dimensions feasible set defined by

(7) $x_h + x_w + x_p \leq \hat{x}$

We restrict our attention to the frontier of the feasible set

(8) $x_h + x_w + x_p = \hat{x}$

and disregard interior allocations because they are not Pareto efficient. With altruistic preferences, whether a frontier allocation is Pareto efficient depends, *inter alia*, on the spouses' preferences for the public good relative to private goods. Nevertheless, with household public goods altruism shrinks the Pareto efficient set and, hence, the set of potential cooperative bargaining solutions. Our proof strategy uses what we know about private goods to investigate Pareto efficiency *conditional* on the quantity of the household public good. If an allocation (x_h, x_w, x_p) is Pareto efficient, then the allocation of private goods (x_h, x_w), must be Pareto efficient conditional on x_p. This strategy allows us to focus on conditional utility functions $U^h(x_h, x_w|x_p)$ and $U^w(x_h, x_w|x_p)$ subject to the conditional resource constraint

(9) $x_h + x_w \leq \hat{x} - x_p$

The quantity of the public good affects both the household's conditional resource constraint for the private goods, (9), and the spouses' preferences over private goods, $U^i(x_h, x_w|x_p)$. Given the conditional resource constraint for private goods, we denote the wife's preferred allocation over (x_h, x_w) conditional on the quantity of the household public good by A*(x_p) and the husband's preferred allocation by B*(x_p). Thus, A*(x_p) and B*(x_p) are analogous to A* and B* in the case when all goods are private. The conditional argument now proceeds as it did when all goods were private. We distinguish in the obvious way between "conditional ordinary altruism" and "conditional excessive altruism."

Proposition 7: With conditional ordinary altruism and a household public good, the set of potential cooperative bargaining solutions in the space of private goods conditional on the quantity of the public good is the central region A*(x_p)B*(x_p). Allocations in the two peripheral regions A(x_p)A*(x_p) and B*(x_p)B(x_p) are not Pareto efficient.

Proof: We assume ordinary altruism conditional on x_p for all x_p. Under this assumptions, the

argument made in the private goods case shows that, conditional on x_p, allocations on the line segment A(x_p)A*(x_p) are not Pareto efficient. By an analogous argument, conditional on x_p, allocations on the line segment B*(x_p) B(x_p) are not Pareto efficient. This proves that with household public goods, altruism shrinks the set of potential cooperative bargaining solutions. (This argument does not assume that with household public goods household bargaining is a two-stage game.)

9. Becker's Voluntary Transfer Model

I now turn to the relationship between cooperative bargaining with altruism and Becker's voluntary transfer model (Becker, 1981, 1991, Chapter 8). Both models address the same consumption allocation problem, both rely on ordinal rather than von Neumann-Morgenstern utilities and both ensure efficient solutions, but their analytical approaches are fundamentally different. Ignoring the substantial literature on noncooperative bargaining in families and elsewhere, I model the consumption allocation problem as a cooperative bargaining game. Becker models the problem as a noncooperative game and insists that it involves no bargaining ("no bargaining, commitment, or threats are allowed" p. 11).

In the cooperative bargaining model with altruism and in the voluntary transfer model, altruism need not be "effective" (the term is Becker's). If we reinterpret Figure 6 in terms of the voluntary transfer model and if the initial allocation lies between A* and B*, then no voluntary transfers take place because neither spouse would benefit from making such a transfer. In both models the region in which altruism is NOT effective coincides with the region of potential cooperative bargaining solutions discussed in section 6.[4] In the voluntary transfer model if altruism is effective, then the solution is the

[4]In Becker's voluntary transfer model the initial consumption allocation is determined by the requirements of equilibrium in the marriage market, but the origin of the initial conditions has no effect on the voluntary transfer model itself.

feasible consumption allocation that maximizes the altruist's utility. In the cooperative bargaining model, efficiency is guaranteed, but "distribution" depends on the location of the threat point and even if altruism is effective, the solution need not maximize the altruist's utility.

In noncooperative bargaining models the rules of the game are crucial, and Becker imposes strong restrictions on the moves available to the players ("no bargaining, commitment, or threats are allowed"). For example, assuming the husband is an altruist and the wife an egoist, the rules of the voluntary transfer game do not allow the wife to reject a proffered transfer from her husband that would make her better off in the belief that rejection would lead to a better offer. One can imagine a noncooperative game in which the altruistic husband would like to commit himself not to make better offer if his opening offer were rejected, but Becker rules out such commitments. If some commitments are allowed, then the rules of the noncooperative game must specify which commitments are possible and which are not. For example, can the husband make "take-it-or-leave it" offers? Can the wife preemptively demand a large transfer and commit herself not to accept less? If so, do the rules specify which spouse moves first? Or is the order of moves determined in the course of the game (e.g., by flipping a coin at the outset)? Cooperative bargaining models by focusing on Pareto efficiency avoid the need to specify the rules of the game.

In cooperative bargaining with altruism and in Becker's voluntary transfer model, in some cases altruism is effective and in others it is not. In both models, super altruism is the boundary between ordinary altruism and excessive altruism, and is also the razor's edge case in which altruism is always effective, resulting in a unique Pareto efficient allocation, $A^* = B^*$.[5]

[5] The results in Barro (1974) depend on super altruism.

10. Conclusion

What have we learned about bargaining with altruism? If all goods are private and both spouses are egoists, then all allocations on the frontier of the feasible set are potential cooperative bargaining solutions. If one or both spouses are altruistic, then some of these frontier allocations are no longer Pareto efficient. Thus, altruism shrinks the set of potential cooperative bargaining solutions and this result holds not only when all goods are private but also in the leading case in which there are also household public goods.

Although noncooperative bargaining does not require Pareto efficient solutions, Pareto efficiency has long been a central concern of economics. Our analysis implies that noncooperative bargaining solutions that lies outside the set of potential cooperative bargaining solutions are not Pareto efficient.

The analysis is ordinal. Pareto efficiency depends on the spouses' preference orderings (i.e., on their indifference maps). Working with ordinal preferences permits a wholesale rather than a retail analysis of cooperative bargaining, allowing us to deal at once with all possible threat points and all possible von Neumann-Morgenstern utility functions consistent with the spouses' ordinal preferences. In the case of Nash bargaining, this implies that the familiar Nash product function, which depends on the spouses' von Neumann-Morgenstern utility functions, plays no role in the analysis. The analysis is straightforward and the conclusions apply not only to Nash bargaining, but to all cooperative bargaining games. The analysis depends on the implications of altruism for Pareto efficiency and the implications of Pareto efficiency for potential cooperative bargaining solutions. Altruism reduces the set of Pareto efficient allocations and, because cooperative bargaining requires Pareto efficiency, altruism shrinks the set of allocations that are potential cooperative bargaining solutions.

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FIGURES

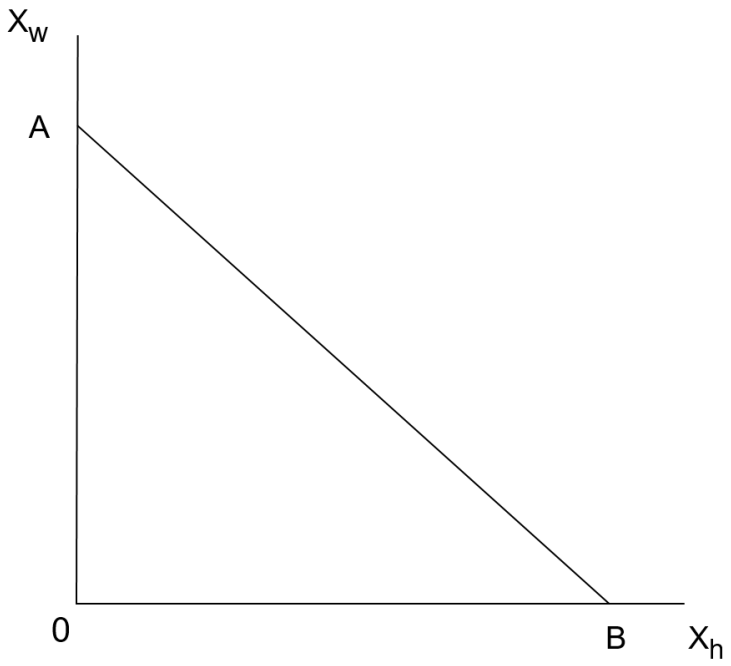

Figure 1: The Feasible Set

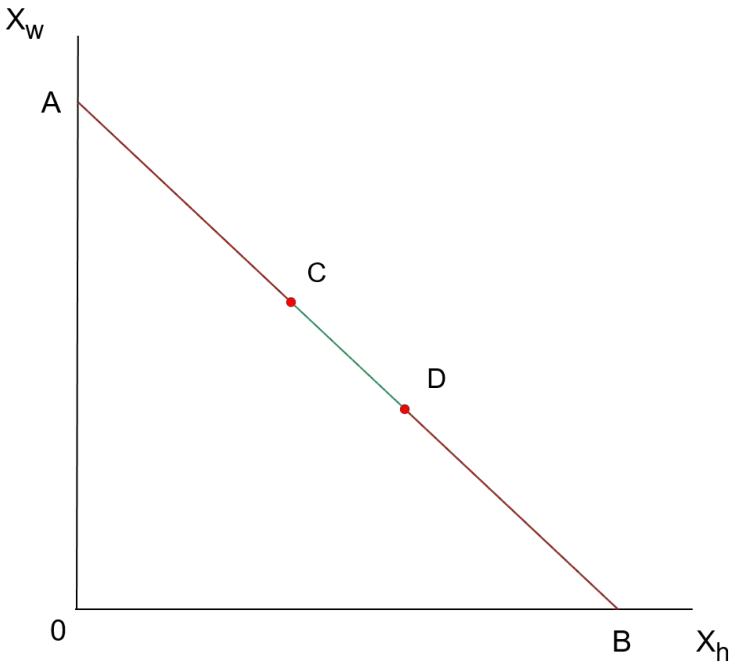

Figure 2: Potential Cooperative Bargaining Solutions

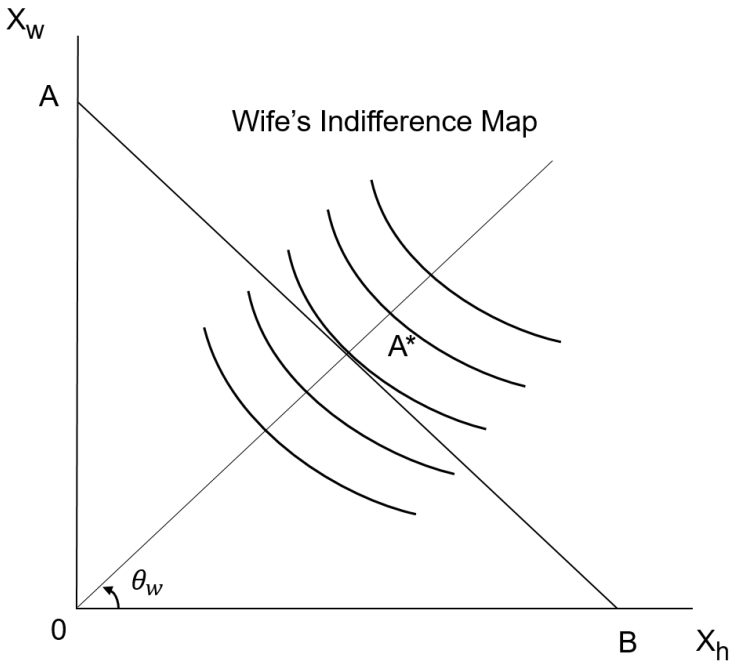

Figure 3: Wife's Cobb-Douglas Indifference Map

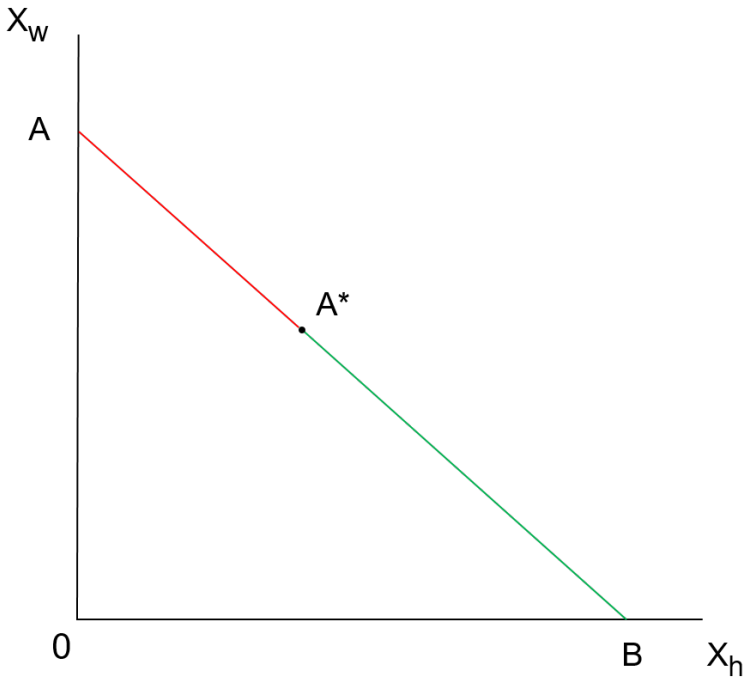

Figure 4: Wife Altruistic, Husband Egoistic: Pareto Efficient Allocations A*B:

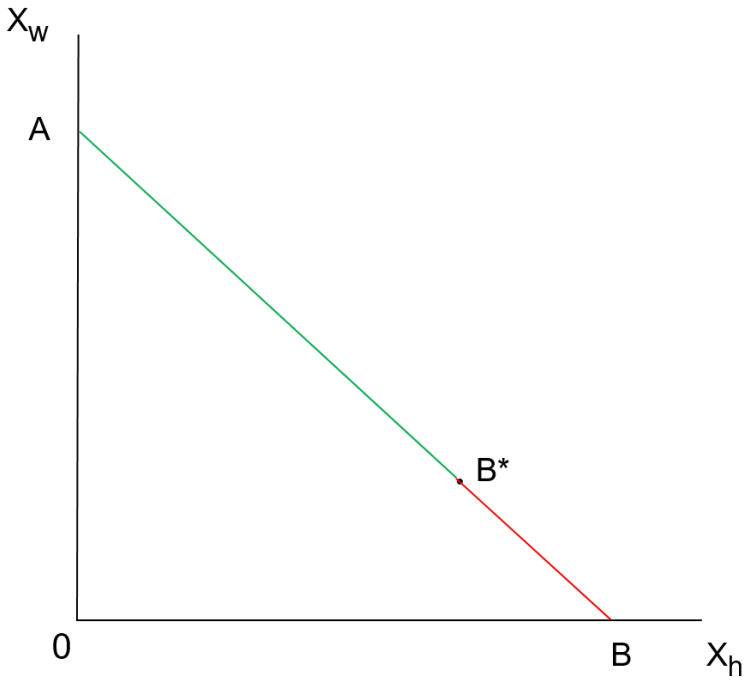

Figure 5: Husband Altruistic, Wife Egoistic: Pareto Efficient Allocations A*B

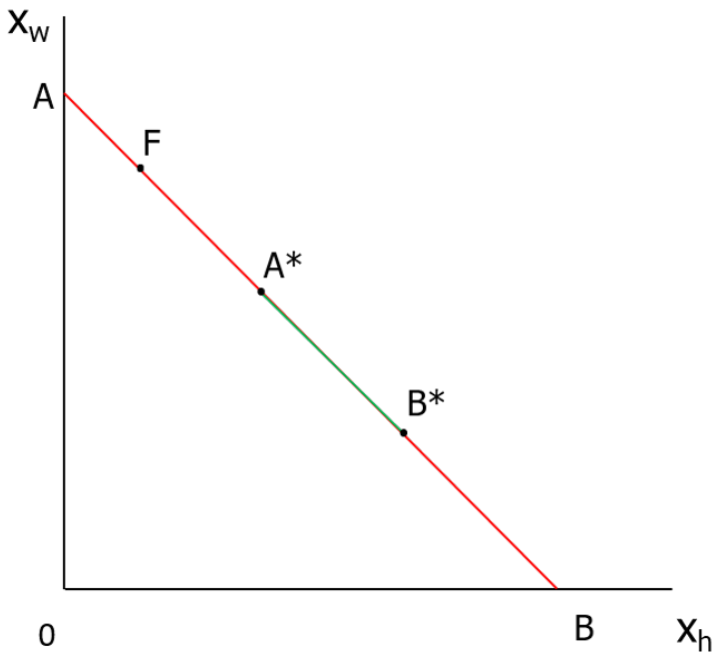

Figure 6: Both Spouses Altruistic, Ordinary Altruism
Pareto Efficient Allocations A*B*
$X_{hB*}>X_{hA*}$ or, equivalently, $X_{wA*}>X_{wB*}$

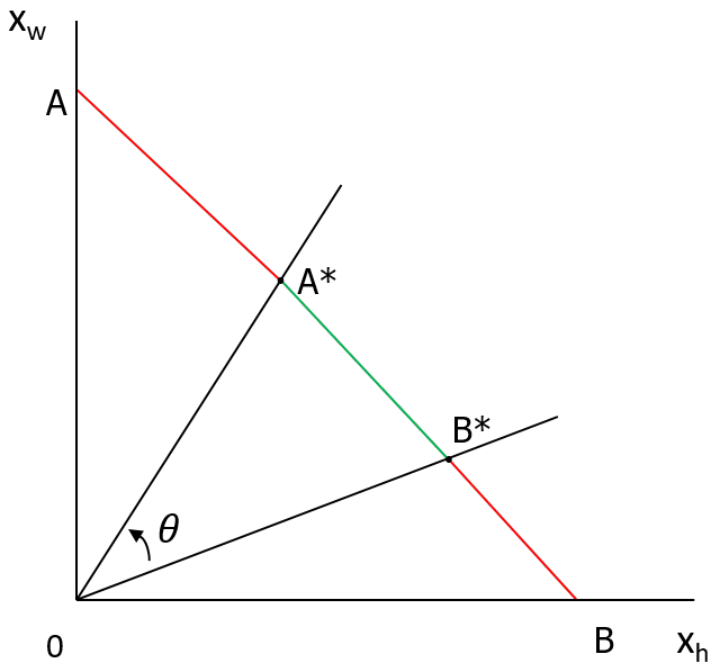

Figure 7: Both Spouses Altruistic; Ordinary Altruism
Measuring the Size of the Set of
Potential Cooperative Bargaining Solutions

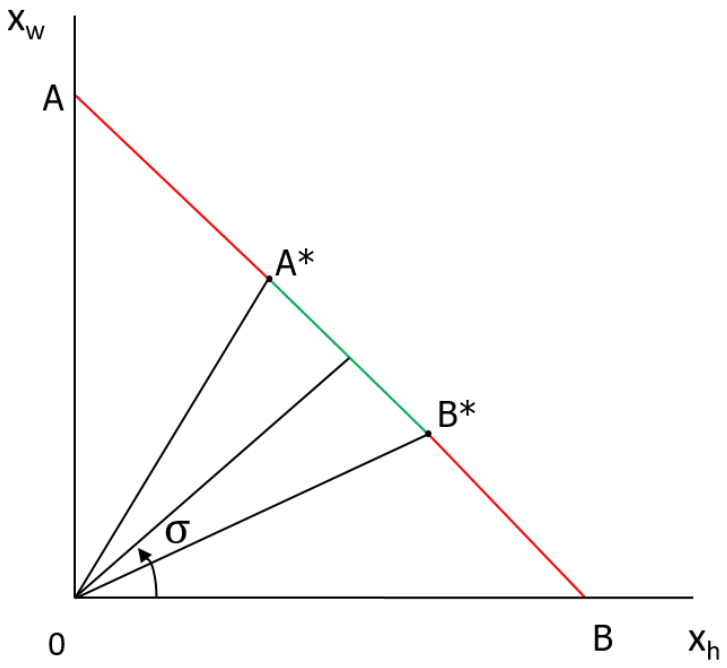

Figure 8: Both Spouses Altruistic; Ordinary Altruism:
Measuring the Location of the Set of
Potential Cooperative Bargaining Solutions