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BARGAINING WITH ALTRUISM

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ABSTRACT

Despite extensive use of bargaining models in economics and despite Becker's insistence on the importance of altruism in families, the theoretical literature on bargaining ignores altruism and assumes that everyone is an egoist. This paper shows that incorporating altruism into cooperative bargaining models shrinks the set potential cooperative bargaining solutions. The analysis depends on the implications of altruism for Pareto efficiency and the implications of Pareto efficiency for potential cooperative bargaining solutions. For noncooperative bargaining, the analysis implies that any noncooperative solution that lies outside the shrunken set of potential cooperative bargaining solutions is not Pareto efficient.

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1. Introduction

Despite extensive use of bargaining models in economics and despite Becker's insistence on the importance of altruism in families, the theoretical literature on bargaining ignores altruism and assumes that everyone is an egoist. This paper shows that incorporating altruism into cooperative bargaining models shrinks the set potential cooperative bargaining solutions. The analysis depends on the implications of altruism for Pareto efficiency and the implications of Pareto efficiency for potential cooperative bargaining solutions. Although Nash bargaining is the workhorse of family bargaining literature, the analysis here applies to all cooperative bargaining models. For noncooperative bargaining, the analysis implies that any noncooperative solution that lies outside the shrunken set of potential cooperative bargaining solutions is not Pareto efficient.

An allocation is a potential cooperative bargaining solution if and only if there exists a threat point for which it is a bargaining solution. Standard cooperative bargaining axioms require solutions to cooperative games to be Pareto efficient. It is easy to show that an allocation is a potential bargaining solution if and only if it is Pareto efficient. For the "if" argument, consider an initial allocation that is Pareto efficient and assume that it coincides with the threat point. For this threat point, the initial allocation must be the corresponding cooperative bargaining equilibrium.

This paper investigates the consequences of incorporating altruism into bargaining models. Although the analysis has applications well beyond family economics, bargaining between spouses in marriage provides a plausible context for two-person bargaining with altruism. The marriage context further simplifies the exposition by allowing the use of gendered pronouns.

I show that incorporating altruism into a cooperative bargaining model shrinks the set potential solutions. More specifically, altruism eliminates some allocations as bargaining solutions that were potential bargaining solutions with egoism without introducing any new solutions (i.e., no allocations

that were not solutions with egoism become solutions with altruism.) With cooperative bargaining, all solutions must lie in the shrunken set of potential cooperative bargaining solutions. With noncooperative bargaining, some or all solutions may lie outside the set of potential cooperative bargaining solutions, but any solution that lies outside this set is not Pareto efficient.

The approach I take in this paper is intuitive and novel. It leads to a simple, nontechnical analysis using arguments that rely heavily on geometry and are supported by simple figures. Although cooperative bargaining models are often described as if they rely entirely on von Neumann-Morgenstern utility functions, this is not the case: the analysis in this paper relies entirely on the ordinal preferences (i.e., the indifference maps) of the bargainers.

The analysis rests on the implications of altruism for Pareto efficiency. Instead of considering separately every possible pair of von Neumann-Morgenstern utility functions corresponding to the spouses' ordinal utility functions, my approach permits a wholesale rather than a retail analysis of potential cooperative bargaining solutions: neither the location of the threat point nor the numbering of bargainers' indifference curves corresponding to their von Neumann-Morgenstern utility functions play any role in the analysis. The price we pay for this "batch processing" is that instead of identifying *the* solution to a particular cooperative bargaining problem, we identify a set that contains all solutions to a well-specified class of bargaining problems -- those corresponding to the bargainers' ordinal preferences but applicable to all von Neumann-Morgenstern utility functions consistent with their preferences.

With altruism, ordinal preferences can be represented in either of two forms. Using the obvious notation, we distinguish between the "allocation representation" in the quantity space

$$\{U^h = U^h(x_h,x_w),\ U^w = U^w(x_h,x_w)\}$$

and the "utility representation"

$$\{U^h = W^h[x_h, U^w],\ U^w = W^w[x_w, U^h]\}.$$

Becker (1981, 1991, Ch. 8) uses both representations. Bergstrom (1999) shows that under very general conditions the utility representation can be solved for the allocation representation, so the choice between them is a matter of convenience. For our purposes, the allocation representation is more convenient because it allows the standard indifference map analysis.

To motivate the analysis of bargaining with altruism, I offer two examples from family economics that are most easily explained by altruism. First, spouses with weak bargaining power often (usually?) do better than egoistic bargaining models would predict. This is true not only in societies long ago and far away, in which women had few legal or political rights, but also in North America and Western Europe. In the 19th century, divorce was difficult in the US and virtually impossible in England and Wales; despite divorce law liberalization in 1857, there were fewer than 1000 divorces per year in England and Wales until 1918; Stone (1990, p. 435). Child custody laws favored fathers. Until the Married Women's Property Acts (Mississippi, 1839; NY, 1848; England and Wales, 1870), married women could not own property and did not have rights to their own earnings. The Nineteenth Amendment to the US Constitution giving women the right to vote was ratified in 1920. In the United Kingdom women over the age of 30 who met certain property qualifications were given the right to vote in 1918; in 1928 women were given the right to vote on the same terms as men. Although anomalous in bargaining models with egoistic spouses, altruism allows us to understand why spouses with little bargaining power do less badly than standard egoistic bargaining models predict.

Second, as Becker (1981, 1991, Ch. 8) and Weiss (1997) argue, spouses provide informal insurance for each other. The prospect of reciprocity allows egoistic spouses to credibly commit to

providing insurance to each other against shocks that are relatively small and likely to recur such as temporary unemployment and temporary disability. The difficulty arises, however, when we consider how egoistic spouses can credibly commit to providing insurance against shocks that are large and nonrecurring such as permanent unemployment and permanent disability.

The organization of the paper is as follows: In section 2, I briefly review the rich but disjoint literatures on bargaining in families. In section 3, I set the stage by considering the standard case in which both spouses are egoists. In section 4, I consider the case in which one spouse is an egoist and the other an altruist. This case features prominently in the literature; Becker (1981, 1991) often assumes that only the "head of the household" is an altruist. I show that in this transparent case altruism shrinks the set of potential cooperative bargaining solutions; although this does not prove my more general claim that altruism shrinks the set of potential cooperative bargaining solutions, it should convince even initially skeptical readers of its plausibility. The assumption that only one spouse is an altruist simplifies the analysis because it avoids the need to distinguish between "ordinary altruism" and "excessive altruism." In section 5, I turn to the cases in which both spouses are altruists, explaining the distinction between ordinary altruism and excessive altruism. Section 6 establishes the basic result -- ordinary altruism shrinks the set of potential cooperative bargaining solutions. Section 7 shows that the basic result also holds with excessive altruism. In section 8, I show that the basic result -- altruism shrinks the set of potential cooperative bargaining solutions -- continues to hold when the model is generalized to include household public goods. Section 9 concludes.

2. Bargaining in Economics, Especially in the Economics of the Family

Apart from the altruism of parents toward their children, egoism is the default assumption in economics. Altruism plays a major role in the analysis of charitable giving (see, for example, Andreoni, 1990), in explaining the motives of individuals who support redistributive policies (see, for example, Fehr, Epper, and Senn, 2022), and in the analysis of ultimatum games, dictator games, and other

experimental games; see Eckel and Grossman, 1996; Levine, 1998; and van Damme, et al., 2014). When economists turn to formal models of cooperative and noncooperative bargaining, however, altruism disappears and we return to the realm of egoistic preferences.

The remainder of this section illustrates the breadth of the economics literature involving bargaining in families. A comprehensive survey of even the subset of the literature on cooperative bargaining models of marriage would be lengthy, but the family bargaining literature now extends far beyond marriage and far beyond cooperative bargaining.

Bargaining models of allocation in marriage were introduced by Manser and Brown (1980) and McElroy and Horney (1981). The early bargaining models were cooperative -- in Manser and Brown, Nash and Kalai-Smorodinsky bargaining; in McElroy and Horney, Nash bargaining. These pathbreaking articles analyzed egoistic spouses bargaining about allocation in marriage with divorce as the threat point. A decade later, Thomas (1990) refocused the bargaining literature by showing that child survival probabilities were much greater in two-parent families in which mothers controlled a larger fraction of household nonlabor income. Thomas's results show that spouses do not "pool" their resources as economists traditionally assumed and that allocation in marriage depends on control over resources.

Lundberg and Pollak (1993) also considered egoistic spouses bargaining over allocation in marriage, but they proposed a model in which the threat point is internal to the marriage and reflects gender norms. Lundberg, Pollak, and Wales (1997), exploiting the natural experiment presented by a change in the British Child Allowance in the late 1970s which transferred resources "from the wallet to the purse." They found that in two-parent families the increase in the wife's share of household resources changed household expenditure patterns (e.g., in favor of women’s clothing and children’s clothing).

Browning, et al., 1994, an early version of Chiappori's "collective model," did not specify an underlying bargaining model but assumed that allocation in marriage was determined by Pareto efficient agreements that spouses made within marriage. In contrast, subsequent versions of the collective model assume that allocation in marriage is determined not by bargaining within marriage but by Pareto efficient agreements that prospective spouses made in the marriage market.[1] In Pollak (2019) I summarize my reservations about the assumption that allocation in marriage is determined in the marriage market. Although I shall not rehearse my reservations here, a key objection is that courts in the United States will not enforce terms of premarital agreements regarding allocation in marriage.

Asymmetric information complicates the analysis of bargaining. Weiss and Willis (1985) investigate bargaining by divorced couples over child support, arguing that a noncooperative model is appropriate because divorced couples face monitoring and enforcement issues fundamentally different from those facing married couples. Relaxing the assumption that each spouse can monitor the other's behavior, Ashraf (2009, 2014) uses field experiments to investigate the role of asymmetric information in marriage; the earlier study investigates fertility control and the later savings behavior. Rangel (2006) investigates the effect on the time allocation of cohabiting couples of legal changes in Brazil that extended to cohabiting couples alimony rights similar to those previously available only to married couples.

Turning from couples to bargaining between parents and adolescent children, Hao, Hotz, and Jin (2008) propose and analyze a noncooperative model in which parents signal their seriousness to younger children by "over-punishing" older children for misbehavior. Using a laboratory experiment, Peters et al., (2004) investigate the extent to which parents and children cooperate in

[1] Browning et al., cite Manner and Brown and McElroy and Horney, but mention the marriage market only once and cite neither Becker's argument that the marriage market determines allocation within marriage (Becker, 1981, 1991, Chs. 3,4) nor any of the marriage market literature.

public goods games and conclude that they cooperate more than strangers but less than Becker's "Rotten Kid Theorem" predicts.

The analysis of bargaining between elderly parents and their adult children focuses on long-term care, inter vivos transfers, and bequests. Pezzin and Schone (1999) emphasize the role of altruism as well as bargaining, but altruism plays no role in their formal analysis. Using survey data, but without an explicit bargaining model, Light and McGarry (2004) evaluate the importance of parents' altruistic, exchange, and "evolutionary" motives (i.e., favoring genetic children over stepchildren and adopted children) for the division of their estates among their children and find that all three motives play substantial roles. Bernheim, Shleifer, and Summers (1986) propose a model in which exchange motives determine bequests; they contrast their analysis with that of Barro (1974), a widely cited paper that builds on Becker's voluntary transfer model and argues that if parents and adult children are "altruistically linked," then government bonds are not net worth to a "dynastic family."[2] Using U.S. data, Altonji, Hayashi, and Kotlikoff (1992) show that parents and children are not "altruistically linked" in the sense require for Barro's result; the phrase "altruistically linked" is theirs, not Barro's. Duflo (2003) analyzes the effect of the South African old age pension program on the grandchildren of black pension recipients. She finds little effect of grandfathers' pensions but finds that grandmothers' pensions had a substantial positive effect on the weight for height and the height for weight of granddaughters but not grandsons. Both Barro and Duflo avoid the word "altruism."

Family insurance extends beyond marriage and beyond parent-child interactions and includes the provision of foster care and adoption by grandparents, uncles, and aunts of their orphaned grandchildren, nieces, and nephews; see Bald, et al., (2022) and Brahm (2021).

[2] Using U.S. data, Altonji, Hayashi, and Kotlikoff (1992) show that parents and children are not "altruistically linked" in the sense require for Barro's result; the phrase "altruistically linked" is theirs, not Barro's.

Governments now incentivize with subsidies the provision of kin foster care and kin adoptions, but eh prevalence of such kin foster care and adoption before government subsidization is evidence of altruism.

Because the current version of Chiappori's collective model assumes that allocation in marriage is determined by binding agreements that prospective spouses make in the marriage market, one might expect bargaining in the marriage market to play a prominent role in the collective model literature. It does not. Other assumptions of the collective model ensure that allocation in marriage is determined by the requirements of equilibrium in a perfectly competitive marriage market in which everyone meets everyone else simultaneously and has full information about prospective spouses. There is no scope for bargaining in the collective model marriage market, just as there is no scope for bargaining in the market for hard red winter wheat.

3. Setting the Stage: Bargaining between Egoists

I begin with the simplest version of bargaining in marriage: spouses must allocate a private good between them, where the set of feasible allocations is given by

$$(1) \quad x_h + x_w \leq \hat{x}$$

where $\hat{x}$ is the resource constraint. Figures are useful and we begin with the feasible set (Fig. 1)

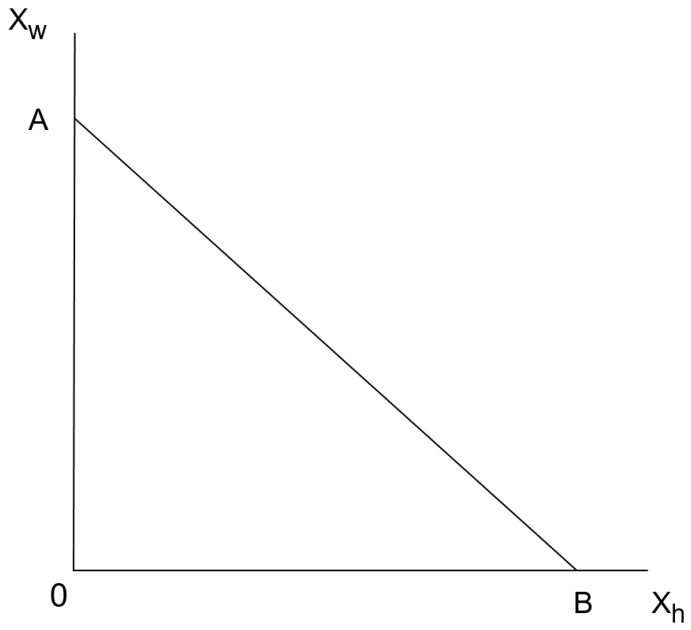

Figure 1: The Feasible Set

We begin with the usual assumption that both spouses are egoists -- that is, each cares only about his or her own consumption. We assume that allocation is determined as the solution to a cooperative bargaining game. For definiteness, it is sometimes useful to think of cooperative Nash bargaining but for our purposes it is more useful to adopt a broader perspective to avoid being distracted by the special features of Nash bargaining and instead to think of cooperative bargaining models.

Without specifying the particular cooperative bargaining model that the spouses use to determine an allocation, we know that by definition the cooperative bargaining solutions must be Pareto efficient. Pareto efficiency, together with egoism and nonsatiation, imply:

Proposition 1: If both spouses are egoists, then the set of potential cooperative bargaining solutions is AB, the entire frontier of the feasible set. Allocations that do not lie on AB are not Pareto efficient.

More specifically, if the threat point coincides with an allocation on the frontier of the feasible set, then the cooperative bargaining solution coincides with that allocation.

The assumption that both spouses are egoists implies that we can represent their preferences by ordinal utility functions of the form

(2a) $U^h(x_h,x_w) = x_h$

(2b) $U^w(x_h,x_w) = x_w$

Let A* denote the allocation that the wife prefers to all other allocations in the feasible set; that is, A* is the allocation the wife would choose if she were a dictator. Similarly, let B* denote the husband's preferred allocation in the feasible set. For egoists, the preferred allocations are at the intersection of the frontier of the feasible set with the y-axis (x-axis). The spouses' indifference maps are shown in Figures 2 and 3.

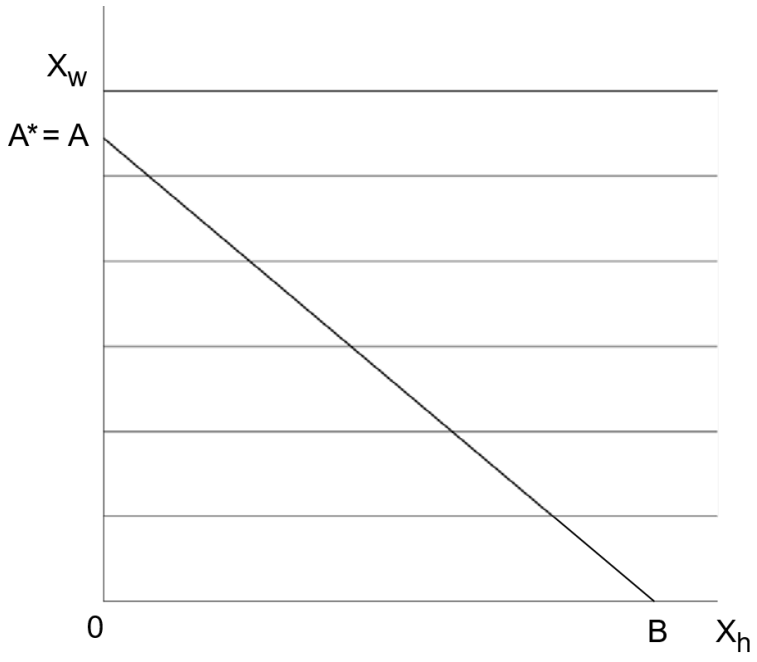

Figure 2: Indifference Map of Egoistic Wife

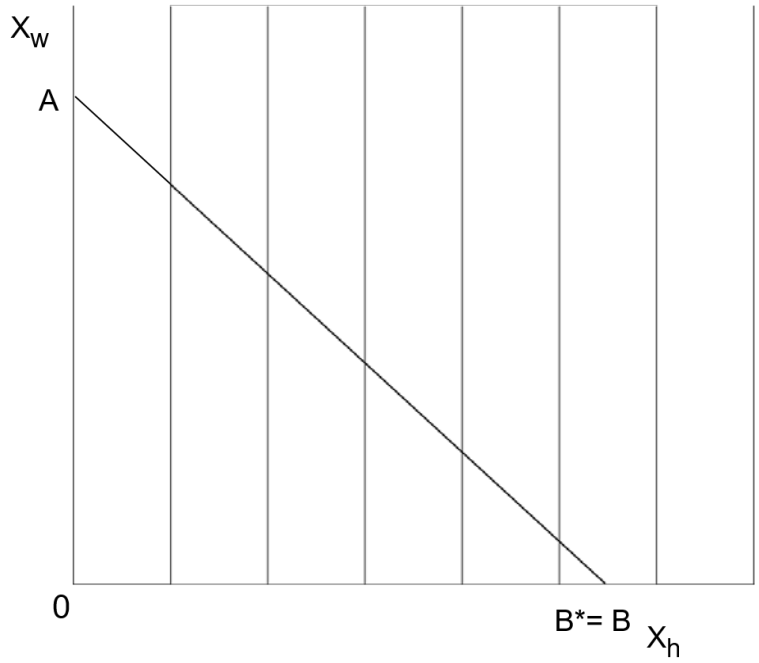

Figure 3: Indifference Map of Egoistic Husband

Proof of Proposition 1: It is easy to see that interior allocations are not Pareto efficient and that with egoistic preferences the Pareto efficient set coincides with the frontier (Fig. 4).

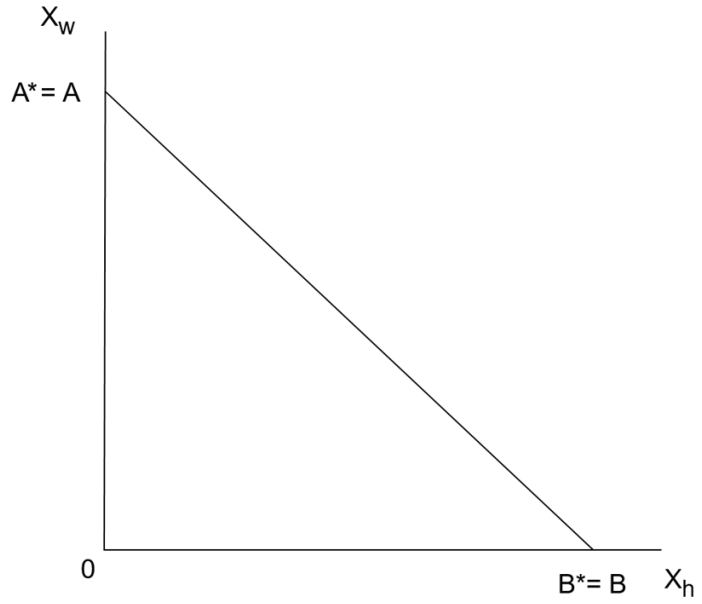

Figure 4: With Egoistic Preferences,
the Pareto Efficient Set Coincides with the Frontier

4a. Both Spouses Altruists: Preview

Altruism changes everything. If both spouses are altruists, then the set of potential cooperative bargaining solutions shrinks from the entire frontier of the feasible set (i.e., the line segment AB), to proper subset of AB that I denote by CD (Fig. 5). The size and location of CD depends on the spouses' ordinal preferences or, equivalently, on the spouses' indifference maps.

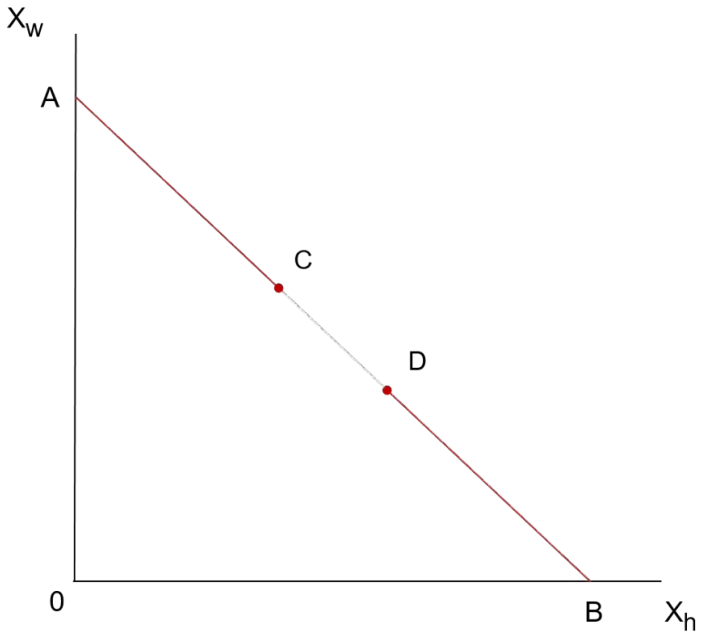

Figure 5: Potential Cooperative Bargaining Solutions

In sections 6 and 7, I provide interpretations of the allocations C and D, which are the end points of the set of potential cooperative bargaining solutions.

The analysis in this paper depends on the implications of altruism for Pareto efficiency and on the implications of Pareto efficiency for potential cooperative bargaining solutions

Thus, from the standpoint of this paper, Pareto efficiency is the key cooperative bargaining axiom. I will show that two-sided altruism implies that

(a) all potential cooperative bargaining solutions lie on CD which is a proper subset of AB, and

(b) any noncooperative bargaining solution that lies outside the subset CD is not Pareto efficient.

4b. One Spouse Is an Altruist

For definiteness, suppose the wife is an altruist and the husband an egoist. I begin by assuming that the wife's preferences over allocations are Cobb-Douglas.

(3a) $U^w(x_h,x_w) = x_h^{\alpha_w} x_w^{1-\alpha_w}$

The Cobb-Douglas parameter α_w represents the share of total resources allocated to the husband at A*, the allocation the wife prefers to all other allocations in the feasible set (Fig. 6).

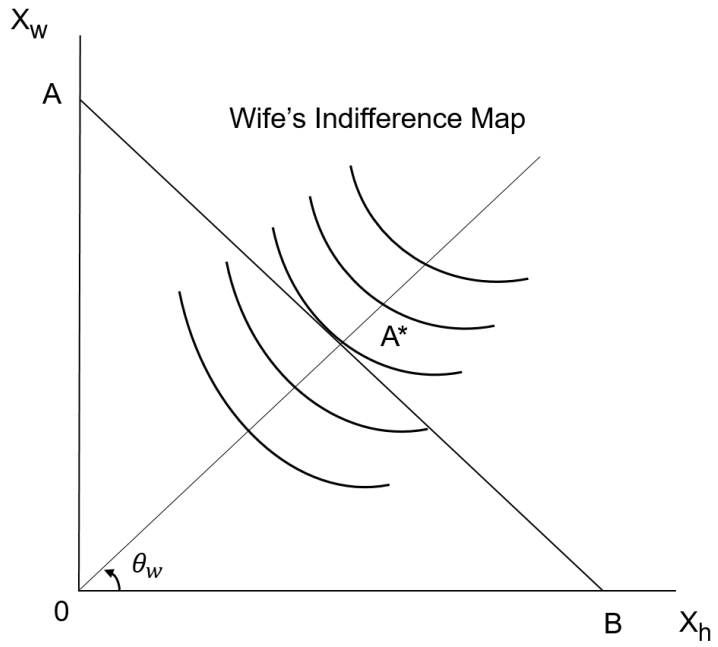

Figure 6: Wife's Cobb-Douglas Indifference Map

Proposition 2: If the wife is an altruist and the husband is an egoist, then the set of potential cooperative bargaining solutions is A*B. Allocations on the peripheral line segment AA* are not Pareto efficient.

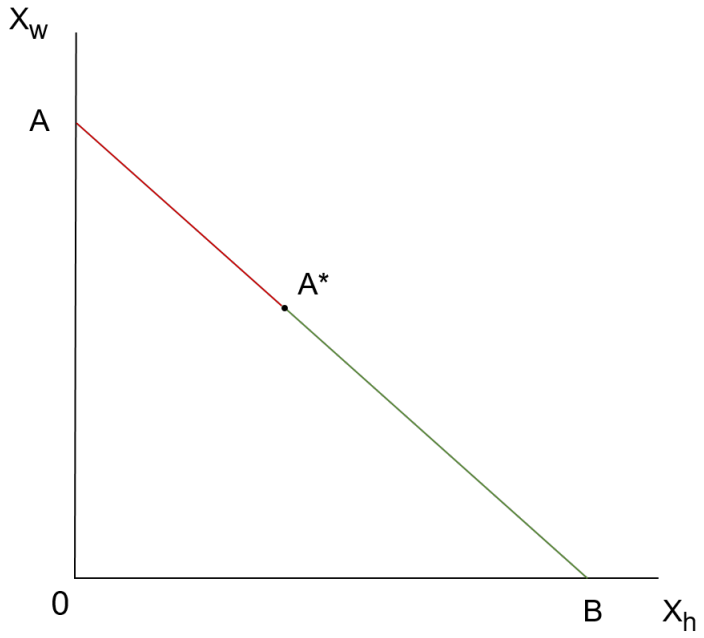

Figure 7: Pareto Efficient Allocations A*B: Wife Altruistic, Husband Egoistic

Proof that any allocation on AB is Pareto efficient: From any allocation on A*B, a move toward A* makes the wife better off and the husband worse off; any move toward B makes the husband better off and the wife worse off.

Proof that, apart from A*, no allocation on AA* is Pareto efficient and, hence, not potential solutions to any cooperative bargaining problem: To show this, select an allocation F on AA* other than A*.

(1) The altruistic wife prefers A* to F. (Indeed, the wife prefers A* to every other allocation in the feasible set.)

(2) The egoistic husband prefers A* to F. (Because the husband is egoistic and his consumption is greater at A* than at F.)

Hence, apart from A*, no allocation on AA* is Pareto efficient. Every allocation on A*B is Pareto efficient and, hence, a potential cooperative bargaining solution.

Now suppose that the wife is an egoist and the husband is a Cobb-Douglas altruist

(3b) $U^h(x_h,x_w) = x_h^{(1-\alpha_h)} x_w^{\alpha_h}$

Recall that B* denotes the husband's preferred allocation in the feasible set. The Cobb-Douglas parameter α_h is the wife's share at B* -- that is, B* is the share of $\hat{x}$ the husband would allocate to the wife if he were a dictator.

Proposition 3: If the husband is an altruist and the wife is an egoist, then the set of potential cooperative bargaining solutions is AB* (Fig. 8). Allocations on the peripheral line segment B*B are not Pareto efficient.

Proof: The proof is the same as that of the previous proposition; only the names differ.

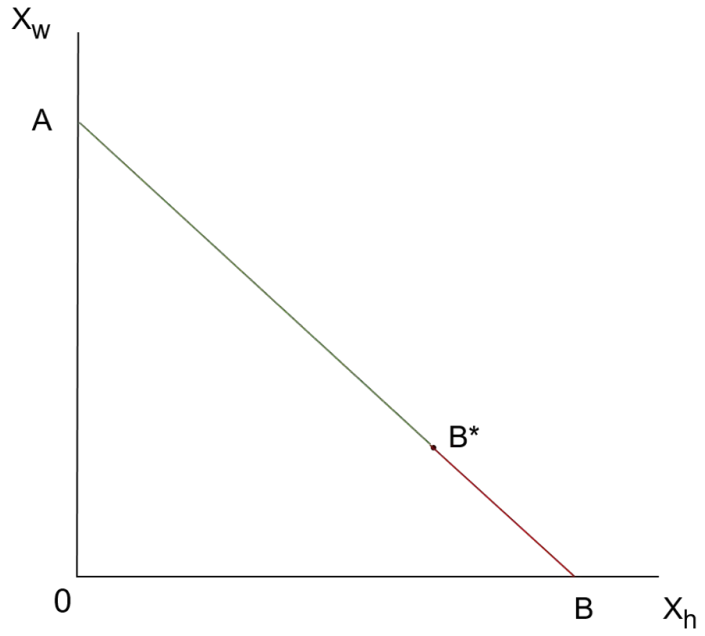

Figure 8: Pareto Efficient Allocations A*B: Husband Altruistic, Wife Egoistic

Taken together, these two propositions lend plausibility to my claim that, if both spouses are altruists, then the set of potential cooperative bargaining solutions shrinks from AB to CD, where C and D are distinct from A and B. The analysis of two-sided altruism is complicated, however, by the need to distinguish between "ordinary altruism" and "excessive altruism."

5. Two-Sided Altruism

Ordinary altruism is the case in which the husband's preferred share for himself (i.e., his share at B*) is greater than the wife's preferred share for him (i.e., his share at A*). That is, $(x_h$ |B*) > $(x_h$ |A*) where $(x_h$ |B*) and $(x_h$ |A*) denote the allocations at B* and A*. Equivalently, ordinary altruism is the case in which the wife's preferred share for herself (i.e., her share at A*) is greater than the husband's preferred share for her (i.e., her share at B*). That is, $(x_w$ |A*) > $(x_w$|B*). Ordinary altruism implies that when spouses disagree, as they do for allocations on the frontier between C and D, each spouse would prefer to consume a greater share of the marital pie.

"Excessive altruism" -- the term is Becker's -- is the case in which for some allocations, each spouse would prefer to consume a smaller share so that the other spouse can consume a greater share. (The 1905 O. Henry short story "The Gift of the Magi" provides a much-anthologized literary example of excessive altruism.) Ordinary altruism and excessive altruism are properties of the preferences of both spouses, not of the preferences of an individual spouse; it makes no sense to say that the wife is an excessive altruist or that she is an ordinary altruist. The relative positions of the spouses' preferred allocations, A* and B*, follow from the definitions of ordinary and excessive altruism (Figs. 9a, 9b). Excessive altruism interchanges the positions of A* and B*.

"Super altruism" – the term is mine – is the razor's edge case in which A* = B*. With super altruism the only Pareto efficient allocation is A* = B* collapses to a single point. Super altruism is the cooperative analogue of the special case of Becker's noncooperative voluntary transfer model in which

both spouses are altruistic and, using Becker's terminology, altruism is always "effective," resulting in the unique Pareto efficient allocation A* = B*.

This is a convenient place to discuss the relationship between my analysis of cooperative bargaining with altruism and Becker's voluntary transfer model. The initial allocation in Becker's voluntary transfer model is determined by the requirements of equilibrium in the marriage market.

I view Becker's voluntary transfer model as a noncooperative game, in contrast to the Nash bargaining model, which, with or without altruism, is a cooperative game. From this perspective, the threat point in the cooperative model is analogous to the initial allocation in Becker's noncooperative model. Becker emphasizes that his model involves no bargaining, but this seems more a semantic point than a substantive one.

The crucial distinction between my model of BWA and Becker's voluntary transfer model is that my analysis does not focus on the cooperative bargaining game corresponding to a particular threat point and a particular pair of von Neumann-Morgenstern utility function, but to a class of cooperative games corresponding to all possible threat points and all possible pairs of von Neumann-Morgenstern utility functions corresponding to a given feasible set and a given pair of ordinal utility functions. I focus on Pareto efficient allocations because these are the only potential solutions to cooperative bargaining games.

In Becker's voluntary transfer model the rules of the game (e.g., under what circumstances, if any, can a spouse reject a proffered transfer) are crucial. With ordinary altruism, a spouse would reject a proffered transfer that would make him better off if he believes that doing so will lead to a better offer. This possibility suggests that the spouse offering the transfer would like to commit herself not to make a better offer in the event that her initial offer is rejected. The rules of the game may or may not allow her to make such a commitment. Becker (1991, p. 286) recognized both of these possibilities and recognized that with excessive altruism, if the rules of the voluntary transfer game allowed him to do so, a spouse

would reject a proffered transfer if accepting would make him worse off than the initial allocation. Cooperative bargaining, with or without altruism, avoids these difficulties by focusing on Pareto efficiency.

With ordinary altruism, the frontier can be partitioned into three regions corresponding to the line segments {AA*, A*B*, B*B} (Fig. 9a). Bargaining takes place only in the central region, (A*B*). At every allocation in the interior of A*B* the wife and the husband prefer to move in opposite directions along the frontier; hence, these allocations, as well and A* and B*, are Pareto efficient.

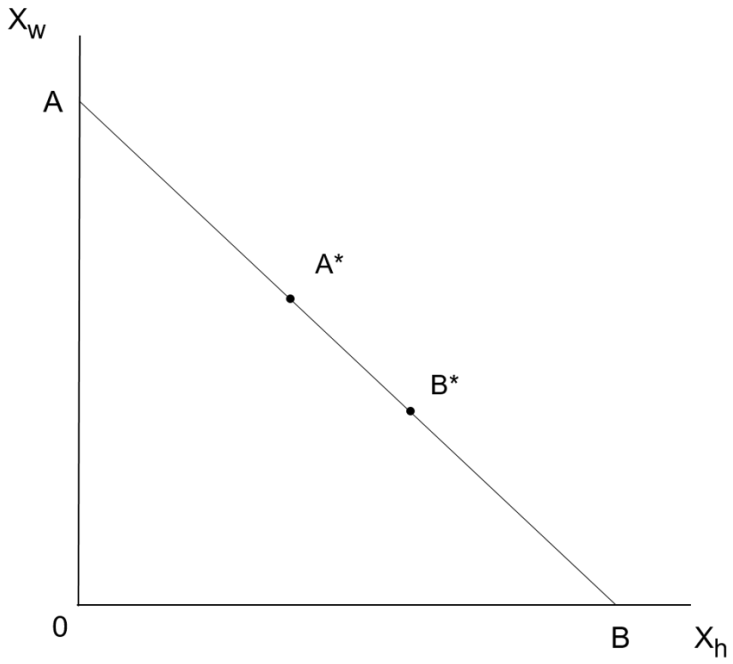

Figure 9a: Pareto Efficient Allocations with Ordinary Altruism

With excessive altruism, the frontier can be partitioned into three regions corresponding to the line segments (AB*, B*A*, A*B) (Fig. 9b). Bargaining takes place only in the central region, (B*A*). At every allocation in the interior of B*A* the wife and the husband prefer to move in opposite directions along the frontier, so these allocations, as well as A* and B*, are Pareto efficient.

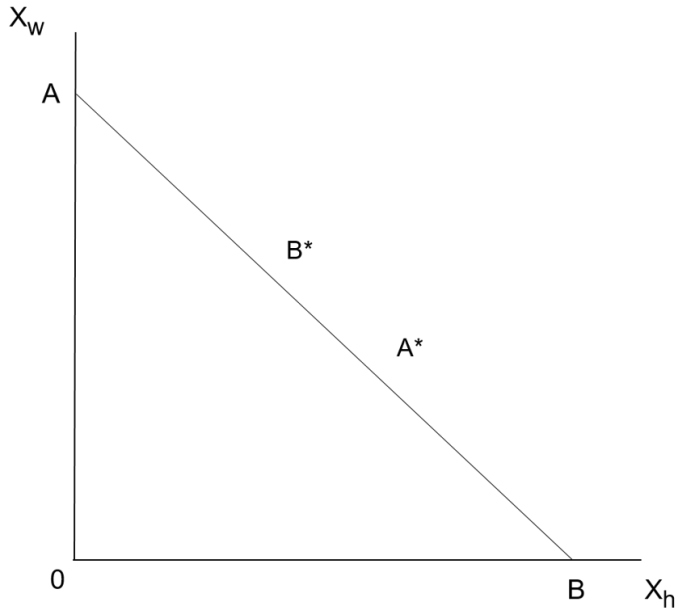

Figure 9b: Pareto Efficient Allocations with Excessive Altruism:
Positions of A* and B* Interchanged

6. Ordinary Altruism

Proposition 4: With ordinary altruism, the set of potential cooperative bargaining solutions is the central region, A*B*. Allocations in the two peripheral regions, AA* and B*B, are not Pareto efficient. With ordinary altruism, C corresponds to A* (the wife's preferred allocation) and D corresponds to B* (the husband's preferred allocation).

Proof of Proposition 4: To show that allocations on A*B* are Pareto efficient, consider a threat point F on A*B*. Any small reallocation in either direction from an interior allocation on A*B* makes one spouse better off and the other worse off (Fig. 10).

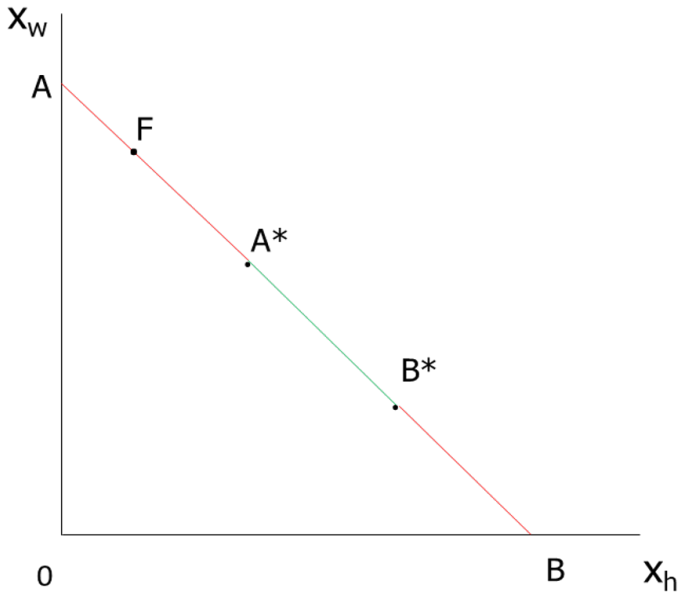

Figure 10: Pareto Efficient Allocations with Ordinary Altruism
$X_{hB*}>X_{hA*}$ *or, equivalently,* $X_{wA*}>X_{wB*}$

To prove that allocations on AA* are not Pareto efficient, we consider a small reallocation from the altruistic wife to her altruistic husband along the linear frontier of the feasible set, AA•. The reallocations we consider are sufficiently small that they remain in the region AA*. If a small reallocation is in the direction of the wife's preferred allocation, then she prefers the reallocation to the original allocation. But

a small reallocation toward A* also makes the husband better off because it is in the direction of his preferred allocation, B*. Because the reallocation makes both spouses better off, the original allocation was not Pareto efficient. An analogous argument shows that allocations on the peripheral line segments B*B are not Pareto efficient; a small reallocation in the direction of B* makes both altruistic spouses better off. This implies that with ordinary altruism the set of Pareto efficient allocations shrinks from AB to A*B*. The argument also shows that, with ordinary altruism, the end points of the bargaining set (C,D) correspond to the wife's preferred allocation and the husband's preferred allocation: C=A* and D=B*.

Compared with egoism, altruism always shrinks the set of potential cooperative bargaining solutions, but the resulting bargaining set may be relatively large (if A* is close to A and B* is close to B), or relatively small (if A* and B* are close to each other). When the set of potential bargaining solutions is small, then all bargaining solutions are bunched together and, hence, imply similar allocations. In this case we could call this paper "altruism with bargaining" rather than "bargaining with altruism".

We can describe the size and the location of the set of potential bargaining solutions using two angles, θ and σ, where θ is the angle between the rays corresponding to A* and B* and σ is the angle of the ray connecting the origin to the midpoint of the bargaining set, the line segment A*B* (Figs. 10a and 10b). The slope of the wife's preferred ray, OA*, is equal to the ratio

$$x_w / x_h = (1 - a_w) / a_w$$

Hence,

$$\tan \theta_w = (1 - a_w) / a_w$$

and, for those who do not remember high school trigonometry,

$$(4a) \quad \theta_w = \arctan (x_w / x_h) = \arctan (1 - a_w) / a_w$$

The slope of the husband's preferred ray, OB*, is equal to the ratio

$$x_w / x_h = a_h / (1 - a_h) \text{ so}$$

$$(4b) \quad \theta_h = \arctan x_w / x_h = \arctan a_h / (1 - a_h)$$

The difference between the slopes of the preferred rays OA* and OA* determines the length of the line segment A*B*. Thus, the angle θ, a measure of the size of the Pareto efficient set, is given by

$$(5) \quad \theta = \theta_w - \theta_h$$

and we can calculate θ_w and θ_h using the arctan formulae (4a) and (4b).

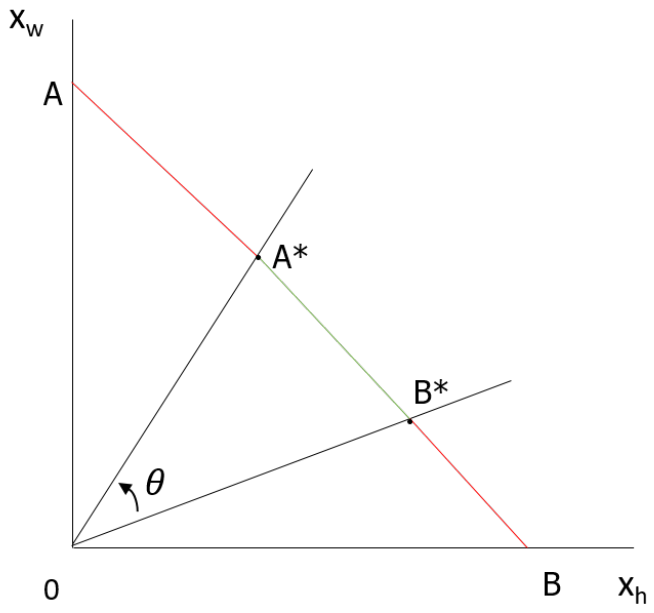

Figure 11a: Pareto Efficient Allocations with Ordinary Altruism - 1

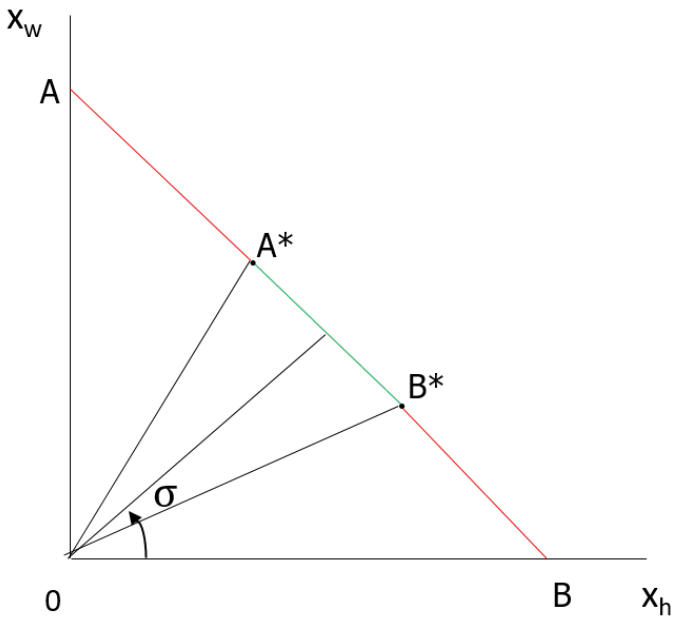

Figure 11b: Pareto Efficient Allocations with Ordinary Altruism - 2

It is easy to verify that σ is the mean of θ_w and θ_h:

$$(6) \quad \sigma = \frac{1}{2}[\theta_w + \theta_h]$$

Super-altruism is the limiting case separating ordinary altruism and excessive altruism, so that

$$\theta_w = \theta_h$$

and the angle $\theta = 0$.

Proposition 5: With super-altruism, the wife's preferred allocation coincides with the husband's preferred the allocation (A* = B*). Hence, the set of Pareto efficient allocations is a singleton and the set of potential cooperative bargaining solutions is a singleton.

7. Excessive Altruism

Despite its limited practical importance, excessive altruism requires discussion both because of its place in the literature and because establishing the results for ordinary altruism (Proposition 4) depended on assuming away excessive altruism. With excessive altruism, at Pareto efficient allocations (i.e., at allocations in the bargaining region) the spouses disagree because each spouse wants a *smaller* share of the marital pie so that the other may have a larger share. The defining characteristic of excessive altruism is that the wife's (husband's) preferred share for herself (himself) is *less* than the share the husband (wife) would prefer her (him) to have. As we have seen, these preferences are reflected in the relative positions of A* and B* (Figs. 9a and 9b).

Proposition 6: With excessive altruism, the set of potential cooperative bargaining solutions is the central region, B*A*. The two peripheral regions, AB* and A*B, are not Pareto efficient. That is, with excessive altruism, C = B* and D = A*.

The proof is essentially the same as the proof of Proposition 5. As in the proof of Proposition 5, the restriction to small reallocations is required to keep the reallocation in the same region as initial allocation.

8. Household Public Goods

Household public goods play a major role in family economics. Sometime economists emphasize children's consumption or investments in children's human capital, while others home produced household public goods such as a clean house and home-cooked meals. Regardless of the emphasis, household public goods complicate the analysis of household bargaining, but it remains true that altruism shrinks the set of potential cooperative bargaining solutions. The argument is straightforward and similar to the argument used when all goods are private.

The argument begins by noting that with a single household public good that must be purchased on the market, x_p, the set of feasible allocations ($x_{h,}\ x_w,\ x_p$) lie in the three dimensions feasible set defined by the equation

(7) $\quad x_h + x_w + x_p \ \leq \hat{x}$

We disregard interior allocations because they are clearly not Pareto efficient, and restrict our attention to the frontier of the feasible set:

(8) $\quad x_h + x_w + x_p = \hat{x}$

With altruistic preferences, whether a frontier allocation is Pareto efficient depends, for example, on the spouses' preferences for the public good relative to private goods. Nevertheless, we can prove that with household public goods altruism shrinks the Pareto efficient set and, hence, the set of potential cooperative bargaining solutions. Our strategy is to use what we know about private goods to investigate Pareto efficiency *conditional* on the quantity of the household public good. If an allocation (x_h, x_w, x_p) is Pareto efficient, then the allocation of private goods (x_h, x_w), must be Pareto efficient conditional on x_p. This strategy allows us to focus on $U^h(x_h, x_w|x_p)$ and $U^w(x_h, x_w|x_p)$ subject to the conditional resource constraint

(9) $x_h + x_w \leq \hat{x} - x_p$

The quantity of the public good affects both the household's conditional resource constraint for the private goods, (9), and the spouses' preferences over private goods. Given the household's conditional resource constraint for private goods, we denote the wife's preferred allocation over (x_h,x_w) conditional on the quantity of the household public good by A*(x_p) and the husband's preferred allocation by B*(x_p).

Thus, A*(x_p) and B*(x_p) are analogous to A* and B* in the case without household public goods. The conditional argument now proceeds as it did when there were only private goods. We must distinguish between "conditional ordinary altruism" and "conditional excessive altruism," where the conditioning is on the quantity of the household public good.

Proposition 7: With ordinary altruism and a household public good, the set of potential cooperative bargaining solutions in the space of private goods conditional on the quantity of the public good is the central region A*(x_p)B*(x_p). Allocations in the two peripheral regions A(x_p)A*(x_p) and B*(x_p)B(x_p) are not Pareto efficient.

Proof: We assume ordinary altruism conditional on x_p for all x_p. Under this assumptions, the argument made in the private goods case shows that, conditional on x_p, allocations on the line segment A(x_p)A*(x_p) are not Pareto efficient. By an analogous argument, conditional on x_p, allocations on the line segment B*(x_p) B(x_p) are not Pareto efficient. This proves that with household public goods, altruism shrinks the set of potential cooperative bargaining solutions. (This argument is not equivalent to modeling household bargaining as a two-stage game.)

An alternative approach to household public goods avoids conditioning on x_p by imposing strong

a prior assumption that make conditioning unnecessary. Suppose that private goods are separable from the household public good in both spouses' preferences.

(10) $U^i\left[V^i(x_h, x_w), x_p\right]\ i = h, w$

Suppose further that for both spouses the subutility functions over private goods are homothetic. The separability assumption ensures that preferences over private goods are independent of the quantity of the household public good, and homotheticity of the subutility functions ensures that each spouse's

preferred ratio of the private goods is independent of total resources allocated to private goods: $\hat{x}$ -x_p. Hence, the preferred ratios are independent of the quantity of the household public good, the Pareto efficient set lies between the two preferred rays, and the peripheral allocations (i.e., those outside the preferred rays) are not Pareto efficient.

9. Conclusion

What have we learned about cooperative bargaining with altruism? When all goods are private, altruism shrinks the set of potential cooperative bargaining solutions and this remains true
when there are also household public goods. In both cases, the argument is straightforward. If both spouses are egoists, all allocations on the linear (conditional) frontier of the feasible set are potential cooperative bargaining solutions. With altruism some of these frontier allocations are not Pareto efficient.

The analysis depends on the implications of altruism for Pareto efficiency and the implications of Pareto efficiency for potential cooperative bargaining solutions. Altruism reduces the set of Pareto efficient allocations and, because cooperative bargaining requires Pareto efficiency, altruism also shrinks the set of allocations that are potential cooperative bargaining solutions.

The analysis is ordinal. Pareto efficiency depends on the spouses' preference orderings (i.e., their indifference maps). The analysis is surprisingly simple because we can analyze potential cooperative

bargaining solutions using ordinal preferences. Working with ordinal preferences permits a wholesale rather than a retail analysis, dealing at once with all von Neumann-Morgenstern utility functions consistent with the spouses' ordinal preferences. In the case of Nash bargaining, this implies that the familiar Nash product function, which depends on the spouses' von Neumann-Morgenstern utility functions, plays no role.

The analysis makes use of utility functions whose arguments are allocations -- that is, on the "allocation representation" of preferences rather than on the more usual "utility representation" of preferences. Because the utility representation can be solved for the allocation representation, basing the analysis on the allocation representation involves no loss of generality.

The analysis applies not only to cooperative Nash bargaining, but to all cooperative bargaining models (e.g., Kalai-Smorodinsky bargaining). The conclusion that altruism shrinks the set of potential cooperative bargaining solutions does not depend on special assumptions about preferences but follows from standard assumptions (e.g., nonsatiation; strictly quasiconcave preferences).

Although noncooperative bargaining does not require Pareto efficient solutions, Pareto efficiency has long been a central concern of economists. Our analysis implies that any noncooperative bargaining solution that lies outside the set of potential cooperative bargaining solutions is not Pareto efficient.

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