NBER WORKING PAPER SERIES

EXPECTATIONS AND THE NEUTRALITY OF INTEREST RATES

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Working Paper 30468
http://www.nber.org/papers/w30468

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
September 2022, Revised February 2024

This paper stems from a talk given at the Foundations of Monetary Policy Conference celebrating 50 years since the publication of Lucas (1972a) "Expectations and the Neutrality of Money," Federal Reserve Bank of Minneapolis, September 2022. I thank Ed Nelson, Greg Kaplan, Loukas Karabarbounis, Christopher Sims, and anonymous referees for helpful suggestions. The views expressed herein are those of the author and do not necessarily reflect the views of the National Bureau of Economic Research.

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NBER Working Paper No. 30468
September 2022, Revised February 2024
JEL No. E4,E5

ABSTRACT

Our central banks set interest rates, and do not even pretend to control money supplies. How do interest rates affect inflation? We finally have a complete theory of inflation under interest rate targets and unconstrained liquidity. Its long-run properties mirror those of monetary theory: Inflation can be stable and determinate under interest rate targets, including a peg, analogous to a k-percent rule. The zero bound era is confirmatory evidence. Uncomfortably, stability means that higher interest rates eventually raise inflation, just as higher money growth eventually raises inflation. Sticky prices generate some short-run non-neutrality as well: Higher nominal interest rates can raise real rates and lower output. A model in which higher nominal interest rates temporarily lower inflation, without a change in fiscal policy, is a harder task. I exhibit one such model, but it paints a much more limited picture than standard beliefs. We either need a model with a stronger effect, or to accept that higher interest rates have limited power to lower inflation. Empirical understanding of how interest rates affect inflation without fiscal help is also a wide-open question.

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1 Introduction

A half-century ago, Robert Lucas (1972a) published the watershed "Expectations and the Neutrality of Money." Lucas studied expectations and the neutrality—and temporary non-neutrality—of, as the title says, *money*. But our central banks set *interest rates*. The Federal Reserve does not even pretend to control money supply, especially inside money. There are no reserve requirements. Super-abundant reserves pay the same or more interest as short-term treasuries and overnight money markets. The Fed controls interest rates by changing the interest it offers on abundant reserves, not by rationing scarce zero-interest reserves. Other central banks follow similar policies. The quantity of M2 is whatever people feel like holding in that form.

We need an analogous theory of inflation under *interest rate targets*. Ideally, the theory should likewise be based on robust and clean economics, starting with a well described frictionless and neutral benchmark, and adding minimal frictions to describe temporary non-neutrality. The theory should be consistent with current institutions, including an interest rate target, ample reserves, no control of inside money, and a plausible description of how central banks operate. Its basic mechanisms and signs should be explainable to undergraduates, central bankers, and intelligent laypeople.

I offer a short history of efforts to develop such a theory. I argue that by integrating monetary and fiscal policy we have at last such a theory. In this theory, inflation is stable and determinate under an interest rate peg, analogous to monetarist analysis of a k% money growth rule. The theory starts with a frictionless and neutral benchmark, and it remains approximately long-run neutral with sticky prices. As appealing as those properties are, however, they inexorably imply that higher nominal interest rates eventually produce *higher* inflation, just as higher money growth is thought to inexorably produce higher inflation.

The final piece, a theory of temporary non-neutrality, the central contribution of Lucas (1972a) for money-based theory, is unfinished — or, if it is finished, it is not commonly accepted that the models we have are the best we can do. It is not settled whether and how by raising interest rates, the central bank can temporarily lower inflation.

That monetary and fiscal policy must always be analyzed together is a truism usually acknowledged but frequently ignored. It rises to first-order importance here. Monetarists long understood that the central bank must be insulated against pressure to print money to cover fiscal deficits, and inflation would result otherwise. Sargent and Wallace (1981) "unpleasant monetarist arithmetic" crystallized this tension in modern intertemporal terms.

Even without such direct fiscal pressure, monetary policy, now interest-rate policy, has fiscal consequences: Higher real interest rates raise interest costs on the debt, and unexpectedly lower inflation is a fiscal transfer to nominal bondholders. If higher interest rates slow economic activity, they induce deficits. Such fiscal-monetary interactions were often glossed over as second-order effects. But that is no longer true, if it ever was. With 100% debt to GDP ratio, a one percentage point higher real interest rate means one percentage point higher debt service costs and one percentage point of GDP larger primary deficit. Each recession sparks trillions in additional deficits.

It is common to assume that fiscal tightening automatically accompanies monetary tightening, to pay these costs and often more. That assumption has two dangers: First, such fiscal tightening may not happen. In the current environment of large debts and structural deficits, fiscal authorities may not wish to or be able to raise surpluses to pay bondholders. Such lack of fiscal support may explain the failure of some past monetary tightenings. (Sims (2024) argues this explains the resurgence of inflation after 1975.) Second, it leaves open just how much of the inflation-fighting power of interest rate rises comes directly, and how much is due to the induced fiscal tightening. Is monetary policy just a carrot in front of the fiscal horse that pulls the inflation cart?

For this reason, I focus on the question, can higher interest rates lower inflation without a change in fiscal surpluses? We can treat this focus as simply an interesting theoretical orthogonalization, to help us to understand how monetary policy works. We also study the effects of fiscal shocks with no interest rate change, and then combine these two policies.

In a range of contemporary models, higher interest rates only lower inflation if they are accompanied by tighter fiscal policy. The best model we have in which higher interest rates lower inflation without tighter fiscal policy includes long-term debt, and temporarily lowers inflation only by raising later inflation in a form of unpleasant arithmetic. This effect is weaker than most economists and central bankers believe, and it is based on a completely different mechanism than they profess. We also lack robust empirical understanding of the effect of interest rate changes, not accompanied by fiscal tightening, on inflation. The hoped-for negative effect may not be there, and experience may have reflected fiscal policy movements.

Ignorance is great news for researchers. The 1970s were a golden decade for macroeconomic research, as much as they were a miserable decade for the economy. The 2020s may well repeat both features.

These questions are also crucial for policy debates. Must central banks substantially raise nominal rates above current inflation, as the Taylor Rule recommends and as we observed in the early 1980s, in order to control inflation? Or can inflation ease without such high nominal interest rates, as stability implies, and as we observed in 2022-2023? If the central bank raises interest rates, and fiscal policy does not or cannot tighten to pay higher interest costs on the debt, does the interest rate rise still lower inflation?

This paper is a synthetic review, a stylized history of thought, and a primer. The contemporary monetary economics literature is full of models that add ingredients. My aim here is to simplify and to see which ingredients are really necessary to the basic properties of monetary policy. Some of these points are also made at longer length with more complex models in Cochrane (2023) and Cochrane (2022b).

2 Theories of Inflation Under Interest Rate Targets

What is the dynamic effect of *interest rates*—not money supply—on inflation? I use a very simple standard model to think about this question, and the historical development of our answers to it,

$$x_t = E_t x_{t+1} - \sigma(i_t - \pi_t^e) \tag{1}$$

$$\pi_t = \pi_t^e + \kappa x_t, \tag{2}$$

where x = output gap, π = inflation, π^e = expected inflation, and i = interest rate. Variables are all deviations from steady state. The central bank sets the interest rate. I include policy rules below.

Equation (1) is the first-order condition for consumption, in an economy without capital so that output equals consumption. Higher real interest rates encourage saving, so depress current consumption relative to future consumption. Equation (2) is the Phillips curve. High output coincides with high inflation, relative to expected inflation.

Lucas specified that expectations enter the Phillips curve as $\pi_t^e = E_{t-1}\pi_t$. Inflation has no permanent output effect, as Friedman (1968) famously explained. But *unexpected* inflation still causes output to rise. Lucas paired that Phillips curve with, essentially, $MV = PY$ and constant V, which determines the price level. Lucas already had in hand a theory of the price level, and needed only to extend that theory to describe non-neutrality. Our challenge is to develop a theory of inflation based on interest rate targets. We have to work to get to Lucas's launch pad. A little historical tour substantiates my claim that until recently we really hadn't got there, but now, with the addition of fiscal underpinnings, we have.

I hesitate to write down such a model without preferences, technology, market structure, definition of equilibrium, and recursive statement. Lucas' most important contribution may have been methodological, to express a monetary economics question with a completely articulated general equilibrium model. But this is well-trod ground and it is well known how to provide those foundations. See, for example, Woodford (2003), Galí (2009).

I simplify further by dropping $E_t x_{t+1}$ on the right hand side of (1), leaving a simple statement that higher real interest rates depress output,

$$x_t = -\sigma(i_t - \pi_t^e). \tag{3}$$

This simplification turns out not to make any difference for the points I make, and leaving it out allows me to do everything with transparent algebra. Equation (1) iterates forward to

$$x_t = -\sigma E_t \sum_{j=0}^{\infty}(i_{t+j} - \pi_{t+j}^e) + \lim_{T\to\infty} E_t x_{t+T},$$

so when output reverts to its mean, both static and dynamic versions give lower output in response to a higher real interest rate. The interpretation of σ changes.

Substituting output out of (2)-(3), we obtain a relationship between interest rates and inflation:

$$\pi_t = (1+\sigma\kappa)\pi_t^e - \sigma\kappa i_t. \tag{4}$$

The dynamic response of inflation to interest rates now depends on how expectations are formed.

2.1 Expectations, Stability, Determinacy, and Neutrality, and Sign

Table 1 summarizes the steady forward march of expectations in the Phillips curve. Each equation is simplified to be emblematic of an era. Actual Phillips curves also include disturbances.

Author	Phillips curve	Expectations
Phillips (1958)	$\pi_t = \pi_0 + \kappa x_t$	Absent
Dynamic empirical (1960s)	$\pi_t = \alpha\pi_{t-1} + \kappa x_t,\ \alpha < 1$	Adaptive
Friedman (1968); ISLM AS/AD (1970s)	$\pi_t = \pi_{t-1} + \kappa x_t$	Adaptive
Lucas (1972)	$\pi_t = E_{t-1}\pi_t + \kappa x_t$	Rational
Calvo (1983), Rotemberg (1982); NK (1990s)	$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t$	Rational

Table 1: The forward march of expectations in the Phillips curve

Phillips didn't have any expectations or other variables to shift the Phillips curve, nor did the Keynesian advocates of inflation in the early 1960s such as Samuelson and Solow (1960). (See Nelson (2020) Ch. 13.)

Dynamic estimates of the Phillips curve in the 1960s and 1970s added lags of unemployment or inflation, depending which variable one put on the right-hand side of the regression. These specifications retained a long-run inflation-output tradeoff, $\alpha < 1$ here, and when thinking theoretically, adaptive expectations.[1]

2.1.1 Adaptive Expectations

Friedman's (1968) address is fundamentally about neutrality. He proclaimed two things that monetary policy *cannot* do: It cannot permanently lower unemployment and it cannot peg an interest rate. On the first point, he proclaimed that the Phillips curve would shift once people come to expect inflation, so the long-run Phillips curve is vertical.

On the way there, Friedman describes explicitly adaptive expectations: "This price expectation effect is slow to develop and also slow to disappear." Phelps (1967) also writes "a sort of Phillips Curve ... that shifts one-for-one with variations in the expected rate of inflation; ... the expected inflation rate adjusts gradually over time to the actual inflation rate."

As a simple model of adaptive expectations, suppose expected inflation is just last period's inflation,

$$\pi_t^e = \pi_{t-1}.$$

[1] Among many others, Lipsey (1960), Gordon (1970). Gordon (1976) p. 192–193 provides a nice summary of the era. Sargent (1971) and Lucas (1972b) also summarize insightfully.

Then from (4) inflation and interest rates are related by

$$\pi_t = (1 + \sigma\kappa)\pi_{t-1} - \sigma\kappa i_t. \tag{5}$$

Steady states have higher inflation with higher interest rates, and no change in output, employment, or real interest rate. In this sense the model displays long-run neutrality. The term $(1 + \sigma\kappa)$ in front of lagged inflation is greater than one. Inflation is *unstable*. Thus, Friedman proclaimed a key doctrine: The central bank cannot peg the nominal interest rate. If it tries to do so, the economy will spiral off to hyperinflation or hyperdeflation. In Friedman's description, the central bank needs to print more and more money to keep the interest rate down. In the ISLM AS/AD tradition, a too-low nominal interest rate lowers the real rate, which boosts demand, which boosts inflation, and around we go. The left panel of Figure 1 illustrates instability and the inflation or deflation spirals that break out under an interest rate peg.

Thus began the tradition in which an interest rate target is a fundamentally incomplete price-level anchor. Friedman' answer, of course, was that the Fed should control the money supply rather than interest rates.

The last term of Equation (5) is negative: Higher interest rates lower inflation. Indeed, higher interest rates set off an unstable deflationary spiral. Friedman said so (in the opposite, inflationary, direction), though quickly adding that the central bank would soon give up the peg and lower money growth to stop the spiraling inflation.

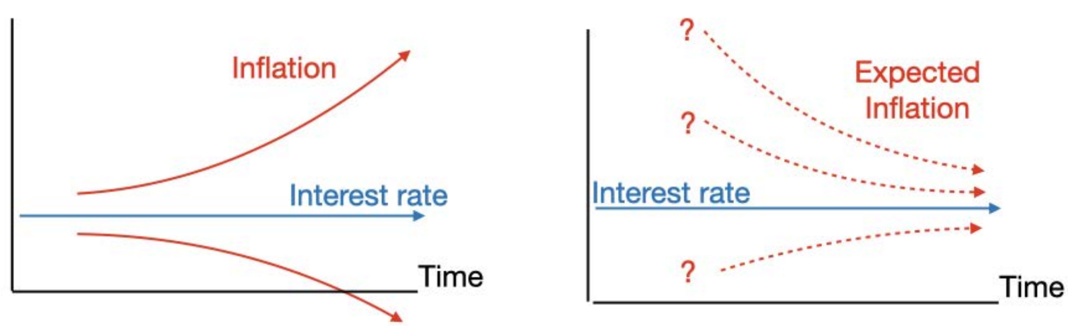

Figure 1: Adaptive (left) vs. Rational Expectations (right) Dynamics

The Taylor rule repairs Friedman's critique of interest rate targets.[2] Let the central bank

[2]McCallum (1981) is the first formal statement that the $\phi > 1$ principle resolves indeterminacy with rational expectations. Taylor (1999) shows how a Taylor rule resolves instability with adaptive expectations. Taylor (1993) is the

systematically respond to inflation with higher interest rates,

$$i_t = \phi \pi_t + u_t \tag{6}$$

with $\phi > 1$. Substituting for i_t in (5), inflation dynamics become

$$\pi_t = \frac{1+\sigma\kappa}{1+\sigma\kappa\phi}\pi_{t-1} - \frac{\sigma\kappa}{1+\sigma\kappa\phi}u_t. \tag{7}$$

Now inflation is stable under an interest rate target. The coefficient on lagged inflation is less than one. The central bank's interest rate policies *stabilize* the economy. It appears the central bank can target interest rates after all. But the economy itself remains fundamentally unstable, with that instability only tamed if the central bank reacts swiftly and more than one for one to inflation, like a seal balancing a ball on its nose.

Higher interest rates still lower inflation. The second term in (7) is negative, so a positive monetary shock lowers inflation. The positive monetary policy shock raises interest rates, so we also see higher interest rates with lower inflation. (This isn't obvious. In some models, a positive monetary shock lowers equilibrium interest rates.) In the period 1 of the shock we have $\pi_1 = -\sigma\kappa/(1+\sigma\kappa\phi)u_1$ and $i_1 = 1/(1+\sigma\kappa\phi)u_1$ so $\pi_1 = -\sigma\kappa i_1$. The higher interest rate sets off a deflation spiral, but the interest rate soon chases inflation downwards and stops the spiral.

The adaptive expectations model captures standard views. However, it would be unfortunate if this is the best we can do.The expectations *of* this model are systematically different from expectations *in* the model. Are economists really that much better forecasters than everyone else? What happens if people catch on? In this model, irrationality is an always and everywhere *necessary* minimal ingredient. If this is our simplest model, we admit that there is *no* simple economic model that delivers the basic sign and operation of monetary policy. Maybe this is where we end up, but it's at least worth considering if another path is possible.

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most influential statement of the rule and its practical implementation. Wicksell (1898), (1965) proposed verbally that a central bank could stabilize the price level by raising and lowering an interest rate target. In the 19th century, the Bank of England systematically raised and lowered the discount rate to defend the gold standard, in part in response to inflation and vice versa.

2.1.2 Rational Expectations and New-Keynesian Models

New-Keynesian models use rational expectations,

$$\pi_t^e = E_t \pi_{t+1}$$

and consciously play by the Lucas rules of how to write and solve intertemporal general equilibrium macroeconomic models. (The standard new-Keynesian model writes the Phillips curve $\pi_t = \beta E_t \pi_{t+1} + \kappa x_t$; Calvo (1983), Rotemberg (1982). I use $\beta = 1$ for simplicity. I show below that $\beta < 1$ makes no important difference to my points.) Now from (4) the dynamic response of inflation to interest rates is

$$E_t \pi_{t+1} = \frac{1}{1+\sigma\kappa}\pi_t + \frac{\sigma\kappa}{1+\sigma\kappa} i_t. \tag{8}$$

This model starts to provide a fully economic model of inflation under nominal interest rate targets. Inflation is still long-run neutral. Inflation is now *stable* since $1/(1+\sigma\kappa) < 1$. Inflation eventually converges to the nominal interest rate, just as higher money growth eventually leads to proportionally higher inflation. This model also has a sensible frictionless $\kappa \to \infty$ limit and limit point,

$$E_t \pi_{t+1} = i_t. \tag{9}$$

Higher interest rates lead to instantly higher inflation, as with flexible prices higher money growth instantly leads to higher inflation. Price stickiness draws out dynamics and adds output effects. People are rational, markets clear.

But this model so far only ties down expected inflation. There are multiple equilibria. Unexpected inflation $\pi_{t+1} - E_t \pi_{t+1}$ can be anything, or wander up and down following sunspots. The right hand panel of Figure 1 illustrates, with the question mark indicating all the many equilibria that could break out at that point. In a related model, Sargent and Wallace (1975) modified Friedman's doctrine: Inflation is *indeterminate* under an interest rate target. Central banks should target money.

Sargent and Wallace's stable and indeterminate is different from Friedman's unstable (and determinate). Both models suggest volatile inflation, and a deep problem with interest rate targets. But spiraling away on a determinate path is different from batting up and down unpredictably around the peg.

The sign on the last term in (8) has also changed. Now higher nominal interest rates *raise* expected inflation, as also in the flexible price model (9), in both the short and long run. We will return to look for additional frictions that can reverse this conclusion.

One often thinks of rational vs. adaptive expectations as a small issue, and adaptive expectations as a small friction that can easily deliver somewhat more realistic dynamics of a baseline model, icing on the cake. That is not true here. Rational vs. adaptive expectations fundamentally change the stability and determinacy properties of the model, and change the sign by which interest rates affect inflation. This is the cake.

New-Keynesian modelers resolve indeterminacy with a novel application of the Taylor principle. If we add $i_t = \phi\pi_t + u_t$ in this case, inflation dynamics (8) become

$$E_t\pi_{t+1} = \frac{1+\sigma\kappa\phi}{1+\sigma\kappa}\pi_t + \frac{\sigma\kappa}{1+\sigma\kappa}u_t. \tag{10}$$

With $\phi > 1$, dynamics are now *unstable*. Adding a rule against nominal explosions, new-Keynesian modelers can now choose the unique initial value of inflation that precludes an explosion, and thus produce determinate inflation; $\pi_t = 0$ with no shocks, and

$$\pi_t = -\frac{\sigma\kappa}{1+\sigma\kappa\phi}\sum_{j=0}^{\infty}\left(\frac{1+\sigma\kappa}{1+\sigma\kappa\phi}\right)^j E_t u_{t+j}$$

with shocks u_t.

The central bank is imagined to deliberately *destabilize* an economy which is already stable on its own, exactly the opposite of the adaptive expectations economy in which the Taylor rule *stabilizes* an otherwise unstable economy. The central bank threatens hyperinflation or hyperdeflation in order to select or "coordinate expectations" on the equilibrium it likes. The Taylor principle is an *equilibrium-selection* policy not a *stabilization* policy. Indeed, the central bank may simply announce its inflation target, announce this threat, and inflation jumps to whatever value the central bank desires.

These statements are easiest to see in the $\kappa = \infty$ case of flexible prices, in which the interest rate directly sets expected inflation via (9), $i_t = E_t\pi_{t+1}$. They are also easiest to see if we rewrite the policy rule equivalently as

$$i_t = \phi\pi_t + u_t = i_t^* + \phi(\pi_t - \pi_t^*) = E_t\pi_{t+1}^* + \phi(\pi_t - \pi_t^*). \tag{11}$$

The second and third equalities define i_t^* and translate between the u_t and i_t^*, π_t^* notation for the monetary policy disturbance. (King (2000) invented this clever notation.) Eliminating i_t from (9)-(11), the model's equilibrium condition is

$$E_t(\pi_{t+1} - \pi^*_{t+1}) = \phi(\pi_t - \pi^*_t). \tag{12}$$

With $\phi > 1$, the unique bounded equilibrium is $\pi_t = \pi^*_t$, $i_t = i^*_t$.

In this notation, we may interpret $\{\pi^*_t\}$ as the central bank's stochastic inflation target, the value of inflation it wishes to produce in each date and state. The central bank implements that target with an interest rate policy i^*_t, which sets the equilibrium interest rate which we observe, and a *separate* equilibrium-selection policy $\phi(\pi_t - \pi^*_t)$, which threatens to produce an expected hyperinflation or deflation should unexpected inflation come out against the bank's desires. The interest rate policy determines expected inflation, the equilibrium-selection policy determines unexpected inflation. For example, if π^*_t is i.i.d., observed interest rates never move. The central bank simply announces each period what inflation it would like to see, and that inflation occurs.

Don't we know from regressions that $\phi > 1$, at least away from the zero bound? No. Since in equilibrium $\pi_t = \pi^*_t$ and $i_t = i^*_t$, we never observe the $\phi(\pi_t - \pi^*_t)$ reaction that generates determinacy. The last example with constant interest rate and i.i.d. inflation gives a vivid illustration: The regression coefficient of i_t on π_t is zero in that case. Estimated Taylor rules measure a correlation between equilibrium interest rates and inflation, not the reaction to off-equilibrium inflation which selects equilibria. For example, suppose the central bank chooses an inflation target with $E_t\pi^*_{t+1} = \theta\pi^*_t + u_t$. Then it would pick $i^*_t = \theta\pi^*_t + u_t$ and again $i_t = i^*_t + \phi(\pi_t - \pi^*_t)$. The equilibrium condition is still $E_t(\pi_{t+1} - \pi^*_{t+1}) = \phi(\pi_t - \pi^*_t)$. We observe $i_t = \theta\pi_t + u_t$; we measure θ not ϕ. The equilibrium-selection parameter ϕ does not appear in observed, equilibrium quantities. (For a fuller treatment of these points, see Cochrane (2011).)

What matters to equilibrium selection is that the central bank makes inflation rise more than one for one with past inflation and hence explode for all but one value; that $\phi > 1$ in (12). Whether interest rates rise more than one for one with inflation is not directly important. In this model, interest rates that rise more than one for one with inflation generate inflation that rises more than one for one with inflation because higher interest rates *raise* inflation. If higher interest rates lowered inflation, following standard intuition, a Taylor principle would no longer select equilibria. And it's clear that in (12) regressions will not measure $\phi > 1$. In larger models, interest rates that rise more than one for one with inflation are often neither necessary nor sufficient for the requisite stable eigenvalue (less than one) to become unstable.

In all these ways, regression evidence and historical episodes in which (by definition observed, equilibrium) interest rates rise more than one for one with inflation are not central evidence for the new-Keynesian equilibrium-selection evidence.

The habit of writing $i_t = \phi\pi_t + v_t$ and restricting v_t to follow an AR(1) has led to a lot of confusion on these points. That superficially plausible specification involves implicit identifying assumptions, in particular that the off-equilibrium threat ϕ is the same as the on-equilibrium relation between interest rate and inflation targets θ. But there is little reason for such an assumption. Historically, authors naturally started with $i_t = \phi\pi_t + u_t$ and AR(1) u_t, and only much later discovered the i_t^*, π_t^* notation that makes the implicit identifying assumptions clear.

To verify the same points in our little sticky price model, again write the policy rule as

$$i_t = \phi\pi_t + u_t = i_t^* + \phi(\pi_t - \pi_t^*), \tag{13}$$

where now

$$i_t^* \equiv \frac{1+\sigma\kappa}{\sigma\kappa} E_t\pi_{t+1}^* - \frac{1}{\sigma\kappa}\pi_t^*. \tag{14}$$

Equation (14) generalizes $i_t^* = E_t\pi_{t+1}^*$, applying the equilibrium condition (8) to the starred variables. If we want to think in terms of an interest rate target and an inflation target, those targets must be compatible with private sector equilibrium conditions. With this policy rule, the equilibrium condition (8) becomes

$$E_t(\pi_{t+1} - \pi_{t+1}^*) = \frac{1+\phi\sigma\kappa}{1+\sigma\kappa}(\pi_t - \pi_t^*). \tag{15}$$

Again, with $\phi > 1$ the central bank destabilizes an economy that is stable on its own. By doing so, it repairs indeterminacy: $\pi_t = \pi_t^*$ and $i_t = i_t^*$ is the unique locally bounded equilibrium.

New-Keynesian modelers can produce lower inflation when interest rates rise, overcoming the positive signs in (8), (9), and (10), by such equilibrium-selection choices. Suppose that coincident with an unexpected rise in interest rates at time t, the central bank also announces a lower stochastic inflation target π_t^*. Now current inflation π_t jumps down. Translate back to $\{u_t\}$ and you have the conventional new-Keynesian model. However, as you can see by this rewriting, the higher interest rate has nothing to do with the lower initial inflation. The central bank could just as easily lower initial inflation without a higher interest rate by equilibrium selection policy alone. Indeed, the higher interest rate, by raising future inflation, requires a larger downward equilibrium-selection jump for the model to produce a protracted disinflation.

The trouble with this approach is that central banks do not have equilibrium-selection policies. They do not threaten hyperinflation or deflation if inflation comes out against their desires. Such threats being contrary to their objectives, nobody would believe them if they tried. Central banks do not intentionally de-stabilize economies that are stable on their own. Ask central banks. Look at central bank websites. They loudly announce that they *stabilize* economies; no matter what inflation does, they promise to act resolutely to bring it back.

As I rejected the beautiful $MV = PY$ because central banks set interest rates and do not limit money supply, I argue that we should also reject the new-Keynesian equilibrium-selection approach, because our monetary institutions simply do not remotely behave as this model specifies.

The new-Keynesian theory also requires an extra rule against non locally bounded equilibria, not a part of the standard definition of Walrasian equilibrium. And yet we see hyperinflations.

From 1968 to the 1990s, then, economists worked to develop a theory of inflation under interest rate targets. So far in my story, they did not succeed. But central banks target interest rates, inflation is something, and indeed was rather successfully controlled by the mid 1990s. How?

2.1.3 Fiscal Theory with Rational Expectations

The fiscal theory of the price level adds an equilibrium condition, or rather recognizes one that was there all along and has been left out of the analysis so far. The log real value of nominal government debt v_t evolves as

$$\rho v_{t+1} = v_t + i_t - \pi_{t+1} - \tilde{s}_{t+1} \tag{16}$$

where $\rho = e^{-r} < 1$ is the steady state discount factor and $\tilde{s}_{t+1}$ is the real primary surplus scaled by the steady state value of debt. Higher real interest rates raise debt, higher primary surpluses lower debt. I use here the simple case of one-period debt and no economic growth; I generalize to long-term debt below. This equation is also linearized. See Cochrane (2023) Ch. 3.5 for a derivation. The consumer's transversality condition also requires

$$\lim_{T\to\infty} E_t \rho^T v_T = 0. \tag{17}$$

We can add these conditions to the VAR(1) statement of the model. But in this simple case, we can solve the model analytically by iterating (16) forward to

$$v_t = E_t \sum_{j=0}^{\infty} \rho^j [\tilde{s}_{t+1+j} - (i_{t+j} - \pi_{t+1+j})]. \tag{18}$$

The real value of debt is the discounted present value of future surpluses. Taking innovations $\Delta E_{t+1} \equiv E_{t+1} - E_t$ of both sides of (18), we obtain.

$$\Delta E_{t+1}\pi_{t+1} = -\Delta E_{t+1} \sum_{j=0}^{\infty} \rho^j \tilde{s}_{t+1+j} + \Delta E_{t+1} \sum_{j=1}^{\infty} \rho^j (i_{t+j} - \pi_{t+j+1}). \tag{19}$$

Unexpected inflation devalues outstanding debt. Thus, unexpected inflation corresponds to the revision in the present value of future primary surpluses. In the second term, higher discount rates lower the value of debt and cause inflation. Equivalently, if interest costs on the debt rise, but current or future surpluses do not rise to pay them, then the resources must come by inflating away outstanding bonds.

The rational expectations model left an indeterminacy indexed by unexpected inflation $\Delta E_{t+1}\pi_{t+1}$. Condition (19) restores that determinacy, in place of central bank equilibrium-selection rules. This point is also easiest to see in the simplest case of flexible prices. Then, (8) and (19) boil down to

$$i_t = E_t \pi_{t+1} \tag{20}$$

$$\Delta E_{t+1}\pi_{t+1} = -\Delta E_{t+1} \sum_{j=0}^{\infty} \rho^j \tilde{s}_{t+1+j}. \tag{21}$$

The interest rate target sets expected inflation but leaves multiple equilibria. Fiscal policy determines unexpected inflation and picks one equilibrium.

With sticky prices, we (8), which I repeat for convenience,

$$E_t \pi_{t+1} = \frac{1}{1+\sigma\kappa}\pi_t + \frac{\sigma\kappa}{1+\sigma\kappa} i_t, \tag{22}$$

along with (19). Equation (22) describes a set of stable paths for expected inflation, and fiscal policy in (19) selects one of the many possible equilibria. It's straightforward to add policy rules in which surpluses and interest rates respond to inflation and output, and all the other complications of DSGE models. (Leeper (1991) is the watershed first article that described fiscal theory with interest rate targets.)

In sum, with (19) to choose unexpected inflation, inflation is stable *and determinate* at an interest rate peg (or Taylor coefficients $\phi < 1$), overcoming Sargent and Wallace's contrary doctrine. The peg can include zero. The economy can follow the Friedman (1969) optimal quantity of money rule and leave us satiated in liquidity, without worry over spirals and sunspots. (Woodford (1994), Woodford (1995) showed that an interest rate peg is determinate and stable with fiscal theory, including a peg.)

The *volatility* of inflation depends on the size of fiscal and discount rate shocks. But economics picks one value for inflation. In addition, we now have a model of the price *level*, a nominal anchor, rather than simply a model of the inflation *rate* at any level. In (18), v_t is the ratio of nominal debt to the price level. The price level adjusts so that (18) holds.

Monetarists, ISLM-AS/AD or adaptive-expectations Keynesians, Sargent and Wallace, and new-Keynesians make explicit assumptions to wipe out fiscal theory. Careful versions of these theories include the government debt equilibrium condition, but they assume that fiscal authorities "passively" adjust surpluses as needed to validate inflation determined elsewhere. In (19), the central bank picks $\Delta E_{t+1}\pi_{t+1}$, and then fiscal authorities supply whatever surpluses $\tilde{s}_{t+1}$ are necessary to satisfy (18) or (19), often via lump-sum taxes. (Leeper (1991), Woodford (2003) section 4.4.)

Since the equilibrium conditions are the same, the fiscal and new-Keynesian ways of completing the sticky price model make predictions for equilibrium time series that are formally observationally equivalent, at least without further assumptions. (See Cochrane (1998) and Cochrane (2023) Ch. 17, 22.) Thus, for the purposes of everything that follows, you may think in terms of the new-Keynesian rather than fiscal-theory version of equilibrium formation, if you disagree with my argument.

However, new-Keynesian modelers typically do not examine what the required surpluses are. For a full evaluation of the theory, we must examine them. Observational equivalence includes model's predictions for "passive" fiscal policies. For example, in the standard new-Keynesian policy experiment, a monetary policy shock that lowers inflation comes with a "passive" fiscal tightening to pay higher interest costs on the debt and a windfall to bondholders. Fiscal theory calls this rise a "fiscal shock" coincident with the interest-rate increase. New-Keynesian theory calls it an induced fiscal policy response to the equilibrium-selection part of the central bank's monetary policy. By either name, however, surpluses must rise, or inflation cannot fall. Standard new-Keynesian analysis doesn't look. So, you can interpret my equations as exploring fiscal underpinnings of conventional new-Keynesian models.

We should also not overstate the consequences of observational equivalence. Formal tests on equilibrium time series are not the only way to distinguish theories. For example, in the fiscal theory model, a fiscal shock (a decline in the present value of surpluses) results in inflation to devalue debt that the central bank cannot completely avoid by any path of interest rates. In the new-Keynesian model, the central bank fully controls inflation, expected and unexpected. Fiscal shocks are ruled out: If the central bank does not choose inflation, fiscal authorities repay all deficits in full. Whether there are inflationary fiscal shocks, and whether the central bank alone can or cannot fully control inflation are pretty important policy and doctrinal issues, which historical analysis can illuminate. Did we suffer inflation in 2021-2022 because of a fiscal shock, or because central banks failed to credibly commit to appropriate equilibrium-selection threat to support a rock-steady 2% inflation target, inducing hyperinflation for anything greater? Finally, knowledge of how our institutions work—that central banks simply do not make explosive equilibrium-selection threats—is information that can help to distinguish theories.

Fiscal theory does not have to be married to with rational expectations. I investigate fiscal foundations of adaptive expectations models below. If the rational expectations Calvo pricing structure turns out to be a dead end, fiscal theory can apply to any macroeconomic theory in which government debt is not a free lunch.

2.2 Lucas's Phillips Curve

In my little Phillips curve history (Table 1), I skipped over Lucas. Lucas (1972a) first made Phillips-curve expectations rational. His Phillips curve relates output to unexpected inflation only, first moving forward the time subscript in the Phillips curve, from $\pi_t^e = \pi_{t-1}$ to $\pi_t^e = E_{t-1}\pi_t$. In the spirit of rational expectations, it makes most sense to pair Lucas' Phillips curve with rational expectations in the bond market and consumption. So let's use Lucas' Phillips curve in an interest-rate model by writing

$$x_t = -\sigma(i_t - E_t\pi_{t+1}) \tag{23}$$

$$\pi_t = E_{t-1}\pi_t + \kappa x_t. \tag{24}$$

Eliminating x_t, inflation dynamics (4) are now

$$E_t\pi_{t+1} = i_t + \frac{1}{\sigma\kappa}(\pi_t - E_{t-1}\pi_t). \tag{25}$$

Further in the future,

$$E_t\pi_{t+j+1} = E_t i_{t+j};\ j \geq 1.$$

Lucas' specification of the rational expectations Phillips curve, along with our IS curve, passive fiscal policy, and an interest rate target, leads to an economy that is stable and indeterminate, like the new-Keynesian model. Relative to the flexible-price model $i_t = E_t\pi_{t+1}$, Lucas's Phillips curve gives one period of additional inflation after a shock, which then reverts to the frictionless value. It gives a one-period output effect. Adding fiscal theory to this model we again restore determinacy, and name the unexpected inflation shock. One could also add new-Keynesian equilibrium selection and policy rules.

2.3 Anchored Expectations

The 1970s adaptive expectations model as I have presented it doesn't fully capture the views of today's policy consensus, as explained for example in central bank analysis and communications. We can capture those views a bit better by writing expectations as

$$\pi_t^e = \begin{cases} \pi^* & \text{if anchored} \\ \sum_{j=0}^{\infty} a_j \pi_{t-j} & \text{if not} \end{cases}$$

along with the model (2)-(3),

$$x_t = -\sigma(i_t - \pi_t^e) + \sigma u_{x,t} \tag{26}$$

$$\pi_t = \pi_t^e + \kappa x_t + u_{\pi,t}. \tag{27}$$

I add here IS ("demand") and Phillips ("supply," "cost") shocks, and use u rather than ε to emphasize that such shocks are persistent. I multiply the demand shock by σ so it has rate of return units. Inflation dynamics are then

$$\pi_t = (1 + \sigma\kappa)\pi_t^e - \sigma\kappa i_t + \sigma\kappa u_{x,t} + u_{\pi,t}. \tag{28}$$

Inflation expectations underlying consumption-saving and price-setting behavior are usually "anchored" at the bank's inflation target, $\pi^* = 2\%$. Those expectations remain anchored even if a rational or rational-expectations (using the model) forecast of inflation temporarily deviates from 2%, for example in response to a u shock. The anchor comes from a belief that eventually the central bank will do something to control inflation, even if inflation breaks out in the meantime. That belief is malleable by speeches, policy "strategies," forward guidance, and other bank communication. If the belief is broken, then expectations become "unanchored," but remain slowly adaptive rather than forward-looking and model-consistent or "rational." Something like a Markov switching process with probabilities that depend on observed inflation, interest rates, and bank communication describes a shift from anchored to unanchored.

In (28), while expectations remain "anchored" with $\pi^e = \pi^*$, a shock to inflation eventually goes away on its own without aggressive bank action. The Fed is still important, though. Higher interest rates can lower inflation and thereby partially offset u shocks. However, this rise comes at the cost of lower output, see (26) with $\pi^e = \pi^*$. The Fed faces a basically static Phillips policy tradeoff between inflation and employment.

Inflation is stable and determinate, as long as expectations remain anchored. We might call it "conditionally stable." Unlike the inflation spiral of the adaptive expectations model, a slow reaction to inflation such as 2021-2022, while harmful, does not threaten immediately to spiral out of control. The damage of delay, while perhaps sorting out just what shocks did hit the economy and what the data mean, is limited.

But as inflation rises, perhaps due to u shocks, with no central bank action, the chance of de-anchoring rises, and then an inflation spiral breaks out. So raising rates eventually is doubly, and perhaps mostly, important via a signaling channel to reanchor expectations. Indeed, if expectations have become a bit unanchored, $\pi_t^e > \pi^*$, then a rate rise that signals a firmer commitment to doing whatever it takes can quickly lower inflation π_t by snapping back expectations, with no Phillips curve output effect, and no period of interest rates above current and past inflation as required by the adaptive expectations model. Thus, the story goes, the beginning of rate rises in 2022 brought inflation down quickly and with no recession, even though interest rates stayed below inflation for a year.

Once expectation become unanchored, the policy model reverts to adaptive expectations, but with expectations that adjust slowly to experience, say

$$\pi_t^e = (1-a)\sum_{j=0}^{\infty} a^j \pi_{t-1-j}$$

with a large value of a. Now inflation dynamics, suppressing the u disturbances for simplicity, are

$$\pi_t = (1+\sigma\kappa)(1-a)\sum_{j=0}^{\infty} a^j \pi_{t-1-j} - \sigma\kappa i_t. \tag{29}$$

We can express this result as[3]

$$\pi_t = [1+\sigma\kappa(1-a)]\pi_{t-1} - \sigma\kappa(i_t - a i_{t-1}).$$

For $0 \le a < 1$, inflation is still unstable, $1+\sigma\kappa(1-a) > 1$. But more slowly adapting expectations, larger a, mean a slower explosion. Slowly adaptive expectations also mean that it takes the central bank longer and with a deeper recession to bring inflation under control.

This branch answers the nagging question of "anchored" expectations, "anchored by what?" The answer is, the central bank will repeat the adaptive-expectations story of 1980, lowering inflation via the $-\sigma\kappa i_t$ term at the cost of recession.

The policy model has some features of ancient fixed-expectations analysis, while expectations are "anchored." But it also takes some of the lessons of the rational-expectations revolution, that expectations are not fixed functions of past experience, and depend on policy "regimes." It takes expectations as something of a free variable, malleable by announcements, but also dependent on the Fed's reputation, and ability, to eventually do what it takes to constrain inflation by higher interest rates. But expectations are still not model-consistent; the expectations of the model are not the expectations in the model. Perhaps most deeply, in both adaptive and policy models, expectations are not *reactive*. In the rational expectations model, expectations react, immediately, to a change in interest rate or other shock. Here, they stay put.

Most policy discussion also sees "long and variable lags" in the response of the real economy to the interest rate (26), which also induces such lags in inflation dynamics (28). Therefore,

[3]

$$\left[1-(1+\sigma\kappa)\frac{1-a}{1-aL}L\right]\pi_t = -\sigma\kappa i_t$$
$$\left[\frac{1-[1+\sigma\kappa(1-a)]L}{1-aL}\right]\pi_t = -\sigma\kappa i_t$$

higher interest rates do not lower *current* inflation as in my (28), but rather slowly lower *future* inflation. But our purpose is to understand basic properties with simple economic models, not to match dynamic beliefs with ad-hoc lags, so I move on.

3 Long-Run Properties

Combining fiscal theory with rational expectations, then, we have, finally, a complete economic (rational agents, Walrasian equilibrium) theory of inflation under interest rate targets, compatible with what we know about how central banks behave. In this theory, and like monetarist theory, the economy is *determinate*, *stable*, starts from a *neutral* flexible-price benchmark, displays (potentially approximate) long-run neutrality, and offers a foundation for models with frictions that create additional short-run non-neutralities.

These properties are on full display with flexible prices,

$$i_t = E_t \pi_{t+1}$$

$$\Delta E_{t+1} \pi_{t+1} = \Delta E_{t+1} \sum_{j=0}^{\infty} \rho^j \tilde{s}_{t+1+j}.$$

There is only one equilibrium value of inflation. The central bank can follow a peg, analogous to a k% rule, and inflation will neither spiral away nor suffer multiple-equilibrium volatility. Like a k% rule, a peg may not be optimal, but it is possible. A one percentage point higher interest rate with no change in fiscal policy produces a one percentage point higher inflation, with no change in real rate or output. That's neutrality.

The sticky-price version of the model (22)-(19) also offers a first step of explicitly modeled short-run non-neutrality, keeping the frictionless model's properties in the long run, again similarly to standard monetary theory. The model remains stable and determinate: Inflation slowly converges to the interest rate. Output and real interest rates vary, but that variation dies away in the long run. A one percentage point higher interest rate leads eventually to one percentage point higher inflation, but not right away.

Higher interest rates raise inflation? Even if qualified by "in the long run," this "Fisherian" property is so contrary to standard doctrine that one looks up in disbelief from the screaming implications of the equations. Really?

Stability is the central property behind the Fisherian result. If inflation is stable under an interest rate peg, it follows that raising the peg must eventually raise inflation. The adaptive expectations model is long-run neutral, in that steady states with higher inflation have higher interest rates. But it is unstable, so raising the interest rate from such a steady state lowers inflation.

How do higher interest rates produce higher inflation? The intuition is not as easy as simply printing up money and handing it out. What about standard intuition that higher interest rates depress aggregate demand, so should *reduce* inflation?

First, consider the full consumer first-order condition $x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1})$, with no pricing frictions. Raise the nominal interest rate i_t. Before prices change, a higher nominal interest rate is a higher real rate, and induces people to demand less today x_t and more next period x_{t+1}. That change in demand pushes down the price level today p_t and hence current inflation $\pi_t = p_t - p_{t-1}$, and it pushes up the expected price level next period p_{t+1} and thus expected future inflation $\pi_{t+1} = p_{t+1} - p_t$. So, standard intuition is correct, and refers to a force that can lower *current* inflation as it lowers the current price level relative to the future. Fisherian intuition is correct too, and refers to a natural force that can raise *expected future* inflation, as it raises the future price level relative to the current one.

But which is it, lower p_t or higher p_{t+1}? This consumer first-order condition, capturing an "intertemporal substitution" effect, or telling us the slope of the price level over time, cannot tell us. Unexpected inflation and the overall price level is determined by a "wealth effect," by the overall level of prices, by the "nominal anchor." Handing money or government debt out for free with no change in fiscal policy generates such a wealth effect. Hence, if we pair the higher interest rate with no change in surpluses and thus no wealth effect, whether "actively" or "passively" produced, then the initial price level p_t does not change and the entire effect of higher interest rates is a rise in p_{t+1}. If instead there is a positive "wealth effect," a concurrent rise in the present value of expected surpluses, then we have a lower price level p_t and less current inflation π_t. Intuition that higher interest rates lower inflation implicitly assumes that fiscal policy tightens in concert with monetary policy.

Second, consider the Phillips curve. Suppose a higher nominal rate means a higher real rate in the IS curve and pushes output down, as it does in classic static ISLM thinking and in my simplified model. Lower output x_t in the Phillips curve,

$$\pi_t = \pi_t^e + \kappa x_t$$

means lower inflation π_t, yes, but *relative* to expected inflation. With adaptive expectations, $\pi_t^e = \pi_{t-1}$, lower inflation π_t means that inflation *decreases* over time. But with rational expectations, $\pi_t^e = E_t\pi_{t+1}$, lower inflation means that inflation *increases* over time. So we might again be talking to cross-purposes, confusing current inflation that is lower relative to expected inflation with inflation that decreases over time. And lower inflation relative to expected inflation may not mean lower inflation, as expected inflation may rise so much that both current and expected inflation are higher than otherwise, as happens in this case.

Third, verbal intuition may also confuse adjustment to equilibrium with the movement of inflation over time in equilibrium. Hold fixed expected inflation, i.e. assume it is "anchored." Then higher real interest rates and lower output lower inflation π_t. But that means lower inflation *today*, right away, than otherwise would have occurred. It does not mean that higher interest rates today provoke lower output in the near future, and slowly drive inflation down in the further future, as most policy discussion describes. These equations do not have "long and variable lags." So part of the contrast with standard intuition may come from confusing the process of adjustment to equilibrium with the evolution of equilibrium over time.

(Since Ball (1994), the fact that the standard new-Keynesian Phillips curve relates high output to *decreasing* inflation has been a persistent concern, attracting a wide variety of additional ingredients or a return to adaptive expectations. The point here is not to pursue this issue, but to understand the intuition of the new-Keynesian result.)

Stability means that higher interest rates must eventually lead to higher inflation. Stability is a natural and robust consequence of forward-looking or rational expectations, while instability is a natural consequence of backward-looking expectations. If you drive a car looking through the rear view mirror, you will veer off the road. If you drive looking forward the car will stably follow the road.

If intuition still rebels, keep in mind that stability and a Fisherian response may take a long time to take hold, and keep in mind the experiment: permanently higher interest rates with no change in fiscal policy. Central bankers seldom abstain from moving interest rates for very long, monetary and fiscal policy both respond to events, monetary policy induces fiscal responses, and lies in the shadow of fiscal events which push inflation around. We do not obviously have a lot of experience that bears on the Fisherian proposition.

Exchange rate pegs and purchasing power parity are a good analogy. If a country wishes a lower nominal exchange rate, a less valuable currency, it can do so by pegging at that rate and waiting long enough. Any peg also requires fiscal backing, the ability to get as much foreign exchange or the will to print as much domestic currency or debt as needed to enforce the peg. Pegs fail when governments give up on these fiscal requirements. One may have to wait a very long time for purchasing power parity and a long slow inflation to kick in. Nonetheless, the proposition that an exchange rate peg is eventually neutral, that relative price levels eventually adjust, if a government can stick to that peg long enough, is intuitively clear, as are the fiscal, foreign exchange, and time-consistency limitations of that proposition for policy and historical analysis. The proposition that higher interest rates lead eventually to higher inflation is analogous. One pegs a cross-sectional nominal price and waits for real prices to return to normal. The other pegs an intertemporal nominal price and waits for real prices to return to normal. Standard intuition holds that an exchange rate peg with adequate fiscal backing is stable, just as it holds that a money growth peg with adequate fiscal backing is stable. We just extend those ideas to a nominal interest rate peg.

In sum, stability and consequent view that higher interest rates eventually raise inflation are logically linked, explainable by simple intuition, and a robust result of forward-looking expectations. They are hard to avoid.

3.1 Neutrality

Stability is the central question for the possibility of an interest-rate peg and the sign of inflation's long-run response to interest rates. Exact long-run neutrality is not important. If a one percentage point interest rate rise corresponds to 0.9 or 1.1 percentage point higher inflation rather than 1.0 higher inflation, no important conclusion has changed. If that interest rate rise corresponds to an 0.1 percentage point permanent output rise or fall, less has changed. It would take a mighty non-neutrality for steady state inflation to *decrease* when steady state interest rates rise – real rates would have to rise more than nominal rates. Similarly, standard monetary theory is also little affected by small non-neutralities, say that inflation goes up 0.9 percentage points when money growth rises one percentage point. It would take a huge non-neutrality for higher money growth to lower inflation.

With a standard new-Keynesian Phillips curve,

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t,$$

the sticky-price model is not exactly long-run neutral. Real variables are eventually independent of the price *level*, which is one definition of neutrality, but steady states

$$x = (1 - \beta)/\kappa \pi, \tag{30}$$

with a higher inflation *rate* have higher output, which violates a more stringent definition of long-run neutrality. Since β is close to one, however, this is a small effect.

This effect is contentious, with several attempts to fix it and critiques of those attempts. Its presence is not a central part of new-Keynesian doctrine, which unlike 1960s Keynesianism does not argue for higher steady state inflation to produce higher steady state employment. Moreover, it is only one of many peculiarities of the Calvo (1983) and related pricing models that are not meant to be taken to extremes. (See Auclert et al. (2022) for theory and Steinsson and Nakamura (2013) for microeconomic evidence).

Even with $\beta < 1$, higher nominal interest rates correspond one for one with higher *inflation* in the standard new-Keynesian model. Moreover, the relationship between interest rates and inflation could also display small non-neutralities without much affecting the analysis. Since nominal interest is taxed, at least one of pre-tax or after-tax nominal interest rates does not move exactly one for one with steady state inflation. Even steady inflation generates distortions via the tax code, and changes the distribution of prices among the monopolistically competitive producers of new-Keynesian models. But these considerations do not affect the *stability*, *determinacy* or the *sign* of the long-run response of inflation to interest rates. It would take a mighty non-neutrality for higher steady state nominal interest rates to correspond to lower steady state inflation.

I use the words "approximate long-run neutrality" to reflect these considerations.

4 Continuous Time

Here I develop the simple model in continuous time. This framework shows many ideas and issues more clearly. Some of the timing conventions that obscure the analysis vanish. Continuous time allows inflation to jump without a price level jump, clarifying how the economy reacts to a shock. In the "primer" function of this essay, expressing the ideas in continuous time opens a range of useful tools. I use continuous time in several calculations below, as they give much simpler formulas.

Write the standard model (1)-(2)

$$E_t(x_{t+\Delta} - x_t) = \sigma(i_t - E_t\pi_{t+\Delta})\Delta \tag{31}$$

$$E_t(\pi_{t+\Delta} - \pi_t) = -\kappa x_t \Delta. \tag{32}$$

This standard model in continuous time is thus

$$E_t dx_t = \sigma(i_t - \pi_t)dt \tag{33}$$

$$E_t d\pi_t = -\kappa x_t dt. \tag{34}$$

Normally a term $-\rho\pi_t dt$ appears on the right of (34). As I simplified the discrete time Phillips curve $\pi_t = \beta E_t \pi_{t+1} + \kappa x_t$ with $\beta = 1$, I simplify here with $\rho = 0$. Nothing important hinges on this simplification.

The price level is continuous, and cannot jump or diffuse. In an instant dt only a fraction λdt of producers may change prices. The inflation rate may have jumps or diffusions. But $E_t\pi_{t+\Delta} - \pi_t$ is still of order Δ, so the relevant inflation in the consumer's first order condition (33) is π_t. The issue whether inflation in that condition should be rationally anticipated or adaptive disappears.

Equations (32) and (34) express the standard rational-expectations Phillips curve. The adaptive-expectations analogue is

$$\pi_t - \pi_{t-\Delta} = \kappa x_t \Delta \tag{35}$$

$$d\pi_t = \kappa x_t dt. \tag{36}$$

Thus, adaptive and rational expectations differ by whether higher output corresponds to increasing (36) or decreasing (34) inflation. Essentially, they differ by the sign of κ. Adaptive expectations also produce a differentiable inflation, with neither jumps nor diffusion terms.

Again I simplify the model so we can see the main points without algebra, by using a static version of the consumption equation,

$$x_t = -\sigma(i_t - \pi_t). \tag{37}$$

Eliminating output from the Phillips curve, we have the dynamic relation between interest rates and inflation. With rational expectations

$$E_t d\pi_t = -\sigma\kappa\pi_t dt + \sigma\kappa i_t dt, \tag{38}$$

while with adaptive expectations

$$d\pi_t = \sigma\kappa\pi_t dt - \sigma\kappa i_t dt. \tag{39}$$

With rational expectations, inflation is stable but indeterminate. With adaptive expectations, inflation is unstable but determinate. “Stable” means that the coefficient in front of π_t on the right hand side is negative. “Indeterminate” means that the model as specified so far, with an exogenous interest rate, does not fully determine inflation.

I simplify here by analyzing impulse-response functions, equivalently how the economy responds to a single unexpected “MIT shock” at time 0. It is straightforward to generalize to responses to jump-diffusion shocks. For rational expectations, such solutions to (38) are

$$\pi_t = \sigma\kappa \int_{\tau=0}^{t} e^{-\sigma\kappa\tau} i_{t-\tau} d\tau + e^{-\sigma\kappa t}\pi_0 \tag{40}$$

and a single undetermined initial condition π_0. In this context, “stability” means that the influence of past interest rates disappears over time, while “indeterminacy” means that there are multiple equilibria.

For adaptive expectations, “unstable” means that the coefficient in front of π_t on the right hand side of (39) is negative. It is “determinate” since $d\pi_t$ not $E_t d\pi_t$ appears on the left. The solutions of (39) are

$$\pi_t = \sigma\kappa \int_{\tau=0}^{t} e^{\sigma\kappa\tau} i_{t-\tau} d\tau + e^{\sigma\kappa t}\pi_0.$$

In this context, “unstable” means that interest rates and initial conditions further in the past have larger effects today. Despite the positive sign $\sigma\kappa\pi_t dt$ on the right hand side of (39), we solve the model backward, because there is no jump or diffusion in inflation. If we try to solve forward,

$$\pi_t = \sigma\kappa \int_{\tau=0}^{\infty} e^{-\sigma\kappa\tau} i_{t+\tau} d\tau,$$

the right hand side can require an inflation jump that the model rules out. You don't always solve unstable roots forward and stable roots backward.

A Taylor rule stabilizes the unstable adaptive expectations model. Adding

$$i_t = \phi\pi_t + u_{i,t},$$

the adaptive-expectations dynamics (39) become

$$\frac{d\pi_t}{dt} = -\sigma\kappa(\phi - 1)\pi_t - \sigma\kappa u_{i,t}$$

With $\phi > 1$, dynamics are now stable and determinate. A monetary policy shock $u_{i,t}$ raises the interest rate and lowers inflation.

In the rational expectations model, adding a policy rule following the Taylor principle in the new-Keynesian tradition, dynamics (38) become

$$E_t d\pi_t = \sigma\kappa(\phi - 1)\pi_t dt + \sigma\kappa u_{i,t} dt.$$

Now $\phi > 1$ induces instability. We can solve the integral forward, and with a rule against nominal explosions recover determinacy,

$$\pi_t = -\sigma\kappa E_t \int_{\tau=0}^{\infty} e^{-\sigma\kappa(\phi-1)\tau} u_{i,t+\tau} d\tau.$$

Fiscal theory offers an alternative route to determinacy in the rational expectations model. Include the linearized evolution of real government debt.

$$E_t dv_t = (rv_t + i_t - \pi_t - \tilde{s}_t)dt. \tag{41}$$

This version specifies instantaneous debt, i.e. a floating rate. Integrating forward and imposing the transversality condition, the real value of debt equals the present value of surpluses.

$$v_t = E_t \int_{\tau=0}^{\infty} e^{-r\tau}[\tilde{s}_{t+\tau} - (i_{t+\tau} - \pi_{t+\tau})]d\tau. \tag{42}$$

Again, I treat here only responses to an unexpected time-0 shock. Then, corresponding to (19),

$$dp_0 = \int_{t=0}^{\infty} e^{-rt}[-\tilde{s}_t + (i_t - \pi_t)]dt. \tag{43}$$

The left hand side dp_0 is a potential jump in the price level at time 0 (or diffusion in a full model). Price stickiness rules out such jumps. Then, of multiple equilibria, we pick the inflation *path* in which the discount rate/interest cost effect, the second term, exactly balances any change in surplus, the first term. In the absence of a surplus change, we pick the inflation path so that the response of total interest costs is zero.

5 Can Higher Interest Rates Temporarily Lower Inflation?

Experience suggests, and it is widely believed, that higher interest rates can lower inflation. Stability, determinacy, and long-run neutrality mean that higher interest rates eventually raise inflation. But there could well be a *temporary* negative sign.

If there is a temporary negative effect, we could also reconcile stability and long-run neutrality with the widespread belief in a uniformly negative effect, as well as the absence of a well-documented long-run positive effect in most econometric estimates. Central banks seldom leave interest rates alone long enough, with a background of unchanged fiscal policy and few other shocks, to see the positive long-run effects. The long-run neutrality of money was similarly not part of common experience-based doctrine until Friedman (1968).

We are, in short, finally where Lucas started. Lucas built on a theory of neutral money, to describe a short-run *non-neutrality* of money. In his model, higher money growth temporarily raises output, the concern and common belief of his day. We finally have a complete theory of neutral nominal interest rates. We want a theory of short-run non-neutrality in which higher interest rates temporarily lower inflation, the concern and common belief today, as well as output.

No matter how one feels about fiscal theory, in its wake it is clear that we have to specify the question, "what are the effects of monetary policy?" in a way that jointly specifies fiscal policy and the nature of monetary-fiscal coordination. Even if one completely adopts the standard new-Keynesian point of view, that central bank equilibrium-selection policies complete the monetary model and fiscal authorities will via lump-sum taxes "passively" adjust fiscal surpluses to pay any fiscal consequences of monetary policy, the size and nature of that fiscal policy adjustment needs to be examined in theory and in data.

I focus on the effects of monetary policy—a change in the nominal interest rate—with no change in primary surpluses. Central banks cannot directly control fiscal surpluses. They cannot change tax rates or spending. Monetary policy may well induce fiscal surpluses or deficits, but our central theoretical question is, what can monetary policy do all by itself? If monetary policy only reduces inflation by inducing a fiscal contraction, that's news for economic theory and practitioners.

In data, history, and policy, monetary and fiscal policy are intertwined. Fiscal policy reacts to the same events as monetary policy reacts to. An interest rate rise leads to higher interest costs on the debt, which may induce a fiscal tightening. If successful at lowering inflation, the monetary tightening induces a fiscal transfer to nominal bondholders who are repaid in more valuable money. The interest rate rise might lead to a recession, which leads to stimulus, bank and business bailouts, and automatic stabilizers. Inflation induced by monetary policy might lead to austerity and fiscal retrenchment. For evaluating history or for predicting the likely course of the economy following a monetary policy shock, we want to include such contemporaneous fiscal shocks and induced fiscal responses. For example, we can include a fiscal rule describing surplus reactions to economic events, and calculate reactions to monetary policy shocks with such a rule in place. That method is common in quantitative fiscal-theory model evaluation. But it does not answer our theoretical question, what are the effects of *monetary* policy? So I focus on monetary policy changes without any change in fiscal surpluses.

One can also regard this focus as an orthogonalization: if we calculate the effects of "monetary policy" as interest rate changes without changes in surpluses, and the effects of "fiscal policy" as changes in surpluses and deficits without interest rate changes, we can add the two to understand history and to evaluate joint fiscal and monetary policies.

There might be no model with all these desiderata in which higher interest rates lower inflation, even temporarily. If so, that fact is important. A search that establishes that standard beliefs require induced fiscal policy responses, irrational people, or an inherently unstable or non-neutral foundation, is just as important as a search that finds a model to confirm standard beliefs.

You might think it's easy. Just add some sticky prices. That turns out not to be the case. You might think that standard models in use for decades satisfy this obvious desire, since the belief is so widespread. That also turns out not to be the case. Both standard new-Keynesian and adaptive expectations models produce a negative effect by positing a fiscal contraction coincident with the interest rate rise. Without that contraction, they don't produce a negative effect of interest rates on inflation.

5.1 Higher Inflation in Simple Sticky Price Models

Figure 2 plots the response of my simplified rational expectations model (22)-(19) to an unexpected permanent interest rate rise, with no change in fiscal surpluses. The "sticky price" line gives the main message of this section: Inflation rises, even in the short run. Sticky prices alone do not give us a negative effect. The real interest rate rises (interest rate higher than following inflation), so output declines (not shown). But sticky prices only draw out the positive response of inflation to interest rates. We maintain long-run neutrality: The inflation rate eventually rises to match the interest rate.

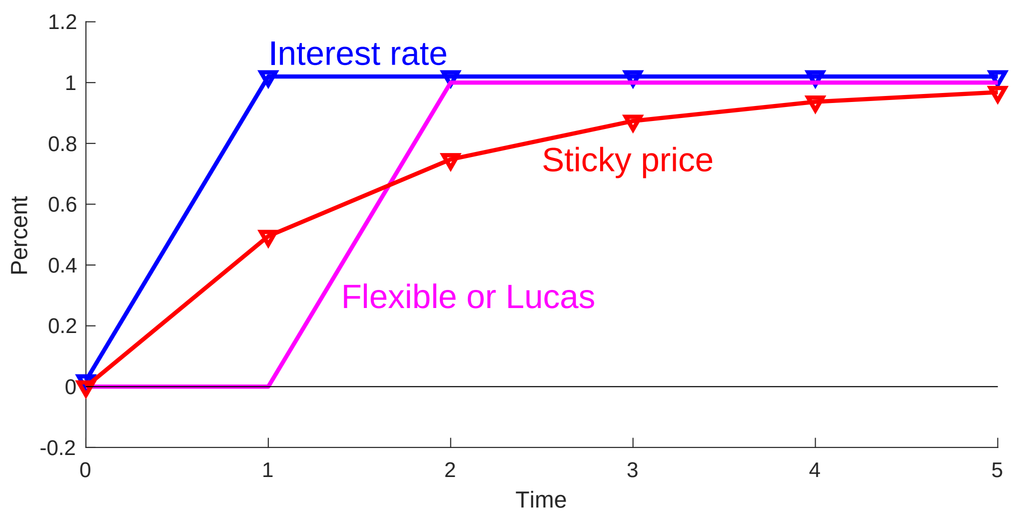

Figure 2: Inflation response to a 1% permanent rise in the interest rate with no change in fiscal policy. Rational expectations model. Parameters $\sigma\kappa = 1$.

Start with the "flexible or Lucas" line. The flexible price model is (20)–(21). The responses solve

$$E_t\pi_{t+1} = i_t \tag{44}$$

$$\Delta E_{t+1}\pi_{t+1} = 0. \tag{45}$$

At time 1, the interest rate rises unexpectedly from 0 to $i_1 = 1$. There is no expected $E_0\pi_1 = i_0 = 0$ or unexpected $\Delta E_1\pi_1$ inflation, so $\pi_1 = 0$. After that, however, inflation rises to meet the higher interest rate path, $\pi_t = i_{t-1} = 1_1 = 1$. The Lucas Phillips curve in this model (25) produces the same inflation result, and a one-period movement in output.[4]

The "sticky price" line uses the forward-looking new-Keynesian Phillips curve. The response of inflation to the interest rate solves (22)-(19):

$$E_t\pi_{t+1} = \frac{1}{1+\sigma\kappa}\pi_t + \frac{\sigma\kappa}{1+\sigma\kappa}i_t \tag{48}$$

$$\Delta E_{t+1}\pi_{t+1} = \Delta E_{t+1}\sum_{j=1}^{\infty}\rho^j(i_{t+j} - \pi_{t+j+1}). \tag{49}$$

In response to the unexpected permanent interest rate shock, $i_t = i_1$, $t = 1, 2, \ldots$, there is a family of solutions to (48) which we can index by π_1,

$$\pi_{t+1} = i_1 + \frac{1}{(1+\sigma\kappa)^t}(\pi_1 - i_1). \tag{50}$$

We can use (49) at time $t = 0$ to determine

$$\pi_1 = \sum_{j=1}^{\infty}\rho^j(i_1 - \pi_{j+1}). \tag{51}$$

Substituting from (50) and simplifying,

$$\pi_1 = \frac{\rho}{1+\sigma\kappa}i_1 \tag{52}$$

[4]The impulse-response solves

$$E_t\pi_{t+1} = i_t + \frac{1}{\sigma\kappa}\Delta E_t\pi_t \tag{46}$$

$$\Delta E_{t+1}\pi_{t+1} = \Delta E_{t+1}\sum_{j=1}^{\infty}\rho^j(i_{t+j} - \pi_{t+j+1}). \tag{47}$$

With only one unexpected movement at time 1 ($t = 0$ in the equation), so $\pi_2 = E_1\pi_2$ and so forth, (46) leaves π_1 arbitrary but then $\pi_2 = i_1 + \pi_1/(\sigma\kappa)$, $\pi_3 = i_1$, $\pi_4 = i_1$, etc. Now, use (47) to find π_1 and the unique path. (47) reduces to $\pi_1 = i_1 - \pi_2 = -\pi_1/(\sigma\kappa)$. The unique solution is $\pi_1 = 0$, and thus $\pi_2 = i_1$, $\pi_3 = i_1$, and so forth.

and the full unique solution is

$$\pi_{t+1} = \left[1 - \frac{1+\sigma\kappa-\rho}{(1+\sigma\kappa)^{t+1}}\right] i_1. \tag{53}$$

Inflation rises to meet the nominal interest rate.

The real interest rate rises and output declines. If your objective is only to produce a model in which the central bank can temporarily cool economic activity with interest rate rises, you have it. You might stop here and say that we have indeed redone Lucas (1972a) with interest rate targets, since his purpose was to understand how money growth affects the real economy.

But inflation still rises uniformly after the interest rate rise. Indeed, in period 1, inflation is *greater* than the (zero) value of the frictionless model. Why? With sticky prices, higher nominal interest rates lead to higher real interest rates. Higher real interest rates mean greater unfunded (by assumption of no change in fiscal policy) interest costs on the debt. With no change in surpluses, higher interest costs must come from a higher unexpected period 1 inflation, which devalues outstanding debt. Equation (51) expresses this result. Higher real interest rates, induced by sticky prices, are an *inflationary* force, since they raise debt service costs. For this reason, as prices become stickier, $\kappa \to 0$, first period inflation π_1 rises further, $\pi_1 \to \rho$ (see (52)) which is just barely less than one. Stickier prices make inflation rise sooner.

Fiscal underpinnings matter crucially to these calculations. If we said that fiscal surpluses would always rise to pay higher interest costs on the debt, then (49) has an extra surplus term on the right hand side, as in the original (19), and the two terms on the right hand side are always equal. Then $\pi_1 = 0$, no immediate inflation rise, though inflation would still rise uniformly after that. If we could pair the interest rate rise with even higher surpluses, we could predict lower inflation, $\pi_1 < 0$. But the question we want to ask is, what can higher interest rates do to lower inflation *without* fiscal support? The answer is, so far, higher interest rates can produce a recession, but they raise inflation.

And even the slow rise of inflation is fragile, really the result of one-period rather than instantaneous (floating-rate) debt. With instantaneous debt, as with very sticky prices, inflation rises immediately. To see this effect, we can compute the same response in continuous time. From (40), for a generic interest rate path $\{i_t\}$, the solution

$$\pi_t = \sigma\kappa \int_{\tau=0}^{t} e^{-\sigma\kappa\tau} i_{t-\tau} d\tau + e^{-\sigma\kappa t}\pi_0 \tag{54}$$

gives us a family of inflation paths indexed by π_0. Inflation is, due to sticky prices, a backward-looking sum of nominal interest rates, plus a decaying transient. Only one of these paths satisfies the valuation equation (42), which is in this case with no change in surpluses $\tilde{s}_t = 0$,

$$0 = \int_{t=0}^{\infty} e^{-rt}(i_t - \pi_t)dt. \tag{55}$$

Average interest costs on the debt must be zero. This condition selects which path $\{\pi_t\}$ is the equilibrium. Any period of positive real rates must be balanced by a period of negative real rates.

In the same case that the interest rate rises from 0 to a new constant value i, then (54) reduces to

$$\pi_t = (1 - e^{-\sigma\kappa t})i + e^{-\sigma\kappa t}\pi_0. \tag{56}$$

Plugging this into the valuation equation (55) with $\tilde{s}_t = 0$ to determine π_0,

$$0 = \frac{i}{r} - \int_{t=0}^{\infty} e^{-rt}[(1 - e^{-\sigma\kappa t})i + e^{-\sigma\kappa t}\pi_0]dt \tag{57}$$

$$\pi_0 = i, \tag{58}$$

and then $\pi_t = 1$, $t \geq 0$ as well. Despite sticky *prices*, we pick the equilibrium in which *inflation* moves instantly to match the interest rate.

How is it that stickier prices make inflation jump up more in the discrete time model, and inflation jumps up instantly in a most non-sticky way, for any amount of price stickiness, in the continuous time model? Sticky *prices* do not imply sticky *inflation*. The few firms which can change price at any instant raise their prices a lot. Much verbal intuition describes sticky *inflation* or inflation momentum. That needs a different set of costs than those specified by current forward-looking sticky *price* models.

Appealing to the new-Keynesian model will not help to produce a negative sign. This *is* the new-Keynesian model. Solving the discrete-time model in new-Keynesian style, the central bank can produce any value of first-period inflation $\pi_1 = \pi_1^*$ it wishes, by following an interest rate policy $i_1 = i_1^* + \phi(\pi_1 - \pi_1^*)$, where $i_1^* = 1$, the first point on the desired interest rate path, and π_1^* is the desired, possibly negative, first-period inflation. Write it up as $i_t = \phi\pi_t + u_t$ with $u_t = i_t^* - \phi\pi_t^*$. But any inflation path other than the one we have already plotted requires a change in surpluses. Lower initial inflation requires positive surpluses. If we phrase the question of a new-Keynesian ($\phi > 1$, passive fiscal policy) model, "What is the response of inflation to a monetary policy disturbance $\{u_t\}$ that produces an unexpected permanent rise in the interest rate, and the associated passive fiscal policy requires no change in surpluses?" we have just calculated the unique answer.[5]

For algebraic simplicity, I have so far used the model with no forward-looking term in the IS curve. I asserted that simplification is not essential to the results, but it's worth verifying that assertion. I add a second important generalization: I allow lagged inflation in the Phillips curve. That is a common modification in models such as Christiano, Eichenbaum, and Evans (2005), and it brings the model back toward the adaptive expectations model which may lead one to hope for something like the latter's short-run dynamics.

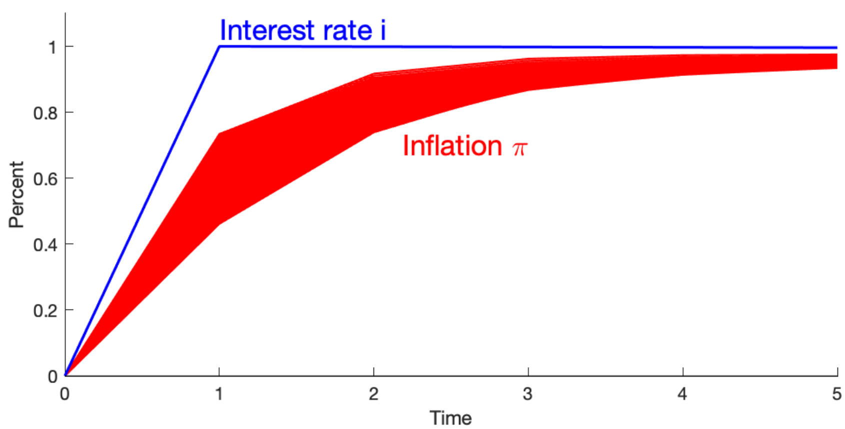

Figure 3: Inflation response to a permanent interest rate rise, with no change in fiscal policy. Full model with forward-looking IS curve and a lag in the Phillips curve. The shaded area shows impulse response functions for $\rho = 0.99$, $\eta = 0.999$, and other parameters in a grid with $\alpha \in [0, 1]$, $\kappa \in [0.01, 2]$, $\sigma \in [0.01 : 5]$, that produce real eigenvalues.

The model is

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1}) \tag{59}$$

$$\pi_t = (1 - \alpha) E_t \pi_{t+1} + \alpha \pi_{t-1} + \kappa x_t \tag{60}$$

$$\rho v_{t+1} = v_t + i_t - \pi_{t+1} \tag{61}$$

[5]Bianchi and Melosi (2019) study a related question. The also study interest rate increases with no fiscal policy reaction, in a more elaborate sticky-price model. They imagine a central bank that goes one step further, following an "active" Taylor rule that reacts to current inflation with coefficient greater than one. Now, higher interest rates raise interest costs on the debt, which leads as here to greater inflation. Their central bank raises interest rates even further, which if left alone leads to explosive inflation. The possibility of a Markov switch back to a coordinated regime saves this asymptotic possibility. Since on the path, future inflation is higher than current inflation, output falls.

$$i_{t+1} = \eta i_t + \varepsilon_{i,t+1}. \tag{62}$$

Equation (59) includes the forward-looking term in the IS equation. Equation (60) allows for lagged inflation in the Phillips curve. Equation (61) specifies short-term debt and no change in surplus.

Figure 3 presents the response of this model to a permanent interest rate rise. I solve the model numerically. The shaded area gives inflation paths for a grid of parameter values. I restrict parameters to those that produce real eigenvalues, however. Sawtooth or sine-wave responses induced by complex eigenvalues can produce some temporarily negative inflation responses, but are clearly unrealistic. As the figure shows, for all such parameter values, this generalized model produces a steady rise in inflation. The static IS curve did capture the results of this more complex model.

5.2 Transitory Interest Rates

The essential failure of the rational expectations sticky price model to produce a negative inflation effect is not tied to the permanent interest rate increase shown in Figure 2 and 3. I only graph permanent interest rate changes so that we can see long-run properties. However, transitory interest rate paths can give a misleading appearance of such an effect.

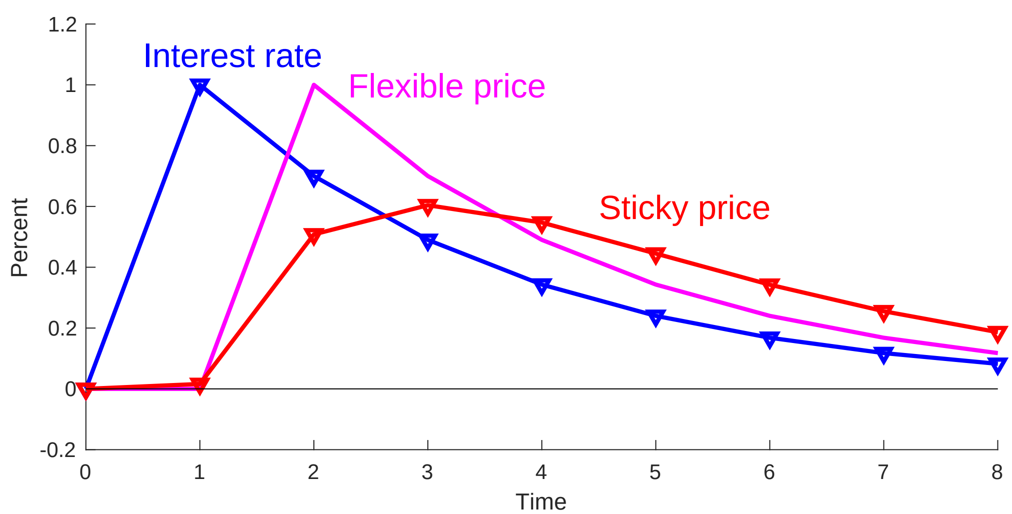

Figure 4: Response of the simple rational expectations sticky price model to a transitory interest rate path, with no change in fiscal policy. Parameters $\sigma\kappa = 1$, $\rho = 0.99$, and $i_t = 0.7 i_{t-1} + \varepsilon_t$.

Figure 4 plots the response of the simplified rational expectations model to a transitory interest rate movement, with $i_t = 0.7 i_{t-1} + \varepsilon_t$. Inflation still rises uniformly. The only difference is that one doesn't notice long-run neutrality with a transitory shock.

At the cost of some algebra, relegated to the Appendix, we can find the response of the rational-expectations sticky-price model (48)-(49) to an arbitrary interest rate path $\{i_t\}$, announced at period 1, with no change in fiscal surplus:

$$\pi_{t+1} = \frac{1-\rho}{(1+\sigma\kappa)^{t+1}} \sum_{j=1}^{\infty} \rho^j i_j + \frac{\sigma\kappa}{1+\sigma\kappa} \sum_{j=1}^{t} \frac{1}{(1+\sigma\kappa)^{t-j}} i_j \tag{63}$$

and in particular

$$\pi_1 = \frac{1-\rho}{1+\sigma\kappa} \sum_{j=1}^{\infty} \rho^j i_j. \tag{64}$$

In (63), all the coefficients are positive. Hence, *any sequence of positive interest rates* $\{i_t\}$ *generates uniformly positive inflation response* $\{\pi_t\}$. In this sense, the positive response of the rational expectations sticky price model is general and does not depend on the time-series process of the interest rates.

As usual, the analogous continuous time version is more elegant. The general solution to (54)-(55), for an arbitrary interest rate path is given by:

$$\pi_t = e^{-\sigma\kappa t} r \int_{\tau=0}^{\infty} e^{-r\tau} i_\tau d\tau + \sigma\kappa \int_{\tau=0}^{t} e^{-\sigma\kappa\tau} i_{t-\tau} d\tau. \tag{65}$$

See the Appendix for algebra. In (65) the coefficients of inflation on the interest rate are also all positive.

We can generate the *appearance* of a negative effect of interest rates on inflation, however. Inflation is a long moving average of interest rates, with forward and backward terms. If interest rates were to rise in the short run, and then turn towards a long-lasting negative value, then the future lower interest rates could pull down even current inflation.

Figure 5 presents an example. As in all figures, this is the response of inflation to the indicated interest rate path, with no change in fiscal surpluses. I create an interest rate path with a hump shape rather than an AR(1) shape to make the graph prettier, and to resemble paths often seen in VARs. I specify $i_t = e^{-0.35t} - 0.6e^{-0.9t} - 0.2$. I then calculate the inflation path from (65).

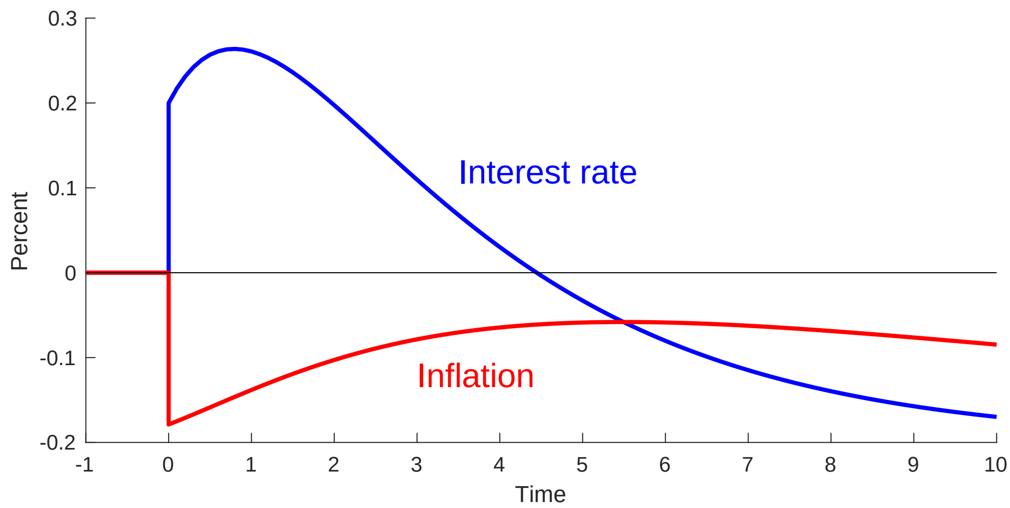

Figure 5: Inflation response to a transitory rise in the interest rate. Parameters $\sigma\kappa = 0.1$, $r = 0.01$.

For an AR(1) interest rate $i_t = i_0 e^{-\eta t}$, (65) reduces to

$$\pi_t = \frac{1}{\sigma\kappa - \eta}\left[\sigma\kappa e^{-\eta t} - \frac{r + \sigma\kappa}{r + \eta}\eta e^{-\sigma\kappa t}\right] i_0. \tag{66}$$

(See the Appendix.) I add three such solutions to calculate Figure 5.

Figure 5 looks appealing. A higher nominal interest rate seems to produce lower inflation. As inflation comes down, so does the nominal interest rate, both ending up lower. This looks a lot like the standard story of a disinflation such as the 1980s. One could even read the plot as an account of 1980 through 2020—high real interest rates (1980-1990), followed by negative real interest rates (2000-2020), which slowly fade away (past the right end of the graph) when the economy settles back permanently lower inflation and nominal interest rates. Until the next shock, that is. (Maybe this graph is right!)

But a reading that higher interest rates *cause* the disinflation would be profoundly misleading. Inflation declines initially because expected interest rates decline in the far future, *despite*, not because of, the short-term interest-rate rise. Indeed, the positive interest rates drag inflation up from even more negative values. If you want less inflation in this model, lowering interest rates immediately—the negative of Figure 2—is an even more powerful tool. There is nothing in the mechanics of this model that resembles standard intuition, that high real interest rates drive inflation down. Beware causal readings of impulse-response functions!

If we want a negative effect of interest rates on inflation, without a contemporaneous fiscal shock that's really doing the work, then, we will need to add ingredients beyond sticky prices.

5.3 Fiscal Requirements for Adaptive Expectations Models

The adaptive expectations model (5),

$$\pi_t = (1 + \sigma\kappa)\pi_{t-1} - \sigma\kappa i_t. \tag{67}$$

produces standard intuition. Higher nominal interest rates lower inflation. Inflation or deflation left unchecked will spiral away.

Debt still accumulates by (16),

$$\rho v_{t+1} = v_t + i_t - \pi_{t+1} - \tilde{s}_{t+1}. \tag{68}$$

This is an identity, unaffected by expectations. Higher ex-post real interest rates $i_t - \pi_{t+1}$ with no change in surpluses would lead debt to spiral upward, violating the transversality condition or other bounds that define an equilibrium.

If we ask the adaptive expectations model, "What is the response of inflation to a permanent unexpected rise in the interest rate, *with no change in fiscal surpluses*?," the downward spiral does not answer that question. There is no answer to that question. The central bank cannot permanently raise interest rates without higher fiscal surpluses. There are no multiple equilibria, which we can select based on fiscal backing. (67) alone determines the inflation path, and if that path does not satisfy the present value debt equation, or (68) and the transversality condition, the path can't be an equilibrium

In reality, central banks would eventually give in and abandon an interest rate policy if a debt spiral emerges. Friedman (1968), describing a reduction in interest rates, wrote that the central bank would eventually give in out of distaste for exploding inflation. Here, it is forced to give in by exploding debt and deficits.

Pegs and spirals are good conceptual experiments, but not reasonable policies. The most standard model that produces a disinflation from a higher interest rate combines adaptive expectations with a Taylor rule. A positive shock to the Taylor rule starts an inflation decline, and then the Taylor rule automatically lowers the interest rate to steady the incipient spiral. However, this model also requires tighter fiscal policy to pay higher interest costs on the debt. Without such support, the disinflation does not occur.

It is easiest to exhibit this behavior in the continuous time version of the model. Uniting the continuous time versions of (67) and (68) with a Taylor rule,

$$\frac{d\pi_t}{dt} = \sigma\kappa(i_t - \pi_t) \tag{69}$$

$$\frac{dv_t}{dt} = rv_t + i_t - \pi_t \tag{70}$$

$$i_t = \phi\pi_t + u_t. \tag{71}$$

(I retain for now the assumption of short-term debt.) At time 0, the monetary policy shock u_t rises suddenly and unexpectedly from 0 and stays at the constant value $u_t = u_0$. Substituting (71) in (69), inflation dynamics become

$$\frac{d\pi_t}{dt} = -\sigma\kappa(\phi - 1)\pi_t - \sigma\kappa u_0.$$

The solutions for inflation, interest rate, real rate, and debt are

$$\pi_t = -\frac{1}{\phi - 1}\left[1 - e^{-\sigma\kappa(\phi-1)t}\right]u_0$$

$$i_t = -\frac{1}{\phi - 1}\left[1 - \phi e^{-\sigma\kappa(\phi-1)t}\right]u_0$$

$$r_t = e^{-\sigma\kappa(\phi-1)t}u_0$$

$$e^{-rt}v_t = \frac{e^{-[r+\sigma\kappa(\phi-1)]t} - 1}{r + \sigma\kappa(\phi - 1)}.$$

Figure 6 presents this result. The nominal and hence real interest rate rises, and inflation starts on its downward spiral. Following the Taylor rule, the interest rate swiftly follows inflation down, and the economy trends to the new lower inflation rate. This is a standard story of the 1980s, for example.

However, the real interest rate is positive throughout the episode. It does not satisfy our fiscal condition that the average real interest rate must be zero. Thus, with no greater surpluses, the greater interest costs on the debt are simply rolled over, and debt increases without bound. The transversality condition $\lim_{T\to\infty} e^{-rT}v_T = 0$ is violated, as shown.

Therefore, this simulation does not answer the question, what can the central bank do by

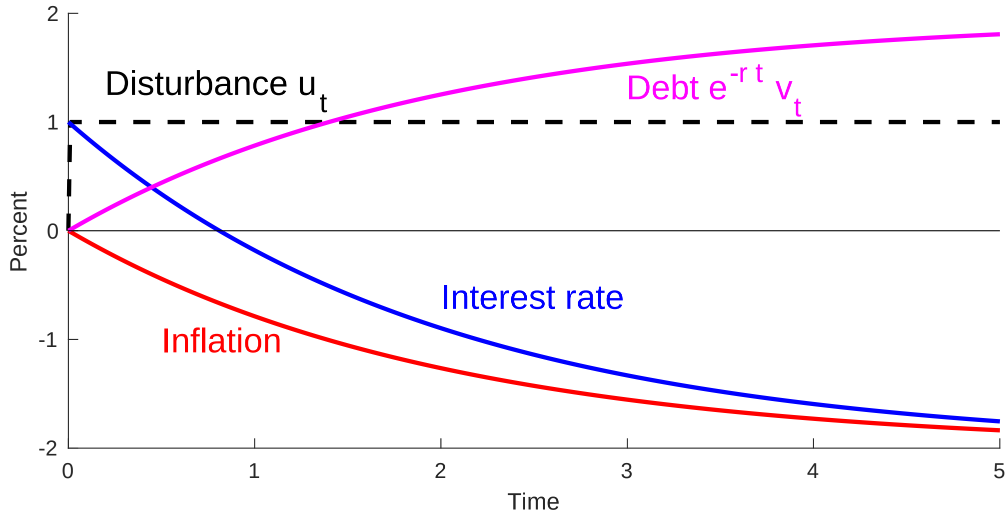

Figure 6: Response to a 1% permanent monetary policy shock in the adaptive expectations model with a Taylor rule. Parameters $\sigma\kappa = 1$, $\phi = 1.5$, $\rho = 0.01$.

itself, without fiscal support? To achieve this outcome, fiscal policy must increase surpluses to pay the interest costs. If this is the story of the 1980s, the story is a joint monetary-fiscal stabilization, with surpluses rising (as they did), not a story of monetary policy acting alone.

5.4 Adaptive Expectations With a Fiscal Constraint

In response to our question, the effect of interest rates on inflation *holding fiscal policy constant*, the adaptive expectations model cannot realistically produce a permanent inflation change, by any path of interest rates .

This point is again easier to see in the continuous time version of the model. Denoting $r_t \equiv i_t - \pi_t$, (69)-(70) and the transversality condition are

$$\frac{d\pi_t}{dt} = -\sigma\kappa r_t \tag{72}$$

$$\frac{dv_t}{dt} = rv_t + r_t \tag{73}$$

$$\lim_{T\to\infty} E_t e^{-rT} v_T = 0. \tag{74}$$

The symbol r without subscript represents the steady state real interest rate, and point of linearization. The symbol r_t represents variation of the real interest rate around this steady state value. Normally the surplus $\tilde{s}_t$ appears on the right hand side of (73) but I omit it as the question holds fiscal policy constant. We are calculating a response function, with all variables zero before time 0, and a single unexpected shock to a new interest rate path at that date.

The adaptive expectations model is determinate: Given the interest rate path, there is only one equilibrium inflation path. Thus, rather than select among equilibria, fiscal policy creates a constraint on the set of interest rate paths that the central bank can follow, in order not to create a debt spiral. A nominal rate path gives rise to one inflation path, so is equivalent to a chosen real rate path. It is simpler to look at the real rate choice and then find the implied inflation and nominal rates.

Equations (73) and (74) imply the familiar restriction that the average interest cost is zero

$$0 = \int_{\tau=0}^{\infty} e^{-r\tau} r_\tau d\tau \tag{75}$$

This is now a restriction on the real interest rates that the central bank may choose.

Given a real interest rate path, the solution to (72) is backward-looking,

$$\pi_t = -\sigma\kappa \int_{\tau=0}^{t} r_\tau d\tau \tag{76}$$

Inflation is proportional to the sum of all past real interest rates.

Define the long-run inflation rate $\pi_\infty \equiv \lim_{T\to\infty} \pi_t$, so

$$\pi_\infty = -\sigma\kappa \int_{\tau=0}^{\infty} r_\tau d\tau. \tag{77}$$

To lower long-run inflation in (77), then, the central bank picks a real rate path $\{r_t\}$, subject to the fiscal constraint (75). Given the real rate path, we find inflation from (76), and the nominal rate from $i_t = r_t + \pi_t$.

We see right away that *in the limit $r \to 0$, the adaptive expectations model cannot produce any permanent inflation or disinflation—a value of π_∞ other than zero—in the absence of a change in fiscal policy.* The right hand sides of (77) and (75) are the same. Positive real interest rates that push down inflation also build up the debt. They must be balanced by negative real rates to bring down the debt, but those real rates raise inflation right back to where it was.

A positive steady state real rate $r > 0$ offers an apparent avenue for permanent disinflation. But, since r is small, this result is fragile, and the resulting policies are unrealistic. Equation

(75) downweights real rates in the future. So, to lower long-run inflation, the central bank must first *lower* real interest rates, building *up* inflation, to produce a period of low interest costs, which lowers the debt. Then the central bank turns around and raises real interest rates, driving inflation down, using the accumulated savings on the debt to pay the higher interest costs. The weighted integral in (75) allows a longer period of future high interest rates than the initial period of low interest rates, which then overall drives inflation down relative to its initial value.

But with small r, the opportunity requires large swings in rates and inflation to produce a small permanent reduction in inflation. Moreover, the period of low rates and high inflation must come first, before the final period of high rates and lower inflation. Imagine a central banker announcing that the plan for combatting inflation is first to make inflation worse, to drive down the debt, and only then turn around to cure the initial inflation plus the deliberate addition to inflation. This is highly unrealistic, and not at all consistent with the classic intuition we are trying to rescue.

When we pose the question as, "Can higher interest rates lower inflation, *without a change in fiscal policy*," not even the most classic adaptive expectations model, which fits the narratives of central bankers and the policy world, can do it.

5.5 A Model with Long-Term Debt that Produces a Negative Effect

Figure 7 offers a simulation of the only current model I know of in which higher interest rates with no change in fiscal policy produce a negative short-run inflation effect.

The model is

$$x_t = E_t x_{t+1} - 0.5(i_t - E_t \pi_{t+1}) \tag{78}$$

$$\pi_t = E_t \pi_{t+1} + 0.5 x_t \tag{79}$$

$$\rho v_{t+1} = v_t + r^n_{t+1} - \pi_{t+1} - \tilde{s}_{t+1} \tag{80}$$

$$E_t r^n_{t+1} = i_t \tag{81}$$

$$r^n_{t+1} = 0.9 q_{t+1} - q_t. \tag{82}$$

Here I use the full rational-expectations model, i.e. including the $E_t x_{t+1}$ term in (78), as analytic solutions are not insightful. I include long-term debt with a geometric maturity structure. The face value of zero coupon bonds of maturity j, $B_t^{(j)} = (1-\omega)\omega^j B_t$ declines at rate $\omega = 0.9$. The

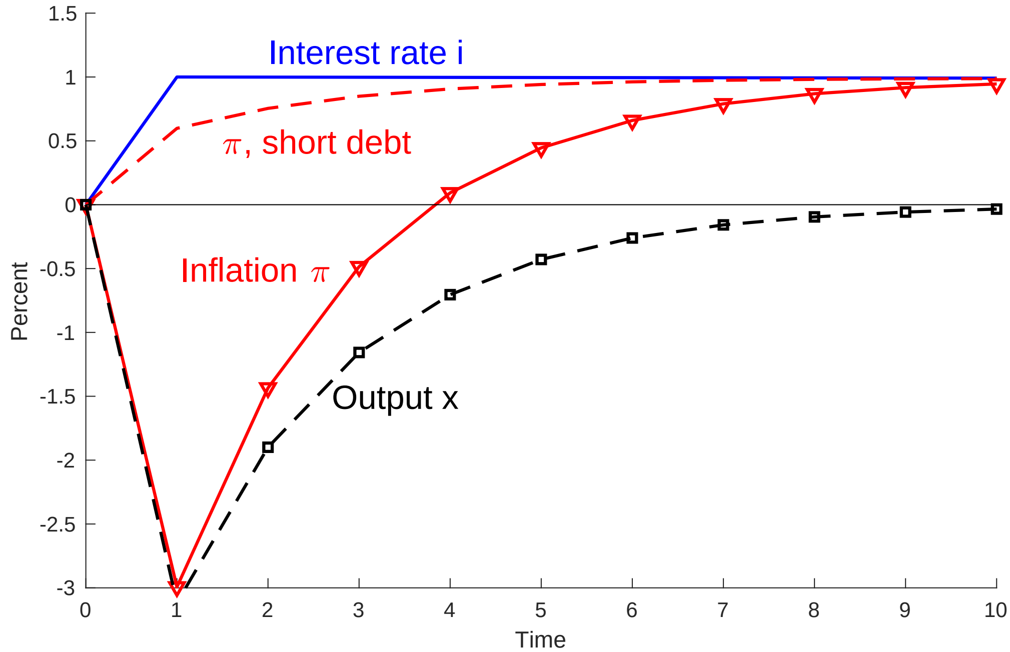

Figure 7: Response of inflation to an interest rate shock, and no change in fiscal policy, with long term debt. In the base case, debt has a geometric maturity structure, decaying at rate 0.9^t with $t =$ maturity. The short-term debt case is one-period debt only.

symbol v_t is the real market value of nominal government debt. The symbol r^n_{t+1} represents the ex-post nominal return on the portfolio of government debt. Equation (81) prices long-term bonds with the expectations hypothesis. Equation (82) links the log price q_t of the government bond portfolio to its rate of return. This is a simplified version of the model in Cochrane (2021), which in turn simplifies Sims (2011).

Again, I raise interest rates, I leave fiscal surpluses unchanged, and I calculate the inflation and output responses. Inflation declines temporarily! Inflation then rises in the long run, fulfilling long-run neutrality. This is the pattern we have been seeking.

Sims (2011) calls the pattern of Figure 7 “stepping on a rake” and offers it as a parable of the 1970s inflation cycles, in which higher interest rates temporarily lowered inflation, but inflation came back. We can call the pattern “unpleasant interest-rate arithmetic,” a successor to Sargent and Wallace (1981) unpleasant monetarist arithmetic. (Sargent and Wallace focus on seignorage in a model with real debt, in which the central bank controls money supply. In this model, there is no seignorage or money, government debt is nominal, and the central bank follows an interest rate target.) Unpleasant interest-rate arithmetic is here a negative sum, or inequality proposition: The central bank gets more long-run inflation than it saves in short-run inflation.

(Woodford (2001) has the first statement of this effect.)

Long-term debt is the crucial innovation relative to the simple models of previous figures, and its inclusion produces the negative sign. With long-term debt, but no ability to change surpluses, the central bank can lower inflation now, but by raising inflation later. Raising long-run interest rates, and thus long-run inflation, devalues long-run debt. Since surpluses haven't changed, lowering the value of long-term debt must raise the value of short-term debt. But short-term debt can only become more valuable via a lower price level.

You can see the mechanism directly with flexible prices. The real interest rate is a constant r, and there is a geometric maturity structure in which debt of maturity j has face value $\varphi Be^{-\varphi j}$. There is a constant nominal interest rate i. The government debt valuation equation, stating that the market value of debt equals the present value of surpluses, reads

$$\frac{\varphi B\int_{j=0}^{\infty}e^{-ij}e^{-\varphi j}dj}{P}=\frac{\varphi}{i+\varphi}\frac{B}{P}=E_t\int_{j=0}^{\infty}e^{-rj}s_{t+j}dj.$$

Imagine an unexpected permanent rise of the nominal interest rate from i to i' with no change in surpluses. The price level then jumps by

$$\frac{P'}{P}=\frac{i+\varphi}{i'+\varphi}$$

With perpetuities, $\varphi=0$, this is a powerful effect. A rise from $i=1\%$ (0.01) to $i=2\%$ (0.02) cuts the price level in half. A 1% larger inflation then starts right away. Shorter maturity debt gives a smaller response. For example, if debt has a 4 year half life (the face value of debt declines to e^{-1} by a 4 year maturity), then $\varphi=0.25$, so a rise from 1% to 2% nominal rate implies $0.26/0.27\approx 10\%$ price level drop. That is still a large cumulative disinflation inflation from a 1% interest rate rise.

As with the short-term debt case, there is nothing essential about a permanent interest rate shock to generating this example. Any persistent shock will also lower long-term bond prices and raise long-term inflation, raising the value of short-term bonds. The initial disinflation is smaller, but still present.

A Taylor-type rule, in which the interest rate reacts to inflation, automatically adds this sort of response to the inflationary effect of fiscal or other shocks. In this way, a Taylor-type rule spreads the inflation of such shocks over time. With the forward-looking Phillips curve, smoothing inflation over time reduces output volatility, so it is a good policy.

Taylor emphasizes that his rule works well in a variety of models, and that robustness rather than strict optimality in a particular model is its virtue. Raising interest rates in response to inflation can eliminate the instability of the adaptive expectations model; it can eliminate the indeterminacy of the rational expectations model; and it can reduce volatility in the rational-expectations fiscal-theory model.

The graph is attractive, but this mechanism of this model is more limited and quite different from conventional intuition about monetary policy.

This negative effect of interest rates on inflation in this model only holds for *unexpected* interest rate rises. Lower inflation breaks out when a higher interest rate is announced, not when it happens. But maybe interest rates also lower inflation when they actually rise, not just when they are announced.

The effect vanishes when governments borrow short term, as illustrated by Figure 7 in the line marked "π, short debt." More generally, the size of the negative effect depends on the maturity structure of debt. Whether debt maturity is long enough to produce quantitatively important effects, and whether the effect of interest rates on inflation varies with the maturity structure of the debt as the model predicts are important unanswered questions.

The negative effect requires long-lasting interest rate increases, that raise long-term nominal bond yields. We should check that higher interest rates lower inflation more when they are persistent, and only when they propagate to the long-term yield curve, lowering long-term nominal bond prices.

Stickier prices *reduce* the negative effect of interest rates on inflation. With sticky prices, the higher real interest rates add interest costs of the debt, an inflationary force. The negative effect of interest rates on inflation is strongest in the flexible price case. In this model there is no connection at all between sticky prices and the central non-neutrality that drives inflation down, contradicting standard intuition in which sticky prices are the central non-neutrality that gives monetary policy its power.

In this model, inflation declines immediately and then recovers, where in the common intuition higher interest rates lower future inflation gradually with "long and variable lags." Perhaps further frictions or another Phillips curve can produce such a response, but that also needs to be shown.

These features are not necessarily counterfactual. They are unknown. Nobody has looked to see if the negative effect of interest rates on inflation is quantitatively linked to announcement, maturity, persistence, price stickiness, and fiscal events as the model predicts. Looking would be a valuable empirical project.

But the model's mechanism and these features are completely different from standard intuition, that higher real interest rates depress demand, and through a Phillips curve lower future inflation. It does not produce something like the adaptive-expectations dynamics in the short run, which then turn around and become stable when some suitable friction or information problem is resolved. And it will be a long time before we write opeds, Fed chairs explain, and we teach to undergraduates that the central mechanism by which the central bank can temporarily lower inflation is to rearrange the real payoffs to different maturities of nominal government debt!

6 Fiscal Constraints

The calculations so far add inflation dynamics produced by particular models together with fiscal constraints on what any model can do. A look directly at those fiscal constraints clarifies the challenge.

With long-term debt, in response to a shock that happens at time 0, and with sticky prices so that there are no jumps in the price level, the decompositions (19) and (43) generalize to

$$\int_{t=0}^{\infty} e^{-(r+\varphi)t}\pi_t dt = -\int_{t=0}^{\infty} e^{-rt}\tilde{s}_t dt + \int_{t=0}^{\infty} e^{-rt}(1-e^{-\varphi t})(i_t - \pi_t)dt. \tag{83}$$

(Cochrane (2023) section 3.5.) Here φ models debt with a geometric maturity structure, $B_t^{(t+j)} = \varphi B_t e^{-\varphi j}$. I impose the expectations hypothesis; more generally the expected nominal return on the government bond portfolio appears in place of the interest rate.

This equation unites the three important fiscal mechanisms. On the left, a rise in expected future inflation devalues the claims of long-term bondholders, and deflation gives them a windfall. This mechanism takes the place of a price level jump, but acts via more plausible drawn out inflation. On the right, we have surpluses, but those are zero for my definition of "monetary policy." Finally, we have the effect of interest costs on the debt. For a given path of interest rates, this equation restricts the path of inflation. For a rational expectations model with multiple equilibria, it determines the equilibrium inflation path. For an adaptive expectations model, it restricts what interest rate paths the central bank may follow.

With no change in surpluses and for short term debt $\varphi = \infty$, expression (83) simplifies to

$$0 = \int_{t=0}^{\infty} e^{-rt}(i_t - \pi_t)dt. \tag{84}$$

With short-term debt and no price-level jumps, inflation cannot devalue outstanding debt directly. Then *the long-run average of real interest rates and interest costs on the debt must be unchanged*. For *any* model of inflation dynamics, the average weighted difference between inflation and interest rates must be zero.

For perpetual debt $\varphi = 0$, (83) simplifies to

$$\int_{t=0}^{\infty} e^{-rt}\pi_t dt = 0. \tag{85}$$

In this case, both government and bondholders are fully insured against changes in real interest rates, as neither has to roll over existing debt. With long-term debt, however, inflation devalues long-term nominal promises, and deflation delivers a windfall to bondholders. Thus, with no change in surpluses, *the long-term average of inflation itself must be unchanged.*

In these expressions, and the intermediate cases in between, *unpleasant interest-rate arithmetic is generic*. *Any* model in which higher interest rates lower short-run inflation must produce higher subsequent inflation. Monetary policy is limited to moving inflation around through time, lower at some time in return for higher at another time.

With short-term debt in (84), inflation averages zero around the nominal interest rate path. With long-term debt in (85), inflation itself averages zero around zero response, which makes it a little easier to get lower inflation in the short run in return for more inflation later, as I did in Figure 5. But *no* model can produce uniformly negative inflation from higher nominal interest rates.

7 Evidence and Experience

Stability and determinacy under an interest rate peg challenge conventional doctrines, but they are not obviously contrary to historical experience. We have just seen something close to an interest rate peg. From 2008 to 2016 in the US, from 2008 to 2022 in Europe, and from 1995 to

2022 in Japan, interest rates were effectively stuck at zero. They could not move much in the downward direction, they did not move in the upward direction. Central bankers gave "forward guidance" that interest rates would not move, at least not promptly and more than one for one with observed inflation.

When these countries hit the zero (or effective lower) bound, many economists, commenters, central banks, and international monetary institutions, armed with thinking that I represent by the adaptive expectations model, predicted a "deflation spiral" as depicted in the left-hand panel of Figure 1. They continued to worry that a spiral would break out during the decades of zero interest rates. New-Keynesians had a different worry. They had warned for 20 years that a zero bound would lead to volatile multiple-equilibrium inflation as depicted in the right-hand panel of Figure 1. Yet neither unstable spirals nor additional multiple-equilibrium inflation volatility broke out. Inflation just batted around 1-2% the whole time, if anything less volatile than it had been when central banks could move interest rates.[6] The view that inflation can be stable and determinate, even under an interest rate peg, looks a lot more plausible after this episode.

Long-run fiscal policy was not in great shape in the 2010s, but there wasn't much bad *news*. Moreover, real interest rates and interest costs on the debt fell, unexpectedly relative to long-term bond prices in 2007, were negative for most of the period, and were forecast or feared to remain negative pretty much permanently. (Recall r^* and $r < g$ debates.) Low interest costs on the debt act like surpluses (see (19)) to hold down inflation. At 100% debt to GDP, 1-2% negative real interest rates in place of 1-2% positive real interest rates have the same deflationary effect as 2-4% of GDP additional primary surpluses.

From 2021 through 2022, analysts similarly worried that by reacting slowly to inflation in 2022, with interest rates below rising inflation, central banks made inflation worse, and a positive inflation spiral threatened. They worried that only a sustained dose of interest rates higher than current and past inflation, with a steep recession, could bring inflation down, repeating the early 1980s. (See, for example, most of Bordo, Cochrane, and Taylor (2022).) Yet, from 2022 to summer 2023, inflation declined with interest rates still below current inflation, as it did previously in 1948, 1951, and 1975. On the other hand, the outbreak of inflation following a huge $5 trillion fiscal stimulus, and its gentle decline afterwards despite interest rates below inflation, mirrors the simplest fiscal-theory response to a fiscal shock.[7]

[6] I summarize here more detailed analysis in Cochrane (2017), Cochrane (2018), and Cochrane (2023) Ch. 20.
[7] See Cochrane (2022a), Cochrane (2024) for simulations.

But if we accept this evidence that inflation is stable around a zero peg, we have to accept its uncomfortable implication that inflation will be stable around a higher peg, and raising the peg, with no change in fiscal policy, will eventually raise inflation.

What about historical pegs that preceded inflation? These episodes are central to Friedman's (1968) argument that pegs are unstable. Well, quiet (the opposite of volatile) inflation under a peg also requires sustainable fiscal policy. Most governments with interest rate pegs and spiraling inflation used the peg to hold down interest costs of the debt in a time of high and rising fiscal problems. One can read such pegs as part of a financial repression regime designed to boost demand for government debt and hold down interest costs, not a market interest rate with solvent fiscal policy. The US interest rate peg of the 1940s and early 1950s was explicit in that aim, and came with restrictions on private interest rates. And even so, that peg lasted remarkably long, relative to models that predict instability or indeterminacy. The peg lasted through nearly 20% inflation in 1947, which also faded away with no spiral. Those interest rate pegs also typically included exchange rate controls and trade restrictions.

Econometric analysis gives weak evidence that higher interest rates lower inflation. When monetary policy shocks reduce inflation at all, such reductions are delayed more than a year and then are small and marginally statistically significant or insignificant. Ramey (2016) provides a comprehensive review and update. See especially Ramey's Figures 1, 2, and 3, with small, sometimes positive, delayed, and insignificant inflation effects, and note the disappearance of any effect after 1983. Romer and Romer (2023) Figure 5 and Figure 7 summarize the results of the narrative approach. After a higher interest rate or monetary tightening, inflation rises slightly for a year, then gently declines, but just about two standard errors below zero. Moreover, these estimates include credit and quantitative shocks in the 1950s and 60s, and the last shocks are in 1981 and 1988 (Table 2). The "price puzzle" that higher interest rates want to raise inflation dogged VAR analysis for decades, requiring careful orthogonalization to produce even this mild inflation reduction. It may have been trying to tell us something.

The empirical evidence that higher interest rates lower output and employment is much stronger. The question here is whether they lower *inflation*.

Several other considerations weaken even this much evidence. Most of all, none of these estimates tries to orthogonalize monetary to fiscal policy, and so do not answer our basic question. That omission is natural, as without a fiscal theory in mind, nobody thought that it was important to do so. It is plausible that fiscal contractions accompany monetary contractions. If nothing else, fiscal policy may tighten to pay higher interest on the debt. But fiscal policy is also likely to tighten to control inflation just as monetary policy does. Fiscal stimulus is a common response to deflation, and austerity is a common response to inflation. The early 1980s included a major Social Security reform, large tax reforms, spending controls, and growth-enhancing deregulation. Sharply rising primary surpluses in the late 1980s and 1990 paid higher interest costs on the debt and paid for the real interest costs and bondholder windfall of the 1980s. Conversely, failed stabilizations around the world often include an attempted monetary tightening that is not accompanied by fiscal and microeconomic reform.

Nakamura and Steinsson (2018) eloquently summarize the hard identification and computation troubles of contemporary empirical work. To identify an interest-rate shock, as opposed to an endogenous response, we must assume that regressions control for all variables that the central bank sees and responds to. But central banks watch more variables than we can ever include. Nakamura and Steinsson use 9/11 as a clear example: The Fed lowered interest rates after the terrorist attack, likely reacting to its consequences for output and inflation. But VARs, even those that include interest rates to capture market expectations, register the event as an exogenous shock.

Nakamura and Steinsson also emphasize the oft-forgotten lesson of 1980s time-series econometrics, that estimating 10-year responses from monthly VARs raised to the 120th power is dangerous. Such responses rely crucially on one having in hand the persistent state variables, and accurately measuring their near-unit-root dynamics. Direct regressions ("local projections") of variables of interest on shocks are potentially more reliable, but less parsimonious and therefore less well measured. They force one to recognize just how few non-overlapping samples we have at the horizons of interest.

Is there even such a thing as an exogenous policy shock? Central banks always say they are reacting to something. At best, central banks might react to something that has no effect on inflation or (separately) output. Such reactions would qualify as exogenous for measuring the latter responses. But nobody has taken this old suggestion seriously, preferring to try to find ephemeral shocks that are orthogonal to everything. (It is neither necessary nor sufficient for an interest rate shock to be unexpected by asset market participants in order to identify the output and inflation response.)

No matter how measured, responses to shocks completely miss the central ingredient of an intervention such as the early 1980s. By design, "shocks" are deviations from a rule, not changes in rule or "regime" itself that may durably change expectations. If the art of reducing inflation is to convince people that a new regime has arrived, then the response to any monetary policy "shock" orthogonal to a stable "rule" completely misses that policy. Clarida, Galí, and Gertler (2000) famously attribute the end of 1970s inflation to a change in the rule, a change in ϕ_π in $i_t = \phi_\pi \pi_t + \phi_x x_t + u_t$, not to a big shock u_t.

The vast majority of current estimates also find that identified monetary policy shocks lead to short-lived interest rate responses. They therefore do not measure the effect of the persistent interest rate movements that we need to evaluate the long-term debt mechanism, or to examine the long-run Fisher prediction that inflation should eventually rise. They do not measure the effect of the persistent high real interest rates of the one great episode, the early 1980s.

Uribe (2022) evaluates via an identified VAR the "neo-Fisherian" hypothesis that persistently higher interest rates raise inflation. He identifies a permanent shock as one that increases both the nominal interest rate and inflation in the long run. He finds that shock *raises* inflation and nominal interest rates in the short run. Similarly, Schmitt-Grohé and Uribe (2022) find that permanent interest rate shocks *depreciate* the currency. In both papers, transitory interest rate movements lead to the standard disinflation and appreciation.

Their results are good news for Figure 2, in which a persistent interest rate rise raises inflation even in the short run. But they are bad news for Figure 4 in which a transitory rate rise also raises inflation, and Figure 7 in which a permanent interest rate rise lowers short-run inflation with long-term debt. They are also bad news for the view that the 1980s succeeded via a highly persistent monetary policy shock. However, Schmitt-Grohé and Uribe also did not attempt to find monetary policy shocks orthogonal to fiscal policy changes, so their results do not directly bear on our question.

In sum, the empirical literature evaluating whether higher interest rates lower inflation is tenuous despite enormous effort, and is not addressed at our central questions. Thus, current empirical work leaves open a possibility: The widespread faith that higher interest rates reliably lower inflation, without fiscal help, may simply not be true.

8 Paths to Follow

Despite 50 years of intertemporal general equilibrium macroeconomics since Lucas (1972a), we still don't have well-agreed-on economic model of the conventional belief that higher nominal interest rates, without a change in fiscal policy, can lower inflation even temporarily. The best I have been able to show here – the rational-expectations fiscal-theory model with long-term debt – is far from the power of and works by utterly different mechanisms than standard intuition, and is evidently very far from widely accepted.

Naturally, one wants to add additional frictions and model ingredients in the hope of producing a stronger effect, and one that perhaps matches standard intuition. But it's not as easy as it looks – or someone would have done it already.

One line of new-Keynesian literature is actively exploring a return to adaptive expectations, or complex learning and expectation formation schemes which have a similar effect.[8] This line of work has not been extended to ask our question – whether higher interest rates can lower inflation without fiscal help – but it is a natural avenue to explore.

Most discouraging, we have learned that even in the traditional adaptive expectations model, higher interest rates cannot lower inflation without a fiscal tightening. Someone has to pay higher interest costs and a disinflation-induced windfall to bondholders.

In my little tour, adaptive or backward-looking expectations change the sign of the response of inflation to interest rates by changing the basic stability and determinacy properties of the model. That's major surgery, not an epicycle. It requires an eigenvalue to cross one. Small parameter changes around the frictionless rational benchmark won't have the required effect. Conversely, the basic sign and stability of the backward-looking expectations model must change along the way back to that benchmark. Thus, if adaptive expectations are the basis for a negative sign, then we have to say that the fundamental operation of monetary policy *cannot* be

[8] Prominent examples include Gabaix (2020) (see comments at Cochrane (2016)), García-Schmidt and Woodford (2019), and Bianchi-Vimercati, Eichenbaum, and Guerreiro (2022). Examples with learning: Marcet and Nicolini (2003) study recurrent hyperinflations with learning. They explain why bouts of hyperinflation break out in the context of large and stable inflation, with no large changes in seigniorage. In Orphanides and Williams (2005), learning contributes to inflation dynamics, and allows private sector inflation expectations to provide useful information to the central bank. In Williams (2006), learning can undermine the desirable properties of a price-level target, that people expect a period of inflation to follow deflation. In Bullard and Mitra (2002), learning changes the optimal policy rule. Sargent (2001) "Conquest of American Inflation" is a classic example in which learning by the Fed takes center stage. Evans and Honkapohja (2001) consider learning models broadly. McCallum (2009a), McCallum (2009b), argue that the new-Keynesian explosive threat equilibrium selection is learnable, and fiscal theory is not; Cochrane (2009) argues the opposite.

understood in a simple fully economic benchmark or anything close to it. There *is no* simple model like $MV = PY$ that explains the basic ides of price level determination under interest rate targets. Less than rational expectations are surely a reasonable ingredient for episodes, regime changes, or transient dynamics. But if they are a minimum necessary ingredient for the basic sign and stability properties of monetary policy, that undermines the robustness of our analysis.

It is possible that more complex models produce the desired sign without changing the basic stability and determinacy properties of the model. But now you can see the challenge, and you know a major question to ask of any apparently successful model.

Whether people are truly "rational" and what that term even means does not really matter here. The issue is expectations in the model, not expectations in the world. The issue is whether expectations should be "model-consistent," whether the expectations *of* a simplified model are the same as the expectations *in* that model. (Lucas (1991)). Related, the key point of the "anchored" expectations view I outlined above is fundamentally whether expectations in the model are *reactive*. Rational expectations react instantly to model shocks, as the model forecast changes. "Anchored" expectations in the policy view do not move.

You can't tell just by looking out the window. Rational expectations may seem adaptive. Expectations must be, in equilibrium, functions of variables that people observe, and past inflation is a likely to be prominent among those variables. The point of "rational expectations" is only that those forecasting rules are likely to change when the policy rule changes, as Lucas (1976) famously pointed out[9] in his "critique." Adaptive expectations may even be model-consistent, until you change the model.

For example, consider the debate over 2022-2023 inflation. Survey, bond market, forecaster, and central bank expectations were slow to move as observed inflation picked up. One might proclaim them adaptive, a slow-moving average of lagged inflation, as Friedman speculated in 1968, $\pi_t^e = \sum_{j=1}^{\infty} \alpha_j \pi_{t-j}$. But the same expectations could well have been rational: Output was temporarily higher than the reduced post-pandemic potential, so current inflation was higher than rationally expected future inflation ($\pi_t = E_t \pi_{t+1} + \kappa x_t$). In both interpretations, expected inflation is lower than current inflation. Tests are hard. You can't just look at in-sample expectations to proclaim them rational or not.

[9]Sargent (1971) and Lucas (1972b) made the point earlier in the Phillips curve context. They showed that in a $\pi_t = \alpha \pi_{t-1} + \kappa x_t + u_t$ regression, $\alpha < 1$ coefficients can easily occur even when the true Phillips curve has rational expectations. The coefficient α shifts if the monetary policy process shifts.

"Rational," "model-consistent," or "reactive" expectations do not mean "clairvoyant," or even particularly good forecasts. Rational people may be quite bad at forecasting inflation, in life and in models. With noisy signals, rational forecast errors can even be serially correlated (Coibon and Gorodnichenko (2012)). Inflation, like stocks, may simply be hard to forecast. The Fed and sophisticated market participants are pretty bad at forecasting inflation too. One can regard rational, model-consistent, expectations as simply a humility principle reflecting the fact that economists armed with structural models do not outperform the supposedly irrational agents we survey.

To move back towards adaptive expectations and consequent instability, we would also need to face the failures of the adaptive expectations / unstable approach during the zero bound era, in 1970s stagflation, in the relatively rapid end of inflation in 1982, in the success of 1990s inflation targets, in the ends of hyperinflations (Sargent (1982)), and in the failure of inflation to spiral upwards in 2022-2023, among other episodes.

One is drawn to add other model ingredients. Surely in the DSGE smorgasbord there are enough ingredients to come up with a temporary negative sign. That is, I think, the right answer, in the sense that we need to know if it works. My point is, it has not yet been done—and especially, it has not been done with the kind of clarity, simplicity, economic rigor, transparency, and tractability that Lucas brought to the non-neutrality of money. Many model-implied monetary-policy response functions have been computed of course, but not many yet hold fiscal policy constant in an interesting way, and few look into the footnote about lump-sum taxes to see just what those are and to what extent inflation reduction comes from an implicit fiscal contraction. The literature that puts fiscal theory in explicit DSGE models is an exception; see for example most recently Bianchi and Melosi (2022), Chen, Leeper, and Leith (2021), and Leeper (2021). This literature is explicit about fiscal-monetary coordination. However, it has focused on switching between active-fiscal and active-money regimes, and so far has not addressed the question in this paper, whether higher interest rates can lower inflation with no change in fiscal policy.

One easily jumps to consider capital with adjustment costs, financial frictions, credit constraints, portfolio adjustment costs, money, liquidity effects in government bonds, additional price and wage-setting frictions, strategic complementarities, individual heterogeneity, and other model complications. But the negative response of inflation to interest rates should be a robust and deeply rooted phenomenon, one that will not vanish if, for example, the US changes working capital financing traditions or the downpayment rules on mortgages. In his Nobel Lecture, Lucas (1996) cites David Hume for understanding the neutrality and non-neutrality of money in

1752. Velde (2009) documents a beautiful non-neutrality episode in 1724 France, with a monetary and financial system utterly unlike our own.

It is likely to be possible to find in these generalizations *sufficient* conditions to deliver the negative sign, with enough model complications. Our goal though is the minimum *necessary* conditions, that apply most broadly and robustly, that we can put at the basis of any good theory of inflation and monetary policy under interest rate targets. That is a harder goal. Again, Lucas (1972a) is a great example. In his economy, the flexible price version leads to super-neutrality: An increase in money just raises the price level. Bob put in *one* "friction," imperfect information about aggregates, leading to a confusion between relative and aggregate price movements.

Adding to the difficulty, standard new-Keynesian models have struggled to produce the kind of response one sees in VAR estimates, and that represent traditional beliefs, even allowing a contemporaneous "passive" fiscal response. In the estimates and common beliefs, after an interest rate rise inflation either rises or stays constant for a year, and then falls over time. Interest rates, like money, act with long and variable lags. In the theory, inflation jumps down, and rises over time (Ball (1994) first drew attention to this contrast. The Phillips curve is $\pi_t = \beta E_t \pi_{t+1} + \kappa x_t$. Higher interest rates lower x_t, which lowers π_t *relative* to $E_t \pi_{t+1}$. Equilibrium selection chooses a π_t lower than it otherwise would have been, but inflation rises over time.)

Current medium-scale new-Keynesian models can reproduce something like standard beliefs and VARs, if not by the standard mechanisms. Christiano, Eichenbaum, and Evans (2005), Christiano, Eichenbaum, and Trabandt (2016), Christiano, Eichenbaum, and Trabandt (2018), and Smets and Wouters (2007), are classic examples. But they include a long list of additional ingredients. Many of those ingredients are also quite different from standard microeconomic specifications, troubling both for the project that microfoundations should be micro founded, and for whether the specifications really are policy invariant. Some of the ingredients which seem crucial to producing the inflation response include: 1) A strong one-period habit in utility, which essentially adds a time derivative on the relation between consumption growth and interest rates. 2) Sticky wages, driven by labor *monopoly* power, in contrast to the usual microeconomic worry about labor monopsony. 3) Firms must borrow money to pay the next period's wage bill in advance. Higher real interest rates then raise marginal costs in the Phillips curve. This raises inflation now π_t relative to expected future inflation $\beta E_t \pi_{t+1}$, but with equilibrium selection and timing rules holding π_t from moving, that means $\beta E_t \pi_{t+1}$ falls. 4) Capital with adjustment costs that depend on investment growth, not the investment to capital ratio as in most microeconomic and asset pricing literature. 5) Wages and prices are indexed, so that those who cannot adjust nonetheless react to inflation. This modification puts lagged inflation in the Phillips curve. But few prices and wages in the US are actually indexed. More recent literature that matches impulse response functions, for example, Auclert, Rognlie, and Straub (2020), adds still more ingredients, in this case heterogeneous agents.

Are all these ingredients *necessary* for the basic sign and pattern of monetary policy in standard New-Keynesian models? And what will we need when we separate monetary and fiscal policy? Those are good and important questions, which have not yet been answered. Authors in this literature (and perhaps editors and referees!) have been good at adding ingredients, not so much in seeing which ingredients can be subtracted but keep the important story intact.

The Phillips curve seems an obvious weak point, and locus of improvement in this quest. In my little models, the Phillips curve is the central source of price dynamics. That point is particularly clear in continuous time, where the IS curve is the same for rational and adaptive expectations. Yet the Phillips curve has not achieved great theoretical and empirical clarity, despite decades of dedicated work.

Its status is ephemeral in policy analysis. The Phillips curve is said to be "flat," $\kappa \approx 0$ and unreliable. Then inflation depends only expected inflation. But expected inflation can't just come from speeches, announcements, or interest rates interpreted as signals of central bank intentions. Signals of what? Eventually some central bank action must lower inflation.

Mankiw and Reis (2002), (2006), (2007) pursue a "sticky information" Phillips curve. It is particularly attractive in this context since they motivate it to reverse the Ball (1994) puzzle and produce inflation that rises over time when output is high. It is essentially a generalization of the Lucas (1972a) model, in which price-setters take more than a period to learn the aggregate price level. They relate output to current inflation relative to a distributed lag of expected inflation rather than the previous period's inflation. It solves the problem that Lucas (1972a) only produces a one-period output movement. However, Mankiw and Reis (2002) merge their Phillips curve with money supply control; Mankiw and Reis (2006) doesn't present any results; and Mankiw and Reis (2007), which merges their Phillips curve with an interest rate rule, doesn't actually produce the desired result. Inflation and output move essentially together, and inflation is high and declining when the level of output is high. Perhaps this attractive ingredient will in the future produce the desired sign and pattern of the inflation response, but it does not yet do so.

In addition to wondering what ingredients to put in, then, perhaps the Phillips curve is one we can take out or avoid. Our goal is to understand the dynamic relationship between interest rates and inflation, $\pi_t = a(L)i_t$. The Phillips curve came from thinking about output and employment *effects* of a given inflation. Lucas (1972a) had a theory of inflation, $MV = PY$. He wanted a theory of how inflation affects output. Interest rate based models reverse the causal logic. They use the IS equation to describe how interest rates move output, and then the Phillips curve to describe how output affects inflation. (With the usual caveats for causal readings of equilibrium conditions.) But the Phillips curve wasn't designed to be the central mechanism for inflation dynamics.

Moreover, Lucas wrote following Friedman, in the view that monetary policy shocks accounted for the majority of business cycle fluctuations. The VAR literature and history since the 1980s emphatically overturn that view. Monetary policy shocks account for tiny forecast errors of output. The 2008 and 2020 recessions were obviously not caused by monetary policy shocks. Understanding how monetary policy contributes to or controls inflation without tying it to a specific output effect is more interesting today than it would have been in 1972.

Consider, then, what kind of inflation dynamics would we observe in an endowment economy? The Phillips curve approach says that with constant output and a constant real interest rate, nominal interest rates translate immediately to expected inflation just as if prices are completely flexible. Are there really no inflation dynamics in an endowment economy, or one with a constant real interest rate? Perhaps somewhere in the new interest in supply chain dynamics, (for example Lane (2022)), reallocation shocks (for example, Guerrieri et al. (2021)) or production networks (for example, Minton and Wheaton (2022), Rubbo (2022), and La'O and Tahbaz-Salehi (2023)) a qualitatively different model of inflation dynamics will emerge.

On the other hand, the goal that higher nominal interest rates reduce inflation, without fiscal help, will still be difficult. First, a 1% higher nominal interest rates must lower the real interest rate by more than 1% to lower expected inflation. A small effect of nominal rates on real rates will not do. Second, the basic fiscal constraints on the path of inflation given interest rates with no change in fiscal policy, (83) remain. Monetary policy without a change in fiscal surpluses is limited to shifting inflation over time.

There is again another possibility: It might not be true. The long-term debt model, with unpleasant interest-rate arithmetic or "stepping on a rake" may be as close as we come to standard intuition. Otherwise, nominal interest rate rises, *with no change in fiscal policy*, may not lower inflation even in the short run.

That leaves us, for practical purposes, to unseat the caveat. Monetary policy has fiscal effects, via seigniorage, via liquidity frictions on government debt generally, via induced fiscal policies to pay interest on debt and windfalls to bondholders, by inducing stimulus in recessions and austerity following inflations. But understanding "monetary policy" only via its contemporaneous or induced fiscal effects obviously leads us to a quite different understanding of its economic mechanism.

9 Conclusion

What is the dynamic effect of *interest rates* on inflation, $\pi_t = a(L)i_t$, in our world of abundant reserves, in which central banks set nominal interest rates, do not control money supplies, do not make equilibrium-selection threats, and cannot directly change fiscal policy? And, of course, after that, how do interest rates then affect output, employment, and other variables?

I have followed one line of thought on these questions to its logically inevitable conclusion: Rational expectations and fiscal underpinnings of monetary policy imply that inflation is stable and determinate in the long run. Those features imply that pegs are possible, and that higher interest rates, without a change in fiscal policy, eventually raise inflation. There may well be a short-run negative effect of interest rates on inflation. I show one such model, but we need to know if there are better yet simple models of that effect. Or, we need to accept the limited power of that model, and that otherwise higher nominal interest rates without changes in fiscal policy raise inflation.

Thus, as I see it, we have made a lot of progress. We're finally at the launch pad, and we have some promising ideas, but elaboration exploration and consensus remain to be achieved.

But all this is novel, sort of known but not widely recognized, or known but controversial. Thus, even these basic questions are, in fact, still up for grabs. These are the good old days. Low-hanging fruit abounds. We're really at the beginning stages where simple models need exploration, not, as it appears, in a mature stage where essentials are settled and all there is to do is to add to the immense stock of complicated epicycles. However, given the state of actual agreed-on knowledge, central banks' proclamations of detailed technocratic ability to manipulate delicate frictions is really not justified.[10]

[10]For a lovely example, see the chart at https://www.ecb.europa.eu/mopo/intro/transmission/html/index.en.html. Lucas (1979) is a good antidote to hubris.

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Online Appendix

A1 Solving the Model for Arbitrary Interest Rates

The model is

$$x_t = -\sigma(i_t - E_t\pi_{t+1}) \tag{A1}$$

$$\pi_t = E_t\pi_{t+1} + \kappa x_t \tag{A2}$$

$$\rho v_{t+1} = v_t + i_t - \pi_{t+1} - \tilde{s}_{t+1} \tag{A3}$$

$$\lim_{T\to\infty} \rho^T v_T = 0 \tag{A4}$$

We want to calculate the impulse-response function for a generic path $\{i_t\}$. All variables are zero until time 1. At time 1 we set off a sequence $\{i_1, i_2, ...\}$. There is no change to surpluses, so $\tilde{s}_t = 0$. Given π_1, the other π_t follow since there is no more uncertainty. Equations (A1)-(A2) give us a set of possible paths of inflation indexed by π_1. We use (A3) and (A4) to choose π_1.

This section establishes the following results for this impulse-response function. For an arbitrary sequence $\{i_1, i_2, ...\}$,

$$\pi_{t+1} = \frac{1}{(1+\sigma\kappa)^{t+1}}(1-\rho)\sum_{j=1}^{\infty}\rho^j i_j + \frac{\sigma\kappa}{1+\sigma\kappa}\sum_{j=1}^{t}\frac{1}{(1+\sigma\kappa)^{t-j}} i_j \,.$$

For an AR(1) $i_t = \eta i_{t-1} + \varepsilon_t$,

$$\pi_{t+1} = \left[\left(\frac{(1-\rho)}{(1+\sigma\kappa)}\frac{\rho}{(1-\rho\eta)} + \frac{\sigma\kappa}{1-\eta(1+\sigma\kappa)}\right)\frac{1}{(1+\sigma\kappa)^t} - \frac{\sigma\kappa}{1-\eta(1+\sigma\kappa)}\eta^t\right] i_1.$$

For $\eta = 1$, i.e. a one-time permanent increase in the interest rate,

$$\pi_{t+1} = \left[1 - \frac{(1+\sigma\kappa-\rho)}{(1+\sigma\kappa)^{t+1}}\right] i_1$$

Now, to derive these results. Eliminating output from (A1)-(A2),

$$E_t\pi_{t+1} = \frac{1}{1+\sigma\kappa}\pi_t + \frac{\sigma\kappa}{1+\sigma\kappa} i_t. \tag{A5}$$

$$\tag{A6}$$

Iterating forward (A5), after the shock at time 1, (for $t \geq 1$),

$$\pi_{t+1} = \frac{1}{(1+\sigma\kappa)^t}\pi_1 + \frac{\sigma\kappa}{1+\sigma\kappa}\sum_{j=1}^{t}\frac{1}{(1+\sigma\kappa)^{t-j}}i_j. \tag{A7}$$

In the case of AR(1), $i_t = \eta i_{t-1} + \varepsilon_t$, we have the not very elegant expression

$$\pi_{t+1} = \frac{1}{(1+\sigma\kappa)^t}\pi_1 + \frac{\sigma\kappa}{1+\sigma\kappa}\sum_{j=1}^{t}\frac{\eta^{j-1}}{(1+\sigma\kappa)^{t-j}}i_1$$

$$\pi_{t+1} = \frac{1}{(1+\sigma\kappa)^t}\pi_1 + \frac{\frac{1}{(1+\sigma\kappa)^t} - \eta^t}{\frac{1}{(1+\sigma\kappa)} - \eta}\frac{\sigma\kappa}{1+\sigma\kappa}i_1$$

If $\eta = 1$, so $\pi_t = \pi_1, t > 1$, this reduces to

$$\pi_{t+1} = i_1 + \frac{1}{(1+\sigma\kappa)^t}\left(\pi_1 - i_1\right).$$

Now, we need to find π_1. Iterating (A3) forward,

$$\rho^t v_t = (0 - \pi_1) + \rho\left(i_1 - \pi_2\right) + \rho^2\left(i_2 - \pi_3\right) + \rho^3\left(i_3 - \pi_4\right) + \ldots$$

Thus, the condition $\rho^t v_t \to 0$ is

$$\pi_1 = \sum_{j=1}^{\infty}\rho^j\left(i_j - \pi_{j+1}\right).$$

Debt is devalued to pay the higher interest costs that result from higher real interest rates. Now plug inflation from (A7),

$$\pi_1 = \sum_{j=1}^{\infty}\rho^j\left(i_j - \left(\frac{1}{(1+\sigma\kappa)^j}\pi_1 + \frac{\sigma\kappa}{1+\sigma\kappa}\sum_{k=1}^{j}\frac{1}{(1+\sigma\kappa)^{j-k}}i_k\right)\right)$$

$$\pi_1 = -\sum_{j=1}^{\infty}\rho^j\frac{1}{(1+\sigma\kappa)^j}\pi_1 + \sum_{j=1}^{\infty}\rho^j i_j - \frac{\sigma\kappa}{1+\sigma\kappa}\sum_{j=1}^{\infty}\rho^j\sum_{k=1}^{j}\frac{1}{(1+\sigma\kappa)^{j-k}}i_k$$

$$\pi_1 = -\sum_{j=1}^{\infty}\rho^j\frac{1}{(1+\sigma\kappa)^j}\pi_1 + \sum_{j=1}^{\infty}\rho^j i_j - \frac{\sigma\kappa}{1+\sigma\kappa}\sum_{k=1}^{\infty}\sum_{j=k}^{\infty}\rho^j\frac{1}{(1+\sigma\kappa)^{j-k}}i_k$$

$$\pi_1 = -\sum_{j=1}^{\infty}\rho^j\frac{1}{(1+\sigma\kappa)^j}\pi_1 + \sum_{j=1}^{\infty}\rho^j i_j - \frac{\sigma\kappa}{1+\sigma\kappa}\sum_{k=1}^{\infty}\rho^k\left(\frac{1}{1-\frac{\rho}{1+\sigma\kappa}}\right)i_k$$

$$\pi_1 = -\frac{\frac{\rho}{(1+\sigma\kappa)}}{1-\frac{\rho}{(1+\sigma\kappa)}}\pi_1 + \left(1-\frac{\sigma\kappa}{1+\sigma\kappa}\left(\frac{1}{1-\frac{\rho}{1+\sigma\kappa}}\right)\right)\sum_{j=1}^{\infty}\rho^j i_j$$

$$\pi_1 = -\frac{\rho}{1+\sigma\kappa-\rho}\pi_1 + \frac{1-\rho}{1+\sigma\kappa-\rho}\sum_{j=1}^{\infty}\rho^j i_j$$

$$(1+\sigma\kappa-\rho)\,\pi_1 = -\rho\pi_1 + (1-\rho)\sum_{j=1}^{\infty}\rho^j i_j$$

$$\pi_1 = \frac{1-\rho}{1+\sigma\kappa}\sum_{j=1}^{\infty}\rho^j i_j. \tag{A8}$$

For an AR(1)

$$\pi_1 = \frac{1-\rho}{1+\sigma\kappa}\sum_{j=1}^{\infty}\rho^j\eta^{j-1}i_1 = \frac{\rho}{1+\sigma\kappa}\frac{1-\rho}{1-\rho\eta}i_1.$$

For $\eta = 1$

$$\pi_1 = \frac{\rho}{1+\sigma\kappa}i_1.$$

With π_1, we now have the general solution. Using (A8) in (A7),

$$\pi_{t+1} = \frac{1}{(1+\sigma\kappa)^{t+1}}(1-\rho)\sum_{j=1}^{\infty}\rho^j i_j + \frac{\sigma\kappa}{1+\sigma\kappa}\sum_{j=1}^{t}\frac{1}{(1+\sigma\kappa)^{t-j}}i_j\ .$$

For the AR(1)

$$\pi_{t+1} = \frac{1}{(1+\sigma\kappa)^t}\left(\frac{1-\rho}{1+\sigma\kappa}\frac{\rho}{1-\rho\eta}i_1\right) + \frac{\sigma\kappa}{1+\sigma\kappa}\sum_{j=1}^{t}\frac{\eta^{j-1}}{(1+\sigma\kappa)^{t-j}}i_1$$

$$\pi_{t+1} = \left(\frac{1-\rho}{1+\sigma\kappa}\frac{\rho}{1-\rho\eta}i_1\right)\frac{1}{(1+\sigma\kappa)^t} + \frac{\sigma\kappa}{1+\sigma\kappa}\frac{\frac{1}{(1+\sigma\kappa)^t}-\eta^t}{\frac{1}{1+\sigma\kappa}-\eta}i_1$$

$$\pi_{t+1} = \left[\left(\frac{(1-\rho)}{(1+\sigma\kappa)}\frac{\rho}{(1-\rho\eta)} + \frac{\sigma\kappa}{1-\eta\,(1+\sigma\kappa)}\right)\frac{1}{(1+\sigma\kappa)^t} - \frac{\sigma\kappa}{1-\eta\,(1+\sigma\kappa)}\eta^t\right]i_1$$

For $\eta = 1$,

$$\pi_{t+1} = \left[1-\frac{(1+\sigma\kappa-\rho)}{(1+\sigma\kappa)^{t+1}}\right]i_1.$$

A1.1 Continuous time solution

Here I derive the general solution for inflation given an arbitrary path of interest rates in continuous time. The model is (38) - (41)

$$E_t d\pi_t = -\sigma\kappa\pi_t dt + \sigma\kappa i_t dt, \tag{A9}$$

$$dv_t = (rv_t + i_t - \pi_t - \tilde{s}_t)dt. \tag{A10}$$

$$\lim_{T\to\infty} E_t \rho^T v_T = 0$$

We study a single shock at time 0, followed by perfect foresight, with $\tilde{s}_t = 0$. Solving the first and second equations separately, we have (54)-(55)

$$\pi_t = \sigma\kappa \int_{\tau=0}^{t} e^{-\sigma\kappa\tau} i_{t-\tau} d\tau + e^{-\sigma\kappa t}\pi_0 \tag{A11}$$

$$0 = \int_{t=0}^{\infty} e^{-rt}(i_t - \pi_t)dt. \tag{A12}$$

Express (A11) as

$$\pi_t = \sigma\kappa \int_{\tau=0}^{t} e^{-\sigma\kappa(t-\tau)} i_\tau d\tau + e^{-\sigma\kappa t}\pi_0. \tag{A13}$$

Substitute in (A12) and solve for π_0:

$$0 = \int_{t=0}^{\infty} e^{-rt} i_t dt - \sigma\kappa \int_{t=0}^{\infty} e^{-rt} \int_{\tau=0}^{t} e^{-\sigma\kappa(t-\tau)} i_\tau d\tau dt - \pi_0 \int_{t=0}^{\infty} e^{-rt} e^{-\sigma\kappa t} dt. \tag{A14}$$

$$0 = \int_{t=0}^{\infty} e^{-rt} i_t dt - \sigma\kappa \int_{t=0}^{\infty} \int_{\tau=0}^{t} e^{-rt} e^{-\sigma\kappa(t-\tau)} i_\tau d\tau dt - \pi_0 \int_{t=0}^{\infty} e^{-rt} e^{-\sigma\kappa t} dt. \tag{A15}$$

$$0 = \int_{t=0}^{\infty} e^{-rt} i_t dt - \sigma\kappa \int_{\tau=0}^{\infty} \int_{t=\tau}^{\infty} e^{-rt} e^{-\sigma\kappa(t-\tau)} i_\tau d\tau dt - \pi_0 \int_{t=0}^{\infty} e^{-rt} e^{-\sigma\kappa t} dt. \tag{A16}$$

$$0 = \int_{t=0}^{\infty} e^{-rt} i_t dt - \sigma\kappa \int_{\tau=0}^{\infty} e^{\sigma\kappa\tau} i_\tau \int_{t=\tau}^{\infty} e^{-rt} e^{-\sigma\kappa t} d\tau dt - \pi_0 \int_{t=0}^{\infty} e^{-rt} e^{-\sigma\kappa t} dt. \tag{A17}$$

$$0 = \int_{t=0}^{\infty} e^{-rt} i_t dt - \sigma\kappa \int_{\tau=0}^{\infty} e^{\sigma\kappa\tau} i_\tau \frac{e^{-(r+\sigma\kappa)\tau}}{r+\sigma\kappa} d\tau - \pi_0 \frac{1}{r+\sigma\kappa}. \tag{A18}$$

$$0 = \int_{t=0}^{\infty} e^{-rt} i_t dt - \sigma\kappa \int_{\tau=0}^{\infty} e^{\sigma\kappa\tau} i_\tau \frac{e^{-(r+\sigma\kappa)\tau}}{r+\sigma\kappa} d\tau - \pi_0 \frac{1}{r+\sigma\kappa}. \tag{A19}$$

$$0 = \int_{t=0}^{\infty} e^{-rt} i_t dt - \frac{\sigma\kappa}{r+\sigma\kappa} \int_{\tau=0}^{\infty} e^{-r\tau} i_\tau d\tau - \frac{1}{r+\sigma\kappa}\pi_0. \tag{A20}$$

$$0 = \frac{r}{r+\sigma\kappa}\int_{t=0}^{\infty} e^{-rt} i_t dt - \frac{1}{r+\sigma\kappa}\pi_0. \tag{A21}$$

$$\pi_0 = r\int_{t=0}^{\infty} e^{-rt} i_t dt. \tag{A22}$$

Then, back to (A11),

$$\pi_t = \sigma\kappa\int_{\tau=0}^{t} e^{-\sigma\kappa\tau} i_{t-\tau} d\tau + e^{-\sigma\kappa t} r\int_{\tau=0}^{\infty} e^{-r\tau} i_\tau d\tau. \tag{A23}$$

or

$$\pi_t = \sigma\kappa\int_{\tau=0}^{t} e^{-\sigma\kappa(t-\tau)} i_\tau d\tau + e^{-\sigma\kappa t} r\int_{\tau=0}^{\infty} e^{-r\tau} i_\tau d\tau. \tag{A24}$$

For an AR(1)

$$i_t = i_0 e^{-\eta t},$$

we have

$$\pi_t = \sigma\kappa\int_{\tau=0}^{t} e^{-\sigma\kappa(t-\tau)} i_0 e^{-\eta\tau} d\tau + e^{-\sigma\kappa t} r\int_{\tau=0}^{\infty} e^{-r\tau} i_0 e^{-\eta\tau} d\tau. \tag{A25}$$

$$\pi_t = \left[\sigma\kappa e^{-\sigma\kappa t}\int_{\tau=0}^{t} e^{(\sigma\kappa-\eta)\tau} d\tau + e^{-\sigma\kappa t} r\int_{\tau=0}^{\infty} e^{-(r+\eta)\tau} d\tau\right] i_0. \tag{A26}$$

$$\pi_t = \left[\sigma\kappa\frac{e^{(\sigma\kappa-\eta)t}-1}{\sigma\kappa-\eta} e^{-\sigma\kappa t} + \frac{r}{r+\eta} e^{-\sigma\kappa t}\right] i_0. \tag{A27}$$

$$\pi_t = \left[\frac{\sigma\kappa}{\sigma\kappa-\eta}[e^{-\eta t} - e^{-\sigma\kappa t}] + \frac{r}{r+\eta} e^{-\sigma\kappa t}\right] i_0. \tag{A28}$$

$$\pi_t = \left[\frac{\sigma\kappa}{\sigma\kappa-\eta} e^{-\eta t} + \left[\frac{r}{r+\eta} - \frac{\sigma\kappa}{\sigma\kappa-\eta}\right] e^{-\sigma\kappa t}\right] i_0. \tag{A29}$$

$$\pi_t = \left[\frac{\sigma\kappa}{\sigma\kappa-\eta} e^{-\eta t} + \left[\frac{r(\sigma\kappa-\eta) - \sigma\kappa(r+\eta)}{(r+\eta)(\sigma\kappa-\eta)}\right] e^{-\sigma\kappa t}\right] i_0. \tag{A30}$$

$$\pi_t = \left[\frac{\sigma\kappa}{\sigma\kappa-\eta} e^{-\eta t} - \frac{\eta(r+\sigma\kappa)}{(r+\eta)(\sigma\kappa-\eta)} e^{-\sigma\kappa t}\right] i_0. \tag{A31}$$

$$\pi_t = \frac{1}{\sigma\kappa-\eta}\left[\sigma\kappa e^{-\eta t} - \frac{r+\sigma\kappa}{r+\eta}\eta e^{-\sigma\kappa t}\right] i_0. \tag{A32}$$