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EXPECTATIONS AND THE NEUTRALITY OF INTEREST RATES

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Expectations and the Neutrality of Interest Rates

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### **ABSTRACT**

51 years ago, Bob Lucas (1972a) published his pathbreaking analysis of the temporary non-neutrality of money. But our central banks set interest rate targets, and do not even pretend to control money supplies. How do interest rates affect inflation? Until recently, we have not had a complete theory of inflation under interest rate targets. Now we have such a theory. It mirrors the long-run properties of monetary theory: Inflation can be stable and determinate under interest rate targets, including a peg, analogous to a  $k$  percent rule. The zero bound era is confirmatory evidence. Uncomfortably, stability means that higher interest rates eventually raise inflation, just as higher money growth eventually raises inflation. Sticky prices generate some short-run non-neutrality as well: Higher nominal interest rates can raise real rates and lower output. A model in which higher nominal interest rates temporarily lower inflation, without a change in fiscal policy, is a harder task. I exhibit one such model, but it paints a much more limited picture than standard beliefs. We either need a model with a stronger effect, or to accept that higher interest rates have quite limited power to lower inflation. Empirical understanding of how interest rates affect inflation without fiscal help is also a wide-open question.

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## 1 Introduction

51 years ago, Bob Lucas (1972a) published the watershed “Expectations and the Neutrality of Money.” Lucas studied expectations and the neutrality—and temporary non-neutrality—of, as the title says, *money*. But our central banks set *interest rates*. The Federal Reserve does not even pretend to control money supply, especially inside money. There are no reserve requirements. Super-abundant reserves pay the same or more interest as short-term treasuries and overnight money markets. The Fed controls interest rates by changing the interest it offers on abundant reserves, not by rationing scarce zero-interest reserves. Other central banks follow similar policies. The quantity of M2 is whatever people feel like holding in that form.

We need an analogous theory of inflation under *interest rate targets*. Ideally, the theory should likewise be based on robust and clean economics, starting with a well described frictionless and neutral benchmark, and adding minimal frictions to describe temporary non-neutrality. The theory should be consistent with current institutions, including an interest rate target, ample reserves, and no control of inside money. Its basic mechanisms and signs should be explainable to undergraduates, central bankers, and intelligent laypeople.

I argue that we have at last such a theory of inflation under interest rate targets. The theory describes inflation that is stable and determinate under an interest rate peg, analogous to monetarist analysis of a  $k\%$  money growth rule. The theory starts with a frictionless and neutral benchmark, and it remains approximately long-run neutral with sticky prices. But even these statements are controversial. Moreover, stability under an interest rate peg and approximate long-run neutrality inexorably imply that higher nominal interest rates eventually produce *higher* inflation.

The final piece, a theory of temporary non-neutrality, the central contribution of Lucas (1972a), is unfinished. It is not settled whether and how by raising interest rates, without a change in fiscal policy, the central bank can temporarily lower inflation. In a range of contemporary models, higher interest rates only lower inflation if they are accompanied by tighter fiscal policy, at least to pay higher interest costs of the debt and to repay bondholders in more valuable dollars, and often more. Without that contemporaneous fiscal tightening, higher interest rates do not lower inflation. The best model we have includes long-term debt, and temporarily lowers inflation only by raising later inflation in a form of unpleasant arithmetic. Its effect is much weaker effect than most economists and all central bankers believe, and based on a completely different mechanism. We also lack robust empirical understanding of the effect of interest rate

changes, not accompanied by fiscal tightening, on inflation. The hoped-for negative effect may not be there.

Ignorance is great news for researchers. The 1970s were a golden decade for macroeconomic research, as much as they were a miserable decade for the economy. The 2020s may well repeat both features.

These questions are also crucial for current policy. The Fed waited a whole year to raise interest rates after inflation emerged in early 2021. Is the Fed's slow reaction partially to blame for 2022 inflation? Must the Fed dramatically raise nominal rates above current inflation, as the Taylor Rule recommends and as we observed in the early 1980s, in order to control inflation? Or can inflation remain contained without such high nominal interest rates, as stability implies? How much does monetary policy depend on fiscal policy? For example, if the central bank raises real interest rates, and fiscal policy does not tighten to pay consequent higher interest costs on the debt, does the interest rate rise still lower inflation? Do interest rate rises without a change in fiscal policy lower inflation, and if so how?

This paper is a synthetic essay, an evaluation of what we know and do not know. I do not present new models. I simplify and connect the dots of existing models, and I present some novel calculations with those models. Some of these points are made at longer length with more algebra and more complex models in Cochrane (2023), Ch. 5, 12, 16, 17, 20, and Cochrane (2022b).

## 2 Theories of Inflation Under Interest Rate Targets

What is the dynamic effect of *interest rates*—not money—on inflation? I use a very simple standard model to think about this question, and the historical development of our answers to it,

$$x_t = E_t x_{t+1} - \sigma(i_t - \pi_t^e) \quad (1)$$

$$\pi_t = \pi_t^e + \kappa x_t, \quad (2)$$

where  $x$  = output gap,  $\pi$  = inflation,  $\pi^e$  = expected inflation, and  $i$  = interest rate. Variables are all deviations from steady state.

Equation (1) is the first-order condition for consumption or dynamic IS curve, in an economy without capital so that output equals consumption. Equation (2) is the Phillips curve.

Lucas's central innovation was, of course, to specify how expectations enter the Phillips curve so that output variation comes from *unexpected* inflation. Lucas paired that Phillips curve with, essentially,  $MV = PY$  and constant  $V$ , which determines the price level. With this structure, Lucas already had in hand a theory of price level determination, and needed only to extend that theory to describe non-neutrality. Our challenge is to develop a theory of price level determination based on interest rates. We have to work to get to Lucas's launch pad.

I hesitate to write down such a model without preferences, technology, market structure, definition of equilibrium, and recursive statement. Lucas' most important contribution may have been methodological, to express a monetary economics question with a completely articulated general equilibrium model. But this is well-trod ground and it is well known how to provide those foundations. See, for example, Woodford (2003).

I simplify further by dropping  $E_t x_{t+1}$  on the right hand side of (1), leaving a simple statement that higher real interest rates depress output,

$$x_t = -\sigma(i_t - \pi_t^e). \quad (3)$$

This simplification turns out not to make any difference for the points I make, and leaving it out allows me to do everything with transparent algebra. Equation (1) iterates forward to

$$x_t = -\sigma E_t \sum_{j=0}^{\infty} (i_{t+j} - \pi_{t+j}^e) + \lim_{T \rightarrow \infty} E_t x_{t+T},$$

so my static version is the same as the dynamic version when the current real interest rate is a sufficient statistic for the expected sum of future real rates, and output is expected to return to steady state. The parameter  $\sigma$  in the simplified model is then larger than the intertemporal elasticity of substitution, as it includes how long the high rates last. Using the static IS curve makes a second point: The troubles I document cannot be fixed just by attenuating the forward-looking part of the IS curve. (For example, Gabaix (2020).)

Substituting output out of (2)-(3), we obtain a relationship between interest rates and inflation:

$$\pi_t = (1 + \sigma\kappa)\pi_t^e - \sigma\kappa i_t. \quad (4)$$

The dynamic response of inflation to interest rates now depends on how expectations are formed.

## 2.1 Expectations, Stability, Determinacy, and Neutrality, and Sign

Table 1 summarizes the steady forward march of expectations in the Phillips curve. Each equation is simplified to be emblematic of an era. Actual Phillips curves also include disturbances. While the table is in chronological order, my discussion will skip around to emphasize the theoretical points.

Author	Phillips curve	Expectations
Phillips (1958)	$\pi_t = \pi_0 + \kappa x_t$	Absent
Dynamic empirical (1960s)	$\pi_t = \alpha \pi_{t-1} + \kappa x_t, \alpha < 1$	Adaptive
Friedman (1968); ISLM AS/AD(1970s)	$\pi_t = \pi_{t-1} + \kappa x_t$	Adaptive
Lucas (1972)	$\pi_t = E_{t-1} \pi_t + \kappa x_t$	Rational
Calvo (1983), Rotemberg (1982); NK (1990s)	$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t$	Rational

Table 1: The steady forward march of expectations in the Phillips curve

Phillips didn't have any expectations or other variables to shift the Phillips curve, nor did the Keynesian advocates of inflation in the early 1960s such as Samuelson and Solow (1960). (See Nelson (2020) Ch. 13.)

Dynamic estimates of the Phillips curve in the 1960s and 1970s added lags of unemployment or inflation, depending which variable one put on the right-hand side of the regression. These specifications retained a long-run inflation-output tradeoff,  $\alpha < 1$  here, and when thinking theoretically, adaptive expectations.<sup>1</sup>

### 2.1.1 Adaptive Expectations

Friedman's (1968) address was fundamentally about neutrality. He proclaimed two things that monetary policy *cannot* do. First, he proclaimed that the Phillips curve would shift once people come to expect inflation, so the central bank cannot permanently lower unemployment. The long-run Phillips curve is vertical. But on the way there, he described explicitly adaptive expectations: "This price expectation effect is slow to develop and also slow to disappear." Phelps (1967) also writes "a sort of Phillips Curve ... that shifts one-for-one with variations in the expected rate of inflation; ... the expected inflation rate adjusts gradually over time to the actual inflation rate."

<sup>1</sup>Among many others, Lipsey (1960), Gordon (1970). Gordon (1976) p. 192–193 provides a nice summary of the era. Sargent (1971) and Lucas (1972b) also summarize insightfully.

Second, Friedman proclaimed that the central bank cannot peg the nominal interest rate. We can see this result in our little model. Let expectations be adaptive,  $\pi_t^e = \pi_{t-1}$ . Then from (4) inflation and interest rates are related by

$$\pi_t = (1 + \sigma\kappa)\pi_{t-1} - \sigma\kappa i_t. \quad (5)$$

Inflation is now *unstable* under an interest rate peg, since  $(1 + \sigma\kappa) > 1$ . In Friedman's description, the central bank needs to print more and more money to keep the interest rate down. The ISLM AS/AD tradition of the 1970s adopted the same adaptive-expectations Phillips curve, in models without money. In that description, a too-low nominal interest rate lowers the real rate, which boosts demand, which boosts inflation, and around we go. The left panel of Figure 1 illustrates instability and the inflation or deflation spirals that break out under an interest rate peg.

The last term of Equation (5) also shows the conventional sign: Higher interest rates lower inflation. Indeed, higher interest rates set off an unstable deflationary spiral. Friedman said so (in the opposite, inflationary, direction), though quickly adding that the central bank would soon give up and raise interest rates (lower money growth) to stop the spiraling inflation.

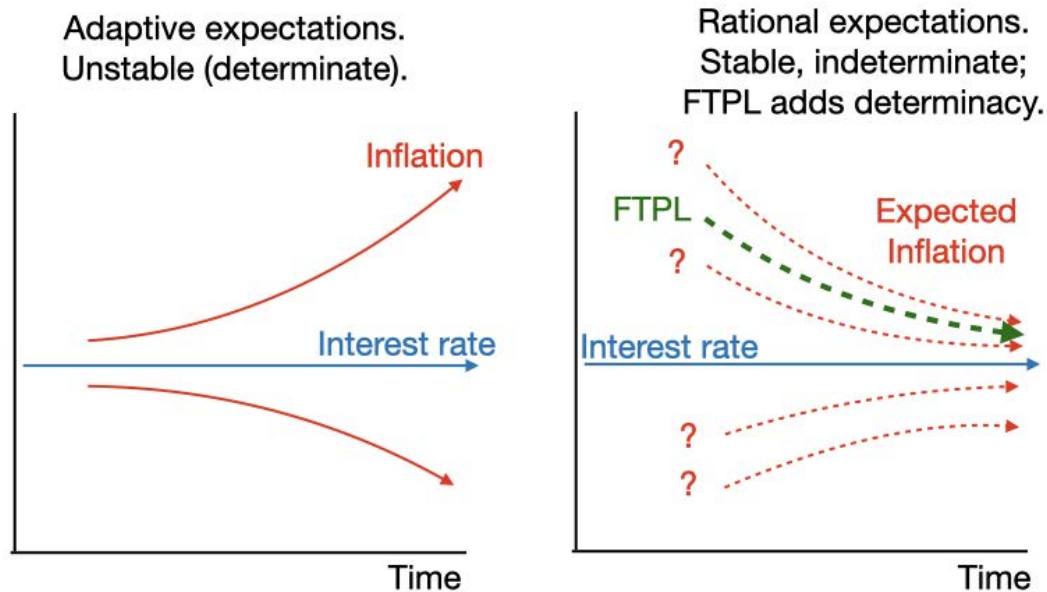


Figure 1: Adaptive vs. Rational Expectations Dynamics

Thus began the long tradition that views an interest rate target as a fundamentally incomplete price-level anchor.

The Taylor rule repairs Friedman's critique of interest rate targets.<sup>2</sup> Let the central bank systematically respond to inflation with higher interest rates,

$$i_t = \phi\pi_t + u_t$$

with  $\phi > 1$ . Substituting for  $i_t$  in (5), inflation dynamics become

$$\pi_t = \frac{1 + \sigma\kappa}{1 + \sigma\kappa\phi}\pi_{t-1} - \frac{\sigma\kappa}{1 + \sigma\kappa\phi}u_t. \quad (6)$$

Now inflation is stable under an interest rate target. Sensibly, the central bank's interest rate policies act to *stabilize* the economy. But the economy itself remains fundamentally unstable, with that instability only tamed if the central bank reacts swiftly and more than one for one to inflation, like a seal balancing a ball on its nose. A peg would swiftly unravel.

With this modification, higher interest rates still lower inflation.<sup>3</sup> Figure 5 below plots a simulation.

Adaptive expectations may seem attractive. This model captures policy maker's world view, and it easily generates the conventional view that higher interest rates lower inflation. However, the adaptive expectations model is an unsatisfactory foundational theory of inflation under interest rate targets, certainly as compared to Lucas (1972a) for monetary targets. Irrationality and sticky prices are *necessary* ingredients here, for us to say anything about the price level. The model does not have a frictionless limit or limit point; as prices become less sticky  $\kappa \rightarrow \infty$ , it blows up. We would like to start with a benchmark in which prices are flexible prices, people are rational, and the Lucas (1976) critique holds, and then see just how far we must modify that benchmark to capture realistic dynamics. If there is no such benchmark, monetary economics is on pretty shaky ground.

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<sup>2</sup>McCallum (1981) is the first formal statement that the  $\phi > 1$  principle resolves indeterminacy with rational expectations. Taylor (1999) shows how a Taylor rule resolves instability with adaptive expectations. Taylor (1993) is the most influential statement of the rule and its practical implementation. Wicksell (1898), (1965) proposed verbally that a central bank could stabilize the price level by raising and lowering an interest rate target. In the 19th century, the Bank of England systematically raised and lowered the discount rate to defend the gold standard, in part in response to inflation and vice versa.

<sup>3</sup>In the period of an unexpected shock  $u_1 = \varepsilon_1$  we have  $\pi_1 = -\sigma\kappa/(1 + \sigma\kappa\phi)\varepsilon_1$  and  $i_1 = 1/(1 + \sigma\kappa\phi)\varepsilon_1$  so  $\pi_1 = -\sigma\kappa i_1$ . Interest rates rise and inflation falls. This is important to check, as a positive shock  $u_1$  can lead to negative interest rates  $i_1$  in new-Keynesian models, and then interest rates and inflation go in the same direction.



### 2.1.2 Rational Expectations and New-Keynesian Models

New-Keynesian models use rational expectations, and consciously play by the Lucas rules of how to write and solve intertemporal general equilibrium macroeconomic models. With price adjustment costs, the standard new-Keynesian model bases the Phillips curve on inflation relative to rationally expected future inflation,  $\pi_t^e = \beta E_t \pi_{t+1}$  (Calvo (1983), Rotemberg (1982)). For simplicity, I use  $\beta = 1$  here. I show below that  $\beta < 1$  makes no important difference to my points.

Now from (4) the dynamic response of inflation to interest rates is

$$E_t \pi_{t+1} = \frac{1}{1 + \sigma \kappa} \pi_t + \frac{\sigma \kappa}{1 + \sigma \kappa} i_t. \quad (7)$$

Inflation is *stable* since  $1/(1 + \sigma \kappa) < 1$ . Unlike the adaptive expectations model, this rational expectations model has a sensible frictionless  $\kappa \rightarrow \infty$  limit and limit point:

$$E_t \pi_{t+1} = i_t. \quad (8)$$

Price stickiness draws out dynamics.

But this model so far only ties down expected inflation. Unexpected inflation  $\pi_{t+1} - E_t \pi_{t+1}$  can be anything, or wander up and down following sunspots. Using rational expectations in a related model, Sargent and Wallace (1975) modified Friedman's doctrine: Inflation is *indeterminate* under an interest rate peg. Moreover, neither this model nor the adaptive expectations model determines the price *level*, the nominal anchor, as opposed to the rate of inflation. The sense continued that we don't have a model of inflation under interest rate targets.

Rational vs. adaptive expectations fundamentally change the stability and determinacy properties of the model. The right hand panel of Figure 1 illustrates, with the question mark indicating all the many equilibria that could break out at that point.

Friedman's unstable (and determinate) is different from Sargent and Wallace's stable and indeterminate. They are frequently confused. Both models suggest volatile inflation. But spiraling away on a determinate path is different from batting up and down unpredictably around the peg.

The sign on the last term in (7) has also changed. Now higher nominal interest rates *raise* expected inflation, as they do in the flexible price model (8).

New-Keynesian modelers resolve indeterminacy with a novel application of the Taylor principle. If we add  $i_t = \phi\pi_t + u_t$  in this case, inflation dynamics (7) become

$$E_t\pi_{t+1} = \frac{1 + \sigma\kappa\phi}{1 + \sigma\kappa}\pi_t + \frac{\sigma\kappa}{1 + \sigma\kappa}u_t. \quad (9)$$

With  $\phi > 1$ , dynamics are now *unstable*. Adding a rule against nominal explosions, new-Keynesian modelers can now choose the unique initial value of inflation that precludes an explosion, and thus produce determinate inflation;  $\pi_t = 0$  with no shocks, and

$$\pi_t = -E_t \sum_{j=0}^{\infty} \left( \frac{\sigma\kappa}{1 + \sigma\kappa} \right)^j u_{t+j}$$

with shocks  $u_t$ .

The central bank is imagined in this vision to deliberately *destabilize* an economy which is already stable on its own, exactly the opposite of the Taylor rule in an adaptive expectations economy which *stabilizes* an otherwise unstable economy. The central bank threatens hyperinflation or hyperdeflation in order to select or “coordinate expectations” on the equilibrium it likes. Indeed, the central bank may simply announce its inflation target, announce this threat, and inflation jumps to whatever value the central bank desires. The Taylor Principle in a new-Keynesian model is an *equilibrium-selection* policy not a *stabilization* policy.

This statement is easiest to see in the  $\kappa = \infty$  case of flexible prices, in which the interest rate directly sets expected inflation (8). Write the policy rule equivalently as

$$i_t = \phi\pi_t + u_t = i_t^* + \phi(\pi_t - \pi_t^*) = E_t\pi_{t+1}^* + \phi(\pi_t - \pi_t^*). \quad (10)$$

Here, we may interpret  $\{\pi_t^*\}$  as the central bank’s stochastic inflation target, the value of inflation it wishes to produce in each date and state, and  $\{i_t^*\}$  as the interest rate target it will use to implement the inflation target. The second and third equalities define  $i_t^*$  and translate between the  $u_t$  and  $i_t^*$ ,  $\pi_t^*$  notation for the monetary policy disturbance. (King (2000) invented this clever notation.) Eliminating  $i_t$  from (8)-(10), the model’s equilibrium condition is

$$E_t(\pi_{t+1} - \pi_{t+1}^*) = \phi(\pi_t - \pi_t^*).$$

With  $\phi > 1$ , the unique bounded equilibrium is  $\pi_t = \pi_t^*$ ,  $i_t = i_t^*$ .

The central bank chooses the inflation it wishes to see  $\{\pi_t^*\}$ . It obtains this value by an inter-

est rate policy  $i_t^*$ , which sets the equilibrium observed interest rate to equal the expected value of the inflation target, and a *separate* equilibrium-selection policy, threatening to produce an expected hyperinflation or deflation should unexpected inflation come out against its desires. For example, if  $\pi_t^*$  is i.i.d., observed interest rates never move. The central bank simply announces each period what inflation it would like to see, and that inflation occurs.

To see the same point in our little sticky price model, again write the policy rule as

$$i_t = \phi\pi_t + u_t = i_t^* + \phi(\pi_t - \pi_t^*), \quad (11)$$

where now

$$i_t^* \equiv \frac{1 + \sigma\kappa}{\sigma\kappa} E_t\pi_{t+1}^* - \frac{1}{\sigma\kappa} \pi_t^*. \quad (12)$$

Equation (12) applies (7) to the starred variables. If we want to think in terms of an interest rate target and an inflation target, those targets must be compatible with private sector equilibrium conditions. With this policy rule, the equilibrium condition (7) becomes

$$E_t(\pi_{t+1} - \pi_{t+1}^*) = \frac{1 + \phi\sigma\kappa}{1 + \sigma\kappa} (\pi_t - \pi_t^*). \quad (13)$$

Again, with  $\phi > 1$  the central bank destabilizes an economy that is stable on its own. By doing so, it repairs indeterminacy:  $\pi_t = \pi_t^*$  and  $i_t = i_t^*$  is the unique locally bounded equilibrium.

New-Keynesian modelers can produce lower inflation when interest rates rise, overcoming the positive signs in (7), (8), and (9), by such equilibrium-selection choices. With higher interest rates, expected inflation  $E_t\pi_{t+1}$  must rise relative to current inflation  $\pi_t$ . But suppose that coincident with an unexpected rise in interest rates at time  $t$ , the central bank also announces a lower stochastic inflation target  $\pi_t^*$ . Now current inflation  $\pi_t$  jumps down. If  $\pi_t$  jumps down enough, then  $E_t\pi_{t+1}$ , though still higher than  $\pi_t$ , can also fall relative to what was expected at time  $t - 1$ . Translate back to  $\{u_t\}$  and you have the conventional new-Keynesian model. However, as you can see by this rewriting, the higher interest rate has nothing to do with the lower initial inflation. The central bank could just as easily lower initial inflation without a higher interest rate.

In reality, central banks do not have equilibrium-selection policies. They do not threaten hyperinflation or deflation if inflation comes out against their desires. Such threats being contrary to their objectives, nobody would believe them if they tried. Central banks do not intentionally de-stabilize economies that are stable on their own. Ask central banks. Look at central bank websites. They loudly announce that they they *stabilize* economies; no matter what inflation

does, they will act resolutely to bring it back.

As I rejected the beautiful  $MV = PY$  because central banks set interest rates, do not limit money supply, and because reserves pay the same interest as bonds, I argue that we should also reject the new-Keynesian solution, because our monetary institutions simply do not remotely behave as this model specifies.

The new-Keynesian theory also requires an extra rule against non locally bounded equilibria, not a part of the standard definition of Walrasian equilibrium (Cochrane (2011)). And yet we see hyperinflations. Also, in the new-Keynesian model, the central bank completely controls inflation, expected and unexpected, and can move inflation with speeches alone and no observed action. Real central banks seem not to have such power. (For all these points, see Cochrane (2011).)

### 2.1.3 Fiscal Theory

The fiscal theory of the price level adds an equilibrium condition, or rather recognizes one that was there all along and has been left out so far. The real value of nominal government debt  $v_t$  evolves as

$$\rho v_{t+1} = v_t + i_t - \pi_{t+1} - \tilde{s}_{t+1} \quad (14)$$

where  $\rho < 1$  and  $\tilde{s}_{t+1}$  is the real primary surplus scaled by the steady state value of debt. I use here the simple case of one-period debt and no economic growth; I generalize to long-term debt below. This equation is also linearized. See Cochrane (2023) Ch. 3.5 for a derivation. The consumer's transversality condition also requires

$$\lim_{T \rightarrow \infty} E_t \rho^T v_T = 0. \quad (15)$$

We can add these conditions to the VAR(1) statement of the model. But in this simple case, we can solve the model analytically by iterating (14) forward to

$$v_t = E_t \sum_{j=0}^{\infty} \rho^j [\tilde{s}_{t+1+j} - (i_{t+j} - \pi_{t+1+j})]. \quad (16)$$

The real value of debt is the discounted present value of future surpluses. Taking innovations  $\Delta E_{t+1} \equiv E_{t+1} - E_t$  of both sides of (16), we obtain.

$$\Delta E_{t+1} \pi_{t+1} = -\Delta E_{t+1} \sum_{j=0}^{\infty} \rho^j \tilde{s}_{t+1+j} + \Delta E_{t+1} \sum_{j=1}^{\infty} \rho^j (i_{t+j} - \pi_{t+j+1}). \quad (17)$$

Unexpected inflation devalues outstanding debt. Thus, unexpected inflation corresponds to the revision in the present value of future primary surpluses. In the second term, higher discount rates lower the value of debt and cause inflation. Equivalently, if interest costs on the debt rise, but current or future surpluses do not rise to pay them, then the resources must come by inflating away outstanding bonds.

Since the rational expectations model left an indeterminacy indexed by unexpected inflation  $\Delta E_{t+1} \pi_{t+1}$ , (17) clearly steps in to restore that determinacy, in place of central bank equilibrium-selection rules.

The point is also easiest to see in the simplest case of flexible prices. Then, (7) and (17) boil down to

$$i_t = E_t \pi_{t+1} \quad (18)$$

$$\Delta E_{t+1} \pi_{t+1} = -\Delta E_{t+1} \sum_{j=0}^{\infty} \rho^j \tilde{s}_{t+1+j}. \quad (19)$$

The interest rate target sets expected inflation; fiscal policy determines unexpected inflation and picks one equilibrium.

With sticky prices, we have the pair (7)-(17), which I repeat for convenience,

$$E_t \pi_{t+1} = \frac{1}{1 + \sigma \kappa} \pi_t + \frac{\sigma \kappa}{1 + \sigma \kappa} i_t \quad (20)$$

$$\Delta E_{t+1} \pi_{t+1} = -\Delta E_{t+1} \sum_{j=0}^{\infty} \rho^j \tilde{s}_{t+1+j} + \Delta E_{t+1} \sum_{j=1}^{\infty} \rho^j (i_{t+j} - \pi_{t+j+1}). \quad (21)$$

Now (20) picks a set of paths for expected inflation, and (21) selects which one is the unique equilibrium. The right hand panel of Figure 1 illustrates this option as well. Fiscal policy determines one of the many possible equilibria.

In sum, with the combination (20) and (21) to choose unexpected inflation, inflation is stable *and determinate* at an interest rate peg (or Taylor coefficients  $\phi < 1$ ), overcoming Sargent

and Wallace's contrary doctrine.<sup>4</sup>

How *volatile* inflation is depends on how much fiscal shock or quiet there is. But economics picks one value. In addition, we now have a model of the price *level*, a nominal anchor. In (16),  $v_t$  is the ratio of nominal debt to the price level. The price level adjusts so that (16) holds.

Monetarists, ISLM-AS/AD or adaptive-expectations Keynesians, Sargent and Wallace, and new-Keynesians make explicit assumptions to wipe out fiscal theory. Careful versions of these theories include the government debt equilibrium condition, but they assume that fiscal authorities “passively” adjust surpluses as needed to validate inflation determined elsewhere. In (21), the central bank picks  $\Delta E_{t+1}\pi_{t+1}$ , and then fiscal authorities supply whatever surpluses  $\tilde{s}_{t+1}$  are necessary to satisfy (16) or (17), often via lump-sum taxes. (Leeper (1991), Woodford (2003) section 4.4.)

Since the equilibrium conditions are the same, the fiscal and new-Keynesian theories make predictions for equilibrium time series that are formally observationally equivalent, at least without further assumptions. (See Cochrane (1998), Cochrane (2023) Ch. 17, 22.) Thus, for the purposes of everything that follows, you may think in terms of the new-Keynesian rather than fiscal-theory version of equilibrium formation, if you disagree with my argument. However, new-Keynesian modelers typically do not examine what the required surpluses are. For a full evaluation of the theory, we must examine them.

For example, in the standard new-Keynesian policy experiment, a monetary policy shock that lowers inflation comes with a “passive” fiscal tightening to pay higher interest costs on the debt and a windfall to bondholders. In the flexible price model, using (19), a surprise decline in initial inflation  $\Delta E_1\pi_1$  *must* come with a rise in primary surpluses. Fiscal theory calls this rise a “fiscal shock” coincident with the interest-rate increase. New-Keynesian theory calls it an induced fiscal policy response to the equilibrium-selection part of the central bank's monetary policy. By either name, however, surpluses must rise, or inflation cannot fall. An interest rate rise without such fiscal backing has different effects, even in totally standard new-Keynesian analysis. So, you can interpret my equations as exploring different fiscal underpinnings of conventional new-Keynesian models.

In the sticky price case, (21) tells us that an unexpected inflation  $\Delta E_1\pi_1$  can also come from lower real interest rates, i.e. lower real interest costs on the debt. But the sign is “wrong.” Higher

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<sup>4</sup>Woodford (1994) shows that with fiscal theory, an interest rate peg is determinate and stable, even a zero rate peg, allowing an optimal quantity of money.

nominal rates that reduce inflation are supposed to do so via higher, not lower, real interest rates. More generally, (19) gives a general preview why we find it so hard for higher interest rates to lower inflation. If the path of real interest rates is on average positive, and with no change in fiscal policy, initial inflation  $\Delta E_1 \pi_1$  must be positive, not negative. With no change in inflation on the date of the shock,  $\Delta E_1 \pi_1 = 0$  (this term is absent in continuous time with sticky prices), the average real interest rate must be zero; we pick an inflation path that on average is equal to the nominal interest rate path.

But we should not overstate observational equivalence. For example, in the fiscal theory model, a fiscal shock (a decline in the present value of surpluses) results in inflation to devalue debt that the central bank cannot completely avoid by any path of interest rates. In the new-Keynesian model, the central bank fully controls inflation, expected and unexpected. Fiscal shocks are ruled out: If the central bank does not choose inflation, deficits are always repaid in full. Whether the central bank alone can or cannot fully control inflation is a pretty important policy and doctrinal issue. Did we suffer inflation in 2021-2022 because of a fiscal shock, or because the Fed failed to announce appropriate equilibrium-selection threats to support its 2% inflation target? And knowledge of how our institutions work—that central banks simply do not make explosive equilibrium-selection threats—is information beyond the stochastic properties of equilibrium time series that help to distinguish theories.

A note on nomenclature: Fiscal theory of the price level does not have to be married to with rational expectations. I investigate fiscal foundations of adaptive expectations models below.

## 2.2 Lucas's Phillips Curve

In my little Phillips curve history (Table 1), I skipped over Lucas. Lucas (1972a) first made Phillips-curve expectations rational. His Phillips curve relates output to unexpected inflation only, first moving forward the time subscript in the Phillips curve, from  $\pi_t^e = \pi_{t-1}$  to  $\pi_t^e = E_{t-1} \pi_t$ . In the spirit of rational expectations, it makes most sense to pair Lucas' Phillips curve with rational expectations in the bond market and consumption. So let's use Lucas' Phillips curve in an interest-rate model by writing

$$x_t = -\sigma(i_t - E_t \pi_{t+1}) \quad (22)$$

$$\pi_t = E_{t-1} \pi_t + \kappa x_t. \quad (23)$$

Eliminating  $x_t$ , inflation dynamics (4) are now

$$E_t \pi_{t+1} = i_t + \frac{1}{\kappa \sigma} (\pi_t - E_{t-1} \pi_t). \quad (24)$$

Iterating forward,

$$E_t \pi_{t+2} = E_t i_{t+1}.$$

Lucas' specification of the rational expectations Phillips curve, along with our IS curve, passive fiscal policy, and an interest rate target, leads to an economy that is stable and indeterminate, like the new-Keynesian model. Relative to the flexible-price model  $i_t = E_t \pi_{t+1}$ , Lucas's Phillips curve gives one period of additional inflation after a shock, which then reverts to the frictionless value. Adding fiscal theory to this model we again restore determinacy, and name the unexpected inflation shock. One could also add new-Keynesian equilibrium selection.

### 3 Long-Run Properties: Neutrality, Stability, and Fisherism

In sum, combining fiscal theory and rational expectations, we have, finally, a complete economic (rational agents, Walrasian equilibrium) theory of inflation under interest rate targets. In it, and like monetarist theory, the economy is *determinate, stable*, starts from a *neutral* benchmark, displays (potentially approximate) long-run neutrality, and offers a foundation for models with frictions that create short-run non-neutralities.

These properties are on full display with flexible prices,

$$i_t = E_t \pi_{t+1}$$

$$\Delta E_{t+1} \pi_{t+1} = \Delta E_{t+1} \sum_{j=0}^{\infty} \rho^j \tilde{s}_{t+1+j}.$$

There is only one equilibrium value of inflation. The central bank can follow a peg, analogous to a k% rule, and inflation will neither spiral away nor suffer multiple-equilibrium volatility. (Like a k% rule, a peg may not be optimal, but it is possible.) A one percentage point higher interest rate with no change in fiscal policy produces a one percentage point higher inflation, with no change in real rate or output.

The sticky-price version of the model also offers a first step of explicitly modeled short-run non-neutrality, keeping the frictionless model's properties in the long run, again similarly to



standard monetary theory. The model remains determinate and stable: Inflation slowly converges to the interest rate. Output and real interest rates vary, but that variation dies away in the long run. A one percentage point higher interest rate leads eventually to one percentage point higher inflation, but not right away.

*Higher interest rates raise inflation?* Even if qualified by “in the long run,” this “Fisherian” property is so contrary to standard doctrine that one looks up in disbelief from the screaming lessons of equations. Really?

*Stability* is the central property behind the Fisherian result. If inflation is stable under an interest rate peg, it follows that raising the peg must eventually raise inflation. The adaptive expectations model is long-run neutral, in that steady states with higher inflation have higher interest rates. But it is unstable, so raising the interest rate from such a steady state lowers inflation.

The equations are clear, but just how do higher interest rates produce higher inflation? What about standard intuition that higher interest rates depress aggregate demand, so should *reduce* inflation?

First, consider the full consumer first-order condition  $x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1})$  with no pricing frictions. Raise the nominal interest rate  $i_t$ . Before prices change, a higher nominal interest rate is a higher real rate, and induces people to demand less today  $x_t$  and more next period  $x_{t+1}$ . That change in demand pushes down the price level today  $p_t$  and hence current inflation  $\pi_t = p_t - p_{t-1}$ , and it pushes up the expected price level next period  $p_{t+1}$  and thus expected future inflation  $\pi_{t+1} = p_{t+1} - p_t$ .

So, standard intuition is correct, and refers to a force that can lower *current* inflation. Fisherian intuition is correct too, and refers to a natural force that can raise *expected future* inflation.

But which is it, lower  $p_t$  or higher  $p_{t+1}$ ? This consumer first-order condition, capturing an intertemporal substitution effect, cannot tell us. Unexpected inflation and the overall price level is determined by a wealth effect. If we pair the higher interest rate with no change in surpluses, and thus no wealth effect, then the initial price level  $p_t$  does not change and the entire effect of higher interest rates is a rise in  $p_{t+1}$ . A concurrent rise in expected surpluses leads to a lower price level  $p_t$  and less current inflation  $\pi_t$ . Thus in this context standard intuition also implicitly assumes that fiscal policy acts in concert with monetary policy.

Second, consider the Phillips curve. Suppose a higher nominal rate means a higher real rate

in the IS curve and pushes output down, as it does in classic static ISLM thinking and in my simplified model. Lower output  $x_t$  in the Phillips curve,

$$\pi_t = \pi_t^e + \kappa x_t$$

means lower inflation  $\pi_t$ , yes, but *relative* to expected inflation. With adaptive expectations,  $\pi_t^e = \pi_{t-1}$ , lower inflation  $\pi_t$  means that inflation *decreases* over time. But with rational expectations,  $\pi_t^e = E_t \pi_{t+1}$ , lower inflation means that inflation *increases* over time. So we might again be talking to cross-purposes, confusing current inflation that is lower relative to expected inflation with inflation that decreases over time. And lower inflation relative to expected inflation may not mean lower inflation, as expected inflation may rise so much that both current and expected inflation are higher than otherwise, as happens in this case.

Third, verbal intuition may also confuse adjustment to equilibrium with the movement of inflation over time in equilibrium. Hold fixed expected inflation, i.e. assume it is “anchored” as Fed discussions often posit. Then higher real interest rates and lower output lower inflation  $\pi_t$ . But that means lower inflation *today*, right away, than otherwise would have occurred. It does not mean that higher interest rates today provoke lower output in the near future, and slowly drive inflation down in the further future, as most policy discussion describes. These equations do not have “long and variable lags.” So part of the contrast with standard intuition may come from confusing the process of adjustment to equilibrium with the evolution of equilibrium over time.

Stability is a natural and robust consequence of forward-looking or rational expectations, while instability is a natural consequence of backward-looking expectations. If you drive a car looking through the rear view mirror, you will veer off the road. If you drive looking forward the car will stably follow the road.

If intuition still rebels, keep in mind that stability and a Fisherian response may take a very long time to take hold, and keep in mind the experiment: permanently higher interest rates with no change in fiscal policy. Central bankers seldom abstain from moving interest rates for very long, monetary and fiscal policy both respond to events, monetary policy induces fiscal responses, and lies in the shadow of fiscal events which push inflation around.

Exchange rate pegs and purchasing power parity are a good analogy. If a country wishes a lower nominal exchange rate, a less valuable currency, it can do so by pegging at that rate and

waiting long enough. Any peg also requires fiscal backing, the ability to get as much foreign exchange or the will to print as much domestic currency or debt as needed to enforce the peg. Pegs fail when governments give up on those fiscal requirements. And one may have to wait a very long time for purchasing power parity and a long slow inflation to kick in. Nonetheless, the proposition that an exchange rate peg is eventually neutral, that relative price levels eventually adjust, if a government can stick to that peg long enough, is intuitively clear, as are the limitations of that proposition for policy and historical analysis. The proposition that higher interest rates lead eventually to higher inflation is analogous. One pegs a cross-sectional nominal price and waits for real prices to return to normal. The other pegs an intertemporal nominal price and waits for real prices to return to normal. Standard intuition holds that an exchange rate peg with adequate fiscal backing is stable, just as it holds that a money growth peg with adequate fiscal backing is stable. We just extend those ideas to a nominal interest rate peg.

In sum, stability and consequent view that higher interest rates eventually raise inflation are logically linked, explainable by simple intuition, and a robust result of forward-looking expectations. They are hard to avoid.

### 3.1 Stability

A few words on the definition of "stability" are in order. I implicitly defined stability by looking at system dynamics, and then wrote that if a model is stable, expected inflation converges under an interest rate peg. We could understand stability more generally by using the latter as a defining property: A model is "stable" if expected inflation converges under an interest rate peg. Generalizations beckon. One could state that if interest rates follow a stationary process, so does inflation; or if interest rates are bounded so is inflation. Nonlinear models can have local instability and global stability. These generalizations are not important for the models I consider here, so I do not belabor the point.

Stability is a property of the whole model. A stable model may contain both stable and unstable eigenvalues. Some equations of the model only describe expectations, as the standard IS and Phillips curve equations (1)–(2) only describe expected output and inflation given their current values and structural shocks. With either a transversality condition or a rule against explosive solutions, though, we can solve explosive eigenvalues forward to determine expectational errors, leaving a stable model. When the number of explosive eigenvalues is equal to the number of expectational equations, the model is stable and determinate. When the number of

explosive eigenvalues is less than the number of expectational equations, the model is stable but indeterminate. When the number of explosive eigenvalues is greater than the number of expectational equations, the model is unstable.

For example, in the full model with  $E_t x_{t+1}$  on the right hand side of the IS equation, the difference equation linking inflation to interest rates generalizing (7) has a root greater than one and a root less than one. (See (26) below.) We can, however, solve the unstable root forward and the model remains stable. Under an interest rate peg, expected inflation converges toward that peg. (See Cochrane (2023) section 5.1.) Likewise,  $p_t = E_t[(p_{t+1} + d_{t+1})/R]$  has an explosive eigenvalue, but we solve forward so the asset price is stable when the transversality condition is satisfied: Prices converge towards a constant dividend, and prices do not explode away from stochastic dividends. By contrast, though the adaptive expectations model  $\pi_t = (1 + \sigma\kappa)\pi_{t-1} - \sigma\kappa i_t$  has an explosive eigenvalue, we still solve it backward, not forward, and we describe explosive dynamics. There is no free unexpected inflation, no free initial condition, no jump variable which can move to offset an explosion.

As above, the policy rule can repair instability or indeterminacy by changing the number of explosive roots. I refer to the “stability” of the underlying economy, with fixed or stochastically varying interest rates, or rates that follow a rule that does not change the stability and determinacy properties of the model.

Traditional monetary doctrine does not worry about stability: the possibility that there are steady states with higher money growth and proportionally higher inflation, but raising money growth would send prices off on a downward spiral. However, when money demand is sensitive to interest rates—it is—that proposition is not so obvious either. Such models can have multiple unstable equilibria. For example, if velocity rises with the interest rate, so log money, price, and output obey<sup>5</sup>

$$m_t + \alpha(p_{t+1} - p_t) = p_t + y_t$$

and with constant output  $y$ , then inflation  $\pi_{t+1} = p_{t+1} - p_t$  follows

$$\pi_{t+1} = \left(1 + \frac{1}{\alpha}\right) \pi_t - \frac{1}{\alpha}(m_t - m_{t-1}).$$

Steady states with higher money growth have higher inflation, but they are unstable. From a steady state, raising money growth leads to spiraling deflation. This issue is usually glossed over

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<sup>5</sup>This is the Sargent and Wallace (1973) rational expectations version of the Cagan (1956) hyperinflation model.

in standard monetarist doctrine, with talk about the "stability" of velocity, where the issue is more contentious in today's debates about interest rate targets. Again, central banks don't target money supplies, so there is not much point in reviving this controversy.

### 3.2 Stability Versus Neutrality

*Stability* is the central question for the possibility of an interest-rate peg and the sign of inflation's long-run response to interest rates. Exact long-run neutrality is not important. If a one percentage point interest rate rise leads to 0.9 or 1.1 percentage point higher inflation, no important conclusion has changed. If that rise leads to an 0.1 percentage point permanent output rise or fall, even less has changed. It would take a mighty non-neutrality for steady state inflation to *decrease* when steady state interest rates rise. Similarly, standard monetary theory is little affected by small non-neutralities. It would take a huge non-neutrality for higher money growth to lower inflation.

With a standard new-Keynesian Phillips curve,

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t$$

the sticky-price model is not exactly long-run neutral. Real variables are eventually independent of the price *level*, which is one definition of neutrality, but steady states

$$x = (1 - \beta)/\kappa\pi, \tag{25}$$

with a higher inflation *rate* have higher output, which violates a more stringent definition of long-run neutrality. Since  $\beta$  is close to one, however, this is a small effect.

This effect is contentious, with several attempts to fix it and critiques of those attempts. Its presence is not a central part of new-Keynesian doctrine, which unlike 1960s Keynesianism does not argue for higher steady state inflation to produce higher steady state employment. Moreover, it is only one of many peculiarities of the Calvo (1983) and related pricing models (Auclert et al. (2022)), that are not meant to be taken to extremes. For example, a fixed rate of price setting opportunities breaks down as inflation is larger. The Calvo fairy visits more frequently in Argentina. And microeconomic evidence on prices does not fit the model well (Steinsson and Nakamura (2013)).

To the central point,  $\beta < 1$  does not change inflation dynamics, it does not change the sign

of the effect of interest rates on inflation, and it does not produce instability instead of stability. In this simple model, with  $\beta < 1$  in the Phillips curve, inflation dynamics become

$$E_t \pi_{t+1} = \frac{1}{\beta + \kappa \sigma} \pi_t + \frac{\kappa \sigma}{\beta + \kappa \sigma} i_t.$$

The last coefficient on the interest rate is positive for any  $\beta > 0$ . With  $\beta < 1$ , a tiny  $\kappa \sigma$  can produce an unstable leading term, i.e., greater than one. But this possibility does not exist in the full model, with  $E_t x_{t+1}$  in the IS curve. In that case, inflation is a two-sided moving average of the nominal interest rate,

$$E_t [(1 - \lambda_1^{-1} L)(1 - \lambda_2 L^{-1}) \pi_{t+1}] = \sigma \kappa \lambda_1^{-1} i_t. \quad (26)$$

where

$$\lambda_{1,2} = \frac{(1 + \beta + \sigma \kappa) \pm \sqrt{(1 + \beta + \sigma \kappa)^2 - 4\beta}}{2}.$$

(Again, see Cochrane (2023) section 5.1. Seeing this formula may also motivate the much simpler model of this paper.) Here  $\lambda_1 > 1$  and  $\lambda_2 < 1$  if  $\sigma \kappa > 0$  and for any  $\beta \in [0, 1]$ . Lowering  $\beta$  cannot change the stability or determinacy of the model.

More importantly, from the IS equation

$$x_t = E_t x_{t+1} - \sigma (i_t - E_t \pi_{t+1}), \quad (27)$$

inflation and interest rates remain long-run neutral even with  $\beta < 1$  and a non-neutral output  $x$ : A 1 percentage point higher nominal interest rate still eventually produces a 1 percentage point higher inflation. The permanent output effect has no impact on the central relation between nominal interest rates and inflation.

The relationship between interest rates and inflation could also display small non-neutralities. Since nominal interest is taxed, at least one of pre-tax or after-tax nominal interest rates does not move exactly one for one with steady state inflation. Even steady inflation generates distortions via the tax code, and changes the distribution of prices among the monopolistically competitive producers of new-Keynesian models. But these considerations do not affect the *stability* or the *sign* of the long-run response of inflation to interest rates.

## 4 Can Higher Interest Rates Temporarily Lower Inflation?

Stability and determinacy mean that higher interest rates eventually raise inflation. Experience suggests, and it is widely believed, that higher interest rates can lower inflation. There is nothing in what we have done so far that rules out a *temporary* negative sign. At a minimum we want a model that can express that belief, and see if the required ingredients and other predictions of such a model make sense.

If that were true, then the central bank could do some good by raising rates. We could then also understand central bankers' and policy commentators' belief in a uniformly negative effect, as well as the absence of a well-documented long-run positive effect in most econometric estimates. Central banks seldom leave rates alone long enough, with a background of stable fiscal policy and no other shocks, to see the positive long-run effect.

We are, in short, finally where Lucas started. Lucas's central contribution was to build on a theory of neutral money, to describe the short-run *non-neutrality* of money.

Here are the rules of the game: We want a model in which the central bank operates via an interest rate target, does not limit money supply, and pays full interest on abundant reserves. We want a higher interest rate, with no change in fiscal policy, to lower inflation, at least for a while. The model should follow the usual rules of economics, including rational expectations. It should build on the frictionless model with well-described frictions. That is likely to leave the frictionless model predictions in the long run, but not necessarily so.

I emphasize the qualifier, "with no change in fiscal policy." Monetary policy can appear to have an effect if it comes with or induces a change in fiscal policy. Fiscal policy might react to the same events as monetary policy reacts to. An interest rate rise leads to higher interest costs on the debt, which might induce a fiscal tightening. It might lead to a recession, which leads to stimulus, bank and business bailouts, and automatic stabilizers. On the other hand, inflation induced by monetary policy might lead to austerity and fiscal retrenchment. For evaluating history or the likely course of the economy following a monetary policy shock, we want to include such contemporaneous and induced fiscal responses. But our question is a theoretical one: What can monetary policy do all by itself? If monetary policy only reduces inflation by inducing a fiscal contraction, then monetary policy by itself is not that effective.

There might be no such model. But knowing that fact is important. A search that establishes no such model exists, that standard beliefs are either wrong, fundamentally rely on induced

fiscal policy responses, or must be founded on an analysis that violates the above rules, is just as important as a search that finds a model to confirm standard beliefs.

You might think it's easy. Just add some sticky prices. That turns out not to be the case. You might think that standard models in use for decades satisfy this desire. That also turns out not to be the case. Both standard new-Keynesian and adaptive expectations models produce a negative effect by positing a fiscal contraction coincident with the interest rate rise. Without that contraction, they don't produce a negative sign either.

#### 4.1 A Failure in Simple Sticky Price Models

Figure 2 plots the response of my simplified rational expectations model (20)-(21) to an unexpected permanent interest rate rise. The “sticky price” line gives the main message of this section: Inflation rises, even in the short run. The real interest rate rises (interest rate higher than following inflation), so output declines (not shown). But sticky prices only draw out the positive response of inflation to interest rates. We maintain long-run neutrality: The inflation rate eventually rises to match the interest rate.

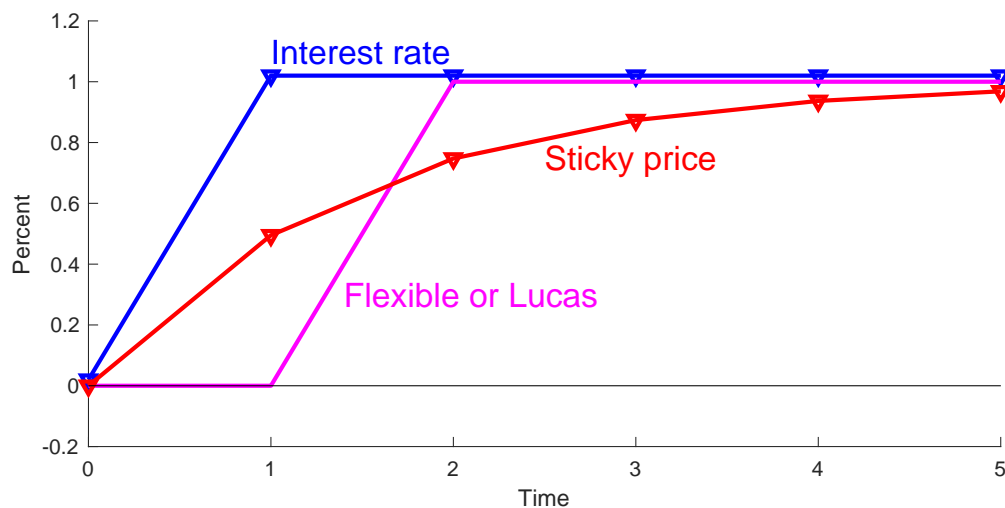


Figure 2: Inflation response to a 1% permanent rise in the interest rate with no change in fiscal policy. Rational expectations model. Parameters  $\sigma\kappa = 1$ .

Start with the “flexible or Lucas” line. The flexible price model is (18)–(19). The responses solve

$$E_t\pi_{t+1} = i_t \quad (28)$$



$$\Delta E_{t+1} \pi_{t+1} = 0. \quad (29)$$

The response starts with all variables 0 at time 0. We want the response to a permanent unexpected interest rate rise to  $i_t = i_1$ ,  $t = 1, 2, \dots$  at time 1, with no change in fiscal surpluses. The first, surplus, term in the general unexpected inflation equation (17) is zero. By (28), the discount rate or interest cost term is zero as well. Hence, as in (29) and as shown in Figure 2, there is no change in inflation on the day the interest rate rises, but inflation fully follows the interest rate with a one-period lag.

The Lucas Phillips curve in this model gives dynamics (24). The surplus innovation term in the unexpected inflation equation (17) is again zero, but now there are potentially interest costs to pay. The impulse-response solves

$$E_t \pi_{t+1} = i_t + \frac{1}{\kappa \sigma} \Delta E_t \pi_t \quad (30)$$

$$\Delta E_{t+1} \pi_{t+1} = \Delta E_{t+1} \sum_{j=1}^{\infty} \rho^j (i_{t+j} - \pi_{t+j+1}). \quad (31)$$

With only one unexpected movement at time 1 ( $t = 0$  in the equation), so  $\pi_2 = E_1 \pi_2$  and so forth, (30) leaves  $\pi_1$  arbitrary but then  $\pi_2 = i_1 + \pi_1 / (\kappa \sigma)$ ,  $\pi_3 = i_1$ ,  $\pi_4 = i_1$ , etc. Now, use (31) to find  $\pi_1$  and the unique path. (31) reduces to  $\pi_1 = i_1 - \pi_2 = -\pi_1 / (\kappa \sigma)$ . The unique solution is  $\pi_1 = 0$ , and thus  $\pi_2 = i_1$ ,  $\pi_3 = i_1$ , and so forth. You can verify that this path solves both (30) and (31). Despite the non-neutrality in the Phillips curve, which produces a one-period rise in output (not shown), inflation follows the flexible-price path, and does not decline.

The “sticky price” line uses the forward-looking new-Keynesian Phillips curve. The response function solves (20)-(21):

$$E_t \pi_{t+1} = \frac{1}{1 + \sigma \kappa} \pi_t + \frac{\sigma \kappa}{1 + \sigma \kappa} i_t \quad (32)$$

$$\Delta E_{t+1} \pi_{t+1} = \Delta E_{t+1} \sum_{j=1}^{\infty} \rho^j (i_{t+j} - \pi_{t+j+1}). \quad (33)$$

Again, surpluses are zero in (33) as that is the question we are asking. In response to the interest rate shock, there is a family of solutions to (32) which we can index by  $\pi_1$ ,

$$\pi_{t+1} = i_1 + \frac{1}{(1 + \sigma \kappa)^t} (\pi_1 - i_1). \quad (34)$$

Now, we use (33) at time  $t = 0$  to determine  $\pi_1$ :

$$\pi_1 = \sum_{j=1}^{\infty} \rho^j (i_1 - \pi_{j+1}). \quad (35)$$

Substituting from (34) and simplifying,

$$\pi_1 = \frac{\rho}{1 + \sigma\kappa} i_1 \quad (36)$$

and the full unique solution is

$$\pi_{t+1} = \left[ 1 - \frac{1 + \sigma\kappa - \rho}{(1 + \sigma\kappa)^{t+1}} \right] i_1. \quad (37)$$

The real interest rate rises and output declines, so if your objective is only to produce a model in which the central bank can cool economic activity in the short run with interest rate rises, you have it. You might stop here and say that we have indeed redone Lucas (1972a) with interest rate targets, since his purpose was to understand how money growth affects the real economy.

But inflation still rises uniformly after the interest rate rise. Indeed, in period 1, inflation is *greater* than the (zero) value of the frictionless model. Why? With sticky prices, higher nominal interest rates lead to higher real interest rates. Higher real interest rates mean greater unfunded (by assumption of no change in fiscal policy) interest costs on the debt. With no change in surplus, these higher interest costs must come from a higher unexpected period 1 inflation, which devalues outstanding debt.

Indeed, as prices become stickier,  $\kappa \rightarrow 0$ ,  $\pi_1 \rightarrow \rho$  (see (36)) which is just barely less than one. Stickier prices lead to *more* period 1 inflation, since they lengthen the period of high real interest costs.

How is it that inflation jumps up in a most non-sticky way, in the limit that prices become stickier? Sticky *prices* do not imply sticky *inflation*. The few firms which can change price at any instant know inflation will be persistently higher, so they raise their prices a lot. Much verbal intuition describes sticky inflation or inflation momentum, but that needs a different set of costs than those specified by most current sticky *price* models. (Lagged inflation in the Phillips curve would add some inflation stickiness; I investigate below and it does not solve the puzzle.)

The fiscal underpinnings of the model matter crucially. If we said that fiscal surpluses would rise to pay higher interest costs on the debt, then we would obtain at least  $\pi_1 (= \Delta E_1 \pi_1) = 0$ , no immediate rise. If we could pair the interest rate rise with even higher surpluses, for example if future inflation led to future fiscal austerity, we could predict lower current inflation,  $\pi_1 < 0$ . But the question we want to ask is, what can higher interest rates do to lower inflation *without* fiscal support. The answer is, so far, higher interest rates can produce a recession, but they raise inflation.

And even this much non-neutrality is fragile, really the result of one-period rather than instantaneous debt. In the continuous-time version of this response, with instantaneous debt, inflation instantly tracks the interest rate, for any price stickiness. Intuitively, the  $\Delta E_1 \pi_1$  term on the left-hand side of the unexpected inflation identity (33) is absent in continuous time with sticky prices, because prices do not jump, so the discounted sum (integral) of interest costs must be zero. With AR(1) dynamics that cannot overshoot and return, that means inflation must jump instantly to match the interest rate and the real interest rate does not move at all. Sticky prices really do not imply sticky inflation! The Appendix says this with equations.

Appealing to the new-Keynesian model will not help. This *is* the new-Keynesian model. Solving the model in new-Keynesian style, the central bank can produce any value of first-period inflation  $\pi_1 = \pi_1^*$  it wishes, by following an interest rate policy  $i_1 = i_1^* + \phi(\pi_1 - \pi_1^*)$ , where  $i_1^* = 1$ , the first point on the desired interest rate path, and  $\pi_1^*$  is the desired, possibly negative, first-period inflation. Write it up as  $i_t = \phi\pi_t + u_t$  with  $u_t = i_t^* - \phi\pi_t^*$ . But any inflation path other than the one we have already plotted requires a change in surplus, and lower initial inflation requires positive surpluses. If we phrase the question of a new-Keynesian ( $\phi > 1$ , passive fiscal policy) model, “What is the response of inflation to a monetary policy disturbance  $\{u_t\}$  that produces an unexpected permanent rise in the interest rate, and the associated passive fiscal policy requires no change in surpluses?” we have just calculated the unique answer.<sup>6</sup>

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<sup>6</sup>Bianchi and Melosi (2019) study a related question. They also study interest rate increases with no fiscal policy reaction, in a more elaborate sticky-price model. They imagine a central bank that goes one step further, following an “active” Taylor rule that reacts to current inflation with coefficient greater than one. Now, higher interest rates raise interest costs on the debt, which leads as here to greater inflation. Their central bank raises interest rates even further, which if left alone leads to explosive inflation. The possibility of a Markov switch back to a coordinated regime saves this asymptotic possibility. Since on the path, future inflation is higher than current inflation, output falls.

## 4.2 A Negative Effect From a Transitory Interest Rate?

The essential failure of the rational expectations sticky price model to produce a negative inflation effect is not tied to the permanent interest rate increase shown in Figure 2. I only graph permanent interest rate changes so that we can see long-run properties.

However, transitory interest rate paths can give a misleading appearance of such an effect. These statements are also important to check as experience with the standard new-Keynesian model and AR(1) disturbances has led to the impression that permanent shocks raise inflation, but transitory shocks lower inflation. That conclusion is an artifact of the restriction to AR(1) shocks, but it motivates a look at transitory shocks here. (Explicit calculations in Cochrane (2023) Section 17.3.1).

To illustrate this point, Figure 3 plots the response of the simplified rational expectations model to a transitory interest rate movement, with  $i_t = 0.7i_{t-1} + \varepsilon_t$ . Inflation still rises uniformly. The only difference is that one doesn't notice long-run neutrality with a transitory shock. Again, the main point does not require permanent interest rate rises.

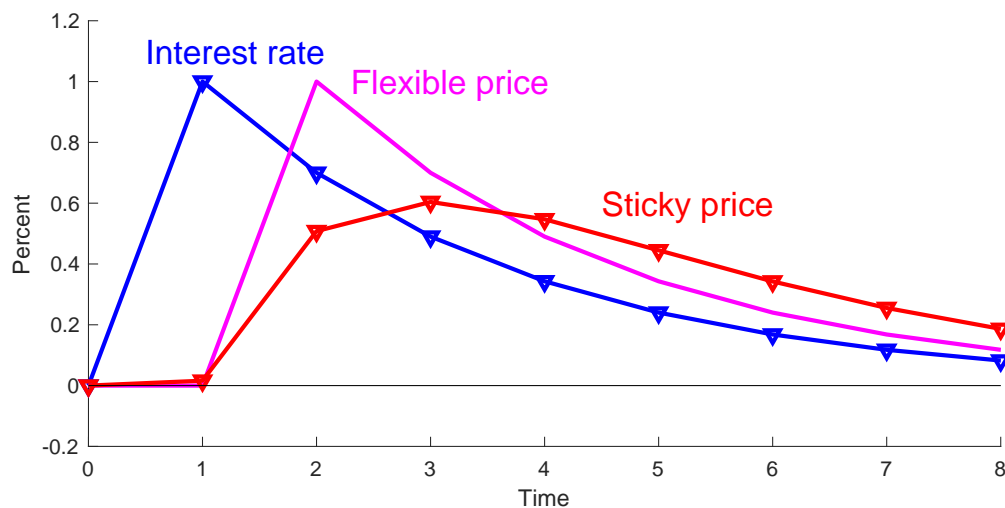


Figure 3: Response of the simple rational expectations sticky price model to a transitory interest rate path, with no change in fiscal policy. Parameters  $\sigma\kappa = 1$ ,  $\rho = 0.99$ , and  $i_t = 0.7i_{t-1} + \varepsilon_t$ .

The standard new-Keynesian specification produces a negative inflation response to an AR(1) monetary policy shock. But it does so by supposing a contemporaneous fiscal tightening (a “passive” response to the central bank’s equilibrium-selection policy). Interest rates rise, inflation falls, so you can see a period of higher real interest costs on the debt. The initial disinflation is also a windfall to bondholders. Surpluses rise to pay these costs. Even looked at

in standard new-Keynesian way, Figure 3 shows the unique equilibrium inflation path with the plotted interest rate path that has no change in fiscal surpluses.

At the cost of some algebra, relegated to the Appendix, we can find the response of the rational-expectations sticky-price model (32)-(33) to an arbitrary interest rate path  $\{i_t\}$ , announced at period 1, with no change in fiscal surplus:

$$\pi_{t+1} = \frac{1-\rho}{(1+\sigma\kappa)^{t+1}} \sum_{j=1}^{\infty} \rho^j i_j + \frac{\sigma\kappa}{1+\sigma\kappa} \sum_{j=1}^t \frac{1}{(1+\sigma\kappa)^{t-j}} i_j \quad (38)$$

and in particular

$$\pi_1 = \frac{1-\rho}{1+\sigma\kappa} \sum_{j=1}^{\infty} \rho^j i_j. \quad (39)$$

The sum in the first term of equation (38) is common to inflation at all dates, and decays at a rate determined by the price-stickiness parameters  $\sigma\kappa$ . It captures the present value of all future interest costs. The second term is a backward-looking moving average. It captures the smoothed version of  $E_t \pi_{t+1} = i_t$ .

In (38), all the coefficients are positive. Hence, *any sequence of positive interest rates  $\{i_t\}$  generates uniformly positive inflation response  $\{\pi_t\}$* . In this sense, the positive response of the rational expectations sticky price model is general and does not depend on the time-series process of the interest rates.

We can generate a negative apparent negative effect of interest rates on inflation, however. If interest rates were to rise in the short run, and then turn towards a long-lasting negative value, we could have a few positive interest rates  $i_t$ , despite a negative value of  $\sum_{j=1}^{\infty} \rho^j i_j$  which drives a negative overall inflation response.

Figure 4 presents an example. As in all figures, this is the response of inflation to the indicated interest rate path, with no change in fiscal surpluses. I create an interest rate path with a hump shape rather than an AR(1) shape to make the graph prettier, and to resemble paths often seen in VARs. I specify  $i_t = 0.7^{(t-1)} - 0.6 \times 0.4^{(t-1)} - 0.2$ . The long-run interest rate response is thus  $-0.2$ . I then calculate the inflation path from (38). For an AR(1)  $i_t = \eta i_{t-1} + \varepsilon_{i,t}$ , (38) becomes

$$\pi_{t+1} = \left\{ \left( \frac{1-\rho}{1+\sigma\kappa} \frac{\rho}{1-\rho\eta} \right) \frac{1}{(1+\sigma\kappa)^t} + \frac{\sigma\kappa}{1-\eta(1+\sigma\kappa)} \left[ \frac{1}{(1+\sigma\kappa)^t} - \eta^t \right] \right\} i_1$$

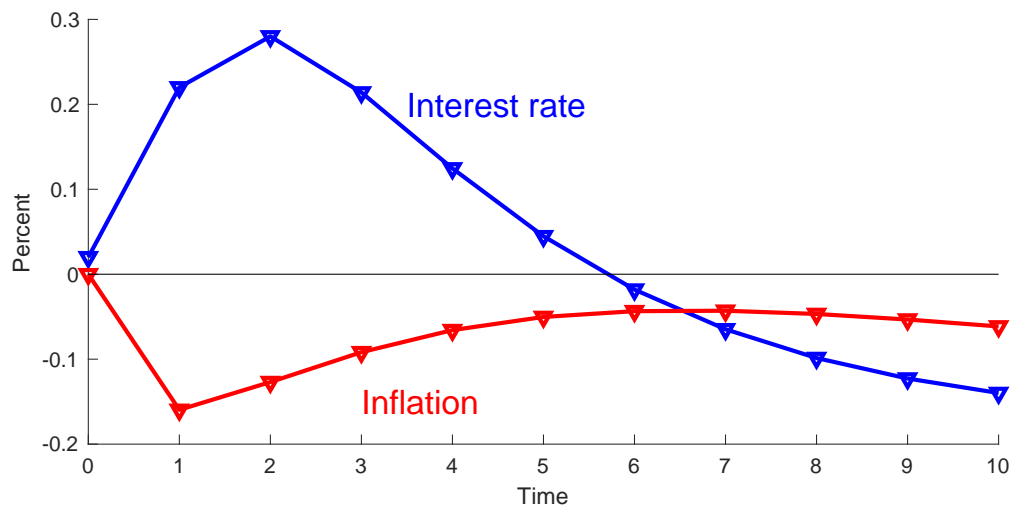


Figure 4: Inflation response to a transitory rise in the interest rate. Parameters  $\sigma\kappa = 0.1$ .

Then, such solutions add for the three AR(1)s that generate the interest rate path.

Figure 4 looks initially appealing. A higher interest rate lowers inflation! In the very long run, of course, both interest rate and inflation end up at -0.2%. But even that is not so unrealistic. We don't often see the long run; and when we do, both interest rates and inflation decline after a successful stabilization such as the 1980s. The plot looks superficially like the standard adaptive expectations model account of that episode (Figure 5 below). The long period in which the interest rate has a slightly greater negative response than inflation on the right-hand side would be easy to miss.

But this reading is profoundly misleading. Inflation declines initially because interest rates decline in the far future, *despite*, not because of, the short-term rise in rates. Indeed, the positive interest rates drag inflation up from even more negative values. If you want less inflation in this model, lowering interest rates immediately—the negative of Figure 2—is an even more powerful tool. There is nothing in the mechanics of this model that resembles standard intuition, that high real interest rates drive inflation down. Beware causal readings of impulse-response functions!

If we want a negative effect of interest rates on inflation, without a contemporaneous fiscal shock that's really doing the work, then, we will need to add ingredients beyond sticky prices.

### 4.3 Fiscal Requirements for the Adaptive Expectations Model

The adaptive expectations model (5),

$$\pi_t = (1 + \sigma\kappa)\pi_{t-1} - \sigma\kappa i_t. \quad (40)$$

produces standard intuition. In response to permanently higher interest rates, inflation declines in a classic downward spiral.

This is not the answer we are looking for, however. In addition to the above objections to irreducibly adaptive expectations and to unstable models, this response also requires fiscal support. Debt still accumulates by (14),

$$\rho v_{t+1} = v_t + i_t - \pi_{t+1} - \tilde{s}_{t+1}. \quad (41)$$

If inflation spirals off downward, real interest costs spiral upward. If there is no change in surpluses, then debt spirals off upward as well. If we phrase the question of the adaptive expectations model, “What is the response of inflation to a permanent unexpected rise in the interest rate, *with no change in fiscal surpluses*,” the downward spiral does not answer that question. Since the model is determinate, there is no answer to that question—the central bank cannot permanently raise interest rates without fiscal support.

In reality, then, as in theory, the central bank must give in and drop the nominal interest rate to stop the nascent deflation spiral. Friedman (1968) recognized this fact, and describing the opposite reduction in interest rates, wrote that the central bank would eventually give in out of distaste for exploding inflation. Here, it is forced to give in by exploding debt and deficits.

The most standard model that produces a disinflation from a higher interest rate combines adaptive expectations with a Taylor rule. A positive shock to the Taylor rule starts an inflation decline, and then the Taylor rule automatically lowers the interest rate to steady the incipient spiral. This model also requires tighter fiscal policy to pay higher interest costs on the debt. Without such support, the disinflation does not occur.

It is easiest to exhibit this behavior in the continuous time version of the model, which avoids timing issues of inflation. Uniting the continuous time versions of (40) and (41) with a Taylor rule,

$$d\pi_t/dt = \sigma\kappa(i_t - \pi_t) \quad (42)$$

$$dv_t/dt = rv_t + i_t - \pi_t \quad (43)$$

$$i_t = \phi\pi_t + u_t. \quad (44)$$

(The Appendix develops the continuous time model more fully. I retain for now the assumption of short-term debt.) At time 0, the monetary policy shock  $u_t$  rises suddenly and unexpectedly from 0 and stays at the constant value  $u_t = u_0$ . Substituting (44) in (42), inflation dynamics become

$$\frac{d\pi_t}{dt} = -\sigma\kappa(\phi - 1)\pi_t - \sigma\kappa u_0.$$

The solution is

$$\begin{aligned} \pi_t &= -\frac{1}{\phi - 1} \left[ 1 - e^{-\sigma\kappa(\phi-1)t} \right] u_0 \\ i_t &= -\frac{1}{\phi - 1} \left[ 1 - \phi e^{-\sigma\kappa(\phi-1)t} \right] u_0 \\ r_t &= e^{-\sigma\kappa(\phi-1)t} u_0 \\ e^{-rt} v_t &= \frac{e^{-[r+\sigma\kappa(\phi-1)]t} - 1}{r + \sigma\kappa(\phi - 1)}. \end{aligned}$$

Figure 5 presents this result. The nominal and hence real interest rate rises, and inflation starts on its downward spiral. Following the Taylor rule, the interest rate swiftly follows inflation down, and we stabilize at a new lower inflation rate. This is a standard story of the 1980s, for example.

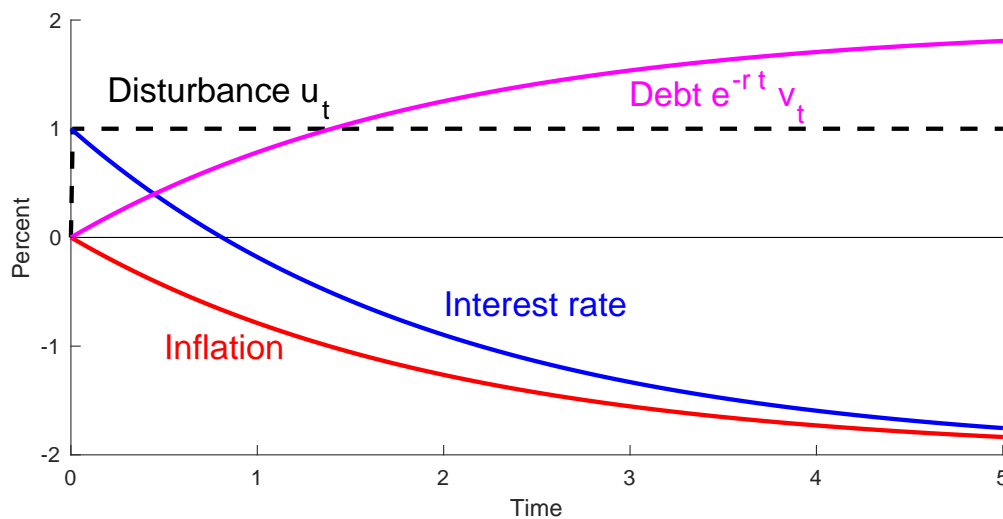


Figure 5: Response to a 1% permanent monetary policy shock in the adaptive expectations model with a Taylor rule. Parameters  $\sigma\kappa = 1$ ,  $\phi = 1.5$ ,  $\rho = 0.01$ .



However, the real interest rate is positive throughout the episode. Thus, with no greater surpluses, the greater interest costs on the debt are simply rolled over, and debt increases without bound. The transversality condition  $\lim_{T \rightarrow \infty} e^{-rT} v_T = 0$  is violated.

This simulation does not answer the question, what can the central bank do by itself, without fiscal support? To achieve this outcome, fiscal policy must increase surpluses to pay the interest costs, and keep debt from exploding. If this is the story of the 1980s, the story is a joint monetary-fiscal stabilization, with surpluses rising (as they did) to pay the higher interest costs of the debt, not a story of monetary policy acting alone.

#### 4.4 Adaptive Expectations With a Fiscal Constraint

In response to our question, the effect of interest rates on inflation *holding fiscal policy constant*, the adaptive expectations model cannot realistically produce a permanent inflation change.

This point is again easier to see in the continuous time version of the model, because the timing of interest rate relative to inflation collapses to  $i_t = \pi_t$ . Denoting  $r_t \equiv i_t - \pi_t$ , (42)-(43) and the transversality condition are

$$d\pi_t/dt = -\sigma\kappa r_t \quad (45)$$

$$dv_t/dt = rv_t + r_t \quad (46)$$

$$\lim_{T \rightarrow \infty} E_t e^{-rT} v_T = 0. \quad (47)$$

The symbol  $r$  without subscript represents the steady state real interest rate, and point of linearization. The symbol  $r_t$  represents variation of the real interest rate around this steady state value. Normally the surplus  $\tilde{s}_t$  appears on the right hand side of (46) but I omit it as the exercise holds fiscal policy constant.

Given a real interest rate path, the solutions to (45)-(46) are

$$\pi_t = -\sigma\kappa \int_0^t r_j dj \quad (48)$$

$$e^{-rt} v_t = \int_0^t e^{-rj} r_j dj. \quad (49)$$

Imposing the transversality condition (47), the discounted sum of interest costs on the debt must

be zero, which is a constraint on the real rate path  $\{r_j\}$ ,

$$0 = \int_0^{\infty} e^{-rj} r_j dj. \quad (50)$$

Define the long-run inflation rate  $\pi_{\infty} \equiv \lim_{T \rightarrow \infty} \pi_t$ , so

$$\pi_{\infty} = -\sigma\kappa \int_0^{\infty} r_j dj. \quad (51)$$

To lower long-run inflation in (51), then, pick a real rate path  $\{r_t\}$ , subject to the fiscal constraint (50), which states that the present value of interest costs is zero, i.e. unchanged. Once we pick the real rate path, and solve for inflation from (48), we can find the nominal rate from  $i_t = r_t + \pi_t$ . One can also express the  $\{\pi_t, v_t\}$  solution directly in terms of the nominal interest rate, but the resulting expressions are not so simple and transparent.

We see right away that *in the limit*  $r \rightarrow 0$ , *the adaptive expectations model cannot produce any permanent inflation or disinflation—a value of  $\pi_{\infty}$  other than zero—in the absence of a change in fiscal policy.* The right hand sides of (51) and (50) are the same. Positive real interest rates that push down inflation also build up the debt. They must be balanced by negative real rates to bring down the debt, but those real rates raise inflation right back to where it was.

A positive steady state real rate  $r > 0$  offers an apparent avenue for permanent disinflation. But, since  $r$  is small, this result is fragile, and the resulting policies are unrealistic. The present value in (50) downweights real rates in the future. So, to lower inflation, the central bank must first *lower* real interest rates, building up inflation, to produce a period of low interest costs, which lowers the debt. Then the central bank turns around and raises real interest rates, driving inflation down, using the accumulated savings on the debt to pay the higher interest costs. The unweighted integral in (51) allows a longer period of future high interest rates than the initial period of low interest rates, and overall to drive inflation down relative to its initial value.

But with small  $r$ , the opportunity requires large swings in rates and inflation to produce a small permanent reduction in inflation. Moreover, the period of low rates and high inflation must come first, before the final period of high rates and lower inflation. This is highly unrealistic, and not at all consistent with the classic intuition we are trying to rescue.

Since I do not pursue adaptive expectations in this baseline model, I do not develop the possibility further. The point of this section: When we pose the question as, “Can higher interest rates lower inflation, *without a change in fiscal policy*,” not even the most classic adaptive

expectations model, which fits the narratives of central bankers and the policy world, can do it.

#### 4.5 The Standard Model with Lagged Inflation in the Phillips Curve

Here I verify that with short-term debt and no change in fiscal policy, a higher interest rate does not raise inflation, even in the standard new-Keynesian model, and even extending that model to include lagged inflation in the Phillips curve. The model is

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1}) \quad (52)$$

$$\pi_t = (1 - \alpha)E_t \pi_{t+1} + \alpha \pi_{t-1} + \kappa x_t \quad (53)$$

$$\rho v_{t+1} = v_t + i_t - \pi_{t+1} \quad (54)$$

$$i_{t+1} = \eta i_t + \varepsilon_{i,t+1}. \quad (55)$$

Equation (52) includes the forward-looking output term. Equation (53) allows for lagged inflation. Perhaps we can get some of the adaptive expectations dynamics? Alas no, as we shall see. Equation (54) specifies short-term debt and zero surplus, as I only calculate the response to a monetary policy shock.

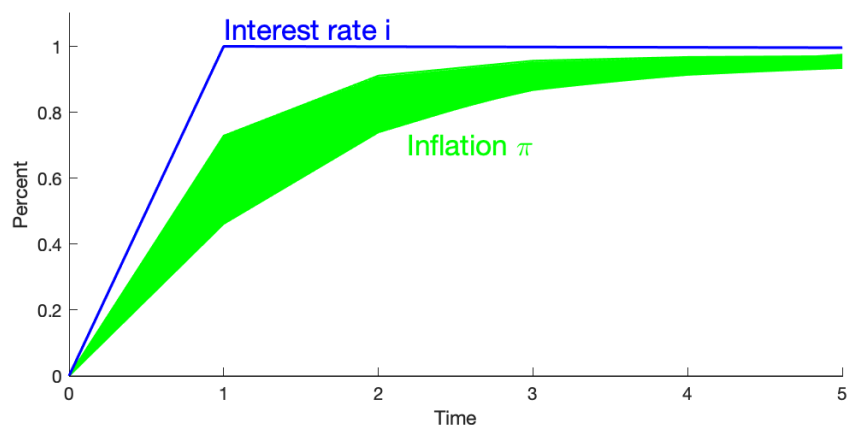


Figure 6: Inflation response to a permanent interest rate rise, with no change in fiscal policy. Full model with forward-looking IS curve and a lag in the Phillips curve. The shaded area shows impulse response functions with all parameters  $\alpha$ ,  $\sigma$ ,  $\kappa$  that produce real eigenvalues.

Figure 6 presents the response of this model to a permanent interest rate rise. I solve the model numerically. The shaded area gives all possible impulse responses, calculated by evaluating the response function for a grid of parameter values. I include all values of  $\alpha \in [0, 1]$ .

I restrict parameters to those that produce real eigenvalues, however. Sawtooth or sine-wave responses induced by complex eigenvalues enlarge the possibilities and are sometimes negative, but clearly unrealistic. As the figure shows, for all such parameter values, this generalized model produces a steady rise in inflation. The static IS curve did capture the results of this more complex model.

#### 4.6 A Model with Long-Term Debt that Produces a Negative Effect

Figure 7 offers a simulation of the only current model I know of in which higher interest rates produce a negative short-run inflation effect, with rational expectations and no change in fiscal policy.

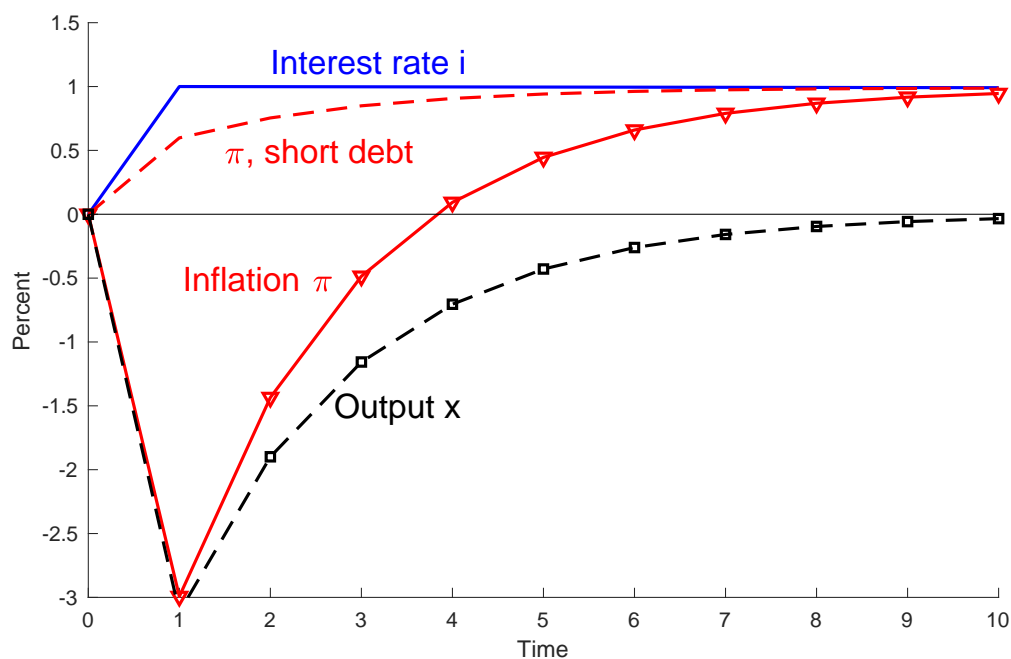


Figure 7: Response of inflation to an interest rate shock, and no change in fiscal policy, with long term debt. In the base case, debt has a geometric maturity structure, decaying at rate  $0.9^t$  with  $t = \text{maturity}$ . The short-term debt case is one-period debt only.

The model is

$$x_t = E_t x_{t+1} - 0.5(i_t - E_t \pi_{t+1}) \quad (56)$$

$$\pi_t = E_t \pi_{t+1} + 0.5x_t \quad (57)$$

$$i_t = i_{t-1} + \varepsilon_{i,t} \quad (58)$$

$$\rho v_{t+1} = v_t + r_{t+1}^n - \pi_{t+1} - \tilde{s}_{t+1} \quad (59)$$

$$E_t r_{t+1}^n = i_t \quad (60)$$

$$r_{t+1}^n = 0.9q_{t+1} - q_t. \quad (61)$$

Here I use the full model, i.e. including the  $E_t x_{t+1}$  term in (56), as analytic solutions are not insightful. I include long-term debt with a geometric maturity structure. The face value of zero coupon bonds of maturity  $j$ ,  $B_t^{(j)} = \omega^j B_t$  declines at rate  $\omega = 0.9$ . The symbol  $r_{t+1}^n$  represents the ex-post nominal return on the portfolio of government debt. Equation (60) prices long-term bonds with the expectations hypothesis. Equation (61) links the log price  $q_t$  of the government bond portfolio to its rate of return. This is a simplified version of the model in Cochrane (2021), which simplifies Sims (2011).

Again, I raise interest rates, but I leave fiscal surpluses unchanged and I calculate the inflation and output responses. Inflation declines temporarily! Inflation then rises in the long run, fulfilling long-run neutrality. This is the pattern we have been seeking.

Long-term debt is the crucial innovation relative to the simple models of previous figures, and its inclusion produces the negative sign. With long-term debt, but no ability to change surpluses, the central bank can lower inflation now, but by raising inflation later. Raising long-run interest rates, and thus long-run inflation, devalues long-run debt. Since surpluses haven't gone *down* either, that action raises the value of short-term debt. But short-term debt can only become more valuable via a lower price level.

Sims (2011) calls the pattern of Figure 7 “stepping on a rake” and offers it as a parable of the 1970s inflation cycles, in which higher interest rates temporarily lowered inflation, but inflation came back larger. We can call the pattern “unpleasant interest-rate arithmetic,” a successor to Sargent and Wallace (1981) unpleasant monetarist arithmetic. (Sargent and Wallace focus on seignorage in a model with real debt, in which the central bank controls money supply. In this model, there is no seignorage or money, government debt is nominal, and the central bank follows an interest rate target.) Unpleasant interest-rate arithmetic is here a negative sum, or inequality proposition: The central bank gets more long-run inflation than it saves in short-run inflation.

Generalizing (17) to long-term debt, we have

$$\sum_{j=0}^{\infty} \omega^j \Delta E_{t+1} \pi_{t+1+j} = -\Delta E_{t+1} \sum_{j=0}^{\infty} \rho^j \tilde{s}_{t+1+j} + \Delta E_{t+1} \sum_{j=1}^{\infty} \rho^j (i_{t+j} - \pi_{t+j+1}), \quad (62)$$

(Cochrane (2023) section 3.5.) By assumption, surpluses on the right hand side do not change. With no change in real interest rates, then, (62) offers the central bank a menu: reduce inflation at one date, in exchange for more inflation at a different date. Devalue bonds at the latter date to repay bondholders at the former date with more valuable dollars. This is unpleasant arithmetic in simplest form. Higher real interest rates on the right hand side are higher interest costs on the debt, and make the tradeoff *worse*. Inflation must rise more on some dates.

You can see the mechanism directly with flexible prices. Let the real interest rate be constant  $r$ , and imagine a geometric maturity structure in which debt of maturity  $j$  has face value  $Be^{-\varphi j}$  ( $e^{-\varphi} = \omega$ ). Imagine a constant nominal interest rate  $i$ . The government debt valuation equation, stating that the market value of debt equals the present value of surpluses, reads

$$\frac{\int_{j=0}^{\infty} e^{-ij} e^{-\varphi j} B}{P} = \frac{1}{i + \varphi} \frac{B}{P} = E_t \int_{j=0}^{\infty} e^{-rj} s_{t+j}.$$

Imagine an unexpected permanent rise of the nominal interest rate from  $i$  to  $i'$ . The price level then jumps by

$$\frac{P'}{P} = \frac{i + \varphi}{i' + \varphi}$$

With perpetuities,  $\varphi = 0$ , this is a powerful effect. A rise from  $i = 1\%$  (0.01) to  $i = 2\%$  cuts the price level in half. A 1% larger inflation then starts right after this price level drop. Shorter debt gives a smaller response. For example, if debt has a 4 year half life (the face value of debt declines to  $e^{-1}$  by a 4 year maturity), then  $\varphi = 0.25$ , so a rise from 1% to 2% nominal rate implies  $0.26/0.27 \approx 10\%$  price level drop. That is still a large cumulative inflation from a 1% interest rate rise.

As with the short-term debt case, there is nothing essential about a permanent interest rate shock to generating this example. Any persistent shock will also lower long-term bond prices and raise long-term inflation, raising the value of short-term bonds. The initial disinflation is smaller, but still present.

A Taylor-type rule, in which the interest rate reacts to inflation, adds this sort of response to the inflationary effect of fiscal or other shocks. In this way, a Taylor-type rule spreads the inflation of such shocks forward, reducing their immediate impact. With the forward-looking Phillips curve, random walk inflation has no output effect, so by smoothing inflation forward such a rule reduces output volatility. In this model, the Taylor-rule coefficient must be less or equal to one, however. Taylor emphasizes that his rule works well in a variety of models, and

that robustness rather than strict optimality in a particular model is its virtue. Raising interest rates in response to inflation can eliminate the instability of the adaptive expectations model; it can eliminate the indeterminacy of the rational expectations model; and it can reduce volatility in the rational-expectations fiscal-theory model.

The graph is attractive, but this mechanism is more limited than conventional intuition about monetary policy. If the world matches this model, monetary policy is a lot weaker than most people think, it is conditioned on variables including the persistence of interest rate changes and the maturity structure of debt that are not part of conventional intuition, and its basic mechanism is utterly different from standard intuition.

This negative effect of interest rates on inflation in this model only holds for *unexpected* interest rate rises. Lower inflation breaks out when a higher interest rate is announced, not when it happens. Thus, we have a lovely continuation of our list by which interest rates inherit many Lucas (1972a) properties of money growth. Maybe interest rates also lower inflation when they actually rise, not just when they are announced. If so, we need a different model.

Since the negative effect depends on long-term debt, the effect vanishes when governments borrow short term (one period), as illustrated by Figure 7 in the line marked “ $\pi$ , short debt.” More generally, the size of the negative effect depends on the maturity structure of debt. US government debt is relatively short-term, and has been shorter in the past. Whether debt maturity is long enough to produce quantitatively important effects, and whether the effect of interest rates on inflation varies with the maturity structure of the debt as the model predicts are important unanswered questions.

The negative effect requires long-lasting interest rate increases, that raise long-term nominal bond yields. It is not obvious that higher interest rates lower inflation more when they are persistent, and only when they propagate to the long-term yield curve.

As above, price stickiness *reduces* the strength of the negative effect. With sticky prices, the higher real interest rates add interest costs of the debt, an inflationary force. (See 62.) The negative effect of interest rates on inflation is strongest in the flexible price case, when real interest costs do not move. In this model there is no connection at all between sticky prices and the central non-neutrality that drives inflation down, completely contradicting standard intuition.

This response function also lowers inflation immediately, where the common intuition lowers inflation gradually. As with the above Phillips curve discussion, do not confuse a response

that lowers today's inflation over what it otherwise would have been with a response that gradually brings equilibrium inflation down over time. Perhaps further frictions can produce such a response, but that also needs to be shown.

These features are not necessarily counterfactual. They are unknown, and not part of conventional wisdom. This model is new. Nobody has looked to see if the negative effect of interest rates on inflation is quantitatively linked to announcement, maturity, persistence, price stickiness, and fiscal events as the model predicts. Looking would be a valuable empirical project, though not an easy one.

I suspect the final conclusion may be that this mechanism works; that with long-term debt, persistently higher interest rates can lower inflation somewhat at the cost of higher future inflation. Yet, it will end up empirically as it is theoretically, as a quantitatively small mechanism that does not revive standard beliefs about the power of higher interest rates to mechanically lower inflation. Thus, standard beliefs are either wrong, or require powerful new frictions.

Even if it can quantitatively replicate the result of standard beliefs, i.e. the impulse-response function, the long-term debt mechanism has nothing to do with standard intuition, that higher real interest rates depress demand, and then work through a Phillips curve to lower inflation. By exploiting the long-term Fisher effect, i.e. by raising interest rates to inflate away long-term debt, the central bank makes short-term debt more valuable. The only way for short-term debt to become more valuable is for the price level to decline. It describes an unpleasant arithmetic, but only an unpleasant arithmetic, shifting inflation between time periods without changing the overall value of the government bond portfolio.

Clearly, this model does not give an always and everywhere, mechanical connection between higher rates and lower inflation, Lucas holy water sprinkled on IS-LM thinking. It does not produce something like the adaptive-expectations dynamics in the short run, which then turn around and become stable when some suitable friction or information problem is resolved. And it will be a long time before we write opeds, Fed chairs explain, and we teach to undergraduates that the central mechanism by which the Fed can temporarily lower inflation is to rearrange the real payoffs to different maturities of nominal government debt!



## 5 Evidence and Experience

Stability and determinacy under an interest rate peg is a sharp break with conventional doctrine, but it is not obviously contrary to historical experience. We have just seen something close to an interest rate peg. From 2008 to 2016 in the US, from 2008 to 2022 in Europe, and from 1995 to 2022 in Japan, interest rates were effectively stuck at zero. They could not move much in the downward direction, they did not move in the upward direction. Central bankers gave “forward guidance” that interest rates would not move, at least not promptly and more than one for one with observed inflation.

When these countries hit the zero bound, many economists, commenters, central banks, and international monetary institutions predicted a “deflation spiral” as depicted in the left-hand panel of Figure 1. They continued to worry that a spiral would break out during the decades of zero interest rates. New-Keynesians had warned for 20 years that a zero bound would lead to volatile multiple-equilibrium inflation as depicted in the right-hand panel of Figure 1. Yet neither unstable spirals nor additional inflation volatility broke out. Inflation just batted around 1-2% the whole time.<sup>7</sup>

The widespread opinion that by reacting slowly to inflation in 2022 the Fed made inflation worse, and that only a sustained dose of interest rates higher than current and past inflation can bring inflation down, reflects the same belief in unstable adaptive-expectations dynamics. (See, for example, most of Bordo, Cochrane, and Taylor (2022).) Yet, as I write, inflation has declined with interest rates still below current inflation, as it did previously in 1948, 1951, and 1975. The outbreak of inflation following a huge \$5 trillion stimulus, and its gentle decline afterwards mirrors the simplest fiscal-theory response to a fiscal shock.

But if we accept this evidence for stability, we have to accept its uncomfortable long-run Fisherian implication.

What about historical pegs that did precede inflation? These episodes are central to Friedman’s (1968) argument that pegs are unstable. Well, quiet (the opposite of volatile) inflation also requires no fiscal news in (17). Most governments with interest rate pegs and spiraling inflation were using the peg to hold down interest costs of the debt in a time of fiscal stress. The US interest rate peg of the 1940s and early 1950s was explicit in that aim. And even so, that peg lasted remarkably long, relative to models that predict instability or indeterminacy. The peg lasted

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<sup>7</sup>I summarize here much longer analysis in Cochrane (2017), Cochrane (2018), and Cochrane (2023) Ch. 20.

through nearly 20% inflation in 1947, which did not set off a spiral. Now, only unexpected fiscal events can spark inflation. But sticky prices draw out the response to such events. And if you pick episodes *ex-post* with large inflation, you also are likely to pick episodes that had a stream of bad fiscal news shocks. By contrast, in the zero bound era, real interest rates and interest costs on the debt fell. Low interest costs on the debt act like surpluses (see (17)). Long-run fiscal policy was not in great shape, but there wasn't much bad *news* during the quiet 2010s.

Empirical analysis of the effects of monetary policy shocks gives weak evidence that higher interest rates lower inflation. Both vector autoregression and narrative approaches to identifying shocks lead to estimates that when monetary policy shocks reduce inflation at all, such reductions are delayed more than a year and then are small and marginally statistically significant or insignificant. (Ramey (2016)) provides a comprehensive review, including replication and extension of the important results. See especially Figures 1, 2, and 3, with small, sometimes positive, delayed, and insignificant inflation effects, and note the disappearance of any effect after 1983. Romer and Romer (2023) Figure 5 and Figure 7 present the latest of the narrative approach. Inflation rises for a year, then gently declines in point estimate, but is just about two standard errors below zero. And these include credit and quantitative shocks rather than interest rate increases in the 1950s and 60s, and the last and only shock since 1981 was in 1988 (Table 2). The “price puzzle” that higher interest rates want to raise inflation has dogged VAR analysis for decades, requiring careful orthogonalization carpentry to produce even this mild inflation reduction. It may have been trying to tell us something.

The empirical evidence that higher interest rates lower output and employment is much stronger. The question is only whether they lower *inflation*. The Phillips curve is the obvious weak point in our understanding.

Several other considerations weaken even this much evidence for our question. None of these estimates tries to orthogonalize monetary to fiscal policy, to answer our basic question. That was natural, as without a fiscal theory in mind, nobody thought that was important to do so. It is plausible that monetary contractions came with fiscal contractions, if nothing else to pay higher interest on the debt, but likely as a coordinated anti-inflation policy response. For example, Cochrane (2022a) outlines the fiscal underpinnings of the most important episode, the early 1980s. Social security reform, a series of massive tax reforms, deregulation, and growth led in fact to sharply rising primary surpluses that paid higher interest costs on the debt and a bondholder windfall. Conversely, failed stabilizations around the world often include an attempted monetary tightening that is not accompanied by fiscal reform.

Even these point estimates produce a result sharply at odds with the new-Keynesian model. That model predicts a sharp drop in current inflation, which then rises. As we have seen, this is an essential feature. Only the sharp drop in current inflation by equilibrium-selection policy overcomes the Fisherian force of the model. Even my long-term debt fiscal theory model produces lower inflation immediately. Yet the estimates show no effect or a small inflation rise for a year or so, followed by a gentle declines in subsequent inflation. The estimates lower expected future inflation; the theory lowers current inflation. At best the estimates are evidence for something like the adaptive expectations result, with additional long and variable lags introduced somehow, and fiscal policy paying the interest cost bills.

Nakamura and Steinsson (2018) eloquently summarize the hard identification and computation troubles. VAR shock identification relies on a complete specification of the Fed's information set. But the Fed watches more variables than we can ever include. They use 9/11 as a clear example: The Fed lowered interest rates after the terrorist attack, likely reacting to the news that event portended for output and inflation. But VARs, even including interest rates to capture market expectations, register it as an exogenous shock. Nakamura and Steinsson also emphasize the oft-forgotten lesson of 1980s time-series econometrics, that estimating 10-year responses from quarterly VARs raised to the 40th power, or a monthly VAR raised to the 120th power, is dangerous. Such responses rely crucially on one having in hand the persistent state variables, and accurately measuring their dynamics. Long run dynamics are driven by the largest eigenvalue of the system, which is typically near one. Direct regressions ("local projections") of variables of interest on shocks are more reliable but less parsimonious. They force one to recognize just how few non-overlapping samples we have at the horizons of interest.

Is there even such a thing as an exogenous policy shock? The Fed always says it is reacting to something. At best, the Fed might react to something that has no effect on inflation or (separately) output, and such reactions would qualify as exogenous for measuring the latter responses. But nobody has taken this old suggestion seriously, preferring to try to find ephemeral shocks that are orthogonal to everything.

The vast majority of current estimates also find that identified monetary policy shocks lead to quite short-lived interest rate responses. They therefore do not measure the effect of the persistent interest rate responses that we need to invoke the long-term debt mechanism, or to examine the long-run Fisher prediction that inflation should eventually rise. They also do not measure the effect of the persistent and determined high interest rates of the one great episode, the early 1980s.

Uribe (2022) addresses this issue. He evaluates the “neo-Fisherian” hypothesis that higher interest rates raise inflation via an identified VAR. He therefore identifies a permanent shock as one that increases both the nominal interest rate and inflation in the long run. He finds that shock *raises* inflation and nominal interest rates even in the short run. Similarly, Schmitt-Grohé and Uribe (2022) find that permanent interest rate shocks *depreciate* the currency. In both cases, transitory interest rate movements lead to the standard disinflation and appreciation.

Their results are good news for Figure 2, in which a permanent interest rate rise raises inflation even in the short run. But they are bad news for Figure 3 in which a transitory rate rise also raises inflation, and Figure 7 in which a permanent interest rate rise lowers short-run inflation with long-term debt. They are also bad news for the view that the 1980s succeeded via a highly persistent monetary policy shock. However, Schmitt-Grohé and Uribe also did not attempt to find monetary policy shocks orthogonal to fiscal policy changes, so their results do not directly bear on our question.

One may also feel that the transitory policy responses isolated by VARs completely miss the central ingredient of an intervention such as the early 1980s. By design, VARs and narratives find idiosyncratic deviations from a rule, not changes in rule or “regime” that may durably change expectations. If the art of reducing inflation is to convince people that a new regime has arrived, then the response to any monetary policy “shock” orthogonal to a stable “rule” completely misses that policy.

In sum, the empirical literature evaluating whether higher interest rates lower inflation is tenuous despite enormous effort, and is not addressed at our central questions. Thus, current empirical work leaves open a possibility: The widespread faith that higher interest rates reliably lower inflation, without fiscal help, may simply not be true.

## 6 Paths to Follow

So, despite 50 years of modern intertemporal general equilibrium macroeconomics since Lucas (1972a), we still don’t have a solid well-agreed on theoretical or empirical answer to the basic questions: Can higher interest rates, without concurrent or induced changes in fiscal policy, lower inflation, even temporarily, following common belief? If higher interest rates can temporarily lower inflation, by what economic mechanism do they do so?

One may be tempted to return to adaptive expectations, or add complex learning and ex-

pectation formation schemes (Gabaix (2020), García-Schmidt and Woodford (2019), or Bianchi-Vimercati, Eichenbaum, and Guerreiro (2022)) to the same effect. There was a widespread hope that the new-Keynesian enterprise would put rational expectations and Lucas general equilibrium foundations under classic intuition, finally answering the call for Keynesian “microfoundations.” Forty years on, as this little review emphasizes, we realize that the rational expectations, market clearing, and forward-looking sticky-price models produce something utterly different. It’s clearer now that if you want to rescue classic intuition, you pretty much have to go back to classic models.

But my little tour should make us even more hesitant to follow this approach. Now we understand just how far away the adaptive expectations model that produces conventional intuition is from the rational benchmark. Adaptive expectations produces the desired short-run sign by changing the basic stability and determinacy properties of the economy, including the long-run sign. It does not produce a transitory negative effect but preserve long-run neutrality.

Somewhat irrational expectations and learning dynamics are often an important ingredient for matching dynamics and episodes, on top of basic models.<sup>8</sup> But here, adaptive expectations are a *necessary*, always-and-everywhere feature to talk about inflation and monetary policy at all, not just a *sufficient* friction to match dynamics on top of a simple economic benchmark. We would have to say that there *is no* simple model like  $MV = PY$  that explains the basic ideas of price level determination under interest rate targets, which we modify with more complex expectation formation. Sign and stability properties only change when an eigenvalue crosses one. Thus, small changes around the frictionless rational benchmark won’t do, and the basic economics of the adaptive expectations model here must change along the way back to that benchmark. (See Cochrane (2016) for a concrete example.)

We would also need to face the failures of the adaptive expectations / unstable approach during the zero bound era, in 1970s stagflation, in the relatively rapid end of inflation in 1982, in the success of 1990s inflation targets, in the ends of hyperinflations (Sargent (1982)) in which inflation fell with no monetary stringency or output consequence, and, as of mid-2023, in the failure of inflation to spiral upwards despite interest rates well below inflation, among other

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<sup>8</sup>A taste: Marcet and Nicolini (2003) study recurrent hyperinflations with learning. They explain why bouts of hyperinflation break out in the context of large and stable inflation, with no large changes in seigniorage. Orphanides and Williams (2005) present a model in which constant learning contributes to inflation dynamics, and allows private sector inflation expectations to provide useful information to the central bank. Williams (2006) shows how learning can undermine the desirable properties of a price-level target, that people expect a period of inflation to follow deflation. Bullard and Mitra (2002) show that learning changes the optimal policy rule. Sargent (2001) “Conquest of American Inflation” is a classic example in which learning by the Fed takes center stage.

episodes.

Whether people are “rational” is a contentious philosophical debate, but the issue is really just whether expectations are model-consistent. The expectations *of* the adaptive-expectations model can be systematically different from expectations *in* the model. (Lucas (1991) refers to rational expectations as a “consistency condition.”) People in the adaptive-expectations model never catch on, no matter how many times they see inflation rise and fall. Again, that’s ok for an episode or transitory phenomena, but should that be a necessary ingredient of our most basic model of inflation?

Expectations may seem adaptive. Expectations must always be, in equilibrium, functions of variables that people observe, and likely weighted to past inflation. The point of “rational expectations” is that those forecasting rules are likely to change as soon as a policy maker changes policy rules, as Lucas (1976) famously pointed out<sup>9</sup> in his “critique.” Adaptive expectations may even be model-consistent, until you change the model.

That observation is important in the current policy debate. The proposition that interest rates must be higher than current inflation in order to lower inflation assumes that expected inflation equals current inflation – the simple one-period lagged adaptive expectations that I have specified here. Through 2021-2022, market and survey expectations were much lower than current (year on year) inflation. Perhaps that means that markets and surveys have rational expectations: Output is temporarily higher than the somewhat reduced post-pandemic potential, so inflation is higher than expected future inflation ( $\pi_t = E_t\pi_{t+1} + \kappa x_t$ ). But that observation could also mean that inflation expectations are a long slow-moving average of lagged inflation, just as Friedman speculated in 1968 ( $\pi_t^e = \sum_{j=1}^{\infty} \alpha_j \pi_{t-j}$ ). In either case, expected inflation is much lower than current inflation, and interest rates only need to be higher than that low expectation to reduce inflation. Tests are hard, and you can’t just look at in-sample expectations to proclaim them rational or not.

Of course, “rational” does not mean “clairvoyant.” People may be quite bad at forecasting inflation. Inflation, like stocks, may simply be hard to forecast. The Fed and market participants are pretty bad at forecasting inflation too.

Much policy discussion recognizes the importance of expectations, treats the Phillips curve

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<sup>9</sup>Sargent (1971) and Lucas (1972b) made the point earlier in the Phillips curve context. They showed that in a  $\pi_t = \alpha\pi_{t-1} + \kappa x_t + u_t$  regression,  $\alpha < 1$  coefficients can easily occur even when the true Phillips curve has rational expectations. The coefficient  $\alpha$  shifts if the monetary policy process shifts.

$\pi_t = \pi_t^e + \kappa x_t$  as a causal relation, and thinks of expectations as a free variable. Expectations can be influenced by speeches and “anchoring,” but they are not tied systematically to experience of how the Fed behaves in similar circumstances, and most of all they do not react to a current policy action. Higher interest rates reduce output, and holding expectations fixed, lower output lowers inflation. The key assumption in this train of thought is not so much *adaptive* expectations, but exogenous expectations. The central feature of the rational expectations model is that policy actions or rules change expectations.

Most of all, ignoring all these philosophical troubles, the adaptive expectations model I pursued here *cannot* lower inflation without a fiscal tightening to pay the higher interest costs on the debt. In response to the question, how can higher interest rates *by themselves* lower inflation, even the adaptive expectations model comes up empty. Rescuing adaptive expectations to justify powerful monetary policy must somehow overcome its reliance on a fiscal tightening.

You may wish to put money back in the model. Raising interest rates means printing less money  $M$ , which lowers nominal income  $PY$  and eventually the price level  $P$ . (Alvarez, Atkeson, and Edmond (2009) is a good example.) But it’s not so easy as a matter of theory, and as in the first paragraph of this essay, our central banks simply do not control money supplies. Adding liquidity effects in government bonds or other financial assets to the model is an attractive generalization, but the supply of such liquidity needs to be constrained just like money for this avenue to produce a basic model of the price level.

You might say that the Fed *should* go back to controlling the money supply, and start cracking down on inside liquid money substitutes. But we need some advice for central banks in the meantime, and at least we should understand how our current system based on interest rate targets works, or doesn’t. From 1982 to February 2021 it looked like a pretty good system! Inflation is *something* while central banks control interest rates and provide and allow ample liquidity. We need a theory of what that something is.

One is drawn to add model ingredients. Surely in the DSGE smorgasbord there are enough ingredients to come up with a temporary negative sign. That is, I think, a plausible answer. My point is, it has not yet been done—and especially, it has not been done with the kind of clarity, simplicity, economic rigor, transparency, and tractability that Lucas brought to the non-neutrality of money. Many model-implied monetary-policy response functions have been computed of course, but not many yet hold fiscal policy constant in an interesting way, and few look into the footnote about lump-sum taxes to see just what those are and to what extent in-

flation reduction comes from an implicit fiscal contraction. The literature that puts fiscal theory in explicit DSGE models is an exception; see for example most recently Bianchi and Melosi (2022), Chen, Leeper, and Leith (2021), and Leeper (2021). This literature is explicit about fiscal-monetary coordination. However, it has focused on switching between active-fiscal and active-money regimes, and so far has not addressed the question in this paper, whether higher interest rates can lower inflation with no change in fiscal policy.

One easily jumps to capital with adjustment costs, financial frictions, credit constraints, portfolio adjustment costs, liquidity effects, additional price and wage-setting frictions, strategic complementarities, individual heterogeneity, or other model complications. But the negative response of inflation to interest rates should be a robust and deeply rooted phenomenon, one that will not vanish if, for example, the US changes the downpayment rules on mortgages. In his Nobel Lecture, Lucas (1996) cites David Hume for understanding the neutrality and non-neutrality of money in 1752. Velde (2009) documents a beautiful non-neutrality episode in 1724 France, with a monetary and financial system utterly unlike our own.

It is likely to be possible to find in this soup *sufficient* conditions to deliver the negative sign, with enough model complications. Our goal though is the minimum *necessary* conditions, that apply most broadly and robustly, that we can put at the basis of any good theory of inflation and monetary policy under interest rate targets. That is a harder goal. Again, Lucas (1972a) is a great example. In his economy, the flexible price version leads to super-neutrality: An increase in money just raises the price level. Bob put in *one* “friction,” imperfect information about aggregates, leading to a confusion between relative and aggregate price movements.

This train of thought brings us back to the Phillips curve. In my little models, the Phillips curve is the central source of inflation dynamics. Yet the Phillips curve has not achieved great theoretical and empirical clarity, despite decades of dedicated work by top macroeconomists. It may make sense that firms sell more when output prices are high, or that workers work harder when wages are high. But these are relative prices, where the Phillips curve states that output and employment increase when all prices and wages rise together. So, any Phillips curve theory needs some confusion or correlation of relative prices with the overall price level.

In addition to wondering what ingredients to put in, then, perhaps this is one we should take out. Perhaps we can start to study the dynamic relationship between inflation and nominal interest rates apart from the Phillips curve.

Our goal is to understand  $\pi_t = a(L)i_t$ , the dynamic relationship between interest rates and



inflation. The Phillips curve came from thinking about output and employment *effects* of inflation. That's the central point of Lucas (1972a). Lucas had a perfectly good theory of inflation,  $MV = PY$ , but he wanted a theory how inflation affects output. We are reversing the causal logic. We use the IS equation to describe how interest rates lower output, and then the Phillips curve to describe how output affects inflation. (With the usual caveats for causal readings of equilibrium conditions.) The Phillips curve wasn't designed to be the central mechanism for nominal dynamics. We will of course still want to understand how inflation affects output and employment, and surely that understanding will feed back on inflation dynamics. But in the spirit of adding ingredients and frictions one at a time, perhaps the price dynamics should come before the Phillips curve.

A disaggregated production approach could be one example. In 2021-2022 most commentary in and around central banks centered on "supply chain" shocks and relative price movements, which particular goods or sectors were going up or down, as both underlying cause and key variables for the dynamics of inflation. Lane (2022) is a good example. In this view, large good- or sector-specific shocks move relative prices. Prices are more sticky downward than upward, so inflation rises. Guerrieri et al. (2021) argue that some inflation is optimal when there are reallocation shocks and downward nominal stickiness. However, this view requires either a compliant or absent nominal anchor, enough money or overvalued government debt for people to pay the higher overall prices, rather than force down prices of goods not in short supply. In this view, relative prices – average vs. marginal rents, house prices vs. rents, core vs. full CPI, etc.—are key state variables that forecast future inflation. The old Phillips curve, with a single output gap or unemployment rate capturing the entire effect of the real economy on inflation, is a dramatic simplification that misses these dynamics. Related, there is new interest in describing inflation dynamics in production networks, for example Minton and Wheaton (2022) and Rubbo (2022). Perhaps reallocations, networks, supply and demand shocks interacted with sticky prices and wages, will take over from the IS and Phillips curve as our basic model of inflation dynamics, on top of our view—fiscal theory in mine—of a nominal anchor.

On the other hand, recognize how high the hill is that we have to climb. First, if we wish to produce the usual view, that higher interest rates lower *future* inflation relative to today's inflation (rather than force today's inflation to jump down), then a 1% higher nominal interest rates must lower the real interest rate by more than 1% to lower expected inflation. A small effect of nominal rates on real rates will not do.

Second, the basic logic of the fiscal identity (17),

$$\Delta E_{t+1} \pi_{t+1} = -\Delta E_{t+1} \sum_{j=0}^{\infty} \rho^j \tilde{s}_{t+1+j} + \Delta E_{t+1} \sum_{j=1}^{\infty} \rho^j (i_{t+j} - \pi_{t+j+1}), \quad (63)$$

poses a robust constraint. With no change in surpluses, the first term on the right hand side is zero. In continuous time with sticky prices, the term on the left is also zero, as the price level cannot jump—we cannot devalue instantaneous debt. What’s left simply says that the long-run average of real interest rates must be zero. Without surpluses, without devaluing short-run debt, there is nobody to pay interest costs, so inflation must adjust to the path that gives no long-run average interest costs – and therefore, in these models, no long-run average real interest rate. This is what made the adaptive expectation model incapable of lowering inflation without fiscal help. It’s going to be hard to get *any* model to lower inflation by higher interest rates when any period of high real interest rates must be matched by a period of lower real interest rates. Perhaps financial frictions that drive a wedge between real interest rates that bring down inflation and the government’s real interest costs will help, but again one hesitates for such a friction to be a *necessary* bedrock ingredient for the basic proposition that higher interest rates lower inflation.

There is again another possibility: It might not be true. The long-term debt model, with unpleasant interest-rate arithmetic or “stepping on a rake” may be as close as we come to standard intuition. Otherwise, nominal interest rate rises, *with no change in fiscal policy*, may not lower inflation even in the short run.

## 7 Conclusion

What is the dynamic effect of *interest rates* on inflation,  $\pi_t = a(L)i_t$ , in our world of abundant reserves, in which central banks set nominal interest rates, do not control money supplies, do not make equilibrium-selection threats, and cannot directly change fiscal policy? And, of course, after that, how do interest rates then affect output, employment, and other variables?

I have followed one line of thought on these questions to its logically inevitable conclusion: Rational expectations and fiscal underpinnings of monetary policy imply that inflation is stable and determinate in the long run. Those features imply that pegs are possible, and that higher interest rates, without a change in fiscal policy, eventually raise inflation. There may well be a short-run negative effect of interest rates on inflation. I show one suggestive model, but we need to know if there are better models of that effect. Or, we need to accept the limited power of

this model, and that outside its narrow scope come to believe that higher nominal interest rates without changes in fiscal policy *raise* inflation.

Thus, as I see it, we have made a lot of progress. We're finally at the launch pad, and we have some promising ideas, but we're still waiting for a new Lucas – and then, perhaps a new Sims on the empirical side – to finish the project.

If this path succeeds, we will still be left with an understanding that central banks are less powerful than we thought. First, fiscal policy remains a central determinant of inflation. When a fiscal shock occurs, when the government borrows or prints and spends and people do not expect the debt to be repaid, and absent explicit default, inflation must rise to devalue the debt. The central bank can choose when and how abruptly that inflation will occur, but inflation is no longer always and everywhere controlled by monetary policy alone. Second, the central bank's ability to lower inflation by higher interest rates, provoking a little bit of recession, remains contingent on the frictions of the model that produces a temporary negative effect. The central bank still fully controls the long-run price level, however, by its ability to drag expected inflation to wherever it sets the nominal interest rate. Third, the simple static story that higher rates lower demand which lowers inflation via a Phillips curve does not even vaguely describe these models.

All this is controversial. Much of the point of this essay is to proclaim and explain the frictionless benchmark for inflation under interest rate targets, which is otherwise not obvious in the equations of new-Keynesian and fiscal theory models. Most academic literature still uses new-Keynesian equilibrium-selection threats and ignores fiscal underpinnings of monetary policy, even just to pay interest costs on the debt. The stability, approximate long-run neutrality, and long-run positive sign of that model is not widely recognized. Most of the policy world uses a muddle with somewhat adaptive expectations, or expectations as an independent force.

At a minimum, basic questions are still up for grabs. In the long run, is inflation stable or unstable, determinate or indeterminate under a peg? If the central bank raises rates persistently, and there is no fiscal news or other shocks, does inflation rise or decline in the long run? If not this, what is the neutral, frictionless benchmark on which we build a theory of inflation under interest rate targets?

The short run non-neutral and disinflationary effects of higher interest rates have more consensus of opinion behind them, but even less well-accepted theory and empirical work behind that opinion. If the central bank raises interest rates, does inflation temporarily decline? If so, by what mechanism, and under what preconditions? And even though most economists seem

to believe in the sign of the effect, the all important magnitude is still contentious. Must the central bank raise interest rates by more than the current rate of inflation, following the Taylor Principle, in order to lower inflation at all? Or will the substantially lower interest rate rises that central banks followed in 2023 be sufficient for inflation to fade away, at least until the next big shock? Are transitory or persistent interest rate increases more effective at lowering inflation?

The question I have posed—what is the effect of interest rates on inflation, with no change in fiscal surpluses—is important for understanding monetary economics. But it is an unlikely scenario with which to understand history, and it is not the right question to ask if one wants to forecast the effects of policy. Monetary and fiscal policies change together in response to events, and fiscal policy responds to economic changes brought about by monetary policy. The right interpretation of leaving fiscal policy unchanged depends on context too. For example, if the tax rate and automatic stabilizer laws are unchanged, and higher interest rates cool the economy, then they will lead to deficits, which can further raise inflation. To forecast the effects of an interest rate rise, allowing such responses might be a better definition of unchanged fiscal policy. One define a monetary policy shock as one that leaves the fiscal rule unchanged but has no disturbance to that rule. Cochrane (2021) and the above-cited fiscal theory literature include such fiscal policy rules, responding to output and inflation. Ask interesting questions and be clear what question you're asking.

How is it that we've been playing with interest-rate based models for at least 40 years, yet such basic questions are still unsettled? As I look at monetary models based on interest rate targets, I think we have been guilty of playing with too-complex models when we don't really understand basics, such as stability, determinacy, and the frictionless limit. But ideas in economics and sciences always start complex, and simplicity only emerges after much hard work.

This is all great news for young researchers. These are the good old days. Low-hanging fruit abounds. We're really at the beginning stages where simple models need exploration, not, as it appears, in a mature stage where essentials are settled and all there is to do is to add to the immense stock of complicated epicycles.

However, given the state of actual agreed-on knowledge, central banks' proclamations of detailed technocratic ability to manipulate delicate frictions is really not justified.<sup>10</sup>

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<sup>10</sup>For a lovely example, see the chart at <https://www.ecb.europa.eu/mopo/intro/transmission/html/index.en.html>. Lucas (1979) is a good antidote to hubris.

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## Online Appendix

### A1 Solving the Model for Arbitrary Interest Rates

The model is

$$x_t = -\sigma(i_t - E_t\pi_{t+1}) \quad (\text{A1})$$

$$\pi_t = E_t\pi_{t+1} + \kappa x_t \quad (\text{A2})$$

$$\rho v_{t+1} = v_t + i_t - \pi_{t+1} - \tilde{s}_{t+1} \quad (\text{A3})$$

$$\lim_{T \rightarrow \infty} \rho^T v_T = 0 \quad (\text{A4})$$

We want to calculate the impulse-response function for a generic path  $\{i_t\}$ . All variables are zero until time 1. At time 1 we set off a sequence  $\{i_1, i_2, \dots\}$ . There is no change to surpluses, so  $\tilde{s}_t = 0$ . Given  $\pi_1$ , the other  $\pi_t$  follow since there is no more uncertainty. Equations (A1)-(A2) give us a set of possible paths of inflation indexed by  $\pi_1$ . We use (A3) and (A4) to choose  $\pi_1$ .

This section establishes the following results for this impulse-response function. For an arbitrary sequence  $\{i_1, i_2, \dots\}$ ,

$$\pi_{t+1} = \frac{1}{(1 + \sigma\kappa)^{t+1}} (1 - \rho) \sum_{j=1}^{\infty} \rho^j i_j + \frac{\sigma\kappa}{1 + \sigma\kappa} \sum_{j=1}^t \frac{1}{(1 + \sigma\kappa)^{t-j}} i_j.$$

For an AR(1)  $i_t = \eta i_{t-1} + \varepsilon_t$ ,

$$\pi_{t+1} = \left[ \left( \frac{(1 - \rho)}{(1 + \sigma\kappa)} \frac{\rho}{(1 - \rho\eta)} + \frac{\sigma\kappa}{1 - \eta(1 + \sigma\kappa)} \right) \frac{1}{(1 + \sigma\kappa)^t} - \frac{\sigma\kappa}{1 - \eta(1 + \sigma\kappa)} \eta^t \right] i_1.$$

For  $\eta = 1$ , i.e. a one-time permanent increase in the interest rate,

$$\pi_{t+1} = \left[ 1 - \frac{(1 + \sigma\kappa - \rho)}{(1 + \sigma\kappa)^{t+1}} \right] i_1$$

Now, to derive these results. Eliminating output from (A1)-(A2),

$$E_t\pi_{t+1} = \frac{1}{1 + \sigma\kappa} \pi_t + \frac{\sigma\kappa}{1 + \sigma\kappa} i_t. \quad (\text{A5})$$

$$(\text{A6})$$

Iterating forward (A5), after the shock at time 1, (for  $t \geq 1$ ),

$$\pi_{t+1} = \frac{1}{(1 + \sigma\kappa)^t} \pi_1 + \frac{\sigma\kappa}{1 + \sigma\kappa} \sum_{j=1}^t \frac{1}{(1 + \sigma\kappa)^{t-j}} i_j. \quad (\text{A7})$$

In the case of AR(1),  $i_t = \eta i_{t-1} + \varepsilon_t$ , we have the not very elegant expression

$$\pi_{t+1} = \frac{1}{(1 + \sigma\kappa)^t} \pi_1 + \frac{\sigma\kappa}{1 + \sigma\kappa} \sum_{j=1}^t \frac{\eta^{j-1}}{(1 + \sigma\kappa)^{t-j}} i_1$$

$$\pi_{t+1} = \frac{1}{(1 + \sigma\kappa)^t} \pi_1 + \frac{\frac{1}{(1 + \sigma\kappa)^t} - \eta^t}{\frac{1}{(1 + \sigma\kappa)} - \eta} \frac{\sigma\kappa}{1 + \sigma\kappa} i_1$$

If  $\eta = 1$ , so  $\pi_t = \pi_1$ ,  $t > 1$ , this reduces to

$$\pi_{t+1} = i_1 + \frac{1}{(1 + \sigma\kappa)^t} (\pi_1 - i_1).$$

Now, we need to find  $\pi_1$ . Iterating (A3) forward,

$$\rho^t v_t = (0 - \pi_1) + \rho(i_1 - \pi_2) + \rho^2(i_2 - \pi_3) + \rho^3(i_3 - \pi_4) + \dots$$

Thus, the condition  $\rho^t v_t \rightarrow 0$  is

$$\pi_1 = \sum_{j=1}^{\infty} \rho^j (i_j - \pi_{j+1}).$$

Debt is devaulted to pay the higher interest costs that result from higher real interest rates. Now plug inflation from (A7),

$$\begin{aligned} \pi_1 &= \sum_{j=1}^{\infty} \rho^j \left( i_j - \left( \frac{1}{(1 + \sigma\kappa)^j} \pi_1 + \frac{\sigma\kappa}{1 + \sigma\kappa} \sum_{k=1}^j \frac{1}{(1 + \sigma\kappa)^{j-k}} i_k \right) \right) \\ \pi_1 &= - \sum_{j=1}^{\infty} \rho^j \frac{1}{(1 + \sigma\kappa)^j} \pi_1 + \sum_{j=1}^{\infty} \rho^j i_j - \frac{\sigma\kappa}{1 + \sigma\kappa} \sum_{j=1}^{\infty} \rho^j \sum_{k=1}^j \frac{1}{(1 + \sigma\kappa)^{j-k}} i_k \\ \pi_1 &= - \sum_{j=1}^{\infty} \rho^j \frac{1}{(1 + \sigma\kappa)^j} \pi_1 + \sum_{j=1}^{\infty} \rho^j i_j - \frac{\sigma\kappa}{1 + \sigma\kappa} \sum_{k=1}^{\infty} \sum_{j=k}^{\infty} \rho^j \frac{1}{(1 + \sigma\kappa)^{j-k}} i_k \\ \pi_1 &= - \sum_{j=1}^{\infty} \rho^j \frac{1}{(1 + \sigma\kappa)^j} \pi_1 + \sum_{j=1}^{\infty} \rho^j i_j - \frac{\sigma\kappa}{1 + \sigma\kappa} \sum_{k=1}^{\infty} \rho^k \left( \frac{1}{1 - \frac{\rho}{1 + \sigma\kappa}} \right) i_k \end{aligned}$$

$$\begin{aligned}
\pi_1 &= -\frac{\frac{\rho}{(1+\sigma\kappa)}}{1 - \frac{\rho}{(1+\sigma\kappa)}}\pi_1 + \left(1 - \frac{\sigma\kappa}{1 + \sigma\kappa} \left(\frac{1}{1 - \frac{\rho}{1+\sigma\kappa}}\right)\right) \sum_{j=1}^{\infty} \rho^j i_j \\
\pi_1 &= -\frac{\rho}{1 + \sigma\kappa - \rho}\pi_1 + \frac{1 - \rho}{1 + \sigma\kappa - \rho} \sum_{j=1}^{\infty} \rho^j i_j \\
(1 + \sigma\kappa - \rho)\pi_1 &= -\rho\pi_1 + (1 - \rho) \sum_{j=1}^{\infty} \rho^j i_j \\
\pi_1 &= \frac{1 - \rho}{1 + \sigma\kappa} \sum_{j=1}^{\infty} \rho^j i_j. \tag{A8}
\end{aligned}$$

For an AR(1)

$$\pi_1 = \frac{1 - \rho}{1 + \sigma\kappa} \sum_{j=1}^{\infty} \rho^j \eta^{j-1} i_1 = \frac{\rho}{1 + \sigma\kappa} \frac{1 - \rho}{1 - \rho\eta} i_1.$$

For  $\eta = 1$

$$\pi_1 = \frac{\rho}{1 + \sigma\kappa} i_1.$$

With  $\pi_1$ , we now have the general solution. Using (A8) in (A7),

$$\pi_{t+1} = \frac{1}{(1 + \sigma\kappa)^{t+1}} (1 - \rho) \sum_{j=1}^{\infty} \rho^j i_j + \frac{\sigma\kappa}{1 + \sigma\kappa} \sum_{j=1}^t \frac{1}{(1 + \sigma\kappa)^{t-j}} i_j.$$

For the AR(1)

$$\begin{aligned}
\pi_{t+1} &= \frac{1}{(1 + \sigma\kappa)^t} \left( \frac{1 - \rho}{1 + \sigma\kappa} \frac{\rho}{1 - \rho\eta} i_1 \right) + \frac{\sigma\kappa}{1 + \sigma\kappa} \sum_{j=1}^t \frac{\eta^{j-1}}{(1 + \sigma\kappa)^{t-j}} i_1 \\
\pi_{t+1} &= \left( \frac{1 - \rho}{1 + \sigma\kappa} \frac{\rho}{1 - \rho\eta} i_1 \right) \frac{1}{(1 + \sigma\kappa)^t} + \frac{\sigma\kappa}{1 + \sigma\kappa} \frac{\frac{1}{(1 + \sigma\kappa)^t} - \eta^t}{\frac{1}{1 + \sigma\kappa} - \eta} i_1 \\
\pi_{t+1} &= \left[ \left( \frac{(1 - \rho)}{(1 + \sigma\kappa)} \frac{\rho}{(1 - \rho\eta)} + \frac{\sigma\kappa}{1 - \eta(1 + \sigma\kappa)} \right) \frac{1}{(1 + \sigma\kappa)^t} - \frac{\sigma\kappa}{1 - \eta(1 + \sigma\kappa)} \eta^t \right] i_1
\end{aligned}$$

For  $\eta = 1$ ,

$$\pi_{t+1} = \left[ 1 - \frac{(1 + \sigma\kappa - \rho)}{(1 + \sigma\kappa)^{t+1}} \right] i_1.$$

## A2 Continuous Time

Here I develop the simple model in continuous time. This is a clearer though less familiar way to see the main points. In particular, we can see here that the central question is really the sign of output in the Phillips curve: Is output high when inflation is *increasing* or *decreasing*? In continuous time, some of the timing conventions that obscure the analysis vanish. In particular, we see that rational expectations in the IS curve are not an issue. Continuous time with sticky prices emphasizes a fundamentally different reinterpretation of the model: The government debt valuation equation does not adjust via price-level jumps on the date of a shock, but by choosing a whole path of inflation that adjusts the discount rate applied to future surpluses, or equivalently adjusts the interest costs on the debt.

Write the standard model (1)-(2)

$$E_t(x_{t+\Delta} - x_t) = \sigma(i_t - E_t\pi_{t+\Delta})\Delta \quad (\text{A9})$$

$$E_t(\pi_{t+\Delta} - \pi_t) = -\kappa x_t \Delta. \quad (\text{A10})$$

This standard model in continuous time is thus

$$E_t dx_t = \sigma(i_t - \pi_t) dt \quad (\text{A11})$$

$$E_t d\pi_t = -\kappa x_t dt. \quad (\text{A12})$$

Normally a term  $-\rho\pi_t dt$  appears on the right of (A12). As I simplified the discrete time Phillips curve from  $\pi_t = \beta E_t \pi_{t+1} + \kappa x_t$  with  $\beta = 1$ , I simplify here with  $\rho = 0$ ; the Phillips curve is centered on expected future inflation, and permanent inflation is fully neutral. Nothing important hinges on this simplification.

The price level is continuous and differentiable, and cannot jump or diffuse. In an instant  $dt$  only a fraction  $\lambda dt$  of producers may change prices. The inflation rate may have jumps or diffusions. But  $E_t \pi_{t+\Delta} - \pi_t$  is still of order  $\Delta$ , so the relevant inflation in the consumer's first order condition (A11) is  $\pi_t$ . The issue whether inflation in that condition should be rationally anticipated or adaptive disappears. This is a useful clarification of continuous time. Expectations in the Phillips curve are the central issue.

Equations (A10) and (A12) express the standard rational-expectations Phillips curve. The

adaptive-expectations analogue is

$$\pi_t - \pi_{t-\Delta} = \kappa x_t \Delta \quad (\text{A13})$$

$$d\pi_t = \kappa x_t dt. \quad (\text{A14})$$

Thus, adaptive and rational expectations differ by whether higher output corresponds to increasing (A14) or decreasing (A12) inflation; by inflation greater than future or past inflation. Essentially, they differ by the sign of  $\kappa$ . Adaptive expectations also produce a differentiable inflation, with neither jumps nor diffusion terms.

Again I simplify the model so we can see the main points without algebra, by using a static version of the consumption equation,

$$x_t = -\sigma(i_t - \pi_t). \quad (\text{A15})$$

Eliminating output from the Phillips curve, we have the dynamic relation between interest rates and inflation. With rational expectations

$$E_t d\pi_t = -\sigma \kappa \pi_t dt + \sigma \kappa i_t dt, \quad (\text{A16})$$

while with adaptive expectations

$$d\pi_t = \sigma \kappa \pi_t dt - \sigma \kappa i_t dt. \quad (\text{A17})$$

We have immediately the results of the discrete-time model: Inflation is stable but indeterminate under rational expectations; while inflation is unstable but determinate under adaptive expectations. “Stable” means that the coefficient in front of  $\pi_t$  on the right hand side is negative. “Indeterminate” means that we do not fully determine inflation. We can write (A16)

$$d\pi_t = -\sigma \kappa \pi_t dt + \sigma \kappa i_t dt + d\delta_t \quad (\text{A18})$$

where

$$d\delta_t = d\pi_t - E_t d\pi_t$$

is an arbitrary random variable (compensated jump or diffusion) with  $E_t d\delta_t = 0$ . The solutions

of (A18) are

$$\pi_t = \sigma\kappa \int_{\tau=0}^t e^{-\sigma\kappa\tau} i_{t-\tau} d\tau + e^{-\sigma\kappa t} \pi_0 + \int_{\tau=0}^t e^{-\sigma\kappa\tau} d\delta_{t-\tau}. \quad (\text{A19})$$

“Stability” means that the influence of past interest rates disappears over time, while “indeterminacy” means that the expectational errors  $d\delta_t$  appear.

For adaptive expectations, “unstable” means that the coefficient in front of  $\pi_t$  on the right hand side is negative. It is “determinate” since  $d\pi_t$  not  $E_t d\pi_t$  appears on the left. The solutions of (A17) are

$$\pi_t = \sigma\kappa \int_{\tau=0}^t e^{\sigma\kappa\tau} i_{t-\tau} d\tau + e^{\sigma\kappa t} \pi_0.$$

“Unstable” means that interest rates and initial conditions further in the past have larger effects today. Despite the  $\sigma\kappa\pi_t dt$  on the right hand side of (A17), we solve the model backward, because there is no jump or diffusion in inflation. If we try to solve forward,

$$\pi_t = \sigma\kappa \int_{\tau=0}^{\infty} e^{-\sigma\kappa\tau} i_{t+\tau} d\tau,$$

the right hand side can require a jump or diffusion that the model rules out. Inflation is predetermined. “Instability” means that for all but one special  $\pi_0$ , inflation or deflation spirals. But  $\pi_0$  is just as predetermined as at other dates, and in particular cannot react to the future realizations of the interest rate.

In the case of a peg,  $i_t = i$ , for rational expectations (A19) becomes

$$\pi_t = (1 - e^{-\sigma\kappa t})i + e^{-\sigma\kappa t} \pi_0 + \int_{\tau=0}^t e^{-\sigma\kappa\tau} d\delta_{t-\tau}. \quad (\text{A20})$$

For adaptive expectations, a peg leads to

$$\pi_t = i + e^{\sigma\kappa t} (\pi_0 - i). \quad (\text{A21})$$

The peg is generically unstable.

As in discrete time, a Taylor rule stabilizes the unstable adaptive expectations model. Adding

$$i_t = \phi\pi_t + u_{i,t}$$

the adaptive-expectations dynamics (A17) become

$$\frac{d\pi_t}{dt} = \sigma\kappa(1 - \phi)\pi_t - \sigma\kappa u_{i,t}$$

With  $\phi > 1$ , dynamics are now stable and determinate. A monetary policy shock  $u_{i,t}$  raises the interest rate and lowers inflation. See Figure 5.

In the rational expectations model with Taylor rule, in the new-Keynesian tradition, rational-expectations dynamics (A17) become

$$E_t d\pi_t = \sigma\kappa(\phi - 1)\pi_t dt - \sigma\kappa u_{i,t} dt.$$

Now  $\phi > 1$  induces instability. This time instability means we can solve the integral forward, and with a rule against nominal explosions recover determinacy,

$$\pi_t = -\sigma\kappa E_t \int_{\tau=0}^{\infty} e^{-\sigma\kappa(\phi-1)\tau} u_{i,t+\tau} d\tau.$$

Define an inflation target  $\{\pi_t^*\}$  and define  $i_t^*$  by

$$E_t d\pi_t^* = -\sigma\kappa\pi_t^* dt + \sigma\kappa i_t^* dt$$

In words,  $i_t^*$  is the interest rate target that implements  $\{\pi_t^*\}$  as an equilibrium. Now write the policy rule as

$$i_t = i_t^* + \phi(\pi_t - \pi_t^*)$$

With this notation, we can write rational-expectations dynamics (A17) as

$$E_t d(\pi_t - \pi_t^*) = \sigma\kappa[-(\pi_t - \pi_t^*) + (i_t - i_t^*)] dt$$

$$E_t d(\pi_t - \pi_t^*) = \sigma\kappa(\phi - 1)(\pi_t - \pi_t^*) dt.$$

Monetary policy has two parts, an interest rate policy  $i_t^*$  which generates the desired path of expected inflation, and an equilibrium-selection policy  $\phi(\pi_t - \pi_t^*)$  which generates explosions unless  $d\pi_t - E_t d\pi_t = d\pi_t^* - E_t d\pi_t^*$ .

Fiscal theory offers an alternative route to determinacy in the rational expectations model. Include the linearized evolution of real government debt, with instantaneous debt and differen-



tiable prices

$$dv_t = (rv_t + i_t - \pi_t - \tilde{s}_t)dt.$$

Integrating forward, taking expectations, and imposing the transversality condition, the real value of debt equals the present value of surpluses.

$$v_t = E_t \int_{\tau=0}^{\infty} e^{-r\tau} [\tilde{s}_{t+\tau} - (i_{t+\tau} - \pi_{t+\tau})] d\tau. \quad (\text{A22})$$

To use this equation in the rational-expectations dynamics (A18) as above, let  $\Delta_t v_t$  isolate the compensated jump or diffusion component of a process. In this case, and unlike discrete time,  $\Delta_t v_t = 0$ . Corresponding to (17),

$$0 = \Delta_t \int_{\tau=0}^{\infty} e^{-r\tau} [\tilde{s}_{t+\tau} - (i_{t+\tau} - \pi_{t+\tau})] d\tau. \quad (\text{A23})$$

Short-term nominal debt is predetermined, and since prices cannot jump or diffuse, the value of debt cannot jump or diffuse. Rather than shock the initial value of debt at all, of the multiple equilibria, we pick the inflation *path* in which the discount rate/interest cost effect, the second term, exactly balances any change in surplus, the first term. In the absence of a surplus change, such as a pure monetary policy shock, we pick the inflation path so that the integral of the discount rate term is zero.

$$0 = \Delta_t \int_{\tau=0}^{\infty} e^{-r\tau} [(i_{t+\tau} - \pi_{t+\tau})] d\tau$$

Continuous time fundamentally changes how we think of the model at high frequency. With flexible prices, a decline in surpluses must be met by a price-level jump which devalues outstanding debt. The discrete-time unexpected inflation equation (17) included some of that intuition along with a discount rate / interest cost effect. Now the latter is everything. In essence, the left-hand side of (17),  $\Delta E_{t+1} \pi_{t+1}$ , is always zero. Now in (A22) a decline in surpluses (first term on the right of (17)) is met by a period of inflation higher than nominal interest rates (second term) which slowly devalues debt. Bondholders lose by a long period of inflation above the nominal interest rate, not by a price-level jump. Monetary policy changes in the path of nominal interest rates generate an inflation path in which the present value of interest costs is zero.

Next, I find the response of inflation to an interest rate shock, Figure 2 in discrete time. We see that with instantaneous debt, even the slow rise of inflation shown in that figure disappears. It was a feature of one-year debt.

We can compute this impulse-response function by supposing there is a shock at time 0, and all variables represent how their expected values respond to that shock. From (A19), for a generic interest rate path  $\{i_t\}$ , the response function

$$\pi_t = \kappa\sigma \int_{\tau=0}^t e^{-\sigma\kappa\tau} i_{t-\tau} d\tau + e^{-\sigma\kappa t} \pi_0 \quad (\text{A24})$$

gives us a family of inflation paths indexed by  $\pi_0$ . Only one of these paths satisfies the valuation equation (A22), which is in this case

$$0 = \int_{t=0}^{\infty} e^{-rt} [\tilde{s}_t - (i_t - \pi_t)] dt. \quad (\text{A25})$$

In the simple case that the interest rate rises at time 0 from 0 to a new value  $i$ , then (A24) reduces to

$$\pi_t = (1 - e^{-\sigma\kappa t})i + e^{-\sigma\kappa t} \pi_0. \quad (\text{A26})$$

Plugging this into the valuation equation (A25) with  $\tilde{s}_t = 0$  to determine  $\pi_0$ ,

$$0 = \frac{i}{r} - \int_{t=0}^{\infty} e^{-rt} [(1 - e^{-\sigma\kappa t})i + e^{-\sigma\kappa t} \pi_0] dt \quad (\text{A27})$$

$$\pi_0 = i. \quad (\text{A28})$$

Despite sticky prices, we pick the equilibrium in which inflation moves instantly to match the interest rate.