NBER WORKING PAPER SERIES

EXPECTATIONS AND THE NEUTRALITY OF INTEREST RATES

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Working Paper 30468
http://www.nber.org/papers/w30468

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
September 2022, Revised December 2022

This paper stems from a talk given at the Foundations of Monetary Policy Conference celebrating 50 years since the publication of Lucas (1972a) “Expectations and the Neutrality of Money,” Federal Reserve Bank of Minneapolis, September 2022. I thank Ed Nelson and two anonymous commenters for helpful suggestions. The views expressed herein are those of the author and do not necessarily reflect the views of the National Bureau of Economic Research.

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JEL No. E4,E5

ABSTRACT

50 years ago, Bob Lucas (1972a) published his pathbreaking analysis of the neutrality and temporary non-neutrality of money. But our central banks set interest rate targets, and do not even pretend to control money supplies. How is inflation determined under an interest rate target?

We finally have a complete theory of inflation under interest rate targets, that mirrors the long-run neutrality and frictionless limit of monetary theory: Inflation can be stable and determinate under interest rate targets, including a k percent rule or a peg. The zero bound era is confirmatory evidence. Uncomfortably, long-run neutrality means that higher interest rates eventually raise inflation.

With a Phillips curve, we have some non-neutrality as well: Higher nominal interest rates raise real rates and lower output. A model in which higher nominal interest rates temporarily lower inflation is a harder task. I exhibit one such model, which adds long-term debt, but has several shortcomings. A better model, and empirical understanding, is as crucial to today's research agenda as Lucas (1972a) was in its day.

Much of this is contentious. The issues are crucial for policy: Does an end to inflation require interest rates substantially above current inflation? Do central bank interest rate hikes, without contemporaneous fiscal tightening, raise or lower inflation? We do not have well-grounded, widely-agreed answers to these questions. Given the state of knowledge, a bit of humility is in order.

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https://www.johnhcochrane.com/research-all/inflation-neutrality

1 Introduction

50 years ago, Bob Lucas (1972a) published the watershed "Expectations and the Neutrality of Money." Lucas studied expectations and the neutrality—and temporary non-neutrality—of, as the title says, *money*. But our central banks set *interest rates*. The Federal Reserve does not even pretend to control money supply, especially inside money. There are no reserve requirements. Super-abundant reserves pay the same or more interest as short-term treasuries and overnight money markets. The Fed controls interest rates by changing the interest it offers on abundant reserves, not by rationing scarce zero-interest reserves. Other central banks follow similar corridors. The quantity of M2 is whatever people feel like holding in that form.

We need a theory of inflation under *interest rate targets*. The theory should express and respect long-run neutrality. The theory should also capture temporary non-neutrality, with robust and clean economics, just as Bob's does. Its basic ideas and signs should be explainable to undergraduates, central bankers, and intelligent laypeople.

We do not have such a theory. We have made a lot of progress. I argue that we have at last a complete theory of inflation under interest rate targets, and that theory expresses long-run neutrality. But even that statement is controversial. Long-run neutrality inexorably implies that higher nominal interest rates must eventually produce *higher* inflation. And the final piece, a satisfactory theory of temporary non-neutrality, the central contribution of Bob's paper, is unfinished. In particular, we do not really know the most basic question, whether and how by raising interest rates, without a contemporaneous tightening of fiscal policy, the central bank can temporarily lower inflation. We also lack robust empirical understanding of the effect of such interest rate changes on inflation.

Ignorance is great news for researchers. The 1970s were a golden decade for macroeconomic research, as much as they were a miserable decade for the economy. The 2020s may well repeat both features.

The question is also crucial for current (late 2022) policy. The Fed waited a whole year to raise interest rates after inflation emerged in early 2021, and interest rates remain far below the rise of inflation. Is the Fed's slow reaction partially to blame for 2022 inflation? Must the Fed dramatically raise rates, so that interest rates exceed current inflation as the Taylor Rule recommends, to keep inflation from spiraling higher? Or is the Fed right that inflation can go away largely on its own without such high nominal interest rates? How much does monetary policy depend on fiscal policy? Given that fiscal tightening is unlikely, even to pay higher interest costs on the debt, and that a Fed-induced slowdown is likely to trigger stimulus of borrowed money, how much can the Fed lower inflation by interest-rate increases?

This paper builds on and synthesizes many points in Cochrane (2023), Ch. 5, 12, 16, 17, 20 and Cochrane (2022b). These also includes more detail, literature, and generalization to more complex models.

2 Inflation under interest rate targets

What is the dynamic effect of *interest rates*—not money—on inflation? I use a very simple standard model to think about this question,

$$x_t = E_t x_{t+1} - \sigma(i_t - \pi_t^e) \tag{1}$$

$$\pi_t = \pi_t^e + \kappa x_t \tag{2}$$

where $x =$ output gap, $\pi =$ inflation, $\pi^e =$ expected inflation, and $i =$ interest rate. Variables are all deviations from steady state.

Equation (1) is the first-order condition for consumption or dynamic IS curve. Equation (2) is the Phillips curve. Lucas's central innovation was, of course, to specify how expectations enter the Phillips curve so that output variation comes from *unexpected* inflation.

Lucas paired that Phillips curve with, essentially, $MV = PY$ and constant V, which determines the price level. With this structure, Lucas already had in hand a theory of price level determination, and one that expresses neutrality to boot. Our challenge is to develop a theory of price level determination based on interest rates, not money supplies. It should express neutrality as a flexible-price market-clearing launch point, and develop non-neutralities from distortions to that ideal. Unlike Lucas, we have to work to get to that launch pad.

I hesitate to write down such a model without preferences, technology, market structure, definition of equilibrium, and recursive statement. Lucas' most important contribution may have been methodological, to express a monetary economics question with a completely articulated general equilibrium model. But this is well-trod ground and it is well known how to provide those foundations. See, for example, Woodford (2003).

I simplify further by dropping $E_t x_{t+1}$ on the right hand side of (1), leaving a simple statement that higher real interest rates depress the level of output,

$$x_t = -\sigma(i_t - \pi_t^e).$$

This simplification turns out not to make any difference for the points I make, and leaving it out allows me to do everything with transparent algebra. Equation (1) iterates forward to

$$x_t = -\sigma E_t \sum_{j=0}^{\infty} (i_{t+j} - \pi_{t+j+1}),$$

so my static version is the same as the dynamic version when the current real interest rate is a sufficient statistic for that sum. The parameter σ in the simplified model is then larger than the intertemporal elasticity of substitution, as it includes how long the high rates last. Using the static IS curve makes a second point: The troubles I document cannot be fixed just by attenuating the forward-looking part of the IS curve.

Substituting output out of (1)-(2), we obtain the relationship between interest rates and inflation which we are after,

$$\pi_t = (1+\sigma\kappa)\pi_t^e - \sigma\kappa i_t. \tag{3}$$

The dynamic response of inflation to interest rates now depends on how expectations are formed.

2.1 Expectations, stability, and determinacy

Table 1 summarizes the steady forward march of expectations in the Phillips curve. (Each equation is simplified, of course, to be emblematic of an era. Actual Phillips curves also include error terms.)

Author	Phillips curve	Expectations
Phillips (1958)	$\pi_t = \pi_0 + \kappa x_t$	Absent
Dynamic empirical (1960s)	$\pi_t = \alpha\pi_{t-1} + \kappa x_t,\ \alpha < 1$	Adaptive
Friedman (1968); ISLM AS/AD(1970s)	$\pi_t = \pi_{t-1} + \kappa x_t$	Adaptive
Lucas (1972)	$\pi_t = E_{t-1}\pi_t + \kappa x_t$	Rational
Calvo (1983), Rotemberg (1982); NK (1990s)	$\pi_t = E_t\pi_{t+1} + \kappa x_t$	Rational

Table 1: The steady forward march of expectations in the Phillips curve

Phillips didn't have any expectations or other variables to shift the Phillips curve, nor did the Keynesian advocates of inflation in the early 1960s such as Samuelson and Solow (1960). (See Nelson (2020) Ch. 13.)

Dynamic estimates of the Phillips curve added lags of unemployment or inflation, depending which variable one put on the right-hand side of the regression. These specifications retained a long-run inflation-output tradeoff, $\alpha < 1$ here, and when thinking theoretically, adaptive expectations.[1]

Friedman's (1968) address was fundamentally about neutrality. He proclaimed two things that monetary policy *cannot* do. First, he proclaimed that the Phillips curve would shift once people come to expect inflation, so the Fed cannot permanently lower unemployment. The long-run Phillips curve is vertical. But on the way there, he described explicitly adaptive expectations: "This price expectation effect is slow to develop and also slow to disappear." Phelps (1967) also writes "a sort of Phillips Curve..that shifts one-for-one with variations in the expected rate of inflation; ...the expected inflation rate adjusts gradually over time to the actual inflation rate."

Second, Friedman proclaimed that the Fed cannot peg the nominal interest rate. We can see this result in our little model. Let expectations be adaptive, $\pi_t^e = \pi_{t-1}$. Then from (3) inflation and interest rates are related by

$$\pi_t = (1 + \sigma\kappa)\pi_{t-1} - \sigma\kappa i_t. \tag{4}$$

Inflation is now *unstable* under an interest rate peg, since $(1+\sigma\kappa) > 1$. In Friedman's description, the Fed needs to print more and more money to keep the interest rate down. The ISLM AS/AD tradition of the 1970s adopted the same adaptive-expectations Phillips curve, without money. In that description, a too-low nominal interest rate lowers the real rate, which boosts demand, which boosts inflation, and around we go. The left panel of Figure 1 illustrates instability and the inflation or deflation spirals that break out under an interest rate peg.

The last term of Equation (4) also shows the conventional sign; higher interest rates lower inflation. Indeed if not quickly reversed, higher interest rates set off an unstable deflationary spiral. Friedman also said so (in the opposite, inflationary, direction), though quickly adding

[1]Among many others, Lipsey (1960), Gordon (1970). Gordon (1976) p. 192-193 provides a nice summary of the era. Sargent (1971) and Lucas (1972b) also summarize insightfully. They point out that $\alpha > 1$ regression coefficients can easily occur even if inflation in the Phillips curve responds one for one to rationally expected inflation, anticipating Lucas's (1976) more comprehensive critique.

that the central bank would soon give up and raise interest rates (lower money growth) to stop the spiraling inflation.

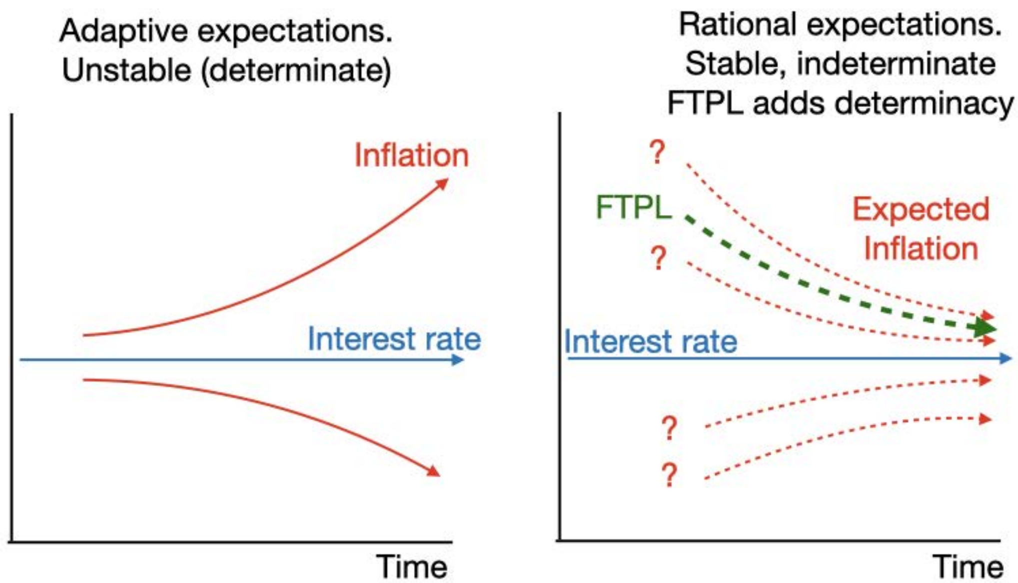

Figure 1: Instability vs. stability with and without indeterminacy

Thus began the long tradition that views an interest rate target as a fundamentally incomplete price-level anchor.

Taylor (1993) repaired Friedman's critique of interest rate targets.[2] Let the Fed systematically respond to inflation with higher interest rates, $i_t = \phi \pi_t$ with $\phi > 1$. Substituting for i_t in (4), inflation dynamics become

$$\pi_t = \frac{1 + \sigma\kappa}{1 + \sigma\kappa\phi} \pi_{t-1}. \tag{5}$$

Now inflation is stable and determinate with an interest rate target. Sensibly, the Fed's interest rate policies act to *stabilize* an inherently unstable economy.

Belief in unstable dynamics is alive and well today (2022), in the widespread opinion that by reacting slowly to inflation, the Fed is making inflation worse, and a sustained dose of interest rates higher than current inflation is the only way to cure inflation. (See, for example, most of Bordo, Cochrane, and Taylor (2022).)

[2]McCallum (1981) is the first formal statement that the $\phi > 1$ principle resolves problems with interest rate targets, and Wicksell (1898), Wicksell (1965) is the verbal historical antecedent. But Taylor, for example Taylor (1993) is the most influential advocate of the rule which justly bears his name.

It is desirable for monetary models to start from a benchmark that expresses neutrality when prices are flexible. With $MV = PY$, flexible prices and constant velocity mean that inflation equals money growth immediately. Prices become more flexible in the interest-rate based model (1)-(2) as κ grows. Sensibly, as κ grows, dynamics happen faster and faster. But these dynamics are unstable. Higher interest rates just lead to more quickly exploding deflation. The model does not have a well-defined frictionless and neutral limit, or limit point. Adaptive expectations and the Phillips curve are *necessary* in this view to have any understanding of inflation determination at all.

The issue is really model-consistent expectations, not the contentious question of rationality. Here, the expectations *of* the model are systematically and permanently different from expectations *in* the model. While less than fully rational expectations are likely a useful ingredient for fitting episodes or transient dynamics, and while actual expectations likely diverge from those of any model because real people see a lot more information than we can include, we, like Lucas, should surely be unsatisfied to require model-inconsistent expectations and sticky prices as a *necessary* ingredient to even be able to talk about price-level determination at all. Perhaps they are, but we should really know that there is *no* underlying supply-and-demand model, that the most basic questions that MV=PY answered in simple form *require* irrationality under interest rate targets. A search that comes up decisively empty-handed is still worthwhile.

New-Keynesian models use rational expectations, and consciously play by the Lucas rules of how to write and solve intertemporal general equilibrium macroeconomic models. With sticky prices, the standard new-Keynesian model bases the Phillips curve on inflation relative to rationally expected future inflation, $\pi_t^e = E_t \pi_{t+1}$ (Calvo (1983), Rotemberg (1982).)

Now from (3) the dynamic response of inflation to interest rates is

$$E_t \pi_{t+1} = \frac{1}{1+\sigma\kappa}\pi_t + \frac{\sigma\kappa}{1+\sigma\kappa} i_t. \tag{6}$$

Inflation is *stable* since $1/(1+\sigma\kappa) < 1$. And this model has a sensible frictionless $\kappa \to \infty$ limit and limit point:

$$E_t \pi_{t+1} = i_t. \tag{7}$$

But this model so far only ties down expected inflation. Unexpected inflation $\pi_{t+1} - E_t \pi_{t+1}$ can be anything, or wander up and down following sunspots. Using rational expectations in a related model, Sargent and Wallace (1975) modified Friedman's doctrine: Inflation is *indetermi-*

nate under an interest rate peg.

Rational vs. adaptive expectations fundamentally change the stability and determinacy properties of the model. The right hand panel of Figure 1 illustrates, with the question mark indicating all the many equilibria that could break out at that point.

Friedman's unstable (and determinate) is different from Sargent and Wallace's stable and indeterminate. They are frequently confused. Both suggest volatile inflation. But spiraling away on a determinate path is different from batting up and down unpredictably around the peg. In different ways, however, both express the view that a model of inflation built around interest rate targets is fundamentally incomplete.

The sign on the last term in (6) has also changed. Now higher nominal interest rates *raise* expected inflation. This observation inaugurates the effort to overturn this result in what follows.

New-Keyensian modelers resolve indeterminacy with a novel application of the Taylor principle. If we add $i_t = \phi\pi_t$ in this case, inflation dynamics (6) become

$$E_t\pi_{t+1} = \frac{1+\sigma\kappa\phi}{1+\sigma\kappa}\pi_t. \tag{8}$$

With $\phi > 1$, dynamics are now *unstable*. Adding a rule against nominal explosions, new-Keynesian modelers can now choose the unique initial value of inflation that precludes an explosion, and thus produce determinate inflation; $\pi_t = 0$ in this model with no shocks.

The central bank is imagined in this vision to deliberately *destabilize* an economy which is already stable on its own, exactly the opposite of the Taylor rule in an adaptive expectations economy which *stabilizes* an otherwise unstable economy. The central bank threatens hyperinflation or hyperdeflation in order to select or "coordinate expectations" on the equilibrium it likes.

Indeed, the central bank may simply announce its inflation target, announce this threat, and inflation jumps to whatever value the central bank desires. The Taylor Principle in a new-Keynesian model is an *equilibrium-selection* policy not a *stabilization* policy.

This statement is easiest to see in the $\kappa = \infty$ case of flexible prices, in which the interest rate directly sets expected inflation (7). Add a disturbance u_t to the policy rule, and write the rule equivalently as

$$i_t = \phi\pi_t + u_t = i_t^* + \phi(\pi_t - \pi_t^*) = E_t\pi_{t+1}^* + \phi(\pi_t - \pi_t^*). \tag{9}$$

Here, $\{\pi_t^*\}$ is the Fed's stochastic inflation target, the value of inflation it wishes to produce in each date and state. The second and third equalities define i_t^* and translate between the u_t and i_t^*, π_t^* notation for the monetary policy disturbance.

Eliminating i_t from (7)-(9), the model's equilibrium condition is

$$E_t(\pi_{t+1} - \pi_{t+1}^*) = \phi(\pi_t - \pi_t^*).$$

With $\phi > 1$, the unique bounded equilibrium is $\pi_t = \pi_t^*$, $i_t = i_t^*$. The central bank chooses the inflation it wishes to see $\{\pi_t^*\}$. It obtains this value by an interest rate policy i_t^*, which sets the equilibrium observed interest rate to equal the expected value of the inflation target, and a *separate* equilibrium-selection policy, threatening to produce an expected hyperinflation or deflation should unexpected inflation come out against its desires. For example, if π_t^* is i.i.d., observed interest rates never move. The central bank simply announces each period what inflation it would like to see, and that inflation occurs.

To see the same point in our little sticky price model, again add a disturbance u_t to the policy rule and write the policy rule equivalently as

$$i_t = \phi\pi_t + u_t = i_t^* + \phi(\pi_t - \pi_t^*), \tag{10}$$

where now

$$i_t^* \equiv \frac{1+\sigma\kappa}{\sigma\kappa} E_t\pi_{t+1}^* - \frac{1}{\sigma\kappa}\pi_t^*. \tag{11}$$

Equation (11) applies (6) to the starred variables. If we want to think in terms of an interest rate target and an inflation target, those targets must be compatible with private sector equilibrium conditions.

With this policy rule, the equilibrium condition (6) becomes

$$E_t(\pi_{t+1} - \pi_{t+1}^*) = \frac{1+\phi\sigma\kappa}{1+\sigma\kappa}(\pi_t - \pi_t^*). \tag{12}$$

By this policy, with $\phi > 1$, $\pi_t = \pi_t^*$ and $i_t = i_t^*$ is the unique bounded equilibrium.

We now have a full economic model of inflation under interest rate targets. The central bank completely controls inflation, expected and unexpected. But in reality, central banks do not have "equilibrium selection" policies. They do not threaten hyperinflation or deflation if inflation comes out against their desires. Such threats being contrary to their objectives, nobody would believe them if they tried. Central banks do not intentionally de-stabilize economies that are stable enough on their own. Ask central banks. Look at central bank websites. They loudly announce that they they *stabilize* economies; no matter what inflation does, they will act resolutely to bring it back.

As I rejected the beautiful $MV = PY$ because central banks set interest rates, do not limit money supply, and because money pays the same interest as bonds, I argue that we should also reject this elegant solution, because our monetary institutions simply do not remotely behave as this model specifies. However, as I will make clear below, you can still follow this path for the remaining analysis if you're really attached to it.

The fiscal theory of the price level adds an equilibrium condition, or rather recognizes one that was there all along and has been left out so far. The real value of nominal government debt v_t evolves as

$$\rho v_{t+1} = v_t + i_t - \pi_{t+1} - \tilde{s}_{t+1} \tag{13}$$

where $\rho < 1$ and $\tilde{s}_{t+1}$ is the real primary surplus scaled by the steady state value of debt. I use here the simple case of one-period debt and no economic growth; I generalize to long-term debt below. This equation is also linearized. See Cochrane (2023) Ch. 3.5 for a derivation. The consumer's transversality condition also requires

$$\lim_{T\to\infty} \rho^T v_T = 0. \tag{14}$$

We can add these conditions to the VAR(1) statement of the model. But in this simple case, we can solve the model analytically by iterating (13) forward to

$$v_t = E_t \sum_{j=0}^{\infty} \rho^j [\tilde{s}_{t+1+j} - (i_{t+j} - \pi_{t+1+j})]. \tag{15}$$

The real value of debt is the discounted present value of future surpluses. Taking innovations $\Delta E_{t+1} \equiv E_{t+1} - E_t$ of both sides of (15), we obtain.

$$\Delta E_{t+1}\pi_{t+1} = -\Delta E_{t+1} \sum_{j=0}^{\infty} \rho^j \tilde{s}_{t+1+j} + \Delta E_{t+1} \sum_{j=1}^{\infty} \rho^j (i_{t+j} - \pi_{t+j+1}). \tag{16}$$

Unexpected inflation devalues outstanding debt. Thus, unexpected inflation corresponds to the revision in the present value of future primary surpluses. Deficits ($\tilde{s}_t < 0$) that are not expected to be repaid by subsequent surpluses cause inflation. In the second term, higher discount rates likewise lower the value of debt and cause inflation. Equivalently, higher interest costs function like lower surpluses. If interest costs on the debt rise, but current or future surpluses do not rise to pay them, then the resources must come by inflating away outstanding bonds.

Since the rational expectations model left an indeterminacy indexed by unexpected inflation $\Delta E_{t+1}\pi_{t+1}$, (16) clearly steps in to restore that determinacy, in place of central bank equilibrium-selection rules.

The point is easiest to see in the simplest case of flexible prices. Then, (6) and (16) boil down to

$$i_t = E_t\pi_{t+1} \tag{17}$$

$$\Delta E_{t+1}\pi_{t+1} = -\Delta E_{t+1}\sum_{j=0}^{\infty}\rho^j\tilde{s}_{t+1+j}. \tag{18}$$

The interest rate target sets expected inflation; fiscal policy determines unexpected inflation.

With sticky prices, we have the pair (6)-(16), which I repeat for convenience,

$$E_t\pi_{t+1} = \frac{1}{1+\sigma\kappa}\pi_t + \frac{\sigma\kappa}{1+\sigma\kappa}i_t \tag{19}$$

$$\Delta E_{t+1}\pi_{t+1} = -\Delta E_{t+1}\sum_{j=0}^{\infty}\rho^j\tilde{s}_{t+1+j} + \Delta E_{t+1}\sum_{j=1}^{\infty}\rho^j(i_{t+j} - \pi_{t+j+1}). \tag{20}$$

Now (19) picks a set of paths for expected inflation, and (20) selects which one is the unique equilibrium. The right hand panel of Figure 1 illustrates this option as well. Fiscal policy determines one of the many possible equilibria.

In sum, with the combination (19) and (20) to choose unexpected inflation, inflation is stable *and determinate* at an interest rate peg (or Taylor coefficients $\phi < 1$), overcoming Sargent and Wallace's contrary doctrine. How *volatile* inflation is depends on how much fiscal shock or quiet there is. But economics picks one value.

The preceding doctrines aren't logically wrong. They just make different assumptions. Friedman and ISLM-AS/AD writers assumed adaptive expectations. They, Sargent and Wallace, new-Keynesians, and their followers make explicit assumptions to wipe out fiscal theory. In an era of small government debt, small interest costs on that debt, and relatively low taxes, this first approximation is sensible. All economic models leave out small effects. But we now live in an era of large debts, potentially large interest costs, and questionable ability to raise additional large amounts of permanent revenue.

Indeed, careful New-Keynesian modelers include the government debt equilibrium condition, though often in footnotes. They assume that fiscal authorities "passively" adjust surpluses as needed to validate the central bank's equilibrium choices. In (20), the central bank picks $\Delta E_{t+1}\pi_{t+1}$, and then fiscal authorities supply whatever surpluses $\tilde{s}_{t+1}$ are necessary, often via lump-sum taxes.

Since the equilibrium conditions are the same, the fiscal and new-Keynesian theories are at this level observationally equivalent. (See Cochrane (2023) Ch. 17, 22.) Thus, for the purposes of everything that follows, you may think in terms of the new-Keynesian rather than fiscal-theory version of equilibrium formation. However, new-Keynesian modelers typically do not examine what the required surpluses are, or if they are reasonable or consistent with data. I simply examine them. For example, in the standard new-Keynesian model, a monetary policy shock that lowers inflation comes with a "passive" fiscal tightening. (Caramp and Silva (2021) document these fiscal foundations.) I ask, let us examine monetary policy shocks that do not come with such a fiscal policy, either because fiscal policy is unwilling or unable to tighten, or just out of intellectual curiosity to understand what monetary policy does on its own and not by triggering a fiscal response to its equilibrium-selection threats.

But we should not overstate observational equivalence. For example, in the new-Keynesian interpretation, the central bank fully controls inflation, expected and unexpected. In the fiscal-theory interpretation, fiscal shocks (a permanent decline in surpluses) result in inflation that the central bank cannot completely avoid. In new-Keynesian models such fiscal shocks are ruled out: The central bank picks unexpected inflation, and the government comes up with the needed surpluses no matter what. Whether the central bank can or cannot fully control inflation is a pretty important policy and doctrinal issue. Did we suffer inflation in 2021-2022 because of a fiscal shock, or because the Fed failed to announce appropriate equilibrium-selection threats to go with its zero interest rates? Episodes such as the zero bound or explicit pegs in which the central bank cannot exercise new-Keynesian equilibrium-selection threats are also illuminating.

2.2 Continuous time

Here I develop the simple model in continuous time. This is a clearer though less familiar way to see the main points. In particular, we can see here that the central question is really the sign of output in the Phillips curve: Is output high when inflation is *increasing* or *decreasing*? In continuous time, some of the timing conventions that obscure the analysis vanish. In particular, we see that rational expectations in the IS curve are not an issue, which suggest that various attempts to modify the IS curve, such as adding hand to mouth consumers, may not change the fundamental sign and stability properties of the model. Continuous time with sticky prices points to a fundamentally different reinterpretation of the model: The government debt valuation equation does not adjust via price-level jumps on the date of a shock, but by choosing a whole path of inflation that adjusts the discount rate applied to future surpluses, or equivalently adjusts the interest costs on the debt.

Write the standard model (1)-(2)

$$E_t(x_{t+\Delta} - x_t) = \sigma(i_t - E_t\pi_{t+\Delta})\Delta \tag{21}$$

$$E_t(\pi_{t+\Delta} - \pi_t) = -\kappa x_t \Delta. \tag{22}$$

This standard model in continuous time is thus

$$E_t dx_t = \sigma(i_t - \pi_t)dt \tag{23}$$

$$E_t d\pi_t = -\kappa x_t dt. \tag{24}$$

Normally a term $-\rho\pi_t dt$ appears on the right of (24). As I simplified the discrete time Phillips curve from $\pi_t = \beta E_t \pi_{t+1} + \kappa x_t$ with $\beta = 1$, I simplify here with $\rho = 0$; the Phillips curve is centered on expected future inflation, and permanent inflation is fully neutral.

The price level is continuous and differentiable, and cannot jump or diffuse. In an instant dt only a fraction λdt of producers may change prices. The inflation rate may have jumps or diffusions. But $E_t\pi_{t+\Delta} - \pi_t$ is still of order Δ, so the relevant inflation in the consumer's first order condition (23) is π_t. The issue whether inflation in that condition should be rationally anticipated or adaptive disappears. This is a useful clarification of continuous time. Expectations in the Phillips curve are the central issue.

Equations (22) and (24) express the standard rational-expectations Phillips curve. The adaptive-expectations analogue is

$$\pi_t - \pi_{t-\Delta} = \kappa x_t \Delta \tag{25}$$

$$d\pi_t = \kappa x_t dt. \tag{26}$$

Thus, adaptive and rational expectations differ by whether higher output corresponds to increasing (26) or decreasing (24) inflation; by inflation greater than future or past inflation. Essentially, they differ by the sign of κ. Adaptive expectations also produce a differentiable inflation, with neither jumps nor diffusion terms.

Again I simplify the model so we can see the main points without algebra, by using a static version of the consumption equation,

$$x_t = -\sigma(i_t - \pi_t). \tag{27}$$

Eliminating output from the Phillips curve, we have the dynamic relation between interest rates and inflation. With rational expectations

$$E_t d\pi_t = -\sigma\kappa\pi_t dt + \sigma\kappa i_t dt, \tag{28}$$

while with adaptive expectations

$$d\pi_t = \sigma\kappa\pi_t dt - \sigma\kappa i_t dt. \tag{29}$$

We have immediately the results of the discrete-time model: Inflation is stable but indeterminate under rational expectations; while inflation is unstable but determinate under adaptive expectations. "Stable" means that the coefficient in front of π_t on the right hand side is negative. "Indeterminate" means that we do not fully determine inflation. We can write (28)

$$d\pi_t = -\sigma\kappa\pi_t dt + \sigma\kappa i_t dt + d\delta_t \tag{30}$$

where

$$d\delta_t = d\pi_t - E_t d\pi_t$$

is an arbitrary random variable (compensated jump or diffusion) with $E_t d\delta_t = 0$. The solutions of (30) are

$$\pi_t = \sigma\kappa \int_{\tau=0}^{t} e^{-\sigma\kappa\tau} i_{t-\tau} d\tau + e^{-\sigma\kappa t}\pi_0 + \int_{\tau=0}^{t} e^{-\sigma\kappa\tau} d\delta_{t-\tau}. \tag{31}$$

"Stability" means that the influence of past interest rates disappears over time, while "indeterminacy" means that the expectational errors $d\delta_t$ appear.

For adaptive expectations, "unstable" means that the coefficient in front of π_t on the right hand side is negative. It is "determinate" since $d\pi_t$ not $E_t d\pi_t$ appears on the left. The solutions of (29) are

$$\pi_t = \sigma\kappa \int_{\tau=0}^{t} e^{\sigma\kappa\tau} i_{t-\tau} d\tau + e^{\sigma\kappa t}\pi_0.$$

"Unstable" means that interest rates and initial conditions further in the past have larger effects today. Despite the $\sigma\kappa\pi_t dt$ on the right hand side of (29), we solve the model backward, because there is no jump or diffusion in inflation. If we try to solve forward,

$$\pi_t = \sigma\kappa \int_{\tau=0}^{\infty} e^{-\sigma\kappa\tau} i_{t+\tau} d\tau,$$

the right hand side can require a jump or diffusion that the model rules out. Inflation is predetermined. "Instability" means that for all but one special π_0, inflation or deflation spirals. But π_0 is just as predetermined as at other dates, and in particular cannot react to the future realizations of the interest rate.

In the case of a peg, $i_t = i$, for rational expectations (31) becomes

$$\pi_t = (1 - e^{-\sigma\kappa t})i + e^{-\sigma\kappa t}\pi_0 + \int_{\tau=0}^{t} e^{-\sigma\kappa\tau} d\delta_{t-\tau}. \tag{32}$$

For adaptive expectations, a peg leads to

$$\pi_t = i + e^{\sigma\kappa t}(\pi_0 - i). \tag{33}$$

The peg is generically unstable.

As in discrete time, a Taylor rule stabilizes the unstable adaptive expectations model. Adding

$$i_t = \phi\pi_t + u_{i,t}$$

the adaptive-expectations dynamics (29) become

$$\frac{d\pi_t}{dt} = \sigma\kappa(1-\phi)\pi_t - \sigma\kappa u_{i,t}$$

With $\phi > 1$, dynamics are now stable and determinate. A monetary policy shock $u_{i,t}$ raises the interest rate and lowers inflation. A simulation follows below.

In the rational expectations model with Taylor rule, in the new-Keynesian tradition, rational-expectations dynamics (29) become

$$E_t d\pi_t = \sigma\kappa(\phi - 1)\pi_t dt - \sigma\kappa u_{i,t} dt.$$

Now $\phi > 1$ induces instability. This time instability means we can solve the integral forward, and with a rule against nominal explosions recover determinacy,

$$\pi_t = -\sigma\kappa E_t \int_{\tau=0}^{\infty} e^{-\sigma\kappa(\phi-1)\tau} u_{i,t+\tau} d\tau.$$

Define an inflation target $\{\pi_t^*\}$ and define i_t^* by

$$E_t d\pi_t^* = -\sigma\kappa\pi_t^* dt + \sigma\kappa i_t^* dt$$

In words, i_t^* is the interest rate target that implements $\{\pi^*\}$ as an equilibrium. Now write the policy rule as

$$i_t = i_t^* + \phi(\pi_t - \pi_t^*)$$

With this notation, we can write rational-expectations dynamics (29) as

$$E_t d(\pi_t - \pi_t^*) = \sigma\kappa[-(\pi_t - \pi_t^*) + (i_t - i_t^*)]dt$$

$$E_t d(\pi_t - \pi_t^*) = \sigma\kappa(\phi - 1)(\pi_t - \pi_t^*)dt.$$

Monetary policy has two parts, an interest rate policy i_t^* which generates the desired path of expected inflation, and an equilibrium-selection policy $\phi(\pi_t - \pi_t^*)$ which generates explosions unless $d\pi_t - E_t d\pi_t = d\pi_t^* - E_t d\pi_t^*$.

Fiscal theory offers an alternative route to determinacy in the rational expectations model. Include the linearized evolution of real government debt, with instantaneous debt and differentiable prices

$$dv_t = (rv_t + i_t - \pi_t - \tilde{s}_t)dt$$

Integrating forward, taking expectations, and imposing the transversality condition, the real value of debt equals the present value of surpluses.

$$v_t = E_t \int_{\tau=0}^{\infty} e^{-r\tau}[\tilde{s}_{t+\tau} - (i_{t+\tau} - \pi_{t+\tau})]d\tau \tag{34}$$

To use this equation in the rational-expectations dynamics (30) as above, let $\Delta_t v_t \equiv v_t - E_t v_t$ isolate the compensated jump or diffusion component of a process. In this case, and unlike discrete time, $\Delta_t v_t = 0$. Corresponding to (16),

$$0 = \Delta_t \int_{\tau=0}^{\infty} e^{-r\tau}[\tilde{s}_{t+\tau} - (i_{t+\tau} - \pi_{t+\tau})]d\tau. \tag{35}$$

Short-term nominal debt is predetermined, and since prices cannot jump or diffuse, the value of debt cannot jump or diffuse. Rather than shock the initial value of debt at all, of the multiple equilibria, we pick the inflation *path* in which the discount rate/interest cost effect, the second term, exactly balances any change in surplus, the first term. In the absence of a surplus change, such as a pure monetary policy shock, we pick the inflation path so that the integral of the discount rate term is zero.

$$0 = \Delta_t \int_{\tau=0}^{\infty} e^{-r\tau}[(i_{t+\tau} - \pi_{t+\tau})]d\tau$$

Continuous time fundamentally changes how we think of the model at high frequency. With flexible prices, a decline in surpluses must be met by a price-level jump which devalues outstanding debt. The discrete-time unexpected inflation equation (16) included some of that intuition along with a discount rate / interest cost effect. Now the latter is everything. In essence, the left-hand side of (16), $\Delta E_{t+1}\pi_{t+1}$, is always zero. Now in (34) a decline in surpluses (first term on the right of (16)) is met by a period of inflation higher than nominal interest rates (second term) which slowly devalues debt. Bondholders lose by a long period of inflation above the nominal interest rate, not by a price-level jump. Monetary policy changes in the path of nominal interest rates generate an inflation path in which the present value of interest costs is zero.

2.3 Lucas's Phillips curve

In my little Phillips curve history, I skipped over Lucas. Lucas (1972a) first made expectations rational or model-consistent, and forward-looking. His Phillips curve relates output to unexpected inflation only, first moving forward the time subscript in the Phillips curve, from $\pi_t^e = \pi_{t-1}$ to $\pi_t^e = E_{t-1}\pi_t^e$. In the spirit of rational expectations, it makes most sense to pair Lucas' Phillips curve with rational expectations in the bond market and consumption. So let's use Lucas' Phillips curve in an interest-rate model by writing

$$x_t = -\sigma(i_t - E_t\pi_{t+1}) \tag{36}$$

$$\pi_t = E_{t-1}\pi_t + \kappa x_t. \tag{37}$$

Eliminating x_t, inflation dynamics (3) are now

$$E_t\pi_{t+1} = i_t + \frac{1}{\kappa\sigma}(\pi_t - E_{t-1}\pi_t) \tag{38}$$

Iterating forward,

$$E_t\pi_{t+2} = E_t i_{t+1}.$$

Lucas' specification of the rational expectations Phillips curve, along with our IS curve, passive fiscal policy, and an interest rate target, is stable and indeterminate, like the new-Keynesian model. Relative to the flexible-price model $i_t = E_t\pi_{t+1}$, Lucas's Phillips curve gives one period of additional inflation after a shock, which then reverts to the frictionless value. This behavior is much like Lucas's monetary model, which produces one period of output. Adding fiscal theory to this model we again restore determinacy, and name the additional shock.

3 Neutrality and its consequences

We have, finally, a complete economic theory of inflation determination under interest rate targets, comparable to $MV = PY$. It includes rational expectations and market clearing. It starts from a simple frictionless model, analogous to the case that money leads instantly to inflation, but it also allows sticky-price dynamics and consequent output effects.

That inflation is stable and determinate under an interest rate peg amounts to a sensible characterization and statement of long-run *neutrality*. Inflation eventually settles down to the nominal interest rate, and does so faster as prices are more flexible, just as under $MV = PY$ inflation eventually settles down to follow the money growth rate, and does so faster as prices are more flexible. Good. Anchoring our understanding of inflation under interest-rate targets in a well-defined sense of neutrality is a desirable characteristic.

Neutrality for this interest-rate based model, as I have defined it, is however a little touchier than neutrality for a monetary model with fixed velocity. From $MV = PY$ it follows quickly that more money M means more nominal income PY, and long-run neutrality means it eventually has to be prices P not real income Y. Likewise, $i_t = r_t + E_t\pi_{t+1}$ mans that, when real r_t and nominal π_t effects decouple, steady states with higher interest rates i_t have commensurately higher inflation π_t.

But neutrality as I have described it also includes stability—that inflation π_t eventually moves to follow the interest rate i_t. The adaptive-expectations model also has steady states in which 1% higher interest rates correspond to 1% higher inflation (see (4), (29) and (33)). But these are unstable steady states, so the model does not display long-run neutrality as I have defined it.

We don't traditionally worry about stability with $MV = PY$, the possibility that there are steady states with higher money M and proportionally higher prices P, but the economy is unstable so that raising money growth would send prices off on a downward spiral. However, when money demand is sensitive to interest rates—it is—that proposition is not so obvious either. Such models have also have multiple unstable equilibria. For example, if velocity rises with the interest rate,

$$m_t + \alpha(p_{t+1} - p_t) = p_t + y_t$$

and constant output y, then inflation $\pi_{t+1} = p_{t+1} - p_t$ follows

$$\pi_{t+1} = \left(1 + \frac{1}{\alpha}\right)\pi_t - \frac{1}{\alpha}(m_t - m_{t-1}).$$

Steady states with higher money growth have higher inflation, but they are unstable. From a steady state, raising money growth leads to spiraling deflation. But this theoretical issue is usually glossed over, where it is more contentious in today's debates about interest rate targets.

As sensible and unavoidable as I hope I have made long-run interest-rate neutrality seem, however, its implications are uncomfortable and initially counterintuitive—as the implications of long-run monetary neutrality were difficult for our predecessors to swallow.

3.1 History and interest rate pegs

Just because a theory is beautiful does not make it true. Again, neutrality implies that *inflation is stable and determinate under an interest rate peg*, contrary to classic doctrines. Does our economy display long-run neutrality?

We have just seen something close to an interest rate peg: The long quiet lower bound. From 2008 to 2016 in the US, from 2008 to 2022 in Europe, and from 1995 to 2022 in Japan, interest rates were effectively stuck at zero. They could not move much in the downward direction, they did not move in the upward direction. Central bankers issued "forward guidance" that interest rates would not move, at least not promptly and more than one for one with observed inflation as required by either interpretation of the Taylor principle.

When interest rates hit zero, the adaptive-expectations model clearly predicts a deflation spiral. Central bankers, oped writers, international institutions, and commenters warned of the danger, correctly given that popular view. But the deflation spiral did not happen.

The new-Keynesian model clearly predictes multiple-equilibrium sunspot volatility. That too simply did not happen. (We can model these events by adding a discount rate shock, replacing i_t with $i_t - r_t$, and modeling the response to discount rate shock in r_t.)

Interest rates were stuck at zero for many years, and inflation just batted around in the 1-2% range the whole time. Inflation was if anything *less* volatile at the zero bound than when central banks could move interest rates in their efforts to control inflation.[3]

Score one for rational expectations with fiscal-monetary coordination. At the level of ingredients, fans of Lucas (1972a) should be pleased, though the prediction and result are novel and perhaps uncomfortable.

What about many historical pegs that did seem to lead to spiraling inflation? These episodes are central to Friedman's (1968) argument that pegs are unstable. Well, stability, determinacy and quiet also require no fiscal news in (16). Most governments with interest rate pegs and spiraling inflation were using the peg to hold down interest costs of the debt while they printed money and other debt to finance out-of-control deficits. Also, if you pick episodes ex-post with large inflation, you also are likely to pick episodes with multiple fiscal shocks. In the zero bound era, for some reason, people were rushing to buy government debt at negative real rates. Unex-

[3] I summarize here much longer analysis in Cochrane (2017), Cochrane (2018), and Cochrane (2023) Ch. 20.

pectedly low interest costs on the debt act like surpluses (see (16)). Long-run fiscal policy was not in great shape, but there wasn't much *news* on that score during the quiet 2010s, at least until 2021.

3.2 k percent rules

If an interest rate peg at zero is stable and determinate, it follows that a peg at a positive interest rate peg is stable and determinate. *The central bank may follow a k-percent interest rate rule.* Inflation will simply bat around a higher interest rate, plus or minus the underlying real rate, as it batted around during the long quiet zero bound.

K percent may not be the *optimal* rule. But it is *possible.* And the quiet of the zero bound era suggests that maybe central banks' active "stabilization" might not have been doing all that much good. Milton Friedman did not argue for a 4% money growth rule because it solves the optimal control problem of a specific dynamic model, but for its robustness in the fog of reality. The central bank won't fiddle with the hot and cold water producing a scalding or freezing shower.

3.3 Current (2022) events

In the mid-2002 policy debate, neutrality implies that if the Fed does nothing, or only gently raises rates, never exceeding inflation ($\phi < 1$), inflation will nonetheless not spiral out of control. Inflation may surge for a while, following other shocks, and as the natural momentum of somewhat sticky prices proceeds. In a fully worked-out sticky price model with a fiscal shock, it takes an extended period of negative ex-post real interest rates to devalue nominal debt. (Cochrane (2022a) Figure 1 plots a simple example.) But inflation eventually comes back on its own, so long as fiscal policy or other shocks do not create more inflation. The same modeling that worries about an inflation spiral today worried about a deflation spiral last time.

3.4 Long-run Fisherism

If a k-percent peg is stable and determinate, then raising the peg from k to k+1 percent must move the economy to a new equilibrium with 1 percent higher stable and determinate inflation. *Raising interest rates will raise inflation* at least in the long run.

Long-run Fisherism is an inescapable logical conclusion of stability, determinacy and neutrality. All of the rational expectations models I have written have this prediction.

3.5 Neutrality intuition

The equations are transparent, but the implications are hard to believe. The frictionless model captures the problem most simply. We have $i_t = E_t\pi_{t+1} + r$, so peg i_t at a higher level and expected inflation $E_t\pi_{t+1}$ must rise as well. With sticky prices or other frictions, $i_t = E_t\pi_{t+1} + r_t$, but once real interest rates r_t settle down and decouple from nominal events, expected inflation must rise. Similarly, real rate declines should not cause problems, even at the zero bound. If the real rate r_t declines, as in a discount rate shock, and the Fed does not or cannot lower nominal rates, inflation should appear endogenously. Really? How?

Consider the full consumer first-order condition $x_t = E_t x_{t+1} - \sigma(i_t - E_t\pi_{t+1})$. Raise the nominal interest rate i_t. Before prices change, a higher nominal interest rate is a higher real rate, and induces people to demand less today x_t and more tomorrow x_{t+1}. That change in demand pushes down the price level today p_t and pushes up the expected price level tomorrow p_{t+1}. If prices are flexible, and the economy has a constant real rate, for example constant endowments, that force continues until $E_t\pi_{t+1} = i_t$. Yes, a higher nominal interest rate naturally pushes the economy to more inflation.

But which is it, lower p_t or higher p_{t+1}? The consumer first-order condition, the intertemporal substitution effect, cannot tell us. As before, that condition alone leaves an indeterminacy, unexpected inflation, in this case the value of $p_t - E_{t-1}p_t$ or $\pi_t - E_{t-1}\pi_t$. Unexpected inflation is determined by a wealth effect. If we pair the higher interest rate with no change in surpluses, and thus no wealth effect, then the initial price level p_t does not change and the entire effect of higher interest rates is a rise in p_{t+1}. A rise in surpluses, actively or passively achieved, leads to a lower price level p_t and less current inflation π_t.

Whether reflected in an unexpectedly lower price level p_t or higher price level p_{t+1}, however, the proposition that higher nominal interest rates raise higher expected *future* expected inflation $E_t(p_{t+1} - p_t) = E_t\pi_{t+1}$ is a natural outcome of the consumer's intertemporal optimization. The common intuition that higher interest rates should lower current demand, and thus lower inflation is also correct. The resolution of this apparent contradiction is to distinguish current, and unexpected inflation, $\pi_t = p_t - p_{t-1}$ from expected future inflation $\pi_{t+1} = p_{t+1} - p_t$.

4 Short run non-neutrality; can higher interest rates temporarily lower inflation?

We finally have a theory with rational (or at least model-consistent) expectations that determines inflation. It displays long-run neutrality. We should stop and smile. Pretty much everything that you used to do with money growth starting with $MV = PY$ and stable velocity you can now do with an interest rate in the place of money growth.

However, long-run neutrality means that higher nominal interest rates lead to *higher* inflation. Extensive experience suggests that higher interest rates can at least *temporarily* lower inflation, under some conditions. At a minimum we want a model that can express that belief, and see if the required ingredients make sense. And there is nothing in what we have done so far that rules out a *temporary* negative sign.

If that were true, then the Fed could do some good by raising rates, and at least temporarily offset the underlying causes (fiscal, I think) of 2022 inflation. We could then also understand central bankers' and policy commentators' belief in a uniformly negative effect, as well as the absence of a well-documented long-run positive effect in econometric estimates. Central banks never left rates alone long enough, with a background of stable fiscal policy and no other shocks, to see the positive long-run effect. Until the lower bound provided the experiment, I would add, but one episode might be excused with epicycles.

We are, in short, where Lucas started. But Lucas's central contribution was to describe the short-run *non-neutrality* of money.

Here are the rules of the game: We want a model in which the central bank operates via an interest rate target. We want a higher interest rate, with no change in fiscal policy, to lower inflation, at least for a while. The model should follow the usual rules of economics, including model-consistent expectations. The model should have a neutral frictionless limit and should display long run neutrality.

I emphasize the qualifier, with no change in fiscal policy. Monetary policy can appear to have an effect if it comes with a change in fiscal policy, or if the economic events that monetary policy produces themselves change fiscal policy. An interest rate rise might lead to a recession, which leads to stimulus and automatic stabilizers and even more inflation. Or inflation induced by monetary policy might lead to austerity and fiscal retrenchment. For evaluating history or the likely course of the economy following a monetary policy shock, we want to include contemporaneous and induced fiscal responses. But our question is a theoretical one. We want to know, what can monetary policy do all by itself? If monetary policy only reduces inflation by inducing a fiscal contraction, then monetary policy by itself is not that effective. To understand monetary policy, then, we want to know if monetary policy all by itself can lower inflation, even if such responses do not capture historical events or the likely future course of the economy after a shock.

There might be no such model. Higher interest rates may need fiscal support to lower inflation. Or a negative response may require extensive frictions such as irrational expectations, credit constraints, etc. But knowing that fact is important to evaluating monetary policy. A search that establishes no such model exists is just as important as a search that finds the model we seek.

You might think it's easy. Just add some sticky prices, for example. That turns out not to be the case. You might think that standard models in use for decades satisfy this desire. That also turns out not to be the case. The standard policy-maker's model is based on adaptive expectations, and does not display long-run neutrality or a neutral limit. The standard new-Keynesian model produces a negative effect by slipping in a fiscal contraction coincident with the interest rate rise.

4.1 A failure in simple sticky price models

Figure 2 plots the response of my simplified rational expectations model (simplified with a static IS equation) to an unexpected permanent interest rate rise. Inflation rises, even in the short run. Sticky prices lead to output effects (not shown), since the real interest rate rises. But they only draw out the positive response of inflation to interest rates.

Start with the "flexible or Lucas" line. The flexible price model is (17)-(18). The responses solve

$$E_t \pi_{t+1} = i_t \tag{39}$$

$$\Delta E_{t+1} \pi_{t+1} = 0. \tag{40}$$

The response starts with all variables 0 at time 0. We want the response to a permanent unex-

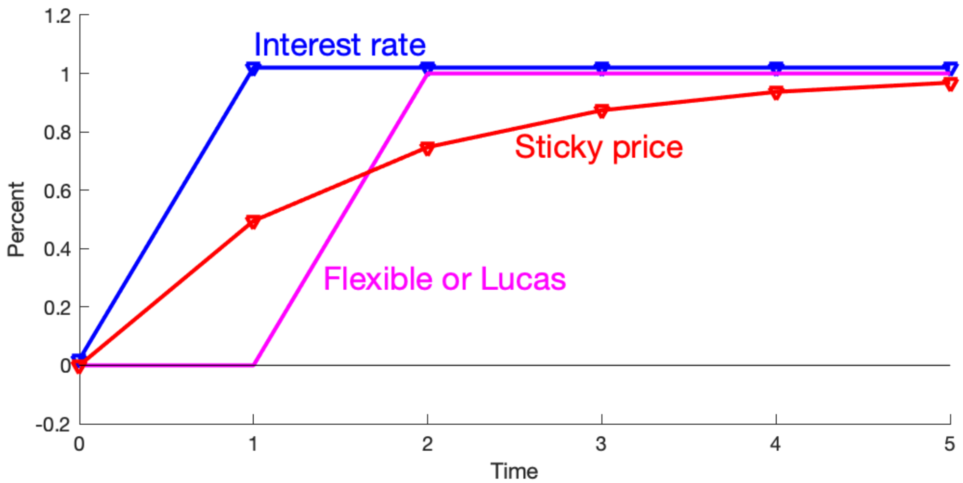

Figure 2: Inflation response to a 1% permanent rise in the interest rate. Parameters $\sigma\kappa = 1$.

pected interest rate rise to i_1 at time 1, with no change in fiscal surpluses. The first, surplus, term in the general unexpected inflation equation (16) is zero. By (39), the discount rate or interest cost term is zero as well. Hence, as in (40) and as shown in Figure 2, there is no change in inflation on the day the interest rate rises, but inflation fully follows the interest rate with a one-period lag.

The Lucas Phillips curve married to the static IS model gives dynamics (38). With no change in surpluses, the surplus innovation term in the unexpected inflation equation (16) is again zero, but now there are potentially interest costs to pay. The impulse-response solves

$$E_t\pi_{t+1} = i_t + \frac{1}{\kappa\sigma}\Delta E_t\pi_t \tag{41}$$

$$\Delta E_{t+1}\pi_{t+1} = \Delta E_{t+1}\sum_{j=1}^{\infty}\rho^j(i_{t+j} - \pi_{t+j+1}). \tag{42}$$

With only one unexpected movement at time 1 ($t = 0$ in the equation) so $\pi_2 = E_1\pi_2$ and so forth, (41) leaves π_1 arbitrary but then $\pi_2 = i_1 + \pi_1/(\kappa\sigma)$, $\pi_3 = i_1$, $\pi_4 = i_1$, etc. Now, use (42) to find π_1 and the unique path. (42) reduces to $\pi_1 = i_1 - \pi_2 = -\pi_1/(\kappa\sigma)$. The unique solution is $\pi_1 = 0$, and thus $\pi_2 = i_1$, $\pi_3 = i_1$, and so forth. You can verify that this path solves both (41) and (42). Despite the non-neutrality in the Phillips curve, which produces a one-period rise in output (not shown), inflation follows the flexible-price path, and does not decline.

The "sticky price" line uses the forward-looking new-Keynesian Phillips curve. The response function solves (19)-(20):

$$E_t\pi_{t+1} = \frac{1}{1+\sigma\kappa}\pi_t + \frac{\sigma\kappa}{1+\sigma\kappa}i_t \tag{43}$$

$$\Delta E_{t+1}\pi_{t+1} = \Delta E_{t+1}\sum_{j=1}^{\infty}\rho^j(i_{t+j} - \pi_{t+j+1}). \tag{44}$$

Again, surpluses are zero in (44) as that is the question we are asking. In response to the interest rate shock, there is a family of solutions to (43) which we can index by π_1,

$$\pi_{t+1} = i_1 - \frac{1}{(1+\sigma\kappa)^t}(i_1 - \pi_1). \tag{45}$$

Now, we use (44) at time $t = 0$ to determine π_1:

$$\pi_1 = \sum_{j=1}^{\infty}\rho^j(i_1 - \pi_{j+1}). \tag{46}$$

Substituting from (45) and simplifying,

$$\pi_1 = \frac{\rho}{1+\sigma\kappa}i_1 \tag{47}$$

and the full unique solution is

$$\pi_{t+1} = \left[1 - \frac{1+\sigma\kappa-\rho}{(1+\sigma\kappa)^{t+1}}\right]i_1. \tag{48}$$

(This solution method is useful to see the intuition of this simple model. For larger models and numerical solutions, it is more convenient to write the standard VAR(1) form,

$$x_t = -\sigma(i_t - E_t\pi_{t+1})$$

$$\pi_t = E_t\pi_{t+1} + \kappa x_t$$

$$\rho v_{t+1} = v_t + i_t - \pi_{t+1} - \tilde{s}_{t+1}$$

and solve by the usual matrix methods.)

As we can see in Figure 2, sticky prices smooth the dynamics. Since the nominal interest rate rises faster than the inflation rate, there is a period of high real interest rates, which depress output (not shown). Thus we have short-run non-neutrality. But we maintain long-run neutrality; the inflation rate eventually rises to fully match the interest rate.

Sticky prices have done their job. If your objective is only to produce a model in which the Fed can cool economic activity in the short run with interest rate rises, you have it. You might stop here and say, we have indeed redone Lucas (1972a) with interest rate targets, since his purpose was to understand how money growth affects the real economy.

But inflation still rises uniformly after the interest rate rise. Indeed, in period 1, inflation is *greater* than the (zero) value of the frictionless model. Why? The answer is the fiscal implications of interest costs. As prices get stickier, real interest rates rise, as you see in the right-hand part of the impulse-response. Higher real interest rates mean greater unfunded (by assumption of no change in fiscal policy) interest costs on the debt. These higher interest costs must come from a higher unexpected period 1 inflation, which devalues outstanding debt.

Indeed, as prices become stickier, $\kappa \to 0$, $\pi_1 \to \rho$ (see (47)) which is just barely less than one. Stickier prices lead to *more* inflation. This is not paradoxical. Sticky *prices* do not imply sticky *inflation*. The few firms who can change price at any instant know inflation will be persistently higher, so they raise their prices a lot. Sticky inflation seems intuitively plausible, but it requires costs to changing inflation that current sticky-price models or empirical work do not recognize.

It's clear here how the fiscal underpinnings of the model matter crucially. If we said that fiscal surpluses would rise to pay higher interest costs on the debt, then we would obtain at least $\pi_1 (= \Delta E_1 \pi_1) = 0$, no immediate rise. If we could pair the interest rate rise with even higher surpluses, for example if future inflation led to future fiscal austerity, we could predict lower current inflation, $\pi_1 < 0$. But the question we want to ask is, what can higher interest rates do to lower inflation *without* such fiscal support. The answer is, so far, they can produce a recession but they cannot lower inflation.

And even this much non-neutrality is fragile, really the result of one-period rather than instantaneous debt. In the continuous-time version of this model, with instantaneous (overnight) debt, inflation instantly tracks the interest rate, for any price stickiness. Intuitively, the $\Delta E_1 \pi_1$ term on the left-hand side of the unexpected inflation identity (44) is absent in continuous time, so interest costs must be zero on net. With AR(1) dynamics that cannot overshoot and return, that means inflation must jump instantly to match the interest rate so the real interest rate does not move at all.

To be specific, we can compute the impulse-response function by supposing there is a shock at time 0, and all variables represent how their expected values respond to that shock. From (31), for a generic interest rate path $\{i_t\}$, the response function

$$\pi_t = \kappa\sigma \int_{\tau=0}^{t} e^{-\sigma\kappa\tau} i_{t-\tau} d\tau + e^{-\sigma\kappa t}\pi_0 \tag{49}$$

gives us a family of inflation paths indexed by π_0. Only one of these paths satisfies the valuation equation (34), which is in this case

$$0 = \int_{t=0}^{\infty} e^{-rt}[\tilde{s}_t - (i_t - \pi_t)]dt. \tag{50}$$

In the simple case that the interest rate rises at time 0 from 0 to a new value i, then (49) reduces to

$$\pi_t = (1 - e^{-\sigma\kappa t})i + e^{-\sigma\kappa t}\pi_0. \tag{51}$$

Plugging this into the valuation equation (50) with $\tilde{s}_t = 0$ to determine π_0,

$$0 = \frac{i}{r} - \int_{t=0}^{\infty} e^{-rt}[(1 - e^{-\sigma\kappa t})i + e^{-\sigma\kappa t}\pi_0]dt \tag{52}$$

$$\pi_0 = i. \tag{53}$$

Despite sticky prices, we pick the equilibrium in which inflation moves instantly to match the interest rate.

Appealing to the new-Keynesian model will not help. This *is* the new-Keynesian model. Solving the model in new-Keynesian style, the central bank can produce any value of first-period inflation $\pi_1 = \pi_1^*$ it wishes, by following an interest rate policy $i_1 = i_1^* + \phi(\pi_1 - \pi_1^*)$, where $i_1^* = 1$, the first point on the desired interest rate path, and π_1^* is the desired, possibly negative, first-period inflation. Inflation then recovers and eventually rises following the interest rate.

But any path other than the one we have already plotted requires a change in surplus, and lower inflation requires positive surpluses. If we phrase the question of a new-Keynesian ($\phi > 1$, passive fiscal policy) model, “What is the response of inflation to a monetary policy shock that produces an unexpected permanent rise in the interest rate, and the associated passive fiscal policy requires no change in surpluses?” we have just calculated the unique answer.

What of the standard intuition that a higher real rate lowers output, and lower output should lower inflation via the Phillips curve? Again, don’t confuse the force that may lower the current price level and unexpected current inflation, from a change in the expected future price level and expected future inflation. Indeed, a higher nominal rate means a higher real rate in the IS curve, pushing output down. Lower output x_t in the Phillips curve,

$$\pi_t = \pi_t^e + \kappa x_t$$

means lower inflation π_t *relative* to expected inflation. With adaptive expectations, $\pi_t^e = \pi_{t-1}$, lower inflation π_t means that inflation is also *decreasing* over time. With rational expectations, $\pi_t^e = E_t \pi_{t+1}$, lower inflation means that inflation is *increasing* over time. Don't confuse inflation that is "lower" than it might otherwise be with inflation that is decreasing over time. And, as with the IS intuition, lower inflation relative to future inflation does not necessarily mean lower inflation, as future inflation may rise so much that both current and future inflation are higher than otherwise, as happens in this case. The outcome depends both on intertemporal and level effects, so just getting the intertemporal intuition right does not tell you the answer.

4.2 A negative effect from a transitory interest rate?

The essential failure of the rational expectations sticky price model to produce a negative inflation effect is not tied to the permanent interest rate increase shown in Figure 2. However, transitory interest rate paths can give a misleading appearance of such an effect. This is also important to check as experience with the standard new-Keynesian model and AR(1) shocks has led to the impression that permanent shocks raise inflation, but transitory shocks lower inflation. That conclusion is an artifact of the restriction to AR(1) shocks. There is no fundamental relationship between the persistence of monetary policy shocks and a negative inflation response in the standard new-Keynesian model. (Explicit calculations on this point in Cochrane (2023) Section 17.3.1).

To illustrate this point, Figure 3 plots the response of the simplified rational expectations model to a transitory interest rate movement, with $i_t = 0.7 i_{t-1} + \varepsilon_t$. Inflation rises uniformly. The only difference is that one doesn't really notice long-run neutrality with a transitory shock. The standard new-Keynesian specification does produce a negative inflation response to this shock. But it does so by supposing a contemporaneous fiscal tightening. The plot shows the unique equilibrium path that has no change in fiscal surpluses.

At the cost of some algebra, relegated to the Appendix, we can find its response of the rational-expectations sticky-price model (43)-(44) to an arbitrary interest rate path $\{i_t\}$, with

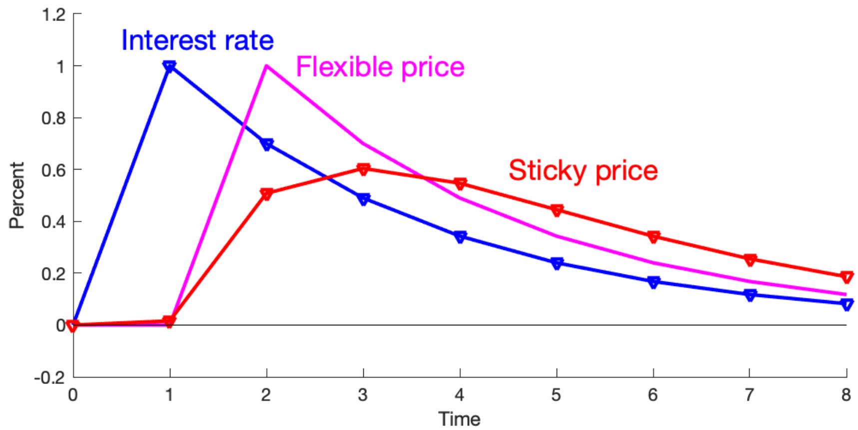

Figure 3: Response of the simple rational expectations sticky price model to a transitory interest rate path, with no change in fiscal policy. Parameters $\sigma\kappa = 1$, $\rho = 0.99$, and $i_t = 0.7 i_{t-1} + \varepsilon_t$.

no change in fiscal surplus:

$$\pi_{t+1} = \frac{1-\rho}{(1+\sigma\kappa)^{t+1}} \sum_{j=1}^{\infty} \rho^j i_j + \frac{\sigma\kappa}{1+\sigma\kappa} \sum_{j=1}^{t} \frac{1}{(1+\sigma\kappa)^{t-j}} i_j \tag{54}$$

and in particular

$$\pi_1 = \frac{1-\rho}{1+\sigma\kappa} \sum_{j=1}^{\infty} \rho^j i_j. \tag{55}$$

The sum in the first term of equation (54) is common to inflation at all dates, and decays at a rate determined by the price-stickiness parameters $\sigma\kappa$. It captures the present value of all future interest costs. The second term is a backward-looking moving average. It captures the smoothed version of $i_t = E_t \pi_{t+1}$.

In (54), all the coefficients are positive. Hence, *any sequence of positive interest rates* $\{i_t\}$ *generates uniformly positive inflation response* $\{\pi_t\}$. In this sense, the positive response of the rational expectations sticky price model is general and does not depend on the time-series process of the interest rates.

We can generate a negative apparent negative effect of interest rates on inflation, however. If interest rates were to rise in the short run, and then plunge to negative values, we could have a few positive interest rates i_t, despite a negative value of $\sum_{j=1}^{\infty} \rho^j i_j$ which drives a negative overall inflation response.

Figure 4 presents an example. As in all figures, this is the response of inflation to the indi-

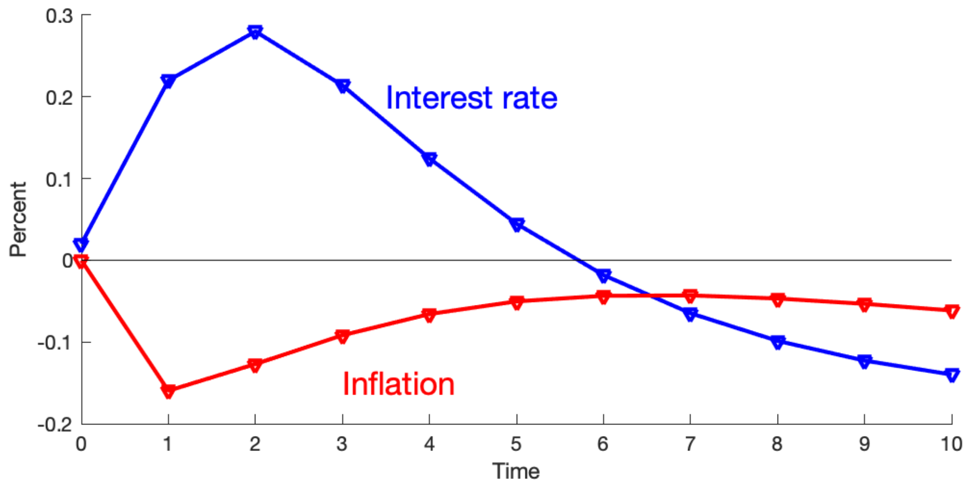

Figure 4: Inflation response to a transitory rise in the interest rate. Parameters $\sigma\kappa = 0.1$.

cated interest rate path, with no change in fiscal surpluses. The interest rate path is arbitrary. I created a path with a hump shape rather than an AR(1) shape just to make the graph prettier, and to resemble paths often seen in VARs. I specify $i_t = 0.7^{(t-1)} - 0.6 \times 0.4^{(t-1)} - 0.2$. The long-run interest rate response is thus -0.2. I then calculate the inflation path from (54). For an AR(1) $i_t = \eta i_{t-1} + \varepsilon_{i,t}$, (54) becomes

$$\pi_{t+1} = \left\{ \left(\frac{1-\rho}{1+\sigma\kappa} \frac{\rho}{1-\rho\eta} \right) \frac{1}{(1+\sigma\kappa)^t} + \frac{\sigma\kappa}{1-\eta(1+\sigma\kappa)} \left[\frac{1}{(1+\sigma\kappa)^t} - \eta^t \right] \right\} i_1$$

Then, such solutions add for the three AR(1)s that generate the interest rate path. This is also how I compute Figure 3.

Figure 4 looks initially appealing. A higher interest rate lowers inflation! In the very long run, of course, the interest rate declines. Both interest rate and inflation end up at -0.2%. But even that is not so unrealistic. We don't often see the long run; and when we do, both interest rates and inflation decline after a successful stabilization such as the 1980s. The plot looks superficially like the standard adaptive expectations model of Figure 5 below. The long period with interest rate slightly below inflation on the right hand side would be easy to miss or not to notice.

But this reading is profoundly misleading. Inflation declines because interest rates decline in the far future, *despite*, not because of, the short-term rise in rates. The positive interest rates drag inflation up from even more negative values. If you want less inflation in this model, lowering interest rates immediately—the negative of Figure 2—is an even more powerful tool. There is absolutely nothing in the mechanics of this model that resembles standard intuition, high real interest rates driving inflation down. Beware causal readings of impulse-response functions!

If we want a negative effect of interest rates on inflation, without a contemporaneous fiscal shock that's really doing the work, then, we will need to add frictions beyond sticky prices.

4.3 Fiscal requirements for the adaptive expectations model

The adaptive expectations model (4),

$$\pi_t = (1 + \sigma\kappa)\pi_{t-1} - \sigma\kappa i_t.$$

produces standard intuition. In response to permanently higher interest rates, inflation declines in a classic downward exploding spiral.

This is not the answer we are looking for, however. Of course, it violates the larger quest in that it places model-inconsistent expectations as an irreducible necessary ingredient for inflation determination and the negative sign. But more importantly, this response violates the two central rules of our quest. First, to produce a short-run negative effect, it induces a long-run negative effect. To produce a short-run non-neutrality, it abandons long-run neutrality. It fundamentally overturns the stability and determinacy properties of the frictionless economy. Our quest is for a model of short-run non-neutrality that respects long-run neutrality.

Second, and more importantly, this response also requires fiscal support. Debt still accumulates by

$$\rho v_{t+1} = v_t + i_t - \pi_{t+1} - \tilde{s}_{t+1}.$$

If inflation spirals off downward, real interest costs spiral upward. If there is no change in surpluses, then debt spirals off upward as well. If we phrase the question of the adaptive expectations model, "What is the response of inflation to a permanent unexpected rise in the interest rate, *with no change in fiscal surpluses*," the downward spiral does not answer that question. Since the model is determinate, there is no answer to that question—the central bank cannot permanently raise interest rates without fiscal support.

In reality, then, as in theory, the Fed must give in and drop the nominal interest rate to stop the spiral. Friedman (1968) recognized this fact, and described a central bank giving in out of distaste for exploding inflation. Here, it is forced to give in by exploding deficits.

The most standard model that produces a disinflation from a higher interest rate combines adaptive expectations with a Taylor rule. However, this model also requires tighter fiscal policy to pay higher interest costs on the debt. Without such support, the disinflation does not occur.

It is easiest to exhibit this behavior in the continuous time version of the model, which avoids timing issues of inflation. Repeating for clarity, model dynamics are

$$d\pi_t/dt = \sigma\kappa(i_t - \pi_t) \tag{56}$$

$$dv_t/dt = rv_t + i_t - \pi_t \tag{57}$$

$$i_t = \phi\pi_t + u_t \tag{58}$$

At time 0, the monetary policy shock u_t rises suddenly and expectedly from 0 and stays at the constant value $u_t = u_0$. Inflation dynamics become

$$\frac{d\pi_t}{dt} = -\sigma\kappa(\phi - 1)\pi_t - \sigma\kappa u_0.$$

The solution is

$$\begin{aligned}
\pi_t &= -\frac{1}{\phi - 1}\left(1 - e^{-\sigma\kappa(\phi-1)t}\right)u_0 \\
i_t &= -\frac{1}{\phi - 1}\left(1 - \phi e^{-\sigma\kappa(\phi-1)t}\right)u_0 \\
r_t &= e^{-\sigma\kappa(\phi-1)t}u_0 \\
e^{-rt}v_t &= \frac{e^{-[r+\sigma\kappa(\phi-1)]t} - 1}{r + \sigma\kappa(\phi - 1)}.
\end{aligned}$$

Figure 5 presents this result. The nominal and hence real interest rate rises, and inflation starts on its downward spiral. But, following the Taylor rule, the interest rate swiftly follows inflation down, and we stabilize at a new lower inflation rate. This is a standard story of the 1980s, for example.

However, the real interest rate is positive throughout the episode. Thus, with no greater surpluses, the greater interest costs on the debt are simply rolled over, and debt increases without bound. The transversality condition $\lim_{T\to\infty} r^T v_T = 0$ is violated. Even granting adaptive expectations, this simulation does not answer the question, what can the central bank do by itself, without fiscal support. If this is the story of the 1980s, the story is a joint monetary-fiscal stabilization, with surpluses rising (as they did) to pay the higher interest costs of the debt, not a story

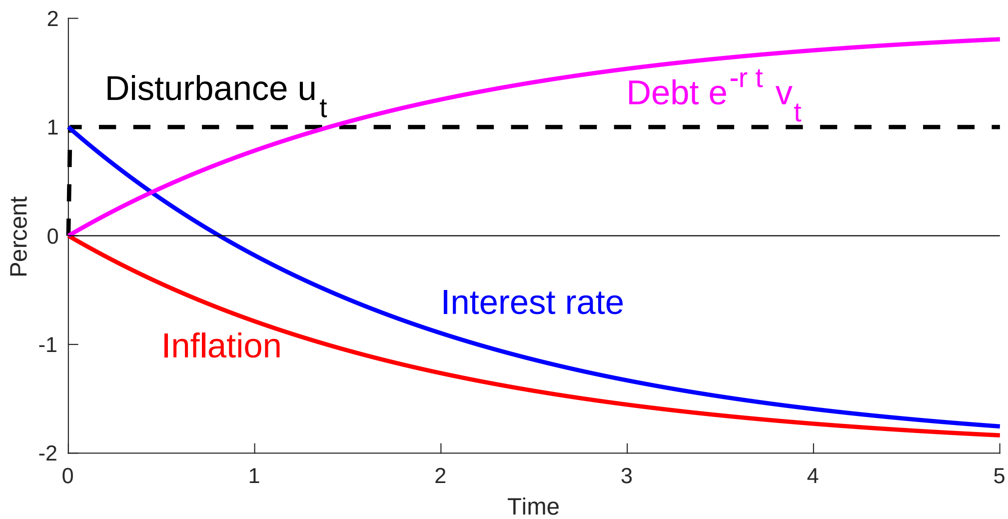

Figure 5: Response to a 1% permanent monetary policy shock in the adaptive expectations model with a Taylor rule. Parameters $\sigma\kappa = 1$, $\phi = 1.5$, $\rho = 0.01$.

of monetary policy acting alone.

4.4 Adaptive expectations with a fiscal constraint

In response to our question, the effect of interest rates on inflation *holding fiscal policy constant*, the adaptive expectations model also has great trouble to produce a permanent inflation change, and to the extent it can do so, the result is far from realistic.

This point is again easier to see (i.e. with much less algebra) in the continuous time version of the model, because the timing of interest rate relative to inflation collapses to $i_t = \pi_t$. Denoting $r_t \equiv i_t - \pi_t$, (56)-(57) and the transversality condition are

$$d\pi_t/dt = -\sigma\kappa r_t \tag{59}$$

$$dv_t/dt = rv_t + r_t \tag{60}$$

$$\lim_{T\to\infty} E_t e^{-rT} v_T = 0. \tag{61}$$

The symbol r without subscript still represents the steady state real interest rate, and point of linearization. r_t represents variation of the real interest rate around this steady state value. Normally the surplus $-s_t$ appears on the right hand side of (60) but I omit it as the exercise holds fiscal policy constant.

Given a real interest rate path, the solution to (59)-(61) is

$$\pi_t = -\sigma\kappa \int_0^t r_j dj \tag{62}$$

$$e^{-rt} v_t = \int_0^t e^{-rj} r_j dj. \tag{63}$$

Define the long-run inflation rate π,

$$\pi = -\sigma\kappa \int_0^\infty r_j dj. \tag{64}$$

Imposing the transversality condition (61) with no change in fiscal policy, we have a constraint on the real rate path $\{r_j\}$,

$$0 = \int_0^\infty e^{-rj} r_j dj. \tag{65}$$

Once we pick the real rate and solve for inflation from (62), we can find the nominal rate from $i_t = r_t + \pi_t$. (One can also express the $\{\pi_t,\ v_t\}$ solution directly in terms of the nominal interest rate, but the resulting expressions are not so simple and transparent.)

We see right away that *in the limit* $r \to 0$*, the adaptive expectations model cannot produce any permanent disinflation at all—a value of* π *other than zero—in the absence of a change in fiscal policy.* The right hand sides of (64) and (65) are the same. Intuitively, the discounted real rate must be zero so that the present value of interest costs is zero. The sum of all negative real rates that push inflation up, plus positive real rates that push inflation down adds up to the terminal inflation. Without discounting, the former must equal the latter.

The more realistic $r > 0$ offers an apparent avenue for permanent disinflation. But, since r is small, this result is fragile, and the resulting policies are unrealistic. The present value in (65) downweights real rates in the far future. So, to lower inflation, we must *lower* interest rates in the short run, building up inflation, producing a period of low interest costs, and lowering the debt. Then we turn around and raise real interest rates driving inflation down, using the accumulated savings on the debt to pay the higher interest costs. The unweighted integral in (64) allows us to have a longer period of future high interest rates than the initial period of low interest rates, and overall to drive inflation down relative to its initial value.

But with small r, the opportunity requires large swings in rates and inflation to produce a small permanent reduction in inflation. And the period of low rates and high inflation must come first, before the final period of high rates and lower inflation. Imagine the Fed announcing, "We are going to lower inflation, without asking Congress to tighten fiscal policy. Here's how we'll do it. First, we are going to lower interest rates, and deliberately raise inflation. That move will inflate away enough of the debt, that we can turn around and raise rates to undo all that inflation, and more. The higher interest costs on the debt during that disinflation will just return us to today's debt to GDP ratio. Hang on, here we go."

Since I do not wish to pursue adaptive expectations in this baseline model, I do not develop the possibility further. The point of this section: When we pose the question as, "Can higher interest rates lower inflation, *without a change in fiscal policy*," not even the most classic adaptive expectations model, which fits the narratives of central bankers and the policy world, can easily do it.

4.5 The standard model with lagged inflation in the Phillips curve

Here I verify that with no change in fiscal policy, a higher interest rate does not raise inflation, even in the standard new-Keynesian model, and even extending that model to include lagged inflation in the Phillips curve. The model is

$$x_t = E_t x_{t+1} - \sigma \left(i_t - E_t \pi_{t+1} \right) \tag{66}$$

$$\pi_t = (1-\alpha) E_t \pi_{t+1} + \alpha \pi_{t-1} + \kappa x_t \tag{67}$$

$$\rho v_{t+1} = v_t + i_t - \pi_{t+1} \tag{68}$$

$$i_{t+1} = \eta i_t + \varepsilon_{i,t+1} \tag{69}$$

Equation (66) includes the forward-looking term. Equation (67) allows for lagged inflation. Perhaps we can get some of the adaptive expectations dynamics? Alas no, as we shall see. Equation (68) specifies short-term debt and zero surplus, as I only calculate the response to a monetary policy shock.

Figure 6 presents the response of this model to a permanent interest rate rise. I solve the model numerically. The shaded area gives all possible impulse responses, calculated by evaluating the response function for a grid of parameter values, including all values of $\alpha \in [0,1]$. I restrict parameters to those that produce real eigenvalues, however. Sawtooth or sine-wave responses induced by complex eigenvalues enlarge the possibilities and are sometimes negative, but clearly unrealistic. As the figure shows, for all parameter values, this generalized model pro-

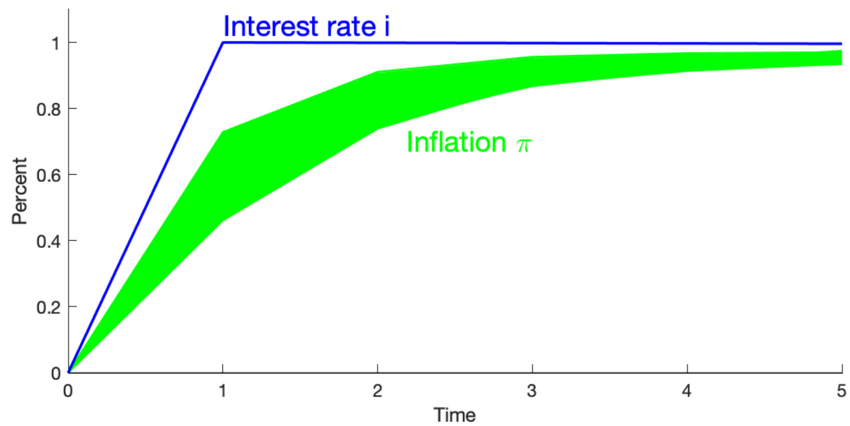

Figure 6: Inflation response to a permanent interest rate rise, with no change in fiscal policy. Full model with forward-looking IS curve and a lag in the Phillips curve. The shaded area shows impulse response functions with all parameters $\alpha,\ \sigma\ \kappa$ that produce real eigenvalues.

duces a steady rise in inflation. The static IS curve did capture the results of this more complex model.

4.6 An imperfect model that produces a negative effect

Figure 7 offers a simulation of the only current model I know of in which higher interest rates produce a negative short-run inflation effect, in a rational expectations model that respects long-run neutrality, and without fiscal help.

The model is

$$x_t = E_t x_{t+1} - 0.5(i_t - E_t \pi_{t+1}) \tag{70}$$

$$\pi_t = E_t \pi_{t+1} + 0.5 x_t \tag{71}$$

$$i_t = i_{t-1} + \varepsilon_{i,t} \tag{72}$$

$$\rho v_{t+1} = v_t + r^n_{t+1} - \pi_{t+1} - \tilde{s}_{t+1} \tag{73}$$

$$E_t r^n_{t+1} = i_t \tag{74}$$

$$r^n_{t+1} = 0.9 q_{t+1} - q_t \tag{75}$$

Here I use the full model, i.e. including the $E_t x_{t+1}$ term in (70), as analytic solutions are not insightful. I include long-term debt with a geometric maturity structure. The face value of zero coupon bonds of maturity j, $B_t^{(j)} = \omega^j B_t$ declines at rate $\omega = 0.9$. The symbol r^n_{t+1} represents

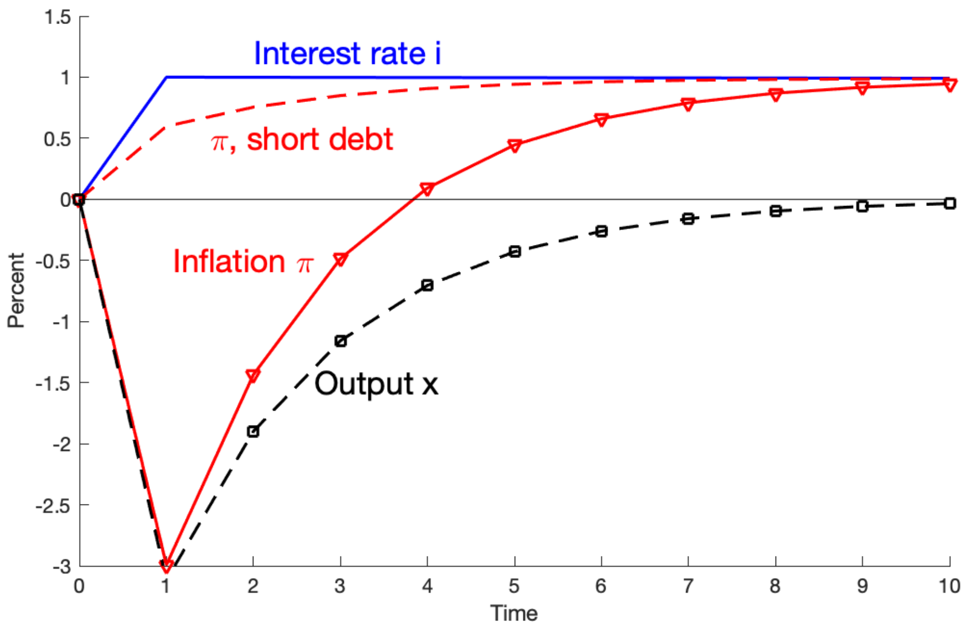

Figure 7: Response of inflation to an interest rate shock, and no change in fiscal policy, with long term debt. In the base case, debt has a geometric maturity structure, decaying at rate 0.9^t with $t =$ maturity. The short-term debt case is one-period debt only.

the ex-post nominal return on the portfolio of government debt. Equation (74) prices long-term bonds with the expectations hypothesis. Equation (75) links the log price q_t of the government bond portfolio to its rate of return. This is a simplified version of the model in Cochrane (2021).

Again, I raise interest rates, but I leave fiscal surpluses unchanged and calculate the inflation and output response. Inflation declines temporarily! Inflation then rises in the long run, fulfilling long-run neutrality. This is the pattern we have been seeking.

Long-term debt is the crucial innovation relative to the simple models of previous figures, and its inclusion produces the negative sign. With long-term debt, but no ability to change surpluses, the central bank can lower inflation now, but by raising inflation later. Raising interest rates, and thus inflation in the long run, devalues long-run debt. Since surpluses haven't gone *down* ether, that action raises the value of short-term debt. But short-term debt can only become more valuable via a lower price level.

Sims (2011) calls the pattern of Figure 7 "stepping on a rake" and offeres it as a parable of the 1970s inflation cycles, in which higher rates temporarily lowered inflation, but inflation came back larger. The pattern also represents a form of unpleasant interest-rate arithmetic, a successor to Sargent and Wallace (1981) unpleasant monetarist arithmetic. (Sargent and Wallace focus on seignorage in a model with money and real debt. In this model, there is no seignorage or money. Instead, inflation devalues nominal debt, and higher real interest payments on the debt can also cause inflation.) Unpleasant interest-rate arithmetic is here a negative sum, or inequality proposition: The Fed gets more long-run inflation than it saves in short-run inflation.

A Taylor-type rule, in which the interest rate reacts to inflation, adds this sort of response to the inflationary effect of fiscal or other shocks. In this way, a Taylor rule spreads the inflation of a such shocks forward, reducing their immediate impact. With the forward-looking Phillips curve, random walk inflation has no output effect, so by smoothing inflation forward the Taylor rule reduces output volatility. In this model, the Taylor-rule coefficient must be slightly less than one, however. Taylor emphasizes that his rule works well in a variety of models, and that robustness rather than strict optimality in a particular model is its virtue. It eliminates instability of the adaptive expectations model; it eliminates indeterminacy of the rational expectations model; and it reduces volatility in the rational-expectations fiscal-theory model.

As much as I would like to trumpet this model as the successor to Lucas (1972a) for interest-rate based inflation economics, however, its limitations lead me to argue that there is more work to do.

This negative effect only holds for *unexpected* interest rate rises. Lower inflation breaks out when a higher interest rate is announced, not when it happens. Thus, we have a lovely continuation of our list by which interest rates inherit many Lucas (1972a) properties of money growth. For fitting the data, however, this may be a limitation. Maybe expected interest rate rises can also lower inflation when the interest rates actually rise, not just when they are announced. If so, we need a different model.

Since the negative effect depends on long-term debt, the effect vanishes when governments borrow short term, as illustrated by Figure 7 in the line marked "π, short debt." More generally, the size of the negative effect depends on the maturity structure of debt. US government debt is relatively short-term, and has been shorter in the past. Whether debt maturity is long enough to produce quantitatively important effects, and whether the effect of interest rates on inflation varies with the maturity structure of the debt as the model predicts are not obvious.

The negative effect requires long-lasting interest rate increases, that raise long-term nominal interest rates. It is not obvious that higher interest rates lower inflation more when they are persistent, and when they propagate to the long-term yield curve.

Price stickiness *reduces* the strength of the effect. With sticky prices, the higher real interest rates add interest costs of the debt, an inflationary force. The negative effect is strongest in the flexible price case.

This response function also lowers inflation immediately, where the common intuition we would like to see if the model can produce lowers inflation gradually. Sims (2011) produces a hump-shaped output response by adding habit persistence in consumption, but still does not produce such an inflation response. One imagines that further frictions, perhaps sticky inflation, can produce such a response, but that also needs to be checked.

These features are not necessarily counterfactual. They are just unknown. This model is new. Nobody has looked to see if the negative effect of interest rates, orthogonalized to fiscal policy, on inflation is quantitatively linked to announcement, maturity, persistence, and price stickiness as the model predicts. Looking would be a valuable empirical project.

This mechanism also offers a novel intuition for the negative effect of interest rates on inflation. It has nothing to do with higher real interest rates that depress demand that works through a Phillips curve to lower inflation. It is a wealth effect; by exploiting the long-run Fisher effect, i.e. by raising interest rates to inflate away long-term debt, it makes short-term debt more valuable. Perhaps that is so, but clearly, this model does not give an always and everywhere, mechanical connection between higher rates and lower inflation, Lucas holy water sprinkled on IS-LM thinking. It does not produce something like the adaptive-expectations dynamics in the short run, which then turn around and become stable when some suitable friction or information problem is resolved. And it will be a long time before we write opeds, Fed chairs explain, and we teach to undergraduates that the central mechanism by which the Fed can temporarily lower inflation is to rearrange the real payoffs to different maturities of nominal government debt.

5 Estimates

The empirical estimates we have, when they indicate that higher interest rates reduce inflation at all, show no immediate effect, and then a slow downward drift of the price level.

Figure 8 presents two estimates of the effect of higher interest rates that have the desired sign, from Valerie Ramey's (2016) comprehensive review. (The plots show the *price level* not the inflation rate.) The top estimate implements the classic Christiano, Eichenbaum, and Evans

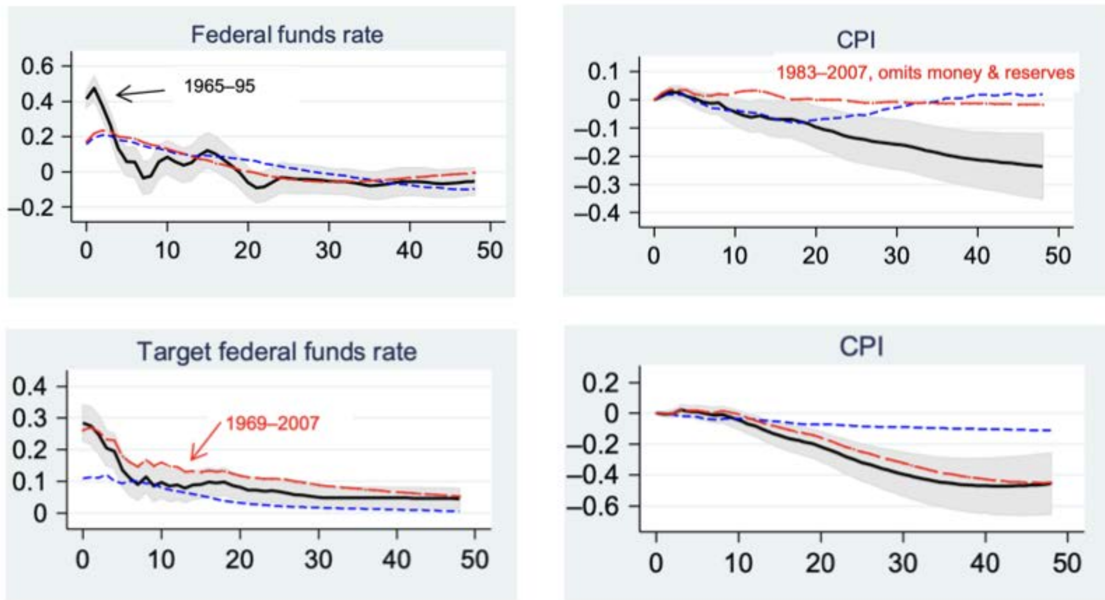

Figure 8: Two estimates of the effect of monetary policy shocks. Top: Christiano et al. (1999) identification. 1965m1–1995m6 full specification: solid black lines; 1983m1–2007m12 full specification: short dashed blue lines; 1983m1–2007m12, omits money and reserves: long-dashed red lines. Light gray bands are 90% confidence bands. Bottom: Romer and Romer monetary shock. Coibion VAR 1969m3–1996m12: solid black lines; 1983m1–2007m12: short dashed blue lines; 1969m3–2007m12: long- dashed red lines. Source: Ramey (2016).

(1999) VAR. The lower estimate is based on the Romer and Romer (2004) narrative identification.

Here and more generally monetary VARs find that higher nominal interest rates raise real interest rates and reduce *output*, but they have slow, small, and uncertain effects on *inflation*. Given that the US currently hopes that higher interest rates will swiftly reduce the current inflation, these plots are sobering.

On the other hand, one may feel that the VARs completely miss the important effect of an intervention such as 1980. The VARs isolate transitory idiosyncratic movements in the federal funds rate, not long-lasting movements that we saw in 1980, or that the last model requires. Most of all, by design, they find idiosyncratic deviations from a rule, not changes in rule or "regime" that may durably change expectations. If the art of reducing inflation is to convince people that something has changed so they should lower inflation expectations, then the response to a monetary policy "shock" orthogonal to a stable "rule" completely misses the successful policy.

The standard new-Keynesian model also predicts that a permanent rise in interest rates raise inflation both in the long run and the short run. Uribe (2022) evaluates this "neo-Fisher" hypothesis, finding that a permanent monetary shock, identified as one that increases both the nominal interest rate and inflation in the long run, raises both inflation and interest rates in the short run. Similarly, Schmitt-Grohé and Uribe (2022) find that such permanent interest rate shocks *depreciate* the currency. In both cases, transitory interest rate movements lead to the standard disinflation and appreciation.

However, no current VAR including these attempts to find monetary policy shocks orthogonal to fiscal policy, so we must read VAR evidence with that additional grain of salt relative to the conceptual experiment we wish to learn about. Empirically evaluating the model of Figure 7, checking if the inflation response varies with debt maturity, shock persistence, and anticipation as it should, remains low-hanging fruit.

6 Paths to follow

So, despite 50 years of modern intertemporal general equilibrium macroeconomics since Lucas (1972a), we still don't have a solid well-agreed on theoretical or empirical answer to the basic questions: Can higher interest rates temporarily lower inflation (on their own, without concurrent changes in fiscal policy)? If higher interest rates can temporarily lower inflation, by what economic mechanism do they do so?

If you want to return to adaptive expectations, or add complex learning and expectation formation schemes (Gabaix (2020), García-Schmidt and Woodford (2019), Bianchi-Vimercati, Eichenbaum, and Guerreiro (2022)), then you have to face the failures of the adaptive expectations approach during the zero bound era—as well as its failures in stagflation, in the relatively rapid end of inflation in 1982, in the success of inflation targets and ends of hyperinflations in which inflation fell with no monetary stringency or output consequence, and, as I write in late 2022, in the failure of inflation to spiral upwards despite interest rates well below inflation.

Yes, expectations might sometimes seem adaptive. But as Bob also taught us in Lucas (1973), apparently-adaptive expectations are ephemeral too. The Calvo fairy visits every day in Argentina, and every hour in Venezuela. Moreover, rational expectations produces adaptive-looking rules, because expectations and decision rules must react to current observables. The point of rational expectations is that rules, say relating expected inflation to the history of inflation, change when policy rules change, not that such rules don't describe observed expectations or hold in sample. (Lucas (1976) "critique" of course, but also Sargent (1971) and Lucas (1972b).)

Testing rational vs. adaptive expectations is hard.

That observation is important in the current policy debate. The proposition that interest rates must be higher than current inflation to lower inflation assumes that expected inflation equals current inflation. Market and survey expectations are much lower than current inflation. Perhaps that means that markets and surveys have rational expectations: Output is temporarily high so inflation is higher than expected future inflation ($\pi_t = E_t\pi_{t+1} + \kappa x_t$.) But it is also possible that current inflation expectations are a long, slow moving average of past inflation, just as Friedman speculated in 1968. Then expected inflation is much lower than current inflation, and interest rates only need to be higher than that low expectation to reduce inflation in an adaptive expectations model. You can't just look at expectations and proclaim them rational or not.

More deeply, returning to adaptive expectations produces the desired sign by changing the stability and determinacy properties of the flexible-price rational economy. Stability and determinacy are basic and robust dynamic properties. To change stability and determinacy, you have to move an eigenvalue across one. Small changes in model structure or parameterization do not work, as they change eigenvalues by small amounts. (One needs an unusual model for the eigenvalue to change discontinuously with a parameter.) That means that this approach needs to make a large deviation from rationality, and rational or flexible price limits eventually cross back across one to ruin the result. (See Cochrane (2016) for a concrete example.)

At a minimum, founding the most basic prediction of monetary economics on the idea that people are permanently, exploitably, immutably and substantially irrational, that the expectations of the model differ from the expectations in the model, to the point that reverses the stability and determinacy of the underlying frictionless economy, makes monetary economics ephemeral. In the absence of $MV = PY$, explaining how monetary policy works to the public, central bankers, and undergraduates will require that we say it only works because people are predictably and immutably dumb. If they only woke up, the Fed would be powerless. (Just why Fed economists should be so much more rational than everyone else to correctly exploit irrationality is another problem with this view.) Perhaps that is so, but then we should be more upfront about it.

A model of temporary, limited or contingently adaptive expectations, that then turn around to become rational in a suitable long run is a possibility. Some verbal commentary distinguishes times that people are paying attention to national attention and quieter times when they mind their own business. Following Lucas, however, we need to describe the transition between the neutral and the non-neutral behavior. I am not aware of such a model.

You may wish to put money back in the model. Raising interest rates means printing less money M, which lowers nominal income PY and eventually the price level P. (Alvarez, Atkeson, and Edmond (2009) is a good example.) But it's not so easy as a matter of theory, and as in the first paragraph of this essay, money supply control simply does not describe our world. Adding liquidity effects in government bonds or other financial assets to the model is an attractive generalization, but the supply of such liquidity needs to be constrained just like money for this avenue to produce anything interesting.

You might say that the Fed *should* go back to controlling the money supply, and start cracking down on inside liquid money substitutes. But we need some advice for central banks in the meantime, and at least we should understand how our current system based on interest rate targets works, or doesn't. From 1982 to February 2021 it looked like a pretty good system! Inflation is *something* while banks control interest rates and provide unlimited liquidity, and we need a theory of what that something is.

One is drawn to add model ingredients. Surely in the DSGE smorgasbord there are enough ingredients to come up with a temporary negative sign. That is, I think, exactly the right answer. My point is, it has not yet been done–and especially, it has not been done with the kind of clarity, simplicity, economic rigor, transparency and tractability that Bob brought to the non-neutrality of money. Many model-implied monetary-policy response functions have been computed of course, but not many yet hold fiscal policy constant in an interesting way, and few look into the footnote about lump-sum taxes to see just what those are and to what extent inflation reduction comes from an implicit fiscal contraction. (The literature that puts fiscal theory in explicit DSGE models is an exception; see for example most recently Bianchi and with Leonardo Melosi (2022) and Chen, Leeper, and Leith (2021), Leeper (2021). This literature is explicit about fiscal-monetary coordination. However, it has focused on switching between active-fiscal and active-money regimes, and so far has not addressed the question in this paper, whether higher interest rates can lower inflation with no change in fiscal policy.)

One easily jumps to capital with adjustment costs, financial frictions, credit constraints, portfolio adjustment costs, liquidity effects, additional price and wage-setting frictions, strategic complementarities, individual heterogeneity, or other model complications. But the negative response of inflation to interest rates should be a robust and deeply rooted phenomenon, one that will not vanish if, for example, the US changes the downpayment rules on mortgages. In his

Nobel Lecture, Lucas (1996), Bob cites David Hume for understanding the neutrality and non-neutrality of money in 1752. Velde (2009) documents a beautiful non-neutrality episode in 1724 France, with a monetary and financial system utterly unlike our own.

It is likely to be possible to find *sufficient* conditions to deliver the negative sign, with enough model complications. Our goal though is the minimum *necessary* conditions, that apply most broadly and robustly.

Again, Lucas (1972a) is a great example. In his economy, the flexible price version leads to super-neutrality: An increase in money just raises the price level. Bob put in *one* "friction," imperfect information about aggregates, leading to a confusion between relative and aggregate price movements. But it is limited: if the information problem is absent, the non-neutrality goes away.

This train of thought brings us back to the Phillips curve. In my little models, the Phillips curve is the central source of inflation dynamics. Yet the Phillips curve has not achieved great theoretical and empirical clarity, despite decades of dedicated work by top macroeconomists. It may make sense that firms sell more when output prices are high, or that worker work harder when wages are high. But these are relative prices, where the Phillips curve states that output and employment increase when all prices and wages rise together. So, any Phillips curve needs some confusion or correlation of relative prices with the overall price level.

In addition to wondering what ingredients to put in, then, perhaps this is one we should take out. Perhaps we can start to study the dynamic relationship between inflation and nominal interest rates apart from the Phillips curve.

Our goal is to understand $\pi_t = a(L)i_t$, the dynamic relationship between interest rates and inflation. The Phillips curve came from thinking about output and employment *effects* of inflation. That's what Lucas (1972a) was all about. Lucas had a perfectly good theory of inflation, $MV = PY$, but wanted a theory how inflation affects output. We are reversing the logic, using the IS equation to describe how interest rates lower output, and then the Phillips curve to describe how output affects inflation. (With usual caveats for causal readings of equilibrium conditions.) The Phillips curve wasn't designed to be the central mechanism for nominal dynamics. We will of course still want to understand how inflation affects output and employment, and surely that understanding will feed back on inflation dynamics, but in the spirit of adding ingredients and frictions one at a time, perhaps the price dynamics should come before the Phillips curve.

For example, in 2021-2022 most commentary in and around central banks centered on "supply chain" shocks and relative price movements, which particular goods or sectors were going up or down, as both underlying cause and key variables for the dynamics of inflation. A good example is Lane (2022). In this view, large good or sector specific supply shocks or demand shocks move relative prices; interacted with prices that are more sticky downward than upward, they and not the immense fiscal expansion, account for inflation. More importantly, these relative prices – average vs. marginal rents, house prices vs. rents, etc.—are key state variables that analysts look at for forecasting future inflation. The old Phillips curve, with a single output gap or unemployment capturing the entire effect of the real economy on inflation, is pushed to the background. This extensive commentary and forecasting is only beginning to enter academic modeling. Related, there is new interest in describing inflation dynamics in production networks, for example Minton and Wheaton (2022) and Rubbo (2022). Guerrieri et al. (2021) argue that some inflation is optimal when there are reallocation shocks and downward nominal stickiness. Perhaps reallocations, networks, supply and demand shocks interacted with sticky prices and wages, will completely take over from the IS and Phillips curve as our basic model of inflation dynamics.

There is, of course, another possibility: It might not be true. A persistent nominal interest rate rise, *with no change in fiscal policy*, may not lower inflation even in the short run. The VAR literature is tenuous despite enormous effort. The "price puzzle" that higher interest rates seem to raise inflation without delicate orthogonalization may have been trying to tell us something. Interest rate rises in the past that have seemed to lower inflation may have come with fiscal tightenings, or pro-growth fiscal and microeconomic policy that raise revenue. Fiscal authorities respond to the same economic and political situations that drive monetary authorities to tighten.

7 Conclusion

What is the dynamic effect of *interest rates* on inflation, $\pi_t = a(L)i_t$, in our world of abundant reserves, in which central banks set nominal interest rates, do not control money supplies, do not make equilibrium-selection threats, and cannot directly change fiscal policy? And, of course, after that, how do interest rates then affect output, employment, and other variables?

I have followed one line of thought on these questions to its logically inevitable conclusion:

Rational expectations and fiscal underpinnings of monetary policy imply that inflation is stable and determinate in the long run. That implies neutrality, that higher interest rates, without a change in fiscal policy, eventually raise inflation, and a k percent rule is possible. There may well be a short-run negative effect of interest rates on inflation. I show one suggestive model, but we need better models of that effect.

Thus, as I see it, we have made a lot of progress. We're finally at the launch pad, and we have some promising ideas, but we're still waiting for a new Lucas – and then, perhaps a new Sims on the empirical side – to finish the project.

If this path succeeds, however, we will be left with an understanding that central banks are a lot less powerful than we thought. First, fiscal policy remains a central determinant of inflation. When a fiscal shock occurs, when the government borrows or prints and spends and people do not expect the debt to be repaid, and absent explicit default, inflation must rise to devalue the debt, sooner or later. The central bank can choose when and how abruptly, but inflation is no longer always and everywhere just a monetary policy phenomenon. Second, the central bank's ability to lower inflation by higher interest rates, provoking a little bit of recession, remains contingent on the frictions of the model that produces a temporary negative effect, just as the central bank's ability to affect output by changing the money supply is contingent in Lucas (1972a). The central bank still fully controls the long-run price level, however, by its ability to drag expected inflation to wherever it sets the nominal interest rate. And the simple static story that higher rates lower demand which lowers inflation via a Phillips curve does not even vaguely describe these rational expectations models.

All this is controversial. Much of the point of this essay is to proclaim and explain the neutral benchmark, which is otherwise a bit implicit in the equations of new-Keynesian and fiscal theory models. Most academic literature still uses new-Keynesian equilibrium-selection threats, ignores fiscal requirements of monetary policy, and the long-run neutrality of even that model is not widely recognized. Most of the policy world uses a muddle with somewhat adaptive expectations, or expectations as an independent force. Thus, across economics today, basic questions are still up for grabs. In the long run, is inflation stable or unstable, determinate or indeterminate under a peg? If the Fed raises rates persistently, and there is no fiscal news or other shocks, does inflation rise or decline in the long run? If not this, what is the neutral, frictionless benchmark on which we build a theory of inflation under interest rate targets?

The short run non-neutral and disinflationary effects of higher interest rates have more consensus of opinion behind them, but even less well-accepted theory behind that opinion. If the Fed raises interest rates, does inflation temporarily decline? If so, by what mechanism, and under what preconditions? And even though most economists seem to believe in the sign of the effect, the all important magnitude is still contentious. Must the Fed raise interest rates by more than the current rate of inflation, following the Taylor Principle, in order to lower inflation at all? Or will the substantially lower interest rate rises the Fed has followed and envisions be sufficient for inflation to fade away, at least until the next big shock?

The question I have posed here—what is the effect of interest rates on inflation, with *no* change in fiscal surpluses?—is an important thought experiment for understanding monetary economics. But it is an unlikely scenario with which to understand history, and it is not the right question to ask if one wants to know the effects of policy. Monetary and fiscal policies change together in response to events, and fiscal policy responds to economic changes brought about by monetary policy. One can even debate the right interpretation of the desire to leave fiscal policy unchanged. For example, if the tax rate and automatic stabilizer laws are unchanged, then recessions will lead to deficits, which might further raise inflation in response to higher interest rates. For many purposes, this might be a better definition of unchanged fiscal policy. Cochrane (2021), like the above-cited fiscal theory literature, includes a model of fiscal policy with a rule, responding to output and inflation, and disturbances. One might define a monetary policy shock as one that leaves the fiscal rule unchanged but has no disturbance to that rule. Here, for the purpose of understanding what monetary policy does by itself and not by induced fiscal changes, and for the goal of utmost simplicity, I hold surpluses constant, but that is not always the right thing to do. Ask interesting questions and be clear what question you're asking.

How is it that we've been playing with interest-rate based models for at least 40 years, yet such basic questions are still unanswered? I think another important Bob Lucas lesson applies. I recall attending a seminar in which Bob was presenting an early draft of Lucas (2009). Bob had been working on it over a year. In response to question after question why Bob had not included some ingredient, he answered to the effect of "I tried that, but it didn't make an important difference," and then explained why. Bob does not so much build models as he sculpts them, removing unneccessary piece after unneccessary piece.

As I look at monetary models based on interest rate targets, I think we have been guilty of playing with too-complex models when we don't really understand basics, such as stability, determinacy, and the frictionless limit. But this is always the way in economics, as it is in the sciences. Ideas start complex and simplicity only emerges after much hard work.

This is all great news for young researchers. These are the good old days. Low-hanging fruit abounds. We're really at the beginning stages where simple models need exploration, not, as it appears, in a mature stage where essentials are settled and all there is to do is to add to the immense stock of complicated epicycles.

However, given the state of actual agreed-on knowledge, central banks' proclamations of detailed technocratic ability to manipulate delicate frictions is laughable. Figure 9 shows in chart form the Rube-Goldberg list of mechanisms the ECB thinks it understands and can manipulate. Central bankers who think they have any idea how all these boxes and arrows work, and how to manipulate them, should reread Bob's unsung classic "on a report to the OECD" Lucas (1979) once a week. A little humility would do us all good.

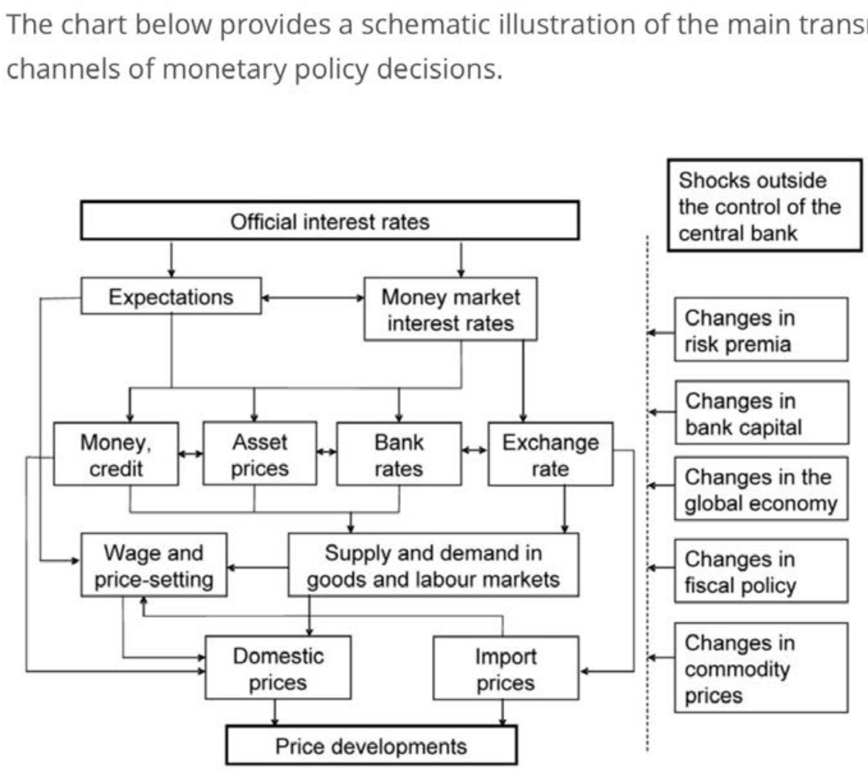

Figure 9: The ECB's view of monetary policy. Source: https://www.ecb.europa.eu/mopo/intro/transmission/html/index.en.html

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Online Appendix.

1 Solving the simple FTPL model analytically

The model is

$$x_t = -\sigma(i_t - E_t\pi_{t+1}) \tag{1}$$

$$\pi_t = E_t\pi_{t+1} + \kappa x_t \tag{2}$$

$$\rho v_{t+1} = v_t + i_t - \pi_{t+1} - \tilde{s}_{t+1} \tag{3}$$

$$\lim_{T\to\infty} \rho^T v_T = 0 \tag{4}$$

We want to calculate the impulse-response function for a generic path $\{i_t\}$. All variables are zero until time 1. At time 1 we set off a sequence $\{i_1, i_2, ...\}$. There is no change to surpluses, so $\tilde{s}_t = 0$. Given π_1, the other π_t follow since there is no more uncertainty. Equations (1)-(2) give us a set of possible paths of inflation indexed by π_1. We use (3) and (4) to choose π_1.

This section establishes the following results for this impulse-response function. For an arbitrary sequence $\{i_1, i_2, ...\}$,

$$\pi_{t+1} = \frac{1}{(1+\sigma\kappa)^{t+1}}(1-\rho)\sum_{j=1}^{\infty}\rho^j i_j + \frac{\sigma\kappa}{1+\sigma\kappa}\sum_{j=1}^{t}\frac{1}{(1+\sigma\kappa)^{t-j}}i_j\ .$$

For an AR(1) $i_t = \eta i_{t-1} + \varepsilon_t$,

$$\pi_{t+1} = \left[\left(\frac{(1-\rho)}{(1+\sigma\kappa)}\frac{\rho}{(1-\rho\eta)} + \frac{\sigma\kappa}{1-\eta(1+\sigma\kappa)}\right)\frac{1}{(1+\sigma\kappa)^t} - \frac{\sigma\kappa}{1-\eta(1+\sigma\kappa)}\eta^t\right] i_1.$$

For $\eta = 1$, i.e. a one-time permanent increase in the interest rate,

$$\pi_{t+1} = \left[1 - \frac{(1+\sigma\kappa-\rho)}{(1+\sigma\kappa)^{t+1}}\right] i_1$$

Now, to derive these results. Eliminating output from (1)-(2),

$$E_t\pi_{t+1} = \frac{1}{1+\sigma\kappa}\pi_t + \frac{\sigma\kappa}{1+\sigma\kappa}i_t. \tag{5}$$

$$\tag{6}$$

Iterating forward (5), after the shock at time 1, (for $t \geq 1$),

$$\pi_{t+1} = \frac{1}{(1+\sigma\kappa)^t}\pi_1 + \frac{\sigma\kappa}{1+\sigma\kappa}\sum_{j=1}^{t}\frac{1}{(1+\sigma\kappa)^{t-j}}i_j. \tag{7}$$

In the case of AR(1), $i_t = \eta i_{t-1} + \varepsilon_t$, we have the not very elegant expression

$$\pi_{t+1} = \frac{1}{(1+\sigma\kappa)^t}\pi_1 + \frac{\sigma\kappa}{1+\sigma\kappa}\sum_{j=1}^{t}\frac{\eta^{j-1}}{(1+\sigma\kappa)^{t-j}}i_1$$

$$\pi_{t+1} = \frac{1}{(1+\sigma\kappa)^t}\pi_1 + \frac{\frac{1}{(1+\sigma\kappa)^t} - \eta^t}{\frac{1}{(1+\sigma\kappa)} - \eta}\frac{\sigma\kappa}{1+\sigma\kappa}i_1$$

If $\eta = 1$, so $\pi_t = \pi_1, t > 1$, this reduces to

$$\pi_{t+1} = i_1 + \frac{1}{(1+\sigma\kappa)^t}\left(\pi_1 - i_1\right).$$

Now, we need to find π_1. Iterating (3) forward,

$$\rho^t v_t = (0 - \pi_1) + \rho\left(i_1 - \pi_2\right) + \rho^2\left(i_2 - \pi_3\right) + \rho^3\left(i_3 - \pi_4\right) + ...$$

Thus, the condition $\rho^t v_t \to 0$ is

$$\pi_1 = \sum_{j=1}^{\infty}\rho^j\left(i_j - \pi_{j+1}\right).$$

Debt is devauled to pay the higher interest costs that result from higher real interest rates. Now plug inflation from (7),

$$\pi_1 = \sum_{j=1}^{\infty}\rho^j\left(i_j - \left(\frac{1}{(1+\sigma\kappa)^j}\pi_1 + \frac{\sigma\kappa}{1+\sigma\kappa}\sum_{k=1}^{j}\frac{1}{(1+\sigma\kappa)^{j-k}}i_k\right)\right)$$

$$\pi_1 = -\sum_{j=1}^{\infty}\rho^j\frac{1}{(1+\sigma\kappa)^j}\pi_1 + \sum_{j=1}^{\infty}\rho^j i_j - \frac{\sigma\kappa}{1+\sigma\kappa}\sum_{j=1}^{\infty}\rho^j\sum_{k=1}^{j}\frac{1}{(1+\sigma\kappa)^{j-k}}i_k$$

$$\pi_1 = -\sum_{j=1}^{\infty}\rho^j\frac{1}{(1+\sigma\kappa)^j}\pi_1 + \sum_{j=1}^{\infty}\rho^j i_j - \frac{\sigma\kappa}{1+\sigma\kappa}\sum_{k=1}^{\infty}\sum_{j=k}^{\infty}\rho^j\frac{1}{(1+\sigma\kappa)^{j-k}}i_k$$

$$\pi_1 = -\sum_{j=1}^{\infty}\rho^j\frac{1}{(1+\sigma\kappa)^j}\pi_1 + \sum_{j=1}^{\infty}\rho^j i_j - \frac{\sigma\kappa}{1+\sigma\kappa}\sum_{k=1}^{\infty}\rho^k\left(\frac{1}{1-\frac{\rho}{1+\sigma\kappa}}\right)i_k$$

$$\pi_1 = -\frac{\frac{\rho}{(1+\sigma\kappa)}}{1-\frac{\rho}{(1+\sigma\kappa)}}\pi_1 + \left(1 - \frac{\sigma\kappa}{1+\sigma\kappa}\left(\frac{1}{1-\frac{\rho}{1+\sigma\kappa}}\right)\right)\sum_{j=1}^{\infty}\rho^j i_j$$

$$\pi_1 = -\frac{\rho}{1+\sigma\kappa-\rho}\pi_1 + \frac{1-\rho}{1+\sigma\kappa-\rho}\sum_{j=1}^{\infty}\rho^j i_j$$

$$(1+\sigma\kappa-\rho)\,\pi_1 = -\rho\pi_1 + (1-\rho)\sum_{j=1}^{\infty}\rho^j i_j$$

$$\pi_1 = \frac{1-\rho}{1+\sigma\kappa}\sum_{j=1}^{\infty}\rho^j i_j. \tag{8}$$

For an AR(1)

$$\pi_1 = \frac{1-\rho}{1+\sigma\kappa}\sum_{j=1}^{\infty}\rho^j \eta^{j-1} i_1 = \frac{\rho}{1+\sigma\kappa}\frac{1-\rho}{1-\rho\eta}i_1.$$

For $\eta = 1$

$$\pi_1 = \frac{\rho}{1+\sigma\kappa}i_1.$$

With π_1, we now have the general solution. Using (8) in (7),

$$\pi_{t+1} = \frac{1}{(1+\sigma\kappa)^{t+1}}(1-\rho)\sum_{j=1}^{\infty}\rho^j i_j + \frac{\sigma\kappa}{1+\sigma\kappa}\sum_{j=1}^{t}\frac{1}{(1+\sigma\kappa)^{t-j}}i_j\ .$$

For the AR(1)

$$\pi_{t+1} = \frac{1}{(1+\sigma\kappa)^t}\left(\frac{1-\rho}{1+\sigma\kappa}\frac{\rho}{1-\rho\eta}i_1\right) + \frac{\sigma\kappa}{1+\sigma\kappa}\sum_{j=1}^{t}\frac{\eta^{j-1}}{(1+\sigma\kappa)^{t-j}}i_1$$

$$\pi_{t+1} = \left(\frac{1-\rho}{1+\sigma\kappa}\frac{\rho}{1-\rho\eta}i_1\right)\frac{1}{(1+\sigma\kappa)^t} + \frac{\sigma\kappa}{1+\sigma\kappa}\frac{\frac{1}{(1+\sigma\kappa)^t}-\eta^t}{\frac{1}{1+\sigma\kappa}-\eta}i_1$$

$$\pi_{t+1} = \left[\left(\frac{(1-\rho)}{(1+\sigma\kappa)}\frac{\rho}{(1-\rho\eta)} + \frac{\sigma\kappa}{1-\eta\,(1+\sigma\kappa)}\right)\frac{1}{(1+\sigma\kappa)^t} - \frac{\sigma\kappa}{1-\eta\,(1+\sigma\kappa)}\eta^t\right]i_1$$

For $\eta = 1$,

$$\pi_{t+1} = \left[1 - \frac{(1+\sigma\kappa-\rho)}{(1+\sigma\kappa)^{t+1}}\right]i_1.$$