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# THE FIRST SALE DOCTRINE AND THE DIGITAL CHALLENGE TO PUBLIC LIBRARIES 

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#### Abstract

Libraries have traditionally provided free communal access to books, facilitated by the first sale doctrine's (FSD) guarantee that libraries may purchase physical books at consumer prices. Increasingly restrictive ebook access terms may imperil libraries, and we compare the welfare cost of higher ebook prices to the welfare benefit of the FSD's guarantee of low physical book prices for libraries and their patrons. Using data on over 8,000 library systems for 2013-2019, we measure the impacts of physical and electronic holdings on the respective formats' circulation. We then build a structural model of the library market, and we rationalize the status quo book holdings with a librarian utility function that attaches higher weights to electronic circulation. While higher counterfactual ebook prices would induce libraries to substitute physical for electronic holdings, this would have little effect on patron CS because of consumer willingness to substitute. By contrast, higher physical book prices, as would prevail absent the first sale doctrine, reduce CS by almost ten times as much as an analogous ebook price increase.


Imke C. Reimers<br>304 Lake Hall<br>Northeastern University<br>360 Huntington Ave<br>Boston, MA 02115<br>United States<br>imke.reimers@gmail.com<br>Joel Waldfogel<br>Frederick R. Kappel Chair in Applied Economics<br>3-177 Carlson School of Management<br>University of Minnesota<br>321 19th Avenue South<br>Minneapolis, MN 55455<br>and NBER<br>jwaldfog@umn.edu

## Introduction

Municipal public libraries in the US have traditionally drawn patrons together to borrow print books without charge, and library distribution accounts for the majority of book consumption. ${ }^{1}$ Libraries have obtained these books at prices kept low by a feature of copyright law known as the first sale doctrine, which allows libraries to buy physical books at the same prices offered to consumers. ${ }^{2}$ Digitization has challenged libraries: With the arrival of electronic books, the function of libraries could in principle be performed without local "bricks and mortar" institutions; but libraries have endured by embracing a hybrid model of distributing both physical and electronic books. Still, the growth of ebooks creates a tension arising from the very different terms on which libraries purchase physical vs electronic books. Under the first sale doctrine, library physical book prices are lower than publishers might prefer, but library electronic book prices face no constraints. Growing reliance on ebooks therefore exposes libraries to a challenge from substantially higher expenses for materials. On average, lending an electronic book costs libraries roughly four times the cost of lending a physical book; and ebooks now account for a third of library expenditures.

Concerned about cannibalization of consumer ebook sales, publishers have recently raised ebook prices further, exacerbating the possible digital challenge to public libraries. Diminished availability of ebooks at libraries could make libraries less useful to patrons or possibly even obsolete; but this fate depends largely on the substitutability of physical and electronic books to patrons. If consumers view ebooks as sufficiently more appealing than physical

[^0]books, then diminished access to ebooks will reduce interest in libraries, at least as sources for books. On the other hand, if library patrons are readily willing to substitute physical for electronic books, then libraries' continued ability to purchase and lend physical books under the first sale doctrine will limit the vulnerability of libraries to the pricing of digital products.

This paper explores the effects of digitization on libraries by asking two broad questions. First, how do libraries' decisions about electronic and physical holdings affect users' tendencies to borrow electronic and physical books, as well as library visits? Related, to what extent are patrons willing to substitute between electronic and physical book formats? Second, how vulnerable are patrons to the publishers' proposed increases in ebook prices; and, analogously, how much protection do patrons derive from low physical book prices guaranteed to libraries by the first sale doctrine?

To answer these questions, we use data on over 8,000 libraries in the US from 2013 to 2019, including the number of volumes held in each format and their respective annual circulations, as well as the annual number of visits to each library. Our study has two parts. The first part is a descriptive analysis measuring the impact of physical and electronic holdings on the circulation of books in the respective formats, as well as visits. We employ two strategies to address the potential endogeneity of holdings. We use library fixed effects to control for time-constant unobservable determinants of circulation; and we also use discontinuous jumps in holdings (which arise frequently for ebooks given bundled purchasing arrangements) to measure causal impacts of holdings on circulation. With both approaches, we find that holdings affect patron demand for circulation: Larger physical or ebook holdings bring about higher physical and ebook circulation, respectively. Moreover, the circulation impact of an additional physical book is much larger than for an ebook. Cross effects, by contrast, are negative, indicating that patrons regard physical and ebooks as substitutes. Finally, physical holdings stimulate library visits, while ebook holdings depress them.

The descriptive findings are indicative of mechanisms that we incorporate into the second part of our analysis, a structural model of consumer demand and library book holdings. We use this model to compare the relative impacts of higher electronic and physical book prices as a means of evaluating alternative availability regimes. On the demand side, consumers decide which types of books to borrow in response to library format holdings. This model, which we implement as a two-level nested logit, has sufficient flexibility to accommodate different possible degrees of substitution among titles within format as well as across formats. Given consumer demand for physical vs electronic books, the supply side of the market consists of libraries choosing how to allocate their book budgets to holdings in each format. We rationalize observed holdings as those that maximize each library's "utility function," which consists of a weighted sum of the physical and electronic circulation that patrons choose, subject to the budget constraint implicit in current library spending on holdings. Our model rationalizes current holdings with library-specific utility functions that generally value an instance of electronic circulation substantially more than physical circulation. Patrons, by contrast, are more willing than librarians to substitute physical for electronic books.

We compare two counterfactual analyses that entail changes in access terms for electronic and print books, respectively. First, we evaluate the threat to patron consumer surplus (CS) from publishers' proposed changes in ebook access terms, which model as increased ebook prices. An increase in the ebook price would induce substantial substitution toward physical holdings. Because patrons are willing to substitute physical for electronic books, depressed ebook holdings would have little effect on circulation; and patron CS would fall only slightly with an increase in the ebook price. We then measure the welfare benefit afforded libraries and their patrons by the first sale doctrine by simulating higher print book prices that might prevail in the doctrine's absence. An increase in the print book price has almost ten times the negative effect on patron CS of an equal-sized increase in the ebook price. Moreover, library visits would be stimulated by higher ebook prices and reduced by higher print book
prices, providing another reason why the threat of higher ebook prices is small relative to the welfare benefit afforded by the first sale doctrine. Finally, we find that low physical, rather than electronic, book prices are particularly important to low-income and rural communities.

The paper proceeds in seven sections. Section 1 provides background on the library and ebook markets, including discussions of the copyright literature, the first sale doctrine, and librarian and publisher perspectives on book pricing. The section also presents a simple microeconomic framework making clear the possible misalignment of library holdings choices and patron preferences for formats, as well as the potentially different impacts of increased electronic vs physical book prices. Section 2 describes the data, and Section 3 provides our estimates of the causal impacts of physical and electronic holdings on circulation and visits. Section 4 introduces our empirical structural model, which combines consumer demand for books and formats at libraries with library choice of physical and electronic holdings, given consumer preferences. Section 5 presents estimates of the structural library patron demand model and the libraries' implied preference parameters. We use the estimated model in Section 6, which presents the counterfactual scenarios comparing higher library prices for electronic vs physical books, discussions of heterogeneous effects by income and urban status, and analyses of robustness of the main results with respect to values of estimated parameters. Finally, Section 7 concludes.

## 1 Background

### 1.1 Libraries, digitization, and ebooks

In the past century and a half, US public libraries have played a growing role in providing universal access to information. ${ }^{3}$ Modern public libraries have their origins in the $19^{\text {th }}$ cen-

[^1]tury, with libraries financed by Andrew Carnegie (Berkes and Nencka, 2020), which were "dedicated to the diffusion of knowledge." ${ }^{4}$ By 1920, the US had 3,500 public libraries, just over half of which had been endowed by Carnegie between 1883 and $1929 .{ }^{5}$ A series of federal Library Services Acts in the 1950s and 1960s subsidized library development in both rural and urban areas. ${ }^{6}$ By 2019 the US had 16,548 libraries. ${ }^{7}$ In addition to distributing books, public libraries have come to play "important roles in community building." 8 According to the American Library Association, library programs help patrons locate reliable health information and learn the skills for navigating the digital economy, while also "helping children and families succeed in school and life." ${ }^{9}$ Libraries have been successful at delivering information to consumers: By 2018, library circulation accounted for the vast majority of US book consumption.

Library sharing of books has long been facilitated by the first sale doctrine, which dictates that libraries can purchase physical books at the same prices offered to individuals, even as they lend the books without charge. ${ }^{10}$ Because an instance of library circulation generates less revenue than a purchase, libraries have long held out the possibility of cannibalizing book sales, but the first sale doctrine prevented publishers from pricing or providing access to print books in ways that might control sales displacement. ${ }^{11}$

[^2]Digitization has rapidly and substantially changed both the consumer and library markets for books. The US trade book market had $\$ 16.19$ billion in 2018 revenue, with 12.4 percent ( $\$ 2.01$ billion) on ebooks. ${ }^{12}$ Researchers have addressed various aspects of ebooks' impact (Gilbert, 2015; De los Santos and Wildenbeest, 2017; Chen et al., 2019) but not their impacts on libraries. While ebooks make it possible for libraries to lend books remotely, libraries have overwhelmingly adopted hybrid approaches, continuing to distribute physical books from their branch locations and distributing ebooks online. Ebooks are now a substantial part of library holdings and acquisitions; and library collections tend to include the most popular titles. ${ }^{13}$ Libraries account for a modest share of book expenditure - about 6.4 percent of print and 24.4 percent of electronic book expenditure in 2018 - but they account for larger shares of unit consumption. In 2018, when US print sales reached 698 million units, library print circulation was three times as high (1.87 billion). Similarly, 2018 saw ebook unit sales of 168 million, while library ebook circulation was 294 million. ${ }^{14}$

### 1.2 The digital challenges to libraries

The emergence of ebooks has created two conflicting challenges for libraries. First, now that ebooks are popular, some major US publishers have further restricted ebook access terms, threatening the ability of libraries to provide continued access. During 2019, Hachette and Simon \& Schuster replaced perpetual licenses with two-year ebook licenses; and audiobook provider Blackstone Publishing announced a new 90-day embargo on sales to libraries, leading

[^3]to a boycott." ${ }^{15}$ Macmillan also announced an embargo in 2019. For the first eight weeks after publication, libraries could purchase only one copy of an ebook. ${ }^{16}$ In response, the American Library Association has opposed efforts "to delay or deny library access to digital content," maintaining that " $[\mathrm{p}]$ roviding perpetual access and reducing the price for a single copy is important to ensuring all people have access to the world's knowledge" at libraries. ${ }^{17}$

The other challenge arising from ebook adoption is that ebook usage may decrease library visits and the sort of community building that libraries foster. Librarians and others have recently emphasized the important role of libraries in building social capital, epitomized by the Klinenberg (2018) designation of libraries as "palaces of the people." Library visits were waning even before the advent of electronic books: After rising for decades, per-capita US library visits began falling in 2009 and by 2012 had fallen more than 10 percent (see Figure 1). ${ }^{18}$ Because patrons can borrow ebooks without visiting libraries, the arrival of ebooks may have hastened the decline in physical library visits and associated community building.

### 1.3 Ebook prices, acquisition, and bundles

Due to the first sale doctrine, libraries purchase physical holdings at the same a la carte price that consumers pay for the books. Libraries obtain ebooks using different arrangements. OverDrive, which supplies ebooks to 76,000 libraries and schools, provides books in four different ways. ${ }^{19}$ First, a book can be purchased outright and made available to one user at a time ("one copy/one user"). Second, a book can be licensed to be made available via "metered access," either a fixed time or a maximum number of checkouts. Third, a

[^4]library can obtain access to each of various publishers' catalogs, and then pay a "cost per circulation" when a user borrows a book. Finally, libraries can purchase "simultaneous use" contracts for either particular titles or collections from each publisher, which allow unlimited borrowing of those titles until contract expiration. ${ }^{20}$

Data on library ebook pricing are not systematically available, but one US librarian has prominently disclosed per-copy price comparisons. Based on an analysis of 645 best-selling titles whose physical editions had suggested retail prices of $\$ 24.78$, the average Amazon print price was $\$ 16.77$, while the physical library editions averaged $\$ 14.14$. Individual book copies on Kindle had an average price of $\$ 12.77$. Libraries, by contrast, faced an average price of $\$ 45.75$ for ebooks, presumably when purchasing metered access. ${ }^{21}$ While the a la carte price comparisons clearly show that libraries face higher ebook prices, the cost-percirculation pricing scheme appears predominant. We show in Section 2 that library ebook expenditures are roughly proportional to ebook circulation rather than holdings and that ebook circulation costs libraries four times what print costs per borrowing instance.

The way that libraries add ebooks to their holdings can give rise to substantial changes in ebook holdings from one year to the next. Rather than purchasing individual copies, as libraries have traditionally done with physical books, ebook cost-per-circulation arrangements allow libraries access to book bundles, and the library incurs a charge when a patron borrows an included title. We exploit the annual jumps in the numbers of ebooks that libraries offer to consumers (their ebook holdings) to measure the impact of holdings on circulation in Section 3.

[^5]
### 1.4 Framework

A simple model is helpful for illustrating the possible misalignment of patron and librarian preferences, as well as for guiding the empirical work below. Given a library's holdings of physical and electronic books ( $N^{P}$ and $N^{E}$ ), consumers collectively maximize their utilities by choosing how many physical and electronic books to borrow. This results in the patron demand functions $Q^{P}\left(N^{P}, N^{E}\right)$ and $Q^{E}\left(N^{P}, N^{E}\right)$, and consumer welfare is described by a utility function over holdings $U\left(N^{P}, N^{E}\right)=U\left(Q^{P}\left(N^{P}, N^{E}\right), Q^{E}\left(N^{P}, N^{E}\right)\right)$.

If libraries charged for books and sought to generate profits, it would be natural to model libraries as maximizing price-weighted sums of physical and electronic circulation, less marginal costs associated with book lending. In reality, libraries derive no revenue from circulation but nevertheless make expenditures on holdings that attract circulation. We model libraries as choosing their holdings of physical and electronic books $N^{P}$ and $N^{E}$ to maximize the notional "welfare" that libraries derive from circulation subject to their budget constraints. That is, the libraries maximize $W\left(N^{P}, N^{E}\right)=W\left(Q^{P}\left(N^{P}, N^{E}\right), Q^{E}\left(N^{P}, N^{E}\right)\right)$ subject to $B=P^{P} N^{P}+P^{E} N^{E}$, where $P^{P}$ and $P^{E}$ are the prices of physical and electronic holdings, respectively, and $B$ is the budget.

It is possible that librarians choose holdings to maximize the welfare of patrons, but given that they make their holdings decisions independent of consumer choices, their holdings decisions need not maximize patron welfare. This potential misalignment is illustrated in Figure 2, which depicts a library's budget constraint, its holdings choice, and a patron indifference curve. In panel A, the librarian's holdings choice (the point $A$ ) happens to maximize CS, suggesting that the librarians' relative valuations of physical and ebook circulation are the same as the consumers'. In panel B, the library holdings choice does not maximize patron CS. Instead, the librarian indifference curve (not shown) is tangent to the budget constraint at $B$. Relative to the depicted holdings at $B$, patrons would prefer more physical books and fewer ebooks.

We rationalize this scenario with librarians who have steeper indifference curves and therefore higher relative valuations of ebook circulation. Relative to the holdings at $B$, increases in electronic and physical book prices would have asymmetric implications for patron welfare: Increases in physical book prices, which move holdings farther from patrons' preferences, harm patrons more than they are harmed by higher ebook prices. The proportional difference in librarian and patron indifference curve slopes indicates the extent of misalignment. We define $\theta=\frac{\text { slope of librarian indifference curve }}{\text { slope of patron indifference curve }}$, where $\theta$ is the supply-side behavioral parameter reflecting the relative librarian preference for ebook circulation. We provide estimates of $\theta$ in Section 5 .

## 2 Data

The data for this study consist of an annual panel of 8,418 US library systems from 2013 to 2019. For each library system and year, we observe the number of physical and electronic volumes in their collection $\left(N^{P}\right.$ and $\left.N^{E}\right)$, the physical and electronic circulation $\left(Q^{P}\right.$ and $Q^{E}$ ), and the number of visits to the library. The data also include the population of the service area (from which we construct market size $M$ ). We obtain the data from the Institute of Museum and Library Services (IMLS). ${ }^{22}$ Because some libraries are not included in all years, our final dataset consists of 50,722 library system-year observations.

We summarize our main variables of interest in Table 1, separately for electronic and print books. On average, libraries hold 73,617 electronic volumes and 105,571 print books. ${ }^{23}$ Table 1 also provides information on trends in electronic and physical holdings. Electronic holdings have increased quickly, while physical holdings are, on average, stable or declining. There is variation across libraries in holdings growth, however. The mean (median) annual

[^6]percentage change in electronic holdings is 26.9 (14.8), while the interquartile range runs from 6.1 to 29.2 percent, or from 849 to 14,477 units. Annual growth in physical holdings is much slower: the mean (median) percentage change is $-0.8(0.2)$ while the interquartile range runs from -3.4 to 2.8 percent, or from $-1,550$ to 1,005 units. In each year, some libraries experience substantial changes in physical or especially electronic holdings, and we use these sharp instances of variation for identification of causal impacts of holdings on circulation (and visits) below.

The information in Table 1 also shows the relative expense on electronic vs physical circulation. For physical books, the ratio of average expenditure to average circulation is $\$ 0.37$, while the ratio is four times as high (\$1.48) for ebooks. This is the sense in which ebooks are more expensive to libraries than physical books. However, books of different formats are purchased by different mechanisms, and this is visible in the relationships between expenditure and either circulation or holdings. While physical book expenditure depends on holdings, a regression of ebook expenditures on ebook circulation and holdings delivers a circulation coefficient seven times higher than the holdings coefficient. This dependence of ebook expenditures on circulation demonstrates the prevalence of the cost-per-circulation model. ${ }^{24}$

In Section 4, we introduce a model in which librarians allocate their budgets to holdings even though, for ebooks, expenditures depend on circulation. Implementing the model requires annual prices per holding for each format. ${ }^{25}$ The ratios of expenditure to holdings, for each format, provide simple measures of these prices, which are $\$ 0.91$ for physical books and $\$ 0.61$ for ebooks. Below, we rely on prices per holding that obey $P^{E}=2 / 3 P^{P}$. While ebooks

[^7]are more expensive than physical books per instance of circulation, ebooks cost libraries just two thirds as much annually per holding. This reversal arises both because ebooks circulate less than physical books and because libraries using cost-per-circulation arrangements incur charges for ebooks only when the titles are borrowed. Note that Table 1 tells much of the story in this paper. Libraries spend almost a third of their acquisition budgets on electronic materials while these materials account for only a tenth of circulation. This suggests both a high librarian marginal valuation of ebook circulation as well as substantial patron enthusiasm for physical books.

Finally, our data also include information about patron visits to libraries. On average, libraries experience $173,109(45,239)$ visits per year, and visits have declined by 2.0 (1.3) percent per year during the sample.

### 2.1 Library usage by patron income and urban/rural status

Libraries offer books without charge, and ebooks are available without travel, so one might expect different effects of changed access terms depending on income levels and population density. Table 2 provides holdings, circulation, and visit data by community income and urban/rural status. ${ }^{26}$ As the top panel shows, holdings and, especially, circulation rise in income levels. Even relative to population, both physical and electronic circulation rise proportionally more than income. Notwithstanding the free nature of library book lending, these patterns indicate that library book usage behaves like a luxury good, suggesting that less favorable book access terms will have larger effects on higher, rather than lower, income patrons.

The bottom panel shows how physical and electronic book holdings and usage vary across library systems according to their urban/rural status. Both physical and ebook holdings

[^8]tend to be larger in more urban systems, although ebook holdings are far less sensitive to population density. Indeed, rural libraries offer their patrons roughly twice as many electronic as physical books, while for urban libraries the ratio is reversed. Despite the large relative availability of ebooks to rural patrons, rural library lending is dominated by physical books: 96 percent of rural library circulation is physical, compared with 90 percent in the most urban systems. This suggests that rural patron well-being will be less sensitive to restrictions on ebook access. We explicitly explore welfare impacts of higher electronic and physical book prices by community income and urban/rural status in Section 6.

## 3 Descriptive evidence

### 3.1 Holdings and circulation

We are ultimately interested in the welfare effects of library resource allocation decisions, in particular, how many physical and electronic books to include in collections. As a first step, we measure the extent to which physical and electronic holdings, respectively, attract library users to consumption and the extent to which patrons regard physical and electronic volumes as substitutes.

To this end, Table 3 presents regressions of physical and ebook circulation on physical and ebook holdings as well as year dummies. In columns 1 and 2, which do not include library fixed effects, coefficients on all holdings variables are positive in both regressions, indicating that libraries with larger holdings have higher circulation. These regressions are vulnerable to a concern that some libraries have populations with greater tastes for borrowing books, so that the coefficients on holdings would not reveal the impact of holdings on circulation. Columns 3 and 4 include library fixed effects, so that coefficients are identified by withinlibrary system changes in physical and electronic holdings. In these specifications, own effects are positive, while cross effects are negative.

While the fixed effects approach deals with time-constant unobservables that may be correlated with both holdings and circulation, one can still be concerned about time-varying unobservables. For example, libraries whose patrons have growing demand for books in a format may respond to this demand growth by increasing their format-specific holdings. This could give rise to positive estimated relationships between holdings and circulation for reasons apart from the causal impact of holdings on circulation.

Circumventing this endogeneity requires some source of exogenous variation in holdings. We use the sudden jumps in holdings that primarily arise from library acquisition of ebook bundles. Even if holdings and circulation are covarying smoothly, we can treat the change in circulation surrounding a large jump in holdings as an "experiment" for measuring the impact of holdings on circulation.

Between 2013 and 2019, the median annual percentage change in ebook holdings is 14.8 percent, but about half of the libraries experience at least one annual ebook change over 50 percent. The median percentage change in physical book holdings is 0.2 percent, but about one fifth of the libraries experience at least one annual instance of 10 percent growth in physical holdings.

For the analysis, we define the top decile of annual percent changes in electronic and physical holdings as discontinuous jumps. We implement this discontinuity idea in two steps, and we use only the two years surrounding the jumps. First, we estimate "first stage" regressions showing the relationship between these discontinuous jumps in holdings and the size of respective holdings. Columns 5 and 6 show first stage regressions of physical and electronic holdings on indicators for their respective jumps. In years in which a library experiences a top-decile physical holdings jump, physical holdings rise by 8,799 , on average. For ebook jumps, the average increase is 56,881 . Columns 7 and 8 report "reduced form" regressions of physical and electronic circulation, respectively, on both jump indicators. Physical circulation rises by 6,233 with a physical holdings jump and declines by 6,827 with an ebook jump.

Ebook circulation rises by 3,866 with an ebook jump and falls negligibly with a physical book jump. The last two columns report IV regressions of circulation on the respective types of holdings, instrumenting holdings with the jumps. ${ }^{27}$

The fixed effects and instrumental variables approaches deliver similar estimates. The positive own-format coefficients - indicating, for example, that larger physical holdings attract more physical book circulation - show that patrons value holdings. The negative cross-format coefficients - showing that libraries with more ebook growth experience depressed physical book circulation and vice versa - indicate that patrons view physical and electronic books as substitutes for one another. The similarity of the IV and the fixed effects estimates lends further credibility to our main causal findings: Main effects are positive and significant, while cross effects are negative and significant; and the physical main effect is much larger than the ebook main effect.

The relative coefficient sizes in Table 3 also offer some clues about library managers' supply side motivations. Ignoring cross effects for illustration, an additional physical book raises physical circulation by 0.676 ( 0.675 ) units in the fixed effects (instrumental variables) estimates, over six times larger than the ebook own effect of 0.108 (0.067). That is, an additional physical holding engenders much more additional circulation than an additional ebook. Because physical books are only 50 percent more costly to libraries per holding (see Table 1), the equilibrium interpretation of this pattern of coefficients is that libraries attach more value to ebook than physical circulation. In our Section 1.4 framework, this implies a librarian library holdings choice resembling that in Panel B of Figure 2 and a librarian relative ebook preference, or $\theta$, above one.

[^9]
### 3.2 Library visits

As Klinenberg (2018) emphasizes, libraries fulfill functions apart from simply distributing books. By bringing people together to physical spaces, libraries help to build social capital in communities. Ebooks can be borrowed without a physical visit to a library. Hence, it is possible that the shift of resources from physical to electronic books discourages library visits, with possible impacts on social capital formation.

Table 4 reports regressions of visits on library physical and ebook holdings. Column 1 includes year fixed effects but does not include library fixed effects. Both physical and ebook holdings have positive coefficients, indicating that libraries with more holdings have more visits. Column 2 adds library fixed effects, and their inclusion changes the results: Physical holdings continue to have a positive coefficient, while the coefficient on ebook holdings is negative. An additional physical holding raises visits by 0.248 (with a standard error of $0.073)$ while an additional ebook holding reduces visits by 0.082 (0.009).

The library fixed effects address the possibility that different communities have timeconstant different preferences for library patronage; but as with the circulation regressions, one can be concerned about time-varying unobservables affecting both holdings and the tendency for patrons to visit libraries. To address this, we apply our holdings jumps IV approach for visits as well. Column 3 reports the reduced form regression of visits on the physical and electronic jump instruments. A jump in physical holdings raises visits by 3,569 per year while a jump in ebook holdings reduces visits by 3,822 per year. The resulting IV estimates are reported in column 4. Using just the interval from two years before until the year after a jump, an additional physical volume raises visits by 0.39 ( 0.11 ), while an additional ebook volume decreases visits by 0.05 (0.02) per year.

How large are the depressing effects of ebook holdings on visits? In 2018, the libraries in the sample had 1.25 billion visits, and these libraries collectively had holdings of 800 million ebooks. This implies that absent ebooks, library visits would have been 6.0 (4.1) percent
higher in 2018, using the ebook coefficients from the fixed effects (instrumental variables) regressions. Visits declined roughly 10.9 percent between 2013 and 2018, suggesting that the growth in ebooks between 2013 and 2018 can explain about a third of this decline. ${ }^{28}$

## 4 Model

Our empirical implementation of the model, which incorporates the findings above, has two broad parts. First, consumers have demand for borrowing physical and electronic books from libraries. Second, given consumer attitudes toward physical and electronic books, along with the respective prices that libraries pay, libraries choose the sizes of their respective physical and electronic holdings to maximize library utility. Although patrons do not pay to borrow, the library's welfare function attaches value to circulation of physical and electronic books, respectively. Given these implicit valuations, library holdings and patron demand circulation - emerge as equilibria. The model delivers these equilibrium outcomes, and we use the model to calculate counterfactual equilibria. We entertain effects of higher electronic and physical book prices on electronic and physical holdings and circulation, on patron and librarian welfare, and (via the descriptive estimates from Table 4) on library visits.

### 4.1 Demand for borrowing

We specify a two-level nested logit model of demand in which consumers at library $l$ first choose between the outside good and obtaining a book from the library. Among library options, they choose between physical and electronic book formats. Then, having chosen to borrow a physical or an electronic book, the consumer chooses a title. ${ }^{29}$ Suppressing the

[^10]library identifier, consumer $i$ 's utility from borrowing book $j$ is given by
\[

$$
\begin{equation*}
u_{i j}=\delta_{j}+\zeta_{i}^{G}+\left(1-\sigma_{2}\right) \zeta_{i}^{f}+\left(1-\sigma_{1}\right)\left(1-\sigma_{2}\right) \epsilon_{i j} \tag{1}
\end{equation*}
$$

\]

In this model, the mean utility of product $j$ is given by $\delta_{j}=x_{j} \beta+\xi_{j}$, and $\epsilon_{i j}$ follows an extreme value distribution. Further, $\zeta_{i}^{G}$ and $\zeta_{i}^{f}$ follow distributions such that $\left[\zeta_{i}^{h}+(1-\right.$ $\left.\left.\sigma_{1}\right)\left(1-\sigma_{2}\right) \epsilon_{i j}\right]$ and $\left[\zeta_{i}^{G}+\left(1-\sigma_{2}\right) \zeta_{i}^{f}+\left(1-\sigma_{1}\right)\left(1-\sigma_{2}\right) \epsilon_{i j}\right]$ are also distributed extreme value. The term $G$ is the set of formats at the library (physical, denoted by $P$, and electronic, denoted by $E$ ), and the term $f$ denotes an individual format (physical or electronic). The parameter $\sigma_{1}$ reflects the correlation of utilities that consumers experience for books within a format, while the parameter $\sigma_{2}$ reflects the correlation of utilities across the two library formats. ${ }^{30}$

Consumer utility maximization results in familiar logit expressions for the shares, which yield quantities of electronic and physical books borrowed from a library when multiplied by market size. Thus,

$$
\begin{equation*}
s_{j}=\frac{e^{\delta_{j} /\left(1-\sigma_{1}\right)}}{D_{f}^{\left(\sigma_{1}-\sigma_{2}\right) /\left(1-\sigma_{2}\right)} D_{G}^{\sigma_{2}}\left(1+D_{G}^{1-\sigma_{2}}\right)}, \tag{2}
\end{equation*}
$$

where $D_{f}=\left[\sum_{j \in \mathcal{J}_{f}} \exp \left(\frac{\delta_{j}}{1-\sigma_{1}}\right)\right]^{\frac{1-\sigma_{1}}{1-\sigma_{2}}}, \mathcal{J}_{f}$ is the set of products in format $f$, and $D_{G}=$ $\sum_{f \in\{P, E\}}\left(D_{f}^{\left(1-\sigma_{1}\right) /\left(1-\sigma_{2}\right)}\right)$. We can use these logit formulas to express circulation as a function of the holdings: $Q^{P}=Q^{P}\left(N^{P}, N^{E}\right)$ and $Q^{E}=Q^{E}\left(N^{P}, N^{E}\right)$, where $N^{P}$ and $N^{E}$ are physical and electronic holdings, respectively.

Given the nested logit structure of the utility function, the consumer surplus that patrons
selected by patrons visiting the library. Data in Section 3 support this nesting structure as well: Library visits have fallen as electronic holdings have risen.
${ }^{30}$ See Berry (1994) and http://www.nathanhmiller.org/nlnotes.pdf.
obtain from a library's holdings is then given by

$$
\begin{equation*}
C S=M \times \ln \left[1+\left(\sum_{f \in\{P, E\}} D_{f}\right)^{\left(1-\sigma_{2}\right)}\right] . \tag{3}
\end{equation*}
$$

Because library books are free to consumers, there is no price in the utility function and therefore no price parameter that translates utility into a dollar figure. As a result, we can only quantify the proportionate change in CS across book price counterfactuals, not the absolute levels of CS.

### 4.2 Library supply of holdings

Libraries do not collect revenue from users. Nevertheless, they operate as if they attached value to instances of circulation. Accordingly, we presume that each library maximizes a welfare function defined over physical and electronic circulation. We describe this function as $W\left(Q^{P}\left(N^{P}, N^{E}\right), Q^{E}\left(N^{P}, N^{E}\right)\right)$, where $W$ reflects the notional value the library obtains from instances of circulation of physical and electronic books.

The library's maximization problem is constrained by its budget: $B=P^{P} N^{P}+P^{E} N^{E}$, where $P^{P}$ and $P^{E}$ are the prices per holdings of physical and electronic volumes, respectively, and $B$ is the library's budget for books. Hence, the Lagrangian for each library's full maximization problem is given by

$$
\begin{equation*}
\mathcal{L}=W\left(Q^{P}\left(N^{P}, N^{E}\right), Q^{E}\left(N^{P}, N^{E}\right)\right)+\lambda\left[B-P^{P} N^{P}-P^{E} N^{E}\right] . \tag{4}
\end{equation*}
$$

We define the derivatives of the librarian's welfare function with respect to circulation as $\theta^{P}$ and $\theta^{E}$ (e.g., $\frac{\partial W}{\partial Q^{P}} \equiv \theta^{P}$ ), so that $\frac{\theta^{E}}{\theta^{P}}=\theta$, as introduced in Section 1.4. Then, suppressing library subscripts and defining $\lambda^{\prime}=\frac{\lambda}{\theta^{P}}$, each library's first-order conditions for this problem
are:

$$
\begin{aligned}
\frac{\partial Q^{P}}{\partial N^{P}}+\theta \frac{\partial Q^{E}}{\partial N^{P}}-\lambda^{\prime} P^{P} & =0 \\
\frac{\partial Q^{P}}{\partial N^{E}}+\theta \frac{\partial Q^{E}}{\partial N^{E}}-\lambda^{\prime} P^{E} & =0 \\
P^{P} N^{P}+P^{E} N^{E} & =B
\end{aligned}
$$

The first two first-order conditions can be rewritten succinctly as

$$
\left[\begin{array}{ll}
\frac{\partial Q^{P}}{\partial N^{P}} & \frac{\partial Q^{E}}{\partial N^{P}}  \tag{5}\\
\frac{\partial Q^{P}}{\partial N^{E}} & \frac{\partial Q^{E}}{\partial N^{E}}
\end{array}\right] \quad\left[\begin{array}{l}
1 / \lambda^{\prime} \\
\theta / \lambda^{\prime}
\end{array}\right]=\left[\begin{array}{l}
P^{P} \\
P^{E}
\end{array}\right] .
$$

Once we have estimated demand, the derivatives of circulation $(Q)$ with respect to holdings $(N)$ are known, as are the prices and the library's budget. We can thus solve for the library's welfare weight $\theta$ using equation (5). ${ }^{31}$ Our inference of library managers' welfare weights determines the librarian indifference curve rationalizing the holdings choice.

### 4.3 Equilibrium

The library-specific parameters $\theta$ show the relative value that each library attaches to electronic vs physical circulation. The values of these parameters that we infer from the status quo holdings choices $\left\{N^{P}, N^{E}\right\}$ provide the baseline solution to the model. We can then calculate counterfactual equilibria arising from different physical and electronic book prices by re-solving the model. We solve the model separately for each library by maximizing the librarian utility with respect to the holdings budget constraint.

[^11]
## 5 Estimates

### 5.1 Demand

Following Berry (1994), we estimate a two-level nested logit model of demand by estimating the following equation across libraries and years:

$$
\begin{equation*}
\ln \left(s_{j}\right)-\ln \left(s_{0}\right)=\gamma \mathbb{1}_{j}(\text { ebook })+\sigma_{2} \ln \left(s_{f \mid G}\right)+\sigma_{1} \ln \left(s_{j \mid f}\right)+\xi_{j} \tag{6}
\end{equation*}
$$

Here, $\ln \left(s_{f \mid G}\right)$ is a format's - electronic or physical book - share of book lending at a library, and $\ln \left(s_{j \mid f}\right)$ is a particular book's share of a format's lending. We also include year and library fixed effects, and we suppress the library and year indices for simplicity.

Our estimation strategy faces two challenges, one related to data and the other to identification. We have as data the total quantity of lending by library and format. That is, we observe $Q^{f}$, so we have a direct measure of $s_{f \mid \mathrm{G}}=\frac{Q^{f}}{Q^{E}+Q^{P}}$. We do not observe the circulation of title $j$, but we do observe library holdings of physical and electronic books, $N^{P}$ and $N^{E}$, respectively. We assume symmetry within format, so that $s_{j \mid f}=\frac{1}{N^{f}}$. Then the overall product-level share is given by $s_{j}=\frac{Q^{f}}{N^{f} M}$.

To identify the substitution parameters, we instrument $s_{f \mid G}$ with group $f$ 's share of total holdings: $\frac{N^{f}}{N^{E}+N^{P}}$. The term $\ln \left(s_{j \mid f}\right)$ would normally require an instrument as well, and $\frac{1}{N^{f}}$ would be a logical candidate. Here, because $\frac{1}{N^{f}}$ is our measure of $s_{j \mid f}$, it can enter directly.

Table 5 reports estimates of the demand parameters. Column 1 reports the model estimated without an instrument. The uninstrumented $\sigma$ coefficients are close to 1 , presumably because we are - in the case of the top-level share - regressing an expression involving $Q^{f}$ (i.e., $\ln \left(\frac{Q^{f}}{N^{f} M}\right)$ ) on another expression involving $Q^{f}$ (i.e., $\ln \left(\frac{Q^{f}}{Q^{E}+Q^{P}}\right)$ ). Column 2 shows the first-stage estimate of $\ln \left(s_{f \mid G}\right)$ on the instrument, where $s_{f \mid G}=\frac{Q^{f}}{Q^{E}+Q^{P}}$. The instrument "works": For example, physical books make up larger shares of circulation in library years
in which physical works make up larger shares of the collections. Column 3 reports our two-level nested logit model of demand. We estimate the parameter $\sigma_{1}$ to be 0.937 (0.006) for substitution among titles within a format, and we estimate $\sigma_{2}$, reflecting substitution across formats, to be 0.671 (0.03). This indicates that titles are closer substitutes for one another within than across formats and more substitutable across formats than to the outside good. ${ }^{32}$ These are the estimates we use in our simulations. We explore the sensitivity of our main results to alternative parameter estimates in Section 6.3. Although the model appears parsimonious, it is important to recall not only that the models include library and year fixed effects but also that both the format-specific mean utilities and the librarian utility parameter $\theta$ are library-specific.

### 5.2 Supply

Given estimates of parameters $\sigma_{1}$ and $\sigma_{2}$, we calculate format-specific $\delta$ 's for each library-year directly from the data, where for each book $j$ in format $f, \delta_{j}=\ln \left(s_{j}\right)-\ln \left(s_{0}\right)-\sigma_{2} \ln \left(s_{f \mid G}\right)-$ $\sigma_{1} \ln \left(s_{j \mid f}\right)$. This, in turn, allows us to create expressions for $Q^{P}$ and $Q^{E}$ as functions of $N^{P}$ and $N^{E}$. Then, using the holdings $\left(N^{P}\right.$ and $\left.N^{E}\right)$ and the average prices of physical and electronic books from Table 1, we calculate the library-specific welfare weights $\theta .{ }^{33}$ The distribution of these weights is skewed: The mean (median) $\theta$ estimate is 5.34 (1.86), and the inter-quartile range runs from 1.28 to 2.69 . Most libraries implicitly attach substantially more value to instances of electronic vs physical circulation: 86.3 percent of libraries have $\theta$ estimates above unity.

One might be concerned that libraries' decisions to hold electronic as opposed to physical books are influenced by space constraints. A few facts allay such concerns. First, physical

[^12]holdings decline over time, both overall and per library square footage. In our sample, the mean (median) physical holdings per square foot fall from 5.34 (4.63) in 2013 to 5.09 (4.31) in 2019. Over the same period, mean (median) absolute physical holdings fall from 125,900 $(44,388)$ to $94,698(36,692)$. Hence, libraries are not increasingly space-constrained by books. Second, the shadow price of space used for physical books appears small. For example, using the Seattle circulation data introduced in footnote 13, we estimate that 80 percent of physical holdings do not circulate each year. ${ }^{34}$ We infer from this that adding physical volumes to the collection would not crowd out holdings that are used much by patrons.

Solving the model leads to two complications that we explore in the Appendix. First, given our parameter estimates, our model does not solve to observed holdings in baseline simulations for 5.7 percent of libraries. Second, the model solves to corners (with zero electronic or physical holdings) for 4.5 (16.2) percent of libraries under a ten (100) percent increase in the respective book prices. Our main results exclude libraries that solve to corners; but Appendix Section A. 2 shows that we obtain very similar results if we include the corner solutions.

## 6 Counterfactuals

This section uses the model for our main counterfactual comparison as well as two additional exercises. First, we compare the welfare cost of higher ebook prices to the welfare benefit of the first sale doctrine's guarantee of low physical book prices for libraries and their patrons. In particular, we compare counterfactuals with equal-sized electronic and print book price increases. Second, we ask how the results vary by income and urban/rural status. Finally,

[^13]we explore the robustness of the results to alternative substitution parameters.

### 6.1 Comparing electronic and print price increases

We do not know the exact size of the print and electronic book price increases that possible policy changes would effect. Our main interest, however, is a comparison of their respective effects on holdings, circulation, and welfare. To this end, we compare equal-sized price increases for books in the two formats. Table 6 reports results for ten percent increases in columns 1 and 2 and for price doubling in columns 3 and 4; and each entry in the table is an average across sample libraries. Increasing a format's price reduces holdings of that format substantially while raising holdings in the other format slightly. Reducing a format's holdings lowers its circulation while raising circulation in the other format. Because of consumer substitution behavior, however, effects on circulation are muted relative to effects on holdings.

As foreshadowed in the descriptive results and the theoretical framework, increases in the electronic and print book prices have asymmetric effects on total circulation and patron welfare. While an increase in the ebook price reduces total circulation by modest amounts, an equal-sized increase in the physical book price would reduce circulation by nearly ten times as much. Similarly, while a doubling of the ebook price would reduce patron CS by 0.37 percent, an equal-sized increase in the physical book price would reduce patron welfare by 9.2 times as much. A ten percent price increase leads to a similar proportionate relationship (8.9). In short, the benefit to patrons from stable ebook prices is small compared with the benefit of the implicit ceiling on print book prices afforded by the first sale doctrine. This arises because patrons have stronger preferences for physical books than do librarians.

Finally, because an increase in the physical book price reduces physical holdings, it also reduces library visits. An increase in the ebook price, by contrast, raises library visits, providing a sense in which the first sale doctrine promotes social capital.

### 6.2 Heterogeneity by income and urban/rural status

To determine the distributional impact of changed access terms, we examine the relative impacts of increased electronic and physical book prices separately by community income as well as urban/rural status. Doing this requires an adjustment. In our baseline approach, we solve the model using a single relationship between $P^{E}$ and $P^{P}$, in particular that $P^{E}=2 / 3 P^{P}$ for all libraries. ${ }^{35}$ This is correct on average, but given the information in Table 2, one can see that the relationship varies across communities according to income and rural/urban status. This occurs because ebook expenditure varies with circulation rather than holdings, and circulation varies across types of communities. When we simulate the model by income or urban status groups, we use group-specific price relationships $P^{E}=\kappa P^{P}$, where $\kappa$ is calculated from Table 2. For income quintiles, $\kappa$ ranges from 0.31 for the lowest income quintile to 1.31 for the highest. In the urban/rural simulation, $\kappa$ varies between 0.14 for rural areas and 1.31 for the most urban. For all groups of libraries, electronic circulation remains more expensive per unit than physical circulation.

Panel A of Table 7 presents price-doubling results separately for groups of libraries by income quintile. Doubling the ebook price has a larger negative effect on CS as community income rises, while doubling the physical book price has a larger negative effect on CS as income falls across communities. Specifically, increased physical book prices reduce CS by 13.2 times as much as increased ebook prices for patrons in the lowest income quintile, compared with 7.8 times as much in the highest income quintile. The difference arises because patrons in low-income areas rely more heavily on physical books.

Panel B of Table 7 presents price-doubling results separately for groups of libraries according to urban/rural status, and the results mirror those by income levels. Given the relatively low ebook usage by rural library patrons, increased ebook prices have smaller negative impacts on CS in rural areas, whereas increased physical book prices have larger

[^14]negative CS impacts. The relative decrease in CS is 7.3 times higher for physical than for electronic book price increases in large cities, while the ratio is 12.2 for rural library patrons. In short, the protections of print books guaranteed by the first sale doctrine are particularly important to rural and low-income communities.

### 6.3 Robustness to substitution parameters

In our baseline estimates, an increase in the physical book prices reduces CS about nine times as much as an increase in the price of ebooks. These results may depend on the estimated substitution parameters ( $\sigma_{1}$ and $\sigma_{2}$ ), and we explore the robustness of our results to these substitution parameters here.

Figure 3 reports the percentage changes in CS for doubling prices of ebooks (dashed line) and print books (solid line), with varying degrees of substitutability within ( $\sigma_{1}$ ) and across $\left(\sigma_{2}\right)$ formats. In particular, we choose $\sigma_{1}$ to be either our estimated value of 0.937 or 0.5 , and we vary $\sigma_{2}$ from 0.1 to 0.8 , keeping $\sigma_{2}<\sigma_{1}$. The baseline results are indicated by the dots. When $\sigma_{1}=0.937$ - and titles are close substitutes within format - the percentage change in CS with a doubling of the ebook price is roughly constant. Whatever the substitutability across formats $\left(\sigma_{2}\right)$, the $\Delta \mathrm{CS}$ ratio is about eight. ${ }^{36}$

The shorter lines describe changes in CS for varying levels of substitutability across formats $\left(\sigma_{2}\right)$ with lower substitutability within format ( $\sigma_{1}=0.5$ ). Here, because withinformat substitution is limited, the degree of substitutability across formats matters more. When across-format substitutability $\left(\sigma_{2}\right)$ is low, the CS effect of an increase in book prices of either format is larger: Consumers are more likely to stop using the library altogether. However, the CS ratio of interest (the effect of doubling print book prices compared to ebooks) is almost unchanged, remaining around eight. While the absolute effects of price

[^15]changes on CS depend on our estimated preference correlation coefficients, the relative effects do not. Across a wide range of possible parameters, the welfare benefit of the first sale doctrine is larger than the potential cost of more restrictive ebook access terms.

## 7 Conclusion

Libraries, which provide the majority of US unit book consumption, have traditionally drawn patrons together to make shared use of physical books. This practice is facilitated by the first sale doctrine, which allows libraries to purchase books at the same prices as consumers. Library ebook prices, not subject to the first sale doctrine, exceed those charged to consumers. Ebooks have become popular at public libraries; and publishers are now raising library ebook prices further. This has raised concerns among librarians because it could undermine the value of public libraries to users, particularly if patrons are unwilling to substitute physical for electronic books.

Our findings on the effects of ebook price increases may provide some comfort to library managers: Because consumers are willing to substitute among books - and continue to attach high value to physical books - restricting ebook access would have limited effects on library patron utility. Moreover, shifting holdings toward physical books would increase library visits. While higher ebook prices pose a limited threat, the first sale doctrine's implicit cap on physical book prices provides valuable protections to library patrons. In contrast to the negligible change in CS arising from an increase in the ebook price, an equal-sized increase in the physical book price would reduce patron CS by almost ten times the impact of higher ebook prices. Moreover, the relative vulnerability of patrons to higher physical book prices is greater for rural and for low-income communities. A continuation of favorable library access to physical books appears important as libraries navigate a more digital society.

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## 8 Figures and Tables

Figure 1: Library visits and circulation over time


Notes: Data on circulation and visits from selected issues of the Digest of Education Statistics.

Figure 2: Librarian and patron preferences and the holdings budget constraint

Panel A: patron-librarian alignment


Panel B: patron-librarian misalignment


Note: The straight line is a library's budget constraint for physical and electronic holdings. The curve is a patron indifference curve over holdings. In the left panel it is tangent to the budget constraint at the observed library holdings $(A)$. The right panel depicts a situation in which the library holds more ebooks and fewer print books than patrons would prefer $(B)$.

Figure 3: Sensitivity of the change in CS to substitution parameters


Notes: Effects of doubling library book prices on consumer surplus for varying substitution parameters $\sigma_{1}$ and $\sigma_{2}$. The dashed lines describe percent changes in CS when doubling the price of ebooks, and the solid lines describe percent changes in CS when doubling the price of print books. The x-axis shows varying levels of $\sigma_{2}$. The longer lines keep $\sigma_{1}$ constant at its estimated level $\left(\sigma_{1}=0.937\right)$ and the shorter lines keep $\sigma_{1}$ constant at 0.5 . Only libraries that solve to interior solutions for both counterfactuals and for all values of $\sigma_{1}$ and $\sigma_{2}$ are included.

Table 1: Summary statistics

|  | Mean | SD | Median |
| :--- | :---: | :---: | :---: |
| Ebooks |  |  |  |
| Holdings | 73616.6 | 135825.4 | 30482.5 |
| \% change in holdings | 26.9 | 86.2 | 14.8 |
| Circulation | 30284.4 | 191922.1 | 3284.0 |
| Expenditure | 44681.6 | 241931.1 | 2950.0 |
| Print books |  |  |  |
| Holdings | 105570.6 | 405702.5 | 39284.5 |
| \% change in holdings | -0.8 | 10.4 | 0.2 |
| Circulation | 258391.5 | 952812.4 | 50681.0 |
| Expenditure | 96050.4 | 377787.0 | 23317.0 |
| Visits |  |  |  |
| Total | 173109.4 | 584098.5 | 45238.5 |
| \% change in visits | -2.0 | 20.8 | -1.3 |
| Observations | 50722 |  |  |

Notes: Summary statistics at the library-year level, for all libraries from 2013 to 2019. Holdings and circulation describe the number of books of each format, and expenditure is in terms of (nominal) dollars.

Table 2: Summary statistics by income and urban/rural status
Panel A: Income quintile

|  | Lowest | $2^{\text {nd }}$ | $3^{\text {rd }}$ | $4^{\text {th }}$ | Highest |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Median income | 19471.5 | 25678.9 | 30422.8 | 36805.0 | 53602.4 |
| Population | 23632.3 | 30948.2 | 42189.8 | 54933.1 | 79161.2 |
| Ebooks |  |  |  |  |  |
| Holdings | 74610.4 | 74387.4 | 77562.0 | 78986.0 | 89695.0 |
| Circulation | 10250.3 | 14885.1 | 33691.5 | 42379.6 | 74797.6 |
| $\quad$ Expenditure | 13920.8 | 22319.4 | 42593.5 | 59498.2 | 123916.0 |
| Print books |  |  |  |  |  |
| Holdings | 69769.6 | 81563.7 | 110561.2 | 115154.4 | 220519.2 |
| $\quad$ Circulation | 97942.4 | 159784.3 | 245936.4 | 340321.9 | 636080.1 |
| $\quad$ Expenditure | 42072.1 | 62739.3 | 95354.9 | 115079.8 | 232665.5 |
| Observations | 39212 |  |  |  |  |

Panel B: Urban/rural

|  | Large city | Medium city | Town | Rural |
| :---: | :---: | :---: | :---: | :---: |
| Population | 97276.0 | 57316.8 | 18841.0 | 9175.7 |
| Ebooks |  |  |  |  |
| Holdings | 96609.0 | 76713.9 | 70434.1 | 61339.8 |
| Circulation | 82760.2 | 40832.2 | 9269.5 | 4637.4 |
| Expenditure | 129202.7 | 57893.0 | 10703.2 | 5335.9 |
| Print books |  |  |  |  |
| Holdings | 237121.0 | 140696.8 | 61217.3 | 33585.2 |
| Circulation | 644008.1 | 379845.9 | 101915.9 | 48316.7 |
| Expenditure | 242368.9 | 130115.0 | 41039.5 | 20784.5 |
| Observations | 50427 |  |  |  |

Notes: Averages across groups of libraries according to community income (Panel A) or urban/rural status (Panel B). In the bottom panel, library systems are grouped by urban status, where our first group includes metropolitan areas whose principal city has more than 250,000 inhabitants. Our second group includes metropolitan areas whose principal city has fewer than 250,000 inhabitants. The third group includes incorporated, or Census-designated, places outside of metropolitan areas and with populations of at least 2,500 . The final group includes Census-designated "rural" places.

Table 3: Holdings and circulation

|  | OLS |  | FE |  | 1st stage |  | Reduced form |  | IV |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{gathered} (1) \\ \text { Phys } \end{gathered}$ | (2) <br> Ebook | (3) <br> Phys | (4) <br> Ebook | (5) <br> Phys | (6) <br> Ebook | (7) <br> Phys | (8) Ebook | (9) <br> Phys | (10) <br> Ebook |
| phys holdings | $\begin{gathered} 1.651 \\ (0.165) \end{gathered}$ | $\begin{gathered} 0.285 \\ (0.0260) \end{gathered}$ | $\begin{gathered} \hline 0.676 \\ (0.152) \end{gathered}$ | $\begin{gathered} \hline-0.229 \\ (0.0959) \end{gathered}$ |  |  |  |  | $\begin{gathered} \hline 0.675 \\ (0.222) \end{gathered}$ | $\begin{aligned} & -0.0219 \\ & (0.135) \end{aligned}$ |
| ebook holdings | $\begin{gathered} 0.540 \\ (0.0524) \end{gathered}$ | $\begin{gathered} 0.132 \\ (0.0145) \end{gathered}$ | $\begin{gathered} -0.164 \\ (0.0240) \end{gathered}$ | $\begin{gathered} 0.108 \\ (0.0156) \end{gathered}$ |  |  |  |  | $\begin{aligned} & -0.0991 \\ & (0.0479) \end{aligned}$ | $\begin{gathered} 0.0671 \\ (0.0282) \end{gathered}$ |
| phys jump |  |  |  |  | $\begin{aligned} & 8799.0 \\ & (618.0) \end{aligned}$ |  | $\begin{gathered} 6233.4 \\ (1629.4) \end{gathered}$ | $\begin{gathered} -281.6 \\ (1016.5) \end{gathered}$ |  |  |
| ebook jump |  |  |  |  |  | $\begin{aligned} & 56880.8 \\ & (2976.5) \end{aligned}$ | $\begin{gathered} -6826.7 \\ (2288.9) \end{gathered}$ | $\begin{gathered} 3865.7 \\ (1249.0) \end{gathered}$ |  |  |
| Library FEs | No | No | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
| Year FEs | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
| Observations | 50722 | 50722 | 50722 | 50722 | 20133 | 20133 | 20133 | 20133 | 20133 | 20133 |
| $\overline{R^{2}}$ | 0.525 | 0.397 | 0.980 | 0.847 | 0.997 | 0.665 | 0.992 | 0.930 |  |  |

Notes: The dependent variable in all columns except 5 and 6 is the total circulation of physical books (odd-numbered columns) or electronic books (even-numbered columns). In columns 5 and 6 the dependent variable is holdings. All specifications include year fixed effects. Columns 3 through 10 also include library fixed effects. Robust standard errors in parentheses.

Table 4: Physical books, ebooks, and library visits

|  | (1) | $(2)$ | $(3)$ | $(4)$ |
| :--- | :---: | :---: | :---: | :---: |
|  | OLS | FE | Red. form | IV |
| phys holdings | 1.194 | 0.248 |  | 0.387 |
|  | $(0.0903)$ | $(0.0733)$ |  | $(0.114)$ |
| ebook holdings | 0.165 | -0.0815 |  | -0.0552 |
|  | $(0.0277)$ | $(0.00930)$ |  | $(0.0249)$ |
| phys jump |  |  | 3568.8 |  |
|  |  |  | $(932.9)$ |  |
| ebook jump |  |  | -3821.8 |  |
|  |  |  | $(1251.5)$ |  |
| Library FEs | No | Yes | Yes | Yes |
| Year FEs | Yes | Yes | Yes | Yes |
| Observations | 50722 | 50722 | 20133 | 20133 |
| $\overline{R^{2}}$ | 0.704 | 0.986 | 0.992 |  |

Notes: The dependent variable in all columns is the number of visits to a library in a year. All specifications include year fixed effects. Columns 2 through 4 also include library fixed effects. Robust standard errors in parentheses.

Table 5: Demand estimates

|  | $(1)$ | $(2)$ | $(3)$ |
| :--- | :---: | :---: | :---: |
|  | $\ln \left(s_{j}\right)-\ln \left(S_{0}\right)$ | $\ln \left(s_{f \mid G}\right)$ | $\ln \left(s_{j}\right)-\ln \left(S_{0}\right)$ |
| $\gamma^{\text {ebook }}$ | -0.0450 | -2.696 | -0.888 |
|  | $(0.00542)$ | $(0.00420)$ | $(0.102)$ |
| $\sigma_{2}$ | 0.983 |  | 0.671 |
|  | $(0.00189)$ |  | $(0.0378)$ |
| $\sigma_{1}$ | 0.995 |  | 0.937 |
|  | $(0.000866)$ |  | $(0.00721)$ |
| $\ln \left(N_{f} / N\right)$ |  | 0.194 |  |
|  |  | $(0.00313)$ |  |
| Library FEs | Yes | Yes | Yes |
| Year FEs | Yes | Yes | Yes |
| Observations | 100846 | 100846 | 100846 |
| $\overline{R^{2}}$ | 0.985 | 0.829 | 0.960 |

Notes: Results from nested logit demand estimations. The dependent variable in columns 1 and 3 is $\ln \left(s_{j}\right)-\ln \left(S_{0}\right)$. Column 2 reports a first stage. All specifications include library and year fixed effects. In column 3 the group share $s_{f \mid \text { inside }}$ is instrumented using the number of products in the format relative to the number of products overall $\left(N_{f} / N\right)$. The parameter $\sigma_{2}$ is the between-format substitution parameter, while $\sigma_{1}$ describes within-format substitution. Robust standard errors in parentheses.

Table 6: Counterfactual prices and \% changes in holdings, circulation, and CS

| raise: | raise 10\% |  | raise 100\% |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $P^{e}$ | $P^{p}$ | $P^{e}$ | $P^{p}$ |
| print holdings | 0.73\% | -9.74\% | 5.03\% | -52.75\% |
| ebook holdings | -10.82\% | 1.84\% | -56.53\% | 14.28\% |
| print circulation | 0.25\% | -0.73\% | 1.72\% | -5.36\% |
| ebook circulation | -2.04\% | 1.60\% | -14.00\% | 12.08\% |
| total circulation | -0.05\% | -0.43\% | -0.34\% | -3.07\% |
| visits | 0.39\% | -2.36\% | 2.31\% | -12.85\% |
| librarian utility | -0.13\% | -0.35\% | -0.91\% | -2.44\% |
| CS | -0.05\% | -0.48\% | -0.37\% | -3.41\% |
| $\Delta$ CS ratio | 8.94 |  | 9.15 |  |

Notes: Cross-library average effects of various changes in electronic or physical book prices on holdings, circulation, and CS. All values are percentage changes from the baseline and based on initial ebook prices related to physical book prices via $P^{E}=2 / 3 P^{P}$. We include all libraries that solved to their observed values for 2018 and solved to interior solutions (positive ebook and physical holdings) at all counterfactual prices.

Table 7: Counterfactual results by income and urban/rural status
Panel A: Income quintile

| double: | Lowest |  | $2^{\text {nd }}$ |  | $3^{r d}$ |  | $4^{\text {th }}$ |  | Highest |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $P^{e}$ | $P^{p}$ | $P^{e}$ | $P^{p}$ | $P^{e}$ | $P^{p}$ | $P^{e}$ | $P^{p}$ | $P^{e}$ | $P^{p}$ |
| visits | 4.67\% | -18.33\% | 3.28\% | -14.79\% | 2.19\% | -12.99\% | 2.00\% | -11.39\% | 1.67\% | -11.33\% |
| CS | -0.23\% | -3.02\% | -0.22\% | -2.96\% | -0.35\% | -2.86\% | -0.38\% | -2.69\% | -0.35\% | -2.72\% |
| $\Delta$ CS ratio | 13.23 |  | 13.70 |  | 8.20 |  | 7.02 |  | 7.82 |  |

Panel B: Urban/rural

| double: | Large city |  | Medium city |  | Town |  | Rural |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $P^{e}$ | $P^{p}$ | $P^{e}$ | $P^{p}$ | $P^{e}$ | $P^{p}$ | $P^{e}$ | $P^{p}$ |
| visits | 1.62\% | -11.74\% | 2.13\% | -11.86\% | 4.41\% | -15.97\% | 8.40\% | -21.75\% |
| CS | -0.45\% | -3.31\% | -0.30\% | -3.49\% | -0.26\% | -3.56\% | -0.29\% | -3.49\% |
| $\Delta$ CS ratio | 7.30 |  | 11.75 |  | 13.62 |  | 12.16 |  |

Notes: Cross-library average effects of various changes in electronic or physical book availability on visits and CS, for groups of libraries based on median patron income (Panel A) and urban status (Panel B). All values are percentage changes from the baseline and based on initial ebook prices calculated as the average expenditure / holdings for the respective library groups reported in Table 2. We include all libraries that solved to their observed values for 2018 and solved to interior solutions (positive ebook and physical holdings) at all counterfactual prices.

## A Appendix

This appendix discusses two aspects of the model solution, constancy of the assumed relationship between $P^{E}$ and $P^{P}$, and corner solutions.

## A. 1 The assumed relationship between $P^{E}$ and $P^{P}$

In our baseline approach, we solve the model using a single relationship between $P^{E}$ and $P^{P}$, in particular that $P^{E}=2 / 3 P^{P}$ for all libraries. This is correct on average, but given the information in Table 2, one can note that the relationship varies across communities according to income and rural/urban status. Accordingly, for the "heterogeneity" simulations, we solve the model using group-specific relationships $P^{E}=\kappa P^{P}$. For income quintiles, $\kappa$ ranges from 0.31 for the lowest income quintile to 1.31 for the highest. In the urban/rural simulation, $\kappa$ varies between 0.14 for the rural areas and 1.31 for the most urban areas.

In each simulation, the choice of the price relationship determines the $\theta$ estimates showing librarians' implied weights on electronic, relative to physical, circulation. Allowing the price relationship to vary across groups of library systems gives rise to both less variation in $\theta$ and a lower distribution of values than in the baseline. While the interquartile range for $\theta$ runs from 1.28 to 2.69 in the baseline, the range is between 1.10 and 2.13 in the income simulation and between 1.04 and 2.25 in the urban/rural simulation.

Although prices and implied $\theta$ parameters vary across initial price choices, the results of interest do not vary. In particular, the ratio of the change in CS from a doubling in physical, relative to electronic, book prices is 9.15 in the baseline; and the estimated ratios are 8.29 and 8.92, respectively, in the income and urban/rural simulations. Hence, our baseline results are not driven by the common assumed relationship between $P^{E}$ and $P^{P}$.

## A. 2 Corner solutions

In our baseline approach (with a common price relationship), the model solves to the observed values of holdings in 94.3 percent of library systems. Of the libraries that solve to baseline observed holdings, 0.27 percent solve to corners (with zero ebook holdings) when doubling the ebook prices, while doubling the physical book price leads to zero physical book corners in 15.92 percent of libraries. For the baseline approach, we exclude all systems that a) do not solve to the observed values in the baseline, and that b) solve to corners with price doubling in either format. This excludes 16.19 percent of library systems.

When we include the systems solving to corners, the results are somewhat larger: for example, rather than reducing CS by 3.41 percent, a doubling of $P^{P}$ reduces CS by 4.79 percent. Still, the ratios are nearly unchanged: The $\Delta \mathrm{CS}$ ratio is 9.59 including corners and 9.15 without them.


[^0]:    ${ }^{1}$ In 2018, US libraries reported print circulation of 1.87 billion units, while Nielsen reported 698 million print unit sales. Libraries reported ebook circulation of 294 million, compared with Nielsen's 168 million ebook sales. See https://www.statista.com/statistics/422595/print-book-sales-usa/, http s://www.statista.com/statistics/191992/sales-of-e-books-in-the-us-since-2006/, and https://www.imls.gov/sites/default/files/2021-02/fy2018_pls_tables.pdf.
    ${ }^{2}$ The first sale doctrine stipulates that "an individual who knowingly purchases a copy of a copyrighted work from the copyright holder receives the right to sell, display or otherwise dispose of that particular copy, notwithstanding the interests of the copyright owner." See https://www.justice.gov/archives/jm/crim inal-resource-manual-1854-copyright-infringement-first-sale-doctrine.

[^1]:    ${ }^{3}$ Recent research documents effects of libraries on education and innovation outcomes. See Gilpin et al. (2021) and Karger (2021) on library impacts on education outcomes and Furman et al. (2021) on patenting.

[^2]:    ${ }^{4}$ The quote comes from the dedication of the 1903 Carnegie Library in Washington DC. See https: //www.npr.org/2013/08/01/207272849/how-andrew-carnegie-turned-his-fortune-into-a-library -legacy.
    ${ }^{5}$ See https://dp.la/exhibitions/history-us-public-libraries/carnegie-libraries.
    ${ }^{6}$ See https://www.presidency.ucsb.edu/documents/statement-the-president-upon-signing-the -library-services-and-construction-act-0.
    ${ }^{7}$ See https://www.ala.org/news/sites/ala.org.news/files/content/2019-soal-report-final-a ccessible.pdf.
    ${ }^{8}$ See https://www.brookings.edu/blog/up-front/2017/03/30/how-public-libraries-help-build -healthy-communities/ as well as Klinenberg (2018).
    ${ }^{9}$ https://www.ala.org/news/sites/ala.org.news/files/content/2019-soal-report-final-accessible.pdf.
    ${ }^{10}$ Shiller (2013) models the effect of the first sale doctrine on the market for video games.
    ${ }^{11}$ The theoretical frameworks relevant to impacts of library lending on purchase mirror those in the literatures on piracy and the sale vs renting of information. See Varian (2000), Bakos et al. (1999), and Smith and Telang (2016). In an empirical study of Japan, Kawaguchi and Kanazawa (2022) find that an instance of library circulation of physical books displaces a quarter to a half of a sale. By contrast, Nagaraj and Reimers (2021) find that library availability via Google Books can stimulate demand.

[^3]:    ${ }^{12}$ See https://www.publishersweekly.com/pw/by-topic/industry-news/bookselling/article/805 92-industry-sales-dipped-in-2018.html and https://publishers.org/data-and-statistics/ind ustry-statistics/.
    ${ }^{13}$ While title-level holdings and circulation data are not generally available, the Seattle Public library releases its statistics. Among top titles, ebook holdings are as extensive as their physical holdings: The library held 48 of the 2018 USA Today top 50 titles as ebooks and 47 as print books. The Seattle data are available at https://data.seattle.gov/widgets/tmmm-ytt6.
    ${ }^{14}$ In 2018, libraries spent $\$ 1.4$ billion on physical and electronic holdings. See https://www.imls.gov/s ites/default/files/2021-02/fy2018_pls_tables.pdf. Based on calculations from our dataset, ebooks accounted for 35.7 percent of the 2018 library holdings costs, or $\$ 490$ million.

[^4]:    ${ }^{15}$ See https://www.libraryjournal.com/?detailStory=publishers-change-ebook-and-audiobook -models-libraries-look-for-answers.
    ${ }^{16}$ See https://d1x9nywezhk0w2.cloudfront.net/wp-content/uploads/2019/10/29160131/A-Letter -from-John-Sargent-.pdf.
    ${ }^{17}$ See https://americanlibrariesmagazine.org/blogs/the-scoop/ala-responds-to-macmillan-le tter/.
    ${ }^{18}$ See https://nces.ed.gov/programs/digest/1999menu_tables.asp and https://www.ala.org/to ols/research/librarystats/public/publiclibraries.
    ${ }^{19}$ See https://company.overdrive.com/company-profile/who-we-are/.

[^5]:    ${ }^{20}$ See the "OverDrive Marketplace User Guide," available online at https://manualzz.com/download/ 7173254.
    ${ }^{21}$ See https://www.forbes.com/sites/rachelkramerbussel/2019/09/20/why-this-librarian-is-u sing-twitter-to-fight-changes-to-library-ebook-pricing-terms/and https://smartbitchestr ashybooks.com/2020/09/hold-on-ebooks-cost-how-much-the-inconvenient-truth-about-libraryecollections/.

[^6]:    ${ }^{22}$ See https://www.imls.gov/research-evaluation/data-collection/public-libraries-survey.
    ${ }^{23}$ Our electronic holdings are primarily ebooks ( 73 percent) but also include audiobooks and video holdings. The vast majority ( 88 percent) of our physical holdings are books.

[^7]:    ${ }^{24} \mathrm{~A}$ comparison with physical books is instructive. Because libraries purchase and possess their physical holdings, we would expect physical expenditures to depend on holdings. A regression of physical expenditures on physical holdings and circulation confirms this, delivering a holdings coefficient more than twice the circulation coefficient.
    ${ }^{25}$ One can view our price per holding as a shorthand for the relationship between ebook holdings and circulation and therefore expenditure.

[^8]:    ${ }^{26}$ We obtain annual place-level median income from the American Community Survey and urban/rural status from the IMLS data.

[^9]:    ${ }^{27}$ The resulting coefficients are quite similar to the Wald IV estimates calculated as ratios of the coefficients in columns 7 and 8 to those in columns 6 and 7 (see Angrist and Krueger, 1991). The coefficients differ from the direct ratios because we have two rather than one instrument per equation.

[^10]:    ${ }^{28}$ The elimination of all ebooks would imply between 4.0 and 6.1 percent more visits in 2018. Actual ebook holdings in 2013 were 23 percent of their level in 2018. Hence, the growth in ebooks between 2013 and 2018 would account for 77 percent times 4.0 to 6.1 percent, or about a third of the 10.9 percent decline in visits.
    ${ }^{29}$ This nesting structure is natural given the bifurcated way that patrons choose books. Without coming to the library, patrons can select among electronic titles. Physical titles, on the other hand, are more easily

[^11]:    ${ }^{31}$ Formally, we obtain the welfare weights using: $\left[\begin{array}{l}1 / \lambda^{\prime} \\ \theta / \lambda^{\prime}\end{array}\right]=\left[\begin{array}{ll}\frac{\partial Q^{P}}{\partial N^{P}} & \frac{\partial Q^{E}}{\partial N^{P}} \\ \frac{\partial Q^{P}}{\partial N^{E}} & \frac{\partial Q^{E}}{\partial N^{E}}\end{array}\right]^{-1}\left[\begin{array}{l}P^{P} \\ P^{E}\end{array}\right]$. Because the $\lambda^{\prime}$ terms cancel, $\theta$ is a function of just the derivatives and prices.

[^12]:    ${ }^{32}$ We also estimate a negative coefficient for ebooks, indicating that ebook holdings have lower mean utility than physical holdings.
    ${ }^{33}$ In our main results, we use a common relationship between $P^{E}$ and $P^{P}\left(P^{E}=2 / 3 P^{P}\right)$. We also solved the model using prices specific to groups of library systems by community income and urban/rural status. Results, which we report in Appendix Section A.1, are nearly identical.

[^13]:    ${ }^{34}$ To calculate the share of print holdings that circulate, we need a measure of the number of print books circulating each year, which is complicated by the facts that a) the Seattle data report circulation at the title level, and b) the library can have multiple copies of each title. Because a copy can circulate no more than 25 times per year, we estimate the number of circulating copies of a title as int(checkouts/25)+1. The sum of this across titles is our measure of circulating print volumes, which we compare with the IMLS holdings measure.

[^14]:    ${ }^{35}$ We show in the Appendix that our main results hold with more refined price ratios.

[^15]:    ${ }^{36}$ These numbers differ slightly from those in Table 6 because these include the same libraries across counterfactuals, all of those that never solve to a corner solution. In Table 6, we keep more libraries, including some of those that solve to corners in counterfactuals with different $\sigma$ s. See Appendix Section A.2.

