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ABSTRACT

I generalize the canonical model--in which relative supply and demand for worker skills shape the skill premium--incorporating monopsony power, minimum wages, and unemployment. I estimate the extended canonical model using national data and, separately, state-level data. I show that incorporating the minimum wage improves the out-of-sample fit of the traditional canonical model. I document that minimum wages--together with supply and demand--play a central role in shaping the evolution of the U.S. college premium and the differential evolution of state-level college premia. Lending credibility to these conclusions, the state and national estimates are not only qualitatively, but also quantitatively consistent.

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1 Introduction

Overview. Why has U.S. inequality changed over time? The voluminous literature studying changes in the returns to skills and the evolution of earnings inequality in the United States can be separated into two essentially distinct strands. The neoclassical perspective highlights the roles of labor supply and demand, viewing the evolution of inequality as the outcome of “The Race Between Education and Technology,” in the words of [Goldin and Katz \(2008\)](#). An alternative perspective emphasizes changes in labor-market institutions, viewing the evolution of inequality as the outcome of “trends in the minimum wage, and other factors such as declining unionization,” in the words of [Card and DiNardo \(2002\)](#). While most economists view labor supply and demand and labor-market institutions as important drivers of inequality, these literatures have remained largely distinct in part because no simple framework synthesizes the two. This paper attempts to develop such a synthesis.

The *canonical model*—introduced by [Tinbergen \(1974\)](#), operationalized by [Katz and Murphy \(1992\)](#), and named by [Acemoglu and Autor \(2011\)](#)—provides the central organizing framework for the neoclassical literature. It straightforwardly relates relative wages of more and less educated workers to their relative supply and demand. Empirically, it has proven remarkably successful. I generalize this theoretical model to incorporate labor market institutions—monopsony power and minimum wages—and unemployment while maintaining its simplicity and tractability. I demonstrate that the extended model generates an estimating equation that is almost identical to the typical empirical implementation of the canonical model except for the presence of one additional term: the change in the real minimum wage. I also show that the reduced-form coefficients of the extended estimating equation have relatively simple theoretical interpretations.

Incorporating institutional features into the canonical model addresses a widely-known empirical shortcoming: I document that the out-of-sample fit of the model including the real minimum wage is improved relative to the model without. More importantly, I document empirically the importance of minimum wages for shaping national and state-level college premia. At the national level, I show that the real minimum wage plays a central role—together with supply and demand—in shaping the evolution of the U.S. college premium between 1963 and 2017. For example, the decline of the minimum wage in the 1980s and its subsequent rise explains a substantial share of the rapid rise in the college premium in the 1980s and its slower growth thereafter. Using the theory to guide a regional estimation of the extended canonical model, I additionally show that differential changes in real minimum wages and relative college supplies across states drive the dif-

ferential evolution of college premia across U.S. states. Minimum wages—together with supply and demand—play a first-order role in shaping national and regional inequality. Results leveraging regional variation are not only qualitatively, but also quantitatively consistent with results leveraging national variation, further lending credibility to each.

Details. In Section 2, I estimate an extended version of the canonical model incorporating the real minimum wage, using data spanning 1963 to 2017. I measure the real minimum wage using either the federal, Fair Labor Standards Acts (FLSA), minimum wage or a weighted average of state-specific real minimum wages. I define skilled workers as those with four or more years of college education, composition adjust the relative wage of skilled to unskilled workers, and construct their relative supply as the dual to exactly match their relative labor incomes in each year.

Estimated over the full sample period, I find that the elasticity of the college premium (the relative wage of college to non-college workers) with respect to the real minimum wage is between -0.14 and -0.20 . The coefficient estimate and its significance are robust to incorporating high-order polynomials of time and instrumenting for relative supply. This range of point estimates implies (for example) that the 27% decline in the real minimum wage between 1979 and 1989 caused between a 3.7% and a 5.3% increase in the skill premium over this time period, which is about a third of the observed increase. I find that the elasticity of the skill premium with respect to the relative supply of college is between -0.53 and -0.60 . This implies that the slowdown in the growth rate of the relative supply of college between 1979 and 1989, relative to its growth rate between 1969 and 1979, raised the skill premium by between 6.4% to 7.3% in the latter period, which is about half of the observed increase. Empirically, supply, demand, and the minimum wage each play important roles in the evolution of the U.S. skill premium in the 1980s and, more generally, across the full sample of years.

Introducing the real minimum wage additionally helps ameliorate a well-known issue: the out-of-sample fit of the canonical model is poor. In particular, the canonical model estimated on data spanning 1963 - 1987 (the set of years used in the seminal work of [Katz and Murphy, 1992](#)) predicts substantially more rapid increases in the skill premium than actually occur in the data thereafter. This issue is mitigated by incorporating the real minimum wage. Its rise in the later period reduces the predicted growth rate of the skill premium.

In Section 3, I present a generalized version of the canonical model that microfound the national regression analysis of Section 2, provides structural interpretations of the reduced-form coefficients, and guides my state-level regression analysis in Section 4. My objective is to maintain the simplicity and tractability of the original framework. In addi-

tion to supply and demand, the model incorporates monopsony power, minimum wages, and unemployment. As in the canonical model, the aggregate production function combines the output of low- and high-skill output and exhibits constant returns to scale. Unlike the canonical model, firms have monopsony power in the labor market. Each worker is characterized not only by an arbitrary worker-specific productivity, but also by an arbitrary vector of amenity values across firms. In equilibrium, if a given worker is employed she works for the firm with which she is best matched. In the absence of a minimum wage, this firm would pay her less than the value of her marginal product, with her *wage markdown* (the ratio of her value marginal product to her wage) determined by the ratio of her best and second-best match qualities. The model additionally incorporates a minimum wage. In the presence of a positive minimum wage, the lowest productivity workers may be unemployed: a worker is unemployed if her value marginal product is below the minimum wage. The minimum wage additionally constrains firms' monopsony power: if employed, a worker's wage is the maximum of the minimum wage and the wage the firm would have paid in its absence.

Theoretically, I consider the impact of changes in the productivities and supplies of low- and high-skill workers, as in the canonical model, as well changes in the real minimum wage. Without imposing any restrictions on worker productivities, worker-firm-specific match qualities, or the aggregate production function, I express changes in the skill premium, to a first-order approximation, as a function of these shocks. The expression is similar to the canonical model except for the inclusion of one additional term: the real minimum wage.¹ I additionally provide structural interpretations of the reduced-form parameters in this equation. In slightly simplified versions of the model, these interpretations are particularly clear.

For example, if the minimum wage has no effect on unemployment locally, then the theory predicts that the elasticity of the average wage across an arbitrary group of workers with respect to the real minimum wage is given by the share of that group's salary income earned by minimum wage workers, which I refer to as the *bite* of the minimum wage for that group.² This implies that the elasticity of the college premium with respect to the real minimum wage is simply its bite among high- minus its bite among low-education workers.

¹To a first-order approximation, the change in the log of the skill premium must be a linear function of the underlying shocks. Because the aggregate production function exhibits constant returns to scale, the ratio of high-to-low skill labor is a sufficient statistic for each (as in the canonical model).

²This result holds regardless of the number of skills (two in the canonical model) and the aggregate production function (CES in the canonical model) that combines them. The result microfound the assumed impact of a change in the minimum wage on the full distribution of income (or the distribution within any arbitrary group of workers) in the famous empirical decomposition of DiNardo et al. (1996).

Finally, in Section 4 I estimate a *regional* extended canonical model leveraging variation across U.S. states. I assume state-specific labor markets, with each state’s own labor supply, labor demand, and minimum wage shaping its own wages. Using the theoretical results of Section 3, I regress changes over time in state-level college premia on changes in state-level relative college supply, a time effect that absorbs national skill-biased technical change, and the interaction between the state-level bite of the minimum wage and the change in the state’s minimum wage (instrumenting for this minimum wage interaction using two distinct approaches).³ I find that differential changes in state-level minimum wages (interacted with their initial bite) and relative college supplies drive the differential evolution of college premia across U.S. states. Moreover, the state-level estimates are qualitatively and quantitatively consistent with the national time series estimates, lending credibility to each.

Additional Literature. My economic conclusions differ from Autor et al. (2008), who estimate a related reduced-form regression including the real minimum wage on the national time series. They find that the coefficient on the real minimum wage is negative, significant, and of a similar magnitude to my estimates in one of their two specifications (with a linear time trend) but smaller and insignificantly different from zero in their other specification (with a cubic time trend). Based on these results, they conclude that the real minimum wage “does not much alter the central role for relative supply growth fluctuations and trend demand growth in explaining the evolution of the college wage premium” and that “institutional factors are insufficient to resolve the puzzle posed by slowing trend relative demand for college workers in the 1990s.” My empirical results are robust to the degree of the polynomial of time in my data (as I show in Section 2 and the Empirical Appendix). These results also hold using Autor et al.’s data across all specifications that vary in the polynomial of time except the cubic one (as I show in the Empirical Appendix). And they additionally hold using state-level variation. Based on these results, I conclude that relative supply growth fluctuations and trend demand growth remain crucial drivers of the skill premium, consistent with Autor et al. (2008), but so too are changes in the real minimum wage. I also conclude that changes in the minimum wage are sufficient to resolve the puzzle posed by slowing trend relative demand for college workers in the 1990s.

A growing literature extends the neoclassical approach to study the allocation of labor (and potentially capital) across heterogeneous tasks (e.g., Caselli and Coleman, 2001; Autor et al., 2003; Autor and Dorn, 2013; Buera et al., 2015; Burstein et al., 2019; Ace-

³I additionally include a state effect and state-level time trends that absorb state-specific linear and higher-order deviations from national skill-biased technical change.

moglu and Restrepo, 2020; Oberfield and Raval, 2021). This approach has the added benefit of linking changes in inequality with changes in factor allocations across sectors, occupations, and/or firms and broadening the nature of technical change. My work largely abstracts from these changes in allocations and maintains a factor-biased perspective on technical change. Complementing this predominantly neoclassical literature is a vast body of work studying the roles of labor-market institutions for shaping inequality. Much of the focus is on minimum wages (e.g., DiNardo et al., 1996; Lee, 1999; Card and DiNardo, 2002; Dube, 2019) and monopsony power (see Manning, 2013 for a summary). My theory is broadly related to a growing macro-labor literature studying the implications of minimum wages in quantitative models of the labor market (e.g., Haanwinckel, 2020; Engbom and Moser, 2021; Ahlfeldt et al., 2022; Berger et al., 2022; Hurst et al., 2022; Trottnner, 2022). Relative to each of these literatures, I theoretically and empirically synthesize the neoclassical and institutional approaches and provide a unified and consistent analysis of the evolution of inequality at the national and regional levels.⁴

This allows me to reach a number of conclusions. First, empirically, the joint evolution of observable supplies and the real minimum wage, together with a linear evolution of skill-biased technology, each play important roles in shaping the evolution of the U.S. college premium between 1963 and 2017. Second, empirically, incorporating the real minimum wage implies that the evolutions of relative demand and relative college supply play not only a central role in shaping the national college premium (as previously noted by the vast literature) but also that this relationship is more stable over time than previously understood. Third, empirically, the differential evolution of observable supplies and the real minimum wage (together with its bite) play important roles in shaping the differential evolution of college premia across U.S. states. Fourth, empirically, the impact of changes in supply and minimum wages on college premia are remarkably consistent at the state and national levels. Finally, theoretically, the minimum wage can help explain changes in real wages (e.g., real wage declines in the 1980s for high school dropouts and high school graduates) and more disaggregated measures of inequality (e.g., wage polarization beginning in the 1990s).

Of course, my synthesis of institutional features into the canonical model is far from exhaustive. Among other omissions, I abstract from unions and changes in unionization rates (Farber et al., 2021).

⁴See Bound and Johnson (1992) and Katz and Autor (1999) for early attempts to analyze changes in wage structure incorporating both institutions and labor supply and demand into integrated empirical frameworks, although not in integrated theoretical models.

2 Empirics leveraging national time series variation

I consider regressions of the form

$$\log \left(\frac{w_{ht}}{w_{\ell t}} \right) = \alpha + \beta_m \log m_t + \beta_L \log \left(\frac{Supply_{ht}}{Supply_{\ell t}} \right) + \gamma_1 t + [\gamma_2 t^2 + \gamma_3 t^3 + \dots] + \iota_t \quad (1)$$

Here, $\log w_{ht}$ and $\log w_{\ell t}$ are measures of average log wages of high- and low-education workers at the national level; $\log Supply_{ht}$ and $\log Supply_{\ell t}$ are measures of high- and low-education labor supply at the national level; m_t is a measure of the real minimum wage at the national level; and t is time with $t = 1$ in the first sample year. I report robust standard errors in all regressions.

Measurement. Here I describe how I measure each variable; see the Empirical Appendix for details. I restrict my sample to the working-age population of those between 16 and 64 years old and define high-education workers as those with 16 or more years of education and low-education workers are those with fewer than 16 years of education. I construct the composition-adjusted college premium by first measuring the average log hourly wage within each of 180 groups (defined by the intersection of 9 age bins, 2 genders, 2 races, and 5 education levels) in year t and then averaging across those groups with college education and those groups without using time-invariant weights. I refer to the relative wage as both the college premium and the skill premium. I measure the supply of college and non-college workers as the dual of these wages, such that the product of composition-adjusted supply and wages equals the observed total income of college and non-college workers in each year. I measure these variables using the March Annual Demographic Files of the Current Population Survey from 1964 to 2018, which report earnings from 1963 to 2017.

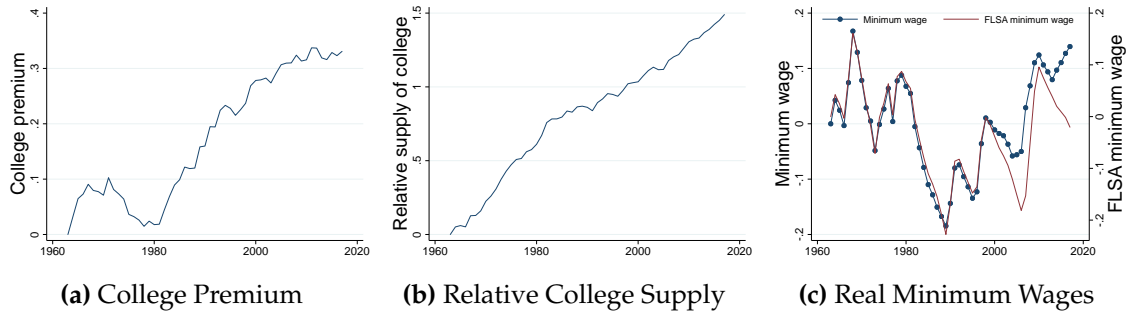
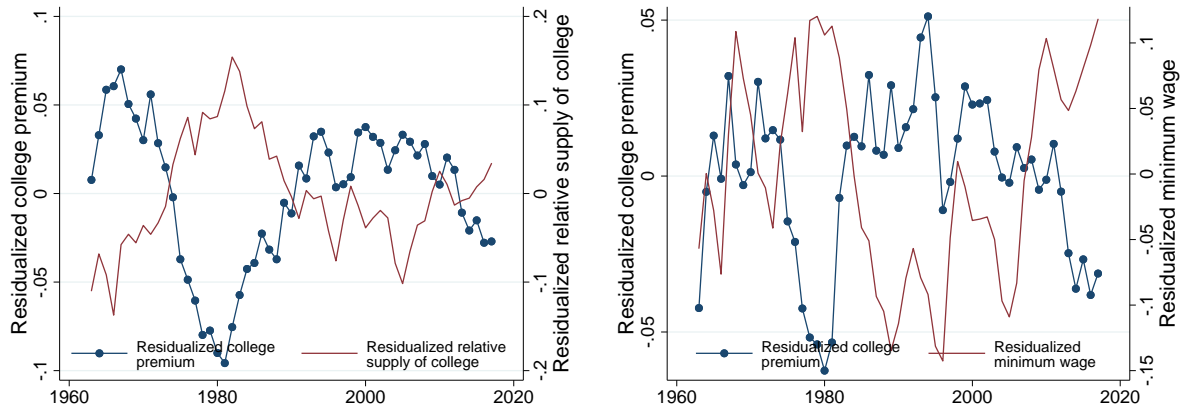


Figure 1: Relative Wages, Relative Supplies, and the Minimum Wage

Notes: Panels (a) and (b) display the composition-adjusted college premium and relative supply of college. Panel (c) displays the real federal (FLSA) minimum wage and the real national minimum wage series, which averages minimum wages across states; both series are deflated by the GDP deflator. All series are normalized to zero in 1963.

I measure the minimum wage in year t two ways. In one approach, I use the federal (FLSA) minimum wage. In my baseline, I use the average minimum wage across states (the maximum of the legislated state and federal minimum wages), using time-invariant weights. I deflate each series using the GDP deflator. I refer to my baseline measure as the real national minimum wage and to my alternative measure as the real federal (or FLSA) minimum wage.

Panels (a) and (b) of Figure 1 display the college premium and relative supply of college workers, both normalized to zero in 1963. The particularly steep rise in the college premium starting in the early 1980s coincides with a decline in the growth rate of the relative supply of college workers, a fact first emphasized in [Katz and Murphy \(1992\)](#). Panel (c) of Figure 1 displays my two real minimum wage series, each normalized to zero in 1963. The real national and FLSA minimum wage series move in lockstep until the late 1980s and diverge thereafter as more states set minimum wages above the federal level. Both real minimum wage series decline dramatically in the 1980s, as fixed nominal minimum wages are eroded by inflation. Finally, there is substantial time variation in both real minimum wage series, which remain even after residualizing on a high-dimensional polynomial of time.



(a) Residualized Relative Supply, College Premium (b) Residualized Minimum Wage, College Premium

Figure 2: Residualized Independent and Dependent Variables

Notes: Panel (a) displays the composition-adjusted college premium and relative supply, both residualized of the real minimum wage and a linear time trend. Panel (b) displays the composition-adjusted college premium and real national minimum wage, both residualized of composition-adjusted relative supply and a linear time trend.

Results. Before providing estimation results, I display the variation in the data that identifies the parameters of interest. Panel (a) of Figure 2 displays the college premium and relative supply of college, each residualized of the real national minimum wage and a linear time trend. Panel (b) displays the college premium and real national minimum wage, each residualized of the relative supply of college and a linear time trend. The variation in Panel (a) identifies β_L and the variation in Panel (b) identifies β_m when estimating regression (1) using OLS. There is a striking negative relationship in each panel. These qualitative patterns are robust to residualizing on high-dimensional polynomials of time.

Column (a) of Table 1 displays results of estimating regression (1) using OLS including a linear time trend and omitting the real national minimum wage on the sample 1963 - 1987. This is the specification and sample years included in the seminal work of [Katz and Murphy \(1992\)](#). Column (b) replicates this analysis, but using the full sample of 1963 - 2017. Consistent with past work, estimates are unstable across samples. The growth rate of skill-biased technical change γ falls from 2.36% per year to 1.81%; and the coefficient on relative supply β_L rises from -0.596 to -0.441 when changing the sample from the Katz and Murphy years to the full sample; in spite of differences in data cleaning and measurement, these results are very similar to [Acemoglu and Autor \(2011\)](#). The model estimated on the 1963 - 1987 sample systematically deviates from the data thereafter, predicting a sharper rise in the college premium than actually occurs.

Introducing the real minimum wage goes some way towards fixing this well-known issue. Columns (c) and (d) replicate Columns (a) and (b), but additionally include the

	1963-1987	1963-2017	1963-1987	1963-2017			
	(a)	(b)	(c)	(d)	(e)	(f)	(g)
Relative supply	-0.596 (0.116)	-0.441 (0.044)	-0.594 (0.137)	-0.556 (0.054)	-0.600 (0.052)	-0.531 (0.102)	-0.502 (0.099)
Real national mw.			-0.237 (0.091)	-0.195 (0.040)	-0.159 (0.059)	-0.136 (0.055)	
Real federal mw.							-0.095 (0.043)
Time	0.024 (0.005)	0.018 (0.001)	0.022 (0.006)	0.021 (0.001)			
Constant	0.013 (0.013)	0.024 (0.007)	0.035 (0.021)	0.032 (0.009)	0.021 (0.014)	0.034 (0.029)	0.035 (0.028)
Time Polynom.	1	1	1	1	2	3	3
Observations	25	55	25	55	55	55	55
R-squared	0.324	0.950	0.517	0.968	0.969	0.969	0.968

Table 1: Regression Models for the College Wage Premium

Notes: Results of estimating (1) using OLS. The dependent variable and “Relative supply” are the logs of the composition-adjusted college premium and relative supply of hours worked. “Real national mw.” and “Real federal mw.” are the logs of the real national (averaging effective minimum wages across states) and real federal (FLSA) minimum wages. The sample is 1963-1987 in columns (a) and (b) and 1963-2017 elsewhere. “Time Polynom.” refers to the degree of the polynomial of time; the coefficient on the linear trend on “Time” is omitted from the table whenever this polynomial is of degree 2 or more.

value of the real national minimum wage. Including the national minimum wage leads to stable parameter estimates across samples, a result that is robust to varying the sample years. The growth rate of skill-biased technical change γ and the coefficient on relative supply β_L are 2.22% and -0.594 when estimated on the 1963 - 1987 sample and are 2.11% and -0.556 when estimated on the full sample.

The remaining columns (e), (f), and (g) are estimated on the full sample of years and include higher-dimensional polynomials of time. Column (g) replaces the real national minimum wage (which averages minimum wages across states) with the federal (FLSA) minimum wage. My baseline specifications leverage the full sample of data and use the real national minimum wage: columns (d), (e), and (f).

The estimates in Table 1 highlight the importance of supply (education), demand (technology), and the minimum wage (institutions) in shaping the evolution of the U.S. college premium. In the baseline specifications, the elasticity of the college premium with respect to the real national minimum wage ranges between -0.136 and -0.195 . This elasticity implies that the 27% decline in the real national minimum wage between 1979 and 1989 caused between a 3.7% and a 5.3% increase in the college premium over this time period, which is between three to four tenths of the observed 13.4% increase. The elasticity of the college premium with respect to relative supply ranges between -0.531 and -0.600 . This elasticity implies that the slowdown in the growth rate of the relative supply of college between 1979 and 1989, relative to the growth rate between 1969 and 1979, raised the college premium by approximately 6.4% to 7.3% in the latter period, which is around one half of the observed 13.4% increase. All three forces—supply, demand, and the minimum wage—play important roles not only in this decade, but also throughout the sample.

Summary. In summary, introducing the real minimum wage into the canonical model’s estimating equation demonstrates that supply, demand, and the minimum wage each play central roles in shaping the evolution of the U.S. college premium. Moreover, incorporating the minimum wage improves the out-of-sample fit of the model.

Robustness. Here, I briefly describe results of four types of robustness exercises. I consider the robustness of results to higher-dimensional polynomials of time, defining high-education workers differently, instrumenting for relative supply, and using data from Autor et al. (2008)’s replication package. Details are provided in the Empirical Appendix.

Appendix Table A.1 extends this analysis including progressively higher-dimensional polynomials of time. Results on the impact of the minimum wage are largely robust up to an eighth degree polynomial (so are unreported results from a double-lasso analysis).

Appendix Table A.2 replicates Appendix Table A.1, but defining high-education work-

ers as those with some college and up and defining low education workers as those with high school degrees and below. Results on the impact of the minimum wage are largely robust, with the only exception being the case of a third-degree polynomial of time.

My measure of relative supply—as well as the standard measure in the literature—depends on (efficiency unit) hours worked. Whereas it may be reasonable to assume that population education levels do not respond to deviations from a time trend (whether linear, quadratic, cubic...) in relative demand for high- and low-education workers, the same need not be true for hours worked. Since these deviations in relative demand enter the residual in equation (1) both in the canonical model and in my extended model in Section 3, I also instrument for relative supply using the relative population of college-educated among the working age population. Appendix Table A.3 displays results of estimating the reduced-form and two-stage least squares specifications. The first stage is strong and instrumenting leaves the results broadly unchanged.⁵

Finally, Appendix Table A.4 shows that my conclusions on the importance of the minimum wage for the U.S. college premium holds using data from Autor et al. (2008)'s replication package. The only exception is the case of a third-degree polynomial of time.

3 Extending the Canonical Model

My objectives are fourfold: (i) to provide a simple extension of the theoretical framework referred to as the canonical model that incorporates monopsony power, minimum wages, and unemployment; (ii) to derive equation (1) estimated above in a general formulation; (iii) to study simple examples in which the resulting reduced-form coefficients have particularly clear and intuitive structural interpretations; and, finally, (iv) to derive a version of equation (1) that can be estimated using regional variation.

Setup. The economy features two labor skills indexed by s , high h and low ℓ , and two types of firm, one hiring only high- and the other hiring only low-skilled labor.⁶ There is a continuum of skill s labor, indexed by $\omega \in \Omega_s$, with measure $L_s \equiv |\Omega_s|$, and there are

⁵Conditional on including a cubic polynomial of time, the first-stage suggests no effect of the real national minimum wage on relative supply. This result is relevant to the structural interpretation of the reduced-form coefficients reported above—according to the model of Section 3—and broadly consistent with the empirical literature; see, e.g., Dube et al. (2010).

⁶The framework can easily be extended to incorporate many skill groups, as in extensions of the canonical model such as Card and Lemieux (2001), as shown in the Theoretical Appendix. If there is no (un)employment effects of the minimum wage, then the impact of the minimum wage across arbitrary groups of workers is invariant to the number of assumed labor skills and the functional form of the aggregate production (two issues that are crucially important in the analysis of the impact of changes in labor supply, as discussed in Card, 2012).

$N_s \geq 2$ skill s firms, indexed by $n \in \mathcal{N}_s$. Each skill s worker is characterized by a vector ε_ω with N_s elements $\varepsilon_{\omega n} > 0$, where $1/\varepsilon_{\omega n}$ denotes a worker-firm-specific amenity or compensating differential. Each worker is also characterized by a scalar $\phi_\omega > 0$ number of efficiency units. I impose no restrictions on the joint distribution of ϕ_ω and the vector ε_ω .

High- and low-skill workers are imperfect substitutes in the production of aggregate output, Y , which is produced by combining skilled and unskilled output according to a constant returns to scale production function,

$$Y = Y(Y_\ell, Y_h)$$

Skill s output, Y_s , is the sum of output across firms, $Y_s = \sum_{n \in \mathcal{N}_s} Y_{sn}$, where Y_{sn} is the output of firm n . Firm n 's output is $Y_{sn} = A_s \sum_{\omega \in \Omega_{sn}} \phi_\omega$ where $A_s > 0$ is skill-specific productivity common across all firms and Ω_{sn} is the set of skill s workers employed by firm n . The price of the final good is the numeraire and P_s is the endogenous real price of each unit of Y_s .

Given a vector of real wages offered to worker ω , one $w_{\omega n}$ for each firm n , her utility if working for firm n is given by $w_{\omega n}/\varepsilon_{\omega n}$. The exogenous real minimum wage is $m \geq 0$. Firm n can employ worker ω only if $w_{\omega n} \geq m$.

All skill s firms observe ε_ω and ϕ_ω for all workers and simultaneously make take-it-or-leave-it wage offers to each worker. Each worker then chooses to work for the firm that maximizes her utility, conditional on that wage being no less than the minimum wage. If no wage offer is as large as the minimum wage, $\max_n \{w_{\omega n}\} < m$, she is unemployed. I look for a SPNE in weakly undominated strategies.

The model reduces to the canonical model under three restrictions: (i) if $\varepsilon_{\omega n} = \varepsilon_{\omega n'}$ for all ω and each n, n' ; (ii) the minimum wage is set to zero, $m = 0$; and (iii) the aggregate production function takes the constant elasticity of substitution form.

Equilibrium. In general equilibrium, skill prices are given by $P_s = \partial Y / \partial Y_s$. Equilibrium characterization then boils down to determining the partial equilibrium allocation of workers across firms and worker wages for a given skill price P_s . I turn to this now.

Define

$$n_1(\omega) \equiv \arg \min_{n \in \mathcal{N}_s} \{\varepsilon_{\omega n}\} \quad \text{and} \quad n_2(\omega) \equiv \arg \min_{n \in \mathcal{N}_s / n_1(\omega)} \{\varepsilon_{\omega n}\}$$

as the firms that are the best and second-best matches for worker ω and define the scalar

$$\epsilon_\omega \equiv \varepsilon_{n_1(\omega)\omega} / \varepsilon_{n_2(\omega)\omega} \leq 1$$

Characterizing equilibrium requires only a joint density over the two scalars, $\epsilon \leq 1$ and $\phi \geq 0$, rather than over all values of the vector ϵ and ϕ . Let $g_s(\epsilon, \phi)$ be the joint density of ϵ and ϕ for skill s . In what follows, I impose the technical restriction that the marginal distribution of ϕ has no mass points.

A worker $\omega \in \Omega_s$ is employed if her value marginal product $P_s A_s \phi_\omega$ is above the minimum wage m and unemployed if it is below. Under the assumption that there are no mass points in the marginal distribution of ϕ , skill s output is

$$Y_s = L_s A_s \int_0^1 \left(\int_{m_s}^\infty \phi g_s(\epsilon, \phi) d\phi \right) d\epsilon \quad (2)$$

where I have defined $m_s \equiv m / (P_s A_s)$, so that worker ω is employed if $\phi_\omega > m_s$.

In any SPNE, if $\omega \in \Omega_s$ is employed, then she works for the firm $n_1(\omega)$ with which she is best matched. And in any SPNE in weakly undominated strategies, firm $n_1(\omega)$ either pays her the lowest wage at which ω weakly prefers $n_1(\omega)$ to $n_2(\omega)$ when $n_2(\omega)$ is offering a wage equal to the worker's value marginal product or, if this is below the minimum wage, it pays the minimum wage.⁷ An employed worker ω earns a wage w_ω given by

$$w_\omega = \max \{ A_s P_s \phi_s \epsilon_\omega, m \} \quad (3)$$

Summary of cross sectional results. In the absence of a minimum wage, $m = 0$, all workers are employed and each worker's wage is determined by her value marginal product and the quality of her second best match relative to her best: $w_\omega = A_s P_s \phi_s \epsilon_\omega$. In this case, each worker's wage markdown, i.e. her value marginal product divided by her wage, is given by $1/\epsilon_\omega \geq 1$.

The equilibrium differs in two respects in the presence of a positive minimum wage. First, some workers may be unemployed. If the minimum wage creates unemployment, it does so only for the least productive workers, whose value marginal product is below the minimum wage, $\phi_\omega < m_s$. Second, the minimum wage may also raise wages for some workers, with wages now being the maximum of $A_s P_s \phi_s \epsilon_\omega$ and m , as shown in equation

⁷In the SPNE in weakly undominated strategies, firm $n_1(\omega)$ offers the wage $w_{\omega f} = w_\omega$ and every other firm $n \in \mathcal{N}_f / n_1(\omega)$ offers a wage equal to the worker's value marginal product. If firm $n_1(\omega)$ offers a wage below $A_f P_f \phi_\omega \epsilon_\omega$ and above m , then firm $n_2(\omega)$ can profitably hire ω away from firm $n_1(\omega)$ by offering a slightly higher wage. If firm $n_1(\omega)$ offers a wage above $A_f P_f \phi_\omega \epsilon_\omega$ and above m , then it can increase its profit by slightly reducing its wage offer, conditional on other firms not playing weakly dominated strategies. Finally, there is no equilibrium in which firms follow their strategies and workers mix with positive probability over choosing any firm but $n_1(\omega)$, since in this case the best matched firm would have an incentive to slightly increase its wage.

(3). The minimum wage raises a worker's wage if it constrains the wage markdown she faces, which is now the minimum of $1/\epsilon_\omega$ and ϕ_s/m_s .

Finally, a worker earns the minimum wage if $\phi_\omega \in [m_s, m_s/\epsilon_\omega]$. Such a worker either has a low productivity (although higher than any unemployed worker) or a relatively high amenity value in her best- relative to her second-best matched firm.

Changes in average wages. I am interested in the average wage paid to employed workers with skill s , which I denote by \bar{w}_s . Given the unemployment cutoff and the determination of worker wages, the average wage can be expressed as

$$\bar{w}_s = \mu_s m + (1 - \mu_s) P_s A_s \mathbb{E} [\phi\epsilon | \phi\epsilon > m_s] \quad (4)$$

where $P_s A_s \mathbb{E} [\phi\epsilon | \phi\epsilon > m_s]$ is the average wage of those earning above the minimum wage and where

$$\mu_s \equiv \Pr [\phi\epsilon \leq m_s | \phi \geq m_s] \quad (5)$$

is the share of skill s workers with wage equated to the real minimum wage ($\phi\epsilon \leq m_s$) conditional on being employed ($\phi \geq m_s$).

I now introduce time t subscripts, so that I can make an assumption on the evolution of factor-biased productivities equivalent to that in the canonical model. Suppose that $\log A_{st}$ follows

$$\log A_{st} = \alpha_s + \gamma_s t + \iota_{st}$$

where γ_s denotes the rate of the skill s specific time trend of technical change and ι_{st} denotes zero mean i.i.d. deviations from this trend. Under this assumption, I obtain the following result about how the skill premium responds to changes in the minimum wage m_t , the supply of more skilled workers L_{ht} , the supply of less skilled workers $L_{\ell t}$, the productivity of more skilled workers A_{ht} , and the productivity of less skilled workers $A_{\ell t}$, to a first-order approximation. In this and all results, I hold fixed the distribution g .

Proposition 1. *The log derivative of the skill premium can be expressed as*

$$d \log \left(\frac{\bar{w}_{ht}}{\bar{w}_{\ell t}} \right) = \beta_m d \log m_t + \beta_L d \log \left(\frac{L_{ht}}{L_{\ell t}} \right) + \beta_A dt + \iota_t \quad (6)$$

The proof of Proposition 1 and the solution for the β parameters are provided in the Theoretical Appendix. By the definition of a first-order approximation, changes in average wages can be expressed as a linear function of the underlying shocks. The only additional result in Proposition 1, besides the solutions for the β coefficients provided in the Theoretical Appendix, is that changes in average wages depend only on the ratio of

the supply of skilled to unskilled workers; this follows because the aggregate production function exhibits constant returns to scale.

The β_m and β_L coefficients in equation (6) are particularly simple in two special examples, to which I now turn. I first consider an example in which the skill premium in the general model simplifies to that in the canonical model.

Example 1 (Canonical Model). Suppose that $m_t = 0$ for all t and that the aggregate production function takes a constant elasticity form, with elasticity of substitution η .

Example 1 is a generalization of the canonical model featuring monopsony power. Yet, with the minimum wage set to zero and no changes in monopsony power over time, the implied skill premium at any moment in time can be expressed as in the canonical model. In particular, I obtain the following corollary of Proposition 1.

Corollary 1. *In Example 1, the skill premium can be expressed as*

$$\log \left(\frac{\bar{w}_{ht}}{\bar{w}_{\ell t}} \right) = \alpha - \frac{1}{\eta} \log \left(\frac{L_{ht}}{L_{\ell t}} \right) + \frac{\eta - 1}{\eta} (\alpha_h - \alpha_\ell) t + \iota_t \quad (7)$$

I next turn to the more interesting example, in which there is a positive minimum wage, yet still no unemployment.

Example 2 (Minimum Wage Without Unemployment). Suppose that $\inf \phi_\omega > m_{st}$ for all $\omega \in \Omega_s$ and for $s = h, \ell$.

Example 2 is a generalization of the canonical model featuring monopsony power, a minimum wage, and a general aggregate production function. The restriction imposed in Example 2 implies that marginal changes in the minimum wage have no impact on unemployment. Now, the skill premium depends on the minimum wage, as shown in the following corollary of Proposition 1.

Corollary 2. *In Example 2, the log derivative of the skill premium can be expressed as in equation (6) where*

$$\beta_m = \sigma_h - \sigma_\ell \quad (8)$$

$$\beta_L = -\frac{1}{\eta} \left(1 - \sigma_h \frac{P_\ell Y_\ell}{Y} - \sigma_\ell \frac{P_h Y_h}{Y} \right) \quad (9)$$

where η is the local elasticity of substitution in the aggregate production function given full employment and $\sigma_s \equiv \mu_s m / \bar{w}_s$ is the share of skill s income earned by workers at the minimum wage.

Consider first a change in the minimum wage m . The direct effect (i.e., holding fixed the set of workers who are employed and the set earning the minimum wage) of a small

increase in the minimum wage m is to raise the wage of minimum wage earners proportionately. The partial elasticity of \bar{w}_s with respect to m is then $\partial \log \bar{w}_s / \partial \log m = \sigma_s$, where $\sigma_s \equiv \mu_s m / \bar{w}_s$ is the share of skill s income earned by workers at the minimum wage, which I refer to as the *bite* of the minimum wage for skill s . The minimum wage also affects the set of employed workers earning the minimum wage; this effect, however, is generically (i.e. in Example 2 and more generally) of second order. Finally, a change in the minimum wage also affects average wages via changes in the set of workers who are employed; however, this channel is inactive by assumption in Example 2. Hence, the coefficient β_m in Corollary 2 reflects only the direct effect of changes in the minimum wage. As long as the share of income earned by workers at the minimum wage is lower for higher than lower skill workers, $\sigma_h < \sigma_\ell$, an increase in the minimum wage will reduce the skill premium.

Consider next a change in the relative supply of more- to less-skilled labor. This matters through its impact on the relative price P_h/P_ℓ , where the elasticity of P_h/P_ℓ with respect to L_h/L_ℓ is given by $-1/\eta$, as in the canonical model. Unlike the canonical model, however, not all worker wages respond to changes in the value marginal product of labor. Instead, only the incomes of workers earning above the minimum wage respond. Hence, the elasticity of the average wage of skill s with respect to P_s is now $1 - \sigma_s$, which—together with the solutions for changes in P_ℓ and P_h —yields β_L in Corollary 2.

Finally, in general—when changes in the minimum wage potentially affect unemployment—the β coefficients in equation (6) depend in addition on the share of workers who are marginally employed (such that a small change in the minimum wage causes them to move into or out of unemployment), the share of employed workers, and the share of workers earning the minimum wage; see the Theoretical Appendix.

4 Empirics leveraging state variation

Section 2 drives a tantalizing empirical stake into the ground, showing that the minimum wage plays a central role—together with supply and demand—in shaping the evolution of the U.S. college premium in the national time series. Nevertheless, however robust are these results to variations in specification and measurement, there is still an obvious limitation: the results are derived from a dataset with 55 observations. In this section, I instead leverage state-level variation and show that the minimum wage and relative skill supply both play central roles in shaping the differential evolution of the college premium across states.

I estimate a regional version of the extended canonical model using the simple insights

of Corollary 2. In particular, I estimate

$$\Delta \log \left(\frac{w_{hrt}}{w_{\ell rt}} \right) = \alpha_t + \beta_m b_{rt} \Delta \log m_{rt} + \beta_L \Delta \log \left(\frac{Supply_{hrt}}{Supply_{\ell rt}} \right) + [\gamma_{r0} + \dots] + \iota_{rt} \quad (10)$$

Here, r indexes region, which correspond to the fifty states, and t indexes time, which correspond to the years 1979 - 2017.⁸ The term m_{rt} is the real minimum wage in state r in year t (the maximum of the legislated state and federal minimum wages) and $b_{rt} \equiv \sigma_{hrt} - \sigma_{\ell rt}$ is the relevant bite of the minimum wage in state r in year t , defined as in Corollary 2 as the share of high-education, σ_{hrt} , minus the share of low-education, $\sigma_{\ell rt}$, worker income earned by minimum wage workers in state r in year t .

Following the theory, I estimate (10) in differences, defining $\Delta x_t \equiv x_{t+T} - x_t$ as the T -period difference in any outcome x starting in year t . As in Corollary 2, changes in the minimum wage matter through the product of the initial bite of the minimum wage, b_{rt} , and the change in the state-level minimum wage starting in that year, $\Delta \log m_{rt}$.⁹ The parameter α_t absorbs national year-specific changes in skill-biased technology, the parameter γ_{r0} absorbs state-specific linear trend deviations from national skill-biased technical change. In various specifications I additionally include the terms $\gamma_{r1}t$ and $\gamma_{r2}t^2$, which absorb state-specific quadratic and cubic deviations.

This approach assumes state-specific labor markets. State wages are set as a function of local supply, local demand, and local minimum wages. Local demand depends both on national time-varying demand shocks with arbitrary skill bias as well as on state-specific skill-biased time trends of varying degrees (from none to cubic in time).

To allow for correlation across time in the error terms ι_{rt} , in all specifications I cluster standard errors by state. To allow for correlation across states within each year, in my baseline I report two-way cluster-robust standard errors by state and year.¹⁰ In all regressions, I weigh each state by the average across years 1963 - 2017 of its share of national population (in the years in which all states are identified in the March CPS).

Measurement and instruments. I measure composition-adjusted state-year wages and

⁸The time period is shorter than the national analysis because the minimum wage bite for each state and year, b_{rt} , can only be measured over this horizon.

⁹In different empirical contexts, Bailey et al. (2021), Derenoncourt and Montialoux (2021), and Chen and Teulings (2022) use similar measures of the minimum wage bite, which they define as the share of workers (rather than the share of income earned by workers) at or below the minimum wage.

¹⁰Since I use the MORG CPS to measure the bite, there are only 38 years in which b_{rt} is measured (1979-2017 excluding 1994, where imputation flags are missing). Given that I take T -year differences, there are only $38 - T$ years of observations. Hence, I have also clustered only by state, which yields similar standard errors.

supplies as in the national specification (using the March CPS), but using state-specific rather than national data. I measure the state-year bite, b_{rt} , defining a worker as earning the minimum wage if her wage is no higher than 1.05 times her state's minimum wage. To measure b_{rt} I use data between 1979 and 2017 from the CPS Merged Outgoing Rotation Groups (MORG), where wages can be measured with less error. I do not measure b_{rt} in 1994 given missing imputation flags, leaving 38 years of data across fifty states.¹¹

The minimum wage bite, b_{rt} , is likely to suffer from measurement error. And this error may be correlated with the dependent variable, since the dependent variable (the change in the college premium between year t and $t + T$) itself depends on measures of wages in state r in year t . To address this endogeneity concern, I consider two distinct instruments for the term $b_{rt}\Delta \log m_{rt}$.

My baseline instrument is $\bar{b}_{rt}^t \bar{b}_{rt}^r \Delta \log m_{rt}$. This instrument replaces b_{rt} with the product of two terms. The first term, $\bar{b}_{rt}^t \equiv \frac{1}{37} \sum_{j \neq t} b_{rj}$, is the leave-out average bite of the minimum wage in state r across all years in the sample other than year t . The second term, $\bar{b}_{rt}^r \equiv \frac{1}{49} \sum_{j \neq r} b_{jt}$, is the leave-out average bite of the minimum wage in year t across all states other than state r . My alternative instrument is simply $b_{rt-1} \Delta \log m_{rt}$. This instrument replaces b_{rt} with its one-year lagged value. The alternative instrument addresses the endogeneity concern if measurement error is uncorrelated across consecutive years whereas the baseline instrument allows measurement error to be correlated across years. It instead requires that measurement error be mean zero on average within each state across years and across states within each year.

Results. In my baseline, I use four-year lags, $T = 4$, yielding 1,700 observations (50 states across 34 time changes). Columns (a) of Table 2 displays results of estimating regression (10) using OLS and not including a state fixed effect. Column (b) replicates this regression, but including a state fixed effect. The coefficient on the change in state-specific relative college supply, of -0.42, is similar to the coefficients in Table 1 from the national time-series regressions, although a bit lower. Comparing the coefficient on the real minimum wage in the state regressions, of 5.48 and 5.33, to those in the national regressions is less direct, since in regression (10) the real minimum wage is multiplied by its bite, whereas in regression (1) it is not. Dividing the real national minimum wage coefficients in Table 1 by the average bite at the national level across years, -0.026 (defining a worker in state r to be at the minimum wage if she earns no higher 1.05 times her state minimum wage, consistent with the construction of b_{rt} in the state-level regressions), yields coefficients in the range of 5.3 - 7.6, depending on the specification, making the OLS state-level estimates

¹¹I clean the MORG CPS data following closely the approach in Lemieux (2006). See the Empirical Appendix for details.

	OLS		Two-stage least squares			
	(a)	(b)	(c)	(d)	(e)	(f)
Δ relative supply	-0.419 (0.040)	-0.424 (0.041)	-0.419 (0.040)	-0.424 (0.041)	-0.406 (0.041)	-0.412 (0.041)
bind x Δ real mw.	5.484 (1.777)	5.325 (1.924)	6.070 (2.236)	6.497 (2.410)	6.847 (1.996)	6.934 (2.307)
State effect		X		X		X
Difference T	4	4	4	4	4	4
Observations	1,700	1,700	1,700	1,700	1,600	1,600
Instrument			BL	BL	Alt	Alt
First-stage F			412	308	1330	1205

Table 2: Regression Models for Changes in State-Level College Wage Premia

Notes: Results of estimating (10) using OLS in columns (a) and (b) and 2SLS in columns (c) - (f). The regression includes a state fixed effect in columns (b), (d), and (f). Changes are defined as $\Delta x_t = x_{t+T} - x_t$ for any variable x , where T is 4 in all columns. The dependent variable and “ Δ relative supply” are the log changes in the composition-adjusted college premium and relative supply at the state level. “bind x Δ real mw.” is the state minimum wage bind in year t times the log change in the state real minimum wage. t is each year in 1979-2017 excluding 1994. The instrument is the baseline (BL) one in columns (c) and (d) and the alternative one in columns (e) and (f). Using the alternative instrument drops $t = 1979$ and $t = 1995$, since $bind_{rt-1}$ is not defined for these years. “First stage F” reports the KP F statistic. Robust standard errors are two-way clustered on state and year.

consistent with the lower values of those obtained in the national time series in Table 1.

Column (c) of Table 2 displays results of estimating regression (10) using two-stage least squares and the baseline instrument, without including a state fixed effect. Column (d) replicates this regression, but including a state fixed effect. In both cases, the first stage is strong, with KP F statistics above 400 and 300 in columns (c) and (d), respectively. The coefficients on relative supply are little changed from the OLS specifications. The coefficients on the minimum wage are now slightly larger, 6.07 and 6.50, and squarely in the range of the national estimates. Columns (e) and (f) of Table 2 replicate Columns (c) and (d) but using the alternative instrument. Here, I lose two observations per state since the alternative instrument uses the one-year lagged value of b_{rt} , thereby dropping the 1979 and 1995 observations (since the bind is not defined in 1994). First stage KP F statistics are very high and second-stage results are slightly larger, 6.85 and 6.93, again squarely in the range of the national estimates.

In summary, changes in the minimum wage and relative supply play central roles in shaping the differential evolution of college premia across U.S. states between 1979 and 2017. Moreover, the parameter estimates leveraging state-level variation are very similar to those leveraging national variation alone.

Robustness. Here, I briefly describe results of two types of robustness exercises: robustness of results to higher-dimensional polynomials of time and varying the length of time differences. Details are provided in the Empirical Appendix.

Table A.5 in the Empirical Appendix reports two-stage least squares results using both instruments (separately), including higher-order polynomials of state-specific time trends. The first stage KP F statistic remains high and the coefficient on the real minimum wage becomes slightly larger.

Table A.6 in the Empirical Appendix displays results of estimating equation (10)—using two-stage least squares, the baseline instrument, and state-specific time trends—varying the length of the time difference, $\Delta x_t \equiv x_{t+T} - x_t$ from $T = 1$ to $T = 7$. Results are broadly similar with $T = 1$, $T = 4$, $T = 5$, $T = 6$, and $T = 7$. The coefficient on $b_{rt}\Delta \log m_{rt}$ is positive yet imprecisely estimated and not statistically different from either zero or the baseline ($T = 4$) with either $T = 2$ or $T = 3$.

Connecting theory and empirics. How do the empirical results align with the theory? According to the theory, if the minimum wage has no unemployment effects—as in Example 2—then the coefficient β_m on $b_{rt}\Delta \log m_{rt}$ in regression (10) should be one if estimated on model-generated data. Instead, I find a coefficient that is approximately five to seven times larger. In the data, the minimum wage has much larger effects on the college premium at the national and state levels than in the theory. Explanations for this gap between theory and data fit into (at least) two broad categories: flawed measurement, flawed theory (inclusive of the assumption in Example 2), or both.

In the theory, there are no wage spillover effects of the minimum wage under the assumption that the minimum wage has no unemployment effects. If in practice there are wage spillover effects that decrease up the wage distribution—as found in Autor et al. (2016) and Chen and Teulings (2022)—then these spillovers will increase the elasticity of the college premium with respect to the minimum wage, since the share of college educated workers declines up the wage distribution.

In the data, I have assumed that workers earning up to 5% above the minimum wage are minimum wage earners. In practice, in the presence of measurement error in wages, many workers earning above this threshold may indeed be minimum wage earners and have their wages rise one-for-one with an increase in the minimum wage. If I assume instead that workers earning up to approximately 55% above the minimum wage are minimum wage earners (averaging across years, this implies that workers are minimum wage earners up to approximately the 23rd percentile of the hourly wage distribution), then the theory and data align well. While seems like an unreasonably large degree of measurement error, the idea that observed wage spillover effects of the minimum wage

may be partially (see, e.g., [Derenoncourt and Montialoux, 2021](#), who assume that the minimum wage increases wages one-for-one up to workers earning 15% above its value) or largely (see, e.g., [Autor et al., 2016](#)) explained by measurement error is well accepted.

Under the admittedly very strong assumption that workers earning up to approximately 55% above the minimum wage are minimum wage earners, in the Theoretical Appendix I use the theory to answer two questions. In both exercises, I use the share of income earned by minimum wage earners (separately for each group of workers analyzed) averaged across all sample years rather than picking a particular year around which to take the first-order approximation.

First, to what extent can the declining real minimum wage explain falling *real* wages among low-education male workers between 1979 and 1993? Since the share of income earned by minimum wage workers (averaged across years) is 27.8%, 11.3%, and 8.1% among male high school dropouts, high school graduates, and those with some college (under my strong assumption on measurement error), the 18.3% decline in the real minimum wage over this time period causes a decline in real wages of 5.1%, 2.1%, and 1.5%, respectively.¹²

Second, to what extent do changes in the evolution of the minimum wage between the 1979 - 1993 and 1996 - 2017 periods explain differences in these periods in the evolution of inequality across male high school dropouts relative to high school graduates? To answer this question, I use my model to solve for the counterfactual changes in these measures of inequality in each period if the real minimum wage had remained unchanged. The 1979 - 1993 period is characterized by a decline in the real minimum wage of 18.3% and an increase in inequality, with high school dropout wages falling by 11.4% relative to high school graduate wages. Both the real minimum wage and inequality at the bottom of the education distribution move in opposite directions in the 1996 - 2017 period: the real minimum wage rises by 26.2% and male high school dropout wages rise by 5.9% relative to high school graduates. Since the share of income earned by minimum wage workers (averaged across years) is 27.8% and 11.3% in these two groups (under my strong assumption on measurement error), I find that high school dropout relative to high school graduate male wages would have fallen by 8.4% instead of 11.4% in the first period and would have risen by 1.5% instead of 5.9% in the latter period. More than two-fifths ($2/5 \approx 1 - (8.4 + 1.5) / (11.4 + 5.9)$) of the differential evolution of inequality across these groups in the earlier and latter periods is then explained by the differential pattern of real

¹²If I assume instead that workers earning up to approximately 27% above the minimum wage are minimum wage earners, then the shares fall to 14.8%, 5.0%, and 3.8%; the resulting real wage declines are about half as large.

minimum wage changes across the periods. This suggests that the pattern of pervasive increases in inequality in the earlier period and wage polarization in the latter period are partially explained by differences in observed changes in the minimum wage (which in my model have little to no effect on higher education groups and, therefore, leave upper tail inequality unaffected).

5 Conclusions

While most economists view each of labor supply, labor demand, and labor-market institutions as an important driver of inequality, these literatures have remained distinct in part because no simple framework synthesizes the two. In this paper I have attempted such a synthesis.

Theoretically, I have provided a generalization of the canonical model featuring monopsony power, minimum wages, and unemployment that maintains the simplicity and tractability of the purely neoclassical model. The model generates a simple estimating equation similar to the canonical model, additionally incorporating the real minimum wage.

Empirically, I have shown that changes in the real minimum wage—together with changes in supply and demand—play a substantial role in generating the observed evolution of the U.S. college premium between 1963 and 2017. I have also documented that incorporating the real minimum wage into the empirical implementation of the canonical model improves its out-of-sample fit. Using the theory to guide my empirical implementation across regions, I have also shown that differential changes in supply and minimum wages each play central roles in shaping the differential evolution of college premia across U.S. states.

Models are simplifications of reality. My goal in constructing this model was to expand the insights of the canonical model to shed light on the impact of institutional changes, placing institutional changes on the same footing as changes in supply and demand, while maintaining the overall structure of the model’s empirical implementation. In many respects this endeavor was successful. Empirically, I shed light on past trends (providing insights on the relative importance of minimum wages, labor supply, and labor demand for the historical evolution of the college premium at the national and state levels) and provide insight into the impacts of counterfactual changes in the minimum wage (showing that the model’s out-of-sample fit is good). The model is also able to provide answers to quantitative questions, shedding light on declining real wages and patterns of wage polarization.

However, the model is less successful in one important respect: the empirical elasticity of the U.S. and state-level college premia with respect to the real minimum wage is higher than predicted by the model (in the absence of counterfactually large unemployment effects or unreasonably large measurement error) because the model implies no wage spillover effects of the minimum wage. Future work should investigate whether alternative models featuring such spillovers—such as search and matching—can match the elasticity of the college premium with respect to the real minimum wage identified in this paper without introducing strong assumptions on measurement error.

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A Empirical Appendix

Basic Processing of the March CPS Data. I use the March Annual Demographic Files of the Current Population Survey from 1964 to 2018, which report earnings from 1963 to 2017, for workers age 16 to 64 during the earnings year. Thus, throughout when I refer to any year, I am using the following year’s March CPS.

I drop respondents with missing schooling, missing or negative earnings, or with missing weeks worked. I additionally drop those who are self employed or engage in unpaid family work and anyone with allocated earnings. Following [Autor et al. \(2008\)](#) I multiply top-coded earnings by 1.5.

In composition adjusting, I bin workers into one of 180 groups, denoted by g , defined by the intersection of 9 age bins, 2 genders, 2 races (white and all other self-reported races), and 5 education levels (high school dropout, high school graduate, some college, college complete, and graduate training).¹³ The lowest three educations—high school dropouts, those with a high school degree, and those with some college—are allocated to non-college; the highest two educations—college graduates and graduate training—are allocated to college.

Constructing Wages and Supplies. In each year t and for each of the 180 groups g I construct total hours worked and total wage and salary income (using sample weights) and, from this, the average wage of each group-year pair, w_{gt} for group g . Within each year I average across the log wages of all groups with at least a college degree (denoting the set of these groups by \mathcal{G}_h) and, separately, across all groups without (denoting the set of these groups by \mathcal{G}_ℓ) using time-invariant weights. For instance, for college-educated workers, I have

$$\log w_{ht} = \sum_{g \in \mathcal{G}_h} \omega_g \log w_{gt}$$

where ω_g is the time-invariant weight applied to group g . These weights are constructed using the average across years of the share of hours worked of each group g within the set of college groups \mathcal{G}_h and, separately, within the set of non-college groups \mathcal{G}_ℓ . The resulting averages are the composition-adjusted wages $\log w_{ht}$ and $\log w_{\ell t}$ used in the analysis. I construct composition-adjusted supplies of college and non-college workers as the dual of these wages. In particular, I set $\log \text{Supply}_{ht} = \log \text{Inc}_{ht} - \log w_{ht}$ where Inc_{ht} is the total income of college-educated workers in raw (weighted) data in year t . I similarly construct

¹³Up to and including 1991, I use the highest grade of school completed; I define college complete as having finished the fourth year of college and graduate training as having more than four years of college. Starting in 1992 I use degree completion, assigning associate’s degrees to some college.

	1963-2017							
	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
Relative supply	-0.556 (0.054)	-0.600 (0.052)	-0.531 (0.102)	-0.477 (0.089)	-0.080 (0.126)	0.046 (0.092)	0.050 (0.094)	0.088 (0.078)
Real national mw.	-0.195 (0.040)	-0.159 (0.059)	-0.136 (0.055)	-0.177 (0.062)	-0.161 (0.041)	-0.117 (0.030)	-0.117 (0.030)	-0.075 (0.029)
Constant	0.032 (0.009)	0.021 (0.014)	0.034 (0.029)	0.025 (0.028)	-0.015 (0.017)	-0.075 (0.012)	-0.073 (0.013)	-0.027 (0.009)
Time Polynom.	1	2	3	4	5	6	7	8
Observations	55	55	55	55	55	55	55	55
R-squared	0.968	0.969	0.969	0.971	0.981	0.990	0.990	0.992

Table A.1: Regression Models for the College Wage Premium (Higher Time Polynomials)

Notes: The estimating equation is (1) and the sample is 1963-2017 in all columns. The dependent variable and “Relative supply” are the logs of the composition-adjusted college premium and relative supply of hours worked. “Real min. wage” is the log of the real national minimum wage. “Time Polynom.” refers to the degree of the polynomial of time. Robust standard errors are reported.

$\log Supply_{\ell t}$. Finally, my instrument used in the analysis displayed in Table A.3 is simply the logarithm of the number of college-educated relative to non-college educated in my sample in year t .

My approach to constructing wages and supplies is the standard approach to constructing consistent price and quantity indices. There are three differences in my approach from typical approaches.

First, I measure the wage and the supply of college (and non-college) labor on a common set of groups. It is standard practice in the literature to measure the non-college wage using only the subset of non-college groups with exactly a high school degree (12 years of completed schooling) and to measure the non-college supply using all workers who do not have a high school diploma, all workers with exactly a high school degree, and half the hours of workers with some college. Similarly, it is standard practice to measure the college wage using only those with a college degree or above, but to measure their supply including half the hours of those with some college. Second, my measure of supply is the dual of the wage, so that the product of the supply of college hours and the hourly wage of college labor exactly equals college income in the data in each year; and the same is true of non-college hours and wages. Third, I do not partially allocate any groups across college and non-college. Instead, I allocate each group either to college or to non-college, but not to both.

	1963-2017							
	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
Relative supply	-0.267 (0.052)	-0.495 (0.102)	-0.228 (0.088)	-0.134 (0.096)	0.005 (0.074)	0.043 (0.058)	0.065 (0.064)	0.100 (0.055)
Real national mw.	-0.181 (0.049)	-0.113 (0.054)	-0.031 (0.043)	-0.092 (0.051)	-0.110 (0.037)	-0.088 (0.030)	-0.091 (0.030)	-0.057 (0.026)
Constant	0.002 (0.008)	-0.033 (0.016)	0.033 (0.016)	0.026 (0.016)	-0.014 (0.013)	-0.052 (0.012)	-0.059 (0.015)	-0.016 (0.016)
Time Polynom.	1	2	3	4	5	6	7	8
Observations	55	55	55	55	55	55	55	55
R-squared	0.895	0.907	0.948	0.952	0.974	0.985	0.985	0.990

Table A.2: Regression Models for the SMC+/HSG- Log Wage Gap

Notes: The table is identical to Table A.1 except workers are allocated into high and low education differently. Here, high-education wages and supplies include those with some college and above (SMC+) and low-education wages and supplies include high school graduates and below (HSG-).

Constructing real minimum wages. I construct real minimum wages as follows. In 1974-2012, I use data from Autor et al. (2016) to measure state-year minimum wages. For each month and state, I define the relevant minimum wage as the maximum of the legislated state and federal (FLSA) minimum wages. For each state and year I then average minimum wages across months to construct the state-year minimum wage. For the remaining years, I measure annual federal minimum wages using data from the Department of Labor (averaging across months) and I measure annual legislated state minimum wages using data from FRED. For each state-year, I take the maximum of these annual numbers to obtain the minimum wage. In measuring state real minimum wages, I deflate using the GDP deflator from FRED.

After constructing the state-year series of annual nominal minimum wages, I average across states in each year using fixed weights to construct the nominal national minimum wage. I deflate this series using the GDP deflator and refer to this as the national real minimum wage. When using federal minimum wages instead, I also deflate using the GDP deflator.

Robustness of Table 1. Table A.1 displays results of estimating equation (1) including progressively higher polynomials of time. The minimum wage is negative and significant in all specifications, up to and including an eighth-degree polynomial of time. Results from a double-lasso analysis are consistent with my baseline result: both supply and the minimum wage are negative and significant.

Table A.2 displays robustness in which I redefine high and low education workers, allocating the top three education levels to high education (some college and above) and the bottom two education levels to low education (high school graduate and below). The coefficient on the real minimum wage is negative and significant in all specifications up to and including an eighth-degree polynomial of time, with one exception: it is insignificant in column (c), which includes a third-degree polynomial of time. As in the baseline case, results from a double-lasso analysis are consistent with my baseline result.

Whereas it may be reasonable to assume that population education levels do not respond to deviations from a time trend (whether linear, quadratic, cubic...) in relative demand for high- and low-education workers, the same need not be true for hours worked. Since these deviations in relative demand enter the residual in equation (1) both in the canonical model and in my extended model in Section 3, I also instrument for relative supply using the relative population of the college educated. Table A.3 displays results of estimating the reduced-form and two-stage least squares specifications. As shown in columns (e) - (h), the first stage is strong and instrumenting leaves the results broadly unchanged, regardless of the degree of the time polynomial. Columns (a) - (c) report the reduced-form results.

Comparison of results with Autor et al. (2008). Autor et al. (2008) estimate a related reduced-form regression including the real minimum wage. They report two specifications including the real minimum wage and relative supply, in columns 6 and 7 of their Table 2. In column 7, they include only a linear time trend and find a significant and negative coefficient on the real minimum wage that is similar to my estimates. In column 6, they include a cubic polynomial of time and find that the coefficient on the real minimum wage is negative, relatively small, and insignificantly different from zero. They conclude that the real minimum wage “does not much alter the central role for relative supply growth fluctuations and trend demand growth in explaining the evolution of the college wage premium” and that “institutional factors are insufficient to resolve the puzzle posed by slowing trend relative demand for college workers in the 1990s.”

I find relative supply growth fluctuations and trend demand growth remain crucial drivers of the skill premium, consistent with Autor et al. (2008), but so too are changes in the real minimum wage. I also find that changes in the minimum wage are sufficient to resolve the puzzle posed by slowing trend relative demand for college workers in the 1990s; that is, the extended canonical model fits well out of sample.

Here, I show that these conclusions can be reached using data from Autor et al.’s replication package. Table A.4 displays results using their data moving from a linear time

	Reduced form				Two-stage least squares			
	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
Relative supply					-0.551 (0.068)	-0.659 (0.066)	-0.666 (0.131)	-0.580 (0.146)
Relative population	-0.583 (0.082)	-0.993 (0.102)	-0.687 (0.176)	-0.594 (0.171)				
Real national mw.	-0.316 (0.060)	-0.269 (0.064)	-0.162 (0.060)	-0.185 (0.070)	-0.193 (0.043)	-0.156 (0.055)	-0.158 (0.056)	-0.180 (0.059)
Time	0.023 (0.002)				0.021 (0.002)			
Constant	0.035 (0.012)	-0.009 (0.016)	0.040 (0.034)	0.036 (0.034)	0.032 (0.009)	0.019 (0.014)	0.018 (0.026)	0.017 (0.026)
Time Polynom.	1	2	3	4	1	2	3	4
Observations	55	55	55	55	55	55	55	55
R-squared	0.941	0.953	0.963	0.963	0.968	0.968	0.968	0.970
First-stage F					145	95	31	30

Table A.3: Instrumented Regression Models for the College Wage Premium

Notes: The estimating equation is (1). Columns (a) - (c) report reduced-form results and Columns (d) - (f) report 2SLS results. The instrument is the log of the working-age population with college degrees or above relative to without. Robust standard errors are reported. "First-stage F" reports the KP F statistic. The dependent variable and "Relative supply" are the logs of the composition-adjusted college premium and relative supply of hours worked. "Real min. wage" is the log of the real national minimum wage. "Time Polynom." refers to the degree of the polynomial of time; the coefficient on the linear trend on "Time" is omitted from the table whenever this polynomial is of degree 2 or more.

	1963-2005					
	(a)	(b)	(c)	(d)	(e)	(f)
Relative supply	-0.431 (0.051)	-0.606 (0.077)	-0.612 (0.091)	-0.216 (0.113)	0.013 (0.104)	0.028 (0.093)
Minimum wage	-0.113 (0.049)	-0.109 (0.049)	-0.064 (0.048)	-0.174 (0.052)	-0.123 (0.040)	-0.119 (0.043)
Constant	0.253 (0.083)	0.071 (0.112)	-0.001 (0.119)	0.523 (0.179)	0.581 (0.134)	0.591 (0.130)
Time Polynom.	1	2	3	4	5	6
Observations	43	43	43	43	43	43
R-squared	0.942	0.947	0.954	0.972	0.979	0.979

Table A.4: Regression Analysis for the College Wage Premium Using Data From the Replication Package for [Autor et al. \(2008\)](#)

Notes: This table estimates (1) using relative wages, relative supply, and the nominal minimum wage from [Autor et al. \(2008\)](#). I deflate their nominal minimum wage using the GDP deflator from FRED rather than the GDP deflator in their replication package. Robust standard errors are reported.

trend in column (a) to a sextic polynomial of time in column (f).¹⁴ Columns (a) and (c) correspond to those reported in [Autor et al. \(2008\)](#). The coefficient on the real minimum wage is negative, significant, and similar to my baseline estimate across all specifications up to and including a sextic polynomial of time except for the case of a third-degree polynomial of time in column (c), in which case the coefficient on the real minimum wage is insignificant and relatively small.

Basic Processing of the Merged Outgoing Rotation Groups CPS Data. I use statistics from the Current Population Survey Merged Outgoing Rotation Groups (MORG CPS) my quantification, but not my estimation. I use the MORG CPS from 1979 to 2018, which reflects current wages. Because of missing imputation flags—the CPS did not flag workers with missing wages—in all of 1994 and up to September in 1995, I do not include these two years. As above, I restrict attention to worker ages 16 to 64.

In the processing the files, I broadly follow the approaches of [Lemieux \(2006\)](#) and [Autor et al. \(2008\)](#), using hourly wages for workers paid by the hour and using usual weekly earnings divided by hours worked last week for non-hourly workers. I multiply top-

¹⁴Whereas they include the male unemployment rate, here I do not. The pattern of coefficient significance is identical including the male unemployment rate and the estimated minimum wage coefficients are almost identical. Given the shorter time period (1963-2005), I stop at a sextic time trend since both the minimum wage and relative supply coefficients are insignificant thereafter.

coded constructed hourly wages by 1.5. I drop respondents with allocated earnings flags. I identify and drop non-flagged allocated observations between 1989 and 1993 using the unedited earnings values.

In constructing the distribution of wages—which I only use in reporting the share of the distribution earning no more than the minimum wage (or no more than some multiple of the minimum wage)—I use the product of earnings weights and hours worked. I am therefore constructing the distribution of hourly wages across hours worked rather than across workers.

Robustness of Table 2. Table A.5 displays results of estimating equation (10) using two-stage least squares including progressively higher order state-specific polynomials of time. Columns (a) - (d) use the baseline instrument whereas columns (e) - (h) use the alternative instrument. The higher state-specific time trends absorb deviations of state-specific skill biased technical change from the arbitrary national skill bias of technological change in each year absorbed by the time effect. Coefficients on changes in state-specific relative college supply and the interaction between the state-year minimum wage bind and the change in the state’s minimum wage are mostly unchanged across specifications.

Table A.6 displays results of estimating equation (10) varying the length of the time difference, $\Delta x_t \equiv x_{t+T} - x_t$ from $T = 1$ to $T = 7$. All specifications are estimated using two-stage least squares, the baseline instrument, and state-specific time trends. Column (d) of Table A.6 exactly replicates column (d) of Table 2, since $T = 4$ in each. Results are very similar with $T = 1$, $T = 4$, $T = 5$, $T = 6$, and $T = 7$. The coefficient on $bind_{rt} \Delta \log m_{rt}$ is positive yet imprecisely estimated and not statistically different either from zero or the baseline with $T = 2$ or $T = 3$.

B Theoretical Appendix

B.1 Partial Equilibrium

To understand the impact of changes in the economic environment on the average skill s wage, I begin in partial equilibrium, taking as given changes in skill prices P_s . In this case, I allow for arbitrarily many skill groups and only restrict attention to two in general equilibrium.

Consider first a change in the minimum wage m . The direct effect (i.e., holding fixed the set of workers who are employed and the set earning the minimum wage) of a small

	Baseline instrument				Alternative instrument			
	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
Δ relative supply	-0.419 (0.040)	-0.424 (0.041)	-0.422 (0.042)	-0.426 (0.044)	-0.406 (0.041)	-0.412 (0.041)	-0.409 (0.042)	-0.411 (0.044)
bind x Δ real mw.	6.070 (2.236)	6.497 (2.410)	6.693 (2.556)	6.957 (2.591)	6.847 (1.996)	6.934 (2.307)	7.702 (2.213)	8.806 (1.895)
State effect		X	X	X		X	X	X
Time Poly.			1	2			1	2
Difference T	4	4	4	4	4	4	4	4
Observations	1,700	1,700	1,700	1,700	1,600	1,600	1,600	1,600
First-stage F	412	308	236	244	1330	1205	1089	465

Table A.5: Regression Models for the State-Level College Wage Premium Including Higher Order State-Specific Time Trends

Notes: Results of estimating (10) using 2SLS using the baseline instrument in columns (a) - (d) and the alternative instrument in columns (e) - (h). The regression includes a state fixed effect in columns indicated with an X. The degree of the state-specific time trend is indicated in the “Time Poly.” row. Changes are defined as $\Delta x_t = x_{t+T} - x_t$ for any variable x , where T is 4 in all columns. The dependent variable and “ Δ relative supply” are the log changes in the composition-adjusted college premium and relative supply at the state level. “bind x Δ real mw.” is the state minimum wage bind in year t times the log change in the state real minimum wage. t is each year in 1979-2017 excluding 1994. Using the alternative instrument drops $t = 1979$ and $t = 1995$, since $bind_{t-1}$ is not defined for these years. “First stage F” reports the KP F statistic.

	2SLS using the baseline instrument, state-specific time trends						
	(a)	(b)	(c)	(d)	(e)	(f)	(g)
Δ relative supply	-0.550 (0.038)	-0.476 (0.040)	-0.448 (0.044)	-0.424 (0.041)	-0.417 (0.041)	-0.412 (0.043)	-0.409 (0.048)
bind x Δ real mw.	7.368 (4.061)	1.930 (2.961)	3.856 (2.722)	6.497 (2.410)	7.225 (2.267)	6.000 (2.250)	5.136 (2.573)
Difference T	1	2	3	4	5	6	7
Observations	1,850	1,800	1,750	1,700	1,650	1,600	1,550
First-stage F	179	188	278	308	196	91	95

Table A.6: Regression Models for the State-Level College Wage Premium Varying the Length of the Time Difference

Notes: This table replicates column (d) of Table 2—all specifications are estimated using 2SLS and the baseline instrument and include a state fixed effect and a state-specific linear time trend—varying the time length of the difference T in $\Delta x_t = x_{t+T} - x_t$ from $T = 1$ to $T = 7$.

increase in the minimum wage m is to raise the wage of minimum wage earners proportionately. The partial elasticity of \bar{w}_s with respect to m is then $\partial \log \bar{w}_s / \partial \log m = \sigma_s$, where $\sigma_s \equiv \mu_s m / \bar{w}_s$ is the share of skill s income earned by workers at the minimum wage.

In partial equilibrium there are also two indirect effects of a small increase in the minimum wage. First, an increase the minimum wage may induce some workers to move from earning above the minimum wage to earning the minimum wage. However, this effect is of second order in the absence of mass points in the joint distribution of ϵ and ϕ , since these workers were already earning wages arbitrarily close to the minimum wage. Second, an increase in the minimum wage may also induce some minimum wage workers to move from employment to unemployment, thereby raising the average wage. The change in the average wage \bar{w}_s induced by an increase in the minimum wage m though its effect on unemployment depends on the impact on the share of employed workers earning the minimum wage μ_s —this is the probability that a worker is on the margin of employment and unemployment, which I denote by

$$\chi_s \equiv \int_0^1 g(\epsilon, \phi = m_s) d\epsilon$$

divided by the probability a worker is employed, which I denote by

$$e_s \equiv \int_{m_s}^{\infty} \int_0^1 g(\epsilon, \phi) d\epsilon d\phi$$

—and the difference between the minimum wage and the average wage of workers earning above the minimum wage—which can be expressed as the percentage difference between the share of employment μ_s and the share of earnings $\sigma_s \leq \mu_s$ of workers at the minimum wage, $(\mu_s - \sigma_s) / \mu_s$.

The intuition for the impact on \bar{w}_s of a small increase in $P_s A_s$ is equivalent. The direct effect of a small increase in $P_s A_s$ is to raise the wage of workers earning above the minimum wage proportionately: $\partial \log \bar{w}_s / \partial \log(P_s A_s) = 1 - \sigma_s$. The indirect effect of a small increase in $P_s A_s$ is to move workers from unemployment into employment, since workers are employed if $w_s P_s A_s > m$; hence, the indirect effect of a small increase in $P_s A_s$ is equivalent to the indirect effect of a proportionate reduction in m . The following proposition summarizes this discussion.

Proposition 2. *The log derivative of the average skill s wage with respect to given changes in P_s ,*

A_s , and m is given by

$$\begin{aligned} d \log \bar{w}_s = & \sigma_s d \log m + (1 - \sigma_s) d \log (P_s A_s) \\ & + \frac{\chi_s (\mu_s - \sigma_s)}{e_s \mu_s} m_s (d \log m - d \log (P_s A_s)) \end{aligned} \quad (\text{B.1})$$

where μ_s and σ_s are the shares of employment and income earned by workers at the minimum wage, e_s is the share of workers who are employed, and χ_s is the probability that a worker is marginally employed.

B.2 General Equilibrium

Of course, the skill price P_s is itself a function of changes in the minimum wage m , productivities A_ℓ and A_h , and factor supplies L_ℓ and L_h . Let η denote the local elasticity of substitution between Y_ℓ and Y_h in the aggregate production function,

$$\eta \equiv -d \log (Y_s / Y_{s'}) / d \log (P_s / P_{s'})$$

Then

$$d \log P_s = \frac{P_{s'} Y_{s'} / Y}{\eta + \zeta_h m_h} \sum_{j=s, s'} \left(\mathbb{I}_{j=s'} - \mathbb{I}_{j=s} \right) \left[d \log L_j + (1 + \zeta_j m_j) d \log A_j - \zeta_j m_j d \log m \right] \quad (\text{B.2})$$

where

$$\zeta_s \equiv \frac{m_s \chi_s}{\int_0^1 \left(\int_{m_s}^\infty \phi g(\epsilon, \phi) d\phi \right) d\epsilon}$$

is the share of output Y_s lost (or gained) when the cutoff m_s marginally increases (or decreases) and $\mathbb{I}_{j=s}$ is an indicator function that takes the value 1 if $j = s$ and 0 otherwise. Since Y is constant returns to scale, the change in the ratio of L_h and L_ℓ is a sufficient statistic for the separate impacts of changes in L_h and L_ℓ on skill prices, the average wage of each skill, and therefore the skill premium, as shown in equation (B.2).

Finally, combining equations (B.1) and (B.2) yields the solution for β_m and β_L in Proposition 1. In particular,

$$\beta_m \equiv \rho_\ell - \rho_h - \frac{(\zeta_\ell m_\ell - \zeta_h m_h)}{\eta + \zeta_h m_h} \left(\rho_h \frac{P_\ell Y_\ell}{Y} + \rho_\ell \frac{P_h Y_h}{Y} \right) \quad (\text{B.3})$$

and

$$\beta_L \equiv -\frac{1}{\eta + \zeta_h m_h} \left(\rho_h \frac{P_\ell Y_\ell}{Y} + \rho_\ell \frac{P_h Y_h}{Y} \right) \quad (\text{B.4})$$

where I have defined

$$\rho_s \equiv (1 - \sigma_s) - \frac{\chi_s (\mu_s - \sigma_s)}{e_s \mu_s} m_s$$

which is simply the elasticity of the average wage of group s with respect to $P_s A_s$, as shown in equation (B.1). In Example 2, $\zeta_s = \chi_s = 0$ for both s . In this case, equations (B.3) and (B.4) simplify to equations (8) and (9), as shown in Corollary 2. Finally, in Example 2, the impact of changes in the minimum wage on relative wages across any two skill groups is invariant to the number of skill groups assumed in the analysis.