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EXPECTATIONS AND THE RATE OF INFLATION

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ABSTRACT

What is the effect of higher expectations of future inflation on current inflation? I compute this passthrough for a series of canonical firm-pricing models, but allowing for arbitrary (non-rational) expectations. In the Calvo model, the expectational-passthrough can be made arbitrarily close to zero for sufficiently high stickiness, but in practice, for reasonable parameters, passthrough is close to its upper bound of 1. In the Taylor model, in contrast, the upper bound for passthrough is ½ instead of 1. For a general time-dependent model I show that: (i) passthrough is given by a measurable sufficient statistic: the ratio of the average duration of ongoing price spells to that of completed price spells; (ii) the lowest theoretically possible passthrough equals ½ by Taylor pricing; and (iii) passthrough can be theoretically greater than 1 with hazards that decrease over time; (iv) breaking down the passthrough across horizons, it is expectations in the near future that matters the most, expectations of long-run inflation are completely irrelevant; (v) I provide a generalized Phillips curve for current inflation as a linear function of expectations of future inflation and realized past inflations; (vi) I show that the sum of all coefficients, both past and future, sums to one, so that the long-run Phillips curve is vertical. Finally, I study state-dependent "menu cost" models and show that passthrough in these models can be extremely low or extremely high, depending on the exact specification and inflation rate. I suggest a model where firms must pay a fixed cost for changing their sS pricing policy bands. This extension gives a passthrough of 0 for small enough changes in expectations.

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1 Introduction

It is widely believed that inflation is strongly affected by the expectations of future inflation, perhaps in a near one-to-one relationship. This tight reverse feedback mechanism justifies considering the management of expectations a crucial part of monetary policy playbook. As the story goes, if inflation expectations are stable or so-called "well anchored" at low levels, then inflation may remain stable and low; on the other hand, if bad news or high inflation rates raise the specter of future inflation, then inflation inevitably follows from the rise in expectations. The tail wags the dog. These are stories, but what do we really know?[1]

Expectations no doubt matter to some degree, but just how direct and powerful is the causal link from inflation expectations to current inflation? Is it long-run or short-run inflation expectations that matters the most? Does the magnitude of this passthrough depend on details of the economy or the way prices are set? Finally, how does past inflation impact current inflation? Must past inflation matter more when expectations matter less?

I explore these questions through the lens of canonical economic models of firm price setting. I calculate the passthrough for the ubiquitous Calvo-pricing setting first (the right expression cannot simply be read off the usual Phillips curve), contrast this to Taylor-pricing and then develop a result for more general time-dependent models. Finally, I also explore a variety of state dependent "sS menu cost" models. As is well appreciated, it is notoriously challenging to identify the effect of expectations on inflation empirically. Studying the theoretically predicted values for passthrough is complementary to empirical approaches to this question.

Although the backbone of the models I employ are standard, the way I solve them is less standard. I solve for the impact of a change in inflation expectations, holding all other relevant variables fixed. Changing expectations in this way requires parting ways with rational expectations, or, indeed, any other particular model of expectations formation. Thus, a key element of my analysis is flexibly allowing for any arbitrary set of expectations and solving for the "temporary equilibrium" at that point in time.

Allowing for arbitrary expectations in this way has additional advantage: the solution for inflation as a function of expectations can always be combined, if desired, with any model of expectations formation, including rational expectations as a special case. Even if this is the ultimate goal, it is useful not to leap to this combination of assumptions in one step: the intermediate temporary equilibrium that does not impose any particular

[1]Theoretically, early rational expectations models relating inflation and output such as Lucas Jr. (1972) justified such a belief and captured the ideas from Friedman (1968) and Phelps (1967). As pointed out below, a cursory approach to the Calvo model also reinforces such a belief.

expectation formation theory sheds light into the economic mechanisms at play.[2]

My stated goal is to shed light on the short-run impact of inflation expectations on current inflation. I go from inflation expectations to inflation. This complements an extensive and important body of theoretical and empirical work going in the opposite direction, studying the formation of expectations, often drawing a link from past inflation to expectations of future inflation.

The metric I focus on is the impact effect on inflation of a change in expectations of future inflation. To keep things simple, I first start by assuming that firms expect a constant inflation rate. This allows me to focus on a single "passthrough" coefficient to measure the immediate impact of a change in expectations. I later complement this by studying the effect of past realized inflation.

My results cast doubt on the firmly held view that the short-run passthrough from inflation expectations to inflation is nearly one-for-one. In particular, I show that the passthrough can take on a wide range of values and is plausibly much lower than unity. On the other hand, for plausible values, the passthrough is greater or equal to 1/2 in time-dependent setups, so one might also say that these theories put a non-trivial lower bound on the effects of expectations. However, I also show that state-dependent setups are capable of lower values.

I start with the widely adopted Calvo pricing model, where firms have a constant probability of getting a chance to reset their price. For this model, it is extremely tempting and common to read the passthrough off the linearized "Phillips curve" equation as being equal to the discount factor, which then leads one to conclude that passthrough is close to one. The logic is correct for computing the "long run slope of the Phillips curve", i.e. the steady solution with constant inflation, but is not right for deriving the independent role of expectations themselves. One way to see this is that solving forward we can also write inflation as a function of current and future output gaps. Should we conclude that the passthrough is zero or one?

The paradox is resolved by recalling that both of these expressions for the Phillips curve impose rational expectations. As a result, inflation expectations are not free and tied down to the future evolution of other variables. With rational expectations one simply cannot consider the thought experiment of modifying expectations without also ma-

[2]This general idea is embraced in many contexts by economists. To take a microeconomic example, the notions of supply and demand curves are beloved concepts, and nobody insist on leaping instead to only solving for the equilibrium. Instead, one often pauses first to think about each curve's determinants, how elastic they are, their shape, etc. This is especially useful if we then wish to move from a competitive analysis to, say, that of a monopolist—we now use the demand curve differently. This is akin to considering different models of expectations formation.

nipulating future primitives. My goal is, instead, to express inflation as a function of the expectation of inflation when treated as free variables, holding constant the expectation of future real marginal costs (which can be expressed in terms of output gaps) and any other determinants of inflation.

Solving for the passthrough from expected inflation to inflation in the Calvo model, I show that it has the potential for being very low. Indeed, for any given discount factor, the passthrough approaches zero as prices become fully rigid. This theoretical possibility notwithstanding, I show that, in practice, for plausible parameter values, the passthrough is close to one. I show this by studying the limit without discounting and arguing that it provides a good approximation.

We find ourselves back with the same conclusion of a near one-for-one passthrough from inflation expectations to realized inflation, but this time on better logical footing. However, my next results show that the one-for-one passthrough is special to Calvo and not a robust economic conclusion.

I turn first to the Taylor-pricing case where firms set prices every fixed number of periods N. This form of price rigidity was initially popular in influential macroeconomic studies of monetary policy and nominal rigidities, introduced by Fischer (1977) and Taylor (1980) for wages and prices. However, these fell out of favor after Calvo (1983) provided a more tractable setup, with a constant probability of changing prices, which simplifies the aggregation and dynamics. Models with nominal rigidities have a large state variable: the entire distribution of prices, but this state can be dispensed with in the special Calvo-pricing model.

Adopting the no discounting limit as an approximation, I show the passthrough of future inflation expectations to current inflation in the Taylor model equals 1/2, rather than 1; discounting gives lower values, but once again in practice this effect is minimal.

What is the economic mechanism behind these results and what explains this difference? Intuitively, when firm plan to have prices fixed for some time they want to set them so that they average out to an ideal price. When firms anticipate positive inflation this ideal price is rising. As a result, to get things right on average, they set their price initially above the ideal price and over time their price ends up below their ideal price. The greater the expected inflation, the greater must be this initial price "overshoot".

This overshooting mechanism is at the heart of the transmission mechanism from expectations of future inflation to current inflation. Overshooting is more aggressive in the Calvo-pricing setup because the constant probability of a price change creates a right-tailed risk, where prices remaining unchanged for very long periods of time. One manifestation of this is that the median price spell duration lower than the mean. In contrast, in the Taylor setup there is no such right-tailed risk; the median duration equals the average age. Overshooting is less aggressive.

In both Calvo and Taylor, I show that the passthrough coefficient does not depend on the frequency of price changes. Lower frequency makes firms overshoot inflation more, but this is exactly offset by the having fewer firms changing prices at any point in time.

The Calvo and Taylor model constitute two historically important benchmarks, yet they are both special. To dive deeper I study a general time-dependent pricing model, with an arbitrary hazard rate schedule. The hazard gives the probability of a price change as a function of the time elapsed since the last price change.

For this relatively general setup, I obtain a surprisingly simple result: passthrough equals the ratio of two duration measures. The numerator is the average duration of ongoing price spells. The denominator is the average duration of completed spells. Revisiting Taylor, it is easy to see that this ratio is 1/2 since the average age of ongoing prices is N/2. In the Calvo case, both averages are equal to each other. More generally, if one thinks of the average duration of ongoing prices as a proxy for that of completed prices there are two sources of "bias". On the one hand, for a given spell, the age of an ongoing price is lower than its eventual duration. On the other hand, short spells are underrepresented relative to longer spells. From the duration literature, it is well known that with an exponential distribution these two effects offset each other, which is why passthrough equals one in Calvo.

I then show that across all time-dependent hazard rates (in the limit with no discounting) the lowest possible passthrough is 1/2 attained by Taylor. Intuitively, all other specifications have greater right-tail risk for firms. Passthroughs above 1 are also possible for distributions with fatter tails than the exponential, such as a Pareto distribution, obtained with falling hazard rates.

I then extend the analysis to allow for general expectations of future inflation, that depend on the horizon of the forecast, as well for nonzero past realized inflation. I produce a general Phillips curve, with two sets of coefficients, those on expected future inflation rates and on past realized inflation rates. I show that the coefficients on expectations are largest at shorter horizons. Thus, expectations of short run inflation dominate. Indeed, expectations of inflation for the very long run are shown to be irrelevant.

The coefficients on past inflation are generally non-zero, except in the Calvo case. Indeed, the sum of both sets of coefficients, past and future, add up to one. In this sense, the "long-run Phillips curve" can be said to be vertical: any steady state inflation is possible (at a "natural" real marginal cost). Although long-run neutrality is sometimes taken for granted, it is not immediately apparent in this general setup.

This more general analysis highlights the spirit of my analysis, as well as its advantage and limits. I am purposefully seeking to elaborate on the determination of inflation, focusing on price setting firms and allowing for flexible expectations of future inflation as well as past inflation, and other real determinants. In this way, my analysis can be seen as characterizing a more flexible Phillips curve. The spirit of the analysis is *not* to "close the model" and combine this condition with a theory of expectation formation, or with other pieces of a greater macroeconomic model and a specification of policy. Any such exercise would be special or open up a plethora of options and can be carried out in other studies. Put starkly, I seek to further our understanding of one important equilibrium condition and leave using it in combination with other equilibrium conditions for another day.

Finally, I turn to state dependent "menu cost" models. I first show that the standard set of these models produce extreme results: inflation can jump discontinuously up or down when expected inflation rises! This has the potential for making passthrough very small (even negative) or very large. However, behind this result is the prediction that the frequency of price changes dramatically in the very short run. In my view, this is an unrealistic feature of these basic models.

Thus, I consider two variations. In the first, the frequency of price changes is assumed fixed in the very short run. The motivation is that the resources of goods and time to change prices is difficult to adjust in the short run. I show that this can produce a more reasonable passthrough, potentially below 1/2.

In the second extension, I elevate the main feature of menu cost models to another level: I consider fixed costs of changing the sS pricing bands. I argue that it is difficult to entertain costs to changing prices, for given pricing rules, and not also consider the costs of changing the bands themselves. If the change in expectations is not too large, the firm will not find it profitable enough to pay the fixed cost to change the bands. If the bands do not change, then there is no change in inflation and the passthrough is zero. Numerical explorations show that reasonable values of the fixed cost produce relatively wide ranges of inaction. For example, if the fixed cost of changing bands is 5 times that of a price change then it takes a 12% rise in expected inflation to trigger a change in the pricing bands.

Related Literature. There is a vast empirical literature on the determinants of inflation related to the estimation or testing of so-called Phillips curve. In the context of a hybrid New Keynesian Phillips Curve Galí and Gertler (1999) estimate time-series regressions coefficients on future inflation and past inflation and find a non trivial role for future inflation (about 0.6–0.7). These findings and their interpretation have been debated; see example Rudd and Whelan, 2005 and a response by Gali, Gertler and David Lopez-Salido (2005).[3] More important, however, for the purposes of the present paper, is that these estimates do not use data on actual expectations, but instead use future realized inflation, as justified under the rational-expectations model they lay out. Thus, they cannot separate the role of expectations of inflation from that of other determinants.

There is a large body of empirical and theoretical work on the formation of expectations, as well as a growing recent body of work on the implications of these expectations.[4] Among the later, the most pertinent attempt to estimate causal impacts of inflation expectation based on surveys of firms. These papers exploit randomized information provision and find relatively small passthrough (Coibion et al., 2020, 2018b) or zero (Rosolia, 2021).

There is a large literature studying departures from rational expectations, of various particular kinds. Perhaps closest in spirit to the approach of the present paper are Preston (2005), García-Schmidt and Woodford (2019) and Farhi and Werning (2019) and others, who use the notion of a temporary equilibrium to allow for arbitrary beliefs. However, these papers do so mostly in passing, as a stepping stone on their way to explore particular departures from rational expectations (adaptive, reflective and level-k expectations, respectively), combining Calvo pricing and consumption decisions to study particular features of monetary policy. They do not summarize features of the passthrough of inflation expectations to inflation or how it depends on the pricing model outside of Calvo.

2 Calvo 1 vs. Taylor ½

It is useful to understand the general spirit of my exercise starting with the most familiar models of price setting. The Calvo model features a constant hazard probability of a price changes each period; it is very tractable and for this reason often used as part of the New Keynesian model. The Taylor model, introduced before Calvo, has a constant interval of time between price changes. The assumption is relatively natural, especially for some goods and labor, but it is somewhat less tractable. For both these models we shall reach very simple and stark conclusions.

2.1 Calvo Pricing

I start with the most familiar form for price stickiness, the Calvo-pricing setup. Before jumping into the calculations it is useful to define what the question is and is not.

[3] Rudd (2021) is a recent paper collecting arguments against the impact of expectations of inflation.

[4] For the former, see For examples, see the surveys by D'acunto, Malmendier and Weber (2022) and Coibion, Gorodnichenko and Kamdar (2018a).

The Passthrough Question. The goal is to compute the passthrough going from expectations of future inflation to current actual inflation. I hold fixed current and future real marginal costs and do not impose rational expectations or any other particular form of expectations (adaptive, learning, inattentive, level-k, imperfect information, etc.). I will also start by imposing that firms expect the inflation rate to be constant over time. This allows me to focus on a single passthrough coefficient. I later extend the analysis relaxing this assumption.

Taking expectations as given, at any point in time, I study the best response of firms and aggregate them to compute a "temporary equilibrium". This allows one to contemplate the impact effect at that point in time of a change in expectations, a comparative static exercise.

Studying a dynamic response may require adopting a particular model of expectations formation, as well as specifying the household and policy side of the model. But these are not required to study the impact effect which is my focus. In any case the Phillips curve I develop must hold at any point in time, even as expectations evolve. Thus, in characterizing this object, I am providing an essential input into any dynamic analysis that must be carried out with a fuller model.

NK Phillips Curve Cannot Provide Answer. To see why this distinction is important, let us review standard practice, which does not separately condition on expectations and other determinants of inflation. With Calvo pricing one can show that in equilibrium inflation satisfies

$$\pi_t = \kappa x_t + \beta \mathbb{E}_t \pi_{t+1}.$$

Here π_t is aggregate inflation and x_t is the "output gap" (departure from the flexible-price value of output), κ is a parameter summarizing price stickiness, and β is the time discount factor. This equilibrium condition is often labeled the "New Keynesian Phillips Curve". From this equation, it is tempting to conclude the the discount factor $\beta \in (0,1)$ is the sought after passthrough going from inflation expectations to inflation. Furthermore, assuming β near 1 one is then led to conclude a near one-for-one passthrough.

However, this reasoning is misleading in that it does not answer the passthrough question as stated above. The reason is that the Phillips curve equilibrium condition is derived under the assumption of rational expectations. Thus, the term $\mathbb{E}_t \pi_{t+1}$ is doing double duty: it is capturing expected inflation but also the effect of future output gaps and these must both be related. Indeed, one can solve forward to express inflation as being proportional to the discounted sum of expected output gaps

$$\pi_t = \kappa \sum_{s=0}^{\infty} \beta^s \mathbb{E}_t[x_{t+s}].$$

which might now, equally misleadingly, suggest a zero passthrough from expected inflation to inflation.[5]

These observations simply help underscore the spirit of our exercise: to separate the role of future primitives from the expectation of future inflation. To do so requires abandoning rational expectations and deriving equilibrium conditions for inflation that allow for any expectation of inflation and other determinants. I now turn to these calculations in the Calvo setting.

Optimal Pricing In the Calvo model firms have a constant hazard probability $1-\lambda$ of getting a price reset opportunity each period. We will be approximating around a zero inflation steady state with a constant (nominal and real) interest rate of $\frac{1}{1+r} = \beta$. The log linearized reset price is then given by

$$p_t^* = \mu + (1-\beta\lambda)\mathbb{E}_{t-1} \sum_{s=0}^{\infty} (\beta\lambda)^s (P_{t+s} + mc_{t+s})$$

The interpretation of this condition is that firms set their price so that they are at their ideal value on average; note that $\omega_s = (\beta\lambda)^s(1-\beta\lambda)$ is indeed a weighted average with $\sum_{s=0}^{\infty} \omega_s = 1$. The ideal price, in turn, equals a constant markup μ over the nominal marginal cost $P_{t+s} + mc_{t+s}$. In this section, I have jumped directly to this relatively standard and familiar condition. But I later justify this log-linearized expression and generalize it in the context of a general time-dependent model, while also allowing for more general shocks, such as shocks to markups.

In what follows, $\mathbb{E}_{t-1}$ must not be interpreted as an objective expectation, but rather

[5] Hazell et al. (2022) use the NK Phillips curve, with rational expectations, and decompose it in a way that a casual reader may interpret as a passthrough of 1, but it is important to understand why this is not the case. They assume inflation and output gaps are stationary and define long run expected inflation and output gaps as $\bar{\pi} = \mathbb{E}_t \pi_{t+s}$ and $\bar{x} = \mathbb{E}_t x_{t+s}$ (which are independent of t because of stationarity). They then write the demeaned condition $\pi_t - \bar{\pi} = \kappa(x_t - \bar{x}) + \beta \mathbb{E}_t(\pi_{t+1} - \bar{\pi})$ and solve it forward $\pi_t = \bar{\pi} + \kappa \sum_{s=0}^{\infty} \beta^s \mathbb{E}_t[\hat{x}_{t+s}]$ where $\hat{x}_t = x_{t+s} - \bar{x}$ is the deviation of the output gap from its long run average. This suggests a zero passthrough for short- or medium-run expectations, but a one-to-one passthrough for long-run inflation. However, here once again a change in expected inflation $\bar{\pi}$ requires a change in future output gaps $\bar{x}$. In other words, this is a correct decomposition that is useful for some purposes, but it does not isolate the expectational passthrough.

In the present paper I explore the impact of changes in expectations even when the expectation of very long-run inflation is constant. Indeed, I find that expectations for the very long-run are entirely irrelevant, but that expectations for the short run do matter.

firms' own subjective expectations. As discussed above, I will purposefully stay away from placing restrictions on expectations or modeling their evolution. Taking expectations as given, at any point in time, I study the best response of firms and aggregate them to compute a "temporary equilibrium". This allows one to contemplate the impact effect at that point in time of a change in expectations, a comparative static exercise.

A brief word about the conditioning information $\mathbb{E}_{t-1}$ instead of $\mathbb{E}_t$: this notation is meant as a reminder that firms do not have the price level P_t in their information set when they set prices: the fraction of firms that are resetting prices for period t are doing so simultaneously and do not observe the prices set by others until after they have set their own. Thus, they know the previous period price level P_{t-1} and have an expectation for the period price level $P_t^e = P_{t-1} + \pi_t^e$. For these reasons, it is useful to imagine price reseting happening at the end of a period and selecting the price that will be in place the at the beginning of the next period.

To focus on the contribution from inflation expectations let I write

$$p_t^* - P_{t-1} = (1 - \beta\lambda)\mathbb{E}_{t-1} \sum_{s=0}^{\infty} (\beta\lambda)^s (P_{t+s} - P_{t-1}) + a_t$$

where $a_t \equiv \mu + \mathbb{E}_{t-1} \sum_{s=0}^{\infty} (\beta\lambda)^s mc_{t+s}$ collects all the non inflation expectation items.

Constant Expectations. To simplify, we first consider expectation of future expectation that are constant at some π^e across all horizons[6]

$$\mathbb{E}_{t-1} P_{t+s} - P_{t-1} = \pi^e (1 + s)$$

We later relax this assumption to break down inflation expectations across different horizons. Substituting this constant expectation into the price setting condition and carrying out the calculations gives

$$p_t^* - P_{t-1} = (1 - \beta\lambda) \sum_{s=0}^{\infty} (\beta\lambda)^s (1 + s)\pi^e + a_t = \frac{1}{1 - \beta\lambda}\pi^e + a_t.$$

Our focus is on the first term involving expected inflation π^e. Thus, from now on whenever it simplifies the discussion set $a_t = 0$; it is trivial to bring back any sequence of $\{a_t\}$. Note that as long as prices are sticky ($\lambda > 0$) the coefficient on π^e is greater than one: the firm *overshoots* its price, relative to the static optimum price at t. The economic rationale for overshooting is that future inflation erodes its price p_t^* relative to the rising ideal price.

[6]This is implied but weaker than assuming the one period ahead inflation expectation is constant and unchanging over time: $\mathbb{E}_{t+s} P_{t+s+1} - P_{t+s} = \pi^e$ and that the law of iterated expectations holds.

In the flexible price limit as $\lambda \to 0$ or as firms become myopic $\beta \to 0$ we see the coefficient becomes one: firms do not overshoot their price, and simply set it at the static optimum $p_t^* = P_t^e = P_{t-1} + \pi^e$. Conversely, as $\beta\lambda \to 1$ the overshooting becomes infinitely large.

Inflation. Inflation is the weighted average of inflation across firms that cannot change their prices 0 with weight λ and those that do reset prices $p_t^* - P_{t-1}$ with weight $1 - \lambda$,

$$\pi_t \equiv P_t - P_{t-1} = (1 - \lambda)(p_t^* - P_{t-1})$$

Combining the two previous equations then gives

$$\pi_t = \phi \pi^e + (1 - \lambda) a_t$$

where

$$\phi = \frac{1 - \lambda}{1 - \beta\lambda}$$

As is well known, inflation is entirely forward-looking in the Calvo model; thus, past inflation does not appear in the above expression. This reflects the fact that reset-pricing behavior is forward looking and firms that get to reset prices are randomly drawn from the pool of all firms, so there is no selection effect. This is a special property of the Calvo assumption of a constant hazard rate.

The next proposition is based on the above expression for passthrough.

Proposition 1. *In the Calvo model passthrough ϕ satisfies*

$$\phi \in (0, 1)$$

and any value can be attained in this interval for some parameters. Indeed, in the limit of no discounting $\beta \to 1$ or flexible prices $\lambda \to 0$: passthrough attains its upper bound $\phi \to 1$; conversely, in the limit of rigid prices $\lambda \to 1$ then passthrough attains its lower bound $\phi \to 0$.

Any value for ϕ between zero and one can be obtained by varying the degree of price stickiness only, for fixed given β. This once again dispels the notion, that passthrough equals β, as in the standard expression for the New Keynesian Phillips curve.

However, we next argue that in practice, for reasonable parameter values, passthrough ϕ is quite close to its upper bound of 1. It is simplest to make this argument in the continuous time limit, to which we now turn.

Continuous Time. The expression for passthrough becomes particularly simple in the continuous-time version of the model. This can be done by using the discrete time expression and setting $\beta = e^{-\rho\Delta}$ and $\lambda = e^{-\delta\Delta}$ and then taking the limit as the period length shrink $\Delta \to 0$ or by setting up the model in continuous time directly.[7] Either way one arrives at

$$\phi = \frac{1}{\rho/\delta + 1},$$

so that passthrough only depends on the ratio ρ/δ.

Using the continuous time condition we easily see that for reasonable values of parameters ϕ will be relatively close to 1. For example, for $\rho \leq 0.05$ and $\delta \geq 1$ (i.e. average duration of a year or less) then $\phi \geq 0.95$. Although theoretically ϕ can be very low, in practice this requires significant amounts of impatience or stickiness. We conclude that the limit with no discounting giving $\phi = 1$ is a good approximation in the Calvo model. This conclusion also applies to the discrete time version of the model.[8]

2.2 Taylor Pricing

In the Taylor setting prices are changed every N periods, with firms staggered over time so that a fraction $1/N$ changes prices each period. Once again, we write the reset price as a weighted average of the expected prices

$$p_t^* = \frac{\sum_{s=0}^{N-1} \beta^s P_{t+s}^e}{\sum_{s=0}^{N-1} \beta^s} + a_t = P_{t-1} + \frac{\sum_{s=0}^{N-1} \beta^s (s+1)}{\sum_{s=0}^{N-1} \beta^s} \pi^e + a_t$$

where $a_t \equiv \mu + \mathbb{E}_{t-1} \sum_{s=0}^{N-1} \beta^s mc_{t+s} / \sum_{s=0}^{N-1} \beta^s$. Combining this with

$$\pi_t = \frac{1}{N}(p_t^* - P_{t-1})$$

we arrive at

$$\pi_t = \phi \pi^e + \frac{1}{N} a_t$$

[7] In continuous time, price setters set

$$p_t^* - P_t = \frac{\int_0^\infty e^{-(\delta+\rho)s} s \, ds}{\int_0^\infty e^{-(\delta+\rho)s} \, ds} \pi^e + a_t = \frac{1}{\delta+\rho} \pi^e + a_t$$

where $a_t = \int_0^\infty e^{-(\delta+\rho)s} mc_t \, ds / \int_0^\infty e^{-(\delta+\rho)s} \, ds$ and the result for ϕ then follows by computing $\pi_t = \delta(p_t^* - P_t)$.

[8] To see this, set a period to a year and use $\beta = 0.95$ and $\lambda = 1/2$ (giving a relatively large average duration of 2 years): then $\phi = \frac{1-0.5}{1-0.95\times 0.5} = 0.5/0.525 = 0.952$. Setting a period to a quarter instead with $\beta = 0.99$ and $\lambda = 0.75$ (i.e. average duration of 4 quarters) gives $\phi = \frac{1-0.75}{1-0.99\times 0.75} = 0.25/0.2575 = 0.97$.

with

$$\phi = \frac{1}{N}\frac{\sum_{s=0}^{N-1}\beta^s(s+1)}{\sum_{s=0}^{N-1}\beta^s}$$

and ϕ is increasing in β.[9] In the limiting case without discounting $\beta \to 1$ then $\phi = \frac{1}{2}(1+\frac{1}{N})$.[10] Note that if we separate expectations current and future inflation then we could write

$$\pi_t = \frac{1}{N}\pi_0^e + \frac{1}{2}\pi^e + \frac{1}{N}a_t.$$

where $\pi_0^e = P_t^e - P_{t-1}$. Thus, the value on expected *future*, rather than *current*, inflation is always $1/2$. Relatedly, if we think of periods as being very short and increase N accordingly to maintain the same calendar time, then as we take the continuous time limit with $N \to \infty$ we see that $\phi \to 1/2$.

Just as in the Calvo case, the continuous time limit offers some advantages. As we see next, in continuous time $1/2$ serves as an upper bound on ϕ, one that also provides a very good approximation in practice. Taking the limit $\Delta \to 0$ to continuous time by setting $\beta = e^{-\rho\Delta}$ and $N\delta = \frac{1}{\delta}$ where $1/\delta$ denotes the time interval between price changes (δ is the frequency of price changes) or directly solving the model in continuous time gives[11]

$$\pi = \phi\left(\frac{\rho}{\delta}\right)\pi^e$$

with the decreasing function ϕ given by

$$\phi(\rho/\delta) = \frac{1}{\rho/\delta} - \frac{1}{e^{\rho/\delta}-1}.$$

Here $1/\delta$ denotes the time interval between price changes and δ is the frequency of price changes. Note that only the ratio ρ/δ matters, just as in the Calvo case.

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[9]The expression $\frac{\sum_{s=0}^{N-1}\beta^s(s+1)}{\sum_{s=0}^{N-1}\beta^s}$ is a weighted average of the sequence $0,1,2,\ldots,N-1$ and an increase in β puts relative more weight on higher values.

[10]Note that with $\beta = 1$ we have $\phi = \frac{1}{N}\frac{\sum_{s=0}^{N-1}(s+1)}{N}$ and $\sum_{s=0}^{N-1}(s+1) = \frac{1}{2}N(N+1)$ so $\phi = \frac{1}{2}(1+\frac{1}{N})$

[11]In the continuous time model price setting is given by

$$p^* - P = \frac{\int_0^{1/\delta} e^{-\rho s} s ds}{\int_0^{1/\delta} e^{-\rho s} ds}\pi^e = \frac{1}{\rho}\frac{(1-e^{-\rho/\delta}(1+\rho/\delta))}{1-e^{-\rho/\delta}}\pi^e$$

where I am using

$$\int_0^{\Delta} e^{-\rho s} s ds = -\frac{1}{\rho}e^{-\rho\Delta}\Delta + \frac{1}{\rho^2}(1-e^{-\rho\Delta}) = \frac{1}{\rho^2}(1-e^{-\rho\Delta}(1+\rho\Delta))$$

and $\int_0^{\Delta} e^{-\rho s} ds = \frac{1}{\rho}(1-e^{-\rho\Delta})$. Inflation is then $\pi = \delta(p^* - P)$.

Proposition 2. *In the Taylor-pricing case*

$$\phi \in (0, \tfrac{1}{2})$$

and $\phi \to \frac{1}{2}$ *in the limit of no discounting or flexible prices* $\rho/\delta \to 0$*; whereas* $\phi \to 0$ *in the limit of rigid prices* $\rho/\delta \to \infty$*.*

Although the result shows that ϕ near zero is a theoretical possibility and that $\frac{1}{2}$ is an upper bound, for reasonable values of parameters, the passthrough ϕ once again lies close to this upper bound. For example, if $\rho < 0.05$ and $\delta \leq 1$ then one calculates that $\phi \geq 0.495$. Once again, as with Calvo, we conclude that the no-discounting case is a good approximation for reasonable parameters.

The coefficient of $\frac{1}{2}$ on future inflation stands in contrast with some well known derivations for the Taylor model in the New Keynesian literature. For the special case of $N = 2$ Roberts (1995) works out a condition for inflation under rational expectations. The expression features the expectation of inflation in the next period with a unit coefficient. In addition to this derivation being carried out under rational expectations, unlike my analysis, the coefficient in Roberts (1995) cannot be easily interpreted to think about the reaction of inflation to shocks, including shocks that affect the expectations term. The reason is that the equation also contains an expectation error term that will systematically co-move with inflation.[12]

2.3 Intuition: Calvo vs Taylor

Why is expectational passthrough lower in the Taylor price setting relative to Calvo?

Firms set their price initially above their ideal price, but over time their price ends up below their ideal price. The greater the expected inflation, the greater must be the price over the currently ideal price. This "overshooting" mechanism is at the heart of the transmission mechanism from expectations of future inflation to current inflation.

[12] Roberts (1995) equation (8) can be written as

$$\pi_t = \mathbb{E}_t \pi_{t+1} + a_t + \eta_t$$

where x_t collects terms related to the real economy and η_t is an expectations error given by

$$\eta_t = \mathbb{E}_{t-1} P_t - P_t = \mathbb{E}_{t-1} \pi_t - \pi_t$$

It follows that

$$\pi_t = \frac{1}{2} \mathbb{E}_t \pi_{t+1} + \frac{1}{2} a_t + \frac{1}{2} \mathbb{E}_{t-1} \pi_t.$$

Intuitively, price overshooting is more aggressive in the Calvo-pricing setup. The reason is that with a constant price-change probability the distribution of price ages is exponential. This creates a non-trivial right-tailed risk of prices remaining unchanged for very long periods of time. In contrast, in the Taylor setup where firms change their prices every fixed number of periods, there is no such right-tailed risk. Firms overshoots their ideal price, but do so less aggressively. Indeed, the price will be too high exactly half the time, too low the rest. This leads to a lower passthrough, that exactly equals 1/2.

It is interesting to note that in both cases the passthrough coefficient does not depend on the frequency of price changes. Lower frequency makes firms overshoot inflation proportionally more, but this is exactly offset by the fact that there are fewer firms changing prices.

3 Sufficient Statistics for General Time Dependent Pricing

I now consider a general time dependent model. As we shall see, the more general results I obtain can still be stated rather simply in terms of two sufficient statistics. The more general formulation also helps sheds further light on the two previous special cases.

3.1 Preliminaries

We take as given a hazard function h_s giving the probability of getting a price reset $s+1$ periods since the previous reset (e.g. h_0 denotes the probability of resetting a price if the price was also reset in the previous period). It is useful to imagine price reseting happening at the end of a period and selecting the price that will be in place the at the beginning of the next period.

The hazard rate determines the survival probability S_s for each age $s = 0, 1, \ldots$

$$S_{s+1} = S_s(1 - h_s)$$

with $S_0 = 1$. Note that $F_s = 1 - S_{s+1}$ ($F_{-1} = 0$) represents the cumulative distribution function for the duration of *completed* spells i.e. the probability a spell will be s or less is given by F_s. The associated density is $f_s = F_s - F_{s-1} = S_s - S_{s+1} = S_s h_s$.

Rescale the survival probability so that it adds up to one defines the distribution of *ongoing* spells

$$\omega_s = \frac{S_s}{\sum_{s=0}^{\infty} S_s},$$

where I assume $\sum_{s=0}^{\infty} S_s < \infty$. This distribution has two economic interpretations. First, it represents the unique invariant distribution under the Markov process for age s, defined by $s' = s + 1$ with probability $1 - h_s$ and $s' = 0$ with probability h_s.[13] Under this interpretation ω_s represents the fraction of firms with age s in a cross-section of firms as well as the "long run" average time spent at age s for a given firm. A second interpretation is also possible, one that holds for a single firm and a single price spell: ω_s represents the expected amount of time spent at age s divided by the expected time spent at all other ages. In this way, it captures the relative importance of age s relative to all other ages for a given spell.

Using the distributions of completed and ongoing spells f_s and ω_s I define the average hazard $\bar{h} \equiv \sum_s h_s \omega_s$, the average duration of completed spells $\bar{d} \equiv \sum_{s=0}^{\infty} f_s(s+1)$ and the average duration of ongoing spells $\hat{d} \equiv \sum_{s=0}^{\infty} \omega_s(s+1)$.[14] I assume all these averages are finite valued. One can show that[15]

$$\bar{h} = \omega_0 = \frac{1}{\bar{d}}.$$

These relations are intuitive. The average frequency of price changes $\bar{h}$ must equal the density of firms that are resetting their price ω_0. Likewise, the average duration of completed price spells $\bar{d}$ equals the reciprocal of the frequency of price changes $1/\bar{h}$, a relation familiar in the special Calvo case with $h_s = \bar{h}$.

Calvo and Taylor Again. For reference, in the Calvo model the probability of a price change is constant so that $h_s = \bar{h}$ and $S_s = (1-\bar{h})^s$, yielding $\omega_s = \bar{h}(1-\bar{h})^s$. In the Taylor model, instead, prices are stuck for N periods (over $t = 0, 1, \ldots N-1$) so that $h_s = 0$ for $s < N-1$ and $h_s = 1$ for $s \geq N-1$ so that $S_s = 1$ for $s \leq N-1$ and $S_s = 0$ for $s \geq N$, yielding $\omega_s = \frac{1}{N}$ for $s \leq N-1$ and $\bar{h} = \frac{1}{N}$.

Note that in the Calvo case the distribution of completed spells is the same as that of

[13]This follows because $\omega_{s+1} = \omega_s(1-h_s)$ and $\omega_0 = \sum_{s=0}^{\infty} h_s \omega_s$. The invariant distribution is unique because $s' = 0$ is a recurrent point. Under relatively weak conditions, so that $S_s \in (0,1)$ for some s, the unique invariant distribution is also stable, i.e. we converge to it starting from any other distribution. The Taylor case is a knife-edged case lacking stability, starting from any distribution we cycle endlessly every N periods.

[14]We take the expectation of $s+1$ not s because a spell that is reset at $s = 0$ is a spell of duration 1.

[15]Using the definitions above we have that $\omega_s = h_s \omega_s + \omega_{s+1}$ implying $\omega_0 = \sum_{s=0}^{T} h_s \omega_s + \omega_{T+1} = \sum_{s=0}^{\infty} h_s \omega_s = \bar{h}$ where I have used that $\omega_{T+1} \to 0$ because $S_{T+1} \to 0$ is implied by the assumption that $\sum_{s=0}^{\infty} S_s < \infty$. Next

$$\bar{d} \equiv \sum_{s=0}^{\infty} f_s(s+1) = \sum_{s=0}^{\infty} (S_s - S_{s+1})(s+1) = \sum_{s=0}^{\infty} S_s(s+1) - \sum_{s=1}^{\infty} S_s s = \sum_{s=0}^{\infty} S_s = \frac{1}{\omega_0}.$$

Note that these same calculations also justify the second interpretation for ω_s mentioned above, since $\omega_s = S_s/\bar{d}$.

ongoing spells, since both are exponential. In contrast, in the Taylor case the distribution of ongoing spells is uniform, whereas that of completed spells has full mass at N. In particular, the average duration of completed spells is greater than that of ongoing price spells in Taylor but identical in Calvo. This will play a role in interpreting our previous results on expectational passthrough.

Prices and Inflation. The price level (in logs) is defined as the average across firms, which equals the weighted average of past reset prices

$$P_t = \sum_{s=0}^{\infty} \omega_s p^*_{t-s}.$$

Inflation is then given by

$$\pi_t = P_t - P_{t-1}.$$

A bit of algebra shows that

$$\pi_t = \sum_{s=0}^{\infty} \omega_s h_s (p^*_t - p^*_{t-1-s})$$

inflation is a weighted average of the inflation rate associated with each firm (note that, mechanically, a fraction $1 - \bar{h}$ firms produce zero inflation).

3.2 Price Setting Approximation

I first provide a simple formal result justifying and generalizing the type of log-linearized calculations used in the Calvo and Taylor settings.

The firm faces a path of interest rates q_{t+s}. And a path of θ_t shocks to its profit function. These shocks can capture changes in their production functions or the demand firms face—leading to variations in desired prices and markups. A firm resetting its price in period t then solves

$$\max_{p^*_t} \mathbb{E}_{t-1} \sum_{s=0}^{\infty} q_{t+s} \omega_s \Pi(p^*_t - P_{t+s}, \theta_{t+s})$$

with first-order condition

$$\mathbb{E}_{t-1} \sum_{s=0}^{\infty} q_{t+s} \omega_s \Pi_p(p^*_t - P_{t+s}, \theta_{t+s}) = 0$$

Consider a small variation in the firms' problem $\{P_{t+s}, \theta_{t+s}, q_{t+s}, \omega_s\}$. Assume the approximation is carried out around constant primitives $\theta_t = \bar{\theta}$ with zero inflation $P_{t+s} = P_t$ and with perfect foresight expectations.

Proposition 3. *To a first-order approximation, around a zero inflation steady state, the reset price satisfies*

$$p_t^* = \frac{\mathbb{E}_{t-1} \sum_{s=0}^{\infty} q_{t+s}\omega_s \, P_t^e}{\mathbb{E}_{t-1} \sum_{s=0}^{\infty} q_{t+s}\omega_s} + a_t$$

where

$$a_t = \frac{\Pi_{p\theta}}{\Pi_{pp}} \frac{\mathbb{E}_{t-1} \sum_{s=0}^{\infty} q_{t+s}\omega_s \, \theta_t}{\mathbb{E}_{t-1} \sum_{s=0}^{\infty} q_{t+s}\omega_s}.$$

Further setting $q_{t+s} = \beta^s$ gives the desired result.

Proof. Totally differentiating gives

$$0 = \mathbb{E}_{t-1} \sum_{s=0}^{\infty} q_{t+s}\omega_s \Pi_{pp,t} \left(dp_t^* - dP_{t+s}^e\right)$$
$$+ \mathbb{E}_{t-1} \sum_{s=0}^{\infty} \Pi_{p,t} (dq_{t+s}\omega_s + q_{t+s} d\omega_s)$$
$$+ \mathbb{E}_{t-1} \sum_{s=0}^{\infty} q_{t+s}\omega_s \Pi_{p\theta,t} \, d\theta_{t+s}$$

Around a steady state with zero inflation $\Pi_{pp,t}$ and $\Pi_{p\theta,t}$ are constant over time and $\Pi_{p,t} = 0$ from the first order condition. Thus, the middle term cancels and rearranging the remaining terms and setting gives

$$dp_t^* = \frac{\mathbb{E}_{t-1} \sum_{s=0}^{\infty} q_{t+s}\omega_s \, dP_t^e}{\mathbb{E}_{t-1} \sum_{s=0}^{\infty} q_{t+s}\omega_s} + da_t$$

At a zero inflation steady state we also have that the constants $p^* = P^e$ and $a = 0$ satisfy

$$p^* = \frac{\mathbb{E}_{t-1} \sum_{s=0}^{\infty} q_{t+s}\omega_s \, P^e}{\mathbb{E}_{t-1} \sum_{s=0}^{\infty} q_{t+s}\omega_s} + a.$$

Adding these two conditions gives the desired result, where $p_t = p^* + dp_t^*$, $P_t^e = P^* + dP_t^e$ and $a_t = a + da_t$ up to first order. □

My analysis takes the sequence for $\{\theta_t\}$ and hence $\{a_t\}$ as given and is focused on the effects of expectations $\{P_t^e\}$.

3.3 Sufficient Statistics for Passthrough

I now calculate the impact of a sudden change in inflation expectations. I do so approximating around a zero inflation rate steady state and also focus on the no discounting limit $\beta \to 1$.

Without discounting the optimal reset price is a weighted average of the marginal cost. Thus, abstracting from shifters to real marginal costs to focus on expected inflation, we have:

$$p_t^* = \sum_{s=0}^{\infty} \omega_s P_{t+s}^e = \sum_{s=0}^{\infty} \omega_s \pi^e (1+s) + P_{t-1}$$

This gap between the reset price and the price level is one element that is needed to calculate inflation.

We assume inflation has been zero in the past (we later study the effects of past inflation) and that all firms have the initial price $p_{t-s}^* = P_{t-1}$ for all $s = 1, 2, \ldots$ and change their price to p_t^* at $t = 0$ when the shock to expectations occurs. Inflation, then is simply $\pi_t = \bar{h}(p_t^* - P_{t-1})$. Combining the above expression then gives the following result.

Proposition 4. *Up to a first-order approximation around a zero inflation steady state with zero past inflation*

$$\pi_t = \phi \pi^e + a_t$$

where

$$\phi = \bar{h} \sum_{s=0}^{\infty} \omega_s (1+s) = \frac{\sum_{s=0}^{\infty} \omega_s (1+s)}{\sum_{s=0}^{\infty} f_s (1+s)} = \frac{\hat{d}}{\bar{d}}$$

The passthrough ϕ equals the ratio of the duration of ongoing spells $\hat{d}$ to that of completed spells $\bar{d}$. This is a very simple formula in terms of two sufficient statistics that are in principle directly observable in the data.

The duration of ongoing spells controls the incentive firms have to overshoot their price, relative to their current ideal price, in the face of expected inflation. Indeed, the weights ω_s for ongoing spells captures the average time firms their price will be at different durations. This average duration determines how much expected inflation will impact its pricing decision today. In contrast, the duration of completed spells is not relevant for this decision, but appears in the numerator because its captures the frequency of price changes.

We now use this general result to revisit the two special cases considered earlier.

Two important cases: Calvo and Taylor. In Calvo $\bar{d} = \hat{d} = \frac{1}{\bar{h}}$ while in Taylor $\bar{d} = N \neq \hat{d} = \frac{N+1}{2}$ implying

$$\phi_{Calvo} = 1$$

$$\phi_{Taylor} \to \frac{1}{2} \qquad N \to \infty$$

Extension with Heterogeneity. The result extends easily with heterogeneity in the hazard rates. Let firms of type i have hazard $h(s;i)$ with associated $\bar{h}(i)$,$\hat{d}(i)$ and $\bar{d}(i) = 1/\bar{h}(i)$, then one can show that

$$\pi = \phi\pi^e + \bar{a}_t$$
$$\phi = \int \frac{\hat{d}(i)}{\bar{d}(i)}\, di = \int \bar{h}(i)\hat{d}(i)\, di$$

Note that ϕ is generally different from $\frac{\int \hat{d}(i)di}{\int \bar{d}(i)di}$ as well as different from $\int \frac{1}{\bar{d}(i)} di \cdot \int \hat{d}(i)\, di = \int \bar{h}(i)di \cdot \int \hat{d}(i)\, di$. Indeed $\bar{h}$ and $\hat{d}$ may be correlated in the population of firms. Indeed, this correlation may be negative so that heterogeneity of $\bar{h}$ and $\hat{d}$ cancels out. For example, heterogeneity in the hazard $\bar{h}$ in Calvo is irrelevant, we always have $\phi = 1$. Likewise, heterogeneity in the length of rigidity N within Taylor is irrelevant and always gives $\phi = 1/2$. So in these cases heterogeneity of the frequency of price changes does not affect ϕ. However, other forms of heterogeneity in the hazard function may matter: for example, if a fraction of firms have a Calvo hazard and another have a Taylor hazard.

How Low Can We Go? How High? What is the range of possible passthrough ϕ? In particular, how low can we make ϕ by choice of the hazard function? I now show that the lowest possible passthrough is $1/2$ achieved by the Taylor pricing case.

Proposition 5. *Let ϕ be given by Proposition 4 then for any $\{h_s\}$ we have*

$$\phi \geq \frac{1}{2}.$$

Moreover, any value of ϕ can be attained by some choice of the hazard function. In particular, $\phi > 1$ and arbitrarily large is possible.

Proof. One can show that

$$\hat{d} = \sum_{s=0}^{\infty}\left(1 - \frac{\sum_{n=0}^{s}(1-F_n)}{\bar{d}}\right)$$

Next we show that for any duration $\bar{d} \in \{0, 1, 2, \dots\}$ the distribution $\{F_n\}$ that minimizes $\hat{d}$ subject to $\bar{d} = \sum_{n=0}^{\infty}(1-F_s)$ is the Dirac distribution $\{F_n^*\}$ with full mass at $\bar{d}$. Any alternative distribution $\{\tilde{F}_n\}$ with $\sum(1-\tilde{F}_n)ds = \bar{d}$ second order dominates F_n^* implying that $\sum_{n=0}^{s}(\tilde{F}_n - F_n^*) \geq 0$ for all s. This then implies

$$\phi_{\tilde{F}} - \phi_{F^*} = \frac{1}{\bar{d}}\sum_{s=0}^{\infty}\sum_{n=0}^{s}(\tilde{F}_n - F_n^*) \geq 0$$

Moreover $\phi_{F^*} = 1/2$. The Dirac F^* corresponds to the Taylor case. For $\bar{d} \notin \{0, 1, 2, \dots\}$

a similar result holds but with a distribution with mass only at the two closest values $s \in \{0, 1, 2, \ldots\}$.

Next I show that arbitrarily large value $\phi > 1$ are possible. Fix the average completed duration $\bar{d}$. Now pick any whole number $\tilde{s} \geq \bar{d}$ and set $f_{\tilde{s}} = \bar{d}/\tilde{s}$ and $f_0 = 1 - \bar{d}/\tilde{s}$ and $f_s = 0$ otherwise; this gives a bimodal distribution with average duration of completed spells $\bar{d}$. Next, we compute the average duration of ongoing spells. Then $S_0 = 1$, $S_s = \bar{d}/\tilde{s}$ for $s = 1, 2, \ldots, \tilde{s}$ and

$$\omega_s = \frac{S_s}{\sum S_s} = \frac{\bar{d}/\tilde{s}}{1 + \bar{d}} > 0$$

so that

$$\hat{d} = 1 + \frac{\bar{d}}{1 + \bar{d}} \frac{\tilde{s} + 1}{2}.$$

The result then follows by choosing $\tilde{s}$ large enough. Note that using the same construction we have

$$\phi = \frac{\hat{d}}{\bar{d}} = \frac{1}{\bar{d}} + \frac{1}{2} \frac{1 + \tilde{s}}{1 + \bar{d}}.$$

This implies that we can attain any value for ϕ in a limiting sense. Choosing any desired value for ϕ we can send $\bar{d} \to \infty$ and $\bar{s} \to \infty$ so that $\frac{1}{2} \frac{1+\tilde{s}}{1+\bar{d}} \to \phi$. Small perturbations of this construction can attain any value of ϕ without taking limits. □

The intuition for this result is as follows. The duration of ongoing spells suffers from two "biases" that make it generally different from that of completed spells. Firstly, for any given spell the age at which we sample an ongoing spell is by definition below that of the completed spell; a downward bias. Secondly, unless spells are all of the same duration there is also an upward bias because we oversample relatively longer spells. Intuitively, Taylor minimizes the ratio at $1/2$ because it has the downward bias, but not the upward bias.

The proposition not only rules out $\phi < 1/2$, but shows that any value $\phi \geq 1/2$ is possible. Since Taylor attains ½ and Calvo 1, it should be intuitive that we can get anything in between by mixing these two models. But $\phi > 1$ arbitrarily large is perhaps less obvious. The intuitive idea is that we can make average duration of ongoing spells very large relative to the completed ones by having most spells end as soon as they start, but have a small fraction end after a very long time. Ongoing spells are then the selected sample of "survivors" with very long durations. Effectively, we can make the second positive "bias" described in the previous paragraph arbitrarily large.

Discounting. What happens away from the no discounting case, when $\beta < 1$? With discounting, firms set prices that are a weighted average including the discounting. As a result, discounting lowers passthrough.

Proposition 6. *The passthrough $\phi_\beta$as a function of β is increasing in β and*

$$\phi_\beta = \frac{\hat{d}_\beta}{\hat{d}}\phi_{\beta=1}$$

where $\hat{d}_\beta = \sum_{s=0}^{\infty} \hat{\omega}_{s,\beta} s$ and $\hat{\omega}_{s,\beta} = \beta^s \omega_s / \sum_{s=0}^{\infty} \beta^s \omega_s$ is a weight with the property that $\hat{\omega}_{s,\beta}$ is increasing in a first order stochastic dominance (FOSD) sense. Thus, $\hat{d}_\beta$ is increasing and $\hat{d}_1 = \hat{d}$.

For Calvo and Taylor we saw that the no-discounting case provided a very good approximation. Based on this proposition, we now see that this conclusion is more general as long as $\hat{d}_\beta / \hat{d}$ lies close to 1. For reasonable discount rates and hazard rates this must be the case: for price stickiness lasting a year or so, discounting within that year does not change the relative weights $\hat{\omega}_{s,\beta}$ significantly, thus it will not change $\hat{d}_\beta$ and ϕ_β significantly.

4 General Phillips Curve: Short vs Long Run Expectations and Past Inflation

I now develop the equilibrium condition for current inflation while relaxing the simplifying assumptions imposed previously. Since this equilibrium condition is informally termed a "Phillips curve" one can see this section as developing a Phillips curve for a general-time dependent model.

I relax two assumptions from the previous section. First, expectations for inflation at different horizons are no longer assumed to be flat: expectations are given by an arbitrary sequence $\{\pi^e_{t+s}\}_{s=0}^{\infty}$ where π^e_{t+s} represents the expectations of inflation for period $t+s$ held at time t; previously $\pi^e_t = \pi^e$ constant. Second, I study the impact of past realized inflation taking any given any sequence $\{\pi_{t-s}\}_{s=0}^{\infty}$; previously $\pi_{t-s} = 0$ for $s > 0$.

The next proposition writes current inflation as a linear function of expectations of future inflation rates and past realized inflation rates. It also provides some properties of the coefficients on these variables. After stating the result, I discuss the results, provide the sketch of the proof and its associated economic intuition.

Proposition 7. *Suppose the hazard function is strictly positive $h_s > 0$ for all s then to a first-order approximation in the limit of no discounting we have*

$$\pi_t = \sum_{s=0}^{\infty} \phi_s \pi^e_{t+s} + \sum_{s=-1}^{-\infty} \phi_s \pi_{t+s} + a_t$$

with ϕ_s decreasing for $s > 0$ and

$$\sum_{s=-\infty}^{\infty} \phi_s = 1$$

and $\{a_t\}$ a sequence dependent on the expectation of future real variables given by $a_t = \alpha \sum_{s=0}^{\infty} \omega_s \theta^e_{t+s}$.

The previous section characterized $\phi = \sum_{s=0}^{\infty} \phi_s$ the sum of the coefficients on expectations of future inflation. This proposition generalizes this result, breaking down ϕ over different horizons. It also considers non-zero past inflation, the most interesting aspect of which is that it allows us to see that the sum of both sets of coefficients is unity. This can be interpreted as saying that the "long-run Phillips curve is vertical", or more precisely that steady-state inflation is not determined.

Proof and Intuition. To see how this result is derived we first compute the reset price. This satisfies the usual condition but this time we do not impose $P^e_{t+s} = \pi^e(1+s)$ (setting $a_t = 0$ to simplify expressions)

$$p^*_t = \sum_{s=0}^{\infty} \omega_s P^e_{t+s} = \sum_{s=0}^{\infty} \omega_s \sum_{j=0}^{s} \pi^e_{t+j} + P_{t-1} = \sum_{s=0}^{\infty} (1 - \Omega_{s-1}) \pi^e_{t+s} + P_{t-1}$$

where the cumulative distribution is $\Omega_s = \sum_{n=0}^{s} \omega_n$ with the convention that $\Omega_{-1} = 0$. Using $\pi_t = \sum_{s=0}^{\infty} \omega_s h_s (p^*_t - p^*_{t-1-s})$ we arrive a

$$\begin{aligned} \pi_t &= \sum_{s=0}^{\infty} \omega_s h_s (p^*_t - P_{t-1}) + \sum_{s=0}^{\infty} \omega_s h_s (P_{t-1} - p^*_{t-1-s}) \\ &= \bar{h} \sum_{s=0}^{\infty} (1 - \Omega_{s-1}) \pi^e_{t+s} + \sum_{s=0}^{\infty} \omega_s (\bar{h} - h_s) p^*_{t-1-s} \end{aligned}$$

The first set of terms provides the required coefficients $\phi_s = \bar{h}(1 - \Omega_{s-1})$ for expectations of future inflation $s \geq 0$. We see that that ϕ_s are decreasing since the cumulative distribution Ω_{s-1} is increasing. Intuitively, when resetting their prices, firms care relatively more about earlier inflation because it affects the price level over more periods. In fact, if $S_s = 0$ for $s \geq N$ for some N then inflation expectations beyond N are completely irrelevant.

The second backward-looking set of terms depend on past reset prices $\{p^*_{t-s-1}\}$. Using ω_s as probabilities we have $\sum_{s=0}^{\infty} \omega_s h_s = \mathbb{E} h_s = \bar{h}$, so that

$$\sum_{s=0}^{\infty} \omega_s (\bar{h} - h_s) p^*_{t-1-s} = -\text{Cov}\left(h_s, p^*_{t-1-s}\right)$$

We see immediately that if $h_s = \bar{h}$ is constant as in Calvo then this backward-looking term is zero. It is also zero if one starts at a zero inflation steady state with constant past reset prices $p^*_{t-1-s} = p^*$; our working assumption in the previous sections. In contrast, if the reset price p^*_t has been rising in the past and the hazard rate h_s is increasing then the covariance is negative, and the new term contributes towards positive current inflation.

To next step in the proof is to convert the expression in terms of past reset prices into one involving past inflation rates. The price level is a weighted average of past reset p**?**rices

$$P_t = \sum_{s=0}^{\infty} \omega_s p^*_{t-s}.$$

The key idea is to prove that this can be inverted to solve for

$$p^*_t = \sum_{s=0}^{\infty} \alpha_s P_{t-s}$$

with $\sum_{s=0}^{\infty} \alpha_s = 1$. This is possible by applying the Eneström–Kakeya Theorem (Gardner and Govil, 2014), noting that the hypothesis are satisfied because $\omega_s \geq 0$ and $\omega_{s+1} \leq \omega_s$. Substituting this expression for p^* and rearranging then provides a linear expression in terms of past inflation. I omit the details here, but collect the relevant expressions in an Appendix.

Finally, to see that $\sum_{s=-\infty}^{\infty} \phi_s = 1$ we can work through the effects of constant past inflation $\pi \neq 0$, noting that this can only emerge from constant inflation in the reset price $p^*_{t-s} = p^*_t - s\pi$.[16] Evaluating the second term then gives

$$\pi \sum_{s=-1}^{\infty} \phi_s = \sum_{s=0}^{\infty} \omega_s(\bar{h} - h_s) p^*_{t-1-s} = \pi \sum_{s=0}^{\infty} \omega_s(\bar{h} - h_s)s = \pi\bar{h} \sum_{s=0}^{\infty} (\omega_s s - f_s s) = \pi(1-\phi).$$

Cancelling π we obtain $\sum_{s=-1}^{\infty} \phi_s = 1 - \phi$ and the result follows.

Economically this result can be interpreted as saying that the "long-run Phillips curve is vertical". That is, if $a_t = 0$ for all t then any constant solution for $\pi_t = \pi$ is possible; if instead $a_t = \bar{a} > 0$ then $\pi = \infty$ and if $\bar{a}_t = \bar{a} < 0$ then $\pi = -\infty$. .

[16] To see this note that if $p^*_{t-s} - p^*_{t-s-1} = \pi$ for all $s = 0, 1, \ldots$ then

$$P_{t+1} - P_t = \sum_{s=0}^{\infty} \omega_s (p^*_{t+1-s} - p^*_{t-s}) = \sum_{s=0}^{\infty} \omega_s \pi = \pi$$

The same is true for all earlier dates. Thus, constant reset price inflation produces constant past inflation. But since we can invert p^* from P this must be the only possible sequence of p^* consistent with constant inflation in P.

5 Menu Costs: State Dependent Models

I now explore the implications of "menu cost" state-dependent models. The key difference between these models and the ones studied earlier is that the frequency of price adjustments is endogenous. In particular, firms can be seen as using so-called "sS bands" characterizing regions of inaction where the price is left unchanged.

I first explore the simplest setting introduced by Sheshinski and Weiss (1977), a deterministic economy with positive inflation, where there are only price increases. As I will show the predictions of this model are quite extreme. In particular, an increase in expected inflation can actually *reduce* inflation all the way to zero. This is due to extreme movements in the frequency of price adjustments brought about by discrete changes in the sS bands. This may not be realistic, so I study a version of the model where the frequency of price changes cannot be changed in the very short run. This model produces a passthrough that is positive but lower than the Taylor one of ½.

5.1 Setup

It is useful to move to a continuous time setup. At any point in time t a firm has price p_t^i and aggregate price $P_t = \int p_t^i dt$. It useful to define the price gap $x_t^i = p_t^i - P_t$.

Firms profits are a function of their price gap $f(x)$. They discounted profits at a constant rate ρ, although we often take the limit as $\rho \to 0$. Firms must pay a fixed cost c each time they change their price, as a result they change their price only at discrete intervals of time and use an (s, S) rule keeping x in the interval $[s, S]$ with $s < 0 < S$ adjusting the price up to $x = S$ when $x = s$. We are interested in characterizing the (s, S) rule and deriving its implications for the passthrough from expected inflation to realized inflation.

Consider a steady state where expected inflation is given by $\pi^e > 0$. Let V^* denote the stationary value of a firm that has just optimally changed its price. Then the firm problem at time t is

$$V(x_t; \pi^e) \equiv \max_T \int_0^T e^{-\rho s} f(x_{t+s}) dt + e^{-\rho T}(V^*(\pi^e) - c)$$

$$x_{t+s} = x_t - \pi^e t$$

with x_t given. The first order condition for T gives

$$f(\underline{x}) = \rho(V^*(\pi) - c)$$

a condition for the lower bound on the sS policy $\underline{x} = x_t e^{-\pi T}$. The upper bound of the sS policy is the optimal reset price $x^* \in \arg\max_x V(x; \pi)$. This reset price x^* actually satisfies a condition akin to the Taylor model: the price is set so that the average marginal profit is zero

$$\frac{1}{\rho}\int_0^{T^*} e^{-\rho s} f'(x^* - \pi s)dt = 0.$$

Taking a quadratic approximation of $f(x)$ around the maximal value $\hat{x} = \arg\max_x f(x)$ gives f' linear, and in the limit $\rho \to 0$ gives $x^* = \frac{1}{T}\int_0^{T^*} \pi t + \hat{x} = \frac{1}{2}T^* + \hat{x}$. The $\hat{x}$ shifter plays the same role as a_t did in the time-dependent model; we set $\hat{x} = 0$. The bands are then symmetric around zero: $\underline{x} = -x^*$.

(Sheshinski and Weiss, 1977) proved that firms will expand their bands, so that $x^*(\pi^e) = -\underline{x}(\pi^e)$ is increasing in π^e. Moreover, firms anticipate that prices will be set for a shorter amount of time $T = \frac{\pi^e}{2x^*(\pi^e)}$ is decreasing in π^e.[17]

5.2 A Shock To Expectations with Full Adjustment

Now suppose we are at a steady state with constant expected and actual inflation that coincide $\pi_0^e = \pi_0$. At a steady state we have an invariant cross-sectional distribution of firms distributed with density $\omega(x)$.[18] In equilibrium we must have $\int x\omega(x) = 0$. The invariant distribution is uniform between $[\underline{x}, x^*]$ with constant density $\omega = \frac{1}{x^* - \underline{x}}$.[19]

The rate of inflation pushes firms down to the boundary $\underline{x}$ making them change prices; the higher is inflation the greater the flow of firms hitting the boundary. The density of firms, or frequency of price changes, equals $\bar{h} = \omega\pi$. These firms change their price by a discrete positive amount $\Delta_+ = x^* - \underline{x}$. Thus, inflation is given by the product

$$\pi = \bar{h}\Delta.$$

One then observes that $\pi = \bar{h}\Delta = \omega\pi\frac{1}{\omega} = \pi$ a consistency condition.[20]

Now from this steady state position, imagine firms anticipate higher inflation $\pi^e > \pi_0$. Thus, when expectations change firms will expand their bands immediately. As a result the distribution of firms is strictly away from the bands and no firms adjust prices, resulting in zero inflation. A decrease in the inflation expectations shrinks the optimal bands, calling on a mass of firms to adjust prices.

Proposition 8. *In the Sheshinski-Weiss menu cost model, starting from a steady state with π_0,*

[17] This latter result requires imposing a condition that is satisfied with our quadratic approximation for f.

[18] In equilibrium we require the consistency condition that $\int \omega(s)xds = 0$

[19] This is the unique invariant distribution and one can ensure stability by perturbing this model slightly, e.g. allowing some small Poisson arrival of free price changes.

[20] For given bounds inflation is indeterminate, but this conclusion is knife-edged and dependent on the simplifying assumptions we have adopted; thus, we will not be concerned with it.

an increase in inflation expectations at $t = 0$ $\pi^e > \pi_0$ lowers realized inflation on impact to zero $\pi = 0$.

Conversely, a decrease in inflation expectations $\pi^e < \pi_0$ induces at a mass of firms to change their price immediately. This results in an upward jump in the price level that is greater the larger the change in expectations $\pi_0 - \pi^e$.

For an increase in π^e the passthrough is initially negative infinity. Once inflation is at zero further increases in π^e have no effect, so the marginal passthrough becomes zero. On the other hand, for a decrease in inflation expectations the immediate passthrough is also negative infinity, but even more extreme since it is the price level, not inflation, that jumps up. These results are obviously extreme, but they illustrate the possibilities when the fraction of firms changing their price becomes endogenous.

5.3 Adjustment Frictions for Price Frequency

The results above are extreme and probably not realistic. Across steady states one can imagine firms adjusting the frequency of price changes, taking as given a "menu cost" of each price change, i.e. the cost is linear in frequency. But in the short run the resources devoted to changing prices may not be perfectly adjustable. For example, if we think of a retail store, managing many products and prices, then changing prices requires employee time devoted to this task. Staffing and training of these employees cannot be immediately adjusted so the retail store cannot suddenly increase the rate at which they change prices arbitrarily; nor will they want to stop adjusting prices altogether since that would leave idle time and resources devoted to that task.

On the other hand, given enough time the frequency of price changes can be changed, staffing rearrangements or hiring can be done.

Now let us revisit the result from the previous section, focusing on a rise in inflation expectations. If inflation expectations rise, the bands are widened, and the frequency of price adjustments was predicted to fall to zero. However, let us now instead entertain that in the very short run this frequency is held constant at its previous value, due to the notion that resources devoted to price changes are fixed in the very short run. On the other hand, the firm anticipates that it will be able to adjust the frequency of price changes over the medium term. Indeed, suppose the firm anticipates that if it is resetting its price today, then by the time it has to reset it again, it will have been able to freely adjust its price adjustment frequency. The next result shows that under these assumptions the passthrough is positive, but below 1/2 the value with Taylor pricing.

Proposition 9. *In a Sheshinski-Weiss menu-cost model where frequency of price adjustment is fixed in the very short run, the marginal passthrough from inflation expectations to inflation satisfies*

$$0 < \phi < 1/2.$$

The calculations behind this result are as follows. If the frequency of price adjustment is fixed then inflation on impact satisfies

$$\pi = \bar{h}(\pi_0)(x^*(\pi^e) - \underline{x}(\pi_0))$$

with $\bar{h}$ and $\underline{x}$ held fixed at its previous value. Now this is the same calculation for the Taylor model except for the value that $x^*(\pi^e) = p^*(\pi^e) - P$. In the Taylor case we have

$$p^* - P = \frac{1}{2}T\pi^e$$

with $T = T^*(\pi_0)$ fixed at its original value. Instead, we now have the same formula but with $\pi^e > \pi_0$ the anticipated value of $T^*(\pi^e)$ is lower. Intuitively, firms anticipate that they do not need to overshoot due to inflation as much because they will increase the frequency of price adjustment in the near future. Thus, the price spell that is just starting is anticipated to be of lower duration. Since the Taylor case gave $1/2$ we now get a passthrough below $1/2$.

5.4 Idiosyncratic Uncertainty

In the simple Sheshinski and Weiss (1977) menu cost model, all price changes are prompted by inflation. The desired relative prices are constant and in the absence of inflation there are no price changes. This is obviously a simplification. These models have been extended in various ways to incorporate idiosyncratic uncertainty at the product level. We now consider these extensions. Departing from the basic benchmark opens a host of opportunities. These models have been extended not just to include uncertainty, but also to allow for free opportunities to change prices, to consider firms managing multiple products and prices, etc.

We take a small step and keep things simple, discussing the setup in Alvarez et al. (2019). The model is cast is once again cast in continuous time, except that now due to shocks to marginal costs we postulate that firms keep track of $x_t = -\pi_t dt + \sigma dW_t$ where W_t is a Brownian motion process, so that dW_t can be interpreted as an iid shock across periods, permanently impacting x_t with standard deviation σ. Once again the firm sets up pricing bands, except that now they are characterized by three numbers: $\underline{x}$, x^* and a new upper bound $\bar{x}$. the firm adjusts prices whenever x_t hits $\underline{x}$ or $\bar{x}$, in which case it resets x to x^*.

This model is more general than the one we studied earlier. The Sheshinski-Weiss case sets $\sigma = 0$, but, intuitively, similar results obtain when σ is small enough relative to inflation π_0. However, when $\pi_0 = 0$ and $\sigma > 0$ or, more generally, when σ is large and π is small then we get different results. First, the distribution is no longer uniform, but it peaks at x^* instead. Secondly and most importantly, an increase in π^e does not widen the sS bands, instead: it shifts them to the right (see the proof of Proposition 1 in Alvarez et al., 2019). This induces an increase in price increases from the bottom at $\underline{x}$—indeed a mass of firms instead of a flow—and a drop to zero in the flow of price adjustments downward, at the upper bound $\bar{x}$. As a result, one obtains an extreme result: a discrete upwards jump in the price level.

Such extreme results can no longer be so easily arrested by freezing the frequency of price adjustments in the short run. If we assume that both the frequency of price increases *and* the frequency of price decreases must remain constant, in the short run, then we can get a passthrough below $1/2$ as before. However, one may assume instead that the total frequency must remain unchanged in the short run, but that the firm can reallocate this frequency between price increases and decreases. It may then be optimal to reallocate all price changes to the price increases, leading to a discrete jump in inflation in response to a small increase in π^e.

6 mc^2

The spirit of state dependent models is that firms are often inactive and do not adjust prices frequently because there is a "menu cost". Sometimes this cost is taken literally in terms of the goods and time cost of printing menus, catalogs or relabeling sticker prices on physical goods in supermarkets. However, an important component is also the managerial decision of changing prices.

In this section I take the idea of managerial costs seriously and push it one step further. During normal times the firm may have converged on certain sS policies that are optimal for some steady state inflation rate. The bands may simply embed the idea that prices should be ±5% of some desired markup over marginal costs. It seems natural to think that there are managerial costs to reconsidering and changing these bands to a new situation. Doing so requires reviewing the available information, weighing the tradeoffs, holding meetings, making marketing decisions and communicating them.

Following the menu cost literature, we formalize these ideas in a stylized way, assuming there is a fixed cost $c_B > 0$ that must be paid to modify some preexisting pricing bands. If the cost is not paid, the firm can continue using its previous pricing bands.

For any arbitrary bands $\underline{x}$ and x^* let us denote by $V(\underline{x}, x^*, \pi^e)$ the anticipated value obtained by a firm with expectations π^e. Let $V^*(\pi^e)$ denote the value function using optimal bands $\underline{x}(\pi^e)$ and $x^*(\pi^e)$ given π^e. Then if

$$V(\underline{x}(\pi_0), x^*(\pi_0), \pi^e) \geq V^*(\pi^e) - c_B$$

the firm will choose to maintain its old bands. If the inequality is violated then the new bands are implemented. For given π_0 this induces a region of inaction for π^e around π_0.

Proposition 10. *Consider the Sheshinski-Weiss menu cost model extended so that, in addition to menu costs for changing prices, there are also fixed costs c_B for changing the pricing bands $(\underline{x}, x^*)$.*

Then starting from an steady-state with inflation and expected inflation equal to π_0 and associated optimized bands $\underline{x}(\pi_0)$ and $x^*(\pi_0)$, there is an interval of inaction $[\underline{\pi}, \bar{\pi}]$ with $\underline{\pi} < \pi_0 < \bar{\pi}$ such that the the firm maintains its bands unchanged. Then for any $\pi^e \in I_0$ inflation remains unchanged, so the passthrough from inflation expectations to inflation is zero.

Moreover, the inaction region increases in c_B and $\frac{\partial}{\partial c_B}\underline{\pi} \to -\infty$ and $\frac{\partial}{\partial c_B}\bar{\pi} \to \infty$ as $c_b \to 0$.

If the bands do not changed then inflation expectations has no effect on firm behavior and hence no effect on inflation. The last part of the proposition suggests that the bands can be quite significant even for small fixed costs. The reasoning is the same as in Akerlof and Yellen (1985) and Mankiw (1985): if the bands were initially optimal the loses from not changing them are only second order.

Next, I perform a quantitative exploration. Annual inflation is initially 2% and the real discount rate is 2%. The size of the menu cost and idiosyncratic shocks is calibrated to match the observed size and frequency of price adjustments (in their sample from Argentina, those values are 10% and 2.7 adjustments per year during low inflation times).

I benchmark the costs of changing the pricing bands relative to the menu costs. It seems natural to imagine that the costs of changing the bands may be significantly higher than changing a single price, following a pre-established rule. Thus, I compute the upper inaction region $\bar{\pi} - \pi_0$ as a function of c_B/c between 1–10. The result with and without idiosyncratic shocks is plotted in Figure (1).

For example, with idiosyncratic uncertainty, if costs are five times greater than menu costs, then the change in inflation expectations must be upwards of 12% for the firm to find it worthwhile to re-optimize the pricing bands. The implied costs of following a sub-optimal policy are small so that large changes in expectations are required to trigger changes in the bands for these range of costs.

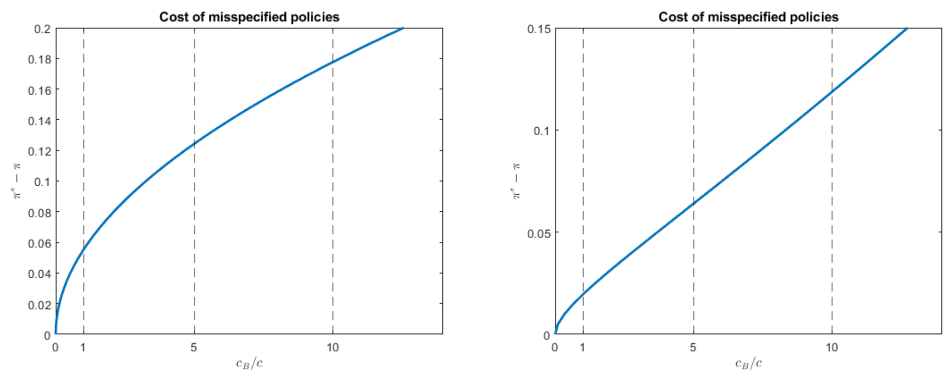

Figure 1: Upper inaction region $\bar{\pi} - \pi_0$ as a function of relative fixed costs c_B/c. Left panel $\sigma = 0.1641$; right panel $\sigma = 0$.

7 Conclusions

In this paper I take a step towards understanding the effect of inflation expectations on pricing and inflation, holding all other determinants of these fixed. I show that the common perception that optimizing models imply a 1-to-1 passthrough is generally misleading. My results uncover that this passthrough depends quite a bit on the pricing model. Exploring a wide range of cases I try to make the case that much lower values of the passthrough are possible and plausible.

Casting aside these results, I believe that a side-product of the analysis is that it lends a greater economic intuition and understanding for the transmission mechanism of the inflationary process, often ignored in formal analyses or the subject of speculation. In particular, expectations matters to the extent that individual firms "overshoot" their ideal relative price or if the frequency of price increases rises in the short run (the overall frequency of price changes is not relevant, as I show). Understanding this mechanism suggests new empirical or theoretical directions. Can we measure this overshooting directly at the microeconomic level? Theoretically, are there other important economic considerations shaping the degree of overshooting such as price complementarities or the shape of demand?

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A First Order Approximation for General Time Dependent Model

Consider a general time-dependent model defined by the hazard rate function $\{h_s\}$. Let $\omega_{s+1} = \omega_s(1-h_s)$ (the value of ω_0 will be inconsequential). The firm faces a path of interest rates q_{t+s}. And a path of θ_t shocks to its profit function.

The firm then solves

$$\max_{p^*} \mathbb{E}_{t-1} \sum_{s=0}^{\infty} q_{t+s}\omega_s \Pi(p_t^* - P_{t+s}, \theta_{t+s})$$

with first-order condition

$$\mathbb{E}_{t-1} \sum_{s=0}^{\infty} q_{t+s}\omega_s \Pi_p(p^* - P_{t+s}, \theta_{t+s}) = 0$$

We consider a small variation in the firms' problem $\{P_{t+s}, \theta_{t+s}, q_{t+s}, \omega_s\}$. Totally differentiating gives

$$\begin{aligned} 0 = \mathbb{E}_{t-1} \sum_{s=0}^{\infty} q_{t+s}\omega_s \Pi_{pp,t}\left(dp^* - dP_t^e\right) \\ + \mathbb{E}_{t-1} \sum_{s=0}^{\infty} \Pi_{p,t}(dq_{t+s}\omega_s + q_{t+s}d\omega_s) \\ + \mathbb{E}_{t-1} \sum_{s=0}^{\infty} q_{t+s}\omega_s \Pi_{p\theta,t}\, d\theta_t \end{aligned}$$

Let us assume the approximation is carried out around a constant $\theta_t = \bar{\theta}$ and with zero inflation $P_{t+s} = P_t$ then $\Pi_{pp,t}$ and $\Pi_{p\theta,t}$ are constant and $\Pi_{p,t} = 0$ (from the first order condition) so the middle term cancels. We are left with

$$dp^* = \frac{\mathbb{E}_{t-1} \sum_{s=0}^{\infty} q_{t+s}\omega_s \, dP_t^e}{\mathbb{E}_{t-1} \sum_{s=0}^{\infty} q_{t+s}\omega_s} + a_t$$

where

$$a_t = \frac{\Pi_{p\theta}}{\Pi_{pp}} \frac{\mathbb{E}_{t-1} \sum_{s=0}^{\infty} q_{t+s}\omega_s \, d\theta_t}{\mathbb{E}_{t-1} \sum_{s=0}^{\infty} q_{t+s}\omega_s}.$$

Further setting $q_{t+s} = \beta^s$ gives the desired result.

B Coefficients on Past Inflation

The inverse has coefficients satisfying the recursion

$$\alpha_s = \begin{cases} \frac{1}{\omega_0} & s = 0 \\ -\sum_{j=1}^{s} S_j \alpha_{s-j} & s \geq 1 \end{cases}$$

Writing p_t^* in terms of past inflation rates gives

$$\begin{aligned} p_t^* =& \sum_{s=0}^{\infty} \alpha_s P_{t-s} \\ =& P_t - \sum_{s=1}^{\infty} \alpha_s \left(P_t - P_{t-s} \right) \\ =& P_t - \sum_{s=1}^{\infty} \alpha_s \left(\sum_{j=0}^{s-1} \pi_{t-j} \right) \\ =& P_t - \sum_{s=1}^{\infty} \left(\sum_{j=s}^{\infty} \alpha_j \right) \pi_{t-s+1} \end{aligned}$$

Define $\gamma_s = \sum_{j=s}^{\infty} \alpha_j$.

$$p_t^* = P_t - \sum_{s=1}^{\infty} \gamma_s \pi_{t-s+1} \tag{1}$$

Equivalently

$$p_{t-s-1}^* - P_{t-1} = p_{t-s-1}^* - P_{t-s-1} + P_{t-s-1} - P_t = -\sum_{z=1}^{\infty} \gamma_z \pi_{t-s-z} - \left(P_{t-1} - P_{t-s-1} \right)$$

Since $\sum_{s=0}^{\infty} \omega_s \left(\bar{h} - h_s \right) = 0$ then

$$\sum_{s=0}^{\infty} \omega_s \left(\bar{h} - h_s \right) p_{t-s-1}^* = \sum_{s=0}^{\infty} \omega_s \left(\bar{h} - h_s \right) \left(p_{t-s-1}^* - P_{t-1} \right) = \sum_{s=0}^{\infty} \delta_s \left(p_{t-s-1}^* - P_{t-1} \right)$$

some calculations

$$\begin{aligned} \sum_{s=0}^{\infty} \delta_s \left(p_{t-s-1}^* - P_t \right) &= \sum_{s=0}^{\infty} \delta_s \left[-\sum_{z=1}^{\infty} \gamma_z \pi_{t-s-z} + P_{t-s-1} - P_{t-1} \right] \\ &= -\sum_{s=0}^{\infty} \sum_{z=1}^{\infty} \delta_s \gamma_z \pi_{t-s-z} - \sum_{s=0}^{\infty} \delta_s \left(P_{t-1} - P_{t-s-1} \right) \end{aligned}$$

define $\theta_s = -\sum_{z=1}^{s} \delta_{s-z}\gamma_z$ and $\mu_s = \sum_{z=s}^{\infty} \delta_z$, then

$$\sum_{s=0}^{\infty} \delta_s \left(p_{t-s-1}^{*} - P_t \right) = \sum_{s=1}^{\infty} \theta_s \pi_{t-s} - \sum_{s=1}^{\infty} \mu_s \pi_{t-s}$$

thus

$$\sum_{s=0} \omega_s \left(\bar{h} - h_s \right) p_{t-s-1}^{*} = \sum_{s=1}^{\infty} \phi_{-s} \pi_{t-s} \tag{2}$$

with

$$\phi_{-s} = \theta_s - \mu_s.$$

This provides the desired expressions to compute ϕ_s for all $s < 0$.