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OPERATING HEDGE AND GROSS PROFITABILITY PREMIUM

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ABSTRACT

We show theoretically that variable production costs lower systematic risk of firms' cash flows if capital and variable inputs are complementary in firms' production and input prices are pro-cyclical. In our dynamic model, this operating hedge effect is weaker for more profitable firms, giving rise to a gross profitability premium. Moreover, gross profitability and value factors are distinct and negatively correlated, and their premia are not captured by the CAPM. We estimate the model by the simulated method of moments, and find that its main implications for stock returns and cash flow dynamics are quantitatively consistent with the data.

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A data appendix is available at http://www.nber.org/data-appendix/w30241

1 Introduction

A firm's fundamental risk is closely linked to the joint dynamics of its revenue and costs. Variable costs, which include expenses on intermediate inputs such as raw materials, energy, and wages of production workers that are directly related to firm's production, are economically large – at approximately two-thirds of revenue on average – and highly correlated with aggregate revenue fluctuations. Moreover, aggregate variable costs are more volatile than aggregate revenue: using data on manufacturing industries from National Bureau of Economic Research - U.S. Census Bureau's Center for Economic Studies (NBER-CES) from 1958 to 2011, we find that a one percentage change in aggregate revenue is associated with 1.14% change in aggregate variable costs, and only 0.74% change in the aggregate gross profit (measured by value added). Thus, variable costs create an operating hedge effect in firms' cash flows, which in turn generates predictable cross-sectional differences in systematic stock return risk.[1]

We analyze quantitative implications of the operating hedge mechanism in a partial-equilibrium dynamic structural model. Firms in our model produce output using a production function with a constant elasticity of substitution (CES) between capital and variable inputs. Capital investment decisions are lumpy and irreversible, while firms can adjust the quantity of their variable inputs flexibly over time in response to both aggregate and firm-specific profitability shocks.[2] If the price of variable inputs is pro-cyclical with respect to the *aggregate* profitability shocks, and variable inputs complement physical capital in the production function, firms reduce their variable inputs in response to a negative aggregate profitability shock to such an extent that their variable costs fall more (in proportion) than their revenue. Variable costs thus offer a natural hedge against aggregate profitability shocks.

The strength of the operating hedge effect in our model is naturally related to firms' gross

[1]In this paper we analyze the operating hedge effect due to variable production costs, which stands in contrast to the operating leverage effect driven by fixed operating costs. This concept is distinct from the strategic "operational hedging" of risks, such as foreign exchange risk.

[2]Because our model is set in partial equilibrium, we do not delineate the exact origins of the aggregate profitability shock affecting the entire population of firms. These shocks reflect the technological shocks affecting profitability across all firms, as well as aggregate "demand" shocks, which may originate as shocks to investors' beliefs or tastes, or be driven by government spending shocks. What is important for our purpose is that such common profitability shocks create correlated movements in firms' profits relative to their output.

profitability. Consider the effect of heterogeneous firm-specific profitability. Because the market price of variable inputs is not affected by the *firm-specific* profitability shocks, firm's revenue rises more with firm-specific profitability shocks than its variable costs when physical capital and variable inputs are complements. Therefore, for more profitable firms (firms with higher gross profit per unit of capital), variable costs constitute a lower fraction of revenue. Such firms thus benefit less from the operating hedge, and have higher exposure of profits to aggregate profitability shocks relative to less profitable firms. This result, combined with a positive premium on aggregate profitability shocks, implies a positive gross profitability premium in our model.

Our empirical analysis confirms that variable costs act as an operating hedge, and that this mechanism helps generate the gross profitability premium in stock returns. In particular, using data from Compustat, we find that the aggregate gross profit is less volatile than aggregate revenue and has elasticity with respect to revenue lower than one. In our model, this is sufficient for the gross profitability premium to arise. We also find that the gross profitability factor in stock returns covaries positively with the difference in profit growth between high- and low-gross-profitability firms, which supports the central implication of the model — that high-gross-profitability firms have higher exposure of their cash flows to the systematic profitability shock. Moreover, we show that this pattern is driven primarily by the operating hedge mechanism: systematic risk of revenue and variable costs is flatter across firms with different profitability, but high-gross-profitability firms tend to have a lower ratio of variable costs to revenue.

In our model, the profitability and value factors, based on standard empirical sorts on gross profitability and book-to-market ratios, are distinct and negatively correlated. This addresses an empirical puzzle that has challenged many earlier models (see Novy-Marx, 2013), where highly profitable firms tend to be growth firms, and the main mechanism for the value premium generates counterfactual implications for the profitability premium. The value premium in our model arises because firms with a higher value of growth opportunities relative to the value of their assets in place, the "growth firms," are more exposed to the investment-specific (or capital-embodied) technology shocks, which follows the development in Kogan and Papanikolaou (2013, 2014). Sorting firms in the model on their market-to-book ratios creates a value factor, which loads heavily on the aggregate investment-specific technology shock. The value factor and the gross profitability factor have different risk exposures. Both factors load negatively on the aggregate investment-specific technology shock. However, the gross profitability factor loads positively on the systematic profitability shock, while the value factor has a negative loading, because growth firms tend to be more profitable on average. This negative loading accounts for the negative correlation between the two factors in our model.

We estimate our model using the simulated method of moments, targeting the aggregate and cross-sectional moments on quantities, firm characteristics, and stock prices and returns. Our structural estimation implies a pro-cyclical variable input price in the model, with a loading on aggregate profitability shocks of 0.309, and an elasticity of substitution between capital and variable inputs of 0.696 — a combination of parameter estimates consistent with the sufficient condition for more profitable firms to have higher systematic cash flow risk, stated in Section 2.1, equation (8). Our model generates economically large and empirically plausible cross-sectional differences in expected stock returns. The annualized gross profitability premium in the model, based on the value-weighted quintile portfolio returns sorted on gross profitability, is 4.32%, which is close to the empirical premium of 3.70% over our 1963-2019 period. Similarly, the value premium is 3.11% in the model, relative to 4.21% in the data. Our model also reproduces the failure of the CAPM to capture these two return premia. Importantly, as in the data, more profitable firms have higher market-to-book ratios in our model, and the two corresponding return factors — the gross profitability and the value factors — are negatively correlated.

Relation to the prior literature

We uncover a novel economic effect in firms' cash flow risk – a natural hedge induced by the cyclicality of variable costs. Our results complement several prior studies in the literature, which analyzed the implications of operating leverage for systematic risk in cash flows and stock returns. Carlson, Fisher, and Giammarino (2004) and Zhang (2005), two prominent early contributions to this literature, show how operating leverage can generate a value spread in stock returns. Novy-Marx (2010) proposes an empirical measure of operating leverage and documents its positive predictive power for the cross-section of stock returns. Novy-Marx (2013) highlights an important tension between the operating-leverage based explanations of the value premium and related return patterns, and the profitability premium. Our paper does not rely on operating leverage as the mechanism for the value premium, and thus does not suffer from the same limitation.

Our study is also related to the literature on the effects of labor costs on stock returns. Danthine and Donaldson (2002) emphasize wage rigidity as an important source of operating leverage, and show that this mechanism helps raise equilibrium equity premium and stock market volatility. Favilukis and Lin (2016) study a dynamic general equilibrium model and find the interaction between wage rigidity, labor-induced operating leverage, and financial leverage is quantitatively important to understand the equity premium and the value premium. Donangelo, Gourio, Kehrig, and Palacios (2019) document that firms with high labor shares have higher expected returns than firms with low labor shares. Favilukis, Lin, and Zhao (2020) document that the labor market frictions play a first-order role in the credit market. They show that wage growth and labor share help forecast aggregate credit spreads and debt growth. All of the above papers emphasize stickiness of wages of existing workers (selling, general, and administrative (SG&A) expenses, which include a labor component, also tend to have low cyclicality). Our study offers a complementary perspective that emphasizes the impact of highly cyclical variable input costs.

Our paper contributes to the growing literature on the relation between firm stock returns and firm characteristics, such as firm profitability and valuation ratios. While the value premium has been extensively studied in the literature,[3] the economic mechanism behind the profitability premium is not as well understood. Kogan and Papanikolaou (2013) show that firm heterogeneity in growth opportunities gives rise to a sizable profitability premium. All cross-sectional return factors in their model are driven by investment-specific technological shocks, and hence their model cannot generate a profitability factor in returns that has a low or even a negative conditional correlation with the value factor. In our model, the operating hedge and the positive exposure

[3]Studies on the value premium include Lakonishok, Shleifer, and Vishny (1994), Berk, Green, and Naik (1999), Gomes, Kogan, and Zhang (2003), Carlson, Fisher, and Giammarino (2004), Zhang (2005), Lettau and Wachter (2007), Garleanu, Kogan, and Panageas (2012), Choi (2013), Ai and Kiku (2013), Ai, Croce, and Li (2013), Kogan and Papanikolaou (2014), Donangelo (2021), Kogan, Papanikolaou, and Stoffman (2020) among many others.

of the gross profitability factor to the aggregate profitability shocks are the primary sources of the gross profitability premium. Ma and Yan (2015) extend the idea of Garlappi and Yan (2011) and find that the performance of the value and gross profitability strategies varies with credit conditions. Their model has a single firm-level state variable, and, like the models of the value premium based on operating leverage (e.g., Carlson, Fisher, and Giammarino, 2004; Zhang, 2005), cannot generate positive gross profitability and value premia simultaneously. Ai, Li, and Tong (2021) find that a general equilibrium model with heteroneity in the persistence of productivity shocks can account for the coexistence of profitability and value premiums. Dou, Ji, and Wu (2020) develop a model for the industry profitability premium and industry value premium by embedding oligopolistic competition within an endowment economy. Dou, Ji, and Wu (2021) study the industry gross profitability premium in an asset pricing model with dynamic strategic competition. In their model, firm's tradeoff between short-term benefits of higher revenue and long-term costs of being in a price war with its competitors gives rise to a higher profitability for industries with more persistent leadership, as well as higher risk exposure to discount rate variations. Wang and Yu (2015) and Lam, Wang, and Wei (2014) compare the risk-based and behavioral explanations of the gross profitability premium and argue that the empirical evidence is more consistent with investors' under-reaction to news about firms' fundamentals. Akbas, Jiang, and Koch (2017) find that recent trajectory of a firm's profits predicts future profitability and stock returns. Bouchaud, Krueger, Landier, and Thesmar (2019) propose a theoretical explanation for the profitability premium based on sticky expectations.

2 Operating hedge and the gross profitability premium

In Section 2.1, we introduce the main element of our model, the production function, and show how variable costs give rise to an operating hedge in firm's profits, and why profits of more profitable firms may have higher systematic risk. Our empirical analysis in Section 2.2 provides evidence in support of the operating hedge mechanism.

2.1 The production function and operating hedge

Consider a firm using two types of inputs to produce output: physical capital K and variable inputs E. The production function features a constant elasticity of substitution (CES) between capital and variable inputs. The firm's gross profit Π is the difference between revenue and input costs:

$$\Pi = \max_E (Y - PE) = \max_E \left[Z\left(E^{\frac{\eta-1}{\eta}} + (XK)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} - PE \right], \tag{1}$$

where Y is the output; P is the price of variable inputs; Z is an idiosyncratic, firm-specific shock; X is the systematic shock to firm profitability, common to all firms, and $\eta > 0$ measures the elasticity of substitution between capital and variable inputs.[4] Firms take the price of variable inputs (P) as given and choose the quantity of variable inputs (E) to maximize their gross profit. We assume that the price of the variable input is a function of the systematic profitability X:

$$\ln P = p_0 + p_1 \ln X, \tag{2}$$

where p_0 and p_1 capture the level and the cyclicality of the variable input price.

Proposition 1 *With the production technology described above and assuming $p_1 > 0$, the risk exposure to the systematic profitability shock is higher for gross profit than revenue, i.e.,*

$$\frac{\partial \ln \Pi}{\partial \ln X} - \frac{\partial \ln Y}{\partial \ln X} < 0, \tag{3}$$

if and only if capital and variable inputs are complements in the production function, i.e., $\eta < 1$.

Proof: See Appendix.

This result describes the operating hedge effect in our model. When variable input price is procyclical ($p_1 > 0$) and the two production inputs are complements ($\eta < 1$), the firm's expenditure

[4] We model the aggregate profitability X multiplying capital stock K to capture inputs being procyclical with respect to aggregate profitability. The qualitative results on the operating hedge and gross profitability premium remain if we use an alternative production function $Y = ZX\left(E^{\frac{\eta-1}{\eta}} + K^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}$.

on the variable input rises more than its revenue in response to a positive systematic profitability shock, lowering the systematic risk of its profit. In this static model, $\beta_X \equiv \frac{\partial \ln \Pi}{\partial \ln X}$ is also the systematic risk exposure of the firm, given by

$$\beta_X \equiv \frac{\partial \ln \Pi}{\partial \ln X} = 1 - \frac{d \ln P}{d \ln X} \left(\frac{E}{XK} \right)^{\frac{\eta - 1}{\eta}}. \tag{4}$$

Next, consider how the firm's revenue, variable cost, and gross profit respond to the *idiosyncratic* profitability shock Z. The following proposition summarizes the results.

Proposition 2 *When there is a positive idiosyncratic profitability shock, a firm's cost of variable input rises less than its revenue, that is,*

$$\frac{\partial \ln(PE)}{\partial \ln Z} - \frac{\partial \ln Y}{\partial \ln Z} < 0, \tag{5}$$

and its gross profit rises more than its revenue, that is,

$$\frac{\partial \ln \Pi}{\partial \ln Z} - \frac{\partial \ln Y}{\partial \ln Z} > 0, \tag{6}$$

if and only if capital and variable inputs are complements in the production function, i.e., $\eta < 1$.

Proof: See the Appendix.

The above result is intuitive: the magnitude by which the firm increases its use of the variable input in response to a favorable firm-specific shock is muted because variable input and capital (a fixed input) act as complements in the firm's production function. Because variable input price does not vary with the idiosyncratic shock, the variable cost responds less than revenue to the firm-specific profitability shock. The gross profit responds more than revenue.

We now establish how the firm's cash flow exposure to the systematic profitability shock is correlated with its gross profitability (GP/A) . Note that gross profitability,

$$\text{GP/A} \equiv \frac{\Pi}{K} = XZ\left[\left(\frac{E}{XK}\right)^{\frac{\eta-1}{\eta}} + 1\right]^{\frac{1}{\eta-1}}, \tag{7}$$

is increasing in the idiosyncratic profitability shock Z. This leads to the following proposition.

Proposition 3 *With the production technology described above and assuming $p_1 > 0$, the risk exposure of a firm's gross profit to the systematic profitability shock increases with its gross profitability, that is,*

$$\frac{\partial \beta_X}{\partial \ln Z} > 0 \tag{8}$$

if and only if capital and variable inputs are complements in the production function, i.e., $\eta < 1$.

Proof: See Appendix.

As we establish above, in Equation (5), a firm with higher gross profitability (higher Z), spends less on the variable input as a fraction of its revenue. As a result, the operating hedge effect from variable costs is weaker for such a firm, resulting in higher systematic risk of its profit.

2.2 Empirical evidence on cash flow risk

In this section we summarize properties of firms' cash flows relevant for our analysis. In particular, in line with the operating hedge mechanism in Proposition 1, firms' profits are less sensitive to systematic profitability shocks than their revenue. The opposite being the case for firm-specific shocks (see Proposition 2). Thus, as implied by the model in Equations (3) and (6), the operating hedge mechanism operates with respect to the systematic profitability shocks, but not the firm-specific shocks.

2.2.1 Data and variable definitions

We use two sources of data in this section. Our data on aggregate revenue, the price and value of variable inputs, and gross profit (value added) are from the NBER-CES Manufacturing Industry

Database, covering the 1958-2011 period. This database includes prices and costs related to materials (Mat), energy (Eng), production worker wages (Prd), and office worker wages (Off) across 459 four-digit 1987 SIC industries.[5] For each variable (revenue, input, gross profit), we compute its aggregate value by summing up the corresponding values across industries, and its aggregate price index as the weighted average of the price indices across industries using the corresponding one-year lagged industry revenue as the weight. These value and price indices are further deflated by the Consumer Price Index from U.S. Bureau of Labor Statistics. In line with the definition of the cost of goods sold (COGS) in Compustat, we categorize material costs, energy costs, and production worker wages that are directly related to the production of finished goods as variable costs, and define the gross profit as the difference between revenue and variable cost.

We also use accounting data on publicly traded firms from the annual Compustat North America, and stock return data from the Center for Research in Security Prices (CRSP) (which we use in our analysis in Section 3). Following Novy-Marx (2013), we define the gross profitability as revenue (Compustat item REVT) minus cost of goods sold (Compustat item COGS), divided by total asset (Compustat item AT), that is, (REVT − COGS)/AT (referred to as GP/A). We define the book-to-market equity ratio (BM) following Fama and French (1992).[6] Consistent with prior studies, we remove firms in the financial industries and only keep in our sample firms with a share code (SHRCD) 10 or 11, and exchange code (EXCHCD) of 1, 2, or 3. Our final sample spans the time period from July 1963 to December 2019.

2.2.2 Systematic profitability shocks and operating hedge

In this section we examine cyclicality of the aggregate revenue, variable costs, and gross profit. These aggregate quantities are not affected by the diversifiable firm-specific shocks, and thus our

[5]We define the office worker wages as the difference between total payroll and the production worker wages.

[6]Other variables include: financial leverage (Flev) is the sum of total debt in current liability (Compustat item DLC) and total long-term debt (Compustat item item DLTT), divided by the sum of DLC, DLTT, and firm's market cap; operating leverage (Olev) is flow-based and is defined as XSGA/(REVT-COGS), where Compustat item XSGA is the selling, general and administrative expense; Tobin's Q (Q) is the sum of market value, long-term debt (Compustat item DLC), preferred stock redemption value (Compustat item PSTKRV), minus the total inventories (Compustat item INVT) and deferred tax in balance sheet (Compustat item TXDB), divided by gross property, plant and equipment (Compustat item PPEGT); gross margin (GM) is the (REVT-COGS)/REVTS.

analysis applies to the effect of systematic profitability shocks on firms' profits, and the basic operating hedge mechanism, as described in Proposition 1. In Panel A of Table 1, we report the main summary statistics – the mean and standard deviation of the ratio of various input values to revenue (the first two rows) and the growth rate of the revenue ($\Delta \ln V(Rev)$), material costs ($\Delta \ln V(Mat)$), energy costs ($\Delta \ln V(Eng)$), production worker wage bills ($\Delta \ln V(Prd)$), total variable costs ($\Delta \ln V(COGS)$), gross profit ($\Delta \ln V(GP)$), and the office worker wage bills ($\Delta \ln V(Off)$) (the next two rows). We compute all of these using the NBER-CES Manufacturing Industry Database. Of all types of inputs, material costs are the largest component, representing on average 54.8% of revenue for the manufacturing industries. Production worker wages account for about 10.8% of revenue, whereas the energy cost is about 1.9%. The ratio of the sum of these three categories of variable costs (COGS) to revenue is about 67.6%, with a standard deviation of 3.6%. Therefore, variable cost is a highly economically important component in firm production. In contrast to variable costs, the office worker wages are only 6.6% of aggregate revenue on average.

[Insert Table 1 Here]

Panel A also shows that, in our sample, variable costs are generally more volatile than revenue. The volatility of aggregate revenue is 5.47% per year, as compared to 6.73% for material costs, 7.95% for energy costs, and 4.99% for production worker wages. The combined variable costs (COGS) have an annual standard deviation of 6.36%, about 16% higher than that for revenue. In contrast, the volatility of office worker wages is only 3.27% per year.

Panel B reports the correlation matrix of the growth rates of aggregate revenue, variable costs, and gross profit. Growth rates of revenue and variable costs have a high correlation coefficient, 0.98. Note that the operating hedge effect requires not only that variable costs are more volatile than revenue, but also that the two are sufficiently highly correlated.

In Panel C, we estimate the elasticity of variable costs and gross profit with respect to aggregate revenue. We regress the growth rate of COGS on the growth rate of revenue. The estimated coefficient suggests that a one percent increase in gross output is associated with a 1.14% increase in variable costs, and this coefficient is significantly higher than one, which explains the low volatility of gross profit relative to revenue (4.79% versus 5.47%, as reported in Panel A). Further, because of the hedge effect from variable costs, the elasticity of aggregate gross profit with respect to aggregate revenue is 0.73. Since $\partial \beta_X / \partial \ln Z$ in Proposition 3 and $\partial \ln Y / \partial \ln X - \partial \ln \Pi / \partial \ln X$ in Proposition 1 share the same sign — they both depend on the sign of p_1 and magnitude of η — the above estimate is consistent with systematic risk of the firm's cash flow increasing with firm-specific profitability.[7]

We find the properties of revenue and variable costs in the NBER-CES database to be quantitatively similar to the public firms covered in Compustat. When we focus on manufacturing firms in Compustat, to better align firm coverage with the NBER-CES database, the average COGS-to-Rev ratio is 69.8%, as compared to 67.6% reported above. The estimated elasticity of the total variable cost (COGS) with respect to revenue in manufacturing firms in Compustat is 1.09, with 0.8 for gross profit (GP), which is again close to what we find in the NBER-CES dataset. Furthermore, the correlation between the growth rates of revenue, variable costs, and gross profit between these two datasets is 0.83, 0.83, and 0.72, respectively (untabulated). Since the Compustat database does not separate variables costs into different sources (material, energy, and production worker wage), we only focus on the total variable cost (COGS) for the remainder of the paper.

In Panel A1 of Table 2, we summarize the statistics of the aggregate sales growth (ΔlogASale) and aggregate gross profit growth (ΔlogAGP), which are aggregated from our sample of Compustat firms with a fiscal year end of December. Consistent with the finding based on the data from the NBER-CES database (Table 1), the aggregate sales growth is more volatile than the aggregate gross profit growth (5.95% versus 5.29% per year) in Compustat. When we regress the aggregate gross profit growth onto the aggregate sales growth, the estimated coefficient from the time-series regression is 0.77 and close to 0.73 in Table 1, although these two databases differ in their coverage.

The above results indicate that, unlike the fixed costs (such as selling, general, and adminis-

[7] Here we estimate the elasticity of aggregate variable cost and gross profit with respect to aggregate revenue. In an untabulated analysis, we find same patterns when we use the utilization-adjusted TFP growth as the proxy for the aggregate profitability shock. For example, the elasticity of aggregate variable costs w.r.t. TFP shocks is 1.39, which is higher than that of aggregate revenues (1.17). The difference in their TFP exposure, 0.22, is statistically significant from zero.

trative costs) that tend to create an operating leverage effect, aggregate variable costs are more cyclical than aggregate revenue and hence induce an operating hedge effect.

2.2.3 Firm-specific profitability shocks

Cash flow elasticities at the firm level behave qualitatively differently from those at the aggregate level, suggesting a qualitatively different effect of firm-specific profitability shocks on firms' profits: relative to firm-specific shocks, variable costs tend to induce an operating leverage effect rather than an operating hedge.

[Insert Table 2 Here]

Panel B1 of Table 2 shows that the firm-level gross profit growth is slightly more volatile than the firm-level sales growth (35.02% versus 34.46%), in contrast to the reverse inequality in the aggregate data in Panel A1. The estimated coefficient in the cross-sectional Fama-MacBeth regression of the firm-level gross profit growth on the firm-level sales growth in Panel B2 is 1.07, greater than one. Again, this stands in contrast to the estimate of 0.77 based on the aggregate time-series.

To interpret the differences between the aggregate- and firm-level results through the lens of our model, note that the idiosyncratic profitability shock is in general more volatile than the aggregate profitability shock. With that, the impact of idiosyncratic shocks may overwhelm the effect of aggregate shocks at the firm level, so the finding that the sales elasticity of gross profit at the firm level is greater than at the aggregate level is consistent with Equation (6) and $\eta < 1$.

3 Quantitative analysis

In this section we formulate a dynamic model of the operating hedge and the profitability premium, and fit it to the data using the simulated method of moments (SMM). We thus address several important questions about the quantitative implications of the proposed operating hedge mechanism. These include whether the operating hedge due to cyclical input costs affects firm cash flows sufficiently to generate cross-sectional differences in the average stock return of the magnitude comparable to the observed empirical patterns, and whether sorting firms in the model on gross profitability and book-to-market ratios gives rise to distinct and negatively correlated factors in returns – the profitability and the value factors – and the corresponding return premia.

3.1 The model

Our model is set in partial equilibrium, and combines the basic structure of the model in Kogan and Papanikolaou (2014) with the CES production function and variable input costs. There is a large number of competitive firms in the economy. Each firm derives its value from existing projects (i.e., assets in place) and growth opportunities associated with adoption of new projects. Firms operate in a complete, frictionless financial market. We first describe the production and value maximization of the existing projects in Section 3.1.1. We then discuss the process of new project arrival and calculate the present value of growth opportunities in Section 3.1.2. We describe the stochastic discount factor in Section 3.1.3.

3.1.1 Assets in place

The basic unit of production is projects. Each project j uses capital $K_{j\tau}$ of vintage τ, which is optimally determined at the installation time τ and remains constant throughout the life of the project, and E_{jt} units of variable inputs optimally chosen for production at time t ($t \geq \tau$). The production function of a project takes the CES form. For each installed project j in firm f, the gross profit Π_{jt} at time t is the difference between revenue and variable input costs:

$$\begin{aligned}\Pi_{jt} &= \max_{E_{jt}} \left(Y_{jt} - P_t E_{jt}\right) \\ &= \max_{E_{jt}} \left[Z_{ft} \left(E_{jt}^{\frac{\eta-1}{\eta}} + (X_t K_{j\tau})^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} - P_t E_{jt} \right] \\ &= \max_{\hat{E}_{jt}} K_{j\tau} \left[Z_{ft} \left(\hat{E}_{jt}^{\frac{\eta-1}{\eta}} + X_t^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} - P_t \hat{E}_{jt} \right],\end{aligned} \qquad (9)$$

where Z_{ft} and X_t are the firm-specific and aggregate profitability processes, respectively. P_t is the price of variable inputs and η measures the elasticity of substitution between capital and variable inputs. The last line of Equation (10), where we have defined $\hat{E}_{jt} = E_{jt}/K_{j\tau}$, indicates that the revenue, variable cost, and gross profit are all proportional to $K_{j\tau}$. Firms take the process for P_t as given and choose variable inputs to maximize profit within each period. Furthermore, all projects of the same vintage within a firm choose the same $\hat{E}_{jt}$, and they differ only in scale $K_{j\tau}$.

We define the lower-case variables x and z as the logarithmic transformation of the productivity processes X and Z, respectively, and assume that they follow independent AR(1) processes:

$$x_t = \rho_x x_{t-1} + \sigma_x \varepsilon_t^x - \frac{1}{2}\sigma_x^2, \tag{10}$$

$$z_{ft} = \rho_z z_{ft-1} + \sigma_z \varepsilon_{ft}^z - \frac{1}{2}\sigma_z^2. \tag{11}$$

Let V_{jt}^{AP} be the value of project j. Given the processes for the input price, $\ln P_t = p_0 + p_1 x_t$, and the aggregate and idiosyncratic profitabilities, the project value normalized by its capital stock ($\hat{V}_{jt}^{AP} \equiv V_{jt}^{AP}/K_{j\tau}$) can be written recursively as

$$\hat{V}_{jt}^{AP} = \hat{\Pi}_{jt} + (1-\delta)\mathrm{E}_t[M_{t+1}\hat{V}_{jt+1}^{AP}], \tag{12}$$

where $\hat{\Pi}_{jt} \equiv \Pi_{jt}/K_{j\tau}$, and $0 < \delta < 1$ is the depreciation rate capturing that projects become obsolete randomly, and M_t is the stochastic discount factor which we specify below. Note that both $\hat{\Pi}_{jt}$ and $\hat{V}_{jt}^{AP}$ are functions of only exogenous state variables x_t and z_{ft}, so they are identical for all projects within a firm. We substitute the subscript j by f and denote them as $\hat{\Pi}_{ft}$ and $\hat{V}_{ft}^{AP}$. $\hat{\Pi}_{ft}$ measures the firm-level gross profitability (GP/A), and $\hat{V}_{ft}^{AP}$ measures the market-to-book ratio of existing projects in firm f.

The total value of assets in place of firm f, V_{ft}^{AP}, is the sum of values across all existing projects: $V_{ft}^{AP} = \sum_{j\in \text{ Projects}(f)} V_{jt}^{AP}$.

3.1.2 Growth opportunities

Each period, identical new projects arrive with a profitability equal to its current firm-level profitability (i.e., $z_{j\tau} = z_{ft}$). Specifically, when a new project j arrives at time τ, a firm needs to choose the size of the project ($K_{j\tau}$). Assuming the cost of creating a project of capital size $K_{j\tau}$ is $K_{j\tau}^{\theta}/\Xi_\tau$, where Ξ_τ captures the aggregate investment-specific technology level, as in Kogan and Papanikolaou (2014), and $\theta > 1$ parameterizes the adjustment costs of investment, the firm's decision is to choose $K_{j\tau}$ to optimize the net project value:

$$\max_{K_{j\tau}} \hat{V}_{j\tau}^{AP}(x_\tau, z_{ft})K_{j\tau} - K_{j\tau}^{\theta}/\Xi_\tau. \tag{13}$$

The first order condition implies the optimal capital stock is:

$$K_{j\tau}^{*} = \left(\frac{\hat{V}_{j\tau}^{AP}(x_t, z_{ft})\Xi_\tau}{\theta}\right)^{1/(\theta-1)} \tag{14}$$

and the maximized project value is $(\theta - 1)\left(\hat{V}_{j\tau}^{AP}(x_\tau, z_{ft})/\theta\right)^{\frac{\theta}{\theta-1}} \Xi_\tau{}^{\frac{1}{\theta-1}}$. Note the optimal capital stock and project value depend only on x_τ, Ξ_τ and z_{ft}.

The gross profitability of project j upon installation at time τ is

$$\text{GP/A}_{j\tau} = \frac{\Pi_{j\tau}}{K_{j\tau}} = \hat{\Pi}_{f\tau}^{AP}(x_\tau, z_{ft}). \tag{15}$$

Note that the gross profitability of a newly installed project only varies with x_τ and z_{ft} and does not depend on Ξ_τ. Therefore, the project and firm gross profitability in the model is stationary despite the stochastic trend in Ξ_t, which we introduce below.

We specify a_{ft}, the logarithm of the project arrival rate for firm f, to follow a mean-reverting process:

$$a_{ft} = \rho_a a_{ft-1} + (1 - \rho_a)\bar{a} + \sigma_a \varepsilon_{ft}^{a} - \frac{1}{2}\sigma_a^2. \tag{16}$$

We assume that $\xi_t \equiv \ln \Xi_t$ follows a random walk:[8]

$$\Delta \xi_t = \mu_\xi + \sigma_\xi \varepsilon_t^\xi - \frac{1}{2}\sigma_\xi^2. \tag{17}$$

The present value of growth opportunities V_{ft}^{GO} is given by the following recursive form:

$$V_{ft}^{GO}(x_t, \Xi_t, z_{ft}, a_{ft})$$
$$= \mathrm{E}_t \left[M_{t+1} \left[\exp(a_{ft+1})(\theta - 1) \left(\frac{\hat{V}_{ft+1}^{AP}(x_{t+1}, z_{ft+1})}{\theta} \right)^{\frac{\theta}{\theta-1}} \Xi_{t+1}{}^{\frac{1}{\theta-1}} + V_{ft+1}^{GO}(x_{t+1}, \Xi_{t+1}, z_{ft+1}, a_{ft+1}) \right] \right], \tag{18}$$

Because V_{ft}^{GO} is linear in $\Xi_t^{\frac{1}{\theta-1}}$, we define $\hat{V}_{ft}^{GO} = V_{ft}^{GO}/\Xi_t^{\frac{1}{\theta-1}}$ and the above equation becomes:

$$\hat{V}_{ft}^{GO}(x_t, z_{ft}, a_{ft}) = \mathrm{E}_t \left[M_{t+1} \left[\exp(a_{ft+1})(\theta - 1) \left(\frac{\hat{V}_{ft+1}^{AP}(x_{t+1}, z_{ft+1})}{\theta} \right)^{\frac{\theta}{\theta-1}} + \hat{V}_{ft+1}^{GO}(x_{t+1}, z_{ft+1}, a_{ft+1}) \right] \exp\left(\frac{\mu_\xi + \sigma_\xi \varepsilon_{t+1}^\xi - \frac{1}{2}\sigma_\xi^2}{\theta - 1} \right) \right]. \tag{19}$$

Taken together, firm value is equal to the sum of the values of assets in place and growth opportunities:

$$V_{ft} = V_{ft}^{AP} + V_{ft}^{GO} = \hat{V}_{ft}^{AP} K_{ft} + \hat{V}_{ft}^{GO} \Xi_t^{\frac{1}{\theta-1}}, \tag{20}$$

where we have defined $K_{ft} = \sum_{j \in \text{ Projects}(f)} K_{j\tau}$. Firm gross profitability is

$$\mathrm{GP/A}_{ft} = \frac{\sum_{j \in \text{ Projects}(f)} \Pi_{jt}}{K_{ft}}, \tag{21}$$

[8]Our parameter μ_ξ is lower than the overall rate of capital-embodied technological change in the economy, because it applies only to the incumbent firms. These firms suffer from displacement due to technological advances introduced by the new entrants into the market, which our model does not capture. Thus, as it affects the value of firms' growth opportunities, μ_ξ captures both the direct impact of investment-specific technological progress and the associated displacement effects.

and firm log book-to-market equity ratio is

$$\text{logBM}_{ft} = \ln\left(\frac{K_{ft}}{V_{ft}}\right). \tag{22}$$

In this model we abstract from modeling financial leverage and real leverage arising from fixed operating costs. All of these effects are subsumed by the effective leverage ratio ϕ, which we use to scale excess stock returns (i.e., $\text{Ret}^{\text{ex}}(\text{Levered}) = \phi \times \text{Ret}^{\text{ex}}(\text{Unlevered})$) – this essentially creates a wedge between asset return volatility and cash flow volatility that we target in the model estimation. We analyze the effect of operating leverage on asset returns and the dynamics of real quantities in Kogan, Li, Zhang, and Zhu (2020).

3.1.3 Stochastic discount factor

We specify the stochastic discount factor (SDF) M_t to be a function of the two aggregate shocks, ε_t^x and ε_t^ξ:

$$M_{t+1} = \exp\left(-r_f - \gamma_x \varepsilon_{t+1}^x - \gamma_\xi \varepsilon_{t+1}^\xi - \frac{1}{2}\gamma_x^2 - \frac{1}{2}\gamma_\xi^2\right), \tag{23}$$

where γ_x and γ_ξ are the prices of risks for the aggregate profitability shock and the aggregate investment shock, respectively, and r_f is the risk-free rate. For simplicity, we assume that the SDF is homoscedastic, and therefore prices of risk associated with the two systematic shocks do not vary over time. Because our analysis is in partial equilibrium, the above specification of the SDF is agnostic about the origins of the premia on the systematic shocks: these could be consistent with a traditional, "rational" model of investor behavior, or with a behavioral model (see Kozak, Nagel, and Santosh, 2018). This SDF simply reflects absence of arbitrage in the market with two systematic shocks, ε_t^x and ε_t^ξ, and imposes constant prices of risk for simplicity.

3.2 Estimation of the model parameters

There are 18 parameters in our model. We calibrate three parameters as reported in Panel A of Table 3. Specifically, we set the constant monthly risk-free rate to be 0.24%, corresponding to an annualized value of 2.92% as in Campbell and Cochrane (1999). The project depreciation rate is set to 1% per month (or 12% per year), consistent with the literature on the real business cycles (e.g., Kydland and Prescott (1982), Cooper and Haltiwanger (2006)). The average of the log of the project arrival rate $\bar{a}$ is set to 0.3, which implies an average of about 83 projects per firm. Since the profitability shocks across projects within a firm are identical, the number of projects has little impact on the asset pricing properties.[9]

[Insert Table 3 Here]

We estimate the remaining 15 parameters, listed in Panel B of Table 3, using the simulated method of moments (Lee and Ingram (1991)). Given a vector Ψ of target moments in the data, we obtain parameter estimates by

$$\hat{p} = \arg\min_p \left(\Psi - \frac{1}{S} \sum_{i=1}^{S} \hat{\Psi}_i(p) \right)' W \left(\Psi - \frac{1}{S} \sum_{i=1}^{S} \hat{\Psi}_i(p) \right), \tag{24}$$

where $\hat{\Psi}_i(p)$ is the vector of moments computed in one out of S simulations of the model. We choose the weighting matrix $W = \text{diag}(\Psi\Psi')^{-1}$ to penalize proportional deviations of the model statistics from their empirical counterparts. We solve the model numerically using value function iterations at a monthly frequency. We simulate the model 100 times ($S = 100$) with each sample representing 1,000 firms and 600 months. Following Bloom (2009), we solve the above minimization problem using an annealing algorithm to find the global minimum.

The 28 target moments in our estimation include moments of asset returns and economic variables at the aggregate, portfolio, and firm levels. Aggregate moments such as the properties of aggregate GP/A, ln(BM), and investment are informative about the processes of aggregate profitability shocks and aggregate investment shocks. The elasticity of aggregate variable input prices with respect to aggregate revenue, the relative volatilities of aggregate sales, variable costs,

[9]In the Online Appendix, we report the moments using alternative values of $\bar{a}$, while keeping other parameters unchanged. The result shows that all target moments are quantitatively close to those from the benchmark parameterization ($\bar{a} = 0.3$), and the difference is mainly numerical due to more or fewer projects being simulated. We chose $\bar{a} = 0.3$ to balance the need for a sufficient number of projects in a firm and computational cost in the SMM estimation.

and gross profit also contain information about the elasticity of substitution between capital and variable inputs (η) and the level and cyclicality of aggregate variable input prices (p_0 and p_1). In the cross-section, the moments of firm-level sales growth, variable cost growth, GP/A, ln(BM), as well as the dispersions in GP/A, ln(BM), and gross margin across different portfolios help identify the dynamics of firm-level and project-level processes. The relation between the gross profit margin and gross profitability, in particular, helps identify η, as can be seen from Equation (6). Specifically, the positive relation between these two variables, which can also be seen in Table 5 below, implies that $\eta < 1$. Lastly, we include the Sharpe ratios and return volatilities of the market factor, gross profitability factor, and value factor, respectively. In particular, we define the value and gross profitability factors as long-short portfolio based on the quintiles of firms sorted on the corresponding characteristic. Because our model abstracts from inter-industry heterogeneity, we construct the empirical gross profitability factor by an intra-industry sort with 30 industries, based on the classification by Fama and French (see Section D of the Online Appendix for a comparison of the intra-industry and the unconditional profitability premia). Restrictions from the Sharpe ratios of the market, gross profitability, and value factors help pin down prices of risk of the two systematic shocks.[10]

The last two columns of Panel B Table 3 report the parameter estimates and standard errors.[11] In the model, the estimated price of risk (Sharpe ratio) for the aggregate profitability shock is 0.297, whereas the price of risk for the aggregate investment shock is -0.403. The signs of the

[10]To shed light on the sources of information about specific model parameters, we plot the Andrews, Gentzkow, and Shapiro (2017) measure of sensitivity of parameters to moment values in Figure A1 of the Online Appendix. We compute this measure as an elasticity, i.e., a percentage change in a parameter associated with a percentage change in a moment. Figure A1 shows, in particular, that the average aggregate sales/COGS ratio and the elasticity of aggregate variable input prices with respect to aggregate revenues are highly informative about p_1 (the cyclicality of variable input prices), and the average aggregate sales/COGS ratio and the relative volatility of aggregate sales and GP are particularly informative about η (the elasticity of substitution between capital and variable inputs). This should not be surprising because these moments are linked to the core mechanism of the operating hedge.

[11]Following Lee and Ingram (1991), the variance-covariance matrix of parameter estimates can be computed using $\left(1+\frac{1}{N_S}\right)(J'WJ)^{-1}J'W\Omega WJ(J'WJ)^{-1'}$, where $N_S = 100$ is the number of simulations, J is the score matrix that measures the sensitivity of simulation moments with respect to parameters, and Ω is the variance-covariance matrix of moments. To estimate the score matrix J, we calculate four values of the numerical derivative from perturbing each parameter by four different amounts in percentage, i.e., ε, and then take the average value of these four numerical derivatives. For the persistence parameters ρ_x, ρ_z, and ρ_a, we choose $\varepsilon = -0.1\%, 0.1\%, -0.05\%, 0.05\%$. For all other parameter, $\varepsilon = -1\%, 1\%, -0.5\%, 0.5\%$. The standard errors of the parameter estimates are the diagonal elements of the variance-covariance matrix of parameter estimates.

prices of risk for these two factors are consistent with the prior results in Kogan and Papanikolaou (2013, 2014) and Kogan, Papanikolaou, and Stoffman (2020). The two critical parameters that affect the relation between gross profitability and systematic risk of cash flows are the elasticity of substitution between capital and variable input (i.e., η), and the elasticity of variable inputs price with respect to aggregate profitability shocks (i.e., p_1). We estimate η to be 0.696 (with a standard error of 0.0175). The estimated elasticity of substitution between capital and variable inputs is lower than one, which means that the variable input and capital are complements in the firms' production function. The estimated value of p_1 is 0.309 and statistically significantly greater than 0, which means that the price of the variable input is procyclical: $p_1 > 0$. Lastly, the estimated effective leverage ratio $\phi = 2.946$. This value is close to 3.0 in Bansal and Yaron (2004).

In Table 4, we compare the values of the targeted moments to their empirical counterparts. Specifically, we compare the values of these moments in the data with the mean, 2.5th, 25th, 75th, and 97.5th percentiles of the corresponding moments in model simulations. Most of the empirical moments are close to their model counterparts, and fall within the 2.5th-97.5th intervals from simulations. The annualized market Sharpe ratio and volatility are 0.473 and 17.3% in our simulations, compared with 0.428 and 15.2% in the data, respectively. In the data, the standard deviation of the aggregate sales growth is 6%, while in simulations this number is 6.3% on average. The model matches well the volatility of gross profit and the volatility of variable costs relative to the aggregate sales (0.82 and 1.11 in the model, compared to 0.89 and 1.12 in the data). This difference in the cyclicality of aggregate sales and variable costs is essential for the operating hedge effect. The elasticity of the aggregate input prices to aggregate revenue is positive at 0.333, which is somewhat lower than the estimate in the data (0.482).

The model has some difficulty matching the large cross-sectional dispersion in book-to-market ratios in the data, with the average value of 0.81 in the model versus the spread of 2.09 in the data. The fit of the gross profitability spread is much closer, with the average of 0.422 for the model compared to 0.54 in the data. Table 4 shows that the model-implied GP/A factor has a Sharpe ratio of 0.532 with the volatility of 8.12% relative to the Sharpe ratio of 0.397 with the volatility of 9.32% in the data. The model-implied value factor has the Sharpe ratio of 0.355, which is comparable to the 0.31 in the data, and a lower volatility (9.06% relative to 13.6% in the data).[12] Overall, however, the model is rejected by the overidentification test (Lee and Ingram (1991)).

[Insert Table 4 Here]

3.3 Implications for the gross profitability and value factors

Our model reproduces the empirical relation between firms' gross profitability and their book-to-market ratios, as we show in Table 5. This table summarizes average firm characteristics and stock returns across the gross-profitability quintile portfolios, and the results of CAPM tests. Panel A uses historical data, and Panel B shows analogous results in the simulated data from the model. Panel B1 shows that high-GP/A firms in our simulations have a ln(BM) of −0.99, as compared to −0.68 for low-GP/A firms. The corresponding empirical values are −0.76 and −0.29, respectively (Panel A1).

[Insert Table 5 Here]

Panel B1 confirms that the cross-sectional variation in GP/A is mainly driven by the idiosyncratic profitability shock and that gross margin increases with GP/A, as it does in the data. The relation between gross profitability and gross margin is central to the operating hedge effect: more profitable firms have higher gross margin, i.e., lower variable costs relative to revenue, than less profitable firms. Because of that, more profitable firms experience lower operating hedge. The higher profitability of high-GP/A firms raises the value of assets in place, giving rise to a lower average VGO/VAP than for low-GP/A firms. In the data (Panel A1), gross profitability has a slightly negative correlation with financial leverage (Flev) and weak correlation with operating leverage (Olev).

[12] The estimated average rate of the aggregate investment-specific technological progress, μ_ξ, is slightly negative. As we discuss above in Footnote 8, our parameter μ_ξ is likely lower than the overall rate of capital-embodied technological change in the economy, because our model abstracts from firm displacement due to investment-specific technological progress. For instance, asset pricing implications of the model remain virtually unchanged if we allow for firm death (displacement) at the annual constant rate of 4%, raising the value of μ_ξ from −0.177% to 0.048% (corresponding to 0.576% per year).

Panels A2 and B2 of Table 5 present the value-weighted returns and CAPM tests of the gross profitability portfolios. The model generates a positive gross profitability premium of 4.32% per year, compared with 3.7% in the data. The pattern in the market beta across GP/A quintiles is weak, and the CAPM alpha of the gross profitability factor — the long-short Hi-Lo quintile portfolio — is 3.61% per year (with a t-statistic of 3.21). The CAPM alpha is 4.98% per year in the data (with a t-statistic of 3.97).

Table 6 shows the results of the same analysis on ln(BM) quintiles. Panel B1 shows that the cross-sectional variation in ln(BM) in the model is primarily associated with the firm-specific project arrival rate a. Compared to value firms, growth firms have a higher project arrival rate on average and a greater value of growth opportunities relative to assets in place (VGO/VAP). The finding that VGO/VAP is higher among growth and low-GP/A (Table 5) firms is consistent with Kogan and Papanikolaou (2014). Our model also reproduces the empirical finding that the gross profit margin tends to be higher in growth firms than in value firms. Intuitively, firms with higher idiosyncratic profitability also have a higher valuation ratio (low ln(BM)) than firms with low profitability.[13]

[Insert Table 6 Here]

Panels A2 and B2 of Table 6 show the average returns and CAPM test results of the ln(BM) quintiles. Our model generates a value premium of 3.11%, slightly lower than 4.21% in the data. The CAPM beta goes in the wrong direction (a negative market beta of the value factor) both in the data and in the model, so that the abnormal return spread is even greater than the raw return spread. Above results show that the profitability premium and the value premium coexist in our model, and the unconditional CAPM fails to explain them.

[13]Panel A of Table 6 also reports the average financial leverage and operating leverage across ln(BM) quintiles. Value stocks have a higher financial leverage than growth stocks, consistent with the findings in Ozdagli (2012) and Favilukis and Lin (2016).

3.4 The economic mechanism

To understand the drivers of the profitability premium and the value premium in our model, we examine the exposures of the GP/A and ln(BM) portfolios to the aggregate profitability shock (X) and the aggregate investment shock (Ξ). We summarize the results in Table 7, for GP/A portfolios in Panel A, and for ln(BM) portfolios in Panel B. We find that high-GP/A firms have higher exposures to the aggregate profitability shock than low-GP/A firms, with a significant difference in β_X of 0.14. Thus, the aggregate profitability shock is an important risk factor behind the positive gross profitability premium in the model. The asset composition channel in Kogan and Papanikolaou (2013) also contributes to the gross profitability premium: firms with low profitability tend to be growth firms, with higher VGO/VAP, and hence are more exposed to the aggregate investment-specific shocks. The difference in the β_Ξ between high- and low-GP/A firms is -0.46.

[Insert Table 7 Here]

For the value premium, our results show that value firms have an exposure to the aggregate investment shock of 0.96, much lower than that of growth firms (1.85), consistent with the asset composition channel in Kogan and Papanikolaou (2014). In addition, the exposure of the book-to-market portfolios to aggregate profitability shocks decreases from 1.28 from growth firms to 1.11 for value firms. There are two competing effects driving the relation between the book-to-market ratio and β_X. First, value firms tend to have lower idiosyncratic profitability (Z) and are therefore more affected by the operating hedge effect. This effect implies lower β_X for value firms. Second, value firms have a lower share of growth opportunities relative to assets in places, which gives rise to a higher exposure to aggregate profitability shocks. This is because assets in place are more sensitive to aggregate profitability shocks than growth opportunities. For our estimated parameter values, the first effect is stronger. Overall, the positive value premium in our model is mostly driven by the differences in firms' exposures to aggregate investment-specific shocks.

Although both the gross profitability factor and the value factor have negative loadings on the aggregate investment shock, Panel B of Table 7 shows that their opposite exposures to the aggregate profitability shock generate a negative correlation of −19% (compared to −40% in the data) between these two factors with a standard error of 1.3% across simulations. Furthermore, because gross profitability and book-to-market ratios are also negatively correlated, portfolios constructed by conditional double sorting based on these two characteristics can further enhance the performance of these factors. Indeed, Table 8 shows that based on sequential double sorts, the gross profitability premium conditional on book-to-market is 6.07% (versus 6.4% in the data), and the value premium conditional on GP/A is 5.59% (versus 7.16% in the data). Therefore, the conditional return premium associated with both the gross profitability and the book-to-market ratio is substantial higher than its unconditional counterpart. Further, the model-implied Sharpe ratio is 0.76 for the conditional gross profitability premium and 0.65 for the conditional value premium. These values are quantitatively close to 0.65 and 0.56, respectively, in the data.

[Insert Table 8 Here]

We find that our production-based model is able to reproduce the interaction between GP/A and logBM in predicting future stock returns, even though we do not explicitly target the conditional return premia in estimation. In the data, the GP/A premium is 13.08% among growth stocks and only 2.5% in value stocks. In the model simulations, the GP/A premium is 8.32% among growth stocks vs. 3.37% among the value stocks. Similarly, our model generates the empirical pattern that the value premium is substantially stronger among low profitability stocks.

3.5 Additional empirical evidence

3.5.1 Cash flow risk of GP/A portfolios

The main mechanism for the profitability premium in our model is that cash flows of profitability-sorted portfolios load differently on systematic profitability shocks, and therefore the gross profitability factor also loads strongly on the systematic profitability shock. Note that this factor also loads on aggregate investment shocks. However, near-term profits are relatively insensitive to the investment-specific shock (which affects investment and long-term cash flows of the firm). We should therefore observe that high-gross-profitability firms have higher cash flow loadings on the gross profitability factor returns.[14]

Table 9 quantifies differences in systematic cash flow risk of profitability-sorted portfolios in the model. It shows the relation between the growth rates of gross profit, revenue, and the cost of goods sold (COGS) of the gross-profitability quintile portfolios, from t to $t+K$ ($K = 0,\ 1,\ 2$), and the gross profitability factor return in year t (we normalize the return process to have a unit standard deviation). Consistent with the model mechanism discussed above, the returns on the gross profitability factor reflect differences in systematic cash flow shocks (gross profit beta) of low- and high-profitability firms, and the gross profit beta increases with gross profitability.

The gross profit beta of the lowest-GP/A portfolio is negative, albeit typically insignificant. To see how the negative exposure arises in the model, note that according to Equation (4), firms with low firm-specific profitability Z may have negative systematic cash flow risk. Since variable inputs and capital stock are complements ($\eta < 1$), as idiosyncratic profitability falls, firms are reluctant to reduce their variable input use, which leads their gross margins to decline. Firms' cash flow exposures to the aggregate profitability shock may then turn negative due to a strong hedge effect from variable costs.

Table 9 also shows that betas of both revenue and variable costs increase with gross profitability, but not as much as the betas of gross profit. These results reflect the main operating hedge mechanism in the model — high-profitability firms have more cyclical profit primarily because their variable costs, which are procyclical, are lower relative to their revenue.[15]

[Insert Table 9 Here]

Next, we examine the relation between gross profit, revenue, and the cost of goods sold of the gross profitability quintile portfolios, and the gross profitability factor return in the data. We find

[14]Our empirical results below are similar when we control for measures of investment shocks such as the investment-minus-consumption portfolio return (Kogan and Papanikolaou, 2014).

[15]The cost of goods sold is slightly less cyclical than revenue in the high GP/A quintile in Table 9. This is due to the fact that the GP/A factor in our model is driven by both aggregate profitability and aggregate investment shocks. In Table A6 (Panel A) of the Online Appendix, we examine cash flow responses to the aggregate profitability shock in simulated data. We find that the cost of goods sold is more cyclical than revenue in the high GP/A quintile, whereas the other results discussed above still hold.

that the difference in cash flow risk between low- and high-gross-profitability firms is economically and statistically significant, and close in magnitude to the corresponding patterns in the model. Table 10 shows that a one-standard-deviation unexpected positive return of the GP/A factor portfolio is associated with a 0.92% contemporaneous decline in the growth rate of gross profit for the low-GP/A portfolio, and a contemporaneous increase of 1.42% in the gross profit growth for the high-GP/A portfolio, so the difference between the high- and low-GP/A portfolio is 2.35% (with a t-statistic of 2.76). The difference in cumulative growth increases further to 4.32% and 5.41% over two-year ($K = 1$) and three-year ($K = 2$) periods, respectively.

[Insert Table 10 Here]

Looking into the source of the cash flow beta difference among the portfolios in Table 10, we find that the pattern in the sales beta and COGS beta is much weaker. The difference in the sales beta between low- and high-GP/A firms is quantitatively similar to that for the COGS beta, and both differences are substantially smaller than the difference in gross profit betas. These patterns are similar to those in the model, as we show in Table 9. Therefore, consistent with the economic mechanism of our model, cross-sectional differences in gross profit betas arise mainly from the compositional difference between revenue and costs, rather than from the difference in cyclicality of sales and COGS across the gross profitability portfolios.

3.5.2 Risk premium on the systematic profitability shock

Next, we provide additional evidence on the risk premium associated with the aggregate profitability shock. Our structural parameter estimates imply a positive price of risk for the systematic profitability shock, which is essential for matching the gross profitability premium in stock returns. To evaluate this result empirically, we start by using the gross-profitability factor as a factor-mimicking portfolio for the gross profitability shock, and estimate its price of risk using an alternative set of test assets: the Fama-French 17 and 30 industry portfolios. We specify the stochastic discount factor as a linear function of the market factor and the profitability factor, and estimate prices of risk of these two shocks in the GMM framework.[16] Panel A of Table 11 shows that the two-factor model describes risk premia on the industry portfolios quite well. The annualized mean absolute error in the risk premia implied by the model across the test assets is close to 1.1% in both cases. The over-identification test fails to reject the two-factor model. Observed good performance of the pricing model is nontrivial since the factor-mimicking portfolio is constructed within industries, whereas the test assets are industry portfolios. More importantly, the estimated price of risk for the aggregate profitability shock is positive and statistically significant. The estimated SDF also matches closely the average excess returns on the stock market and the gross profitability factor in our sample.

Our analysis above builds on the profitability factor in stock returns loading strongly on the systematic profitability shock, which is supported by the evidence on cash flow risk in Table 9. Next, we approximate the aggregate profitability shock directly using the utilization-adjusted total factor productivity shock (dTFP) from Basu, Fernald, and Kimball (2006) and Fernald (2014). Panel B of Table 11 confirms that returns on the profitability-sorted portfolios have an increasing profile of betas with respect to the dTFP series.[17] This implies, in particular, that a positive risk premium on TFP shocks would help generate a positive profitability premium in stock returns.

In Panel C of Table 11, we follow the same design as in Panel A. We find that TFP growth shocks command a positive risk premium, which is close in magnitude to the premium in Panel A estimated using the profitability factor returns as the second factor. Collectively, results in Table 11 help support an important exogenous element of our model — that systematic profitability shocks enter the SDF with a positive price of risk.

[Insert Table 11 Here]

[16] We only report the results using the identity weighting matrix, but results are quantitatively similar with the efficient GMM weighting matrix.

[17] We use TFP growth over a two-year period, because we find that stock returns on the profitability-sorted portfolios partly reflect the news about TFP growth over the next year. Because of their forward-looking nature, stock returns commonly lead the realization of growth in real variables. In a closely related context, this idea has been advanced in Parker and Julliard (2005), who use the multi-quarter consumption growth (which they term the ultimate consumption risk) to measure SDF shocks.

4 Conclusion

In this paper we explore a novel economic channel for heterogeneity in cash flow risk among firms — the operating hedge effect arising from procyclical variable input costs. This operating hedge effect implies that firms' exposures to aggregate profitability shocks correlate with their firm-specific profitability: less profitable firms benefit more from risk reduction due to variable costs, and thus exhibit lower cash flow risk and lower average stock returns.

We analyze this phenomenon quantitatively using a dynamic structural model, in which the profitability premium coexists with the value premium. The two premia are generated by different economic channels. The value premium largely reflects cross-sectional differences in firms' growth opportunities, and thus their different exposures to the aggregate investment-specific shock. The profitability factor is driven primarily by the systematic profitability shocks and the profitability premium reflects differential operating hedge effect of firms with different profitability. Our model produces gross profitability and value factors that are negatively correlated with each other, and reproduces the failure of the CAPM to explain expected returns on the two factors — these patterns have been difficult to reconcile within the existing structural models of stock returns.

Our results complement the existing literature focused on operating leverage as the source of the cross-sectional differences in expected stock returns. We show that the impact of production costs on firms' cash flow risk is more nuanced than suggested by the operating leverage channel alone, and variable costs give rise to a first-order operating hedge effect on firm cash flows. Further research is needed to better understand the properties of firms' costs, including their relation to the input-output structure of the economy and cross-sectional differences in production technologies and market power. This is likely to offer useful insights into the fundamental properties of stock returns and firm dynamics.

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Appendix

Proof of Proposition 1:

The first order condition of Equation (1) implies that

$$P = XZE^{\frac{-1}{\eta}} \times (E^{\frac{\eta-1}{\eta}} + K^{\frac{\eta-1}{\eta}})^{\frac{1}{\eta-1}}. \tag{A.1}$$

At the optimum, the share of the variable inputs (ES) is given by

$$ES \equiv \frac{PE}{Y} = \frac{E^{\frac{\eta-1}{\eta}}}{E^{\frac{\eta-1}{\eta}} + (XK)^{\frac{\eta-1}{\eta}}}. \tag{A.2}$$

This implies that the firm's gross profit satisfies

$$\Pi = Z\left(E^{\frac{\eta-1}{\eta}} + (XK)^{\frac{\eta-1}{\eta}}\right)^{\frac{1}{\eta-1}} (XK)^{\frac{\eta-1}{\eta}}, \tag{A.3}$$

and therefore the gross profitability is given by

$$\text{GP/A} \equiv \frac{\Pi}{K} = XZ\left[\left(\frac{E}{XK}\right)^{\frac{\eta-1}{\eta}} + 1\right]^{\frac{1}{\eta-1}}. \tag{A.4}$$

Taking the partial derivative of the logarithm of both sides of Equation (A.1) with respect to $\ln X$, we have:

$$\frac{\partial \ln E}{\partial \ln X} = 1 - p_1\eta\left[1 + \left(\frac{E}{XK}\right)^{\frac{\eta-1}{\eta}}\right]. \tag{A.5}$$

Differentiating the firm's gross profit (Π) and revenue (Y) in logarithm with respect to the logarithm of profitability shock (X) and using Equation (A.5), we obtain the profitability shock elasticities of firm's gross profit and revenue as follows:

$$\frac{\partial \ln \Pi}{\partial \ln X} = 1 - p_1\left(\frac{E}{XK}\right)^{\frac{\eta-1}{\eta}}, \tag{A.6}$$

$$\frac{\partial \ln Y}{\partial \ln X} = 1 - p_1 \eta \left(\frac{E}{XK} \right)^{\frac{\eta-1}{\eta}}. \tag{A.7}$$

In this static model, $\beta_X \equiv \frac{\partial \ln \Pi}{\partial \ln X}$ is also the firm's systematic risk exposure.

When $p_1 > 0$ and $\eta < 1$,

$$\frac{\partial \ln \Pi}{\partial \ln X} - \frac{\partial \ln Y}{\partial \ln X} = p_1(\eta - 1) \left(\frac{E}{XK} \right)^{\frac{\eta-1}{\eta}} < 0. \tag{A.8}$$

Q.E.D.

Proof of Proposition 2:

To prove Proposition 2, we take the partial derivative of the logarithm of both sides of Equation (A.1) with respect to $\ln Z$:

$$\frac{\partial \ln E}{\partial \ln Z} = \eta \left[1 + \left(\frac{E}{XK} \right)^{\frac{\eta-1}{\eta}} \right]. \tag{A.9}$$

Then differentiating the logarithms of revenue, variable cost, and gross profit with respect to the logarithm of idiosyncratic profitability Z and using Equation (A.9), and we have

$$\frac{\partial \ln Y}{\partial \ln Z} = 1 + \eta \left(\frac{E}{XK} \right)^{\frac{\eta-1}{\eta}}, \tag{A.10}$$

$$\frac{\partial \ln(PE)}{\partial \ln Z} = \eta \left[1 + \left(\frac{E}{XK} \right)^{\frac{\eta-1}{\eta}} \right], \tag{A.11}$$

and

$$\frac{\partial \ln \Pi}{\partial \ln Z} = 1 + \left(\frac{E}{XK} \right)^{\frac{\eta-1}{\eta}}. \tag{A.12}$$

Therefore, when $\eta < 1$,

$$\frac{\partial \ln(PE)}{\partial \ln Z} - \frac{\partial \ln Y}{\partial \ln Z} = \eta - 1 < 0, \tag{A.13}$$

and

$$\frac{\partial \ln \Pi}{\partial \ln Z} - \frac{\partial \ln Y}{\partial \ln Z} = (1 - \eta) \left(\frac{E}{XK} \right)^{\frac{\eta-1}{\eta}} > 0. \tag{A.14}$$

Q.E.D.

Proof of Proposition 3:

The elasticity of firm GP/A with respect to Z is

$$\frac{\partial \ln(\mathrm{GP/A})}{\partial \ln Z} = 1 + \left(\frac{E}{XK}\right)^{\frac{\eta-1}{\eta}} > 0. \tag{A.15}$$

Therefore, gross profitability always increases with idiosyncratic profitability Z. To examine how a firm's risk exposure of gross profit to the aggregate profitability shock varies with its gross profitability, we inspect $\frac{\partial \beta_X}{\partial \ln Z}$:

$$\frac{\partial \beta_X}{\partial \ln Z} = p_1(1-\eta)\left(\frac{E}{XK}\right)^{\frac{\eta-1}{\eta}}\left[1+\left(\frac{E}{XK}\right)^{\frac{\eta-1}{\eta}}\right]. \tag{A.16}$$

The last two terms in Equation (A.16) are always positive. When $p_1 > 0$ and $\eta < 1$, $\frac{\partial \beta_X}{\partial \ln Z} > 0$, that is, profitable firms are more exposed to systematic profitability shocks. Q.E.D.

Table 1: The cyclicality of aggregate revenue, costs, and gross profit

This table reports the cyclicality of aggregate revenue, costs, and gross profit. The values of aggregate revenue, costs, and gross profit are from the NBER-CES Manufacturing Industry Database. Revenue (Rev) is measured as the total value of shipments. Variable costs (COGS) are defined as the sum of costs for materials (Mat), energy (Eng), and production worker wages (Prd). As a comparison, we also consider office worker wages (Off). These values are further deflated by the Consumer Price Index. Panel A reports the mean and standard deviation of the ratio of each type of costs and gross profit to total revenue, as well as the annual growth rates of revenue (ΔlogV(Rev)), material costs (ΔlogV(Mat)), energy costs (ΔlogV(Eng)), the total variable costs (ΔlogV(COGS)), gross profit (ΔlogV(GP)), and office worker wages (ΔlogV(Off)) in percentages. Panel B reports the correlation matrix of the growth rates in the values of aggregate revenue, variable costs, and gross profit. Panel C reports the elasticity of V(COGS) and V(GP) with respect to V(Rev) (β^{V}_{Rev}(COGS) and β^{V}_{Rev}(GP), respectively), which are estimated from univariate time-series regressions. In parentheses are the Newey-West t-statistics computed with 4 lags to control for heteroskedasticity and autocorrelation. The data are annual from 1958 to 2011.

Panel A: Summary statistics

	V(Mat)/V(Rev)	V(Eng)/V(Rev)	V(Prd)/V(Rev)	V(COGS)/V(Rev)	V(Off)/V(Rev)
Mean (%)	54.83	1.93	10.80	67.56	6.60
Std (%)	2.11	0.41	2.87	3.62	0.87

	ΔlogV(Rev)	ΔlogV(Mat)	ΔlogV(Eng)	ΔlogV(Prd)	ΔlogV(COGS)	ΔlogV(GP)	ΔlogV(Off)
Mean (%)	1.87	1.96	20.8	0.0	1.69	2.27	0.86
Std (%)	5.47	6.73	7.95	4.99	6.36	4.79	3.27

Panel B: Correlation matrix

	ΔlogV(Rev)	ΔlogV(COGS)	ΔlogV(GP)
ΔlogV(Rev)	1.00	0.98	0.83
ΔlogV(COGS)	0.98	1.00	0.71
ΔlogV(GP)	0.83	0.71	1.00

Panel C: Elasticity of variable costs and gross profit

	β^{V}_{Rev}(COGS)	β^{V}_{Rev}(GP)
Est.	1.14	0.73
t-stat	(26.83)	(9.06)

Table 2: Cash flow elasticities at aggregate and firm levels

This table reports the cash flow elasticities at the aggregate level and the firm level. In Panel A1, we report the mean and standard deviation of the aggregate-level sales growth (ΔlogASale) and the aggregate-level gross profit growth (ΔlogAGP). In Panel A2, we estimate the elasticity of AGP with respect to ASale by running the time series regression: $\Delta\text{logAGP} = a + b \times \Delta\text{logASale}$. In Panel B1, we report the mean and standard deviation of the firm-level sales growth (ΔlogSale) and firm-level gross profit growth (ΔlogGP). In Panel B2, we report the elasticity of GP with respect to Sale by running value-weighted Fama-MacBeth regressions $\Delta\text{logGP}_{it} = a_t + b_t \times \Delta\text{logSale}_{it}$ using lagged revenue as the weight and report the time series average of b_t. The Newey-West t-statistics computed with 4 lags in Panels A2 and B2 control heteroskedasticity and autocorrelation. The sample is annual from 1964 to 2019.

Panel A: Sales growth and gross profit growth at the aggregate level

Panel A1

	ΔlogASale	ΔlogAGP
Mean (%)	2.82	3.12
Std (%)	5.95	5.29

Panel A2

β_{ASale}(AGP)	
Est.	0.77
t-stat	(14.61)

Panel B: Sales growth and gross profit growth at the firm level

Panel B1

	ΔlogSale	ΔlogGP
Mean (%)	7.9	7.06
Std (%)	34.46	35.02

Panel B2

β_{Sale}(GP)	
Est.	1.07
t-stat	(13.01)

Table 3: Model parameters

This table reports model parameters. Panel A lists the calibrated parameter values. Panel B lists the parameters that are estimated from the simulated method of moments (SMM). We solve and simulate the model at a monthly frequency.

Panel A: Calibrated parameters

Parameter	Symbol	value
Risk-free rate	r_f	0.243%
Depreciation rate	δ	0.01
Average log of the project arrival rate	$\bar{a}$	0.3

Panel B: Parameters estimated via SMM

Parameter	Symbol	Est.	S.E.
Price of risk for aggregate profitability shocks	γ_x	0.297	0.0744
Price of risk for aggregate investment shocks	γ_ξ	-0.403	0.183
Persistence of aggregate profitability shocks	ρ_x	0.996	0.0015
Conditional volatility of aggregate profitability shocks	σ_x	0.037	0.0047
Average growth rate of aggregate investment shocks	μ_ξ	-0.26%	0.07%
Conditional volatility of aggregate investment shocks	σ_ξ	0.013	0.0048
Logarithm of the level of variable inputs price	p_0	0.298	0.040
Elasticity of variable inputs price w.r.t. aggregate profitability shocks	p_1	0.309	0.0354
Persistence of idiosyncratic profitability shocks	ρ_z	0.969	0.0015
Conditional volatility of idiosyncratic profitability shocks	σ_z	0.059	0.0032
Persistence of project arrival rate	ρ_a	0.996	0.0019
Conditional volatility of project arrival rates	σ_a	0.115	0.0219
Capital adjustment cost coefficient	θ	1.857	0.143
Elasticity of substitution between capital and variable inputs	η	0.696	0.0175
Effective leverage ratio	ϕ	2.946	0.400

Table 4: Moments used in SMM estimation of the model

This table reports the moments used in the SMM estimation of the model. We solve and simulate the model at a monthly frequency, and report all statistics at an annual frequency. We simulate 100 independent samples from the model, with each sample representing 1,000 firms over 600 months. We report the moments in the data, and the mean, 2.5th, 25th, 75th, and 97.5th percentiles of the corresponding moments in simulations. Means and standard deviations of all returns are in percent.

		Model				
Moments	Data	Mean	2.5^{th}	25^{th}	75^{th}	97.5^{th}
Elasticity of aggregate variable input price w.r.t. aggregate revenue	0.482	0.333	0.197	0.297	0.373	0.438
Average aggregate gross profitability	0.249	0.255	0.204	0.237	0.277	0.309
AR(1) coefficient of aggregate gross profitability	0.922	0.892	0.745	0.857	0.937	0.977
Average aggregate ln(BM)	-0.622	-0.858	-1.011	-0.915	-0.797	-0.670
AR(1) coefficient of aggregate ln(BM)	0.912	0.852	0.679	0.792	0.921	0.952
Average aggregate investment rate	0.107	0.059	0.042	0.052	0.065	0.077
AR(1) coefficient of aggregate investment rate	0.796	0.884	0.672	0.856	0.934	0.962
Volatility of aggregate investment rate	0.019	0.013	0.007	0.010	0.015	0.020
Volatility of aggregate sales growth	0.060	0.063	0.051	0.058	0.067	0.078
Volatility of aggregate gross profit growth/volatility of aggregate sales growth	0.890	0.820	0.753	0.796	0.839	0.895
Volatility of aggregate variable cost growth/volatility of aggregate sales growth	1.115	1.110	1.077	1.099	1.119	1.143
Average ratio of aggregate sales to aggregate variable costs	1.450	1.504	1.431	1.476	1.531	1.586
AR(1) coefficient of firm-level gross profitability	0.886	0.754	0.746	0.751	0.756	0.762
AR(1) coefficient of firm-level Log(BM)	0.833	0.887	0.855	0.877	0.898	0.918
Volatility of firm-level sales growth	0.345	0.418	0.376	0.401	0.437	0.475
Volatility of firm-level variable cost growth	0.350	0.377	0.335	0.359	0.395	0.433
Gross profitability spread of gross profitability quintiles	0.540	0.422	0.307	0.377	0.473	0.557
Log(BM) spread of gross profitability quintiles	-0.470	-0.314	-0.375	-0.338	-0.297	-0.233
Gross margin spread of gross profitability quintiles	0.270	0.120	0.114	0.118	0.122	0.126
Log(BM) spread of Log(BM) quintiles	2.090	0.810	0.761	0.790	0.828	0.865
Gross profitability spread of Log(BM) quintiles	-0.150	-0.149	-0.237	-0.174	-0.120	-0.075
Gross margin spread of Log(BM) quintiles	-0.150	-0.048	-0.062	-0.053	-0.044	-0.033
Sharpe ratio of market factor	0.428	0.473	0.242	0.407	0.555	0.724
Volatility of market factor (%)	15.190	17.263	13.620	16.121	18.448	21.432
Sharpe ratio of GPA factor	0.397	0.532	0.295	0.453	0.614	0.764
Volatility of GPA factor (%)	9.315	8.117	6.876	7.632	8.611	9.292
Sharpe ratio of value factor	0.310	0.355	-0.087	0.221	0.496	0.725
Volatility of value factor (%)	13.598	9.060	7.808	8.481	9.448	10.735

Table 5: Characteristics and returns of gross profitability quintiles: data vs. model

This table reports the characteristics and returns of GP/A quintiles in the data (Panel A) and in simulated model output (Panel B). Panels A1 and B1 report the time series average of the cross-sectional median of firm characteristics for each GP/A quintile, including the gross profitability (GP/A), the logarithm of the book-to-market ratio (ln(BM)), and the gross profit margin (GM). The empirical characteristics in Panel A1 also include financial leverage (Flev), operating leverage (Olev), and Tobin's Q (Q), and the simulated characteristics in Panel B1 also include average logarithm of project profitability (z), the logarithm of the firm-level project arrival rate (a), and the ratio of the value of growth opportunities to the value of assets in place (VGO/VAP). Panels A2 and B2 report the mean and standard deviation of the value-weighted excess returns and the CAPM test results. The historical sample is from July 1963 to December of 2019, and GP/A quintiles are created by sorting firms within the Fama-French 30 industries. We simulate 100 independent samples from the model, with each sample representing 1,000 firms over 600 months. The portfolio returns, standard deviations, and the abnormal returns are annualized. The Newey-West t-statistics are computed with 4 lags to adjust for heteroskedasticity and autocorrelation.

Panel A: Data							Panel B: Model						
Panel A1: Characteristics							Panel B1: Characteristics						
	GP/A	ln(BM)	GM	Flev	Olev	Q		GP/A	ln(BM)	GM	z	a	VGO/VAP
Lo	0.11	-0.29	0.17	0.26	0.64	2.14	Lo	0.07	-0.68	0.25	-0.34	-0.92	1.10
2	0.25	-0.30	0.28	0.26	0.61	1.77	2	0.14	-0.75	0.28	-0.17	-0.93	0.93
3	0.34	-0.41	0.32	0.22	0.64	1.76	3	0.21	-0.81	0.31	-0.05	-0.94	0.82
4	0.45	-0.55	0.37	0.16	0.66	2.10	4	0.30	-0.87	0.33	0.06	-0.96	0.72
Hi	0.65	-0.76	0.44	0.10	0.72	2.53	Hi	0.50	-0.99	0.37	0.23	-1.00	0.58
Panel A2: Returns and CAPM							Panel B2: Returns and CAPM						
	Lo	2	3	4	Hi	Hi-Lo		Lo	2	3	4	Hi	Hi-Lo
Mean	4.59	6.36	7.08	6.31	8.29	3.70	Mean	5.57	6.59	7.42	8.41	9.89	4.32
Std	17.74	16.49	15.43	15.17	14.66	9.32	Std	16.35	16.61	16.87	17.23	17.88	8.12
α	-2.60	-0.47	0.72	0.05	2.38	4.98	α	-1.91	-1.02	-0.30	0.52	1.69	3.61
	(-3.04)	(-0.86)	(1.30)	(0.09)	(3.65)	(3.97)		(-2.94)	(-1.52)	(-0.45)	(0.75)	(2.36)	(3.21)
MKT	1.11	1.05	0.98	0.96	0.91	-0.20	MKT	0.96	0.97	0.99	1.01	1.05	0.09
	(48.74)	(72.23)	(73.46)	(65.72)	(51.43)	(-5.47)		(83.52)	(84.76)	(83.84)	(84.48)	(83.17)	(4.41)

Table 6: Characteristics and returns of ln(BM) quintiles: data vs. model

This table reports the characteristics and returns of ln(BM) quintiles in the data (Panel A) and in simulated model output (Panel B). Panels A1 and B1 report the time series average of the cross-sectional median of firm characteristics for each ln(BM) quintile, including the gross profitability (GP/A), the logarithm of the book-to-market ratio (ln(BM)), and the gross profit margin (GM). The empirical characteristics in Panel A1 also include financial leverage (Flev), operating leverage (Olev), and Tobin's Q (Q), and the simulated characteristics in Panel B1 also include average logarithm of project profitability z, the logarithm of the firm-level project arrival rate (a), and the ratio of value of growth option to value of assets in place (VGO/VAP). Panels A2 and B2 report the mean and standard deviation of the value-weighted excess returns and the CAPM test results. The historical sample is from July of 1963 to December of 2019. We simulate 100 independent samples from the model, with each sample representing 1,000 firms over 600 months. The portfolio returns, standard deviations, and the abnormal returns are annualized. The Newey-West t-statistics are computed with 4 lags to adjust for heteroskedasticity and autocorrelation.

Panel A: Data							Panel B: Model						
Panel A1: Characteristics							Panel B1: Characteristics						
	GP/A	ln(BM)	GM	Flev	Olev	Q		GP/A	ln(BM)	GM	z	a	VGO/VAP
Lo	0.41	-1.63	0.41	0.05	0.66	9.32	Lo	0.28	-1.28	0.33	0.06	-1.42	1.72
2	0.37	-0.89	0.36	0.13	0.62	3.53	2	0.26	-1.00	0.32	0.04	-1.02	1.04
3	0.34	-0.46	0.32	0.21	0.63	1.87	3	0.23	-0.83	0.31	-0.01	-0.87	0.81
4	0.29	-0.07	0.29	0.28	0.66	1.07	4	0.19	-0.68	0.30	-0.08	-0.77	0.65
Hi	0.26	0.45	0.26	0.41	0.72	0.52	Hi	0.13	-0.47	0.28	-0.22	-0.66	0.47
Panel A2: Returns and CAPM							Panel B2: Returns and CAPM						
	Lo	2	3	4	Hi	Hi-Lo		Lo	2	3	4	Hi	Hi-Lo
Mean	5.73	6.34	6.94	8.32	9.94	4.21	Mean	5.50	7.26	7.81	8.24	8.61	3.11
Std	17.35	15.44	15.25	15.09	17.32	13.60	Std	19.11	18.27	17.51	16.80	15.48	9.06
α	-1.23	-0.02	0.86	2.52	3.70	4.93	α	-3.15	-1.08	-0.21	0.54	1.51	4.66
	(-1.25)	(-0.04)	(1.11)	(2.52)	(2.74)	(2.35)		(-3.63)	(-1.44)	(-0.31)	(0.83)	(2.57)	(3.94)
MKT	1.07	0.98	0.94	0.89	0.96	-0.11	MKT	1.10	1.07	1.03	0.98	0.91	-0.19
	(46.49)	(56.61)	(42.11)	(30.19)	(23.83)	(-1.92)		(69.42)	(78.89)	(84.03)	(85.89)	(87.42)	(-8.84)

Table 7: Risk exposures and factor correlation: model-based simulations

Panel A reports the risk factor exposures to the aggregate profitability shock (β_X) and aggregate investment shock (β_Ξ) of the GP/A quintiles and ln(BM) quintiles from simulations of the model. Panel B reports the correlation between the gross profitability factor and the value factor from the empirical data and from the model. The standard error (S.E.) for the model is estimated from the standard deviation across simulations. We simulate 100 independent samples from the model, with each sample representing 1,000 firms over 600 months. The historical sample is from July 1963 to December 2019. The Newey-West t-statistics are computed with 4 lags to adjust for heteroskedasticity and autocorrelation.

Panel A: Risk exposures

GP/A quintiles	Lo	2	3	4	Hi	Hi-Lo
β_X	1.13	1.16	1.18	1.21	1.27	0.14
	(70.87)	(66.63)	(64.51)	(63.48)	(60.44)	(5.89)
β_Ξ	1.52	1.40	1.31	1.22	1.06	-0.46
	(33.24)	(28.64)	(25.15)	(22.36)	(17.69)	(-6.28)

ln(BM) quintiles	Lo	2	3	4	Hi	Hi-Lo
β_X	1.28	1.26	1.23	1.19	1.11	-0.17
	(64.45)	(63.10)	(63.61)	(62.72)	(62.80)	(-6.68)
β_Ξ	1.85	1.51	1.33	1.17	0.96	-0.89
	(30.85)	(25.74)	(24.12)	(22.24)	(19.88)	(-11.95)

Panel B: Correlation between the gross profitability factor and the value factor

	Data	Model
Correlation	-0.40	-0.19
S.E.		0.013

Table 8: Double sorts and conditional factor premiums: data vs. model

This table reports the average excess returns of 5-by-5 portfolios double-sorted on logBM and then GP/A (Panel A) and double-sorted on GP/A and logBM (Panel B) and the average annualized conditional GP/A and value premiums, both in the data and in the simulations. The conditional GP/A premium is the average Hi-Lo GP/A premium across the five logBM groups, and the conditional value premium is the average Hi-Lo value premium across the five GP/A groups. Besides the average conditional premiums, we also report their annualized Sharpe ratios (SR). The historical sample is from July 1963 to December 2019. We simulate 100 independent samples from the model, with each sample representing 1,000 firms over 600 months.

Panel A: Double sorts on logBM and then GP/A

	Data							Model					
	Lo	2	GP/A	4	Hi	Hi-Lo		Lo	2	GP/A	4	Hi	Hi-Lo
Lo	-4.89	2.77	3.98	6.50	8.19	13.08	Lo	0.98	2.30	4.11	5.99	9.30	8.32
2	1.89	5.13	6.43	7.63	9.60	7.71	2	2.86	4.65	6.43	8.10	10.13	7.27
logBM	4.91	5.47	9.00	10.22	10.55	5.64	logBM	4.18	5.51	6.85	8.82	10.26	6.08
4	6.81	6.94	11.04	13.02	9.87	3.06	4	4.97	6.41	7.99	8.82	10.26	5.29
Hi	8.74	9.05	12.40	13.32	11.24	2.50	Hi	6.65	7.65	8.45	8.94	10.02	3.37
			Conditional GP/A Prm.			6.40				Conditional GP/A Prm.			6.07
					SR	0.65						SR	0.76

Panel B: Double sorts on GP/A and then logBM

	Data							Model					
	Lo	2	logBM	4	Hi	Hi-Lo		Lo	2	logBM	4	Hi	Hi-Lo
Lo	-2.44	1.61	4.78	6.49	9.06	11.50	Lo	1.15	3.53	4.98	6.15	7.68	6.53
2	3.52	4.07	6.57	8.28	10.10	6.58	2	2.02	4.65	5.91	6.95	8.84	6.82
GP/A	4.75	5.73	9.68	11.14	12.87	8.11	GP/A	2.97	5.48	6.92	8.27	9.17	6.20
4	4.63	8.84	10.18	12.30	11.83	7.20	4	4.51	6.96	7.93	9.07	9.95	5.44
Hi	8.71	9.16	10.16	10.86	11.11	2.40	Hi	7.60	9.37	10.15	10.04	10.54	2.94
			Conditional value Prm.			7.16				Conditional value Prm.			5.59
					SR	0.56						SR	0.65

Table 9: Cash flow betas of gross profitability quintiles: model-based simulations

This table reports the cash flow exposures of GP/A quintile portfolios to the gross profitability factor return using the simulated model output. We regress the cumulative growth rate of gross profit, sales, and cost of goods sold of the quintile portfolios from year t to $t+K$ onto the gross profitability factor return in year t. We consider $K = 0$, 1, and 2, where $K = 0$ corresponds to contemporaneous annual regressions. We simulate 100 independent samples from the model, with each sample representing 1,000 firms over 600 months. We standardize the gross profitability factor return to have a unit standard deviation. The Newey-West t-statistics are computed with $K + 4$ lags to adjust for heteroskedasticity and autocorrelation.

	Exposures of gross profit					
K=	Lo	2	3	4	Hi	Hi-Lo
0	-1.79	0.62	0.73	0.81	3.11	4.90
	(-1.81)	(0.70)	(0.89)	(0.91)	(3.76)	(4.80)
1	-2.34	1.04	0.89	0.99	4.77	7.12
	(-1.68)	(0.84)	(0.71)	(0.74)	(4.06)	(5.03)
2	-1.55	0.95	0.73	0.77	3.48	5.03
	(-0.95)	(0.62)	(0.47)	(0.47)	(2.36)	(2.72)
	Exposures of sales					
0	-0.88	0.96	1.02	1.08	2.92	3.81
	(-0.96)	(1.08)	(1.16)	(1.13)	(3.11)	(4.97)
1	-1.17	1.50	1.39	1.46	4.52	5.69
	(-0.87)	(1.17)	(1.02)	(0.99)	(3.32)	(5.37)
2	-0.69	1.32	1.15	1.18	3.42	4.11
	(-0.48)	(0.80)	(0.66)	(0.64)	(2.01)	(2.91)
	Exposures of cost of goods sold					
0	-0.56	1.10	1.15	1.22	2.82	3.38
	(-0.58)	(1.17)	(1.22)	(1.18)	(2.75)	(4.79)
1	-0.71	1.70	1.62	1.70	4.39	5.10
	(-0.50)	(1.25)	(1.12)	(1.08)	(2.94)	(5.16)
2	-0.34	1.49	1.35	1.39	3.40	3.74
	(-0.26)	(0.84)	(0.72)	(0.70)	(1.82)	(2.87)

Table 10: Cash flow betas of GP/A quintiles

This table is an empirical counterpart of Table 9, and reports the cash flow exposures of the GP/A quintile portfolios within the Fama-French 30 industries to the gross profitability factor return. We regress the cumulative growth rate of gross profit, sales, and cost of goods sold of the quintile portfolios from year t to $t+K$ onto the gross profitability factor return in year t. We consider $K = 0,\ 1$, and 2, where $K = 0$ corresponds to contemporaneous annual regressions. We standardize the gross profitability factor return to have a unit standard deviation. The Newey-West t-statistics are computed with $K+4$ lags to adjust for heteroskedasticity and autocorrelation. The sample is annual from 1964 to 2019.

	Exposures of gross profit					
$K =$	Lo	2	3	4	Hi	Hi-Lo
0	-0.92	0.78	0.69	0.69	1.42	2.35
	(-0.80)	(1.11)	(1.13)	(1.09)	(1.98)	(2.76)
1	-3.72	-0.26	-0.73	0.04	0.59	4.32
	(-2.31)	(-0.25)	(-0.96)	(0.04)	(0.80)	(2.99)
2	-4.53	-0.37	-1.50	0.40	0.87	5.41
	(-1.88)	(-0.28)	(-1.66)	(0.34)	(1.05)	(2.53)
	Exposures of sales					
0	0.36	1.26	1.61	1.38	1.70	1.34
	(0.44)	(1.93)	(1.90)	(1.78)	(1.73)	(2.46)
1	-0.92	0.60	0.48	0.84	0.51	1.43
	(-0.95)	(0.61)	(0.57)	(1.04)	(0.59)	(1.80)
2	-1.21	0.21	0.10	1.31	0.57	1.78
	(-1.01)	(0.15)	(0.08)	(1.05)	(0.60)	(2.32)
	Exposures of cost of goods sold					
0	0.64	1.45	2.06	1.80	1.93	1.29
	(0.76)	(2.18)	(1.93)	(1.89)	(1.64)	(1.91)
1	-0.31	0.91	1.03	1.30	0.54	0.85
	(-0.33)	(0.92)	(0.99)	(1.36)	(0.52)	(0.98)
2	-0.49	0.42	0.81	1.83	0.48	0.97
	(-0.41)	(0.29)	(0.60)	(1.35)	(0.41)	(0.99)

Table 11: Stock return betas of GP/A quintiles and pricing of aggregate profitability shocks

This table reports the stock return betas of GP/A quintiles and pricing of the aggregate profitability shock. In Panel A, we estimate a two-factor linear SDF model, with the market portfolio and the gross profitability factor, defined by sorting firms on GP/A within the Fama-French 30 industries. In Panels B and C, the second factor is the total factor productivity shock (dTFP), defined as the cumulative utilization-adjusted TFP growth (e.g., Basu, Fernald, and Kimball, 2006; Fernald, 2014) over the current and subsequent year. In Panels A and C, we report the results from a GMM test. We use the Fama-French 17 and 30 industry portfolios as test assets. We normalize the intercept of the SDF to one, and standardize the gross profitability factor returns and dTFP shocks to a unit standard deviation. We report the annualized mean absolute pricing errors (MAE) in percent, the p-value associated with the over-identification test, and the estimated annualized price of risk (b). Panel B reports the stock return betas of GP/A quintile portfolios. The Newey-West t-statistics are computed with 4 lags to adjust for heteroskedasticity and autocorrelation. The sample is monthly from January 1964 to December 2019 in Panel A, and annual from 1964 to 2019 in Panels B and C.

Panel A: GMM with Profitability factor

Industry	17	30
MAE	1.04	1.15
p-value	0.85	0.70
b(MKT)	0.66	0.71
	(3.78)	(3.88)
b(GP/A factor)	0.55	0.64
	(2.36)	(2.45)

Panel B: Stock return betas

	Lo	2	3	4	Hi	Hi-Lo
MKT	1.14	1.00	0.95	0.96	0.92	-0.22
	(18.23)	(26.31)	(31.85)	(31.47)	(25.54)	(-2.53)
dTFP	-1.03	-0.36	-0.20	-0.10	0.81	1.84
	(-1.54)	(-1.34)	(-1.00)	(-0.51)	(2.14)	(2.20)

Panel C: GMM with dTFP

Industry	17	30
MAE	0.63	0.95
p-value	0.92	1.00
b(MKT)	0.20	0.22
	(2.02)	(1.91)
b(dTFP)	0.58	0.48
	(2.77)	(4.51)