NBER WORKING PAPER SERIES

FOUNDATION OF THE SMALL OPEN ECONOMY MODEL WITH PRODUCT DIFFERENTIATION

Lorenzo Caliendo
Robert C. Feenstra

Working Paper 30223
http://www.nber.org/papers/w30223

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
July 2022, Revised August 2022

We thank Kyle Bagwell, Fernando Parro, Steve Redding, Andres Rodriguez-Clare, Bob Staiger and participants at a conference at UC Davis on June 3, 2022, for their helpful comments. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

Foundation of the Small Open Economy Model with Product Differentiation
Lorenzo Caliendo and Robert C. Feenstra
NBER Working Paper No. 30223
July 2022, Revised August 2022
JEL No. F12,F13

ABSTRACT

We derive a small open economy (SOE) as the limit of an economy as the number or size of its trading partners goes to infinity and trade costs also go to infinity. We obtain this limit in the Armington, Eaton-Kortum, Krugman, and Melitz models. In all cases, the trade of the SOE with the foreign countries approaches a finite limit, and the domestic expenditure share for the SOE approaches a limit that is not zero or unity. The foreign countries can be either infinitely many SOEs, or alternatively, one or many large countries with domestic expenditure shares that approach unity. We illustrate the usefulness of this framework by obtaining a formula for the optimal tariff in the SOE -- depending on the elasticity of domestic wages with respect to the tariff -- that is consistent with all models.

Lorenzo Caliendo
Yale University
School of Management
135 Prospect Street
New Haven, CT 06520
and NBER
lorenzo.caliendo@yale.edu

Robert C. Feenstra
Department of Economics
University of California, Davis
One Shields Avenue
Davis, CA 95616
and NBER
rcfeenstra@ucdavis.edu

1 Introduction

Models with product differentiation hold strong promise for the analysis of trade policy. We include in this framework the simplest model of product differentiation due to Armington (1969); the Ricardian model with stochastic technology due to Eaton and Kortum (2002); the monopolistic competition model with homogeneous firms due to Krugman (1979, 1980); and with heterogeneous firms due to Melitz (2003), while using a Pareto distribution as in Chaney (2008). Arkolakis, Costinot and Rodríguez-Clare (2012) established the similarity of these models for the analysis of the gains from trade, and here we show their similarity – and differences – for the analysis of tariffs in a *small open economy (SOE).*

Demidova and Rodríguez-Clare (2009, 2013) were the first to explore a SOE in the context of the Melitz model. In this setting, the large foreign country has a demand curve for each home export variety of the form $B^*p^{-\eta}$, where $\eta > 1$ is the elasticity of demand for an export variety with the price p. The export demand curve is fixed by the location parameter $B^* > 0$ but has a negative slope. As stressed by Demidova and Rodríguez-Clare (2013), the wage in the SOE is endogenous through trade balance, and since the wage influences the export price, then a tariff applied by the small country has a "terms of trade" impact. Demidova and Rodríguez-Clare (2009) explore trade policy in this setting. They derive a new formula for the optimal tariff that corrects for both the domestic monopoly distortion and the externality present because changes in the home wage impacts import variety (through trade balance). Their formula modifies that from Gros (1987) obtained with homogeneous firms under monopolistic competition. Felbermayr, Jung, and Larch (2013), in turn, extend that formula of Demidova and Rodríguez-Clare (2009) to a large economy, and Costinot, Rodríguez-Clare and Werning (2020) extend the analysis further to allow for nonuniform tariffs (depending on the productivity of firms) and in many other directions. In addition to other theoretical contributions, there is a large quantitative literature that analyzes the impact of tariff reductions in actual economies.[1]

[1]Balistreri, Hillberry and Rutherford (2011) and Costinot and Rodríguez-Clare (2014) examined uniform reductions or increases in tariffs across products and countries in a quantitative Melitz-Chaney model with multiple sectors. They find greater welfare gains from tariff cuts in the presence of input-output (IO) linkages. Caliendo and Parro (2015) used a quantitative Eaton-Kortum model with IO linkages to analyze the NAFTA tariff cuts. Spearot (2016) examined Uruguay Round tariff cuts over 1994-2000 in a quantitative Melitz-Ottaviano (2008) model, but in the absence of any IO structure. Caliendo, Feenstra, Romalis and Taylor (2015, 2020) use a multisector quantitative

We believe that the small country analysis of Demidova and Rodríguez-Clare (2009, 2013) is very important as a simplifying device, but that it is lacking in its theoretical foundation. In particular, if the foreign country grows large enough then it appears that the country 1 expenditure share on domestically-produced varieties, or λ_{11} in the usual notation, should approach zero in the limit, as we demonstrate in section 2.1. We refer to this outcome as a "truly small" economy.[2] Demidova and Rodríguez-Clare (2009, 2013) do not impose this share as zero in their analysis of a SOE, however, but treat it as an endogenous variable that responds to tariffs. Likewise, in quantitative models of a SOE, such as Caliendo, Feenstra, Romalis and Taylor (2021), the model is calibrated to the observed domestic expenditure shares that are never zero.

Our main theoretical result in this paper is to establish conditions under which a SOE has a limiting value of the home expenditure share that is neither zero nor unity. We obtain this solution by allowing the number or size of its trading partners to approach infinity while trade costs also approach infinity. These trade costs can represent the distance between countries or any other factors that lead to trade costs of the iceberg type (i.e. we do not include tariffs within the trade costs that approach infinity). The trade of the SOE with the foreign countries approaches a finite limit, with export demand per variety of $B^* p^{-\eta}$, where the *effective* export price p is the ratio of the c.i.f. export price to the trade costs.[3] The foreign countries can be either infinitely many SOEs, or alternatively, one or many large countries with domestic expenditure shares that approach unity. In sections 2 and 4 we establish these results for the Armington model and the Krugman model, respectively, where we show that $\eta = \sigma$ reflects the elasticity of substitution. In section 3 we consider the two-country Eaton-Kortum model and find that $\eta = \theta + 1$ where θ reflects the Fréchet parameter. In section 5 we examine the Melitz model assuming that exporter fixed costs use labor in the source country, in which case $\eta = \frac{\theta\sigma}{\sigma-1} > \sigma$ where $\theta > \sigma - 1$ is the Pareto parameter.

model with IO linkages to analyze both Uruguay Round tariff cuts over 1990-2010 and optimal tariffs, and they found some negative optimal tariffs. Lashkaripour and Lugovskyy (2020) theoretically analyze optimal first-best tariffs with multiple sectors and input-output linkages, and second-best tariffs in the absence of input-output linkages. Recently, Balistreri and Tarr (2022) analyze a quantitative model with input-output linkages under the Armington, Krugman and Melitz market structures, for both global tariff cuts and optimal tariffs, which they find are low under monopolistic competition. Models with product differentiation have also been used to analyze quantitative models of tariff negotiations and wars, as in Ossa (2014), Bagwell, Staiger and Yurukoglu (2021) and Lashkaripour (2021).

[2]We thank Elhanan Helpman for this phrase.

[3]As we shall show, the effective export price still includes a component of iceberg costs denoted by τ_0, so those iceberg costs can be incorporated into the SOE model.

As an application of our results, in section 6, we consider the optimal tariff for a SOE. When export demand takes the form $B^*p^{-\eta}$ then we establish that the optimal *ad valorem* tariff is $\frac{1}{\eta-1} > 0$. So in the Armington model, we therefore obtain a positive optimal tariff purely due to the downward slope of the export demand curve, leading to a "terms of trade" effect, and without any monopoly distortion. A similar positive optimal tariff is found in the small-country Eaton-Kortum model, where θ reflects the Fréchet parameter and $\eta = \theta + 1$, so the optimal *ad valorem* tariff is $\frac{1}{\theta}$ as shown by Caliendo and Parro (2022).[4] In the monopolistic competition model with homogeneous firms we have $\eta = \sigma$, so the optimal *ad valorem* tariff for the SOE is $\frac{1}{\sigma-1} > 0$, which is the result found by Gros (1987). But for the monopolistic competition model with heterogeneous firms, when the fixed costs of exporting is paid using domestic labor, then we find that $\eta = \frac{\theta\sigma}{\sigma-1}$. So in this case we find that the optimal *ad valorem* tariff for a SOE is $\frac{1}{\eta-1} = \frac{\rho}{\theta-\rho}$ with $\rho \equiv \frac{\sigma-1}{\sigma}$, which is the formula found by Demidova and Rodríguez-Clare (2009). These results integrate the SOE optimal tariff formulas found in the literature. Further conclusions are discussed in section 7.

2 Armington Model

We suppose that there are N countries, and in each country i the representative consumer has a CES utility function defined over consumption q_{ji} of the variety purchased from each country $j = 1, ..., N$:

$$U_i = \left[\sum_{j=1}^{N} q_{ji}^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)}, \quad \sigma > 1. \tag{1}$$

Each country has a labor endowment of L_i and labor productivity of φ_i, so their "effective" labor endowments are $\varphi_i L_i$. The wage in each country is w_i and the cost of producing each unity is w_i/φ_i. With one plus the *ad valorem* tariff denoted by t_{ji} and iceberg costs of $\tau_{ji} \geq 1$, then the price of the variety sold from country j to i is

$$p_{ji} = \tau_{ji} t_{ji} \left(\frac{w_j}{\varphi_j}\right). \tag{2}$$

Denote total income in country i by $I_i = w_i L_i + T_i$, where T_i is tariff revenue. Then using

[4] See note 17.

the CES utility function, we can solve for the share of expenditure in country i on the country j variety, which is given by

$$\lambda_{ji} = \frac{\tau_{ji} t_{ji} w_j q_{ji}}{I_i \varphi_j} = \left(\frac{p_{ji}}{P_i}\right)^{1-\sigma}, \tag{3}$$

where

$$P_i = \left[\sum_{j=1}^{N} p_{ji}^{(1-\sigma)}\right]^{1/(1-\sigma)}. \tag{4}$$

2.1 One Large and One "Truly Small" Economy

Suppose that $N = 2$, and for the moment ignore tariffs. We allow country 2 to grow without bound by letting $\varphi_2 \to +\infty$. We can normalize $w_1 \equiv 1$ and also choose $\varphi_1 = 1$ for simplicity, but then we need to solve for w_2 from trade balance, which is

$$(1 - \lambda_{11}) L_1 = \lambda_{12} w_2 L_2.$$

Using (3), trade balance is

$$\left\{1 - \frac{1}{\left[1 + \left(\frac{\tau_{21} w_2}{\varphi_2}\right)^{(1-\sigma)}\right]}\right\} L_1 = \left\{\frac{\tau_{12}^{(1-\sigma)}}{\left[\tau_{12}^{(1-\sigma)} + \left(\frac{w_2}{\varphi_2}\right)^{(1-\sigma)}\right]}\right\} w_2 L_2.$$

If w_2 remains bounded above as $\varphi_2 \to +\infty$, then it is readily seen that the left side of this expression approaches L_1 but the right side approaches zero, which is a contradiction. So it must be that $w_2 \to +\infty$. On the other hand, if w_2/φ_2 remains bounded away from zero as $\varphi_2 \to +\infty$, then the left side approaches a finite value (zero if $w_2/\varphi_2 \to +\infty$) but the right side approaches $+\infty$ (because $w_2 \to +\infty$), which is another contradiction. We conclude that as $\varphi_2 \to +\infty$, then $w_2 \to +\infty$ but at a slower rate than φ_2, so that $w_2/\varphi_2 \to 0$. It follows immediately that $\lambda_{11} = \left[1 + (\tau_{21} w_2/\varphi_2)^{(1-\sigma)}\right]^{-1} \to 0$, which is what we call a "truly small" economy.[5] This result can be avoided by letting trade costs approach infinity, as we discuss next for various Armington (denoted by A) cases.

[5]We have also found that $\lambda_{11} \to 0$ as $L_2 \to +\infty$ while holding foreign productivity φ_2 fixed.

2.2 CASE A1: N Small and Distant Economies

We now model the iceberg costs of trade as

$$\tau_{ji} \equiv \tau_0 \, \tau \text{ for } \tau_0, \tau \geq 1 \text{ and } j \neq i, \tag{5}$$

with $\tau_{ii} = 1$. We will let $\tau \to +\infty$, which we can think of as growing distance between countries, as measured by geography or any other differences such as language or culture. For convenience in this section we set $\varphi_i = 1$ for all countries. Country 1 has a labor endowment of L_1 and the wage w_1. The labor endowment of the other countries is $L_j = L_F$, and by symmetry, the wage in the other countries will be identical and is denoted by $w_j = w_F$ for $j = 2, ..., N$. We can choose w_F as the numeraire and hold it fixed (though we do not insist that $w_F \equiv 1$). We suppose that only county 1 imposes a tariff of $t_{j1} = t_1$ for $j \neq 1$, with $t_{ji} = 1$ otherwise.

To provide a first illustration of how a SOE can be obtained, we suppose that the number of foreign countries $j = 2, ..., N$ is growing with τ according to

$$N - 1 = \tau^{\beta}, \quad \beta > 0. \tag{6}$$

In the limit, the price index in country 1 is then given by

$$\begin{aligned}
\lim_{\tau \to +\infty} P_1 &= \lim_{\tau \to +\infty} \left[w_1^{(1-\sigma)} + \sum\nolimits_{j=2}^{N} \left(\tau_0 \tau t_1 w_F \right)^{(1-\sigma)} \right]^{1/(1-\sigma)} \\
&= \lim_{\tau \to +\infty} \left[w_1^{(1-\sigma)} + (N-1) \left(\tau_0 \tau t_1 w_F \right)^{(1-\sigma)} \right]^{1/(1-\sigma)} \\
&= \lim_{\tau \to +\infty} \left[w_1^{(1-\sigma)} + \tau^{\beta + (1-\sigma)} \left(\tau_0 t_1 w_F \right)^{(1-\sigma)} \right]^{1/(1-\sigma)} .
\end{aligned}$$

It is immediate that this price index is finite if: $$\beta = (\sigma - 1) . \tag{7}$$

After imposing this condition, we obtain the domestic price index

$$\lim_{\tau \to +\infty} P_1 = \left[w_1^{(1-\sigma)} + \left(\tau_0 t_1 w_F \right)^{(1-\sigma)} \right]^{1/(1-\sigma)} .$$

Notice that this domestic price index looks like that for country 1 facing a *single* foreign country with the home tariff t_1 and variable trade costs τ_0, so that the *effective* foreign price net of the tariff is $\tau_0 w_F$. Then the country 1 expenditure share on the domestic variety is given by

$$\lambda_{11} = \frac{w_1^{(1-\sigma)}}{w_1^{(1-\sigma)} + (\tau_0 t_1 w_F)^{(1-\sigma)}}. \tag{8}$$

Thus, in this simple example we have shown how country 1 can face an infinite number of trading partners but still maintain a *strictly positive* domestic share. This is our first requirement for a country that is "small" but not "truly small".

A second requirement for a SOE is that the location parameter of its export demand is fixed. That condition is satisfied in our example because the limit of the foreign price index is independent of the home wage w_1. For any foreign country F, its price index is:

$$\begin{aligned}
\lim_{\tau \to +\infty} P_F &= \lim_{\tau \to +\infty} \left[(\tau_0 \tau w_1)^{(1-\sigma)} + (N-2)\left(\tau_0 \tau w_F\right)^{(1-\sigma)} + w_F^{(1-\sigma)} \right]^{1/(1-\sigma)} \\
&= \left[0 + \lim_{\tau \to +\infty} \tfrac{(N-2)}{(N-1)} \tau^{\beta+(1-\sigma)} \left(\tau_0 w_F\right)^{(1-\sigma)} + w_F^{(1-\sigma)} \right]^{1/(1-\sigma)} \qquad \text{using (6),} \\
&= \tau_0 w_F \left[1 + \tau_0^{(\sigma-1)} \right]^{1/(1-\sigma)}, \quad \text{using (7).}
\end{aligned}$$

To see how this fixes the location parameter of the foreign export demand curve, use (6) and (7) and the above to take the limiting value of home exports:

$$\lim_{\tau \to +\infty} (N-1) w_F L_F \left(\frac{\tau_0 \tau w_1}{P_F} \right)^{1-\sigma} = \frac{w_F L_F}{1 + \tau_0^{(\sigma-1)}} \left(\frac{\tau_0 w_1}{\tau_0 w_F} \right)^{1-\sigma}.$$

We have already noted that $\tau_0 w_F$ is the effective foreign price for exports, and likewise, $\tau_0 w_1$ is the effective country 1 price for exports. It follows that the limiting value for the quantity of home exports is obtained by dividing the above by $\tau_0 w_1$, obtaining

$$\frac{w_F L_F}{\left[1 + \tau_0^{(\sigma-1)}\right]} \frac{(\tau_0 w_1)^{-\sigma}}{(\tau_0 w_F)^{1-\sigma}} = B^* \left(\tau_0 w_1\right)^{-\sigma} \quad \text{with} \quad B^* \equiv \frac{L_F}{[\tau_0^{(1-\sigma)} + 1] w_F^{-\sigma}}. \tag{9}$$

Thus, we have established the limiting value of export demand facing country 1, and since B^* is independent of the home wage, then country 1 is an SOE. Actually, all N countries in this example are SOEs, with each foreign country having a domestic expenditure share that is strictly positive can differ from (8) because they purchase from themselves at the price determined by their own wage and they do not impose any tariffs.[6] Export demand for country 1 in (9) compares with the SOE export demand of $B^*(\tau_0 w_1)^{-\eta}$ as stated in section 1. It follows that for the Armington model, the export elasticity is $\eta = \sigma$.

2.3 Infinite Trade Costs: A Taxonomy of Cases

We continue with the specification of trade costs in (5), and with assuming that only country 1 imposes a tariff. For simplicity we continue to set $\varphi_1 = 1$, but we now allow the foreign countries $j = 2, ..., N$ to have the identical labor productivities of φ_F. We generalize our illustrative example and allow both the number and size of foreign countries to grow with τ according to:

$$N \;=\; 1 + \tau^{\beta}, \quad \beta \geq 0, \tag{10}$$

$$\varphi_j \;\equiv\; \varphi_F = \tau^{\gamma} \quad \gamma \geq 0 \text{ and } j = 2, ..., N, \tag{11}$$

$$L_j \;\equiv\; L_F = L_{F0}\tau^{\delta} \quad \delta \geq 0 \text{ and } j = 2, ..., N. \tag{12}$$

If we choose $\beta = 0$ then there are only two countries, $N = 2$. If we also choose $\gamma = \delta = 0$ then it is easily shown that with allowing trade costs to approaching infinity leads to autarky in both countries, so that $\lambda_{11} \to 1$. We want to avoid that limiting value for the domestic share just as much as avoiding $\lambda_{11} \to 0$, so at least one of the parameters β, γ, δ must be strictly positive. We will show that having any one of these parameters positive and equal to a certain value will ensure that $\lim_{\tau \to +\infty} \lambda_{11} \in (0, 1)$, and more generally, a linear combination of them must equal a certain value (related to σ).

The share of country 1 expenditure on the domestic product is

$$\lambda_{11} = \frac{w_1^{(1-\sigma)}}{\left[w_1^{(1-\sigma)} + (N-1)\left(\tau_0 \tau t_1 \frac{w_F}{\varphi_F}\right)^{(1-\sigma)} \right]} = \frac{1}{1 + \tau^{\beta - (1-\gamma)(\sigma-1)} \left(\frac{\tau_0 t_1 w_F}{w_1}\right)^{(1-\sigma)}}, \tag{13}$$

[6] Specifically, the domestic expenditure share in each foreign country j equals $w_j^{(1-\sigma)}/[w_j^{(1-\sigma)} + (\tau_0 w_F)^{(1-\sigma)}]$, where w_j is their own wage and w_F is wage in the other $N-2$ foreign countries. Export demand facing each foreign country is identical to that shown in (9), but using their own wage w_j rather than w_1.

where in the second equality we have made use of (10) and (11). In order to have $\lim_{\tau\to+\infty}\lambda_{11}\in(0,1)$ we therefore require

$$k \equiv \lim_{\tau\to+\infty} \tau^{\beta-(1-\gamma)(\sigma-1)}\, w_F^{(1-\sigma)} \in (0,+\infty), \tag{14}$$

where this notation indicates that the limiting value of the expression must be strictly greater than zero and finite.

Equation (14) is the first limit that we will use to solve for the parameters β,γ,δ consistent with a SOE in country 1. We consider only economic environments in which $\lim_{\tau\to+\infty} w_F > 0$, though we allow the foreign wage to approach infinity (as we have seen in section 2.1). It follows from (14) that we must have $\beta \geq (1-\gamma)(\sigma-1)$. If $\beta = (1-\gamma)(\sigma-1)$ then we see immediately from (14) that $w_F = k^{\frac{1}{(1-\sigma)}}$, which suggests that specifying a value for k is equivalent to choosing the foreign wage as numeraire. More generally, as $\beta > (1-\gamma)(\sigma-1)$ then we find from (14) that $w_F \to +\infty$, but along a path that still depends on k. So we will argue below that specifying k is like choosing a numeraire.

The implication of (14) for the foreign wage in various cases can be summarized in the following taxonomy:

$$\lim_{\tau\to+\infty}\lambda_{11}\in(0,1) \iff \begin{cases} & \textbf{Parameters} & \textbf{Foreign Wage} \\ \text{Case A1:} & \beta=(\sigma-1), \gamma=\delta=0 & w_F = k^{\frac{1}{(1-\sigma)}} \\ \text{Case A2:} & \beta=0, \gamma=1, \delta>0 & w_F = k^{\frac{1}{(1-\sigma)}} \\ \text{Case A3:} & \beta=0, \gamma>1, \delta\geq 0 & w_F \to \tau^{(\gamma-1)} k^{\frac{1}{(1-\sigma)}} \\ \text{Case A4:} & \beta=(1-\gamma)(\sigma-1)>0, \gamma>0 & w_F = k^{\frac{1}{(1-\sigma)}} \\ \text{Case A5:} & \beta>(1-\gamma)(\sigma-1)>0, \gamma>0 & w_F \to \tau^{\frac{\beta}{(\sigma-1)}+(\gamma-1)} k^{\frac{1}{(1-\sigma)}} \end{cases}$$

Case A1 corresponds to our example of section 2.2 where we assumed $\gamma=\delta=0$, and we found that $\beta=(\sigma-1)>0$ is needed to keep the domestic price index finite and the domestic expenditure share strictly positive. Cases A2 and A3 have $\beta=0$ and so there are only two countries, $N=2$. In that case, it is immediate from (14) that w_F remains finite if and only if $\gamma=1$, in Case A2.[7]

[7]We will show in section 2.4 that $\delta>0$ is required in Case A2, so we add that condition into the taxonomy.

Otherwise, if $\gamma > 1$ then (14) implies that w_F approaches infinity at the rate shown in Case A3. In Cases A4 and A5 we return to the case where $\beta > 0$ so there are many countries, $N \to +\infty$ from (10), but now we allow for $\gamma, \delta > 0$ so that foreign countries grow in their number and size. Then w_F remains finite if and only if $\beta = (1-\gamma)(\sigma-1) > 0$, while for $\beta > (1-\gamma)(\sigma-1) > 0$ then the foreign wage approaches infinity as $\tau \to +\infty$.[8]

We stress three points. First, requiring $k \in (0,1)$, which is equivalent to $\lim_{\tau\to+\infty} \lambda_{11} \in (0,1)$, imposes restrictions on the parameters of our model: we have begun to identify these restrictions in the various cases outlined in the taxonomy and we add further restrictions throughout this section. Given that k is non-zero and finite, however, then specifying its exact value is like choosing a numeraire in the model. We have already seen this result from the taxonomy in the case where the foreign wage remains finite, so $w_F = k^{\frac{1}{(1-\sigma)}}$ and choosing a value for w_F or k is equivalent. More generally, even if w_F approaches infinity then notice that by using (14), the limiting domestic share in (13) can be expressed as

$$\lim_{\tau\to+\infty} \lambda_{11} = \frac{w_1^{(1-\sigma)}}{\left[w_1^{(1-\sigma)} + k\,(\tau_0 t_1)^{(1-\sigma)}\right]}. \tag{15}$$

It is apparent that this limiting value for the share is homogeneous of degree zero in the variables $\left(w_1^{(1-\sigma)}, k\right)$, and this property will hold in all other equilibrium expressions, such as trade balance. So in this sense, even when $w_2 \to +\infty$, we are free to specify k as the numeraire and then $w_1^{(1-\sigma)}$ will adjust in proportion to it.

Second, while the foreign wages shown in the above taxonomy are needed to obtain a limiting value for $\lambda_{11} \in (0,1)$, we have not yet proved that the *equilibrium* value of the foreign wage is finite (in Cases A2 and A4) or infinite (in Cases A3 and A5). To determine the foreign wage, we use trade balance. Tariff revenue in country 1 is T_1 and income is $I_1 = w_1 L_1 + T_1$. Imports valued at their price inclusive of trade costs and tariffs are $(1-\lambda_{11})I_1$, so that imports net of tariffs are $(1-\lambda_{11})I_1/t_1$ and tariff revenue is $T_1 = (t_1-1)(1-\lambda_{11})I_1/t_1$. It follows that income is

$$I_1 = \frac{t_1 w_1 L_1}{1 + (t_1 - 1)\,\lambda_{11}}. \tag{16}$$

[8] We add $\gamma > 0$ as a condition in Case A4 to distinguish it from Case A1. In addition, we shall argue in section 2.5 that $\gamma > 0$ is also needed in Case A5 in order for that case to not be empty.

We must divide $(1-\lambda_{11})$ by t_1 to obtain the duty-free import share, and then multiplying by income in (16), trade balance is

$$(1-\lambda_{11})\,\frac{w_1 L_1}{1+(t_1-1)\,\lambda_{11}} = (N-1)\lambda_{F1} w_F L_F,$$

where λ_{F1} is the expenditure share from each of the $(N-1)$ foreign countries on country 1 imports. This share is readily obtained from (2) and (3), so that trade balance is

$$(1-\lambda_{11})\,\frac{w_1 L_1}{1+(t_1-1)\,\lambda_{11}} = B(\tau_0 w_1)^{(1-\sigma)}, \quad \text{with} \tag{17}$$

$$B = \frac{(N-1)\,\tau^{(1-\sigma)} w_F L_F}{(w_1\tau_0\tau)^{(1-\sigma)} + (N-2)\left(\tau_0\tau\frac{w_F}{\varphi_F}\right)^{(1-\sigma)} + \left(\frac{w_F}{\varphi_F}\right)^{(1-\sigma)}}. \tag{18}$$

Notice that on the right of (17) we have demand for the country 1 variety, with the location parameter B and the downward slope generated by $(\tau_0 w_1)^{(1-\sigma)}$. Even though the trade costs and the price of a good exported from country 1 approach infinity, we absorb τ within the location parameter and we treat the *effective export price* as $\tau_0 w_1$. The SOE requires a fixed location parameter for its export demand curve. So dividing the numerator and denominator of B by $\tau^{(1-\sigma)}$ and using (10)–(11) and k from (14), we require that

$$B^* \equiv \lim_{\tau\to+\infty} B = \lim_{\tau\to+\infty}\left\{\frac{(N-1)\,w_F L_F}{(w_1\tau_0)^{(1-\sigma)} + \frac{(N-2)}{(N-1)}k\left(\frac{\tau}{\tau_0}\right)^{(\sigma-1)} + k\tau^{2(\sigma-1)-\beta}}\right\} \in (0,+\infty), \tag{19}$$

and we also require that this limiting value is independent of the home wage w_1, so that the first term in the denominator must vanish. This expression gives us a second limit that must be satisfied to obtain a fixed location parameter for export demand. We need to prove that this limit is positive, finite, and independent of w_1 by constraining the parameters $\beta, \gamma, \delta \geq 0$.

Third, instead of using the trade balance condition at home, as in (17), we could have taken the limiting value of the trade balance condition(s) abroad. We have already argued that we can use the foreign wage as the numeraire, either because it is finite or – even if it is infinite – by choosing the parameter k. In either case, it is immediate from how the tariff enters the *effective import price*

$\tau_0 t_1$ in (15) that there is *full pass-through* of the tariff to import price, much as we would expect in a conventional small-country model with a zero optimal tariff. Nevertheless, we will argue that the optimal tariff for the SOE model with product differentiation is positive. That outcome occurs because even with full pass-through of the tariff, the *wage* in the SOE will *increase* due to the tariff, which raises the effective export price along the export demand curve on the right of (17).[9] This terms of trade gain does not depend on whether we examine the trade balance condition at home or abroad (and by Walras' Law, imposing this condition at home ensures that it also holds in the symmetric foreign countries).

We generalize the discussion of this section – so that it applies beyond the Armington model – with the following definition:

Definition 1. *The small open economy (SOE) model with product differentiation has a trade balance condition given by*

$$(1-\lambda_{11})\,\frac{w_1 L_1}{1+(t_1-1)\,\lambda_{11}} = (M_1^e)^{1-\alpha} B^* \tau_0^{1-\varepsilon} w_1^{1-\eta}, \quad \varepsilon, \eta > 1, \tag{20}$$

where M_1^e denotes the mass of entering firms/varieties in country 1, and the location parameter B^* is independent of the country 1 variables but depends on initial foreign labor L_{F0}. The domestic expenditure share is

$$\lambda_{11} = \frac{L_1^{1-\alpha} w_1^{1-\eta}}{\left(L_1^{1-\alpha} w_1^{1-\eta} + L_{F0}^{1-\alpha} \tau_0^{1-\varepsilon} t_1^{1-\eta} k\right)}, \tag{21}$$

where $k = w_F^{1-\eta}$ if foreign wages are finite, and if they are infinite then k can be still chosen as a numeraire. The parameter $\alpha = 1$ in the Armington and Eaton-Kortum models, and $\alpha = 0$ in the standard monopolistic competition (Krugman or Melitz) model.

We have already found that $\varepsilon = \eta = \sigma$ in the Armington model, though more generally we allow ε and η to differ. We introduced the export demand curve $B^* p^{-\eta}$ in section 1, and in (20) the elasticity η applies to an increase in the home wage while in (21) it applies to increases in the home

[9] To use the apt phrase of Bartelme, Costinot, Donaldson and Rodríguez-Clare (2019), a small country is "an economy that is large enough to affect the price of its own good relative to goods from other countries, but too small to affect relative prices in the rest of the world".

wage *and* the tariff.[10] We will find in all models that the parameter B^* depends on initial foreign labor as measured by L_{F0} and also on k. With monopolistic competition, variety M_1^e enters on the right of (20). We also see that (21) depends on home and foreign labor with the parameter α, which we introduce to nest the competitive (Armington and Eaton-Kortum) models, where $\alpha = 1$, and the monopolistically competitive (Krugman and Melitz) models, where $\alpha = 0$, and labor determines product variety. We will give a further interpretation to α under monopolistic competition by drawing on Arkolakis (2010), but now, we turn to the various cases for the Armington model.

2.4 CASES A2 and A3: One Small and One Large Economy

In Cases A2 and A3 we assume that $\beta = 0$ so $N = 2$. Using this in (19), the second term in the denominator vanishes and the third term (which approaches infinity) certainly dominates the first term (which remains finite) as $\tau \to +\infty$. That result is important because it shows that the home wage – which appears in the first term – has *no impact* on the limiting value of the location parameter $\lim_{\tau\to+\infty} B$. Therefore, the limit in (19) becomes

$$B^* = \lim_{\tau\to+\infty} \frac{w_F L_F}{k\tau^{2(\sigma-1)}} \in (0, +\infty). \tag{22}$$

It follows that $w_F L_F \to +\infty$, but this can happen through $L_F \to +\infty$ or $w_F \to +\infty$ (or both).

We need to show that the limits (14) and (22) are consistent with some parameter values for γ and δ, given that $\beta = 0$. In Case A2, with a finite foreign wage $w_F = k^{\frac{1}{(1-\sigma)}}$, we require $\gamma = 1$. Then using (12) in (22) we obtain

$$B^* = \lim_{\tau\to+\infty} \frac{k^{\frac{1}{(1-\sigma)}} L_{F0}\tau^{\delta}}{k\tau^{2(\sigma-1)}} \in (0, +\infty) \iff \delta = 2(\sigma - 1). \tag{23}$$

We have therefore solved for the growth in the foreign labor force in (12) required to obtain a SOE in country 1, namely:

$$L_F = L_{F0}\tau^{2(\sigma-1)} \implies B^* = L_{F0}k^{\frac{-\sigma}{(\sigma-1)}}. \tag{24}$$

To summarize these results, specify a value for k which pins down the foreign wage, and also specify

[10] If $\varepsilon = \eta$ then the natural definition of the SOE effective export price is $p = \tau_0 w_1$, but we will find that $\varepsilon \neq \eta$ in the Melitz model, so the elasticity applying to the export price depends on the source of its change.

the foreign labor force as shown in the first equation of (24). The initial value L_{F0} combined with k determine the limiting location parameter B^* as shown in the second equation. Then in the limit as $\tau \to +\infty$, we obtain a SOE in country 1 with the home wage obtained from trade balance.

Alternatively, consider case Case A3 where now we specify $\gamma > 1$ which implies that $w_F \to \tau^{(\gamma-1)} k^{\frac{1}{(1-\sigma)}}$. We consider solutions where L_F maybe be finite or not. The limit (19) becomes

$$B^* = \lim_{\tau \to +\infty} \frac{\tau^{(\gamma-1)} k^{\frac{1}{(1-\sigma)}} L_{F0} \tau^{\delta}}{k \tau^{2(\sigma-1)}} \in (0, +\infty) \iff \gamma + \delta = 2\sigma - 1, \tag{25}$$

in which case we obtain the same value for B^* as in (24). Note that if $\delta = 0$, so that L_F remains fixed at L_{F0}, then we still obtain a SOE provided that foreign productivity grows fast enough, i.e. with $\gamma = 2\sigma - 1 > 1$. As we allow $\delta > 0$ so that L_F grows with trade costs, then the combination of foreign productivity and population growth at a combined rate of $\gamma + \delta = 2\sigma - 1$ is required to obtain a SOE in country 1.

Finally, in both Cases A2 and A3 we have argued that $w_2 L_2 \to +\infty$ but $\lim \lambda_{11} \in (0, 1)$. From trade balance $(1 - \lambda_{11}) I_1/t_1 = (1 - \lambda_{22}) w_2 L_2$ it follows that $\lambda_{22} \to 1$, so the foreign country is large in two senses: its value of GDP approaches infinity; and its domestic share approaches unity.[11] Because of it immense size combined with infinite trade costs, however, we still obtain trade balance with the SOE that has a *finite* value of trade flowing between the two countries.

2.5 CASES A4 and A5: One Small and Many Large Economies

Now consider the case $\beta > 0$, so from $N = 1 + \tau^{\beta}$ the number of countries grows without bound as $\tau \to +\infty$. Then $\frac{(N-2)}{(N-1)} \to 1$ and the limit in (19) becomes

$$B^* = \lim_{\tau \to +\infty} \left\{ \frac{\tau^{\beta} w_F L_F}{(w_1 \tau_0)^{(1-\sigma)} + k \left(\frac{\tau}{\tau_0}\right)^{(\sigma-1)} + k \tau^{2(\sigma-1)-\beta}} \right\} \in (0, +\infty). \tag{26}$$

The middle term in the denominator approaches infinity as $\tau \to +\infty$, so it dominates the first term which remains finite. Once again, this shows that the home wage has no impact on the limiting

[11] Having a domestic share approaching unity is also a feature of the large foreign country in Demidova and Rodríguez-Clare (2009, 2013), who analyze the SOE in a Melitz model. So we should not be surprised to find the same outcome in the Armington model.

value of the location parameter for export demand. Dropping the first term in the denominator and also substituting for L_F from (12), the above limit becomes

$$B^* = \lim_{\tau \to +\infty} \left\{ \frac{\tau^{\beta} w_F L_{F0} \tau^{\delta}}{k\left(\frac{\tau}{\tau_0}\right)^{(\sigma-1)} + k\tau^{2(\sigma-1)-\beta}} \right\} \in (0, +\infty). \tag{27}$$

Consider first Case A4, where $\beta = (1-\gamma)(\sigma-1) > 0$ and $w_F = k^{\frac{1}{(1-\sigma)}}$ is finite. In this case we obtain solutions where L_F maybe be finite or not. We see that the second term in the denominator above has the exponent $2(\sigma-1) - \beta = (1+\gamma)(\sigma-1)$, which equals the exponent on the first term if $\gamma = 0$ and dominates it if $\gamma > 0$. So in either case, the denominator grows at the rate $\tau^{(1+\gamma)(\sigma-1)}$ for large τ. So the above limit holds with finite w_F if and only if $\beta + \delta = (1+\gamma)(\sigma-1)$, so that using our initial specification $\beta = (1-\gamma)(\sigma-1)$ we obtain

$$\beta = (1-\gamma)(\sigma-1) \text{ and } \delta = 2\gamma(\sigma-1) \implies \frac{\beta}{\sigma-1} + \frac{\gamma+\delta}{2\sigma-1} = 1. \tag{28}$$

Since $\gamma > 0$ in Case A4 (to distinguish it from Case A1), then foreign productivity rises with trade costs and from (28) we see that $\beta < (\sigma-1)$ and $\delta > 0$. In this case, all three parameters are positive and work together to ensure that the limiting location parameter is $B^* \in (0, +\infty)$, and from (26) it turns out to be $B^* = k^{\frac{-\sigma}{(\sigma-1)}} L_{F0}$ once again, as we found in (24).

To check the foreign economies in Case A4, we express trade balance for each *foreign* country by modifying (17) and (18) to become

$$(1-\lambda_{FF}) w_F L_F = B_F \left(\frac{\tau_0 w_F}{\varphi_F}\right)^{(1-\sigma)}, \quad \text{with} \tag{29}$$

$$B_F = \frac{\tau^{(1-\sigma)}\left[w_1 L_1 + (N-2) w_F L_F\right]}{(w_1 \tau_0 \tau)^{(1-\sigma)} + (N-2)\left(\tau_0 \tau \frac{w_F}{\varphi_F}\right)^{(1-\sigma)} + \left(\frac{w_F}{\varphi_F}\right)^{(1-\sigma)}} = B\left[\frac{w_1 L_1 + (N-2) w_F L_F}{(N-1) w_F L_F}\right]. \tag{30}$$

Notice that B_F is changed only in the numerator as compared with B, because each foreign country sells to country 1 (with income $w_1 L_1$) and to $N-2$ other foreign countries (each with income $w_F L_F$). But the difference between B_F and B – which is the term in brackets at the end of (30) – approaches unity as $N \to +\infty$. Therefore, the limit of B and B_F both equal B^*. The foreign wage w_F is finite in Case A4, but because $\gamma > 0$ so that $\varphi_F \to +\infty$, it follows that the right of (29) approaches infinity. Of course, $L_F \to +\infty$ on the left, too, because $\delta > 0$ and the foreign labor force is rising with trade costs. Therefore, to evaluate the limit of λ_{FF}, we move $w_F L_F$ from the left of (29) to the right:

$$\lim_{\tau\to+\infty} (1-\lambda_{FF}) = \lim_{\tau\to+\infty} \frac{B_F}{w_F L_F}\left(\frac{\tau_0 w_F}{\varphi_F}\right)^{(1-\sigma)} = \lim_{\tau\to+\infty} \frac{B^* \tau_0^{(1-\sigma)} w_F^{-\sigma}}{L_{F0}\tau^{\delta}\tau^{\gamma(1-\sigma)}}.$$

From (28) we had $\delta = 2\gamma(\sigma-1)$ and it follows that the above limit is zero, so that $\lambda_{FF} \to 1$. Thus, in contrast to Case A1 where the foreign countries are all SOEs, in Case A4 we see that the foreign countries are all large, with $w_F L_F \to +\infty$ and domestic shares approaching unity.

Finally, consider Case A5 where $\beta > (1-\gamma)(\sigma-1) > 0$ and $w_F \to \tau^{\frac{\beta}{(\sigma-1)}-(1-\gamma)} k^{\frac{1}{(1-\sigma)}} \to +\infty$ as $\tau \to +\infty$. Then ignoring the first term in the denominator of (26) because it is small compared to the other terms, this limit becomes

$$B^* = \lim_{\tau\to+\infty}\left\{\frac{\tau^{\beta}\tau^{\frac{\beta}{(\sigma-1)}-(1-\gamma)} k^{\frac{1}{(1-\sigma)}} L_{F0}\tau^{\delta}}{k\left(\frac{\tau}{\tau_0}\right)^{(\sigma-1)} + k\tau^{2(\sigma-1)-\beta}}\right\} \in (0,+\infty).$$

At first glance, it seems that either of the two terms in the denominator can dominate, depending on whether $\beta >< \sigma-1$. For example, suppose that $\beta \leq (\sigma-1)$ so that the second term dominates or has an equal exponent to the first term. Then the exponents on τ in the numerator and denominator are equal when

$$\frac{\beta}{\sigma-1} + \frac{\gamma+\delta}{2\sigma-1} = 1 \quad \text{if} \quad \beta \leq (\sigma-1). \tag{31}$$

Alternatively, if $\beta > (\sigma-1)$ then the first term in the denominator dominates, and it has the same exponent as on τ in the numerator when

$$\frac{\beta}{\sigma-1} + \frac{\gamma+\delta}{\sigma} = 1 \quad \text{if} \quad \beta > (\sigma-1). \tag{32}$$

If $\beta > (\sigma-1)$, however, then the above condition can be satisfied only if $\gamma+\delta < 0$, which we have ruled out in our initial specification that $\beta, \gamma, \delta \geq 0$. It follows that the correct parameter restriction in Case A5 is (31). It can also be confirmed that the foreign countries are large once again.

2.6 Summary of Cases

We have considered an environment where the number of foreign countries and their size and productivity are weakly increasing in trade costs, i.e. $\beta, \gamma, \delta \geq 0$ in (10)–(12). The parameter restrictions introduced in the taxonomy of cases ensured that $\lim_{\tau \to +\infty} \lambda_{11} \in (0, 1)$, but we also require that the limiting location parameter satisfy $\lim_{\tau \to +\infty} B \in (0, +\infty)$ and independent of any country 1 variables. This second condition has led to parameter restrictions that we have developed for the three cases. All of these parameter restrictions are encompassed by the following, general restriction:

$$\frac{\beta}{\sigma - 1} + \frac{\gamma + \delta}{2\sigma - 1} = 1 \quad \text{and} \quad (1 - \gamma) \leq \frac{\beta}{\sigma - 1} \leq 1. \tag{33}$$

The first condition above encompasses our results for all Cases as shown by (7), (23), (25), (28) and (31). The second condition has been used to restrict our attention to cases where the foreign wage does not approach zero as trade costs rise. The differences between these cases occur because as we vary parameters, then we can have an infinite number of foreign countries that are themselves SOEs (in Case A1), or just one large foreign country (in Cases A2 and A3), or an infinite number of large foreign countries (in Cases A4 and A5). Note that this range of possibilities has been limited because we have assumed symmetry in the foreign countries. We expect that by allowing for asymmetric foreign countries (and perhaps some with wages that might approach zero) then we could generate a wider range of outcomes as trade costs approach infinity. Rather than explore this generalization, however, we turn next to results for other models.

3 Two-Country Eaton-Kortum Model

For convenience, we consider the two-country version of the Eaton and Kortum model (2002) model that becomes a specific form of Dornbusch, Fischer and Samuelson (DFS, 1977). In those models, there is a continuum of goods indexed by z over the interval $z \in [0, 1]$, and we assume a CES utility function defined over consumption $q_i(z)$ in each country $j = 1, 2$:

$$U_i = \left[\int_0^1 q_i(z)^{(\sigma-1)/\sigma} dz \right]^{\sigma/(\sigma-1)}, \quad \sigma \geq 1. \tag{34}$$

Note that we allow for $\sigma = 1$ which is the Cobb-Douglas specification used by DFS. We let $\varphi_i(z)$ denote the productivity of labor in producing product z in country $i = 1, 2$, which is distributed according to the Fréchet distribution

$$F_i(\varphi) = 1 - e^{-T_i \varphi^{-\theta}}, \quad \varphi > 0 \text{ with } F(0) = 0. \tag{35}$$

The Fréchet shape parameter $\theta > 1$ is common across countries but the location parameter T_i differs, with higher T_i indicating higher mean productivity in country i.

Recall that in DFS, $a_i(z)$ is the amount of labor needed for one unit of output in country i, so this is related to productivity by $a_i(z) = 1/\varphi_i(z)$. DFS measure comparative advantage by the ratio $A(z) \equiv a_2(z)/a_1(z)$, where goods are ordered so that A is a non-increasing function of z. Thus, the lower values of z have higher relative productivity in country 1. Under the Fréchet distribution in (35), Eaton and Kortum (2002, note 15) note that the functional form of $A(z)$ is:[12]

$$A(z) = \left(\frac{T_1(1-z)}{T_2\, z} \right)^{1/\theta}. \tag{36}$$

We model the the iceberg costs of trade as in (5), with $\tau_{ii} = 1$, and again suppose that only county 1 imposes a tariff of t_1. As explained by DFS, in the presence of the iceberg costs and the tariff, country 1 only will produce goods and export in the range $[0, z_1)$, both countries will produce goods in the nontraded range $[z_1, z_2]$, and country 2 only will produce and export goods in the range $(z_2, 1]$. The value z_1 is determined by the equality of the export price from country 1 and the local price in country 2:

$$\tau_0 \tau w_1 a_1(z_1) = w_2 a_2(z_1) \iff \frac{\tau_0 \tau w_1}{w_2} = A(z_1),$$

while value z_2 is determined by the equality of the export price from country 2 and the local price in country 1:

$$w_1 a_1(z_2) = \tau_0 \tau w_2 a_2(z_2) \iff \frac{w_1}{\tau_0 \tau w_2} = A(z_2).$$

[12]The steps to derive (36) are outlined in Feenstra (2016, pp. 184-185, problem 6.6) and we thank Andrés Rodríguez-Clare for providing this derivation.

Notice that we can use (36) to solve for these values $z_1 < z_2$, given the country wages:

$$z_1 = \frac{T_1(\tau_0 \tau w_1)^{-\theta}}{\left[T_1(\tau_0 \tau w_1)^{-\theta} + T_2 w_2^{-\theta}\right]}, \quad z_2 = \frac{T_1 w_1^{-\theta}}{\left[T_1 w_1^{-\theta} + T_2(\tau_0 \tau w_2)^{-\theta}\right]}. \tag{37}$$

With country 1 consuming its own goods in the range $[0, z_2)$, and symmetric demand across goods from the CES utility function in (34), then this solution for z_2 equals the country 1 domestic share λ_{11} as derived by Eaton and Kortum (2002). Likewise, the country 2 domestic share λ_{22} equals $1 - z_1$. To close the model we use trade balance between the countries, which is given by $(1 - \lambda_{11})\, I_1/t_1 = (1 - \lambda_{22})\, w_2 L_2$, with country 1 income I_1 as in (16). Making use of $\lambda_{22} = 1 - z_1$, we can write trade balance in the notation of Definition 1 as:

$$(1 - \lambda_{11}) \frac{w_1 L_1}{1 + (t_1 - 1)\, \lambda_{11}} = z_1 w_2 L_2 = B(\tau_0 w_1)^{-\theta}, \tag{38}$$

with

$$\lambda_{11} = \frac{w_1^{-\theta}}{\left[w_1^{-\theta} + \frac{T_2}{T_1}(\tau_0 \tau w_2)^{-\theta}\right]}, \quad B = \frac{\tau^{-\theta} w_2 L_2}{(\tau_0 \tau w_1)^{-\theta} + \frac{T_2}{T_1} w_2^{-\theta}}. \tag{39}$$

We can compare these equations from the Eaton-Kortum model to the Armington model with $N = 2$, so that from (13) and (17)-(18) we have:

$$(1 - \lambda_{11}) \frac{w_1 L_1}{1 + (t_1 - 1)\, \lambda_{11}} = B(\tau_0 w_1)^{(1-\sigma)},$$

with,

$$\lambda_{11} = \frac{w_1^{(1-\sigma)}}{\left[w_1^{(1-\sigma)} + \left(\tau_0 \tau t_1 \frac{w_2}{\varphi_2}\right)^{(1-\sigma)}\right]}, \quad B = \frac{\tau^{(1-\sigma)} w_2 L_2}{(w_1 \tau_0 \tau)^{(1-\sigma)} + \left(\frac{w_2}{\varphi_2}\right)^{(1-\sigma)}}.$$

It is apparent that if $\theta = \sigma - 1$ and $\frac{T_2}{T_1} = \varphi_2^{\sigma-1}$, then these sets of equations are isomorphic, so the SOE Eaton-Kortum model is obtained under the same parameter restrictions as the SOE Armington model. That is, with $\beta = 0$ because we are assuming $N = 2$, we specify that relative technologies grow according to $\frac{T_2}{T_1} = \varphi_2^{\sigma-1} = \tau^{\gamma(\sigma-1)}$ from (11), and that L_2 grows according to $L_2 = L_{F0}\tau^{\delta}$ from (12). Then by imposing the restriction in (25), i.e. $\gamma + \delta = 2\sigma - 1$, country 1 will approach a SOE as trade costs become very large, $\tau \to +\infty$.

To understand where these results are coming from, let us start with the simplest case where *only* $\frac{T_2}{T_1}$ grows, keeping trade costs fixed (and also assuming $t_1 = 1$ for simplicity). Then the trade balance equation in (38)-(39) is simplified as:

$$\frac{\frac{T_2}{T_1}(\tau_0 \tau w_2)^{-\theta}}{\left[w_1^{-\theta} + \frac{T_2}{T_1}(\tau_0 \tau w_2)^{-\theta}\right]} w_1 L_1 = \frac{\tau^{-\theta} w_2 L_2}{\left[(\tau_0 \tau w_1)^{-\theta} + \frac{T_2}{T_1} w_2^{-\theta}\right]} (\tau_0 w_1)^{-\theta}. \tag{40}$$

We normalize $w_1 \equiv 1$. With $\frac{T_2}{T_1} \to +\infty$, if w_2 approaches a finite limit that the right of this equation approaches zero but the left approaches $w_1 L_1$, which is a contradiction. So $w_2 \to +\infty$. On the other hand, if $\frac{T_2}{T_1} w_2^{-\theta}$ approaches a finite limit then the right of this equation approaches infinity along with the country 2 wage w_2, but the left approaches a finite limit, which is another contradiction. We therefore conclude that $\frac{T_2}{T_1} w_2^{-\theta} \to +\infty$, which indicates the country 2 goods are becoming extremely inexpensive. Substituting this condition back into (37), it is immediate that $z_1 \to 0$ and $z_2 \to 0$, so that the range of goods $[0, z_2)$ that country 1 specializes in vanishes, $\lambda_{11} \to 0$, while country 2 is producing and exporting small amounts of all the goods, with $\lambda_{22} \to 1$. So country 1 has become a "truly small" economy.

We avoid this outcome by allowing trade costs to approach infinity, along with the relative technologies and the country 2 labor force growing at the rates described above. Then the condition $\gamma + \delta = 2\sigma - 1$ ensures that the country 1 domestic share approaches a limit strictly between zero and unity, while B approaches a positive and finite limit B^*.

4 Monopolistic Competition with Homogeneous Firms

Under monopolistic competition with homogeneous firms, the number of entering firms and varieties in country 1 is M_1^e and in the other countries is $M_j^e = M_F^e$ for $j \neq 1$, and all the varieties from each country are exported. Then utility is

$$U_i = \left[\sum_{j=1}^{N} M_j^e q_{ji}^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)}, \quad \sigma > 1.$$

The equilibrium market shares are given by

$$\lambda_{ji} = \frac{M_j^e \tau_{ji} t_{ji} w_j q_{ji}}{I_i} = M_j^e \left(\frac{p_{ji}}{P_i}\right)^{1-\sigma},$$

where the prices include a markup:

$$p_{ji} = \left(\frac{\sigma}{\sigma-1}\right)\frac{\tau_{ji} t_{ji} w_j}{\varphi_j}, j \neq i, \text{ and } p_{ii} = \left(\frac{\sigma}{\sigma-1}\right)\frac{w_i}{\varphi_i}.$$

The price index is then

$$P_i = \left[\sum\nolimits_{j=1}^{N} M_j^e \left(\frac{\sigma}{\sigma-1}\frac{\tau_{ji} t_{ji} w_j}{\varphi_j}\right)^{(1-\sigma)}\right]^{1/(1-\sigma)}.$$

As before, we treat county 1 as having its own tariff $t_{j1} = t_1$ for $j \neq 1$ and for simplicity we suppose that $t_{ji} = 1$ for $i \neq 1$. The labor endowment of country 1 is L_1 and the labor endowment of the other countries is $L_j = L_F$ for $j \neq 1$. We continue to assume the parameters in (10)–(12), so that all foreign countries are treated symmetrically. We let w_1 denote the wage of country 1 and the wage in the other countries will be identical and is denoted by $w_j = w_F$ for $j \neq 1$.

We allow fixed costs to be a function of country size. In particular we let

$$f_1 = f_0 L_1^{\alpha}, \text{ and } f_i = f_F = f_0 L_F^{\alpha} \quad \text{for } i = 2, ..., N, \text{ with } 0 \leq \alpha \leq 1. \tag{41}$$

This specification is motivated by the simplest case of advertising costs considered by Arkolakis (2010).[13] He assumes the advertising costs depend on the population of the destination market. Since there are no fixed costs of exporting in the Krugman model, we are treated fixed costs in (41) as proportional to the domestic market size, but we will re-specify this in the next section as we consider the Melitz model. In both cases we refer to $\alpha = 0$ as the *standard* monopolistic competition model, but allowing for the non-standard parameters $0 < \alpha \leq 1$ from Arkolakis (2010) will enable us to nest the Armington and Eaton-Kortum models into our analysis, when $\alpha = 1$ so that variety M_1^e and labor L_1, L_{F0} do not enter Definition 1.

[13] Arkolakis has a second parameter β that influences the function form for advertising costs, which we are treating as unity in the simple specification (41).

To solve for the number of varieties, we use the labor market clearing condition for country 1:

$$L_1 = M_1^e f_1 + (\sigma - 1) M_1^e \frac{\pi_1}{w_1}, \tag{42}$$

where the first term on the right is the labor used for fixed costs, and the second term is the labor used for variable costs. Because profits are zero in equilibrium we have $\frac{\pi_1}{w_1} = f_1$. Then we obtain product variety in country 1 and in the other countries as

$$M_1^e = \frac{L_1}{f_1\sigma} = \frac{L_1^{1-\alpha}}{f_0\sigma}, \quad M_F^e = \frac{L_F}{f_F\sigma} = \frac{L_F^{1-\alpha}}{f_0\sigma}. \tag{43}$$

Income in county 1 is still given by (16), and the share of country 1 expenditure on domestic varieties is

$$\lambda_{11} = M_1^e \left(\frac{\frac{\sigma}{\sigma-1} w_1}{P_1}\right)^{1-\sigma} = \frac{\frac{L_1}{f_1}(w_1)^{1-\sigma}}{\frac{L_1}{f_1}(w_1)^{(1-\sigma)} + (N-1)\frac{L_F}{f_F}\left(\frac{\tau_0\tau t_1 w_F}{\varphi_F}\right)^{(1-\sigma)}}.$$

Using (10)–(12), (41) and (43), the domestic share in country 1 becomes

$$\lambda_{11} = \frac{1}{1 + \left(\frac{L_{F0}}{L_1}\right)^{1-\alpha} \tau^{\beta+(1-\alpha)\delta-(1-\gamma)(\sigma-1)} \left(t_1 \frac{\tau_0 w_F}{w_1}\right)^{(1-\sigma)}}. \tag{44}$$

Notice that if we set $\alpha = 1$ then the share equation above is identical to (13) from the Armington model, and all our results from the Armington model will carry through. More generally, for all $0 \leq \alpha \leq 1$, we re-specify the limit (14) as

$$k \equiv \lim_{\tau \to +\infty} \tau^{\beta+(1-\alpha)\delta-(1-\gamma)(\sigma-1)} w_F^{(1-\sigma)} \in (0, +\infty). \tag{45}$$

When (45) holds then $\lim_{\tau\to+\infty} \lambda_{11} \in (0,1)$, which is the first limit that must hold for country 1 to be a SOE. The second limit is obtained from export demand. Trade balance under monopolistic competition with homogeneous firms is

$$(1-\lambda_{11}) \frac{t_1 w_1 L_1}{1 + (t_1 - 1)\lambda_{11}} = M_1^e B (\tau_0 w_1)^{(1-\sigma)}, \quad \text{with} \tag{46}$$

$$B = \frac{(N-1)\,\tau^{(1-\sigma)} w_F L_F}{M_1^e (w_1 \tau_0 \tau)^{(1-\sigma)} + M_F^e\,(N-2)\left(\tau_0 \tau \frac{w_F}{\varphi_F}\right)^{(1-\sigma)} + M_F^e \left(\frac{w_F}{\varphi_F}\right)^{(1-\sigma)}}. \tag{47}$$

Export demand on the right of (46) will be the same as in Definition 1 once we establish the limiting value B^*, provided that we use the parameter values $\varepsilon = \eta = \sigma$ Using (10)–(12), (41), (43) and k, we obtain the limiting value of B:

$$B^* = \lim_{\tau \to +\infty} \frac{\sigma f_0 w_F L_{F0} \tau^{\beta+\delta}}{L_1^{1-\alpha}(w_1 \tau_0)^{(1-\sigma)} + L_{F0}^{1-\alpha}\left[\frac{(N-2)}{(N-1)} k \left(\frac{\tau}{\tau_0}\right)^{(\sigma-1)} + k\tau^{2(\sigma-1)-\beta}\right]}. \tag{48}$$

This limit looks quite similar to (19) in the Armington model, except that $L_{F0}^{1-\alpha}$ and $L_1^{1-\alpha}$ appear in the denominator reflecting product variety.

The solution for the foreign wage is summarized in the following taxonomy for the Krugman model:

$$\lim_{\tau \to +\infty} \lambda_{11} \in (0,1) \iff \left\{ \begin{array}{lcl} & \textbf{Parameters} & \textbf{Foreign Wage} \\ \text{Case K1:} & \beta = (\sigma-1), \gamma = \delta = 0 & w_F = k^{\frac{1}{(1-\sigma)}} \\ \text{Case K2:} & \beta = 0, \gamma = 1, \delta > 0 & w_F \to \tau^{\frac{(1-\alpha)\delta}{(\sigma-1)}} k^{\frac{1}{(1-\sigma)}} \\ \text{Case K3:} & \beta = 0, \gamma > 1, \delta \geq 0 & w_F \to \tau^{\frac{(1-\alpha)\delta}{(\sigma-1)}+\gamma-1} k^{\frac{1}{(1-\sigma)}} \\ \text{Case K4:} & \beta = (1-\gamma)(\sigma-1) > 0, \gamma > 0 & w_F \to \tau^{\frac{(1-\alpha)\delta}{(\sigma-1)}} k^{\frac{1}{(1-\sigma)}} \\ \text{Case K5:} & \beta > (1-\gamma)(\sigma-1) > 0, \gamma > 0 & w_F \to \tau^{\frac{\beta+(1-\alpha)\delta}{(\sigma-1)}+\gamma-1} k^{\frac{1}{(1-\sigma)}} \end{array} \right.$$

Case K1 is identical to Case A1, because with no growth in the foreign labor force or productivity, then the number of varieties at home and abroad are all constant. So the condition $\beta = (\sigma-1)$ on the growth of countries still applies in the Krugman model, and guarantees a SOE in country 1 and in all the foreign countries. Note that the first term in the denominator of (48) is vanishingly small as compared to the other terms, as also occurs in cases K2-K5.

In cases K2 and K3, $N = 2$ so there is a single foreign country, and its wage approaches infinity in either case provided that $\alpha < 1$ (so there is growth in foreign varieties). Combining these cases by specifying $\gamma \geq 1$, we substitute the solution for foreign wages into (48), and find that $B^* \in (0, +\infty)$

provided that the exponents on τ sum to zero:

$$\frac{(1-\alpha)\,\delta}{(\sigma-1)}+\gamma-1+\delta-2(\sigma-1)=0 \iff \gamma+\delta+\frac{(1-\alpha)\,\delta}{(\sigma-1)}=(2\sigma-1), \tag{49}$$

which is identical to (25) when $\alpha=1$.

Finally, in cases K4 and K5, the foreign wage again approaches infinity provided that $\alpha<1$. Combining these two cases by specifying $\beta\geq(1-\gamma)(\sigma-1)>0$, we again substitute the solution for foreign wages into (48), and find that $B^{*}\in(0,+\infty)$ provided that the exponents on τ sum to zero. After some simplification, this general condition is stated as:

$$\frac{\beta}{(\sigma-1)}+\frac{\gamma+\delta}{(2\sigma-1)}+\frac{(1-\alpha)\,\delta}{(2\sigma-1)(\sigma-1)}=1 \quad \text{and} \quad (1-\gamma)\leq\frac{\beta}{\sigma-1}\leq 1, \tag{50}$$

which is identical to our general condition for the Armington model in (33) when $\alpha=1$. Notice that in the *standard* Krugman model with $\alpha=0$, we can rewrite (50) as

$$\frac{\beta}{(\sigma-1)}+\frac{\gamma+\frac{\sigma}{(\sigma-1)}\delta}{(2\sigma-1)}=1 \quad \text{and} \quad (1-\gamma)\leq\frac{\beta}{\sigma-1}\leq 1. \tag{51}$$

This condition looks different from that obtained in the Armington model, because increases in the effective foreign labor through productivity (the parameter γ) versus population (the parameter δ) have differing impacts when the latter brings with it an increase in the number of varieties in the Krugman model.

5 Monopolistic Competition with Heterogeneous Firms

We continue to assume (10)–(12), with $\beta,\gamma,\delta\geq 0$. With heterogeneous firms, we specify the Pareto distribution for productivity draws in country j as:

$$G_j(\varphi)=1-\left(\frac{\varphi}{\varphi_j}\right)^{-\theta},\varphi\geq\varphi_j \text{ with } \varphi_1=1 \text{ and } \varphi_j=\varphi_F, j=2,...,N. \tag{52}$$

Thus, the specification $\varphi_F=\tau^{\gamma}$ in (11) becomes the *lower bound* to the foreign productivity draw, and for simplicity we treat this lower bound as unity for country 1. We modify the specification of fixed costs that were used in the previous section by now assuming that the fixed costs of domestic sales and exporting are:

$$f_{ij} = f_0 L_j^{\alpha}, \quad i, j = 1, ..., N, \ j \neq i. \tag{53}$$

These specifications for advertising costs at home and abroad are similar to (41), except that now we more accurately reflect the fixed cost specification from Arkolakis (2010) which are *fixed operating* rather than entry costs. Arkolakis further assumes that fixed exporting costs use labor of both country i and j in a Cobb-Douglas fashion. In contrast, we restrict ourselves to the case where fixed exporting costs use labor of the source country.[14] Finally, we assume that entry costs f^e are identical across countries and use labor of the source country, as is standard in the Melitz model.

Given expenditure I_i in country i, CES demand for variety φ sold from country j to i is

$$q_{ji}(\varphi) = \left(\frac{p_{ji}(\varphi)}{P_i}\right)^{-\sigma} \frac{I_i}{P_i}. \tag{54}$$

The CES price indexes over the varieties purchased in country i from all sources $j = 1, ..., N$ are

$$P_{ji} \equiv \left(M_j^e \int\limits_{\varphi_{ji}^*}^{\infty} p_{ji}(\varphi)^{1-\sigma} g(\varphi)\, d\varphi \right)^{\frac{1}{1-\sigma}} \text{ and } P_i \equiv \left(\sum_{j=1}^{N} P_{ji}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}, \tag{55}$$

where M_j^e is the mass of firms *entering* in country j and φ_{ji}^* is the cutoff productivity for sales to country i. The profits in country j from supplying varieties to country i are

$$\pi_{ji}(\varphi) = \max_{p_{ji}(\varphi) \geq 0} \left\{ \frac{p_{ji}(\varphi)}{t_{ji}} q_{ji}(\varphi) - \frac{w_j}{\varphi} \tau_{ji}\, q_{ji}(\varphi) - w_j\, f_{ji} \right\}, \tag{56}$$

where $\tau_{ji} = \tau_0 \tau$ for $j \neq i$, $\tau_{ii} = 1$, and the only tariff is for sales to country 1, $t_{ji} = t_1$ for $i = 1, j \neq 1$ and $t_{ji} = 1$ otherwise. Solving this problem, we obtain as usual the price as a markup over marginal costs, $p_{ji}(\varphi) = \left(\frac{\sigma}{\sigma-1}\right) \frac{w_j \tau_{ji} t_{ji}}{\varphi}$. Using this, we readily obtain the cutoff productivity φ_{ji}^* at which

[14] The alternative case where the fixed costs of exporting use labor of the destination country is analyzed by Demidova, Naito and Rodríguez-Clare (2022).

profits are zero:

$$\pi_{ji}(\varphi_{ji}^*) = 0 \implies \varphi_{ji}^* = \left(\frac{\sigma}{\sigma-1}\right)\left(\frac{\sigma w_j f_{ji} t_{ji}}{I_i P_i^{\sigma-1}}\right)^{\frac{1}{(\sigma-1)}} w_j\, \tau_{ji} t_{ji}. \tag{57}$$

Expected profits in each country must equal the fixed costs of entry, so for a country i firm:

$$\sum_{j=1}^{N} \int_{\varphi_{ij}^*}^{\infty} \pi_{ij}(\varphi) g_j(\varphi) d\varphi = w_i f^e. \tag{58}$$

To evaluate this integral we follow the approach of Melitz and Redding (2014) to obtain

$$\sum_{j=1}^{N} J_j(\varphi_{ij}^*) f_{ij} = f^e \qquad \text{with} \qquad J_j(\varphi^*) \equiv \int_{\varphi^*}^{\infty} \left[\left(\frac{\varphi}{\varphi^*}\right)^{\sigma-1} - 1\right] g_j(\varphi) d\varphi.$$

Using the Pareto distribution in (52) and fixed costs in (53), for country 1 we have

$$\left(\frac{\sigma-1}{\theta-\sigma+1}\right)\left(\varphi_{11}^{*-\theta} f_0 L_1^{\alpha} + \sum_{j=2}^{N} \varphi_{1j}^{*-\theta} f_0 L_F^{\alpha}\right) = f^e. \tag{59}$$

The share of country i expenditure on the differentiated good coming from country j is obtained from CES demand as $\lambda_{ji} = (P_{ji}/P_i)^{1-\sigma}$. We obtain these shares by computing the integral in (55):

$$\begin{aligned} P_{ji}^{1-\sigma} &= M_j^e \left(\frac{\varphi_{ji}^*}{\varphi_j}\right)^{-\theta} \int_{\varphi_{ji}^*}^{\infty} p_{ji}(\varphi)^{1-\sigma} \frac{g_j(\varphi)\, d\varphi}{1-G_j(\varphi)} \\ &= M_j^e \left(\frac{\varphi_{ji}^*}{\varphi_j}\right)^{-\theta} \left[\frac{\sigma w_j\, \tau_{ji} t_{ji}}{(\sigma-1)\varphi_{ji}^*}\right]^{1-\sigma} \left(\frac{\theta}{\theta-\sigma+1}\right). \end{aligned} \tag{60}$$

Then substituting for φ_{ji}^* from (57) we have

$$\lambda_{ji} = M_j^e \left(\frac{\varphi_{ji}^*}{\varphi_j}\right)^{-\theta} \left(\frac{w_j\, f_{ji} t_{ji}}{I_i}\right) \left(\frac{\sigma\theta}{\theta+1-\sigma}\right). \tag{61}$$

From the labor market clearing condition for country 1, we have that

$$L_1 = M_1^e f^e + M_1^e \sum_{j=1}^{N} f_{1j} \int_{\varphi_{1j}^*}^{\infty} g_1(\varphi)\, d\varphi + (\sigma-1) M_1^e \sum_{j=1}^{N} \left[\int_{\varphi_{ij}^*}^{\infty} \frac{\pi_{1j}}{w_1}(\varphi)\, g_1(\varphi)\, d\varphi + f_{1j} \int_{\varphi_{ij}^*}^{\infty} g_1(\varphi)\, d\varphi \right],$$

which has the interpretation of summing the labor used in fixed and variable costs, as in (42). Using (58) to eliminate π_{1j} and then (59) to further simplify, we quickly obtain

$$M_1^e = \frac{(\sigma-1) L_1}{\sigma\theta f^e}. \tag{62}$$

Thus, entry in the Melitz model is proportional to the labor force, just as we found in the Krugman model in (43) if we used $\alpha = 0$ there.

We can now substitute entry and the cut-off threshold into the domestic share in (61), to obtain after some simplification:

$$\lambda_{11} = C_{11} \left(\frac{w_1}{P_1}\right)^{-\theta} \text{ with } C_{11} = M_1^e \left(\frac{\sigma}{\sigma-1}\right)^{-\theta} \left(\frac{\sigma w_1 f_{11}}{I_1}\right)^{1-\frac{\theta}{(\sigma-1)}} \left(\frac{\theta}{\theta+1-\sigma}\right). \tag{63}$$

Similarly, we obtain the import share of each foreign country in country 1 expenditure

$$\lambda_{F1} = C_{F1} \left(\frac{w_F \tau_0 \tau t_1}{\varphi_F P_1}\right)^{-\theta} \text{ with } C_{F1} = M_F^e \left(\frac{\sigma}{\sigma-1}\right)^{-\theta} \left(\frac{\sigma w_F f_{F1} t_1}{I_1}\right)^{1-\frac{\theta}{(\sigma-1)}} \left(\frac{\theta}{\theta+1-\sigma}\right), \tag{64}$$

where $f_{F1} = f_0 L_1^{\alpha}$ from (53) are the foreign fixed costs of exporting. The domestic and import shares sum to unity, so that $\lambda_{11} + (N-1)\lambda_{F1} = 1$. Summing the above equations in this way, we can solve for the price index in country 1 as

$$P_1 = \left(C_{11} (w_1)^{-\theta} + (N-1) C_{F1} \left(\frac{w_F \tau_0 \tau t_1}{\varphi_F}\right)^{-\theta} \right)^{-1/\theta}. \tag{65}$$

Using these various equation for the Melitz model, we can now proceed to bound the limit of

λ_{11}. From (10)–(12), (53) and (63)-(64), the country 1 domestic share is

$$\begin{aligned}
\lambda_{11} &= C_{11}\left(\frac{w_1}{P_1}\right)^{-\theta} = \frac{1}{1+(N-1)\frac{C_{F1}}{C_{11}}\left(\frac{w_F\tau_0\tau t_1}{w_1\varphi_F}\right)^{-\theta}} \\
&= \frac{1}{1+\tau_0^{-\theta}\left(\frac{t_1}{w_1}\right)^{1-\eta}\frac{L_{F0}}{L_1}\tau^{\beta+\delta-(1-\gamma)\theta}w_F^{1-\eta}},
\end{aligned} \tag{66}$$

where

$$\eta \equiv \frac{\theta\sigma}{\sigma-1} = \theta+1+\frac{\theta-\sigma+1}{\sigma-1} > \theta+1 > \sigma \text{ since } \theta > \sigma-1. \tag{67}$$

Then $\lim_{\tau\to+\infty}\lambda_{11} \in (0,1)$ if and only if

$$k \equiv \lim_{\tau\to+\infty}\tau^{\beta+\delta-(1-\gamma)\theta}w_F^{1-\eta} \in (0,+\infty). \tag{68}$$

Notice that the exponent on foreign wages in (68) is $1-\eta$, whereas in (45) and (14) for the Krugman and Armington models, respectively, it is $1-\sigma$. To interpret the coefficient $1-\eta$, notice from (66) that the ratio of imports relative to domestic consumption is

$$\frac{1-\lambda_{11}}{\lambda_{11}} = \tau_0^{-\theta}\left(\frac{t_1}{w_1}\right)^{1-\eta}\frac{L_{F0}}{L_1}\tau^{\beta+\delta-(1-\gamma)\theta}w_F^{1-\eta}.$$

The elasticity of this ratio with respect to t_1/w is $1-\eta$, and because the import share $1-\lambda_{11}$ is measured using duty-inclusive prices, we interpret η as the (absolute value of the) tariff elasticity of duty-free imports relative to domestic consumption. In the Melitz model (with export fixed costs paid in source country labor), we see from (67) that η exceeds $\theta+1$ and σ. The tariff elasticity in the Armington and Krugman models is just σ, so selection in the Melitz model leads to import demand that is *more elastic* than obtained in the Armington or Krugman models. Anticipating our results of the next subsection, we will show that $1-\eta$ is also the elasticity of foreign export demand with respect to the home wage, which is again more elastic in the Melitz model than in the Armington or Krugman models where $\eta=\sigma$, or than in the Eaton-Kortum model where $\eta=\theta+1$. However, if the models were calibrated to an estimated value of the tariff elasticity from the data,[15]

[15] These tariff elasticities are estimated by Caliendo and Parro (2015), for example. It can be expected that the method of Feenstra (1994) would also measure η if the primary variation in import prices comes from tariffs.

which would fix η, then we can conclude that the implied values of $\theta+1$ and σ in the Melitz model, with exporter fixed costs using labor of the source country, would be *less than* the implied value of σ in the Krugman or Armington models, in which case $\eta=\sigma$. These relationships should be taken into account in any quantitative comparison between models.

We can solve for foreign wages in various cases to ensure that the limit in (68) holds. Then for the Melitz model we obtain the taxonomy:

$$\lim_{\tau\to+\infty}\lambda_{11}\in(0,1)\iff\begin{cases} & \textbf{Parameters} & \textbf{Foreign Wage} \\ \text{Case M1:} & \beta=\theta,\gamma=\delta=0 & w_F=k^{\frac{1}{1-\eta}} \\ \text{Case M2:} & \beta=0,\gamma=1,\delta>0 & w_F\to\tau^{\frac{\delta}{\eta-1}}k^{\frac{1}{1-\eta}} \\ \text{Case M3:} & \beta=0,\gamma>1,\delta\geq 0 & w_F\to\tau^{\frac{\delta-(1-\gamma)\theta}{\eta-1}}k^{\frac{1}{1-\eta}} \\ \text{Case M4:} & \beta+\delta=(1-\gamma)\theta>0,\gamma>0 & w_F=k^{\frac{1}{1-\eta}} \\ \text{Case M5:} & \beta+\delta>(1-\gamma)\theta>0,\gamma>0 & w_F\to\tau^{\frac{\beta+\delta-(1-\gamma)\theta}{\eta-1}}k^{\frac{1}{1-\eta}} \end{cases}$$

We find finite foreign wages in Cases M1 and M4. We still need to confirm that the wages shown are consistent with trade balance, and that the location parameter for export demand does not depend on the country 1 wage or tariff.

5.1 Trade Balance in the Melitz Model

Duty-free imports in sector 1 of country 1 are $(\sum_{j=2}^{N}\lambda_{j1}I_1)/t_1$ while exports are $\sum_{j=2}^{N}\lambda_{1j}I_j$. Trade balance therefore requires

$$\sum_{j=2}^{N}\frac{\lambda_{j1}I_1}{t_1}=\sum_{j=2}^{N}\lambda_{1j}I_j.$$

Using the shares in (61) and also the symmetry of the foreign countries, trade balance becomes

$$\frac{(N-1)\theta}{\theta+1-\sigma}{\varphi^{*}_{F1}}^{-\theta}\,M^e_F\,w_F\,f_{F1}=\frac{(N-1)\theta}{\theta+1-\sigma}{\varphi^{*}_{1F}}^{-\theta}\,M^e_1\,w_1\,f_{1F}. \tag{69}$$

Two features of this trade balance equation should be highlighted.

First, the term ${\varphi^{*}_{1F}}^{-\theta}\,M^e_1$ on the right is the mass or variety of the country 1 firms who export to the $(N-1)$ foreign countries. In cases M1, M4 and M5 where the number of foreign countries approaches infinity, export demand will remain finite only if this mass of export variety approaches zero, so that $(N-1){\varphi_{1F}^{*}}^{-\theta} M_1^e$ approaches a finite value. Because M_1^e is fixed from the home labor supply in (62), this finite limit is obtained only if $\varphi_{1F} \to +\infty$, so that only the very most productive country 1 firms export. That result is not surprising because we are assuming that trade costs approach infinity, so it would take an infinitely productive exporter to overcome these costs. But it begs the question of how to interpret the *finite* productivity of an exporter in the SOE model. Recall that in the Armington and Krugman models we found that the effective export price is the actual price divided by the trade costs τ. Likewise, in the Melitz model we will find that the effective export productivity of the SOE is the actual productivity divided by τ, or φ_{1F}/τ, which we will find approaches a finite value.

Second, because exporting uses fixed costs in the Melitz model, the value of country 1 imports on the left of (69) and the value of exports on the right, can be expressed fully in terms of those fixed costs and the cutoff productivities.[16] We have assumed that the fixed costs of exporting use source country labor, so it is foreign wage w_F appearing on the left (country 1 imports) and the country 1 wage w_1 that appears on the right (its exports). Of course, wages affect the cutoff productivities, too. We can use $\varphi_{1F}^{*} = \left(\frac{\sigma}{\sigma-1}\right)\left(\frac{\sigma w_1 f_{1F}}{Y_F P_F^{\sigma-1}}\right)^{\frac{1}{(\sigma-1)}} w_1 \tau_0 \tau$ from (57) to re-express export demand as:

$$
\begin{aligned}
(1-\lambda_{11})\,\frac{I_1}{t_1} &= \frac{(N-1)\theta}{\theta+1-\sigma}\left(\frac{\sigma}{\sigma-1} w_1 \tau_0 \tau \left(\frac{\sigma w_1 f_{1F}}{w_F L_F P_F^{\sigma-1}}\right)^{\frac{1}{(\sigma-1)}}\right)^{-\theta} M_1^e w_1 f_{1F} \qquad (70) \\
&= M_1^e\, B\, \tau_0^{-\theta} w_1^{1-\eta},
\end{aligned}
$$

where

$$
B = \frac{(N-1)\theta}{\theta+1-\sigma}\left(\left(\frac{\sigma}{\sigma-1}\right)\left(\frac{\sigma f_{1F}}{w_F L_F P_F^{\sigma-1}}\right)^{\frac{1}{(\sigma-1)}} \tau\right)^{-\theta} f_{1F}. \qquad (71)
$$

Notice that the country 1 wage w_1 appears three times on the right of (70): first multiplying τ_0 with exponent θ, and then twice multiplying the fixed costs of exporting f_{1F}. Using the notation of Definition 1, the *combined* exponent on w_1 is $1-\eta = 1-\frac{\theta\sigma}{\sigma-1}$ and the component on the iceberg

[16] Note that the left and right of (69) exactly equal country 1 imports and exports, respectively, by multiplying both sides by the constant $\frac{\sigma\theta}{\theta+1-\sigma}$, which is the last term appearing in (61).

transports cost is $1-\varepsilon=-\theta$ so $\varepsilon=\theta+1$.

In order for country 1 to be a SOE, we require that the limit of B is positive and finite, and independent of the country 1 variables. To obtain this limit, we first express the foreign price index in a manner analogous to (63)–(65):

$$\begin{aligned} P_F &= \left(C_{1F}\left(w_1\tau_0\tau\right)^{-\theta}+C_{FF}\left(\frac{w_F}{\varphi_F}\right)^{-\theta}+C_{FF}(N-2)\left(\frac{\tau_0\tau w_F}{\varphi_F}\right)^{-\theta}\right)^{-1/\theta}, &(72)\\ C_{FF} &= M_F^e\left(\frac{\sigma}{\sigma-1}\right)^{-\theta}\left(\frac{\sigma w_F f_0 L_F^{\alpha}}{w_F L_F}\right)^{1-\frac{\theta}{(\sigma-1)}}\left(\frac{\theta}{\theta+1-\sigma}\right), \text{ and}\\ C_{1F} &= M_1^e\left(\frac{\sigma}{\sigma-1}\right)^{-\theta}\left(\frac{\sigma w_1 f_0 L_F^{\alpha}}{w_F L_F}\right)^{1-\frac{\theta}{(\sigma-1)}}\left(\frac{\theta}{\theta+1-\sigma}\right). \end{aligned}$$

Notice that w_1 appears within C_{1F} since it affects the cutoff export productivity, and it also appears in the first term of the price index P_F. So to ensure that $B^*\equiv\lim_{\tau\to+\infty}B$ does not depend on w_1, this first term will have to vanish. Substituting P_F into B, dividing the numerator and denominator by $\tau^{-\theta}$, and using (10)–(12), (53), (63)-(64), and k from (68), we obtain:

$$B^*=\lim_{\tau\to+\infty}\frac{\frac{\theta f^e}{(\sigma-1)}w_F L_{F0}\tau^{\beta+\delta}}{L_1\tau_0^{-\theta}w_1^{1-\eta}+L_{F0}\left[\frac{(N-2)}{(N-1)}k\left(\frac{\tau}{\tau_0}\right)^{\theta}+\tau_0^{-2\theta}k\tau^{2\theta-\beta}\right]}. \tag{73}$$

5.2 Cases in the Melitz Model

Case M1 with $\delta=0$ is identical to Cases A1 and K1, because with no growth in the foreign labor force or productivity, then the number of varieties at home and abroad are all constant. From (73) we see that the the condition $\beta=\theta$ on the growth of foreign countries guarantees a SOE in the Melitz model, because then τ^{θ} appears in both the numerator and the second and third term of the denominator. Furthermore, we can solve for the home productivity that appears on the right of (69) to obtain

$$M_1^e\left(\frac{\varphi_{1F}^*}{\tau}\right)^{-\theta}=\frac{(1-\lambda_{11})I_1}{t_1\sigma\, w_F\, f_{1F}}\left(\frac{\theta-\sigma+1}{\theta}\right).$$

The limiting values of $(1-\lambda_{11})$ and w_F on the right are both positive and finite, so we see that φ_{1F}^*/τ on the left has a finite limit, even though $\varphi_{1F}^*\to+\infty$ as $\tau\to+\infty$. This demonstrates our earlier claim that only the very most productive home firms export, but the *ratio* of their cutoff productivity to trade costs approaches a finite limit.

In cases M2 and M3, $N = 2$ so there is a single foreign country, and its wage approaches infinity in either case. Combining these cases by specifying $\gamma \geq 1$, we substitute the solution for foreign wages into (73), and find that $B^* \in (0, +\infty)$ provided that the exponents on τ sum to zero:

$$\gamma + \frac{\eta\delta}{\theta} = (2\eta - 1).$$

This condition is very similar to that obtained in the Krugman model in (49) provided that we use $\alpha = 0$ to obtain the standard monopolistic competition framework, and it is identical to the standard Krugman model when we set $\eta = \sigma$ and $\theta = \sigma - 1$.

Finally, we combine cases M4 and M5 by specifying $\beta \geq (1-\gamma)(\sigma - 1) > 0$, we again substitute the solution for foreign wages into (73), and find that $B^* \in (0, +\infty)$ provided that the exponents on τ sum to zero, After some simplification, this general condition is stated as:

$$\frac{\beta}{\theta} + \frac{\gamma\theta + \delta\eta}{\theta(2\eta - 1)} = 1 \quad \text{and} \quad (1-\gamma) \leq \frac{\beta}{\sigma - 1} \leq 1.$$

This parameter restriction is identical to the general condition (51) obtained in the standard Krugman model when we set $\eta = \sigma$ and $\theta = \sigma - 1$.

6 Application: Optimal Tariffs

We have shown that trade balance in a SOE is given by Definition 1. As stressed by Demidova and Rodríguez-Clare (2009, 2013), using trade balance we can obtain an upward sloping relationship between the wage in the SOE and the import tariff t_1 (using the foreign wage or k as the numeraire). We now solve for the optimal tariff, which balances this terms-of-trade gain from a tariff against the change in tariff revenue so as to maximize utility.

We begin with several preliminary expression that summarize the general model. First, we simplify the domestic expenditure share shown in Definition 1 as

$$\lambda_{11} = \frac{L_1^{1-\alpha} w_1^{1-\eta}}{\left(L_1^{1-\alpha} w_1^{1-\eta} + L_{F0}^{1-\alpha} \tau_0^{1-\varepsilon} t_1^{1-\eta} k\right)} = \frac{1}{\left(1 + (t_1/w_1)^{1-\eta} \tilde{k}\right)},$$

with $\tilde{k} \equiv k(L_{F0}/L_1)^{1-\alpha}\tau_0^{1-\varepsilon}$. Totally differentiating this share, we can solve for

$$\frac{d\ln\lambda_{11}}{d\ln t_1} = (1-\lambda_{11})(\eta-1)\left(1-\frac{d\ln w_1}{d\ln t_1}\right). \tag{74}$$

Using the above share, we can rewrite trade balance from Definition 1 as

$$(t_1/w_1)^{1-\eta}\tilde{k}w_1L_1 = \tilde{B}w_1^{1-\eta}\left(t_1 + (t_1/w_1)^{1-\eta}\tilde{k}\right).$$

with $\tilde{B} \equiv M_1^e B^* \tau_0^{1-\varepsilon}$. It follows that the equilibrium wage solves

$$w_1^\eta = \tilde{B}\left(1 + \frac{1}{\tilde{k}}t_1^\eta w_1^{1-\eta}\right). \tag{75}$$

This equation neatly summarizes the relationship between the wage and the tariff, or the terms of trade effect in the Armington, Eaton-Kortum, Krugman and Melitz models. Totally differentiating, we obtain

$$\frac{d\ln w_1}{d\ln t_1} = \frac{\eta t_1\lambda_{11}}{\eta(1-\lambda_{11}) + (2\eta-1)t_1\lambda_{11}} > 0. \tag{76}$$

This positive slope shows that there is a unique solution for the SOE wage given any tariff.

Per-capita utility in country 1 is given by

$$U_1 = \frac{I_1}{P_1L_1}.$$

We solve for the price index in the Melitz model by substituting $I_1 = U_1P_1L_1$ into (63) and simplifying to obtain:

$$P_1 = w_1\left(\frac{M_1^e}{\lambda_{11}}\right)^{\frac{1}{1-\eta}}\left(\frac{\sigma}{\sigma-1}\right)^{\frac{\theta}{\eta-1}}\left(\frac{U_1L_1}{\sigma f_{11}}\right)^{-\frac{\theta-\sigma+1}{(\eta-1)(\sigma-1)}}\left(\frac{\theta}{\theta+1-\sigma}\right)^{\frac{1}{1-\eta}}.$$

Also using income I_1 from (16) we can then solve for utility as:

$$U_1 = \left[\frac{w_1t_1}{1+(t_1-1)\lambda_{11}}\Big/ w_1\left(\frac{M_1^e}{\lambda_{11}}\right)^{\frac{-1}{\eta-1}}\right]^{\frac{\eta-1}{\theta}}\left(\frac{L_1}{\sigma f_{11}}\right)^{\frac{\theta-\sigma+1}{\theta(\sigma-1)}}\frac{(\sigma-1)\theta^{\frac{1}{\theta}}}{\sigma(\theta+1-\sigma)^{\frac{1}{\theta}}}. \tag{77}$$

This solution from the Melitz model shows that utility is a monotonic transformation of the term in brackets. The tariff term appearing in that numerator adds per-capita tariff revenue to the wage, as in (16). The term in the denominator is readily interpreted as the price index as it would appear in a Krugman model, with $\eta = \sigma$, or in the Armington model if we further specified $M_1^e = 1$. In the Krugman model, the domestic expenditure share is $\lambda_{11} = M_1^e \left(\frac{w_1 \sigma}{(\sigma-1)}/P_1\right)^{1-\sigma}$, from which we quickly solve for $P_1 = \left(\frac{M_1^e}{\lambda_{11}}\right)^{\frac{-1}{\sigma-1}} \frac{w_1 \sigma}{(\sigma-1)}$. Ignoring the markup and replacing σ with η, this gives us the price index shown in the denominator in (77). Remarkably, we have therefore shown that all four models have an isomorphic measure of utility in (77), which uses the parameter $\eta = \sigma$ under the Armington and Krugman models, and η as in (67) in the Melitz model under our assumption that exporter fixed costs uses labor in the source market.

Taking the natural log of (77), setting $d \ln U / d \ln t_1 = 0$ and using (74), we obtain the first-order condition for the optimal tariff:

$$t_1 - 1 \quad = \quad \frac{\frac{d \ln w_1}{d \ln t_1}}{\eta \lambda_{11} \left(1 - \frac{d \ln w_1}{d \ln t_1}\right)}. \tag{78}$$

This expression shows the tight relationship between the terms of trade effect $d \ln w / d \ln t_1$ in all of our four models and the optimal tariff: if $d \ln w / d \ln t_1 = 0$ then the optimal tariff is zero, and we obtain a positive *ad valorem* tariff if $0 < d \ln w / d \ln t_1 < 1$. We can substitute for the wage elasticity from (76) to obtain

$$t_1 - 1 = \frac{t_1}{\eta (1 - \lambda_{11}) + (\eta - 1) t_1 \lambda_{11}} \implies t_1 - 1 = \frac{1}{\eta - 1}. \tag{79}$$

The above quadratic equation in t_1 has two solutions: $t_1 - 1 = \frac{1}{\eta - 1}$ and $t_1 = \frac{-(1-\lambda_{11})}{\lambda_{11}}$. The negative solution for t_1 can be ignored since it implies negative prices, so we conclude that there is a unique maximum for utility at the optimal *ad valorem* tariff $\frac{1}{\eta-1}$.

This general formula incorporates several existing results for the optimal tariff in a SOE as mentioned in section 1: the *ad valorem* tariff of $\frac{1}{\sigma-1} > 0$ found by Gros (1987), which also holds in the Armington model; the optimal *ad valorem* tariff of $\frac{1}{\theta}$ in the Eaton-Kortum model where θ is the Fréchet parameter, as shown by Caliendo and Parro (2022);[17] and the optimal *ad valorem* tariff in the Melitz model (with fixed costs of exporting using home labor) of $\frac{1}{\eta-1} = \frac{\rho}{\theta-\rho}$ with $\rho \equiv \frac{\sigma-1}{\sigma}$, which is the formula found by Demidova and Rodríguez-Clare (2009).

7 Conclusions

Our goal in this paper was to provide a firm justification for the small open economy (SOE) model in a general context with product differentiation. The issue that arises when the foreign country grows very large is the the home country devotes a greater share of expenditure to imports, so that the domestic share potentially grows vanishingly small: what we have called a "truly small" economy, with a domestic share of zero. To avoid this outcome, we have introduced trade costs that grow to infinity as either the number or size of foreign countries (or both) grow to infinity. Specifically, we have modeled the the number, productivity, and labor force size of the symmetric foreign countries as depending on trade costs with exponents $\beta, \gamma, \delta \geq 0$. We have found that regardless of the model considered – Armington, Eaton-Kortum, Krugman or Melitz – a SOE can be obtained as trade costs go to infinity when these parameters satisfy a linear restriction between them.

Generally, the restriction between the parameters allows for either a single foreign country ($\beta = 0$ and $N = 2$) that must grow in either productivity or labor size (γ or δ or both), or the number of foreign countries growing to infinity with less growth in productivity or labor size. The precise nature of this restriction between differs slightly across models. Under Armington or Eaton-Kortum, the growth in foreign productivity and the foreign labor force enter the parameter restriction symmetrically. Under monopolistic competition, however, growth in the foreign labor force leads to greater entry of firms, but growth of productivity – which we have applied to only the *variable costs* of production – does not impact entry. The entry of firms leads to great product variety and an "effective" price drop for that reason, which is different from the price drop that occurs from the growth in productivity. Therefore, these two parameters (γ and δ) enter with slightly different weights in the parameter restriction for the monopolistic competition model.

[17] Caliendo and Parro (2022) define the Fréchet parameter θ inversely, so they actually find the optimal *ad valorem* tariff of θ for a SOE.

We have shown that all four models fit the SOE export demand for each variety proposed in section 1, i.e. $B^*p^{-\eta}$, with the following features. First, the location parameter B^* is fixed as trade costs approach infinity and is independent of any SOE variable. While the growth in foreign productivity can imply that foreign wages approach infinity, when we treat B^* as fixed then we are using the foreign country (i.e. its wage if finite or otherwise k) as the numeraire. Second, the export elasticity $\eta > 1$ equals σ in the Armington and Krugman models and it equals $\theta\sigma/(\sigma-1)$ under our assumptions in the Melitz model, with fixed exporting costs using source country labor. Third, the effective export price p of the SOE equals the actual c.i.f. export price divided by the trade costs: with the c.i.f. export price approaching infinity along with trade costs, it is their *ratio* that governs exports of the SOE. Specifically, modeling trade costs as $\tau_0\tau$ with $\tau \to +\infty$, then: (a) in the Armington and Eaton-Kortum models the effective export price is $\tau_0 w_1$ where w_1 is the SOE wage; (b) in the Krugman model the effective export price is $\tau_0 w_1 \frac{\sigma}{\sigma-1}$, which includes the markup $\frac{\sigma}{\sigma-1}$ that we have absorbed into the location parameter B^*; (c) in the Melitz model, the *value* of export demand becomes $B^*\tau_0^{1-\theta}w_1^{1-\eta}$, so the effective export price (while ignoring the markup), reflects the iceberg trade costs and the home wage with differing elasticities. In the Melitz model the threshold productivity φ_{1F}^* of SOE exporters also approaches infinity – since they must be infinitely productive to overcome the trade costs – but the ratio φ_{1F}^*/τ approaches a finite value as $\tau \to +\infty$, which is the effective cutoff productivity of exporters.

We conclude by mentioning two directions in which our results could be extended. First, it would be desirable to consider functional forms besides the CES. Consider, for example, symmetric translog preferences (Feenstra, 2003; Feenstra and Weinstein, 2017; Diewert, 2022). As the number or size of foreign countries grow then we can expect that the number of varieties available to the SOE will also grow. Under translog preferences, however, product space can become "crowded" and the elasticity of demand approaches infinity as the share of any product approaches zero. So that feature of translog preferences may be difficult to incorporate into a formal justification of the small country model. Instead, however, we could consider the homothetic Kimball (1995) preferences studied by Errico and Lashkari (2022). They propose specific preferences within this family that have the property that the elasticity of substitution between varieties is variable but bounded between a low and high value, even as the number of varieties grows without limit. Those preferences may prove to be quite amenable to incorporate into the SOE framework.

Second, we could extend the analysis of the optimal tariff, as discussed in section 6, to the case where there is roundabout production in the economy, so that the variable costs of production rely on both labor *and* a bundle of the differentiated goods themselves. In this case, the cost index of production in the SOE could be a Cobb-Douglas function of the wage w_1 and the price index P_1, such as $c_1 = w_1^{1-\mu} P_1^{\mu}$, with $0 \leq \mu < 1$ indicating the degree of roundabout production. In preliminary analysis, we have explored how the results of this paper would be affected. The domestic share appearing in Definition 1 would be changed, since instead of the wage w_1 determining the domestic share, it would be the cost index c_1. In addition, we have found that the effective export price used within export demand in Definition 1 would become a more complicated function of the wage *and* the price index. Finally, many of the equations of section 6 for the impact of a tariff on the domestic share, on the wage, and ultimately on welfare, would be impacted. Caliendo, Feenstra, Romalis and Taylor (2021) have obtained a formula for the optimal second-best tariff in a two-sector SOE model that incorporates roundabout production, and while they argue that the terms-of-trade impact of the tariff on the home wage is important, it is not the only determinant of the optimal tariff. In contrast, in (78) we found a direct connection between the terms-of-trade impact and the optimal tariff. Following our methods of section 6 – while incorporating roundabout production – could give new insight into the second-best optimal tariff in a more general setting.

References

[1] Armington, Paul S. 1969. A Theory of Demand for Products Distinguished by Place and Production. *IMF Staff Papers* 16: 159-178.

[2] Arkolakis, Costas, Arnaud Costinot, and Andrés Rodríguez-Clare. 2012. New Trade Models, Same Old Gains? *American Economic Review* 102(1): 94–130.

[3] Bagwell, Kyle, Robert Staiger and Ali Yurukoglu, 2021. Quantitative Analysis of Multi-Party Tariff Negotiations. *Econometrica* 89(4) July: 1595-1631.

[4] Balistreri, Edward J., Russell H. Hillberry, and Thomas F. Rutherford. 2011. Structural Estimation and Solution of International Trade Models with Heterogeneous Firms. *Journal of International Economics* 83(2): 98–108.

[5] Balistreri, Edward J. and David G. Tarr. 2022. Welfare Gains in the Armington, Krugman and Melitz models: Comparisons Grounded on Gravity. *Economic Inquiry*: https://doi.org/10.1111/ecin.13082.

[6] Bartelme, Dominick G., Arnaud Costinot, Dave Donaldson and Andr és Rodríguez-Clare. 2019. The Textbook Case for Industrial Policy: Theory Meets Data. NBER Working Paper no. 26193.

[7] Caliendo, Lorenzo, Robert C. Feenstra, John Romalis, and Alan M. Taylor. 2015, 2020. Tariff Reductions, Entry, and Welfare: Theory and Evidence for the Last Two Decades. December 2015 version of NBER Working Paper 21768, Revised 2020, available at `http://faculty.som.yale.edu/lorenzocaliendo/Caliendo_Feenstra_Romalis_Taylor_NBER.pdf`.

[8] Caliendo, Lorenzo, Robert C. Feenstra, John Romalis, and Alan M. Taylor. 2021. A Second-Best Argument for Low Optimal Tariffs. NBER Working Paper 28380.

[9] Caliendo, Lorenzo, and Fernando Parro. 2015. Estimates of the Trade and Welfare Effects of NAFTA. *Review of Economic Studies* 82 (1): 1–44.

[10] Caliendo, Lorenzo, and Fernando Parro. 2022. Trade Policy. In Gita Gopinath, Elhanan Helpman, and Kenneth Rogoff, eds. *Handbook of International Economics, vol. V*. Elsevier Science.

[11] Chaney, Thomas. 2008. Distorted Gravity: The Intensive and Extensive Margins of International Trade. *American Economic Review* 98(4): 1707–21.

[12] Costinot, Arnaud, and Andrés Rodríguez-Clare. 2014. Trade Theory with Numbers: Quantifying the Consequences of Globalization. In *Handbook of International Economics*, volume 4, edited by Gita Gopinath, Elhanan Helpman, and Kenneth Rogoff. Amsterdam: Elsevier, pp. 197–262.

[13] Costinot, Arnaud, Andrés Rodríguez-Clare, and Iván Werning. 2020. Micro to Macro: Optimal Trade Policy with Firm Heterogeneity. *Econometrica* 88(6): 2739–776.

[14] Diewert, Erwin W. 2022. Generalization of the Symmetric Translog Functional Form. Vancouver School of Economics.

[15] Demidova, Svetlana, Takumi Naito and Andrés Rodríguez-Clare. 2022. The Small Open Economy in a Generalized Gravity Model. McMaster University, Waseda University and UC Berkeley.

[16] Demidova, Svetlana and Andrés Rodríguez-Clare. 2009. Trade Policy under Firm-Level Heterogeneity in a Small Economy. *Journal of International Economics* 78(1), June: 100–112.

[17] Demidova, Svetlana and Andrés Rodríguez-Clare. 2013. The Simple Analytics of the Melitz Model in a Small Economy. *Journal of International Economics* 90(2), July: 266–272.

[18] Dornbusch, Rudiger, Stanley Fischer and Paul A. Samuelson. 1977. Comparative Advantage, Trade, and Payments in a Ricardian Model with a Continuum of Goods. *American Economic Review* 67(5): December, 823-39.

[19] Eaton, Jonathan, Samuel Kortum. 2002. Technology, Geography and Trade. *Econometrica* 70(5), September: 1741–1779.

[20] Errico, Marco and Danial Lashkari. 2022. The Quality of US Imports and the Consumption Gains from Globalization. Boston College.

[21] Feenstra, Robert C. 1994. New Product Varieties and the Measurement of International Prices, *American Economic Review* 84(1): March, 157-177.

[22] Feenstra, Robert C. 2003. A Homothetic Utility Function for Monopolistic Competition Models, Without Constant Price Elasticity. *Economic Letters* 78: 79-86.

[23] Feenstra, Robert C. 2016. *Advance International Trade.* MIT Press, 2^{nd} edition.

[24] Feenstra, Robert C. and David Weinstein. 2017. Globalization, Markups, and U.S. Welfare. *Journal of Political Economy* 125(4), August: 1041-1074.

[25] Felbermayr, Gabriel, Benjamin Jung, and Mario Larch. 2013. Optimal Tariffs, Retaliation and the Welfare Loss from Tariff Wars in the Melitz Model. *Journal of International Economics* 89: 13-25.

[26] Gros, Daniel. 1987. A Note on the Optimal Tariff, Retaliation and the Welfare Loss from Tariff Wars in a Framework with Intra-Industry Trade. *Journal of International Economics* 23: 357–367.

[27] Kimball, Miles S. 1995. The Quantitative Analytics of the Basic Neomonetarist Model. *Journal of Money, Credit and Banking* 27(4): November,1241-1277.

[28] Krugman, Paul r. 1979. Increasing Returns, Monopolistic Competition and International Trade. *Journal of International Economics* 9: 469-479.

[29] Krugman, Paul R. 1980. Scale Economies, Product Differentiation, and the Pattern of Trade. *American Economic Review* 70: December, 950-959.

[30] Lashkaripour, Ahmad. 2021. The Cost of a Global Trade War: A Sufficient Statistics Approach. *Journal of International Economics* 131, July, 103419.

[31] Lashkaripour, Ahmad and Volodymyr Lugovskyy. 2020. Profits, Scale Economies, and the Gains from Trade and Industrial Policy. Indiana University, Working Paper, April, https://alashkar.pages.iu.edu/Lashkaripour_Lugovskyy_2020.pdf.

[32] Melitz, Marc J. 2003. The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity. *Econometrica* 71(6):1695–1725.

[33] Melitz, Marc J. and Gianmarco I. P. Ottaviano. 2008. Market Size, Trade, and Productivity. *Review of Economic Studies* 75: 295–316.

[34] Melitz, Marc J., and Stephen J. Redding. 2014. Heterogeneous Firms and Trade. In Gita Gopinath, Elhanan Helpman and Kenneth Rogoff, eds. *Handbook of International Trade*, Volume 4. Amsterdam: Elsevier, 1–54.

[35] Ossa, Ralph. 2014. Trade Wars and Trade Talks with Data. *American Economic Review* 104(12): 4104–46.

[36] Spearot, Alan. 2016. Unpacking the Long Run Effects of Tariff Shocks: New Structural Implications from Firm Heterogeneity Models. *American Economic Journal: Microeconomics* 8(2): 128–167.