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Sebastian Galiani
Jose Manuel Paz y Miño
Gustavo Torrens

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ABSTRACT

We develop a simple model to explain why a powerful importer country like the United States may provide political support for international collusive agreements concerning certain commodities (e.g., coffee). This behavior raises questions due to the fact that an importer country should have strong economic incentives to avoid the cartelization of its suppliers. We show that an importer country sometimes helps producer countries organize and enforce collusion to advance important geopolitical goals, e.g., by reducing the chances that the producer countries will align with a rival global power (e.g., the Soviet Union). Moreover, using this practice, a powerful importer country can immediately share the cost of collusion with other importers (including allies). Thus, a powerful importer country may see collusion as a superior strategy to foreign aid (a priori a more direct and efficient instrument), which is riddled with free riding problems. The model sheds light on why the United States supported (or failed to support) international commodity agreements for coffee, sugar, and oil during and immediately after the Cold War period.

Sebastian Galiani
Department of Economics
University of Maryland
3105 Tydings Hall
College Park, MD 20742
and NBER
sgaliani@umd.edu

Jose Manuel Paz y Miño
Department of Social Sciences
Universidad Católica del Uruguay
Av. 8 de Octubre 2738
Of. 408
Montevideo CP11600
Uruguay
jose.pazymino@ucu.edu.uy

Gustavo Torrens
Department of Economics
Indiana University
Wylie Hall, 100 S Woodland Ave
Bloomington, IN 47405-7104
gtorrens@indiana.edu

Mr. Curtis (Member of the U.S. House of Representatives - MO, 2nd congressional district): *We are all interested in protecting the consumer and that is the ultimate purpose of my interrogation, to find out just what might be done, because seen in its very bare bones, this agreement establishes an international cartel arrangement. [...]*

Mr. Mann (Under Secretary of Economic Affairs): *I think the key question is whether it is in the U.S. interest to allow these countries, a large number of them, to go through the wringer, as it were, at a time where populations are doubling every 18 to 20 years and take a chance that they would stay on our side of the curtain which divides the free and the Communist world.*

April, 13, 1965. Executive Hearings before the Committee On Ways and Means - House of Representatives Eighty - Eighty-Ninth Congress - First Session - On S.701 An act to carry out the obligations of the United States under the International Coffee Agreement, 1962, signed at New York on September 28, 1962. Government Printing Office Washington, 1965

1 Introduction

During the Cold War period, the United States government helped coffee producers in developing countries organize collusion (not explicitly, but through International Commodity Agreements, see Gilbert (1996)). From an economic perspective, this behavior is puzzling, as the United States was an important importer of coffee at the time. Indeed, standard international trade arguments imply that the United States should have actually imposed a tariff on coffee to improve its terms of trade (Feenstra, 2015). Moreover, while the U.S. might have been able to tolerate collusion (for example, if the cost of prosecuting collusion was too high), there is clear evidence that the U.S. went much further by helping producers to form and sustain the cartel, i.e., by monitoring the agreements and punishing deviators (Koremenos, 2002). High prosecution costs cannot account for this choice. Further complicating matters, during the same period, the United States actively pushed to disarticulate other international commodity cartels, such as OPEC (Painter, 2014). The U.S. also exhibited a more ambivalent attitude towards the sugar cartel, first backing it and then withdrawing support (Gilbert, 1996). After the Cold War ended, the U.S. government stopped supporting or tolerating international cartels of imported commodities.

We propose a political economy explanation based on: (i) the U.S.'s geopolitical interests during the Cold War; and (ii) internal political issues in both the U.S. and in commodity exporting countries. Geopolitical interests provide a compelling explanation for why the U.S. was willing to transfer resources to some developing countries. Namely, this behavior comprised part of a broader international strategy to contain the spread of communism in Africa, Asia, and Latin America. Nevertheless, this explanation fails to account for why the U.S. used collusion, a relatively costly policy instrument, rather than foreign aid, a likely more efficient policy instrument. Three elements help explain this choice:

First, foreign aid was fully funded by domestic agents (i.e., U.S. taxpayers) and was subject to considerable free riding by U.S. allies. By contrast, collusion allowed the U.S. to share the burden with foreign consumers. In particular, U.S. consumers paid less than one dollar in consumer surplus to transfer one extra dollar in profits to producer countries. The main reason is that collusion increased commodity prices both domestically and abroad. In other words, collusion helped the U.S. transfer a share of the burden of fighting the spread of communism to other countries. From this perspective, forming an international cartel was a superior strategy to foreign aid, even foreign aid financed with taxes that did not generate any deadweight loss. However, the cartel strategy was only superior due to the fact that other countries shared a sufficient portion of the global demand. If United States were the only international consumer of the commodity, foreign aid would always be a superior strategy to supporting collusion.

The second explanation for the U.S. favoring collusion hinged on internal political constraints. Namely, it was politically costly for the U.S. to increase foreign aid in the federal budget. The reason is that many voters in the U.S. saw (and still see) foreign aid as representing a significant fraction of federal government spending (see, for example, Caplan (2011)). By contrast, having consumers pay a higher price for a cup of coffee served as a veiled means of transferring aid to foreign countries.

The third explanation focuses on those most directly affected by the spread of communism: landowners. Specifically, while foreign aid would mostly benefit governments, a higher export price for coffee could directly benefit landowners in coffee producer regions. This was important because landowners had the capacity and incentives to organize paramilitary groups to defend their land and fight communists in rural areas. On the contrary, it was almost impossible for the United States to monitor how corrupt governments were using foreign aid. Moreover, governments run big budgets, and, at the margin, they have more opportunities to neutralize the effects of foreign aid. For example, if aid is supposed to finance military modernization, governments can always neutralize it by quietly reducing other military items in the defense budget. Thus, it is certainly plausible that helping coffee producers form and sustain their cartel was more effective at fighting communism than standard foreign aid.

Finally, why did the United States employ a different strategy for other cartelized commodities, especially oil? First, the U.S. experienced significant economic losses associated with the cartelization of oil exporters (Hamilton, 2010; Kilian, 2008). Second, rising gas prices in the U.S. presented a serious political issue (Knittel, 2014). Third, the Soviet Union was a natural gas and oil exporter; as a consequence, it benefited from any rise in the international price of oil. Finally, in most developing countries, oil was controlled by the governments (i.e., through national oil companies). Thus, increases in the price of oil were captured by the government or groups such as unions or public employees. In short, the political economy logic for the coffee cartel did not apply to the oil cartel.

To formalize these ideas and further explore the political calculus behind international commodity cartels, we build a simple game theoretic model. In the model, the key player is a global power (e.g., the United States) facing a geopolitical challenge (e.g., the spread of communism supported by the Soviet Union) in a developing country whose economy depends heavily on an export commodity. The global power has two economic instruments it can use to address the geopolitical challenge: (i) foreign aid; and (ii) helping commodity exporters form and/or sustain a cartel. We characterize the global power's choices. In particular, we show that supporting collusion might comprise part of the global power's optimal toolkit. The reason is that collusion allows for sharing the burden with consumers all over the world, while foreign aid is fully borne by domestic taxpayers.

We extend the baseline model in several directions. First, in the baseline model, the global power has an ally, who can, in principle, also contribute foreign aid to the developing country. However, in equilibrium, the ally has strong incentives to free ride the global power. To remedy this problem, we explore a scenario in which the global power and its ally cooperatively determine foreign aid. We find that even in this scenario, collusion might still be part of the optimal toolkit due to the advantage in having foreign consumers bear some of the burden.

Second, we consider the possibility that voters in the global power do not perceive foreign aid and collusion equally. In particular, from a political standpoint, the connection between domestic policy decisions and rising commodity prices is easier to camouflage than, e.g., foreign aid in the national budget. In this case, visibility becomes an extra reason for the global power's policy makers to choose collusion.

Third, in the baseline model, all producer countries other than the developing country involved in the geopolitical threat are implicitly assumed to be geopolitically neutral or irrelevant. In other words, for the global power facing the geopolitical threat, the other producer countries are pure lucky economic winners of the cartel. They have no geopolitical interest. In an extension, we consider a situation where some of the producer countries are geopolitical rivals. During the Cold War period, this environment was relevant, for commodities like oil, as the Soviet Union was an important producer and exporter of natural gas and oil. Naturally, when some producer countries are geopolitical rivals, the incentives for collusion diminish. However, they do not disappear entirely.

Fourth, in the baseline model, producers directly sell the commodity to consumers. In many cases, however, there are wholesale companies that specialize in importing the commodity and distributing it among final consumers or retail companies. In an extension, we introduce a wholesale industry that competes a la Cournot. Per se, this modification does not change the results in any relevant way. Final consumers and the wholesale industry experience the negative effects of cartelization because, from their perspective, collusion is equivalent to a rise in production costs. However, if we also introduce heterogeneity among wholesale companies, novel results emerge. In particular, we assume that some wholesale companies are politically connected, and thus find a way to avoid paying the collusive price (for example, producers can offer them a discount). By contrast, non-connected wholesale companies bear the full brunt of collusion, along with consumers. This extension helps explain why politically connected United States coffee roasters supported the International Coffee Agreement before the United States Congress.

Fifth, in the baseline model, an extra dollar of foreign aid and an extra dollar of profits for the commodity producers are perfect substitutes, in the sense that they induce the same effect on the probability that the geopolitically contested developing country aligns with the United States. This might not be the case for several reasons. For example, while foreign aid is often received by the government, collusive profits go to commodity producers, who might be more or less willing to fight communism. Naturally, when commodity producers are more willing or in a better position to influence geopolitical outcomes in the contested developing country, the global power is more likely to use collusion (being an instrument that allows the US to interfere in the domestic affairs of the producer country).

1.1 Related Literature

Our paper relates to two strands of literature: (i) government-sponsored collusion; and (ii) the economics of foreign aid.

Classical treatments of collusion (Tirole, 1988; Fudenberg and Tirole, 1991) assume that firms use monitoring mechanisms to sustain collusive agreements, while the government tries to dissuade firms from engaging in collusive practices, e.g., by prosecuting those practices (Harrington Jr, 2017). However, on some occasions, governments have sponsored collusion. Five rationalizations for government-sponsored collusion have been: coordination of excess capacity when demand decreases (Okazaki et al., 2018); coordination in prices during recessions (Taylor, 2007,?; Vickers and Ziebarth, 2014); technology transfers (Hu et al., 2014); political favors (Libecap, 1989); and protecting collusive profits of national firms in foreign markets (Garcia et al., 2018). However, none of these rationales explains why an importer country might support an international commodity agreement leading to the cartelization of its suppliers. Our model provides a rationale for sponsoring international commodity collusion as a tool for advancing geopolitical goals while transferring some of the burden to foreign consumers.

The economics of foreign aid has mainly focused on two issues: (i) how foreign aid is targeted; and (ii) its effectiveness. Regarding the first issue, (Alesina and Dollar, 2000) found that political factors are just as significant as economic factors in determining which country receives foreign aid. Similarly, (Fleck and Kilby, 2010) examined US foreign aid determinants and found that during the Cold War, anti-communist regimes received substantially more funds. Regarding foreign aid's effectiveness, (Boone, 1996) found no evidence that foreign aid improves human development indicators. By contrast, (Bearce and Tirone, 2010) argue that when donors obtain smaller strategic benefits from foreign aid, its effectiveness increases. The reason is that it creates a more credible threat for recipients. Our paper contributes to this literature by extending the analysis of the political economy of foreign support in at least two ways: First, rather than explaining which countries are targeted to receive support, we focus on the donor country's strategic choice of instrument. Second, rather than focusing on how economic and social effectiveness of foreign support affect recipient countries, we stress its effect on the geopolitical alignment between donor and recipients.

2 A Simple Model of Collusion and Geopolitical Influence

Consider 2 groups of countries: consumers and producers of a particular commodity c (e.g., coffee). Consumer countries are integrated by a global power 1 (e.g., United States), its geopolitical ally 2 (e.g., Europe), and the rest of the world consumers 3. The utility function of a consumer in country $i \in I = \{1, 2, 3\}$ is $u_i(c_i, m_i) = 2\alpha_i\sqrt{c_i} + m_i$, where $c_i \geq 0$ is the consumption of commodity c, $m_i \geq 0$ is the consumption of other commodities, and $\alpha_i > 0$ is a parameter that captures the intensity of preferences for commodity c in country i. Let $p \geq 0$ denote the price of c, $y_i > 0$ the income of country i, and $T_i \geq 0$ a tax imposed by country i. Assume that $y_i > (\alpha_i)^2/p + T_i$. Then, country i's demand for c is $c_i = (\alpha_i/p)^2$ and, hence, the indirect function utility is:

$$v_i(p, T_i) = \frac{(\alpha_i)^2}{p} + y_i - T_i.$$

Producers of commodity c are integrated by a number of developing countries J (e.g., Colombia and Brazil). All producers have the same cost function with constant marginal cost given by $m_c > 0$. Under competition, the commodity is priced at marginal cost (i.e., in equilibrium, $p = m_c$) and, hence, each producer obtains zero profits. There exists, however, the possibility of obtaining positive profits through collusion, which requires the support of global power 1 (otherwise, collusion would not be sustainable). If a cartel is formed, industry profits will be given by:

$$\pi(p) = \left(\frac{p - m_c}{p^2}\right) \sum_{i \in I} (\alpha_i)^2 .$$

Moreover, assume that producer country $j \in J$ will obtain $\pi_j(p) = \beta_j \pi(p)$, where $\beta_j \in [0, 1]$ and $\sum_{j \in J} \beta_j = 1$. Note that $\pi(m_c) = 0$, the price that maximizes $\pi(p)$ is $p = 2m_c$, and $\pi(p)$ is increasing in p for all $p \in [m_c, 2m_c]$.

Global power 1 and its ally 2 (e.g., the United States and Europe) face the geopolitical challenge of another global power (e.g., the Soviet Union), which includes geopolitical rivalry on influencing some of the producer countries. We focus on modeling how 1 and 2 react to this challenge. Let $G \subset J$ denote the set of geopolitically relevant producers and $S \geq 0$ the strength of the geopolitical challenge in those countries. To deal with this challenge 1 and 2 count on two policies. First, they can employ conventional foreign aid, with $(T_1, T_2) \geq (0, 0)$ being the foreign aid provided by countries 1 and 2. Second, global power 1 can help producer countries to organize and enforce collusion, thereby inducing an equilibrium price above the marginal cost and profits given by:

$$\pi_G(p) = \beta_G \pi(p) \;\; with \; \beta_G = \sum_{j \in G} \beta_j$$

Then, the probability that producer countries in G align with 1 and 2 is given by:

$$\mu = \frac{\pi_G(p) + T_1 + T_2}{\pi_G(p) + T_1 + T_2 + S}.$$

That is, the greater the amount of foreign aid provided by 1 and 2 or the profits obtained by producer countries, the greater the probability that geopolitically relevant producer countries align with 1 and 2. The greater the strength of the geopolitical challenge, the lower the probability that geopolitically relevant producer countries align with 1 and 2. Alternatively, we can interpret μ as the probability that communism will be deterred.

The payoff functions of global power 1 and its ally 2 are given by[1]:

$$W_i = v_i(p, T_i) + \mu B_i = \frac{(\alpha_i)^2}{p} + y_i - T_i + \frac{\pi_G(p) + T_1 + T_2}{\pi_G(p) + T_1 + T_2 + S} B_i \; for \; i \in \{1, 2\}.$$

That is, country i takes into account, when deciding its foreign policy, the economic consumer surplus of i's consumers (formally, $v_i(p, T_i)$) as well as the expected geopolitical benefits from having producer countries aligned (formally, μB_i).

The timing of events is as follows.

1. Global power 1 selects a price $p \in [m_c, 2m_c]$.[2]

2. Global power 1 and its ally 2 simultaneously and independently select foreign aid (T_1, T_2), where $T_i \in [0, \bar{T}]$ with $\bar{T} < y_i - (\alpha_i)^2 / m_c$.[3]

We characterize the equilibrium as a subgame perfect Nash equilibrium.

2.1 Equilibrium Analysis

The following lemma characterizes equilibrium transfers for any price selected by global power 1.

[1] For a similar approach modelling the payoff function of a global power see Galiani et al. (2021).

[2] $p = 2m_c$ is the monopoly price. Since we are not considering the possibility of price discrimination, $p = 2m_c$ maximizes industry c's profits and, hence, it is never optimal for 1 to select $p > 2m_c$.

[3] $\bar{T} < y_i - (\alpha_i)^2 / m_c$ ensures that $c_i = (\alpha_i / p)^2$ for all $p \geq m_c$. That is, it is never the case that all income is spent on commodity c.

Lemma 1 *Assume that $B_1 > B_2$ and $0 < \sqrt{SB_1} - S < \bar{T}$. Suppose that 1 has selected $p \in [m_c, 2m_c]$. Then, the unique Nash equilibrium profile of transfers is given by:*

$$T_1(p) = \max\left\{\sqrt{SB_1} - S - \pi_G(p), 0\right\} \text{ and } T_2(p) = 0.$$

Moreover, if $\sqrt{SB_1} - S > \pi_G(2m_c)$, then $T_1(p) > 0$ for all $p \in [m_c, 2m_c]$; while if $\sqrt{SB_1} - S \leq \pi_G(2m_c)$, then $T_1(p) > 0$ for all $p \in [m_c, \bar{p})$ and $T_1(p) = 0$ for all $p \in [\bar{p}, 2m_c]$, where

$$\bar{p} = \frac{\beta_G \sum_{i \in I} (\alpha_i)^2}{2\left(\sqrt{SB_1} - S\right)} \left[1 - \sqrt{1 - \frac{4m_c\left(\sqrt{SB_1} - S\right)}{\beta_G \sum_{i \in I} (\alpha_i)^2}}\right]$$

Proof: *see Online Appendix A.1.* ■

The intuition behind Lemma 1 is simple. Regardless of the price selected by global power 1, player 2 never contributes (formally, $T_2(p) = 0$ for all $p \in [m_c, 2m_c]$). The reason is that player 2 obtains less geopolitical benefits from keeping producer countries aligned than player 1. Therefore, it has incentives to free ride player 1. Regarding the effect of p on T_1, note that the higher the price selected by player 1, the greater the profits obtained by producers and, hence, the less transfers player 1 needs to do to induce its optimal level of deterrence μ (formally, $T_1(p)$ is decreasing in p). When the price is low, the profits obtained by producers are low (formally, $\sqrt{SB_1} - S > \pi_G(p)$), and global power 1 has no choice but to select positive transfers $T_1(p) = \sqrt{SB_1} - S - \pi_G(p) > 0$ to reach its optimal deterrence level $\mu = \left(\sqrt{SB_1} - S\right)/\sqrt{SB_1}$. By contrast, if the price is high, profits for producers are also high ($\sqrt{SB_1} - S \leq \pi_G(p)$). Country 1 does not need to use transfers, as the level of deterrence achieved with profits ($\mu = \pi_G(p)/(\pi_G(p) + S)$) is already greater than player 1's optimal level of deterrence ($\mu = \left(\sqrt{SB_1} - S\right)/\sqrt{SB_1}$). In other words, p and T_1 are substitute instruments for achieving the same goal (increase μ).

Proposition 1 fully characterizes the equilibrium. To do so, it is useful to define s_1, the share of commodity c demanded by global power 1, which is given by $s_1 = c_1 p / \sum_{i \in I} c_i p = (\alpha_1)^2 / \sum_{i \in I} (\alpha_i)^2$.

Proposition 1 *Assume that $B_1 > B_2$ and $0 < \sqrt{SB_1} - S < \bar{T}$.*

1. *Suppose that $\sqrt{SB_1} - S > \pi_G(2m_c)$.*
   - (a) *If $s_1 \geq \beta_G$, then the unique subgame perfect Nash equilibrium outcome is $(p, T_1, T_2) = \left(m_c, \sqrt{SB_1} - S, 0\right)$.*
   - (b) *If $s_1 < \beta_G$, then the unique subgame perfect Nash equilibrium outcome is $(p, T_1, T_2) = \left(\hat{p}^1, \sqrt{SB_1} - S - \pi_G\left(\hat{p}^1\right), 0\right)$, where $\hat{p}^1 = \frac{2m_c\beta_G}{s_1 + \beta_G} \in (m, \bar{p})$.*
2. *Suppose that $\sqrt{SB_1} - S \leq \pi_G(2m_c)$.*
   - (a) *If $s_1 \geq \beta_G$, then the unique subgame perfect Nash equilibrium is outcome $(p, T_1, T_2) = \left(m_c, \sqrt{SB_1} - S, 0\right)$.*
   - (b) *If $\beta_G\left(\frac{2m_c - \bar{p}}{\bar{p}}\right) < s_1 < \beta_G$, then the unique subgame perfect Nash equilibrium outcome is $(p, T_1, T_2) = \left(\hat{p}^1, \sqrt{SB_1} - S - \pi_G\left(\hat{p}^1\right), 0\right)$.*

(c) If $s_1 \leq \beta_G \left(\frac{2m_c - \bar{p}}{\bar{p}}\right)$*, then the unique subgame perfect Nash equilibrium outcome is* $(p, T_1, T_2) = \left(\hat{p}^2, 0, 0\right)$*, where* $\hat{p}^2 \in [\bar{p}, 2m_c)$ *is the unique solution to* $\left(\frac{2m_c - p}{p}\right)\beta_G = \frac{s_1[\pi_G(p)+S]^2}{SB_1}$.

Proof*: see Online Appendix A.1.* ■

The intuition behind Proposition 1 is as follows. When $\sqrt{SB_1} - S > \pi_G(2m_c)$, even monopoly profits are not enough to achieve the desired level of deterrence. Then, global power 1 must rely at least partially on foreign aid. If its market share is relatively great (formally, $s_1 \geq \beta_G$), then most of the burden of allowing collusion is paid by consumers in country 1. Thus, it is better to use foreign aid, which is a more efficient policy instrument. By contrast, if the market share of country 1 is relatively low (formally, $s_1 < \beta_G$), collusion is less burdersome for consumers in country 1. In this case, country 1 uses both instruments. The profits induced by helping producers sustain collusion allow country 1 to reduce foreign aid, keeping aggregate transfers to the producer countries and keeping deterrence constant. The advantage for country 1 is that the required increase in the producers' profits is partially funded by foreign consumers.

When $\sqrt{SB_1} - S \leq \pi_G(2m_c)$, country 1 can only achieve the desired level of deterrence through collusion. This does not immediately imply that this is the best course of action for country 1. Indeed, country 1 only employs collusion when market share is relatively low (formally, $s_1 < \beta_G$). And it only relies on collusion to influence producers when its market share is *very* low (formally, $s_1 \leq \beta_G(2m_c - \bar{p})/\bar{p}$).

So, why did the U.S. support a coffee cartel? Proposition 1 provides a preliminary answer to this puzzle. The geopolitical goal of the U.S. was to fight communism in some geopolitically important coffee producer countries and keep them politically aligned with the U.S. To do so, the U.S. had to somehow bribe these countries. While foreign aid (in theory a more efficient instrument) was fully paid by U.S. tax payers, the burden of collusion was partially share with consumers from other countries.

Proposition 1 raises several concerns. First, Proposition 1 assumes that foreign aid is determined in a non-cooperative game and, thus, suffers from free riding. What if foreign aid is cooperatively determined between the U.S. and its allies? Does the U.S. still have incentives to use collusion? Second, in Proposition 1, the U.S. employs collusion only when its market share is below some threshold (formally, $s_1 < \beta_G$). This would present problems due to the fact that a key reason the U.S. was in the position of helping coffee producers sustain collusion is that the U.S. represented an important share of the global demand of coffee. In the next section, we study several extensions of the model that deal with these and other concerns. Overall, the incentives to use collusion (in combination with foreign aid) to deal with geopolitical deterrence persist.

3 Extensions

There are several ways to enrich the analysis. In this section we explore the following extensions of the model:

1. We consider the case where countries 1 and 2 select transfers cooperatively rather than non-cooperatively.

2. We explore two possible voter biases in country 1. First, we assume that the policy maker in country 1 assigns a lower weigh to the utility that consumers obtain from consuming the commodity. This could capture a situation in which voters in country 1 do not understand that the increased price for

the commodity results from country 1's policies. Second, we assume that voters restrict the maximum amount of foreign aid that the policy maker can choose. This could capture a situation in which voters have a bias against foreign aid, e.g., because they systematically overestimate the proportion of the budget used to financed foreign aid.

3. We introduce wholesale companies that act as intermediaries between producers and final consumers. We show how politically connected wholesale companies in country 1 can take advantage of commodity trade agreements to gain a cost edge over rivals.

4. We explore what happen when some of the profits go to the Soviet Union or its allies. This is relevant for some important commodities, such as oil and sugar.

5. We introduce internal factors in geopolitically relevant producer countries that change the effectiveness of sustaining collusion relative to foreign aid.

3.1 Cooperatively Determined Foreign Aid

Suppose that global power 1 and its ally 2 cooperate to determine foreign aid. In particular, suppose that for each price $p \in [m, 2m_c]$ chosen by player 1, T_1 and T_2 are determined according to the Nash bargaining solution, taking the equilibrium payoff of each player as its outside option. Thus, negotiated transfers are given by:

$$\left(T_1^C(p), T_2^C(p)\right) = \arg\max_{T_1, T_2} \left\{ W^C = \left[W_1^C(T_1, T_2) - W_1\right]^{\gamma_1} \left[W_2^C(T_1, T_2) - W_2\right]^{1-\gamma_1} \right\},$$

where $\gamma_1 \in (0,1)$ is the bargaining power of player 1 and W_i is the equilibrium payoff of player $i \in \{1,2\}$ from lemma 1.

The following lemma characterizes negotiated transfers for any price selected by global power 1.

Lemma 2 *Assume that* $B_1 > B_2$ *and* $0 < \sqrt{SB_1} - S < \bar{T}$, $\pi_G(2m_c) < \sqrt{S(B_1+B_2)} - S < \bar{T}$ *and* $\gamma_1^L < \gamma_1 < \gamma_1^H$.[4] *Suppose that* 1 *has selected* $p \in [m_c, 2m_c]$. *Then, negotiated transfers are given by:*

$$T_1^C(p) = \theta(p, \gamma_1)\left[\sqrt{S(B_1+B_2)} - S - \pi_G(p)\right]$$
$$T_2^C(p) = [1 - \theta(p, \gamma_1)]\left[\sqrt{S(B_1+B_2)} - S - \pi_G(p)\right]$$

where $\theta(p, \gamma_1)$ *is the share paid by player* 1. ***Proof:*** *see Online Appendix A.2.* ■

Comparing Lemmas 1 and 2, we observe two important differences. First, when transfers are determined cooperatively, total transfers are related to the aggregate geopolitical benefits rather than only to the geopolitical benefits accruing to player 1 (as is the case when transfers are determined non-cooperatively). Second, when transfers are determined cooperatively, both players make positive contributions. The intuition behind these differences is that negotiated transfers solve the free rider problem. It is also worth noting that players are better off when transfers are determined cooperatively. Technically speaking, the reason is that in the bargaining problem the outside options are given by equilibrium payoffs induced by lemma 1 and, hence, players can never perform worse than under equilibrium.

[4] For details on these thresholds, refer to the proof of Lemma 2 in Online Appendix A.

More substantially, the idea is that players 1 and 2 will not be willing to enter into negotiations about foreign aid if they expect to obtain a lower payoff than when foreign aid is determined non-cooperatively.

Proposition 2 fully characterizes the equilibrium when transfers are determined cooperatively.

Proposition 2 *Assume that* $B_1 > B_2$, $0 < \sqrt{SB_1} - S < \bar{T}$, $\pi_G(2m_c) < \sqrt{S(B_1+B_2)} - S < \bar{T}$ *and* $\gamma_1^L < \gamma_1 < \gamma_1^H$.

1. *Suppose that* $\sqrt{SB_1} - S > \pi_G(2m_c)$.

   (a) *If* $s_1 \geq \beta_G$, *then* $(p, T_1+T_2) = \left(m_c, \sqrt{S(B_1+B_2)} - S\right)$.

   (b) *If* $s_1 < \beta_G$, *then* $(p, T_1+T_2) = \left(\hat{p}^1, \sqrt{S(B_1+B_2)} - S - \pi_G(\hat{p}^1)\right)$.

2. *Suppose that* $\sqrt{SB_1} - S \leq \pi_G(2m_c)$.

   (a) *If* $s_1 \geq \beta_G$, *then* $(p, T_1+T_2) = \left(m_c, \sqrt{S(B_1+B_2)} - S\right)$.

   (b) *If* $\beta_G\left(\frac{2m_c-\bar{p}}{\bar{p}}\right) < s_1 < \beta_G$, *then* $(p, T_1+T_2) = \left(\hat{p}^1, \sqrt{S(B_1+B_2)} - S - \pi_G(\hat{p}^1)\right)$.

   (c) *If* $\beta_G\left(\frac{B_1-\gamma_1 B_2}{B_1}\right)\left(\frac{2m_c-\bar{p}}{\bar{p}}\right) \leq s_1 \leq \beta_G\left(\frac{2m_c-\bar{p}}{\bar{p}}\right)$, *then* $(p, T_1+T_2) = \left(\bar{p}, \sqrt{S(B_1+B_2)} - S - \pi_G(\bar{p})\right)$ *or* $(p, T_1+T_2) = \left(\hat{p}^3, \sqrt{S(B_1+B_2)} - S - \pi_G(\hat{p}^3)\right)$, *where* $\hat{p}^3 \in [\bar{p}, 2m_c)$ *is the unique solution to* $\left(\frac{2m_c-p}{p}\right)\beta_G = \frac{s_1[\pi(p)+S]^2}{\gamma_1[\pi(p)+S]^2 + S[(1-\gamma_1)B_1 - \gamma_1 B_2]}$.

   (d) *If* $s_1 < \beta_G\left(\frac{B_1-\gamma_1 B_2}{B_1}\right)\left(\frac{2m_c-\bar{p}}{\bar{p}}\right)$, *then* $(p, T_1+T_2) = \left(\hat{p}^3, \sqrt{S(B_1+B_2)} - S - \pi_G(\hat{p}^3)\right)$.

   Proof: *see Online Appendix A.2.* ■

Table 1.1 compares the results in Propositions 1.1 and 2.1 (i.e., when $\sqrt{SB_1} - S > \pi_G(2m_c)$). We can observe that the price selected by player 1 is not affected by how transfers are determined. Regardless of whether transfers are determined non-cooperatively or cooperatively, player 1 does not use collusion when its market share is high (formally, $s_1 \geq \beta_G$) and it chooses $p = \hat{p}^1 > m_c$ when its market share is low (formally, $s_1 < \beta_G$). Thus, the only difference between Propositions 1.1 and 2.1 is that when transfers are determined cooperatively, the free rider problem is solved and, hence, total transfers are higher. This is interesting, as it implies that incentives for collusion do not necessarily vanish after implementing cooperative decisions on foreign aid.

Table 1.2 compares the results in Proposition 2.1 and 2.2 (i.e., when $\sqrt{SB_1} - S \leq \pi_G(2m_c)$). In this case, how transfers are determined may affect the price selected by player 1. In particular, we observe that when $s_1 \leq \beta_G\left(\frac{2m_c-\bar{p}}{\bar{p}}\right)$, player 1 is less prone to use collusion when transfers are determined cooperatively. Formally, $\bar{p}, \hat{p}^3 < \hat{p}^2$ (see the proof of Proposition 2 for details), i.e., the equilibrium price when transfers are determined cooperatively is always lower than the equilibrium price when transfers are determined non-cooperatively. Moreover, for $s_1 \leq \beta_G\left(\frac{2m_c-\bar{p}}{\bar{p}}\right)$, foreign aid is not employed at all when it is determined non-cooperatively (Proposition 1.2.c), while foreign aid is always part of the policy mix when transfers are determine cooperatively (Proposition 2.2.c, and 2.2.d). The intuition behind this result is clear. Player 1 is less willing to use collusion when transfers are determined cooperatively because it only pays a share of total transfer. By contrast, when transfers are determined non-cooperatively, all foreign aid is paid by player 1.

Condition	Proposition 1.1 $(p, T_1 + T_2)$	Proposition 2.1 $(p, T_1^C + T_2^C)$	Comparison
$s_1 \geq \beta_G$	$(m_c, \sqrt{SB_1} - S)$	$\left(m_c, \sqrt{S(B_1 + B_2)} - S\right)$	$\sqrt{S(B_1 + B_2)} - S > \sqrt{SB_1} - S$
$s_1 < \beta_G$	$(\hat{p}^1, T(\hat{p}^1))$	$(\hat{p}^1, T^C(\hat{p}^1))$	$T^C(\hat{p}^1) > T(\hat{p}^1)$

Table 1.1: Proposition 1.1 versus Proposition 2.1.

Note: $T(p) = \sqrt{SB_1} - S - \pi_G(p)$ and $T^C(p) = \sqrt{S(B_1 + B_2)} - S - \pi_G(p)$

Condition	Proposition 1.1 $(p, T_1 + T_2)$	Proposition 2.1 $(p, T_1^C + T_2^C)$	Comparison
$s_1 \geq \beta_G$	$\begin{pmatrix} m_c, \\ \sqrt{SB_1} - S \end{pmatrix}$	$\begin{pmatrix} m_c, \\ \sqrt{S(B_1 + B_2)} - S \end{pmatrix}$	$B_1 + B_2 > B_1$
$\beta_G \left(\frac{2m_c - \bar{p}}{\bar{p}}\right) < s_1 < \beta_G$	$(\hat{p}^1, T(\hat{p}^1))$	$(\hat{p}^1, T^C(\hat{p}^1))$	$T^C(\hat{p}^1) > T(\hat{p}^1) > 0$
$\beta_G B \left(\frac{2m_c - \bar{p}}{\bar{p}}\right) \leq s_1 \leq \beta_G \left(\frac{2m_c - \bar{p}}{\bar{p}}\right)$	$(\hat{p}^2, 0)$	$(\bar{p}, T^C(\bar{p}))$ *or* $(\hat{p}^3, T^C(\hat{p}^3))$	$\hat{p}^2 > \bar{p}$, $T^C(\bar{p}) > 0$ $\hat{p}^2 > \hat{p}^3$, $T^C(\hat{p}^3) > 0$
$s_1 < \beta_G B \left(\frac{2m_c - \bar{p}}{\bar{p}}\right)$	$(\hat{p}^2, 0)$	$(\hat{p}^3, T^C(\hat{p}^3))$	$\hat{p}^2 > \hat{p}^3$ $T^C(\hat{p}^3) > 0$

Table 1.2: Proposition 2.1 versus Proposition 2.1.

Note: $T(p) = \sqrt{SB_1} - S - \pi_G(p)$, $T^C(p) = \sqrt{S(B_1 + B_2)} - S - \pi_G(p)$, $B = \left(\frac{B_1 - \gamma_1 B_2}{B_1}\right)$

Proposition 2 helps explain why the U.S. decided to fight communism by helping producer countries organize collusion rather than by using foreign aid. Part of the problem was that U.S. allies were able to free ride U.S. foreign aid, but it was more complicated for them to escape the burden of collusive prices. In other words, supporting collusion seemed to offer a partial solution to the free rider problem, by forcing allies to contribute to the common geopolitical goal. Note however, that even when transfers are determined cooperatively, the U.S.'s incentives to use collusion do not disappear completely. One reason is that part of the burden of collusion is paid by third party countries, i.e., neither U.S. nor U.S. allies (formally, $\alpha_3 > 0$).

3.2 Internal Politics in the U.S. I: Voters' Biases

Suppose that the policy maker in country 1 only takes into account a fraction of the utility that consumers obtain from consuming commodity c. Formally, assume that the payoff function of country 1 is given by:

$$W_1^B = \frac{(1-b)(\alpha_1)^2}{p} + y_1 - T_1 + \frac{\pi_G(p) + T_1 + T_2}{\pi_G(p) + T_1 + T_2 + S} B_1,$$

where $b \in [0, 1]$ is a measure of the political bias against utility from consuming commodity c. One possible reason for this bias is that voters in country 1 do not fully understand that the price of commodity c is affected by their own government, but they fully understand that foreign aid is financed with tax revenues. Politicians simply internalize this information bias in their policy choices.

It is easy to verify that introducing this political bias only produces a minor change in Proposition 1. Indeed, all we need to do is to replace s_1 by $(1-b)\, s_1$ and Proposition 1 holds. As a consequence, the greater the political bias, the more likely that, in equilibrium, $p > m_c$. More formally, the greater the value of b, the more likely that $(1-b)\, s_1 < \beta_G$ holds. Moreover, when collusion is used to influence producers, it is employed more intensively as the political bias increases. More formally, the greater the value of b, the greater $\hat{p}^1$ and $\hat{p}^2$, where $\hat{p}^1 = \frac{2m_c\beta_G}{(1-b)s_1+\beta_G}$ and $\hat{p}^2$ is the unique solution to $\left(\frac{2m_c-p}{p}\right)\beta_G = \frac{(1-b)s_1[\pi_G(p)+S]^2}{SB_1}$. The intuition behind these results is simple. If voters fail to hold the policy maker fully responsible for a rise in the price of commodity c, the policy maker is more prone to choose a higher price.

Although relatively straightforward, this extension is important, as it implies that even if the market share of global power 1 (i.e., s_1) was significant, global power 1 might still select high prices for commodity c. Thus, the extension formalizes another channel that explains why the U.S. supported the formation of cartels for some commodities. The idea is that the U.S. was in a position to support those cartels because it was an important consumer (formally, s_1 was relatively high) and in spite of that, the geopolitical motivation dominated its decision. The political bias considered here, if present, reinforce the incentives of the US to support the formation of cartels for some imported commodities. According to this perspective, collusion was an attractive instrument, as it was easier to hide from voters than conventional foreign aid. In other words, the politically discounted market share of the U.S. (i.e., $(1-b)\, s_1$) was much smaller than its actual market share (i.e., s_1).

Another way to introduce a political bias in the determination of the instrument choice is to assume that voters restrict the maximum amount of foreign aid that the policy maker can choose. This could capture a situation in which voters are biased against foreign aid, for example, because they systematically overestimate the proportion of the budget used to financed foreign aid (see, for example, Caplan (2011)). Lemma 3 and Proposition 3 characterize the equilibrium when $\sqrt{SB_2} - S < \bar{T} < \sqrt{SB_1} - S$.

Lemma 3 *Assume that $\sqrt{SB_2} - S < \bar{T} < \sqrt{SB_1} - S$. Suppose that 1 has selected $p \in [m_c, 2m_c]$. Then, the unique Nash equilibrium profile of transfers is given by:*

$$T_1^R(p) = \min\left\{\max\left\{\sqrt{SB_1} - S - \pi_G(p), 0\right\}, \bar{T}\right\} \text{ and } T_2^R(p) = 0$$

***Proof**: see Online Appendix A.3.* ■

The intuition behind Lemma 3 is the same as in Lemma 1, with the exception that now for low values of p, $\bar{T}$ is binding.

Proposition 3 *Assume that $\sqrt{SB_2} - S < \bar{T} < \sqrt{SB_1} - S$.*

1. *Suppose that $\sqrt{SB_1} - S > \pi_G(2m_c)$ and $0 < \bar{T} \leq \sqrt{SB_1} - S - \pi_G(2m_c)$.*

   (a) *If $s_1 \geq \frac{\beta_G SB_1}{(\bar{T}+S)^2}$, then the unique subgame perfect Nash equilibrium outcome is $\left(p, T_1^R, T_2^R\right) = \left(m_c, \bar{T}, 0\right)$.*

(b) *If* $s_1 < \frac{\beta_G SB_1}{(\bar{T}+S)^2}$*, then the unique subgame perfect Nash equilibrium outcome is* $\left(p, T_1^R, T_2^R\right) = \left(\hat{p}^4, \bar{T}, 0\right)$*, where* $\hat{p}^4$ *is the unique solution to* $\left(\frac{2m_c - p}{p}\right)\beta_G = \frac{s_1\left[\pi_G(p)+\bar{T}+S\right]^2}{SB_1}$.

2. *Suppose that* $\sqrt{SB_1} - S > \pi_G(2m_c)$ *and* $\bar{T} > \sqrt{SB_1} - S - \pi_G(2m_c)$ *and let*

$$\bar{p}_{\bar{T}} = \frac{\beta_G \sum_{i\in I}(\alpha_i)^2}{2\left(\sqrt{SB_1} - S - \bar{T}\right)}\left[1 - \sqrt{1 - \frac{4m_c\left(\sqrt{SB_1} - S - \bar{T}\right)}{\beta_G \sum_{i\in I}(\alpha_i)^2}}\right]$$

(a) *If* $s_1 \geq \frac{\beta_G SB_1}{(\bar{T}+S)^2}$*, then the unique subgame perfect Nash equilibrium outcome is* $\left(p, T_1^R, T_2^R\right) = \left(m_c, \bar{T}, 0\right)$.

(b) *If* $\frac{(2m_c - \bar{p}_{\bar{T}})\beta_G}{\bar{p}_{\bar{T}}} \leq s_1 < \frac{\beta_G SB_1}{(\bar{T}+S)^2}$*, then the unique subgame perfect Nash equilibrium outcome is* $\left(p, T_1^R, T_2^R\right) = \left(\hat{p}^4, \bar{T}, 0\right)$.

(c) *If* $s_1 < \frac{(2m_c - \bar{p}_{\bar{T}})\beta_G}{\bar{p}_{\bar{T}}}$*, then the unique subgame perfect Nash equilibrium outcome is* $\left(p, T_1^R, T_2^R\right) = \left(\hat{p}^1, \sqrt{SB_1} - S - \pi_G\left(\hat{p}^1\right), 0\right)$.

3. *Suppose that*$\sqrt{SB_1} - S \leq \pi_G(2m_c)$.

(a) *If* $s_1 \geq \frac{\beta_G SB_1}{(\bar{T}+S)^2}$*, then the unique subgame perfect Nash equilibrium outcome is* $\left(p, T_1^R, T_2^R\right) = \left(m_c, \bar{T}, 0\right)$.

(b) *If* $\frac{(2m_c - \bar{p}_{\bar{T}})}{\bar{p}_{\bar{T}}}\beta_G \leq s_1 < \frac{\beta_G SB_1}{(\bar{T}+S)^2}$*, then the unique subgame perfect Nash equilibrium outcome is* $\left(p, T_1^R, T_2^R\right) = \left(\hat{p}^4, \bar{T}, 0\right)$.

(c) *If* $\left(\frac{2m_c - \bar{p}}{\bar{p}}\right)\beta_G < s_1 < \frac{(2m_c - \bar{p}_{\bar{T}})}{\bar{p}_{\bar{T}}}\beta_G$*, then the unique subgame perfect Nash equilibrium outcome is* $\left(p, T_1^R, T_2^R\right) = \left(\hat{p}^1, \sqrt{SB_1} - S - \pi_G\left(\hat{p}^1\right), 0\right)$

(d) *If* $s_1 \leq \left(\frac{2m_c - \bar{p}}{\bar{p}}\right)\beta_G$*, then the unique subgame perfect Nash equilibrium outcome is* $\left(p, T_1^R, T_2^R\right) = \left(\hat{p}^2, 0, 0\right)$.

Proof: *see Online Appendix A.3.* ■

Tables 2.1 and 2.2 compare the results in Proposition 1.1 with those in Propositions 3.1 and 3.2. We observe that it is more likely for player 1 to select $p > m_c$ when voters restrict the maximum amount of foreign aid. Formally, if $s_1 < \beta_G$ holds, then $s_1 < \frac{\beta_G SB_1}{(\bar{T}+S)^2}$ also holds. Moreover, when collusion is used to influence producers, it tends to be employed more intensively. Formally, $\hat{p}^4$ is higher than $\hat{p}^1$ (see the proof of Proposition 3 for details). Summing up, Proposition 3 formalizes the effects of a political constraint on the use of foreign aid that forces the policy maker to rely more intensively on collusion.

Condition	Proposition 1.1 (p, T_1)	Proposition 3.1 (p, T_1)	Comparison
$s_1 \geq \frac{\beta_G SB_1}{(\bar{T}+S)^2}$	$\left(m_c, \sqrt{SB_1} - S\right)$	$\left(m_c, \bar{T}\right)$	$\bar{T} < \sqrt{SB_1} - S$
$\beta_G \leq s_1 < \frac{\beta_G SB_1}{(\bar{T}+S)^2}$	$\left(m_c, \sqrt{SB_1} - S\right)$	$\left(\hat{p}^4, \bar{T}\right)$	$\hat{p}^4 > m_c$ *and* $\bar{T} < \sqrt{SB_1} - S$
$s_1 < \beta_G$	$\left(\hat{p}^1, T\left(\hat{p}^1\right)\right)$	$\left(\hat{p}^4, \bar{T}\right)$	$\hat{p}^4 > \hat{p}^1$ *and* $\bar{T} < T\left(\hat{p}^1\right)$

Table 2.1: Proposition 1.1 versus Proposition 3.1.

Note: $T(p) = \sqrt{SB_1} - S - \pi_G(p)$.

Condition	Proposition 1.1 (p, T_1)	Proposition 3.2 (p, T_1)	Comparison
$s_1 \geq \frac{\beta_G SB_1}{(\bar{T}+S)^2}$	$\left(m_c, \sqrt{SB_1} - S\right)$	$\left(m_c, \bar{T}\right)$	$\bar{T} < \sqrt{SB_1} - S$
$\beta_G \leq s_1 < \frac{\beta_G SB_1}{(\bar{T}+S)^2}$	$\left(m_c, \sqrt{SB_1} - S\right)$	$\left(\hat{p}^4, \bar{T}\right)$	$\hat{p}^4 > m_c$ *and* $\bar{T} < \sqrt{SB_1} - S$
$\frac{(2m_c - \bar{p}_{\bar{T}})\beta_G}{\bar{p}_{\bar{T}}} \leq s_1 < \beta_G$	$\left(\hat{p}^1, T\left(\hat{p}^1\right)\right)$	$\left(\hat{p}^4, \bar{T}\right)$	$\hat{p}^4 \geq \hat{p}^1$ *and* $\bar{T} \leq T\left(\hat{p}^1\right)$
$s_1 < \frac{(2m_c - \bar{p}_{\bar{T}})\beta_G}{\bar{p}_{\bar{T}}}$	$\left(\hat{p}^1, T\left(\hat{p}^1\right)\right)$	$\left(\hat{p}^1, T\left(\hat{p}^1\right)\right)$	*Same*

Table 2.2: Proposition 1.1 versus Proposition 3.2.

Note: $T(p) = \sqrt{SB_1} - S - \pi_G(p)$.

Table 2.3 compares the results in Proposition 1.2 with those in Propositions 3.3. Once again, it is more likely that player 1 selects $p > m_c$ when voters restrict the maximum amount of foreign aid and, conditional on selecting $p > m_c$, higher prices are used. Formally, $\hat{p}^4 \geq \hat{p}^1$ and $\hat{p}^2 \geq \hat{p}^1$ (see the proof of Proposition 3 for details).

Condition	Proposition 1.1 (p, T_1)	Proposition 3.3 (p, T_1)	Comparison
$s_1 \geq \frac{\beta_G S B_1}{(\bar{T}+S)^2}$	$(m_c, \sqrt{SB_1} - S)$	$(m_c, \bar{T})$	$\bar{T} < \sqrt{SB_1} - S$
$\beta_G \leq s_1 < \frac{\beta_G S B_1}{(\bar{T}+S)^2}$	$(m_c, \sqrt{SB_1} - S)$	$(\hat{p}^4, \bar{T})$	$\hat{p}^4 > m_c$ *and* $\bar{T} < \sqrt{SB_1} - S$
$\frac{(2m_c - \bar{p}_{\bar{T}})\beta_G}{\bar{p}_{\bar{T}}} \leq s_1 < \beta_G$	$(\hat{p}^1, T(\hat{p}^1))$	$(\hat{p}^4, \bar{T})$	$\hat{p}^4 \geq \hat{p}^1$ *and* $\bar{T} \leq T(\hat{p}^1)$
$\frac{(2m_c - \bar{p})\beta_G}{\bar{p}} < s_1 < \frac{(2m_c - \bar{p}_{\bar{T}})\beta_G}{\bar{p}_{\bar{T}}}$	$(\hat{p}^1, T(\hat{p}^1))$	$(\hat{p}^1, T(\hat{p}^1))$	*Same*
$s_1 \leq \frac{(2m_c - \bar{p})\beta_G}{\bar{p}}$	$(\hat{p}^1, T(\hat{p}^1))$	$(p, T_1^R, T_2^R) = (\hat{p}^2, 0)$	$\hat{p}^2 \geq \hat{p}^1$ *and* $T(\hat{p}^1) > 0$

Table 2.1: Proposition 1.1 versus Proposition 3.3.
Note: $T(p) = \sqrt{SB_1} - S - \pi_G(p)$.

3.3 Internal Politics in the U.S. II: Connected Roasters

Suppose that in country 1 there are 2 wholesale companies that import commodity c and resell it to final consumers. For example, in the case of coffee, these companies are called roasters. Let p_1^d denote the retail price paid by consumers, which implies that the final demand of commodity c in country 1 is $c_1^d = (\alpha_1/p_1^d)^2$. Wholesale companies compete a la Cournot, i.e., they simultaneously and independently select the quantity they import and resell, which we denote by $q_{r,1}$. The cost function of wholesale company $r \in \{1, 2\}$ is given by:

$$C_{r,1}(q_{r,1}) = (p_{r,1}^s + m_d)\, q_{r,1}$$

where $m_d > 0$ is the marginal cost of distribution and $p_{r,1}^s \geq m_c$ is the price that company r pays per unit of c it imports. Each wholesale company can be politically connected or not. A politically connected company always pays the marginal cost of c for each unit it imports (formally, if r is connected, then $p_{r,1}^s = m_c$). A non-connected company may pay a higher price (formally, if r is non-connected, then $p_{r,1}^s = p^s \geq m_c$). The idea is that, unlike consumers and non-connected companies, politically connected domestic companies lobby to be excluded from paying a higher import price for commodity c induced by the government's decision to support foreign producers of c. One possibility is that politically connected companies only support the commodity agreement if they find a way to be excluded from (or even profit from) its effects. For example, in the case of coffee agreements, powerful US rosters supported the agreements in the Congress but simultaneously signed long term contracts with coffee producers.

The payoff function of global power 1 is given by:

$$W_1^{CR} = \frac{(\alpha_1)^2}{p_1^d} + y_1 - T_1 + \pi_1^W + \frac{\pi_G^P + T_1}{\pi_G^P + T_1 + S} B_1.$$

where $v_1 = (\alpha_1)^2 / p_1^d + y_1 - T_1$ is the consumer surplus of country 1's consumers, π_1^W are the aggregate profits of wholesale companies in country 1, π_G^P are the aggregate profits of commodity c's producers, and $B_1 > 0$ are the geopolitical benefits enjoyed by country 1. Note that we do not consider the transfers of country 2 (the geopolitical ally of global power 1). There are two reasons. First, if transfers are selected non-cooperatively, country 2 always prefers to be a free rider (i.e., in equilibrium, $T_2 = 0$). Second, there is no interesting conceptual gain in treating cooperative transfers and internal lobby simultaneously. The key point of this extension is to explore how internal political forces can change the incentives to use collusion versus transfers to deal with geopolitical challenges.

The timing of events is as follows.

1. Global power 1 selects a price $p^s \in [m_c, 2m_c + m_d]$ and transfers $T_1 \in \left[0, \bar{T}\right]$ with $\bar{T} < y_1 - (\alpha_1)^2 / m_c$.

2. Wholesale companies simultaneously and independently select $q_{r,1}$ for $r \in N_1$.

As in previous sections, we characterize the equilibrium as a subgame perfect Nash equilibrium. We consider three possible scenarios. In all scenarios, wholesale companies in all countries except country 1 are non-connected, thus must pay p^s for each unit of commodity c they import. In other words, only wholesale companies from global power 1 can avoid paying p^s. In scenario 1, we assume that none of the wholesale companies in country 1 are connected. Thus, scenario 1 is just our baseline model with the addition of an intermediary duopolistic domestic sector that imports commodity c and distribute it among final consumers. In scenario 2, we assume that both wholesale companies in country 1 are connected. This is an extreme and unlikely situation, as it assumes that country 1 is in a position to fully isolate its firms and consumers from the effects of higher prices of commodity c and, at the same time, country 1 is the key player to organize the collusive agreement required to induce such rise in prices. Nevertheless, this is an interesting scenario to study, as it generates sharp incentives for country 1 to use collusion as an instrument for advancing its geopolitical goals. Finally, in scenario 3 we assume that one wholesale company in country 1 is politically connected while the other is not. This is the most realistic and interesting case. For example, in the case of coffee, some rosters were powerful and well-connected while other were not.

Scenario 1: Suppose that both wholesale companies in country 1 are non-connected. Given country 1's policy choices, it is easy to compute the equilibrium price of commodity c in each country as well as the equilibrium quantity of c imported by each wholesale company (see Online Appendix A for details). Thus, we can compute the consumer surplus in country 1 ($v_1(p^s)$), the aggregate profits of the wholesale industry in country 1 ($\pi_1^W(p^s)$) and the profits of geopolitically relevant producers ($\pi_G^P(p^s)$).

$$v_1(p^s) = \frac{3(\alpha_1)^2}{4(p^s + m_d)} + y_1 - T_1,\ \pi_1^W(p^s) = \frac{3(\alpha_1)^2}{16(p^s + m_d)},\ \pi_G^P(p^s) = \frac{\beta_G 9(p^s - m_c)\sum_{i\in I}(\alpha_i)^2}{16(p^s + m_d)^2}$$

Proposition 4 *(Scenario* 1*) Assume that* $B_1 > 0$ *and* $0 < \sqrt{SB_1} - S < \bar{T}$*.*

1. *Suppose that* $\sqrt{SB_1} - S > \pi_G^P(2m_c + m_d)$*.*

   (a) *If* $s_1 \geq \frac{\beta_G 3(m_d + 2m_c)}{5m_d}$*, then* $(p, T_1) = \left(m_c, \sqrt{SB_1} - S\right)$*.*

   (b) *If* $s_1 < \frac{\beta_G 3(m_d + 2m_c)}{5m_d}$*, then* $(p, T_1) = \left(\hat{p}^{s,1}, \sqrt{SB_1} - S - \pi_G^P\left(\hat{p}^{s,1}\right)\right)$*, where* $\hat{p}^{s,1} = \frac{\beta_G 3(m_d + 2m_c) - 5s_1 m_d}{5s_1 + 3\beta_G} \in (m, \bar{p}^s)$*.*

2. *Suppose that* $\sqrt{SB_1} - S \leq \pi_G^P(2m_c + m_d)$*.*

(a) *If* $s_1 \geq \frac{\beta_G 3(m_d+2m_c)}{5m_d}$, *then* $(p, T_1) = \left(m_c, \sqrt{SB_1} - S\right)$.

(b) *If* $\frac{\beta_G 3(m_d+2m_c-\bar{p}^s)}{5(\bar{p}+m_d)} < s_1 < \frac{\beta_G 3(m_d+2m_c)}{5m_d}$, *then* $(p, T_1) = \left(\hat{p}^{s,1}, \sqrt{SB_1} - S - \pi_G^P\left(\hat{p}^{s,1}\right)\right)$, *where* $\bar{p}^s \in (m_c, 2m_c + m_d)$ *is the unique solution to* $\sqrt{SB_1} - S - \pi_G^P(\bar{p}^s) = 0$.

(c) *If* $s_1 \leq \frac{\beta_G 3(m_d+2m_c-\bar{p})}{5(\bar{p}+m_d)}$, *then* $(p^s, T_1) = \left(\hat{p}^{s,2}, 0\right)$, *where* $\hat{p}^2 \in [\bar{p}^s, 2m_c + m_d)$ *is the unique solution to*

$$\frac{\partial v_1(p^s)}{\partial p^s} + \frac{\partial \pi_1^W(p^s)}{\partial p^s} + \frac{SB_1}{\left[\pi_G^P(p^s) + S\right]^2} \frac{\partial \pi_G^P(p^s)}{\partial p^s} = 0$$

Proof: *see Online Appendix A.4.* ■

Proposition 4 is very similar to Proposition 1. The thresholds are slightly different, but the overall interpretations and implications are identical. Thus, introducing wholesale companies per se does not affect the analysis. In particular, note that the profits of each wholesale company are decreasing in p^s (formally, each whole company in country 1 gets $\pi_{r,1}^W(p^s) = \pi_1^W(p^s)/2 = 3(\alpha_1)^2/32(p^s + m_d))$. Thus, just as consumers, each company in the wholesale industry is a net loser from collusion.

Scenario 2: Suppose that both wholesale companies in country 1 are connected. Once again, given country 1's policy choices we can compute the consumer surplus in country 1, the aggregate profits of the wholesale industry in country 1 and the profits of geopolitically relevant producers.

$$v_1(p^s) = \frac{3(\alpha_1)^2}{4(m_c + m_d)} + y_1 - T_1,\ \pi_1^W(p^s) = \frac{3(\alpha_1)^2}{16(m_c + m_d)},\ \pi_G^P(p^s) = \frac{9\beta_G\left[\sum_{i\in I, i\neq 1}(\alpha_i)^2\right](p^s - m_c)}{16(p^s + m_d)^2}$$

Proposition 5 *(Scenario 2) Assume that* $B_1 > 0$ *and* $0 < \sqrt{SB_1} - S < \bar{T}$.

1. *Suppose that* $\sqrt{SB_1} - S > \pi_G^P(2m_c + m_d)$, *then* $(p^s, T_1) = \left(2m_c + m_d, \sqrt{SB_1} - S - \pi_G^P(2m_c + m_d)\right)$.
2. *Suppose that* $\sqrt{SB_1} - S \leq \pi_G^P(2m_c + m_d)$, *then* $(p^s, T_1) = (2m_c + m_d, 0)$.

Proof: *see Online Appendix A.4.* ■

The intuition behind Proposition 5 is as follows. If domestic wholesale companies in country 1 are excluded from the effects of collusion among producers of commodity c, then the best choice for country 1 is to advance its geopolitical goals by supporting collusion and only supplement this policy with foreign aid when collusive profits are not insufficient to implement the optimal level of deterrence. Thus, consumers from other countries end up paying to advance the geopolitical goals of country 1. By promoting collusion, country 1 successfully implements 'a passing the buck strategy'. In other words, the free rider problem associated with foreign aid is fully reversed. Now country 1 is essentially free riding other countries.

Scenario 3: Suppose that one wholesale company in country 1 is connected and the other is non-connected. Without loss of generality, assume that wholesale company 1 is connected. Then:

$$v_1(p^s) = \frac{3(\alpha_1)^2}{2(p^s + m_c + 2m_d)} + y_1 - T_1,$$

$$\pi_1^W(p^s) = \pi_{1,1}^W(p^s) + \pi_{2,1}^W(p^s) = \frac{3(\alpha_1)^2\left[(2p^s - m_c + m_d)^2 + (-p^s + 2m_c + m_d)^2\right]}{4(p^s + m_c + 2m_d)^3},$$

$$\pi_{1,1}^W(p^s) = \frac{(\alpha_1)^2\, 3(2p^s - m_c + m_d)^2}{4(p^s + m_c + 2m_d)^3},\ \pi_{2,1}^W(p^s) = \frac{(\alpha_1)^2\, 3(-p^s + 2m_c + m_d)^2}{4(p^s + m_c + 2m_d)^3}$$

$$\pi_G^P(p^s) = \pi_{G,1}^P(p^s) + \pi_{G,-1}^P(p^s)$$

$$\pi_{G,1}^P(p^s) = \frac{9\beta_G(\alpha_1)^2(p^s - m_c)(-p^s + 2m_c + m_d)}{4(p^s + m_c + 2m_d)^3},\ \pi_{G,-1}^P(p^s) = \frac{9\beta_G\left[\sum_{i\in I, i\neq 1}(\alpha_i)^2\right](p^s - m_c)}{16(p^s + m_d)^2}$$

where $\pi_{r,1}^W(p^s)$ are the profits of wholesale company r in country 1, $\pi_{G,1}^P(p^s)$ are the profits that producers of commodity c obtained in country 1 and $\pi_{G,-1}^P(p^s)$ are the profits that producers of commodity c obtained in countries other than 1.

Proposition 6 *(Scenario 3) Assume that $B_1 > 0$, $0 < \sqrt{SB_1} - S < \bar{T}$ and $\beta_G > 4/9$.*

1. *Suppose that $\sqrt{SB_1} - S > \pi_G^P(p^{s,*})$, where $p^{s,*} \in (m_c, 2m_c + m_d)$ is the unique solution to $\frac{\partial \pi_G^P(p^s)}{\partial p^s} = 0$.*

   (a) If $s_1 \geq \frac{6\beta_G}{5+3\beta_G}$, then $(p, T_1) = \left(m_c, \sqrt{SB_1} - S\right)$.

   (b) If $s_1 < \frac{6\beta_G}{5+3\beta_G}$, then $(p, T_1) = \left(\hat{p}^{s,1}, \sqrt{SB_1} - S - \pi_G^P\left(\hat{p}^{s,1}\right)\right)$, where $\hat{p}^{s,1} \in (m_c, p^{s,*})$ is the unique solution to:

$$\frac{\partial v_1(p^s)}{\partial p^s} + \frac{\partial \pi_1^W(p^s)}{\partial p^s} + \frac{\partial \pi_G^P(p^s)}{\partial p^s} = 0$$

2. *Suppose that $\sqrt{SB_1} - S \leq \pi_G^P(p^{s,*})$.*

   (a) If $s_1 \geq \frac{6\beta_G}{5+3\beta_G}$, then $(p, T_1) = \left(m_c, \sqrt{SB_1} - S\right)$.

   (b) If $\sigma(\bar{p}^s) < s_1 < \frac{6\beta_G}{5+3\beta_G}$, then $(p, T_1) = \left(\hat{p}^{s,1}, \sqrt{SB_1} - S - \pi_G^P\left(\hat{p}^{s,1}\right)\right)$, where $\bar{p}^s \in (m_c, p^{s,*})$ is the unique solution to $\sqrt{SB_1} - S - \pi_G^P(\bar{p}^s) = 0$.

   (c) If $s_1 \leq \sigma(\bar{p}^s)$, then $(p^s, T_1) = \left(\hat{p}^{s,2}, 0\right)$, where $\hat{p}^2 \in [\bar{p}^s, p^{s,*})$ is the unique solution to:

$$\frac{\partial v_1(p^s)}{\partial p^s} + \frac{\partial \pi_1^W(p^s)}{\partial p^s} + \frac{SB_1}{\left[\pi_G^P(p^s) + S\right]^2}\frac{\partial \pi_G^P(p^s)}{\partial p^s} = 0$$

Proof: *see Online Appendix A.4.* ■

Proposition 6 is very similar to Proposition 4. Note, however, an important difference. While $\pi_{1,2}^{W}(p^s)$ is decreasing in p^s, $\pi_{1,1}^{W}(p^s)$ is increasing in p^s. Thus, the politically connected wholosale company is a winner from collusion. The reason is that the connected company is not affected by a rise in p^s, but its rival (i.e., the non-connected company) is. In other words, collusion operates as a rise in the marginal cost of a competitor. This is important because it helps explain why some local importers supported international commodity agreements. The policy was detrimental to consumers and non-connected wholesale companies, and might have even be detrimental for the wholesale industry overall, but politically connected companies profited from it gaining a competitive cost edge over their domestic rivals.

3.4 Geopolitical Rivals Among Producers

Suppose that the payoff function of country 1 is given by:

$$W_1^{GP} = \frac{(\alpha_1)^2}{p} + y_1 - T_1 + \frac{\pi_G(p) + T_1}{\pi_G(p) + T_1 + S} B_1 - \lambda \pi_S(p)$$

where $\pi_G(p) = \beta_G \pi(p)$ are the profits accruing to geopolitically relevant producers, $\pi_S(p) = \beta_S \pi(p)$ are the profits accruing to the Soviet Union with $0 < \beta_S \leq 1 - \beta_G$, $\lambda > 0$ is the weight that the policy maker of country 1 puts on Soviet Union' profits, and recall that $\pi(p) = \left(\frac{p-m_c}{p^2}\right) \sum_{i \in I} (\alpha_i)^2$.

Proposition 7 *Assume that* $B_1 > 0$ *and* $0 < \sqrt{SB_1} - S < \bar{T}$*.*

1. *Suppose that* $\sqrt{SB_1} - S > \pi_G(2m_c)$*.*
   (a) *If* $s_1 \geq \beta_G - \lambda\beta_S$*, then* $(p, T_1) = \left(m_c, \sqrt{SB_1} - S\right)$*.*
   (b) *If* $s_1 < \beta_G - \lambda\beta_S$*, then* $(p, T_1) = \left(\hat{p}^1, \sqrt{SB_1} - S - \pi_G\left(\hat{p}^1\right)\right)$*, where* $\hat{p}^1 = \frac{2m_c(\beta_G - \lambda\beta_S)}{s_1 + \beta_G - \lambda\beta_S} \in (m, \bar{p})$*.*
2. *Suppose that* $\sqrt{SB_1} - S \leq \pi_G(2m_c)$*.*
   (a) *If* $s_1 \geq \beta_G - \lambda\beta_S$*, then* $(p, T_1, T_2) = \left(m_c, \sqrt{SB_1} - S\right)$*.*
   (b) *If* $(\beta_G - \lambda\beta_S)\left(\frac{2m_c - \bar{p}}{\bar{p}}\right) < s_1 < \beta_G - \lambda\beta_S$*, then* $(p, T_1) = \left(\hat{p}^1, \sqrt{SB_1} - S - \pi_G\left(\hat{p}^1\right)\right)$*.*
   (c) *If* $s_1 \leq (\beta_G - \lambda\beta_S)\left(\frac{2m_c - \bar{p}}{\bar{p}}\right)$*, then* $(p, T_1) = \left(\hat{p}^2, 0\right)$*, where* $\hat{p}^2 \in [\bar{p}, 2m_c)$ *is the unique solution to* $\frac{s_1 p}{(2m_c - p)} + \lambda\beta_S = \frac{\beta_G S B_1}{[\pi_G(p) + S]^2}$

Proof*: see Online Appendix A.5.* ■

It is easy to verify that as the profits accruing to the Soviet Union and/or the geopolitical importance of those profits for global power 1 rises (formally, $\lambda\beta_S$ higher), global power 1 is less willing to use collusion. More formally, the threshold of s_1 below which global power 1 supports collusion (selects $p > m_c$) is $\beta_G - \lambda\beta_S$, which is decreasing in $\lambda\beta_S$. Moreover,

$$\frac{\partial \hat{p}^1}{\partial(\lambda\beta_S)} = \frac{-2m_c s_1}{(s_1 + \beta_G - \lambda\beta_S)^2} < 0$$
$$\frac{\partial \hat{p}^2}{\partial(\lambda\beta_S)} = \frac{-1}{\frac{s_1 2m_c}{(2m_c - p)^2} + \frac{2\beta_G SB_1}{[\pi_G(p)+S]^3}\frac{\partial \pi_G(p)}{\partial p}} < 0$$

Thus, even when collusion is employed, it is less intensively used as $\lambda\beta_S$ rises.

3.5 Internal Politics in Producer Countries

Suppose that due to internal forces in the producer countries one dollar of foreign aid is not equivalent to one dollar of producer's profits. Formally, assume that the payoff function of country 1 is given by:

$$W_1^I = \frac{(\alpha_1)^2}{p} + y_1 - T_1 + \frac{b\pi_G(p) + T_1 + T_2}{b\pi_G(p) + T_1 + T_2 + S}B_1,$$

where $b > 0$ measures the relative effectiveness of profits relative to foreign aid. That is, $b < 1$ ($b > 1$) means that an extra dollar of profits is less (more) effective at fighting the spread of communism than one extra dollar of foreign aid.

It is easy to verify that introducing this internal bias in the geopolitically relevant producer countries only leads to a minor change in Proposition 1. Indeed, all we need to do is to replace β_G by $b\beta_G$ and $\bar{p}$ for $\bar{p}_b = \frac{b\beta_G \sum_{i\in I}(\alpha_i)^2}{2(\sqrt{SB_1} - S)}\left[1 - \sqrt{1 - \frac{4m_c(\sqrt{SB_1}-S)}{b\beta_G\sum_{i\in I}(\alpha_i)^2}}\right]$ and Proposition 1 holds. More importantly, we can study the effect that a rise in b has on the equilibrium outcome. The higher the effectiveness of profits, the more likely that, in equilibrium, $p > m_c$. Formally, an increase in b, makes $s_1 < b\beta_G$ easier to hold. Likewise, the greater the effectiveness of profits the more likely that foreign aid is not used at all. Formally, an increase in b makes $\sqrt{SB_1} - S \leq b\pi_G(2m_c)$ and $s_1 \leq \beta_G\left(\frac{2m_c - \bar{p}_b}{\bar{p}_b}\right)$ easier to hold. When foreign aid and collusion are both employed, collusion is more intensively used when b rises. Formally,

$$\frac{\partial \hat{p}^1}{\partial b} = \frac{2m_c\beta_G s_1}{(s_1 + b\beta_G)^2} > 0$$

Only when only collusion is employed, a rise in b has an ambigouous effect on the equilibrium price. Formally,

$$\frac{\partial \hat{p}^2}{\partial b} = \frac{(p)^2 s_1\left\{S^2 - [b\pi_G(p)]^2\right\}}{SB_1\beta_G\left[2m_c + \frac{2s_1[b\pi_G(p)+S]}{SB_1}\left(\frac{2m_c - p}{p}\right)\sum_{i\in I}(\alpha_i)^2\right]}$$

which is positive if and only if $S > b\pi_G(\hat{p}^2)$.

4 International Commodity Agreements

After the Second World War, several international commodity agreements (ICAs) were signed.[5] The stated goal of ICAs was to deal with declining and fluctuating prices of commodities. These agreements received the support of multilateral organizations (e.g., UNCTAD) and the United States. In this section, we review the history of one ICA, the International Coffee Agreement (ICOA). We use the model to rationalize its rise and demise. Then, we briefly discuss why other international commodity cartels (e.g., the OPEC), received very different treatment by the United States.

4.1 International Coffee Agreement (ICOA)

It is not surprising that important coffee producers like Brazil and Colombia have always had strong incentives to control and/or coordinate their production to increase the price of a major export (Bates, 1999).[6] For example, Brazil experimented with different types of market controls (Johnson, 1983; Nunberg, 1986). There are also precedents for international agreements between coffee consumer and producer countries. For example, to deal with a significant drop in coffee demand during the Second World War, the United States promoted the short-lived Inter American Coffee Agreement, part of Roosevelt's 'Good Neighbor' administration policiy toward Latin America. However, in the post war period, the agreement ended as the United States shifted its attention to Europe with the Plan Marshall (Wickizer, 1964).

In the 1960s, interest in regulating international trade of coffee again came to the forefront. The International Coffee Agreement (ICOA) of 1963 was an agreement between producer and consumer countries implemented through export quotas activated if prices were inside a price band -based on a composite coffee price index- (Gilbert, 1996). The agreement established a board (denoted the Coffee Organization) with voting rights to each producer and consumer country proportional to its volume of exports or imports, respectively. Consumer countries agreed to purchase coffee from member countries and to monitor the quotas by requiring a certificate of origin for products and sending this information to the ICOA offices. This allowed for credible sanctions and eventual suspension from the agreement of deviating producers (Koremenos, 2002). For consumer countries, membership was voluntary as shown by departures of New Zealand and Israel in the 1980s (Gilbert, 1996). Overall, the public intent behind the agreement was to reduce price volatility by stabilizing prices at a level higher than competitive ones (Gilbert, 1987). ICOA was renewed in 1968, 1976, 1983, 1994, 2001 and 2007. However, the agreements after 1989 did not contain any serious economic provision.

It has been well documented that ICOA was a de-facto cartel, and its primary effect was to increase average prices rather than reduce price fluctuations (Palm and Vogelvang, 1991). It also induced a series of distortions and misallocations of resources (Wickizer, 1964; Bohman and Jarvis, 1990).[7] As Gilbert

[5] The international commodity agreements during the Cold War period include the International Coffee Agreement (ICOA), the International Sugar Agreement (ISA), the International Tin Agreement (ITA), the International Cocoa Agreement (ICCA), and International Natural Rubber Agreement (INRA). For a complete list of all ICAs see Gilbert (1987) and Gilbert (1996).

[6] In comparison to other non-oil commodities, many coffee producer countries were highly dependent on coffee exports. For example, in 1971, coffee was the source of 71% of Colombia's export earnings (Koremenos, 2002).

[7] ICOA has been extensively studied. Igami (2015) studies the evolution of market power by producing countries. Bohman and Jarvis (1990) explores the effect on nonmember countries. Bohman et al. (1996) focuses on rent seeking behavior by producing countries. Mehta and Chavas (2008) studies price dynamics along the coffee supply vertical chain. Palm and Vogelvang (1991) focuses on the role of inventories. Koremenos (2002) studies the changes in bargaining power among

(1996) puts it: "[ICOA] was controversial because, since it operated entirely through export controls, it laid itself open to the charge of being an internationally sanctioned cartel whose objectives were primarily raising rather than stabilizing the coffee price".

An important way in which ICOA helped sustain collusion was supporting the monitoring of the agreement. Producer countries faced the usual challenges associated with sustaining collusion: They had a common interest in restricting their production levels, but each producer also had strong incentives to free ride other producers unilaterally deviating from the collusive agreement (Olson 1965). Moreover, producers could not directly observe the quantities selected by other producers and had to rely on an imperfect public signal (i.e., prices). Given that prices can also fluctuate due to market demand shifters, a producer does not know with certainty if low prices indicate a deviation from the collusive agreement or just a low demand state. As a consequence, imperfect public monitoring made collusion more complicated to sustain (Green and Porter, 1984). In this context, the role of consumer countries, and in particular the US leadership, took on greater importance. In particular, the US implemented certificates of origin for coffee shipments, which allowed for identifying quota violations and blocked coffee shipments that violated the export quotas. The following extract from an Executive Hearing on the matter is revealing: "[...] These certificates, like a custom document, identify the source of the coffee and enable the Coffee Organization to maintain an accurate and timely statistical check on exports. Thus, quota violations are easily and quickly detected. Our requiring certificate helps each exporting country to police its quota system".[8] Another mechanism that the US employed to support ICOA was to condition foreign aid on joining the agreement.

It is clear that ICOA benefited producing countries (and even more as coffee was an important share of their exports) but its support from consumer countries is difficult to rationalize on an economic basis. As ICOA resulted in higher prices for coffee, consumers in importing countries were undoubtedly hurt. For this reason, several studies have argued that political factors were critical in explaining the support of important consumer countries. In particular, the US Department of State, recognizing the strategic threat posed by the Cuban Revolution, considered it necessary to raise and stabilize world coffee prices to promote political and economic stability in Latin American coffee producer countries and prevent the spread of communism in the region.[9] As Krasner (1973) states: "... The Agreement served the foreign policy objectives of American officials. Economic growth and stability were perceived as conducive to the creation of regime types favored by the United States. Department of State officials identified political development with economic growth. In more specific terms, the American government saw economic payoffs as a device for securing Latin American diplomatic support, particularly for action against Cuba." In the same vein, Wickizer (1964) summarizes the international context of ICOA as follows: "... there was a renewed emphasis on political aspects of the problem, as the United States took steps to improve and solidify its position in Latin America because of the threat of Communist infiltration in restless countries ripe for some form of revolution. Pressures upon the United States became severe after Castro's take-over in Cuba, but they had started earlier".

producers. Coggins (1995) focuses on the internal structure and implementation of the agreement. Rettberg (2010) explores the rise of violence associated with the breakdown of the agreement.

[8] See Executive Hearing before the Committee "On Ways and Means", House of Representatives, Eighty-Ninth Congress on S.701: An Act to carry out the obligations of the United States under the International Coffee Agreement, 1962, signed at New York on September 28, 1962, and for other purposes that took place between April 13 and 14, 1965 -statement by Thomas Mann (Secretary of State)-.

[9] The European Community had similar objectives with regard to Africa (Gilbert, 1996).

The tensions between advancing a geopolitical goal at the expense of allowing a coffee cartel led to delays in the implementation of the agreement. Indeed, the initial 1962 agreement (signed by the executive branch) had a 2-year delay as the White House had to build enough support in the House of Representatives and Senate to pass the required legislation (Bates, 1999). More significantly, ICOA also opened the door for domestic lobbying. Given the limited knowledge about coffee production and distribution in the US Congress, legislators relied on The National Coffee Association -dominated by large US roasters- as their main source of information. Large rosters immediately spotted a great opportunity. In exchange for supporting the agreement before the US Congress, they negotiated discounts in their coffee supplies to make those discounts credible, they established bilateral long-term contracts with producers. This strategy secured lower coffee prices for large rosters and forced smaller rivals (and, of course, consumers) to face the entire burden of the agreement (Bates, 1999).[10] Moreover, it created incentives for large rosters to keep supporting the agreements (Bates, 1999). The National Coffee Association, an organization representing American large coffee importers and distributers, became the fundamental interest group lobbying Congress to support an international cartel of coffee exporters.

Eventually, ICOA met its demise. The 1983's ICOA was due to end in late 1989. Conversations about the terms of a new agreement began in 1989, but the parties could not reach an agreement on economic clauses. On the producer side there was disagreement about how to assign quotas (especially how to readjust Brazil's reducing market share). In 1990, Brazilian president Mello abolished the Brazilian Institute of Coffee, which was the country's main coffee organization -and receiver of rents-. Mello pushed support to domestic farmers and roasters, who were, at best, indifferent by future agreements. On the consumers' side, preferences shifted toward coffee beans, which faced greater distortions from the agreement. This increased the economic cost for consumer countries. There were also complains that producers regularly offered discounted prices to nonmember consumer countries. However, the most likely reason for the demise of ICOA was the lack of support from the United States for any agreement that included economic provision, such as quotas (Gilbert, 1996). As a result, the post-1989 agreements did not include any economic clauses. Additionally, producers internalized that without the U.S. support, future agreements could not be sustainable. This further decreased their involvement.

Our model helps explain the main features of ICOA, including its origin and demise. Why did the US, an important coffee importer, support an international agreement leading to the cartelization of coffee producers? Our model points to geopolitical considerations in the context of the Cold War. After the Cuban Revolution, the U.S.'s main geopolitical goal in Latin America was containing the spread of communism. Allowing some coffee producer countries in Latin America to collect collusive profits helped keep them geopolitically aligned with the US. However, geopolitical considerations fail to explain the choice of instrument. In particular, why did the US try to fight communism with a coffee cartel rather than with foreign aid or other less distortionary mechanisms? The most reasonable explanation seem to be avoiding free riding problems. While foreign aid was subject to free riding by US allies, a coffee cartel allowed the US to partially share the burden of supporting coffee producer countries with other coffee importers. Opacity and U.S. internal politics also played a role. Many American voters have strong views against foreign aid, but they would never suspect that they are indirectly supporting third world countries when they are buying a cup of coffee. Note that Olson's logic of collective action is not enough to explain the choice of instrument. Since consumers constitutes a disorganized large group while political

[10] Krasner (1973) argues that long-run contracts were also beneficial for large American roaster companies because a more stable and secure supply of coffee reduces risk of undersupply and generates performance-based incentives to managers.

elites with diplomatic goals form an organized small group, Olson suggests that the later will be more influential, which it is indeed the case. However, this is an equally valid argument for foreign aid and the coffee cartel, while differential voter perceptions and observability distinguish the policy instruments.

Another puzzling question about the ICOA is why organized American importers supported the agreement? Our model shows how large rosters (who dominated industry organizations and had political connections) used ICOA as an opportunity to gain rents and reduce competition from smaller/non-politically connected rosters. They relied on long-term contracts with producers to isolate themselves from any rise in import prices induced by collusion. In other words, they sold their political support to the collusive agreement in exchange for a credible promise by coffee producers that they would not face the consequences of the agreement, which would be borne entirely by their competitors and customers. This behavior sharply contrasts with the idea that long-run contracts always promote economic efficiency (Williamson). Indeed, from this perspective, long-run contracts are merely an instrument to deal with opportunism and credibility problems. They can be employed to support efficient transactions, as well as facilitate rent seeking. The net losers from ICOA were the small and non-politically connected rosters (who had to pay higher import prices) and American consumers. In some sense, this is Olson's logic of collective action reloaded. A small, organized group of firms is capable of completely overturning the negative effects of a policy by transferring all its costs to consumers and non-organized firms in the industry.

Finally, our model helps explain why ICOA collapsed in the 1990s. The answer is simple. The fall of communism completely eliminated the threat that gave rise to ICOA. Without any geopolitical goal, U.S. had no incentive to accept an economically costly agreement. The puzzling (and encouraging) development is how fast the mechanism crumbled as soon as the geopolitical issue disappeared. This is not at all obvious, given that ICOA generated a powerful interest group in favor of sustaining it (i.e., the large rosters).

4.2 Other International Cartels

4.2.1 Sugar

Sugar is produced from cane or beet. Cane, a tropical crop, has qualitatively similar harvesting cycles as coffee. Beet can be grown in non-tropical areas, but at a higher cost. At the beginning of the 20th century, Cuba served as the main sugar producer and exporter and overall lowest-cost producer Bender (1974). Given its dependence on sugar exports, Cuba, with other key exporters, led multiple attempts to control the price of sugar cane, including the Brussels Convention of Sugar of 1902, Cuba's production controls of the 1920s, and the Chadbourne Plan of 1931. These attempts incentivized importing countries to begin producing (or increase production) of beet sugar, restricting the ability of sugar cane exporters to rise prices. Afterward, it became clear that any attempt to regulate the price of sugar should involve the cooperation of importer countries Mahler (1984).

As part of the New Deal Policies, the US implemented the Sugar Act of 1934[11]. The Sugar Act aimed to protect domestic sugar producers by imposing high import tariffs. However, the main exporters of sugar to the US were American companies that controlled the majority of the Cuban sugar industry. Consequently, the act was modified to allow only for domestic and international quotas (mostly com-

[11] Also known as the Jones-Costigan Act

ing from Cuba, and the rest from the Philippines) to supply US national needs at a preferential price substantially higher than the prevailing world price Gerber (1976).[12]

The first International Sugar Agreement (ISA) was signed in 1937. However, it was short-lived , as World War II began soon thereafter. The main tenet of the agreement was an export quota system. The system was aimed at reducing price volatility. The agreement, however, only applied to sugar exports that were not included in existing bilateral or multilateral agreements (i.e., preferential agreements) by the member countries Hagelberg and Hannah (1994). This is particularly important, as the US Sugar Act, and its quota system, significantly limited the amount of sugar exports governed by ISA. At the time, such exports comprised approximately 30% of all sugar production Hoegle (1977). Moreover, although the US signed ISA, the agreement barely impacted its sugar provision.

A subsequent ISA was signed in 1952. It introduced a price indicator that allowed for activating the export quota system (deactivated) when prices dropped below (went above) a lower (upper) bound price. The same agreement was extended in 1958. However, the Cuban Revolution of 1959 significantly impacted the market and future ISA renegotiations. Due to Cuba's nationalization of the sugar industry in 1960, the US Congress modified the US Sugar Act. The modification allowed the President to block the sugar US Act quota at will. Nominally, Cuba still received a quota, but the US President blocked it every year. Sugar imports from other Latin American countries served as a replacement. The US used the fractionated quotas as a foreign policy instrument. Indeed, the US assigned the former Cuban sugar quota to other countries in exchange for their geopolitical allegiance Bender (1974). Eliminating Cuba from the US sugar quota also had a significant impact on ISA. In 1962, the agreement was set to be renewed; however, Cuba, now without access to the US market, pressed for a higher quota. This ended up destabilizing the agreement Gilbert (1996). Moreover, after blocking Cuban imports, the US decided to stop supporting any further ISA agreements.

The Cuban Revolution and the subsequent elimination of Cuba from the US sugar quota opened the door for a dramatic geopolitical realignment. Cuba's proximity to the US gave it enormous geopolitical value to the Soviet Union, who sought to add Cuba to its sphere of influence. Unsurprisingly, the Soviet Union was willing to support Cuba in exchange for its geopolitical alignment. The format the Soviet Union chose for implementing such support is interesting. The USSR and other communist countries signed bilateral sugar agreements with Cuba. As a result, approximately 75% of Cuban sugar production ended up destined to the Communist Bloc. In these bilateral agreements, around 80% of payments were barter trades, but the implicit price paid to Cuban sugar was somehow between the world price and the preferential price in the US market. In any event, the agreements partially compensated Cuba for losing the US sugar quota. They were also key for the Cuban economy during the Cold War period Bender (1974).

Our model sheds light on certain features of the sugar international market during the Cold War period. For the US, prior to the Cuban Revolution, the sugar quota for Cuba had no serious geopolitical relevance. It served merely as a mechanism to exclude American companies that controlled the Cuban sugar industry from the negative impacts of domestic protectionism. The quota was a political compromise for an unusual situation, namely, the fact that domestic companies controlled foreign exporters and would thus be negatively affected by a protectionist tariff. After the Cuban Revolution, continuing the sugar quota did not make sense for the US, given that American companies in Cuba had been nationalized. Moreover, as Cuba began its realignment toward the Soviet Union, the US sugar quota suddenly

[12]This preferential price could be twice the world price (Bender, 1974).

gained geopolitical importance. Maintaining the Cuban quota would have led to supporting a country aligned with a geopolitical rival. Similarly, supporting ISA would have allowed Cuba to sell its sugar at a reasonable price in international markets and the Soviet Union to reduce the implicit subsidy it paid for Cuban sugar. Thus, the US incentives to directly or indirectly support the price of sugar received by Cuba changed dramatically after the Cuban Revolution. Conditional on keeping the same level of protection for domestic sugar producers, the best alternative for the US was to reassign the Cuban sugar quota to other countries. Indeed, as suggested by our model, geopolitical considerations (i.e., containing the spread of communism) played an important role in reallocating the Cuban quota to other sugar producer countries willing to geopolitically align with the US.

Finally, our model is particularly suitable to explain the Soviet Union's reactions to the Cuban Revolution. There are few doubts that losing the American sugar quota was a serious problem for Cuba, and geopolitical reasons explain the interest of the Soviet Union in supporting Cuba. Our model goes a step further to explain the choice of instrument. Given that the Soviet Union was able to resell to other communist countries part of Cuban sugar, the burden of helping Cuba was partially shared with other members in the communist bloc.

4.2.2 Oil

After World War II, the oil market was controlled by western oil companies, the so-called, "Seven Sisters."[13] These companies owned most of the concessions in the Middle East and paid a percentage (also called "split") of the profits to Middle East countries. This percentage which was based on a listed price. In the late 1950s, as the USSR recovered from World War II, it increased its oil production, surpassing its domestic needs. The USSR's entrance into the international oil market created a scenario for cheaper oil (Yergin, 2011). Western oil companies reacted by reducing listed prices for Middle East oil. This generated enormous negative fiscal effects on Middle East countries and triggered the need for a common front among oil producers.

The Saudi Arabian and Venezuelan leaders (Abdullah Tariki and Juan Pablo Perez Alonso, respectively) were eager for greater cooperation. They were outraged by the behavior of western oil companies, and thus advanced agreements that led to the creation of the Organization of Petroleum Exporting Countries (OPEC) in 1960. The founding member of OPEC[14] agreed to gain greater control of oil assets. Specific measures included: (i) rejecting the usual 50-50 split of profits with western oil companies, leaving each country to negotiate the split individually; (ii) OPEC countries beginning to push western oil companies to increase listed prices (Yergin, 2011). While OPEC's official purpose was to act as a counteracting force on western oil companies, it has nevertheless acted in a coordinated (cartel) fashion to advance its economic and political goals. Such actions include: the oil embargo related to the Six-Day War of 1967[15], the Tehran and Tripoli agreements to increase the split of shares in profits among member countries and related price increases, participation in western oil Companies production facilities; and the oil embargo related to the Yom Kippur War of 1973.[16] However, given that OPEC members are fairly

[13]Anglo-Iranian Oil Company (now BP), Royal Dutch Shell (now Shell), Standard Oil of California (now Chevron), Gulf Oil (later merged with Chevron), Texaco (later merged with Chevron), Standard Oil of New Jersey (later Esso and now merger into ExxonMobil) and Standard Oil of New York (later Mobil and now merged into ExxonMobil).

[14]Founding countries: Iran, Iraq, Saudi Arabia, Kuwait, and Venezuela.

[15]Not implemented as part of the OPEC but related as several member countries implemented.

[16]Non-Arab members Iran and Venezuela did not join the embargo (Painter, 2014).

heterogeneous (Arab/Non-Arab, production capacities, closeness to western powers), there is evidence that, beyond these special circumstances, OPEC members have usually had strong incentives to deviate from proposed quotas or prices Colgan (2014).

The United States has never supported OPEC efforts, but it has not directly challenged OPEC. In practice, however, the United States has taken several actions to limit the influence of OPEC. During the Six-Day War in 1967, the United States did its best to counter the consequences of the embargo, organizing non-OPEC resources, using its own production, and providing tankers. During the oil embargo of 1973, the United States avoided bilateral oil agreements with OPEC countries by creating International Energy Agency, which operated as a political counterbalance to OPEC. There is also evidence that the United States incentivized Iran's deviation actions during the ruling of the Shah (given that Iran was one political leader in the Middle East region) and by Saudi Arabia (given that it is a key producer, capable of ameliorating any price increase) Yergin (2011).

Our model sheds light on certain features of OPEC and the US position toward it during the Cold War period. The Middle East was undoubtedly a geopolitical strategic region which, according to our model, helped oil become a target for an international cartel supported by the US. Moreover, OPEC was in dire need of cohesive power to monitor and enforce collusive agreements. Indeed, empirical evidence indicates that OPEC was often incapable of effectively implementing coordinated measures among its members Griffin (1989); Alhajji and Huettner (2000); Radetzki (2012). So why did the US not help OPEC sustain an oil cartel in exchange for Middle East geopolitical alignment? Our model suggests several reasons.

First, while there were immense geopolitical benefits in controlling the region, an oil cartel would have helped the USSR through several channels. Higher oil prices directly benefited the USSR, an important oil exporter. Greater oil revenues allowed the USSR to buy more grain (alleviating a pressing domestic problem) and import western technology Painter (2014). Higher oil prices also allowed some Middle East countries to buy more weapons from the USSR, which indeed happened during the Yom Kippur War. Second, as demonstrated by oil price increases in the 1970s (and its inflationary consequences), the impact of supporting OPEC would have been significant and politically problematic for US consumers. Finally, while coffee and sugar producers operated separately from their governments to some extent, oil producers did not. Oil producers were mostly mixed and nationalized companies rather than small or medium size farmers.

5 Conclusions: The Anatomy of Inefficiency

Our paper discusses the U.S.'s apparently self-defeating support for international commodity agreements during the Cold War period, revealing the economic and political logic behind this decision. We develop a simple model that formalizes the basic choice between foreign aid and supporting collusion as alternative instruments to advance geopolitical goals. We also explore several extensions of the baseline model to capture key factors that shape this choice. Finally, we apply the model and its results to the example of the International Coffee Agreement.

We conclude with a brief discussion of more general points suggested by this paper. First, regarding the choice of instruments, our results suggest reexamining the economic and political calculus of different policy instruments when the cost of some instruments are borne by foreign agents. This step is likely critical when dealing with foreign policy. It is also important to understand what voters observe/believe about different instruments. If certain instruments are easier for policy makers to hide from voters, policy makers will be tempted to use them more extensively. Once again, foreign policy seems a case in point.

Second, the extension to wholesale companies suggests a reconsideration of the political economy of interest groups. Interest groups can seize opportunities when there are policy changes and use their muscle and superior information to redistribute the costs and benefits from the policy change. This fact is critical to understand the final distribution of winner and losers. For example, in the case of ICOA, large United States roasters went from pure losers to clear winners, transferring the entire burden of the ICOA to non-connected roasters and final consumers. The good news is that when the geopolitical goal disappeared (i.e., the fall of the Soviet Union), the system was dismantled. At least in this case, there was no path dependence, in the sense that a lobby group gains power due to a geopolitical need and then the distortionary policy persists because of special interest politics.

Finally, we offer a comment on geopolitical versus economic goals: While the tradeoff between geopolitical and economic goals is undeniable, economic goals tend to be more objectively defined, while geopolitical goals could be vaguer. Our opening cite clearly illustrates this point. While Mr. Curtis clearly identified the fact that international commodity agreements were organizing global cartels (and negatively affecting consumers), Mr. Mann wrote about the threat to the United States if some developing countries were to suffer an economic downturn and consequently turn to communism. An important future contribution would be to provide micro-foundations for the cost and benefits associated with geopolitical issues.

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Online Appendix to Fighting Communism Supporting Collusion

This appendix presents the proofs of all lemmas and propositions.

A.1 Baseline Model (Lemma 1 and Proposition 1)

Lemma 1 *Assume that* $B_1 > B_2$ *and* $0 < \sqrt{SB_1} - S < \bar{T}$. *Suppose that* 1 *has selected* $p \in [m_c, 2m_c]$.

1. *Suppose that* $\sqrt{SB_1} - S > \pi_G(2m_c)$. *Then, the unique Nash equilibrium profile of transfers is* $T_1(p) = \sqrt{SB_1} - S - \pi_G(p)$ *and* $T_2(p) = 0$ *for all* $p \in [m_c, 2m_c]$.

2. *Suppose that* $\sqrt{SB_1} - S \leq \pi_G(2m_c)$. *The unique Nash equilibrium profile of transfers is:*

$$T_1(p) = \begin{cases} \sqrt{SB_1} - S - \pi_G(p) & \text{if } m_c \leq p < \bar{p} \\ 0 & \text{if } \bar{p} \leq p \leq 2m_c \end{cases} \quad \text{and } T_2(p) = 0$$

where $\bar{p} = \frac{\beta_G \sum_{i \in N_c} (\alpha_i)^2}{2\left(\sqrt{SB_1} - S\right)} \left[1 - \sqrt{1 - \frac{4m_c\left(\sqrt{SB_1} - S\right)}{\beta_G \sum_{i \in N_c} (\alpha_i)^2}}\right] \in (m_c, 2m_c]$.

Proof: We first compute the best response function for each player. Then, we derive all the Nash equilibria.

Best response function of player $i \in \{1, 2\}$: Fix $p \in [m_c, 2m_c]$. Then, the best response function of player 1 is the solution to the following optimization problem:

$$\max_{0 \leq T_i \leq \bar{T}} \left\{ W_i = \frac{(\alpha_i)^2}{p} + y_i - T_i + \frac{\pi_G(p) + T_1 + T_2}{\pi_G(p) + T_1 + T_2 + S} B_i \right\}$$

Note that:

$$\frac{\partial W_i}{\partial T_i} = -1 + \frac{SB_i}{[\pi_G(p) + T_1 + T_2 + S]^2} \text{ and } \frac{\partial^2 W_i}{(\partial T_i)^2} = \frac{-2SB_i}{[\pi_G(p) + T_1 + T_2 + S]^3} < 0$$

Therefore, the following Kuhn-Tucker conditions are necessary and sufficient for a unique global maximum:

$$-1 + \frac{SB_i}{(\pi_G(p) + T_1 + T_2 + S)^2} + \lambda^1 - \lambda^2 = 0$$
$$\lambda^1 T_i = 0,\ \lambda^1 \geq 0,\ T_i \geq 0$$
$$\lambda^2 \left(\bar{T} - T_i\right) \geq 0,\ \lambda^2 \geq 0,\ \bar{T} \geq T_i$$

Solving these Kuhn-Tucker conditions, we obtain:

$$T_i = \begin{cases} \bar{T} & \text{if } \sqrt{SB_i} - S - \pi_G(p) - T_{-i} \geq \bar{T} \\ \sqrt{SB_i} - S - \pi_G(p) - T_i & \text{if } 0 < \sqrt{SB_i} - S - \pi_G(p) - T_{-i} < \bar{T} \\ 0 & \text{if } \sqrt{SB_i} - S - \pi_G(p) - T_{-i} \leq 0 \end{cases}$$

Nash equilibrium transfers: To determine the Nash equilibrium profiles of transfers we must consider the following 9 possible cases:

Case N1: $(T_1, T_2) = (\bar{T}, \bar{T})$ is a Nash equilibrium profile of transfers if and only if

$$\sqrt{SB_1} - S - \pi_G(p) \geq 2\bar{T} \textit{ and} \sqrt{SB_2} - S - \pi_G(p) \geq 2\bar{T}$$

Case N2:$(T_1, T_2) = (\bar{T}, \sqrt{SB_2} - S - \pi_G(p) - \bar{T})$ is a Nash equilibrium profile of transfers if and only if

$$B_1 \geq B_2 \textit{ and } \bar{T} < \sqrt{SB_2} - S - \pi_G(p) < 2\bar{T}$$

Case N3: $(T_1, T_2) = (\bar{T}, 0)$ is a Nash equilibrium profile of transfers if and only if

$$\sqrt{SB_2} - S - \pi_G(p) \leq \bar{T} \leq \sqrt{SB_1} - S - \pi_G(p)$$

Case N4: $(T_1, T_2) = (\sqrt{SB_1} - S - \pi_G(p) - \bar{T}, \bar{T})$ is a Nash equilibrium profile of transfers if and only if

$$B_2 \geq B_1 \textit{ and } \bar{T} < \sqrt{SB_1} - S - \pi_G(p) < 2\bar{T}$$

Case N5: $T_1 = T_2 = \left[\sqrt{SB_1} - S - \pi_G(p)\right]/2$ is a Nash equilibrium profile of transfers if and only if

$$B_1 = B_2$$

Case N6: $(T_1, T_2) = (\sqrt{SB_1} - S - \pi_G(p), 0)$ is a Nash equilibrium profile of transfers if and only if

$$0 < \sqrt{SB_1} - S - \pi_G(p) < \bar{T} \textit{ and } B_2 \leq B_1$$

Case N7: $(T_1, T_2) = (0, \bar{T})$ is a Nash equilibrium profile of transfers if and only if

$$\sqrt{SB_1} - S - \pi_G(p) \leq \bar{T} \leq \sqrt{SB_2} - S - \pi_G(p)$$

Case N8: $(T_1, T_2) = (0, \sqrt{SB_2} - S - \pi_G(p))$ is a Nash equilibrium profile of transfers if and only if

$$B_1 \leq B_2 \textit{ and } 0 < \sqrt{SB_2} - S - \pi_G(p) < \bar{T}$$

Case N9: $(T_1, T_2) = (0, 0)$ is a Nash equilibrium profile of transfers if and only if

$$\sqrt{SB_1} - S - \pi_G(p) \leq 0 \textit{ and} \sqrt{SB_2} - S - \pi_G(p) \leq 0$$

Since $B_1 > B_2$ and $0 < \sqrt{SB_1} - S < \bar{T}$, the conditions required in cases N1-N5, N7, and N8 never hold. Thus, if $\sqrt{SB_1} - S - \pi_G(p) > 0$, then the unique Nash equilibrium profile of transfers is $(T_1, T_2) = (\sqrt{SB_1} - S - \pi_G(p), 0)$ (case N6), while if $\sqrt{SB_1} - S - \pi_G(p) \leq 0$, then the unique Nash equilibrium profile of transfers is $(T_1, T_2) = (0, 0)$ (case N9). Summing up, the unique Nash equilibrium profile of transfers is given by:

$$T_1 = \max\left\{\sqrt{SB_1} - S - \pi_G(p), 0\right\} \textit{ and } T_2 = 0$$

Moreover, note that

$$\frac{\partial \pi_G(p)}{\partial p} = \frac{(2m_c - p)}{p^3}\beta_G \sum\nolimits_{i \in I} (\alpha_i)^2 > 0 \textit{ for } p < 2m_c$$

Therefore, there are two possible situations:

Case 1: Suppose that $\sqrt{SB_1}-S > \pi_G(2m_c)$ or, which is equivalent, $\sqrt{SB_1}-S > \beta_G \sum_{i\in I}(\alpha_i)^2/4m_c$. Since $\pi_G(p)$ is strictly increasing in p for all $p \in [m_c, 2m_c]$, we have $\sqrt{SB_1}-S-\pi_G(p) > 0$ for all $p \in [m_c, 2m_c]$.

Case 2: Suppose that $\sqrt{SB_1}-S \leq \pi_G(2m_c)$ or, which is equivalent, $\sqrt{SB_1}-S \leq \beta_G \sum_{i\in I}(\alpha_i)^2/4m_c$. Since $\pi_G(m_c) = 0 < \sqrt{SB_1}-S \leq \pi_G(2m_c)$ and $\pi_G(p)$ is strictly increasing in p for all $p \in [m_c, 2m_c]$, there exists a unique $\bar{p} \in (m_c, 2m_c]$ such that $\sqrt{SB_1}-S-\pi_G(p) > 0$ for all $p \in [m_c, \bar{p})$, $\sqrt{SB_1}-S-\pi_G(\bar{p}) = 0$, and $\sqrt{SB_1}-S-\pi_G(p) < 0$ for all $p \in (\bar{p}, 2m_c]$. Finally, since $\pi_G(p) = \frac{(p-m_c)}{(p)^2}\beta_G \sum_{i\in I}(\alpha_i)^2$, $\sqrt{SB_1}-S-\pi_G(\bar{p}) = 0$ if and only if

$$(\bar{p})^2 - \frac{\beta_G \sum_{i\in I}(\alpha_i)^2}{\sqrt{SB_1}-S}\bar{p} + \frac{m_c\beta_G \sum_{i\in I}(\alpha_i)^2}{\sqrt{SB_1}-S} = 0$$

Hence:

$$\bar{p} = \frac{\beta_G \sum_{i\in N_c}(\alpha_i)^2}{2\left(\sqrt{SB_1}-S\right)}\left[1-\sqrt{1-\frac{4m_c\left(\sqrt{SB_1}-S\right)}{\beta_G \sum_{i\in I}(\alpha_i)^2}}\right] \in (m_c, 2m_c]$$

This completes the proof of Lemma 1. ■

Proposition 1 *Assume that $B_1 > B_2$ and $0 < \sqrt{SB_1}-S < \bar{T}$.*

1. *Suppose that $\sqrt{SB_1}-S > \pi_G(2m_c)$.*

   (a) *If $s_1 \geq \beta_G$, then the unique subgame perfect Nash equilibrium outcome is $(p, T_1, T_2) = \left(m_c, \sqrt{SB_1}-S, 0\right)$.*

   (b) *If $s_1 < \beta_G$, then the unique subgame perfect Nash equilibrium outcome is $(p, T_1, T_2) = \left(\hat{p}^1, \sqrt{SB_1}-S-\pi_G\left(\hat{p}^1\right), 0\right)$, where $\hat{p}^1 = \frac{2m_c\beta_G}{s_1+\beta_G}$.*

2. *Suppose that $\sqrt{SB_1}-S \leq \pi_G(2m_c)$.*

   (a) *If $s_1 \geq \beta_G$, then the unique subgame perfect Nash equilibrium is outcome $(p, T_1, T_2) = \left(m_c, \sqrt{SB_1}-S, 0\right)$.*

   (b) *If $\frac{2m_c-\bar{p}}{\bar{p}}\beta_G < s_1 < \beta_G$, then the unique subgame perfect Nash equilibrium outcome is $(p, T_1, T_2) = \left(\hat{p}^1, \sqrt{SB_1}-S-\pi_G\left(\hat{p}^1\right), 0\right)$.*

   (c) *If $s_1 \leq \frac{2m_c-\bar{p}}{\bar{p}}\beta_G$, then the unique subgame perfect Nash equilibrium outcome is $(p, T_1, T_2) = \left(\hat{p}^2, 0, 0\right)$, where $\hat{p}^2$ is the unique solution to $\left(\frac{2m_c-p}{p}\right)\beta_G = \frac{s_1[\pi_G(p)+S]^2}{SB_1}$.*

Proof of Part 1: Suppose that $\sqrt{SB_1}-S > \pi_G(2m_c) = \beta_G \sum_{i\in I}(\alpha_i)^2/4m_c$. Then, from Lemma 1, $(T_1(p), T_2(p)) = \left(\sqrt{SB_1}-S-\pi_G(p), 0\right)$ for all $p \in [m_c, 2m_c]$ and, hence, the price selected by player 1 is the solution to the following optimization problem:

$$\max_{p\in[m_c,2m_c]}\left\{W_1(p) = \frac{(\alpha_1)^2}{p} + y_1 - \sqrt{SB_1} + S + \pi_G(p) + \frac{\sqrt{SB_1}-S}{\sqrt{SB_1}}B_1\right\}$$

Take the derivative of W_1 with respect to p:

$$\frac{\partial W_1(p)}{\partial p} = \frac{-s_1 p + (2m_c - p)\beta_G}{(p)^3 \left[\sum_{i\in I}(\alpha_i)^2\right]^{-1}}$$

where $s_1 = \frac{(\alpha_1)^2}{\sum_{i\in I}(\alpha_i)^2}$. The numerator of $\frac{\partial W_1(p)}{\partial p}$ is decreasing in p. Thus, there are two possible cases to consider:

Case 1.a: Suppose that $s_1 \geq \beta_G$. Then, W_1 is strictly decreasing in p for all $p \in [m_c, 2m_c]$. Thus, the price that maximizes W_1 is $p = m_c$. Therefore, the unique subgame perfect Nash equilibrium outcome is $(p, T_1, T_2) = \left(m_c, \sqrt{SB_1} - S, 0\right)$.

Case 1.b: Suppose that $s_1 < \beta_G$. Then, W_1 is strictly increasing in p for all $p \in \left[m_c, \hat{p}^1\right]$ and strictly decreasing in p for all $p \in \left[\hat{p}^1, 2m_c\right]$, where $\hat{p}^1 = \frac{2m_c\beta_G}{s_1+\beta_G}$. Thus, W_1 adopts its maximum at $p = \hat{p}^1$. Therefore, the unique subgame perfect Nash equilibrium outcome is $(p, T_1, T_2) = \left(\hat{p}^1, \sqrt{SB_1} - S - \pi_G\left(\hat{p}^1\right), 0\right)$.

Proof of Part 2: Suppose that $\sqrt{SB_1} - S \leq \pi_G(2m_c)$. Then, from Lemma 1, if $p \in [m_c, \bar{p})$, then $(T_1, T_2) = \left(\sqrt{SB_1} - S - \pi_G(p), 0\right)$, while if $p \in [\bar{p}, 2m_c]$, then $(T_1, T_2) = (0, 0)$. Hence, the price selected by player 1 is the solution to the following optimization problem:

$$\max_{p\in[m_c,2m_c]} \left\{ W_1(p) = \frac{(\alpha_1)^2}{p} + y_1 + \left\{ \begin{array}{ll} -\left[\sqrt{SB_1} + S - \pi_G(p)\right] + \frac{\sqrt{SB_1}-S}{\sqrt{SB_1}} B_1 & if\ p \in [m_c, \bar{p}) \\ \frac{\pi_G(p)}{\pi_G(p)+S} B_1 & if\ p \in [\bar{p}, 2m_c] \end{array} \right\} \right.$$

W_1 has the following properties:

- W_1 is a continuous function of p for all $p \in [m_c, 2m_c]$. In particular, it is continuous at $p = \bar{p}$. To prove this, note that:

$$\begin{aligned} \lim_{p\to\bar{p}^-} W_1(p) &= \frac{(\alpha_1)^2}{\bar{p}} + y_1 - \left[\sqrt{SB_1} + S - \pi_G(\bar{p})\right] + \frac{\pi_G(\bar{p})}{\pi_G(\bar{p}) + S} B_1 \\ &= \frac{(\alpha_1)^2}{\bar{p}} + y_1 + \frac{\sqrt{SB_1} - S}{\sqrt{SB_1}} B_1 = \lim_{p\to\bar{p}^+} W_1(p), \end{aligned}$$

  where we have employed that $\sqrt{SB_1} - S - \pi_G(\bar{p}) = 0$.

- Take the derivative of W_1 with respect to p for $p \in [m_c, \bar{p})$:

$$\frac{\partial W_1(p)}{\partial p} = \frac{-s_1 p + (2m_c - p)\beta_G}{(p)^3 \left[\sum_{i\in I}(\alpha_i)^2\right]^{-1}}$$

  Let $N(p) = -s_1 p + (2m_c - p)\beta_G$ be the numerator of $\frac{\partial W_1(p)}{\partial p}$. $N(p)$ is decreasing in p. Thus, there are three possible cases to consider:

  - Suppose that $s_1 \geq \beta_G$. Then, W_1 is strictly decreasing in p for all $p \in [m_c, \bar{p})$:
  - Suppose that $\left(\frac{2m_c-\bar{p}}{\bar{p}}\right)\beta_G < s_1 < \beta_G$. Then, W_1 is strictly decreasing in p for all $p \in \left[m_c, \hat{p}^1\right]$ and strictly decreasing in p for $p \in \left[\hat{p}^1, \bar{p}\right)$, where $\hat{p}^1 = \frac{2m_c\beta_G}{s_1+\beta_G} \in (m_c, \bar{p})$

– Suppose that $s_1 \leq \left(\frac{2m_c-\bar{p}}{\bar{p}}\right)\beta_G$. Then, W_1 is strictly increasing in p for all $p \in [m_c, \bar{p})$.

- Take the derivative of W_1 with respect to p for $p \in [\bar{p}, 2m_c]$:

$$\frac{\partial W_1(p)}{\partial p} = \frac{-s_1 p + \frac{(2m_c-p)\beta_G SB_1}{[\pi_G(p)+S]^2}}{(p)^3\left[\sum_{i\in I}(\alpha_i)^2\right]^{-1}}$$

Let $N(p) = -s_1 p + \frac{(2m_c-p)\beta_G SB_1}{[\pi_G(p)+S]^2}$ be the numerator of $\frac{\partial W_1(p)}{\partial p}$. $N(p)$ is decreasing in p and $N(2m_c) = -2s_1 m_c < 0$. Thus, there are two possible cases to consider:

  – Suppose that $s_1 \geq \left(\frac{2m_c-\bar{p}}{\bar{p}}\right)\beta_G$. Then, W_1 is strictly decreasing in p for all $p \in [\bar{p}, 2m_c]$.

  – Suppose that $s_1 < \left(\frac{2m_c-\bar{p}}{\bar{p}}\right)\beta_G$. Then, W_1 is strictly increasing in p for all $p \in \left[\bar{p}, \hat{p}^2\right]$ and strictly decreasing in p for all $p \in \left[\hat{p}^2, 2m_c\right]$, where $\hat{p}^2 \in (\bar{p}, 2m_c)$ is the unique solution to $\left(\frac{2m_c-p}{p}\right)\beta_G = \frac{s_1[\pi_G(p)+S]^2}{SB_1}$.

Employing the above characterization of $W_1(p)$ we have the following possible cases:

Case 2.a: Suppose that $s_1 \geq \beta_G$. Then, W_1 is strictly decreasing in p for all $p \in [m_c, 2m_c]$. Thus, W_1 adopts its maximum at $p = m_c$. Therefore, the unique subgame perfect Nash equilibrium outcome is $(p, T_1, T_2) = \left(m_c, \sqrt{SB_1} - S, 0\right)$.

Case 2.b: Suppose that $\left(\frac{2m_c-\bar{p}}{\bar{p}}\right)\beta_G < s_1 < \beta_G$. Then, W_1 is strictly increasing in p for all $p \in \left[m_c, \hat{p}^1\right]$ and W_1 is strictly decreasing in p for all $p \in \left[\hat{p}^1, 2m_c\right]$. Thus, W_1 adopts its maximum at $p = \hat{p}^1 \in (m_c, \bar{p})$. Therefore, the unique subgame perfect Nash equilibrium outcome is $(p, T_1, T_2) = \left(\hat{p}^1, \sqrt{SB_1} - S - \pi_G\left(\hat{p}^1\right), 0\right)$.

Case 2.c: Suppose that $s_1 \leq \left(\frac{2m_c-\bar{p}}{\bar{p}}\right)\beta_G$. W_1 is strictly increasing in p for all $p \in \left[m_c, \hat{p}^2\right]$ and W_1 is strictly decreasing in p for all $p \in \left[\hat{p}^2, 2m_c\right]$. Thus, W_1 adopts its maximum at $p = \hat{p}^2 \in [\bar{p}, 2m_c)$. Therefore, the unique subgame perfect Nash equilibrium outcome is $(p, T_1, T_2) = \left(\hat{p}^2, 0, 0\right)$.

This completes the proof of Proposition 1. ■

A.2 Cooperative Foreign Aid (Lemma 2 and Proposition 2)

Lemma 2 *Assume that* $B_1 > B_2$, $0 < \sqrt{SB_1} - S < \bar{T}$, $\pi_G(2m_c) < \sqrt{S(B_1+B_2)} - S < \bar{T}$ *and* $\gamma_1^L < \gamma_1 < \gamma_1^H$. *Suppose that 1 has selected* $p \in [m_c, 2m_c]$. *Then, negotiated total transfers are given by:*

$$T^C(p) = \sqrt{S(B_1+B_2)} - S - \pi_G(p)$$

Moreover:

1. *Suppose that* $[\sqrt{SB_1} - S > \pi_G(2m_c)]$ *or* $[\sqrt{SB_1} - S \leq \pi_G(2m_c)$ *and* $p < \bar{p}]$. *Then, negotiated transfers are given by:*

$$T_1^C(p) = \gamma_1 \left\{ \sqrt{S(B_1+B_2)} - S - \pi_G(p) - \left[1 - \sqrt{\frac{B_1}{B_1+B_2}}\right] \sqrt{SB_1}\frac{B_2}{B_1} \right\}$$
$$+ (1-\gamma_1) \left\{ \left[1 - \sqrt{\frac{B_1}{B_1+B_2}}\right] \sqrt{SB_1} + \sqrt{SB_1} - S - \pi_G(p) \right\}$$
$$T_2^C(p) = \gamma_1 \left[1 - \sqrt{\frac{B_1}{B_1+B_2}}\right] \sqrt{SB_1}\frac{B_2}{B_1}$$
$$+ (1-\gamma_1) \left\{ \sqrt{S(B_1+B_2)} - \sqrt{SB_1} - \left[1 - \sqrt{\frac{B_1}{B_1+B_2}}\right] \sqrt{SB_1} \right\}$$

2. *Suppose that* $\sqrt{SB_1} - S \leq \pi_G(2m_c)$ *and* $p \geq \bar{p}$. *Then, negotiated transfers are given by:*

$$T_1^C(p) = \left\{ \gamma_1 + \frac{S[(1-\gamma_1)B_1 - \gamma_1 B_2]}{[\pi_G(p) + S]\sqrt{S(B_1+B_2)}} \right\} \left[\sqrt{S(B_1+B_2)} - S - \pi_G(p)\right]$$
$$T_2^C(p) = \left\{ 1 - \gamma_1 - \frac{S[(1-\gamma_1)B_1 - \gamma_1 B_2]}{[\pi_G(p) + S]\sqrt{S(B_1+B_2)}} \right\} \left[\sqrt{S(B_1+B_2)} - S - \pi_G(p)\right]$$

Proof: Given $p \in [m_c, 2m_c]$, players 1 and 2 select the transfers that solve the following optimization problem:

$$\max_{T_1,T_2 \in [0,\bar{T}]} \left\{ \begin{array}{c} W^N = \left(W_1^C - W_1\right)^{\gamma_1} \left(W_2^C - W_2\right)^{1-\gamma_1} = \\ = \left[\frac{(\alpha_1)^2}{p} + y_1 - T_1 + \frac{\pi_G(p)+T}{\pi_G(p)+T+S}B_1 - W_1\right]^{\gamma_1} \left[\frac{(\alpha_2)^2}{p} + y_2 - T_2 + \frac{\pi_G(p)+T}{\pi_G(p)+T+S}B_2 - W_2\right]^{1-\gamma_1} \end{array} \right\}$$

where $T = T_1 + T_2$, W_1, W_2 are the equilibrium payoffs of players 1 and 2 (that is, their outside options if they do not cooperate), and $\gamma_1 \in (0,1)$ is the bargaining power of player 1.

The following Kuhn-Tucker conditions are sufficient for a global maximum:

$$\left(\frac{W_1^C - W_1}{W_2^C - W_2}\right)^{\gamma_1} \left\{ -\gamma_1 \frac{W_2^C - W_2}{W_1^C - W_1} + \frac{\gamma_1 \frac{W_2^C - W_2}{W_1^C - W_1} SB_1 + (1-\gamma_1) SB_2}{[\pi_G(p) + T + S]^2} \right\} + \lambda_1 - \lambda_2 = 0$$
$$\lambda_1 \geq 0,\ T_1 \geq 0,\ \lambda_1 T_1 = 0,\ \lambda_2 \geq 0,\ \left(\bar{T} - T_1\right) \geq 0,\ \lambda_2\left(\bar{T} - T_1\right) = 0$$
$$\left(\frac{W_1^C - W_1}{W_2^C - W_2}\right)^{\gamma_1} \left\{ -(1-\gamma_1) + \frac{\gamma_1 \frac{W_2^C - W_2}{W_1^C - W_1} SB_1 + (1-\gamma_1) SB_2}{[\pi_G(p) + T + S]^2} \right\} + \lambda_3 - \lambda_4 = 0$$
$$\lambda_3 \geq 0,\ T_2 \geq 0,\ \lambda_3 T_2 = 0,\ \lambda_4 \geq 0,\ \left(\bar{T} - T_2\right) \geq 0,\ \lambda_4\left(\bar{T} - T_2\right) = 0$$

We look for interior solutions in which $T_1, T_2 \in (0, \bar{T})$. Then, Kuhn-Tucker conditions becomes:

$$-1 + \frac{S(B_1 + B_2)}{[\pi_G(p) + T + S]^2} = 0 \text{ and } \gamma_1 \left(W_2^C - W_2\right) = (1 - \gamma_1)\left(W_1^C - W_1\right)$$

Solving the first equation we obtain $T^C = \sqrt{S(B_1 + B_2)} - S - \pi_G(p) > 0$, which always holds. Therefore, the payoffs of players 1 and 2 as a function of p will be given by:

$$W_1^C(p) = \frac{(\alpha_1)^2}{p} + y_1 - T_1 + \frac{\sqrt{S(B_1 + B_2)} - S}{\sqrt{S(B_1 + B_2)}} B_1$$

$$W_2^C(p) = \frac{(\alpha_2)^2}{p} + y_2 - T_2 + \frac{\sqrt{S(B_1 + B_2)} - S}{\sqrt{S(B_1 + B_2)}} B_2$$

To determine T_1 and T_2 we must consider two possible cases:

Case 1: Suppose that $[\sqrt{SB_1} - S > \pi_G(2m_c)]$ or $[\sqrt{SB_1} - S \leq \pi_G(2m_c)$ and $p < \bar{p}]$. Then, if players do not cooperate, Lemma 1 implies that the payoff of players 1 and 2 will be given by:

$$W_1 = \frac{(\alpha_1)^2}{p} + y_1 - \left[\sqrt{SB_1} - S - \pi_G(p)\right] + \frac{\sqrt{SB_1} - S}{\sqrt{SB_1}} B_1$$

$$W_2 = \frac{(\alpha_2)^2}{p} + y_2 + \frac{\sqrt{SB_1} - S}{\sqrt{SB_1}} B_2$$

Introducing these expressions into the Kuhn-Tucker condition $\gamma_1 \left(W_2^C - W_2\right) = (1 - \gamma_1)\left(W_1^C - W_1\right)$ and using that $T = \sqrt{S(B_1 + B_2)} - S - \pi_G(p)$ we obtain:

$$\begin{aligned} T_1^C &= \gamma_1 \left\{ \sqrt{S(B_1 + B_2)} - S - \pi_G(p) - \left[1 - \sqrt{\frac{B_1}{B_1 + B_2}}\right] \sqrt{SB_1} \frac{B_2}{B_1} \right\} \\ &+ (1 - \gamma_1) \left\{ \left[1 - \sqrt{\frac{B_1}{B_1 + B_2}}\right] \sqrt{SB_1} + \sqrt{SB_1} - S - \pi_G(p) \right\} \\ T_2^C &= \gamma_1 \left[1 - \sqrt{\frac{B_1}{B_1 + B_2}}\right] \sqrt{SB_1} \frac{B_2}{B_1} \\ &+ (1 - \gamma_1) \left\{ \sqrt{S(B_1 + B_2)} - \sqrt{SB_1} - \left[1 - \sqrt{\frac{B_1}{B_1 + B_2}}\right] \sqrt{SB_1} \right\} \end{aligned}$$

Finally, we must check that $T_1^C, T_2^C > 0$. $T_1^C > 0$ if and only if

$$\gamma_1 < \frac{\left[2 - \sqrt{\frac{B_1}{B_1 + B_2}}\right] \sqrt{SB_1} - S - \pi_G(p)}{\left[1 - \sqrt{\frac{B_1}{B_1 + B_2}}\right] \sqrt{SB_1} \frac{B_2}{B_1} + \left[2 - \sqrt{\frac{B_1}{B_1 + B_2}}\right] \sqrt{SB_1} - \sqrt{S(B_1 + B_2)}}$$

This inequality holds for any $p \in [m_c, 2m_c]$ whenever

$$\gamma_1 < \gamma_1^H = \frac{\left[2 - \sqrt{\frac{B_1}{B_1 + B_2}}\right] \sqrt{SB_1} - S - \pi_G(2m_c)}{\left[1 - \sqrt{\frac{B_1}{B_1 + B_2}}\right] \sqrt{SB_1} \frac{B_2}{B_1} + \left[2 - \sqrt{\frac{B_1}{B_1 + B_2}}\right] \sqrt{SB_1} - \sqrt{S(B_1 + B_2)}}$$

$T_2^C > 0$ if and only if

$$\gamma_1 > \gamma_1^L = \frac{\left[2 - \sqrt{\frac{B_1}{B_1+B_2}}\right]\sqrt{SB_1} - \sqrt{S\left(B_1+B_2\right)}}{\left[1 - \sqrt{\frac{B_1}{B_1+B_2}}\right]\sqrt{SB_1}\frac{B_2}{B_1} + \left[2 - \sqrt{\frac{B_1}{B_1+B_2}}\right]\sqrt{SB_1} - \sqrt{S\left(B_1+B_2\right)}}$$

Therefore, we need $\gamma_1^L < \gamma_1 < \gamma_1^H$.

Case 2: Suppose that $\sqrt{SB_1} - S \leq \pi_G\left(2m_c\right)$ and $p \geq \bar{p}$. Then, if players do not cooperate, Lemma 1 implies that the payoff of players 1 and 2 will be given by:

$$W_1\left(p\right) = \frac{\left(\alpha_1\right)^2}{p} + y_1 + \frac{\pi_G\left(p\right)}{\pi_G\left(p\right)+S}B_1$$
$$W_2\left(p\right) = \frac{\left(\alpha_2\right)^2}{p} + y_2 + \frac{\pi_G\left(p\right)}{\pi_G\left(p\right)+S}B_2$$

Introducing these expressions into the Kuhn-Tucker condition $\gamma_1\left(W_2^C - W_2\right) = \left(1-\gamma_1\right)\left(W_1^C - W_1\right)$ and using $T = \sqrt{S\left(B_1+B_2\right)} - S - \pi_G\left(p\right)$ we obtain:

$$T_1^C = \left\{\gamma_1 + \frac{S\left[\left(1-\gamma_1\right)B_1 - \gamma_1 B_2\right]}{\left[\pi_G\left(p\right)+S\right]\sqrt{S\left(B_1+B_2\right)}}\right\}\left[\sqrt{S\left(B_1+B_2\right)} - S - \pi_G\left(p\right)\right]$$
$$T_2^C = \left\{1 - \gamma_1 - \frac{S\left[\left(1-\gamma_1\right)B_1 - \gamma_1 B_2\right]}{\left[\pi_G\left(p\right)+S\right]\sqrt{S\left(B_1+B_2\right)}}\right\}\left[\sqrt{S\left(B_1+B_2\right)} - S - \pi_G\left(p\right)\right]$$

Finally, it is easy to verify that $T_1^C, T_2^C > 0$ for all $\gamma_1 \in (0,1)$. This completes the proof lemma 2. ■

Proposition 2 *Assume that* $B_1 > B_2$, $0 < \sqrt{SB_1} - S < \bar{T}$, $\pi_G\left(2m_c\right) < \sqrt{S\left(B_1+B_2\right)} - S < \bar{T}$ *and* $\gamma_1^L < \gamma_1 < \gamma_1^H$.

1. *Suppose that* $\sqrt{SB_1} - S > \pi_G\left(2m_c\right)$.

   (a) *If* $s_1 \geq \beta_G$, *then* $\left(p, T_1+T_2\right) = \left(m_c, \sqrt{S\left(B_1+B_2\right)} - S\right)$.

   (b) *If* $s_1 < \beta_G$, *then* $\left(p, T_1+T_2\right) = \left(\hat{p}^1, \sqrt{S\left(B_1+B_2\right)} - S - \pi_G\left(\hat{p}^1\right)\right)$, *where* $\hat{p}^1 = \frac{2m_c\beta_G}{s_1+\beta_G}$.

2. *Suppose that* $\sqrt{SB_1} - S \leq \pi_G\left(2m_c\right)$.

   (a) *If* $s_1 \geq \beta_G$, *then* $\left(p, T_1+T_2\right) = \left(m_c, \sqrt{S\left(B_1+B_2\right)} - S\right)$.

   (b) *If* $\left(\frac{2m_c-\bar{p}}{\bar{p}}\right)\beta_G < s_1 < \beta_G$, *then* $\left(p, T_1+T_2\right) = \left(\hat{p}^1, \sqrt{S\left(B_1+B_2\right)} - S - \pi_G\left(\hat{p}^1\right)\right)$.

   (c) *If* $\beta_G\left(\frac{B_1-\gamma_1 B_2}{B_1}\right)\left(\frac{2m_c-\bar{p}}{\bar{p}}\right) \leq s_1 \leq \left(\frac{2m_c-\bar{p}}{\bar{p}}\right)\beta_G$, *then* $\left(p, T_1+T_2\right) = \left(\bar{p}, \sqrt{S\left(B_1+B_2\right)} - S - \pi_G\left(\bar{p}\right)\right)$ *or* $\left(\hat{p}^3, T_1+T_2\right) = \left(\hat{p}^3, \sqrt{S\left(B_1+B_2\right)} - S - \pi_G\left(\hat{p}^3\right)\right)$, *where* $\hat{p}^3$ *is the unique solution to:*

$$\left(\frac{2m_c - p}{p}\right)\beta_G = \frac{s_1\left[\pi\left(p\right)+S\right]^2}{\gamma_1\left[\pi\left(p\right)+S\right]^2 + S\left[\left(1-\gamma_1\right)B_1 - \gamma_1 B_2\right]}$$

(d) *If* $s_1 < \beta_G \left(\frac{B_1 - \gamma_1 B_2}{B_1}\right)\left(\frac{2m_c - \bar{p}}{\bar{p}}\right)$, *then* $(p, T_1 + T_2) = \left(\hat{p}^3, \sqrt{S(B_1 + B_2)} - S - \pi_G\left(p^{T,bis}\right)\right)$.

Proof of Part 1: Suppose that $\sqrt{SB_1} - S > \pi_G(2m_c)$. Then, from Lemma 2, we have:

$$\begin{aligned} T_1^C(p) &= \gamma_1 \left\{ \sqrt{S(B_1+B_2)} - S - \pi_G(p) - \left[1 - \sqrt{\frac{B_1}{B_1+B_2}}\right]\sqrt{SB_1}\frac{B_2}{B_1} \right\} \\ &\quad + (1-\gamma_1)\left\{ \left[1 - \sqrt{\frac{B_1}{B_1+B_2}}\right]\sqrt{SB_1} + \sqrt{SB_1} - S - \pi_G(p) \right\} \\ T_2^C(p) &= \gamma_1 \left[1 - \sqrt{\frac{B_1}{B_1+B_2}}\right]\sqrt{SB_1}\frac{B_2}{B_1} \\ &\quad + (1-\gamma_1)\left\{ \sqrt{S(B_1+B_2)} - \sqrt{SB_1} - \left[1 - \sqrt{\frac{B_1}{B_1+B_2}}\right]\sqrt{SB_1} \right\} \end{aligned}$$

Hence, the price selected by player 1 is the solution to the following optimization problem:

$$\max_{p \in [m_c, 2m_c]} \left\{ W_1^C(p) = \frac{(\alpha_1)^2}{p} + y_1 - T_1^C(p) + \frac{\sqrt{S(B_1+B_2)} - S}{\sqrt{S(B_1+B_2)}} B_1 \right\}$$

Take the derivative of W_1^C with respect to p:

$$\frac{\partial W_1^C(p)}{\partial p} = \frac{-s_1 p + (2m_c - p)\beta_G}{\sum_{i \in I}(\alpha_i)^2 (p)^3}$$

There are two possible cases to consider:

Case 1.a: Suppose that $s_1 \geq \beta_G$. Then, W_1^C is strictly decreasing in p for all $p \in [m_c, 2m_c]$. Thus, the price that maximizes W_1^C is $p = m_c$. Therefore, the unique equilibrium outcome is $(p, T_1, T_2) = \left(m_c, T_1^C(m_c), T_2^C(m_c)\right)$.

Case 1.b: Suppose that $s_1 < \beta_G$. Then, W_1^C is strictly increasing in p for all $p \in \left[m_c, \hat{p}^1\right]$ and strictly decreasing in p for all $p \in \left[\hat{p}^1, 2m_c\right]$, where $\hat{p}^1 = \frac{2m_c\beta_G}{s_1 + \beta_G}$. Thus, W_1^C adopts its maximum at $p = \hat{p}^1$. Therefore, the unique equilibrium outcome is $(p, T_1, T_2) = (p, T_1, T_2) = \left(p^T, T_1^C\left(\hat{p}^1\right), T_2^C\left(\hat{p}^1\right)\right)$.

Proof of Part 2: Suppose that $\sqrt{SB_1} - S \leq \pi_G(2m_c)$. Then, from Lemma 2, we have:

$$T_1^C(p) = \begin{cases} \gamma_1 \left\{ \sqrt{S(B_1+B_2)} - S - \pi_G(p) - \left[1 - \sqrt{\frac{B_1}{B_1+B_2}}\right]\sqrt{SB_1}\frac{B_2}{B_1} \right\} & \\ + (1-\gamma_1)\left\{ \left[1 - \sqrt{\frac{B_1}{B_1+B_2}}\right]\sqrt{SB_1} + \sqrt{SB_1} - S - \pi_G(p) \right\} & \text{if } m_c \leq p < \bar{p} \\ \left\{ \gamma_1 + \frac{S[(1-\gamma_1)B_1 - \gamma_1 B_2]}{[\pi_G(p)+S]\sqrt{S(B_1+B_2)}} \right\} \left[\sqrt{S(B_1+B_2)} - S - \pi_G(p)\right] & \text{if } \bar{p} \leq p \leq 2m_c \end{cases}$$

$$T_2^C(p) = \begin{cases} \gamma_1 \left[1 - \sqrt{\frac{B_1}{B_1+B_2}}\right]\sqrt{SB_1}\frac{B_2}{B_1} & \\ + (1-\gamma_1)\left\{ \sqrt{S(B_1+B_2)} - \sqrt{SB_1} - \left[1 - \sqrt{\frac{B_1}{B_1+B_2}}\right]\sqrt{SB_1} \right\} & \text{if } m_c \leq p < \bar{p} \\ \left\{ 1 - \gamma_1 - \frac{S[(1-\gamma_1)B_1 - \gamma_1 B_2]}{[\pi_G(p)+S]\sqrt{S(B_1+B_2)}} \right\} \left[\sqrt{S(B_1+B_2)} - S - \pi_G(p)\right] & \text{if } \bar{p} \leq p \leq 2m_c \end{cases}$$

Hence, the price selected by player 1 is the solution to the following optimization problem:

$$\max_{p\in[m_c,2m_c]}\left\{W_1^C(p)=\frac{(\alpha_1)^2}{p}+y_1-T_1^C(p)+\frac{\sqrt{S(B_1+B_2)}-S}{\sqrt{S(B_1+B_2)}}B_1\right\}$$

W_1^C has the following properties:

- W_1^C is a continuous function of p for all $p\in[m_c,2m_c]$. In particular, it is continuous at $p=\bar{p}$. To prove this, note that:

$$\begin{aligned}\lim_{p\to\bar{p}^-}W_1^C(p)&=\frac{(\alpha_1)^2}{\bar{p}}+y_1-T_1^C(\bar{p})+\frac{\sqrt{S(B_1+B_2)}-S}{\sqrt{S(B_1+B_2)}}B_1\\&=\frac{(\alpha_1)^2}{\bar{p}}+y_1+\frac{\sqrt{S(B_1+B_2)}-S}{\sqrt{S(B_1+B_2)}}B_1\\&-\gamma_1\left[\sqrt{S(B_1+B_2)}-\sqrt{SB_1}\right]-[(1-\gamma_1)B_1-\gamma_1B_2]\left[1-\sqrt{\frac{B_1}{B_1+B_2}}\right]\\&=\lim_{p\to\bar{p}^+}W_1^C(p),\end{aligned}$$

  where we have employed that $\sqrt{SB_1}-S-\pi_G(\bar{p})=0$.

- Take the derivative of W_1^C with respect to p for $p\in[m_c,\bar{p})$:

$$\frac{\partial W_1^C(p)}{\partial p}=\frac{-s_1p+(2m_c-p)\beta_G}{(p)^3\sum_{i\in I}(\alpha_i)^2}$$

  Let $N(p)=-s_1p+(2m_c-p)\beta_G$ be the numerator of $\frac{\partial W_1^C(p)}{\partial p}$. Since $N(p)$ is strictly decreasing in p, there are three possible cases to consider:

  - Suppose that $s_1\geq\beta_G$. Then, $N(m_c)\leq 0$ and, hence, W_1^C is strictly decreasing in p for all $p\in[m_c,\bar{p})$.
  - Suppose that $\left(\frac{2m_c-\bar{p}}{\bar{p}}\right)\beta_G<s_1<\beta_G$. Then, $N(m_c)>0>\lim_{p\to\bar{p}^-}N(p)$ and, hence, W_1^C is strictly increasing in p for all $p\in\left[m_c,\hat{p}^1\right]$ and strictly decreasing in p for $p\in\left[\hat{p}^1,\bar{p}\right)$, where $\hat{p}^1=\frac{2m_c\beta_G}{s_1+\beta_G}\in(m_c,\bar{p})$.
  - Suppose that $s_1\leq\left(\frac{2m_c-\bar{p}}{\bar{p}}\right)\beta_G$. Then, $\lim_{p\to\bar{p}^-}N(p)\geq 0$ and, hence, W_1^C is strictly increasing in p for all $p\in[m_c,\bar{p})$.

- Take the derivative of W_1^C with respect to p for $p\in[\bar{p},2m_c]$:

$$\frac{\partial W_1^C(p)}{\partial p}=\frac{-s_1p+\left\{\gamma_1+\frac{S[B_1-\gamma_1(B_1+B_2)]}{[\pi_G(p)+S]^2}\right\}(2m_c-p)\beta_G}{(p)^3\sum_{i\in I}(\alpha_i)^2}$$

Let $N(p) = -s_1 p + \left\{\gamma_1 + \frac{S[B_1-\gamma_1(B_1+B_2)]}{[\pi_G(p)+S]^2}\right\}(2m_c - p)\beta_G$ be the numerator of $\frac{\partial W_1^C(p)}{\partial p}$. Then:

$$N(2m_c) = -s_1 p < 0$$

$$\frac{\partial N(p)}{\partial p} = -s_1 - \frac{2S[B_1-\gamma_1(B_1+B_2)](2m_c-p)^2(\beta_G)^2\sum_{i\in I}(\alpha_i)^2}{[\pi_G(p)+S]^3 p^3} - \left\{\gamma_1 + \frac{S[B_1-\gamma_1(B_1+B_2)]}{[\pi_G(p)+S]^2}\right\}\beta_G$$

$$\frac{\partial^2 N(p)}{(\partial p)^2} < 0 \textit{ if and only if } \gamma_1 > B_1/(B_1+B_2)$$

It is clear that whenever $\gamma_1 \leq \frac{B_1}{B_1+B_2}$, $N(p)$ is strictly decreasing in p. For $\gamma_1 > \frac{B_1}{B_1+B_2}$, $N(p)$ is strictly concave in p. Moreover, if $\frac{B_1}{B_1+B_2} < \gamma_1 \leq \bar{\gamma} = \frac{B_1 2S\beta_G(2m_c-\bar{p})^2\sum_{i\in I}(\alpha_i)^2 + B_1\sqrt{SB_1}\bar{p}^3}{(B_1+B_2)2S\beta_G\sum_{i\in I}(\alpha_i)^2(2m_c-\bar{p})^2 + B_2\sqrt{SB_1}\bar{p}^3}$, then $\frac{\partial N(\bar{p})}{\partial p} \leq 0$ and, hence, $N(p)$ is strictly decreasing in p. If $\gamma_1 > \bar{\gamma}$, then there exists $p^* \in (\bar{p}, 2m_c)$ such that $N(p)$ is strictly increasing in p for all $p \in [\bar{p}, p^*]$ and strictly decreasing in p for all $p \in [p^*, 2m_c]$. Overall, we have two possible cases to consider:

- Suppose that $s_1 \geq \beta_G\left(\frac{B_1-\gamma_1 B_2}{B_1}\right)\left(\frac{2m_c-\bar{p}}{\bar{p}}\right)$. Then, $N(\bar{p}) \leq 0$. If $\gamma_1 \leq \bar{\gamma}$, then $N(p)$ is strictly decreasing in p. Therefore, W_1^C is strictly decreasing in p for all $p \in [\bar{p}, 2m_c]$. If $\gamma_1 > \bar{\gamma}$, $N(p)$ is strictly increasing in p for all $p \in [\bar{p}, p^*]$ and strictly decreasing in p for all $p \in [p^*, 2m_c]$. There are two possible cases. If $N(p^*) \leq 0$, then W_1^C is strictly decreasing in p for all $p \in [\bar{p}, 2m_c]$. If $N(p^*) > 0$, then W_1^C is strictly decreasing in p for $p \in [\bar{p}, p']$, strictly increasing in p for $p \in [p', \hat{p}^3]$ and strictly decreasing in p for $p \in [\hat{p}^3, 2m_c]$, where p' is the solution to $N(p) = 0$ that satisfies $\frac{\partial N(p)}{\partial p} > 0$ and $\hat{p}^3 \in (\bar{p}, 2m_c)$ is the solution to $N(p) = 0$ that satisfies $\frac{\partial N(p)}{\partial p} < 0$.
- Suppose that $s_1 < \beta_G\left(\frac{B_1-\gamma_1 B_2}{B_1}\right)\left(\frac{2m_c-\bar{p}}{\bar{p}}\right)$. Then, $N(\bar{p}) > 0$. $N(p)$ is either strictly decreasing in p (when $\gamma_1 \leq \bar{\gamma}$) or it is strictly increasing in p for all $p \in [\bar{p}, p^*]$ and strictly decreasing in p for all $p \in [p^*, 2m_c]$ (when $\gamma_1 > \bar{\gamma}$). Moreover, $N(2m_c) < 0$. Therefore, W_1^C is strictly increasing in p for all $p \in [\bar{p}, \hat{p}^3]$ and strictly decreasing in p for all $p \in [\hat{p}^3, 2m_c]$, where $\hat{p}^3 \in (\bar{p}, 2m_c)$ is the unique solution to $N(p) = 0$.

• Finally, note that if $\frac{\partial W_1^C(\bar{p})}{\partial p} = \frac{-s_1\bar{p} + (2m_c-\bar{p})\beta_G}{(\bar{p})^3\sum_{i\in I}(\alpha_i)^2} < 0$, then $\frac{\partial W_1^C(p)}{\partial p} = \frac{-s_1 p + \left\{\gamma_1 + \frac{S[B_1-\gamma_1(B_1+B_2)]}{[\pi_G(p)+S]^2}\right\}(2m_c-p)\beta_G}{(p)^3\sum_{i\in I}(\alpha_i)^2} < 0$ for all $p \in [\bar{p}, 2m_c]$. The reason is that $\gamma_1 + \frac{S[B_1-\gamma_1(B_1+B_2)]}{[\pi_G(p)+S]^2} < 1$.

Employing the above characterization of $W_1^C(p)$ we have the following possible cases:

Case 2.a: Suppose that $s_1 \geq \beta_G$. Then, W_1^C is strictly decreasing in p for all $p \in [m_c, 2m_c]$. Thus, $(p, T_1, T_2) = \left(m_c, T_1^C(m_c), T_2^C(m_c)\right)$.

Case 2.b: Suppose that $\left(\frac{2m_c-\bar{p}}{\bar{p}}\right)\beta_G < s_1 < \beta_G$. Then, W_1^C is strictly decreasing in p for all $p \in \left[m_c, \hat{p}^1\right]$ and strictly decreasing in p for $p \in \left[\hat{p}^1, 2m_c\right]$, where $\hat{p}^1 = \frac{2m_c\beta_G}{s_1+\beta_G} \in (m_c, \bar{p})$. Thus, $(p, T_1, T_2) = \left(\hat{p}^1, T_1^C\left(\hat{p}^1\right), T_2^C\left(\hat{p}^1\right)\right)$.

Case 2.c: Suppose that $\beta_G\left(\frac{B_1-\gamma_1 B_2}{B_1}\right)\left(\frac{2m_c-\bar{p}}{\bar{p}}\right) \leq s_1 \leq \left(\frac{2m_c-\bar{p}}{\bar{p}}\right)\beta_G$. If $[\gamma_1 \leq \bar{\gamma}]$ or $[\gamma_1 > \bar{\gamma}$ and $N(p^*) \leq 0]$, then W_1^C is strictly increasing in p for all $p \in [m_c, \bar{p}]$ and strictly decreasing in p for $p \in [\bar{p}, 2m_c]$. Thus, $(p, T_1, T_2) = \left(\bar{p}, T_1^C(\bar{p}), T_2^C(\bar{p})\right)$. If $\gamma_1 > \bar{\gamma}$ and $N(p^*) > 0$. then W_1^C adopts its maximum either at $p = \bar{p}$ or at $p = \hat{p}^3$, where $\hat{p}^3 \in (\bar{p}, 2m_c)$ is the unique solution to $N(p) = 0$. That is, $\hat{p}^3 \in (\bar{p}, 2m_c)$ is the unique solution to:

$$\left(\frac{2m_c - p}{p}\right)\beta_G = \frac{s_1\left[\pi(p) + S\right]^2}{\gamma_1\left[\pi(p) + S\right]^2 + S\left[(1-\gamma_1)B_1 - \gamma_1 B_2\right]}$$

Thus, $(p, T_1, T_2) = \left(\bar{p}, T_1^C(\bar{p}), T_2^C(\bar{p})\right)$ or $(p, T_1, T_2) = \left(\hat{p}^3, T_1^C\left(\hat{p}^3\right), T_2^C\left(\hat{p}^3\right)\right)$

Case 2.d: Suppose that $s_1 < \beta_G\left(\frac{B_1-\gamma_1 B_2}{B_1}\right)\left(\frac{2m_c-\bar{p}}{\bar{p}}\right)$. Then, W_1^C is strictly increasing in p for all $p \in \left[0, \hat{p}^3\right]$ and strictly decreasing in p for all $p \in \left[\hat{p}^3, 2m_c\right]$, where $\hat{p}^3 \in (\bar{p}, 2m_c)$ is the unique solution to $N(p) = 0$. Thus, $(p, T_1, T_2) = \left(\hat{p}^3, T_1^C\left(\hat{p}^3\right), T_2^C\left(\hat{p}^3\right)\right)$.

This completes the proof of Proposition 2.

Proof that $[\hat{p}^2 > \hat{p}^3, \bar{p}]$. We have already proved that $\hat{p}^3 \in (\bar{p}, 2m_c)$. Thus, we only need to prove that $\hat{p}^2 > \hat{p}^3$. $\hat{p}^2$ is the unique solution to:

$$\left(\frac{2m_c - p}{p}\right)\beta_G = \frac{s_1\left[\pi_G(p) + S\right]^2}{SB_1}$$

while $\hat{p}^3$ is the unique solution to:

$$\left(\frac{2m_c - p}{p}\right)\beta_G = \frac{s_1\left[\pi(p) + S\right]^2}{\gamma_1\left[\pi(p) + S\right]^2 + S\left[(1-\gamma_1)B_1 - \gamma_1 B_2\right]}$$

It is easy to verify if

$$\frac{1}{\gamma_1\left[\pi(p) + S\right]^2 + S\left[(1-\gamma_1)B_1 - \gamma_1 B_2\right]} > \frac{1}{SB_1}$$

$SB_1 > \gamma_1\left[\pi(p) + S\right]^2 + S\left[(1-\gamma_1)B_1 - \gamma_1 B_2\right]$, then $\hat{p}^2 > \hat{p}^3$. Moreover, since $\sqrt{S(B_1 + B_2)} - S > \pi(p)$ for all $p \in [m_c, 2m_c]$, it is always the case that $SB_1 > \gamma_1\left[\pi(p) + S\right]^2 + S\left[(1-\gamma_1)B_1 - \gamma_1 B_2\right]$. ■

A.3 Voters' Biases (Lemma 3 and Proposition 3)

Lemma 3 *Assume that* $\sqrt{SB_2} - S < \bar{T} < \sqrt{SB_1} - S$. *Suppose that* 1 *has selected* $p \in [m_c, 2m_c]$.

1. *Suppose that* $\sqrt{SB_1} - S > \pi_G(2m_c)$.

   (a) *Suppose that* $0 < \bar{T} \leq \sqrt{SB_1} - S - \pi_G(2m_c)$. *Then, the unique Nash equilibrium profile of transfers is* $\left(T_1^R(p), T_1^R(p)\right) = \left(\bar{T}, 0\right)$ *for all* $p \in [m_c, 2m_c]$.

(b) *Suppose that* $\bar{T} > \sqrt{SB_1} - S - \pi_G(2m_c)$. *Then, the unique Nash equilibrium profile of transfers is*

$$\left(T_1^R(p), T_2^R(p)\right) = \begin{cases} \left(\bar{T}, 0\right) & \text{if } m_c \le p \le \bar{p}_{\bar{T}} \\ \left(\sqrt{SB_1} - S - \pi_G(p), 0\right) & \text{if } \bar{p}_{\bar{T}} < p \le 2m_c \end{cases}$$

where $\bar{p}_{\bar{T}} = \frac{\beta_G \sum_{i\in I}(\alpha_i)^2}{2\left(\sqrt{SB_1}-S-\bar{T}\right)}\left[1 - \sqrt{1 - \frac{4m_c\left(\sqrt{SB_1}-S-\bar{T}\right)}{\beta_G \sum_{i\in I}(\alpha_i)^2}}\right]$.

2. *Suppose that* $\sqrt{SB_1} - S \le \pi_G(2m_c)$. *Then, the unique Nash equilibrium profile of transfers is*

$$\left(T_1^R(p), T_2^R(p)\right) = \begin{cases} \left(\bar{T}, 0\right) & \text{if } m_c \le p \le \bar{p}_{\bar{T}} \\ \left(\sqrt{SB_1} - S - \pi_G(p), 0\right) & \text{if } \bar{p}_{\bar{T}} < p < \bar{p} \\ (0,0) & \text{if } \bar{p} \le p \le 2m_c \end{cases}$$

Proof: Following the procedure employed in the proof of Lemma 1 we obtain the same 9 candidates for a Nash equilibrium profile of transfers. Since $\sqrt{SB_2} - S < \bar{T} < \sqrt{SB_1} - S$, the conditions required in cases N1, N2, N4, N5, N7, and N8, never hold. Therefore, we have:

Case N3 (from Lemma 1): $(T_1, T_2) = \left(\bar{T}, 0\right)$ is a Nash equilibrium profile of transfers if and only if $\sqrt{SB_1} - S - \pi_G(p) \ge \bar{T}$.

Case N6 (from Lemma 1): $(T_1, T_2) = \left(\sqrt{SB_1} - S - \pi_G(p), 0\right)$ is a Nash equilibrium profile of transfers if and only if $0 < \sqrt{SB_1} - S - \pi_G(p) < \bar{T}$.

Case N9 (from Lemma 1): $(T_1, T_2) = (0,0)$ is a Nash equilibrium profile of transfers if and only if $\sqrt{SB_1} - S - \pi_G(p) \le 0$.

Recall that

$$\bar{p} = \frac{\beta_G \sum_{i\in I}(\alpha_i)^2}{2\left(\sqrt{SB_1} - S\right)}\left[1 - \sqrt{1 - \frac{4m_c\left(\sqrt{SB_1} - S\right)}{\beta_G \sum_{i\in I}(\alpha_i)^2}}\right]$$

and define:

$$\bar{p}_{\bar{T}} = \frac{\beta_G \sum_{i\in I}(\alpha_i)^2}{2\left(\sqrt{SB_1} - S - \bar{T}\right)}\left[1 - \sqrt{1 - \frac{4m_c\left(\sqrt{SB_1} - S - \bar{T}\right)}{\beta_G \sum_{i\in I}(\alpha_i)^2}}\right]$$

There are several cases to consider:

Case 1.a: Suppose that $\sqrt{SB_1} - S > \pi_G(2m_c)$ and $0 < \bar{T} \le \sqrt{SB_1} - S - \pi_G(2m_c)$. Then, the conditions in cases 6 and 9 never hold. Therefore, $(T_1, T_2) = \left(\bar{T}, 0\right)$.

Case 1.b: Suppose that $\sqrt{SB_1} - S > \pi_G(2m_c)$ and $\bar{T} > \sqrt{SB_1} - S - \pi_G(2m_c)$ Then, the condition in case 9 never holds. If $\sqrt{SB_1} - S - \pi_G(p) \ge \bar{T}$ (equivalently, $p \le \bar{p}_{\bar{T}}$), then $(T_1, T_2) = \left(\bar{T}, 0\right)$. If $\sqrt{SB_1} - S - \pi_G(p) < \bar{T}$ (equivalently, $p > \bar{p}_{\bar{T}}$), then $(T_1, T_2) = \left(\sqrt{SB_1} - S - \pi_G(p), 0\right)$.

Case 2: Suppose that$\sqrt{SB_1} - S \le \pi_G(2m_c)$. If $\sqrt{SB_1} - S - \pi_G(p) \ge \bar{T}$ (equivalently, $p \le \bar{p}_{\bar{T}}$), then $(T_1, T_2) = \left(\bar{T}, 0\right)$. If $0 < \sqrt{SB_1} - S - \pi_G(p) < \bar{T}$ (equivalently, $\bar{p}_{\bar{T}} < p < \bar{p}$), then $(T_1, T_2) = \left(\sqrt{SB_1} - S - \pi_G(p), 0\right)$. Finally, if $\sqrt{SB_1} - S - \pi_G(p) \le 0$ (equivalently, $p \ge \bar{p}$), then $(T_1, T_2) = (0,0)$.

This completes the proof of Lemma 3. ■

Proposition 3 *Assume that* $\sqrt{SB_2} - S < \bar{T} < \sqrt{SB_1} - S$.

1. *Suppose that* $\sqrt{SB_1} - S > \pi_G(2m_c)$ *and* $0 < \bar{T} \le \sqrt{SB_1} - S - \pi_G(2m_c)$.

(a) *If* $s_1 \geq \frac{\beta_G SB_1}{(\bar{T}+S)^2}$, *then the unique subgame perfect Nash equilibrium outcome is* $\left(p, T_1^R, T_2^R\right) = \left(m_c, \bar{T}, 0\right)$.

(b) *If* $s_1 < \frac{\beta_G SB_1}{(\bar{T}+S)^2}$, *then the unique subgame perfect Nash equilibrium outcome is* $\left(p, T_1^R, T_2^R\right) = \left(\hat{p}^4, \bar{T}, 0\right)$, *where* $\hat{p}^4$ *is the unique solution to* $\left(\frac{2m_c - p}{p}\right)\beta_G = \frac{s_1\left[\pi_G(p)+\bar{T}+S\right]^2}{SB_1}$.

2. *Suppose that* $\sqrt{SB_1} - S > \pi_G(2m_c)$ *and* $\bar{T} > \sqrt{SB_1} - S - \pi_G(2m_c)$ *and let*

$$\bar{p}_{\bar{T}} = \frac{\beta_G \sum_{i\in I}(\alpha_i)^2}{2\left(\sqrt{SB_1} - S - \bar{T}\right)}\left[1 - \sqrt{1 - \frac{4m_c\left(\sqrt{SB_1} - S - \bar{T}\right)}{\beta_G \sum_{i\in I}(\alpha_i)^2}}\right]$$

(a) *If* $s_1 \geq \frac{\beta_G SB_1}{(\bar{T}+S)^2}$, *then the unique subgame perfect Nash equilibrium outcome is* $\left(p, T_1^R, T_2^R\right) = \left(m_c, \bar{T}, 0\right)$.

(b) *If* $\frac{(2m_c - \bar{p}_{\bar{T}})\beta_G}{\bar{p}_{\bar{T}}} \leq s_1 < \frac{\beta_G SB_1}{(\bar{T}+S)^2}$, *then the unique subgame perfect Nash equilibrium outcome is* $\left(p, T_1^R, T_2^R\right) = \left(\hat{p}^4, \bar{T}, 0\right)$.

(c) *If* $s_1 < \frac{(2m_c - \bar{p}_{\bar{T}})\beta_G}{\bar{p}_{\bar{T}}}$, *then the unique subgame perfect Nash equilibrium outcome is* $\left(p, T_1^R, T_2^R\right) = \left(\hat{p}^1, \sqrt{SB_1} - S - \pi_G\left(\hat{p}^1\right), 0\right)$

3. *Suppose that* $\sqrt{SB_1} - S \leq \pi_G(2m_c)$.

(a) *If* $s_1 \geq \frac{\beta_G SB_1}{(\bar{T}+S)^2}$, *then the unique subgame perfect Nash equilibrium outcome is* $\left(p, T_1^R, T_2^R\right) = \left(m_c, \bar{T}, 0\right)$.

(b) *If* $\frac{(2m_c - \bar{p}_{\bar{T}})}{\bar{p}_{\bar{T}}}\beta_G \leq s_1 < \frac{\beta_G SB_1}{(\bar{T}+S)^2}$, *then the unique subgame perfect Nash equilibrium outcome is* $\left(p, T_1^R, T_2^R\right) = \left(\hat{p}^4, \bar{T}, 0\right)$.

(c) *If* $\left(\frac{2m_c - \bar{p}}{\bar{p}}\right)\beta_G < s_1 < \frac{(2m_c - \bar{p}_{\bar{T}})}{\bar{p}_{\bar{T}}}\beta_G$, *then the unique subgame perfect Nash equilibrium outcome is* $\left(p, T_1^R, T_2^R\right) = \left(\hat{p}^1, \sqrt{SB_1} - S - \pi_G\left(\hat{p}^1\right), 0\right)$

(d) *If* $s_1 \leq \left(\frac{2m_c - \bar{p}}{\bar{p}}\right)\beta_G$, *then the unique subgame perfect Nash equilibrium outcome is* $\left(p, T_1^R, T_2^R\right) = \left(\hat{p}^2, 0, 0\right)$.

Proof of Part 1: Suppose that $\sqrt{SB_1} - S > \pi_G(2m_c)$ and $0 < \bar{T} \leq \sqrt{SB_1} - S - \pi_G(2m_c)$. Then, from Lemma 3, $\left(T_1^R(p), T_1^R(p)\right) = \left(\bar{T}, 0\right)$ for all $p \in [m_c, 2m_c]$. Hence, the price selected by player 1 is the solution to the following optimization problem:

$$\max_{p\in[m_c,2m_c]}\left\{W_1^R(p) = \frac{(\alpha_1)^2}{p} + y_1 - \bar{T} + \frac{\pi_G(p) + \bar{T}}{\pi_G(p) + \bar{T} + S}B_1\right\}$$

- Take the derivative of $W_1^R(p)$ with respect to p:

$$\frac{\partial W_1^R(p)}{\partial p} = \frac{-s_1 p + \frac{(2m_c - p)\beta_G S B_1}{\left[\pi_G(p) + \bar{T} + S\right]^2}}{p^3 \sum_{i\in I} (\alpha_i)^2}$$

  Let $N(p) = -s_1 p + \frac{(2m_c - p)\beta_G S B_1}{\left[\pi_G(p) + \bar{T} + S\right]^2}$ be the numerator of $\frac{\partial W_1(p)}{\partial p}$. $N(p)$ is decreasing in p and $N(2m_c) < 0$. Thus, there are two possible cases to consider.

  – Suppose that $s_1 \geq \frac{\beta_G S B_1}{\left(\bar{T}+S\right)^2}$. Then, W_1^R is strictly decreasing in p for all $p \in [m_c, 2m_c]$.

  – Suppose that $s_1 < \frac{\beta_G S B_1}{\left(\bar{T}+S\right)^2}$. Then, W_1^R is strictly increasing in p for all $p \in \left[m_c, \hat{p}^4\right]$ and strictly decreasing in p for all $p \in \left[\hat{p}^4, 2m_c\right]$, where $\hat{p}^4 \in (m_c, 2m_c)$ is the unique solution to $\left(\frac{2m_c - p}{p}\right)\beta_G = \frac{s_1\left[\pi_G(p) + \bar{T} + S\right]^2}{SB_1}$.

Employing the above characterization of $W_1^R(p)$ we have the following possible cases:

Case 1.a: Suppose that $s_1 \geq \frac{\beta_G S B_1}{\left(\bar{T}+S\right)^2}$. Then, W_1^R is strictly decreasing in p for all $p \in [m_c, 2m_c]$. Thus, the price that maximizes W_1^R is $p = m_c$. Therefore, the unique subgame perfect Nash equilibrium outcome is $\left(p, T_1^R, T_2^R\right) = \left(m_c, \bar{T}, 0\right)$.

Case 1.b: Suppose that $s_1 < \frac{\beta_G S B_1}{\left(\bar{T}+S\right)^2}$. Then, W_1^R adopts its maximum at $p = \hat{p}^4$. Therefore, the unique subgame perfect Nash equilibrium outcome is $\left(p, T_1^R, T_2^R\right) = \left(\hat{p}^4, \bar{T}, 0\right)$.

Proof of Part 2: Suppose that $\sqrt{SB_1} - S > \pi_G(2m_c)$ and $\bar{T} > \sqrt{SB_1} - S - \pi_G(2m_c)$. Then, from Lemma 3, we have:

$$\left(T_1^R(p), T_2^R(p)\right) = \begin{cases} \left(\bar{T}, 0\right) & \text{if } m_c \leq p \leq \bar{p}_{\bar{T}} \\ \left(\sqrt{SB_1} - S - \pi_G(p), 0\right) & \text{if } \bar{p}_{\bar{T}} < p \leq 2m_c \end{cases}$$

Hence, the price selected by player 1 is the solution to the following optimization problem:

$$\max_{p\in[m_c, 2m_c]} \left\{ W_1^R(p) = \frac{(\alpha_1)^2}{p} + y_1 + \begin{cases} -\bar{T} + \frac{\pi_G(p)+\bar{T}}{\pi_G(p)+\bar{T}+S} B_1 & \text{if } m_c \leq p \leq \bar{p}_{\bar{T}} \\ -\left[\sqrt{SB_1} - S - \pi_G(p)\right] + \frac{\sqrt{SB_1} - S}{\sqrt{SB_1}} B_1 & \text{if } \bar{p}_{\bar{T}} < p \leq 2m_c \end{cases} \right\}$$

W_1^R has the following properties:

- W_1^R is continuous for all $p \in [m_c, 2m_c]$. In particular, it is continuous at $p = \bar{p}_{\bar{T}}$. To prove this, note that:

$$\begin{aligned} \lim_{p\to(\bar{p}_{\bar{T}})^-} W_1^R(p) &= \frac{(\alpha_1)^2}{\bar{p}_{\bar{T}}} + y_1 - \bar{T} + \frac{\pi_G(\bar{p}_{\bar{T}}) + \bar{T}}{\pi_G(\bar{p}_{\bar{T}}) + \bar{T} + S} B_1 \\ &= \frac{(\alpha_1)^2}{\bar{p}_{\bar{T}}} + y_1 - \left[\sqrt{SB_1} - S - \pi_G(\bar{p}_{\bar{T}})\right] + \frac{\sqrt{SB_1} - S}{\sqrt{SB_1}} B_1 \\ &= \lim_{p\to(\bar{p}_{\bar{T}})^+} W_1^R(p) \end{aligned}$$

  where we have employed that $\sqrt{SB_1} - S - \pi_G(\bar{p}_{\bar{T}}) = \bar{T}$.

- Take the derivative of W_1^R with respect to p for $p \in [m_c, \bar{p}_{\bar{T}}]$:

$$\frac{\partial W_1^R(p)}{\partial p} = \frac{-s_1 p + \frac{(2m_c - p)\beta_G S B_1}{\left[\pi_G(p) + \bar{T} + S\right]^2}}{p^3 \sum_{i \in I} (\alpha_i)^2}$$

Let $N(p) = -s_1 p + \frac{(2m_c - p)\beta_G S B_1}{\left[\pi_G(p) + \bar{T} + S\right]^2}$ be the numerator of $\frac{\partial W_1^R(p)}{\partial p}$. $N(p)$ is decreasing in p. Thus, there are three possible cases to consider.

  - Suppose that $s_1 \geq \frac{\beta_G S B_1}{(\bar{T}+S)^2}$. Then, W_1^R is strictly decreasing in p for all $p \in [m_c, \bar{p}_{\bar{T}}]$.
  - Suppose that $\frac{(2m_c - \bar{p}_{\bar{T}})\beta_G}{\bar{p}_{\bar{T}}} < s_1 < \frac{\beta_G S B_1}{(\bar{T}+S)^2}$. Then, W_1^R is strictly increasing in p for all $p \in \left[m_c, \hat{p}^4\right]$ and strictly decreasing in p for all $p \in \left[\hat{p}^4, \bar{p}_{\bar{T}}\right]$, $\hat{p}^4 \in (m_c, \bar{p}_{\bar{T}})$ is the unique solution to $\left(\frac{2m_c - p}{p}\right)\beta_G = \frac{s_1\left[\pi_G(p) + \bar{T} + S\right]^2}{S B_1}$.
  - Suppose that $s_1 \leq \frac{(2m_c - \bar{p}_{\bar{T}})\beta_G}{\bar{p}_{\bar{T}}}$. Then, W_1^R is strictly increasing in p for all $p \in [m_c, \bar{p}_{\bar{T}}]$.

- Take the derivative of W_1^R with respect to p for $p \in (\bar{p}_{\bar{T}}, 2m_c]$:

$$\frac{\partial W_1^R(p)}{\partial p} = \frac{-s_1 p + (2m_c - p)\beta_G}{p^3 \sum_{i \in I} (\alpha_i)^2}$$

Let $N(p) = -s_1 p + (2m_c - p)\beta_G$ be the numerator of $\frac{\partial W_1^R(p)}{\partial p}$. $N(p)$ is decreasing in p and $N(2m_c) < 0$. Thus, there are two possible cases to consider.

  - Suppose that $s_1 \geq \frac{(2m_c - \bar{p}_{\bar{T}})}{\bar{p}_{\bar{T}}}\beta_G$. Then, W_1^R is decreasing in p for all $p \in (\bar{p}_{\bar{T}}, 2m_c]$.
  - Suppose that $s_1 < \frac{(2m_c - \bar{p}_{\bar{T}})}{\bar{p}_{\bar{T}}}\beta_G$. Then, W_1^R is strictly increasing in p for all $p \in \left(\bar{p}_{\bar{T}}, \hat{p}^1\right]$ and strictly decreasing in p for all $p \in \left[\hat{p}^1, 2m_c\right]$, where $\hat{p}^1 = \frac{2m_c\beta_G}{s_1 + \beta_G} \in (\bar{p}_{\bar{T}}, 2m_c)$.

Employing the above characterization of W_1^R we have the following possible cases:

Case 2.a: Suppose that $s_1 \geq \frac{\beta_G S B_1}{(\bar{T}+S)^2}$. Then, W_1^R is strictly decreasing in p for all $p \in [m_c, 2m_c]$. Thus, the price that maximizes W_1^R is $p = m_c$. Therefore, the unique subgame perfect Nash equilibrium outcome is $\left(p, T_1^R, T_2^R\right) = \left(m_c, \bar{T}, 0\right)$.

Case 2.b: Suppose that $\frac{(2m_c - \bar{p}_{\bar{T}})\beta_G}{\bar{p}_{\bar{T}}} \leq s_1 < \frac{\beta_G S B_1}{(\bar{T}+S)^2}$. Then, W_1^R is strictly increasing in p for all $p \in \left[m_c, \hat{p}^4\right]$ and strictly decreasing in p for all $p \in \left[\hat{p}^4, 2m_c\right]$. Thus, the price that maximizes W_1^R is $p = \hat{p}^4 \in (m_c, \bar{p}_{\bar{T}}]$. Therefore, the unique subgame perfect Nash equilibrium outcome is $\left(p, T_1^R, T_2^R\right) = \left(\hat{p}^4, \bar{T}, 0\right)$.

Case 2.c: Suppose that $s_1 < \frac{(2m_c - \bar{p}_{\bar{T}})\beta_G}{\bar{p}_{\bar{T}}}$. Then, W_1^R is strictly increasing in p for all $p \in \left[m_c, \hat{p}^1\right]$ and strictly decreasing for all $p \in \left[\hat{p}^1, 2m_c\right]$. Thus, the price that maximizes W_1^R is $p = \hat{p}^1 \in (\bar{p}_{\bar{T}}, 2m_c)$. Therefore, the unique subgame perfect Nash equilibrium outcome is $\left(p, T_1^R, T_2^R\right) = \left(\hat{p}^1, \sqrt{S B_1} - S - \pi_G\left(\hat{p}^1\right), 0\right)$.

Proof of Part 3: Suppose that $\sqrt{SB_1} - S \leq \pi_G(2m_c)$. Then, from Lemma 3, we have:

$$\left(T_1^R(p), T_2^R(p)\right) = \begin{cases} \left(\bar{T}, 0\right) & \text{if } m_c \leq p \leq \bar{p}_{\bar{T}} \\ \left(\sqrt{SB_1} - S - \pi_G(p), 0\right) & \text{if } \bar{p}_{\bar{T}} < p < \bar{p} \\ (0,0) & \text{if } \bar{p} \leq p \leq 2m_c \end{cases}$$

Hence, the price selected by player 1 is the solution to the following optimization problem:

$$\max_{p\in[m_c,2m_c]} \left\{ W_1^R(p) = \frac{(\alpha_1)^2}{p} + y_1 + \begin{cases} -\bar{T} + \frac{\pi_G(p)+\bar{T}}{\pi_G(p)+\bar{T}+S}B_1 & \text{if } m_c \leq p \leq \bar{p}_{\bar{T}} \\ -\left[\sqrt{SB_1} - S - \pi_G(p)\right] + \frac{\sqrt{SB_1}-S}{\sqrt{SB_1}}B_1 & \text{if } \bar{p}_{\bar{T}} < p < \bar{p} \\ \frac{\pi_G(p)}{\pi_G(p)+S}B_1 & \text{if } \bar{p} \leq p \leq 2m_c \end{cases} \right\}$$

W_1^R has the following properties:

- W_1^R is continuous for all $p \in [m_c, 2m_c]$. In particular, it is continuous at $p = \bar{p}_{\bar{T}}$ and $p = \bar{p}$. To prove that it is continuous at $p = \bar{p}_{\bar{T}}$, note that:

$$\begin{aligned} \lim_{p\to(\bar{p}_{\bar{T}})^-} W_1^R(p) &= \frac{(\alpha_1)^2}{\bar{p}_{\bar{T}}} + y_1 - \bar{T} + \frac{\pi_G(\bar{p}_{\bar{T}}) + \bar{T}}{\pi_G(\bar{p}_{\bar{T}}) + \bar{T} + S}B_1 \\ &= \frac{(\alpha_1)^2}{\bar{p}_{\bar{T}}} + y_1 - \left[\sqrt{SB_1} - S - \pi_G(\bar{p}_{\bar{T}})\right] + \frac{\sqrt{SB_1} - S}{\sqrt{SB_1}}B_1 = \lim_{p\to(\bar{p}_{\bar{T}})^+} W_1^R(p) \end{aligned}$$

  where we have employed that $\sqrt{SB_1} - S - \pi_G(\bar{p}_{\bar{T}}) = \bar{T}$. To prove that it is continuous at $p = \bar{p}$, note that:

$$\begin{aligned} \lim_{p\to(\bar{p})^-} W_1^R(p) &= \frac{(\alpha_1)^2}{\bar{p}} + y_1 - \left[\sqrt{SB_1} - S - \pi_G(\bar{p})\right] + \frac{\sqrt{SB_1} - S}{\sqrt{SB_1}}B_1 \\ &= \frac{(\alpha_1)^2}{\bar{p}} + y_1 + \frac{\pi_G(\bar{p})}{\pi_G(\bar{p}) + S}B_1 = \lim_{p\to(\bar{p})^+} W_1^R(p) \end{aligned}$$

  where we have employed that $\sqrt{SB_1} - S - \pi_G(\bar{p}) = 0$.

- Take the derivative of W_1^R with respect to p for $p \in [m_c, \bar{p}_{\bar{T}}]$:

$$\frac{\partial W_1^R(p)}{\partial p} = \frac{-s_1 p + \frac{(2m_c-p)\beta_G SB_1}{\left[\pi_G(p)+\bar{T}+S\right]^2}}{p^3 \sum_{i\in I}(\alpha_i)^2}$$

  Let $N(p) = -s_1 p + \frac{(2m_c-p)\beta_G SB_1}{\left[\pi_G(p)+\bar{T}+S\right]^2}$ be the numerator of $\frac{\partial W_1^R(p)}{\partial p}$. $N(p)$ is decreasing in p. Thus, there are three possible cases to consider.

  – Suppose that $s_1 \geq \frac{\beta_G SB_1}{(\bar{T}+S)^2}$. Then, W_1^R is strictly decreasing in p for all $p \in [m_c, \bar{p}_{\bar{T}}]$.

– Suppose that $\frac{(2m_c-\bar{p}_{\bar{T}})\beta_G}{\bar{p}_{\bar{T}}} < s_1 < \frac{\beta_G SB_1}{(\bar{T}+S)^2}$. Then, W_1^R is strictly increasing in p for all $p \in [m_c, \hat{p}^4]$ and strictly decreasing in p for all $p \in [\hat{p}^4, \bar{p}_{\bar{T}}]$, $\hat{p}^4 \in (m_c, \bar{p}_{\bar{T}})$ is the unique solution to $\left(\frac{2m_c-p}{p}\right)\beta_G = \frac{s_1[\pi_G(p)+\bar{T}+S]^2}{SB_1}$.

– Suppose that $s_1 \leq \frac{(2m_c-\bar{p}_{\bar{T}})\beta_G}{\bar{p}_{\bar{T}}}$. Then, W_1^R is strictly increasing in p for all $p \in [m_c, \bar{p}_{\bar{T}}]$.

- Take the derivative of W_1^R with respect to p for $p \in (\bar{p}_{\bar{T}}, \bar{p})$:

$$\frac{\partial W_1^R(p)}{\partial p} = \frac{-s_1 p + (2m_c - p)\beta_G}{p^3 \sum_{i\in I}(\alpha_i)^2}$$

Let $N(p) = -s_1 p + (2m_c - p)\beta_G$ be the numerator of $\frac{\partial W_1^R(p)}{\partial p}$. $N(p)$ is decreasing in p. Thus, there are three possible cases to consider.

– Suppose that $s_1 \geq \frac{(2m_c-\bar{p}_{\bar{T}})\beta_G}{\bar{p}_{\bar{T}}}$. Then, W_1^R is decreasing in p for all $p \in (\bar{p}_{\bar{T}}, \bar{p})$.

– Suppose that $\frac{(2m_c-\bar{p})\beta_G}{\bar{p}} < s_1 < \frac{(2m_c-\bar{p}_{\bar{T}})}{\bar{p}_{\bar{T}}}\beta_G$. Then, W_1^R is strictly increasing in p for all $p \in (\bar{p}_{\bar{T}}, \hat{p}^1]$ and strictly decreasing in p for all $p \in [\hat{p}^1, \bar{p})$, where $\hat{p}^1 = \frac{2m_c\beta_G}{s_1\beta_G}$.

– Suppose that $s_1 \leq \frac{(2m_c-\bar{p})}{\bar{p}}\beta_G$. Then, W_1^R is increasing in p for all $p \in (\bar{p}_{\bar{T}}, \bar{p})$.

- Take the derivative of W_1^R with respect to p for $p \in [\bar{p}, 2m_c]$.

$$\frac{\partial W_1^R(p)}{\partial p} = \frac{-s_1 p + \frac{(2m_c-p)\beta_G SB_1}{[\pi_G(p)+S]^2}}{(p)^3 \sum_{i\in I}(\alpha_i)^2}$$

Let $N(p) = -s_1 p + \frac{(2m_c-p)\beta_G SB_1}{[\pi_G(p)+S]^2}$ be the numerator of $\frac{\partial W_1^R(p)}{\partial p}$. $N(p)$ is decreasing in p and $N(2m_c) = -2s_1 m_c < 0$. Thus, there are two possible cases to consider:

– Suppose that $s_1 \geq \left(\frac{2m_c-\bar{p}}{\bar{p}}\right)\beta_G$. Then, W_1^R is strictly decreasing in p for all $p \in [\bar{p}, 2m_c]$.

– Suppose that $s_1 < \left(\frac{2m_c-\bar{p}}{\bar{p}}\right)\beta_G$. Then, W_1^R is strictly increasing in p for all $p \in [\bar{p}, \hat{p}^2]$ and strictly decreasing in p for all $p \in [\hat{p}^2, 2m_c]$, where $\hat{p}^2 \in (\bar{p}, 2m_c)$ is the unique solution to: $\left(\frac{2m_c-p}{p}\right)\beta_G = \frac{s_1[\pi_G(p)+S]^2}{SB_1}$.

Employing the above characterization of $W_1^R(p)$ we have the following possible cases:

Case 3.a: Suppose that $s_1 \geq \frac{\beta_G SB_1}{(\bar{T}+S)^2}$. Then, W_1^R is strictly decreasing in p for all $p \in [m_c, 2m_c]$. Thus, the price that maximizes W_1^R is $p = m_c$. Therefore, the unique subgame perfect Nash equilibrium outcome is $(p, T_1^R, T_2^R) = (m_c, \bar{T}, 0)$.

Case 3.b: Suppose that $\frac{(2m_c-\bar{p}_{\bar{T}})}{\bar{p}_{\bar{T}}}\beta_G \leq s_1 < \frac{\beta_G SB_1}{(\bar{T}+S)^2}$. Then, W_1^R is strictly increasing in p for all $p \in [m_c, \hat{p}^4]$ and strictly decreasing in p for all $p \in [\hat{p}^4, 2m_c]$. Thus, the price that maximizes W_1^R is

$p = \hat{p}^4 \in (m_c, \bar{p}_{\bar{T}}]$. Therefore, the unique subgame perfect Nash equilibrium outcome is $\left(p, T_1^R, T_2^R\right) = \left(\hat{p}^4, \bar{T}, 0\right)$.

Case 3.c: Suppose that $\left(\frac{2m_c-\bar{p}}{\bar{p}}\right)\beta_G < s_1 < \frac{(2m_c-\bar{p}_{\bar{T}})}{\bar{p}_{\bar{T}}}\beta_G$. Then, W_1^R is strictly increasing in p for all $p \in \left[m_c, \hat{p}^1\right]$ and strictly decreasing in p for all $p \in \left[\hat{p}^1, 2m_c\right]$. Thus, the price that maximizes W_1^R is $p = \hat{p}^1 \in (\bar{p}_{\bar{T}}, \bar{p})$. Therefore, the unique subgame perfect Nash equilibrium outcome is $\left(p, T_1^R, T_2^R\right) = \left(\hat{p}^1, \sqrt{SB_1} - S - \pi_G\left(\hat{p}^1\right), 0\right)$.

Case 3.d: Suppose that $s_1 \leq \left(\frac{2m_c-\bar{p}}{\bar{p}}\right)\beta_G$. Then, W_1^R is strictly increasing in p for all $p \in \left[m_c, \hat{p}^2\right]$ and strictly decreasing in p for all $p \in \left[\hat{p}^2, 2m_c\right]$. Thus, the price that maximizes W_1^R is $p = \hat{p}^2 \in [\bar{p}, 2m_c)$. Therefore, the unique subgame perfect Nash equilibrium outcome is $\left(p, T_1^R, T_2^R\right) = \left(\hat{p}^2, 0, 0\right)$.

This completes the proof of Proposition 3.

Proof that $[s_1 \geq \frac{(2m_c-\bar{p}_{\bar{T}})\beta_G}{\bar{p}_{\bar{T}}}$ implies $\hat{p}^4 \geq \hat{p}^1]$: $\hat{p}^4$ is the unique solution to:

$$\left(\frac{2m_c - p}{p}\right)\beta_G = \frac{s_1\left[\pi_G(p) + \bar{T} + S\right]^2}{SB_1}$$

It is easy to verify that $\hat{p}^4 \geq \hat{p}^1$ if and only if $\bar{T} \leq \sqrt{SB_1} - S - \pi_G\left(\hat{p}^1\right)$ (with strict inequality if $\bar{T} < \sqrt{SB_1} - S - \pi_G\left(\hat{p}^1\right)$). We have to consider three possible cases, corresponding to Propositions 3.1, 3.2, and 3.3, respectively.

Case 1 : Suppose that $\sqrt{SB_1} - S > \pi_G(2m_c)$ and $0 < \bar{T} \leq \sqrt{SB_1} - S - \pi_G(2m_c)$. Then, $\bar{T} < \sqrt{SB_1} - S - \pi_G(p)$ for all $p \in [m_c, 2m_c]$. Thus, $\bar{T} < \sqrt{SB_1} - S - \pi_G\left(\hat{p}^1\right)$ always holds.

Case 2: Suppose that $\sqrt{SB_1} - S > \pi_G(2m_c)$ and $\bar{T} > \sqrt{SB_1} - S - \pi_G(2m_c)$. Note that if $s_1 \geq \frac{(2m_c-\bar{p}_{\bar{T}})\beta_G}{\bar{p}_{\bar{T}}}$, then $\bar{p}_{\bar{T}} \geq \frac{2m_c\beta_G}{s_1+\beta_G}$ (with strict inequality if $s_1 > \frac{(2m_c-\bar{p}_{\bar{T}})\beta_G}{\bar{p}_{\bar{T}}}$). Therefore, $\bar{p}_{\bar{T}} \geq \hat{p}^1$ (with strict inequality if $s_1 > \frac{(2m_c-\bar{p}_{\bar{T}})\beta_G}{\bar{p}_{\bar{T}}}$). $\bar{p}_{\bar{T}} \geq \hat{p}^1$ if and only if $\pi_G(\bar{p}_{\bar{T}}) = \sqrt{SB_1} - S - \bar{T} \geq \pi_G\left(\hat{p}^1\right)$ (with strict inequality if $\bar{p}_{\bar{T}} > \hat{p}^1$). Thus, if $s_1 \geq \frac{(2m_c-\bar{p}_{\bar{T}})\beta_G}{\bar{p}_{\bar{T}}}$, then $\bar{T} \leq \sqrt{SB_1} - S - \pi_G\left(\hat{p}^1\right)$ (with strict inequality if $s_1 > \frac{(2m_c-\bar{p}_{\bar{T}})\beta_G}{\bar{p}_{\bar{T}}}$).

Case 3: Suppose that $\sqrt{SB_1} - S \leq \pi_G(2m_c)$. The proof is identical to case 2. ■

A.4 Connected Rosters (Propositions 4-6)

Cournot competition among wholesale companies in country i: The final demand of commodity c in country i is $c_i^d = \left(\alpha_i/p_i^d\right)^2$, which implies that the inverse demand of commodity c in country i is $p_i^d = \alpha_i/\sqrt{c_i^d}$. Therefore, the profits of wholesale company $r \in \{1, 2\}$ in country i are given by:

$$\pi_{r,i}^W = \left(\frac{\alpha_i}{\sqrt{q_{r,i} + q_{-r,i}}} - m_d - p_{r,i}^s\right) q_{r,i},$$

where $p^s_{r,i} = m_c$ if r is a connected company and $p^s_{r,i} = p^s$ if r is a non-connected company. Take the first and second derivatives of $\pi^W_{r,i}$ with respect to $q_{r,i}$:

$$\begin{aligned}
\frac{\partial \pi^W_{r,i}}{\partial q_{r,i}} &= \frac{\alpha_i \left(q_{r,i} + 2q_{-r,i}\right)}{2\left(q_{r,i} + q_{-r,i}\right)^{3/2}} - m_d - p^s_{r,i} \\
\frac{\partial^2 \pi^W_{r,i}}{\left(\partial q_{r,i}\right)^2} &= \frac{-\alpha_i \left(\frac{q_{r,i}}{4} + q_{-r,i}\right)}{\left(q_{r,i} + q_{-r,i}\right)^{5/2}} < 0
\end{aligned}$$

Then, the best response function of wholesale company r is implicitly given by the following Kuhn-Tucker condition:

$$\frac{\alpha_i \left(q_{r,i} + 2q_{-r,i}\right)}{2\left(q_{r,i} + q_{-r,i}\right)^{3/2}} - m_d - p^s_{r,i} + \mu_{r,i} = 0,\ \mu_{r,i} \geq 0,\ q_{r,i}\mu_{r,i} = 0$$

Without loss of generality, assume that $p^s_{r,i} \leq p^s_{-r,i}$. Then, we have three possible situations to consider.

- Assume that $q_{r,i} > 0$ for $r \in \{1, 2\}$. Then, $\mu_{r,i} > 0$ for all r and, hence:

  $$\alpha_i \left(q_{r,i} + 2q_{-r,i}\right) = \left(q_{r,i} + q_{-r,i}\right)^{3/2} \left(2m_d + 2p^s_{r,i}\right) \text{ for all } r$$

  Adding for all r we have:

  $$3\alpha_i \left(q_{r,i} + q_{-r,i}\right) = 2\left(q_{r,i} + q_{-r,i}\right)^{3/2} \left(2m_d + p^s_{r,i} + p^s_{-r,i}\right),$$

  which implies

  $$q_{r,i} + q_{-r,i} = \frac{9\left(\alpha_i\right)^2}{4\left(2m_d + p^s_{r,i} + p^s_{-r,i}\right)^2}$$

  Therefore:

  $$q_{r,i} = \frac{9\left(\alpha_i\right)^2 \left(m_d + 2p^s_{-r,i} - p^s_{r,i}\right)}{4\left(2m_d + p^s_{r,i} + p^s_{-r,i}\right)^3},\ q_{-r,i} = \frac{9\left(\alpha_i\right)^2 \left(m_d + 2p^s_{r,i} - p^s_{-r,i}\right)}{4\left(2m_d + p^s_{r,i} + p^s_{-r,i}\right)^3}$$

  Finally, $q_{r,i} > 0$ for $r \in \{1, 2\}$ if and only if $p^s_{-r,i} < 2p^s_{r,i} + m_d$.

- Assume that $q_{r,i} > 0$ and $q_{-r,i} = 0$. Then, $\mu_{r,i} > 0$ and, hence:

  $$q_{r,i} = \frac{\left(\alpha_i\right)^2}{4\left(m_d + p^s_{r,i}\right)^2},\ \mu_{-r,i} = p^s_{-r,i} - m_d - 2p^s_{r,i}$$

  Finally, $\mu_{-r,i} \geq 0$ if and only if $p^s_{-r,i} \geq 2p^s_{r,i} + m_d$.

- Assume that $q_{-r,i} > 0$ and $q_{r,i} = 0$. Then, $\mu_{-r,i} > 0$ and, hence:

  $$q_{-r,i} = \frac{\left(\alpha_i\right)^2}{4\left(m_d + p^s_{r,i}\right)^2},\ \mu_{r,i} = p^s_{r,i} - m_d - 2p^s_{-r,i}$$

  Finally, $\mu_{r,i} \geq 0$ if and only if $p^s_{r,i} \geq 2p^s_{-r,i} + m_d$, which never holds because $p^s_{r,i} \leq p^s_{-r,i}$.

Using the analysis above we can distinguish three possible cases:

Case 1: Suppose that both wholesale companies in country i are non-connected. Formally, $p^s_{r,i} = p^s$ for all r. Then, the equilibrium quantities supplied by wholesale companies in country i are given by:

$$q_{r,i} = \frac{9\left(\alpha_i\right)^2}{32\left(p^s + m_d\right)^2} \text{ } for \text{ } r \in \{0,1\}$$

Introducing these expressions into $\pi^W_{r,i}$ we obtain the equilibrium profits of wholesale companies in country i:

$$\pi^W_{r,i} = \frac{3\left(\alpha_i\right)^2}{32\left(p^s + m_d\right)} \text{ } for \text{ } r \in \{0,1\} \text{ } and \text{ } \pi^W_i = \pi^W_{1,i} + \pi^W_{2,i} = \frac{3\left(\alpha_i\right)^2}{16\left(p^s + m_d\right)}$$

Since $c_i = \sum_{r\in\{1,2\}} q_{r,i}$ and $p^d_i = \alpha_i/\sqrt{c^d_i}$, the equilibrium quantity and price of commodity c in country i are given by:

$$c_i = \frac{9\left(\alpha_i\right)^2}{16\left(p^s + m_d\right)^2} \text{ } and \text{ } p^d_i = \frac{4\left(p^s + m_d\right)}{3}$$

Finally, the profits that geopolitically relevant producers obtained in country i are $\pi^P_{G,i} = \beta_G \sum_{r\in\{1,2\}} \left(p^s_{r,i} - m_c\right) q_{r,i}$. Thus, in equilibrium,

$$\pi^P_{G,i} = \frac{\beta_G\left(\alpha_i\right)^2 9\left(p^s - m_c\right)}{16\left(p^s + m_d\right)^2}$$

Case 2: Suppose that both wholesale companies in country i are connected. Formally, $p^s_{r,i} = m_c$ for all r. Then, the equilibrium quantities supplied by wholesale companies in country i are given by:

$$q_{r,i} = \frac{9\left(\alpha_i\right)^2}{32\left(m_c + m_d\right)^2} \text{ } for \text{ } r \in \{0,1\}$$

Introducing these expressions into $\pi^W_{r,i}$ we obtain the equilibrium profits of wholesale companies in country i:

$$\pi^W_{r,i} = \frac{3\left(\alpha_i\right)^2}{32\left(m_c + m_d\right)} \text{ } for \text{ } r \in \{0,1\} \text{ } and \text{ } \pi^W_i = \pi^W_{1,i} + \pi^W_{2,i} = \frac{3\left(\alpha_i\right)^2}{16\left(m_c + m_d\right)}$$

Since $c_i = \sum_{r\in\{1,2\}} q_{r,i}$ and $p^d_i = \alpha_i/\sqrt{c^d_i}$, the equilibrium quantity and price of commodity c in country i are given by:

$$c_i = \frac{9\left(\alpha_i\right)^2}{16\left(m_c + m_d\right)^2} \text{ } and \text{ } p^d_i = \frac{4\left(m_c + m_d\right)}{3}$$

Finally, the profits that geopolitically relevant producers obtained in country i are $\pi^P_{G,i} = \beta_G \sum_{r\in\{1,2\}} \left(p^s_{r,i} - m_c\right) q_{r,i}$. Thus, in equilibrium,

$$\pi^P_{G,i} = 0$$

Case 3: Suppose that one wholesale company in country i is connected while the other is non-connected. Without loss of generality, assume that $p^s_{1,i} = m_c$ and $p^s_{2,i} = p^s$. Then, we must distinguish two possible situations:

- Suppose that $p^s \leq 2m_c + m_d$. Then, the equilibrium quantities supplied by wholesale companies in country i are:

$$q_{1,i} = \frac{(\alpha_i)^2 9(2p^s - m_c + m_d)}{4(p^s + m_c + 2m_d)^3} \text{ and } q_{2,i} = \frac{(\alpha_i)^2 9(-p^s + 2m_c + m_d)}{4(p^s + m_c + 2m_d)^3}$$

Introducing these expressions into $\pi_{r,i}^W$ we obtain the equilibrium profits of wholesale companies in country i:

$$\pi_{1,i}^W = \frac{(\alpha_i)^2 3(2p^s - m_c + m_d)^2}{4(p^s + m_c + 2m_d)^3}, \ \pi_{2,i}^W = \frac{(\alpha_i)^2 3(-p^s + 2m_c + m_d)^2}{4(p^s + m_c + 2m_d)^3}, \text{ and}$$
$$\pi_i^W = \pi_{1,i}^W + \pi_{2,i}^W = \frac{(\alpha_i)^2 3\left[(2p^s - m_c + m_d)^2 + (-p^s + 2m_c + m_d)^2\right]}{4(p^s + m_c + 2m_d)^3}$$

Since $c_i = \sum_{r\in\{1,2\}} q_{r,i}$ and $p_i^d = \alpha_i/\sqrt{c_i^d}$, the equilibrium quantity and price of commodity c in country i are given by:

$$c_i = \frac{9(\alpha_i)^2}{4(p^s + m_c + 2m_d)^2} \text{ and } p_i^d = \frac{2(p^s + m_c + 2m_d)}{3}$$

- Suppose that $p^s \geq 2m_c + m_d$. Then, the equilibrium quantities supplied by wholesale companies in country i are:

$$q_{1,i} = \frac{(\alpha_i)^2}{4(m_c + m_d)^2} \text{ and } q_{2,i} = 0$$

Introducing these expressions into $\pi_{r,i}^W$ we obtain the equilibrium profits of wholesale companies in country i:

$$\pi_{1,i}^W = \frac{(\alpha_i)^2}{4(m_c + m_d)}, \ \pi_{2,i}^W = 0 \text{ and } \pi_i^W = \frac{(\alpha_i)^2}{4(m_c + m_d)}$$

Since $c_i = \sum_{r\in\{1,2\}} q_{r,i}$ and $p_i^d = \alpha_i/\sqrt{c_i^d}$, the equilibrium quantity and price of commodity c in country i are given by:

$$c_i = \frac{(\alpha_i)^2}{4(m_c + m_d)^2} \text{ and } p_i^d = 2(m_c + m_d).$$

The profits that geopolitically relevant producers obtained in country i are $\pi_{G,i}^P = \beta_G \sum_{r\in\{1,2\}} \left(p_{r,i}^s - m_c\right) q_{r,i}$. Thus, in equilibrium,

$$\pi_{G,i}^P = \begin{cases} \frac{\beta_G (p^s - m_c)(\alpha_i)^2 9(-p^s + 2m_c + m_d)}{4(p^s + m_c + 2m_d)^3} & \text{if } p^s \leq 2m_c + m_d \\ 0 & \text{if } p^s \geq 2m_c + m_d \end{cases}$$

Player* 1's *decisions: The decision of player 1 is the solution of the following optimization problem:

$$\max_{p^s \geq m_c, T_1 \in [0,\bar{T}]} \left\{ W_1(p^s, T_1) = \frac{(\alpha_1)^2}{p_1^d(p^s)} + y_1 - T_1 + \pi_1^W(p^s) + \frac{\pi_G^P(p^s) + T_1}{\pi_G^P(p^s) + T_1 + S} B_1 \right\}$$

where $\pi_1^W(p^s) = \pi_{1,1}^W(p^s) + \pi_{2,1}^W(p^s)$ and $\pi_G^P(p^s) = \sum_{i\in I}\pi_{G,i}^P$. We must study three possible scenarios.

Scenario 1: Suppose that both wholesale companies in country 1 are non-connected. Then:

$$p_1^d(p^s) = \frac{4(p^s + m_d)}{3},\ \pi_1^W(p^s) = \frac{3(\alpha_1)^2}{16(p^s + m_d)},\ \pi_G^P(p^s) = \frac{\beta_G 9(p^s - m_c)\sum_{i\in I}(\alpha_i)^2}{16(p^s + m_d)^2}.$$

Therefore, the problem of player 1 becomes:

$$\max_{p^s \geq m_c, T_1 \in [0,\bar{T}]} \left\{ W_1(p^s, T_1) = \frac{15(\alpha_1)^2}{16(p^s + m_d)} + y_1 - T_1 + \frac{\pi_G^P(p^s) + T_1}{\pi_G^P(p^s) + T_1 + S} B_1 \right\}$$

Take the first and second derivatives of $W_1(p^s, T_1)$ with respect to T_1:

$$\frac{\partial W_1(p^s, T_1)}{\partial T_1} = -1 + \frac{SB_1}{\left[\pi_G^P(p^s) + T_1 + S\right]^2}$$
$$\frac{\partial^2 W_1(p^s, T_1)}{(\partial T_1)^2} = \frac{-2SB_1}{\left[\pi_G^P(p^s) + T_1 + S\right]^3} < 0$$

Since $0 < \sqrt{SB_1} - S < \bar{T}$, player 1 always selects:

$$T_1 = \max\left\{\sqrt{SB_1} - S - \pi_G^P(p^s), 0\right\}$$

Moreover, note that

$$\frac{\partial \pi_G^P(p^s)}{\partial p^s} = \frac{9\beta_G\left[\sum_{i\in I}(\alpha_i)^2\right](2m_c - m_d - p^s)}{16(p^s + m_d)^3}$$

Thus, π_G^P is strictly increasing in p^s for $p^s \leq 2m_c + m_d$ and strictly decreasing in p^s for $p^s \geq 2m_c + m_d$. Therefore, there are two possible situations:

Part 1: Suppose that $\sqrt{SB_1} - S - \pi_G^P(2m_c + m_d) > 0$ or, which is equivalent, $\sqrt{SB_1} - S > \frac{9\beta_G\sum_{i\in I}(\alpha_i)^2}{64(m_c+m_d)}$. Then, $T_1 = \sqrt{SB_1} - S - \pi_G^P(p^s)$ for all $p^s \geq m_c$ and, hence, player 1's problem becomes:

$$\max_{p^s \in [m_c, 2m_c + m_d]} \left\{ W_1(p^s) = \frac{15(\alpha_1)^2}{16(p^s + m_d)} + y_1 - \sqrt{SB_1} + S + \pi_G^P(p^s) + \frac{\sqrt{SB_1} - S}{\sqrt{SB_1}} B_1 \right\}$$

Take the derivative of W_1 with respect to p^s:

$$\frac{\partial W_1(p^s)}{\partial p^s} = \frac{\left[-15s_1(p^s + m_d) + \beta_G 9(m_d + 2m_c - p^s)\right]\sum_{i\in I}(\alpha_i)^2}{16(p^s + m_d)^3}$$

where $s_1 = \frac{(\alpha_1)^2}{\sum_{i\in I}(\alpha_i)^2}$. There are two possible cases to consider:

Case 1.a: Suppose that $s_1 \geq \frac{\beta_G 3(m_d + 2m_c)}{5m_d}$. Then, W_1 is decreasing in p^s for all $p^s \geq m_c$. Thus, the price that maximizes W_1 is $p^s = m_c$. Therefore, player 1 selects $(p^s, T_1) = \left(m_c, \sqrt{SB_1} - S\right)$.

Case 1.b: Suppose that $s_1 < \frac{\beta_G 3(m_d+2m_c)}{5m_d}$. Then, W_1 is strictly increasing in p^s for all $p^s \in \left[m_c, \hat{p}^{s,1}\right]$ and strictly decreasing in p^s for all $p \in \left[\hat{p}^{s,1}, 2m_c+m_d\right]$, where $\hat{p}^{s,1} = \frac{\beta_G 3(m_d+2m_c)-5s_1 m_d}{5s_1+3\beta_G} \in (m_c, 2m_c+m_d)$. Therefore, player 1 selects $(p^s, T_1) = \left(\hat{p}^{s,1}, \sqrt{SB_1} - S - \pi_G^P\left(\hat{p}^{s,1}\right)\right)$.

Part 2: Suppose that $\sqrt{SB_1} - S - \pi_G(2m_c+m_d) \leq 0$ or, which is equivalent, $\sqrt{SB_1} - S \leq \frac{9\beta_G \sum_{i\in I}(\alpha_i)^2}{64(m_c+m_d)}$. Since $\pi_G^P(m_c) = 0 < \sqrt{SB_1} - S \leq \pi_G(2m_c+m_d)$ and π_G^P is strictly increasing in p^s for all $p^s \in [m_c, 2m_c+m_d]$, there exists a unique $\bar{p}^s \in (m_c, 2m_c+m_d]$ such that $\sqrt{SB_1} - S - \pi_G^P(p^s) > 0$ for all $p^s \in [m_c, \bar{p}^s)$, $\sqrt{SB_1} - S - \pi_G(\bar{p}^s) = 0$, and $\sqrt{SB_1} - S - \pi_G(p^s) < 0$ for all $p^s \in (\bar{p}^s, 2m_c+m_d]$. Then, player 1's problem becomes:

$$\max_{p^s\in[m_c,2m_c+m_d]} \left\{ W_1(p^s) = \left\{ \begin{array}{ll} \frac{15(\alpha_1)^2}{16(p^s+m_d)} + \pi_G^P(p^s) + y_1 - \sqrt{SB_1} + S + \frac{\sqrt{SB_1}-S}{\sqrt{SB_1}} B_1 & if\ p^s \leq \bar{p}^s \\ \frac{15(\alpha_1)^2}{16(p^s+m_d)} + \frac{\pi_G^P(p^s)}{\pi_G^P(p^s)+S} B_1 + y_1 & if\ p^s \geq \bar{p}^s \end{array} \right. \right\}$$

W_1 has the following properties:

- W_1 is a continuous function of p^s for all $p^s \in [m_c, 2m_c+m_d]$. In particular, it is continuous at $p^s = \bar{p}^s$. To prove this, note that:

$$\begin{aligned} \lim_{p^s\to(\bar{p}^s)^-} W_1(p^s) &= \frac{15(\alpha_1)^2}{16(\bar{p}^s+m_d)} + y_1 - \left[\sqrt{SB_1} + S - \pi_G(\bar{p}^s)\right] + \frac{\pi_G(\bar{p}^s)}{\pi_G(\bar{p}^s)+S} B_1 \\ &= \frac{15(\alpha_1)^2}{16(\bar{p}^s+m_d)} + y_1 + \frac{\sqrt{SB_1}-S}{\sqrt{SB_1}} B_1 = \lim_{p^s\to(\bar{p}^s)^+} W_1(p^s), \end{aligned}$$

  where we have employed that $\sqrt{SB_1} - S - \pi_G(\bar{p}^s) = 0$.

- Take the derivative of W_1 with respect to p^s for $p^s \in [m_c, \bar{p}^s)$:

$$\frac{\partial W_1(p^s)}{\partial p^s} = \frac{\left[-5s_1(p^s+m_d) + \beta_G 3(m_d+2m_c-p^s)\right]}{16(p^s+m_d)^3 \left[\sum_{i\in I}(\alpha_i)^2\right]^{-1}}$$

  Let $N(p^s) = -15s_1(p^s+m_d) + \beta_G 9(m_d+2m_c-p^s)$ be the numerator of $\frac{\partial W_1(p^s)}{\partial p^s}$. $N(p^s)$ is strictly decreasing in p^s. Thus, there are three possible cases to consider:

  - Suppose that $s_1 \geq \frac{\beta_G 3(m_d+2m_c)}{5m_d}$. Then, W_1 is strictly decreasing in p^s for all $p^s \in [m_c, \bar{p}^s)$.
  - Suppose that $\frac{\beta_G 3(m_d+2m_c-\bar{p})}{5(\bar{p}+m_d)} < s_1 < \frac{\beta_G 3(m_d+2m_c)}{5m_d}$. Then, W_1 is strictly decreasing in p^s for all $p^s \in \left[m_c, \hat{p}^{s,1}\right]$ and strictly decreasing in p^s for $p^s \in \left[\hat{p}^{s,1}, \bar{p}^s\right)$, where $\hat{p}^{s,1} = \frac{\beta_G 3(m_d+2m_c)-5s_1 m_d}{5s_1+3\beta_G} \in (m_c, \bar{p}^s)$.
  - Suppose that $s_1 \leq \frac{\beta_G 3(m_d+2m_c-\bar{p})}{5(\bar{p}+m_d)}$. Then, W_1 is strictly increasing in p^s for all $p^s \in [m_c, \bar{p}^s)$.

- Take the derivative of W_1 with respect to p^s for $p^s \in [\bar{p}^s, 2m_c+m_d]$:

$$\frac{\partial W_1(p^s)}{\partial p^s} = \frac{-15s_1(p^s+m_d) + \frac{9SB_1\beta_G(m_d+2m_c-p^s)}{\left[\pi_G^P(p^s)+S\right]^2}}{16(p^s+m_d)^3 \left[\sum_{i\in I}(\alpha_i)^2\right]^{-1}}$$

Let $N(p^s) = -15s_1(p^s + m_d) + \frac{9SB_1\beta_G(m_d+2m_c-p^s)}{\left[\pi_G^P(p^s)+S\right]^2}$ be the numerator of $\frac{\partial W_1(p^s)}{\partial p^s}$. $N(p^s)$ is decreasing in p^s and $N(2m_c + m_d) = -30s_1(m_c + m_d) < 0$. Thus, there are two possible cases to consider:

- Suppose that $s_1 \geq \frac{\beta_G 3(m_d+2m_c-\bar{p})}{5(\bar{p}+m_d)}$. Then, W_1 is strictly decreasing in p^s for all $p^s \in [\bar{p}^s, 2m_c + m_d]$.
- Suppose that $s_1 < \frac{\beta_G 3(m_d+2m_c-\bar{p})}{5(\bar{p}+m_d)}$. Then, W_1 is strictly increasing in p^s for all $p^s \in \left[\bar{p}^s, \hat{p}^{s,2}\right]$ and strictly decreasing in p^s for all $p^s \in \left[\hat{p}^{s,2}, 2m_c + m_d\right]$, where $\hat{p}^{s,2} \in (\bar{p}^s, 2m_c + m_d)$ is the unique solution to $-15s_1(p^s + m_d) + \frac{SB_1\beta_G 9}{\left[\pi_G^P(p^s)+S\right]^2}(m_d + 2m_c - p^s) = 0$.

Employing the above characterization of W_1 we have the following possible cases:

Case 2.a: Suppose that $s_1 \geq \frac{\beta_G 3(m_d+2m_c)}{5m_d}$. Then, W_1 is strictly decreasing in p^s for all $p^s \in [m_c, 2m_c + m_d]$. Thus, W_1 adopts its maximum at $p^s = m_c$. Therefore, player 1 selects $(p, T_1) = \left(m_c, \sqrt{SB_1} - S\right)$.

Case 2.b: Suppose that $\frac{\beta_G 3(m_d+2m_c-\bar{p})}{5(\bar{p}+m_d)} < s_1 < \frac{\beta_G 3(m_d+2m_c)}{5m_d}$. Then, W_1 is strictly increasing in p^s for all $p^s \in \left[m_c, \hat{p}^{s,1}\right]$ and W_1 is strictly decreasing in p^s for all $p^s \in \left[\hat{p}^{s,1}, 2m_c + m_d\right]$. Thus, W_1 adopts its maximum at $p = \hat{p}^{s,1} \in (m_c, \bar{p}^s)$. Therefore, player 1 selects $(p^s, T_1) = \left(\hat{p}^{s,1}, \sqrt{SB_1} - S - \pi_G\left(\hat{p}^{s,1}\right)\right)$.

Case 2.c: Suppose that $s_1 \leq \frac{\beta_G 3(m_d+2m_c-\bar{p}^s)}{5(\bar{p}+m_d)}$. W_1 is strictly increasing in p^s for all $p^s \in \left[m_c, \hat{p}^{s,2}\right]$ and W_1 is strictly decreasing in p^s for all $p^s \in \left[\hat{p}^{s,2}, 2m_c + m_d\right]$. Thus, W_1 adopts its maximum at $p^s = \hat{p}^{s,2} \in [\bar{p}^s, 2m_c + m_d)$. Therefore, player 1 selects $(p^s, T_1) = \left(\hat{p}^{s,2}, 0\right)$.

Scenario 2: Suppose that both wholesale companies in country 1 are connected. Then:

$$p_1^d = \frac{4(m_c + m_d)}{3}, \; \pi_1^W = \frac{3(\alpha_1)^2}{16(m_c + m_d)}, \; \pi_G^P(p^s) = \frac{9\beta_G\left[\sum_{i\in I, i\neq 1}(\alpha_i)^2\right](p^s - m_c)}{16(p^s + m_d)^2}$$

Therefore, the problem of player 1 becomes:

$$\max_{p^s \geq m_c, T_1 \in [0,\bar{T}]} \left\{ W_1(p^s, T_1) = \frac{15(\alpha_1)^2}{16(m_c + m_d)} + y_1 - T_1 + \frac{\pi_G^P(p^s) + T_1}{\pi_G^P(p^s) + T_1 + S} B_1 \right\}$$

Take the first and second derivatives of $W_1(p^s, T_1)$ with respect to T_1:

$$\frac{\partial W_1(p^s, T_1)}{\partial T_1} = -1 + \frac{SB_1}{\left[\pi_G^P(p^s) + T_1 + S\right]^2}$$
$$\frac{\partial^2 W_1(p^s, T_1)}{(\partial T_1)^2} = \frac{-2SB_1}{\left[\pi_G^P(p^s) + T_1 + S\right]^3} < 0$$

Since $0 < \sqrt{SB_1} - S < \bar{T}$, player 1 always selects:

$$T_1 = \max\left\{\sqrt{SB_1} - S - \pi_G^P(p^s), 0\right\}$$

Moreover, note that

$$\frac{\partial \pi_G^P(p^s)}{\partial p^s} = \frac{9\beta_G\left[\sum_{i\in I, i\neq 1}(\alpha_i)^2\right](2m_c - m_d - p^s)}{16(p^s+m_d)^3}$$

Thus, π_G^P is strictly increasing in p^s for $p^s \leq 2m_c + m_d$ and strictly decreasing in p^s for $p^s \geq 2m_c + m_d$. Therefore, there are two possible situations:

Part 1: Suppose that $\sqrt{SB_1} - S - \pi_G^P(2m_c + m_d) > 0$ or, which is equivalent, $\sqrt{SB_1} - S > \frac{9\beta_G \sum_{i\in I, i\neq 1}(\alpha_i)^2}{64(m_c+m_d)}$. Then, $T_1 = \sqrt{SB_1} - S - \pi_G^P(p^s)$ for all $p^s \geq m_c$ and, hence, player 1's problem becomes:

$$\max_{p^s\in[m_c, 2m_c+m_d]}\left\{W_1(p^s) = \frac{15(\alpha_1)^2}{16(m_c+m_d)} + y_1 - \sqrt{SB_1} + S + \pi_G^P(p^s) + \frac{\sqrt{SB_1}-S}{\sqrt{SB_1}}B_1\right\}$$

Since π_G^P adopts its maximum at $p^s = 2m_c + m_d$, W_1 must also adopts its maximum at $p^s = 2m_c + m_d$. Therefore, player 1 selects $(p^s, T_1) = \left(2m_c + m_d, \sqrt{SB_1} - S - \pi_G(2m_c + m_d)\right)$.

Part 2: Suppose that $\sqrt{SB_1} - S - \pi_G(2m_c + m_d) \leq 0$ or, which is equivalent, $\sqrt{SB_1} - S \leq \frac{9\beta_G \sum_{i\in I, i\neq 1}(\alpha_i)^2}{64(m_c+m_d)}$. Since $\pi_G^P(m_c) = 0 < \sqrt{SB_1} - S \leq \pi_G(2m_c + m_d)$ and π_G^P is strictly increasing in p^s for all $p^s \in [m_c, 2m_c + m_d]$, there exists a unique $\bar{p}^s \in (m_c, 2m_c + m_d]$ such that $\sqrt{SB_1} - S - \pi_G^P(p^s) > 0$ for all $p^s \in [m_c, \bar{p}^s)$, $\sqrt{SB_1} - S - \pi_G(\bar{p}^s) = 0$, and $\sqrt{SB_1} - S - \pi_G(p^s) < 0$ for all $p^s \in (\bar{p}^s, 2m_c + m_d]$. Then, player 1's problem becomes:

$$\max_{p^s\in[m_c, 2m_c+m_d]}\left\{W_1(p^s) = \left\{\begin{array}{ll} \frac{15(\alpha_1)^2}{16(m_c+m_d)} + \pi_G^P(p^s) + y_1 - \sqrt{SB_1} + S + \frac{\sqrt{SB_1}-S}{\sqrt{SB_1}}B_1 & \textit{if } p^s \leq \bar{p}^s \\ \frac{15(\alpha_1)^2}{16(m_c+m_d)} + \frac{\pi_G^P(p^s)}{\pi_G^P(p^s)+S}B_1 + y_1 & \textit{if } p^s \geq \bar{p}^s \end{array}\right.\right\}$$

Since π_G^P adopts its maximum at $p^s = 2m_c + m_d \geq \bar{p}^s$, and W_1 is a continuous and strictly increasing function of $\pi_G^P(p^s)$, W_1 must also adopts its maximum at $p^s = 2m_c + m_d$. Therefore, player 1 selects $(p^s, T_1) = (2m_c + m_d, 0)$.

Scenario 3: Suppose that one wholesale company in country 1 is connected while the other is non-connected. Without loss of generality, assume that $p_{1,1}^s = m_c$ and $p_{2,1}^s = p^s$. Then, the problem of player 1 becomes:

$$\max_{p^s\geq m_c, T_1\in[0,\bar{T}]}\left\{W_1(p^s, T_1) = \frac{(\alpha_1)^2}{p_1^d(p^s)} + y_1 - T_1 + \pi_1^W(p^s) + \frac{\pi_G^P(p^s)+T_1}{\pi_G^P(p^s)+T_1+S}B_1\right\}$$

where:

$$p_1^d(p^s) = \begin{cases} \frac{2(p^s+m_c+2m_d)}{3} & \textit{if } p^s \leq 2m_c+m_d \\ 2(m_c+m_d) & \textit{if } p^s \geq 2m_c+m_d \end{cases}$$

$$\pi_1^W(p^s) = \begin{cases} \frac{3(\alpha_1)^2\left[(2p^s-m_c+m_d)^2+(-p^s+2m_c+m_d)^2\right]}{4(p^s+m_c+2m_d)^3} & \textit{if } p^s \leq 2m_c+m_d \\ \frac{(\alpha_1)^2}{4(m_c+m_d)} & \textit{if } p^s \geq 2m_c+m_d \end{cases}$$

$$\pi_G^P(p^s) = \begin{cases} \pi_{G,1}^P(p^s) + \pi_{G,-1}^P(p^s) & \textit{if } p^s \leq 2m_c+m_d \\ \pi_{G,-1}^P(p^s) & \textit{if } p^s \geq 2m_c+m_d \end{cases}$$

$$\pi_{G,1}^P(p^s) = \frac{9(\alpha_1)^2\beta_G(p^s-m_c)(-p^s+2m_c+m_d)}{4(p^s+m_c+2m_d)^3}$$

$$\pi_{G,-1}^P(p^s) = \frac{9\beta_G\left[\sum_{i\in I, i\neq 1}(\alpha_i)^2\right](p^s-m_c)}{16(p^s+m_d)^2}$$

Result 1: It never optimal to set $p^s > 2m_c+m_d$. To prove this, assume that 1 selects T_1 and $p^s \geq 2m_c+m_d$. Then:

$$W_1(p^s, T_1) = \frac{(\alpha_1)^2}{2(m_c+m_d)} + y_1 - T_1 + \frac{(\alpha_1)^2}{4(m_c+m_d)} + \frac{\pi_{G,-1}^P(p^s)+T_1}{\pi_{G,-1}^P(p^s)+T_1+S}B_1$$

Take the derivative of $\pi_{G,-1}^P$ with respect to p^s:

$$\frac{\partial \pi_{G,-1}^P(p^s)}{\partial p^s} = \frac{9\beta_G\left[\sum_{i\in I, i\neq 1}(\alpha_i)^2\right](2m_c-m_d-p^s)}{16(p^s+m_d)^3}$$

Thus, $\pi_{G,-1}^P$ is strictly decreasing in p^s for $p^s \geq 2m_c+m_d$, which implies that $W_1(p^s, T_1)$ is also strictly decreasing in p^s for $p^s \geq 2m_c+m_d$ and T_1.

Result 2: $\frac{(\alpha_1)^2}{p_1^d(p^s)} + \pi_1^W(p^s)$ is strictly decreasing in p^s for all $p^s \in [m_c, 2m_c+m_d]$. To prove this, note that for $p^s \in [m_c, 2m_c+m_d]$ we have:

$$\frac{(\alpha_1)^2}{p_1^d(p^s)} + \pi_1^W(p^s) = \frac{3(\alpha_1)^2\left[7(p^s-m_c)^2+10(p^s-m_c)(m_c+m_d)+10(m_c+m_d)^2\right]}{4(p^s+m_c+2m_d)^3}$$

Take the derivative of $\frac{(\alpha_1)^2}{p_1^d(p^s)} + \pi_1^W(p^s)$ with respect to p^s for $p^s \in [m_c, 2m_c+m_d]$:

$$\frac{\partial\left[\frac{(\alpha_1)^2}{p_1^d(p^s)} + \pi_1^W(p^s)\right]}{\partial p^s} = \frac{3(\alpha_1)^2\left[-7(p^s-m_c)^2+8(m_c+m_d)(p^s-m_c)-10(m_c+m_d)^2\right]}{4(p^s+m_c+2m_d)^4}$$

It is easy to verify that the numerator of this expression is always negative for $p^s \in [m_c, 2m_c+m_d]$.

Result 3: $\pi_G^P(p^s)$ is an strictly concave function of p^s for $p^s \in [m_c, 2m_c + m_d]$. Moreover, $\pi_G^P(p^s)$ has a unique global maximum at $p^s \in (m_c, 2m_c + m_d)$. To prove this, note that for $p^s \in [m_c, 2m_c + m_d]$ we have:

$$\pi_G^P(p^s) = \pi_{G,1}^P(p^s) + \pi_{G,-1}^P(p^s)$$
$$\pi_{G,1}^P(p^s) = \frac{9(\alpha_1)^2 \beta_G (p^s - m_c)(-p^s + 2m_c + m_d)}{4(p^s + m_c + 2m_d)^3}$$
$$\pi_{G,-1}^P(p^s) = \frac{9\beta_G \left[\sum_{i\in I, i\neq 1} (\alpha_i)^2\right](p^s - m_c)}{16(p^s + m_d)^2}$$

Take the first and second derivatives of $\pi_{G,1}^P(p^s)$ with respect to p^s:

$$\frac{\partial \pi_{G,1}^P(p^s)}{\partial p^s} = \frac{9\beta_G(\alpha_1)^2\left[(p^s - m_c)^2 - 6(m_c + m_d)(p^s - m_c) + 2(m_c + m_d)^2\right]}{4\left[(p^s - m_c) + 2(m_c + m_d)\right]^4}$$
$$\frac{\partial^2 \pi_{G,1}^P(p^s)}{(\partial p^s)^2} = \frac{-9\beta_G(\alpha_1)^2\left[(p^s - m_c)^2 - 11(m_c + m_d)(p^s - m_c) + 10(m_c + m_d)^2\right]}{2\left[(p^s - m_c) + 2(m_c + m_d)\right]^5}$$

It is easy to verify that $\frac{\partial^2 \pi_{G,1}^P(p^s)}{(\partial p^s)^2} < 0$ for $p^s \in [m_c, 2m_c + m_d)$ and $\frac{\partial^2 \pi_{G,1}^P(p^s)}{(\partial p^s)^2} = 0$ for $p^s = 2m_c + m_d$. Take the first and second derivatives of $\pi_{G,-1}^P(p^s)$ with respect to p^s:

$$\frac{\partial \pi_{G,-1}^P(p^s)}{\partial p^s} = \frac{9\beta_G\left[\sum_{i\in I, i\neq 1}(\alpha_i)^2\right]\left[-(p^s - m_c) + (m_c + m_d)\right]}{16\left[(p^s - m_c) + (m_c + m_d)\right]^3}$$
$$\frac{\partial^2 \pi_{G,-1}^P(p^s)}{(\partial p^s)^2} = \frac{-9\beta_G\left[\sum_{i\in I, i\neq 1}(\alpha_i)^2\right](2m_d + 3m_c - p^s)}{8(p^s + m_d)^4}$$

It is easy to verify that $\frac{\partial^2 \pi_{G,-1}^P(p^s)}{(\partial p^s)^2} < 0$ for $p^s \in [m_c, 2m_c + m_d]$. Thus, $\frac{\partial^2 \pi_G^P(p^s)}{(\partial p^s)^2} < 0$ for $p^s \in [m_c, 2m_c + m_d]$, which implies that $\pi_G^P(p^s)$ is an strictly concave function of p^s for $p^s \in [m_c, 2m_c + m_d]$. Finally, note that

$$\frac{\partial \pi_G^P(m_c)}{\partial p^s} = \frac{9\beta_G\left\{(\alpha_1)^2 + 2\left[\sum_{i\in I, i\neq 1}(\alpha_i)^2\right]\right\}}{32(m_c + m_d)^2} > 0$$
$$\frac{\partial \pi_G^P(2m_c + m_d)}{\partial p^s} = \frac{-\beta_G(\alpha_1)^2}{12(m_c + m_d)^2} < 0$$

Thus, π_G^P has a unique interior global maximum at $p^{s,*}$ given by $\frac{\partial \pi_G^P(p^{s,*})}{\partial p^s} = 0$. Moreover, π_G^P is strictly increasing in p^s for all $p^s \in [m_c, p^{s,*}]$ and strictly decreasing in p^s for all $p^s \in [p^{s,*}, 2m_c + m_d]$.

From Results 1-3, we obtain that the problem of player 1 becomes:

$$\max_{p^s \in [m_c, p^{s,*}], T_1 \in [0, \bar{T}]} \left\{ \frac{(\alpha_1)^2}{p_1^d(p^s)} + \pi_1^W(p^s) + y_1 - T_1 + \frac{\pi_G^P(p^s) + T_1}{\pi_G^P(p^s) + T_1 + S} B_1 \right\}$$

where

$$\frac{(\alpha_1)^2}{p_1^d(p^s)} + \pi_1^W(p^s) = \frac{3(\alpha_1)^2\left[7(p^s - m_c)^2 + 10(m_c + m_d)(p^s - m_c) + 10(m_c + m_d)^2\right]}{4(p^s + m_c + 2m_d)^3}$$

$$\pi_G^P(p^s) = \pi_{G,1}^P(p^s) + \pi_{G,-1}^P(p^s)$$

$$\pi_{G,1}^P(p^s) = \frac{9(\alpha_1)^2\beta_G(p^s - m_c)(-p^s + 2m_c + m_d)}{4(p^s + m_c + 2m_d)^3}$$

$$\pi_{G,-1}^P(p^s) = \frac{9\beta_G\left[\sum_{i\in I, i\neq 1}(\alpha_i)^2\right](p^s - m_c)}{16(p^s + m_d)^2}$$

Take the first and second derivatives of $W_1(p^s, T_1)$ with respect to T_1:

$$\frac{\partial W_1(p^s, T_1)}{\partial T_1} = -1 + \frac{SB_1}{\left[\pi_G^P(p^s) + T_1 + S\right]^2}$$

$$\frac{\partial^2 W_1(p^s, T_1)}{(\partial T_1)^2} = \frac{-2SB_1}{\left[\pi_G^P(p^s) + T_1 + S\right]^3} < 0$$

Since $0 < \sqrt{SB_1} - S < \bar{T}$, player 1 always selects:

$$T_1 = \max\left\{\sqrt{SB_1} - S - \pi_G^P(p^s), 0\right\}$$

Since π_G^P is strictly increasing in p^s for all $p^s \in [m_c, p^{s,*}]$, there are two possible cases to consider:

Part 1: Suppose that $\sqrt{SB_1} - S - \pi_G^P(p^{s,*}) > 0$. Then, $T_1 = \sqrt{SB_1} - S - \pi_G^P(p^s)$ for all $p^s \in [m_c, p^{s,*}]$ and, hence, player 1's problem becomes:

$$\max_{p^s\in[m_c,p^{s,*}]}\left\{W_1(p^s) = \frac{(\alpha_1)^2}{p_1^d(p^s)} + \pi_1^W(p^s) + y_1 - \sqrt{SB_1} + S + \pi_G^P(p^s) + \frac{\sqrt{SB_1} - S}{\sqrt{SB_1}}B_1\right\}$$

where

$$\frac{(\alpha_1)^2}{p_1^d(p^s)} + \pi_1^W(p^s) = \frac{3(\alpha_1)^2\left[7(p^s - m_c)^2 + 10(m_c + m_d)(p^s - m_c) + 10(m_c + m_d)^2\right]}{4(p^s + m_c + 2m_d)^3}$$

$$\pi_G^P(p^s) = \pi_{G,1}^P(p^s) + \pi_{G,-1}^P(p^s)$$

$$\pi_{G,1}^P(p^s) = \frac{9\beta_G(\alpha_1)^2(p^s - m_c)(-p^s + 2m_c + m_d)}{4(p^s + m_c + 2m_d)^3}$$

$$\pi_{G,-1}^P(p^s) = \frac{9\beta_G\left[\sum_{i\in I, i\neq 1}(\alpha_i)^2\right](p^s - m_c)}{16(p^s + m_d)^2}$$

Take the derivative of W_1 with respect to p^s:

$$\begin{aligned}\frac{\partial W_1(p^s)}{\partial p^s} &= \frac{\partial\left[\frac{(\alpha_1)^2}{p_1^d(p^s)} + \pi_1^W(p^s)\right]}{\partial p^s} + \frac{\partial \pi_{G,1}^P(p^s)}{\partial p^s} + \frac{\partial \pi_{G,-1}^P(p^s)}{\partial p^s} \\ &= \frac{N(p^s)}{4\left[(p^s - m_c) + 2(m_c + m_d)\right]^4 \left[\sum_{i \in I} (\alpha_i)^2\right]^{-1}}\end{aligned}$$

where

$$N(p^s) = \left\{\begin{array}{l} 3\left[-7(p^s - m_c)^2 + 8(m_c + m_d)(p^s - m_c) - 10(m_c + m_d)^2\right] s_1 \\ 9\beta_G\left[(p^s - m_c)^2 - 6(m_c + m_d)(p^s - m_c) + 2(m_c + m_d)^2\right] s_1 \\ \frac{9\beta_G}{4}\left[(m_c + m_d)^2 - (p^s - m_c)^2\right]\left[1 + \frac{m_c + m_d}{p^s + m_d}\right]^4 (1 - s_1) \end{array}\right\}$$

is the numerator of $\frac{\partial W_1(p^s)}{\partial p^s}$. It is easy to verify that whenever $\beta_G \geq 4/9$, $N(p^s)$ is strictly decreasing in p^s. Moreover:

$$\begin{aligned} N(m_c) &= 6\left[6\beta_G - (5 + 3\beta_G)s_1\right](m_c + m_d)^2 \\ N(p^{s,*}) &= 3\left[-7(p^{s,*} - m_c)^2 + 8(m_c + m_d)(p^{s,*} - m_c) - 10(m_c + m_d)^2\right] s_1 < 0 \end{aligned}$$

Therefore, there are two possible cases to consider:

Case 1.a: Suppose that $s_1 \geq \frac{6\beta_G}{5+3\beta_G}$. Then, $N(m_c) \leq 0$ and, hence, W_1 is strictly decreasing in p^s for all $p^s \geq m_c$. Thus, the price that maximizes W_1 is $p^s = m_c$. Therefore, player 1 selects $(p^s, T_1) = \left(m_c, \sqrt{SB_1} - S\right)$.

Case 1.b: Suppose that $s_1 < \frac{6\beta_G}{5+3\beta_G}$. Then, W_1 is strictly increasing in p^s for all $p^s \in \left[m_c, \hat{p}^{s,1}\right]$ and strictly decreasing in p^s for all $p \in \left[\hat{p}^{s,1}, 2m_c + m_d\right]$, where $\hat{p}^{s,1} \in (m_c, p^{s,*})$ is the unique solution to $N(p^s) = 0$. Therefore, player 1 selects $(p^s, T_1) = \left(\hat{p}^{s,1}, \sqrt{SB_1} - S - \pi_G^P\left(\hat{p}^{s,1}\right)\right)$.

Part 2: Suppose that $\sqrt{SB_1} - S - \pi_G(p^{s,*}) \leq 0$. Since $\pi_G^P(m_c) = 0 < \sqrt{SB_1} - S \leq \pi_G(p^{s,*})$ and π_G^P is strictly increasing in p^s for all $p^s \in [m_c, p^{s,*}]$, there exists a unique $\bar{p}^s \in (m_c, p^{s,*}]$ such that $\sqrt{SB_1} - S - \pi_G^P(p^s) > 0$ for all $p^s \in [m_c, \bar{p}^s)$, $\sqrt{SB_1} - S - \pi_G(\bar{p}^s) = 0$, and $\sqrt{SB_1} - S - \pi_G(p^s) < 0$ for all $p^s \in (\bar{p}^s, p^{s,*}]$. Then, player 1's problem becomes:

$$\max_{p^s \in [m_c, p^{s,*}]} \left\{ W_1(p^s) = \left\{\begin{array}{ll} \frac{(\alpha_1)^2}{p_1^d(p^s)} + \pi_1^W(p^s) + y_1 - \sqrt{SB_1} + S + \pi_G^P(p^s) + \frac{\sqrt{SB_1} - S}{\sqrt{SB_1}} B_1 & \textit{if } p^s \leq \bar{p}^s \\ \frac{(\alpha_1)^2}{p_1^d(p^s)} + \pi_1^W(p^s) + y_1 + \frac{\pi_G^P(p^s)}{\pi_G^P(p^s) + S} B_1 & \textit{if } p^s \geq \bar{p}^s \end{array}\right\}\right.$$

where

$$\frac{(\alpha_1)^2}{p_1^d(p^s)} + \pi_1^W(p^s) = \frac{3(\alpha_1)^2\left[7(p^s-m_c)^2+10(m_c+m_d)(p^s-m_c)+10(m_c+m_d)^2\right]}{4(p^s+m_c+2m_d)^3}$$

$$\pi_G^P(p^s) = \pi_{G,1}^P(p^s) + \pi_{G,-1}^P(p^s)$$

$$\pi_{G,1}^P(p^s) = \frac{9(\alpha_1)^2\beta_G(p^s-m_c)(-p^s+2m_c+m_d)}{4(p^s+m_c+2m_d)^3}$$

$$\pi_{G,-1}^P(p^s) = \frac{9\beta_G\left[\sum_{i\in I,i\neq 1}(\alpha_i)^2\right](p^s-m_c)}{16(p^s+m_d)^2}$$

W_1 has the following properties:

- W_1 is a continuous function of p^s for all $p^s \in [m_c, p^{s,*}]$. In particular, it is continuous at $p^s = \bar{p}^s$. To prove this, note that:

$$\lim_{p^s\to(\bar{p}^s)^-} W_1(p^s) = \frac{(\alpha_1)^2}{p_1^d(\bar{p}^s)} + \pi_1^W(\bar{p}^s) + y_1 - \left[\sqrt{SB_1} + S - \pi_G(\bar{p}^s)\right] + \frac{\pi_G(\bar{p}^s)}{\pi_G(\bar{p}^s)+S}B_1$$
$$= \frac{(\alpha_1)^2}{p_1^d(\bar{p}^s)} + \pi_1^W(\bar{p}^s) + y_1 + \frac{\sqrt{SB_1}-S}{\sqrt{SB_1}}B_1 = \lim_{p^s\to(\bar{p}^s)^+} W_1(p^s),$$

  where we have employed that $\sqrt{SB_1} - S - \pi_G(\bar{p}^s) = 0$.

- Take the derivative of W_1 with respect to p^s for $p^s \in [m_c, \bar{p}^s)$:

$$\frac{\partial W_1(p^s)}{\partial p^s} = \frac{\partial}{\partial p^s}\left[\frac{(\alpha_1)^2}{p_1^d(p^s)} + \pi_1^W(p^s)\right] + \frac{\partial \pi_G^P(p^s)}{\partial p^s}$$
$$= \frac{N(p^s)}{4\left[(p^s-m_c)+2(m_c+m_d)\right]^4\left[\sum_{i\in I}(\alpha_i)^2\right]^{-1}}$$

  where

$$N(p^s) = \left\{\begin{array}{l} 3\left[-7(p^s-m_c)^2+8(m_c+m_d)(p^s-m_c)-10(m_c+m_d)^2\right]s_1 \\ 9\beta_G\left[(p^s-m_c)^2-6(m_c+m_d)(p^s-m_c)+2(m_c+m_d)^2\right]s_1 \\ \frac{9\beta_G}{4}\left[(m_c+m_d)^2-(p^s-m_c)^2\right]\left[1+\frac{m_c+m_d}{p^s+m_d}\right]^4(1-s_1) \end{array}\right\}$$

  It is easy to verify that whenever $\beta_G \geq 4/9$, $N(p^s)$ is strictly decreasing in p^s. Moreover,

$$N(m_c) = 6\left[6\beta_G - (5+3\beta_G)s_1\right](m_c+m_d)^2$$

  and $N(\bar{p}^s) < 0$ if and only if $s_1 > \sigma(\bar{p}^s)$, where

$$\sigma(\bar{p}^s) = \frac{\frac{9\beta_G}{4}\left[(m_c+m_d)^2-(\bar{p}^s-m_c)^2\right]\left[1+\frac{m_c+m_d}{\bar{p}^s+m_d}\right]^4}{\left\{\begin{array}{c} \frac{9\beta_G}{4}\left[(m_c+m_d)^2-(\bar{p}^s-m_c)^2\right]\left[1+\frac{m_c+m_d}{\bar{p}^s+m_d}\right]^4 \\ +3\left[(-7+3\beta_G)(\bar{p}^s-m_c)^2+2(4-9\beta_G)(m_c+m_d)(\bar{p}^s-m_c)-2(5-3\beta_G)(m_c+m_d)^2\right] \end{array}\right\}}$$

Thus, there are three possible cases to consider:

- Suppose that $s_1 \geq \frac{6\beta_G}{5+3\beta_G}$. Then, $N(m_c) \leq 0$ and, hence, W_1 is strictly decreasing in p^s for all $p^s \in [m_c, \bar{p}^s)$.
- Suppose that $\sigma(\bar{p}^s) < s_1 < \frac{6\beta_G}{5+3\beta_G}$. Then, W_1 is strictly increasing in p^s for all $p^s \in \left[m_c, \hat{p}^{s,1}\right]$ and strictly decreasing in p^s for all $p \in \left[\hat{p}^{s,1}, \bar{p}^s\right]$, where $\hat{p}^{s,1} \in (m_c, \bar{p}^s)$ is the unique solution to $N(p^s) = 0$.
- Suppose that $s_1 \leq \sigma(\bar{p}^s)$. Then, W_1 is strictly increasing in p^s for all $p^s \in [m_c, \bar{p}^s)$.

- Take the derivative of W_1 with respect to p^s for $p^s \in [\bar{p}^s, p^{s,*}]$:

$$\frac{\partial W_1(p^s)}{\partial p^s} = \frac{\partial}{\partial p^s}\left[\frac{(\alpha_1)^2}{p_1^d(p^s)} + \pi_1^W(p^s)\right] + \frac{SB_1}{\left[\pi_G^P(p^s) + S\right]^2}\frac{\partial \pi_G^P(p^s)}{\partial p^s}$$
$$= \frac{N(p^s)}{4\left[(p^s - m_c) + 2(m_c + m_d)\right]^4\left[\sum_{i\in I}(\alpha_i)^2\right]^{-1}}$$

where

$$N(p^s) = \left\{\begin{array}{l} 3\left[-7(p^s - m_c)^2 + 8(m_c + m_d)(p^s - m_c) - 10(m_c + m_d)^2\right]s_1 \\ \frac{SB_1}{\left[\pi_G^P(p^s)+S\right]^2}9\beta_G\left[(p^s - m_c)^2 - 6(m_c + m_d)(p^s - m_c) + 2(m_c + m_d)^2\right]s_1 \\ \frac{SB_1}{\left[\pi_G^P(p^s)+S\right]^2}\frac{9\beta_G}{4}\left[(m_c + m_d)^2 - (p^s - m_c)^2\right]\left[1 + \frac{m_c+m_d}{p^s+m_d}\right]^4(1 - s_1) \end{array}\right\}$$

It is tedious but easy to verify that whenever $\beta_G \geq 4/9$, $N(p^s)$ is strictly decreasing in p^s. Moverover,

$$N(p^{s,*}) = 3\left[-7(p^{s,*} - m_c)^2 + 8(m_c + m_d)(p^{s,*} - m_c) - 10(m_c + m_d)^2\right]s_1 < 0$$

Thus, there are two possible cases to consider:

- Suppose that $s_1 \geq \sigma(\bar{p}^s)$. Then, W_1 is strictly decreasing in p^s for all $p^s \in [\bar{p}^s, p^{s,*}]$.
- Suppose that $s_1 < \sigma(\bar{p}^s)$. Then, W_1 is strictly increasing in p^s for all $p^s \in \left[\bar{p}^s, \hat{p}^{s,2}\right]$ and strictly decreasing in p^s for all $p^s \in \left[\hat{p}^{s,2}, p^{s,*}\right]$, where $\hat{p}^{s,2} \in (\bar{p}^s, p^{s,*})$ is the unique solution to $N(p^s) = 0$

Employing the above characterization of W_1 we have the following possible cases:

Case 2.a: Suppose that $s_1 \geq \frac{6\beta_G}{5+3\beta_G}$. Then, W_1 is strictly decreasing in p^s for all $p^s \in [m_c, p^{s,*}]$. Thus, W_1 adopts its maximum at $p^s = m_c$. Therefore, player 1 selects $(p, T_1) = \left(m_c, \sqrt{SB_1} - S\right)$.

Case 2.b: Suppose that $\sigma(\bar{p}^s) < s_1 < \frac{6\beta_G}{5+3\beta_G}$. Then, W_1 is strictly increasing in p^s for all $p^s \in \left[m_c, \hat{p}^{s,1}\right]$ and W_1 is strictly decreasing in p^s for all $p^s \in \left[\hat{p}^{s,1}, p^{s,*}\right]$. Thus, W_1 adopts its maximum at $p = \hat{p}^{s,1} \in (m_c, \bar{p}^s)$. Therefore, player 1 selects $(p^s, T_1) = \left(\hat{p}^1, \sqrt{SB_1} - S - \pi_G(\hat{p}^1)\right)$.

Case 2.c: Suppose that $s_1 \leq \sigma(\bar{p}^s)$. W_1 is strictly increasing in p^s for all $p^s \in \left[m_c, \hat{p}^{s,2}\right]$ and W_1 is strictly decreasing in p^s for all $p^s \in \left[\hat{p}^{s,2}, p^{s,*}\right]$. Thus, W_1 adopts its maximum at $p^s = \hat{p}^{s,2} \in [\bar{p}^s, p^{s,*})$. Therefore, player 1 selects $(p^s, T_1) = \left(\hat{p}^{s,2}, 0\right)$. ■

A.5 Geopolitical Rivals Among Producers (Proposition 7)

Proposition 7 *Assume that* $B_1 > 0$ *and* $0 < \sqrt{SB_1} - S < \bar{T}$.

1. *Suppose that* $\sqrt{SB_1} - S > \pi_G(2m_c)$.
   (a) *If* $s_1 \geq \beta_G - \lambda\beta_S$, *then* $(p, T_1) = \left(m_c, \sqrt{SB_1} - S\right)$.
   (b) *If* $s_1 < \beta_G - \lambda\beta_S$, *then* $(p, T_1) = \left(\hat{p}^1, \sqrt{SB_1} - S - \pi_G\left(\hat{p}^1\right)\right)$, *where* $\hat{p}^1 = \frac{2m_c(\beta_G - \lambda\beta_S)}{s_1 + \beta_G - \lambda\beta_S} \in (m, \bar{p})$.
2. *Suppose that* $\sqrt{SB_1} - S \leq \pi_G(2m_c)$.
   (a) *If* $s_1 \geq \beta_G - \lambda\beta_S$, *then* $(p, T_1, T_2) = \left(m_c, \sqrt{SB_1} - S\right)$.
   (b) *If* $(\beta_G - \lambda\beta_S)\left(\frac{2m_c - \bar{p}}{\bar{p}}\right) < s_1 < \beta_G - \lambda\beta_S$, *then* $(p, T_1) = \left(\hat{p}^1, \sqrt{SB_1} - S - \pi_G\left(\hat{p}^1\right)\right)$.
   (c) *If* $s_1 \leq (\beta_G - \lambda\beta_S)\left(\frac{2m_c - \bar{p}}{\bar{p}}\right)$, *then* $(p, T_1) = \left(\hat{p}^2, 0\right)$, *where* $\hat{p}^2 \in [\bar{p}, 2m_c)$ *is the unique solution to* $\frac{s_1 p}{(2m_c - p)} + \lambda\beta_S = \frac{\beta_G S B_1}{[\pi_G(p) + S]^2}$

Proof: Following the same argument employed in the proof of lemma 1, we have that $T_1 = \max\left\{\sqrt{SB_1} - S - \pi_G(p), 0\right\}$. Since $\pi_G(p)$ is an strictly increasing function of p for all $p \in [m_c, 2m_c]$ there are two possible situations.

Case 1: Suppose that $\sqrt{SB_1} - S > \pi_G(2m_c) = \beta_G \sum_{i\in I}(\alpha_i)^2 / 4m_c$. Then, the price selected by player 1 is the solution to the following optimization problem:

$$\max_{p\in[m_c,2m_c]}\left\{W_1(p) = \frac{(\alpha_1)^2}{p} - \lambda\pi_S(p) + y_1 - \sqrt{SB_1} + S + \pi_G(p) + \frac{\sqrt{SB_1} - S}{\sqrt{SB_1}}B_1\right\}$$

Take the derivative of W_1 with respect to p:

$$\frac{\partial W_1(p)}{\partial p} = \frac{-s_1 p + (\beta_G - \lambda\beta_S)(2m_c - p)}{p^3\left[\sum_{i\in I}(\alpha_i)^2\right]^{-1}}$$

where $s_1 = \frac{(\alpha_1)^2}{\sum_{i\in I}(\alpha_i)^2}$. The numerator of $\frac{\partial W_1(p)}{\partial p}$ is decreasing in p. Thus, there are two possible cases to consider:

Case 1.a: Suppose that $s_1 \geq \beta_G - \lambda\beta_S$. Then, W_1 is strictly decreasing in p for all $p \in [m_c, 2m_c]$. Thus, the price that maximizes W_1 is $p = m_c$. Therefore, the unique subgame perfect Nash equilibrium outcome is $(p, T_1, T_2) = \left(m_c, \sqrt{SB_1} - S, 0\right)$.

Case 1.b: Suppose that $s_1 < \beta_G - \lambda\beta_S$. Then, W_1 is strictly increasing in p for all $p \in \left[m_c, \hat{p}^1\right]$ and strictly decreasing in p for all $p \in \left[\hat{p}^1, 2m_c\right]$, where $\hat{p}^1 = \frac{2m_c(\beta_G - \lambda\beta_S)}{s_1 + \beta_G - \lambda\beta_S}$. Thus, W_1 adopts its maximum at $p = \hat{p}^1$. Therefore, $(p, T_1) = \left(\hat{p}^1, \sqrt{SB_1} - S - \pi_G\left(\hat{p}^1\right)\right)$.

Case 2: Suppose that $\sqrt{SB_1} - S \leq \pi_G(2m_c) = \beta_G \sum_{i\in I}(\alpha_i)^2 / 4m_c$. Then, the price selected by player 1 is the solution to the following optimization problem:

$$\max_{p\in[m_c,2m_c]}\left\{W_1(p) = \frac{(\alpha_1)^2}{p} - \lambda\pi_S(p) + y_1 + \left\{\begin{array}{ll} -\left[\sqrt{SB_1} + S - \pi_G(p)\right] + \frac{\sqrt{SB_1} - S}{\sqrt{SB_1}}B_1 & \text{if } p \in [m_c, \bar{p}) \\ \frac{\pi_G(p)}{\pi_G(p) + S}B_1 & \text{if } p \in [\bar{p}, 2m_c] \end{array}\right.\right\}$$

where $\bar{p}$ is the unique solution to $\sqrt{SB_1} - S = \pi_G(\bar{p})$.

W_1 has the following properties:

- W_1 is a continuous function of p for all $p \in [m_c, 2m_c]$. In particular, it is continuous at $p = \bar{p}$. To prove this, note that:

$$\lim_{p \to \bar{p}^-} W_1(p) = \frac{(\alpha_1)^2}{\bar{p}} - \lambda \pi_S(\bar{p}) + y_1 - \left[\sqrt{SB_1} + S - \pi_G(\bar{p})\right] + \frac{\pi_G(\bar{p})}{\pi_G(\bar{p}) + S} B_1$$
$$= \frac{(\alpha_1)^2}{\bar{p}} - \lambda \pi_S(\bar{p}) + y_1 + \frac{\sqrt{SB_1} - S}{\sqrt{SB_1}} B_1 = \lim_{p \to \bar{p}^+} W_1(p),$$

  where we have employed that $\sqrt{SB_1} - S - \pi_G(\bar{p}) = 0$.

- Take the derivative of W_1 with respect to p for $p \in [m_c, \bar{p})$:

$$\frac{\partial W_1(p)}{\partial p} = \frac{-s_1 p + (\beta_G - \lambda \beta_S)(2m_c - p)}{p^3 \left[\sum_{i \in I} (\alpha_i)^2\right]^{-1}}$$

  Let $N(p) = -s_1 p + (\beta_G - \lambda \beta_S)(2m_c - p)$ be the numerator of $\frac{\partial W_1(p)}{\partial p}$. $N(p)$ is decreasing in p. Thus, there are three possible cases to consider:

  - Suppose that $s_1 \geq (\beta_G - \lambda \beta_S)$. Then, W_1 is strictly decreasing in p for all $p \in [m_c, \bar{p})$:
  - Suppose that $\left(\frac{2m_c - \bar{p}}{\bar{p}}\right)(\beta_G - \lambda \beta_S) < s_1 < (\beta_G - \lambda \beta_S)$. Then, W_1 is strictly increasing in p for all $p \in \left[m_c, \hat{p}^1\right]$ and strictly decreasing in p for $p \in \left[\hat{p}^1, \bar{p}\right)$, where $\hat{p}^1 = \frac{2m_c(\beta_G - \lambda \beta_S)}{s_1 + \beta_G - \lambda \beta_S} \in (m_c, \bar{p})$.
  - Suppose that $s_1 \leq \left(\frac{2m_c - \bar{p}}{\bar{p}}\right)(\beta_G - \lambda \beta_S)$. Then, W_1 is strictly increasing in p for all $p \in [m_c, \bar{p})$.

- Take the derivative of W_1 with respect to p for $p \in [\bar{p}, 2m_c]$:

$$\frac{\partial W_1(p)}{\partial p} = \frac{-s_1 p - \lambda \beta_S (2m_c - p) + \frac{(2m_c - p)\beta_G S B_1}{[\pi_G(p) + S]^2}}{(p)^3 \left[\sum_{i \in I} (\alpha_i)^2\right]^{-1}}$$

  Let $N(p) = -s_1 p - \lambda \beta_S (2m_c - p) + \frac{(2m_c - p)\beta_G S B_1}{[\pi_G(p) + S]^2}$ be the numerator of $\frac{\partial W_1(p)}{\partial p}$. There are two possible cases to consider:

  - Suppose that $s_1 \geq \left(\frac{2m_c - \bar{p}}{\bar{p}}\right)(\beta_G - \lambda \beta_S)$. Then, W_1 is strictly decreasing in p for all $p \in [\bar{p}, 2m_c]$.
  - Suppose that $s_1 < \left(\frac{2m_c - \bar{p}}{\bar{p}}\right)(\beta_G - \lambda \beta_S)$. Then, W_1 is strictly increasing in p for all $p \in \left[\bar{p}, \hat{p}^2\right]$ and strictly decreasing in p for all $p \in \left[\hat{p}^2, 2m_c\right]$, where $\hat{p}^2 \in (\bar{p}, 2m_c)$ is the unique solution to $\frac{p s_1}{(2m_c - p)} + \lambda \beta_S = \frac{\beta_G S B_1}{[\pi_G(p) + S]^2}$.

Employing the above characterization of $W_1(p)$ we have the following possible cases:

Case 2.a: Suppose that $s_1 \geq (\beta_G - \lambda\beta_S)$. Then, W_1 is strictly decreasing in p for all $p \in [m_c, 2m_c]$. Thus, W_1 adopts its maximum at $p = m_c$. Therefore, $(p, T_1) = \left(m_c, \sqrt{SB_1} - S, \right)$.

Case 2.b: Suppose that $\left(\frac{2m_c - \bar{p}}{\bar{p}}\right)(\beta_G - \lambda\beta_S) < s_1 < (\beta_G - \lambda\beta_S)$. Then, W_1 is strictly increasing in p for all $p \in \left[m_c, \hat{p}^1\right]$ and W_1 is strictly decreasing in p for all $p \in \left[\hat{p}^1, 2m_c\right]$. Thus, W_1 adopts its maximum at $p = \hat{p}^1 \in (m_c, \bar{p})$. Therefore, $(p, T_1) = \left(\hat{p}^1, \sqrt{SB_1} - S - \pi_G\left(\hat{p}^1\right)\right)$.

Case 2.c: Suppose that $s_1 \leq \left(\frac{2m_c - \bar{p}}{\bar{p}}\right)(\beta_G - \lambda\beta_S)$. Then, W_1 is strictly increasing in p for all $p \in \left[m_c, \hat{p}^2\right]$ and W_1 is strictly decreasing in p for all $p \in \left[\hat{p}^2, 2m_c\right]$. Thus, W_1 adopts its maximum at $p = \hat{p}^2 \in [\bar{p}, 2m_c)$. Therefore, $(p, T_1) = \left(\hat{p}^2, 0\right)$.

This completes the proof of Proposition 7. ■