NBER WORKING PAPER SERIES

DEMAND ANALYSIS UNDER LATENT CHOICE CONSTRAINTS

Nikhil Agarwal
Paulo J. Somaini

Working Paper 29993
http://www.nber.org/papers/w29993

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
April 2022, Revised May 2025

We are grateful to the USRDS for facilitating access to the data. The authors acknowledge support from the NIH (P30AG012810), the Sloan Foundation (FG-2019-11484), and the MIT SHASS Dean's Research Fund. The data reported here have been supplied by the United States Renal Data System (USRDS). We thank Mert Demirer, Liran Einav, Alessandro Iaria, Francesca Molinari, Aviv Nevo, Peter Reiss and participants at several seminars and conferences for helpful feedback, and Chris Conlon and Alex McKay for their constructive discussions of our paper. We also thank Felipe Barbieri, Sichang Chen, Alden Cheng, Idaliya Grigoryeva, Lia Petrose, Ricardo Ruiz, and Yucheng Shang for their excellent research assistance. The interpretation and reporting of these data are the responsibility of the author(s) and in no way should be seen as an official policy or interpretation of the U.S. government, nor do they necessarily reflect the views of the National Bureau of Economic Research.

Demand Analysis under Latent Choice Constraints
Nikhil Agarwal and Paulo J. Somaini
NBER Working Paper No. 29993
April 2022, Revised May 2025
JEL No. C50, I11, L0

ABSTRACT

Consumer choices are constrained in many markets due to either supply-side rationing or information frictions. Examples include matching markets for schools and colleges; entry-level labor markets; limited brand awareness and inattention in consumer markets; and selective admissions to healthcare services. We analyze a general random utility model for consumer preferences that allows for endogenous characteristics and a reduced-form choice-set formation rule that can be derived from models of the examples described above. We show non-parametric identification of this model, propose an estimator, and apply these methods to study admissions in the market for kidney dialysis in California. Our identification results require two sets of instruments, one that only affects consumer preferences and the other that only affects choice sets. We show that both instruments are necessary for identification. These results also suggest tests of choice-set constraints, which we apply to the dialysis market. We find that dialysis facilities are less likely to admit new patients when they have a higher-than-normal caseload and that patients are more likely to travel further when nearby facilities have high caseloads. Finally, we estimate consumers' preferences and facilities' rationing rules using a Gibbs sampler.

Nikhil Agarwal
Massachusetts Institute of Technology
Department of Economics
and NBER
agarwaln@mit.edu

Paulo J. Somaini
Stanford University
Stanford Graduate School of Business
Department of Economics
and NBER
soma@stanford.edu

1 Introduction

In textbook discrete choice models, consumers pick their preferred option from an observed choice set at posted prices. Moreover, prices are the sole instrument that clears the market. In many instances, demand is rationed by information frictions or by supply-side policies other than prices: schools and colleges select which students to admit, healthcare providers may be capacity-constrained or selectively admit patients, and consumers may be unaware of some products due to information frictions. The final allocation, in these cases, depends on the constraints on the choice sets in addition to preferences and prices.

With the few exceptions that are discussed below, existing approaches for estimating preferences with latent choice constraints assume specific models of choice set formation. In two-sided matching models – school or college admissions (e.g. Agarwal and Somaini, 2018; Fack et al., 2019), and certain labor markets (e.g. Agarwal, 2015) – choice sets are determined by supply-side preferences, whereas search costs and incomplete information limit choice sets in models of consumer search (Hortaçsu et al., 2017; Heiss et al., 2021) or consideration sets (e.g. Manski, 1977; Swait and Ben-Akiva, 1987; Goeree, 2008; Abaluck and Adams-Prassl, 2021; Barseghyan et al., 2021a,b). Perhaps the only apparent similarity between these models is that consumers cannot choose from the full set of options.

This paper unifies the analysis of a large class of empirical models of consumer choice with latent choice-set constraints. Our model combines a general random utility model (Block and Marshak, 1960; Matzkin, 1993) with a reduced-form model for choice set formation. We show, by way of examples, that many commonly used models of latent choice sets are consistent with this general reduced form. We derive conditions under which this general model is non-parametrically identified using data on final allocations when preference and choice-set shifters are available. We also propose a tractable estimation procedure. Finally, we apply our methods to the market for kidney dialysis to test for supply-side rationing and to describe the potential biases from ignoring choice set constraints.

The random utility model for consumer preferences allows for rich observed and unobserved heterogeneity in consumer preferences and nests both product space and characteristic space models. It also allows for product unobserved attributes to be correlated with observed product characteristics (e.g. Berry, 1994; Berry et al., 1995). The reduced-form model of latent choice sets is grounded on structural models of constrained choices, including models of two-sided matching; dynamic models in which profit motives induce firms to be selective in their admission policies; and certain models of consideration sets, consumer search, and informational advertising.

The empirical challenge is that the observed allocations depend both on the preferences of agents and the choice set formation process, making it hard to disentangle the two. In particular, standard methods based on inverting market shares to estimate key demand parameters (e.g. Berry, 1994; Berry et al., 2013) are inapplicable in the presence of product-specific unobservables that influence choice sets. Intuitively, the product chosen most often need not be the one most preferred by the largest proportion of customers.[1] We show that our model is non-parametrically identified in the presence of two sources of variation. The first is an observable that affects choice-set constraints but is excluded from consumer preferences. The second is an observable that influences consumer choices but is excluded from the choice-set constraints. We show how to use these shifters to trace out the joint distribution of consumer preferences and latent choice sets. Moreover, we show that these shifters are necessary–our model is not identified if either is not available.

At the cost of requiring shifters on both sides, our results place minimal functional form and statistical restrictions on preferences and latent choice sets. The preference shifter may enter non-linearly in utility; functional form restrictions on the choice-set shifters are similarly weak; and unobservables that affect the choice sets can be arbitrarily correlated with preferences. Specific models of choice set formation and other approaches typically require stronger restrictions. The non-identification result in the absence of our shifters implies that these restrictions are necessary, and substitute for exogenous variation in the data.

We also allow for unobserved product characteristics that are correlated with observable characteristics to influence preferences or choice sets in our identification analysis, which creates an endogeneity problem similar to the one in demand analysis (see Berry et al., 1995, for example). We adapt methods from Berry and Haile (2014) to our model to show that across-market variation in instruments can be used to solve this problem.

We apply our methods to the kidney dialysis market in California. Patients with low enough kidney function need to undergo regular dialysis, typically thrice weekly for several hours at a time. The procedure requires the use of expensive machines, nursing care, and physical space to accommodate a patient. These resource constraints can limit the number of patients a facility can serve. Most of the costs of dialysis are borne by the taxpayer since Medicare provides near-universal coverage for costs related to kidney failure, irrespective of age. With

[1] A salient example is colleges – the largest colleges need not be the most desirable. Consider that Stanford University has an undergraduate enrollment higher than that of MIT. Tuition at Stanford is also higher. One of the authors of this study claims that MIT has a lower enrollment only because it has a lower capacity and is therefore more selective. Even when confronted with Stanford's lower overall acceptance rates, the author rebuts by suggesting that acceptance rates are a biased measure of selectivity because the applicant pools are endogenous and different.

approximately 750,000 patients on dialysis currently in the US, these costs approach 1% of the national healthcare expenditure (Chapter 10, U.S. Renal Data System, 2020).

Our choice-set shifter measures a facility's occupancy when patient i begins dialysis using the difference between the number of patients being treated at the facility when patient i begins dialysis and an estimated target. We exclude short-term variation in this measure from patient preferences while controlling for facility fixed effects. As hypothesized, our measure of caseload predicts whether or not a new patient is admitted into a facility even after controlling for facility-quarter fixed effects, suggesting that supply-side rationing due to capacity constraints can constrain a consumer's choice set. Gandhi (2021) relies on a similar argument to estimate preferences in nursing homes.

The shifter of consumer preferences is the distance between a facility and a patient's residence. We exclude this variable from the choice set formation process but include it in consumer preferences because dialysis involves several weekly visits. Consistent with the hypothesized effects, we document that distance to the facility chosen by a patient is higher if nearby facilities have higher than usual caseloads.

The main challenge in estimating our model is that the number of potential choice sets is large, even in markets with a few facilities. This curse of dimensionality creates a computational burden for approaches that integrate over all possible choice sets when computing the likelihood.[2] We solve this problem by estimating a parametric version of our model using a Gibbs sampler (see also He et al., 2024), which uses data augmentation to condition iteratively on either choice sets or utilities to address the curse of dimensionality. The Bernstein-von Mises Theorem implies that the posterior mean of the sampling chain we generate is asymptotically equivalent to a maximum likelihood estimator (van der Vaart, 2000, Theorem 10.1).

Our estimates indicate that selective admissions practices are important in the dialysis market. The probability that a patient is accepted at their first-choice facility is only 59.1%, and this probability varies by facility. Because selective admissions push patients to less desirable facilities, models that do not account for choice set constraints yield biased estimates. Abstracting away from selective admissions would estimate that the largest facilities are also the most desirable. We compare our approach to alternatives that naively correct for capacity constraints by including our measure of occupancy in the utility function, and show that naive corrections yield biased estimates of diversion ratios.

[2] Prior approaches have either assumed additional restrictions to reduce dimension (e.g., Gandhi, 2021) or have used non-likelihood-based methods that use a first-stage approximation to a market share function (e.g. Abaluck et al., 2020).

Related Literature

A large literature – dating back to Block and Marshak (1960) and Manski (1977) – presents several specific models with latent constraints on choice sets. A much more recent literature has analyzed identification in these models, including models of consideration sets (Abaluck and Adams-Prassl, 2021; Barseghyan et al., 2021b,a); two-sided matching (Diamond and Agarwal, 2017; He et al., 2024); and consumer search (Abaluck et al., 2020). Our approach covers models in each of these three groups but is not nested. At the cost of requiring both shifters of choice sets and of preferences, our results achieve point identification using fewer functional-form restrictions on preferences (cf. Diamond and Agarwal, 2017; Abaluck and Adams-Prassl, 2021; Barseghyan et al., 2021b,a; Abaluck et al., 2020; He et al., 2024; Barseghyan and Molinari, 2023) or on the dependence between preferences and choice-sets (cf. Abaluck et al., 2020; Abaluck and Adams-Prassl, 2021). It is worth reiterating that our non-identification results show that either two sets of shifters or these additional restrictions are necessary to achieve identification. Our results also hold under more general conditions than those in He et al. (2024), which requires additional shifters and non-primitive rank restrictions. We provide a more detailed comparison as we develop our results.

In addition, we also address endogeneity concerns with estimating demand models, by extending results in Berry and Haile (2014) to allow for constrained choice sets. This solution can be useful for a number of applications, such as estimating school demand to study equilibrium effects (e.g. Neilson, 2020), which has so far abstracted away from selective admission due to capacity constraints. Similar issues are likely important in other settings where prices are not the sole market-clearing mechanism.

A small recent literature studies the industrial organization of the dialysis industry. Many of these studies are based on quasi-experimental research designs (e.g. Dafny et al., 2018; Wollmann, 2022) or focus on supply-side issues such as investment or quality choice (Grieco and McDevitt, 2017; Eliason, 2019; Eliason et al., 2020; Kepler et al., 2022). In contrast, our focus is on estimating demand and the supply-side rationing policies in response to shorter-term capacity constraints while keeping investment and quality decisions fixed. Previous approaches to estimating demand in this setting have abstracted away from supply-side rationing.

Our empirical model is closest to those of selective admission practices in nursing homes (Ching et al., 2015; Gandhi, 2021), although these papers do not formally consider the identification. Our identification results also cover models of two-sided matching with fixed prices (e.g. Agarwal, 2015; Azevedo and Leshno, 2016); models of consumer choice with incomplete consideration sets (e.g. Manski, 1977; Swait and Ben-Akiva, 1987; Goeree, 2008); models with strict capacity constraints (de Palma et al., 2007); and models of consumer stock-outs (Conlon and Mortimer, 2013; Hickman and Mortimer, 2016). Estimating a more primitive model than our reduced form supply side requires additional application-specific assumptions on the structural model. We discuss the interpretation of our model in these specific applications in further detail in Section 2.2.

Overview

The paper proceeds as follows. Section 2 presents our model with attention to various models of supply-side rationing that yield the reduced form we consider. Section 3 presents the identification results and the estimator. Section 4 describes the dialysis industry and presents descriptive evidence on supply-side rationing. Section 5 presents the results from our estimates. Section 6 concludes. All proofs not included in the main text are in the appendix.

2 Model

We will consider markets, indexed by t, in which agents can be divided into two sets, I_t and J_t. We will refer to I_t as *consumers* and J_t as *products*. Consumers, indexed by $i \in I_t$, have unit demand. We will say that consumer i is *matched* with product $j \in J_t$ if it is in the consumer's choice set and the consumer chooses it. A product can match with many consumers. The outside option, denoted with 0, is always in the consumer's choice set.[3] Each consumer i participates in only one market t.

2.1 Preferences and Choices

We adopt a random utility model for consumer preferences. The indirect utility of consumer i for matching with product j is given by

$$v_{ij} = u_{jt}\left(d_i, \omega_i\right) - g_{jt}\left(d_i, y_{ij}\right), \tag{1}$$

where d_i is a vector of observed consumer attributes; y_{ij} is a scalar observed attribute that varies at the consumer-product level; and ω_i is a random vector of arbitrary dimension that introduces unobserved consumer-specific preference heterogeneity. We impose the following normalizations, which are without loss of generality (Matzkin, 2007): we normalize the utility of the outside option v_{i0} to zero for each i; for some known value y_0 and a fixed j in each t, we

[3] This avoids empty choice sets. The loss of generality is limited because the outside option can be defined as a composite of alternatives outside the market.

set $\left|\frac{\partial g_{jt}}{\partial y}\left(d_i, y_0\right)\right| = 1$ for all d_i; and we set $g_{jt}\left(d_i, y_0\right) = 0$ for every j, t and d_i. The restrictions on v_{i0t} and the partial derivative of $g_{jt}\left(\cdot\right)$ are familiar location and scale normalizations. The restriction that $g_{jt}\left(d_i, y_0\right) = 0$ is without loss because a constant shift in $g_{jt}\left(\cdot\right)$ can be subsumed in $u_{jt}\left(\cdot\right)$.

This model places minimal restrictions on the representation of preferences. The term ω_i allows for multi-dimensional unobserved heterogeneity, including idiosyncratic product-specific preference shocks. The functions $u_{jt}\left(\cdot\right)$ and $g_{jt}\left(\cdot\right)$ are indexed by product and market, allowing them to vary arbitrarily due to both observed and unobserved market-product-specific attributes. The term d_i may include attributes that vary at the consumer-product level in addition to those that only vary at the consumer level. The main distinction between y_{ij} and observables included in d_i is that y_{ij} only affects the indirect utility of product j and is separable from ω_i.

Unlike standard consumer choice models, consumers in our model cannot simply choose their most preferred product. In education markets, students must be accepted by the school; in healthcare markets, patients need appointments; in labor markets, applicants need job offers; in models of consumer search or consideration sets, choice sets are incomplete. For uniformity in nomenclature, we personify products and say that they must accept the consumer. Let

$$\sigma_{ijt} = \sigma_{jt}\left(d_i, \omega_i, z_{ij}\right) \in \{0, 1\} \tag{2}$$

denote this latent acceptance decision, where $\sigma_{jt}\left(d_i, \omega_i, z_{ij}\right) = 1$ denotes that consumer i was accepted by product j in market t. We refer to the function $\sigma_{jt}\left(\cdot\right)$ as the *acceptance policy function*. It is indexed by product and market, allowing it to depend on market-product-specific observables and unobservables. The product's decision to accept the consumer depends arbitrarily on ω_i as well. Therefore, utilities and acceptance decisions may be correlated due to unobservables.[4]

The term z_{ij} is a consumer-product-specific observable scalar that affects the decision of the product to accept the consumer but is excluded from the consumer's utility. As opposed to d_i, the scalar characteristic z_{ij} can only affect acceptances by product j, not product k. This implicitly rules out strategic interactions between products based on knowledge of competitor's z_{ij}, but allows for strategic interactions arising from aggregate market conditions or from consumer i's characteristics via the dependence of $\sigma_{jt}(\cdot)$ on t and on d_i.

We assume that each consumer is matched with their most preferred product that accepts

[4]This formulation and the results do not impose restrictions on the dependence between indirect utilities and choice sets. One can write $\omega_i = (\omega_i^u, \omega_i^\sigma)$ each of arbitrary dimension, and $u_{jt}\left(\cdot\right)$ and $\sigma_{jt}\left(\cdot\right)$ only depend on ω_i^u and ω_i^σ. At the extremes, ω_i^u and ω_i^σ could be independent or perfectly correlated.

them. Formally, consumer i's (latent) choice set is given by

$$O_i = \{j \in J_t : \sigma_{ijt} = 1\} \cup \{0\}$$

and she picks a product with the highest indirect utility within this set. Let $c_{ij} \in \{0, 1\}$ be an indicator for consumer i matching with $j \in O_i$. We assume that $\sum_{j \in O_i} c_{ij} = 1$ and $c_{ij} = 1$ only if $j \in \arg\max_{k \in O_i} v_{ik}$. If $\arg\max_{k \in O_i} v_{ik}$ is not a singleton, then the tie between the products with the highest indirect utilities is broken independently of $y_i = (y_{ij})_{j \in J_t}$ where t is the market to which i belongs. Thus, the choice set formation process is the only source of friction.

We will make the following assumption throughout the paper:

Assumption 1. *In each market t, the unobserved term ω_i is conditionally independent of the vector (y_i, z_i) given d_i.*

This assumption places two substantive restrictions. First, the conditional independence of ω_i from y_i implies that each component y_{ij} shifts preferences for j without interacting with consumer-specific unobservables that affect either preferences or choice sets. Second, z_i is similarly an instrument that shifts choice sets without affecting the distribution of preferences. The assumption does not rule out correlation between y_i and z_i conditional on d_i. The plausibility of these restrictions is specific to the empirical application. For now, we defer the discussion of these issues for specific context.

We assume that the data are generated by sampling the random vector ω_i independent and identically across consumers. Therefore, for each market t, the choice set and preferences of consumer i are independent from those of other consumers in market t conditional on the observables (d_i, y_i, z_i), where $y_i = (y_{ij})_{j \in J_t}$ and $z_i = (z_{ij})_{j \in J_t}$. However, consumer preferences and choice sets may be correlated within a market via the functions $u_{jt}(\cdot)$, $g_{jt}(\cdot)$ and $\sigma_{jt}(\cdot)$. These assumptions imply that the share of consumers with observables (d_i, y_i, z_i) that are matched with product j in market t is given by

$$s_{jt}(d_i, y_i, z_i) = \sum_{O \in \mathcal{O}} P\left(O_i = O, c_{ij} = 1 \middle| t, d_i, y_i, z_i\right),$$

where $\mathcal{O}$is the set of all possible choice sets. The data consists only of these market shares for each value of (d_i, y_i, z_i) in its support. Assumption 1 implies that the shares $s_{jt}(\cdot)$ can be re-written as

$$s_{jt}\left(d_i, y_i, z_i\right) = \sum_{O \in \mathcal{O}} P\left(c_{ij} = 1 \middle| O_i = O, t, d_i, y_i, z_i\right) P\left(O_i = O \middle| t, d_i, z_i\right). \tag{3}$$

The first term in the summand is the probability that a consumer with attributes (d_i, y_i, z_i) is matched with product j when faced with the choice set O, whereas the second term is the probability of choice set O given (d_i, z_i). Assumption 1 allows us to omit the conditioning on y_i when writing the second term. However, we cannot omit z_i from the first term because of dependence between preferences and choice sets due to ω_i, which is often not allowed in the prior literature. Since the distribution of ω_i conditional on $O_i = O$ depends on z_i, the distribution of c_{ij} conditional on $O_i = O$ also depends on z_i.

We assume that product j is more likely to be in consumer i's choice set if the shifter z_{ij} is lower:

Assumption 2. *The function $\sigma_{jt}\left(d_i, \omega_i, z_{ij}\right)$ is non-increasing in z_{ij}.*

Define the cutoff quantity, $\pi_{jt}\left(d_i, \omega_i\right) = \sup\left\{z : \sigma_{jt}\left(d_i, \omega_i, z\right) = 1\right\}$ where we adopt the convention that $\pi_{jt}\left(d_i, \omega_i\right) = \infty$ if $\sigma_{jt}\left(d_i, \omega_i, z\right) = 0$ for all z and $\pi_{jt}\left(d_i, \omega_i\right) = -\infty$ if $\sigma_{jt}\left(d_i, \omega_i, z\right) = 1$ for all z. Under assumption 2, the function $\pi_{jt}\left(\cdot\right)$ determines product j's acceptance policy for almost every z since $z < \pi_{jt}\left(d_i, \omega_i\right)$ implies $\sigma_{jt}\left(d_i, \omega_i, z\right) = 1$, and $z > \pi_{jt}\left(d_i, \omega_i\right)$ implies $\sigma_{jt}\left(d_i, \omega_i, z\right) = 0$. However, the acceptance policy function can take any value when $z = \pi_{jt}\left(d_i, \omega_i\right)$.

Our target primitive for each market t is the joint distribution of the random vector $(u_{it}, \pi_{it}) = \left(u_{1t}\left(d_i, \omega_i\right), \ldots, u_{J_t t}\left(d_i, \omega_i\right), \pi_{1t}\left(d_i, \omega_i\right), \ldots, \pi_{J_t, t}\left(d_i, \omega_i\right)\right)$ conditional on d_i and t, and the function $g_{jt}\left(\cdot\right)$. To see why, consider the special case in which (u_{it}, π_{it}) admits a density. Re-write equation (3) noting that ties in utility and acceptance cutoffs are zero-probability events:

$$\begin{aligned} s_{jt}\left(d_i, y_i, z_i\right) = \sum_{O \in \{O \in \mathcal{O} : j \in O\}} \int\int 1\left\{u_{ijt} - g_{jt}\left(d_i, y_{ij}\right) \geq u_{ikt} - g_{kt}\left(d_i, y_{ik}\right) \forall k \in O\right\} \\ \times \left[\prod\nolimits_{k \notin O} 1\left\{\pi_{ikt} < z_{ik}\right\} \prod\nolimits_{k \in O} 1\{\pi_{ikt} > z_{ik}\}\right] f_{U,\Pi|d_i,t}\left(u_{it}, \pi_{it}\right) \mathrm{d}u_{it} \mathrm{d}\pi_{it}. \end{aligned} \tag{4}$$

Hence, the vector of market shares in t is determined by $F_{U,\Pi|d_i,t}\left(u_{it}, \pi_{it}\right)$ and the functions $g_{jt}\left(\cdot\right)$. This joint distribution also determines the effects of changes in y_{ij} and z_{ij} on consumer and producer surplus.

This equation also shows that market shares depend both on the preferences of the consumers and the acceptance policies. Thus, unlike in standard models of consumer demand, the market share of product j is not equal to the fraction of consumers who prefer j to all other products. Therefore, commonly used demand-inversion methods (cf. Berry, 1994; Berry et al., 1995, 2013) are not applicable in our model.

2.2 Examples

We start by showing that our preference model accommodates commonly used random utility models. Then, we work out several examples that yield constrained consumer choice sets that are compatible with our acceptance policy function.

Example 1. (Preference Model) We encompass widely used discrete choice models with random coefficients and product-specific unobservables ξ_{jt} (e.g. Berry et al., 1995):

$$v_{ij} = d_i \Gamma x_{jt} + x_{jt}\beta_i + y_{ij} + \xi_{jt} + \varepsilon_{ij},$$

where each individual i belongs to only one market t. We can nest this specification by setting $u_{jt}(d_i, \omega_i) = d_i \Gamma x_{jt} + x_{jt}\beta_i + \xi_{jt} + \varepsilon_{ij}$, $\omega_i = (\beta_i, \varepsilon_{i1}, \ldots, \varepsilon_{iJ_t})$ and $g_{jt}(d_i, y_{ij}) = -y_{ij}$. The price of good j in market t can be included as an observed characteristic in x_{jt}. Our identification results will accommodate most commonly used distributional assumptions on ε_{ij}, including those for the logit or nested-logit models. We can also the pure characteristics model of Berry and Pakes (2007).

Example 2. (Selective Admission in Healthcare) Our acceptance policy function accommodates the supply-side model for skilled nursing facilities in Gandhi (2021). Facility j accepts a new patient i if the patient's profitability exceeds a threshold that depends on the facility's current caseload:

$$\sigma_{ijt}(d_i, \omega_i, z_{ij}) = 1\left\{NPV_{jt}(\omega_i, d_i) + V_j(z_{ij} + 1) - V_j(z_{ij}) > 0\right\},$$

where $NPV_{jt}(\omega_i, d_i)$ denotes the present value of variable profits from patient i at facility j, and $V_j(z_{ij}+1) - V_j(z_{ij})$ is the change in the continuation value given current caseload z_{ij}. The difference $V_j(z_{ij}) - V_j(z_{ij}+1)$ is the opportunity cost of accepting a new patient, which Gandhi (2021) shows, is increasing in z_{ij}.

Example 3. (Two-Sided Matching) Our framework encompasses empirical models of two-sided matching markets with non-transferable utility. Examples include the matching of students to schools or colleges, and entry-level labor markets with fixed pay scales (e.g. Agarwal, 2015). Let $e_{jt}(d_i, \omega_i, z_{ij})$ be an unknown rule that school or college j employs in market t to evaluate candidates. For example, in the case of college acceptances, d_i may contain demographic information and observable exam scores, ω_i includes unobservable essay quality or other hard-to-codify aspects of an application, and z_{ij} is an observed characteristic that varies at the student-school level. Azevedo and Leshno (2016) showed that a pairwise stable allocation in a many-to-one model can be described by school- and market-specific cutoffs p_{jt} such that each agent i is assigned to her most preferred facility in the set $O_i = \{j \in J_t : e_{jt}(d_i, \omega_i, z_{ij}) \geq p_{jt}\} \cup \{0\}$. Thus, $\sigma_{ijt} = 1\left\{e_{jt}(d_i, \omega_i, z_{ij}) - p_{jt} > 0\right\}$.The identification of a similar many-to-one matching model was studied in He et al. (2024). Our results require fewer exogenous shifters and place fewer restrictions on primitives, a comparison that we further flesh out in section 3.

Example 4. (Consideration Sets) Several models in marketing and economics assume that consumers choose among the subset of products in the market (see Manski, 1977; Swait and Ben-Akiva, 1987; Goeree, 2008). In our framework, product j belongs to the latent consideration set O_i if $\sigma_{jt}(d_i, \omega_i, z_{ij}) = 1$. Since d_i and ω_i are arguments in $u_{jt}(\cdot)$, consideration sets can be correlated with utilities. The main requirement of our model is that there are consumer-product-specific characteristics z_{ij} that affect the probability that product j belongs to i's consideration set. This requirement is satisfied by a number of microfoundations. We discuss a few below:

<u>Brand Awareness:</u> Butters (1977) models advertising as affecting the probability with which a consumer is informed about a product. Goeree (2008) estimates an empirical model that uses the interaction between a product's advertising expenditure and a consumer's exposure to advertising to construct z_{ij}. Another example is Gaynor et al. (2016), which models a physician who determines a patient's consideration set. Consideration sets are likely to be correlated with preferences in this setting, as is allowed in our framework.

<u>Inattention and Defaults:</u> Consumers in some models are inattentive and choose a default unless sprung into action (e.g. Hortaçsu et al., 2017). These models often feature strong defaults where only the characteristics or utility of the default option influences attention (e.g. Abaluck and Adams-Prassl, 2021). In some of these models, attention is binary where a consumer either pays attention to all products or none. Our framework allows some products to be more likely to be considered than others, but requires product-specific consideration with z_{ij} as a shifter.

<u>Fixed Sample Search:</u> Models of fixed sample search often feature choice over a latent subset of heterogeneous products (see Honka, 2014; Honka et al., 2017, for example). Assume that consumers know their preferences for each product except for the prices. The consumer pays a search cost to obtain price quotes for a set of products determined based on ex-ante beliefs

(Chade and Smith, 2006). Thus, the decision to search for a product is given by the search policy function $\sigma_{jt}(\cdot)$.

In our framework, let y_{ij} be the price that is unobserved by the consumer prior to search. The realized values of σ_{ijt} can depend on the ex-ante price distribution, the other components of indirect utility, and search costs. Thus, σ_{ijt} can be correlated with v_{ijt}, but it is not a deterministic function of v_{ijt}.[5] We also require an observable z_{ij} that is excluded from preferences but shifts the probability that consumer i searches for product j. Examples may include informative advertising or distance to the product – the former through awareness and the latter through search costs – while being independent of preferences.

Stock-outs: Consider a case in which a product may be stocked out when a consumer arrives. Hickman and Mortimer (2016) distinguish two data environments depending on whether stock-out events are observed or not. When stock-out events are observed, they are useful for estimating demand cross-elasticities as in Conlon and Mortimer (2013). Alternatively, a product may or may not be available for all consumers within a market in which case, the aggregate market share of the product will be zero in that market (see Dube et al. (2021)). In this case, choice sets are effectively observed.[6] However, when the dataset does not directly yield specific stock-out events, consumer choice sets are latent and cross-elasticities are generally not identified. We model latent choice sets by letting σ_{ijt} denote whether product j was available at the time agent i arrived. The choice set shifter z_{ij} may be the time lag between when product j was last restocked and when consumer i checked out. Our results imply that variation in z_{ij} can restore identification of demand.

3 Identification

Subsection 3.1 shows identification of the joint distribution $F_{U,\Pi|d_i,t}$ and the function $g_{jt}(\cdot)$. Identifying the distribution of u_{it} in a neighborhood and the derivative of the function $g_{jt}(\cdot)$ is sufficient for identifying changes in demand in response to changes in y_i and for performing welfare analysis if $g_j(d_i, y_i)$ is an appropriate numeraire. Identifying π_{it} allows us to obtain $\sigma_{jt}(\cdot)$, which are product-specific acceptance policy functions. Subsection 3.2 shows that choice-set shifters are necessary for the aforementioned identification results.

[5] Models of sequential search do not naturally fit our framework because the decision to continue searching depends on the highest utility amongst the goods already searched (Weitzman, 1979). In this case, y_{ij} cannot be excluded from $\sigma_{kt}(\cdot)$.

[6] Dube et al. (2021) also require an observable that shifts choice sets that is excluded from demand, but make progress using a shifter that is product-specific because choice sets are common to all consumers within a market.

This analysis will condition on d_i and t, focusing on within-market variation in (y_i, z_i). The conditioning on t fixes product-level observables and unobservables for all products in a market. In subsection 3.3, we will micro-found the dependence of $u_{jt}(\cdot)$, $\pi_{jt}(\cdot)$ and $g_{jt}(\cdot)$ on observed and unobserved product attributes x_{jt} and ξ_{jt}, allowing for endogeneity in x_{jt}. We will then show how instruments can be used to address this endogeneity, which allows us to identify the effects of changing x_{jt} while holding ξ_{jt} fixed on market shares.

3.1 Identification within a market

We will build the main result of this subsection (theorem 1) in two steps. First, lemma 1 shows identification given that the functions $g_j(\cdot)$ are known (section 3.1.1). Second, lemma 2 shows that the functions $g_j(\cdot)$ are identified under slightly stronger assumptions (section 3.1.2). These two results together will imply our main theorem (section 3.1.3). Throughout this subsection, we omit the market subscript t because we condition on it.

3.1.1 Identification with known $g(\cdot)$

Let $u_i = (u_j(d_i, \omega_i))_{j \in J}$, $\pi_i = (\pi_j(d_i, \omega_i))_{j \in J}$ and $\sigma_i = (\sigma_j(d_i, \omega_i, z_{ij}))_{j \in J}$. If $g(\cdot) = (g_j(\cdot))_{j \in J}$ is known, identification of the joint distribution of (u_i, π_i) given d_i, which implies identification of the joint distribution of (v_i, σ_i) given (d_i, y_i, z_i), can be achieved without any further assumptions.

Lemma 1. *Fix d_i. Suppose that assumptions 1 – 2 are satisfied, and $g(\cdot)$ is known. Let χ be the interior of the support of (g, z) given d_i. The joint distribution of (u_i, π_i) conditional on $(u_i, \pi_i) \in \chi$ and d_i is identified.*

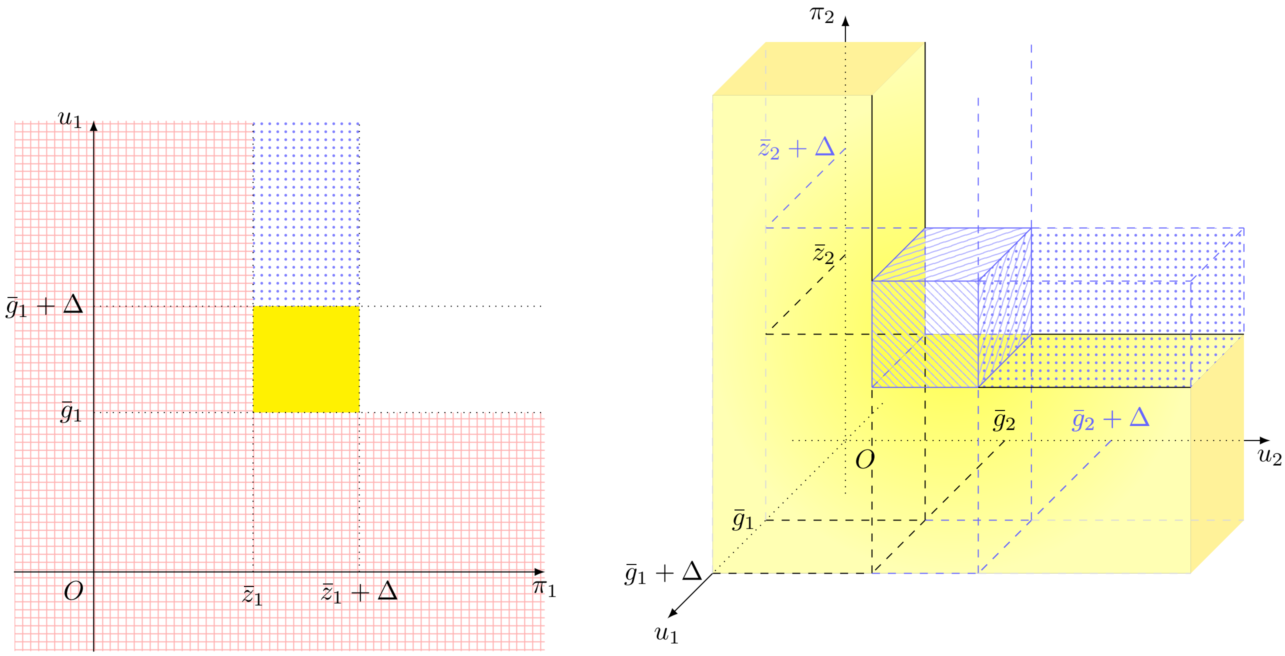
Proofs are in appendix A. The idea is best described with the aid of two figures. Assume for this illustration that (u, π) admits a density, a requirement that is not necessary for our formal results. Consider the probability that a consumer is not matched to any of the products in the market. This probability, which is observed, is equal to the probability that for every product j either $u_j < g_j$ or $\pi_j < z_j$, where ties are zero probability events. The cross-hashed region in figure 1(a) shows this set projected on the $u_1 - \pi_1-$hyperplane. That is, the random variables $u_2, \ldots, u_J$ and $\pi_2, \ldots, \pi_J$ are marginalized conditional on $u_j < g_j$ or $\pi_j < z_j$ for $j > 1$. The point $(\bar{g}_1, \bar{z}_1)$ collects the first components of any vector $(\bar{g}, \bar{z}) \in \chi$, which we fix in the remainder of the argument. Now, consider a small $\Delta > 0$ such that all points that are at most Δ away from each component of $(\bar{g}, \bar{z})$ belong to the interior of the support of $g(\cdot)$ and z. Perturb $\bar{z}_1$ by Δ to obtain the region between $\bar{z}_1$ and $\bar{z}_1 + \Delta$ that lies above $\bar{g}_1$. The probability that (u_1, π_1) falls within this region is equal to the increase in the probability from a consumer remaining unmatched at $(\bar{g}, \bar{z})$ to remaining unmatched when $\bar{z}_1$ is increased by Δ. This is because the change from $\bar{z}_1$ to $\bar{z}_1 + \Delta$ only affects consumers who would like to match with product 1 but the product drops out of the choice set due to this change. Since this increase in probability is observed, we can determine the probability that (u_1, π_1) belongs to the set $[\bar{g}_1, \infty) \times [\bar{z}_1, \bar{z}_1 + \Delta]$. Using a similar argument and subtracting observed probabilities, we can determine the probability that (u_1, π_1) belongs to the yellow square, with $u_2, \ldots, u_J$ and $\pi_2, \ldots, \pi_J$ marginalized as before. We can determine the density at the point $(\bar{g}_1, \bar{z}_1)$, marginalized over the other components, by considering an arbitrary small Δ.

In the special case when $J = 1$ so that there is only one inside option, the perturbations above have intuitive interpretations. Specifically, variation in $\bar{g}_1$ only affects the match of consumers on the margin between choosing the sole inside option and the outside option, and variation in $\bar{z}_1$ affects the match of consumers that are on the margin of being acceptable for product 1. Together, these two perturbations yield the density at the point $(\bar{g}_1, \bar{z}_1)$.

The argument outlined above only provides us with the marginal density of (u_1, π_1). This is because the shaded yellow box in figure 1(a) is a projection on the $u_1 - \pi_1 -$hyperplane. When projected on the $u_2 - \pi_2$ hyperplane, the set still has the L-shape implied by the conditions $u_2 < \bar{g}_2$ or $\pi_2 < \bar{z}_2$. The yellow region in figure 1(b) illustrates this set projected on the $u_1 - u_2 - \pi_2$ hyperplane for a particular value of $(\bar{g}, \bar{z})$. Observe that this region conditions on the event that $u_2 < \bar{g}_2$ or $\pi_2 < \bar{z}_2$ in order to focus on the set of consumers that would not be matched with product 2 if $\bar{g}_1$ or $\bar{z}_1$ were perturbed.

Our approach uses mathematical induction to extend this argument to higher dimensions, ultimately recovering the joint distribution of (u, π). The inductive step is also illustrated in figure 1(b). We can perturb $\bar{z}_2$ to $\bar{z}_2 + \Delta$ and repeat the steps of perturbing $\bar{z}_1$ and $\bar{g}_1$ at the value $\bar{z}_2 + \Delta$ to obtain the probability that $u_2 < \bar{g}_2$ or $\pi_2 < \bar{z}_2 + \Delta$, while focusing on consumers such that $(u_1, \pi_1) \in [\bar{g}_1, \bar{g}_1 + \Delta] \times [\bar{z}_1, \bar{z}_1 + \Delta]$. Similarly, we can perturb $\bar{g}_2$ to $\bar{g}_2 + \Delta$ to obtain the analogous quantity at $\bar{g}_2 + \Delta$. Subtracting these two quantities yields the probability that $(u_2, \pi_2) \in [\bar{g}_2, \bar{g}_2 + \Delta] \times [\bar{z}_2, \bar{z}_2 + \Delta]$, $(u_1, \pi_1) \in [\bar{g}_1, \bar{g}_1 + \Delta] \times [\bar{z}_1, \bar{z}_1 + \Delta]$ and for $j > 3$, $u_j < \bar{g}_j$ or $\pi_j < \bar{z}_j$. This set is the cross-hashed cube in Figure 1(b).

Although an illustration in higher dimensions is challenging, this process can be used to determine the probability that (u, π) belongs to $\prod_{j=1}^{J} [\bar{g}_j, \bar{g}_j + \Delta] \times [\bar{z}_j, \bar{z}_j + \Delta]$. This probability, for arbitrarily small Δ, yields the density of (u, π) if it exists. The proof formalizes this intuition without requiring that (u, π) admits a density by identifying the mass accumulated in sets that generate the Borel sigma algebra.



((a)) Two Dimensions

((b)) Three Dimensions

Figure 1: Identification

The message of the result is intuitive. Two sets of instruments, one that shifts choice sets and one that shifts preferences, can be used together to identify the distribution of utilities and acceptance decisions. The argument uses the variation in match probabilities with respect to the shifters g and z for preferences and acceptance decisions, respectively. Assumption 1 implies that each shift leaves the joint distribution of (u, π) unchanged. And, since π implies the vector of acceptance decisions σ for a given z, the result implies the identification of acceptance decisions jointly with the distribution of indirect utilities, u.

This argument is closely related to prior work in He et al. (2024), which shows identification in models of two-sided matching markets while relaxing previous restrictions on preference heterogeneity (e.g. Diamond and Agarwal, 2017). Although He et al. (2024) (henceforth HSS) show a result similar to lemma 1, there are three ways in which the results in HSS require stronger assumptions. First, HSS requires exogenous continuous variation in d_i, y_i and z_i (see Condition 3.3 and Proposition 3.4 in HSS), while we dispense with any requirement of variation (continuous or not) in d_i. Second, HSS identifies the functions $u_j(d_i)$, $g_j(y_{ij})$ and $\pi_j(d_i)$ in a first step using a non-primitive rank condition.[7] While we take $g(\cdot)$ to be given

[7] In our notation HSS assume $v_{ijt} = u_{jt}(d_i) - g_{jt}(y_{ij}) + \omega_{ijt}$, $\sigma_{ijt} = 1\{\pi_{jt}(d_i) - h_{jt}(z_{ij}) + \eta_{ijt} > 0\}$. Their results assume a rank condition on the matrix of derivatives of market shares with respect to each of the observable characteristics (Condition 3.3, HSS). One interpretation of lemma 1 is that it provides a primitive condition for their results in a more general model. In appendix B, HSS also show identification of

for now, our assumptions in section 3.1.2 for identifying this function can be verified from model primitives. Third, HSS require that the unobservable ω_i is separable from both d_i and y_{ij} so that $v_{ij} = u_j(d_i) + \omega_{ij}^u - g_j(y_{ij})$ and $\pi_{ij} = \pi_j(d_i) + \omega_{ij}^\pi$. This restriction rules out models with non-separable unobserved heterogeneity, which include characteristic space models (e.g. Berry and Pakes, 2007) and certain other random coefficient models.[8] Our approach allows d_i and ω_i to be non-separable. Finally, section 3.3 considers a model with endogenous product characteristics.

3.1.2 Identification of $g(\cdot)$

The results above assume that the functions $g_{jt}(\cdot)$ are known. We will now show that $g_j(\cdot)$ is also non-parametrically identified under weak assumptions.

Definition 1. Goods j and k are strict substitutes in y at (d_i, y_i, z_i) if $\frac{\partial}{\partial y_{ik}} s_j(d_i, y_i, z_i)$ and $\frac{\partial}{\partial y_{ij}} s_k(d_i, y_i, z_i)$ exist and are non-zero.

This notion is a mild strengthening of requirements imposed in equation (1) and assumption 1, which together imply that the market share of each good k is weakly increasing (decreasing) in y_{ij} if $g_j(d_i, y_{ij})$ is weakly increasing (decreasing) in y_{ij}. It further requires the existence of cross-partials and assumes that they are non-zero.

Define $\Sigma_{j,k}(d_i, y_{ij}, y_{ik}) = 1$ if there is $\bar{y}_i$ and z_i in their respective supports such that goods j and k are strict substitutes in y at $(d_i, \bar{y}_i, z_i)$, $\bar{y}_{ij} = y_{ij}$, and $\bar{y}_{ik} = y_{ik}$. For two goods j and k, we say that there is a path connecting two values y_j and y_k, respectively, if there is a sequence of goods m_l and values of y_l, $(j, y_j) = (m_1, y_1), (m_2, y_2), ..., (m_n, y_n) = (k, y_k)$, such that for all $l = 2, ..., n$, $\Sigma_{m_{l-1}, m_l}(d_i, y_{l-1}, y_l) = 1$.

Assumption 3. *For every d_i, every good k, and almost all values of y_{ik} in its support, there exists a path connecting good k and value y_{ik}, (k, y_{ik}), to the reference good j and the reference value y_0, (j, y_0), for which we have normalized* $\left|\frac{\partial g_j(d_i, y_0)}{\partial y}\right| = 1$.

This substitutes assumption is weaker than requiring strict substitution between every pair of goods at all values of y_i and z_i. Moreover, the condition is testable. In our model, there are at least two important reasons why a given pair of goods j and k may not be substitutes. First,

certain derivatives of indirect utility functions with non-separable unobserved heterogeneity. However, these results are not sufficient for identification of the distribution of preferences.

[8] In appendix A, He et al. (2024) also show identification of certain derivatives of indirect utility functions with a particular form of non-separable unobserved heterogeneity that is not nested in our model. The results in that appendix are not sufficient for identification of the distribution of preferences.

preferences for goods may restrict substitution patterns between goods that are considered. Salient examples include models with vertical preferences where consumers only substitute to goods that are adjacent in quality ranking or the pure characteristics model of Berry and Pakes (2007). Nonetheless, these models often admit a path connecting any pair of goods, thereby satisfying assumption 3 (see Berry et al., 2013, for related ideas). Second, choice sets may restrict substitution in demand. For example, if latent choice sets are of the form that goods j and k never appear in the choice set together, then the relevant cross-partials of the shares of these goods would be zero. However, there will still be a path connecting two values of their shifters y_j and y_k, if there is a third good, l and a shifter value y_l, such that the pairs $\{j, l\}$ and $\{l, k\}$ are strict substitutes. Furthermore, the values of y_i and z_i at which $\{j, l\}$ are strict substitutes can be different than those at which $\{l, k\}$ are strict substitutes.

Proposition 2 in the appendix shows weak primitive conditions under which goods j and k are strict substitutes. It shows that goods j and k are strict substitutes in y_i at (d_i, y_i, z_i) if the pair of goods $\{j, k\}$ belong to the choice set O_i with non-zero probability, the derivatives of $g_j(d_i, \cdot)$ and $g_k(d_i, \cdot)$ are non-zero, and the joint distribution of indirect utilities implies substitution between the goods in demand. This requirement is satisfied for well-behaved pure characteristics models, including versions with vertical preferences. Hence, a researcher may justify assumption 3 by either evaluating the assumption directly in the data or arguing for the sufficient conditions based on proposition 2 or corollary 3.

While assumptions 1 and 2 have allowed for atoms in the joint distribution of (u_i, π_i), assumption 3 requires that some regions admit a density between pairs of components of u_i. If the distribution of u_i (conditional on d_i) has an atom at $g(d_i, y_i)$, then $s(d_i, y_i, z_i)$ may not be differentiable with respect to y_i at that value even if the function $g(d_i, y_i)$ is differentiable. We view this restriction as mild.

Finally, we require support and regularity conditions to identify $g(\cdot)$:

Assumption 4. *(i) The support of the random vector y_i, denoted Y, is rectangular with a non-empty interior.*

(ii) For each d_i and j, the function $g_j(d_i, y_j)$ is continuously differentiable in y_j.

Part (i) places a weak requirement on the support of Y that is used mostly for tractability and allows us to write $Y = \prod_j Y_j$ where Y_j is a non-empty closed interval. Part (ii) implies that the functions $g_j(d_i, y_j)$ are smooth with respect to the second argument.

Lemma 2. *Suppose that assumptions 1, 3 and 4 hold and $|J| > 1$. Then, for every $j \in J$, the function $g_j(d_i, \cdot)$ is identified for all $y_j \in Y_j$.*

The argument first identifies the ratio of $g_k'(d_i, y_{ik})$ and $g_j'(d_i, y_{ij})$ for goods j and k that are strict substitutes in y at (d_i, y_i, z_i). Consider the inclusive value of a consumer conditional on (d_i, z_i, y_i). Dropping the conditioning on d_i and z_i, this inclusive value is given by

$$V^*(g(y_i)) = \sum_{O \in \mathcal{O}} E\left(\max_{j \in O} u_j(\omega_i) - g_j(y_{ij}) \middle| O, g(y_i)\right) P(O),$$

where $g(y_i) = \left(g_1(y_{i1}), \ldots, g_J\left(y_{i|J|}\right)\right)$ and assumption 1 implies that the probability that $P(O)$ does not depend on y_i. The envelope theorem implies that $\frac{\partial V^*(g(y_i))}{\partial g_j} = -s_j(y_i)$. This result is a version of Roy's identity for stochastic choice models (see McFadden, 1981), but for models with latent choice set constraints.[9] Assume that $V^*(g)$ is twice-continuously differentiable, a requirement that our proof dispenses with but is useful for exposition. Then, the partial derivative of this equation with respect to y_{ik} yields that $\frac{\partial s_j(y_i)}{\partial y_{ik}} = -\frac{\partial^2 V^*(g(y_i))}{\partial g_j \partial g_k} g_k'(y_{ik})$. Taking the ratio of the partial derivatives of $s_j(\cdot)$ with respect to y_{ik} and of $s_k(\cdot)$ with respect to y_{ij}, we identify the following ratio by applying Young's theorem:

$$\frac{g_k'(y_{ik})}{g_j'(y_{ij})} = \frac{\partial s_j(y_i)}{\partial y_{ik}} / \frac{\partial s_k(y_i)}{\partial y_{ij}}. \tag{5}$$

If all pairs of goods are strict substitutes at all values of (y_i, z_i) (for each d_i), we could directly use the normalizations that $g_j(y_0) = 0$, $\left|\frac{\partial g_j(y_0)}{\partial y}\right| = 1$ and assumption 4 to solve for $g_k(\cdot)$ and $g_j(\cdot)$. Although not all pairs of good are strict substitutes, assumption 3 guarantees that there is a path connecting good k for almost all values of y_{ik} to the reference good j at the reference value y_0. Thus, the ratio of derivatives $\frac{g_k'(y_{ik})}{g_j'(y_{ij})} = \prod_{l=1}^{n} \frac{g_{j_l}'\left(y_{ij_l}\right)}{g_{j_{l-1}}'\left(y_{ij_{l-1}}\right)}$ is identified. The normalizations that $g_j(y_0) = 0$, $\left|g_j'(y)\right| = 1$ and assumption 4 can again be used to solve for $g_k(\cdot)$ and $g_j(\cdot)$.

As argued above, each function $g_{jt}(d_i, \cdot)$ can be identified when $J > 1$ under the assumptions outlined earlier. If $|J| = 1$, we can assume without loss that $g_j(d_i, \cdot)$ is known as long as it is monotonic since the outside option is normalized to zero.[10]

This result, which shows the identification of $g(\cdot)$, allows us to achieve identification without relying on quasi-linear special regressors. It differentiates our approach from that of Abaluck and Adams-Prassl (2021), which identifies three specific models of consideration set formation using departures from Slutsky symmetry of choice probabilities with respect to y. Instead, our model allows for asymmetries to arise from the non-linearity of indirect utilities in y_{ij}. The cost of this generality is the need for choice-set shifters.

[9] This proof technique is related to but not derivative of the methods used in Allen and Rehbeck (2019) to consider latent utility models with additive heterogeneity but without latent choice sets.

[10] To prove this claim, assume that $g_1(\cdot)$ is increasing and note that the market share of good 1 conditional on (y, z) is $s(y, z) = \int 1\{u_1(\omega) > g_1(y_1), \pi(\omega) > z_1\} \mathrm{d}F_\omega = \int 1\left\{g_1^{-1}(u_1(\omega)) > y_1, \pi(\omega) > z_1\right\} \mathrm{d}F_\omega$ where the equality follows because $g_1(\cdot)$ is monotonically increasing. Thus, the model is observationally equivalent to one in which $u_1(\cdot)$ is replaced with $g_1^{-1}(u_1(\cdot))$, and y_1 enters linearly.

3.1.3 Main Result

Lemmas 1 and 2 above yield the main identification result of the paper:

Theorem 1. *If assumptions 1 – 4 hold and $|J| > 1$, then for every d_i, (i) the function $g_j(d_i, \cdot)$ is identified for every $j \in J$ and $y_j \in Y_j$, and (ii) the joint distribution of u_i and π_i conditional on d_i is identified for every value (u, π) in the interior of $g(d_i, Y) \times Z = \prod_{j=1}^{J} g_j(d_i, Y_j) \times Z$, where $g_j(d_i, Y_j)$ is the image of the set Y_j under $g_j(d_i, \cdot)$ and Z is the support of the random vector z_i.*

Proof. The result follows immediately from Lemmas 2 and 1. The techniques used in this section rely only on local variation in the shifters y_i and z_i. The benefit of this approach is that it does not lean on "identification at infinity" arguments (see He et al., 2024, for example). For example, an alternative method for identifying the distribution of indirect utilities would be to focus on extreme values of z_i under which consumers can choose any product in the market and then rely on previous results. Such an argument would extrapolate the preferences of all consumers from a subset. □

Of course, we can learn about the distributions of u_i and π_i in only the regions that correspond to the support of the observables. When the observables have full support, we can identify the joint distribution of (π_i, u_i) conditional on d_i everywhere:

Corollary 1. *Suppose the hypotheses of theorem 1 hold. If the support of (u_i, π_i) is a subset of $int\,(g(d_i, Y) \times Z)$,* the *joint distribution of u_i and π_i conditional on d_i is identified.*

This joint distribution of u_i and π_i contains information about a host of economic phenomena based on unobservable factors. For example, the correlation between u_{ij} and $u_{ij'}$ implies that products j and j' are close substitutes., i.e. consumers who like one tend to also like the other one. Correlation between π_{ji} and $\pi_{j'i}$ suggests that products j and j' tend to prefer the same set of consumers. Moreover, the correlation between u_{ij} and π_{ji} suggests that consumers tend to prefer products that are likely to admit them.

Although local variation in (y_i, z_i) is useful, the effects on the probability that j is chosen from the set $O \supseteq \{j\}$ or on the probability that O is the choice set requires full or large support assumptions on $(g(d, Y), Z)$. An alternative approach to these support assumptions is to further restrict the model (see, for example, Barseghyan and Molinari, 2023). The trade-off between these strategies is context-specific.

3.2 Necessity of Choice Set Shifters for Identification

One might conjecture that choice set shifters are not necessary because a model with full choice sets is testable as long as an additively separable shifter of preferences is available. One rationale goes as follows: suppose that assumption 1 is satisfied, the joint distribution of (π_i, u_i) admits a continuous density function, and $P(O = J) = 1$. The density of indirect utilities at a point $g \in \mathbb{R}^J$ can be recovered either by using only local variation in g in the market share of the outside good or the market share in any good j.[11] Since the densities recovered in these two alternative ways must be equal to each other, the model is over-identified. Thus, it may be possible for the restrictions implicit in the model to discriminate between preferences and latent choice sets.

Our next result shows that this conjecture is false. That is, without further restrictions, it is not possible to identify both the distribution of latent choice sets and indirect utilities unless both sets of shifters are available.

Proposition 1. *Suppose assumption 1 is satisfied, and the joint distribution of u_i admits a density function. Further, assume that the support of z_i is a singleton $\{\bar{z}\}$ and $g(d_i, y_i)$ is observed and has full support on $\mathbb{R}^{|J|}$. If there exists an open set $B \subset \mathbb{R}^{|J|}$ and a choice set $O \subsetneq J$ such that for all $u \in B$, $f_U(u) > 0$ and $P(O|\, u) > \kappa > 0$, then $f_U(u)$ is not identified.*

The result shows that if variation from a shifter of choice sets is not available, then we cannot recover the distribution of utilities if we allow for incomplete latent choice sets. Therefore, the conclusions of lemma 1 and theorem 1 do not hold. Our proof explicitly constructs an alternative distribution of indirect utilities and latent choice set probabilities that result in an identical market share function. Intuitively, we can explain the probability that a product is chosen either using preferences conditional on a choice set or using the probability that a product is in the choice set.

The non-identification argument assumes that the preference shifter has full support. The under-identification issue would be more severe if the support of $g(d_i, y_i)$ is more limited or if $g(\cdot)$ were unknown. The main assumption is that choice sets cannot be complete for all values of u. As discussed above, if latent choice sets are complete, the distribution of preferences is over-identified under the remaining assumptions.

The result indicates that the conditions in theorem 1 are sharp. Any alternative to using shifters of choice sets would require further restrictions on the model. There are two such

[11] Observe that $s_0(g) = \int 1\{u \leq g\} f_U(u)\, \mathrm{d}u$ and $s_j(g) = \int 1\{u_j - g_j > 0\} \prod_{k \neq j} 1\{u_k \leq u_j + \tilde{g}_k\} f_U(u)\, \mathrm{d}u$ where $\tilde{g}_k = g_k - g_j$. Thus, $\frac{\partial^{|J|} s_0}{\partial g_1 \ldots \partial g_{|J|}}(g) = \frac{\partial^{|J|} s_0}{\partial \tilde{g}_1 \ldots \partial \tilde{g}_{j-1} \partial g_j \partial \tilde{g}_k \ldots \partial \tilde{g}_{|J|}}(g) = f_U(g)$.

approaches that we are aware of. The first, proposed in Abaluck and Adams-Prassl (2021), uses specific models of choice set formation. The second approach, proposed in Barseghyan et al. (2021a) and Barseghyan and Molinari (2023), uses a characteristic space model for the distribution of preferences (e.g. Berry and Pakes, 2007). In this approach the distribution of indirect utilities lies in a lower-dimensional manifold of $\mathbb{R}^{|J|}$, which cannot allow for idiosyncratic product-specific preferences. Our approach does not require these *a priori* restrictions.

3.3 Introducing Endogeneity

A challenge in estimating discrete choice demand systems is that unobserved characteristics lead to bias (Berry, 1994; Berry et al., 1995). Following this literature, assume that indirect utilities and the selectivity threshold can be written, with a slight abuse of notation, as:

$$u_{ijt} = \tilde{u}\left(x_{jt}, \xi^u_{jt}, \omega_i\right) \qquad \pi_{ijt} = \tilde{\pi}\left(x_{jt}, \xi^\pi_{jt}, \omega_i\right),$$

where ξ^u_{jt} and ξ^π_{jt} are scalar unobservables, x_{jt} denotes a vector of observable product characteristics that are potentially correlated with the unobservables $\xi_{jt} = \left(\xi^u_{jt}, \xi^\pi_{jt}\right)$, and $u(\cdot)$ and $\pi(\cdot)$ are unknown functions. We have dropped d_i from the notation because our arguments condition on it. Thus, the unobservable ξ_{jt} is implicitly d_i-specific. The combination of the assumptions that (i) the unobservables are scalars and (ii) the unknown functions are not indexed by j and t, makes this specification more restrictive than the ones analyzed in the prior subsections.

[12]

We assume that the data-generating process starts by sampling markets with characteristics of all the products in market t, namely $(x_t, \xi_t) = \{x_{jt}, \xi_{jt}\}_j$, drawn i.i.d. from a joint distribution that is common across markets. Then, ω_i and (y_i, z_i) are drawn i.i.d. across consumers in a market, with $\omega_i \perp (y_i, z_i)$ as before. Unlike prior subsections, the results in this subsection will exploit both cross-product and cross-market variation. We will therefore include the market index t for clarity.

Our goal is to identify the joint distribution $u_{it}, \pi_{it}| \, x_t, \xi_t$. Our previous results could not separate the effects of observables and unobservables because they conditioned on t. Now we aim to identify how the distribution of (u_{it}, π_{it}) varies with x_t and ξ_t. Knowledge of these

[12]We can also allow for observables in $g_{jt}(\cdot)$ that may be correlated with unobservables ξ^g_{jt}. Theorem 1 and corollary 1 imply that $g_{jt}(y_{ij})$ is identified on the support of y_{ij} in each market t. When $g_{jt}(y_{ij})$ takes the form $g\left(x_{jt}, \xi^g_{jt}, y_{ij}\right)$ and x_{jt} is potentially correlated with ξ^g_{jt}, then identification of the function $g(\cdot, \cdot, \bar{y})$ for a fixed value of $\bar{y}$ follows from existent results for non-linear IV models (e.g. Chernozhukov and Hansen, 2005).

distributions is necessary for identifying counterfactual choices or choice sets with exogenous changes in x_t.

We make the following restriction on $u(\cdot)$ and $\pi(\cdot)$:

Assumption 5. *Index restrictions.* x_{jt} *can be partitioned into* $\left(x^*_{jt}, \left(x^\delta_{jt}, x^\gamma_{jt}\right)\right)$ *such that indirect utility and acceptance thresholds take the form* $u_{ijt} = u\left(x^*_{jt}, \delta_{jt}, \omega_i\right)$ *and* $\pi_{ijt} = \pi\left(x^*_{jt}, \gamma_{jt}, \omega_i\right)$, *where* $\delta_{jt} = x^\delta_{jt} + \xi^u_{jt}$ *and* $\gamma_{jt} = x^\gamma_{jt} + \xi^\pi_{jt}$.

The index restrictions above are similar to those imposed in Berry and Haile (2014) to identify demand without choice-set constraints. Although the observable components x^δ_{jt} and x^γ_{jt} are one-dimensional, this restriction is inessential in a linear model as long as one of the components is known to have a non-zero coefficient because the model can be re-normalized. In other words, the observables x^δ_{jt} and x^γ_{jt} set the units for ξ_{jt}. [13] Finally, the observables x^δ_{jt} and x^γ_{jt} may be the same although this is not required as long as x^*_{jt} does not contain $\left(x^\delta_{jt}, x^\gamma_{jt}\right)$.

We now turn to the key assumption that forms the basis of our solution:

Assumption 6. *Invertibility. There exists a function* $\psi(\cdot, \cdot; x^*)$ *such that for any two markets* t *and* t' *with* $x^*_t = x^*_{t'} = x^*$, $\psi(\delta_t, \gamma_t; x^*_t) = \psi(\delta_{t'}, \gamma_{t'}; x^*_{t'})$ *implies* $(\delta_t, \gamma_t) = (\delta_{t'}, \gamma_{t'})$. *Moreover, for each market* t, $\phi_t = \psi(\delta_t, \gamma_t; x^*_t)$ *is known.*

We need that (δ_t, γ_t) is invertible in the observable quantity ϕ_t. It is worth emphasizing that the analyst need not know the function $\psi(\cdot)$, only the realized value of ϕ_t for any market. This assumption parallels the literature on the identification of demand. Specifically, Berry and Haile (2014) assume that the index of demand – δ_t in our case – is invertible in the vector of market shares – ϕ_t in our notation – and the unknown function maps δ_t to market shares – $\psi(\cdot)$ in our notation. Primitive conditions for invertibility in the case of demand (without constraints) are studied in Berry et al. (2013).

Our approach is similar. Recall that theorem 1 and corollary 1 show that the joint distribution of (u_{it}, π_{it}) is identified on the support of $(g(Y), Z)$. To solve the endogeneity problem, we will require the analyst to place sufficient primitive restrictions on the model to guarantee that these features identify ϕ_t.

We present two examples that satisfy assumption 6 below:

[13] Linearity can also be relaxed. The case when $\delta_{jt} = \tilde{\delta}(x_{jt}) + \xi^u_{jt}$ and likewise for γ_{jt} follows from an extension based on results in Matzkin (2007). The non-separable case follows from Chernozhukov and Hansen (2005), which requires strengthening the mean-independence restriction in assumption 1(i) below.

Example 5. Suppose that $(\delta_{jt}, \gamma_{jt})$ and x^*_{jt} are additively separable in both utility and acceptance,

$$u\left(x^*_{jt}, \delta_{jt}, \omega_i\right) = u_0\left(\delta_{jt}, \omega_i\right) + u_1\left(x^*_{jt}, \omega_i\right)$$
$$\pi\left(x^*_{jt}, \gamma_{jt}, \omega_i\right) = \pi_0\left(\gamma_{jt}, \omega_i\right) + \pi_1\left(x^*_{jt}, \omega_i\right),$$

and that $E\left[u_0\left(\delta_{jt}, \omega_i\right) \middle| \delta_{jt}\right]$ and $E\left[\pi_0\left(\gamma_{jt}, \omega_i\right) \middle| \gamma_{jt}\right]$ are strictly monotonic in δ_{jt} and γ_{jt}, respectively. The linear random coefficients preference model (example 2.2) satisfies these assumptions. Taking expectations conditional on market observed characteristics and indices, we get that

$$E\left[u_{ijt} \middle| \delta_t, x^*_t\right] = E\left[u_0\left(\delta_{jt}, \omega_i\right) \middle| \delta_{jt}\right] + E\left[u_1\left(x^*_{jt}, \omega_i\right) \middle| x^*_{jt}\right]$$
$$E\left[\pi_{ijt} \middle| \gamma_t, x^*_t\right] = E\left[\pi_0\left(\gamma_{jt}, \omega_i\right) \middle| \gamma_{jt}\right] + E\left[\pi_1\left(x^*_{jt}, \omega_i\right) \middle| x^*_{jt}\right],$$

where the equality follows because ω_i is independent of (δ, γ, x^*). This model satisfies assumption 6 with $\psi\left(\delta_t, \gamma_t; x^*_t\right) = \left\{E\left[u_{ijt} \middle| \delta_t, x^*_t\right], E\left[\pi_{ijt} \middle| \gamma_t, x^*_t\right]\right\}$. Large support of (Y, Z) is sufficient to identify these expectations (see corollary 1).

Example 6. Assumption 6 also holds under weaker requirements on support but stronger functional form assumptions. Consider the following vertical model:

$$u\left(x^*_{jt}, \delta_{jt}, \omega_i\right) = \alpha_i u_0\left(\delta_{jt}, x^*_{jt}\right) \qquad \pi\left(x^*_{jt}, \gamma_{jt}, \omega_i\right) = \beta_i \pi_0\left(\gamma_{jt}, x^*_{jt}\right),$$

for positive valued functions u_0 and π_0 that are strictly monotone in their first argument. Assume that $\omega_i = (\alpha_i, \beta_i)$ has support on $\mathbb{R}^2_+$. If the joint distribution of (α_i, β_i) is unimodal and the support of (Y, Z) in market each t identifies the mode of $\left(u\left(x^*_{jt}, \delta_{jt}, \omega_i\right), \pi\left(x^*_{jt}, \gamma_{jt}, \omega_i\right)\right)$ (via corollary 1), then assumption 6 follows with $\psi\left(\delta_t, \gamma_t; x^*_t\right)$ equal to the $2J$ vector with the mode of the joint distribution of u_{ijt} and π_{ijt} in the j and $j + J$ positions. Note that this support condition on (Y, Z) is weaker than those needed to identify expectations. Moreover, the assumption that ω_i is unimodal is testable.

Finally, we require the availability of instruments for x_t, which may be endogenous:

Assumption 7. *(i) Availability of instruments.* $E\left[\xi_t \middle| r_t\right] = 0$ *for all* r_t.[14]

(ii) Completeness. For any function $B\left(\phi_t, x^*_t\right)$ *with finite expectation,* $E\left[B\left(\phi_t, x^*_t\right) \middle| r_t\right] = 0$ *a.e. in* r_t *implies that* $B\left(\phi_t, x^*_t\right)$*=0 a.e. in* $\left(\phi_t, x^*_t\right)$.

This assumption is standard in the analysis of non-parametric instrumental variable models (see Newey and Powell, 2003) and is also required by Berry and Haile (2014). The completeness condition in part (ii) is the non-parametric analog to a rank condition in linear

[14] We sample r_t jointly with (x_t, ξ_t).

instrumental variable models. It implicitly requires that the dimension of r_t is at least equal to the dimension of (ϕ_t, x_t^*).[15]

We are now ready to prove our main result for the section:

Theorem 2. *If assumptions 5-7 are satisfied, then the pair of vectors* (δ_t, γ_t) *and* $\xi_t = (\xi_t^u, \xi_t^\pi)$ *are identified. In particular, the conditional distribution of* $u_{it}, \pi_{it}| x_t, \xi_t$ *is identified on the interior of the support of* $(g(Y_t), Z_t)$.

Assumption 6, the key hypothesis of Theorem 2, represents the main difference relative to those used for identifying and estimating models of demand without choice set constraints (e.g. Berry and Haile, 2014; Berry et al., 1995). In these analyses, identification arguments are based on a $J-$dimensional vector of indices containing product-level unobservables to be invertible in the $J-$dimensional vector of market shares. However, the existence of such an inverse – proved in Berry et al. (2013) for the case of demand – is not available in our case because we need to invert a $2J-$dimensional vector, (δ_t, γ_t), whereas market shares only have dimension J.

Our use of an inversion based on the $2J$-dimensional vector ϕ_t is motivated by our identification results that use within-market variation in preference and choice-set shifters, Y and Z. Corollary 1 shows that this variation allows us to identify the distribution of the $2J$-dimensional random variable (u_{it}, π_{it}) on the relevant support (corollary 1). To apply results in this subsection, the researcher needs to place sufficient restrictions on the model so that ϕ_t is a known function of this joint distribution and is identified for each market.

These results, which allow for endogenous characteristics in the presence of constrained choices, can be relevant for a number of applications. For example, a growing literature uses estimates of school demand to study the effects of competition between schools(e.g. Neilson, 2020). While this work incorporates unobserved factors that affect school demand, it abstracts away from the possibility that schools select students by assuming that each student is matched with their most preferred school in equilibrium, an assumption that may not be reasonable in markets with selective school admissions. Our framework, to our knowledge, is the first to accommodate both these features.

[15] The observables (x_t^δ, x_t^γ) and x_t^* may serve as instruments if they are mean-independent of ξ_t.

4 Data and Descriptive Analysis

4.1 Background

Dialysis, which removes toxins typically filtered by a kidney, is the predominant form of treatment for patients with End Stage Renal Disease (ESRD). Even with dialysis, the median survival for ESRD patients is about five years (Figure 5.7, U.S. Renal Data System, 2020).

The most commonly used method in the US is hemodialysis, accounting for about 90% of dialysis patients (Figure 1.2, U.S. Renal Data System, 2020). This method circulates the patient's blood through an extracorporeal artificial kidney. Hemodialysis is usually performed in an outpatient facility that focuses exclusively on dialysis treatments. It lasts between three to four hours and is performed two to three times a week, depending on the patient's residual kidney function. The second method, peritoneal dialysis, requires a surgically inserted catheter which can be used to administer a cleansing fluid and to collect waste. A patient's choice between the two modalities depends on numerous medical and lifestyle factors. We focus on facility-based hemodialysis patients, considering the choice of alternative treatment modalities as part of the outside option.

Facilities performing hemodialysis are highly regulated (Department of Health and Human Services: Centers for Medicare and Medicaid Services, 2008). The most binding constraint in the medium-term is the number of kidney dialysis stations in the facility. Dialysis machines are large and dedicated to a single patient at a time. Short-term inputs influencing capacity include nursing staff and technicians. Staffing, capital and space requirements make capacity adjustments to demand fluctuations a slow response (Eliason, 2019; Grieco and McDevitt, 2017).

Medicare provides insurance for costs related to ESRD for all US patients, irrespective of age. This coverage is secondary for patients with a private or employer health insurance plan during first 30 months after diagnosis of ESRD, called the coordination period. Each patient-year on hemodialysis costs approximately $90,000 at Medicare rates, and higher at private rates (Chapter 10, U.S. Renal Data System, 2020). With approximately 750,000 patients suffering from ESRD in the US, Medicare costs of patients with kidney failure totaled to $49.2 billion in 2018 (Chapter 1 and 10, U.S. Renal Data System, 2020). This figure is more than 7% of all Medicare claims and more than 1% of national healthcare spending (Chapter 10, U.S. Renal Data System, 2020).

4.2 Data

The data for this study are taken from the US Renal Data System (U.S. Renal Data System, 2020). These data are assembled from various sources, including Medicare claims, facility reports and data on patient outcomes. There are two important features of the data. First, we observe the residential zip-code, demographics, employment status and comorbidities of each patient, as well as the facility where each patient is being treated. These data also include patients who are initially covered by a private or employer health insurance plan because the start of dialysis determines eligibility for Medicare coverage. Second, the role of Medicare as the near-universal insurer allows us to track the number of patients being treated in each facility on any given day.

Our analysis sample focuses on patients whose first treatment commenced at a facility in California between 2015 and 2018. There are two main restrictions imposed by this choice. First, the restriction to a single state is for tractability. The vast majority of Californians do not live close to a neighboring state. Given the role of Medicare in this part of the healthcare sector, idiosyncrasies regarding California's healthcare sector are less relevant to our study. Our sample selection procedure is further described in appendix B. Second, we focus on the first facility where a patient begins dialysis to abstract away from considerations that are unique to switching facilities, which include interference with continuity of care and administrative or financial barriers.[16] In our sample, 74.2% of patients are treated at only one facility, and the average patient only visits 1.30 facilities. Our approach is consistent with facility moves being unexpected at the time when the patient begins dialysis.

4.3 Description of Sample and Choices

Table 1 describes the hemodialysis facilities in our sample. There are 552 facilities, most of them owned by one of the two large chains, Fresenius and DaVita. These and the vast majority of other facilities are for-profit and freestanding (not associated with a hospital). The average facility cares for just under 100 patients at a time, with chains and freestanding facilities caring for more patients per facility. The ratio of the number of stations to the number of patients is approximately five. This ratio is consistent with an average of two four-hour treatments per station per day since most patients require three treatments per week. Indeed, figure 2 shows that the number of patients per station is almost constant at five patients per station over the size distribution of facilities.

[16]We drop the certain quarters in which a facility enters, exits, moves or rapidly expands or contracts. See appendix B for further details. Patients matched to one of these facilities during this time-period are

Table 1: Facility Sample

	All facilities	Ownership		
		Fresenius and Davita	Other chains	Independent
Facility				
N	553	377	114	78
Facility-year	2093	1418	385	290
Number of patients				
Mean	108.6	113.0	100.2	98.2
Std. dev	46.8	46.3	38.9	54.9
Number of stations				
Mean	22.3	22.3	22.0	22.5
Std. dev	7.6	7.2	7.2	9.6

Notes: Sample of all facility-year observations, as described in table B.1. The number of patients at a facility is the daily average of enrolled patients undergoing hemodialysis.

Table 2 describes the patient sample, which contains 41,913 new patients. Most of these patients choose hemodialysis at a facility in our facility sample. The patients are predominantly white, and the incidence of hypertension and diabetes is high. The majority of patients are on Medicare, an HMO, or in the waiting period. The HMO group primarily consists of patients over the age of 65 who are covered by a Medicare Advantage plan. Going forward, we pool all patients who are Medicare eligible. The table also shows that the majority of patients begin dialysis in a freestanding facility.

Table 3 describes the facilities near the patients in our sample and the chosen facility. The average patient has 6.5 facilities within 5 miles of their home zip- code and 17 facilities within 10 miles. The typical patient receives dialysis at a facility with an average distance of 6.8 miles, but the median is lower, at 4.4 miles.

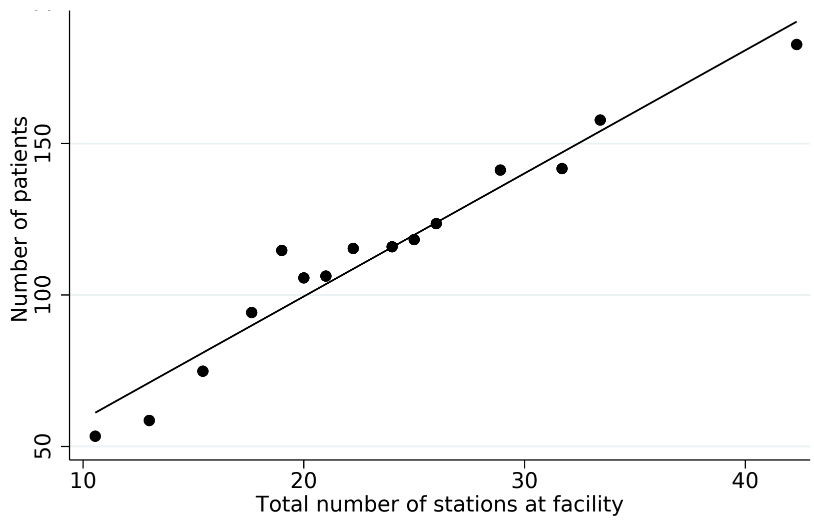
4.4 Evidence on Supply-Side Rationing

We now argue that capacity constraints affect the choice sets of patients. First, we show that facilities that have an unusually high caseload at a given point in time relative to their baseline are less likely to accept new patients for a while. Second, we show that the distance to the chosen facility is higher if nearby facilities are more constrained. Moreover, the effects of constraints at facilities of different qualities are different. This latter finding suggests that patients also have preferences over our measures of quality.

Effects on flow of new patients: We hypothesize that the current caseload at a facility

considered to be matched to the outside option.

Figure 2: Patients per Dialysis Station



Notes: Binscatter with all facility-year observations as described in table B.1. The number of patients at a facility is the daily average of enrolled patients undergoing hemodialysis. The number of stations is taken from the CMS Annual Facility Survey for the corresponding year.

influences the facility's decision to accept a new patient. Let z_{ij} be a measure of the excess occupancy (relative to a target) in facility j when patient i enters the dialysis market. If excess occupancy is excludable from the patients' utility, then the inflow of new patients into facility j should be conditionally independent of the facility's caseload, given controls for preferences. To see this, consider a model without capacity constraints in which $\sigma_{ij} = 1$ for all i and j. Assuming that the patient arrival is exogenous, the probability that a new patient arrives at facility j is given by the probability that $u_{ij} > u_{ij'}$ for all j', which is independent of z_{ij}. However, if facilities are less likely to accept a patient when z_{ij} is high, then the inflow of new patients will be negatively correlated with caseload. Gandhi (2021) presents one micro-foundation for this relationship.

We will test this hypothesis using regressions of two sets of dependent variables measuring patient inflow on occupancy and excess occupancy. In the first set, the dependent variable is whether a facility j accepts a new patient on day t. We use all days a facility is operating during our sample period for this set. The dependent variable in the second set is the number of the days until the next patient begins treatment at facility j. We use the subset of days in which a new patient began treatment for this set. The regressions control for either facility-year or facility-month level fixed effects, and cluster standard errors at the facility level. In a subset of regressions, we also control for the average occupancy in other facilities within five miles of facility j.

Facility occupancy is measured as the number of patients being treated on date t at facility

Table 2: Patient Sample

	All patients	Treated at an in-sample facility	Ownership		
			Fresenius and Davita	Other chains	Independen
			Panel A: Patient characteristics		
Patient Count	50002	43423	28647	7853	6923
Age (Mean)	63.1	63.7	63.6	64.9	63.1
Age (Std. Dev.)	15.0	14.8	14.8	14.9	15.1
Employed (%)	11.5	8.9	9.1	9.2	7.7
White (%)	71.3	72.1	73.0	67.4	73.4
Black (%)	10.7	10.8	11.1	11.2	9.2
BMI (Mean)	28.4	28.4	28.4	28.4	28.4
BMI (Std. Dev.)	7.3	7.4	7.4	7.6	7.5
Diabetes (%)	39.6	40.4	40.4	40.0	40.8
Hypertension (%)	86.5	86.4	85.8	86.9	88.2
			Panel B: Insurance type at admission		
Medicare (%)	32.9	32.8	32.4	37.0	29.6
Medicare Advantage (%)	24.5	24.6	24.7	24.0	24.5
Medicare waiting period (%)	12.5	13.1	12.7	12.8	15.2
Other (%)	30.1	29.6	30.2	26.3	30.8

Notes: Sample of patients, as described in patient Table B.2. BMI is Body Mass Index (kg/m^2). Medicare Waiting Period is the 90-day period before Medicare covers hemodialysis. Other represents patients not covered by Medicare when they begin dialysis.

j and the excess occupancy is the difference between occupancy and a measure of target occupancy. The target occupancy is motivated by an examination of the time series of the number of patients at a facility, which reveals that several facilities undergo periods of expansion or contraction. These periods may correspond to investment in capital, increases in staffing or restructuring of the facility's operations. One way to estimate target occupancy would be to use high-frequency data on facility inputs and investments in order to estimate facility capacity. Unfortunately, labor inputs and capital investment are recorded only annually, and their timing is unknown. Instead, we estimate target occupancy using a regime-switching autoregressive model with a linear trend on the occupancy time series for each facility. The model detects breaks in each facility's occupancy trend to identify points at which the facility's occupancy process changes. We construct the target occupancy on a given date as the expected value on a given day.[17] We do not detect any breaks in trends

[17] Specifically, let $n_{j\tau}$ be the number of patients being treated at facility j on day τ. Assume that $n_{j\tau}$ follows the following time series model with $m \geq 1$ regimes $n_{j\tau} = \alpha_{jk(\tau)} + \beta_{jk(\tau)}\tau + \gamma_{jk(\tau)} n_{j\tau-1} + e_{j\tau}$,where $k(\tau)$ is a weakly increasing function that maps days $\tau = 1, ..., T$ to regimes $k = 1, ..., m$. The disturbance $e_{j\tau}$ has mean zero, constant variance, and follows an ergodic process. This model is consistent with a birth-death process in which departure rates are proportional to n_{jt} and arrival rates are a function of $n_{j\tau} - n^*_{j\tau}$. The target occupancy on date τ is defined as $n^*_{j\tau} = \frac{\alpha_{jk(\tau)} + \beta_{jk(\tau)}\tau}{1-\gamma_{jk(\tau)}}$. The regime changes for each facility are estimated using a modified Schwartz criterion proposed in Liu et al. (1997). We winsorize $n_{j\tau} - n^*_{j\tau}$ by censoring the top and bottom 5% for each facility j in order to limit the influence of outliers.

Table 3: Patient Choices

	Facilities			
	Chosen	Within 5 miles	Within 10 miles	Within 25 miles
Number of facilities				
Mean	---	6.6	18.0	59.4
Std. dev	---	5.2	17.5	55.2
Median	---	5.0	11.0	32.0
Distance to facility				
Mean	6.7	3.2	6.0	14.1
Std. dev	7.3	0.7	1.3	3.1
Median	4.3	3.2	6.1	14.3
95th percentile	21.5	4.4	8.3	18.7
Number of patients at facility				
Mean	124.1	120.9	118.0	114.9
Std. dev	47.9	27.4	24.5	18.5
Median	120.0	121.7	120.1	120.9
Total stations				
Mean	24.1	23.4	23.1	22.8
Std. dev	7.8	4.2	3.3	2.3
Median	24.0	23.2	23.3	23.4
Chain (%)				
Overall mean	87.1	89.7	89.2	88.2
Fresenius	20.4	23.3	22.7	22.2
Davita	48.3	48.6	48.1	48.7

Notes: Sample of patient-facility pairs. Distance is measured in miles from the facility to the centroid of a patient's zip-code. The number of patients counts all patients enrolled at a facility that are undergoing hemodialysis.

for 500 of 553 facilities. Conditional on finding a break in the trend, the average number of breaks is 2.02. Therefore, while not rare, the breaks in trend are not relevant for the vast majority of facilities. Table B.3 in the appendix shows that our estimate of target occupancy is positively correlated with the (low-frequency) measures of facility inputs available in our dataset, even conditional on facility fixed effects. The daily within-facility standard deviation of excess occupancy is 4.22.

There are three notable findings from the regressions of patient inflows on occupancy (see table 4). First, controlling for facility-year fixed effects, higher occupancy is negatively correlated with the probability of a new patient beginning dialysis at the facility and positively correlated with the expected waiting time until the next patient (columns 3 and 4). Although not reported, the relationships are robust to the inclusion of occupancy at other nearby facilities or of finer controls, such as facility-quarter or facility-month fixed effects.

Second, including facility-time controls appears to be important. The results in columns (1) and (2) are analogous to those in columns (3) and (4), but use only facility-specific fixed effects instead of facility-year fixed effects. The estimated relationship between the probability of new patient beginning dialysis and the facility's occupancy is now positive. Thus, fluctuations in a facility's target occupancy may be important.

Third, our measure of excess occupancy purges some of the confounding variation in the raw

Table 4: Evidence of Capacity Constraints

	Any new patient	Log(days to next patient)	Any new patient	Log(days to next patient)	Any new patient		Log(days to next patient)	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Occupancy	0.0002**	0.004***	-0.0012***	0.019***				
	(0.0001)	(0.001)	(0.0001)	(0.002)				
Excess occupancy					-0.0004***	-0.0006***	0.016***	0.016***
					(0.0001)	(0.0001)	(0.002)	(0.002)
Occupancy within 5 miles						0.0007***		0.002*
						(0.0001)		(0.001)
Facility FE	X	X			X	X	X	X
Facily-Year FE			X	X				
Observations	724,946	35,332	724,946	35,332	724,946	706,690	35,332	35,332
R-squared	0.0128	0.112	0.0264	0.158	0.0128	0.0119	0.116	0.116

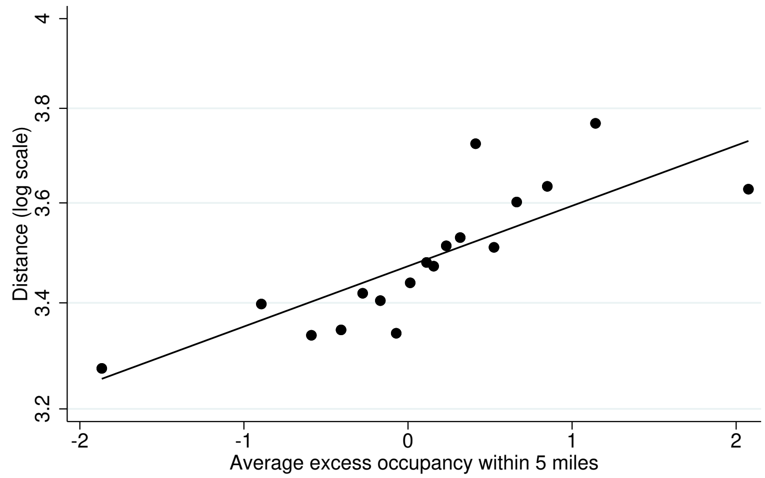
Notes: Sample of facilities as described in table 1. An observation is a day-facility pair, where the facility is open over the entire sample. Regressions with Log(days to next patient) consider the subset of days on which a facility admitted a new patient. $*p < 0.1$, $**p < 0.05$, $***p < 0.01$. Standard errors are clustered at the facility and county level.

measure of occupancy that resulted in a positive coefficient in column (1). This variation was absorbed in specifications that employed fixed effects at the facility-year or finer levels. Since including a richer set of fixed effects will not be feasible in the non-linear model that we will ultimately estimate, our empirical specifications will use this measure of excess occupancy in the acceptance policy function.

Fourth, these regressions also speak to the effect of capacity constraints at other facilities close to facility j. There are two opposing forces. Constraints at other facilities close to j can increase the demand for facility j. But, this force can also push facility j to be more selective and turn away less profitable patients because it expects a higher flow of patients, allowing the facility to cream-skim. Our results show that the number of patients being treated at other facilities close to facility j increases the probability that new patients start treatment at facility j (see columns 6 and 8 in table 4). This evidence suggests that increased demand at the facility dominates greater selectivity induced by constraints at nearby facilities. Because our results are consistent with facilities not responding to short-term constraints faced by competitors, we will ignore strategic interactions of this nature in our model as done also in Gandhi (2021) for the case of nursing homes.

Effects on where patients are treated: Having shown that capacity constraints affect the inflow of patients, we now investigate the effects of capacity constraints on where patients receive treatment. Figure 3 presents a binscatter indicating that the distance to the chosen facility is increasing in the average excess occupancy of facilities within five miles of the

Figure 3: Distance to Chosen Facility



Notes: Binscatter of incoming patients, residualized against patient zip-code and quarter-year fixed effects.

patient's zip-code centroid. This exhibit residualizes fixed effects at the zip-code-quarter level in order to control for confounding trends in the facilities' target occupancy.

Discussion: We argue that the evidence above suggests that supply-side rationing due to capacity constraints is important. The main potential threat is that crowded facilities are undesirable. However, this concern is limited if patient preferences depend on longer-term crowding than the finer variation that we leverage in these estimates. The annual within-facility autocorrelation in excess occupancy is 0.06, suggesting that utilization on a specific day is not strongly correlated with long-term occupancy.

An alternative interpretation is that capacity constraints result in waiting times rather than accept/reject decisions. Dialysis, however, is a time-sensitive treatment and delaying treatment even by a few days can pose substantial health costs. We therefore favor our interpretation over rationing by waiting time.

5 Estimates

5.1 Parametric Specification and Estimation

Although the arguments showing Theorem 1 are non-parametric and constructive, there are important challenges in using a non-parametric estimator. Our proof suggests estimating the market shares in equation (3) and then recovering $g(\cdot)$ and the joint distribution of (u, π) in a second step. This approach is challenging because of dimensionality – the share equation has a $J-$dimensional range and at least a $2J-$dimensional domain. Even in models without choice constraints, the curse of dimensionality limits the ability to scale the method for analyzing markets with many products (see also Compiani, 2021, for example). The typical solution directly estimates a parametric distribution of preferences and, in our case, also a parametric distribution of choice sets. Estimating such a model with latent choice sets using likelihood methods is exceedingly difficult because the number of potential choice sets is large even for relatively small J.

[18] Thus, enumerating all possible latent choice sets in order to compute the likelihood is computationally infeasible.

Instead, following much of the literature on discrete choice demand models, we parametrize the distribution of (u, π). First, we will address the curse of dimensionality due to the large number of potential choice sets using a Gibbs sampler (see also He et al., 2024). It modifies the sampler from McCulloch and Rossi (1994) with a data augmentation step to accommodate the case with latent choice sets. This will motivate distributional assumptions that admit closed-form solutions of certain conditional distributions. Second, we allow for correlations between preferences and choice sets via unobservables (ω_i in our notation). Third, we include random coefficients on the agent's preferences for facility characteristics, which allows for more flexible substitution patterns.

Based on these considerations, we make the following parametric assumptions:

$$v_{ij} = \delta_j + \beta_d d_i - g\left(d_i, y_{ij}\right) + \beta_i x_j + \varepsilon_{i0} + \varepsilon_{ij} \tag{6}$$

$$\sigma_{ij} = 1\left\{\gamma_j + \alpha d_i - z_{ij} + \nu_{ij} > 0\right\}, \tag{7}$$

where x_j are observed product characteristics, δ_j and γ_j are product fixed effects, and $\beta_i, \varepsilon_{i0}$, ε_{ij} and ν_{ij} are idiosyncratic shocks. We adopt the normalizations that $g'\left(d_i, y_{ij}\right) = 1$ at $y_{ij} = 1$ and $g\left(d_i, y_{ij}\right) = 0$ at $y_{ij} = 0$ for all d_i, and that the admission index is expressed in units of z_{ij}. As before, d_i is a vector of agent i's characteristics. We parametrize $g(\cdot)$ as a quadratic function given d_i, with parameters β_g, and collect $\beta = (\beta_w, \beta_g)$.

We allow for unobserved match-specific correlations by allowing ε_{ij} and ν_{ij} to be jointly normally distributed with mean zero and an estimated covariance matrix Σ. The term ε_{i0} captures individual heterogeneity in preferences for the outside option. A restriction relative to the non-parametric identification result is that we do not allow ν_{ij} and $\nu_{ij'}$ to be correlated

[18] The number of terms in the sum in equation (3) is equal to the number of possible choice sets, which is equal to $2^{|J|}$. With only fourteen facilities, which is approximately the average number of facilities within ten miles for a patient, the number of choice sets is 16,384.

with each other nor do we allow random coefficients on the acceptance functions.[19]

We will use the measure of excess occupancy presented in section 4 as the choice-set shifter, z_{ij}. Although Gandhi (2021) provides a micro-foundation, the model can accommodate other unspecified reasons why a facility may not be in a patient's choice-set via error terms in the specification of π_{ij}. Decomposing specific reasons for a facility not belonging to patient i's choice set requires additional structure and is, therefore, beyond the scope of this paper.

The parametric assumptions on the error terms allow us to use a Gibbs sampler for estimation because, under conjugate prior distributions, the conditional distributions of any of the latent error terms and random coefficients given the other terms can be obtained in closed form. Moreover, the conditional distributions of each of the parameters $(\alpha, \beta, \Sigma, \delta, \gamma)$ given the errors, random coefficients, and the other parameters can be obtained in closed form. The procedure iterates through each of these parameters, obtaining draws from their conditional posteriors to obtain a Markov Chain of draws of $(\alpha, \beta, \Sigma, \delta, \gamma)$. The draws of the chain converge to the posterior distribution, which is asymptotically equivalent to the maximum likelihood estimator (see van der Vaart, 2000, Theorem 10.1 (Bernstein-von-Mises)). Thus, the mean of the chains' draws yields our point estimate and the covariance of the draws consistently estimates the asymptotic covariance. We check for convergence by ensuring that the number of effective draws is large, the potential scale reduction factor is close to 1, and by visually inspecting the chains.

The key modification from McCulloch and Rossi (1994) involves a data augmentation step in order to avoid calculating the likelihood of choices for each possible latent choice set. Given our model, the likelihood of consumer (henceforth patient) i matching with product (henceforth facility) j is equal to the probability of the event that $\pi_{ij} \geq z_{ij}$, $v_{ij} \geq 0$ and that for all $j' \in J_t$, either $\pi_{ij} < z_{ij}$ or $v_{ij} \geq v_{ij'}$. That is, facility j admits patient i, patient i finds facility j acceptable, and every other facility in the market satisfies at least one of two conditions: either it does not admit i or i prefers j to it. To the best of our knowledge, closed-form solutions for this probability are not known. However, the problem is standard and tractable once we condition on either the vector $\pi_i = (\pi_{i1}, \dots, \pi_{iJ})$ or $u_i = (u_{i1}, \dots, u_{iJ})$. This is because π_i determines the latent choice set, making the remaining problem a standard discrete choice problem. And, conditional on u_i, i matches with j if and only if $\pi_{ij} > 0$ and $\pi_{ij'} < 0$ for all j' with $u_{ij'} > u_{ij'}$. This set of π_i is a standard orthant. Thus, our sampler will iterate between data augmentation steps for π_i and u_i. Further details on the Gibbs sampler are provided in appendix C.

[19]We found specifications that included such correlations to be difficult to estimate and unstable in our empirical application. This problem did not exist in Monte Carlo simulations.

Our approach differs from most methods for estimating models with latent choice sets, which typically simulate latent choice sets and choice probabilities (Honka, 2014; Gandhi, 2021). Simulation bias can create particularly large computational challenges in these models because of dimensionality. For similar reasons, Barseghyan et al. (2021a) also utilize an estimation procedure that avoids simulating all the latent choice sets by integrating instead over the distribution of preference parameters and evaluating the probabilities of latent choice sets.

Our estimation procedure yields estimates of the pair of product-specific fixed effects δ_j and γ_j. The empirical questions we consider do not require decomposing these fixed effects in terms of product observables x_j and unobservables ξ_j, e.g. $\delta_j = x_j\beta_X + \xi_j$. A researcher concerned about potential endogeneity of x_j can consistently estimate β_X in a second step if instruments that are mean independent of ξ_j are available. This two-step approach has been used in a number of prior papers estimating demand (e.g. Goolsbee and Petrin, 2004).

We conducted Monte Carlo exercises to assess the performance of our estimator, and also to study the consequences of estimating a mis-specified model. Specifically, we consider variations that omit random coefficients, incorrectly assume that choice sets are unconstrained, or include z_{ij} in the utility function as a naive correction for constrained choices. As expected, the resulting bias on the remaining parameter estimates is substantial. Perhaps more importantly, these mis-specifications bias economic quantities of interest such as the diversion ratios that we consider in further detail in Section 5.2.4 below. The results from these exercises are discussed in Appendix D.

5.2 Estimates

We start by describing various specifications before discussing potential biases in section 5.2.3 and implications on diversion ratios in section 5.2.4.

5.2.1 Empirical Specifications

In all the specifications we consider, the unconstrained choice set for each patient is the set of facilities within a 50-mile radius of their home zip-code centroid. The patient's utility for the inside versus the outside option depends on whether the patient has part-time or full-time employment, and whether she is Medicare-eligible. The term $g(\cdot)$ is a quadratic function of distance y_{ij} between the patient's zip code and the facility, with a slope that is allowed to depend on employment status and on the population density of the county where the patient lives. The variable z_{ij} is the excess occupancy of facility j when patient i begins dialysis. Fixed effects are included for each facility.

We compare estimates from three specifications. The first specification – our preferred specification – models both preferences and acceptance policies (equations 6 and 7). Patient characteristics that affect acceptance policies include Medicare eligibility when she begins dialysis, bins of body mass index, age, diabetic status and hypertension. We also include patient-specific random coefficients for chain and non-chain facilities in the preferences equation. The second specification, which we call the unconstrained demand model, omits capacity constraints and sets $\sigma_{ij} = 1$ for all i and j in equation (7). Finally, the third specification, which we refer to as the naive model, modifies the second specification by adding a term $\beta_z z_{ij}$ in equation (6), where β_z is to be estimated. There are two interpretations of this specification. The first is that patients do not face choice constraints, but dislike facilities with high values of z_{ij} (if β_z is negative). Since this interpretation does away with capacity constraints, access to desirable facilities is not influenced by supply-side rationing. The second interpretation is that the specification represents a reduced-form approach that corrects for latent choice set constraints. An undesirable implication of this specification is that changes in z_{ij} affect the welfare of patients matched to facility j, whereas z_{ij} should only influence selective admissions.

5.2.2 Parameter Estimates

Table 5 presents the estimates from the three specifications. As expected, the marginal disutility of distance is decreasing with distance. This and several other estimates are robust across specifications. Consistent with the descriptive evidence in section 4.4, the coefficient on excess occupancy in the naive specification is negative.

There are some notable differences between our preferred specification and the rest. First, the mean utility of chain and non-chain facilities is higher in our preferred specification than in the others. This gap reflects bias if some patients are forced to an outside option because of capacity constraints. Second, the standard deviations of the facility mean utilities, the outside option utility ε_{i0}, and preference shocks ε_{ij} are lower in the preferred specification. This result is consistent with the unconstrained demand model attributing latent choice constraints to unobserved preference heterogeneity.

Turning to the acceptance policy function, we find that measures of patient health conditions and insurance status are correlated with acceptance. The propensity of facilities to accept patients increases with the patient's BMI and whether the patient is insured by Medicare Advantage or a private insurer, and therefore, is in the waiting period. Figure 4 shows the estimated distribution of acceptance probabilities for each facility, averaged over all patients within a 50-mile radius. The probability of acceptance is calculated based on the excess

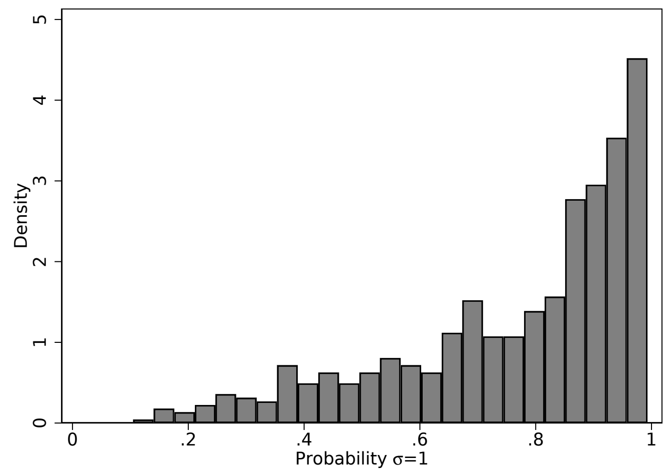
Table 5: Parameter Estimates

	Preferred Specification		Unconstrained	Naïve Model
	(1)		(2)	(3)
	Acceptance	Utility	Utility	Utility
Chain	6.637	5.345	2.391	2.405
	(5.054)	(0.780)	(0.829)	(0.822)
No Chain	2.207	5.379	2.028	2.048
	(6.076)	(0.840)	(0.879)	(0.872)
Diabetes	0.570	0.724	0.809	0.821
	(2.121)	(0.148)	(0.138)	(0.141)
Hypertension	-5.873	-0.244	-0.501	-0.502
	(2.708)	(0.201)	(0.191)	(0.194)
BMI<20	-4.232	-0.038	-0.232	-0.230
	(1.880)	(0.254)	(0.265)	(0.268)
25<=BMI<30	1.038	-0.367	-0.360	-0.357
	(1.239)	(0.161)	(0.170)	(0.173)
30<=BMI	5.996	0.024	0.266	0.268
	(1.398)	(0.161)	(0.169)	(0.172)
Age	0.470	0.001	0.001	0.001
	(0.198)	(0.000)	(0.000)	(0.000)
Age squared	-0.001	-1.275	-1.399	-1.409
	(0.002)	(0.188)	(0.192)	(0.198)
Medicare	-2.411	0.008	0.034	0.035
	(1.656)	(0.026)	(0.027)	(0.027)
Medicare Advantage	26.929	-2.652	-1.923	-1.937
	(3.169)	(0.213)	(0.218)	(0.226)
Medicare waiting period	8.618	2.183	2.772	2.797
	(1.690)	(0.225)	(0.242)	(0.244)
Employed		-5.304	-5.764	-5.802
		(0.230)	(0.262)	(0.269)
Employed x distance		0.004	0.006	0.006
		(0.007)	(0.006)	(0.006)
Population density x distance		0.000	0.001	0.001
		(0.000)	(0.001)	(0.001)
Distance squared		0.013	0.013	0.013
		(0.000)	(0.000)	(0.000)
Excess Occupancy				-0.042
				(0.003)
Standard deviation of δ_j		2.574	2.414	2.402
		(0.100)	(0.094)	(0.095)
Standard deviation of ε_{i0}		8.715	9.550	9.663
		(0.264)	(0.327)	(0.360)
Standard deviation of ε_{ij}		4.274	4.799	4.796
		(0.042)	(0.028)	(0.028)
Standard deviation of γ_j	38.799			
	(3.950)			
Standard deviation of random coef on Chain		1.827	2.350	2.358
		(0.264)	(0.219)	(0.217)
Standard deviation of random coef on No Chain		0.688	0.704	0.697
		(0.223)	(0.247)	(0.240)
Standard deviation of v_{ij}	37.398			
	(3.314)			
Correlation between ε_{ij} and v_{ij}	-0.118			
	(0.036)			

Notes: All specifications include distance with a coefficient normalized to -1 in the utility equation. Specification (1) includes "excess occupancy" in the acceptance equation. Specific intercepts for Chain and No Chain facilities obviate the need of a constant term. Standard errors in parentheses.

occupancy at the facility on the date when the patient begins dialysis. Our results indicate that while the acceptance probabilities are close to 1 for a significant portion of facilities, there are a large number of facilities where the average acceptance probability is much lower than 1. Thus, constraints on choices due to supply-side rationing are non-trivial.

Figure 4: Acceptance Probabilities



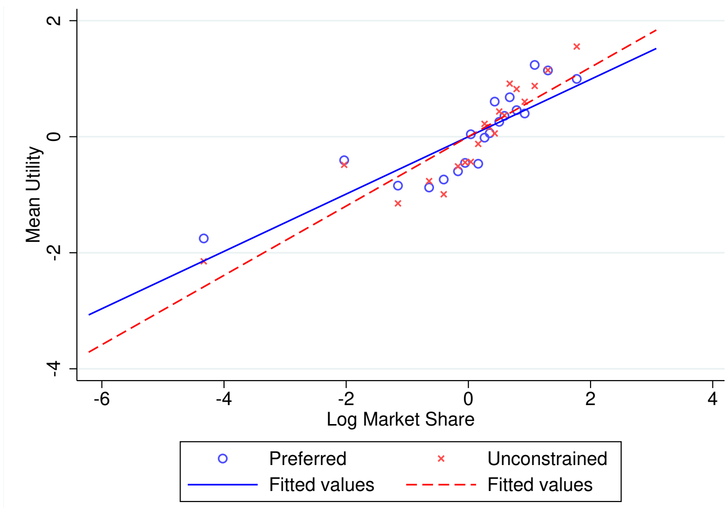
Notes: Histogram of facility acceptance probability. The facility acceptance probability is calculated based on the excess occupancy on the date when each incoming patient living within 50 miles started dialysis.

5.2.3 Biases in demand estimates

The capacity constraints estimated above imply a bias in estimated demand using standard approaches. In particular, estimates of demand based on observed market share have the property that, within a market, the product with the highest market share provides the highest indirect utility to the average consumer. Figure 5 shows the estimated relationship between (the log of) market shares and the estimated mean utility (in miles) for our preferred and unconstrained specifications. The relationship between these two quantities is positive in both specifications, but steeper in the unconstrained specification. This difference occurs because constraints at desirable facilities can force patients to choose less desirable ones.

The biased relationship between market shares and utility reflects into a bias in the estimated demand for a facility. For instance, the number of patients for which the facility is the patients' first choice will be misestimated. Figure E.3(a) in the appendix shows that the latent demand for some facilities is higher for some and lower for others. The former bias is clear – a desirable facility may have to turn away some patients for whom the facility is their first choice. The latter bias occurs because these patients then start treatment at

Figure 5: Willingness to Travel and Market Shares



Notes: Regression line and binscatter with twenty bins of the mean utility estimated by our preferred and unconstrained specifications and the observed market share for the year 2015.

a facility that is not their first choice. The results from the naive correction are similar, suggesting that the correction does little to reduce this bias. This bias is also reflected in the estimated willingness to travel for various dialysis facilities relative to the outside option (Figure E.3(b)).

5.2.4 Implications of choice constraints on diversion ratios

We close this section by noting that there can also be economic grounds on which naive corrections for latent choice constraints are unappealing. We illustrate this point by showing that the naive specification (of the form in specification 3) restricts the comparison between diversion ratios arising from demand-side factors and acceptance decisions.

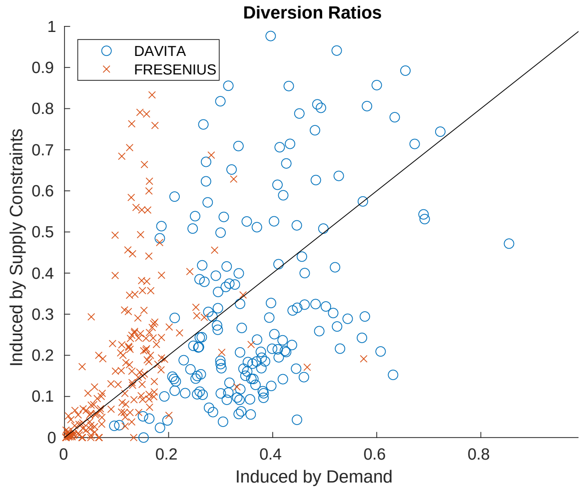
The diversion ratio of j with respect to k, in principle, depends on whether j loses a customer because of changes in choice constraints, equivalently z, or changes in preferences, equivalently y. Dropping the market subscript t, the two diversion ratios are

$$\frac{\partial s_k}{\partial z_{ij}} / \frac{\partial s_j}{\partial z_{ij}} \quad \text{and} \quad \frac{\partial s_k}{\partial y_{ij}} / \frac{\partial s_j}{\partial y_{ij}}.$$

In our empirical specification, the latter diversion ratio is equal to the diversion ratio obtained based on changes in mean utility δ_j.

Notice that there are no a priori reasons why these two diversion ratios are the same. Following a marginal change in y_{ij}, product j loses customers that are indifferent between j and another good. The consumers that switch between k and j following a change in y_{ij} are

Figure 6: Diversion Ratios



Notes: Scatterplot of diversion ratios induced by changes in demand and by supply. Each circle (cross) represents diversion ratios for independent facilities with respect to DaVita (Fresenius) facilities.

consumers that (i) are indifferent between j and k, (ii) have both j and k in their choice sets, and (iii) do not have any other more preferable options in their choice set. Contrast this with consumers that switch between these two products following a change in z_{ij}. These consumers (i) strictly prefer j to k, (ii) are on the margin of being accepted by j, and (iii) do not have any other more preferable options in their choice set. Notice that the changes select consumers on different dimensions – on the preference margin following changes in y_{ij} and on the acceptance margin following changes in z_{ij}. Thus, the diversion ratios on these two margins may be different.

Figure 6 compares the two types of diversion ratios. Each point represents an independent facility j, where we sum the diversion ratios over all facilities k that are either run by DaVita or Fresenius, the two largest dialysis chains in the US. We find that the two diversion ratios above are substantially different. The diversion with respect to demand factors is usually higher than diversion with respect to factors affecting supply constraints, with larger diversion with respect to demand for DaVita than for Fresenius. These differences reflect into competitive incentives when strategically choosing capacity or quality. A merger between two firms with high diversion ratios driven by demand factors is likely to reduce competitive pressure on quality. In contrast, a merger between firms with high diversion ratios on supply constraints could incentivize them to implement stricter admission policies because the merged entity can sort patients according to facility-specific profitability.

Prior prospective merger analyses in the dialysis industry abstract away from potential effects on selective admissions. For example, Wollmann (2022) focuses on the effects of mergers on the quality of care. Our analysis points to changes in capacity constraints and selective admission practices as another important effect of a merger.[20] We leave a thorough investigation to future work.

6 Conclusion

Consumers often face restricted choice sets for reasons such as information or search frictions, preferences of the other side in two-sided matching markets, and selective admission practices. These constraints are typically unobserved. We developed a unified model for analyzing discrete choice demand in the presence of latent choice constraints such as the ones above.

We show how to point identify the joint distribution of preferences and latent choice sets in the presence of two sets of observable shifters, one that influences preferences and the other that influences choice sets. Each set of shifters must be excluded from the other side of the model. Relative to the prior literature, our approach achieves point identification while placing minimal restrictions on functional forms, on the statistical dependence between choice sets and preferences, and allows for the endogeneity of product characteristics. The cost is that our results require the shifters mentioned above. However, we show that these shifters are necessary for identification without further restrictions on the model.

As an illustrative example, we estimate the demand for hemodialysis facilities. The data shows clear evidence of supply-side rationing – facilities with a higher than usual occupancy are less likely to admit new patients, and patients that begin dialysis when nearby centers are constraints are observed to travel further. Next, we use data on patient enrollment to estimate a joint model of preferences and supply-side rationing using a Gibbs sampler. Our results show that ignoring supply-side constraints can lead to significant bias in estimates and yield misleading answers to important economic quantities.

Our approach stops at specifying a reduced form for the supply-side acceptance decision. This reduced form immediately yields a structural object in certain models, such as in empirical models of two-sided matching (Agarwal, 2015; He et al., 2024). It also yields a first-stage estimate in models with more complex supply-side behavior. For example, Gandhi (2021) interprets acceptance probabilities as conditional choice probabilities (Hotz and Miller, 1993) to estimate a dynamic model of selective admissions. Micro-foundations for our reduced form are application-specific, but are important for evaluating counterfactuals that involve changes

[20] As shown earlier, failing to account for selective admissions may bias demand estimates and therefore the measured incentives for the merging facilities to change quality.

in equilibrium supply-side behavior.

References

Abaluck, Jason and Abi Adams-Prassl, "What Do Consumers Consider Before They Choose? Identification from Asymmetric Demand Responses," *The Quarterly Journal of Economics*, 2021, pp. 1611–1663.

_ **, Giovanni Compiani, and Fan Zhang**, "A Method to Estimate Discrete Choice Models that is Robust to Consumer Search," *Working Paper*, 2020.

Agarwal, Nikhil, "An Empirical Model of the Medical Match," *American Economic Review*, 2015, *105* (7), 1939–78.

_ **and Paulo Somaini**, "Demand Analysis Using Strategic Reports: An Application to a School Choice Mechanism," *Econometrica*, 2018, *86* (2), 391–444.

Allen, Roy and John Rehbeck, "Identification With Additively Separable Heterogeneity," *Econometrica*, 2019, *87* (3), 1021–1054.

Azevedo, Eduardo M. and Jacob Leshno, "A Supply and Demand Framework for Two-Sided Matching Markets," *Journal of Political Economy*, 2016, *124* (5), 1235–1268.

Barseghyan, Levon and Francesca Molinari, "Identification and Inference for Pure Random Coefficients Models with Limited Consideration," *Working Paper*, 2023.

_ **,** _ **, and Matthew Thirkettle**, "Discrete Choice under Risk with Limited Consideration," *American Economic Review*, 2021, *111* (6), 1972–2006.

_ **,** _ **, Maura Coughlin, and Joshua C. Teitelbaum**, "Heterogeneous Choice Sets and Preferences," *Econometrica*, 2021, *89* (5), 2015–2048.

Berry, Steven and Ariel Pakes, "The Pure Characteristics Demand Model," *International Economic Review*, 2007, *48* (4), 1193–1225.

Berry, Steven T., "Estimating Discrete-Choice Models of Product Differentiation," *The RAND Journal of Economics*, 1994, *25* (2), 262.

_ **, Amit Gandhi, and Philip A. Haile**, "Connected Substitutes and Invertibility of Demand," *Econometrica*, 2013, *81* (5), 2087–2111.

Berry, Steven T and Philip A Haile, "Identification in Differentiated Products Markets Using Market Level Data," *Econometrica*, 2014, *82* (5), 1749–1797.

Berry, Steven T. and Phillip A. Haile, "Nonparametric Identification of Multinomial Choice Demand Models with Heterogeneous Consumers," 2010.

_ , **James Levinsohn, and Ariel Pakes**, "Automobile Prices in Market Equilibrium," *Econometrica*, 1995, *63* (4), 841 – 890.

Block, H and J Marshak, "Random Orderings and Stochastic Theories of Responses," in I Olkin, S Ghurye, W Hoeffding, W.G. Mado, and H.B. Mann, eds., *Contributions to Probability and Statistics: Essays in Honor of Harold Hotelling*, Stanford, CA: Stanford University Press, 1960, pp. 97–132.

Butters, Gerard R., "Equilibrium Distributions of Sales and Advertising Prices," *Review of Economic Studies*, 1977, *44* (3), 465–491.

Chade, Hector and Lones Smith, "Simultaneous Search," *Econometrica*, 2006, *74* (5), 1293–1307.

Chernozhukov, Victor and Christian Hansen, "An IV Model of Quantile Treatment Effects," *Econometrica*, 2005, *73* (1), 245–261.

Ching, Andrew T., Fumiko Hayashi, and Hui Wang, "Quantifying the Impact of Limited Supply: The Case of Nursing Homes," *International Economic Review*, 2015, *56* (4), 1291–1322.

Compiani, Giovanni, "Market Counterfactuals and the Specification of Multi-Product Demand: A Nonparametric Approach," *Working Paper*, 2021.

Conlon, Christopher T. and Julie Holland Mortimer, "Demand Estimation under Incomplete Product Availability," *American Economic Journal: Microeconomics*, 2013, *5* (4), 1–30.

Dafny, Leemore S., David Cutler, and Christopher Ody, "How Does Competition Impact Quality of Care? A Case Study of the U.S. Dialysis Industry," *Working Paper*, 2018.

de Palma, André, Nathalie Picard, and Paul Waddell, "Discrete Choice Models with Capacity Constraints: An Empirical Analysis of the Housing Market of the Greater Paris Region," *Journal of Urban Economics*, 2007, *62* (2), 204–230.

Department of Health and Human Services: Centers for Medicare and Medicaid Services, "Medicare and Medicaid Programs; Conditions for Coverage for End-Stage Renal Disease Facilities, 42 Federal Register," 2008.

Diamond, W. and N. Agarwal, "Latent indices in assortative matching models," *Quantitative Economics*, 2017, *8* (3), 685–728.

Dube, Jean-Pierre, Ali Hortaçsu, and Joonhwi Joo, "Random-Coefficients Logit Demand Estimation with Zero-Valued Market Shares," *Marketing Science*, 7 2021, *40*, 637–660.

Eliason, Paul, "Market Power and Quality: Congestion and Spatial Competition in the Dialysis Industry," *Working Paper*, 2019.

Eliason, Paul J, Benjamin Heebsh, Ryan C McDevitt, and James W Roberts, "How Acquisitions Affect Firm Behavior and Performance: Evidence from the Dialysis Industry," *The Quarterly Journal of Economics*, 2020, *135* (1), 221–267.

Fack, Gabrielle, Julien Grenet, and Yinghua He, "Beyond Truth-Telling: Preference Estimation with Centralized School Choice and College Admissions," *American Economic Review*, 2019, *109* (4), 1486–1529.

Gandhi, Ashvin, "Picking Your Patients: Selective Admissions in the Nursing Home Industry," *Working Paper*, 2021.

Gaynor, Martin, Carol Propper, and Stephan Seiler, "Free to Choose? Reform, Choice, and Consideration Sets in the English National Health Service," *American Economic Review*, 2016, *106* (11), 3521–3557.

Gelman, Andrew, John B. Carlin, Hal S. Stern, David B. Dunson, Aki Vehtari, and Donald B. Rubin, *Bayesian Data Analysis*, 3 ed., Boca Raton, FL: CRC Press, 2014.

Goeree, Michelle Sovinsky, "Limited Information and Advertising in the U.S. Personal Computer Industry," *Econometrica*, 2008, *76* (5), 1017–1074.

Goolsbee, Austan and Amil Petrin, "The Consumer Gains from Direct Broadcast Satellites and the Competition with Cable TV," *Econometrica*, 2004.

Grieco, Paul L. E. and Ryan C. McDevitt, "Productivity and Quality in Health Care: Evidence from the Dialysis Industry," *The Review of Economic Studies*, 2017, *84* (3), 1071–1105.

He, YingHua, Shruti Sinha, and Xiaoting Sun, "Identification and Estimation in Many-to-one Two-sided Matching without Transfers," *Econometrica*, 5 2024, *92*, 749–774.

Heiss, Florian, Daniel McFadden, Joachim Winter, Amelie Wuppermann, and Bo Zhou, "Inattention and Switching Costs as Sources of Inertia in Medicare Part D," *American Economic Review*, 2021, *111* (9), 2737–2781.

Hickman, William and Julie Holland Mortimer, "Demand Estimation with Availability Variation," in Emek Basker, ed., *Handbook on the Economics of Retailing and Distribution*, Cheltenham, UK: Edward Elgar Publishing Ltd., 2016, chapter 13, pp. 306–339.

Honka, Elisabeth, "Quantifying search and switching costs in the US auto insurance industry," *The RAND Journal of Economics*, 2014, *45* (4), 847–884.

_ , Ali Hortaçsu, and Maria Ana Vitorino, "Advertising, Consumer Awareness, and Choice: Evidence from the U.S. Banking Industry," *The RAND Journal of Economics*, 2017, *48* (3), 611–646.

Hortaçsu, Ali, Seyed Ali Madanizadeh, and Steven L. Puller, "Power to Choose? An Analysis of Consumer Inertia in the Residential Electricity Market," *American Economic Journal: Economic Policy*, 2017, *9* (4), 192–226.

Hotz, V Joseph and Robert A Miller, "Conditional Choice Probabilities and the Estimation of Dynamic Models," *Review of Economic Studies*, 1993, *60* (3), 497–529.

Kepler, John D., Valeri V. Nikolaev, Nicholas Scott-Hearn, and Christopher R. Stewart, "Quality Transparency and Healthcare Competition," *Working Paper*, 2022.

Liu, Jian, Shiying Wu, and James V. Zidek, "On Segmented Multivariate Regression," *Statistica Sinica*, 1997, *7* (2), 497–525.

Manski, Charles F., "The structure of random utility models," *Theory and Decision*, 1977, *8* (3), 229–254.

Matzkin, Rosa L., "Nonparametric identification and estimation of polychotomous choice models," *Journal of Econometrics*, 1993, *58* (1-2), 137–168.

_ , "Nonparametric identification," in James J. Heckman and Edward E. Leamer, eds., *Handbook of Econometrics*, Vol. 6B, Amsterdam: Elsevier, 2007, chapter 73, pp. 5307–5368.

McCulloch, Robert and Peter E Rossi, "An exact likelihood analysis of the multinomial probit model," *Journal of Econometrics*, 1994, *64* (1-2), 207–240.

McFadden, Daniel, "Econometric Models of Probabilistic Choice," in Charles F. Manski and Daniel L. McFadden, eds., *Structural Analysis of Discrete Data and Econometric Applications*, Cambridge: The MIT Press, 1981, chapter 5.

Neilson, Christopher, "Targeted Vouchers, Competition Among Schools, and the Academic Achievement of Poor Students," *Working Paper*, 2020.

Newey, Whitney K. and James L. Powell, "Instrumental Variable Estimation of Nonparametric Models," *Econometrica*, 2003, *71* (5), 1565–1578.

Swait, Joffre and Moshe Ben-Akiva, "Incorporating random constraints in discrete models of choice set generation," *Transportation Research Part B: Methodological*, 1987, *21* (2), 91–102.

U.S. Renal Data System, "2020 USRDS Annual Data Report: Epidemiology of kidney disease in the United States — End Stage Renal Disease," 2020.

van der Vaart, A. W., *Asymptotic Statistics*, Cambridge:: Cambridge University Press, 2000.

Weitzman, Martin L., "Optimal Search for the Best Alternative," *Econometrica*, 1979, *47* (3), 641–654.

Wollmann, Thomas, "How to Get Away with Merger: Stealth Consolidation and Its Real Effects on US Healthcare," *Working Paper*, 2022.

Appendix

A Proofs

A.1 Proof of Lemma 1

Because ties are allowed, it must be that

$$s_{jt}\left(d_i, y_i, z_i\right) \leq \sum_{O \in \mathcal{O}} P\left(O_i = O, j \in \arg\max_{k \in O} v_{ikt} \middle| t, d_i, y_i, z_i\right)$$

The inequality follows because $c_{ij} = 1$ only if $j \in \arg\max_{k \in O_i} v_{ikt}$. Conditioning on d_i and dropping it from the notation, we rewrite preferences as

$$v_{ij} = u_j\left(\omega_i\right) - g_{ij}$$

and we treat g_{ij} as observable. Consumer i remains unmatched if for every facility $j \in O_i$ $u_j\left(\omega_i\right) < g_{ij}$ and only if for every facility $j \in O_i$ $u_j\left(\omega_i\right) \leq g_{ij}$. Similarly, facility $j \in O_i$ if $\pi_j\left(\omega_i\right) < z_j$ and only if $\pi_j\left(\omega_i\right) \leq z_j$. Let $s_0\left(g, z\right)$ be the share of consumers that are unmatched conditional on g and z, define $\bar{s}_0\left(\bar{g}, \bar{z}\right)$ as $\lim_{(g,z)\downarrow(\bar{g},\bar{z})} s_0\left(g, z\right)$, where $(g, z) \downarrow (\bar{g}, \bar{z})$ if there exists a sequence $g_n > \bar{g}$ and $z_n > \bar{z}$ with $g_n \to \bar{g}$ and $z_n \to \bar{z}$. If $s_0\left(g, z\right)$ is continuous at $(\bar{g}, \bar{z})$, $\bar{s}_0\left(g, z\right) = s_0\left(g, z\right)$; otherwise, $\bar{s}_0\left(g, z\right) > s_0\left(g, z\right)$. By assumption (1) and by set inclusion,

$$\begin{aligned}\lim_{(g,z)\downarrow(\bar{g},\bar{z})} s_0\left(g, z\right) &\geq \lim_{(g,z)\downarrow(\bar{g},\bar{z})} P\left(\cap_j \left\{u_j\left(\omega_i\right) < g_j \vee \pi_j\left(\omega_i\right) < z_j\right\}\right) \\ &\geq P\left(\cap_j \left\{u_j\left(\omega_i\right) \leq \bar{g}_j \vee \pi_j\left(\omega_i\right) \leq \bar{z}_j\right\}\right).\end{aligned}$$

Moreover,

$$\begin{aligned}\lim_{(g,z)\downarrow(\bar{g},\bar{z})} s_0\left(g, z\right) &\leq \lim_{(g,z)\downarrow(\bar{g},\bar{z})} P\left(\cap_j \left\{u_j\left(\omega_i\right) \leq g_j \vee \pi_j\left(\omega_i\right) \leq z_j\right\}\right) \\ &= P\left(\cap_j \left\{u_j\left(\omega_i\right) \leq \bar{g}_j \vee \pi_j\left(\omega_i\right) \leq \bar{z}_j\right\}\right),\end{aligned}$$

where the inequality follows from set inclusion and the equality follows because the probability of a sequence of nested events converges to the probability of the limiting event. Thus,

$$\bar{s}_0\left(\bar{g}, \bar{z}\right) = \lim_{(g,z)\downarrow(\bar{g},\bar{z})} s_0\left(g, z\right) = P\left(\cap_j \left\{u_j\left(\omega_i\right) \leq \bar{g}_j \vee \pi_j\left(\omega_i\right) \leq \bar{z}_j\right\}\right).$$

Let $\mathcal{B}_\chi$ be the collection of sets that are a Cartesian product of half-open intervals of the form $B = \{(u, \pi) : \underline{u} < u \leq \bar{u}, \underline{\pi} < \pi \leq \bar{\pi}\}$ with $B \subseteq \chi$. Consider some $B \in \mathcal{B}_\chi$ and let $\underline{g} = \underline{u}$, $\overline{g} = \overline{u}$,$\underline{z} = \underline{\pi}$ and $\bar{z} = \bar{\pi}$. Define g^j such that $g^j_k = \bar{g}_k$ for $j = k$ and $g^j_k = \underline{g}_k$ for $j \neq k$. Likewise, define $\bar{z}^j$ such that $z^j_k = 1\{j = k\}\,\bar{z}_k + 1\{j \neq k\}\,\underline{z}_k$. Define:

$$\Lambda_1\left(g, z\right) \equiv \left[\bar{s}_0\left(g^1, z\right) - \bar{s}_0\left(g, z\right)\right] - \left[\bar{s}_0\left(g^1, z^1\right) - \bar{s}_0\left(g, z^1\right)\right],$$

and for $j > 1$,

$$\Lambda_j\left(g, z\right) \equiv \left[\Lambda_{j-1}\left(g^j, z\right) - \Lambda_{j-1}\left(g, z\right)\right] - \left[\Lambda_{j-1}\left(g^j, z^j\right) - \Lambda_{j-1}\left(g, z^j\right)\right].$$

Observe that each $\Lambda_j\left(\underline{g},\underline{z}\right)$ is identified. We will now calculate $\Lambda_J(\underline{g},\underline{z})$. To do this, observe that $\bar{s}_0\left(g^1,\underline{z}\right)-\bar{s}_0\left(\underline{g},\underline{z}\right)$ is equal to

$$P\left(\left\{\underline{g}_1<u_1\left(\omega_i\right)\leq\bar{g}_1\wedge\pi_1\left(\omega_i\right)>\underline{z}_1\right\}\cap_{k>1}\left\{u_k\left(\omega_i\right)\leq\underline{g}_k\vee\pi_k\left(\omega_i\right)\leq\underline{z}_k\right\}\right).$$

Similarly, $\bar{s}_0\left(g^1,z^1\right)-\bar{s}_0\left(\underline{g},z^1\right)$ equals

$$P\left(\left\{\underline{g}_1<u_1\left(\omega_i\right)\leq\bar{g}_1\wedge\pi_1\left(\omega_i\right)>\bar{z}_1\right\}\cap_{k>1}\left\{u_k\left(\omega_i\right)\leq\underline{g}_k\vee\pi_k\left(\omega_i\right)\leq\underline{z}_k\right\}\right).$$

By set inclusion, the probability

$$P\left(\left\{\underline{g}_1<u_j\left(\omega_i\right)\leq\bar{g}_1\wedge\underline{z}_1<\pi\left(\omega_i\right)\leq\bar{z}_1\right\}\cap_{k>1}\left\{u_k\left(\omega_i\right)\leq\underline{g}_k\vee\pi_k\left(\omega_i\right)\leq\underline{z}_k\right\}\right)$$

is equal to $\Lambda_1\left(\underline{g},\underline{z}\right)$. By an identical argument and induction, for any $j>1$, we have that $\Lambda_j\left(\underline{g},\underline{z}\right)$ equals

$$P\left(\cap_{k\leq j}\left\{\underline{g}_j<u_j\left(\omega_i\right)\leq\bar{g}_j\wedge\underline{z}_j<\pi\left(\omega_i\right)\leq\bar{z}_j\right\}\cap_{k>j}\left\{u_j\left(\omega_i\right)\leq\underline{g}_k\vee\pi\left(\omega_i\right)\leq\underline{z}_k\right\}\right).$$

In particular,

$$\begin{aligned}\Lambda_J\left(\underline{g},\underline{z}\right)&=P\left(\cap_j\left\{\underline{g}_j<u_j\left(\omega_i\right)\leq\bar{g}_j\wedge\underline{z}_j<\pi\left(\omega_i\right)\leq\bar{z}_j\right\}\right)\\&=P\left(\left(u\left(\omega_i\right),\pi\left(\omega_i\right)\right)\in B\right).\end{aligned}$$

Thus, we can identify the probability that $\left(u\left(\omega_i\right),\pi\left(\omega_i\right)\right)$ belongs to any set $B\in\mathcal{B}_\chi$, i.e., sets that are a Cartesian product of half-open intervals and are subsets of the interior of the support of (g,z).

We will show that conditional cumulative distribution function of (u_i,π_i) given $(u_i,\pi_i)\in\chi$, $P\left(u_i\leq\bar{u},\pi_i\leq\bar{\pi}|\left(u_i,\pi_i\right)\in\chi\right)$, is identified. There are two cases. The first case is when $P\left((u_i,\pi_i)\in\chi\right)>0$. Then, we have that

$$P\left(u_i\leq\bar{u},\pi_i\leq\bar{\pi}|\left(u_i,\pi_i\right)\in\chi\right)=P\left(\left(u_i,\pi_i\right)\in\bar{B}\cap\chi\right)/P\left((u_i,\pi_i)\in\chi\right)$$

where $\bar{B}=\{(u,\pi):u\leq\bar{u},\pi\leq\bar{\pi}\}$. It would suffice to show that we can identify $P\left(\left(u_i,\pi_i\right)\in\bar{B}\cap\chi\right)$ and $P\left((u_i,\pi_i)\in\chi\right)$. In the second case, $P\left((u_i,\pi_i)\in\chi\right)=0$. In this case, we will still be able to identify $P\left((u_i,\pi_i)\in\chi\right)$, but notice that the statement is vacuous and thus completes the proof.

To identify $P\left((u_i,\pi_i)\in\chi\right)$, we will show that $\chi=\bigcup_{k=1}^\infty B_k'$ for a countable collection of $B_k'\in\mathcal{B}_\chi$ and $B_k'\cap B_{k'}'=\emptyset$. This would imply that $P\left((u_i,\pi_i)\in\chi\right)=\sum_{k=1}^\infty P\left((u_i,\pi_i)\in B_k'\right)$ is identified since each term in the summand is identified. Towards this, we first show that there exists a countable collection of half-open cartesian products of intervals $B_k=\{(u,\pi):\underline{u}_k<u\leq\bar{u}_k,\underline{\pi}_k<\pi\leq\bar{\pi}_k\}\in\mathcal{B}_\chi$ such that $\chi=\bigcup_{k=1}^\infty B_k$. To do this, let $x\in\chi$ and note that there exist vectors of rational numbers $\underline{u}_k,\bar{u}_k,\underline{\pi}_k$ and $\bar{\pi}_k$ such that

$$x\in B_k=\{(u,\pi):\underline{u}_k<u\leq\bar{u}_k,\underline{\pi}_k<\pi\leq\bar{\pi}_k\}$$

and $B_k\subseteq\chi$. Since the set of rational numbers is countable, we have that there exists a countable collection of B_k with $\chi=\bigcup_{k=1}^\infty B_k$ and $B_k\subseteq\chi$. Now, notice that for any two elements of this collection B_k and $B_{k'}$, $B_k \cap B_{k'} \in \mathcal{B}_\chi$. And, $B_k \backslash B_{k'}$ is a union of at most $2^{2J} - 1$ sets in $\mathcal{B}_\chi$. Therefore, there exists an at most a countable number of disjoint sets $B'_k \in \mathcal{B}_\chi$ such that $\bigcup_k B'_k = \bigcup_k B_k = \chi$. Hence, $P\left((u_i, \pi_i) \in \chi\right)$ is identified.

Next, we show that $P\left((u_i, \pi_i) \in \bar{B} \cap \chi\right)$ is identified. Notice that $\bar{B} \cap \chi = \bigcup_k \left(\bar{B} \cap B'_k\right)$. Since $\bar{B} \cap B'_k \in \mathcal{B}_\chi$, the quantity $P\left((u_i, \pi_i) \in \bar{B} \cap B'_k\right)$ is identified. Since $B'_k \cap B'_{k'} = \emptyset$, we have that $P\left((u_i, \pi_i) \in \bar{B} \cap \chi\right) = \sum_k P\left((u_i, \pi_i) \in \bar{B} \cap B'_k\right)$ is identified. Hence, the conditional cumulative distribution function of (u_i, π_i) conditional on $(u_i, \pi_i) \in \chi$ is identified.

A.2 Primitive Conditions for Assumption 3

Condition on d_i and drop it from the notation for simplicity. Fix $\{j, k\}$. For each y_i, define the set

$$U_{jk}\left(y_i, O_i\right) = \left\{u\left(\omega_i\right) : \min_{l \in \{j,k\}} \left\{u_l\left(\omega\right) - g_l\left(y_{il}\right)\right\} \geq \max_{l \in O_i \backslash \{j,k\}} \left\{u_l\left(\omega\right) - g_l\left(y_{il}\right)\right\}\right\}.$$

Definition 2. The pair of goods $\{j, k\}$ is relevant at characteristics (y_i, z_i) and choice set O if

$$P\left(O, u\left(\omega_i\right) \in U_{jk}\left(y_i, O\right) \middle| z_i\right) > 0.$$

Proposition 2. *Suppose assumption 1 is satisfied. If (i) the pair of goods $\{j, k\}$ is relevant at characteristics (y_i, z_i) and choice set O_i for some $O_i \in \mathcal{O}$, (ii) the distribution of*

$$u_j\left(\omega\right) - u_k\left(\omega\right)$$

conditional on $u\left(\omega\right) \in U_{jk}\left(y_i, O_i\right)$ and O_i admits a density f_{jk}, (iii) $f_{jk}\left(g_j\left(y_{ij}\right) - g_k\left(y_{ik}\right)\right) > 0$, and (iv) for each O and all y in a neighborhood of y_i, $P\left(\left|\arg\max_{j \in O}\left\{u_j\left(\omega\right) - g_j\left(y_{ij}\right)\right\}\right| > 1 \middle| O, y\right) = 0$ then (i) $g_j\left(y_{ij}\right)$ is differentiable if and only if $s_k\left(y_i, z_i\right)$ is differentiable with respect to y_{ij}, (ii) the sign of $\frac{\partial s_k\left(y_i, z_i\right)}{\partial y_{ij}}$ coincides with the sign of $\frac{\partial g_j\left(y_{ij}\right)}{\partial y_{ij}}$ provided that these derivatives exist, and (iii) a symmetric relation exists between $g_k\left(y_{ik}\right)$ and $s_j\left(y_i, z_i\right)$. Consequently, j and k are strict substitutes if and only if $g_j\left(y_{ij}\right)$ and $g_k\left(y_{ik}\right)$ are differentiable with non-zero derivatives.

Proof. Fix specific values of y_i and z_i. Observe that

$$\begin{aligned} s_j\left(y_i, z_i\right) &= \sum_{O \in \mathcal{O}} P\left(c_{ij} = 1 \middle| O, y_i, z_i\right) P\left(O \middle| y_i, z_i\right) \\ &= \sum_{O \in \mathcal{O}} P\left(j \in \arg\max_{l \in O} u_l\left(\omega\right) - g_l\left(y_{il}\right) \middle| O, z_i\right) P\left(O \middle| z_i\right) \end{aligned}$$

since requirement (iv) implies that $\arg\max_{l \in O}\left\{u_l\left(\omega\right) - g_l\left(y_{il}\right)\right\}$ has at most one element with probability 1 and assumption 1 allow us to drop the conditioning on y_i. Equation 3 implies that

$$
\begin{aligned}
&\frac{\partial s_j\left(y_i, z_i\right)}{\partial y_{ik}} \\
&= \sum_{O\in\mathcal{O}} \frac{\partial P\left(j \in \arg\max_{l\in O} u_l\left(\omega\right) - g_l\left(y_{il}\right) \mid O, z_i\right)}{\partial y_{ik}} P\left(O \mid z_i\right) \\
&= \sum_{O\in\mathcal{O}} \left.\frac{\partial P\left(u_j\left(\omega_i\right) - \bar{g}_{ij} \geq u_k\left(\omega_i\right) - \bar{g}_{ik} \mid O, u\left(\omega_i\right) \in U_{jk}\left(y_i, O\right), z_i\right)}{\partial g_{ik}}\right|_{\bar{g}_{ik}=g_k\left(y_{ik}\right)} \\
&\frac{\partial g_k\left(y_{ik}\right)}{\partial y_{ik}} P\left(O, u\left(\omega_i\right) \in U_{jk}\left(y_i, O\right) \mid z_i\right) \\
&= \frac{\partial g_k\left(y_{ik}\right)}{\partial y_{ik}} \sum_{O\in\mathcal{O}} \frac{\partial \int_{g_{ij}-g_{ik}}^{\infty} f_{jk}\left(v\right) dv}{\partial g_{ik}} P\left(O, u\left(\omega_i\right) \in U_{jk}\left(y_i, O\right) \mid z_i\right) \\
&= \frac{\partial g_k\left(y_{ik}\right)}{\partial y_{ik}} \sum_{O\in\mathcal{O}} f_{jk}\left(g_{ij} - g_{ik}\right) P\left(O, u\left(\omega_i\right) \in U_{jk}\left(y_i, O\right) \mid z_i\right)
\end{aligned}
$$

where the derivatives in the summands exist since f_{jk} is a density. The hypotheses ensure the existence of $O_i \in \mathcal{O}$ such that its corresponding summand is strictly positive. Thus, if $g_k\left(y_{ik}\right)$ is differentiable, $\frac{\partial s_j\left(y_i, z_i\right)}{\partial y_{ik}}$ exists and it has the same sign as $\frac{\partial g_k\left(y_{ik}\right)}{\partial y_{ik}}$. Conversely, if $g_k\left(y_{ik}\right)$ is not differentiable, the limit $\frac{g_k\left(y_{ik}\right) - g_k\left(y_{ik}+\Delta\right)}{\Delta}$ as $\Delta \to 0$ does not exist; thus, $\frac{\partial s_j\left(y_i, z_i\right)}{\partial y_{ik}}$ does not exist. This completes the proof of parts (i) and (ii). Part (iii) follow immediately from a symmetric argument. $\square$

Corollary 2. *Suppose assumption 1 is satisfied. If there exists $z_i^* \in Z$ such that (i) $\cup_{O:\{j,k\}\subseteq O} P\left(O \mid z_i^*\right) > 0$, and (ii) for each each O with $\{j,k\} \subseteq O$ and $P\left(O \mid z_i^*\right) > 0$, the joint distribution of $\left(u_{ij}\right)_{j\in O}$ conditional O on has full support on an open neighborhood $B \subseteq \mathbb{R}^{|O|}$ of $\left(g_j\left(y_{ij}\right)\right)_{j\in O}$ and is absolutely continuous with respect to Lebesgue measure on B, then the functions $s_j\left(y_i, z_i^*\right)$ and $s_k\left(y_i, z_i^*\right)$ are differentiable at y_{ik} and y_{ij} respectively with non-zero derivatives if and only if $g_j\left(y_{ij}\right)$ and $g_k\left(y_{ik}\right)$ are differentiable at y_{ij} and y_{ik} with non-zero derivatives.*

As another corollary, we state stronger but simpler to interpret conditions.

Corollary 3. *Suppose assumption 1 is satisfied. If the joint distribution of u_i conditional on each O admits a density conditional on each O and there exists O with $\{j,k\} \subseteq O$ and $P\left(O \mid z_i^*\right) > 0$ for some z_i^*, then the functions $s_j\left(y_i, z_i^*\right)$ and $s_k\left(y_i, z_i^*\right)$ are strictly increasing and differentiable at y_{ik} and y_{ij} respectively if and only if $g_j\left(y_{ij}\right)$ and $g_k\left(y_{ik}\right)$ are strictly increasing and differentiable at y_{ij} and y_{ik}.*

A.3 Proof of Lemma 2

The proof of lemma 2 requires the following intermediate result.

Lemma 3. *Suppose that assumption 1 holds and $|J|>1$. If j* and *k* are strict substitutes in *y* at *some (d_i,y_i,z_i^*) in the support of the data and $g_j'(d_i,y_i)\neq 0$, then (i) $g_k'(d_i,y_i)\neq 0$, (ii) the sign of $g_k'(d_i,y_i)$ coincides with the sign of $\frac{\partial s_k(d_i,y_i,z_i^*)}{\partial y_{ij}}$, and (iii)*

$$\frac{g_k'(d_i,y_i)}{g_j'(d_i,y_i)}=\frac{\partial s_j(d_i,y_i,z_i^*)}{\partial y_{ik}}/\frac{\partial s_k(d_i,y_i,z_i^*)}{\partial y_{ij}},$$

which implies that $\frac{g_k'(d_i,y_i)}{g_j'(d_i,y_i)}$ is identified and bounded.

Proof. Because j and k are strict substitutes in y at (d_i,y_i,z_i^*) $\frac{\partial s_j\left(y_i,d_i,z_i^*\right)}{\partial y_{ik}}$ and $\frac{\partial s_k\left(y_i,d_i,z_i^*\right)}{\partial y_{ij}}$ exist and are non-zero. For notational simplicity, we omit z_i^*, d_i, y_l and g_l for $l\notin\{j,k\}$ from the notation as they are fixed throughout the proof.

Since $P\left(c_{ij}=1|\,O_i,t,d_i,y_i=y,z_i\right)=P\left(c_{ij}=1|\,O_i,t,d_i,y_i=y',z_i\right)$ if $g(y)=g(y')$, equation (3) and Assumption 1 implies that there exists a function $\hat{s}(\cdot):\mathbb{R}^2\to\mathbb{R}^2$ such that

$$s_k(y_{ik},y_{ij})=\hat{s}_k\left(g_k(y_{ik}),g_j(y_{ij})\right).$$

Moreover, the function $\hat{s}_k(g_k,g_j)$ is weakly increasing in g_j and weakly decreasing in g_k.

The proof consists of four steps. The first step shows that the function $\hat{s}_k(g_k,g_j)$ is differentiable with respect to g_j at $g_k=g_k(y_{ik})$ and $g_j=g_j(y_{ij})$. Therefore, we can use the chain rule to calculate the cross partials of $s_k(y_{ik},y_{ij})$ and $s_j(y_{ik},y_{ij})$. The second step proves part (i): the derivative of $g_k(\cdot)$ at y_{ik} is not zero. The third step shows symmetry of the cross-partial derivatives $\frac{\partial\hat{s}_j(g_k,g_j)}{\partial g_k}=\frac{\partial\hat{s}_k(g_k,g_j)}{\partial g_j}$ without requiring continuity of $\frac{\partial\hat{s}_j(g_k,g_j)}{\partial g_j}$ and $\frac{\partial\hat{s}_k(g_k,g_j)}{\partial g_k}$, a key requirement for Young's Theorem. The fourth and final step applies the chain rule and employs the symmetry of cross-partial derivatives to derive parts (ii) and (iii).

First step: For any $\Delta\neq 0$,

$$\frac{\hat{s}_k\left(g_k,g_j(y_{ij}+\Delta)\right)-\hat{s}_k\left(g_k,g_j(y_{ij})\right)}{g_j(y_{ij}+\Delta)-g_j(y_{ij})}=\frac{s_k(y_{ik},y_{ij}+\Delta)-s_k(y_{ik},y_{ij})}{\Delta}/\frac{g_j(y_{ij}+\Delta)-g_j(y_{ij})}{\Delta}.$$

The limit of the right-hand side as $\Delta\to 0$ exists because $\frac{\partial s_k(y_{ik},y_{ij})}{\partial y_{ij}}$ and $\frac{\partial g_j(y_{ij})}{\partial y_{ij}}$ exist, and the latter is non-zero. Thus, the limit on the left hand side as $\Delta\to 0$ also exists and it is finite. Moreover, $\frac{\partial s_k(y_{ik},y_{ij})}{\partial y_{ij}}\neq 0$, and weak monotonicity of $\hat{s}_k(g_k,g_j)$ with respect to g_j implies that

$$\frac{\partial\hat{s}_k\left(g_k,g_j(y_{ij})\right)}{\partial g_j}=\frac{\partial s_k(y_{ik},y_{ij})}{\partial y_{ij}}/\frac{\partial g_j(y_{ij})}{\partial y_{ij}}>0,\tag{8}$$

where the strict inequality follows because each term in the RHS is non-zero. By a symmetric argument, $\frac{\partial g_k(y_{ik})}{\partial y_{ik}}\neq 0$ implies that $\frac{\partial\hat{s}_j(g_k,g_j)}{\partial g_k}$ exists at $g_k=g_k(y_{ik})$ and $g_j=g_j(y_{ij})$. We will show in the second step below that $\frac{\partial g_k(y_{ik})}{\partial y_{ik}}\neq 0$ without assuming that $\frac{\partial\hat{s}_j(g_k,g_j)}{\partial g_k}$ exists.

Second step: Consider $\Delta>0$. The difference $\hat{s}_j(g_k+\Delta,g_j)-\hat{s}_j(g_k,g_j)$ is equal to the mass of consumers whose match switches from k to j due to an increase in g_k. Since the distribution of choice sets O is independent of y, and therefore $g(y)$, the switchers have both k and j

in their choice sets at both $(g_k+\Delta, g_j)$ and (g_k, g_j). These consumers would switch to j if, instead of g_k increasing by Δ, g_j decreased by the same amount. Thus, by set inclusion,

$$0 \leq \hat{s}_j\left(g_k+\Delta, g_j\right)-\hat{s}_j\left(g_k, g_j\right) \leq \hat{s}_k\left(g_k, g_j\right)-\hat{s}_k\left(g_k, g_j-\Delta\right). \tag{9}$$

By definition of $\frac{\partial s_j\left(y_{ij}, y_{ik}\right)}{\partial y_{ik}}$,

$$\frac{\partial s_j\left(y_{ij}, y_{ik}\right)}{\partial y_{ik}}=\lim_{\Delta \downarrow 0}\left(\frac{\hat{s}_j\left(g_k\left(y_{ik}+\Delta\right), g_j\right)-\hat{s}_j\left(g_k\left(y_{ik}\right), g_j\right)}{g_k\left(y_{ik}+\Delta\right)-g_k\left(y_{ik}\right)} \times \frac{g_k\left(y_{ik}+\Delta\right)-g_k\left(y_{ik}\right)}{\Delta}\right) \neq 0. \tag{10}$$

The limit on the right-hand side exists because $\frac{\partial s_j\left(y_{ij}, y_{ik}\right)}{\partial y_{ik}}$ is well-defined. Taking the absolute value of the terms in parenthesis and using the inequalities in (9) yields

$$\begin{aligned} & \lim_{\Delta \downarrow 0}\left|\frac{\hat{s}_j\left(g_k\left(y_{ik}+\Delta\right), g_j\right)-\hat{s}_j\left(g_k\left(y_{ik}\right), g_j\right)}{g_k\left(y_{ik}+\Delta\right)-g_k\left(y_{ik}\right)} \times \frac{g_k\left(y_{ik}+\Delta\right)-g_k\left(y_{ik}\right)}{\Delta}\right| \\ & \leq \lim_{\Delta \downarrow 0} \frac{\hat{s}_k\left(g_k\left(y_{ik}\right), g_j\right)-\hat{s}_k\left(g_k\left(y_{ik}\right), g_j-\tilde{\Delta}\right)}{\tilde{\Delta}} \times\left|\frac{g_k\left(y_{ik}+\Delta\right)-g_k\left(y_{ik}\right)}{\Delta}\right|\end{aligned}$$

where $\tilde{\Delta}=g_k\left(y_{ik}+\Delta\right)-g_k\left(y_{ik}\right)$. Both terms converge as $\Delta \downarrow 0$: the first one converges to $\frac{\partial \hat{s}_k\left(g_k, g_j\right)}{\partial g_j}$ and the second one to the absolute value of $\frac{\partial g_k(y_{ik})}{\partial y_{ik}}$. Therefore, $\frac{\partial g_k(y_{ik})}{\partial y_{ik}} \neq 0$ because otherwise, $\frac{\partial s_j\left(y_{ij}, y_{ik}\right)}{\partial y_{ik}}=0$ contradicting equation (10). This proves part (i).

Third step: The arguments above imply that $\frac{\partial \hat{s}_j\left(g_k, g_j\right)}{\partial g_k}$ exists at $g_k=g_k\left(y_{ik}\right)$ and $g_j=g_j\left(y_{ij}\right)$. By a symmetric argument to the one that yields equation (9), for any $\Delta>0$

$$0 \leq \hat{s}_k\left(g_k, g_j+\Delta\right)-\hat{s}_k\left(g_k, g_j\right) \leq \hat{s}_j\left(g_k, g_j\right)-\hat{s}_j\left(g_k-\Delta, g_j\right). \tag{11}$$

Dividing (9) and (11) by Δ and taking the limit $\Delta \downarrow 0$, yields:

$$0<\frac{\partial \hat{s}_j\left(g_k, g_j\right)}{\partial g_k}=\frac{\partial \hat{s}_k\left(g_k, g_j\right)}{\partial g_j}. \tag{12}$$

Fourth step: We have shown that $\hat{s}_k\left(g_k, g_j\right)$ is differentiable with respect to g_j and that $\hat{s}_j\left(g_k, g_j\right)$ is differentiable with respect to g_k. Applying the chain rule yields:

$$\frac{\partial s_j\left(y_{ij}, y_{ik}\right)}{\partial y_{ik}}=\frac{\partial \hat{s}_j\left(g_k\left(y_{ik}\right), g_j\right)}{\partial g_k} \times \frac{\partial g_k\left(y_{ik}\right)}{\partial y_{ik}} \tag{13}$$

and

$$\frac{\partial s_k\left(y_{ij}, y_{ik}\right)}{\partial y_{ij}}=\frac{\partial \hat{s}_k\left(g_k, g_j\left(y_{ij}\right)\right)}{\partial g_j} \times \frac{\partial g_j\left(y_{ij}\right)}{\partial y_{ij}}. \tag{14}$$

Parts (ii) and (iii) follow immediately from equations (12), (13), and (14). □

We are now ready to prove lemma 2. Fix d_i and omit it from notation. Let j be the reference good and recall the normalization that $\left|g_j'\left(y_0\right)\right|=1$ and $g_j\left(y_0\right)=0$ for some y_0. Take any pair (k, y_k) such that there is a path connecting it with (j, y_0) where j is the reference good and y_0 is the value for which we have normalized $\left|\frac{\partial g_j(d_i, y_0)}{\partial y}\right|=1$. Let this path be $(j, y_0)=(m_0, y_1),(m_2, y_2), \ldots.,(m_n, y_n)=(k, y_k)$ where for all $l=2, \ldots, n$, m_l and m_{l-1}

are strict substitutes in y at some (d_i, y_i, z_i^*) in the support of the data with $y_{im_l} = y_l$ and $y_{im_{l-1}} = y_{l-1}$. Lemma 3 implies that $\frac{g'_{m_l}(y_l)}{g'_{m_{l-1}}(y_{l-1})}$ is identified for each $l \in \{2, ...n\}$. Moreover, $g'_{m_l}(y_l)$ and $g'_{m_{l-1}}(y_{l-1})$ are bounded and non-zero. Thus, $g'_k(y_k) = \frac{g'_k(y_k)}{g'_j(y_0)} = \prod_{l=2}^n \frac{g'_{m_l}(y_l)}{g'_{m_{l-1}}(y_{l-1})}$ is identified. Since $g_k(y_0) = 0$ and $g_k(\cdot)$ is continuously differentiable, $g_k(y_k) = \int_{y_0}^{y_k} g'_k(\tau)\,\mathrm{d}\tau$ is identified as the argument above and assumption 3 imply that $g'_k(\tau)$ is identified for almost all τ in the support of y_{ik}. [21]

A.4 Proof of Proposition 1

To simplify notation, we drop the conditioning on d_i. Since the function $g(\cdot)$ is known, in a minor abuse of notation we write $g = g(y)$ and $s(g) = \{s_j(g)\}_{j\in J}$. We also drop z_i from the notation because its support is a singleton. With this simplification, the function $s_j(g)$ can be re-written as follows:

$$\begin{aligned}
s_j(g) &= \sum_{O\in\mathcal{O}} P\left(O, j \in \arg\max_{k\in O} u_k - g_k \middle| g\right)\\
&= \sum_{O\in\mathcal{O}} \int 1\left\{j \in \arg\max_{k\in O} u_k - g_k\right\} P(O|\, u, g)\, f_U(u)\,\mathrm{d}u\\
&= \sum_{O\in\mathcal{O}} \int 1\left\{j \in \arg\max_{k\in O} u_k - g_k\right\} P(O|\, u)\, f_U(u)\,\mathrm{d}u\\
&= \sum_{O\in\mathcal{O}} \int 1\left\{j \in \arg\max_{k\in O} u_k - g_k\right\} \left(\int_{O^c} P(O|\, u)\, f_U(u)\,\mathrm{d}u_{O^c}\right)\mathrm{d}u_O\\
&= \sum_{O\in\mathcal{O}} \int_{g_j}^{\infty} \left(\int_{-\infty}^{u_j - g_j + g_k} \dots \int_{-\infty}^{u_j - g_j + g_{k'}} h_O(u_O)\,\mathrm{d}u_{O-\{j\}}\right) du_j,
\end{aligned}$$

where $O^c = J\backslash O$, $u_O = (u_j)_{j\in O}$, $u_{O^c} = (u_j)_{j\in J\backslash O}$ and $h_O(u_O) = \int P(O|\, u)\, f_U(u)\,\mathrm{d}u_{O^c}$. The third equality follows from assumption 1 whereas the others simply re-write the problem. Since $s(g)$ is the only observable when the support of z is a singleton, under assumption 1, identification of the model is equivalent to identification of $P(O|\, u)$ and $f_U(u)$.

We use a standard definition of identification (Matzkin, 2007). Define a model as a collection of admissible structures $\{P(\cdot|\,\cdot), f_U(\cdot)\}$. A pair of structures is observationally equivalent if they yield the same observable market share functions $s(\cdot)$. In particular, since the functions $\{h_O(\cdot)\}_{O\in\mathcal{O}}$ determine the functions $s_j(g)$, two structures that yield the same functions $h_O(\cdot)$ are also observationally equivalent. Thus, the function $f_U(\cdot)$ is identified if and only if for any pair of observationally equivalent admissible structures $\{P(\cdot|\,\cdot), f_U(\cdot)\}$ and $\left\{\tilde{P}(O|\,\cdot), \tilde{f}_U(\cdot)\right\}$, $f_U(\cdot) = \tilde{f}_U(\cdot)$.

[21] Footnote 15 of HSS refers to a previous version of our paper that employed a more restrictive version of assumption 3.

To complete the proof of the proposition, define admissible structures as pairs $\{P(\cdot|\cdot), f_U(\cdot)\}$ such that (i) $f_U(u)$ is a density, (ii) $0 < \tilde{P}(O|u) < 1$ for all $O \in \mathcal{O}$ and all $u \in \mathbb{R}^{|J|}$, and (iii) the choice set probabilities add to one for each u: $\sum_{O \in \mathcal{O}} \tilde{P}(O|u) = 1$. The first conditions follow from the assumptions in the proposition. The second and third conditions ensure that $P(O|u)$ is a proper probability for any pair (O, u). The distribution of indirect utilities is not identified if there are two observationally equivalent admissible structures $\{P(\cdot|\cdot), f_U(\cdot)\}$ and $\left\{\tilde{P}(O|\cdot), \tilde{f}_U(\cdot)\right\}$ with $f_U(\cdot) \neq \tilde{f}_U(\cdot)$. The following lemma shows that this is the case under the hypothesis of the proposition.

Lemma 4. *If for the admissible structure $\{P(\cdot|\cdot), f_U(\cdot)\}$ there exists an open set $B \subset \mathbb{R}^{|J|}$ and a choice set $O \subsetneq J$ such that for all $u \in B$, $f_U(u) > 0$ and $P(O|u) > \kappa > 0$, then there exist an alternative admissible structure $\left\{\tilde{P}(\cdot|\cdot), \tilde{f}_U(\cdot)\right\}$ with $f_U(\cdot) \neq \tilde{f}_U(\cdot)$ and for all u_O,*

$$h_O(u_O) = \int P(O|u) f_U(u) \, du_{O^c} = \int \tilde{P}(O|u) \tilde{f}_U(u) \, du_{O^c}.$$

Proof. Fix an open set $U \subset \mathbb{R}^{|J|}$, a choice set $O \subsetneq J$ such that for all $u \in U$, $f_U(\boldsymbol{u}) > 0$ and $P(O|\boldsymbol{u}) > \kappa > 0$. These quantities exist by assumption. Let $R = \prod_{j \in \mathcal{J}} \left[\underline{u}_j, \bar{u}_j\right] \subset U$ be a closed cartesian product of $|J|$ intervals, one for each good. Define an arbitrary absolutely continuous function $c(u_{O^c})$ such that (i) $c(u_{O^c}) \neq 0$, (ii) $\|c(u_{O^c})\|_\infty < \frac{\kappa}{2}$, (iii) $c(u_{O^c}) = 0$ for $u_{O^c} \notin R_{O^c}$, where $R_{O^c} = \prod_{j \in O^c} \left[\underline{u}_j, \bar{u}_j\right]$ denotes the product of the intervals in R corresponding to the products in O^c.

Define a family of functions $\{a_{O'}(u)\}_{O' \in \mathcal{O}}$ as follows. Let $a_{O'}(u) = 0$ for $O' \neq O$ and

$$a_O(u) = 1\{u \in R\} \left[c(u_{O^c}) - \frac{\int_{R_{O^c}} c(u_{O^c}) f_U(u) \, du_{O^c}}{\int_{R_{O^c}} f_U(u) \, du_{O^c}}\right].$$

Note that each $\|a_O(u)\| < \kappa$, and that

$$\int a_O(u) f(u) \, du_{O^c} = \int_{R_{O^c}} \left[c(u_{O^c}) - \frac{\int_{R_{O^c}} c(u_{O^c}) f_U(u) \, du_{O^c}}{\int_{R_{O^c}} f_U(u) \, du_{O^c}}\right] f(u) \, du_{O^c} = 0.$$

Moreover, for every $O' \subset O$

$$\int a_O(u) f(u) \, du_{O'^c} = \int \int a_O(u) f(u) \, du_{O^c} du_{O \setminus O'} = 0.$$

Define the alternative structure as

$$\tilde{f}(u) = (1 - a_O(u)) f(u)$$

$$\tilde{P}(O'|u) = \frac{P(O'|u) - a_{O'}(u)}{1 - a_O(u)}$$

for every $O' \in \mathcal{O}$. Now we verify that $\left\{\tilde{P}(\cdot|\cdot), \tilde{f}(\cdot)\right\}$ is an admissible structure. First, $\tilde{f}(u)$ is a density because $(1 - a_O(u)) f(u) \geq 0$ and

$$\int (1 - a_O(u)) f(u) \, du = 1 - \int_O \int_{O^c} a_O(u) f(u) \, du_{O^c} du_O = 1.$$

Second, the choice set probabilities satisfy $0 < \tilde{P}(O'|u) < 1$ for all $O' \in \mathcal{O}$.Third, the choice set probabilities add to one for each u:

$$\sum_{O'\in\mathcal{O}} \tilde{P}\left(O'\middle| u\right) = \frac{\sum_{O'\in\mathcal{O}} P\left(O'\middle| u\right) - a_O\left(u\right)}{1-a_O\left(u\right)} = 1.$$

Now we verify that the alternative structure is observationally equivalent to the original one. Note that $\int_{O'^c} \tilde{P}\left(O'\middle| u\right)\tilde{f}\left(u\right) du_{O'^c} = \int_{O'^c} P\left(O'\middle| u\right) f\left(u\right) du_{O'^c} = h_{O'}\left(u_{O'}\right)$ for all $O' \neq O$. And, finally

$$\begin{aligned}\int_{O^c} \tilde{P}\left(O\middle| u\right)\tilde{f}\left(u\right) du_{O^c} &= \int_{O^c} \left(P\left(O\middle| u\right) - a_O\left(u\right)\right) f\left(u\right) du_{O^c} \\ &= \int_{O^c} P\left(O\middle| u\right) f\left(u\right) du_{O^c} - \int_{O^c} a_O\left(u\right) f\left(u\right) du_{O^c} \\ &= h_O\left(u_O\right).\end{aligned}$$

$\square$

A.5 Proof of Theorem 2

Assumptions 5 and 6 imply that there exists a function $\psi^{-1}\left(\cdot;x^*\right)$ such that $\left(\delta_t,\gamma_t\right) = \psi^{-1}\left(\phi_t;x_t^*\right)$. Assumption 7(i) implies that $E\left[\psi^{-1}\left(\phi_t;x_t^*\right) - \left(x_t^\delta, x_t^\gamma\right)\middle| r_t\right] = E\left[\xi_t\middle| r_t\right] = 0$. Let $\tilde{\psi}^{-1}\left(\cdot;x^*\right)$ be an alternative function such that$E\left[\psi^{-1}\left(\phi_t;x_t^*\right) - \tilde{\psi}^{-1}\left(\phi_t;x_t^*\right)\middle| r_t\right] = 0$ almost everywhere. Assumption 7(ii) implies that $\psi^{-1}\left(\phi_t;x^*\right) = \tilde{\psi}^{-1}\left(\phi_t;x^*\right)$ almost everywhere. Therefore, $\psi^{-1}\left(\cdot;x^*\right)$ is identified. Since ϕ_t is known, $\left(\delta_t,\gamma_t\right) = \psi^{-1}\left(\phi_t;x_t^*\right)$ and $\xi_t = \left(\delta_t,\gamma_t\right) - \left(x_t^\delta, x_t^\gamma\right)$ are identified.

A.6 Additional Results on Identification across Markets

We show results analogous to those in Proposition 2 for non-separable models. These results follow Theorem 2 in Berry and Haile (2010). Let

$$\delta_{jt} = \tilde{u}_j\left(x_{jt},\xi_{jt}\right) \equiv \text{med}\ \left(u_{ijt}\middle| x_{jt},\xi_{jt}\right),$$

and let $f_{\delta_j}\left(\cdot\middle| x_{jt}, r_{jt}\right)$ be the conditional density of δ_j, where r_{jt} are a set of instruments.

Fix $\varepsilon_\tau > 0$ and $\varepsilon_f > 0$, small. For $\tau \in (0,1)$, let $\mathcal{L}_j\left(\tau\right)$ be the convex hull of functions $m_j\left(\cdot,\tau\right)$ such that for all r_{jt}, $P\left(\delta_{jt} \leq m_j\left(x_{jt},\tau\right)\middle| r_{jt}\right) \in \left[\tau-\varepsilon_\tau, \tau+\varepsilon_\tau\right]$, and for all x_{jt}, $m_j\left(x_{jt},\tau\right) \in s_j\left(x_{jt}\right) \equiv \left\{\delta : f_{\delta_j}\left(\delta\middle| x_{jt}, r\right) \geq \varepsilon_f,\ \forall r \text{ with } f_X\left(x_{jt}\middle| r\right) > 0\right\}$.

Assumption 8. $\xi_{jt} \perp r_{jt}$

Assumption 9. *For all j and $\tau \in (0,1)$, (i) for any bounded function $B_j\left(x,\tau\right) = m_j\left(x,\tau\right) - \tilde{u}_j\left(x,\tau\right)$ with $m_j\left(\cdot,\tau\right) \in \mathcal{L}_j\left(\tau\right)$ and $\varepsilon_{jt} \equiv \delta_{jt} - \tilde{u}_j\left(x_{jt},\tau\right)$, $E\left[B_j\left(x_{jt},\tau\right)\psi_j\left(x_{jt},r_{jt},\tau\right)\middle| r_{jt}\right] = 0$ a.s. only if $B_j\left(x_{jt},\tau\right) = 0$ a.s. for $\psi_j\left(x,r,\tau\right) = \int_0^1 f_{\varepsilon_j}\left(\sigma B_j\left(x,\tau\right)\middle| x, r\right) \mathrm{d}\sigma > 0$. (ii) the density $f_{\varepsilon_j}\left(e\middle| x,w\right)$ of ε_{jt} is continuous and bounded for all $e \in \mathbb{R}$, and (iii) $\tilde{u}_j(x_{jt},\tau) \subset s_j(x_{jt})$ for all x_{jt}.*

Proposition 3. *(Berry and Haile, 2014; Chernozhukov and Hansen, 2005). If δ_{jt} is identified and assumptions 8 and 9 are satisfied, then the functions $\tilde{u}(\cdot)$ and ξ_{jt} are identified for each j and t.*

Proof. Follows from theorem 4 in Chernozhukov and Hansen (2005) since δ_{jt} is identified. □

An analogous results holds for identification of $\tilde{g}_j$ since

$$g_{jt} = \tilde{g}_j\left(x_{jt}, \zeta_{jt}\right)$$

is known. Here, we switch g_{jt} for δ_{jt} and $\tilde{g}_j\left(\cdot\right)$ for $\tilde{u}_j\left(\cdot\right)$.

B Data Appendix

The data reported here have been supplied by the United States Renal Data System (USRDS) and the Centers for Medicare & Medicaid Services (CMS). These sources provide us with data on all dialysis facilities and the near universe of kidney patients in the US. Patient characteristics include the residence zip-code, co-morbidities and the facility that they attend. For each facility, we observe their address, ownership status and the number of stations. Patients and facilities are uniquely identified by a USRDS generated identifier that can be used to link records across separate datasets. We geocode patient zip-codes and facility addresses to calculate the straight line distance between a given facility and a patient's zip-code centroid.

We will retain copies of the data until permitted by our Data Use Agreement with the United States Renal Data System (USRDS). Researchers interested in using our dataset should directly contact USRDS to obtain permission.

B.1 Data Description

Our data on patient profiles and treatment history come from the USRDS Researcher Standard Analysis File (SAF) which combines information from ESRD claims filed to CMS and data from the Consolidated Renal Operations in a Web-Enabled Network System (CROWN), a mandatory data system used by dialysis facilities to collect information on all patients, regardless of payer type. The main SAF datasets used in this analysis are Medical Evidence (medevid), which includes patient health information like co-morbidities and the whether a nephrologist was already caring for a patient when dialysis commenced, Treatment History (rxhist), where we obtain the sequence of facilities in which a patient was treated, Payer

History (payhist) for insurance information, Residence History for the residence zip code and the Facility dataset from the USRDS.

Though the patient information is sourced from claims, facility data come from the CMS Annual Facility Survey and the CMS Facility Compare dataset maintained separately by CMS. These includes identifiers for the facility, years of operation, profit status, chain status, and setting status. The facility and patient identifiers allow us to link the patient information from claims and the facility information from Facility Compare, providing a complete overview of the patient-facility interaction.

We also geocoded facility addresses and obtained the geocodes for the centroid of each patient's zip code. These coordinates are used to estimate the distance from the facility to the patient, calculated as the distance from the patients' reported zip code centriod to the facility. Geo-coordinates are obtained via queries sent to the Google Maps API; these queries have as an input the facility addresses included in the Facility Compare dataset provided by CMS and return as an output the associated longitude and latitude for each facility. Zip-code centroids are also obtained using Google Maps.

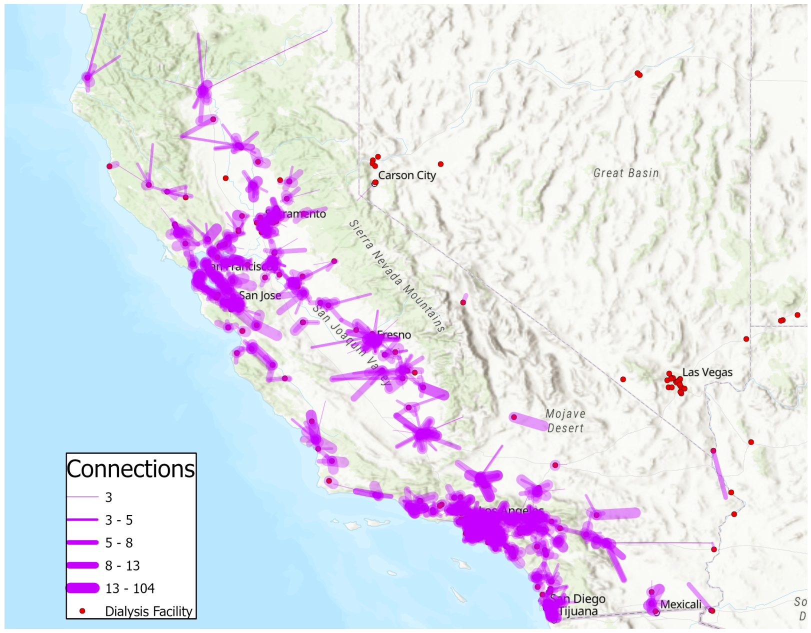
We use the Treatment History files to construct the number of patients receiving care at each facility at a given point in time. This file contains the start date and the end date of each patient's treatment at each facility where they receive care. We use this information to compute the number of patients undergoing in-patient hemodialysis at each facility on each day during our sample period. These calculations will include all patients, irrespective of whether they are in the sample of patients that we use to estimate our model (see section B.2.2 below).

B.2 Sample Selection

We consider first-time admissions in California facilities between Jan 1, 2015 and December 31, 2018. As mentioned in the main text, moving costs and other considerations can be important in subsequent stays, which complicates the analysis. Nonetheless, the first facility a patient chooses is consequential as the median and average patient is treated at 1 and 1.30 facilities respectively.

California is essentially an isolated market, with few outgoing or incoming patient-facility connections across its state borders. Figure B.1 shows the linkages between all facilities in the US and zip-code centroids in California. The thickness of each edge connecting a facility with a zip-code centroid indicates the number of patients residing in a zip-code that started dialysis at a given facility. We omit edges with fewer than three patients. Only in rare instances does a patient living in California attend a facility outside the state. When they do, our approach will treat the patient as choosing the outside option.

Figure B.1: California Connections



B.2.1 Facility Sample Selection

Table B.1 describes the facility sample. All facilities in California during our sample period were successfully geocoded. From this universe of facilities, we restrict attention to facilities that focus on in-center care and are non-pediatrics. Both variables are calculated using the admissions data for facilities during our sample period; a facility is said to focus on in-facility care if more than 50% of its admitted patients enroll in facility-based hemodialysis. We classify a facility as pediatrics if the average age of the patients they admit is less than or equal to 18. Patients living in California who receive dialysis but do not attend one of these facilities are considered as being treated at a composite outside option.

We restricted to facilities that focus on non-pediatric and in-center care for two reasons. First, we want to focus on the interactions for individuals that are going to facilities to receive treatment, as opposed to receiving home dialysis in which case the distance to the facility is not as salient in the patient's choice of facility. Only a small minority of patients receive home dialysis and are likely selected on health condition and income. Second, we restrict to non-pediatric facilities because the baseline differences in co-morbidities and clinical indications for pediatric and adult dialysis can be substantial, creating significantly different needs and operational setups for pediatric facilities.

We only include the quarters for which the facility operation was relatively stable, excluding periods around entry, exit, capacity changes, or moves as these events could substantially affect a facility's demand and acceptance policies. In particular, we include in the inside option facility-quarters in years with no changes in the number of stations or address. We remove the quarter of and the quarter after a facility entered. Similarly, we remove the quarter before and the quarter of a facility exit.

Table B.1: Facility Sample

Restrictions	Facilities
Restricted to 2015 - 2018 and California	721
Restricted to facilities with geocoordinates	721
Restricted to facilities specializing in facility-based hemodialysis and are non-pediatric	640
Facilities with at least one stable quarter	553

B.2.2 Patient Sample Selection

Table B.2 describes the patient sample. We make three major restrictions on the patient sample, starting from the universe of patients with a residential zip-code in California that started dialysis in the years 2015 - 2018. First, and analogously to the focus on non-pediatric facilities, we keep only adults in our sample, defined as at least 18 years of age when they first started dialysis. Second, we drop patients for whom we weren't able to compute a distance to the facility attended; practically, this means that we drop a handful of patients for whom we did not observe a valid zip-code. These two restrictions together result in a couple hundred patients being dropped from our sample. The biggest cut in the sample comes from dropping patients that chose facilities greater than 50 miles from their reported zip-code centroid. Based on an inspection of these observations, we suspect that the residential zip-code is incorrectly recorded for these patients. One indication is that the 95th percentile of distance, conditional on the chosen facility being is less than 50 miles away, is less than 20 miles.

Table B.2: Patient Sample

Restriction	Patients
Restricted to 2015 - 2018 and California	53,074
Restricted to adults (>=18 years old)	52,768
Restricted to admissions with distance between patient and facility	52,751
Restricted to those that chose a facility within 50 miles	50,002

B.2.3 Target Capacity

Table B.3 presents estimates of a regression of the estimated target capacity on facility inputs measured annually, controlling for facility fixed effects. The result shows that univariate regressions of facility inputs are positively correlated with target capacity. This includes both capital and labor inputs. The relationship holds even though (i) target capacity varies at a higher frequency level than the recorded inputs and (ii) the inputs are measured only annually.

Table B.3: Correlation Between Target Capacity and Facility Inputs

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Total Number of Dialysis Stations	-0.006***								-0.019***
	(0.002)								(0.004)
Late Shift		-0.016							-0.063
		(0.029)							(0.044)
Registered Nurses on staff full-time			0.014						0.001
			(0.009)						(0.021)
Licensed Practical/Visiting Nurses FTime				0.060					0.058
				(0.048)					(0.051)
Patient Care Technicians on staff FTime					0.006				0.005
					(0.006)				(0.017)
Advanced Practice Nurses on staff FTme						0.107			0.096
						(0.131)			(0.135)
Dieticians on staff full-time							0.087		-0.144
							(0.074)		(0.158)
Social Workers on staff full-time								0.215***	0.381***
								(0.062)	(0.140)
Constant	0.093***	-0.049***	-0.135**	-0.081***	-0.110	-0.056***	-0.136*	-0.265***	0.043
	(0.030)	(0.013)	(0.056)	(0.029)	(0.070)	(0.014)	(0.075)	(0.066)	(0.042)
Observations	2,061	2,038	2,061	2,061	2,061	2,061	2,061	2,061	2,038
R-squared	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.002	0.005

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

C Estimation Appendix: Gibbs Sampler

Our sampler starts with an initial guess for the parameters $(\alpha, \beta, \Sigma, \delta, \gamma)$, variances $\left(\sigma_\gamma^2, \sigma_\delta^2, \sigma_\beta^2, \sigma_{\varepsilon 0}^2\right)$ and the latent variables $(\beta_i, \varepsilon_{i0}, v_i, \pi_i)$ for every i. We denote this guess by $\theta^{(0)}$. For each draw k, we perform the following steps:

1. Data augmentation:

   (a) Draw the latent acceptance index $\pi_{ij}|\theta^{(k-1)}$ for every i and j in the sample. The posterior distribution of π_{ij} conditional on all the parameters $\theta^{(k-1)}$ is normal. If i was allocated to facility j, then we draw π_{ij} from the conditional posterior truncated by $\pi_{ij} \geq z_{ij}$. If i was allocated to facility $j^* \neq j$ and $v_{ij}^{(k-1)} > v_{ij^*}^{(k-1)}$, then we draw π_{ij} from the conditional posterior truncated by $\pi_{ij} < z_{ij}$. Otherwise, we draw it from the conditional posterior without any truncation. Let $\pi^{(k)}$ denote the vector of draws and let $O_i^{(k)}$ be $\{j \in J : \pi_{ij} \geq z_{ij}\}$.

   (b) Draw the latent utility $v_{ij}|\,\theta^{(k-1)}, \pi^{(k)}$ for every i and j. The posterior distribution of v_{ij} conditional on all the parameters $\theta^{(k-1)}$ and on $\pi^{(k)}$ is normal. Let j^* be the facility chosen by i. Draw v_{ij^*} from the conditional posterior truncated at $v_{ij^*} \geq \max_{j \in O_i^{(k)}/\{j^*\}} v_{ij}$. Denote it by $v_{ij^*}^{(k)}$. Then, draw v_{ijt} for $j \in O_i^{(k)} \setminus \{j^*\}$ from the conditional posterior truncated at $v_{ij} \leq v_{ij^*}^{(k)}$. Lastly, draw v_{ij} for $j \notin O_i^{(k)}$from its unconditional posterior without any truncation. Let $v^{(k)}$ denote the vector of draws.

2. Seemingly unrelated Bayesian regression: with the draws of $v^{(k)}$and $\pi^{(k)}$ and for fixed value of $\delta_j^{(k-1)}, \gamma_j^{(k-1)}, \beta_i^{(k-1)}$ and $\varepsilon_{i0}^{(k-1)}$; the equations above form a system of seemingly unrelated regressions. The posterior distributions of the parameters α, β are normal and the posterior distribution of Σ is inverse Wishart. We draw these parameters and obtain the resulting residuals $\hat{\varepsilon}_{ij}^{(k)}$ and $\hat{\nu}_{ij}^{(k)}$.

3. Update random effects:

   (a) Draw $\beta_i|\hat{\varepsilon}_{ij}^{(k)}, \hat{\nu}_{ij}^{(k)}, \Sigma^{(k)}, \sigma_\beta^{2,(k-1)}$. The posterior distribution of β_i conditional on the residuals $\hat{\varepsilon}_{ij}^{(k)}$ and $\hat{\nu}_{ij}^{(k)}$ and the previous variance draws $\Sigma^{(k)}$ and $\sigma_\beta^{2,(k-1)}$ is normal. We draw β_i from this conditional posterior. Let $\beta_i^{(k)}$ denote these draws and obtain the updated residuals $\bar{\varepsilon}_{ij}^{(k)} = \hat{\varepsilon}_{ij}^{(k)} + \beta_i^{(k)} x_j - \beta_i^{(k-1)} x_j$.

   (b) Draw $\varepsilon_{i0}|\hat{\bar{\varepsilon}}_{ij}^{(k)}, \hat{\nu}_{ij}^{(k)}, \Sigma^{(k)}, \sigma_{\varepsilon 0}^{2,(k-1)}$. The posterior distribution of ε_{i0} conditional on the residuals $\bar{\varepsilon}_{ij}^{(k)}$ and $\hat{\nu}_{ij}^{(k)}$ and the previous variance draws $\Sigma^{(k)}$ and $\sigma_{\varepsilon 0}^{2,(k-1)}$ is normal. We draw ε_{i0} from this conditional posterior. Let $\varepsilon_{i0}^{(k)}$ denote these draws and obtain the updated residuals $\tilde{\varepsilon}_{ij}^{(k)} = \bar{\varepsilon}_{ij}^{(k)} + \hat{\varepsilon}_{i0}^{(k-1)} - \hat{\varepsilon}_{i0}^{(k)}$.

   (c) Draw $\gamma_j|\hat{\tilde{\varepsilon}}_{ij}^{(k)}, \hat{\nu}_{ij}^{(k)}, \Sigma^{(k)}, \sigma_\gamma^{2,(k-1)}$. The posterior distribution of γ_j conditional on the residuals $\tilde{\varepsilon}_{ij}^{(k)}$ and $\hat{\nu}_{ij}^{(k)}$ and the previous variance draws $\Sigma^{(k)}$ and $\sigma_\gamma^{2,(k-1)}$ is normal. We draw γ_j from this conditional posterior. Let $\gamma_j^{(k)}$ denote these draws and obtain the updated residuals $\tilde{\nu}_{ij}^{(k)} = \hat{\nu}_{ij}^{(k)} + \gamma_j^{(k-1)} - \gamma_j^{(k)}$.

(d) Draw $\delta_j|\tilde{\varepsilon}_{ij}^{(k)}, \tilde{\nu}_{ij}^{(k)}, \Sigma^{(k)}, \sigma_\delta^{2,(k-1)}$. The posterior distribution of δ_j conditional on the residuals $\tilde{\varepsilon}_{ij}^{(k)}$ and $\tilde{\nu}_{ij}^{(k)}$ and the previous variance draws $\Sigma^{(k)}$ and $\sigma_\delta^{2,(k-1)}$ is normal. We draw δ_j from this conditional posterior. Let $\delta_j^{(k)}$ denote these draws.

4. Update the variance of the random effects:

   (a) Draw $\sigma_{\varepsilon 0}^2|\varepsilon_{i0}^{(k)}$. The posterior distribution of $\sigma_{\varepsilon 0}^2$ conditional on $\varepsilon_{i0}^{(k)}$ is inverse-gamma. Similarly, draw $\sigma_\beta^2|\beta_i^{(k)}$, $\sigma_\gamma^2|\gamma_j^{(k)}$ and $\sigma_\delta^2|\delta_j^{(k)}$. Let $\sigma_{\varepsilon 0}^{2,(k)}$, $\sigma_\beta^{2,(k)}$, $\sigma_\gamma^{2,(k)}$and $\sigma_\gamma^{2,(k)}$denote these draws.

5. Finally, collect all parameter draws in step k and denote them by $\theta^{(k)}$.

We specify a set of diffuse conjugate priors to each set of parameters, following recommendations in McCulloch and Rossi (1994). The priors for $\alpha, \beta, \delta, \gamma$ are normal with zero mean and covariance equal to the identity matrix times a large constant: 1000. The prior of Σ is an inverse Wishart with a 2×2 identity matrix as its scale matrix and 3 degrees of freedom. Similarly, the priors of $\sigma_{\varepsilon 0}^2$, σ_β^2, σ_γ^2 and σ_δ^2 are four independent inverse-gamma distributions with scale and shape parameters equal to 1/2. These priors are uninformative relative to the size of our dataset and thus, the estimation results are unlikely to change substantially should we make them even less precise.

We start a chain from random starting points and run the Gibbs sampler for 4 million draws, discarding the first million draws. We summarize the draws for each parameter and verify that the Potential Scale Reduction Factor for each parameter is close to one, which indicates that letting the chain run for longer is not likely to change the results (Gelman et al., 2014).

D Monte Carlo Exercises

This section presents Monte Carlo evidence to assess the properties of the Gibbs sampler described in the main text, and to assess bias arising from model mis-specification. Our experiments focus on a single market with $J = 5$ products and vary the number of consumers in the market, $N \in \{5000, 20000\}$.

To simulate a dataset, we begin by simulating observed characteristics. Consumer and product locations are drawn uniformly at random from a unit square to generate distances x_{ij}; an observable preference shifter y_{ij} is drawn from a standard normal; a choice-set shifter z_{ij} is drawn from the Poisson distribution with parameter 10; a consumer-specific binary observable d_i is drawn from the Bernoulli distribution with parameter 0.5.

Next, we then simulate indirect utilities and choice sets by drawing

$$v_{ij} = \delta_j + \beta_i x_{ij} - y_{ij} + \varepsilon_{i0} + \varepsilon_{ij},$$
$$\sigma_{ij} = 1\left\{\gamma_j + \alpha_i d_i + \nu_{i0} + \nu_{ij} > z_{ij}\right\}$$

where ε_{i0}, ν_{i0} and $(\epsilon_{ij}, \nu_{ij})$ are mutually independent (multivariate) normal distributions with mean zero and variance $\sigma^2_{\varepsilon 0}$, $\sigma^2_{\nu 0}$, and Σ respectively; the random coefficients β_i and α_i are normally distributed, mutually independent of each other and other random variables in the model with means and variances $\left(\bar{\beta}, \sigma^2_{\beta}\right)$ and $(\bar{\alpha}, \sigma^2_{\alpha})$ respectively; and the facility fixed-effects γ_j and δ_j are generated from independent mean-zero normal distributions with variances σ^2_{γ} and σ^2_{δ} respectively. These latent variables provides an a product that each consumer is matched with.

We repeat this simulation procedure to produce 100 datasets that are then used to estimate the model using a Gibbs' sampler. Our sampler uses 1 million iterations, a burn-in of 25% of the chain, and one-in-ten thinning. For each dataset, we estimate four different models:

1. The correct specification
2. The "No Random Coefficients" model, which sets $\beta_i = \bar{\beta}$ and $\alpha_i = \bar{\alpha}$ for all i
3. The model with "Choice Set Shifter in Utility," which sets $\sigma_{ij} = 1$ and $v_{ij} = \delta_j + \beta_i x_{ij} + \beta_z z_{ij} - y_{ij} + \varepsilon_{i0} + \varepsilon_{ij}$,
4. The "Unconstrained Demand" model, which sets $\sigma_{ij} = 1$.

The second model assess the importance of random coefficients whereas the third and fourth assess whether mis-specification by omitting choice-set constraints are important, whether with or without the "naive" correction in the third model.

The estimated parameters and the coverage of the 95% confidence sets are presented in Tables D.4 and D.5 respectively for the case with 5000 and 20000 patients. As expected, the correct specification exhibits appropriate coverage of the true parameters. The omission of random coefficients not only creates a substantial bias in the coverage of $\bar{\beta}$ and $\bar{\alpha}$, but also in other parameters such as $\sigma_{\nu 0}$. Models that omit choice-set constraints are particularly problematic with extremely low coverage ratios.

Table D.4: Monte Carlo Summary with 5000 patients

	True Value	Correct Specification (1)			No Random Coefficient (2)			Choice Set Shifter in Utility (3)			Unconstrained Demand (4)		
		Bias	RMSE	95% cov	Bias	RMSE	95% cov	Bias	RMSE	95% cov	Bias	RMSE	95% cov
Mean γ	10	-0.138	0.724	82	-0.133	0.735	83	---	---	---	---	---	---
Mean α	-2	0.014	0.095	94	0.019	0.102	91	---	---	---	---	---	---
Mean δ	10	-0.001	0.446	91	0.069	0.457	91	6.923	7.043	0	-6.409	6.739	0
Mean β	-2	0.005	0.077	97	0.187	0.212	28	0.303	0.323	11	0.312	0.333	15
Coef on z_{ij} in Utility	0	---	---	---	---	---	---	-1.440	1.465	0	---	---	---
Sd ν	1	-0.014	0.094	97	0.049	0.130	93	---	---	---	---	---	---
Sd ε	1	-0.032	0.109	97	0.992	1.002	0	2.720	2.761	0	4.089	4.161	0
Corr (ν,ε)	0	0.075	0.256	99	0.018	0.155	95	---	---	---	---	---	---
Sd ε_0	1.22	0.041	0.127	94	0.188	0.274	80	1.620	1.686	2	3.553	5.218	4
Sd ν_0	1.22	0.033	0.087	92	0.167	0.187	55	---	---	---	---	---	---
Sd δ	1	-0.090	0.290	91	-0.089	0.289	93	0.721	0.955	52	0.607	0.839	61
Sd γ	1.41	-0.213	0.425	88	-0.213	0.428	86	---	---	---	---	---	---
Sd β	2	-0.025	0.104	96	---	---	---	-0.238	0.326	68	-0.336	0.490	65
Sd α	1	-0.098	0.173	94	---	---	---	---	---	---	---	---	---

Notes: Bias is the difference between true parameter and mean estimates. RMSE is the root mean squared error of the estimates. The 95% coverage probability is the number of simulations (out of 100) for which the true parameter lies in the 95% credible interval derived from the Gibbs sampler.

Table D.5: Monte Carlo Summary with 20000 patients

	True Value	Correct Specification (1)			No Random Coefficient (2)			Choice Set Shifter in Utility (3)			Unconstrained Demand (4)		
		Bias	RMSE	95% cov	Bias	RMSE	95% cov	Bias	RMSE	95% cov	Bias	RMSE	95% cov
Mean γ	10	0.023	0.610	94	0.043	0.628	96	---	---	---	---	---	---
Mean α	-2	0.000	0.038	96	-0.003	0.044	97	---	---	---	---	---	---
Mean δ	10	-0.027	0.393	94	0.031	0.386	94	6.762	6.844	0	-6.050	6.201	0
Mean β	-2	-0.002	0.046	92	0.173	0.185	6	0.301	0.312	2	0.300	0.310	2
Coef on z_{ij} in Utility	0	---	---	---	---	---	---	-1.389	1.404	0	---	---	---
Sd ν	1	-0.005	0.053	93	0.071	0.097	76	---	---	---	---	---	---
Sd ε	1	-0.007	0.066	95	1.037	1.041	0	2.654	2.680	0	3.878	3.927	0
Corr (ν,ε)	0	0.027	0.146	95	-0.029	0.076	94	---	---	---	---	---	---
Sd ε_0	1.22	0.011	0.074	93	0.143	0.194	64	1.560	1.611	1	3.586	4.192	0
Sd ν_0	1.22	0.010	0.051	94	0.157	0.164	11	---	---	---	---	---	---
Sd δ	1	-0.043	0.250	94	-0.048	0.256	95	0.776	0.963	47	0.641	0.820	57
Sd γ	1.41	-0.131	0.384	93	-0.135	0.388	92	---	---	---	---	---	---
Sd β	2	-0.006	0.064	94	---	---	---	-0.184	0.220	39	-0.206	0.266	42
Sd α	1	-0.032	0.086	92	---	---	---	---	---	---	---	---	---

Table D.6: Monte Carlo Diversion Ratio

	Average True Value	Correct Specification (1)	No Random Coefficient (2)	Choice Set Shifter in Utility (3)	Unconstrained Demand (4)
Demand Side	0.221				
Mean Bias		0.000	0.027	0.016	0.011
RMSE		0.003	0.035	0.095	0.093
Supply Side	0.625				
Mean Bias		0.024	-0.001	-0.388	---
RMSE		0.473	0.551	1.480	---

Notes: Demand side diversion ratio is defined as $\frac{\partial s_k}{\partial y_{ij}} / \frac{\partial s_j}{\partial y_{ij}}$. Supply side diversion ratio is defined as $\frac{\partial s_k}{\partial z_{ij}} / \frac{\partial s_j}{\partial z_{ij}}$.

Perhaps an economically more important estimand on which to compare the specifications are the estimated diversion ratios. The "demand-side" diversion ratios are computed using marginal changes in y_{ij} and the "supply-side" diversion ratios are computed using marginal changes in z_{ij}. The mean bias and the root mean squared errors are reported in Table D.6. As expected, the mean bias and the RMSE are the lowest for the correct specification. The omission of random coefficients does increase the size of the biases and the RMSE, but less so than misspecified models that omit choice-set constraints altogether.

E Appendix of Exhibits

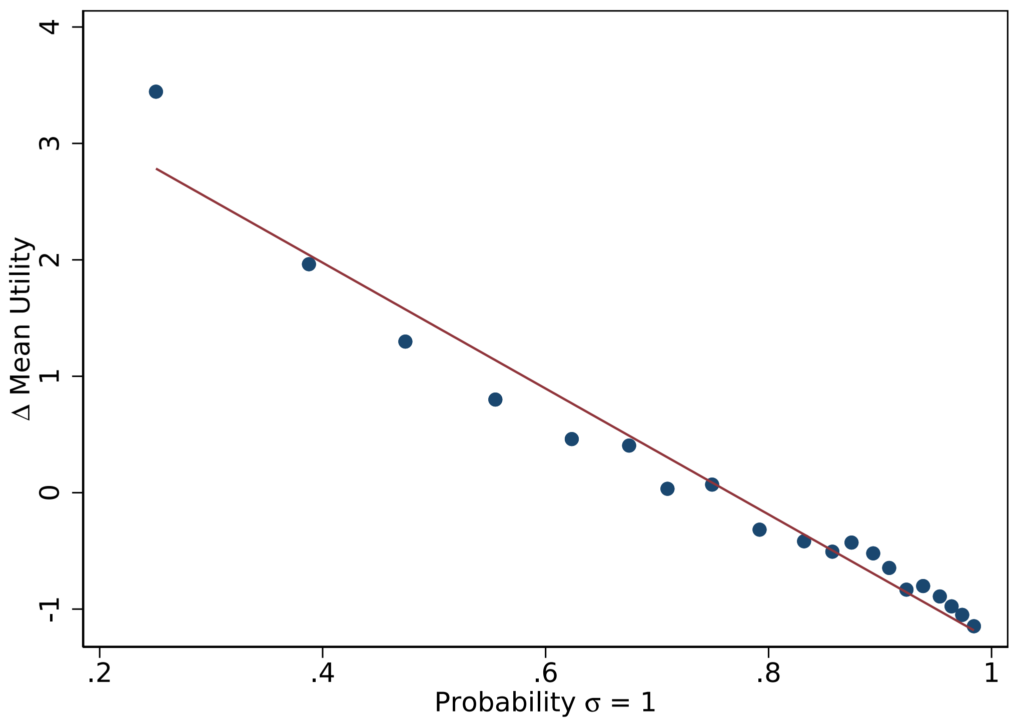
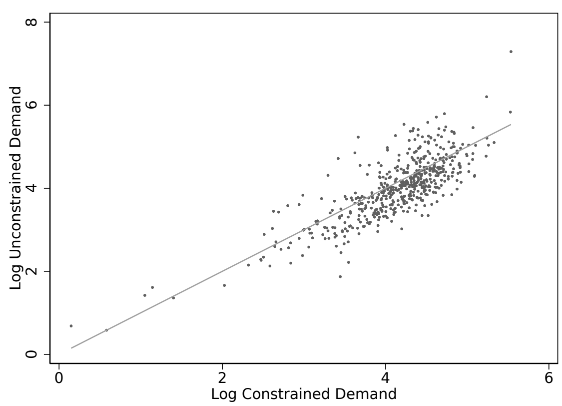
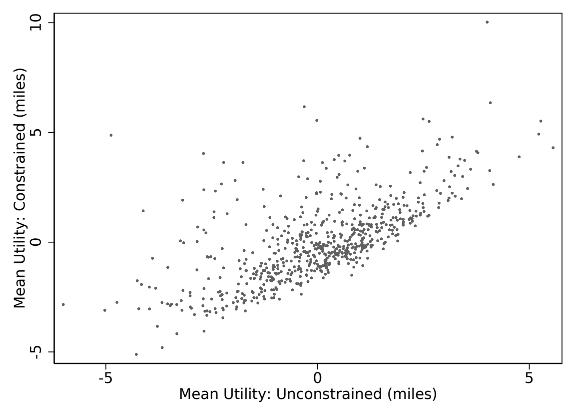


Figure E.2: Mean Utility vs Acceptance Probability



((a)) Effects of constraints on demand



((b)) Comparing estimated mean utilities

Figure E.3: Bias in Demand

Notes: (a) Scatterplot of latent demand by facility-year estimated using the preferred and unconstrained specifications. (b) Scatterplot of mean utility by facility-year estimated using the preferred and unconstrained specifications.