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COLLECTIVE HOLD-UP

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ABSTRACT

We consider dynamic processes of coalition formation in which a principal bargains sequentially with a group of agents. This problem is at the core of a variety of applications in economics and politics, including a lobbyist seeking to pass a bill, an entrepreneur setting up a start-up, or a firm seeking the approval of corrupt bureaucrats. We show that when the principal's willingness to pay is high, strengthening the bargaining position of the agents generates delay and reduces agents' welfare. This occurs in spite of the lack of informational asymmetries or discriminatory offers. When this collective action problem is severe enough, agents prefer to give up considerable bargaining power in favor of the principal.

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1. Introduction

In this paper, we study dynamic processes of coalition formation in which a principal bargains sequentially with a group of agents. This type of problem, in which one of the players takes a central role in organizing collective action, is pervasive in applications. Consider, for instance, a lobbyist seeking to influence legislators to pass a policy proposal, an entrepreneur seeking to form a start-up, or a firm negotiating with buyers the adoption of a new technology with network externalities.

A salient feature of these problems is that the principal often bargains with agents sequentially. In legislative politics, for instance, lobbyists and party principals rarely make proposals simultaneously to all members in the floor of the chamber. Instead, they typically strike individual deals with committee members, gradually accumulating support in favor of their preferred alternative. Whenever this is the case, the offers the principal makes to, or receives from, an agent, will generally depend on how advanced the negotiation process is. This consideration becomes important when agents are strategically farsighted, because the principal's ability to successfully negotiate with each agent depends on their expectations about the nature of future trades.

We show that in this context, efficiency requires power to be sufficiently concentrated in the principal. When instead agents have a relatively stronger say in bilateral negotiations, the equilibrium of the decentralized bargaining process entails inefficient delay. Moreover, due to the destruction of surplus caused by delay, decentralizing bargaining power from the principal to the agents reduces their welfare. These results hold irrespective of the presence or the direction of externalities on uncommitted agents, and without asymmetric information, discriminatory contracts or deadlines.

In our model, a principal negotiates sequentially with a group of n agents over an infinite horizon. In each meeting, the principal bargains with an agent over the terms by which the agent would support the principal. If an agreement is reached, the agent commits his support to the principal and exits negotiations, otherwise he remains uncommitted. The principal needs to obtain the agreement of $q < n$ agents to implement a reform, action, or policy change which affects the payoffs of all players. When this happens, the principal obtains a payoff $v > 0$, agents who committed their support to the principal obtain $z > 0$, and agents who remained uncommitted obtain $w \in \mathbb{R}$, where $w > 0$ ($w < 0$) implies that there are positive (negative) externalities on uncommitted agents. All players have a discount factor $\delta \in (0, 1)$.

One of the key innovations of our model is to consider arbitrary allocations of bargaining power between the principal and each agent (while maintaining the structure of the game fixed).[1] To do this, we assume that in a bilateral meeting the principal makes an offer with probability $\phi \in [0, 1]$, and the agent makes an offer with probability $1 - \phi$.[2] This introduces a technical complexity absent with the usual assumption that the principal has full bargaining power, because both principal and agents can extract rents from each other. As a result, the system of difference equations characterizing equilibrium payoffs can not be decoupled. We solve for symmetric Markov perfect equilibria of this game, in order to rule out "discriminatory contracts" that exploit coordination failures among agents.[3]

In our main theorem we show that decentralizing bargaining power induces inefficient delay in reaching agreements. The inefficiency results from the intersection of two elements. One resembles the traditional hold-up problem, where the initial transactions take the role of investments for the principal, who is carrying the coalition formation process. This happens because when agents have a strong bargaining position, agents trading late in the process can extract a large fraction of the rents from the principal. As a result, the principal is not willing to pay much to agents trading early on. The second key component is inter-temporal competition among agents. In fact, it is this form of competition among agents which can lead to delay instead of a sequence of increasing prices, as in Blanchard and Kremer (1997) and Olken and Barron (2009).

Interestingly, delay can arise with positive externalities, no externalities, or even negative externalities on uncommitted agents. All else equal, a larger negative externality on uncommitted agents lowers the value of holding-out. But when agents have sufficient bargaining power and the principal's willingness to pay is sufficiently large, holding out can still be attractive, and the main logic for delay is unaltered. This illustrates the possibly surprising effect of increasing the principal's willingness to pay

[1]The literature on sequential contracting has generally maintained the assumption that the principal has full bargaining power. See Rasmusen, Ramseyer, and Wiley Jr (1991), Rasmusen and Ramseyer (1994), Jehiel and Moldovanu (1995a,b), Segal and Whinston (2000), Genicot and Ray (2006), Iaryczower and Oliveros (2017)). Cai (2000) considered an alternating offer protocol. Galasso (2008) compared a finite horizon alternating-offers game with a one-shot game in which the principal makes a TIOLI offer to agents.

[2]This formulation is formally equivalent to nesting an infinite horizon bilateral bargaining in our game, where one of the sides decides whether to enter in negotiations or not, and in any period of the negotiation phase after a proposal is rejected, the principal (agent) makes offers with probability ϕ (respectively, $1 - \phi$).

[3]We show in Section 5.5 that the unique symmetric MPE we identify is the unique symmetric subgame perfect Nash equilibrium with bounded recall, as in Jehiel and Moldovanu (1995a).

on equilibrium outcomes. In fact, for any given allocation of bargaining power for which there is delay, the expected delay grows continuously with v, and in the limit as $v \to \infty$, the expected time for completion goes to infinity.

In addition to our main results on concentration of power and efficiency, the model yields rich empirical implications. Consider first the properties of delay for a fixed allocation of bargaining power. In principle, delay could occur in the early stages or in the late stages of the bargaining process. Or there could be regions of delay followed by states in which trade is efficient. Moreover, the probability of trade could be non-monotonic, having stages in which the negotiation process accelerates after every trade followed by periods in which it slows down with subsequent commitments. In fact, we show that whenever there is delay in negotiations, delay occurs from the beginning of the bargaining process, but the pace of negotiations slows down as the process of negotiations move forward, possibly until some point in which the process unravels and all further transactions occur without delay.

The fact that delay is front-loaded for any given allocation of bargaining power ϕ means that delay occurs in early transactions, i.e., when there principal still needs more than $\overline{m}(\phi)$ agents in order to win. We show that the cutpoint $\overline{m}(\cdot)$ is non-increasing in ϕ and has full range. Thus, the range of transaction situations (states) with positive expected delay is decreasing in the principal's bargaining power. As bargaining power is transferred from agents to the principal, we move from an equilibrium in which there is delay in all but the critical state, to more efficient outcomes, eventually reaching a fully efficient equilibrium when power is sufficiently concentrated in the principal.

The efficiency loss induces a tradeoff for agents. Keeping the strategy profile fixed, agents would prefer to retain as much power as possible. However, decentralizing power to agents also increases the range of states in which negotiations suffer delay. We show that (also for large v), agents' welfare is maximized when they relinquish significant bargaining power to the principal.

After presenting our main results, we consider two important special cases of the model. In section 5.1, we study equilibria as bargaining frictions vanish ($\delta \to 1$), and in section 5.2, we consider the special case in which bargaining power is fully decentralized to agents. Both cases provide important lessons. We then consider two extensions of the model. In section 5.3, we allow the payoff of agents who commit to be non-positive, $z \leq 0$, as it would be the case in corporate takeovers ($z = 0$). In section 5.4, we consider a version of the model in which transfers between principal and agent are contingent on the completion of the project. All proofs are in the Appendix.

2. Related Literature

Our paper touches on three sets of papers. Most directly, it contributes to the literature on sequential contracting between a principal and a group of agents (in particular Jehiel and Moldovanu (1995a,b), Segal and Whinston (2000), Cai (2000), Genicot and Ray (2006), Chowdhury and Sengupta (2012), Iaryczower and Oliveros (2017)). It also contributes to, and is informed by, papers on multilateral bargaining and non-cooperative dynamic coalition formation.

Beginning with Grossman and Hart (1980), one of the central contributions of the literature that focused on understanding contracting problems between a principal and a group of agents is to emphasize the role of externalities.[4] The importance of positive and negative externalities in contracting models was further highlighted in a static setting by Segal (1999, 2003), and is also a central component in the dynamic setup of Jehiel and Moldovanu (1995a,b). It has also emerged as a key consideration in the literature on non-cooperative coalitional bargaining games, which has shown that externalities can lead to breakdown of efficiency (see Bloch (1996), Ray and Vohra (1999), Ray and Vohra (2001), Gomes (2005), and Gomes and Jehiel (2005)). Coalitional bargaining games without externalities, instead, generally have efficient equilibria.[5]

A second important lesson from the literature is that the principal's ability to treat agents asymmetrically (either building on primitive heterogeneity among agents, or simply using "discriminating contracts" that treat similar agents differently) *can* allow the principal to exploit a subset of agents. Segal and Whinston (2000) show that discrimination allows the incumbent firm to take advantage of the externalities that exist across buyers without relying on coordination failures. Instead, the firm can turn buyers against one another offering an exclusionary contract to only a subset of the buyers, who then impose the externality of no entry on the other buyers (see

[4]This is indeed the key point of Grossman and Hart (1980), which showed that externalities across shareholders (here free-riding) can prevent takeovers that add value to the company.

[5]See however Chatterjee, Dutta, Ray, and Sengupta (1993), Ray and Vohra (1999) and Gomes (2005), which provide examples featuring delay in general bargaining models

also Chowdhury and Sengupta (2012)). A fundamental lesson from Genicot and Ray (2006) is that the reason the principal can exploit the agents in the Rasmusen et al and Segal papers – as opposed to say in efficient takeovers – is that by signing an exclusive contract a buyer imposes a *negative* externality on other buyers.[6,7]

With negative externalities, delay can be welfare improving for agents. In fact, Jehiel and Moldovanu (1995a,b) and Genicot and Ray (2006) show that heterogeneity and discriminatory contracts can lead to (efficient) delay in these settings.[8] Cai (2000) shows that discriminatory contracts can also lead to inefficient delay. In his model, the principal bargains with n agents in a pre-specified order, and has to obtain unanimous support from all agents, i.e., $q = n$. When players are sufficiently patient, there are multiple equilibria, including equilibria with and without delay. Differently than in our paper, delay here appears as a result of discriminating offers, which can be constructed using the predetermined order of meetings.

The bargaining literature provided other explanations for delay with complete information, less directly related to this paper. Fershtman and Seidmann (1993) and Ma and Manove (1993) show that deadlines can lead to delay. Merlo and Wilson (1995) show that efficient delay can emerge when the size of the surplus to be divided evolves stochastically over time. Yildiz (2004) and Ali (2006) show delay in bargaining with heterogeneous priors, and Acharya and Ortner (2013) show that delay can arise in bargaining over multiple issues with partial agreements.

In recent years there has been a growing literature focusing on understanding the nature of inefficiencies in legislative bargaining. On the one hand, Banks and Duggan (2006) show that in a general version of the Baron Ferejohn model, equilibria will

[6]This is also the case in the single-principal vote buying model by Dal Bo (2007), which also imposes negative externalities among agents. In this context, the principal can exploit agents even in a static setting, by using *pivotal* contracts. Relatedly, Segal (2003) and Segal and Whinston (2000) show that the principal can exploit agents in a static setting if she can make discriminatory offers.

[7]This is also the case in Galasso (2008), who shows that when there are negative externalities across agents, and agents are sufficiently patient, the principal prefers to enter a finite horizon bargaining game in which she is the last mover, to a one-shot game in which she makes a TIOLI offer to agents.

[8]In Genicot and Ray (2006) delay can be supported in equilibrium through history dependent strategies when the principal can make offers to multiple agents at a time. Jehiel and Moldovanu (1995b) considers a setup in which non-traders suffer a negative externality and there is a finite deadline. Jehiel and Moldovanu (1995a) extend the model to allow for positive externalities and an infinite horizon. They show that without deadlines, delay can occur with negative externalities (when it is welfare-improving for agents), but not with positive externalities (when it would be inefficient).

have no delay.[9] On the other hand, several papers have shown that inefficiencies arise when the allocation of power changes over time because of voters behavior (Battaglini and Coate (2008)), when present actions have effects on future available actions (Acemoglu, Egorov, and Sonin (2008)), when players disagree on what to do given the evolving environment (Dziuda and Loeper (2016)), when an agent can act as an intermediary in legislative bargaining (Iaryczower and Oliveros (2016)), or because coalition principals face a trade-off between efficiency and surplus extraction (Battaglini (2019)). We contribute to this literature by highlighting a novel dynamic effect that changes willingness to trade of all players.

3. The Model

There is a principal and a group of n agents who interact in an infinite horizon, $t = 1, 2, \ldots$. We say the principal wins if and when she obtains the support of $q < n$ agents. In each period t before the principal wins, any one of the $k(t)$ agents who remain uncommitted at time t meets the principal with probability $1/k(t) > 0$. In each meeting, principal and agent bargain over the terms of a deal by which i would support the principal. With probability $\phi \in [0, 1]$ the principal makes an offer $p \in \mathbb{R}$ to the agent, and with probability $1 - \phi$ the agent makes an offer $b \in \mathbb{R}$ to the principal. In both cases, the offer is a transfer from the principal to the agent (which can be positive or negative). If the recipient of the offer accepts it, i commits his support for the principal and the transfer takes place; if the offer is rejected, i remains uncommitted. Upon completion, the principal gets a payoff $v \in \mathbb{R}_+$, committed agents get $z \in \mathbb{R}_+$, and uncommitted agents get $w \in \mathbb{R}$. In any period before completion, all players get a payoff of zero, not including any transfer they have received or paid. Principal and agents have a discount factor $\delta \in (0, 1)$.

The solution concept is symmetric Markov perfect equilibria (MPE). The restriction to symmetric MPE rules out discriminatory contracts, in the spirit of Genicot and Ray (2006). In particular, the strategies of principal and agents only condition on the number of agents $m \leq q$ the principal still needs to obtain for completion. We let the state space be $M \equiv \{1, \ldots, q\}$. The offers when the principal and agents propose in state m are denoted $p(m)$ and $b(m)$, respectively. We let $w(m)$ and $w_{out}(m)$ denote

[9] A stationary equilibrium with delay can only exist if the status quo is in the core, which is generally empty in multidimensional policy spaces, or when transfers are possible.

the continuation values of an uncommitted and a committed agent in state $m \in M$, and $v(m)$ denote the principal's continuation value in state $m \in M$.

Although quite simple, the model has a number of applications. To fix ideas, we sketch some of these here.

Corruption. Consider a firm or agent bribing corrupt bureaucrats, in the spirit of Olken and Barron (2009). Olken and Barron observe bribes paid by truck drivers to police, soldiers, and weigh station attendants in Indonesia. They model checkpoints as a chain of vertical monopolies, where the sequence of meetings is exogenously given, and the agreement of each checkpoint is needed for completion. In our model, instead, a firm needs to get the approval of q out of n bureaucrats, and does not have to get these approvals in a given sequence.[10] We assume that if the project is greenlighted, the firm gets an expected payoff $v > 0$, and the bureaucrat who supports the project obtains $z > 0$ (possibly due to more benefits down the line), while $w \geq 0$ or $w \leq 0$ depending on whether the project benefits or hurts the population at large.

New Technologies with Increasing Returns to Scale. Consider exclusive deals contracts for the introduction of a new product with network externalities (Katz and Shapiro (1992), Segal and Whinston (2000)). Suppose there are n buyers and an incumbent producing with an old technology, in a market that can accommodate at most one supplier due to increasing returns to scale or network externalities. Under the incumbent supplier, buyers obtain a per period payoff which we normalize to zero. A challenger P can supply the market with a new technology, but entry is profitable only if it can serve at least q buyers. In each period, the challenger negotiates with a potential buyer an exclusive deal contract, which can include some advantage in service or tailored design. If q buyers sign exclusive deals, the challenger enters and the incumbent drops out. In this case the challenger firm gets a payoff $v > 0$, buyers who didn't sign get $w > 0$ and buyers who signed agreements get $z \geq w$.

Start-Ups. A firm needs to hire q specialized workers to produce a new product. Upon starting production, the firm obtains an expected payoff of $v > 0$, while each of the workers gets profit participation leading to an expected value $z > 0$. To sign the workers to the company, the firm negotiates with each worker a sign-up bonus.

[10] McMillan and Zoido (2004) – which documents the details of corruption in Peru in the 1990s under President Alberto Fujimori – show that bribes typically involve a subset of bureaucrats and politicians in the relevant organizations. McMillan and Zoido (2004) and Olken and Barron (2009) show that equilibrium bribes are in part set through ex post bargaining.

Workers that are not hired by the firm do not benefit (or suffer) from the company's activities, so $w = 0$. Notice that in this application there are no externalities towards uncommitted agents.

In Section 5.3 we consider the case $z \leq 0$. This allows us to extend our analysis to other applications, including corporate takeovers ($z = 0$) or vote buying with audience costs ($z < 0$). We show that – differently to the case of $z > 0$, in this case there is a breakdown of negotiations.

4. Existence and Uniqueness of Equilibrium Outcomes

We begin by establishing some basic properties of equilibria. Suppose the principal has the opportunity to make an offer to agent i in state $m \in M$. Note that the agent will accept an offer $p(m)$ from the principal only if his continuation value after accepting the offer, $\delta w_{out}(m-1) + p(m)$, is at least as large as his continuation value after rejecting the offer, $\delta w(m)$, and will accept the offer with probability one if this inequality holds strictly. Thus, whenever the principal makes an offer to agent i in state m, she offers

$$p(m) = -\delta[w_{out}(m-1) - w(m)]. \tag{1}$$

Similarly, whenever the agent makes an offer to the principal he offers

$$b(m) = \delta[v(m-1) - v(m)]. \tag{2}$$

While the offers are pinned down, equilibrium strategies can differ in the probability of trade in each state; i.e., the probability that the proposer makes an offer and that this offer is accepted. Let λ_m (resp., $\hat{\lambda}_m$) denote the probability of trade in state $m \in M$ when the principal (resp, the agent) has the opportunity to propose, and let $\mu_m \equiv \phi\lambda_m + (1-\phi)\hat{\lambda}_m$ denote the ex ante probability of trade in state $m \in M$. Note that when she has the opportunity to propose, the principal is willing to make the offer (1) iff $p(m) \leq \delta[v(m-1) - v(m)]$, or (substituting) iff the bilateral surplus of moving forward is nonnegative:

$$s(m) \equiv [v(m-1) - v(m)] + [w_{out}(m-1) - w(m)] \geq 0. \tag{3}$$

As usual, if in equilibrium $s(m) > 0$, we must have trade with probability one; i.e., $\lambda_m = 1$.[11] However if $s(m) = 0$, both the principal and the agent are indifferent between trading with transfer $p(m)$ or not trading. Thus $\lambda_m \in (0,1)$ only if $s(m) = 0$. By the same logic, when the agent has the opportunity to propose, we must have trade with probability one when $s(m) > 0$, no transaction when $s(m) < 0$, and $\hat{\lambda}_m \in (0,1)$ only if $s(m) = 0$.

Since the surplus $s(m)$ depends on the continuation values of principal and agent, pinning down the equilibrium probability of trade requires that we learn more about these equilibrium payoffs. Using (1) and (2), and letting $s^+(m) = \max\{s(m), 0\}$, the values of principal and uncommitted agents can be written recursively as follows (see Appendix A.1 for details).

$$v(m) = \left(\frac{\delta}{1-\delta}\right)\phi s^+(m), \tag{4}$$

$$w(m) = \left[\frac{\delta\beta(m)}{1-\delta\beta(m)}\right](1-\phi)s^+(m) + \left[1 + \left(\frac{1-\delta}{1-\beta(m)}\right)\frac{1}{\delta\mu_m}\right]^{-1} w(m-1), \tag{5}$$

where $\beta(m)$ denotes the probability that an agent meets the principal in state $m \in M$. As equation (4) shows, the principal's equilibrium payoff in state m is proportional to the surplus $s(m)$ by a factor that increases with the principal's nominal bargaining power ϕ.[12] Because delay can only occur in equilibrium if $s(m) = 0$, this means that if there is delay in state m in equilibrium, then $v(m) = 0$. The agent's equilibrium payoff in state m, on the other hand, has two components. The first comes from the events in which the agent is negotiating with the principal, and is proportional to the surplus $s(m)$ by a factor that increases with the agents' bargaining power $1-\phi$. But differently to the principal's value, the agent's value $w(m)$ is positive even when $s(m) = 0$. This second component is increasing in the probability of trade μ_m and the lagged value $w(m-1)$, and is due to the fact that as long as the negotiation process moves forward in state m with positive probability, the agent receives some value even when he does not meet the principal in that state.

[11]First, note that the agent must accept the offer (1) with probability one, for otherwise the principal could obtain a discrete gain in payoffs by increasing her offer slightly, as any such offer would be accepted. Then it must be that the principal makes this offer with probability one.

[12]The expression eliminates the dependency on the probability of trade λ_m using the fact that if $s(m) > 0$ then $\lambda_m = 1$, if $s(m) < 0$ then $\lambda_m = 0$, and that $s(m) = 0$ when $\lambda_m \in (0,1)$.

The value of a committed agent, on the other hand, only depends on the probability that the process moves forward or not: if there is a transaction (with probability μ_m) the committed agent gets a continuation payoff $\delta w_{out}(m-1)$, and otherwise gets $\delta w_{out}(m)$. Solving recursively, we obtain

$$w_{out}(m) = \left[\prod_{k=1}^{m}\left(\frac{\delta\mu_k}{1-\delta(1-\mu_k)}\right)\right] z \tag{6}$$

We can now present the main result of this section. We show that equilibrium exists, is unique up to the probability of trade μ, and that trade never collapses. We characterize the probability of trade in each state $m \leq q$ as a function of continuation values $w_{out}(m-1), v(m-1)$ and $w(m-1)$. Letting $\Gamma(m) \equiv w(m)/(v(m)+w_{out}(m))$ for any state $m \in M$, we have:

Theorem 4.1. *There exists an essentially unique equilibrium, characterized by trade probabilities*

$$\mu_m = \min\left\{1, \left(\frac{1-\delta}{\delta}\right)\left(\frac{1}{1-\beta(m)}\right)\left(\frac{1}{\Gamma(m-1)-1}\right)\right\} > 0 \quad \forall m \in M. \tag{7}$$

The proof is by induction. Note first that with $v, z > 0$, a critical meeting ($m = 1$) must have trade with positive probability, and thus $v(1) + w_{out}(1) > 0$. In fact, for large v, a critical meeting must result in trade with probability one (lemma A.1). Now suppose that for all $k < m$ there are transactions with positive probability, and take the implied continuation values $w_{out}(m-1), v(m-1)$ and $w(m-1)$ as given. Note that since in all states $k < m$ there is trade with positive probability, the values of a committed agent and of the principal in state $m-1$ are positive. Thus inaction at m is not an equilibrium, for then $v(m) = w(m) = 0$ and $s(m) = v(m-1)+w_{out}(m-1) > 0$, giving principal and agent an incentive to trade. We then show that the "one-shot" game in state m, in which payoffs are given by the continuation payoffs, has a unique SPE. This game has delay if $\Gamma(m-1)$ is large enough.

Note that the denominator of $\Gamma(m-1)$, $v(m-1)+w_{out}(m-1)$, is the surplus in state m when in equilibrium $\mu_m = 0$ (as in this case $v(m) = w(m) = 0$), and therefore the largest $s(m)$ could possibly be given continuation values in $m-1$. Call this $s^0(m)$. Equation (7) says that there will be inefficient delay in a state m when the amount of resources available to compensate the agent for trading in state m is small relative to his payoff of remaining uncommitted, $w(m-1)$, even when he anticipates a very low probability of trade μ_m.

This mechanism is at the core of the paper. For an inefficiency to appear in state m, the agent anticipates that the gains from trade will be larger when the negotiation is further along, and that he will be able to capture a large enough fraction of this surplus in equilibrium. In this case, the principal anticipates that agents trading late in the process will extract a large fraction of the surplus, and as a result is not willing to pay much to agents trading early on.

The key problem results from the intersection of two elements. One is a manifestation, in this setting, of the traditional hold-up problem, where the initial transactions are investments for the principal, who is carrying the coalition formation process. The second is the inter-temporal competition among agents. In fact, note that with fixed trading positions for the agents, this mechanism can lead to increasing prices, as in Blanchard and Kremer (1997) and Olken and Barron (2009), but no inefficiency. Here, however, agents have a chance to change their trading position by not accepting the deal. It is this form of *competition among agents* which can lead to delay, and what we call a collective hold-up problem.

In the next section, we explore this collective hold-up problem in depth. We focus on the case in which the principal's willingness to pay is high, which induces a strong competition among agents to capture these rents, and results in a severe collective hold-up problem.

5. Main Results

In this section, we characterize equilibrium outcomes when the collective hold-up problem is severe. We address how the allocation of bargaining power between principal and agents affect the efficiency of collective decisions, how the characteristics of the collective decision affect efficiency, how delay appears in the negotiation process, and how agents' bargaining power affect their welfare. We also reconsider these questions in the limiting cases when frictions vanish, and when all bargaining power is concentrated in the agents.

Our main result, Theorem 5.4, provides a complete characterization of equilibria for large v.[13] This shows that redistributing bargaining power from the principal to the agents creates inefficient delay, and reduces agents' welfare.

Theorem 5.4 builds on three key results. First, we provide a necessary and sufficient condition for trading efficiency in any state $m \in M$ given an arbitrary probability of trade of the $m-1$ subgame. As the discussion of the previous section illustrates, a key step to be able to do this is to solve the value functions of the principal and of an uncommitted agent. The difficulty here comes from the fact that – differently to the standard assumption in the literature – in our model both principal and agents make proposals with positive probability. When the principal has all the bargaining power, one can solve for agents' values independently, use these values to express transfers as a function of primitives, and then solve for the principal's equilibrium payoffs. In our case, instead, the principal can extract rents from agents, agents can extract rents from the principal, and agents can extract rents from other agents. As a result, the system of difference equations characterizing equilibrium payoffs can not be decoupled.

To tackle this difficulty, we use a transformation to express the system of value functions as a second order difference equation, which we then solve. With this, we are able to characterize agents' equilibrium payoffs in each state $m \in M$ as a function of primitives, for any given probability of trade $\vec{\mu}^m \equiv (\mu_1, \ldots, \mu_m)$ in the m-subgame, $\tilde{w}(m|\vec{\mu}^m)$ (see Lemma A.2 in the Appendix). Armed with this result, we can then provide a necessary and sufficient condition for full trade in any state m for an arbitrary probability of trade of the $m-1$ subgame.

Lemma 5.1. *Consider any $m \in M$. For any trading probabilities $\vec{\mu}^m$ in the m subgame, $s(m) \geq (\leq) 0$ if and only if*

$$T(m|\vec{\mu}^m) \equiv \frac{\tilde{w}(m|\vec{\mu}^m)}{\beta(m)} - \left(\prod_{j=1}^{m} \frac{\delta \mu_j}{1 - \delta(1 - \mu_j)} \right) (v + mz + (n-q)w) \leq (\geq) 0$$

As a direct application of lemma 5.1, we obtain a necessary and sufficient condition for inefficiency of equilibrium outcomes. To do this, we first use Lemma A.2 (with $\mu_j = 1$ for all $j \in M$) to obtain an expression for payoffs with efficient trading $w^\dagger(m), v^\dagger(m)$ in terms of primitives of the model. We then use Lemma 5.1, with $T^\dagger(m) \equiv T(m||\vec{\mu}^m =$

[13]Here and in the rest of the paper, we write the statement "for v large, [A] is true" to mean that for fixed parameters other than v, there exists a $\overline{v} > 0$ such that if $v \geq \overline{v}$, [A] is true.

1), to provide a condition for existence of an efficient equilibrium. We refer to this as a full trading equilibrium (FTE), and to $w^\dagger(m), v^\dagger(m)$ as FTE payoffs.

Proposition 5.2 (Efficiency). *There exists a FTE in the subgame starting in state m' iff $T^\dagger(m) \leq 0 \quad \forall m \leq m'$. Moreover, for large v the following is true: for any state $m > 1$, there exist $\overline{\phi}(m) < 1$ and $\underline{\phi}(m) > 0$ such that:*

(1) *if $\phi > \overline{\phi}(m)$, the unique MPE of the m-subgame is a FTE, and*
(2) *if $\phi < \underline{\phi}(m)$, the unique MPE of the m-subgame entails delay.*[14]

Proposition 5.2 shows that the allocation of bargaining power has efficiency consequences. As long as the principal has enough bargaining power, the unique equilibrium for any $z \in \mathbb{R}_+$ and $w \in \mathbb{R}$ is efficient. But when bargaining power is decentralized to agents, on the other hand, the equilibrium of any m-subgame, $m > 1$, involves inefficient delay.[15]

While this is an important result, the proposition is silent about many of the questions we seek to answer. In particular, we still know very little about the properties of delay for a fixed allocation of bargaining power. In principle, delay could be front-loaded (occur at the beginning of the bargaining process), back-loaded, or occur in some interior subset of states. Or the set of states with delay could potentially be unconnected, with regions of delay followed by states in which trade is efficient.[16] Moreover, even if delay occurs in connected sets of states, the probability of trade could be non-monotonic, having stages in which the negotiation process accelerates after every trade followed by periods in which it slows down with subsequent commitments.

Our third building block sharply restricts the kinds of outcomes we can observe in equilibrium for large v. First, we show that delay is in fact front-loaded: if in equilibrium there is delay in a state $m' < q$, then there is delay in all states $m > m'$. Second, we show that while delay is front-loaded, the pace of negotiations must *slow down* as

[14]Note that this does not say that if there is delay in a state $m > 1$, there must be delay in all states $m' < m$, but only that for any subgame $m \in M$, some transaction involves delay.

[15]Recall that for large v, in a critical state $m = 1$ there is trade with probability one (see the discussion following Theorem 4.1, or lemma A.1 in the Appendix).

[16]For instance, in the context of negotiations between the seller of a good and several potential buyers, Jehiel and Moldovanu (1995a) show that when the seller is sufficiently patient and externalities between buyers are negative, SPNE in pure strategies with bounded recall have the property that long periods of waiting alternate with short periods of activity (When externalities are positive there is no delay in equilibrium within this class).

the process of negotiations move forward. In fact, the probability of trade decreases at a rate that is independent of the discount factor, the principal's bargaining power or her willingness to pay. We state the general version of this result in the proposition.

Proposition 5.3 (Inefficiency for Fixed ϕ)**.**

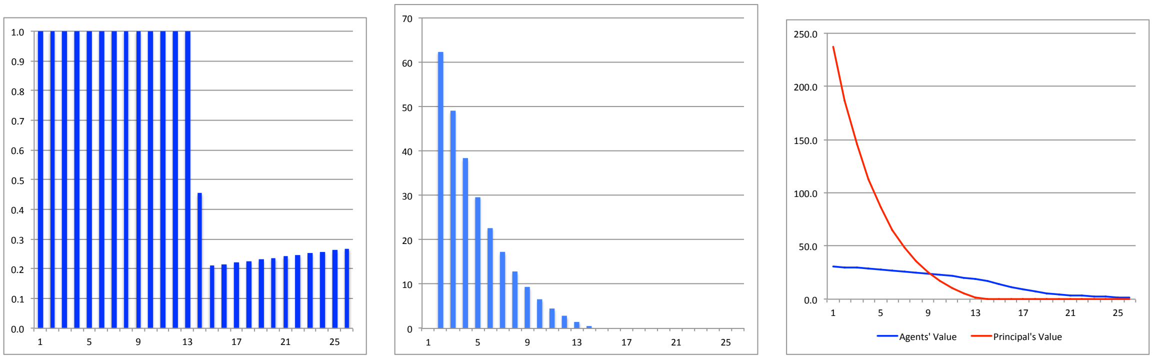
(1) *Suppose in equilibrium* $\mu_m \in (0, 1)$ *for all* $m \in \{\underline{m}, \ldots, \overline{m}\}$. *Then*
$$\frac{\mu_{m+1} - \mu_m}{\mu_m} = \beta(m) \quad \forall m \in \{\underline{m} + 1, \ldots, \overline{m} - 1\},$$
and thus $\mu_{m+1} > \mu_m$ *for all* $m \in \{\underline{m} + 1, \ldots, \overline{m} - 1\}$.

(2) *For large* v*, if there is inefficient delay in a state* $m' < q$*, there must also be inefficient delay in all states* $m > m'$.

In other words, whenever there is delay in negotiations, this occurs from the beginning of the bargaining process, and negotiations get harder as the negotiations move forward, possibly until some point in which the process unravels and all further transactions occur without delay.

The fact that for large v delay is front-loaded might be surprising at first. However, note that the reason there is delay in a state m is that – given what she anticipates paying in the future – the principal is not willing to pay the agent trading in that position enough to prevent him from passing on the deal in order to put himself in the position of being one of the agents trading later. But then backloaded delay cannot be consistent with equilibrium, for the farsighted principal would only be willing to pay agents trading early if she could appropriate enough of the surplus to pay agents trading later.

Proposition 5.3 takes as given the existence of delay for a fixed allocation of bargaining power between principal and agents, and characterizes how the inefficiency would manifest itself in the negotiation process. We already know that this is not a vacuous result, because proposition 5.2 says that for any $m > 1$, in equilibrium there is delay in a state $m' \leq m$ if agents have enough bargaining power. Moreover, since this is true for state $m = 2$, the second part of proposition 5.3 implies that when agents have enough bargaining power, in equilibrium there is delay in all but the critical state. Since we already know that the equilibrium is efficient when the principal has enough bargaining power, this gives us a full characterization of equilibrium outcomes for extreme allocations of bargaining power. However, our previous results leave open



trade probability | surplus | values

FIGURE 1. Trade probability, equilibrium payoffs, surplus in an example ($v = 300$, $z = w = 30$, $\delta = 0.95$, $n = 51$, $q = 26$, $\phi = 0.2$.)

the possibility that the range of states in which negotiations are inefficient varies non-monotonically with the principal's bargaining power for intermediate values of ϕ. Theorem 5.4 shows that this is not the case.

The proof of this result builds on the fact that the agents' FTE payoff $w^\dagger(m)$ is decreasing in the principal's bargaining power ϕ whenever the equilibrium of the m-subgame is a FTE. This result is intuitive, because the direct effect of reducing ϕ is to increase the ability of agents to extract from the principal (all else equal, this increases the value of agents, even those trading in earlier states). The general argument is slightly more involved, because reducing ϕ has the indirect effect of lowering the principal's willingness to pay in earlier states, thus reducing the value of agents transacting early. We show, however, that *in a full trading equilibrium*, the direct effect dominates.

To see this, recall that the value of the principal is proportional to the surplus, by a factor that is increasing in ϕ: $v(m) = \left(\frac{\delta}{1-\delta}\right)\phi s^+(m)$ (eq. (4)). Since in a FTE the surplus $s(m)$ does not change with ϕ, it follows that $v^\dagger(m)$ is increasing in ϕ. Now, total welfare in state m is

$$J(m) \equiv v(m) + (n - q + m)w(m) + (q - m)w_{out}(m)$$

Since in a FTE both $J(m)$ and $w_{out}(m)$ are constant in ϕ, it follows that

$$v^\dagger(m;\phi) - v^\dagger(m;\phi') = -(n - q + m)[w^\dagger(m;\phi) - w^\dagger(m;\phi')],$$

and thus $w^\dagger(m)$ is decreasing in ϕ.

We can now establish the main result of the paper:

Theorem 5.4 (Characterization for large v)**.** *For any $\phi \in [0,1]$ there exists a unique cutpoint $\overline{m}(\phi) \in M$ such that, in equilibrium, there is delay in each state $m \in M$ s.t. $m > \overline{m}(\phi)$, and full trading in any $m \leq \overline{m}(\phi)$. The cutpoint $\overline{m}(\cdot)$ is weakly increasing in ϕ and has range M. Moreover, for any $m > \overline{m}(\phi) + 1$,*

$$\mu_m = \frac{1-\delta}{\delta\beta(m)} \left(\frac{\beta(\overline{m})\delta^{\overline{m}} z}{w^\dagger(\overline{m}) - \delta^{\overline{m}} z + \beta(\overline{m}) v^\dagger(\overline{m})} \right) \tag{8}$$

is decreasing in v and goes to zero as $v \to +\infty$.

Theorem 5.4 unifies our previous results and provides a complete characterization of equilibria when the collective hold-up problem is severe.[17] We are now in a position to answer the questions we posed in the introduction.

How does the allocation of bargaining power between principal and agents affect the efficiency of collective decisions? The theorem shows that redistributing bargaining power from the principal to the agents creates delay and reduces agents' welfare. In particular, the number of transactions with positive expected delay is decreasing in ϕ, so that giving more power to the agents increases the number of bargaining positions in which transactions fail with positive probability.

How do the characteristics of the collective decision affect this inefficiency? Theorem 5.4 confirms the intuitions regarding the effect of changes in the agents' preferences: a higher value for belonging to the coalition (z large) reduces delay, as it increases the incentive to trade, while a large positive externality on uncommitted agents (large w) has the opposite effect. However, the theorem also shows the possibly surprising effect of increasing the principal's willingness to pay. As we have shown, for any given ϕ for which there is delay in more than one state, expected delay grows continuously with v and in the limit with $v \to \infty$, the expected time for completion goes to infinity.

How does this delay appear in the negotiation process? For a given allocation of bargaining power ϕ inducing delay, the expected delay for each transaction increases

[17] While Theorem 5.4 focuses on the case in which the collective hold-up problem is severe, most of our results apply generically, for all values of v. In particular, there is still trade with positive probability in all states, the equilibrium exists and is still essentially unique, the characterization of values is unchanged, as is the condition for no delay, and the growth of the probability of trade in contiguous states. The result that holds for large v but *does not* hold in general is the second part of Proposition 5.3. In fact, we have constructed examples in which, for low v, delay is backloaded, or occurs in an intermediate set of states.

as we move further along the process in the first $q - \overline{m} - 1$ transactions, possibly decreasing in the last transaction with delay. But once the principal obtains the support of $q - \overline{m}$ agents, the remaining transactions occur without delay. In the special case in which the agents have full or almost full bargaining power, delay occurs in all but the critical state, and the expected delay is monotonically increasing until the critical state as we move further along the process.

How does agents' bargaining power affect their welfare? As we showed above, in the efficient equilibrium, agents' welfare increases with their bargaining power. As a result, keeping the strategy profile fixed, agents would prefer to retain as much power as possible. However, as we have seen, decentralizing power to agents also increases the range of states in which negotiations suffer delay. Moreover, in each of these states, delay is increasing in the principal's willingness to pay, v. This poses a tradeoff for agents' welfare: a larger v increases the total surplus from transacting, but also leads to larger delay.

Using our previous results, it is easy to show that the larger delay more than compensates for the increase in total surplus, and leads to a loss of welfare for the agents. This leads to the counterintuitive result that, for large v, agents' welfare is maximized when they relinquish significant bargaining power to the principal. Note that from (5) we can express the equilibrium payoff of an uncommitted agent as

$$w(q) = \left[\prod_{k=\overline{m}+1}^{q} \left(1 + \left(\frac{1-\delta}{1-\beta(k)} \right) \frac{1}{\delta \mu_k} \right)^{-1} \right] w^{\dagger}(\overline{m}) \leq w^{\dagger}(\overline{m})$$

It follows directly from the expression above that if there are at least two states with delay, the probability of trade vanishes as $v \to \infty$, and thus $w(q) \to 0$. Thus, we have:

Corollary 5.5. *For large enough v, any ϕ such that $\overline{m}(\phi) < q - 1$ leads to lower equilibrium payoffs for the agents than giving complete bargaining power to the principal, $\phi = 1$. In particular, agents are better off if the principal has full bargaining power than if agents have full bargaining power.*

In turn, since $w^{\dagger}(q)$ is decreasing in ϕ when a FTE exists, agents prefer the smallest ϕ such that a FTE exists to $\phi = 1$. It follows that for large enough v, agents prefer ϕ such that either $\overline{m}(\phi) = q - 1$ or $\overline{m}(\phi) = q$, granting considerable bargaining power to the principal.

Overall, the basic intuition of how bargaining power affects outcomes works well in a FTE, but when the collective hold up problem is severe, the intuition breaks down. Instead, concentrating bargaining power in the principal reduces inefficiencies and improves agents' welfare.

In the next sections, we consider two important special cases of the model. In section 5.1, we study equilibria as bargaining frictions vanish ($\delta \to 1$), and in section 5.2, we consider the special case in which bargaining power is fully decentralized to agents ($\phi = 0$). Both cases provide important lessons. We then consider two extensions of the model. In section 5.3, we allow the payoff of agents who commit to be non-positive, $z \leq 0$, as it would be the case in corporate takeovers ($z = 0$) and vote buying with audience costs ($z < 0$). In section 5.4, we consider a version of the model in which transfers between principal and agent are contingent on the completion of the project.

5.1. **Vanishing Frictions.** In Theorem 5.4, we characterized equilibrium outcomes for fixed $\delta < 1$, and sufficiently large v. A natural question is how do equilibrium outcomes change for fixed v as frictions vanish. In fact, the results in the literature on delay in bargaining with complete information have generally been established for large δ. This is the case for delay caused by deadlines in Fershtman and Seidmann (1993), for the monopolist selling a good to heterogeneous buyers in Jehiel and Moldovanu (1995a), for delay through discriminatory contracts in Cai (2000), and in the example provided by Gomes (2005) in a general model of coalitional bargaining.

From the expression for the trading probability μ_m in the theorem, one might be tempted to conclude that for fixed v, the probability of trade goes to zero as $\delta \to 1$, so that when bargaining frictions vanish negotiations slow down almost to a halt. This would be incorrect, for the threshold $\overline{m}(\phi)$ is itself a function of δ. Making the dependence of $\overline{m}(\phi)$ on δ explicit, we have that as long as $z \geq w$, $\overline{m}_\delta(\phi) \to q$ as $\delta \to 1$. Thus for any given $\phi > 0$ and $v > 0$ there is a $\overline{\delta} > 0$ such that if $\delta \geq \overline{\delta}$, the unique equilibrium is a FTE.

To see this more directly, note that from (27), for any $m \in M$,

$$\lim_{\delta \to 1} w^\dagger(m) = w \quad \text{and} \quad \lim_{\delta \to 1} v^\dagger(m) = v + m(z - w).$$

Thus, from proposition 5.2, the condition for existence of a FTE boils down to

$$v \geq -m(z-w) \quad \forall m \in M.$$

and is therefore satisfied for any $v > 0$ whenever $z \geq w$.

To understand the result, consider the critical state $m = 1$. Note that $p(1) = -[z - w(1)] = -(z-w)$, so that when the principal can make an offer, she keeps v and can extract the differential $z - w > 0$. But even when the agent proposes, the agent gets $b(1) = v - v(1) = -(z-w) = p(1)$. Thus, the critical agent cannot extract δv from the principal, and there are no incentives to hold out, and no collective hold-up problem. The result is due to simple economics. When both principal and agents do not discount the future, both principal and agents are willing to wait to get a better deal, but the principal is a monopolist, while the agent faces competition from other agents. This means that the critical agent cannot extract any surplus from the principal. Because agents are willing to wait, all agents are guaranteed w. But the principal, being the short side of the market, gets the differential $z - w$ entirely. And once this happens in $m = 1$, then by the same logic $b(m) = p(m) = -(z-w)$ for all $m \in M$, and thus $w^\dagger(m) = w$ and $v^\dagger(m) = v + m(z-w)$.

Note that the necessary and sufficient condition $v \geq -m(z-w)$ for all $m \in M$ for existence of a FTE is independent of ϕ, provided $\phi > 0$. This is because for any $\phi > 0$, the probability that the principal gets to propose within T periods is arbitrarily close to one for sufficiently large T. Thus, these cases are strategically similar for the very patient principal.

5.2. **Full Decentralization.** In this section, we consider the limiting case in which bargaining power is fully decentralized to agents. There are two reasons to do this. First, for fixed $\delta < 1$, the $\phi = 0$ case allows us to present the key arguments in a stark and simple manner. We are also able to provide a full analysis of equilibrium outcomes for all v. Second, we want to shed light on the model for $\phi = 0$ in the limit case as $\delta \to 1$, which as we discussed in the previous section, is a special case. We show that in this case, the model yields massive delay, with negotiations almost breaking down, and the expected time to completion going to infinity.

The case in which agents have all bargaining power offers two useful simplifications. First, because the principal is never able to make a proposal, the agent negotiating in the critical state extracts all the principal's surplus from completion of the project.

Since the continuation value of the principal in state m is $\delta v(m)$ independently of whether the agent makes an offer or not. It follows that $v(m) = \delta v(m)$, which implies that $v(m) = 0$ for all $m \geq 1$. As a result, the principal is not willing to pay in previous states to move the process forward; i.e., $b(1) = \delta v$, and $b(m) = 0$ for $m \geq 2$. In the absence of side payments, the probability of trade depends on the relative value for an agent of moving the process along supporting the principal for free, $w_{out}(m-1)$, versus holding out support with the goal of extracting the rent δv in late trading, $w(m)$.

The second simplification is that because the principal is not able to capture rents from the agents, the value function of an uncommitted agent becomes a stand alone difference equation, which is considerably easier to solve. Note that the payoff of an *uncommitted* agent in state $m > 1$ in this case is simply

$$w(m) = \hat{\lambda}_m \left[\beta(m)\delta w_{out}(m-1) + (1-\beta(m))\,\delta w(m-1)\right] + (1-\hat{\lambda}_m)\delta w(m).$$

Thus, solving recursively,

$$w(m) = \left[\prod_{j=1}^{m} \frac{\delta\hat{\lambda}_j}{1-\delta(1-\hat{\lambda}_j)}\right](w + \beta(m)v)\,. \tag{9}$$

With (9) – and since the value of committed agents is still given by (6) – the condition for trade with positive probability at $m > 1$ that $w_{out}(m-1) \geq w(m)$ boils down to

$$w \geq \left[\frac{\delta\hat{\lambda}_m}{1-\delta(1-\hat{\lambda}_m)}\right](w + \beta(m)v) \tag{10}$$

For delay to occur with positive probability at m, we need (10) to hold with equality. Now, note that the right hand side is a continuous increasing function $f(\cdot; m)$ of $\hat{\lambda}_m$ such that $f(0; m) = 0$ and $f(1; m) = \delta\,(w + \beta(m)v)$. Since (10) is satisfied with $\hat{\lambda}_m = 0$, this implies that in equilibrium there is always trade with positive probability in all states $m > 1$. On the other hand, there exists a (unique) solution $\hat{\lambda}_m \in (0, 1)$ satisfying (10) with equality if and only if

$$w < \delta\,(w + \beta(m)v) \quad\iff\quad m < \frac{\delta}{(1-\delta)}\frac{v}{w} - (n-q) \equiv \hat{m} \tag{11}$$

It follows immediately from this that there exists a unique cutpoint $\hat{m} > 2$ such that, in equilibrium, there is delay in each state $m \in M : 2 \leq m < \hat{m}$, and trade with probability one in any $m \geq \hat{m}$.[18]

From eq. (11), the set of states in which there is delay is weakly increasing in v/w, which captures the relative value of holding out, and for any $m \in M$ there is a v/w large enough such that $m < \overline{m}$. The ratio v/w also increases the probability of delay in states below the cutpoint. In fact, note that when there is delay in state m, the probability of trade is given by $\hat{\lambda}_m \in (0,1)$ solving $w_{out}(m-1) = w(m)$, or

$$\hat{\lambda}_m = \left(\frac{1-\delta}{\delta}\right)\frac{w}{v}\frac{1}{\beta(m)} \tag{12}$$

Note that from (12), the probability of trade is increasing in m. Therefore, we expect transactions to occur at a faster pace initially, with the process of negotiations slowing down as it goes along, as in proposition 5.3.

Now, note that as $\delta \to 1$, the threshold $\overline{m}$ in (11) goes to $+\infty$, so that in equilibrium there is delay in all non-critical states. Moreover, from (12), $\lim_{\delta\to 1}\hat{\lambda}_m = 0$, so that in each state $m > 2$, negotiations slow down almost to a halt. This contrasts with our discussion in Section 5.2, were we showed that for any $\phi > 0$, the equilibrium for large enough δ is a FTE. The difference in outcomes reflects the fact that for any $\phi > 0$, the probability that the principal gets to propose within T periods is arbitrarily close to one for sufficiently large T, but is still zero for any T when $\phi = 0$.

To conclude this section, note that in this case it is easy to compute agents' equilibrium payoffs (in terms of primitives). For v large enough so that in equilibrium there is delay in all but the critical state (i.e., $\overline{m} = q+1$), this is

$$w(q) = \left(\prod_{j=2}^{q-1}\frac{w}{w+\beta(j)v}\right)\delta w, \quad \text{and} \quad \lim_{v\to\infty} w(q) = 0.$$

[18]At first sight, this result seems to run against our earlier results for large v, where we showed that whenever there is delay, this must happen at the beginning of the negotiation process. However, it should be clear that strictly back-loaded delay can only occur for v sufficiently small. For large v, $\hat{m} \to \infty$, and the unique equilibrium of the $\phi = 0$ case entails delay in all non-critical states, consistent with the result for small ϕ in proposition 5.3 (see footnote 17 for additional details).

When the principal has all the bargaining power, instead, agents' equilibrium payoff is (use (27) with $\phi = 1$)

$$w^{\dagger}(q) = \left(\prod_{j=1}^{q} \frac{1}{1 - \delta\beta(j))} \right) \left(\frac{n-q}{n} \right) \delta^q w > 0$$

It follows that for v sufficiently large, agents are better off when the principal has full bargaining power than when agents have full bargaining power.[19]

5.3. **Breakdown of Negotiations.** Up to this point, we maintained the assumption that in the event the principal obtains the support of q agents, an agent who committed his support to the principal obtains a positive payoff $z > 0$. In some applications, however, it is reasonable to assume that $z = 0$ (e.g., corporate takeovers) or even $z < 0$ (e.g., vote buying with audience costs). Here we consider the case $z \leq 0$.

Consider for example a dynamic version of corporate takeovers model of Grossman and Hart (1980). Grossman and Hart analyze a problem in which a company (the raider) acquires shares of a target company to control its board of directors. It is assumed that the raider can improve the value of the company. To capture this feature, we assume that under the raider's control, the value of a share is $w > 0$, and we normalize the value of a share under the incumbent management to zero. We distinguish the payoff that a shareholder obtains when the raider wins if the shareholder does not sell to the raider ($w > 0$) from the payoff he obtains if he does sell to the raider ($z = 0$).[20]

We show that whenever there are positive externalities on uncommitted agents ($w > 0$), the condition $z > 0$ is necessary for robust delay. In particular, we show that when contracting with the principal leads to a negative payoff for the agent when the principal wins, in equilibrium there can only be delay in the initial state $m = q$, a result which holds for a "small" (but not measure zero) set of parameter values. With

[19] As the equation shows, this particular conclusion does *not* hold under unanimity, which is the classic railroad-farmers example considered by Coase (see Cai (2000), Olken and Barron (2009), Chowdhury and Sengupta (2012)). This is because with $q = n$, $\beta(1) = 1$, so in the critical state the agent cannot free ride on others. Thus $w(1) = \delta w(1)$, which implies $w(1) = 0$. But then, recursively, $w(m) = 0$ for all $m \in M$. Thus, while the agents' equilibrium payoff when $\phi = 0$ approaches 0 as $v \to \infty$, agents are still better off by retaining bargaining power.

[20] As in Grossman and Hart (1980) and Segal (2003), we assume that shareholders are homogeneous. Unlike Grossman and Hart, we suppose that shareholders are fully aware of the effect of their action on the outcome of the raid attempt.

this exception, equilibrium is either a FTE or is such that there are no transactions in the initial state and thus $w(q) = 0$.

The result follows from Lemma 5.6 below. In it we establish two results. First we show that if $z \leq 0 < w$, there cannot be cycles of trade with probability one and trade failure with positive probability; in fact, if in equilibrium there is trade with probability one in a state m', then this also has to be the case in all states $m < m'$. This means that if there is delay, delay is front-loaded. The second part of the proposition establishes that there cannot be delay in two contiguous states m and $m+1$. Together, the two results imply that with the exception of possibly mixing in the initial state, the equilibrium is either a FTE, involves no transactions in any state, or has a FTE in a m'-subgame off the equilibrium path for some $m' < q$, with no trade for $m > m'$, which implies that the process of transactions never starts.

Lemma 5.6. *Suppose $z \leq 0 < w$. Then (i) $s(m-1) \leq 0 \Rightarrow s(m) \leq 0$. Moreover, (ii) if $s(m') \leq 0$ for some $m' < q$, then $\mu_m = 0$ for all $m > m'$ and $w(q) = v(q) = 0$.*

Why no delay in contiguous states? Suppose there is delay in m' in equilibrium. Since $s(m'+1) \leq 0$, either trade collapses in $m'+1$ or again there is delay. If there is delay in both m' and $m'+1$, $v(m') = v(m'+1) = 0$, so $s(m'+1) = 0$ if and only if $w(m'+1) = w_{out}(m')$. But $w(m'+1) \geq 0$, as it is the value obtained from being uncommitted and depends on $w > 0$, while $w_{out}(m') = \left[\prod_{k=1}^{m'} \left(\frac{\delta\mu_k}{1-\delta(1-\mu_k)}\right)\right] z < 0$, so this is impossible. With no possible payments from the principal, all incentives to trade have to come from diminishing the value of holding out through delay. But delay can only lower the (positive) value of not trading, and thus by itself is insufficient to induce agents to trade when $z \leq 0$.

The next proposition builds on Lemma 5.6 to provide a characterization of equilibria with $z \leq 0$. To do this, we first show that if there exists a FTE, this is the unique MPE. We then provide a necessary and sufficient condition for existence of a FTE. This condition follows as a corollary of previous results. An examination of the proof of Lemma 5.2 shows that these results do not require the assumption that $z > 0$, and thus also hold for $z \leq 0$. Thus, agents' FTE payoffs are still given by $w^{\dagger}(\cdot)$ as defined by (27), and there exists a FTE in the m'-subgame if and only if $T^{\dagger}(m) \leq 0$ for all $m \leq m'$. Moreover, we know from Lemma 5.6 that when $z \leq 0$, $s(m) > 0 \Rightarrow s(m-1) > 0$. As a result, a necessary and sufficient condition for existence of a FTE when $z \leq 0$ is that $T^{\dagger}(q) \leq 0$.

Proposition 5.7. *Suppose $z \leq 0 < w$. The (unique) equilibrium, (i) is a FTE iff $T^{\dagger}(q) \leq 0$, and (ii) has breakdown of negotiations iff $T^{\dagger}(q-1) > 0$. Otherwise, in equilibrium there is delay in the initial state q, and trade with probability one for all $m < q$.*

Note that adopting the project without delay is efficient for members of the coalition if $v+qz \geq 0$, and is efficient for the group as a whole if $v+qz+(n-q)w \geq 0$. Thus, for large enough v, it is efficient to adopt the project even if $w, z < 0$. In fact, we know from part (ii) of Lemma 5.2 that for large v, if the principal has enough bargaining power the unique MPE of the m-subgame is a FTE. So here the coalition should form, and it does form in equilibrium when the principal has enough bargaining power. On the other hand, part (iii) of the same lemma shows that when the agents have enough bargaining power there is no full trading equilibrium for large v, even when this would be efficient.

The main point of the GH paper is that externalities across shareholders can prevent takeovers that add value to the company. The idea is that since shareholders that do not sell can capture the increase in value brought by the *raider*, no shareholder will tender his shares at a price that would allow the raider to profit from the takeover. GH work with a static model, and assume that shareholders ignore the impact of their actions on the outcome of the bid. In our version of the GH model – where the principal buys shares one at a time and shareholders are fully forward looking and strategic – efficient takeovers are not prevented by externalities when $\delta < 1$ *as long as the raider has enough nominal bargaining power.*[21] But when agents do have enough bargaining power, efficient takeovers can fail to occur due to the collective hold-up problem: with $z \leq 0$ the collective hold-up problem still exists, but leads not to delay but to breakdown of negotiations.

5.4. **Robustness I: Contingent Offers.** In the model, we assumed that the transfers between principal and agent are a quid pro quo contingent on the behavior of the agent transacting with the principal, but not contingent on the completion of the project. This assumption is by far the most prevalent in the literature, and fits many

[21] Holmstrom and Nalebuff (1992) show that when shareholdings are divisible the free-riding problem does not prevent the takeover process in the GH model. In our model with $\phi = 1$, the raider's profit goes to zero as $\delta \to 1$. Thus, with fixed costs, efficient raids would be prevented in the limit. This result is similar to that of Harrington and Prokop (1993), who consider a dynamic version of GH in which the raider can re-approach the shareholders who have not sold (taking all offers at the posted price in each period).

applications well. In some other cases, however, the transfers between principal and agents only occur if and when the principal attains the prize.

An interesting example where this occurs is corporate restructuring in bankruptcy proceedings. In these cases, the firm or government in distress often negotiates new terms with creditors bilaterally and sequentially, as in our model.[22] But the debt shaving that each creditor agrees to is only realized upon completion of the entire restructuring package.

A natural question is whether our results hold in this modified setting. In fact, the standard hold-up logic would suggest that they would not: since agents contracting early can commit the principal's rents and leave nothing up for other agents to grab later on in the bargaining process, incentives to hold out disappear. We show, however, that while contingent contracts allow other equilibria, collective hold up can still occur. When agents have all the bargaining power, the unique equilibrium outcomes in the benchmark model are still an equilibrium when transfers are contingent on completion of the project. We state this result formally for ease of reference (the proof is included in Appendix B.1).

Remark 5.8. *Consider a variant of the model in which transfers are contingent on completion of the project, and suppose $\phi = 0$ and $z = w > 0$. For v large, there is an equilibrium with trading at $m = 1$ and delay in all $m : 2 \leq m \leq q$, given by trading probabilities (12).*

A fundamental difference in the contingent transfer model is that by affecting the amount of standing promises, agents can affect the equilibrium play of agents contracting later.[23] As a result, it is not generally optimal for an agent to propose a transfer that extracts all the principal's willingness to pay as it is the case in the main model. Instead, the optimal transfer in a non-critical state is the one that maximizes the continuation value of the committed agent subject to the constraints that

[22]Consider for instance the city of Detroit's bankruptcy restructuring. On December 2014, Detroit exited bankruptcy protection, 18 months after the city filed for Chapter 9 bankruptcy. The city negotiated a settlement with Bank of America and UBS in December 13', with several bond insurers in January of 14', pension plans in May 14', reached a deal with three Michigan counties over regional water and sewer services in September, and bond insurers Syncora and FGIC in September and October 14'.

[23]This implies, in particular, that the payoff-relevant state has to be extended to include both the number of agents required for completion and the amount of standing promises.

the principal accepts the offer and the agent is better off than remaining uncommitted. However, *in a critical state*, the amount of the transfer does not affect future behavior, and as a result incentives for the principal and agent are the same as in the cash transfer model (corrected by using the principal's value net of outstanding promises). Because of this, the agent contracting in a critical state will extract any surplus the principal arrives to that state with in equilibrium, and as a result, the principal will not be willing to reward agents contracting in earlier states. It follows that the decision of whether to transact or not with the principal is determined by the same tradeoffs than in the cash transfers model, so if an agent anticipates the same delay as in the equilibrium of the main model, he will likewise be indifferent between trading and not trading with the principal.

While the equilibrium outcomes in the benchmark model are still an equilibrium when transfers are contingent on completion of the project, the contingent payment model allows other equilibria. The reason is that agents can use the principal as a vessel to extract payments from agents contracting later. This requires the principal to transitory carry a positive or negative balance even when she will not ultimately benefit from monetary flows. Analyzing all equilibria of the contingent transfer model is both interesting and important to understand sequential contracting in applications such as bankruptcy restructuring, but it is also fundamentally different than the analysis in this paper. We leave this for future research.

5.5. **Robustness II: History Independence.** We have shown that the equilibrium of Theorem 5.4 is the unique symmetric markov perfect equilibrium. As we argued above, in our main analysis we focus on symmetric MPE because we know from Cai (2000) that discriminatory contracting can sustain inefficient equilibria if players are sufficiently patient. The question still remains of which outcomes survive if we relax the markovian assumption and focus on symmetric subgame perfect equilibria. In particular, allowing for symmetric but non-anonimous treatment of the agents allows the principal to make her strategy a function of agents' past actions, as in Genicot and Ray (2006). In our case, this may reduce the incentives to hold-out and thus affect our conclusions about delay.[24]

[24] Cai (2000) rules this out by imposing the refinement that offers cannot depend on previously rejected offers. However, discriminatory contracts can still be constructed using the predetermined order of meetings.

Analogous to our definition of a critical state in the paper ($m = 1$), we define a *critical history* as a history of play following which the principal only needs the support of one additional player in order to win. Suppose for instance that the principal's strategy calls for the principal not to trade with agent i in any critical history for which the principal met player i in the past, and the meeting resulted in no agreement. If this profile were sustainable in equilibrium, delay may disappear. This is because the principal's strategy reduces the value of remaining uncommitted. As a result, all agents may now may be willing to trade early, reducing delay and eliminating collective hold-up.

While a characterization of the set of all subgame perfect Nash equilibria is beyond the scope of the paper, we follow Jehiel and Moldovanu (1995a,b) and consider SPE with bounded recall; i.e., we allow for history dependent strategies but restrict the dependance to a finite number of rounds $k > 0$. This allows us to study the impact of history and the possibility of history-dependent punishments in a manegeable way. In the next proposition we show that, under bounded recall, the equilibrium of Theorem 5.4 is the unique symmetric SPE with trade.

Proposition 5.9. *If strategies have bounded recall, the MPE of Theorem 5.4 is the unique symmetric subgame perfect equilibrium with trade.*

The key idea for the proof is the following. Consider an equilibrium strategy profile and an arbitrary period t in a non-terminal history h_t. By sequential rationality, when player i makes an offer to player j, the offer will leave player j indifferent between accepting the current offer or rejecting it and moving to the next period. The values of all players in $t + 1$ following h_t are a function of the continuation strategies from $t + 1$ on. Since these continuation strategies can only depend on the preceding k periods, they are a function of the actions that occurred between $t - k + 1$ and $t + 1$. Since the values at $t + 1$ determine the offers and acceptance rules at t, the strategies at t do not depend on the $t - k$ period actions. Recursively, it follows that strategies are history independent.

Proposition 5.9 shows that bounded recall is *sufficient* for our MPE to be the unique symmetric SPE. Establishing whether this assumption is necessary for this result is beyond the scope of the paper. We do believe that some restriction regarding the ability of histories to determine strategies is needed to make significant progress, as characterizing all SPE in these setups is generally a daunting task (see the discussion in Jehiel and Moldovanu (1995a)).

6. Conclusion

In this paper, we consider a dynamic process of coalition formation in which a principal bargains sequentially with a group of agents. We provide a complete characterization of equilibrium outcomes when the principal's willingness to pay is high, and uncover new tradeoffs absent in bilateral bargaining models. We show that redistributing bargaining power from the principal to the agents generates delay and reduces agents' welfare, even in the absence of informational asymmetries or discriminatory offers, and even with negative externalities on uncommitted agents. Concentrating bargaining power on the principal, instead, leads to efficient collective decision-making and, for any non-unanimous decision rule, does not lead to complete rent extraction by the principal.

Our results have implications for a number of diverse applications in economics and politics, including lobbying, exclusive deals, start-ups, endorsements and corruption. While the model abstracts away from some of the details pertinent to each application, the results shed light on a common idea behind these apparently diverse problems: bargaining institutions that decentralize power to agents can be detrimental to agents' welfare by making the coalition formation process inefficient.

The source of the inefficiency has two parts. The first is a form of the traditional hold-up problem: when agents have significant bargaining power relative to the principal, the principal anticipates that agents trading late in the process will extract a large fraction of the surplus, and as a result is not willing to pay much to agents trading early on. This is similar to Blanchard and Kremer (1997) and Olken and Barron (2009), where sequential bargaining under unanimity leads to increasing prices. But when agents are not excluded from the negotiation process after rejecting a proposal, *inter-temporal competition among agents leads to delay* whenever agents have too much bargaining power. This is what we call a collective hold-up problem.

The collective hold-up problem emerges in our model in the absence of discriminatory contracts or asymmetric equilibria, and do not require a particular form of externalities on uncommitted agents (non-traders). While we do not allow the principal to bargain with multiple agents simultaneously, we can show that this is not crucial for our results. In fact it is sufficient to assume that the principal cannot contract with q agents at once.

Other extensions of the model are more challenging, and are left for future work. First, as we discussed in Section 5.4, we believe it is both interesting and important to study sequential contracting in applications such as bankruptcy restructuring, where contingent transfers are paramount. As we have shown in an example, in this case a form of collective hold-up will still appear in equilibrium. Second, our model does not allow for more general payoff structures in which payoffs depend on the size of the coalition that supports the principal, and can accrue before the coalition is formed. For example, in industries in which new technologies have a component of learning by doing, earlier sales affect later payoffs. Here the incentives to hold out compete with the benefits of joining early. This presents an interesting problem, where the principal may optimally front payments and sell at a loss. In that sense, collective hold-up may manifest itself in delayed learning.

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Appendix A. Proofs

A.1. **Values.** Consider the value of the principal in state m, $v(m)$. With probability $\phi\lambda_m$, the principal has agenda setting power and makes an offer that is accepted by the agent, getting a payoff $\delta v(m-1) - p(m)$. With probability $1-\phi\lambda_m$ either there is no transaction in m or there is a transaction following a proposal by the agent, and the principal obtains a discounted continuation value $\delta v(m)$. Thus

$$v(m) = \phi\lambda_m \left(\delta v(m-1) - p(m)\right) + (1-\phi\lambda_m)\delta v(m).$$

Using (1), and subtracing $\phi\lambda_m \delta v(m)$ on both sides, we have

$$v(m) = \left(\frac{\delta}{1-\delta}\right)\phi s^+(m) \qquad (4.\text{A}),$$

where $s^+(m) = \max\{s(m), 0\}$. Equation (4.A) says that the value of the principal in state m is proportional to the surplus in state m whenever this is positive, and zero otherwise. The expression eliminates the dependency on the probability of trade λ_m using the fact that if $s(m) > 0$ then $\lambda_m = 1$, if $s(m) < 0$ then $\lambda_m = 0$, and that $s(m) = 0$ when $\lambda_m \in (0, 1)$.

Consider instead the value of an uncommitted agent i in state m, $w(m)$, recalling that $\beta(m) \equiv 1/(n+m-q)$ denotes the probability that agent i meets the principal. With probability $\beta(m)(1-\phi)\hat{\lambda}_m$, agent i meets the principal, has agenda setting power, and makes an offer $b(m)$ (which is accepted), leading to a payoff $\delta w_{out}(m-1) + b(m)$. With probability $(1-\beta(m))\,\mu_m$ another agent $j \neq i$ meets the principal, and the meeting results in a transaction, leading to a payoff $\delta w(m-1)$ for player i. In all other cases (i meets the principal but either the principal has agenda setting power or the transactions falls through, or some other agent $j \neq i$ meets the principal but the transaction falls through), agent i gets a continuation payoff $\delta w(m)$:

$$\begin{aligned} w(m) &= \beta(m)(1-\phi)\hat{\lambda}_m \left[\delta w_{out}(m-1) + b(m)\right] + (1-\beta(m))\,\mu_m \delta w(m-1) \\ &\quad + \Big[\beta(m)[\phi + (1-\phi)(1-\hat{\lambda}_m)] + (1-\beta(m))\,(1-\mu_m)\Big]\,\delta w(m) \end{aligned}$$

Using (2) for the transfer $b(m)$ and simplifying, we have that for all $m \geq 2$,[25]

$$w(m) = \left[\frac{\delta\beta(m)}{1-\delta\beta(m)}\right](1-\phi)s^+(m) + \left[1 + \left(\frac{1-\delta}{1-\beta(m)}\right)\frac{1}{\delta\mu_m}\right]^{-1} w(m-1). \quad (5.\text{A})$$

[25]As before, we have used the fact that if $s(m) > 0$ then $\hat{\lambda}_m = \mu_m = 1$, if $s(m) < 0$ then $\hat{\lambda}_m = \mu_m = 0$, and that $s(m) = 0$ when $\mu_m \in (0, 1)$.

Using (5.A), we can express the current value for an uncommitted agent as a function of the final payoff w and the sequence of surpluses $[s_k]$ for $k \leq m$:

$$w(m) = (1-\phi)\sum_{k=1}^{m}\left(\frac{\beta(k)}{1-\beta(k)}\right)e_{km}s^{+}(k) + e_{1m}w \quad \forall m \geq 1, \tag{13}$$

where we have defined

$$e_{km} \equiv \left[\prod_{j=k}^{m}\left(1+\left(\frac{1-\delta}{1-\beta(j)}\right)\frac{1}{\delta\mu_j}\right)\right]^{-1}$$

A.2. **Proofs.**

Lemma A.1 (Equilibrium Trade in state $m=1$)**.** *The equilibrium probability of trade in state $m=1$ is uniquely determined by the following conditions:*

(1) *If $v+z \leq 0$, $\mu_1 = 0$ (no trade at $m=1$),*
(2) *If $0 < v+z < \delta\left(\frac{n-q}{n-q+(1-\delta)}\right)w$, $\mu_1 \in (0,1)$ (probabilistic trade at $m=1$),*
(3) *If $v+z \geq \delta\left(\frac{n-q}{n-q+(1-\delta)}\right)w$, $\mu_1 = 1$ (trade w.p. 1 at $m=1$).*

Proof of Lemma A.1. Fix a MPE σ. Since the principal only makes an offer if $s(m) \geq 0$, (4) implies $v(m) \geq 0$ for all m, and in particular $v(1) \geq 0$. Similarly, since the agent only makes an offer if $s(m) \geq 0$, if $s(m) < 0$ then $\hat{\lambda}_m = 0$. Therefore (13) implies $w(m) \geq 0$, and in particular $w(1) \geq 0$. Since $s(1) = v+z-v(1)-w(1)$, $w(1), v(1) \geq 0$ imply $s(1) \leq v+z$. It follows that if $v+z < 0$ then $s(1) < 0$ and there is no trade in equilibrium at $m=1$. Now suppose $v+z=0$. Then $s(1) = -[v(1)+w(1)]$. If $\mu_1 > 0$, then $v(1), w(1) > 0$, and thus $s(1) < 0$, which implies $\mu_1 = 0$, a contradiction. Thus $\mu_1 = 0$ and $v(1) = w(1) = 0$. It follows that if $v+z \leq 0$, in equilibrium there is no trade in state $m=1$.

Now suppose $v+z > 0$. If $\mu_1 = 0$ (no trade), then $v(1) = w(1) = 0$ and $s(1) > 0$, which implies $\lambda_1 > 0$, a contradiction. Suppose $\mu_1 = 1$. Then (4) gives $v(1) = \frac{\delta}{1-\delta}\phi s(1)$ and (13) gives

$$w(1) = \frac{\delta(1-\phi)\beta(1)}{(1-\delta\beta(1))}s(1) + \frac{\delta(1-\beta(1))}{(1-\delta\beta(1))}w$$

Substituting,

$$s(1)\left[1+\frac{\delta\phi}{1-\delta}+\frac{\delta(1-\phi)}{(1-\delta\beta(1))}\beta(1)\right] = v+z-\frac{\delta(1-\beta(1))}{(1-\delta\beta(1))}w$$

Thus $s(1) \geq 0$, consistent with equilibrium, iff

$$v + z \geq \frac{\delta(1-\beta(1))}{(1-\delta\beta(1))} w$$

If instead

$$0 < v + z \leq \frac{\delta(1-\beta(1))}{(1-\delta\beta(1))} w = \delta \left(\frac{n-q}{n-q+(1-\delta)} \right) w \tag{14}$$

we have $\mu_1 \in (0,1)$. Note that with $s(1) = 0$, (4) implies $v(1) = 0$, and (13) implies that

$$w(1) = \left(\frac{\delta\mu_1}{\left(\frac{1-\delta}{1-\beta(1)}\right) + \delta\mu_1} \right) w \tag{15}$$

Substituting in (3), the equilibrium probability of trade is given by

$$\mu_1 = \left(\frac{1-\delta}{\delta}\right) \frac{1}{1-\beta(1)} \left(\frac{v+z}{w-(v+z)}\right) \tag{16}$$

Note that the RHS of (16) $\in (0,1)$ iff (14) holds. □

Proof of Proposition 4.1. Fix an equilibrium in the subgame starting in state $m-1$. This produces continuation values $\tilde{v}(m-1)$, $\tilde{w}(m-1)$ and $\tilde{w}_{out}(m-1)$. Given these continuation values, let $v(m;\mu_m)$ and $w(m;\mu_m)$ denote the values of the principal and uncommited agent in state m when transaction probability μ_m, and let $s(m;\mu_m)$ denote the surplus in state m when transaction probability μ_m.

From (4) and (5), $v(m;0) = w(m;0) = 0$. Thus $s(m;0) \equiv [\tilde{v}(m-1) - v(m;0)] + [\tilde{w}_{out}(m-1) - w(m;0)] = \tilde{v}(m-1) + \tilde{w}_{out}(m-1)$. It follows that if $\tilde{v}(m-1) + \tilde{w}_{out}(m-1) \geq 0$, inaction at m is not an equilibrium. But note that $\tilde{v}(m-1) \geq 0$, and by (6), if $z > 0$ and $\mu_k > 0$ for all $k < m$, then $w_{out}(m) = \left[\prod_{k=1}^{m} \left(\frac{\delta\mu_k}{1-\delta(1-\mu_k)}\right)\right] z > 0$. Thus $\mu_m = 0$ is not part of an equilibrium if $\mu_k > 0$ for all $k < m$.

Suppose $\mu_m = 1$. Using the expression for the principal's value (4) and the expression for the uncommitted agent's value (5) in the definition of the surplus (3), we have

$$s(m)\left[1+\frac{\delta}{1-\delta}\phi+\frac{\left(\frac{\delta}{1-\delta}\right)\beta(m)(1-\phi)}{\left(1+\left(\frac{\delta}{1-\delta}\right)(1-\beta(m))\right)}\right]$$
$$=\tilde{w}_{out}(m-1)+\tilde{v}(m-1)-\frac{1}{\left[1+\left(\frac{1-\delta}{\delta}\right)\left(\frac{1}{1-\beta(m)}\right)\right]}\tilde{w}(m-1).$$

Equilibrium requires $s(m)>0$. From the previous expression, $s(m)>0$ iff

$$1+\left(\frac{1-\delta}{\delta}\right)\left(\frac{1}{1-\beta(m)}\right)>\frac{\tilde{w}(m-1)}{\tilde{w}_{out}(m-1)+\tilde{v}(m-1)}. \tag{17}$$

Next, suppose $\mu_m\in(0,1)$. Equilibrium then requires $s(m)=0$, which in turn implies $v(m)=0$ and then $w(m)=\tilde{v}(m-1)+\tilde{w}_{out}(m-1)$. Also with $s(m)=0$, (5) gives

$$w(m)=\left(\frac{\delta\mu_m}{\left(\frac{1-\delta}{1-\beta(m)}\right)+\delta\mu_m}\right)\tilde{w}(m-1)$$

Substituting in $w(m)=\tilde{v}(m-1)+\tilde{w}_{out}(m-1)$, and then solving for μ_m gives

$$\mu_m=\left(\frac{1-\delta}{\delta}\right)\left(\frac{1}{1-\beta(m)}\right)\left(\frac{\tilde{v}(m-1)+\tilde{w}_{out}(m-1)}{\tilde{w}(m-1)-(\tilde{v}(m-1)+\tilde{w}_{out}(m-1))}\right), \tag{18}$$

which is the statement in the proposition. This is less than one iff (17) doesn't hold.

We have shown that if $\mu_k>0$ for all $k<m$, equilibrium play in state m is uniquely determined, and is either $\mu_m=1$ if (17) holds or $\mu_m\in(0,1)$ given in (18) if (17) doesn't hold. Finally note that by Lemma A.1, if $v,z>0$ then $\mu_1>0$. An induction argument then completes the proof. □

Lemma A.2. *Let $\tilde{w}(m|\vec{\mu}^m)$ denote agents' payoffs in state $m\in M$ given trade probabilities $\vec{\mu}^m$ in the m-subgame, and define $\theta_{km}\equiv\prod_{j=k}^m\left(\frac{\delta\phi\mu_j}{1-\delta+\delta\mu_j\phi(1-\beta(j))}\right)$. Then*

$$\frac{\tilde{w}(m|\vec{\mu}^m)}{\beta(m)}\equiv\theta_{1m}(n-q)w+\sum_{k=1}^m\left(\frac{1-\phi}{\phi}\frac{1-\delta}{\delta}\right)\frac{\theta_{km}}{\mu_k}\left(\prod_{j=1}^k\frac{\delta\mu_j}{1-\delta(1-\mu_j)}\right)(v+kz+(n-q)w) \tag{19}$$

Proof of Lemma A.2. The value functions of the principal and agents satisfy

$$v(m)=\mu_m\frac{\delta}{1-\delta}\phi s(m) \tag{20}$$

and

$$w(m)=\frac{\delta\beta(m)(1-\phi)\mu_m}{1-\delta+\delta(1-\beta(m))\mu_m}s(m)+\frac{\delta(1-\beta(m))\mu_m}{1-\delta+\delta(1-\beta(m))\mu_m}w(m-1) \tag{21}$$

Substituting (20) in the surplus condition (3) and using that $\frac{1-\beta(m)}{\beta(m)} = \frac{1}{\beta(m-1)}$ we have the system of difference equations:

$$(1-\phi)s(m) = \left(\frac{1-\delta}{\delta\mu_m} + 1 - \beta(m)\right)\frac{w(m)}{\beta(m)} - \frac{w(m-1)}{\beta(m-1)} \tag{22}$$

$$\frac{1-\delta+\delta\phi\mu_m}{1-\delta}s(m) = \mu_{m-1}\frac{\delta}{1-\delta}\phi s(m-1) + w_{out}(m-1) - w(m)$$

Solving the first equation for $s(m)$ and substituting in the second equation, we transform the system of first order difference equations into a second order difference equation. Letting $\alpha_m \equiv \frac{\delta\mu_m}{1-\delta(1-\mu_m)}$, and defining

$$H(m) \equiv \frac{\phi}{1-\phi}\frac{\delta}{1-\delta}\left[\left(\frac{1-\delta}{\delta\phi} + \mu_m(1-\beta(m))\right)\frac{w(m)}{\beta(m)} - \mu_m\frac{w(m-1)}{\beta(m-1)}\right], \tag{23}$$

we can write this recursion as

$$H(m) = \alpha_m H(m-1) + \alpha_m w_{out}(m-1) \quad \text{for } m : 3 \leq m \leq m' \tag{24}$$

Solving recursively, and using that $w_{out}(m) = \alpha_m w_{out}(m-1)$ we have

$$H(m) = \left(\prod_{j=3}^{m}\alpha_j\right)H(2) + (m-2)w_{out}(m)$$

Therefore, letting $\tau_m = \frac{1-\delta}{1-\delta+\delta\mu_m\phi(1-\beta(m))}$ for convenience,

$$\frac{w(m)}{\beta(m)} = \frac{1-\delta(1-\mu_m)}{1-\delta}\phi\tau_m\alpha_m\frac{w(m-1)}{\beta(m-1)} + \tau_m(1-\phi)\left[\left(\prod_{j=3}^{m}\alpha_j\right)H(2) + (m-2)w_{out}(m)\right]$$

The boundary conditions follow by (22) for $m = 1, 2$ and (23) for $H(2)$, which give

$$H(2) = \alpha_2\alpha_1\left(v + 2z + \frac{w}{\beta(0)}\right)$$

$$\frac{w(2)}{\beta(2)} = \tau_2\left(\alpha_2\frac{1}{\tau_1} + \frac{\delta}{1-\delta}\mu_2\phi\right)\frac{w(1)}{\beta(1)} - \alpha_2\tau_2\mu_1\frac{\delta}{1-\delta}\phi\frac{w}{\beta(0)} + \alpha_2\tau_2(1-\phi)w_{out}(1)$$

$$\frac{w(1)}{\beta(1)} = \tau_1\phi\left[\frac{\delta}{1-\delta}\mu_1\frac{w}{\beta(0)} + \alpha_1\frac{1-\phi}{\phi}\left(v + z + \frac{w}{\beta(0)}\right)\right]$$

Using these initial conditions together with $w_{out}(m) = \left(\prod_{j=1}^{m} \alpha_j\right) z$, we obtain a simple recursive representation of the value functions

(25)
$$\frac{w(m)}{\beta(m)} = \frac{1-\delta(1-\mu_m)}{1-\delta}\phi\tau_m\alpha_m\frac{w(m-1)}{\beta(m-1)} + \tau_m(1-\phi)\left(\prod_{j=1}^{m}\alpha_j\right)(v+mz+(n-q)w)$$

Solving recursively, we obtain

(26)
$$\begin{aligned}\frac{w(m)}{\beta(m)} &= \left(\prod_{j=1}^{m}\alpha_j\right)\left[\prod_{j=1}^{m}\left(\frac{1-\delta(1-\mu_j)}{1-\delta}\phi\tau_j\right)\right](n-q)w \\ &+ (1-\phi)\left(\prod_{j=1}^{m}\alpha_j\right)\sum_{k=1}^{m-1}\left\{\left[\prod_{j=k+1}^{m}\left(\frac{1-\delta(1-\mu_j)}{1-\delta}\phi\tau_j\right)\right]\tau_k\left(v+kz+(n-q)w\right)\right\} \\ &+ \tau_m(1-\phi)\left(\prod_{j=1}^{m}\alpha_j\right)(v+mz+(n-q)w)\,,\end{aligned}$$

which is equivalent to (19). □

Proof of Lemma 5.1. Using (25) we get that the surplus condition (22) is equivalent to

$$\text{(22b)} \qquad \left(\frac{\delta}{1-\delta}\right)\phi\mu_m s(m) = \left(\prod_{j=1}^{m}\frac{\delta\mu_j}{1-\delta(1-\mu_j)}\right)(v+mz+(n-q)w) - \frac{w(m)}{\beta(m)}$$

Therefore $s(m) > (<) 0$ if and only if

$$\left(\prod_{j=1}^{m}\frac{\delta\mu_j}{1-\delta(1-\mu_j)}\right)(v+mz+(n-q)w) > (<)\frac{w(m)}{\beta(m)}$$

□

Proof of Proposition 5.2. Part 0. From Lemma A.2, with $\mu_j = 1$ for all $j \in M$, it follows that the equilibrium payoffs of an uncommitted agent in a FTE, $w^{\dagger}(m)$, are

$$\text{(27)} \qquad \frac{w^{\dagger}(m)}{\beta(m)} \equiv \overline{\theta}_{1m}(n-q)w + \sum_{k=1}^{m}\frac{1-\delta}{\delta}\frac{1-\phi}{\phi}\overline{\theta}_{km}\delta^k\left(v+kz+(n-q)w\right)$$

where for convenience we have defined $\overline{\theta}_{km} \equiv \prod_{j=k}^{m} \left(\frac{\delta\phi}{1-\delta+\delta\phi(1-\beta(j))}\right)$. From (6)

$$w_{out}^{\dagger}(m) = \delta^m z \tag{28}$$

Substituting in (4), and solving the difference equation, we then have

$$v^{\dagger}(m) = \left(\frac{\delta\phi}{1-\delta(1-\phi)}\right)^m v - \left(\sum_{r=1}^{m} \left(\frac{\delta\phi}{1-\delta(1-\phi)}\right)^r\right)(w^{\dagger}(m) - \delta^{m-1}z) \tag{29}$$

From Lemma 5.1 with $T^{\dagger}(m) \equiv T(m||\vec{\mu}^m = 1)$, it follows that the equilibrium of the m-subgame is a FTE if and only if

$$T^{\dagger}(m) \equiv \frac{w^{\dagger}(m)}{\beta(m)} - \delta^m \left(v + mz + (n-q)w\right) \le 0 \quad \forall m \le m'. \tag{30}$$

Part 1. Consider $m \in M$, and suppose $v \ge m(w-z)$. We show that there is a $\overline{\phi} < 1$ such that if $\phi > \overline{\phi}(m)$, the unique MPE of the m-subgame is a FTE.

From expression (27),

$$\lim_{\phi \to 1} \frac{w^{\dagger}(m)}{\beta(m)} = \left(\prod_{j=1}^{m} \frac{\delta}{1-\delta\beta(j)}\right)(n-q)w$$

So in the limit $T^{\dagger}(m) \le 0$ iff

$$\left(\prod_{j=1}^{m} \frac{\delta}{1-\delta\beta(j)}\right)(n-q)w \le \delta^m \left(v + mz + (n-q)w\right)$$

or iff

$$\left(\prod_{j=1}^{m} \frac{n+j-q}{n+j-q-\delta}\right) \le \left(\frac{v+mz+(n-q)w}{(n-q)w}\right)$$

Expanding the product, the LHS is smaller than $\frac{n+m-q}{n+1-\delta-q} < \frac{n+m-q}{n-q}$, so it is sufficient that $v \ge m(w-z)$. Thus, for large ϕ, a sufficient condition for a FTE in the m-subgame is $v \ge q(w-z)$.

Part 2. We show that: for any $m \in M$, there exists $\underline{\phi}(m) > 0$ and $\overline{v}(m) > 0$ such that if $\phi < \underline{\phi}(m)$ and $v > \overline{v}(m)$, the unique MPE of the m-subgame entails delay.

From expression (27),

$$\frac{w^\dagger(m)}{\beta(m)} = \sum_{j=1}^{m} \left(\prod_{k=j}^{m} \frac{\delta\phi}{(1-\delta)+\delta\phi(1-\beta(k))} \right) \frac{1-\delta}{\delta} \frac{(1-\phi)}{\phi} \delta^j \left(v + jz + (n-q)w\right)$$
$$+ \left(\prod_{j=1}^{m} \frac{\delta\phi}{(1-\delta)+\delta\phi(1-\beta(j))} \right)(n-q)w$$

Note that for large v all terms are positive, except possibly the last one. Dropping the first $m-2$ terms of the summation, and denoting the last term C for convenience, we have

$$\frac{w^\dagger(m)}{\beta(m)} > \left(\frac{\delta\phi}{1-\delta+\delta\phi(1-\beta(m-1))} \right) \left(\frac{(1-\delta)(1-\phi)}{1-\delta+\delta\phi(1-\beta(m))} \right) \delta^{m-1} \left(v + (m-1)z + (n-q)w\right)$$
$$+ \left(\frac{(1-\delta)(1-\phi)}{1-\delta+\delta\phi(1-\beta(m))} \right) \delta^{m} \left(v + mz + (n-q)w\right) + C$$

So $T^\dagger(m) \equiv \frac{w^\dagger(m)}{\beta(m)} - \delta^m \left(v + mz + (n-q)w\right) > 0$ iff

$$\left(\frac{(1-\delta)(1-\phi)}{[1-\delta\beta(m)]} \right) \left(\frac{1}{1-\delta+\delta\phi(1-\beta(m-1))} \right) \left(v + (m-1)z + (n-q)w\right) + \tilde{C}$$
$$\geq \left(v + mz + (n-q)w\right).$$

where $\tilde{C}$ is a term, possibly negative, that does not depend on v.

Taking derivatives of both sides with respect to v, the LHS increases faster than the RHS iff

$$\phi \leq \frac{(1-\delta)\delta\beta(m)}{(1-\delta)+\delta(1-\delta\beta(m))(1-\beta(m-1))} \equiv \overline{\phi}(m)$$

It follows that if $\phi < \overline{\phi}$, for v large enough $T^\dagger(m) > 0$. □

Proof of Proposition 5.3. Part 1. Here we prove that if in equilibrium $\mu_m \in (0,1)$ for all $m \in J \equiv \{m_\ell, \ldots, m_u\}$, then

$$\frac{\mu_{m+1} - \mu_m}{\mu_m} = \beta(m) \quad \forall m \in \{m_\ell + 1, \ldots, m_u - 1\},$$

and moreover, for all $m \in \{m_\ell + 1, \ldots, m_u\}$,

$$\mu_m = \left(\frac{n+m-q}{n+m_\ell-q} \right) \left(\frac{1-\delta}{\delta} \right) \left(\frac{1}{w(m_\ell)/w_{out}(m_\ell) - 1} \right).$$

(This second result will be useful in the proof of Theorem 5.4).

Suppose in equilibrium $\mu_m \in (0,1)$ for all $m \in J \equiv \{m_\ell, \ldots, m_u\}$. Then $s(m) = v(m) = 0$ for all $m \in M$. Since $s(m) = 0$ for all $m \in J$, by (5),

$$w(m) = \left(\frac{\delta \mu_m}{\left(\frac{1-\delta}{1-\beta(m)}\right) + \delta \mu_m} \right) w(m-1) \quad \forall m \in J \tag{31}$$

Note that for all $m \in \{m_\ell + 1, \ldots, m_u\}$, $v(m) = v(m-1) = 0$, and then $s(m) = 0$ implies $w(m) = w_{out}(m-1)$. Then

$$\frac{w(m)}{w(m-1)} = \frac{w_{out}(m-1)}{w_{out}(m-2)} \quad \forall m \in \{m_\ell + 2, \ldots, m_u\},$$

Using (31) and (6), this is

$$\left(\frac{\delta \mu_m}{\left(\frac{1-\delta}{1-\beta(m)}\right) + \delta \mu_m} \right) = \left(\frac{\delta \mu_{m-1}}{(1-\delta) + \delta \mu_{m-1}} \right) \quad \forall m \in \{m_\ell + 2, \ldots, m_u\}, \tag{32}$$

which implies that

$$\mu_m = \left(\frac{1}{1-\beta(m)} \right) \mu_{m-1} \quad \forall m \in \{m_\ell + 2, \ldots, m_u\}, \tag{33}$$

This gives the first result using the definition of $\beta(m)$. This result directly implies

$$\mu_m = \left[\prod_{k=m_\ell+2}^{m} \left(\frac{1}{1-\beta(k)} \right) \right] \mu_{m_\ell+1} \quad \forall m \in \{m_\ell + 2, \ldots, m_u\}.$$

Now, by (7), and noting that $v(m_\ell) = 0$,

$$\mu_{m_\ell+1} = \left(\frac{1-\delta}{\delta} \right) \left(\frac{1}{1-\beta(m_\ell+1)} \right) \left(\frac{w_{out}(m_\ell)}{w(m_\ell) - w_{out}(m_\ell)} \right).$$

Substituting gives

$$\mu_m = \left[\prod_{k=m_\ell+1}^{m} \left(\frac{1}{1-\beta(k)} \right) \right] \left(\frac{1-\delta}{\delta} \right) \left(\frac{1}{w(m_\ell)/w_{out}(m_\ell) - 1} \right),$$

for all $m \in \{m_\ell + 1, \ldots, m_u\}$. Noting that $1 - \beta(k) = \frac{n+m-q-1}{n+m-q}$, and simplifying, gives the result in the lemma.

Part 2. In Lemma 5.1 we showed that for any $m \leq q$, and for *any equilibrium* $\mu_1, \ldots, \mu_{m-1}$ of the $m-1$ subgame, $s(m) \geq (\leq) 0$ given $\mu_m \in [0,1]$ if and only if

$T(m) \leq (\geq) 0$. We now show that for large v, $T(m) \leq 0 \Rightarrow T(m-1) < 0$. Note that

$$T(m-1) = \left(\frac{1-\delta+\delta\mu_m\phi(1-\beta(m))}{\phi[1-\delta(1-\mu_m)]}\right) T(m) + z - \left(\frac{\delta\beta(m)\mu_m}{1-\delta(1-\mu_m)}\right)(v+mz+(n-q)w)$$

so if $T(m) \leq 0$, we have

$$T(m-1) \leq z - \delta\beta(m)\left(v+mz+(n-q)w\right),$$

where we have used the fact that $T(m) \leq 0$ implies $\mu_m = 1$. Since the RHS is decreasing in v and goes to $-\infty$ as $v \to \infty$, for sufficiently large v, then $T(m) \leq 0 \Rightarrow T(m-1) < 0$. □

Proof of Theorem 5.4. By Proposition 5.2 (part 2), for any $m \in M$, there exists $\underline{\phi}(m) > 0$ and $\underline{v}(m) > 0$ such that if $\phi < \underline{\phi}(m)$ and $v > \underline{v}(m)$, then $T^\dagger(m|\phi) > 0$. From part 1 of Proposition 5.2, on the other hand, we know that for any $m \in M$, there exists $\overline{\phi}(m) > 0$ and $\overline{v}(m) > 0$ such that if $\phi > \overline{\phi}(m)$ and $v > \overline{v}(m)$, then $T^\dagger(m|\phi) < 0$. Since $T^\dagger(m|\phi)$ is continuous in ϕ, for any m there is a $c_m \in (0,1)$ such that $T^\dagger(m|c_m) = 0$ (for v large, fixed). By part 2 of proposition 5.3, for large v in equilibrium $T(m'|c_m) > 0$ for all $m' > m$. It follows that in the unique MPE for $\phi = c_m$, we have $\mu_k = 1$ for all $k \leq m$ and (provided $m < q$), $\mu_k \in (0,1)$ for $k > m$.

We have shown before that $T(m'|c_m) > 0 = T^\dagger(m|c_m)$ for all $m' > m$, and that if $T^\dagger(m|\phi) \leq 0$ then $T^\dagger(m'|\phi)$ is decreasing in ϕ for all $m' \leq m$ (also for v large). This implies that $c_{m+1} > c_m$ for all $m \leq q-1$, and that for any $\phi \in (c_m, c_{m+1})$, $T^\dagger(m+1|\phi) > 0$ and $T^\dagger(m|\phi) \leq 0$. It follows that the equilibrium characterization above for $\phi = c_m$ applies unchanged to all $\phi \in [c_m, c_{m+1})$.

Now take $\phi \in [0,1]$ given, and let $\overline{m} \in M$ denote the cutpoint such that, in equilibrium, there is delay in each state $m \in M$ s.t. $m > \overline{m}$, and full trading in any $m \leq \overline{m}$. In the proof of part 1 of proposition 5.3 we show that if in equilibrium $\mu_m \in (0,1)$ for all $m \in J \equiv \{m_\ell, \dots, m_u\}$, then

$$\mu_m = \left(\frac{n+m-q}{n+m_\ell-q}\right)\left(\frac{1-\delta}{\delta}\right)\left(\frac{1}{w(m_\ell)/w_{out}(m_\ell)-1}\right) \quad \forall m \in \{m_\ell+1, \dots, m_u\}$$

It follows that here (with $m_\ell = \overline{m}+1$ and $m_u = q$), we have

(34)

$$\mu_m = \left(\frac{n+m-q}{n+\overline{m}+1-q}\right)\left(\frac{1-\delta}{\delta}\right)\left(\frac{1}{w(\overline{m}+1)/w_{out}(\overline{m}+1)-1}\right) \quad \forall m > \overline{m}+1$$

Note that the probability of trade in each state where there is delay is decreasing in the ratio $w(\overline{m}+1)/w_{out}(\overline{m}+1)$. We now argue that this ratio is increasing in v, and that $\mu_m \to 0$ as $v \to \infty$. Note that by (6),

$$\frac{w_{out}(\overline{m}+1)}{w_{out}(\overline{m})} = \left(\frac{\delta\mu_{\overline{m}+1}}{1-\delta(1-\mu_{\overline{m}+1})}\right),$$

and by Proposition 4.1,

$$\delta\mu_{\overline{m}+1} = (1-\delta)\left(\frac{1}{1-\beta(\overline{m}+1)}\right)\left(\frac{v(\overline{m})+w_{out}(\overline{m})}{w(\overline{m})-(v(\overline{m})+w_{out}(\overline{m}))}\right).$$

Substituting,

$$\frac{w_{out}(\overline{m}+1)}{w_{out}(\overline{m})} = \left(\frac{(v(\overline{m})+w_{out}(\overline{m}))}{\beta(\overline{m}+1)(v(\overline{m})+w_{out}(\overline{m}))+(1-\beta(\overline{m}+1))w(\overline{m})}\right).$$

Now, since $\mu_{\overline{m}+1} \in (0,1)$, then $v(\overline{m})+w_{out}(\overline{m}) = w(\overline{m}+1)$. Substituting, and noting that the equilibrium of the $\overline{m}$ subgame is a FTE,

$$\frac{w_{out}(\overline{m}+1)}{w(\overline{m}+1)} = \frac{w^{\dagger}_{out}(\overline{m})}{(1-\beta(\overline{m}+1))w^{\dagger}(\overline{m})+\beta(\overline{m}+1)[(v^{\dagger}(\overline{m})+w^{\dagger}_{out}(\overline{m}))]}.$$

It follows that for $m > \overline{m}+1$,

$$\mu_m = \left(\frac{n+m-q}{n+\overline{m}+1-q}\right)\left(\frac{1-\delta}{\delta}\right)\left(\frac{w^{\dagger}_{out}(\overline{m})}{(1-\beta(\overline{m}+1))(w^{\dagger}(\overline{m})-w^{\dagger}_{out}(\overline{m}))+\beta(\overline{m}+1)(v^{\dagger}(\overline{m}))}\right)$$

Now, $w^{\dagger}_{out}(\overline{m}) = \delta^{\overline{m}}z$ is independent of v, while both $v^{\dagger}(\overline{m})$ and $w^{\dagger}(\overline{m})$ are increasing in v, and unbounded. Thus for $m > \overline{m}+1$, μ_m is decreasing in v and goes to zero as $v \to +\infty$. This completes the proof. □

Proof of Proposition 5.6. Using (4), (13), (6) in (3) we obtain, for all $m \geq 2$

$$\left(1+\phi\left(\frac{\delta}{1-\delta}\right)+(1-\phi)\frac{\delta\beta(m)}{1-\delta\beta(m)}\right)s(m) = \phi\left(\frac{\delta}{1-\delta}\right)s(m-1)+\left[\prod_{k=1}^{m-1}\left(\frac{\delta\mu_k}{1-\delta(1-\mu_k)}\right)\right]z-\pi(1)w - (1-\phi)\sum_{k=1}^{m-1}\left(\frac{\beta(k)}{1-\beta(k)}\right)\pi(k)s(k) \tag{35}$$

Since $w > 0$, $z \leq 0$, and $\pi(k)s(k) \geq 0$, (35) implies

$$(36) \qquad \left(1+\phi\left(\frac{\delta}{1-\delta}\right)+(1-\phi)\frac{\delta\beta(m)}{1-\delta\beta(m)}\right)s(m)\leq\phi\left(\frac{\delta}{1-\delta}\right)s(m-1)$$

It follows that in any equilibrium, $s(m-1)\leq 0 \Rightarrow s(m)\leq 0$. So suppose $s(m')<0$ for some $m'<q$. Then $\mu_{m'}=0$, and thus $w(m)=v(m)=0$ for all $m\geq m'$ with no transactions in equilibrium for $m\geq m'$. Suppose instead $s(m')=0$ for some $m'<q$. If $\mu_{m'}=0$, the same conclusion holds, so suppose in equilibrium $\mu_{m'}\in(0,1)$. Because $s(m'+1)\leq 0$, in equilibrium either $\mu_{m'+1}=0$ or $s(m'+1)=0$ and $\mu_{m'+1}\in(0,1)$. If $\mu_{m'+1}\in(0,1)$, then $v(m')=v(m'+1)=0$, and then $s(m'+1)=0$ implies $w(m'+1)=w_{out}(m')$. But $w(m'+1)\geq 0$, while $w_{out}(m')=\left[\prod_{k=1}^{m'}\left(\frac{\delta\mu_k}{1-\delta(1-\mu_k)}\right)\right]z<0$ by (6), which is a contradiction. It follows that if $s(m')\leq 0$ for some $m'<q$, then $\mu_m=0$ for all $m>m'$ and $w(q)=v(q)=0$. □

Corollary A.3. *Suppose $z\leq 0$. If $v+z<\delta\left(\frac{n-q}{n-q+(1-\delta)}\right)w$, then $s(m)=w(m)=w_{out}(m)=v(m)=0$ for all $m\geq 2$.*

Proof of Corollary A.3. In Lemma A.1 we showed that a necessary condition for trade with probability one at $m=1$ is that $v+z\geq\delta\left(\frac{n-q}{n-q+(1-\delta)}\right)w$. Thus, when this condition is violated, $\mu_1<1$. The result then follows from Proposition 5.6. □

Proof of Proposition 5.7. First we show that $T^\dagger(q)\leq 0$ is a necessary and sufficient condition for existence of a FTE. This follows as a corollary of previous results. First, an examination of the proof of parts (i) and (ii) of Lemma 5.2 shows that these results do not require the assumption that $z>0$, and thus also hold for $z\leq 0$. Thus, agents' FTE payoffs are still given by $w^\dagger(\cdot)$ as defined by (27), and there exists a FTE in the m'-subgame if and only if $T^\dagger(m)\leq 0$ for all $m\leq m'$. Moreover, we know from Lemma 5.6 that when $z\leq 0$, $s(m)>0\Rightarrow s(m-1)>0$. As a result, a necessary and sufficient condition for existence of a FTE when $z\leq 0$ is that at the FTE profile, $s(q)>0$, or $T^\dagger(q)\leq 0$.

Second, we show that if there exists a FTE, this is the unique MPE. Fix an equilibrium in the subgame starting in state $m-1$. This produces continuation values $\tilde{v}(m-1)$ and $\tilde{w}_{out}(m-1)$. Given these continuation values, let $v(m;\mu_m)$ and $w(m;\mu_m)$ denote the values of the principal and uncommited agent in state m when transaction probability

μ_m, and let $s(m;\mu_m)$ denote the surplus in state m when transaction probability μ_m. From (4) and (5), if $\tilde{v}(m-1) > 0$ and $\tilde{w}_{out}(m-1) > 0$, then $v(m;\mu_m)$ and $w(m;\mu_m)$ are both increasing in μ_m, and therefore $s(m;\mu_m) = [\tilde{v}(m-1) - v(m;\mu_m)] + [\tilde{w}_{out}(m-1) - w(m;\mu_m)]$ is decreasing in μ_m. It follows that if $s(m;1) > 0$, then $s(m;\mu_m) > 0$ for any $\mu_m \in (0,1)$, and as a result, any such $\mu_m \in (0,1)$ would not be consistent with equilibrium.

We finish the proof of this step with an induction argument. First, note that if the conditions for existence of a FTE are met, then by Lemma A.1 $\mu_1 = 1$ (the unique MPE of the subgame starting at $m = 1$ is a FTE). Second, we argue that if the unique MPE of the subgame starting in state $m-1$ is a FTE, then $\mu_m = 1$. The two conditions establish the result. To prove the induction step, note that if the unique MPE of the subgame starting in state $m-1$ is a FTE, then existence of a FTE in $m \leq q$ (guaranteed by Lemma 5.6 given the existence of a FTE) implies that $s(m;1) > 0$. Then our previous argument implies that $s(m;\mu_m) > 0$ for any $\mu_m \in (0,1)$, and as a result we must have $\mu_m = 1$.

Finally, from Proposition 5.6 we know that if $T^{\dagger}(q) > 0$ there are two possibilities: either trade stops at some $m < q$ and then $w(q) = v(q) = 0$, or there is a FTE in the $(q-1)$-subgame and delay in the initial state q. The first case holds if $T^{\dagger}(q-1) > 0$, and the latter in the intermediate case in which $T^{\dagger}(q-1) \leq 0 < T^{\dagger}(q)$. This concludes the proof. □

Appendix B. Additional Results (Not for Publication)

B.1. **Contingent Transfers.** Suppose now that transfers are contingent on winning. In this case the state is not just the number of agents remaining in order to win, but also the total accumulated promises before each move, call it B. So the state is a pair $(m, B) \in \mathbb{N} \times \mathbb{R}$. We let $w_{out}(m, B; b)$ denote the eq. payoff in state (m, B) of an agent who committed for the principal with offer b. Thus, if an agent committed for the principal in state (m', B') with offer $b(m', B')$, his value in state (m, B) for $m < m'$ is $w_{out}(m, B; b(m', B'))$.

Note that since offers in each state can affect the probability of trade in subsequent states, it is not necessarily optimal for the agent to propose the largest offer the principal is willing to accept. This makes solving for equilibrium transfers more involved.

Note first that the principal accepts an offer b in state (m, B) only if

$$\delta v(m-1, B+b) \geq \delta v(m, B).$$

Let $A(m, B)$ denote the set of all proposals b that the principal accepts in state (m, B). Note that if an offer b is accepted in state (m, B), then the agent gets a payoff $\delta w_{out}(m-1, B+b; b)$, and if the offer is rejected, or if the agent does not make an offer, the agent gets $\delta w(m, B)$. Let $\overline{A}(m, B) \equiv \{b \in A(m, B) : w_{out}(m-1, B+b; b) \geq w(m, B)\}$. If $\overline{A}(m, B) = \emptyset$, the agent makes no offer, or equivalently, offers $b = \infty$. If $\overline{A}(m, B) \neq \emptyset$, the agent makes an offer $b(m, B)$, given by the b which solves

$$\max_{b \in \overline{A}(m,B)} \delta w_{out}(m-1, B+b; b)$$

Critical states are special, however, because conditional on the transaction being successful, new promises don't affect future play, as before. Consider a critical state $(1, B)$ for some $B > 0$, and suppose the principal accepts if indifferent (this actually has to be the case in equilibrium if $\overline{A}(1, B)$ is not a singleton, for the usual reason in bargaining models). Since $w_{out}(0, B+b; b) = w + b$, and $v(0, B+b) = v - B - b$, the principal accepts iff $b \leq v - B - v(1, B)$, and the agent is better off making an offer b iff $b \geq w(1, B) - w$. Thus $\overline{A}(1, B) = [w(1, B) - w, v - B - v(1, B)]$. This is nonempty iff $s_1(B) = v - B - v(1, B) + w - w(1, B) \geq 0$. Since $w_{out}(0, B+b; b) = w + b$, the optimal offer when $s_1(B) \geq 0$ is the largest b in $\overline{A}(1, B)$. i.e.,

$$b(m, B) = v - B - v(1, B).$$

Note that if $\mu_1(B) = 0$, then $\overline{A}(1, B) = [-w, v-B]$, so to the extent that $w+v-B \geq 0$, no trade in $(1, B)$ is not an equilibrium. Conversely, if $w+v-B < 0$, the equilibrium has no trade in $(1, B)$.

Next, note that

$$v(m, B) = \mu_m(B)\delta v(m-1, B+b(m, B)) + (1-\mu_m(B))\delta v(m, B).$$

Then

$$v(m, B) = \left(\frac{\delta\mu_m(B)}{1-\delta(1-\mu_m(B))}\right) v(m-1, B+b(m, B)). \tag{37}$$

so

$$v(1, B) = \left(\frac{\delta\mu_1(B)}{1-\delta(1-\mu_1(B))}\right) [v - B - b(1, B)].$$

Suppose for now that $w+v-B \geq 0$. Then $b(1, B) = v - B - v(1, B)$, and

$$v(1, B) = \left(\frac{\delta\mu_1(B)}{1-\delta(1-\mu_1(B))}\right) v(1, B) \Rightarrow v(1, B) = 0.$$

Then

$$b(1, B) = v - B$$

If instead $w+v-B < 0$, so $\mu_1(B) = 0$, then also $v(1, B) = 0$.

Note moreover that by (37), $v(1, B) = 0$ for all B implies

$$v(m, B) = 0 \quad \text{for all } B \text{ and } m \geq 1$$

This implies, in particular, that in any state (m, B) with $m > 1$ the principal is indifferent between accepting or rejecting any offer.

Now consider the welfare of an uncommitted agent in state (m, B). Suppose in state (m, B) there is trade with probability $\mu_m(B)$. Then

(38)
$$\begin{aligned} w(m, B) = {} & \beta(m)\left[\mu_m(B)\delta w_{out}(m-1, B+b(m, B); b(m, B)) + (1-\mu_m(B))\delta w(m, B)\right] \\ & + (1-\beta(m))\left[\mu_m(B)\delta w(m-1, B+b(m, B)) + (1-\mu_m(B))\delta w(m, B)\right]. \end{aligned}$$

or

$$w(m, B) = \frac{\mu_m(B)\delta}{1-\delta(1-\mu_m(B))}\left[\beta(m) w_{out}(m-1, B+b(m, B); b(m, B)) + (1-\beta(m)) w(m-1, B+b(m, B))\right].$$

We showed before that if $w+v-B < 0$, there is no trade in $(1, B)$. Suppose instead $B \leq w+v$. Since $w_{out}(0, B+b(1, B); b(1, B)) = w + b(1, B) = w + v - B - v(1, B)$,

$w(0, B + b(m, B)) = w$, and $v(1, B) = 0$, this is

$$w(1, B) = \frac{\mu_1(B)\delta}{1 - \delta(1 - \mu_1(B))} \left[w + \beta(1)(v - B)\right].$$

Note that since $v(1, B) = 0$,

$$s_1(B) = w + v - B - v(1, B) - w(1, B) = w + v - B - \frac{\mu_1(B)\delta}{1 - \delta + \delta\mu_1(B)} \left[w + \beta(1)(v - B)\right]$$

In equilibrium, $\mu_1(B) \geq 0$ iff $s_1(B) \geq 0$ and $\mu_1(B) = 1$ if $s_1(B) > 0$, and similarly $\mu_1(B) \leq 1$ iff $s_1(B) = v - B - v(1, B) + w - w(1, B) \leq 0$, and $\mu_1(B) = 0$ if $s_1(B) < 0$, so that $\mu_1(B) \in (0, 1)$ only if $s_1(B) = 0$. For an equilibrium with $\mu_1(B) = 1$, we need

$$(1 - \delta)w + (1 - \delta\beta(1))(v - B) > 0 \Leftrightarrow B < v + \left(\frac{1 - \delta}{1 - \delta\beta(1)}\right) w$$

Otherwise, i.e., if $v + w\left(\frac{1-\delta}{1-\delta\beta(1)}\right) \leq B \leq v + w$ (recall we have assumed that $B \leq w + v$), in equilibrium $\mu_1(B) \in (0, 1)$, given by

$$\mu_1(B) = \frac{(1 - \delta)[w + v - B]}{\delta(1 - \beta(1))(B - v)} \in (0, 1).$$

Recall that we have shown that $v(m, B) = 0$ for all B and $m \geq 1$. Then for any $m > 1$, $A(m, B)$ can be an arbitrary subset of the reals, $A(m, B) \subset \mathbb{R}$, and $\overline{A}(m, B) = \{b \in A(m, B) : w_{out}(m - 1, B + b; b) \geq w(m, B)\}$.

Consider an equilibrium in which the principal only accepts offers $b \leq 0$ (i.e., $A(m, B) = (-\infty, 0]$) in any state (m, B) such that $m > 1$. Since it is never optimal in this case for the agent to propose $b < 0$ (doesn't affect the probability of trade and leads to a lower payoff), all equilibrium transactions in such states have $b(m, B) = 0$. Note that in this case for large v the unique equilibrium in the critical state $(1, 0)$ has trade with probability one.

Suppose in all other states $(m, 0)$, the probability of trade is the same as in the benchmark model with cash transfers. Since the value functions for committed and uncommitted agents in each state $(m, 0)$ are exactly as in the benchmark model with cash transfers, agents are indifferent between trading and not trading at zero transfers. Since any positive offer is rejected, requesting a positive offer in any such state is not a profitable deviation. It follows that the equilibrium of the benchmark model with cash transfers is still an equilibrium with conditional payments. There are of course other equilibria, including no trade.

B.2. **History Independence .**

Proof of Proposition 5.9. We first define some elements of the game that allow us to prove our result.

Histories: Let H_t be a history defined recursively as $H_t = H_{t-1} \cup h_t$ with $H_0 = \emptyset$, and $h_t = \{i_t, x_t, p_t^{x_t, y_t}, a_t\}$ be an observed event where i_t is the agent meeting the principal, $x_t \in \{0, i\}$ is the player making the offer, $p_t^{x_t, y_t}$ is the offer made by player x_t to player $y_t = \{0, i|x_t\}$, and $a_t \in \{y, n\}$ is the answer by player y_t. We define a k-length history $\hat{H}_t = \{h_t, h_{t-1}, ..., h_{t-k}\}$.

Strategies: A $k-$bounded recall strategy is a behavioral strategy that depends on histories of size k. In our model these are

(1) $p_t^{0,j}\left(\hat{H}_{t-1}, j, 0\right)$ is the offer made by the principal if she is in charge of making an offer in period t to player j
(2) $\underline{p}_t^{j,0}\left(\hat{H}_{t-1}, j, j\right)$ is the principal's reservation price such that any offer by player $j \neq 0$ above this price is accepted, any offer by j below that price is rejected, and any offer by j at that price is accepted with probability $\sigma_t^{0,j}\left(\hat{H}_{t-1}, j, 0\right)$
(3) $p_t^{j}\left(\hat{H}_{t-1}, j, j\right)$ is the offer made by player $j = 1, ...N$ if she is in charge of making an offer in period t
(4) $\underline{p}_t^{j}\left(\hat{H}_{t}, j, 0\right)$ is player's $j \neq 0$ reservation price such that any offer by the principal above this price is accepted, any offer below that price is rejected, and any offer at that price is accepted with probability $\sigma_t^{j}\left(\hat{H}_{t-1}, 0, j\right)$

Continuations: Let $V_{t+1}^{j}\left(\hat{H}_t\right)$ be the SPNE value for player $j = 0, ...n_{t+1}$ at $t+1$ when there are n_{t+1} players remaining and the principal, after history $\hat{H}_t$. $V_{t+1}^{j}\left(\hat{H}_t\right)$ it is the ex ante continuation value at $t+1$ before the random matching. Let $v_{t+1}^{j}\left(\hat{H}_t, x_{t+1}, y_{t+1}\right)$ denote the value after meeting the principal, before trading at $t+1$ when player $x_{t+1} \neq 0$ has been chosen and player $y_{t+1} \in \{0, x_{t+1}\}$ makes the offer.

Consider now a critical history. We now derive the optimal strategies as function of the continuation values. Note that by optimality we have that the principal makes an offer to agent j at t given the history $\hat{H}_t$,

$$\delta z + p_t^{0,j} = \delta V_{t+1}^{j}\left(\hat{H}_t\right)$$

as long as

$$\delta v - p_t^{0,j} \geq \delta V_{t+1}^{0}\left(\hat{H}_t\right)$$

or letting

$$S_t^j\left(\hat{H}_t\right) \equiv v + z - V_{t+1}^{j}\left(\hat{H}_t\right) - V_{t+1}^{0}\left(\hat{H}_t\right) \geq 0$$

It follows then that

$$p_t^{0,j}\left(\hat{H}_{t-1}, j, 0\right) = \underline{p}_t^{0,j}\left(\hat{H}_{t-1}, j, 0\right) = \delta\left(V_{t+1}^{j}\left(\hat{H}_t\right) - z\right) \tag{39}$$

Analogously we have

$$p_t^{j}\left(\hat{H}_{t-1}, j, j\right) = \underline{p}_t^{j}\left(\hat{H}_{t-1}, j, j\right) = \delta\left(V_{t+1}^{0}\left(\hat{H}_t\right) - w\right) \tag{40}$$

Using the offers and acceptance rules as a function of the continuation values we can compute the values as a function of the histories. Considering now the matching process we have that for the principal it must be that

$$\begin{aligned} V_{t+1}^{0}\left(\hat{H}_t\right) &= \frac{1}{|n_{t+1}|}\left[\sum_{j=1}^{n_{t+1}} \phi\sigma_{t+1}^{j}\left(\hat{H}_t, 0, j\right)\left[\delta v - p_{t+1}^{0,j}\left(\hat{H}_t, j, 0\right)\right]\right. \\ &\qquad \left. + \sum_{j=1}^{n_{t+1}}\left(1 - \phi\sigma_{t+1}^{j}\left(\hat{H}_t, 0, j\right)\right)\delta V_{t+2}^{0}\left(\hat{H}_{t+1}\right)\right] \\ &= \frac{1}{|n_{t+1}|}\left[\sum_{j=1}^{n_{t+1}} \delta\hat{S}_{t+2}\left(\hat{H}_{t+1}\right) + \sum_{j=1}^{n_{t+1}} \delta V_{t+2}^{0}\left(\hat{H}_{t+1}\right)\right] \end{aligned} \tag{41}$$

where we define

$$\hat{S}_{t+2}^{j}\left(\hat{H}_{t+1}\right) = \sigma_{t+1}^{j} S_{t+2}^{j}\left(\hat{H}_{t+1}\right)$$

and is either 0 or equal to $S^j_{t+2}\left(\hat{H}_{t+1}\right)$. For the remaining active players we have

(42)

$$\begin{aligned}V^j_{t+1}\left(\hat{H}_t\right) = & \frac{|n_{t+1}|-1}{|n_{t+1}|}\sum_{j=1,j\neq j}^{n_{t+1}}\Big[\left(\phi\sigma^0_{t+1}\left(\hat{H}_t,j,0\right)+(1-\phi)\,\sigma^j_{t+1}\left(\hat{H}_t,j,j\right)\right)\delta w \\ & \quad +\phi\left(1-\sigma^0_{t+1}\left(\hat{H}_t,j,0\right)\right)\delta V^j_{t+2}\left(\hat{H}_t,j,0,p^{0,j}_{t+1}\left(\hat{H}_t,j,0\right),n\right) \\ & \quad +(1-\phi)\left(1-\sigma^j_{t+1}\left(\hat{H}_t,j,j\right)\right)\delta V^j_{t+2}\left(\hat{H}_t,j,j,p^j_{t+1}\left(\hat{H}_t,j,j\right),n\right)\Big] \\ & +\frac{1}{|n_{t+1}|}\Big[\left(1-(1-\phi)\,\sigma^0_{t+1}\left(\hat{H}_t,j,0\right)\right)\delta V^j_{t+2}\left(\hat{H}_{t+1}\right) \\ & \quad +(1-\phi)\,\sigma^0_{t+1}\left(\hat{H}_t,j,0\right)\left(\delta z-p^j_{t+1}\left(\hat{H}_t,j,j\right)\right)\Big] \\ = & \frac{|n_{t+1}|-1}{|n_{t+1}|}\sum_{j=1,j\neq j}^{n_{t+1}}\Big[\delta w+\phi\left(1-\sigma^0_{t+1}\left(\hat{H}_t,j,0\right)\right)\delta\left(V^j_{t+2}\left(\hat{H}_t,j,0,p^{0,j}_{t+1}\left(\hat{H}_t,j,0\right),n\right)-w\right) \\ & \quad +(1-\phi)\left(1-\sigma^j_{t+1}\left(\hat{H}_t,j,j\right)\right)\delta\left(V^j_{t+2}\left(\hat{H}_t,j,j,p^j_{t+1}\left(\hat{H}_t,j,j\right),n\right)-w\right)\Big] \\ & +\frac{1}{|n_{t+1}|}\left[\delta V^j_{t+2}\left(\hat{H}_{t+1}\right)-(1-\phi)\,\delta\hat{S}^j_{t+2}\left(\hat{H}_{t+1}\right)\right]\end{aligned}$$

Note that in $\hat{H}_{t+1}=\left\{h_t,\hat{H}_t|h_{t-k}\right\}$ we must have that h_{t-k} does not affect $V^0_{t+1}\left(\hat{H}_t\right)$ in (41) or $V^j_{t+1}\left(\hat{H}_t\right)$ in (42) and therefore

$$\begin{aligned}V^0_{t+1}\left(\hat{H}_t\right) &= V^0_{t+1}\left(\hat{H}_t|h_{t-k}\right)\\ V^j_{t+1}\left(\hat{H}_t\right) &= V^j_{t+1}\left(\hat{H}_t|h_{t-k}\right)\end{aligned}$$

The same arguments can be used for all players and hence

$$V^j_{t+1}\left(\hat{H}_t\right) = V^j_{t+1}\left(\hat{H}_t|h_{t-k}\right)$$

Replacing now in (39) and (40) it follows that

$$p^{0,j}_t\left(\hat{H}_{t-1},j,0\right)=\underline{p}^{0,j}_t\left(\hat{H}_{t-1},j,0\right)=p^{0,j}_t\left(\hat{H}_{t-1}|h_{t-1-k},j,0\right) \tag{39j}$$

and

$$p^{j,0}_t\left(\hat{H}_{t-1},j,j\right)=\underline{p}^{j,0}_t\left(\hat{H}_{t-1},j,j\right)=p^{j,0}_t\left(\hat{H}_{t-1}|h_{t-1-k},j,j\right) \tag{40j}$$

Therefore, all prices and reservation prices that are $k-$bounded recall are also $k-1-$bounded recall. Applying the same argument recursively, it follows that all strategies are history independent.

This proves history independence in all critical histories. Since w, v and z are arbitrary values, they can be replaced by continuation values. In particular, if we use the continuation values that emerge in all critical histories, using the same logic, we can prove history independence of all strategies in histories that are one step before a critical history. Applying this recursively, it follows that the $k-$bounded recall equilibrium strategies are history independent and, hence, we can find the values recursively as in the main text. □