THE SAVINGS GLUT OF THE OLD:
POPULATION AGING, THE RISK PREMIUM,
AND THE MURDER-SUICIDE OF THE RENTIER

Joseph Kopecky
Alan M. Taylor

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ABSTRACT

Population aging has been linked to a global savings glut and a decline in safe real interest rates.
Conversely, risky real returns have not fallen as much, if at all, with equity risk premia on the
rise. An existing literature can explain changes in safe rates using demographics. We go further to
account for divergent returns on different assets as well as the underlying surge in the wealth-
income ratio and its asset composition. Empirical evidence from historical panel data shows that
demographic shifts are correlated with asset returns and risk premia. We build a heterogeneous
agent life-cycle model with two assets (a safe bond and equity) and with aggregate risk. Aging
demographics can help to simultaneously explain three key trends: the rising wealth-income ratio,
the falling risk free rate, and an increasing risk premium. The shifts exert less pressure on risky
returns as high-wealth elderly reallocate away from equities: aging makes retirement saving a
"crowded trade" but more so for bonds. Projecting our model to 2050, aging pushes the safe rate
below zero, but the risk premium remains elevated, as post-boomer demographics push asset
returns to unprecedented and persistently low levels.

Joseph Kopecky
Department of Economics
Room 3014 Arts Building
Trinity College
Dublin
Ireland
jkopecky@tcd.ie

Alan M. Taylor
Department of Economics and
Graduate School of Management
University of California
One Shields Ave
Davis, CA 95616-8578
and CEPR
and also NBER
amtaylor@ucdavis.edu

1. Introduction

Essential contrasts between different classes of investors and investments have played an important role over the long sweep of macroeconomic thought. In the 19th century, a distinction between a coupon-clipping rentier, vested in fixed-income bonds and debentures, and a risk-taking re-investing capitalist owning mostly private equity, was a crucial one for Marx. Subsequently, in the interwar years of the 20th century Keynes sensed tectonic shifts in the ownership, control, and financing of business, as inflation wracked the traditional rentier class, and a more distributed-ownership public equity model took shape.[1] Looking to the future, and anticipating exogenous technological and political pressures which would force lower real rates of return, Keynes then famously spoke of "the euthanasia of the rentier, of the functionless investor" and thought that, in parallel, even risky returns would be destined to fall, pari passu, except for some premium "to cover risk and the exercise of skill and judgment."[2] Yet this is not exactly the future that came to pass, either in Keynes's time or our own, even if a phase of declining real rates then and now has suggested a renewed tendency to secular stagnation (Hansen, 1939; Summers, 2014).

Turning to recent trends over the last 30–40 years, benchmark real safe rates $r^{safe} = r^*$ in Advanced Economies have fallen by 400–500 bps (Rachel and Smith, 2015), so the classic coupon-clipping rentiers have suffered ever lower yields. An existing literature can explain some of the decline using demographics, but the theme of this paper is subtly different and focuses on accounting for divergent risky and safe rates of return, as well the rise in the wealth-income ratio, in a unified modeling framework. In the last four decades, a massive expansion in the aggregate wealth-income ratio W/Y has emerged, by a factor of 1.5 to 2, another defining feature of our age which demands explanation (Auclert, Malmberg, Martenet, and Rognlie, 2020; Mian, Straub, and Sufi, 2020; Piketty and Zucman, 2014).[3] However, not all asset classes have been affected equally—at the same time, returns to risky equity capital $r^{risky} = r^e$ have not fallen as much, if at all (Damodaran, 2013; Duarte and Rosa, 2015), thus posing yet another puzzle which is, by definition, beyond the scope of standard macroeconomic models with only one return on one type of asset

We introduce an overlapping generations (OLG) model with both safe and risky assets, with labor income uncertainty and saving for retirement in a world with aggregate risk. The model is calibrated to resemble observed patterns in wealth accumulation and portfolio choice in the U.S. microeconomic data. We then study this model's equilibrium under

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[1]Discussion of the economics of the rentier dates back to, e.g., Marx (1844 [1932]), and Keynes (1936). For critical discussion, see Crotty (1990), and McKibbin (2013). On the evolution of financial systems in those eras see Kindleberger (1984).

[2]The quotes are taken from Chapter 24 of Keynes (1936).

[3]See Waldenström (2021) for updated long-run data on W/Y through 2020 in Advanced Economies.

exogenously changing mortality and age structures observed in recent decades, and in current projections out to 2050. In this setup, exogenous demographic changes like those seen in the last 30 years produce a growing mass of traditional rentier types in older age cohorts (i.e., boomers) who compete to demand mostly safe assets for their retirement in ever larger numbers, killing the safe rate of return and widening the risk premium. We therefore describe a phenomenon influencing returns which is not so much an exogenous euthanasia, but rather an endogenous murder-suicide of the rentier. Projecting demographics into future decades, our model predicts that these effects will intensify: savers' efforts to get a return *on* capital will be replaced by a struggle to get even a return *of* capital.

We have three main findings. First, as in current research, our model generates a decline in the safe rate due to population aging; however, the impact in our model is much larger than the existing literature. Second, our model can also match the rise in the equity risk premium in the data, and delivers a risky return where the change over time is flatter. Third, population aging in the model also triggers a large increase in the wealth-income ratio, consistent with observed trends in the data.

The key mechanism driving our results is endogenous age-specific portfolio choice: young workers initially accumulate mainly risky assets ("equity"), but switch to a portfolio with a greater allocation to safe assets ("bonds") as they approach and enter retirement. Firstly, we document this phenomenon empirically, noting that it exists in several established measures of the risk premium. Secondly, the idea is validated using long-run historical panel data where we show that safe and risky asset returns do indeed co-move with demographic variables in line with the proposed mechanism, building on existing studies (Fair and Dominguez, 1991; Poterba, 2001). Finally, we calibrate our OLG heterogeneous-agent life-cycle model to match the U.S., and compare its equilibria under different demographic structures, showing that the model matches the observed trends.

Literature This paper ties into multiple, larger macrofinance literatures, and a truly comprehensive survey is not possible in limited space. Our model shows that a large portion of the observed effects of aging can be explained by portfolio allocation decisions of households. These models have their roots in the seminal work of Bodie, Merton, and Samuelson (1992), the key mechanism being that individuals will rebalance their assets away from risky equity towards safer bonds as they age. As individuals age their relatively safe human capital endowment shrinks, giving them an incentive to move their financial wealth away from risk, so as to keep their overall risk level constant.[4]

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[4]Cocco, Gomes, and Maenhout (2005) build on this result and find that the utility costs of failing to re-balance one's portfolio to account for declining human capital assets is potentially quite large. In related work, Benzoni, Collin-Dufresne, and Goldstein (2007) show that equity dividends are cointegrated with the

The past decade has seen a period of unprecedented low real interest rates and lackluster growth. Recognizing the apparent persistence of these trends, much has been written on the secular stagnation hypothesis reinvigorated in Summers (2014) and most recently explored in Eggertsson, Lancastre, and Summers (2019a). Indeed a large number of recent works have shown that aging, both through falling fertility rates and rising life expectancy is a significant factor in the long-run decline in real rates since as early as the 1980s. One important mechanism linking these demographics to asset prices is the concentration of population into older, higher saving, age groups, as well as increasing life expectancy.

The literature linking demographics to macroeconomic trends has grown in recent years, but as we have noted the framing is almost exclusively in terms of a single rate of return. In their work on the United States, Gagnon, Johannsen, and López-Salido (2016) find that, since 1980, population aging can account for a 125 basis point fall in the long run real interest rates and economic growth. Carvalho, Ferrero, and Nechio (2016) find larger effects, suggesting that the decline may be as high as 200 basis points. They also investigate the strengths of each potential demographic channel and suggest that most of this decline comes from rising life expectancy. While both models differ in some important ways from ours, one of our key mechanisms will be the same.[5]

Earlier work such as Abel (2003) suggested that aging boomers' demand for assets should bid up the price of capital, increasing stock market values, and this is an implication that we can flesh out more clearly in our model. Daly (2016) argues the decline in global bond yields from 1985 to 2000 was driven by this savings channel, but that the subsequent decline came from "equity risk premium shocks" and he proposes population aging as a potentially important channel. In one of the few other papers to study aging and relative asset returns, Geppert, Ludwig, and Abiry (2016) have a related modeling approach, but they focus on projecting forward (to 2050) rather than documenting and accounting for historical effects as well. They find much smaller future effects on asset returns, but our simulated results going back to the 1970s suggest that most of the important demographic influences on shifting asset returns mostly occured in past decades, as the baby boomers have moved through the labor force, with little effect in years to come. Marx, Mojon, and Velde (2019) attempt to replicate movements in the risk premium in a simpler OLG model.

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labor markets. The implication of their work is that the expected portfolio shifting takes on a hump-shape over the life cycle as the human capital aspect of labor income acts as a "stock-like" asset for young investors and a "bond-like" asset for those nearer retirement.

[5]Lisack, Sajedi, and Thwaites (2017) argue that demographic forces can account for roughly half of the roughly 450 basis global real interest rates since 1980 documented in Rachel and Smith (2015), while also accounting for a large fraction of the simultaneous rise in housing prices and debt. Eggertsson, Mehrotra, Singh, and Summers (2016) extend the idea to an open-economy setting showing that global capital markets can be a transmission mechanism for low natural rates and policy spillovers.

They can achieve meaningful movements in the simulated risk premium, but only in their quantitative exercises that allow for either time varying borrowing constraints, or time varying productivity risk. We view our results as complementary to theirs, but we hold these parameters fixed and find large endogenous movements due to demography alone.

In the very long run, the data clearly show that risky and safe returns do not move together. Jordà, Knoll, Kuvshinov, Schularick, and Taylor (2017a) show that the risk premium has been rising due to secular trends in the safe rate. Kuvshinov and Zimmermann (2020) show that risky returns have a negative trend with a risky-safe rate "disconnect" suggesting secular movements in the ex-ante risk premium. Our analysis shows that demographics may drive some of these more medium term differences in asset returns. Caballero, Farhi, and Gourinchas (2017) document the equity risk premium for the U.S., finding that, from 1980 to 2000, the expected rate of return on equities fell in tandem with risk free rates, at which point equity returns stabilized while safe rates continued to fall. They identify the global savings glut as one potential explanation, with a rise in reserve accumulation in emerging markets driving up demand for safe assets relative to risky.

A relative growth in the mass of "risk-averse wealth" in the world is a broader point emphasized by Hall (2016) and aligns with the age-dependent portfolio mechanism we explore. This trend is also observed in Rachel and Smith (2015), who discuss the growing spread between IMF measures of global real interest rates and their global measure of the return on (risky) capital. In very recent work, Auclert, Malmberg, Martenet, and Rognlie (2020) bridge the gap between empirical estimates of the effect on aging and structural models that have been more widely used. They use a shift-share approach to identify both the *compositional* effect (due to changing population shares) as well as a *behavioral* effect (changing decisions across the age profile) on wealth-to-GDP ratios. This allows the authors to study the effect of aging on interest rates, finding strong negative effects, as well as global wealth imbalances. They suggest that a net increase in the demand for safety may affect relative asset prices, as in our model. Other research has identified robust returns or profits in a broader portfolio including risky assets, including observations on flows in the national accounts (Ravikumar, Rupert, and Gomme, 2015) and inferences drawn from summary measures such as $r - g$ (Piketty, 2014).

The contribution of our work is twofold. We document the effect of aging on different asset returns in macrohistorical data, showing that the potential for demographics to act as head/tailwinds for asset returns is quite large. We then quantitatively examine the effects of aging in a financial model of life-cycle portfolio allocation with risky and safe assets. More broadly, our work provides further evidence that macroeconomic theories with more than one asset are needed to gain a fuller understanding of important secular trends.

2. Stylized facts: returns, wealth accumulation, and demographics

As motivation, to see what is to be explained and the driving forces in our model, we present three sets of stylized facts in Figure 1, Figure 2, and Figure 3.
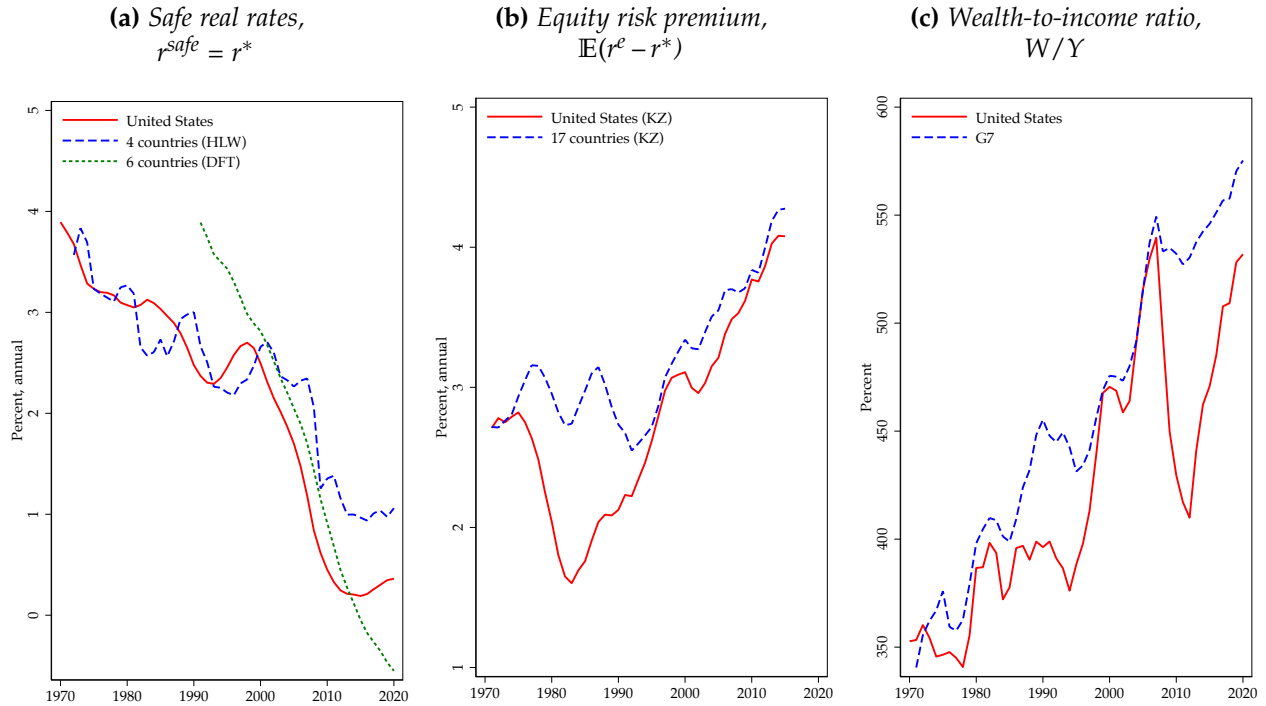
Returns and wealth accumulation trends   The first stylized facts relate to trends in the safe rate, the risk premium, and wealth accumulation, and these constitute the explicandum. Figure 1 displays the key shifts our model aims to match: a decline of several hundred basis points (300–400 bps) in the safe rate, a smaller offsetting increase in the risk premium (100–150 bps), and a substantial increase in the wealth-to-income ratio, with the bulk of these changes after 1990.

Firstly, Figure 1a shows that real safe rates, using the filtered measure of the natural rate, $r^{safe} = r^*$, have been declining for the last 40 years, for U.S. estimates in Laubach and Williams (2003), for 4 Advanced Economies in Holston, Laubach, and Williams (2017) and 6 Advanced Economies in Davis, Fuenzalida, and Taylor (2019). In 1970 real safe rates in the U.S. were around 400 bps. By the year 2000 they had fallen to 200 bps, and are near zero today. Data for other Advanced Economies show the global nature of this trend. It is thus acknowledged that the real safe rate has fallen by a large amount. This is the first fact that our model tries to explain.

Secondly, Figure 1b shows the equity risk premium, *ERP*, the expected difference between the equity rate of return and the safe rate, $r^{risky} = r^{equity}$, for the U.S. and for an average of 17 advanced economies by Kuvshinov and Zimmermann (2020). There is clear secular rise of 150–200 bps, which is particularly strong after the mid-1980s both in the U.S. and the rest of the world. A similar upward trend in *ERP* is shown for the U.S. using alternative methods by Damodaran (2013) and Duarte and Rosa (2015). And Kuvshinov and Zimmermann (2020) show that this result is robust to alternative methodologies, e.g. the measure of the safe rate chosen and the assumptions made about future cashflows and how to discount them. Because the risk premium has risen, the equity rate of return has fallen by less than the safe rate. This is the second fact that our model tries to explain.

Lastly, Figure 1c shows the wealth-income ratio, W/Y, the ratio of total national wealth to total national income. These data series are taken from the World Inequality Database, built on the work of Piketty and Zucman (2014) and others, and we report trends for the U.S. and for an average of the G7 group of Advanced Economies. The central fact here is the large upward trend in this ratio, which is common across the world, among to an upward multiple by a factor of 1.5 over 50 years from around 350% to 500%–550%. Clearly, today's societies are characterize by much more pronounced wealth accumulation relative to the recent past. This is the third and final fact that our model tries to explain.

Figure 1: *Returns and wealth accumulation trends in the U.S. and other Advanced Economies*

(a) *Safe real rates,*
$r^{safe} = r^*$

(b) *Equity risk premium,*
$\mathbb{E}(r^e - r^*)$

(c) *Wealth-to-income ratio,*
W/Y

Sources: In panel a: Laubach and Williams (2003), Holston, Laubach, and Williams (2017) , and Davis, Fuenzalida, and Taylor (2019). In panel b: Kuvshinov and Zimmermann (2020). In panel c: World Inequality Database.

Figure 2: *Demographics and wealth-by-age trends in the U.S.*

(a) *Number of households*
in old and young groups

(b) *Net wealth per person*
in old and young groups

(c) *Total net wealth*
in old and young groups
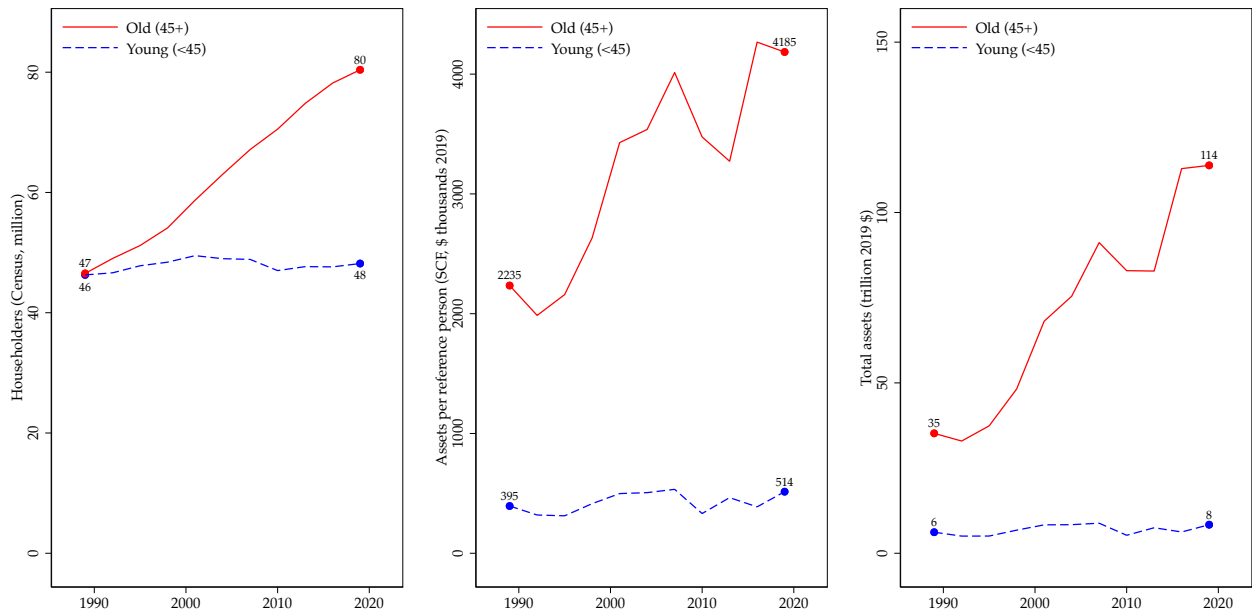
Sources: Federal Reserve and Survey of Consumer Finances data. Similar trends apply to financial assets only, see Appendix and Figure A.1.

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Figure 3: *Portfolio-by-age pattern in the U.S.*

(a) *All households* | **(b)** *Households participating in equity market*

Source: Survey of Consumer Finances data for 2016.

Demographics and wealth-by-age trends The first mechanism we explore is aging, and Figure 2a shows SCF data for U.S. household he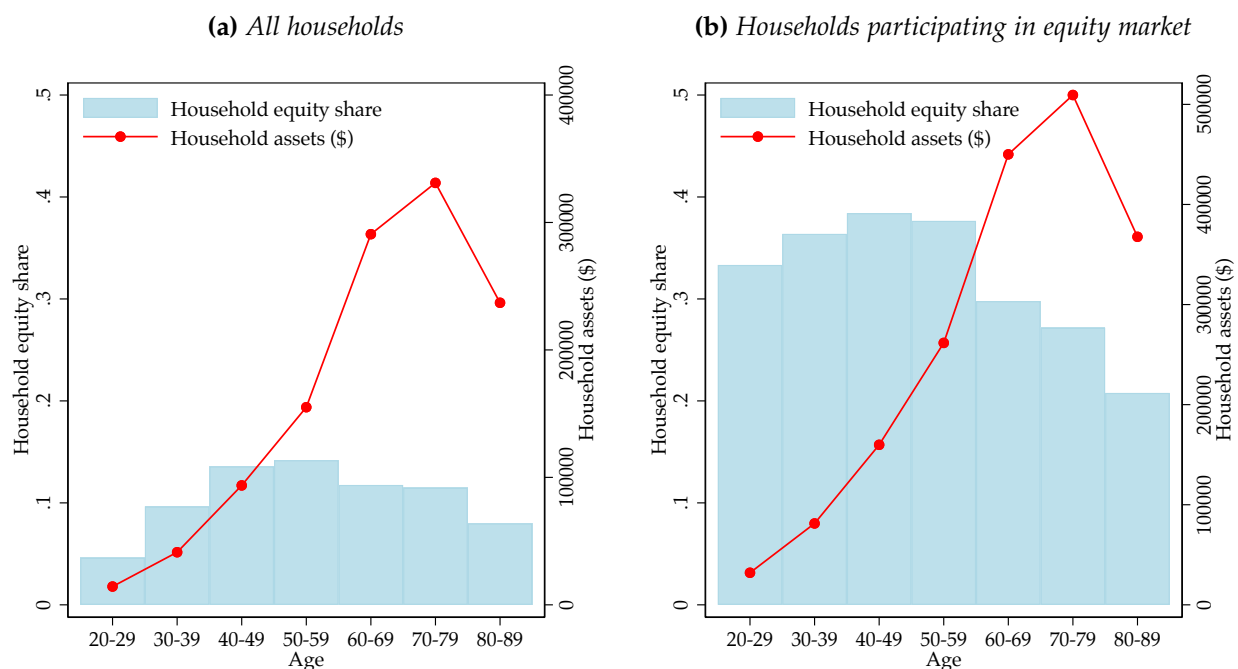ads since 1989. Over time the old (45+) age group has doubled in size, while the size of the young (<45) group was unchanged.

The second aspect of this mechanism is life-cycle accumulation, and Figure 2b shows how the wealth of the old drives almost all changes in wealth over time. The old (45+) age group has almost doubled its wealth per person, from a high level, while for the young (<45) group it has increased by a fifth, and remains at a very low level. Similar trends apply to holdings financial assets only, as shown in Appendix Figure A.1.

Combining these two trends, Figure 2c shows the first key force at work: twice as many old people holding twice as much wealth per person implies a 4-fold increase in the wealth of this group from $35 trillion to $114 trillion, compared to a trivial shift in the young group from $6 to $8 trillion. This concisely sums up the savings glut of the old.

Portfolio-by-age pattern Finally, we add a second driver to the mix, noting not just the magnitude of the savings glut of the old, but also its composition. As U.S. households age they accumulate more wealth, but then they also tend to shift its composition towards safe and away from risky assets (i.e., into bonds and out of equities). Figure 3a shows this pattern clearly. Household assets by age peak in the 70s in the 2016 SCF, but the equity share peaks at a much younger age in the 50s.

Our model employs this second driver to explain why safe and risky assets react differently to aging, with the goal of accounting for observed trends in a quantitative model. Our model will also feature an equity participation constraint, a typical feature needed to match the observed data, so we also check that the same portfolio-by-age pattern holds among the subsample of equity market participants, and it does as shown in Figure 3b.

In sum, aging shoves more weight into age bins with heavier masses of wealth, but comparatively smaller allocations to risky assets. The intuition is now clear, but are the magnitudes large enough to matter? We address this question with empirics and theory.

3. Empirical evidence on the link between aging and asset returns

In this section, to motivate the mechanisms that underpin our modeling approach, we seek to document the empirical link between demographic change and rates of return with the largest cross-country, long-run historical dataset yet assembled for such a study.

An earlier literature from a couple of decades ago had sought to understand this relationship in advance of the retirement of the baby boomers, and one particularly central paper is Poterba (2001), which studies the effect of aging on various asset returns.[6] The paper finds generally only a weak correlation with age structure, but some negative effects for population aged 40–64 and safe returns, but not so much with other asset returns. However, strong effects on safe returns and weak effects on equity returns turns out to be in line with our motivation and model. More recent work by Lunsford and West (2019) offers a methodology for isolating the stable components of long run trends, and suggests a strong relationship between aging and falling interest rates, but to our knowledge the method cannot be applied in a panel setting, and would be computationally burdensome with multivariate controls.

Another exercise in Poterba (2001) uses micro data from the SCF to study demand for assets by age. Here there is little supportive evidence—due, in part, to the muted (if any) decline in observed asset holdings in retirement in the SCF. However, recent findings using better quality data—and often big data—can help corroborate the mechanism. Using virtually-complete Norwegian administrative tax data, Fagereng, Gottlieb, and Guiso (2017) find much stronger evidence of life-cycle retirement dis-saving, while Auclert, Malmberg, Martenet, and Rognlie (2020) show that these wealth profiles in a cross-section of countries (including the U.S.) do appear to decline for older retired populations, even if peak savings may come later than the onset of retirement itself. While we will not in this section be able to get at questions regarding this micro mechanism, we provided some stylized evidence in

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[6]Another notable paper is Mankiw and Weil (1989), which considers the demographic effect on housing.

[Figure 3](#) that with regard to risk balance this is partly a story of participants. Our goal is simply to document the apparent strength and size of the empirical relationship between aging and asset returns using data from the maximal long-run historical period.

3.1. Data

We use annual country data for advanced economies, with historical asset returns from the JST macrohistory dataset ([Jordà, Knoll, Kuvshinov, Schularick, and Taylor, 2017a](#)). This provides our three primary returns of interest: bill rates, long term government bond returns, and total returns on equity.[7] We take the safe rate series as given but smooth equity returns as a 10-year rolling average of current and past returns. Not doing so would allow short-term volatility in this series to dominate any long term affect from aging and makes equity results extremely volatile. The JST dataset also includes a number of useful macroeconomic variables we use to construct macroeconomic controls (see, e.g., [Jordà, Schularick, and Taylor, 2017b](#)). We use CPI inflation to calculate real returns in all cases.

We then merge the macrohistory data with population data at each age from the [Human Mortality Database](#) ([2019](#), henceforth, HMD). We use this source to construct population shares across the age distribution. The data coverage of HMD only allow us to make use of the full sample of the JST data for five countries, but we can go back as far as 1956 for all countries in the sample, or to 1900 for nine countries using this source. Since our later model focus will be on the United States we also use information from the U.S. Census to include U.S. population shares from 1900–1933, the earliest date in the HMD data. This leaves us with a sample of 16 advanced economies from a long sample up to a maximum span of years 1870 until 2017.

The availability of these historical data yields a number of benefits. For one, while the demographic trends in these economies share many common characteristics, there is substantial variation in the timing and intensity of the aging trends that they experience. Second, being able to go far back in time allows us to take advantage of more variation than just the baby boom/bust era which has been extensively examined. Many studies focus exclusively on either the postwar period or the last four decades, capturing primarily this one large demographic shift. We will see in our sample not only the effect of boomers as babies, but also for many countries a range of demographic trends before this time period.

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[7]The JST dataset is built from a wide variety of well-documented sources and is freely available at https://www.macrohistory.net/database.

3.2. The effect of the population age structure on returns

Our goal is to understand the empirical relationship between population structure and asset returns. A reasonable first step is to estimate

$$R_{it} = f_i + \sum_{j=1}^{J} \alpha_j p_{j,it} + X_{it}\theta^h + \epsilon_{it}, \tag{1}$$

where $p_{j,it}$ is a set of population age shares in a given country i at time t, where the population is split into j subgroups that covers the entire population. Additionally we include a number of country-time specific macroeconomic controls in $X_{i,t}$. Of course we cannot jointly estimate all of the population shares simultaneously as they sum to one, but we can omit one and consider the remaining shares relative to this. In the time series estimates of Poterba (2001), estimates are of the form of Equation 1, but as univariate regressions with a single demographic regressor; in Appendix A1, we show that applying this simpler specification yields some evidence for an effect of aging on returns in our large panel dataset, but as with findings in that study these are weaker than what we report here.

To make a start, we split the population into four groups. Those under 20, workers aged 20–39, workers aged 40–64, and retirees over 65. We also include as a demographic control the length of life expectancy at birth. We choose to omit the younger working-age category 20–39 for this estimation and thus interpret coefficients as relative to that group. For country controls we then include debt to GDP ratios, the growth rate of both per-capita real consumption and investment as well as dummies for WW1, WW2, and financial crises. We then run this regression for all three of the real asset returns as above, as well as the equity risk premium as defined by the total real equity return minus the real return on long-term government bonds.[8]

The results of these regressions are reported in Table 1 and they generally accord with intuition. The two older age groups are seen to have strong negative effects on safe rates, with a smaller and less precise negative effect on equity returns. The only group for which the opposite relationship between safe and risky returns appears is the retirees, but the effect is weakly estimated across all returns. Since the 20–40 age group is missing these are all relative to the effect of that group, which theoretically could be positive. Given that we find effects for the prime savings age group 40–64, in the expected negative direction, these results seem reasonably in line with what theory would predict. Even so, there are still potential drawbacks with this approach inspired by Poterba (2001). First, omitting any age group is undesirable as it's not obvious a priori which groups are relatively more

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[8]Results are insensitive to using the bill rate, or a composite "safe rate" (a weighted average of the two).

Table 1: *Real asset returns and population structure, by asset type, full sample*

	(1) Bill rate	(2) Bond total return	(3) Equity total return	(4) Equity risk premium
% population <20	-0.307**	-0.737**	-0.757**	0.052
	(0.0829)	(0.143)	(0.210)	(0.245)
% population 40–64	-0.667**	-0.957**	-0.546[+]	0.441
	(0.110)	(0.214)	(0.299)	(0.269)
% population 65+	-0.054	-0.256	-0.475[+]	-0.131
	(0.142)	(0.258)	(0.244)	(0.335)
Controls $X_{i,t}$	Yes	Yes	Yes	Yes
Country fixed effects	Yes	Yes	Yes	Yes
Adjusted R^2	0.183	0.125	0.074	0.072
Observations	1653	1654	1650	1624

Notes: [+] $p < 0.1$, [*] $p < 0.05$, [**] $p < 0.01$. Clustered standard errors in parentheses. See text.

important. Further, since we expect that there might be a "turning point" during working life it is not ideal to have to impose where that cutoff might be.

As such we wish to take the logic of Equation 1 further and increase the number of age groups to map out exactly what this relationship looks like, letting the data speak for itself. However, doing so poses a number of challenges. First, as already mentioned including population age shares always necessitates at least one omission in order to identify all of the parameters and a regression constant. Secondly, there is an issue of increasing collinearity between population shares as we cut the bins into finer groups. Additionally the number of estimated parameters can become quite large. A solution to this problem that allows dimension reduction and the flexible estimation of parameters over the age distribution was proposed by Fair and Dominguez (1991) and has been used in work such as Higgins (1998) to study the effect of age structure on capital flows.[9] The idea is to estimate these parameters by fitting the coefficients on each age group with a low-order polynomial (here, a cubic). This requires two additional assumptions at Equation 1:

1. The coefficients on population age shares, α_j sum to one.

2. The coefficients on population age shares, α_j are fit with a polynomial

$$\alpha_j = \gamma_0 + \gamma_1 j + \gamma_2 j^2 + \gamma_3 j^3 \,. \tag{2}$$

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[9]To our knowledge the only other recent paper to use these estimation methods on asset returns is Juselius and Takats (2015) who show similar effects to ours on safe rates.

Essentially the approach of Fair and Dominguez (1991) involves creating three variables from the population age structure (one for each degree of the polynomial) and translates the problem from one of estimating J coefficients on J age shares to estimating three coefficients for each degree of the fitted polynomial. Adding this kind of structure to this estimation allows us to estimate some kind of effect akin to those reported in Table 1 without having to take a stance on which parts of the age distribution matter, or perhaps more importantly, where in the age distribution a turning point might appear.

To accomplish this estimation we follow Fair and Dominguez (1991) in generating three demographic variables given by

$$
\begin{aligned}
D_{1,it} &= \left(\sum_{j=1}^{J} j\, p_{j,it} - (1/J) \sum_{j=1}^{J} j \right), \\
D_{2,it} &= \left(\sum_{j=1}^{J} j^2\, p_{j,it} - (1/J) \sum_{j=1}^{J} j^2 \right), \\
D_{3,it} &= \left(\sum_{j=1}^{J} j^3\, p_{j,it} - (1/J) \sum_{j=1}^{J} j^3 \right).
\end{aligned}
\tag{3}
$$

We now split the population into $J = 13$ age groups: the share under 15, then eleven five-year age groups, and finally the share over 65. Using these population shares we can include demographic controls given by Equation 3, along with other controls from Equation 1, with

$$
R_{it} = f_i + \sum_{k=1}^{3} \gamma_k D_{k,it} + X_{it}\theta^h + \epsilon_{it}.
\tag{4}
$$

We have now replaced the problem of estimating the effect for each population share, with estimating the coefficients on the polynomial of Equation 2. Equation 4 provides us with estimates of these γ coefficients, which in turn we can use these to back out the effect across the entire age distribution given by the α_j terms from Equation 1.[10]

The resulting regressions are shown in Table 2. Of course with these regressions using the Fair and Dominguez (1991) approach, there is little to interpret in the reported coefficients. We can however see that the demographic variation is significant for all three variables for bills, bonds, and the ERP. Equity return coefficients are not individually significant, but they are jointly, with an F-statistic of 8.78. But to see how these effects net out across the age distribution we need to construct the age effects α_j.

---

[10]Note that γ_0 is a function of the other estimated γ_k due to the assumption that the α_j sum to zero, whereby $\hat{\gamma}_0 = -(\hat{\gamma}_1/J)\sum_{j=1}^{J} j - (\hat{\gamma}_2/J)\sum_{j=1}^{J} j^2 - (\hat{\gamma}_3/J)\sum_{j=1}^{J} j^3$.

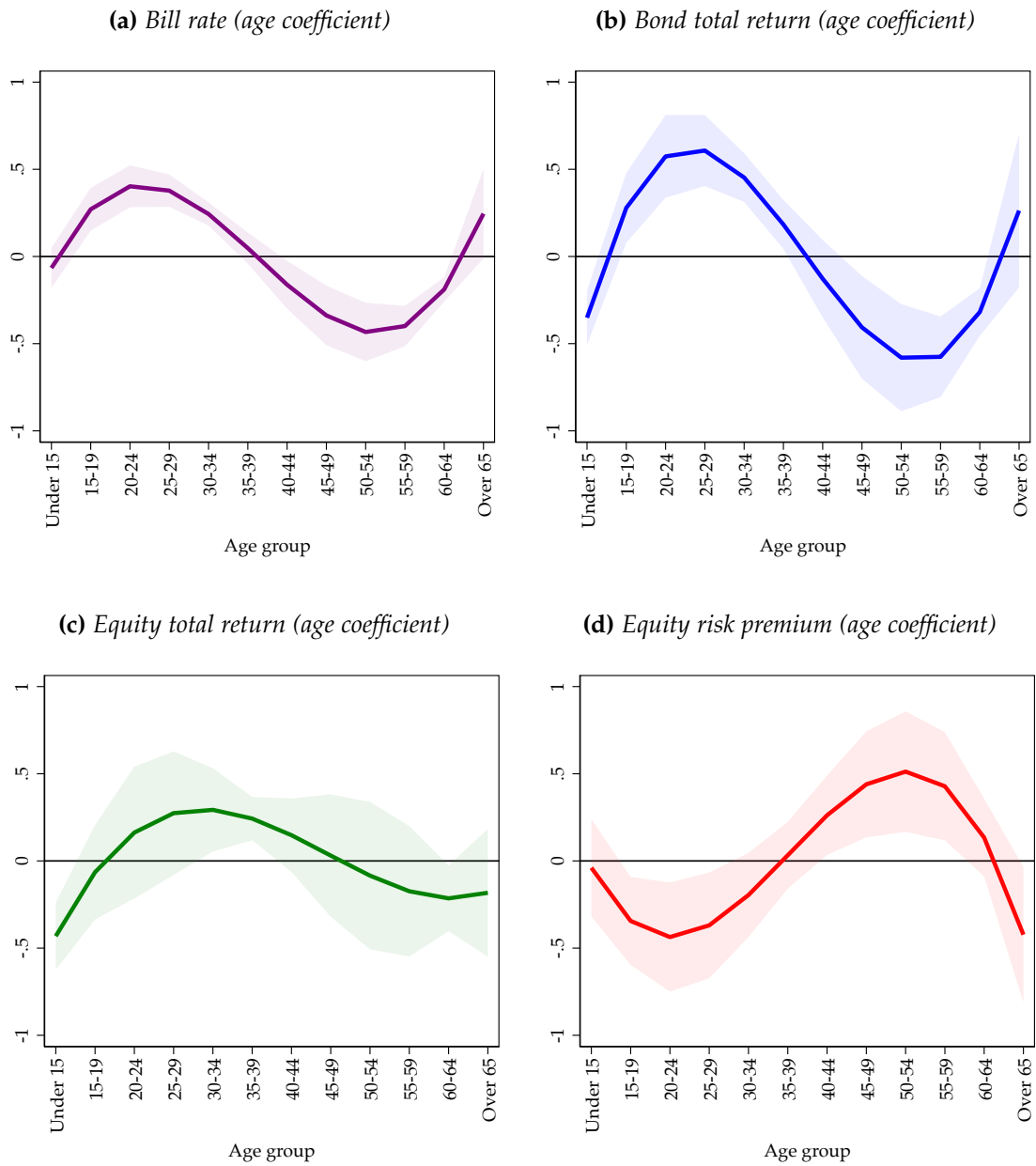**Table 2:** *Real asset returns and population structure, by asset type, Fair and Dominguez (1991) methodology*

	(1) Bill rate	(2) Bond total return	(3) Equity total return	(4) Equity risk premium
D_1	0.730**	1.267**	0.619	-0.720*
	(0.101)	(0.220)	(0.360)	(0.293)
D_2	-0.150**	-0.241**	-0.0933	0.159*
	(0.0235)	(0.0483)	(0.0737)	(0.0578)
D_3	0.00794**	0.0122**	0.00393	-0.00880*
	(0.00139)	(0.00274)	(0.00394)	(0.00305)
Controls $X_{i,t}$	Yes	Yes	Yes	Yes
Country fixed effects	Yes	Yes	Yes	Yes
Adjusted R^2	0.189	0.134	0.0755	0.0825
Observations	1653	1654	1650	1624

Notes: $^+ p < 0.1$, $^* p < 0.05$, $^{**} p < 0.01$. Clustered standard errors in parentheses. See text.

Obtaining the desired α_j effects for each age group can be achieved by using the estimated coefficients in Equation 2 and calculating standard errors using the delta method. These age-specific estimates are plotted in Figure 4. Given the significance of the demographic variables in Table 2 it is unsurprising that we now see strong age effects here. The first two panels of Figure 4 show an effect across the age distribution that is consistent with incomplete markets life-cycle theory, where high savers late in the working stages of life contribute a negative drag on safe returns, while young borrowing workers and dis-saving retirees contribute positively. We see a similar pattern, but muted in amplitude, when looking at the effect on equity returns, with smaller effects and a wider confidence band. The ERP, constructed from these safe and risky returns, is thus inverted across the age distribution. While it does not appear that our simpler equation above (with three age groups and one omitted) had the "turning point" in the wrong place, we still prefer the ability here to let the data tell us both where that ought to be and the size of the effects at each age group. These patterns are fairly robust to model specification, particularly among the two safe rates. We also find similar results if we use the *total risky* (*total safe*) returns from the JST data which weight equity and housing (bills and bonds).

These age effects are suggestive of a potentially strong channel for aging to affect asset returns. Though the charts in Figure 4 give us a good qualitative idea of these forces, they are still somewhat difficult to interpret quantitatively. Movements in a five-year age group's population share are typically small and gradual. To understand the overall strength of

Figure 4: *Implied age effects on real asset returns, by asset type, Fair and Dominguez (1991) methodology*

(a) *Bill rate (age coefficient)*

(b) *Bond total return (age coefficient)*

(c) *Equity total return (age coefficient)*

(d) *Equity risk premium (age coefficient)*

Notes: Shaded areas are 95% confidence intervals. Estimates of Equation 4. See text.

these demographic head- and tail-winds we calculate the predicted effect of the population coefficients from Table 2 holding all other covariates constant. Essentially we estimate how the demographic contribution affects predicted asset returns if we take the age specific relationships from Figure 4, and allow the population weight on each group to change.
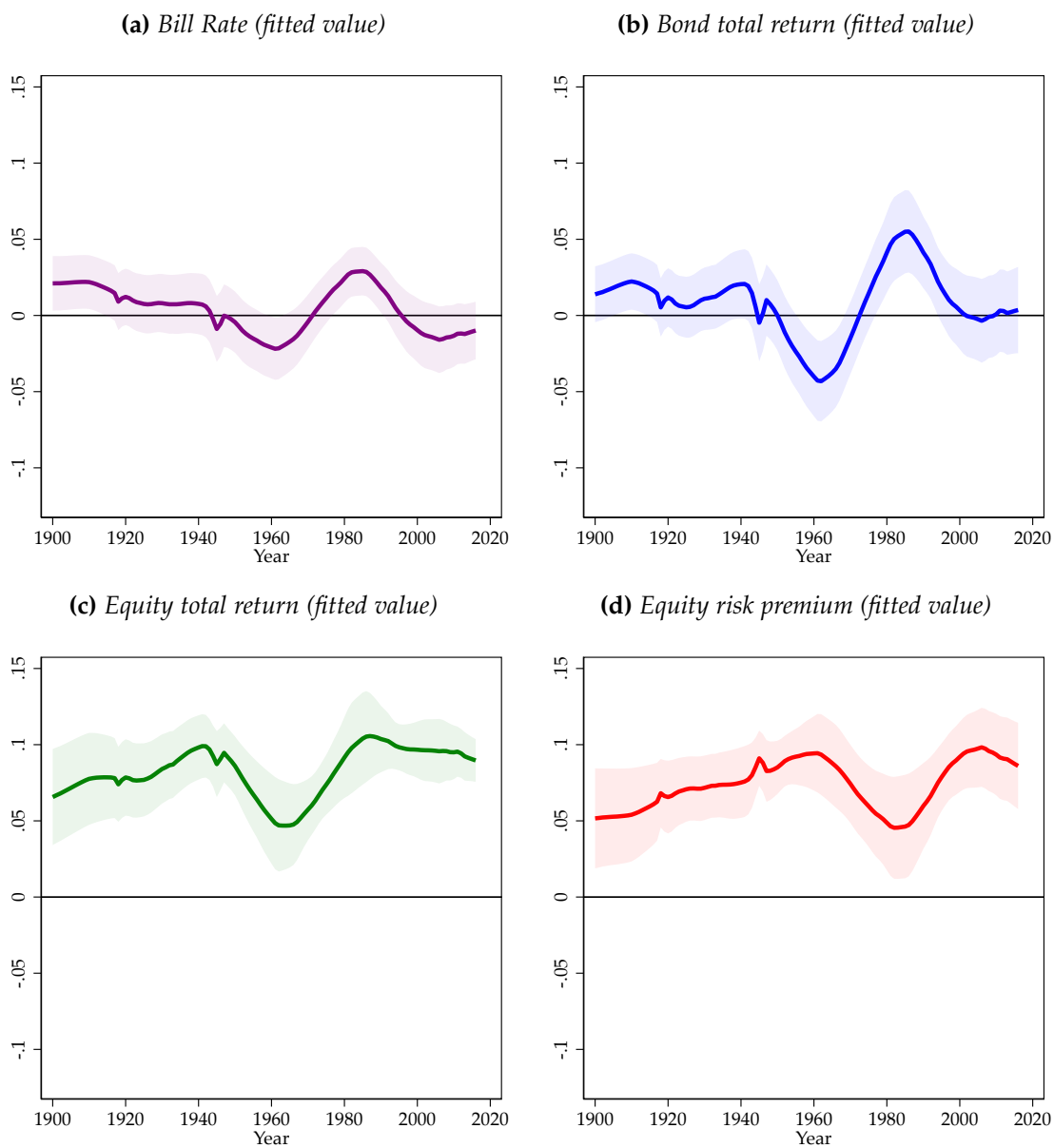
Figure 5 shows the results of this exercise for each of these four asset returns. The takeaway from the trends in Figure 5 is that the implied effects we find in this empirical exercise are quite large, suggesting in fact that demographic forces have provided significant *tailwinds* on safe rates coming off a peak in the 1980s, with much smaller and gradual effects on equity returns. We also see that all asset returns saw a large decline early in the post-war period largely due to a large crop of young dependents and relatively small working age cohorts (due to both world wars and the 1918 Spanish Flu pandemic).

Clearly, a very important message from these results is that the current weight of demographic forces on asset returns appear to be much closer to a return to an *old normal* rather than a *new normal* that some have described. While flattening population age structures are distinct from those of the pre-WW2 economies, it is clear from these figures that the boomer-driven demographic forces from the 1960s to the 2010s were a truly abnormal feature by historical standards.

To see the implication for safe returns, risky returns, and the ERP together, we graph all four point estimates in Figure 6. The chart is dominated by the story of the boomers and the swings they induce in the late 20th century. Impacts on safe returns followed an inverted-U shape. As boomers entered the workforce in the late 60s/70s both equity and safe returns rose sharply, with the latter dominating, depressing the ERP. As older boomers looked towards retirement in the 90s/00s their increased savings, with shifting preferences toward safety, pushed safe rates back down with little overall effect on risky equity returns, pushing up the ERP. Consequently the impacts on ERP followed a U-shape as secular trends in safe rates dominate.

We emphasize that our aim is not to *forecasts* returns in the short-run, where many other factors play a role, but to pinpoint some slow-moving trend components. However, we think our results suggest that a large share of observed secular shifts in asset returns in the post-WW2 era can be accounted for by demographic trends. The mechanisms in a growing literature describing how aging can drive safe rates should be taken seriously and further studied. We contribute to this literature by here documenting and, next, modeling how *relative* asset returns and the ERP are shaped by life-cycle forces. Figure 6 shows that, in our empirical analysis, purely demographic factors can account for a recent large swing each way in safe rates and a similar inverse swing in the risk premium. The goal in the rest of the paper is to see how well a calibrated theoretical model can match these kinds of shifts.

Figure 5: *Demographic head/tail-winds on U.S. real asset returns: model-predicted effects, by asset type*

(a) *Bill Rate (fitted value)*

(b) *Bond total return (fitted value)*

(c) *Equity total return (fitted value)*

(d) *Equity risk premium (fitted value)*

Notes: Shaded areas are 95% confidence intervals. Dotted lines use UN projections to 2050. See text.

Figure 6: *Shifting demographic head- and tail-winds for returns*
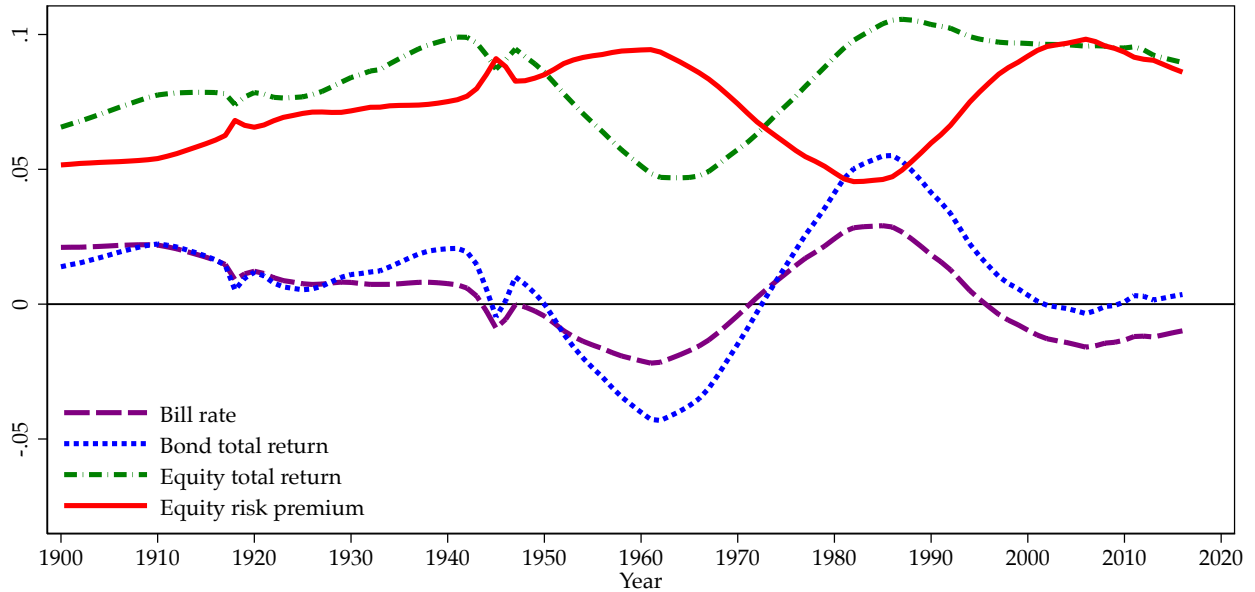
Notes: See text.

4. MODEL

Constructing a model with plausible equity risk premia is a challenging task in itself. For this reason our model closely follows prior work of Gomes and Michaelides (2008), who are able to do so in a heterogeneous-agent general equilibrium model. In particular their work contains a life-cycle portfolio decision rule, while being able to perform reasonably well at matching the mean and volatility of safe and risky returns, consumption volatility, and wealth-to-income ratios. We study the effect of changing the population age structure in a similar setup to see the extent to which the endogenous portfolio decisions of households may affect relative asset prices when the age composition of an economy changes.

We will focus our current analysis on stationary equilibria under various demographic structures. It might be ideal to study the transition path of this model economy from one demographic structure to another. To generate plausible equity risk premia we require aggregate uncertainty, which in turn requires that agents forecast aggregate state variables. While many recent innovations have improved efficiency of heterogeneous agent macroeconomic models,[11] to our knowledge doing so to solve for a transition path containing both of these features remains an open challenge. We think this is a useful step in understanding the potential for demographic forces to affect relative asset prices. Further, if households make investment decisions taking their own life-cycle factors into consideration, while

---

[11]See for example Auclert, Bardóczy, Rognlie, and Straub (2019) or Boppart, Krusell, and Mitman (2018).

largely ignoring the equilibrium effects of aging then these results may be a reasonably good approximation of the demographic head and tail winds that have prevailed.

4.1. Environment

Throughout working life, households (also referred to interchangably as individuals) earn wage income that is subject to idiosyncratic shocks. These households have access to two investment assets. The first is a riskless government bond, and the second is an equity asset that takes the form of a claim on a risky capital stock. In addition, we follow both Gomes and Michaelides (2008), as well as Fagereng, Gottlieb, and Guiso (2017), in requiring that households must pay a fixed participation cost in order to be able to participate in these equity markets. For simplicity we require that this cost only be paid once upon the first access to these markets. The retirement age is exogenously fixed at $R = 65$.

Perfectly competitive, homogeneous firms produce the consumption good using capital and labor in a constant returns to scale technology. There is a government sector that runs a social security scheme that is financed through taxes on wages, while also financing government expenditures and debt interest payments through taxes on capital gains.

4.2. Production

Technology is characterized by a Cobb-Douglas production function with total output at time t given by

$$Y_t = Z_t K_t^\alpha L_t^{1-\alpha} \, ,$$

where K is the total capital stock in the economy, L_t is the total labor supply, and Z_t a stochastic productivity shock, which follows the process

$$Z_t = G_t \, U_t \, ,$$
$$G_t = (1 + g)^t \, .$$

The variable U_t represents productivity shocks that follow a two-state Markov chain and matches the average business-cycle duration. Exogenous secular growth is determined by g. After observing the aggregate shock, firms make decisions. With δ the depreciation rate of capital, factor prices can be determined by the profit maximization problem as

$$W_t \;=\; (1-\alpha) \, Z_t \, \frac{K_t^\alpha}{L_t} \, ,$$
$$R_t^K \;=\; \alpha Z_t \left(\frac{L_t}{K_t} \right)^{(1-\alpha)} - \, \delta_t \, .$$

To allow for return volatility we include a stochastic depreciation rate. This can generate similar effects to adjustment costs while sidestepping complications that would arise in an incomplete markets model. This is used extensively in this literature and is given by

$$\delta_t = \delta + \varsigma \eta_t \, ,$$

where η_t is a standard normal shock and ς is a scaling parameter. This depreciation shock is uncorrelated with our productivity shocks.

4.3. Government

Social security is commonly used in life-cycle models as a means of generating realistic labor income processes. In our case it is also crucial in that it has meaningful impacts on the stockpiling, and drawing down, of wealth as households age.

A specification of this model without social security payments would have the effect of increasing the age-specific risk households are exposed to and exaggerate the mechanism that delivers our results. The government is also responsible for supplying the risk free assets to households. We will follow Gomes and Michaelides (2008) and model the government sector as supplying a positive net supply of bonds. It would be difficult to match portfolios found in data if the government is restricted to a zero net supply of bonds in a way that is more common in the life-cycle literature, while modeling an endogenous government supply is beyond the scope of our work and might muddy the effect of demographics.

We assume

$$SS_t^{pay} + G_t + R_t^B B_t = B_{t+1} - B_t + T_t + SS_t^{rev} \, , \tag{5}$$

where G is government consumption, B government debt, R^B the interest rate on government bonds, T tax revenues from non social security taxes, and the social security payments and revenues are given by the SS terms, which are separated here because the system is always in balance and they drop out of this budget constraint. The social security system is funded through taxes on labor income, τ_{ss}, and payments to retired individuals are given as a fraction of their lifetime earnings, λ_{ss}.

Note that our model will abstract from problems of social security imbalance that are both critical to the actual situation of the United States, and also possibly a channel that could be important to the long run implications of the model. Kitao (2014) and İmrohoroğlu and Kitao (2009) provide an excellent reference for how social security operates in life-cycle models in general, the former giving an in depth exploration of the various policy levers that can solve these imbalances.

4.4. Households

4.4.1 Preferences and labor

The household sector is populated by ex-ante identical individuals, facing finite and uncertain lives. In order to generate sufficiently large risk premiums, we adopt dynamic preferences developed by Epstein and Zin (1989). Given that ρ_i is the coefficient of relative risk aversion (CRRA), ψ_i is the intertemporal elasticity of substitution (IES), and β is the discount factor, these preferences at age, a, can be defined as

$$V_a = \left\{ (1-\beta) C_{a,t}^{1-\frac{1}{\psi_i}} + \beta \left(\mathbb{E}_a \left[s_a V_{a+1}^{1-\rho_i} \right]^{\frac{1-1/\psi_i}{1-\rho_i}} \right) \right\}^{\frac{1}{1-1/\psi_i}}.$$

There are two agent types, differing in both their relative risk aversion ρ_i as well as their EIS, ψ_i, so there is heterogeneity in both their risk aversion and willingness to substitute consumption inter-temporally. Conditional annual survival probabilities, $s_{a,i}$ are uniform across these two types such that the demographic forces act equally on both groups.

In the baseline specification all households supply labor inelastically. The labor income of individual i follows a stochastic process such that their labor income is given by $W_t \ell_{a,t}^i$, where W_t is the aggregate wage and $\ell_{a,t}^i$ is the idiosyncratic and permanent random components to their wages with

$$\ell_{a,t}^i = \exp\left(\xi_{a,t}^i \right) N_{a,t}^i. \tag{6}$$

The term $N_{a,t}^i$ represents the household's permanent idiosyncratic wage shock, which contains a deterministic age-specific trend $n_{a,t}$ and a transitory shock $\xi_{a,t}^i$. The two shocks will be described as follows,

$$N_{a,t}^i = N_{a-1,t-1}^i \exp(n_a \varepsilon_{a,t}^\epsilon) ; \qquad \ln \varepsilon_{a,t}^\epsilon \sim N(0, \sigma_\epsilon^2) ; \qquad \ln \xi_{a,t}^i \sim N(0, \sigma_\epsilon^2). \tag{7}$$

4.4.2 Demographics

Individuals live for a maximum of N periods, with conditional survival probability for individuals aged a in period t given by $s_{a,t}$. This is the probability that an individual lives to age $a+1$ conditional on having reached age a. Thus $1 - s_a$ represents the probability that an individual will die before moving to the next period. If life expectancy falls this will appear in the conditional mortality and individuals will more heavily discount the future retirement due to the lowered expectations that they will survive to enjoy consumption in later periods.

Empirically fertility rates also change every year. The relative size of cohorts are a function of both the fertility rates as well as survival probabilities. Simply put each cohort is born a certain size and dies off at a certain rate. In our model not only are demographics exogenous, but since we run simulations for households in a fixed demographic period without simulating the transition we do not actually simulate a change in the underlying population from one steady state to the next.[12] Rather we opt to keep population weights fixed in their respective years on their empirical values. This means that we lose the ability to study how agent's expectations regarding the future effect of population aging on asset returns may affect aggregates prices. While we believe that household's own expectations about their life-cycle and earnings within it matter (and we capture these in this model), we don't believe that households necessarily use expectations regarding the general equilibrium implications of aging in their investment decisions such that they would have a meaningful impact relative asset prices. In this sense it's possible that this simplification might improve the model's realism relative to one with more realistic dynamics if households do not perfectly respond to these slow moving structural trends, which seems plausible.

The Human Mortality Database (2019) provides information both on age-specific mortality as well as population sizes by age. The size of each age group is the share of that age in the HMD for a particular simulation year, normalized such that the sum of ages in model years (from the start of working life to N) sums to one. This normalization means that while we consider the effect of aging on the relative size of the workforce, we abstract from the effect of population growth more generally. We do this so as to emphasize the role of age specific life-cycle investment decisions, rather than long run population changes more broadly. For reasons described above, each simulation is a steady state that assumes these population shares and their age specific mortality rates. For future simulations we use five-year United Nations population projection data to generate these weights. We denote cohort sizes in a given period as χ_t, which is a vector containing individuals of every age group at time t.

4.4.3 Financial markets

Households have access to two financial assets, a one-period riskless asset and a risky investment opportunity. Agents buy the risk-free asset for price P_t^b, which returns one unit of the consumption good in the following period. Thus,

$$R_t^B = \frac{1}{P_{t-1}^B} - 1.$$

---

[12]As has been done very carefully in work such as Gagnon, Johannsen, and López-Salido (2016)

The return on the risky asset is denoted by R_t^K. Additionally, investors must pay a one time fixed cost, F the first time they invest in equity markets which scales with their permanent component of income $N_{a,t}^i W_t$. As in Gomes and Michaelides (2008) we interpret this cost as a combination of explicit costs associated with entering the market (e.g., brokerage fees) as well as an opportunity cost associated with acquiring information about the investment. This fixed cost is scaled by the permanent component of labor income, N_a^i and the aggregate wage, W_t. Mechanically this is useful to ensure non-participants in the model. This is a quantitatively useful way to achieve this separation mechanism.

4.4.4 Household wealth

Total liquid wealth can be consumed or invested in these two assets. Denote household wealth as cash-on-hand $X_{a,t}$ and an indicator I_p^i to denote as 1 if the individual has not yet paid the participation cost, and has positive equity holdings, and zero otherwise. Then denote the wealth of a working individual of age a and at time t as

$$X_{a,t}^i = K_{a,t}^i(1 + (1 - \tau_K)R_t^K) + B_{a,t}^i(1 + (1 - \tau_K)R_t^B) + \ell_{a,t}^i W_t - I_p^i FN_{a,t}^i W_t, \qquad (8)$$

where $K_{a,t}^i$ is the equity holding, $B_{a,t}^i$ is the bond holding, and $FN_{a,t}^i W_t$ is the cost of entering equity markets, scaled by the permanent component of the income process. This is paid only the first time an equity investment is made (when $I_p^i = 1$).

After retirement, individuals' wage income is replaced by the social security income, given by a fraction of their wage income at retirement. Additionally households are not able to borrow against future labor income, and cannot short any asset. During retirement years ($a > R$), the individual's cash-on-hand is given by

$$X_{a,t}^i = K_{a,t}^i(1 + (1 - \tau_K)R_t^K) + B_{a,t}^i(1 + (1 - \tau_K)R_t^B) + \lambda_{ss} \ell_{a,R}^i(1 - \tau_{ss})W_t - I^i FN_a^i W_t, \qquad (9)$$

where the inability to borrow or short is imposed by the constraints

$$B_{a,t}^i \geq 0, \quad K_{a,t}^i \geq 0.$$

4.5. Individual optimization

Households take prices as given and maximize utility of consumption and leisure given expectations about future aggregate wages and asset returns. A rational expectations equilibrium requires that agents accurately predict the values for wages and rental rates. In heterogeneous-agent models *without* aggregate risk in the style of Aiyagari (1994) this is not

a problem as mean zero idiosyncratic risk does not affect aggregate wages and rental rates. Since labor supply and capital stock are endogenous to household investment decisions in the presence of risky equity, we must employ an algorithm similar to that of Krusell and Smith (1998).

The household optimization problem needs to include state variables that allow agents to forecast values for K_t and P_t^B. While doing so exactly requires the infinite-dimensional wealth distribution, Krusell and Smith (1998) show that it is possible to approximate this with a small set of moments. This can be accomplished in our context using lagged values of aggregate variables K_t and P_{t+1}^B as well as realizations of the aggregate shock draw U_t and the stochastic depreciation draw η_t. These variables must now be state variables in the household value function, so

$$
\begin{aligned}
K_{t+1} &= \Gamma^K(K_t, P_t^B, U_t, \eta_{t+1}), \\
P_{t+1}^B &= \Gamma^L(K_t, P_t^B, U_t, \eta_{t+1}).
\end{aligned}
\tag{10}
$$

4.6. Solving the household's problem

The individual's problem is solved for a stationary equilibrium where individual variables are normalized to the permanent component of household labor income $N_a(G^{\frac{1}{1-\alpha}})$ and aggregate variables normalized by aggregate productivity growth $(G_t^{\frac{1}{1-\alpha}})$. Normalized variables are denoted by lowercase letters. The problem is

$$
V_a\left(x_{a,t}^i, I_p^i; k_t, \eta_t, U_t, P_t^B\right) =
$$

$$
\max_{c_{a,t}^i, h_a^{i,t}, k_{a+1,t+1}^i, b_{a+1,t+1}^i} u_a(c_a, h_a) + \beta s_i \mathbb{E}_{a,t}\left[\left(\frac{N_{a+1}^i}{N_a^i}G^{\frac{1}{1-\alpha}}\right)^{-1} V_{a+1}\left(x_{a+1,t+1}^i, I_p^i; k_{t+1}, \eta_{t+1}, U_{t+1}, P_{t+1}^B\right)\right],
$$

subject to
$$
k_{a+1,t+1}^i \geq 0,
$$
$$
b_{a+1,t+1}^i \geq 0,
$$
$$
x_{a,t}^i = k_{a+1,t+1}^i + b_{a+1,t+1}^i + c_{a,t}^i,
$$
$$
x_{a+1,t+1}^i = \frac{k_{a+1,t+1}^i(1+R_{t+1}^K) + b_{a+1,t+1}^i(1+R_{t+1}^B)}{(N_{a+1}^i/N_a^i)(G^{\frac{1}{1-\alpha}})} + w_t e^{\epsilon_i} - I_p^i FN_a^i W_t,
$$
$$
R_{t+1}^K = R(k_{t+1}, U_{t+1}),
$$
$$
w_{t+1} = W(k_{t+1}, U_{t+1}),
$$
$$
k_{t+1} = \Gamma^K(k_t, P_t^B, U_t, \eta_t),
$$
$$
P_{t+1}^B = \Gamma^L(k_t, P_t^B, U_t, \eta_t).
\tag{11}
$$

4.7. Equilibrium

A steady-state equilibrium is a set of endogenously determined prices, value functions, and policy rules that are specific to age cohorts, and rational expectations by individual agents over the evolution of all endogenously determined variables. We then have:

Households optimize: Households follow cohort specific policy rules $\{V_a, b_a, k_a\}_{a=1}^N$ which are consistent with their dynamic programming problem given by Equation 11.

Firm optimize: Firms maximize profits by setting their MPK and MPL equal to their marginal costs R_t and W_t.

Markets clear: Aggregates are equal to the sum of individual decisions, with

$$
\begin{aligned}
K_t &= \int_i \int_a N_{a-1} k^i_{a,t} \chi_i \, da \, di, \\
B_t &= \int_i \int_a N_{a-1} b^i_{a,t} \chi_i \, da \, di, \\
L_t &= \int_i \int_a N_{a-1} \ell^i_{a,t} \chi_i \, da \, di, \\
U_t K_t^\alpha L_t^{1-\alpha} &= \frac{C_t^G}{G_t^{\frac{1}{1-\alpha}}} + (1+g)^{\frac{1}{1-\alpha}} K_t - (1-\delta) K_t + \int_i \int_a P_a^i c^i_{a,t} \chi_i \, da \, di.
\end{aligned}
\tag{12}
$$

Government balance: Government obeys its own budget constraint each period, maintaining a given level of debt to GDP, as well the social security system at all times:

$$
\int_i \int_{a=0}^{a=R} \tau_{ss} l_a^i \chi_i \, da \, di = \int_i \int_{a=R}^{a=N} \lambda_{ss} \ell_{R,t} W_t \chi_i \, da \, di.
\tag{13}
$$

Prices: Prices are verified in equilibrium.

Analytical solutions are not possible in in this model. In the following section we describe the solution method to solve for a stationary equilibrium computationally.

4.8. Solution method

1. Specify forecasting equations: Γ^K and Γ^{P^B}.

2. Solve the household's problem, generating decision rules for each agent type taking prices as given and using forecasting equations to form expectations. All state variables are mapped into a discrete state space and optimal policy rules are solved by backwards induction from the final year of life.

3. Given policy functions in part 2, simulate the model for market clearing bond prices (2000 periods).

4. Use the simulated time series to update forecasting equations

5. Repeat steps 2–4 until convergence:

   - Markets clear;

   - Stable coefficients in the forecasting equations;

   - Stable equilibrium bond supply;

   - Forecasting with regression R^2 above 99%.

4.9. Simulation

Realizations of the aggregate random shock are drawn from its two state Markov distribution and individual agents' decisions are simulated conditional on their individual draws from the log-normal productivity shock.

For each time period individual behavior is simulated for every possible bond price. Then individual demands are aggregated and linear interpolation is used to determine the market clearing bond price. This determines simulated state variables for next period decisions and the process is repeated. We simulate the long-run steady state of the economy under each demographic regime. While this abstracts from the real world transition dynamics it allows for comparison of demographic effects in a way that is computationally much less burdensome.

4.10. Updating the forecasting equations

Using the simulated time series, forecasting equations are estimated using OLS regressions. For each realization of the productivity shock U_t, and given known change in the employment population ratio λ_{t+1}, we simulate the following

$$
\begin{aligned}
\ln(k_{t+1}) &= \beta_{1,0} + \beta_{1,1} \ln(k_t), \\
\ln(P^B_{t+1}) &= \beta_{2,0} + \beta_{2,1} \ln(k_t) + \beta_{2,2} P^B_t .
\end{aligned}
\tag{14}
$$

which for the baseline specification yields eight equations with separate coefficients to be estimated. Convergence of our simulation requires both that the R^2 of each of these forecasting equations is greater than 99% under each set of aggregate states and equilibrium bond supply and all coefficients in forecasting equations converge.

5. CALIBRATION

5.1. Demographics

Our key dimension of analysis is changing population structure. We take retirement age as exogenous, fixed at $R = 65$. We use annual conditional mortality rates as shown in Figure 7. Conditional annual survival probabilities could be calculated from five-year groups using the method described in Henriksen (2015) in 2050. For historical values we use the annual conditional moralities for the United States in Human Mortality Database (2019). For demographic cohort weights, we similarly use HMD for historical data while using the United Nations data interpolated to annual frequency for future projections.

As noted above, for our quantitative estimation we calculate the general equilibrium conditional on the demographic state in a given year as if it were fixed permanently. Implicitly this assumes that individuals in our economy believe that the current demographic structure, and any impact it has on prices, will persist into the future. Quantitatively estimating the model along the entire transition path would be computationally burdensome and pose a set of challenges without, in our estimation, providing particularly insightful results relative to our current approach. Particularly if one believes that the effects that household expectations regarding the general equilibrium consequences of aging are a second order effect.

Figure 7: *Survival probabilities by age*
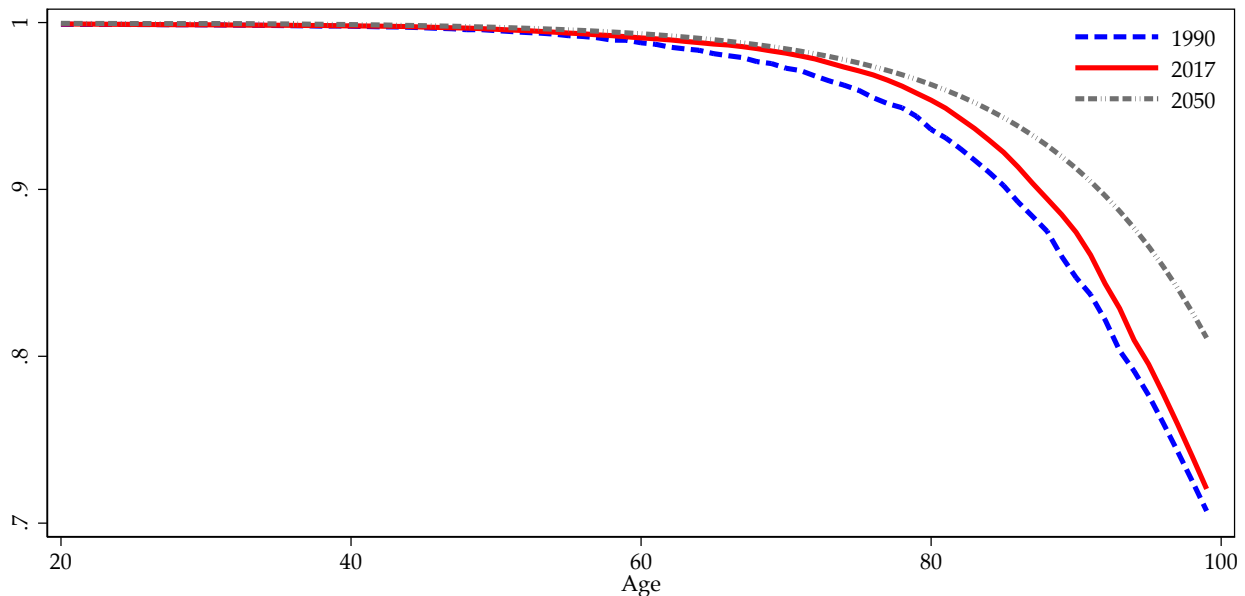
We will show results for the model calibrated to four "steady state" demographic structures in: 1970, 1990, 2017, and 2050. These reflect the age structure and life expectancy (through age specific mortality) in these years. The results in 1970 provide something of a pre-boomer baseline as the oldest members of that cohort will be 24 at that time and small players in asset markets. Our focus will be on results from 1990 to 2017, as well as forecasts to 2050. These three are particularly important because 1990 represents an early year where the entire baby boomer cohort is participating in the labor force (at this point the youngest boomer is 26 years old). In 2017 boomers were at peak savings age with the youngest at age 53 and the oldest 71. By 2050 the age range of baby boomers is 86–104, almost completely aged out of the model, and the relatively "flat" age structure by then should not see dramatic change according to the UN predictions.

5.2. Household variables

We next describe the household, production and government calibration, with details shown in Table 3.

There are two agent *types* in the economy who differ along two dimensions. The first type has low risk aversion, with CRRA $\rho_A = 1.1$, and low elasticity of inter-temporal substitution, with EIS $\psi_A = 0.05$. The second agent has higher CRRA, with $\rho_B = 12$, as well as higher EIS, with $\psi_B = 0.15$. Both agents have the same discount factor $\beta = 0.98$. Giving type A agents low risk aversion causes a reduction in early life savings due to lowered precautionary motive. A low EIS also limits life-cycle savings to smooth consumption. They thus accumulate relatively little in mid-life in preparation for retirement, with some small savings around the retirement age in an effort to smooth the consumption path from working wages to social security income. These effects induce them to endogenously accumulate little in the way of savings, with the members of this type generally being "hand-to-mouth". For those with savings a fixed participation cost induces them to rarely hold equity.[13] Type B agents will endogenously act as the major participants in both asset markets and, in most specifications, are the only holders of equity. We assume an equal share of these agent types for simplicity.

We believe these are reasonable parameters. Guvenen (2006) shows that limited stock market participation along with EZ preferences can reconcile disagreement with the macro literature on EIS parameters. This disagreement stems from micro consumption data often implying EIS close to zero and macro correlations implying a value close to 1. Models with limited participation can remedy this by separating the effect of the average consumer from
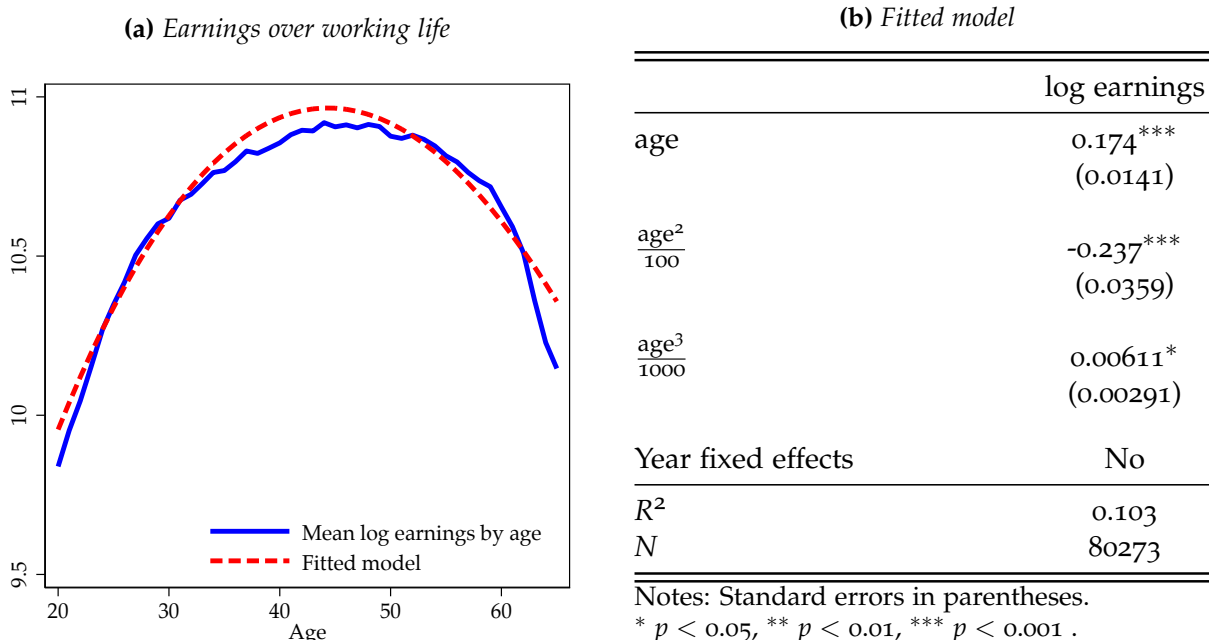
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[13]Some do in later year simulations as the ERP rises and safe assets return negative rates, but still a small share of this agent type, and in all simulations they make up a trivial size of aggregate equity holdings.

Table 3: *Calibration*

Household		
β	Time discount rate	0.98
ρ^A, ρ^B	Risk aversion type-A, type-B	1.1, 12
ψ^A, ψ^B	EIS type-A, type-B	0.05, 0.15
n_a	Age-specific trend parameters	{ 0.174 , -0.237 , 0.00611 }
s_i	Age-specific mortality rate	HMD or UN Forecasts
χ_i	Cohort size	HMD or UN Forecasts
Production		
α	Capital share	0.36
δ	Depreciation	0.10
σ_U	Volatility of aggregate productivity	0.01
ς	Depreciation shock (scaling std. normal)	0.15
Π	Aggregate shock process	$\begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix}$
Government		
B^S	Supply of government bonds, fraction of output	0.60
λ_{ss}	Social security replacement rate	0.40

that of the average investor. Both of our preference parameters for the EIS across agents are quite conservative (0.05 and 0.15), and they are consistent with the empirical literature. In particular we choose them to match Best, Cloyne, Ilzetzki, and Kleven (2020), who show that the average EIS is roughly 0.1, with little variation across the population. Given we have equal shares across types our economy has an average EIS that is exactly 0.10, and is not dramatically different between agents. Barsky, Juster, Kimball, and Shapiro (1997) find a lower bound of the EIS of close to zero and an average upper bound of roughly 0.36 with a mean of 0.18, so our estimates are conservative by their measure. Increasing the EIS for type-B agents would increase the size of the equity risk premium across all demographic specifications. Barsky, Juster, Kimball, and Shapiro (1997) also measure a mean risk tolerance of 0.24. The reciprocal of this, 4.2 is the harmonic mean risk aversion in their sample. This is much smaller than the arithmetic mean estimate for risk aversion, 12.1, due to substantial heterogeneity in their estimated risk tolerance across groups. We choose $\rho^B = 12$ for Type-B agents to match this average risk aversion estimated among stockholders in their sample, setting $\rho^A = 1.1$. With equal weights the average risk aversion in our model is 6.55.

Figure 8: *Age-specific wage calibration*

(a) *Earnings over working life*

(b) *Fitted model*

	log earnings
age	0.174^{***}
	(0.0141)
$\frac{\text{age}^2}{100}$	-0.237^{***}
	(0.0359)
$\frac{\text{age}^3}{1000}$	0.00611^{*}
	(0.00291)
Year fixed effects	No
R^2	0.103
N	80273

Notes: Standard errors in parentheses.
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

The household earnings process has an age-specific trend, n_a, which is calibrated to match the average for the PSID similar to Cocco, Gomes, and Maenhout (2005), Huggett and Kaplan (2016), and others. We fit a third order polynomial on the life-cycle income process with year fixed effects. Figure 8 shows both the sample mean of log earnings over the working life as well as our fitted model. The volatility of idiosyncratic income shocks are set to 10% per year, in line with estimates used in Cocco, Gomes, and Maenhout (2005).

The fixed cost of participation is set such that it corresponds to 8% the household's expected annual income. Moskowitz and Vissing-Jørgensen (2002) suggest per-period costs that are approximately $75–$200 each year. Our one time fixed cost is quite large, relative to that, but may capture other non-pecuniary costs that agents face.[14] Our estimate is similar to other estimates using limited participation, particularly Gomes and Michaelides (2008) whose baseline cost is 6%, though they show that a choice of 2.5% also works with a change in preference parameters. As in their work our average participation is endogenous to the choice of agents to enter equity markets. Under 1990 demographics, this will slit evenly across the two agent times and so 50% of households will participate in equity markets over their lifetime, this increases as risk free rates are pushed toward zero and it becomes optimal for the some of the higher income low EIS agents to pay the fixed cost to participate in equity markets.

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[14]Such as information costs.

Figure 9: *Government debt to GDP*
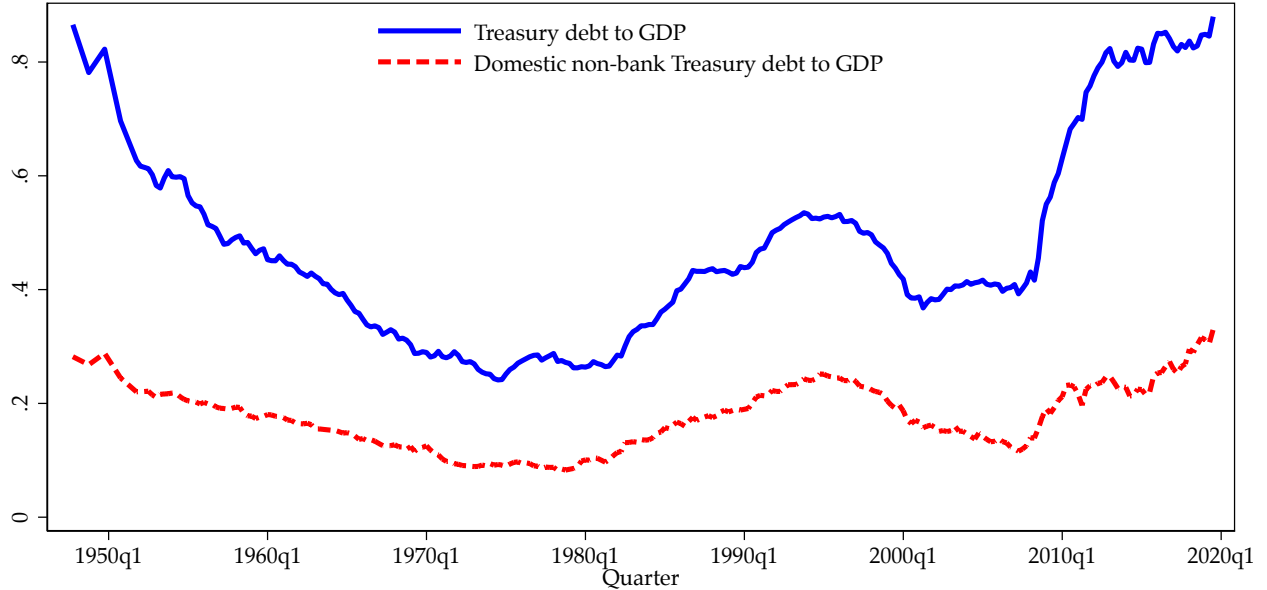
5.3. Aggregate and government variables

In order to match a business cycle duration that is on average six years, the two state Markov process for aggregate productivity is given by

$$\Pi = \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix}.$$ (15)

The volatility of aggregate productivity, (σ_u), is set at 1%. Capital's share of output is set to 36%, and depreciation is 10%. The volatility of asset returns is predominantly determined by ς, which is set at 15%.

We set the net positive supply of government bonds to be a constant share of GDP. While government debt has risen substantially over the period of study, the share held by the domestic non-bank public has remained relatively low and has seen substantially less variation. We calibrate this using Treasury debt held by the domestic non-bank investors using data from the Federal Reserve, Financial Accounts of the United States. We sum all debt held by: households, government retirement accounts, private pension funds, money market funds, mutual funds, ETFs, and closed end funds.

The observed postwar path of this non-bank treasury debt measure is shown in Figure 9 along with the overall level of treasury debt to GDP. For our benchmark model we set this at 60% of equilibrium GDP and constant over time. This is roughly the average of total treasury debt to GDP from 1990 to 2017, which is 57%. A lower value would put further

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downward pressure on safe rates in all model simulations as safe assets become scarcer.[15]

Conversely, allowing time variation with a rising supply of government debt, as has been the case since the 2000s, would put further upward pressure on safe rates in the recent years, relative to our benchmark, which would allow our model to more closely match reality (below, in the benchmark results, our model over-predicts the fall in safe rates). We nonetheless prefer our benchmark, as it provides transparency as to what can be achieved in a model without assuming a path for the bond-supply response.

While a fixed relative bond supply is a strong and unrealistic assumption, we feel it is the only way to understand the impact of demographics directly, all else equal. We also tested the model using lower values for bond supply.[16] While this has level effects on safe and risky returns, the effect on ERP is small. We note that while we fix bonds as a share of output, our model simulations will see changes in the bond supply, indeed fairly large increases over time. These come from endogenous demographic effects on output that arise from changes in capital holdings.

The retirement social security transfer: λ_{ss} is set at 0.4 using the same parameter as the benchmark model in Kitao (2014): roughly the average benefits over average earnings. The social security tax is set to clear the government's requirement to balance the social security budget. This comes to about 15%, but increases as the employment-population ratio shrinks. A model with more sophisticated government actions might wish to understand how aging affects the increased supply of safe debt to finance social security into the future. For simplicity we also abstract from accidental bequests, which occur due to early death, by assuming that the government taxes assets at 100% upon death.

6. Results

We now describe the results that come from the calibration of this model. We solve and simulate the model under a number of different demographic structures. As mentioned these represent a steady state where the demographic structure is stable.

6.1. Assets

In Figure 10a, we show the average cash-on-hand wealth across the age distribution in model simulations under the 1990 and 2017 demographic structures. We see that overall households are saving more in 2017. This is largely due to significant increases survival

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[15]Such scarcity can induce dramatic changes in stock market participation among type-A (non-participating) agents into equity markets as safe rates are pushed strongly negative territory.

[16]Specifically testing our B^S parameter in the range of: 45%–55%

probabilities in 2017 relative to 1990, particularly among old households. The general "hump" shape reflects the accumulation of wealth as individuals both buffer against idiosyncratic shocks and build up savings that can then be drawn down in retirement.

Figure 10b and Figure 10c show a striking result. Average stock and bond holdings across ages differ dramatically between these two demographic structures as households accumulate more financial wealth by retirement. The life-cycle preference to hold more safe assets is quite strong in the periods just before, and in, retirement. We see that older age households now even hold on average *fewer* safe assets relative to equity in 2017. The change in shape of these average holdings should work against our expected effect of rising equity risk premia, but is also an expected response if the safe rate is falling. As the safe rate is driven down (as we shall see), with large fractions of households in old age, general equilibrium effects will push households to endogenously choose to hold higher fractions of relatively more attractive equity. Given we don't change the riskiness of equity between the two periods, it is natural that these equilibrium forces would work in this direction.[17] Rising life expectancy likely mutes some of this effect as households desire to self-finance part of their retirement income for a longer expected period in 2017 than in 1990.

It would be tempting to look at Figure 10b and Figure 10c as a sign that demand for safe assets has fallen. This is not the case. The increasing *cohort effects* of rising population weights in these high saver groups offset the declines in average bond holding by age. The supply of bonds is fixed as a share of GDP in this model. Because of the large increase in equity holdings between the periods, capital increases, which is clear from Figure 10b. It turns out in equilibrium that this offsets the slight decline in labor force between these periods such that output rises. As such the total supply of bonds is actually *higher* in the 2017 calibration of the model. Once again, this is an equilibrium force that works against our expected result, as higher supply of bonds from the government (due to higher output and a fixed Debt/GDP) should work to increase safe return, all things equal. To use partial equilibrium logic, the demand for bonds has risen, while the supply has also risen. If the price (return) rises (falls) then the increase in demand must have dominated.

Figure 10d shows the share of equity in the financial wealth of individuals. For clarity this is the average across age of *total wealth* held in equity. Since most type-A households do not participate, if we averaged across agents this would be closer to 0.5, however this would be deceptive as type-B households also hold the vast majority of wealth in the model economy, particularly early in working life. As with all of our financial wealth dynamics, these patterns are strongly dominated by the type-B agents who are endogenously the only

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[17]Unlike Marx, Mojon, and Velde (2019) who find little variation in the ERP without allowing for time varying changes to the relative riskiness of equity.

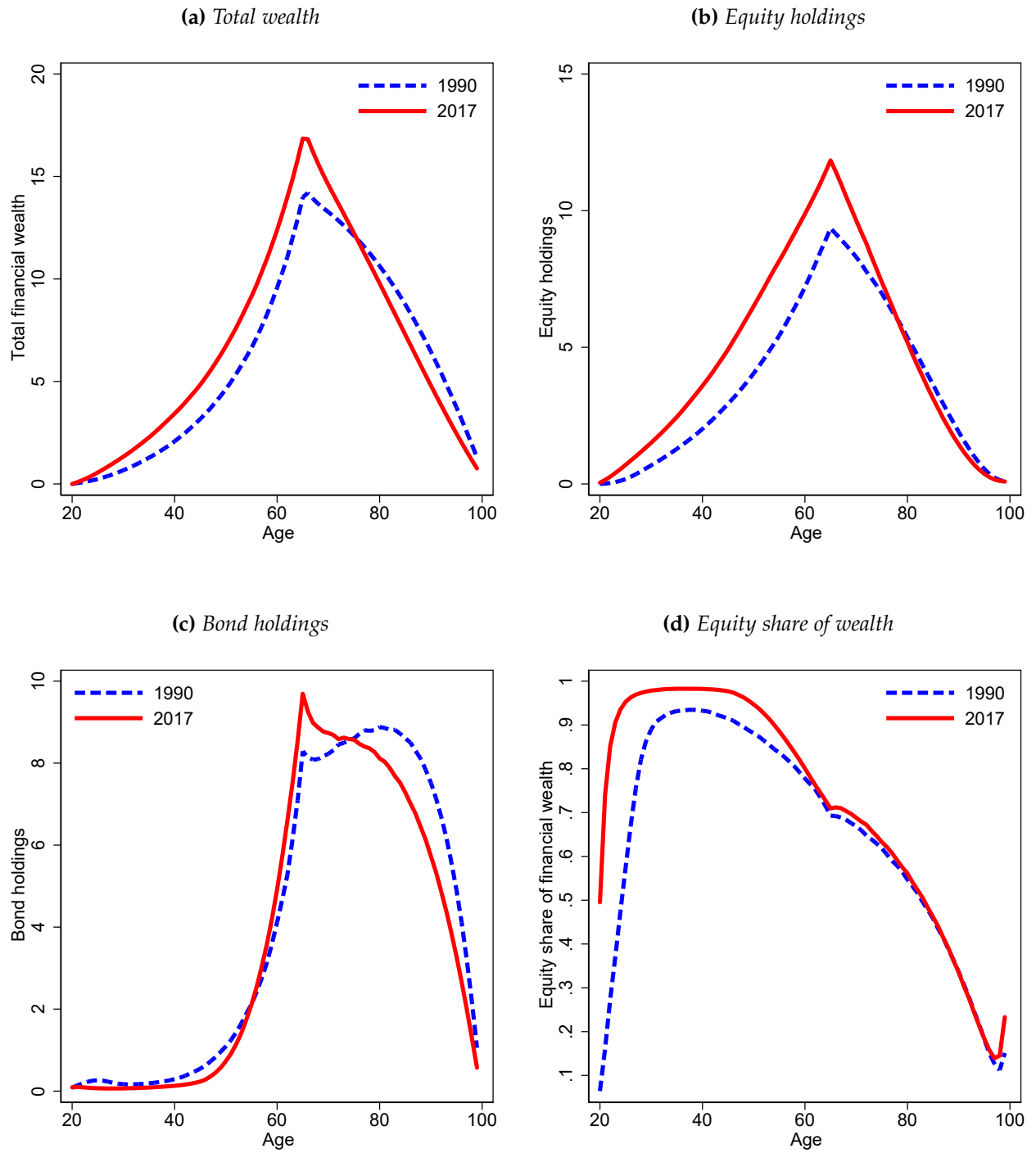**Figure 10:** *Financial wealth in the model: average by age*

(a) *Total wealth*

(b) *Equity holdings*

(c) *Bond holdings*

(d) *Equity share of wealth*

Table 4: *Returns and risk premiums in the model*

		Model			
		1970	1990	2017	2050 (projected)
Equity return, mean	$\bar{r}_e$	7.05%	8.10%	2.57%	0.89 %
s.d.	σ_e	15.41%	15.44%	15.20%	15.19%
Safe return, mean	$\bar{r}_f$	4.93%	6.00%	-0.28%	-2.49%
s.d.	σ_f	4.19%	4.26%	4.11%	3.60%
ERP	$\overline{rp}$	2.12%	2.10%	2.85%	3.38%

(with brief exception) holders of equity, and who have considerably more asset wealth than type-A agents. This is extremely high in early years as individuals have relatively small wealth and are accumulating large amounts of equity, and because the participation rate is near zero for type-A agents

This share falls somewhat dramatically as retirement approaches with a brief reversal driven by the peak in bond holdings seen in Figure 10d. On closer inspection this seems to be due to the resolution of labor income uncertainty, which becomes a certain social security payment upon retirement. All precautionary saving motives against labor income risk disappear and households reallocate accordingly. After this households resume drawing down their share of equity, though at a faster rate in 1990. In 2017 this is more pronounced due to both general equilibrium forces, as above, and the fact that lower mortality increases the net present value of this stream of social security payments substantially making a household's average future income relatively "safer." Once again this might work against our expected result, encouraging a slower draw-down of risky asset holdings. If we saw this share fixed at 1990s level our effects on the ERP and safe rates should be amplified, though the difference between these curves is small in our preferred calibration.

6.2. Returns

In Table 4 we present the baseline results for returns from our model simulations. The table reports the mean and standard deviation of equity return and the risk free rate in each simulated economy under four past demographic conditions and also under projected population demographics in 2050.

The model clearly replicates the rise in the ERP seen in the data since 1990. From a level of 212 bps in the 1970 calibration, the ERP remains level under 1990 demographics (210

bps), then rises to 285 bps in the model under 2017 demographics. In 1970 the model's safe real rate is 493 bps, and rises substantially to 600 bps in 1990 once the large boomer cohort is represented in our working age population (ages 20–64). But after 1990 our baseline calibration then finds a large decline in the equilibrium safe real rate, with a negative value of –28 bps under 2017 demographics.

The model decline in equilibrium safe interest rates is 521 bps from 1970 to 2017. This is substantially higher than estimates found in prior literature, which tend to underestimate this change, but large relative to the observed change we document in Figure 1a. This is even larger looking at the 1990–2017 change of 628 bps. As noted above, allowing bond supply relative to output to rise would likely work to mitigate some of this change, and bring this result closer in line with the data, but would make less clear the demographic channel.

We emphasize that these results represent steady state values calculated as if the empirical demographics in each year were permanent. Calculating the transition path of our model might yield slightly different results at any point in time, but the overall picture would likely be similar. Existing perfect foresight computations using methods similar to Auerbach, Kotlikoff et al (1987), which solve for a path of prices completely known to agents from one equilibrium to another after an exogenous shock, are not compatible with our solution method, which simulates the equilibrium economy many times under new realizations of aggregate shocks.[18] We require that agents accurately forecast under these shocks. Requiring such a forecast to be consistent over an entire transition is perhaps plausible, but to our knowledge is computationally infeasible. It is also unclear how aggregate shocks in such a transition should take place. A technique for solving models with aggregate risk under long run transitions is a challenging endeavor for future work, and we view our results as motivation for more development in this area.

To understand some of the forces driving our results we present model equilibrium values in Table 5. Of particular interest are the relative increases and decreases in the weights of workers and retirees in the model. these are substantial from 1990 to 2017, but accelerate dramatically in 2050. As a large weight of individuals enter peak savings age by 2017, we see the glut of savings pushes capital up, pushing up output in spite of a slightly smaller workforce. As a result the bond supply, as a fixed fraction of output, loosens. We also present consumption volatility, which though increasing as a result of households holding relatively more capital, is still not too far from empirical estimates.[19] In 2050 the population shares in high savings ages are slightly larger, in part due to normalizing

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[18]Here the exogenous shock would be to a demographic distribution

[19]Malloy, Moskowitz, and Vissing-Jørgensen (2009) show this to be about 1.7% in a 1982-2004 sample so we are quite close for our 1970 and 1990 baselines.

Table 5: *Other Model Equilibrium Outcomes*

		Model			
		1970	1990	2017	2050
Labor Supply (Productivity Adjusted)	$\bar{L} = N_a \ell_a$	1.78	1.75	1.69	1.59
Retiree Weight (Productivity Adjusted)	$N_a \ell_R$	0.262	0.279	0.333	0.467
Bonds	$B^S \times y$	1.66	1.56	1.88	1.81
Capital Stock	k	6.25	5.46	9.60	10.35
Output	y	2.75	2.59	3.15	3.00
Consumption	c	1.83	1.79	1.87	1.73
Std. Dev. Consumption Growth	σ_c	1.7%	1.69%	1.99%	2.24%

Notes: Aggregate variables are their stationary equilibrium values, defined above. Retirees weight adjusted by productivity to reflect their weight relative to labor supply which includes life-cycle average earnings.

population to one across simulations, but because there are also now large shares in old age drawing down their savings. The net effect is that capital remains somewhat flat, with the labor force falling substantially. This results in not only significant demand for the safe asset, as before, but now limited supply even relative to the 1970 baseline.

6.3. Model without aggregate risk

Our results stand out in a literature which finds much smaller demographic impacts on safe rates. Our bond return falls by a large 628 bps, much larger than the size of demographic effects in much of the existing literature. One notable feature is our ability to generate negative equilibrium real safe rates. Abel, Mankiw, Summers, and Zeckhauser (1989) show that such negative rates do not imply dynamic inefficiency and are possible in the presence of aggregate risk. Therefore one important contribution of our work is to show that such channels can be quantitatively quite important in generating larger falls in safe rates than models which include only a single safe asset. To see what happens to our model when we remove this channel we keep all other parameters fixed at their values above, while shutting down both the productivity and depreciation shocks. These returns are shown in Table 6.

Without aggregate risk our finding of rising equity risk premium disappears. Indeed there is a small decrease in the ERP in 2017. This is due to the fact that assets are not completely equivalent as equity markets still have fixed participation costs, while the supply of bonds are fixed at 60% of output, so as savings increase (and lower returns) there is some effect on relative holdings. As these are otherwise equivalent assets, there is no

Table 6: *Model Returns: No Aggregate Risk*

		1990	2017	$\Delta_{2017-1990}$
Equity return, mean	$\bar{r}_e$	6.65%	4.22%	-2.43%
Safe return, mean	$\bar{r}_f$	6.31%	4.23%	-2.08%
ERP	$\overline{rp}$	0.33%	-0.01%	-0.28%

downward sloping movement out of equity as agents age as we saw in Figure 10d, and the only difference is due to selection from the fixed entry cost. The key takeaway from this exercise is that removal of aggregate volatility still allows for demographics to push down interest rates, but with much smaller effects than found above on our baseline.

To draw a comparison between our findings and other work, we show these results along with a selection of other papers. The equity return in our *risk-free* model, with no aggregate uncertainty, is comparable to much of the existing literature modeling a fall in r^* due to demography, where the return on capital is modeled as a risk free safe asset from the perspective of the household.[20] By this metric we find a change of –243 basis points over the period, which is much closer to the results in the existing literature, than the fall in the safe rate we document above. In Table 7 we show our baseline results as well as results from this *risk-free* change relative to notable work on the subject. We report the period studied as well as the baseline effect unless otherwise noted. In general, most studies find larger decline post-1990, when the baby boomer cohort first enters peak savings age, in line with our results above.

This exercise tells us two things. The first is that our calibration has not relied on any extreme parametric choices relative to the literature to generate such dramatic results with respect to falling safe rates. When removing aggregate risk, our decline is of a similar magnitude, and by our reading fits in sensibly when doing an apples-to-apples comparison with similar work. The second is that the introduction of both risky and safe assets to this class of models can have qualitatively large impacts on the magnitude of the implied decline in safe rates due to demographic change.

This latter point is mentioned in Eggertsson, Mehrotra, and Robbins (2019b) who discuss it as one possible response to the claim that safe rates cannot be negative in equilibrium.[21] As we have shown here, the inclusion of aggregate risk can lead to an equilibrium negative risk-free interest rate as it does in our benchmark model. By including aggregate risks

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[20]By "risk free" we mean that there is still idiosyncratic risk, on labor earnings in this model as well as mortality risk, just no aggregate uncertainty. Similar to many of the papers we cite here.

[21]The other, which their paper addresses directly, is monopoly rents.

Table 7: *Falling safe real rates: model and literature versus data*

	Period	Change in real safe rate
This model		
Baseline model: safe rate, $\bar{r}_f$	1990–2017	-6.28
Risk-free model: natural rate, $r^* = \bar{r}_e$	1990–2017	-2.43
Risk-free model: bond return, $\bar{r}_f$	1990–2017	-2.08
Other single-return models		
Gagnon, Johannsen, and López-Salido (2016)	1980–2016	-1.25
Carvalho, Ferrero, and Nechio (2016)	1990–2014	$\approx$ -2 *
Lisack, Sajedi, and Thwaites (2017)	1980–2015	-1.60
Eggertsson, Mehrotra, and Robbins (2019b)[†]	1970–2015	-4.02
Summers and Rachel (2019)[‡]	1970–2019	-1.70
Data		
Rachel and Smith (2015)	1990–2015	-4.50

Notes: See text. *Measure that includes social security. [†]Their transition dynamics show much of this fall happening from the late 1980s/early 1990s. [‡]They find a 700 basis point decline in the "private" neutral rate as counterbalancing public programs have offset much of the demographic declines.

along with the safe rate we are able to open up a channel for negative interest rates that is not possible for models such as Gagnon, Johannsen, and López-Salido (2016). Our work suggests that not only are demographics quantitatively important for understanding the risk premium, but that accounting for risk will be a crucial part of understanding the possibly very large role that demographics may play in secular stagnation and the long-run downward trend in the natural rate.

6.4. Rising Wealth-to-Income Ratios

As a final check on our model, we refer to one feature also highlighted in Mian, Straub, and Sufi (2021), the rising age-wealth profile. In addition to describing movements in relative asset prices, our model is able to generate a similar steepening in wealth-to-income by age from 1990 to 2017. In particular we calculate:

$$\omega_{a,t} = \frac{W_{a,t}}{Y_{a,t}} = \frac{\sum_i (1 - \tau_K) \left(P_{i,t}^B B_{a,t}^i + P_t^K K_{a,t}^i \right)}{\sum_i \left[\ell_{a,t}^i W_t + (1 - \tau_K)(R_t^K K_{a,t}^i + R_t^B B_{a,t}^i) \right]} \tag{16}$$

Where $\omega_{a,t}$ is the age specific ratio of total wealth to total income, across all individuals i,

Figure 11: *Wealth-to-Income Across the Age Distribution*
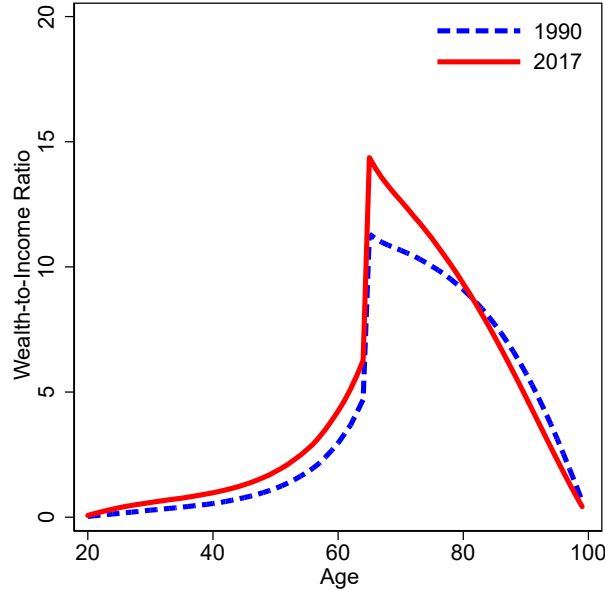
who differ by the two ex ante *types* as well as by their ex-post idiosyncratic income shocks.[22] We plot this value across the age distribution in Figure 11. This ratio is much larger in 2017 up until roughly age 80. The dramatic increase at retirement age is due to labor income being replaced by social security, shrinking the denominator.

To see the overall change in aggregate wealth-to-income in these economies we calculate:

$$\Omega_t = \frac{\sum_a W_{a,t}}{\sum_a Y_{a,t}} \tag{17}$$

As is clear from Figure 11 this value is larger in 2017 than in 1990. This is driven both by increases in the total amount of financial wealth saved over the life cycle, as well as valuation effects.

Overall we calculate that the aggregate wealth-to-income ratio in this economy rises from 2.53 to 3.11 across our simulations in 1990 and 2017 respectively. This reflects a roughly 23.1% rise over the period. This is similar to, but a bit larger than, the observed 18.3% change reported in the SCF+ data by Mian, Straub, and Sufi (2021) for the period of 1983 to 2019, and is smaller than the increase we document in Figure 1c. We see this as an additional moment that future models of demographic change and asset market outcomes will need to match, and our model does a respectable job here as well, and again this is absent any exogenously-imposed bond supply shocks.

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[22]Idiosyncratic shocks are mean zero so we sum the average across the two types, with the aggregate population normalized to one in all simulations.

7. Conclusion

Population aging has a role to play in explaining a large number of long run macroeconomic trends. In the face of evidence of a rising equity risk premium and falling safe rates, we show that there is a plausible demographic channel that may be driving *both* of these trends. In the model, a large mass of aging households—the boomers—drive a savings glut which, as they near retirement, especially depresses real interest rates on safe assets. As they age, they reduce portfolio allocations to equity, so they do not have the same effect on risky equity returns, widening the equity risk premium. Our new long-run econometric evidence accords with this interpretation.

Our work takes a simplified approach to the role of aging, but the results show that studying demographic channels is a fruitful approach to understanding the drivers of long-run trends in both safe and risky returns, and hence risk premia. Aging may operate not only through changing cohort sizes, as with the aging boomers, but also through rising life expectancy, which can decrease individuals' willingness to take on risky assets as they age, an amplifying mechanism. It is crucial to better understand the ways in which demographics affect asset returns since advanced economies will continue to face aging populations for the foreseeable future—and to help us evaluate what, if any, policy options may be placed on the table.

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A1. Replication of Poterba (2001) Findings

The core findings of Poterba (2001) with regard to the time series relationship of aging and asset returns are squarely in line with the intuition highlighted in our earlier discussion. Empirically, negative relationships were found between population in peak saving years (ages 40–64) and returns on safe assets, but in contrast there was no measured impact on returns for common stock (using the S&P 500 index). Evidence from outside the United States, using data from Canada and the UK, was unclear and weakened any conclusions drawn there. In his analysis of the historical macro data he presents a number of purely *univariate* regressions of the form:

$$R_t = \alpha + \beta D_t + \epsilon_t, \tag{18}$$

where R_t is a particular asset return (Treasury bills and long-term government bonds, and common stock), and D_t is a demographic variable of interest.

When running these tests, there is found to be a significant negative relationships for T-bills and long-term government bonds with respect to the share of population aged 40–64, and the relationship is stronger when using five-year age groups to minimize short-term variation unlikely to be related to demographic factors.

We wish revisit these well known results from Poterba (2001) using our larger dataset. Two decades have passed since this influential study, and, tapping into more data from JST and HMD, it is timely to update the analysis to include the last twenty years of boomers transitioning through peak saving years, and exploit a longer and wider panel of countries. We replicate univariate regressions like Equation 18, with country fixed effects, of the form

$$R_{it} = f_i + \beta D_{it} + \epsilon_{it}, \tag{19}$$

and *separately* for each regressor and each asset class, clustering standard errors at the country level. Additionally, we compare these estimates across various splits of our sample, first considering the entire 1870–2016 period, then the post-war period both up to 1999 and through 2017. The pre-1999 split is for ease of comparison with past results, and in particular Table 6 of Poterba (2001) which we follow closely in structuring this exercise.

We report these results in Table A.1, where we scale demographic shares and returns by 100 for ease of reading. Each row and column correspond to a univariate panel regression with one independent variable. There are some interesting differences with Poterba (2001) in these relationships between returns and age structure. The first is that we fail to find a persistently negative relationship between the share aged 40–64 with bill rates and bond returns in our 1947–1999 sub-sample, and there is even a puzzlingly significant *positive* relationships for long term bonds. These correlations across the full sample for Bills are consistent with both his findings and theory, though we find a stronger and more persistent effect for young age groups, which is consistent with theory. Otherwise this appears to be quite a mixed bag.

Similar to past work we find generally insignificant and implausibly large equity results. Notable is that while we generally fail to find a negative correlation between the share of population age 40–64 as a whole we see a very persistent correlation between this group relative to either retirees or the rest of the adult population. This is a result not consistently found in Poterba (2001).

The sign switching between the general share and these relative shares is likely suggestive that single correlations between an age group and asset returns are picking up potentially conflicting

Table A.1: *Real asset returns and demographics, by asset type, univariate regressions as in Poterba (2001)*

| Dependent Variable	Independent Variable				
	(1)	(2)	(3)	(4)	(5)
Return	Median Age	% Pop 20–39	% Pop 40–64	$\frac{\text{Pop 40–64}}{\text{Pop 65+}}$	$\frac{\text{Pop 40–64}}{\text{Pop 20+}}$
Sample: 1870–2016					
Bill rate	-0.045[*]	0.136	-0.139[+]	-0.001	-0.359[*]
	(0.041)	(0.087)	(0.078)	(0.003)	(0.128)
Bond total return	0.213[**]	0.256	0.203[+]	-0.013[**]	-0.226[*]
	(0.050)	(0.136)	(0.092)	(0.003)	(0.203)
Equity total return	0.201	41.63	22.14	-1.136	-7.520
	(0.115)	(21.34)	(17.88)	(0.807)	(23.52)
Sample: 1947–1999					
Bill rate	0.485[**]	0.486[**]	0.257	-0.042[**]	-0.466[*]
	(0.0797)	(0.118)	(0.206)	(0.006)	(0.171)
Bond total return	0.940[**]	1.062[*]	0.471	-0.067[**]	-0.900[+]
	(0.162)	(0.361)	(0.288)	(0.01)	(0.351)
Equity total return	0.390	83.32[+]	-4.024	-2.074	-65.37[+]
	(0.402)	(34.10)	(64.13)	(1.801)	(29.58)
Sample: 1947–2016					
Bill rate	0.139[**]	0.414[**]	0.027	-0.026[**]	-0.406[*]
	(0.038)	(0.087)	(0.097)	(0.005)	(0.127)
Bond total return	0.581[**]	0.460[+]	0.550[***]	-0.06[**]	-0.475[+]
	(0.104)	(0.230)	(0.116)	(0.01)	(0.242)
Equity total return	0.091	75.09[*]	-6.863	-0.868	-54.86[+]
	(0.238)	(25.24)	(35.80)	(2.207)	(25.80)

Notes: [+] $p < 0.1$, [*] $p < 0.05$, [**] $p < 0.01$. Clustered standard errors in parentheses. Each row and column correspond to a panel regression with one independent variable. Both population shares and asset returns are in percentage points. See text.

relationships with other parts of the age distribution. Because changes in relative size of age groups progress through the distribution movements of these shares are highly colinear. Thus the inclusion of just one share is quite difficult to interpret with any clarity as trends in other shares cannot be fully separated when estimated in this way. We believe the empirical results from section 3 provide a much better test of the significance of age-specific relationships in the macro data.
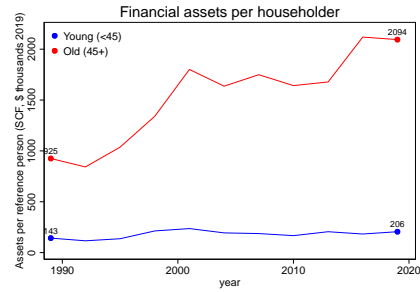
A2. Demographics and financial assets-by-age

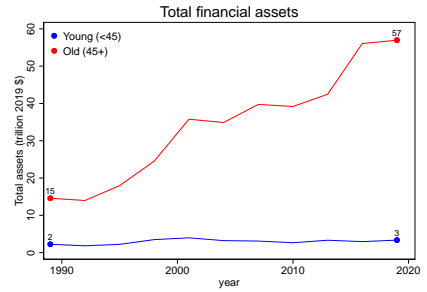**Figure A.1:** *Demographics and financial assets-by-age trends in the U.S.*

(a) *Number of households*
in old and young groups
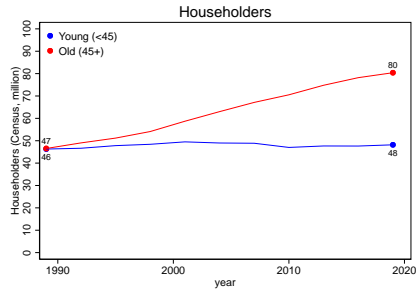
(b) *Financial assets per person*
in old and young groups

(c) *Total financial assets*
in old and young groups

Sources: Federal Reserve and Survey of Consumer Finances data.