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ABSTRACT

We study optimal monetary policy during temporary supply contractions when aggregate demand has inertia and expansionary policy is constrained. In this environment, it is optimal to run the economy hot until supply recovers. Positive output gaps in the low-supply phase lessen the negative output gaps expected to emerge once supply recovers. However, the policy does not remain loose throughout the low-supply phase: The central bank undoes the initial interest rate cuts once aggregate demand gains momentum. If inflation also has inertia, the central bank still overheats the economy during the low-supply phase but gradually cools it down over time.

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https://www.dropbox.com/s/x2i8a1o26ql69pa/SupplyShocks_public.pdf?dl=0

1. Introduction

The U.S. headline inflation reached 7 percent during 2021, vastly exceeding the Fed's stated average inflation target. Similar inflation gaps were observed all around the world. These gaps emerged primarily from the clash between the brisk recovery in aggregate demand, supported by expansionary policies, and a weaker recovery in aggregate supply, due to Covid-related bottlenecks. The static picture was one of overheating, which triggered widespread concern that central banks were falling behind the curve. Throughout most of 2021, major central banks were reluctant to heed the advice to tighten monetary policy, arguing that the supply bottlenecks were only *temporary*, and hence unlikely to generate lasting overheating.

In this note, we characterize the *optimal* monetary policy response to a temporary supply contraction. As a benchmark, observe that in the standard New Keynesian (NK) model, the optimal policy in response to a supply shock is to *raise* interest rates. This is done to reduce aggregate demand to match the lower aggregate supply. Only once aggregate supply recovers, it is optimal to lower the interest rate and boost aggregate demand.

Set against this benchmark, we analyze the optimal policy with two realistic frictions. First, we assume *aggregate demand inertia*: past spending decisions affect future spending. This type of inertia can emerge from several frictions, e.g., habit formation or infrequent spending adjustments. Second, we assume *expansionary policy constraints:* when the output gap is negative, the central bank cannot instantly raise aggregate demand to its desired level. This might be because the central bank cannot cut the interest rate sufficiently (e.g., due to the zero lower bound) or because it prefers to adjust the interest rate gradually due to frictions such as policy uncertainty or concerns with financial stability (see, e.g., Bernanke (2004)). We capture these constraints in reduced form by requiring the central bank to follow a standard Taylor rule *once* aggregate supply recovers (but we allow the central bank to set the optimal policy while supply is temporarily low).

Our main result is that, with aggregate demand inertia and constraints on expansionary policy, it is *optimal for the central bank to run the economy hot during a temporary supply contraction*. When aggregate demand has inertia, overheating the economy in the low-supply phase ensures that the economy has higher aggregate demand once aggregate supply recovers. Having a higher aggregate demand in the high-supply phase is useful because it alleviates the constraints on expansionary policy and accelerates the recovery. The optimal policy balances the costs of positive output gaps during the low-supply phase with the benefits of faster recovery and less negative output gaps in the high-supply phase.

Our analysis does *not* suggest that monetary policy should remain loose *throughout* the low-supply phase. Tempering our main result, we find that in the low-supply phase the central bank *frontloads* the interest rate cuts and then quickly normalizes the interest rate once the output gap reaches its desired (positive) level. This second result is also driven by the inertia in aggregate demand. With inertia, the initial expansionary monetary policy creates aggregate demand momentum, which keeps the output gap close to its desired (positive) level without the need for low interest rates. Keeping the interest rates "too low for too long" overheats the economy beyond the optimal output gap.

While our baseline model assumes fully sticky prices, our main results hold also when prices are partially flexible and inflation responds to output gaps. When inflation is determined according to the standard New-Keynesian Phillips Curve (NKPC), our analysis is mostly unchanged. In a temporary supply contraction, the central bank (typically) induces positive inflation gaps along with positive output gaps. Once aggregate supply recovers, the inflationary pressure flips sign and the central bank fights disinflation and negative output gaps. As before, the central bank runs the economy hot in the low-supply phase, to mitigate the future negative gaps it expects in the high-supply phase.

When inflation is determined by an inertial Phillips curve (e.g., because price setters have backward-looking expectations), the optimal policy features richer dynamics. With inflation inertia, the central bank initially overheats the economy and gradually cools it down while it waits for the aggregate supply recovery. As the recovery is delayed, inflation gradually builds up and the central bank faces a more severe trade-off between inflation and output gaps. Running the economy hot becomes increasingly costly and the central bank optimally "undoes" some of the overheating it has initially induced.

Literature. This note applies and extends our earlier analysis in Caballero and Simsek (2021). In that paper, we provide a microfoundation for the inertial behavior of aggregate demand that we assume in this paper. We characterize the optimal monetary policy with inertial aggregate demand, and show that a central bank facing a negative output gap frontloads interest rate cuts and "overshoots" asset prices. In an appendix, we also show that the central bank might *preemptively* overshoot asset prices when it expects aggregate demand to be below potential output in the future, e.g., because of a temporary supply shock. Here, we focus on temporary supply shocks and characterize the optimal policy in greater detail. We also use a more standard model (a minor modification of the textbook New-Keynesian model) and we focus on the optimal path of output, inflation, and interest rates—rather than on the path of asset prices.

Our note is related to a New-Keynesian literature that investigates the limits of the "divine coincidence" of monetary policy: the idea that central banks do not face a trade-off between stabilizing inflation and output. In the textbook model, the divine coincidence applies not only for aggregate demand shocks but also for standard supply shocks (such as oil shocks). A large literature circumvents the divine coincidence by introducing "cost-push shocks" that affect the wedge between the second-best output (which features real rigidities but no nominal rigidities) and the first-best output (which features no rigidity). With these shocks, the central bank can still replicate the second-best output by stabilizing inflation, but this is no longer desirable from a welfare perspective. As highlighted by Blanchard and Galí (2007), while cost-push shocks help break the divine coincidence, they are not directly related to supply shocks. Blanchard and Galí (2007) focus on supply shocks in an environment with real wage rigidities. In this context, a contractionary supply shock reduces the second-best output more than the first-best output, creating cost-push-like effects. The central bank faces a trade-off between allowing for some inflation and stabilizing the "excessive" decline in output. We also focus on supply shocks but our mechanism is different. In our model, there is an *intertemporal* breakdown of the "divine coincidence": a positive output gap during the low-supply phase shrinks the negative output gap that is expected to emerge once aggregate supply recovers.

Motivated by the Covid-19 episode, Guerrieri et al. (2021) build a model with a *cross-sectional* breakdown of the divine coincidence. They study a multisector economy with downward wage rigidity subject to a reallocation shock. The contracting sectors experience high unemployment but no wage or price decline, due to the downward rigidity, while the expanding sectors experience positive output gaps with high inflation. Guerrieri et al. (2021) show that, under some conditions, the central bank may want to run the economy hot in order to accelerate the reallocation process. We propose a different and complementary rationale for overheating. Our mechanism suggests more overheating when the supply shock is temporary, whereas the mechanism in Guerrieri et al. (2021) would suggest more overheating (under appropriate conditions) when the reallocation shock is permanent.[1]

A central feature of our model is aggregate demand inertia. This type of inertia emerges from various sources, such as infrequent adjustment of spending decisions or habit formation. An extensive literature documents the infrequent adjustment of durables consumption and investment (see Bertola and Caballero (1990) for an early survey). There is also a literature that emphasizes infrequent re-optimization for broader consumption

[1]See Aoki (2001); Benigno (2004); Woodford (2005); Rubbo (2020); Fornaro and Romei (2022) for other analyses of how sectoral heterogeneity affects optimal monetary policy. The common theme in this literature is that monetary policy is also concerned with relative prices.

categories—due to behavioral or informational frictions—and uses this feature to explain the inertial behavior of aggregate consumption (e.g., Caballero (1995); Reis (2006)) as well as asset pricing puzzles (e.g., Lynch (1996); Marshall and Parekh (1999); Gabaix and Laibson (2001)). Habit formation also introduces inertia into aggregate spending (see Woodford (2005) for an exposition). Fuhrer (2000); Amato and Laubach (2004) embed habit formation into standard business cycle models used for monetary policy analysis. We contribute to this line of work by analyzing the optimal monetary policy response to a temporary supply shock when there is demand inertia.

The rest of this note is organized as follows. Section 2 introduces our baseline model, with fully fixed prices. Section 3 characterizes the optimal monetary policy in this environment and establishes our main results. Section 4 extends our baseline model to add partially flexible prices and inflation. This section corroborates the monetary policy implications of the simpler model and establishes additional results when the inflation block of the model also features inertia. Section 5 provides final remarks. The online appendix contains the omitted proofs and results as well as the extensions of the baseline model.

2. A simple model with aggregate demand inertia

In this section, we describe our model's environment. It features a temporary supply shock, inertial aggregate demand, and constraints on expansionary policy in the high-supply state—that we capture with a standard Taylor rule. We also characterize the equilibrium in a benchmark case with *no* reason for overheating the economy during the low-supply phase.

For our baseline model, we assume that goods' prices are fixed. In this inflation*less* context, *overheating* simply means a positive output gap. We introduce a Phillips curve in Section 4, where we confirm that our main results hold in that richer environment.

IS curve with inertial aggregate demand. Consider a discrete time model and let $y_t = \log Y_t$ denote log output, which is determined by aggregate demand. Suppose the log-linearized IS curve is given by

$$y_t = \eta y_{t-1} + (1-\eta)\left(-\left(i_t - \rho\right) + E_t\left[y_{t+1}\right]\right), \tag{1}$$

where ρ is the households' discount rate and i_t is the interest rate at time t (the nominal and real interest rates are the same since prices are fixed). When $\eta = 0$, this reduces to the standard IS curve of the benchmark New Keynesian model. We assume $\eta > 0$, which captures inertia in spending decisions. This kind of inertia in the IS curve is broadly found in, e.g., models with consumption habits (see, e.g., Woodford (2005)), or in models with sluggish consumption adjustment (see, e.g., Caballero and Simsek (2021)).[2] Our IS curve is parsimonious and abstracts from many other factors that might affect aggregate demand (see the concluding section for how fiscal policy would affect our analysis).

Temporary supply shocks. There are two states $s_t \in \{L, H\}$ with potential outputs $y_L^* < y_H^*$. The economy starts in state L and transitions to state H with probability λ in each period. Once the economy is in state H, it stays there (i.e., H is an absorbing state).

Taylor-rule constraint on expansionary policy. After the economy switches to the high-supply state, the central bank sets the interest rate according to a standard Taylor rule:

$$i_t = \rho + \phi \left(y_t - y_H^* \right) \qquad \text{if } s_t = H. \tag{2}$$

Here, ρ is the long-run "rstar" for this economy and $\phi > 0$ is a non-negative coefficient that captures the strength of the interest rate reaction to the output gap. As we will see, this Taylor rule is not fully optimal in our context: it implements a zero output gap in the long-run but not necessarily in the short run. We view this rule as a stand-in for *unmodeled constraints on expansionary policy,* such as restrictions on interest rate cuts or frictions that induce the central bank to adjust the interest rate gradually. In Appendix B, we show that our main result applies to a scenario in which the central bank sets the interest rate optimally also in the high-supply phase but subject to a zero lower bound (ZLB) constraint.

Central bank's problem in the low-supply phase. In the low-supply state, the central bank sets the policy interest rate to minimize the present discounted value of quadratic output gaps, $E_t \left[\sum_{h=0}^{\infty} \beta^h \frac{\left(y_{t+h} - y_{s_{t+h}}^* \right)^2}{2} \right]$. We assume the central bank sets the interest rate *without commitment.* We can then formulate the policy problem recursively as

$$\begin{aligned} V_L \left(y_{t-1} \right) &= \max_{i_t, y_t} - \frac{\left(y_t - y_L^* \right)^2}{2} + \beta E_t \left[V_{s_{t+1}} \left(y_t \right) \right] \\ y_t &= \eta y_{t-1} + (1 - \eta) \left(- \left(i_t - \rho \right) + E_t \left[Y_{s_{t+1}} \left(y_t \right) \right] \right). \end{aligned} \tag{3}$$

Here, $Y_s \left(y_{-1} \right)$ and $V_s \left(y_{-1} \right)$ denote the output and the central bank's value, respectively,

[2] Large-scale New-Keynesian models, e.g., the Fed's FRB/US model, assume inertia because it helps match the observed gradual response of spending to a variety of exogenous shocks (see Brayton et al. (2014)).

when the current state is $s \in \{H, L\}$ and the most recent output is y_{-1}. The central bank takes its future decisions as given and sets the current interest rate and output to minimize quadratic gaps, subject to the inertial IS curve.

Benchmark without constraints on expansionary policy. Let us start with a "first-best" benchmark case in which the central bank faces no constraints on expansionary policy. Specifically, suppose the central bank minimizes output gaps *also* in the high-supply state $s = H$ by solving an analogue of problem (3). In this benchmark, the central bank can achieve zero output gaps in every period and state, $y_t = y^*_{s_t}$, since there is always a feasible interest rate that ensures a zero output gap. Let us solve for these interest rates.

Consider the high-supply state H. Using $y_t = y_{t+1} = y^*_H$, the IS curve (1) implies

$$i_{t,H} = \rho - \frac{\eta}{1-\eta}\left(y^*_H - y_{t-1}\right).$$

If aggregate demand has recently been weak, $y_{t-1} < y^*_H$, *the interest rate needs to be cut below its steady-state level* to ensure the economy operates at its potential. In particular, for the first period in which the economy transitions to the high-supply state, we obtain

$$i_{tran,H} = \rho - \frac{\eta}{1-\eta}\left(y^*_H - y^*_L\right). \tag{4}$$

The central bank needs to *cut* the rate by a greater amount after the transition when aggregate demand has more inertia (higher η), and when the temporary supply shock is more severe (larger $y^*_H - y^*_L$).

Next consider the temporary supply shock state L. Using $y_t = y^*_L$ and $E[y_{t+1}] = \lambda y^*_H + (1-\lambda) y^*_L$, the IS curve (1) implies

$$i_{t,L} = \rho + \lambda\left(y^*_H - y^*_L\right) - \frac{\eta}{1-\eta}\left(y^*_L - y_{t-1}\right). \tag{5}$$

When recent output is equal to potential, $y_{t-1} = y^*_L$, the interest rate in state L is *above* its steady-state level, ρ. Since supply is temporarily low but is expected to recover (and this expectation raises current demand), the central bank raises the interest rate to ensure that current demand is in line with the reduced supply. When $y_{t-1} \neq y^*_L$, the interest rate also accounts for the inertia in aggregate demand.

These interest rate expressions hint that constraints on expansionary policy has the potential to cause problems (especially) during the transition from state L to state H. We next turn to our main case.

3. Overheating with inertia and constrained expansionary policy

In this section, we establish our main result that the optimal policy *overheats* the economy during the temporary supply shock state. The central bank achieves this by frontloading monetary policy (i.e., cutting rates early and normalizing rates quickly), which generates aggregate demand *momentum*. The reason for optimally overheating the economy during the supply-shock phase is to increase the starting level of aggregate demand once supply constraints dissipate and the expansionary policy constraints become binding.

We start by characterizing the equilibrium in the high-supply state $s = H$.

Lemma 1. *Suppose the economy has switched to the high-supply state, $s = H$, with past output y_{t-1}. Then, the output gap converges to zero at a constant rate:*

$$Y_H(y_{t-1}) - y_H^* = \gamma_H (y_{t-1} - y_H^*), \tag{6}$$

where $\gamma_H \in (0,1)$ is the smallest root of the polynomial $P(x) = x^2 - x\left(\frac{1}{1-\eta} + \phi\right) + \frac{\eta}{1-\eta}$. Current output is increasing *in past output, $\frac{dY_H(y_{t-1})}{y_{t-1}} = \gamma_H > 0$. We also have $\frac{d\gamma_H}{d\eta} > 0, \frac{d\gamma_H}{d\phi} < 0$: more inertia or less reactive expansionary policy (greater η or smaller ϕ) makes output more sensitive to past output (greater γ_H).*

The central bank's value function is given by

$$V_H(y_{t-1}) = -\theta_H \frac{(y_{t-1} - y_H^*)^2}{2}, \qquad \text{where } \theta_H = \frac{\gamma_H^2}{1 - \beta\gamma_H^2}. \tag{7}$$

Over the range, $y_{t-1} < y_H^*$, the value function is increasing in past output $\frac{dV_H(y_{t-1})}{y_{t-1}} > 0$.

The first part of the lemma shows that output *eventually* reaches its potential level, y_H^*. However, the convergence is not immediate and output is influenced by demand. Importantly, output is increasing in past output, and more so if aggregate demand has more inertia or the expansionary policy is more constrained (less reactive to output gaps). The last part shows that, as expected, the value function depends on the initial output gap and on the convergence rate.

For intuition, consider the relevant case with $y_{t-1} < y_H^*$. The recent decline in output along with inertia keeps current demand low, and the Taylor rule cannot immediately bring demand back to potential. A greater past output increases current demand, accelerates the recovery, and increases the central bank's value. Naturally, these effects are stronger when aggregate demand has more inertia and the policy is more constrained.[3]

Next consider the equilibrium in the low-supply state $s = L$. Using (7), we can rewrite problem (3) as

$$V_L(y_{t-1}) = \max_{y_t} -\frac{(y_t - y_L^*)^2}{2} + \beta\left((1-\lambda)V_L(y_t) - \lambda\theta_H\frac{(y_t - y_H^*)^2}{2}\right). \tag{8}$$

We dropped the IS curve, which determines the interest rate the central bank needs to set to implement the optimal output level. Note that the value function does not depend on past output: $V_L(y_{t-1}) \equiv V_L$ is constant. The optimality condition is:

$$y_L - y_L^* = \beta\lambda\theta_H(y_H^* - y_L). \tag{9}$$

This leads to our main result.

Proposition 1. *Suppose the economy is in the temporary supply shock state, $s = L$, with past output y_{t-1}. The central bank implements the constant output level $y_L \in (y_L^*, y_H^*)$ that solves (9). The central bank chooses a level of output that induces positive output gaps in the current low-supply state (current overheating), $y_L > y_L^*$, and negative output gaps after transition to the high-supply state (future demand shortages), $Y_H(y_L) < y_H^*$.*

The central bank targets the optimal output by setting the interest rate

$$i_{t,L} = \rho + \lambda(Y_H(y_L) - y_L) - \frac{\eta}{1-\eta}(y_L - y_{t-1}). \tag{10}$$

Starting with $y_{-1} < y_L$, we have $i_{0,L} < i_{t,L} \equiv i_L$ for $t \geq 1$, where $i_L = \rho + \lambda(Y_H(y_L) - y_L)$. The central bank initially sets a low interest rate and then normalizes the interest rate and keeps it at a constant level until the transition to the high-supply state.

The first part characterizes the optimal output in the low-supply state. For intuition, observe that the left side of (9) captures the marginal cost of overheating and the right side of (9) captures the marginal benefit from overheating. When output is at its potential level, $y_L = y_L^*$, the marginal cost of overheating is zero but the marginal benefit is strictly positive. Therefore, the central bank optimally induces some overheating. Overheating *in the current period* mitigates the demand shortage and accelerates the recovery *in future periods* after the transition to high supply. Observe also that the marginal benefit from overheating declines as y_L rises toward y_H^* and it becomes zero when $y_L = y_H^*$. Therefore,

[3] In the limit without inertia or with unconstrained policy, output immediately converges to its potential and is not influenced by past output, $\lim_{\eta\to 0}\gamma_H = \lim_{\phi\to\infty}\gamma_H = 0$.

there is a unique interior optimum $y_L \in (y_L^*, y_H^*)$. The central bank stops short of overheating to the point that the economy would have no (negative) output gaps after the transition to the high-supply state.

The second part of Proposition 1 shows that the central bank does *not* keep the interest rate low *throughout* the low-supply phase. Rather, the central bank frontloads the interest rate cuts and then quickly normalizes the interest rate once the output gap reaches its target level (y_L). This feature is also driven by the inertia in aggregate demand. Recall the IS curve (1)

$$y_t = \eta y_{t-1} + (1-\eta)\left(-\left(i_t - \rho\right) + E_t\left[y_{t+1}\right]\right).$$

With inertia ($\eta > 0$), a greater past output y_{t-1} supports a greater current output y_t for any given interest rate i_t (and expected output $E_t[y_{t+1}]$). Therefore, once the central bank raises the output to its target level, it does not need to keep the interest rate low to keep the output at this level. The initial expansionary monetary policy creates aggregate demand *momentum*. This demand momentum keeps the output gap close to its desired (positive) level without the need for low interest rates. Keeping the interest rates "too low for too long" would overheat the economy beyond the optimal output gap.

Numerical illustration. Figure 1 illustrates the equilibrium in a numerical example. In this simulation, the economy starts in the temporary low-supply state with the most recent output equal to potential output in the low-supply state, $y_{-1} = y_L^*$. The economy transitions to the high-supply state in period seven.

The (black) dotted lines plot the equilibrium in the first-best benchmark case, where the planner does not face any constraints on expansionary monetary policy. In this benchmark, the policy keeps output at its potential level both before and after the transition to the high-supply state. Before the transition, the policy sets a relatively *high* interest rate (see (5)). After the transition, the policy aggressively cuts the interest rate (for one period) to raise aggregate demand to match the higher aggregate supply level (see (4)).

The (blue) solid lines plot the equilibrium characterized in Proposition 1, where the planner faces the Taylor-rule constraint on expansionary policy. Unlike in the first-best benchmark, the policy induces *overheating* in the low-supply state. The policy achieves this by *cutting* the rate aggressively in the first period while the economy is in the low-supply state. Once the policy brings output in the low-supply state to the optimal level of overheating, $y_L > y_L^*$, it raises the interest rate to keep output constant until the economy transitions to the high-supply state. After the transition, the policy cuts the interest rate once again to raise aggregate demand toward the higher aggregate supply level. Due to

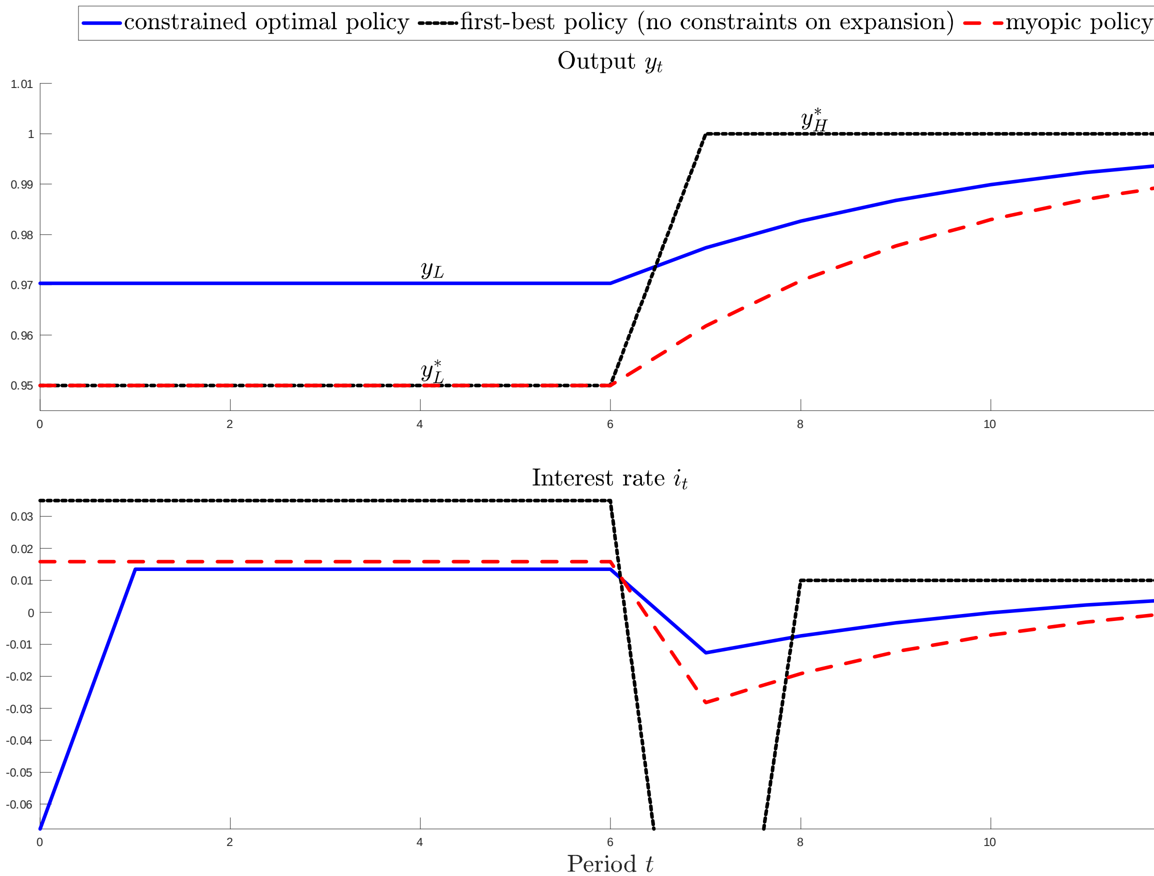

Figure 1: A simulation of the equilibrium starting in the low-supply state, $s_0 = L$, with the most recent output equal to potential output in the low-supply state, $y_{-1} = y_L^*$. The solid lines correspond to the equilibrium with optimal policy. The dotted lines correspond to a first-best benchmark case in which the policy is not subject to the Taylor-rule constraint. The dashed lines correspond to another benchmark in which the policy is myopic and minimizes the output gap in the current period.

the constraints on expansionary policy, the recovery in the high-supply state takes several periods to complete.

Why does the optimal policy cut the interest rates in the low-supply state and induce overheating? As Figure 1 illustrates, the central bank anticipates that the transition to the high-supply state will start with low aggregate demand. Because aggregate demand has inertia, the central bank recognizes that a greater aggregate demand in the low-supply state will accelerate the recovery after the economy transitions to the high-supply state. Therefore, the central bank optimally *frontloads* interest rate cuts and raises output in the low-supply state above its potential. The optimal policy induces some overheating in the low-supply state, but it also reduces the output gaps and accelerates the recovery once the economy switches to the high-supply state.

To further illustrate the dynamic aspects of the optimal policy, Figure 1 plots another benchmark in which the central bank is myopic and focuses on closing *current* gaps (red dashed lines). Formally, the central bank solves Problem (3) with the period-by-period objective function $-\frac{\left(y_t-y_L^*\right)^2}{2}$. In this myopic benchmark, the central bank keeps output in the low-supply state equal to its potential. Consequently, the economy transitions to the high-supply state with a lower aggregate demand than when the central bank implements the optimal (dynamic) policy. Since the expansionary policy in the high-supply state is constrained and aggregate demand is partly backward looking, the recovery takes longer. The myopic policy does not induce overheating in the low-supply state, but it leads to more negative output gaps and a slower recovery once the economy switches to the high-supply state.

4. Overheating with inflation

In this section we extend our setup to allow for partially flexible prices and an inflation rate that is responsive to overheating. We start with the textbook case in which inflation is determined by a New-Keynesian Phillips Curve (NKPC) without *inflation* inertia. In this case, our substantive conclusion remains the same: the central bank overheats the economy in the temporary supply shock state to fight the negative output gaps and *the disinflation* that it expects to emerge after the supply recovers. We then assume that the inflation block of the model also features inertia. This case leads to richer dynamics within the temporary supply shock state: *With inflation inertia, the central bank initially overheats the economy, as before, but gradually cools it down as the supply contraction continues.*

We first modify the baseline setup in Section 2 to incorporate inflation. Let P_t denote the nominal price level and $\pi_t = \log\left(P_t/P_{t-1}\right)$ denote (log) inflation. With inflation, the IS curve (1) becomes

$$\begin{aligned} y_t &= \eta y_{t-1} + (1-\eta)\left(-\left(r_t-\rho\right) + E_t\left[y_{t+1}\right]\right) \\ \text{where } r_t &= i_t - E_t\left[\pi_{t+1}\right]. \end{aligned} \tag{11}$$

Here, r_t denotes *the real interest rate.* We also modify the Taylor rule in the high-supply state to allow the central bank to react to *the inflation gap* as well as the output gap,

$$i_t = \rho + \phi_y \left(y_t - y_H^*\right) + \phi_\pi \pi_t \text{ if } s_t = H. \tag{12}$$

We normalize the inflation target to zero so that the inflation gap (the deviation of the inflation from the target) is the same as inflation. Finally, we adjust the central bank's objective function to incorporate the costs of inflation:

$$E_t \left[\sum_{h=0}^{\infty} \beta^h \left(\frac{\left(y_{t+h} - y_{s_{t+h}}^*\right)^2}{2} + \psi_{t+h} \frac{\pi_{t+h}^2}{2} \right) \right]. \tag{13}$$

Here, ψ_t denotes the relative welfare weight for the inflation gaps in period t. For the analysis with the NKPC in Section 4.1, we simply assume constant welfare weights, $\psi_t \equiv \psi$ for each t. For the analysis with an inertial Phillips curve in Section 4.1, we specify state-dependent welfare weights (as we describe subsequently) to simplify the analysis.

We next describe the inflation block and characterize the optimal policy. We consider two specifications that differ on whether the Phillips curve features inertia or not.

4.1. Overheating with the New-Keynesian Phillips curve

First, suppose inflation is determined by the standard NKPC (see Galí (2015) for a derivation):

$$\pi_t = \kappa \left(y_t - y_{s_t}^*\right) + \beta E_t \left[\pi_{t+1}\right]. \tag{14}$$

Inflation depends on the current output gap, $y_t - y_{s_t}^*$, as well as the expectations for future inflation. The coefficient κ captures the extent of price flexibility. The equation does not feature inertia because inflation expectations are rational and forward-looking.

Consider the first-best benchmark without constraints on expansionary policy. As before, the central bank achieves zero output gaps throughout. In view of Eq. (14), this implies zero inflation throughout. In this benchmark, *the divine coincidence* applies and introducing inflation does not change the analysis.

Next, suppose the central bank is constrained to follow the Taylor rule (12) in the high-supply state. The analysis closely parallels the baseline analysis in Section 3. Therefore, we relegate the formal results to Appendix A.2.1 and discuss the intuition.

In the high-supply state, $s = H$, the equilibrium is characterized by the IS curve (11), the NKPC (14), and the Taylor rule (12). Under appropriate parametric restrictions, the

Taylor rule ensures that the output and inflation gaps *eventually* converge to zero (see Lemma 2 in the appendix). However, the convergence is not immediate. Starting with $y_{t-1} < y_H^*$, the economy experiences a period of negative output gaps and disinflation. As before, increasing y_{t-1} mitigates these gaps and increases the value function.

In the low-supply state, $s = L$, the central bank solves a modified version of problem (8). Proposition 3 in the appendix characterizes the solution and shows that our main result extends to this setup. *The central bank chooses a level of output that induces positive output gaps in the low-supply state* (current overheating), $y_{t,L} \equiv y_L > y_L^*$, *and negative output gaps and disinflation after transition to the high-supply state (future demand shortages),* $Y_H(y_L) < y_H^*, \Pi_H(y_L) < 0$.

Figure 2 illustrates this result in a numerical example. As before, the economy starts in the low-supply state and with past output $y_{-1} = y_L^*$. The dotted lines show the first-best case with unconstrained policy. The solid lines show the equilibrium with constrained optimal policy. The central bank cuts the interest rate (both nominal and real) aggressively in the temporary supply shock state to bring output above its potential level and inflation above its target level. Similar to before, the central bank anticipates that the recovery will start with low aggregate demand *and disinflation*. Therefore, it frontloads interest rate cuts and overheats the economy to ensure that the recovery starts with a greater aggregate demand *and a smaller inflation gap*.

As the figure shows, the central bank implements a relatively low inflation despite the fact that it sets a positive output gap. This aspect is driven by the forward looking nature of the NKPC. Price setters recognize that the supply recovery will start with negative output gaps and disinflation. The expected disinflation puts downward pressure on current inflation. While this result makes overheating relatively less costly, empirical studies of the Phillips curve suggest that inflation is not influenced by the rationally expected future output gaps as much as predicted by the NKPC (see, e.g., Rudd and Whelan (2005)). We next turn to an alternative setup in which inflation is backward-looking.

4.2. Overheating with an inertial Phillips curve

Next suppose inflation is determined by an inertial Phillips curve:

$$\pi_t = \kappa \left(y_t - y_{s_t}^*\right) + b\pi_{t-1}. \tag{15}$$

Here, $b \in (0, 1)$ is a parameter that captures the strength of inflation inertia. In theory, inflation inertia can emerge from several frictions, e.g., backward-looking indexation of

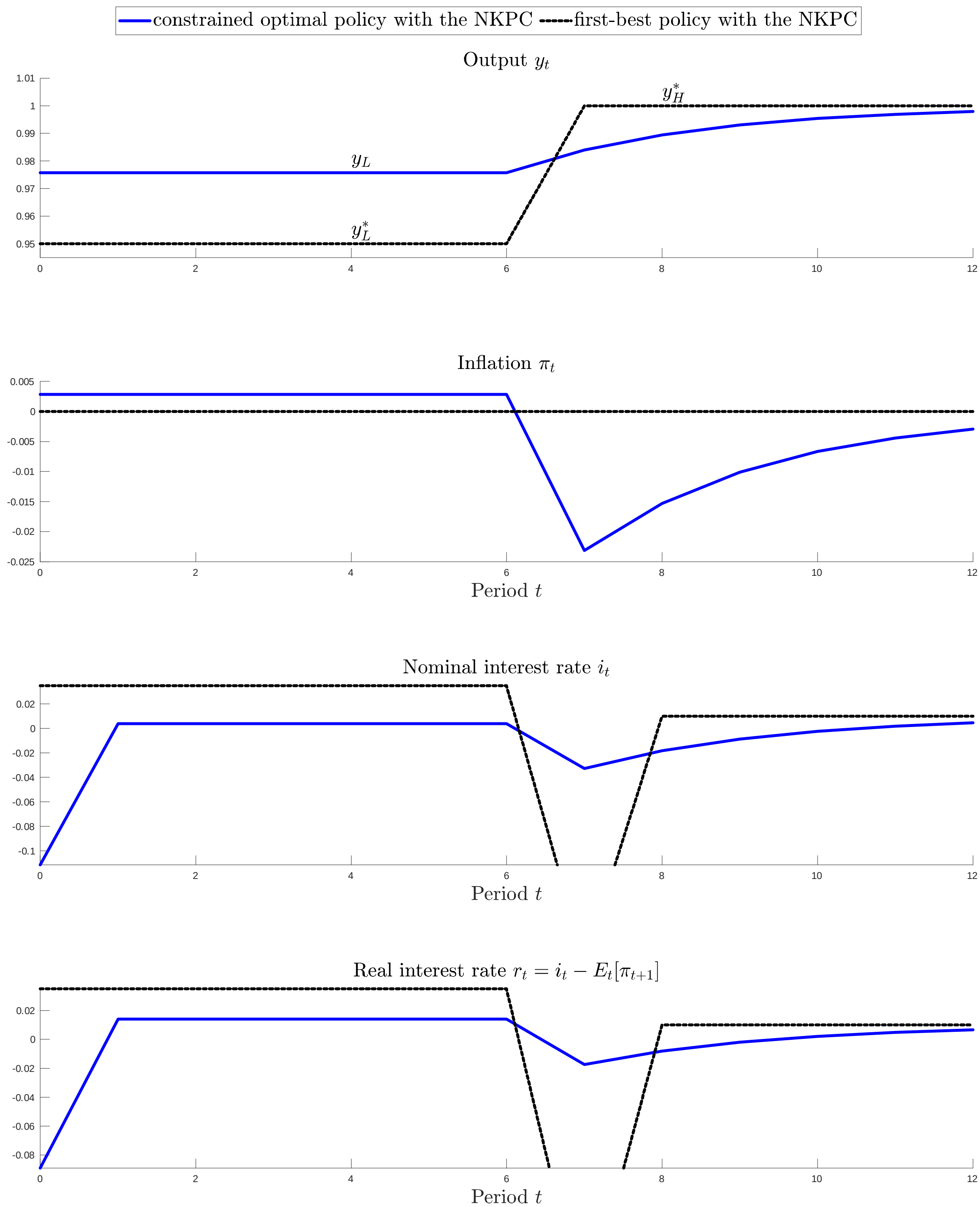

Figure 2: A simulation of the equilibrium in which inflation is determined acording to the NKPC. The solid lines show the equilibrium with optimal policy. The dotted lines illustrate a first-best benchmark case in which the policy is not subject to the Taylor-rule constraint.

prices or wages (e.g., Galı and Gertler (1999)) or adaptive inflation expectations (e.g., Blanchard (2016)). For analytical tractability, we assume inflation is fully backward-looking.

First, consider the first-best benchmark setup without constraints on expansionary policy. As long as the central bank does not inherit past inflation, $\pi_{-1} = 0$, it is easy to check that the equilibrium is the same as before. In particular, the central bank achieves zero output gaps and zero inflation throughout. As long as $\pi_{-1} = 0$, *the divine coincidence* still applies with a backward-looking Phillips curve.

Next, consider the main setup with constraints on expansionary policy. In the high-supply state, $s = H$, the equilibrium is characterized by the IS curve (11), the Phillips curve (15), and the Taylor rule (12). This equilibrium is more complicated than before since there are two state variables: past output y_{t-1} and past inflation π_{t-1}. To simplify this characterization (which is not our focus), we make two assumptions. First, the Taylor-rule coefficient on inflation satisfies $\phi_\pi = b$. With this assumption, Lemma 3 in the appendix shows that the output gaps converge to zero at a constant rate γ_H. Second, we assume the planner's welfare weight on inflation, ψ_t, is constant within the low-supply state (denoted by ψ_L), positive in the first period after the transition to the high-supply state (denoted by Ψ_H), and zero in the remaining periods. With this assumption, Lemma 3 calculates the value function in the first period after the transition to the high-supply state as:[4]

$$
\begin{aligned}
\tilde{V}_H\left(y_{t-1}, \pi_{t-1}\right) &= -\theta_H \frac{\left(y_{t-1} - y_H^*\right)^2}{2} - \Psi_H \frac{\pi_t^2}{2} \\
\text{with } \theta_H &= \frac{\gamma_H^2}{1 - \beta\gamma_H^2} \text{ and } \pi_t = \kappa\gamma_H\left(y_{t-1} - y_H^*\right) + b\pi_{t-1}.
\end{aligned}
$$

In this case, the value function depends on the past inflation, π_{t-1}, as well as on the past output gap, $y_{t-1} - y_H^*$ [cf. Eq. (7)].

In the low-supply state, $s = L$, the central bank solves a version of problem (8):

$$
\begin{aligned}
V_L\left(y_{t-1}, \pi_{t-1}\right) &= \max_{y_t, \pi_t} -\frac{\left(y_t - y_L^*\right)^2}{2} - \psi_L \frac{\pi_t^2}{2} + \beta\left(\left(1-\lambda\right) V_L\left(y_t, \pi_t\right) + \lambda\tilde{V}_H\left(y_{t-1}, \pi_{t-1}\right)\right) \\
\pi_t &= \kappa\left(y_t - y_L^*\right) + b\pi_{t-1} \qquad (16)
\end{aligned}
$$

Our main result characterizes the optimal policy and extends Proposition 1 to this setting.

[4] The assumption on ψ_t is innocuous as long as we interpret the parameter Ψ_H as capturing the *total* cost of inflation after transition to state H (as opposed to the cost in a single period).

To state the result, we define the following derived parameters:

$$\begin{aligned} A &= 1+\beta\lambda\left(\theta_H+\Psi_H\kappa^2\gamma_H\left(\gamma_H+b\right)\right) \\ B &= \left(\psi_L+\beta\lambda\Psi_H b\left(\gamma_H+b\right)\right)\kappa \\ C &= \beta\left(1-\lambda\right)\kappa b\psi_L \\ D &= \beta\lambda\left(\theta_H+\Psi_H\kappa^2\gamma_H\left(\gamma_H+b\right)\right). \end{aligned} \tag{17}$$

Proposition 2. *Consider the setup with inertial Phillips curve. Suppose* $\phi_\pi = b, \phi_y > \kappa$ *and* ψ_t *satisfies the simplifying assumptions described earlier. Let* $A, B, C, D > 0$ *denote the parameters in* (17). *Let* $\gamma_L \in (0, b)$ *denote the positive root of the polynomial* $P(x) = x^2 + x\left(\frac{A/\kappa+B}{C}\right) - \frac{Ab/\kappa}{C}$. *As long as the economy remains in the low-supply state, the optimal choice of output and inflation,* (y_t, π_t), *converge to a steady-state,* $(\overline{y}_L, \overline{\pi}_L)$, *where*

$$\overline{y}_L = y_L^* + \frac{D}{A+(B+C)\frac{\kappa}{1-b}}\left(y_H^* - y_L^*\right) \in \left(y_L^*, y_H^*\right) \tag{18}$$

$$\overline{\pi}_L = \frac{\kappa}{1-b}\left(\overline{y}_L - y_L^*\right) > 0. \tag{19}$$

Along the transition path, the optimal output and inflation are given by

$$y_t - \overline{y}_L = -\frac{b-\gamma_L}{\kappa}\left(\pi_{t-1} - \overline{\pi}_L\right) \tag{20}$$

$$\pi_t - \overline{\pi}_L = \gamma_L\left(\pi_{t-1} - \overline{\pi}_L\right). \tag{21}$$

The associated real and nominal interest rates are given by $(A.14 - A.15)$ *in the appendix. Starting with zero past inflation* $\pi_{-1} = 0$, *the planner implements a relatively high initial output gap,* $y_0 > \overline{y}_L > y_L^*$. *Absent transition to the high supply state, the planner gradually decreases the output gap toward its steady state value,* $y_t - y_L^* \downarrow \overline{y}_L - y_L^*$, *and increases the inflation toward its steady state value,* $\pi_t \uparrow \overline{\pi}_L$.

With inertia in the Phillips curve, the central bank still implements high output gaps in the low-supply state, as before, but it also reduces the output gaps as the supply recovery is delayed. For intuition, note that the output gaps increase inflation. With inflation inertia, past inflation shifts the Phillips curve and worsens the trade-off between increasing the output (to accelerate the *future* recovery) and raising inflation. As time passes and the recovery is delayed, it becomes increasingly costly to induce positive output gaps. The central bank optimally "undoes" some of the overheating it has initially induced.

Figure 3 illustrates the result in a numerical example. The (blue) solid lines show the equilibrium with optimal policy. As before, the central bank cuts the interest rate

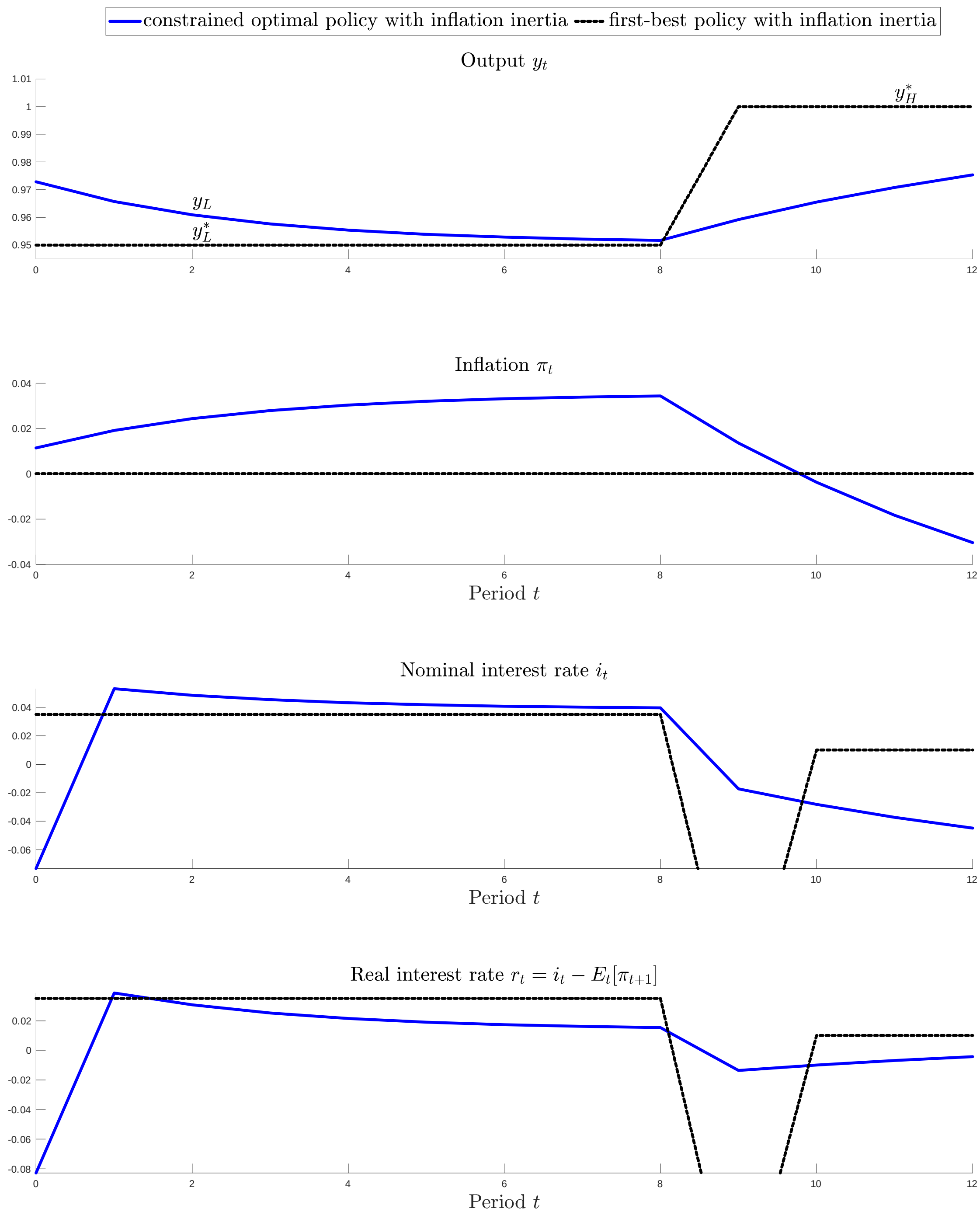

Figure 3: A simulation of the equilibrium in which inflation is determined acording to an inertial Phillips curve. The solid lines show the equilibrium with optimal policy. The dotted lines illustrate a first-best benchmark case in which the policy is not subject to the Taylor-rule constraint.

aggressively in the temporary supply shock state to bring output above its potential level. As time passes and the recovery is delayed, this policy raises inflation. To keep inflation under control, the central bank gradually brings output closer to its potential—undoing some of the initial overheating.

The last two panels of Figure 3 show the nominal and real interest rates the central bank targets. As before, after the initial interest rate cut, the central bank raises the interest rate. Unlike before, the nominal rate can exceed the corresponding nominal interest rate in the first-best benchmark (cf. Figures 1 and 2). In this model, the level of the nominal interest rate is also influenced by *expected* inflation. As inflation increases over time, *expected* inflation increases (we assume *consumers'* inflation expectations are rational and forward looking). Thus, the central bank raises the nominal rate to keep the real rate relatively stable.

5. Final Remarks

Summary. In this note, we developed a model to address two substantive questions: *Should central banks tolerate some degree of overheating during a temporary supply contraction?* And if the answer is yes, as we find, *does this imply that optimal monetary policy should remain ultra-loose throughout the supply constrained phase?*

Our answers to these questions build on the realistic modeling ingredient that aggregate demand has inertia. Inertia implies that the level of aggregate demand in the future, once aggregate supply recovers, is increasing in the level of aggregate demand in the current low-supply state. This dynamic linkage across states implies that a policymaker that anticipates being constrained and facing a negative output gap in the future, once aggregate supply recovers, overheats the economy in the current low-supply state.

Aggregate demand inertia also implies that, within the low-supply phase, the optimal policy frontloads the interest rate cuts and then quickly normalizes the interest rate. The reason is that the initial expansion generates aggregate demand momentum. This momentum supports aggregate demand and ensures that output stays at the optimal level of overheating without the initially low interest rate. In this context, keeping the interest rate "too low for too long" overheats the economy beyond the optimal level.

If the inflation block of the model also features inertia, then the optimal policy features richer dynamics. The initial expansion in the low supply state gradually increases inflation, which makes it increasingly costly to run the economy hot. As the recovery is delayed, the central bank optimally "undoes" some of the initial overheating. The build-up of inflation also raises expected inflation, which induces the central bank to keep hiking the nominal interest rate until supply recovers. In this context, adjusting the nominal interest rate too slowly lowers the real interest rate and overheats the economy beyond the optimal level.

Clarifications and extensions. We assumed that potential output immediately recovers to a high level once the temporary supply contraction ends. This feature is meant to capture a Covid-19 style shock, where supply remains depressed mainly due to virus-related developments (e.g., whether there will be a new variant) and can be expected to recover rapidly once the virus is under control. For other supply shocks, where the expected supply recovery is more gradual, the first-best benchmark implies smaller interest rate cuts during the recovery, which also reduces the need for frontloading interest rate cuts (see Figure 1). In this sense, our results are more relevant for temporary supply shocks driven by highly disruptive but short-lived events, such as epidemics, wars, or political conflicts.

We also abstracted from fiscal policy and focused on the optimal path of monetary policy. In the Covid-19 recession, fiscal policy played an important role and it was *front-loaded*. We could capture frontloaded fiscal policy by assuming a higher level of past output at the onset of the model (y_{-1}). This would not affect the central bank's output gap target in the low-supply phase, but it would reduce the initial interest rate cut necessary to achieve this target (see Eq. (10)). More broadly, fiscal and monetary policy are substitutes in terms of their impact on aggregate demand, which suggests that our results can also speak to the optimal timing of fiscal policy. In particular, aggregate demand inertia provides a rationale for frontloading and then normalizing fiscal policy, along the lines we observed after the Covid-19 shock. We leave a more complete analysis of the optimal fiscal policy response to temporary supply shocks for future work.

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Online Appendix: Not for Publication

A. Omitted Proofs

This appendix contains the results and the derivations omitted from the main text.

A.1. Omitted proofs for Section 3

Proof of Lemma 1. Combining the IS curve in (1) and the Taylor rule in (2), output follows the difference equation,

$$y_t = \eta y_{t-1} + (1-\eta)\left(-\phi\left(y_t - y_H^*\right) + y_{t+1}\right).$$

We drop the expectations since there is no (residual) uncertainty. Let $\tilde{y}_t = y_t - y_H^*$ denote the output gap. Then, we can rewrite the difference equation as,

$$\tilde{y}_t = \eta \tilde{y}_{t-1} + (1-\eta)\left(-\phi \tilde{y}_t + \tilde{y}_{t+1}\right).$$

In matrix notation, we have the system,

$$\begin{bmatrix} \tilde{y}_{t+1} \\ \tilde{y}_t \end{bmatrix} = M \begin{bmatrix} \tilde{y}_t \\ \tilde{y}_{t-1} \end{bmatrix} \text{ where } M = \begin{bmatrix} \frac{1}{1-\eta} + \phi & -\frac{\eta}{1-\eta} \\ 1 & 0 \end{bmatrix}.$$

The characteristic polynomial of the matrix M is given by

$$P(x) = x^2 - x\left(\frac{1}{1-\eta} + \phi\right) + \frac{\eta}{1-\eta}.$$

This polynomial has two roots that satisfy

$$0 < \gamma_1 < 1 < \gamma_2.$$

Since $\tilde{y}_{t-1}$ is predetermined and $\tilde{y}_t$ is not, this condition ensures the system is saddle path stable. Moreover, letting $\gamma_H \equiv \gamma_1 \in (0,1)$ denote the stable eigenvalue, the solution converges to zero at a constant rate:

$$\tilde{y}_{t+h} = \gamma_H \tilde{y}_{t+h-1} = \gamma_H^{h+1} \tilde{y}_{t-1}.$$

This proves (6).

We can then solve for the value function as:

$$V_H = \sum_{h=0}^{\infty} -\beta^h \frac{(\tilde{y}_{t+h})^2}{2} = \sum_{h=0}^{\infty} -\beta^h \frac{\left(\gamma_H^{h+1} \tilde{y}_{t-1}\right)^2}{2} = -\frac{\gamma_H^2}{1-\beta\gamma_H^2} \frac{(\tilde{y}_{t-1})^2}{2}.$$

This establishes (7).

To establish the comparative statics of γ_H, let $\tilde{\eta} = \frac{\eta}{1-\eta}$ and note that γ_H is the solution to the following equation over the range $(0, 1)$:

$$P(x, \tilde{\eta}, \phi) = x^2 - x(1 + \tilde{\eta} + \phi) + \tilde{\eta} = 0.$$

Implicitly differentiating with respect to $\tilde{\eta}$ and evaluating around $x = \gamma_H$, we obtain

$$\frac{dx}{d\tilde{\eta}} = -\left.\frac{\partial P/\partial \tilde{\eta}}{\partial P/\partial x}\right|_{x=\gamma_H} = \frac{1-\gamma_H}{1+\tilde{\eta}+\phi-2\gamma_H} > 0.$$

Here, the inequality follows since $\gamma_H < 1$ and $2\gamma_H < \gamma_1 + \gamma_2 = 1 + \tilde{\eta} + \phi$ (since γ_H is the smaller of the two roots γ_1, γ_2). Since $\tilde{\eta} = \frac{\eta}{1-\eta}$ is increasing in η, we also have $\frac{dx}{d\eta} > 0$. Likewise, we obtain

$$\frac{dx}{d\phi} = -\left.\frac{\partial P/\partial \phi}{\partial P/\partial x}\right|_{x=\gamma_H} = \frac{-\gamma_H}{1+\tilde{\eta}+\phi-2\gamma_H} < 0.$$

Finally, note that we also have the closed-form solution

$$\gamma_H = \frac{1+\tilde{\eta}+\phi-\sqrt{(1+\tilde{\eta}+\phi)^2 - 4\tilde{\eta}}}{2}.$$

As $\eta \to 0$ (and $\tilde{\eta} \to 0$), we have $\gamma_H \to 0$. Likewise, as $\phi \to \infty$, it can be checked that $\gamma_H \to 0$. This completes the proof. □

Proof of Proposition 1. The proof is mostly provided in the main text. To characterize the interest rate, note that the IS curve (1) implies

$$i_{t,L} = \rho + \lambda (Y_H(y_t) - y_{t,L}) + (1-\lambda)(y_{t+1,L} - y_{t,L}) - \frac{\eta}{1-\eta}(y_{t,L} - y_{t-1}).$$

After substituting $y_{t,L} = y_{t+1,L} = y_L$, we obtain (10). □

A.2. Omitted results and proofs for Section 4

We first consider the case with the NKPC and present the formal results omitted from Section 4.1 along with their proofs. We then consider the case with an inertial Phillips curve analyzed in Section 4.2 and present the omitted results and proofs.

A.2.1. Overheating with a New-Keynesian Phillips Curve

Suppose inflation is determined according to the NKPC (14)

$$\pi_t = \kappa\left(y_t - y^*_{s_t}\right) + \beta E_t\left[\pi_{t+1}\right].$$

Let $\Pi_s(y_{t-1}), Y_s(y_{t-1}), V_s(y_{t-1})$ denote the inflation, the output, and the value function level when the current state is $s \in \{H, L\}$, and the most recent output is y_{t-1}. Suppose also that the relative weight of inflation in the planner's objective function is constant throughout, $\psi_t \equiv \psi$ for each t (see (13)).

We first characterize the equilibrium in the high supply state $s = H$. To state the result, we define the polynomial:

$$P(x) = x^3 - x^2\left(\frac{1}{1-\eta} + \phi_y + \frac{1+\kappa}{\beta}\right) + x\left(\left(\frac{1}{1-\eta} + \phi_y\right)\frac{1}{\beta} + \phi_\pi\frac{\kappa}{\beta} + \frac{\eta}{1-\eta}\right) - \frac{1}{\beta}\frac{\eta}{1-\eta}. \quad (A.1)$$

Lemma 2. *Consider the setup with inflation determined by the NKPC* (14)*. Suppose* $\psi_t \equiv \psi$ *and the polynomial in* (A.1) *has exactly one stable root that satisfies* $\gamma_H \in (0,1)$ *(a sufficient condition is* $\phi_y(1-\beta) + (\phi_\pi - 1)\kappa > 0$ *and* $\beta\phi_\pi \leq 1$*). Suppose the economy has switched to the high-supply state,* $s = H$*, with past output* y_{t-1}*. Then, the output gap and the inflation functions are given by:*

$$Y_H(y_{t-1}) - y^*_H = \gamma_H\left(y_{t-1} - y^*_H\right) \quad (A.2)$$
$$\Pi_H(y_{t-1}) = \boldsymbol{\pi}_h\left(y_{t-1} - y^*_H\right) \text{ where } \boldsymbol{\pi}_h = \frac{\kappa\gamma_H}{1-\beta\gamma_H}. \quad (A.3)$$

The output gap and inflation both converge to zero at a constant rate γ_H*. The value function is given by*

$$V_H(y_{t-1}) = -\theta_H\frac{\left(y_{t-1} - y^*_H\right)^2}{2} \text{ where } \theta_H = \frac{1}{1-\beta\gamma_H^2}\left(\gamma_H^2 + \psi\left(\frac{\kappa\gamma_H}{1-\beta\gamma_H}\right)^2\right). \quad (A.4)$$

In the high-supply state, the equilibrium is determined by the IS curve, the NKPC, and the Taylor rule. Under appropriate parametric conditions, the Taylor rule ensures that the output and inflation gaps converge to zero. As before, the convergence is not immediate. Due to inertial demand, *past* output, y_{t-1}, affects the output and inflation gaps in the high-supply state.

Next consider the equilibrium in the low supply state $s = L$. Using Lemma 2, the planner solves the following version of problem (8):

$$\begin{aligned} V_L(y_{t-1}) &= \max_{y_t,\pi_t} -\frac{(y_t - y_L^*)^2}{2} - \psi\frac{\pi_t^2}{2} + \beta\left((1-\lambda)V_L(y_t) - \lambda\theta_H\frac{(y_t - y_H^*)^2}{2}\right) \quad \text{(A.5)} \\ \text{s.t. } \pi_t &= \kappa(y_t - y_L^*) + \beta\left((1-\lambda)\Pi_L(y_t) + \lambda\boldsymbol{\pi}_H(y_t - y_H^*)\right). \end{aligned}$$

Here, the functions, $V_L(y_{t-1})$ and $\Pi_L(y_{t-1}) \equiv \pi_L$, are also both independent of y_{t-1}. Using this observation, the optimality condition is given by

$$\begin{aligned} y_L - y_L^* + \psi\frac{d\pi_t}{dy_t}\pi_L &= \beta\lambda\theta_H(y_H^* - y_L) \\ \text{where } \frac{d\pi_t}{dy_t} &= \kappa + \beta\lambda\boldsymbol{\pi}_H \\ \text{and } \pi_L &= \frac{\kappa(y_L - y_L^*) + \beta\lambda\boldsymbol{\pi}_H(y_L - y_H^*)}{1 - \beta(1-\lambda)}. \end{aligned}$$

Here, the last line uses the NKPC to solve for the inflation in the low-supply state. Combining these observations, the optimum is given by the unique solution to:

$$\left[1 + \frac{\psi(\kappa + \beta\lambda\boldsymbol{\pi}_H)\kappa}{1-\beta(1-\lambda)}\right](y_L - y_L^*) = \beta\lambda\left[\theta_H + \frac{\psi(\kappa+\beta\lambda\boldsymbol{\pi}_H)\boldsymbol{\pi}_H}{1-\beta(1-\lambda)}\right](y_H^* - y_L). \quad \text{(A.6)}$$

This leads to the following result, which generalizes Proposition 1 to this setting.

Proposition 3. *Consider the setup with inflation determined by the NKPC* (14) *and the parametric conditions described in Lemma 2. Suppose the economy is in the temporary supply shock state, $s = L$, with past output y_{t-1}. The central bank implements the constant output level $y_L \in (y_L^*, y_H^*)$ that solves* (A.6) *along with the constant inflation*

$$\pi_{t,L} = \pi_L \equiv \frac{\kappa(y_L - y_L^*) + \beta\lambda\boldsymbol{\pi}_H(y_L - y_H^*)}{1-\beta(1-\lambda)}. \quad \text{(A.7)}$$

The associated real and nominal interest rates are given by

$$r_{t,L} = \rho + \lambda \left(Y_H \left(y_L\right) - y_L\right) - \frac{\eta}{1-\eta}\left(y_L - y_{t-1}\right) \tag{A.8}$$

$$i_{t,L} = r_{t,L} + \lambda \Pi_H \left(y_L\right) + \left(1-\lambda\right)\pi_L. \tag{A.9}$$

The central bank chooses a level of output that induces positive output gaps in the current low-supply state (current overheating), $y_L > y_L^*$, and negative output gaps and disinflation after transition to the high-supply state (future demand shortages), $Y_H(y_L) < y_H^*$ and $\Pi_H(y_L) < 0$.

Comparing (A.6) and (9) shows that inflation affects the policy trade-off in two ways. On the one hand, positive output gaps in the low-supply phase increase current inflation. This raises the cost of overheating, captured by the second term inside the brackets on the left side of (A.6). On the other hand, negative output gaps expected in the *future* high-supply phase reduce *current* inflation. Since overheating helps shrink future gaps, this effect raises the benefit of overheating, captured by the second term inside the brackets on the right side of (A.6). It follows that inflation affects the cost as well as the benefit of overheating, but it does not change the *qualitative* aspects of optimal policy.

The equilibrium with the NKPC has one subtlety: The central bank does not *necessarily* induce positive inflation in the low-supply state: that is, π_L is not necessarily positive (even though $y_L > y_L^*$). This effect is driven by the forward-looking term in the NKPC, together with the fact that the economy experiences disinflation after transition to the high-supply state, $\boldsymbol{\pi}_H\left(y_L - y_H^*\right) < 0$ (see (A.7)). Nonetheless, in our simulations this effects is typically weak and the central bank implements $\pi_L > 0$ along with $y_L > y_L^*$.

Proof of Lemma 2. Combining the NKPC, the IS curve, and the policy rule, the dynamic system that characterizes the equilibrium is given by

$$\begin{aligned} y_t &= \eta y_{t-1} + (1-\eta)\left(-\phi_y\left(y_t - y_H^*\right) - \phi_\pi \pi_t + E_t\left[\pi_{t+1}\right] + E_t\left[y_{t+1}\right]\right) \\ \pi_t &= \kappa\left(y_t - y_H^*\right) + \beta E_t\left[\pi_{t+1}\right]. \end{aligned}$$

We drop the expectations since there is no (residual) uncertainty. Let $\tilde{y}_t = y_t - y_H^*$ denote the output gap. Then, we can rewrite the system as

$$\begin{aligned} \tilde{y}_t &= \eta \tilde{y}_{t-1} + (1-\eta)\left(-\phi_y \tilde{y}_t - \phi_\pi \pi_t + \pi_{t+1} + \tilde{y}_{t+1}\right) \\ \pi_t &= \kappa \tilde{y}_t + \beta \pi_{t+1}. \end{aligned}$$

In matrix notation, we have

$$\begin{bmatrix} \tilde{y}_{t+1} \\ \pi_{t+1} \\ \tilde{y}_t \end{bmatrix} = M \begin{bmatrix} \tilde{y}_t \\ \pi_t \\ \tilde{y}_{t-1} \end{bmatrix} \text{ where } M = \begin{bmatrix} \frac{1}{1-\eta} + \phi_y + \frac{\kappa}{\beta} & \phi_\pi - \frac{1}{\beta} & -\frac{\eta}{1-\eta} \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

The characteristic polynomial of the matrix M is

$$\begin{aligned} P(x) &= -\det\left(\begin{bmatrix} \frac{1}{1-\eta} + \phi_y + \frac{\kappa}{\beta} - x & \phi_\pi - \frac{1}{\beta} & -\frac{\eta}{1-\eta} \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} - x & 0 \\ 1 & 0 & -x \end{bmatrix} \right) \\ &= x^3 - x^2 \left(\frac{1}{1-\eta} + \phi_y + \frac{1+\kappa}{\beta} \right) + x \left(\left(\frac{1}{1-\eta} + \phi_y \right) \frac{1}{\beta} + \phi_\pi \frac{\kappa}{\beta} + \frac{\eta}{1-\eta} \right) - \frac{1}{\beta} \frac{\eta}{1-\eta}. \end{aligned}$$

This is the polynomial we define in $(A.1)$. We assume the parameters are such that this polynomial has a single stable root that satisfies $\gamma_H \in (0,1)$. The conditions in the propositions are sufficient (but not necessary). To check sufficiency, note that we have $P(0) < 0$. We also have

$$P(1) = \frac{\phi_y (1-\beta) + (\phi_\pi - 1)\kappa}{\beta} > 0$$

in view of the first part of the sufficient condition, $\phi_y (1-\beta) + (\phi_\pi - 1)\kappa$. We also have

$$P\left(\frac{1}{\beta}\right) = -\frac{\kappa}{\beta^3} + \phi_\pi \frac{\kappa}{\beta^2} \leq 0$$

in view of the second part of the sufficient condition, $\beta \phi_\pi \leq 1$. Thus, with these conditions the roots of the polynomial satisfy

$$0 < \gamma_1 < 1 < \gamma_2 \leq \frac{1}{\beta} \leq \gamma_3.$$

In particular, the polynomial has exactly one stable root that satisfies $\gamma_H \equiv \gamma_1 \in (0,1)$.

Since $\tilde{y}_{t-1}$ is predetermined but $\tilde{y}_t, \pi_t$ are not, the system is saddle path stable. Moreover, the solution converges to zero at the constant rate $\gamma_H \in (0,1)$, that is:

$$\begin{aligned} \tilde{y}_{t+h} &= \gamma_H \tilde{y}_{t+h-1} = \gamma_H^{h+1} \tilde{y}_{t-1} \\ \tilde{\pi}_{t+h} &= \gamma_H \tilde{\pi}_{t+h-1}. \end{aligned}$$

This establishes $(A.2)$. To solve for the initial inflation, we use the NKPC to obtain

$$\pi_t = \sum_{h=0}^{\infty} \beta^h \kappa \tilde{y}_{t+h} = \sum_{h=0}^{\infty} \beta^h \gamma_H^h \kappa \gamma_H \tilde{y}_{t-1} = \frac{\kappa \gamma_H \tilde{y}_{t-1}}{1-\beta\gamma_H}.$$

This establishes $(A.3)$.

Finally, we calculate the value function as:

$$\begin{aligned} V_H &= -\sum_{h=0}^{\infty} \beta^h \left(\frac{\tilde{y}_{t+h}^2}{2} + \psi \frac{{\pi_{t+h}}^2}{2} \right) \\ &= -\sum_{h=0}^{\infty} \left(\beta\gamma_H^2\right)^h \left(\frac{{\tilde{y}_t}^2}{2} + \psi \frac{{\pi_t}^2}{2} \right) \\ &= -\frac{1}{1-\beta\gamma_H^2} \left(\gamma_H^2 + \psi \left(\frac{\kappa\gamma_H}{1-\beta\gamma_H} \right)^2 \right) \frac{{\tilde{y}_{t-1}}^2}{2}. \end{aligned}$$

Here, the second line uses the fact that inflation and the output gap converge to zero at rate $\gamma_H \in (0,1)$ and the last line substitutes $\tilde{y}_t$ and π_t in terms of the past output gap $\tilde{y}_{t-1}$. This establishes $(A.4)$ and completes the proof of the lemma. □

Proof of Proposition 3. The proof is mostly presented earlier in the section. To solve for the real interest rate, note that the IS curve (11) implies

$$r_{t,L} = \rho + \lambda \left(Y_H\left(y_t\right) - y_{t,L}\right) + (1-\lambda)\left(y_{t+1,L} - y_{t,L}\right) - \frac{\eta}{1-\eta}\left(y_{t,L} - y_{t-1}\right).$$

After substituting $y_{t,L} = y_{t+1,L} = y_L$, this implies $(A.8)$. The nominal interest rate is then

$$i_{t,L} = r_{t,L} + E_t\left[\pi_{t+1}\right] = r_{t,L} + \lambda \Pi_H\left(y_L\right) + (1-\lambda)\pi_L.$$

This establishes $(A.9)$ and completes the proof. □

A.2.2. Overheating with an inertial Phillips Curve

Suppose inflation is determined according to the inertial Phillips curve (15)

$$\pi_t = \kappa\left(y_t - y_{s_t}^*\right) + b\pi_{t-1}.$$

Suppose also that the parameters satisfy the simplifying assumptions described in the main text. We first state the lemma that characterizes the equilibrium in the high supply-state $s = H$. We then present the proof of Proposition 2, which characterizes the optimal policy in the low-supply state $s = L$.

Lemma 3. *Consider the setup with an inertial Phillips curve. Suppose the parameters satisfy* $\phi_\pi = b, \phi_y > \kappa$ *and* $\psi_t = \begin{cases} \psi_L & \text{if } s_t = L \\ \Psi_H & \text{if } s_t = H, s_{t-1} = L \\ 0 & \text{otherwise} \end{cases}$.

Suppose the economy has switched to the high-supply state, $s = H$, *with past output* y_{t-1}. *Let* $\gamma_H \in (0,1)$ *denote the smaller root of the polynomial* $P(x) = (1+\kappa)x^2 - \left(\frac{1}{1-\eta} + \phi_y\right)x + \frac{\eta}{1-\eta}$. *Then the output gap and the inflation functions are given by:*

$$Y_H(y_{t-1}, \pi_{t-1}) - y_H^* = \gamma_H(y_{t-1} - y_H^*) \tag{A.10}$$

$$\Pi_H(y_{t-1}, \pi_{t-1}) = \kappa\gamma_H(y_{t-1} - y_H^*) + b\pi_{t-1}. \tag{A.11}$$

The value function in the first period after transition (with $s_{t-1} = L$*) is given by:*

$$\begin{aligned} \tilde{V}_H(y_{t-1,}\pi_{t-1}) &= -\theta_H \frac{(y_{t-1} - y_H^*)^2}{2} - \Psi_H \frac{\pi_t^2}{2} \\ \text{with } \theta_H &= \frac{\gamma_H^2}{1 - \beta\gamma_H^2} \text{ and } \pi_t = \kappa\gamma_H(y_{t-1} - y_H^*) + b\pi_{t-1}. \end{aligned} \tag{A.12}$$

Proof of Lemma 3. Combining the inertial Phillips curve, the IS curve, and the policy rule, the dynamic system that characterizes the equilibrium is given by

$$\begin{aligned} y_t &= \eta y_{t-1} + (1-\eta)\left(-\phi_y(y_t - y_H^*) - \phi_\pi \pi_t + E_t[\pi_{t+1}] + E_t[y_{t+1}]\right) \\ \pi_t &= \kappa(y_t - y_H^*) + b\pi_{t-1}. \end{aligned}$$

We drop the expectations since there is no (residual) uncertainty. Let $\tilde{y}_t = y_t - y_H^*$ denote the output gap. Then, we can rewrite the system as

$$\begin{aligned} \tilde{y}_t &= \eta\tilde{y}_{t-1} + (1-\eta)\left(-\phi_y\tilde{y}_t - \phi_\pi\pi_t + \pi_{t+1} + \tilde{y}_{t+1}\right) \\ \pi_t &= \kappa\tilde{y}_t + b\pi_{t-1}. \end{aligned}$$

After rewriting the second equation and substituting the first equation, we obtain

$$\begin{aligned} \tilde{y}_{t+1} &= \frac{1}{1+\kappa}\left(\frac{\tilde{y}_t - \eta\tilde{y}_{t-1}}{1-\eta} + \phi_y\tilde{y}_t + (\phi_\pi - b)\pi_t\right) \\ \pi_t &= \kappa\tilde{y}_t + b\pi_{t-1}. \end{aligned}$$

This system is in general complicated, because there are two state variables $\tilde{y}_{t-1}, \pi_{t-1}$.

However, in the special case $\phi_\pi = b$, inflation drops out of the first equation and the system becomes block-recursive. In particular, the output gap satisfies the difference equation:

$$\tilde{y}_{t+1} = \frac{1}{1+\kappa}\left(\left(\frac{1}{1-\eta}+\phi_y\right)\tilde{y}_t - \frac{\eta}{1-\eta}\tilde{y}_{t-1}\right).$$

This is a standard difference equation with the characteristic polynomial given by

$$P(x) = (1+\kappa)x^2 - \left(\frac{1}{1-\eta}+\phi_y\right)x + \frac{\eta}{1-\eta} = 0.$$

Note that $P(0) > 0$ and $P(1) < 0$ in view of the parametric condition $\phi_y > \kappa$. Thus, the polynomial has a single stable root that satisfies $\gamma_H \in (0,1)$. It follows that the output gap converges to zero at a constant rate

$$\tilde{y}_{t+h} = \gamma_H \tilde{y}_{t+h-1} = \gamma_H^{h+1}\tilde{y}_{t-1}.$$

This establishes $(A.10)$. Substituting $\tilde{y}_t$ into the inertial Phillips curve, we solve for inflation as:

$$\pi_t = \kappa\tilde{y}_t + b\pi_{t-1} = \kappa\gamma_H\tilde{y}_{t-1} + b\pi_{t-1}.$$

This establishes $(A.11)$.

Finally, consider the value function in the first-period after transition. Using the simplifying assumption on ψ_t, we obtain

$$\begin{aligned}
\tilde{V}_H(y_{t-1},\pi_{t-1}) &= -\sum_{h=0}^{\infty}\beta^h\frac{\tilde{y}_{t+h}^2}{2} - \Psi_H\frac{\pi_t^2}{2} \\
&= -\sum_{h=0}^{\infty}\beta^h\frac{\left(\gamma_H^{h+1}\tilde{y}_{t-1}\right)^2}{2} - \Psi_H\frac{\left(\kappa\tilde{y}_t + b\pi_{t-1}\right)^2}{2} \\
&= -\frac{\gamma_H^2}{1-\beta\gamma_H^2}\frac{\tilde{y}_{t-1}^2}{2} - \Psi_H\frac{\left(\kappa\gamma_H\tilde{y}_{t-1} + b\pi_{t-1}\right)^2}{2}.
\end{aligned}$$

This establishes $(A.12)$ and completes the proof of the lemma. □

Proof of Proposition 2. Consider problem (16), which we replicate here

$$\begin{aligned}
V_L(y_{t-1},\pi_{t-1}) &= \max_{y_t,\pi_t} -\frac{\left(y_t - y_L^*\right)^2}{2} - \psi_L\frac{\pi_t^2}{2} + \beta\left((1-\lambda)V_L(y_t,\pi_t) + \lambda\tilde{V}_H(y_{t-1},\pi_{t-1})\right) \\
\pi_t &= \kappa\left(y_t - y_L^*\right) + b\pi_{t-1}
\end{aligned}$$

In this case, the value function $V_L(y_{t-1}, \pi_{t-1})$ depends on past inflation, π_{t-1}, but it is still independent of past output, y_{t-1}. Using this observation, the optimality condition is given by

$$y_t - y_L^* + \psi_L \frac{d\pi_t}{dy_t}\pi_t = \beta \begin{pmatrix} (1-\lambda) \frac{d\pi_t}{dy_t} \frac{\partial V_L(y_t,\pi_t)}{\partial \pi_t} + \\ \lambda \left(\frac{\partial \tilde{V}_H(y_t,\pi_t)}{\partial y_t} + \frac{d\pi_t}{dy_t} \frac{\partial \tilde{V}_H(y_t,\pi_t)}{\partial \pi_t} \right) \end{pmatrix}.$$

Using Eq. $(A.12)$ to calculate the partial derivatives of $\tilde{V}_H(y_t, \pi_t)$, we obtain:

$$y_t - y_L^* + \psi_L \frac{d\pi_t}{dy_t}\pi_t = \beta \begin{pmatrix} (1-\lambda) \frac{d\pi_t}{dy_t} \frac{\partial V_L(y_t,\pi_t)}{\partial \pi_t} + \\ \lambda \left(\theta_H (y_H^* - y_t) - \Psi_H \left(\kappa\gamma_H + \frac{d\pi_t}{dy_t} b \right) (\kappa\gamma_H (y_t - y_H^*) + b\pi_t) \right) \end{pmatrix}.$$

Substituting $\frac{d\pi_t}{dy_t} = \kappa$ and using the Envelope Theorem, $\frac{\partial V_L(y_{t-1},\pi_{t-1})}{\partial \pi_{t-1}} = -b\psi_L \pi_t$, we obtain

$$y_t - y_L^* + \psi_L \kappa \pi_t = \beta \begin{pmatrix} -(1-\lambda)\kappa b \psi_L \pi_{t+1} + \\ \lambda \begin{pmatrix} (\theta_H + \Psi_H \kappa^2 \gamma_H (\gamma_H + b)) (y_H^* - y_t) \\ -\Psi_H \kappa b (\gamma_H + b) \pi_t \end{pmatrix} \end{pmatrix}.$$

Here, $\pi_{t+1} = \pi_{t+1,L}$ denotes inflation when the economy stays in state L. After rearranging terms, we obtain:

$$A(y_t - y_L^*) + B\pi_t + C\pi_{t+1} = D(y_H^* - y_L^*).$$

Here, $A, B, C, D > 0$ are the derived parameters in (17):

$$\begin{aligned} A &= 1 + \beta\lambda \left(\theta_H + \Psi_H \kappa^2 \gamma_H (\gamma_H + b) \right) \\ B &= (\psi_L + \beta\lambda\Psi_H b (\gamma_H + b)) \kappa \\ C &= \beta (1-\lambda) \kappa b \psi_L \\ D &= \beta\lambda \left(\theta_H + \Psi_H \kappa^2 \gamma_H (\gamma_H + b) \right). \end{aligned}$$

Combining the equation for y_t with the NKPC, we obtain the system:

$$\begin{aligned} A(y_t - y_L^*) + B\pi_t + C\pi_{t+1} &= D(y_H^* - y_L^*) \\ \pi_t &= \kappa (y_t - y_L^*) + b\pi_{t-1}. \end{aligned} \tag{A.13}$$

We next calculate the steady-state, denoted by $(\overline{y}_L, \overline{\pi}_L)$. From the second equation, the steady-state inflation satisfies $\overline{\pi}_L = \frac{\kappa(\overline{y}_L - y_L^*)}{1-b}$. Substituting this into the first equation, we solve for the steady-state output as:

$$\overline{y}_t = y_L^* + \frac{D\left(y_H^* - y_L^*\right)}{A + (B + C)\frac{\kappa}{1-b}}.$$

We next characterize the transition dynamics away from the steady-state. Let $\tilde{y}_t = y_t - \overline{y}_L$ and $\tilde{\pi}_t = \pi_t - \overline{\pi}_t$ denote the deviations from the steady state (these variables are different than the output and inflation gaps). With this notation, we write $(A.13)$ as

$$\begin{aligned} A\tilde{y}_t + B\tilde{\pi}_t + C\tilde{\pi}_{t+1} &= 0 \\ \tilde{\pi}_t &= \kappa\tilde{y}_t + b\tilde{\pi}_{t-1}. \end{aligned}$$

After substituting for $\tilde{\pi}_{t+1}$ and $\tilde{\pi}_t$ in terms of $\tilde{\pi}_{t-1}$, we can write this system as

$$\begin{aligned} \left(A + (B + bC)\kappa\right)\tilde{y}_t + C\kappa\tilde{y}_{t+1} + (B + bC)\, b\tilde{\pi}_{t-1} &= 0 \\ \tilde{\pi}_t &= \kappa\tilde{y}_t + b\tilde{\pi}_{t-1} \end{aligned}$$

In matrix notation, we have

$$\begin{bmatrix} \tilde{y}_{t+1} \\ \tilde{\pi}_t \end{bmatrix} = \begin{bmatrix} -\frac{A+(B+bC)\kappa}{C\kappa} & -\frac{(B+bC)b}{C\kappa} \\ \kappa & b \end{bmatrix} \begin{bmatrix} \tilde{y}_t \\ \tilde{\pi}_{t-1} \end{bmatrix}.$$

The characteristic polynomial is given by

$$\begin{aligned} P(x) &= \det\left(\begin{bmatrix} -\frac{A+(B+bC)\kappa}{C\kappa} - x & -\frac{(B+bC)b}{C\kappa} \\ \kappa & b - x \end{bmatrix}\right) \\ &= x^2 + x\left(\frac{A + (B + bC)\kappa}{C\kappa} - b\right) + \frac{(B + bC)\, b}{C\kappa}\kappa - \frac{A + (B + bC)\kappa}{C\kappa}b \\ &= x^2 + x\left(\frac{A/\kappa + B}{C}\right) - \frac{Ab/\kappa}{C}. \end{aligned}$$

Note that $P(0) < 0$ and

$$P(b) = b^2 + b\frac{B}{C} > 0.$$

This implies there is a stable root that satisfies $\gamma_L \equiv \gamma_1 \in (0, b)$. Note also that the sum of the roots satisfy

$$\begin{aligned}
\gamma_1+\gamma_2=-\frac{A/\kappa+B}{C} &= -\frac{A/\kappa+(\psi_L+\beta\lambda\Psi_H b(\gamma_H+b))\kappa}{\beta(1-\lambda)\kappa b\psi_L} \\
&= -\frac{A/\kappa+\beta\lambda\Psi_H b(\gamma_H+b)\kappa}{\beta(1-\lambda)\kappa b\psi_L}-\frac{1}{\beta(1-\lambda)b} \\
&< -\frac{1}{\beta(1-\lambda)b}<-1.
\end{aligned}$$

Here, the first line substitutes B and C from (17) and the last line follows since $\beta, 1-\lambda, b \in (0,1)$. It follows that the other root is unstable and satisfies $\gamma_2 < -1$.

These observations prove that the system is saddle path stable. Starting with the inflation deviation $\tilde{\pi}_{t-1}$, both the output deviation and inflation deviation converge to zero at a constant rate γ_L

$$\tilde{y}_{t+1}=\gamma_L\tilde{y}_t \text{ and } \tilde{\pi}_t=\gamma_L\tilde{\pi}_{t-1} \text{ for each } t.$$

To characterize the output in terms of past inflation, note the Phillips curve implies

$$\tilde{\pi}_t=\gamma_L\tilde{\pi}_{t-1}=\kappa\tilde{y}_t+b\tilde{\pi}_{t-1} \Longrightarrow \tilde{y}_t=-\left(\frac{b-\gamma_L}{\kappa}\right)\tilde{\pi}_{t-1}.$$

This establishes $(20-21)$.

Finally, we calculate the interest rate the planner needs to set to implement the optimal output and inflation path. First consider the real interest rate. Using the IS curve (11),

$$\begin{aligned}
r_t &= \rho+E_t[y_{t+1}]-\frac{y_t}{1-\eta}+\frac{\eta}{1-\eta}y_{t-1} \\
&= \rho+\lambda Y_H(y_t)+(1-\lambda)y_{t+1}-\frac{y_t}{1-\eta}+\frac{\eta}{1-\eta}y_{t-1} \\
&= \rho+\lambda(Y_H(y_t)-y_t)+(1-\lambda)(y_{t+1}-y_t)-\frac{\eta}{1-\eta}(y_t-y_{t-1}) \qquad \text{(A.14)}
\end{aligned}$$

Here, y_{t+1} denote the future output if the economy stays in the low-supply state (characterized earlier). Likewise, the nominal interest rate is given by

$$\begin{aligned}
i_t &= E_t[\pi_{t+1}]+r_t \\
&= \begin{array}{c}\lambda\Pi_H(y_t)+(1-\lambda)\pi_{t+1}+ \\ \rho+\lambda(Y_H(y_t)-y_t)+(1-\lambda)(y_{t+1}-y_t)-\frac{\eta}{1-\eta}(y_t-y_{t-1}).\end{array} \qquad \text{(A.15)}
\end{aligned}$$

Here, π_{t+1} is the inflation if the economy stays in state L. This completes the proof. □

B. Alternative model with a ZLB constraint

In the main text, we formalize expansionary policy constraints by requiring the central bank to follow a Taylor rule after transition to state $s = H$. In this appendix, we show that our results are robust to an alternative setup in which the central bank sets the policy optimally in both states, but the expansionary policy is constrained by the zero lower bound (ZLB) on the nominal interest rate. We relegate the proofs to the end of the appendix.

Environment with a ZLB constraint. Consider the setup in Section 2 with two differences. First, there is a ZLB constraint on the nominal interest rate, $i_t \geq 0$. Since prices are fully sticky in our baseline model, the nominal and the real interest rates are the same. Thus, the ZLB also implies a lower bound constraint on the real rate.

Second, the central bank does not follow the Taylor rule (2) in state $s = H$. Instead, the central bank sets the policy optimally in all states. As before, the central bank sets policy without commitment, and it minimizes the present discounted value of quadratic output gaps. We can then formulate the policy problem recursively as

$$\begin{aligned} V_{s_t}\left(y_{t-1}\right) &= \max_{i_t, y_t} -\frac{\left(y_t - y_{s_t}^*\right)^2}{2} + \beta E_t\left[V_{s_{t+1}}\left(y_t\right)\right] \\ \text{s.t. } y_t &= \eta y_{t-1} + (1-\eta)\left(-\left(i_t - \rho\right) + E_t\left[Y_{s_{t+1}}\left(y_t\right)\right]\right) \\ i_t &\geq 0. \end{aligned} \tag{B.1}$$

As in our main setup, $Y_s\left(y_{-1}\right)$ and $V_s\left(y_{-1}\right)$ denote the central bank's optimal output choice and optimal value, respectively, when the current state is $s \in \{H, L\}$ and the most recent output is y_{-1}. The central bank takes its future interest rate decisions and output choices as given and sets the current interest rate and output to minimize quadratic gaps, subject to the inertial IS curve and the ZLB constraint.

Overheating with a ZLB constraint. Recall that, in the first-best benchmark without expansionary policy constraints, the central bank sets a relatively low interest rate in the first period after transition to the high-supply state [see (4)]. We assume the parameters are such that this interest rate is negative: In the first-best benchmark, the ZLB constraint is violated in the first period after transition. Thus, a central bank that is subject to a ZLB constraint cannot achieve zero gaps in all periods and states.

Assumption 1. $\rho - \frac{\eta}{1-\eta}\left(y_H^* - y_L^*\right) < 0.$

Our first result characterizes the equilibrium after the economy transitions to the absorbing state $s = H$.

Lemma 4. *Suppose Assumption 1 holds and the economy has switched to the high-supply state, $s = H$, with past output $y_{-1} \equiv y_{t-1}$. Let $\overline{y}_H = y_H^* - \frac{1-\eta}{\eta}\rho \in (y_L^*, y_H^*)$.*

- *If $y_{-1} \geq \overline{y}_H$, then the ZLB constraint does not bind and the central bank can achieve zero gaps, $Y_H(y_{-1}) = y_H^*$ and $V_H(y_{-1}) = 0$. The interest rate is given by*

$$i_{t,H} = \rho - \frac{\eta}{1-\eta}\left(y_H^* - y_{t-1}\right). \tag{B.2}$$

- *If $y_{-1} < \overline{y}_H$, then the ZLB constraint binds and the output gap is negative for at least one period, $Y_H(y_{-1}) < y_H^*$ and $V_H(y_{-1}) < 0$. The output and the value functions are characterized in the proof and satisfy the following:*
  - *$Y_H(y_{-1}) \geq y_{-1}$ is continuous, strictly increasing, and piecewise linear (it is linear except for a finite number of kink points). Output converges to the efficient level y_H^* after finitely many periods.*
  - *$V_H(y_{-1})$ is continuous, strictly concave and increasing, and piecewise differentiable. At the ZLB cutoff, $y_{-1} = \overline{y}_H$, the value function is differentiable with a zero derivative, $\frac{dV_H(\overline{y}_H)}{dy_{-1}} = 0$.*

Lemma 4 says that, after the supply recovers, the ZLB constraint binds when output is sufficiently low relative to potential. Technically, the ZLB constraint introduces a finite number of kink points into the solution, but the optimal output and the value function satisfy intuitive properties. Starting with a sufficiently low output level, the output gradually recovers and eventually reaches its potential level, y_H^*. Similar to our baseline analysis in Lemma 1, a greater past output increases the current output as well as the value function (over the relevant range $y_{-1} < \overline{y}_H$).

We next establish the analogue of our main result (Proposition 1) in this alternative setup with a ZLB constraint. Consider the optimal policy in the temporary low-supply state, $s = L$. For now, suppose past output y_{-1} is high enough so that the ZLB constraint does not bind in the low-supply state (we consider the case with a binding ZLB in this state subsequently). Then, we can rewrite problem (3) as

$$V_L(y_{-1}) = \max_y -\frac{\left(y - y_L^*\right)^2}{2} + \beta\left((1-\lambda)V_L(y) + \lambda V_H(y)\right). \tag{B.3}$$

The value function in the future low-supply state does not depend on past output, $V_L(y) \equiv V_L$ (as long as the ZLB does not bind, which we will verify). The value function in the future high-supply state $V_H(y)$ is concave. Therefore, the optimality condition is

$$y - y_L^* = \beta\lambda\delta; \quad \text{where } \delta \in \nabla V_H(y). \tag{B.4}$$

Here, δ is a subgradient of the value function. It is equal to the derivative, except possibly at kink points, where it lies in an interval between the left and the right derivatives. Let y_L denote the optimum that solves $(B.4)$.

Eq. $(B.4)$ establishes our main result with the ZLB constraint: the (unique) optimum satisfies $y_L \in (y_L^*, \overline{y}_H)$ and thus $y_L > y_L^*$ and $Y_H(y_L) < Y_H(\overline{y}_H) = y_H^*$. *In the temporary low-supply state, the central bank chooses a level of output that induces positive output gaps in the current low-supply state (current overheating), and negative output gaps after transition to the high-supply state (future demand shortages).* The intuition is the same as in Section 3. As before, the central bank overheats the current output to accelerate the recovery in future periods after transition to high supply.

We can now solve for the associated interest rate:

$$i_t = \rho + \lambda(Y_H(y_L) - y_L) - \frac{\eta}{1-\eta}(y_L - y_{t-1}). \tag{B.5}$$

Recall that $Y_H(y_L) > y_L$. This shows that the ZLB constraint does not bind in the low-supply state ($i_t > \rho > 0$) when past output is already equal to the target level, $y_{t-1} = y_L$. However, there is a sufficiently low level of past output (y_{t-1}) below which the ZLB constraint binds in the low-supply state for at least one period:

$$\overline{y}_L = y_L - \frac{1-\eta}{\eta}(\rho + \lambda(Y_H(y_L) - y_L)). \tag{B.6}$$

The following proposition summarizes the discussion in this appendix and completes the characterization of equilibrium in $s = L$.

Proposition 4. *Suppose Assumption 1 holds and the economy is in the temporary supply shock state, $s = L$, with past output $y_{-1} \equiv y_{t-1}$. Let $\overline{y}_L$ be given by $(B.6)$.*

- *If $y_{-1} \geq \overline{y}_L$, then the ZLB constraint does not bind in $s = L$ and the central bank chooses the output level y_L that is the unique solution to $(B.4)$. The output choice satisfies $y_L \in (y_L^*, \overline{y}_H)$. In the temporary supply shock state, the economy experiences overheating, $y_L > y_L^*$. At the transition to the high-supply state, the*

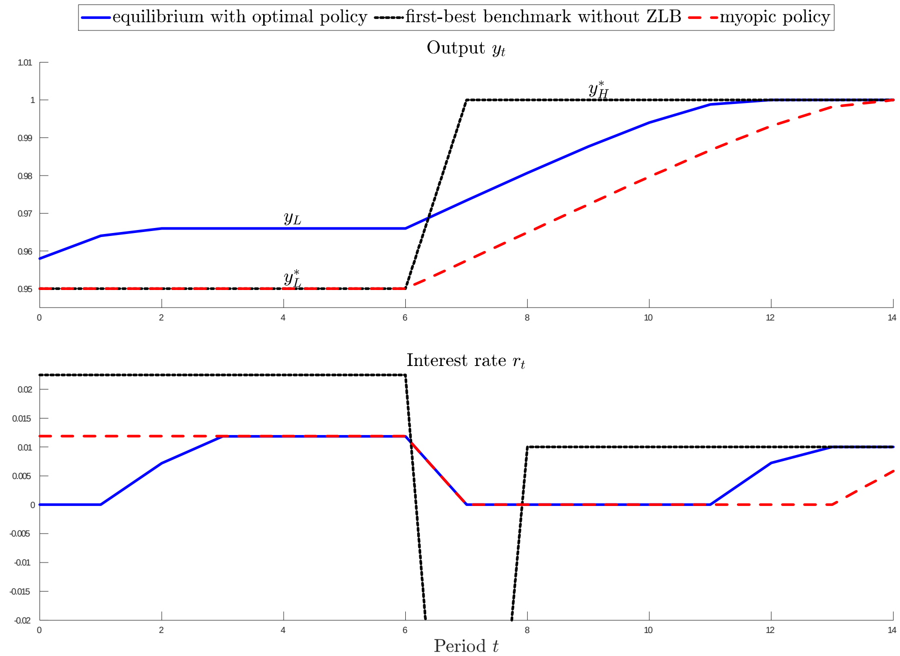

Figure B.1: A simulation of the equilibrium with a ZLB constraint starting in the low-supply state, $s_0 = L$, with the most recent output equal to potential output in the low-supply state, $y_{-1} = y_L^*$. The solid lines correspond to the equilibrium with optimal policy. The dotted lines correspond to a first-best benchmark case in which the policy is not subject to the ZLB constraint. The dashed lines correspond to another benchmark in which the policy is myopic and minimizes gaps in the current period.

economy experiences demand shortages, $Y_H(y_L) < Y_H(\overline{y}_H) = y_H^*$. *The interest rate in* $s = L$ *is given by* $(B.5)$.

- *If* $y_{-1} < \overline{y}_L$, *then the ZLB constraint binds in* $s = L$ *for at least one period. The initial interest rate is zero,* $i_t = 0$, *and the initial output is below its unconstrained level,* $Y_L(y_{-1}) < y_L$. *The output function* $Y_L(y_{-1})$ *(characterized in the appendix) is continuous and strictly increasing. Absent a transition to the high-supply state, output converges to the target level* y_L *after finitely many periods.*

Numerical illustration. Figure B.1 simulates the equilibrium for a numerical example. The dotted lines plot the first-best optimal policy without the ZLB constraint. The solid lines plot the optimal policy with the ZLB constraint and illustrate the main result. The dashed lines plot another benchmark in which the central bank is myopic and focuses on closing the *current* output gap.

The figure resembles Figure 1 in the main text. As before, the optimal policy induces *overheating* in the low-supply state. The policy achieves this by *cutting* the rate aggressively in the earlier periods while the economy is in the low-supply state. In fact, in this simulation the policy runs into the ZLB constraint in the earlier periods. Once the policy brings the output in the low-supply state to a target level above the potential (denoted by $y_L > y_L^*$ in the figure), it raises the interest rate to keep the output constant until the economy transitions to the high-supply state. After the transition, the policy cuts the interest rate once again to raise aggregate demand toward the higher aggregate supply level. However, the policy runs into the ZLB constraint. Due to the binding ZLB, the recovery in the high-supply state takes several periods to complete.

As before, the central bank anticipates that the transition to the high-supply state will start with low aggregate demand and a binding constraint on expansionary policy. Therefore, the central bank optimally *frontloads* interest rate cuts and raises the output in the low-supply state above its potential. Compared to the myopic benchmark (the dashed line), the optimal policy induces some overheating in the low-supply state, but it also reduces the output gaps and accelerates the recovery once the economy switches to the high-supply state.

Proof of Lemma 4. If $y_{-1} \geq \overline{y}_H$, then the central bank can achieve a zero gap, $Y_H(y_{-1}) = y_H^*$ and $V_H(y_{-1}) = 0$. Using the IS curve (1) with $y_t = y_{t+1} = y_H^*$, the interest rate is given by $(B.2)$. The interest rate is nonnegative, $i_{t,H} \geq 0$. In this case, the ZLB constraint does not bind.

In contrast, if $y_{-1} < \overline{y}_H$, then the ZLB constraint binds and the output gap is negative for at least one period, $Y_H(y_{-1}) < y_H^*$ and $V_H(y_{-1}) < 0$.

Consider the constrained range, $y_{-1} \leq \overline{y}_H$. In this range, the IS curve with $i_{t,H} = 0$ implies that output satisfies the recursive relation

$$Y_H(y_{-1}) = \eta y_{-1} + (1-\eta)(\rho + Y_H(Y_H(y_{-1}))). \tag{B.7}$$

We first solve this relation over a sequence of cutoff points for past output. Given $\overline{y}_{H,-1} \equiv y_H^*$ and $\overline{y}_{H,0} = \overline{y}_H$, we recursively define a sequence of cutoffs with:

$$\overline{y}_{H,k+1} = \overline{y}_{H,k} - \frac{1-\eta}{\eta}\left(\rho + \overline{y}_{H,k-1} - \overline{y}_{H,k}\right). \tag{B.8}$$

Using $(B.7)$, it is easy to check that the output function maps a lower cutoff into the higher cutoff:

$$Y_H\left(\overline{y}_{H,k+1}\right) = \overline{y}_{H,k}. \tag{B.9}$$

Note also that the cutoffs satisfy $\overline{y}_{H,k+1} \leq \overline{y}_{H,k} - \frac{(1-\eta)\rho}{\eta}$. Therefore, there exists K_H such that $\overline{y}_{H,K_H} < 0$. Then, the cutoffs $\left\{\overline{y}_{H,k}\right\}_{k=-1}^{K_H}$ cover the entire region $[0, y_H^*]$.

We next extend the solution to the intervals, $\left[\overline{y}_{H,k}, \overline{y}_{H,k-1}\right]$. Specifically, we claim that the output function is piecewise linear and strictly increasing. That is, there exist $\{a_k, b_k\}_{k=0}^{K_H}$ such that

$$Y_H\left(y_{-1}\right) = a_k y_{-1} + b_k \text{ for } y_{-1} \in \left[\overline{y}_{H,k}, \overline{y}_{H,k-1}\right]. \tag{B.10}$$

We also claim that the slope coefficients satisfy $a_k > a_{k-1} \geq 0$ and $a_k < \min\left(1, \frac{\eta}{1-\eta}\right)$.

Using the characterization for the unconstrained region, the claim holds for $k = 0$ with the coefficients

$$a_0 = 0 \text{ and } b_0 = y_H^*. \tag{B.11}$$

Suppose the claim holds for $k - 1$ and consider it for k. Using Eq. $(B.7)$, we have

$$a_k y_{-1} + b_k = \eta y_{-1} + (1-\eta)\left(\rho + a_{k-1}\left(a_k y_{-1} + b_k\right) + b_{k-1}\right).$$

After rearranging terms, we obtain a recursive characterization for the coefficients

$$\begin{aligned} a_k &= \eta + (1-\eta)\, a_{k-1} a_k \\ &\Longrightarrow a_k = \frac{\eta}{1-(1-\eta)\, a_{k-1}} \\ b_k &= (1-\eta)\left(\rho + a_{k-1} b_k + b_{k-1}\right) \\ &\Longrightarrow b_k = \frac{(1-\eta)\left(\rho + b_{k-1}\right)}{1-(1-\eta)\, a_{k-1}} = a_k \frac{1-\eta}{\eta}\left(\rho + b_{k-1}\right). \end{aligned} \tag{B.12}$$

Note that $a_{k-1} < 1$ implies $a_k = \frac{\eta}{1-(1-\eta)a_{k-1}} \in (0,1)$. Likewise, $a_{k-1} < \frac{\eta}{1-\eta}$ implies $a_k = \frac{\eta}{1-(1-\eta)a_{k-1}} < \frac{\eta}{1-\eta}$. We also need to check $a_k = \frac{\eta}{1-(1-\eta)a_{k-1}} > a_{k-1}$. Note that this is equivalent to $P\left(a_{k-1}\right) > 0$ where $P(x) = x^2 - \frac{1}{1-\eta}x + \frac{\eta}{1-\eta}$. This polynomial has roots $\frac{\eta}{1-\eta}$ and 1. Since $a_{k-1} < \min\left(1, \frac{\eta}{1-\eta}\right)$, we have $P\left(a_{k-1}\right) > 0$ and thus $a_k > a_{k-1}$. This proves the claim in $(B.10)$ by induction.

Eqs. $(B.9)$ and $(B.10)$ imply that the output function maps each interval $\left[\overline{y}_{H,k}, \overline{y}_{H,k-1}\right]$ into the higher interval $\left[\overline{y}_{H,k-1}, \overline{y}_{H,k-2}\right]$. This establishes the claim in the proposition that output converges to y_H^* after finitely many periods (at most $K_H + 1$ periods).

We next consider the value function $V_H\left(y_{-1}\right)$. Following similar steps, we can define the value function recursively over the intervals $\left[\overline{y}_{H,k}, \overline{y}_{H,k-1}\right]$. Let $V_{H,0}\left(y_{-1}\right) = 0$ and define a sequence of functions with:

$$V_{H,k}\left(y_{-1}\right)=-\frac{1}{2}\left(a_k y_{-1}+b_k-y_H^*\right)^2+\beta V_{H,k-1}\left(a_k y_{-1}+b_k\right). \tag{B.13}$$

For each interval, the value function agrees with the corresponding function in the sequence:

$$V_H\left(y_{-1}\right)=V_{H,k}\left(y_{-1}\right) \text{ for } y_{-1}\in\left[\overline{y}_{H,k},\overline{y}_{H,k-1}\right].$$

Note also that the functions in the sequence are differentiable with derivatives that satisfy:

$$\frac{dV_{H,k}\left(y_{-1}\right)}{dy_{-1}}=-\left(a_k y_{-1}+b_k-y_H^*\right)a_k+\beta\frac{dV_{H,k-1}\left(a_k y_{-1}+b_k\right)}{dy_{-1}}a_k. \tag{B.14}$$

Therefore, *inside* each interval, the value function is differentiable and its derivative agrees with the derivative of the corresponding function in the sequence:

$$\frac{dV_H\left(y_{-1}\right)}{dy_{-1}}=\frac{dV_{H,k}\left(y_{-1}\right)}{dy_{-1}} \text{ for } y_{-1}\in\left(\overline{y}_{H,k},\overline{y}_{H,k-1}\right).$$

At each cutoff $\overline{y}_{H,k}$, the value function is left and right-differentiable with derivatives respectively given by $\frac{dV_{H,k+1}\left(\overline{y}_{H,k}\right)}{dy_{-1}}$ and $\frac{dV_{H,k}\left(\overline{y}_{H,k}\right)}{dy_{-1}}$.

We next prove that the value function, $V_H\left(y_{-1}\right)$, is strictly concave over the constrained range, $y_{-1}\leq\overline{y}_{H,0}$. For the interior points, $\left(\overline{y}_{H,k},\overline{y}_{H,k-1}\right)$, it is easy to check that the derivative, $\frac{dV_H\left(y_{-1}\right)}{dy_{-1}}$, is strictly decreasing. Consider the cutoff points, $\overline{y}_{H,k}$. It suffices to check that the left derivative is greater than the right derivative:

$$\frac{dV_{H,k+1}\left(\overline{y}_{H,k}\right)}{dy_{-1}}>\frac{dV_{H,k}\left(\overline{y}_{H,k}\right)}{dy_{-1}}.$$

This claim is true for $k=0$. Suppose it is true for $k-1$. Using Eq. $(B.14)$, we have

$$\begin{aligned}\frac{dV_{H,k+1}\left(\overline{y}_{H,k}\right)}{dy_{-1}} &= -\left(\overline{y}_{H,k-1}-y_H^*\right)a_{k+1}+\beta\frac{dV_{H,k}\left(\overline{y}_{H,k-1}\right)}{dy_{-1}}a_{k+1}\\ \frac{dV_{H,k}\left(\overline{y}_{H,k}\right)}{dy_{-1}} &= -\left(\overline{y}_{H,k-1}-y_H^*\right)a_k+\beta\frac{dV_{H,k-1}\left(\overline{y}_{H,k-1}\right)}{dy_{-1}}a_k.\end{aligned}$$

Since $\frac{dV_{H,k}\left(\overline{y}_{H,k-1}\right)}{dy_{-1}}>\frac{dV_{H,k-1}\left(\overline{y}_{H,k-1}\right)}{dy_{-1}}$ and $a_{k+1}>a_k$, we also have $\frac{dV_{H,k+1}\left(\overline{y}_{H,k}\right)}{dy_{-1}}>\frac{dV_{H,k}\left(\overline{y}_{H,k}\right)}{dy_{-1}}$. This proves the claim and shows that $V_H\left(y_{-1}\right)$ is strictly concave over the constrained range.

Finally, we prove that the value function is differentiable at the cutoff point at which starts to bind, $y_{-1} = \overline{y}_H = \overline{y}_{H,0}$, with derivative equal to zero, $\frac{dV_H(\overline{y}_{H,0})}{dy_{-1}} = 0$. The right derivative is zero since $V_{H,0}(y_{-1}) = 0$. Recall that $Y_H\left(\overline{y}_{H,0}\right) = y_H^*$. Therefore, using Eq. $(B.14)$ for $k = 1$, we have

$$\frac{dV_{H,1}\left(\overline{y}_{H,0}\right)}{dy_{-1}} = -\left(Y_H\left(\overline{y}_{H,0}\right) - y_H^*\right) a_1 = 0.$$

This completes the proof of the proposition. Note also that Eqs. $(B.8 - B.14)$ enable a numerical characterization of equilibrium in the high-supply state. □

Proof of Proposition 4. The case $y_{-1} > \overline{y}_L$ is analyzed before the proposition. Suppose $y_{-1} < \overline{y}_L$ so that the ZLB constraint binds. In this case, the IS curve with $i_{t,L} = 0$ implies the output function satisfies the recursive relation

$$Y_L(y_{-1}) = \eta y_{-1} + (1-\eta)\left(\rho + \lambda Y_H\left(Y_L(y_{-1})\right) + (1-\lambda) Y_L\left(Y_L(y_{-1})\right)\right). \tag{B.15}$$

The analysis follows similar steps as in the proof of Lemma 4. Given $\overline{y}_{L,0} = \overline{y}_L$ and $\overline{y}_{L,-1} \equiv y_L$, we recursively define a sequence of cutoffs with:

$$\overline{y}_{L,k+1} = \overline{y}_{L,k} - \frac{1-\eta}{\eta}\left(\rho + \lambda Y_H\left(\overline{y}_{L,k}\right) + (1-\lambda)\overline{y}_{L,k-1} - \overline{y}_{L,k}\right). \tag{B.16}$$

Using $(B.15)$, it is easy to check that the output function maps a lower cutoff into the higher cutoff:

$$Y_L\left(\overline{y}_{L,k+1}\right) = \overline{y}_{L,k}. \tag{B.17}$$

Using $Y_H(y_L) > y_L$, we also obtain $\overline{y}_{L,k+1} < \overline{y}_{L,k} - \frac{(1-\eta)\rho}{\eta}$. Therefore, there exists K_L such that $\overline{y}_{L,K_L} < 0$. Then, the cutoffs $\left\{\overline{y}_{L,k}\right\}_{k=-1}^{K_L}$ cover the entire region $[0, y_L]$.

We can then define the output function recursively over the intervals $\left[\overline{y}_{L,k}, \overline{y}_{L,k-1}\right]$. Let $Y_{L,0}(y_{-1}) = y_L$ and define a sequence of functions with:

$$Y_{L,k}(y_{-1}) = \eta y_{-1} + (1-\eta)\left(\rho + \lambda Y_H\left(Y_{L,k}(y_{-1})\right) + (1-\lambda) Y_{L,k-1}(y_{-1})\right) \text{ for } y_{-1} \in \left[\overline{y}_{L,k}, \overline{y}_{L,k-1}\right]. \tag{B.18}$$

These functions are uniquely defined and increasing over $[0, \overline{y}_L]$ (since the output function in the high-supply state, $Y_H(\cdot)$, is piecewise linear with slopes strictly less than one, as we characterized earlier). Then, Eq. $(B.17)$ implies that for each interval the output function agrees with the corresponding function in the sequence

$$Y_L(y_{-1}) = Y_{L,k}(y_{-1}) \text{ for } y_{-1} \in \left[\overline{y}_{L,k}, \overline{y}_{L,k-1}\right].$$

In particular, the output function maps each interval $[\overline{y}_{L,k}, \overline{y}_{L,k-1}]$ into the higher interval $[\overline{y}_{L,k-1}, \overline{y}_{L,k-2}]$. This establishes the claim in the proposition that, absent transition to the high-supply state, output converges to the target level y_L after finitely many periods (at most $K_L + 1$ periods). This completes the proof of the proposition. Note also that Eqs. $(B.16 - B.18)$ enable a numerical characterization of equilibrium in the low-supply state. □