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ABSTRACT

The concentration of risk within the financial system leads to systemic instability. We propose a theory to explain the structure of the financial system and show how it alters the risk taking incentives of financial institutions when the government optimally intervenes during crises. By issuing interbank claims, risky institutions endogenously become too interconnected to fail. This concentrated structure enables institutions to share the risk of systemic crises in a privately optimal way, but leads to excessive risk taking even by peripheral institutions. Interconnectedness and excessive risk taking reinforce one another. Macroprudential regulation which limits the interconnectedness of risky institutions improves welfare.

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A data appendix is available at http://www.nber.org/data-appendix/w29807

1. INTRODUCTION

A salient feature of the financial systems of advanced economies is the predominance of a few large financial institutions who are highly interconnected with many smaller institutions. This structure, sometimes referred to as core-periphery or hub-and-spoke, is a source of systemic instability as it concentrates resources in systemically important financial institutions (henceforth SIFIs), leaving the rest of the system vulnerable to their failure. Indeed, this concentrated structure is widely seen as a contributing factor for financial crises of late, and has received considerable attention from policymakers as a result. What leads to such a structure to arise in the first place? Why do financial institutions concentrate risk in a manner that generates systemic instability, rather than spreading risk across the financial system?

Addressing these questions requires an understanding of the interaction between the portfolio choices and risk sharing incentives of financial institutions in a setting with heterogeneity and bilateral contracts. Moreover, explaining the predominance of interconnected SIFIs, who benefit from implicit guarantees, requires an understanding of how the anticipation of government intervention shapes the financial system.[1] However, modeling these elements jointly poses methodological challenges, and as a result the literature has typically sought to make progress on one dimension or the other.

The literature on collective moral hazard (e.g., Davila and Walther (2020), Morrison and Walther (2019), Bornstein and Lorenzoni (2018), Farhi and Tirole (2012), Acharya and Yorulmazer (2007)) has shed important light on how government intervention affects the portfolio choices of financial institutions, while the literature on fire sale externalities (e.g., Davila and Korinek (2018), Caballero and Simsek (2013), Lorenzoni (2008)) has established how these choices produce financial fragility. However, both literatures have stopped short of explaining the structural features of the financial system, or take as given the existence of institutions which are 'too big to fail'. By contrast, the literature on endogenous network formation has broken new ground in this regard but often takes financial contracts to be exogenous, or does not consider how this structure affects agents' portfolio choices.

In this paper, we study the interaction between the portfolio choices of financial institutions and their risk sharing incentives. To this end, we construct a parsimonious model with two key ingredients: a government which optimally intervenes during crises and lacks commitment; and an interbank market through which financial institutions can exchange endogenous financial contracts. While these individual elements have been extensively analyzed in the literatures on collective

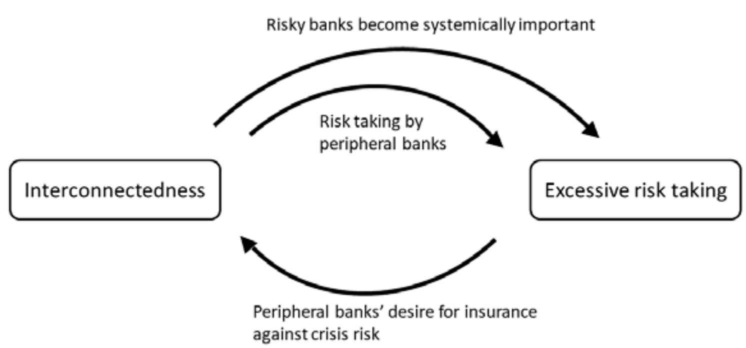
[1] We discuss the empirical evidence that SIFIs benefit from implicit government guarantees in Section 1.2.

moral hazard and financial networks, the contributions of our paper derive from the interaction of risk choices and network formation. This interaction will yield the key mechanism of the model: bank risk taking affects the structure of the interbank market, and also vice versa. While our model features stark results, with a high degree of symmetry in the interbank network and perfect correlation of risk taking, the basic insights would hold in a more general environment.

Our paper makes three contributions to the literature. Our first contribution is to provide a new theory of the structure of the financial system, which puts systemic crises and risk sharing at the core of the analysis.[2] In our setting, the concentrated structure that emerges in equilibrium enables financial institutions to share the risk of systemic crisis in a privately optimal way. During a crisis, the government optimally bails out any institution which is sufficiently large or interconnected in equilibrium. Interconnected SIFIs arise endogenously because they are the only private agents that can insure other financial institutions against systemic crises, since their liabilities are implicitly guaranteed by the government. Risk sharing between these SIFIs and non-SIFIs generates a core-periphery structure in financial markets. Ironically, it is the government's attempt to reduce moral hazard by *only* bailing out systemically important bank that gives rise to the core-periphery structure, which, we show, itself gives rise to systemic moral hazard. Despite its parsimony, the predictions of our model are therefore consistent with this and other important qualitative features of the data, discussed in Section 1.2.

Our second contribution is to show that interconnectedness and excessive risk taking reinforce one another – a dynamic which exacerbates systemic risk. This adverse feedback loop arises in the model due to three channels, each of which is new to the literature. Figure 1 shows a stylized representation of this two-way interaction.

Figure 1:



The first channel through which interconnectedness alters risk taking is represented by the top arrow in Figure 1. Since downside risk is implicitly insured by the government, the interbank market channels funds to investment opportunities with high upside risk. As a result, the institutions

[2]Undoubtedly, there are a multitude of factors which likely play a role in shaping the structure of the financial system, including economies of scale, regulation, and technology, to name a few. We set aside these relevant considerations to focus on the role of risk sharing and systemic crises.

which become large and interconnected (SIFIs) are relatively risky. This is in contrast to both the literature on collective moral hazard (e.g., Davila and Walther (2020)) in which the presence of SIFIs is assumed exogenously, and the literature on network formation in which the identity of SIFIs is typically indeterminate. In our paper, the result that it is relatively risky institutions which become systemically important in equilibrium has important implications for how interconnectedness generates systemic risk, and for the design of optimal policy.

The second channel through which interconnectedness alters risk taking, represented by the middle arrow in Figure 1, is that the implicit insurance offered by a SIFI's liabilities enables smaller, peripheral institutions to hold excessively risky assets, even though they do not directly benefit from bailouts themselves. By contrast, excessive risk taking in the literature is either limited to SIFIs themselves, or the government is assumed to bail out smaller, non-systemic banks. In our model, this channel causes the resources of smaller, peripheral banks to be concentrated in excessively risky projects. Through these two channels, interconnectedness may lead to widespread excessive risk taking, increasing the risk of crisis.

In turn, crisis risk reinforces interconnectedness as a result of interbank risk sharing, represented by the bottom arrow in Figure 1. To obtain insurance against crisis risk, smaller, non-systemic institutions in the periphery invest in the liabilities of SIFIs since they are implicitly insured by the government.

Taken together, these three channels of risk taking and risk sharing imply that interconnectedness and excessive risk taking reinforce one another – a dynamic which exacerbates systemic risk and has implications for the design of optimal policy. Note that the channels which give rise to this dynamic arise precisely from jointly modeling network formation and portfolio choice.

The inefficiencies associated with interconnectedness identified in this paper could aptly be characterized as a too-interconnected-to-fail problem (TITF). Notably, there are several important distinctions between the TITF problem expounded upon in this paper and the too-correlated-to-fail (TCTF) (or too-many-to-fail) problem in which banks can take correlated risks but do not share risk or interact in interbank market, as in Acharya and Yorulmazer (2007). We discuss these distinctions in the literature review. These distinctions have important welfare and policy implications, and stem directly from the interaction between interconnectedness and risk taking that is at the heart of the paper.

Our normative contribution is to show that the concentrated structure of the financial system is suboptimal from a welfare point of view, and to characterize optimal policy. We show that optimal macroprudential policy discourages interbank lending to institutions with risky portfolios. In contrast, policies which only reduce leverage are insufficient in this environment since inefficiencies derive from the asset side of bank balance sheets and do not depend on their capital structure per se. We thus provide a rationale for some post-crisis regulations designed to reduce interconnectedness, but show that they may be inadequate in important respects.

The model also sheds light on the conditions under which interconnectedness leads to financial instability. In contrast to the literature on financial networks in which interconnectedness leads to inefficiencies due to domino effects ex post, in our setting interconnectedness leads to inefficiencies when it concentrates risk and alters risk taking behavior ex ante. As a result, interconnectedness warrants policy intervention when some institutions are too interconnected to fail, and when these institutions engage in risky activities.

In our parsimonious model, the only aspect of the network structure that matters is the core-periphery relationship – beyond this, the network topology does not play an important role determining the equilibrium allocation. While the network topology would play a richer role in a model which featured interbank frictions or idiosyncratic risk, our main insights would hold nevertheless.

Our theory is guided in part by important qualitative features of the data, which we discuss in Section 1.2. Overall, the empirical literature suggests that the financial system is highly concentrated and features a small number of large and interconnected institutions who hold riskier assets, and who benefit from an implicit government guarantee which lowers the cost of their liabilities.[3] These empirical observations suggest an important role for implicit guarantees for understanding the structure of the financial system.

An open question is to what extent interbank lending simply reflects the desire of financial institutions to share risks efficiently. In the absence of bankruptcy costs, the diversification benefit associated with interbank risk sharing could just as well be handled by share ownership rather than the interbank market. An alternative possibility is that the core-periphery structure in interbank markets may facilitate rent seeking behavior in which large, interconnected banks benefit from a too-interconnected-to-fail subsidy, and smaller, peripheral banks share in this bailout premium through interbank financial markets. Our model captures this mechanism.

A key new element of our model is an interbank market through which financial institutions can exchange endogenous financial contracts. We view this very broadly as capturing the various markets through which financial institutions interact, such as derivatives, insurance, equity, syndicated loan, deposit, or money markets. We nevertheless adopt the term 'interbank market' for ease of exposition.

We consider an environment with three dates (0, 1, and 2) in which a risk averse household owns many financial institutions, which we call banks. Banks have access to two projects at date 0 which convert physical capital into a consumption good after one period: a 'prudent' project which is common to all banks, and a risky project which is specific to each bank. While the prudent project has a risk-free return, the returns to the risky project are subject to an aggregate

[3] This lower cost is consistent with the evidence that, historically, the creditors and counterparties of SIFIs have been among the main beneficiaries of government bailouts, despite not being bailed out directly themselves.

shock at date 1. All risky projects are excessively risky from a social perspective: any bank's risky project offers the same expected return as the prudent project, but with higher risk. We assume banks are heterogeneous with respect to the projects to which they have access. Specifically, each bank differs in how exposed its risky project is to the aggregate shock. To simplify the analysis, we assume that the expected return to each project is the same. This makes the welfare analysis easy: clearly, the prudent project is socially preferred. Yet equilibria will emerge in which banks invest in the risky project, and not just the banks that are bailed out. In the more general case, of course, there may be a risk-return tradeoff; the critical issue that our analysis has brought to the fore—the risk of excessive risk-taking—remains.

To invest in either project at date 0, a bank can raise funds from two sources – the household and other banks. On the household side, the model is similar to Lorenzoni (2008): banks can issue state-contingent debt to the household, subject to a limited commitment problem which imposes constraints on borrowing between banks and the household. This constraint creates the possibility of a fire sale of capital to a less productive 'traditional' sector in some states of the world.

In addition to borrowing from the household, banks can exchange bilateral contracts with one another at date 0 in a frictionless interbank market – meant broadly to capture any of the financial markets through which financial institutions exchange claims.[4] The interbank market allows banks to access one another's projects indirectly, and to share aggregate risk. Since the interbank market is frictionless, it channels funds to the projects which generate the largest private surplus.

At date 0, each bank decides how to allocate its portfolio across the available investment projects (prudent and risky) and financial claims issued by other banks on the interbank market. Assets are priced by the stochastic discount factor of the risk averse household and accordingly reflect a risk premium.

At date 1, the aggregate shock to risky projects is realized and financial claims are settled. In the bad state of the world, a bank holding risky assets incurs losses which it must finance by selling physical capital. If the aggregate losses of the banking sector as a whole are sufficiently large – that is, if banks' collective exposure to risky assets is sufficiently high – then the economy enters into a crisis in which banks are forced to fire sell their capital holdings to the less productive traditional sector.

Next, we introduce a government which seeks to maximize household welfare at date 1, subject to information frictions which imply bailouts can be only imperfectly targeted across banks. In a crisis, the government's optimal policy is to bail out banks to prevent the fire sale of capital. The government cannot commit to a policy which is suboptimal ex post. It bails out only the most

[4]While our broad results would hold with incomplete interbank contracts, the assumption that these contracts are restricted only by the limited commitment problem is useful to highlight that any constrained inefficiency of private risk sharing between banks does not derive from imperfections in interbank financial markets.

critical banks so as not to incentivize excessive risk taking by other banks. Moreover, since a bailout occurs only when many banks are failing at the same time, the bailout policy introduces a strategic complementarity in banks' date 0 portfolio choices.

As a result of the strategic complementarity, there are two subgame perfect Nash equilibria. In the 'prudent equilibrium', all banks undertake prudent investments, and so crises and bailouts never occur in equilibrium. In the 'risky equilibrium', both risk sharing and risk taking are constrained inefficient. The interbank market channels funds to the investment opportunities with the highest upside risk. As a result, the banks with relatively risky investment opportunities become excessively large and interconnected, such that they benefit from an implicit government guarantee on their assets. The safety provided by the implicit guarantee drives a wedge between the private and social value of financial claims issued by risky banks. In turn, these SIFIs invest in their risky project, indirectly exposing all other banks to precisely the riskiest projects in the economy through their holdings of SIFI liabilities.

The core-periphery structure of the interbank market plays a crucial role in insuring non-SIFI banks against systemic crises. When a bank holds a risky asset, it bears crisis risk – the risk that it incurs a loss during a crisis, precisely when the stochastic discount factor is highest. Banks are unwilling to hold excessively risky assets in the absence of some form of insurance against this risk.[5] The government provides such insurance in the form of bailouts, but only to SIFIs.

While smaller, peripheral banks do not directly benefit from government guarantees, they benefit indirectly through the interbank market by investing in the liabilities of SIFIs. In a crisis, a SIFI forgoes some of the bailout funds it receives from the government to pay its claimholders a higher rate of return than what it earns on its own assets. This insures claimholders against losses from the SIFI's investments during crises, making risky assets appear safer from the perspective of each individual bank. The insurance value provided by SIFI liabilities is reflected in a lower risk premium, consistent with empirical evidence. As a result of this insurance, even smaller banks who do not directly benefit from the government guarantee may take excessive risks.

While the fact that the SIFIs are both the largest banks and the riskiest financial institutions drops out naturally in our model, it is perhaps not a surprise: their access to bailouts both gives them an advantage over other institutions (some of the benefits of which in our model with competitive banks they share with the periphery) and encourages them to engage in risk taking.

To elucidate the role of the interbank market, we analyze two benchmark variants of the model. In a benchmark without an interbank market in which banks can share the proceeds of bailouts widely, the government's optimal bailout policy is sufficient to eliminate the risky equilibrium. In the second variant of the model, we vary the degree of household risk aversion and show that,

[5] Banks might be willing to hold these risky assets if their expected returns were sufficiently high to offset the risk of losses. This does not occur in our model because we have assumed this possibility away.

under risk neutrality, SIFIs never arise in equilibrium—there is no need to share risks—and in the absence of bailouts, banks never undertake excessive risk. It is the insurance provided by SIFI claims that makes excessively risky investments worthwhile for other banks. It is perhaps ironic that risk aversion leads to systematically more risk which is especially costly given the risk aversion.

The risky equilibrium is associated with strictly lower household welfare due to excessive consumption volatility. A regulator can address the constrained inefficiency using macroprudential interventions in interbank financial markets which prevent banks with risky portfolios from becoming excessively interconnected. These interventions depend on the portfolio risks of borrowing banks, and reflect the higher order counterparty exposures that banks have to one another through the interbank network. By contrast, policies designed to alter the capital structure of banks are inadequate, since implicit guarantees shift risk to the government rather than to creditors. Overall, the interaction between interconnectedness and risk taking that emerges from our model yields new insight into the nature of systemic risk and the design of optimal policy.

1.1. Related literature

Our paper relates most closely to two literatures: the literature on collective moral hazard (especially Davila and Walther (2020), Bornstein and Lorenzoni (2018), Morrison and Walther (2019), Keister (2016), Farhi and Tirole (2012), and Acharya and Yorulmazer (2007)); and the literature on fire sales and financial fragility (especially Lorenzoni (2008), Caballero and Simsek (2013), and Davila and Korinek (2018)).

Relative to literature on collective moral hazard, we examine a different margin of agents' portfolio decisions: their risk sharing incentives. The key new element in our framework is an interbank market through which banks can exchange endogenous financial contracts. The two-way feedback between the structure of the interbank market and risk taking incentives is central to our results. Davila and Walther (2020) were the first to show that the presence of SIFIs can lead to collective moral hazard. In our model, SIFIs emerge endogenously as the result of banks' private risk sharing arrangements, and smaller banks have incentive to hold excessively risky assets despite never being bailed out directly by the government in equilibrium. A canonical paper which inspired much of this literature is Acharya and Yorulmazer (2007). Relative to that paper, the interaction between the structure of the interbank market and risk taking incentives plays a key role in our model. Other related papers in this literature include Freixas (1999), Chari and Kehoe (2016), Schneider and Tornell (2004), Nosal and Ordonez (2016), Acharya (2009), Acharya, Iyer and Sundaram (2020), and Dell'Ariccia and Ratnovski (2019). Other papers, such as Freixas, Parigi and Rochet (2000), Freixas and Rochet (2013), examine optimal regulatory policy taking as given the existence of SIFIs.

There are several important distinctions between the too-interconnected-to-fail problem outlined in this paper and the too-correlated-to-fail problem outlined in Acharya and Yorulmazer (2007), in which banks can take correlated risks but do not share risk or interact in the interbank market. First, in contrast to the allocation under TCTF, the allocation in the TITF economy features maximal exposure to excessive risk, since the interbank market channels resources to SIFIs, who are precisely the riskiest banks in equilibrium. Second, even in the absence of ex ante regulation, the TCTF problem could be eliminated with selective bailouts, as long as this is a credible policy. (We show this in section 6.1.) But this is not possible with the TITF problem; the TITF problem requires regulation ex ante. Third, although we do not formalize this in this paper, the TITF problem can emerge even in the absence of aggregate risk (i.e. when there is only idiosyncratic risk). In that case, interconnectedness is the means by which banks correlate their risk taking even in the absence of common shocks. The TITF gives rise to what *appears* as a TCTF problem. These distinctions have important welfare and policy implications, and stem directly from the interaction between interconnectedness and risk taking that is at the heart of the paper.

Our paper is also related to the large literature on pecuniary externalities (Lorenzoni (2008), Caballero and Simsek (2013), Davila and Korinek (2018), Bornstein and Lorenzoni (2018), Kara and Ozsoy (2020)). In our model, the government's optimal bailout policy completely eliminates inefficiencies related to pecuniary externalities in equilibrium, but replaces them with an inefficiency deriving from the effect of strategic complementarities on the softness of the household budget constraint, a common externality in models of collective moral hazard. In a similar spirit to our paper, Zawadowski (2013) considers how interbank risk sharing leads to moral hazard, using a model in which the network of exposures is fixed exogenously. However, in our setting, the endogenous network is crucial to the model's mechanism, and inefficiencies do not derive from an incompleteness of interbank contracts.

There is a growing literature which analyzes the endogenous formation of financial networks, including Leitner (2005), Acemoglu and Ozdaglar (2014), Di Maggio and Tahbaz-Salehi (2014), Elliott, Golub and Jackson (2014), Chang (2020), Shu (2019), Kanik (2020), and Gai, Haldane and Kapadia (2011). Erol (2018) was the first to show how government bailouts may lead to concentration in an interbank network, but the identity of SIFIs is indeterminate and banks do not face a portfolio choice, elements which are central to our mechanism. Elliott, Georg and Hazell (2021) and Chang and Zhang (2021) also examine the nexus of interconnectedness and risk taking, but our focus is on the role of collective moral hazard and the endogenous emergence of SIFIs. While the network topology plays a richer role in most of these papers, we do not need to keep track of the full structure of the underlying network to characterize allocations in our model; solving for a few features of the network is sufficient to characterize allocations.

1.2 Motivating empirical evidence

In Appendix 14, we present a brief review of the empirical evidence on the structure of interbank markets, with a focus on three 'stylized facts'. The overall picture painted by these facts is one of a highly concentrated financial system in which a small number of large and interconnected institutions hold riskier assets, and benefit from an implicit government guarantee which lowers the cost of their liabilities.

Consistent with these three features of the data, our model will endogenously feature a core-periphery structure in the interbank market in which large, interconnected banks at the core benefit from an implicit government subsidy and undertake riskier investments. In addition, the liabilities of these SIFIs will command a lower risk premium, reflecting the insurance value provided by the implicit government guarantee.

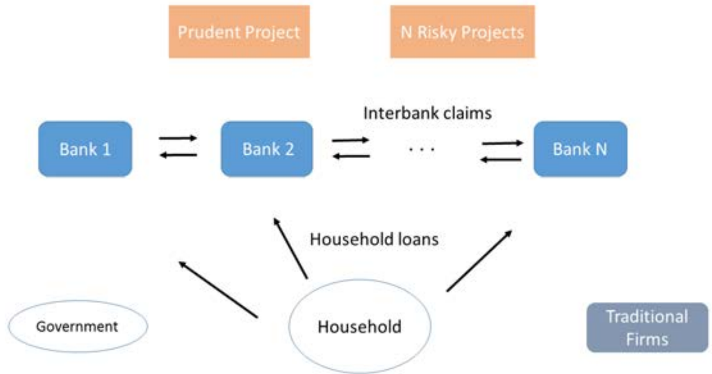
2. MODEL

Figure 2 illustrates the model environment. There are three periods: dates 0, 1, and 2. All uncertainty is resolved at date 1. There are four types of agents: a representative, risk-averse household, banks, traditional firms, and we later introduce a government. The household owns the banks and traditional firms. There are two goods, a consumption good and capital. The consumption good can be costlessly converted one-for-one into capital at any date. Capital can be converted into the consumption good only via investment projects, which are available only to banks. Each bank has access to a prudent (risk-free) project, and a risky project which is subject to an aggregate shock at date 1.

There are N different representative banks, indexed by i. These banks are heterogeneous in how exposed their risky projects are to the aggregate shock. Each representative bank i consists of a continuum of identical, atomistic banks. We henceforth refer to the representative bank of type i simply as bank i.

To invest in their projects, banks can raise funds from the household via an optimal state-dependent debt contract, and from one another by exchanging optimal bilateral contracts on an interbank market. Banks pay out dividends to the household only at date 2. In addition to banks, the household owns traditional firms who make less productive use of capital. We now discuss each agent in more detail.

Figure 2: Model environment



2.1. Household

The representative, risk averse household gets utility from consuming the consumption good according to $u(\cdot)$, where $u(\cdot)$ is twice-differentiable, $u'(\cdot) > 0$, $u''(\cdot) < 0$, and $u(\cdot)$ satisfies the Inada conditions. Each period the household is endowed with e units of the consumption good. At date 1 (after uncertainty is resolved), the household also has access to a riskless storage technology, which we call bond B_1.[6] At date 0, the household is offered a state-contingent financial contract $\left(d_0^i, \left\{d_1^i(s), d_2^i(s)\right\}_s\right)$ by each bank i, which consists of a loan d_0^i from the household to bank i at date 0, and a set of state-contingent payments $\left\{d_1^i(s), d_2^i(s)\right\}_s$ from the bank back to the household at dates 1 and 2, where states are indexed by s. Let f_0^i be an indicator function taking a value of 1 if the household accepts bank i's contract. In addition, the household faces lump-sum taxes T_t in each period t.

Appendix 1 specifies the household's problem in more detail. The household solves a consumption-saving and portfolio allocation problem, taking as given the financial contracts that banks offer, to maximize expected utility $E\left[u\left(c_0\right)+u\left(c_1(s)\right)+u\left(c_2(s)\right)\right]$, subject to each period's budget constraints. The first-order conditions for f_0^i and the date 1 bond holdings B_1 are

$$u'(c_0)d_0^i \geq E\left[u'\left(c_1(s)\right)d_1^i(s)+u'\left(c_2(s)\right)d_2^i(s)\right] \quad (1)$$

$$u'\left(c_1(s)\right)=u'\left(c_2(s)\right). \quad (2)$$

Condition (1) implies that the household accepts bank i's contract only if the expected discounted return promised by the contract exceeds the marginal utility of consumption at date 0. Condition (2) equates marginal utility across dates 1 and, in any state.

[6]The date 1 risk-free bond is not necessary for the model's results, but improves tractability by allowing the household to completely smooth consumption ex post between dates 1 and 2.

2.2. Investment projects and aggregate risk

At date 0, banks have access to investment projects which convert capital at date 0 into the consumption good at date 1. Each bank has access to a prudent project, which is common to all banks, and a risky project which is specific to each bank. We assume that a bank cannot directly invest in another bank's risky project; rather, each bank can directly invest in the prudent project or their own risky project.[7]

The prudent project yields a risk-free return at date 1 of $R_C > 0$ units of the consumption good for each unit of capital invested in the project. On the other hand, each bank's risky project yields a risky return $R_A^i(s) > 0$ at date 1, which varies across states of the world s and across banks i. Our assumptions will imply that risky projects are *excessively* risky: their expected returns do not sufficiently compensate investors for the risk they entail. We elaborate on this below.

The only source of risk in the economy is an aggregate shock $R_A(s)$ to the returns on all banks' risky projects at date 1. The aggregate shock can take a high value or a low value $s \in H,\ L$, where $R_A(H) > R_A(L) > 0$ and $E\left[R_A(s)\right] = R_C$. More precisely, the return on bank i's risky project at date 1 is given by

$$R_A^i(s) = \rho^i R_A(s) - \mu^i. \tag{3}$$

μ^i is a constant which we assume to be $\mu^i = R_C\left(\rho^i - 1\right)$. This constant simply adjusts the return of i's risky project to ensure that all risky projects have an expected return of $E\left[R_A^i(s)\right] = R_C$ (the return on the prudent project). Thus, each bank's risky project entails more risk than the prudent project, but does not compensate the investor for that risk. By construction, risky projects are therefore 'excessively risky' from a social perspective.

Banks are heterogeneous in the riskiness of projects to which they have access, through the parameters $\rho^i > 1$. The parameter ρ^i determines how exposed the returns of bank i's risky project are to the aggregate shock. A bank with a higher ρ^i is riskier in the sense that the risky project to which it has access has a higher variance. One may interpret ρ^i as capturing factors inherent to bank i's business model which expose it to aggregate risk.[8] The parameters ρ^i are thus a reduced-form way to capture the heterogeneity in the inherent 'riskiness' of banks.

[7] This assumption captures the notion that there may be limitations in a bank's ability to replicate the business model or investment opportunities of other banks, due to considerations related to banks' business models, geographic exposures, regulatory constraints, etc. However, as we will see, banks in the model will be able to generate exposure to each others' risky projects by trading financial claims.

[8] In practice, this could arise from any behavior which increases the bank's portfolio returns in good states of the world but magnifies losses in bad states, such as maturity mismatch, leverage, reliance on wholesale funding, having runnable liabilities, or simply having access to projects or assets with a higher market beta.

2.3. Market for physical capital

The spot market for capital features the potential for fire sales due the presence of the less productive traditional firms. At date 0, each bank decides how much capital k_0^i to invest in projects and how to allocate its capital across the prudent and risky projects. The fraction of its portfolio bank i invests in the prudent project is denoted ω^i, with the $1-\omega^i$ being invested in i's risky project.

Turning to date 1, bank i must pay a unit maintenance cost $\gamma < 1$ on its capital holdings k_0^i for the capital to remain productive at date 1. At date 1, each bank i has access to a riskless investment, which we call the *continuation project*, which transforms one unit of capital into one unit of the consumption good at date 2. Each household also owns a so-called *traditional firm* which has access to a less productive, but riskless investment technology at date 1 only. An investment of $k_1^T(s)$ of capital in a traditional firm at date 1 produces $F(k_1^T(s))$ units of the consumption good at date 2, where $F(\cdot)$ is strictly concave, and $F'(\cdot)$ is bounded from above by 1 and below by $\underline{q} < 1$. Since $F(\cdot)$ is strictly concave and $F'(0) < 1$, traditional firms make less productive use of capital at date 1 than investment banks, for any level of investment.

Assumption 1: We assume that $F(0) = 0$, $F'(\cdot) > 0$, $F'(0) < 1$, and $F'(\cdot) \geq \underline{q}$, where $\gamma < \underline{q} < 1$.

Let $q(s)$ denote the price of capital in date 1 state s, which is determined in a competitive market. Because the consumption good can be costlessly converted one-for-one into the capital good at any date, $q(s) \leq 1$ by arbitrage.[9] At date 1 state s, traditional firms choose their date 1 capital holdings $k_1^T(s)$ to maximize $F(k_1^T(s)) - q(s)k_1^T(s)$. If $q(s) \geq 1$, the manager optimally chooses $k_1^T(s) = 0$, whereas if $q(s) < 1$, then $k_1^T(s)$ is chosen to satisfy the first order condition[10]

$$F'(k_1^T(s)) = q(s). \tag{4}$$

Finally, we allow for the possibility of government transfers to banks at date 1. Let $g^i(s, k_0^i, \omega^i)$ denote a subsidy to i's date 1 return on its capital, as a function of the state of the world and bank i's date 0 portfolio.

[9] We assume that the economy begins with 0 units of the capital goods at date 0, which pins down the date 0 price of capital at 1, while the price of capital at date 2 is 0.

[10] More precisely, traditional firms' objective function is to maximize the household's utility $E_0\left[m_2(s)\left(F(k_1^T(s)) - q(s)k_1^T(s)\right)\right]$. But since traditional firms only buy capital at date 1 after uncertainty is resolved, the household's stochastic discount factor does not affect its investment decision, and its objective simplifies to maximizing $F(k_1^T(s)) - q(s)k_1^T(s)$. The first order condition follows from the assumptions that $F(\cdot)$ is strictly concave and $F'(0) = 1$.

2.4. Financial markets

We allow agents to interact in two financial markets: a household debt market and an interbank market. Agents make decisions about borrowing in these two markets simultaneously. In the household debt market, the household can save in the consumption good by lending it to banks at date 0 through the use of an optimal state-contingent debt contract. A limited commitment problem between the household and banks constrains the allocation of funds between these agents – this is a key friction in the model.

In addition to borrowing from the household, we allow banks to borrow from one another in a frictionless interbank market that opens at date 0. In this market, banks can trade claims on each other's portfolio returns in the form of state-contingent financial contracts. An interbank financial contract issued by bank j to bank i is a debt contract which specifies a date 0 loan ℓ^{ij} from i to j, and a state-contingent repayment $r^{ij}(s)$ at date 1 from j to i, per unit of ℓ^{ij}. Appendix 2 discusses further how the two contracting problems interact.

2.5. Banks

Bank budget constraints at date 0 At date 0, each bank is endowed with n units of the consumption good. It can borrow from the household by offering the household a financial contract $\left(d_0^i, \left\{b_1^i(s), b_2^i(s)\right\}_s\right)$ which consists of a date 0 loan d_0^i from the household to bank i, and a set of state-contingent returns $\left\{b_1^i(s), b_2^i(s)\right\}_s$ at dates 1 and 2 from the bank back to the household.[11] In addition, the bank may choose to raise funds from other banks at date 0 by offering another bank an interbank financial contract. An interbank financial contract between bank j and bank i specifies the date 0 initial investment ℓ^{ji} from j to i in units of capital, and a set of state-contingent repayments $r^{ji}(s)$ at date 1 from i back to j, which are chosen optimally.

At date 0, bank i can use its internal funds and debt to finance capital holdings of size k_0^i and make loans ℓ^{ij} to other banks j, subject to a date 0 budget constraint given by

$$k_0^i + \sum_j \ell^{ij} \leq n + d_0^i + \sum_j \ell^{ji}. \tag{5}$$

Given its capital holdings, the bank also decides how to allocate its capital across the prudent project versus its risky project. Let $\omega^i \in [0,1]$ denote the fraction of bank i's capital holdings k_0^i invested in the prudent project; then $1-\omega^i$ is the fraction of i's capital invested in its risky

[11] More precisely, the contract defines state-contingent payments $\left\{d_1^i(s), d_2^i(s)\right\}_s$ at dates 1 and 2 from the bank back to the household. To simplify the notation, we redefine the contract in terms of returns $b_1^i(s) \equiv \frac{d_1^i(s)+d_2^i(s)}{n+d_0^i}$ and $b_2^i(s) \equiv \frac{d_2^i(s)}{k_1^i(s)}$, which scale these repayments by the bank's total net liabilities at dates 0 and 1, respectively.

project. Let $\boldsymbol{\omega}^{\mathbf{i}}$ denote the one-by-two vector $[\omega^i \;\; 1-\omega^i]$ and let $\mathbf{R}^{\mathbf{i}}(s)$ denote the two-by-one vector $[R_C \;\; R_A^i(s)]^T$, so that the date 1 return on bank i's projects are given by the scalar $\boldsymbol{\omega}^{\mathbf{i}}\mathbf{R}^{\mathbf{i}}(s)$.

Bank budget constraints at date 1 At date 1, bank i must pay the unit maintenance cost $\gamma < 1$ on its capital holdings, where $R_C \geq \gamma$. Once the state of the world is realized at date 1, bank i's date 1 funds are given by the sum of its portfolio returns $\boldsymbol{\omega}^{\mathbf{i}}\mathbf{R}^{\mathbf{i}}(s)k_0^i$, the value of its capital holdings $q(s)k_0^i$, and any government transfers $g^i(s,\omega^i)k_0^i$ net of its capital maintenance costs and its debt repayment to the household and other banks. The bank then chooses how much capital $k_1^i(s)$ to hold and invest in the continuation project at date 1 subject to its date 1 budget constraint in state s.

Let $\theta_k^i\left(s,\omega^i,g^i\right) \equiv q(s) + \boldsymbol{\omega}^{\mathbf{i}}\mathbf{R}^{\mathbf{i}}(s) - \gamma - b_1^i(s) + g^i(s,\omega^i)$ denote bank i's rate of return on it's own physical capital holdings in state s, given the allocation ω^i of its capital across projects and any government g^i subsidy to i. Let $\theta_\ell^i(s,j) \equiv r^{ij}(s) - b_1^i(s)$ denote the rate of return on bank i's loan to bank j in state s, given the contract $\left\{r^{ij}(s)\right\}_s$. Define the total rate of return on bank i's assets at date 1 in state s as $\theta^i(s) \equiv \frac{\theta_k^i(s,\omega^i,g^i)k_0^i + \sum_j \theta_\ell^i(s,j)\ell^{ij}}{k_0^i + \sum_j \ell^{ij}}$. We can write bank i's date 1 budget constraint in state s as

$$q(s)k_1^i(s) \leq \theta_k^i\left(s,\omega^i,g^i\right)k_0^i + \sum_j \theta_\ell^i(s,j)\ell^{ij} - \sum_h \left(r^{hi}(s) - b_1^i(s)\right)\ell^{hi} + b_2^i(s)k_1^i(s) \tag{6}$$

Finally, in period 2, investment bank i pays dividends $\pi_2^i(s) = k_1^i(s) - d_2^i(s)$ back to the household, which are determined by bank i's final profits at date 2 net of debt repayments to the household.

2.5.1. Contracting environment between the household and banks

Agents make decisions about household debt and interbank contracts simultaneously. In this section, we begin with household debt. At date 0, each bank i may offer the household a contract which specifies an initial loan d_0^i from the household and a set of state-contingent payments $\left\{d_1^i(s), d_2^i(s)\right\}_s$ to the household at dates 1 and 2. We assume that both the household and banks have a limited ability to commit to honoring the contract at dates 1 and 2. Namely, at dates 1 and 2, the bank chooses whether to honor the contract or not. If the bank does not pay, it makes the household a take-it-or-leave-it offer regarding the date 1 and 2 payments. If the household refuses the offer, the bank is liquidated. The liquidation value of a bank depends on its date 0 portfolio choice, the price of capital in the state, and the banks' net exposures in the interbank market. The contracting problem, and how it depends on banks' interbank exposures, is spelled out in further detail in Appendix 2.

This limited commitment problem imposes no-default constraints on the optimal contracts which ensure that agents never default in equilibrium. The no-default constraints are given by

$$0 \leq d_1^i(s) + d_2^i(s) \leq (q(s) - \gamma) \left(k_0^i + \sum_j \left[\ell^{ij} - \ell^{ji} \right] \right) \tag{7}$$

$$0 \leq d_2^i(s) \leq \Gamma k_1^i(s). \tag{8}$$

Notice from (7) that the limited enforcement constraint depends on the bank's interbank exposures ℓ^{ij}. To entice the household to accept the contract, bank i's contract must satisfy a participation constraint, which is the household's optimality condition (1).

For algebraic simplicity, we scale the state-contingent payments defined in the contract by the bank's net assets, and henceforth use $b_1^i(s) \equiv \frac{d_1^i(s)+d_2^i(s)}{k_0^i - \sum_h \ell^{hi} + \sum_j \ell^{ij}}$ and $b_2^i(s) \equiv \frac{d_2^i(s)}{k_1^i(s)}$ to represent these payments. In addition, the following set of assumptions will be useful.

Assumption 2: We assume that
a) $\gamma - \Gamma - R_A^L > 0$, $R_C + \Gamma \geq 1$, and $R_C - \gamma + \Gamma < \underline{\rho} \left(R_C - R_A(L) \right)$ for $\underline{\rho} \equiv \min_i \left\{ \rho^i \right\}$.
b) $1 - \Gamma > F'(0)$
c) $\left(F'(k_1^T(s)) - \Gamma \right) k_1^T(s)$ is increasing in $k_1^T(s)$.

Assumption 2(a) puts bounds on size of project returns relative to costs; assumption 2(b) ensures that the banks always make more productive use of capital than traditional firms at date 1; and assumption 2(c) will help us rule out multiple equilibria in the date 1 market for capital in any given state.

2.5.2. Contracting environment between banks

Each bank i may raise funds from any other bank h by offering an interbank financial contract at date 0. Recall that an interbank financial contract between bank h and bank i consists of a date 0 investment ℓ^{hi} of capital from h to i, and a set of state-contingent repayments at date 1 given by $r^{hi}(s)\ell^{hi}$.

To simplify the exposition, we assume there are no incentive or enforcement problems or other frictions between banks. Because banks never have an incentive to default on interbank claims, there are no no-default conditions that needed to be imposed on the interbank contract. The interbank contract between banks h and i simply has to satisfy a bank participation constraint to incentivize bank h to lend to bank i.

$$u^{hi} \left(\ell^{hi}, \left\{ r^{hi}(s) \right\}_s \right) \geq \bar{u}^h \tag{9}$$

The variable u^{hi} is the value of bank h if it accepts bank i's contract, while $\bar{u}^h$ is bank h's reservation value – i.e. the value of bank h if it invests in its next best alternative (either lending to another bank, or investing in a project on its own behalf). Therefore, the participation constraint says that bank i must choose a set of state contingent returns to bank h which yields a benefit at least equal to h's outside option. The exact form of this constraint will be derived later from each bank's first order condition for lending to another bank.[12]

2.6. Banks' optimizing behavior

We can now put these elements together to solve each bank's optimization problem. At date 0, each bank i chooses the financial contract $\left(d_0^i, \left\{b_1^i(s), b_2^i(s)\right\}\right)$ with the household, the financial contract $\left\{\ell^{ji}, r^{ji}(s)\right\}_s$ with each other bank j, how much to lend to other banks $\left\{\ell^{ij}\right\}_j$, investment levels k_0^i,$k_1^i(s)$, and portfolio allocation ω^i across projects, to maximize the value of its investment bank $E_0\left[m_2(s)\left(1-b_2^i(s)\right)k_1^i(s)\right]$. Here, $m_2(s)$ denotes the stochastic discount factor at date 2 given state s, and reflects the risk aversion of the household. This problem is subject to budget constraints (5) and (6), no-default constraints for the household contract (7) and (8), the household participation constraint (1), the other banks' participation constraints for each j (9), and non-negativity constraints on capital holdings and interbank loans k_0^i, $k_1^i(s)$, $\ell^{ij} \geq 0\ \forall\, j$.

The full optimization problem and its solution are given in detail in Appendix 3. In what follows, we characterize banks' optimizing behavior.

2.6.1. Date 0 portfolio choice

At date 0, bank i decides how to allocate its funds across its available investment opportunities: claims issued by other banks, or capital to be invested in the prudent project or its risky project. In deciding which assets to hold at date 0, the bank compares the expected discounted value of each asset. Let $\theta_a^i(s)$ denote the state-dependent return to i from investing in some asset a (a project or an interbank claim). Because each bank is owned by a risk averse household, the bank discounts the returns by the household's stochastic discount factor at date 1. Therefore, an asset's value can be decomposed into the expected discounted return plus a risk premium component.

$$\underbrace{E\left[m_1(s)\theta_a^i(s)\right]}_{value\ of\ asset} = \underbrace{E\left[m_1(s)\right]E\left[\theta_a^i(s)\right]}_{expected\ discounted\ return} + \underbrace{Cov\left(m_1(s), \theta_a^i(s)\right)}_{risk\ adjustment}$$

Bank i prefers to invest in asset a rather than another asset b if and only the expected discounted returns to investing in a exceeds that of b.

[12] If a bank defaults on its obligations to the household at date 1 and its assets are seized, the bank's remaining assets must be sufficient to meet its interbank obligations. As a result, optimal interbank contracts are contingent both on the state of the world and on whether the bank defaults on the household at date 1. In equilibrium, however, banks will never default on the household.

$$E\left[m_1(s)\theta_a^i(s)\right] \geq E\left[m_1(s)\theta_b^i(s)\right]. \tag{10}$$

In what follows, we lay out a few results regarding the investment behavior of banks that will help characterize equilibria later on. Lemma 1 shows that each bank is always at a corner solution in its date 0 portfolio allocation decision.

Lemma 1: *Corner solutions in portfolio choice*

Each bank's portfolio allocation choice is a corner solution. Namely, for any bank i,

a) Either $k_0^i = 0$ and $\ell^{ij} > 0$ for some j; *or* $k_0^i > 0$ and $\ell^{ij} = 0$ for all j.

b) If $k_0^i > 0$, then either $\omega^i = 0$ or $\omega^i = 1$.

Proof: For a formal proof, see Appendix 11.

Part (a) of Lemma 1 says that bank i either invests in capital on its own behalf and does not lend funds to any other bank, or the bank lends funds to at least one other bank and does not invest in capital on its own behalf. Part (b) says that if the bank chooses to invest in capital on its own behalf, then it either invests all of its capital in the risky project or it invests all of its capital in the prudent project. Lemma 1 follows from the linearity of the bank's portfolio allocation problem, which arises from the constant returns-to-scale of each bank's investment technologies the absence of idiosyncratic risk, which implies that there is no diversification benefit from investing in different assets.

Intermediaries and investing banks A corollary of this Lemma is that, in equilibrium, all banks can be divided into two groups: banks who invest all of their funds in a project in their own behalf, and banks who forgo their own projects in order to intermediate funds to those investing banks. Henceforth, we refer to banks who invest as *investing banks*, and those banks who intermediate funds between the household and investing banks as *intermediaries*. J and L are defined as the sets of investing banks and intermediaries, respectively. Because (12) holds for all investing banks, investing banks all hold identical portfolios in equilibrium.

Banks endogenously sort themselves into these groups in general equilibrium based on the investment opportunities available to them. In equilibrium, the only banks who invest in a project are those with access to the projects with the highest expected discounted returns $E\left[m_1(s)\theta_k^i\left(s,\omega^i,g^i\right)\right]$. Define this set of banks by W.[13] Therefore, the set of investing banks J is given by $J = W$. All other banks become intermediaries who forgo their own investment projects in favor of intermediating funds between the household and these investing banks (or other intermediaries).

[13] More precisely, $W \equiv \left\{w \mid E\left[z_1(s)\theta_k^w\left(s,\omega^w,g^w\right)\right] \geq E\left[z_1(s)\theta_k^i\left(s,\omega^i,g^i\right)\right] \; \forall\, i \in I\right\}$, where I denotes the set of all N banks.

2.6.2. Choice of which interbank contracts to offer

The analysis above implies that any bank h's participation constraint (9) takes the form

$$E\left[z_1(s)\theta_\ell^h(s,i)\right] \geq E\left[z_1(s)\bar{o}^h(s)\right] \tag{11}$$

where $\bar{o}^h$ is the bank h's opportunity cost of funds, which is determined in general equilibrium. Thus, bank h accepts an interbank contract offered by bank i if and only if the value of contract exceeds that of h's opportunity cost of funds. This opportunity cost of funds will depend on all of its investment opportunities (including interbank contracts offered by other banks) and government transfers.

2.6.3. Optimal contract with household Lemma 2 characterizes the optimal contract between banks and the household. Intuitively, the optimal contract provides some risk sharing between banks and the household, whereby the state-contingent return $b_1^i(s)$ paid to the household is high if the bank's portfolio return is high. However, this risk sharing is limited by the no-default constraints. Moreover, since each representative bank i consists of a continuum of atomistic banks, we have that $z_1^i(s)$ is the same across all banks.

Lemma 2: *Optimal financial contracts with household*

Given a vector of equilibrium prices, an individually optimal financial contract satisfies the conditions for each s: $b_2^i(s)=\Gamma$, $b_1^i(s)=0$ if $z_0^i<\frac{z_1^i(s)}{m_2(s)}$, $b_1^i(s)\in[0,q(s)-\gamma]$ if $z_0^i<\frac{z_1^i(s)}{m_2(s)}$, and $b_1^i(s)=q(s)-\gamma$ if $z_0^i>\frac{z_1^i(s)}{m_2(s)}$, where $\frac{z_1^i(s)}{m_1(s)}=\frac{1-\Gamma}{q(s)-\Gamma}$, and $z_0^i=\frac{\max\left\{E\left[z_1^i(s)\theta_k^i\left(s,\omega^i,g^i\right)\right],\ \max_j\left\{E\left[z_1^i(s)\theta_\ell^i(s,j)\right]\right\}\right\}}{1-E\left[m_1(s)b_1^i(s)\right]}$.

Proof: See Appendix 4.

2.7. Privately optimal interbank financial contracts

We now characterize interbank financial contracts in partial equilibrium. To obtain funds on the interbank market, banks must essentially compete with one another for funds by offering contracts with the most favorable terms. We suppose that the market for interbank funds at date 0 is perfectly competitive.[14] Given perfect competition, interbank contracts are pinned down by the opportunity cost of banks' funds in each state. The lemma below states characterizes interbank contracts as a function of banks' collective choices $\left\{\omega^i\right\}_{i\in I}$, which will be determined in general equilibrium.[15]

[14] Recall that each bank i consists of a continuum of identical atomistic banks. We model competition between banks as a static game between these atomistic banks banks who offer a contract $\left\{\ell^{hi},\, r^{hi}(s)\right\}_s$ to a prospective investor h at date 0. Bank h then evaluates each offered contract based on its risk-return profile and accepts that which has the highest expected discounted returns. We solve for the Nash equilibrium of this game and summarize the results here.

[15] For simplicity of exposition, here we characterize the optimal interbank contracts only for the case in which banks do not default on the household, while we omit the off-equilibrium case in which the banks default on the household.

To put it briefly, interbank contracts are pinned down by the opportunity cost of funds of the investing banks $w \in W$.

Lemma 3: *Interbank financial contracts in partial equilibrium*

For each intermediary bank $h \in L$, interbank contracts are pinned down by the return on capital $\theta_k^w(s, \omega^w, g^w)$ of investing banks $w \in W$, so that $\theta_k^w(s, \omega^w, g^w) = \theta_\ell^h(s)$ for all states of the world. Moreover, the state-contingent return paid by this contract s is given by the unit return on bank w's investment project $r(s) = \omega^{\mathbf{w}}\mathbf{R}^{\mathbf{w}}(s) + q(s) - \gamma + g^w(s, \omega^w)$.

Proof: See Appendix 5.

To understand intuitively how we arrive at Lemma 3, recall that bank h only accepts a contract offered by bank i if the present discounted value of the contract exceeds h's opportunity cost of funds. Because there is only aggregate risk in the economy, a bank w with access to the most privately valuable investment project can always design an interbank contract which incentivizes the prospective lender to accept. In this way, these banks w can out-compete all others for funds on the interbank market.[16]

The optimal contract outlined in Lemma 3 resembles an equity contract in which an investor purchases a claim on the portfolio returns of the issuing bank, where the return on the claim perfectly reflects the portfolio risks of the issuer. Interbank risk sharing is therefore efficient in a partial equilibrium sense: the interbank market channels funds to the most privately valuable assets. However, we will see that in general equilibrium, the private value of risky assets can differ from their social value.

2.8. Investment at date 1

In order to evaluate the date 1 spot market for capital, we first characterize aggregate net investment in capital at date 1. Define $K_0 \equiv \sum_i k_0^i$ and $K_1(s) \equiv \sum_i k_1^i(s)$ to be the aggregate capital holdings of the banking sector at dates 0 and 1, respectively. In Appendix 6, we show that banks' net aggregate investment in capital at date 1 is given by

$$K_1(s) - K_0 = K_0 \left[\frac{\theta_k^w(s, \omega^w, g^w)}{q(s) - \Gamma} - 1 \right]. \tag{12}$$

Equation (12) says that the banking sector's net aggregate investment is given by the aggregate rate of return on capital holdings at date 1, discounted by the cost of capital at date 1. At date 1, the

[16]The results in Lemma 3 imply that we do not have to keep track of the full structure of the underlying network of interbank claims in general equilibrium in order to solve for the allocation of resources and welfare, a feature which greatly improves the model's tractability. The equilibrium network structure of interbank claims matters only insofar as it determines which banks are investing banks in equilibrium.

aggregate rate of return on banks' date 0 capital holdings K_0 is given by the rate of return earned by bank w's assets $\theta_k^w(s, \omega^w, g^w)$. Since banks do not pay out dividends at date 1, this return is invested in capital at date 1. The cost of capital at date 1 is given by the spot price $q(s)$ net of the date 2 repayment to the household $b_2^i = \Gamma$.

2.8.1. Date 1 spot market for capital

We now analyze in partial equilibrium the date 1 spot market for capital. The market for capital features two possible regimes at date 1: *normal times* or a *crisis*. Normal times are characterized by a net positive investment in capital by the banking sector as a whole. This occurs only if the aggregate losses of the banking sector are not too high. In the good state, banks' portfolio returns are high and so they increase their investment in capital. In the bad state, capital is reallocated from banks facing net losses to those not, but otherwise remains entirely within the banking sector.

In contrast, a crisis is characterized by a net sale of capital from the banking sector to traditional firms due to a fire sale externality. This occurs in the bad state when the banking sector's aggregate losses are high – therefore, the precondition for a crisis to occur in the bad state is for the banking sector to have large holdings of risky assets ex ante. In the bad state at date 1, the banking sector is facing a net aggregate loss which needs to be financed. Because there are insufficient funds in the banking sector to cover banks' aggregate losses, banks are forced to liquidate their capital holdings to the traditional sector at a fire sale price. These results are summarized in the lemma below.

Lemma 4: *Date 1 market for capital*

A) In equilibrium we have

$$q(s) = F'(k_1^T(s))$$

$$k_1^T(s) = \max\{0, K_1(s) - K_0\}.$$

B) Moreover, $K_1(s) - K_0 < 0$ if and only if $s = L$ and $\omega^w = 0$.

Proof: See Appendix 7.

2.8.2 Date 1 bond market clearing The date 1 market for bonds clears when supply equals demand. The demand is derived from the household's date 1 budget constraint: $D_B(s) = e_1 + \sum_i f_0^i d_1^i(s) - T_1 - c_1(s)$. Then the supply of bonds $B_1(s)$ adjusts to clear the market.

2.9. Government's problem

We now introduce a benevolent government which seeks to maximize household welfare using unit transfers $g^i(s, \omega^i)$ to each bank, which are financed by lump-sum taxes $T_1(s)$ on the household at date 1. We analyze the government's optimal policy at date 1 after the resolution of uncertainty; the government therefore takes all date 0 variables as given. The government chooses these taxes and transfers to maximize household welfare subject to a budget constraint each period, which at date 1 is given by

$$\sum_i k_0^i g^i(s,\omega^i) = T_1(s). \tag{13}$$

The government fully internalizes how its actions at date 1 affect those of private agents, and therefore takes the equilibrium conditions which determine agents' date 1 and date 2 choices as constraints when solving its problem.

We assume that the government faces two types of frictions, which we discuss in greater detail in Appendix 9A. First, we rule out counterfactual situations in which the government bails out banks even in the absence of a crisis, and we assume that the government cannot bail out a bank unless it can verify that the bank is facing a net loss.[17] These assumptions, common in the literature, prevent the government from enabling banks to circumvent their financial constraints in normal times.

Second, we assume that the government and banks have asymmetric information about the returns on banks' portfolios, which will imply that government transfers can be only imperfectly targeted across banks at date 1. More precisely, the government can only verify a bank's returns on its own investments, but cannot verify a bank's returns from its interbank claims.[18] See Appendix 9A for more discussion.

The lemma below characterizes the solution to the government's problem, which we derive formally in Appendix 8.

Lemma 5: *Government's ex post optimal bailout policy*

A) *Optimal size of total bailout:* The total size of the optimal bailout is given by $G(s,\omega^w)$:

$$G(s,\omega^w) = \begin{cases} K_0\left(q(s) - \Gamma - R_A^w(s)\right) & for\ \ s = L\, and\ \omega^w = 0 \\ 0 & otherwise \end{cases}.$$

B) *Optimal distribution of bailout funds across banks:* Let $g^i(s,\omega^w)$ denote the government

[17] These assumptions can be interpreted as a stand-in for the political constraints that governments face when considering direct transfers to the private sector, or for the the distortionary effect of government intervention or the taxes required to finance bailouts. For instance, in 2008, the US Treasury faced considerable political opposition and pressure from the press against the fiscal measures it had proposed for the rescue of the financial sector.

[18] This assumption is similar to that in Farhi and Tirole (2012) in which the government has an imperfect ability to verify bank losses, and can be interpreted as capturing the greater difficulty that bank regulators and supervisors often have in verifying a financial institution's losses from off-balance sheet exposures, which are often complex and opaque in practice and are frequently associated with interbank financial claims.

transfer to bank i, per unit of i's capital. Any arbitrary set $\left\{g^i(s,\omega^w)\right\}_{i\in W}$ satisfying $\sum_{i\in W} g^i(s,\omega^w)k_0^i = G(s,\omega^w)$ is optimal at date 1. Banks outside the set W of investing banks do not receive a bailout, so that $g^i(s,\omega^w) = 0$ for all $i \notin W$.

Proof: See Appendix 8.

Part (A) of the lemma establishes the size of the aggregate bailout to the banking sector, whereas part (B) establishes how these funds are distributed across banks. First consider part (A). The optimal aggregate bailout $G(s,\omega^w)$ is the minimum aggregate transfer to the banking sector to ensure $K_1(s) = K_0$, i.e. that all capital remains within the banking sector rather than being fire-sold to the traditional sector. Combining this optimal policy with the expression (12) for net aggregate investment in capital at date 1 shows that the optimal bailout puts a floor on the aggregate return on aggregate capital K_0 equal to $q(s) - \Gamma$. Therefore, capital is never misallocated and we always have $q(s) = 1$ and $k_1^T(s) = 0$ in equilibrium.

Part (B) indicates that the government is indifferent at date 1 about how the total bailout is distributed across investing banks. This is because this distribution has no effect on welfare ex post, and is to a large extent inconsequential for the equilibrium allocation at date 0. While we sketch a proof of this in Appendix 9B, the intuition is that perfect interbank risk sharing at date 0 ensures that the benefits of bailouts are shared widely across all banks in equilibrium, regardless of how the government initially distributes the bailout across investing banks at date 1. Nevertheless, for concreteness, we focus on the case in which the government bails out investing banks in proportion to their capital holdings, so that $g^i(s,\omega^w)k_0^i = \frac{k_0^i}{K_0}G(s,\omega^w)$ for each investing bank $i \in W$.

Alternative bailout policies In Appendix 9B, we show that the general equilibrium results we present below are robust to alternative assumptions about the government's behavior, including randomized bailouts as in Nosal and Ordonez (2016).

The government cannot credibly commit at date 0 to an alternative, suboptimal policy. Note that the optimal bailout policy features a kink: a bailout occurs if and only the banking sector's aggregate losses at date 1 are sufficiently large to cause a crisis. This kink introduces strategic complementarity in banks' date 0 portfolio choices.

3. GENERAL EQUILIBRIUM

In the preceding sections, we characterized the equilibrium conditions for all date 1 and date 2 variables as a function of banks' date 0 portfolio choices. It remains to jointly determine the set of investing banks W and their date 0 investment choices ω^w $\forall w \in W$ in general equilibrium. The government's optimal bailout policy introduces a strategic complementarity in banks' date 0 investment actions: a bank's expected discounted payoff to investing in an asset at date 0 depends on the portfolio choices of all other banks due to the possibility of a bailout.[19] In this section, we characterize each bank's best response functions and solve for the subgame perfect Nash equilibria.

An equilibrium is given by a vector of prices $\{q(s)\}_s$, financial contracts $\left\{d_0^i,\ \left\{d_1^i(s),\ d_2^i(s)\right\}_s\right\}_i$, portfolio and investment decisions for the investment banks $\left\{k_0^i,\ \omega^i,\ \left\{k_1^i(s)\right\}_s\right\}_i$, consumption and investment decisions for the household c_0, $\left\{c_1(s),\ c_2(s),\ k_1^T(s)\right\}_s$, $\left\{f_0^i\right\}_i$, bonds $\{B_1(s)\}_s$, and bailout policy and lump-sum taxes $\left\{\left\{g_1^i(s)\right\}_i, \{T_1(s))\}\right\}_s$, such that the household's, banks', traditional firm's, and government's behavior is optimal given their constraints, and capital and goods markets clear in all periods and states.

3.1. Bank best response functions at date 0

Given the government's optimal policy, we now characterize each investing bank i's best response function for its date 0 portfolio choices. We begin with bank i's choice ω^i – the fraction of its capital that bank i invests in the prudent project as opposed to its risky project. Recall from (10) that bank i chooses $\omega^i = 0$ if and only if the discounted value of returns from investing in the risky project exceed those of the prudent project. In equilibrium, this condition reduces to

$$E\left[m_1(s)\right] R_C < E\left[m_1(s)\left(R_A^i(s) + g^w(s)\right)\right] \tag{14}$$

where the return $R_A^i(s) + g^w(s)$ on i's risky project in state s consists of the fundamental return $R_A^i(s)$ and the government subsidy to investing banks $g^w(s)$.[20]

We can split the investment decision of a bank into two cases: when conditions for a bailout to occur in the bad state of the world are satisfied or not, where these conditions are defined in Lemma 4. First, supposed that the conditions for a bailout in the bad state do not hold. Then $g^w(s) = 0$, and banks are forced to fully internalize the riskiness of risky projects. Since the risky project is excessively risky, (14) does not hold and banks choose $\omega^i = 1$ to invest only in the prudent project.

Now suppose that the conditions for bailout to occur during a crisis hold. In this case, the government's optimal bailout policy in Lemma 5 implies that (14) holds, and so banks choose $\omega^i = 0$ to invest only in their risky projects. The implicit put option on the risky asset provided by the government places a floor on the risky asset's return. Therefore, we can summarize bank i's

[19] This is due to a kink in the government's optimal policy outlined in Lemma 5: a bailout is positive if and only if aggregate banking losses are large enough to cause a crisis.

[20] Recall that, because of perfect interbank risk sharing, regardless of how the government initially distributes bailout funds across investing banks, all ultimately receive the same unit subsidy $g^w(s)$ in equilibrium.

best response function for ω^i as follows.

$$\omega^i\left(\{\omega^w\}_{w\in W}\right) = \begin{cases} 1 & if\ g^w(L,\omega^w) = 0 \\ 0 & otherwise \end{cases} \tag{15}$$

We now turn to a bank's choice of whether to invest in an interbank claim or invests in a project. In equilibrium, bank i chooses to invest in a project if and only its expected discounted return from investing in a project is at least as great as that of other investing banks $w \in W$.

$$E\left[m_1(s)\theta_k^i\left(s,\omega^i,g^i\right)\right] \geq E\left[m_1(s)\theta_k^w\left(s,\omega^w,g^w\right)\right] \tag{16}$$

Moreover, given the definition of $\theta_k^w(s,\omega^w,g^w)$ and the government's optimal bailout policy, this condition is satisfied if and only if $\boldsymbol{\omega^i R^i}(s) = \boldsymbol{\omega^w R^w}(s)$. Therefore, bank i is an investing bank in equilibrium if and only if its portfolio returns are equal to that of other investing banks. This pins down the set W of investing banks.

3.2. Subgame perfect Nash equilibria

We first use the banks' best response functions to show that there are two equilibria which vary by banks' date 0 portfolio decisions ω^i, both of which are characterized by herding behavior in date 0 investment. The following proposition shows that there is an equilibrium in which all banks adopt only the prudent investment, and an equilibrium in which all investors adopt only the risky investment.

Proposition 1: *Two subgame perfect Nash equilibria*

There are two subgame perfect Nash equilibria: a 'prudent' equilibrium in which all banks invest in only safe assets (the prudent project or interbank claims on the prudent project), and a 'risky' equilibrium in which all banks invest only in the riskiest assets. Given equilibrium conditions established so far, each equilibrium can be fully described by the investment choice of investing banks ω^w and the set of investing banks W.

Prudent equilibrium: $\omega^w = 1 \quad \forall w \in W$; and the set W is non-empty.

Risky equilibrium: $\omega^w = 0 \quad \forall w \in W$; and $w \in W \iff \rho^w = \bar{\rho}$, where $\bar{\rho} \equiv \max_j\left\{\rho^j\right\}$.

Proof: See Appendix 10.

3.2.1. Prudent equilibrium

In the prudent equilibrium, all banks minimize their portfolio exposures to the aggregate shock by investing only in safe assets, i.e. the prudent project on interbank claims which yield the same return as the prudent project in each state. As a result, in this equilibrium, neither crises nor bailouts ever occur in equilibrium. Moreover, no bank is systemically important from the government's perspective. As a result, no bank has incentive to deviate from investing in safe assets at date 0 to investing in a risky project, since any losses incurred during in the bad state would be fully borne by the bank. Moreover, in the prudent equilibrium, which banks are in the set W of investing banks in equilibrium is both indeterminate and inconsequential for output and welfare (beyond that W is non-empty).

3.2.2. Risky equilibrium

In the risky equilibrium, on the other hand, all banks maximize their exposure to the aggregate shock. The set of investing banks W is given by only those banks who have access to risky projects with the greatest exposure to the aggregate shock – i.e. W consists only of banks i for whom $\rho^i = \bar{\rho} \equiv \max_j \left\{\rho^j\right\}$. These banks invest exclusively in their risky projects. In turn, they finance these investments by issuing claims on these risky investments which are held by all other banks in the economy. In this way, all other banks become exposed to the riskiest project available by forgoing their own projects in favor of buying claims on the riskiest banks' portfolios.[21]

The risky equilibrium thus features a concentration of funds and capital at date 0 in the riskiest banks in the economy. These banks endogenously become highly interconnected with the rest of the banking sector through interbank financial contracts. From the perspective of the government, these risky banks are 'systemically important' in that they are too interconnected to fail.[22] Namely, in the bad state, since losses incurred by these banks on their risky assets are sufficiently large to cause a crisis, the government always bails them out.

We show in Section 4 that the emergence of SIFIs is necessary and sufficient to support excessive risk taking in equilibrium: non-systemic banks will not hold claims on excessively risky assets without insurance against crisis risk; and SIFIs are the only private agents that can provide this insurance, since they are only agents who benefit directly from the government guarantee.

[21] Given our characterization of equilibrium interbank contracts in Lemma 3, the structure ℓ^{ij} for all i and j of interbank claims are neither determinate nor allocatively relevant beyond describing which banks are in set W.

[22] In our parsimonious setting, we abstract from determinants of a bank's size other than interbank borrowing, and as a result, the size of a bank's assets corresponds one for one with the extent to which it is interconnected. In reality, a bank's size can be affected by factors abstracted from in the paper (such as economies of scale, market power etc.) in a way which does not correspond directly either with interconnectedness or risk taking.

3.2.3. Risk sharing and the structure of the interbank market

In the risky equilibrium, banks who hold risky assets bear crisis risk – the risk that the bank incurs a loss during a crisis, precisely when the stochastic discount factor is highest.[23] The core-periphery structure of the interbank market emerges out of banks' desire to share crisis risk in a privately optimal way.

In the risky equilibrium, some banks (SIFIs) benefit directly from the government guarantee while the rest do not. The government guarantee enjoyed by a SIFI w has two effects on its portfolio: the guarantee not only increases the expected return of its assets $E\left[\theta_k^w(s)\right]$ by putting a floor on its losses in the bad state, but it also lowers the risk premium of its assets – that is, it increases the covariance between the household's stochastic discount factor and the return on the SIFI's portfolio, $Cov\left(m_1(s), \theta_k^w(s)\right)$. The lower risk on the portfolios of SIFIs creates scope for risk sharing between SIFIs and peripheral banks in the risky equilibrium.

Proposition 2 shows how this risk sharing occurs. In the bad state of the world, a SIFI pays more on its interbank liabilities than what it earns on its assets by giving up some of the bailout funds it receives from the government. This partially insures its claimholders against losses from the SIFI's investments during crises.

Proposition 2: *Interbank risk sharing in the risky equilibrium*

During a crisis, a SIFI pays a return of $R_A^w(L) + g^w(L, \omega^w)$ on its interbank liabilities, which exceeds the return on its own assets $R_A^w(L)$. This excess return $g^w(s, \omega^w)$ increases $Cov\left(m_1(s), \theta_\ell^i(s, w)\right)$ and hence reduces the portfolio risk of each of the SIFIs' claimholders i.

Proof: From Lemmas 3 and 5, we immediately have $\theta_\ell^i(s, w) = \theta_k^w(s, \omega^w, g^w) = R_A^w(s) + g^w(s, \omega^w)$.

SIFI liabilities are relatively safe assets which command a low risk premium, despite being backed by the SIFI's excessively risky investments. In this manner, SIFIs act as intermediary insurers whereby they benefit directly from the government guarantee, and insure other banks against crisis risk through the interbank market.

3.2.4. Interconnectedness and excessive risk taking reinforce one another

In the risky equilibrium, interconnectedness and excessive risk taking reinforce one another. In particular, the preceding results in Sections 3.2.2. and 3.2.3. indicate that the concentrated

[23] Although the government's optimal bailout policy eliminates the possibility of fire sales, it does not eliminate fluctuations in the household's stochastic discount factor. On the contrary, the aggregate losses that the banking sector incurs in the bad state are absorbed by low household consumption due to the taxes needed to finance the bailout. This manifests as a high stochastic discount factor in the bad state.

structure of the interbank market leads to excessive risk taking through two channels, represented by the top two arrows in Figure 1 in the introduction. First, recall that in the risky equilibrium, it is precisely the riskiest banks who become too interconnected to fail. This is because, since the implicit guarantee insures downside risk, the interbank market channels funds to projects with the highest upside risk. As a result, the implicit guarantee enjoyed by SIFIs induces them to invest in precisely the riskiest projects.

Second, risk sharing between SIFIs and peripheral banks in the interbank market creates incentive for peripheral banks to hold assets which are excessively risky from a social perspective. In particular, peripheral banks hold interbank claims issued by SIFIs even though these claims are backed by the SIFIs' excessively risky investments. The result of this is that the funds of all banks, and not just those of the SIFIs themselves, are ultimately invested in excessively risky projects. This amplifies the aggregate exposure of the banking sector to excessive risks.

In turn, excessive risk taking reinforces the interconnectedness of the financial system due to the mechanism described in Section 3.2.3. Bank risk taking generates crisis risk for which non-systemic, peripheral banks want insurance. Interbank claims issued by SIFIs provide precisely this insurance. Hence, peripheral banks' desire for insurance against crises reinforces the interconnectedness of SIFIs.

The result of these three channels is a two-way feedback effect through which interconnectedness of the financial system and excessive risk taking reinforce one another, represented by Figure 1. Importantly, this dynamic is precisely what sustains excessive risk taking in the risky equilibrium: without this two-way interaction, there could be no equilibrium featuring excessive risk taking.

3.3. Welfare-ranking the equilibria

Let ex ante welfare in the prudent and risky equilibria respectively be denoted by $\bar{\Phi}$ and $\tilde{\Phi}$, so that $\bar{\Phi} \equiv u(\bar{c}_0) + u(\bar{c}_1) + u(\bar{c}_2)$ and $\tilde{\Phi} \equiv u(\tilde{c}_0) + E[u(\tilde{c}_1(s))] + E[u(\tilde{c}_2(s))]$. It is straightforward to show that household welfare is strictly greater in the prudent equilibrium, $\tilde{\Phi} < \bar{\Phi}$. The risky equilibrium is associated with lower welfare because the household's consumption is more volatile. This simply reflects that aggregate output at date 1 is more exposed to aggregate risk in the risky equilibrium (without having a higher mean), and is therefore second-order stochastically dominated by aggregate output in the prudent equilibrium. In Section 5, we show that this welfare loss is due to a soft budget constraint externality which derives from the strategic complementarity in banks' portfolio choices.

4. TWO BENCHMARK ECONOMIES

To further elucidate the role of the interbank market in facilitating risk sharing and creating the collective excessive risk taking, we analyze two variants of the model.

Benchmark 1: Model without interbank market In the first benchmark, we consider a special case of the baseline model in which we shut down the market for interbank financial claims. In this setting, there is a unique subgame perfect Nash equilibrium in which all banks undertake only prudent investments. The reason for this is that, to support risk taking in equilibrium, the insurance benefits of government guarantees need to be shared widely across banks – otherwise not enough banks will be exposed to the aggregate shock to trigger a crisis and bailout in the bad state.

Without an interbank market to facilitate risk sharing between banks, the sole beneficiaries of a bailout are those that the government bails out directly. By concentrating the bailout on small number of investing banks, the government is able to force the majority of banks to internalize the riskiness of their investments, eliminating the risky equilibrium. Thus, the interbank market is the means by which banks can ensure that the benefits of implicit guarantee are shared widely enough to support collective investment in risky assets.

Benchmark 2: Varying the degree of risk aversion In the second benchmark, we illustrate how the risk sharing between SIFIs and non-SIFIs *per se* leads to excessive risk taking. (This benchmark is analyzed in detail in Appendix 12.) We modify the model so that only the risk sharing role of the interbank market affects banks portfolio choices, and then perform a comparative static exercise in which we vary the degree of risk aversion of the household.

Under risk neutrality, each bank i's best response function is to always invest in prudent assets. Since agents do not value risk sharing, the insurance provided by claims on SIFIs has no value. As a result, no bank ever undertakes a risky investment in equilibrium. As the household's risk aversion increases, the safety offered by an interbank claim issued by a SIFI is valued more highly, increasing the safety premium $Cov\left(m_1(s), \theta_\ell^i(s,w)\right)$ commanded by these claims. If the insurance value of these claims is sufficiently high, then other banks forgo their prudent projects in favor of investing in these claims. Protected by the government guarantee, SIFIs in turn invest in risky projects. Hence, banks undertake excessive risks only when the insurance provided by these SIFI claims is sufficiently high.

5. SOCIAL PLANNER'S PROBLEM

In this section, we characterize the constrained efficient allocation. Consider the problem of a social planner who seeks to maximize household welfare and faces the same constraints as private agents and the government. The planner's problem is to choose consumption and investment plans for all agents, the allocation of funds across banks, and taxes and transfers, subject to the limited commitment problem between banks and the household, and the constraint that the allocation of capital at date 1 is determined in a spot market.[24] Moreover, the planner must respect the inability of the government to credibly commit at date 0 to policies which are suboptimal at date 1, which implies the planner faces the same time consistency constraint faced by the government discussed in the section on the government's problem.

Therefore, the only ways in which the planner's problem differs from those of private agents is that the planner internalizes the effect of contracts and portfolio choices on the price of capital, and also internalizes the effect of government transfers on the softness of the household budget constraint via taxes. We formalize and solve the full planner's problem in Appendix 13.

5.1. Social planner's solution

There are three margins through which the planner's optimality conditions differ from those of private agents in the competitive equilibrium: the aggregate leverage (or date 0 investment) of the banking sector, the exposure of investing banks to the aggregate shock, and the risk sharing arrangements of banks. The planner internalizes the effect of each margin on the likelihood and size of a bailout, and the effects that a bailout has on the softness of the household budget constraint through lump-sum taxes, which are required to finance any bailout. In addition, the planner internalizes the network effects of interbank lending on each bank's date 0 investment. We characterize the constrained efficient allocation below; the full planner's solution is analyzed in detail in Appendix 13.

5.1.1. Social valuation of assets

We discussed in Section 2.6.1. how private agents value assets based on the standard asset pricing condition. From the planner's first order conditions, we can see the value to the planner of an asset with state-dependent returns $R(s)$ is given by

[24] The planner's choices are subject to banks' participation constraints for interbank claims, asymmetric information between the household and banks which prevents households from contracting directly on banks' portfolio choices and forces them to contract on banks' ex post returns instead.

$$\underbrace{E\left[m_1(s)R(s)\right]}_{private\ valuation} - \underbrace{E\left[m_1(s)\frac{\partial T_1(s)}{\partial \omega^i}\right]}_{social\ cost}. \tag{17}$$

Hence, the social value of an asset adjusts the value to private agents for the social cost of investing in the asset. In turn, this social cost reflects how investing in the asset tightens the household budget constraint in the bad state through the lump-sum taxes needed to finance a government bailout.

In the competitive economy, the prudent equilibrium is constrained efficient – the allocation lines up with that of the planner. On the other hand, relative to the constrained efficient allocation, the risky competitive equilibrium features three margins of inefficiency: over-borrowing, excessive risk taking, and constrained inefficient risk sharing, the last of which is the focus of our paper.

5.2. Constrained inefficient risk sharing

In this section, we show that risk sharing is constrained inefficient in the risky equilibrium. The planner cares about interbank risk sharing to the extent that it influences the allocation of risk across heterogeneous banks at date 0.[25] Recall that we defined the total rate of return on bank i's portfolio by $\theta^i(s)$, given above equation (6). The ratio $\frac{\theta^i(H)}{\theta^i(L)}$ of its portfolio returns in each state corresponds the extent to which bank i's portfolio of assets is exposed to the aggregate shock. (More precisely, $\frac{\theta^i(H)}{\theta^i(L)} = 1$ if and only if bank i has no net exposure to the aggregate shock, while this ratio diverges from 1 as this net exposure increases.) Let M and m denote the 'riskiest' and 'safest' banks, i.e. the banks with the greatest and least net exposure to the aggregate shock, respectively.[26] It follows that $\frac{\theta^M(H)}{\theta^M(L)} \geq \frac{\theta^m(H)}{\theta^m(L)}$.

Condition for efficient risk sharing The condition for constrained efficient risk sharing is that all banks have the same exposure to the aggregate shock as the safest bank. More formally, taking as given each bank i's choice of risk taking ω^i, this condition is

$$\forall i \in I \quad \frac{\theta^i(H,\omega^i)}{\theta^i(L,\omega^i)} = \frac{\theta^m(H,\omega^m)}{\theta^m(L,\omega^m)} \geq 1. \tag{18}$$

Intuitively, constrained efficiency requires that each bank use interbank contracts to minimize its exposure to the aggregate shock. Moreover, the planner would have banks undertake only prudent investments, implying that under constrained efficiency, $\frac{\theta^i(H)}{\theta^i(L)} = 1$ for all i. This condition is satisfied in the prudent equilibrium.

In the risky equilibrium, by contrast, private risk sharing is constrained inefficient. Indeed, there is a drastic divergence between the risk sharing behavior of banks and that desired by the

[25] The planner does not care about interbank risk sharing beyond its implications for date 0 portfolio choices since the ex post distribution of net worth across banks is irrelevant for welfare.

[26] These banks are define by $M \equiv \left\{i:\ \max_{j\in I}\ \frac{\theta^j(H)}{\theta^j(L)}\right\}$ and $m \equiv \left\{i:\ \min_{j\in I}\ \frac{\theta^j(H)}{\theta^j(L)}\right\}$.

planner: private risk sharing arrangements in the risky equilibrium *maximize* banks' exposure to the aggregate shock, rather than minimizing it as in the planner's solution. The proposition below conveys this stark result.

Proposition 3: *Constrained inefficient risk sharing in the risky competitive equilibrium*

Fix each bank i's portfolio choice of ω^i. Then risk sharing in the risky equilibrium is characterized by

$$\forall i \in I \qquad \frac{\theta^i(H,\omega^i)}{\theta^i(L,\omega^i)} = \frac{\theta^M(H,\omega^M)}{\theta^M(L,\omega^M)} \tag{19}$$

where $\frac{\theta^M(H,\omega^M)}{\theta^M(L,\omega^M)} > \frac{\theta^m(H,\omega^m)}{\theta^m(L,\omega^m)}$.

Moreover, in the risky equilibrium each investing bank undertakes only risky projects ($\omega^i = 0$), implying that $\frac{\theta^i(H)}{\theta^i(L)} > 1$ for all i.

Importantly, the constrained inefficiency of interbank risk sharing does not derive from any imperfections in interbank financial markets, but rather derives from a soft budget constraint externality due to the strategic complementarity in banks' portfolio choices. A bailout is ultimately funded by lump sum taxes on the household, which reduces household consumption in the bad state. Ex ante, private agents do not internalize how their exposure to the aggregate shock, whether through their holdings of risky interbank claims or from investing in their own risky projects, affects the budget constraint of the household in the bad state of the world. The result is excessively high consumption volatility.

In addition to inefficient risk sharing, the risky equilibrium features over-borrowing and excessive risk taking at date 0 because agents do not internalize how aggregate leverage and their asset holdings affect the softness of the household budget constraint in a crisis through the taxes needed to finance the bailout.[27]

6. OPTIMAL MACROPRUDENTIAL POLICY

6.1. Regulation of the interbank market

The model sheds light on the conditions under which interconnectedness leads to financial instability – an issue which has proven challenging for policymakers. In much of the literature on financial networks, interconnectedness leads to inefficiencies due to network contagion ex post. By contrast, in our setting interconnectedness leads to inefficiencies when it concentrates risk and

[27] These results are reminiscent of Lorenzoni (2008), Farhi and Tirole (2012) and Acharya and Yorulmazer (2007).

alters risk taking behavior ex ante. As a result, interconnectedness warrants policy intervention when some institutions are too interconnected to fail, and when these institutions engage in risky activities.

We now discuss how the constrained efficient allocation can be implemented using portfolio taxes and interventions in interbank financial markets at date 0. A regulator can implement constrained efficiency of risk sharing through appropriate taxes on holdings of claims on banks with risky portfolios, in order to prevent these banks from becoming excessively interconnected or large at date 0. Together with a slightly modified optimal bailout policy, described below, the taxes characterized in Lemma 6 are necessary and sufficient to implement constrained efficiency.

Lemma 6: *Optimal taxes on interbank claims*

Taxes τ_ℓ^{ij} on each bank i's holdings ℓ^{ij} of claims on each bank j's portfolio can implement constrained efficiency of interbank risk sharing, where τ_ℓ^{ij} is given by

$$\tau_\ell^{ij}\left(s, g^j(s,\omega^j)\right) = E\left[u'\left(c_1(s)\right) g^j(s,\omega^j)\right] - \frac{E\left[m_1(s)\theta^j\left(s,\omega^j,g^j\right)\right]}{1-(1-\gamma)E\left[m_1(s)\right]}. \tag{20}$$

These taxes distort private portfolio choices away from investing in claims issued by banks with risky portfolios, to prevent these banks from become too interconnected and large to fail from the perspective of the government. The first term of τ_ℓ^{ij} reflects the welfare costs of bank i holding a claim on j's portfolio from increasing the bailout to bank j. The second term is the shadow value of bank j's funds at date 0.

Three features of these taxes on interbank claims are worth emphasizing. First, the tax on i for investing in a claim issued by j is increasing in the riskiness of issuer j of the claim, captured by the exposure of j to the aggregate shock through the dependence of g^j on ω^j. Second, the full set of taxes $\left\{\left\{\tau_\ell^{ij}\right\}_j\right\}_i$ between all bank pairs addresses the indirect exposures of each bank to the aggregate shock through the network effects of their higher order interbank linkages. Third, the taxes are *macroprudential* in nature, and depend on the aggregate exposure of the banking sector as a whole in general equilibrium.[28]

These results rationalize the use of macroprudential tools designed to reduce the systemic consequences of interconnectedness in the financial system, such as the Single-Counterparty Credit Limits adopted by the Federal Reserve in 2018. However, unlike the taxes given in (20), the many policies that have thus far been proposed or implemented often do not take into account the full network of higher-order exposures to risky assets, nor are they generally macroprudential in nature.

[28]These taxes are non-zero only when aggregate exposures of the banking sector as a whole to the aggregate shock are sufficiently large to trigger a crisis and a bailout in the bad state, since otherwise $g^j(s) = 0$ for all s.

While the risky equilibrium features excessive borrowing, correcting this distortion is neither necessary nor sufficient to implement constrained efficiency, highlighting that the inefficiencies in our setting derive from banks' exposures on the asset side of their balance sheets to risky investments and do not depend on their capital structure per se.

Macroprudential policy and selective bailouts The Pigouvian taxes described above prevents SIFIs from arising in equilibrium by ensuring that no bank wants to lend to any bank with risky portfolios. However, it could still be the case that each bank invests in its own risky project, and since these are perfectly correlated, that would give rise to systemic risk and a bailout. This would be a too-correlated-to-fail equilibrium (e.g. Acharya and Yorulmazer (2007)). This TCTF equilibrium can be eliminated with a bailout policy in which the government bails out only some investing banks, and lets others fail. In other words, the optimal bailout policy would still be the same size in aggregate, but it would give a larger bailout to some failing banks, and force other failing banks to cover their losses by selling their capital to these bailed out banks. (Effectively, this bailout would entail a transfer of capital from failing banks to non-failing banks).

This policy would eliminate the TCTF equilibrium. Ex ante, some banks would know that they will never be bailed out in a crisis. And so they would never invest in their risky project in the first place. As a result, the total exposure of the banking sector to the aggregate shock would not be sufficient to trigger a bailout in the bad state. Therefore, even banks who would have benefited from a bailout were it have occurred have no incentive to undertake risky projects. So the unique equilibrium would be the one in which banks invest only in prudent projects. We prove this in Appendix 15.

Note that the government can credibly commit to such a bailout policy, because, ex post, all that matters for household welfare is that capital remains in the banking sector as whole. But which banks hold the capital ex post is irrelevant for future output due to constant returns-to-scale. This bailout policy ensures that capital remains with banks (rather than unproductive traditional firms), but affects the distribution of capital across these banks.

Thus, the Pigouvian tax on interconnectedness in Lemma 6 can implement the constrained efficient *risk sharing* allocation. But to achieve the efficient outcome, we also have to eliminate the TCTF equilibrium, and to do that the government has to commit to a bailout policy in which only some of the financial institutions are bailed out.

7. CONCLUSION

We offered a new theory of the structure of the financial system based on risk sharing, systemic crises, and government intervention. The core-periphery structure enables financial institutions to share risk in a privately optimal way. However, this structure can lead to excessive risk taking even for smaller banks who do not benefit from implicit guarantees, and channels funds to excessively risky investments. Therefore, taking into account how this structure arises shows the collective moral hazard problem generated by government intervention to be more pervasive and severe than previously understood. Macroprudential regulations which limit the interconnectedness of risky institutions can improve welfare, but may face practical challenges. We argued that post-crisis regulations may be inadequate in important respects.

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APPENDICES

APPENDIX 1: Household optimization problem

At date 0, the household solves a consumption-saving and portfolio allocation problem, given the financial contracts available to it. Namely, it chooses consumption at each date and in each state $\{\{c_t(s)\}_s\}_t$, and how to allocate its date 0 savings across investment banks i, described by weights the indicator functions f_0^i which take the value of 1 if the household accepts bank i's contract and 0 otherwise. Given d_0^i, the total amount of funds the household invests in bank i is given by $f_0^i d_0^i$, and aggregate date 0 saving is then $\Sigma_i f_0^i d_0^i$.[29] We further assume that banks cannot commit at date 0 to investing in particular projects at date 1. Therefore, the household has no information on which projects each bank will invest in at date 1. As a result, the household chooses f_0^i based only on the contract $\left(d_0^i, \left\{d_1^i(s), d_2^i(s)\right\}_s\right)$ offered by each bank.

At date 1, the household also chooses its date 1 bond holdings to maximize expected utility subject to its budget constraint each period.

$$\max_{\{\{c_t(s)\}_s\}_t, \left\{f_0^i\right\}_i, \{B_1(s)\}_s} E\left[u(c_0) + u(c_1(s)) + u(c_2(s))\right]$$

[29]The household's problem is equivalent to a consumption CAPM in which the household simultaneously solves a consumption-savings and portfolio allocation problem, in which it chooses total savings and the share of savings allocated to each bank i.

$$c_0 + \Sigma_i f_0^i d_0^i \leq e_0 - T_0 \tag{21}$$

$$c_1(s) + B_1(s) \leq e_1 + \sum_i f_0^i d_1^i(s) - T_1 - q(s) k_1^T(s) \tag{22}$$

$$c_2(s) \leq e_2 + B_1(s) + \sum_i f_0^i d_2^i(s) + \Pi_2(s) - T_2 \tag{23}$$

Here, $\Pi_2(s)$ is the date 2 profits of all banks and traditional firms in state s, and T are lump-sum taxes. Let $e_0 = e_1 = e_2$. Also assume that e is sufficiently large that non-negativity constraints for c_0,c_1, and c_2 are never binding. The first-order conditions for f_0^i and the date 1 bond holdings are

$$u'(c_0) d_0^i \geq E\left[u'\left(c_1(s)\right) d_1^i(s) + u'\left(c_2(s)\right) d_2^i(s)\right] \tag{24}$$

$$u'\left(c_1(s)\right) = u'\left(c_2(s)\right) \tag{25}$$

APPENDIX 2: Contracting environment between the household and banks

Timing of the contracting problems Recall that there are two types of contracting problems: one between households and banks, and another between banks. These contracting problems are solved simultaneously at date 0: at the same time that consumers agree with banks about state contingent payments $\left\{d_1^i(s), d_2^i(s)\right\}_s$, the banks make loans ℓ^{ij} to one another in exchange for state contingent payments $\ell^{ij} r^{ij}(s)$ at date 1. Below we describe the contracting environment between households and banks, and how it interacts with the the contracting problem between banks on the interbank market.

Contracting problem between the household and banks At date 0, each bank i may offer the household a contract which specifies an initial loan d_0^i from the household and a set of state-contingent repayments $\left\{d_1^i(s), d_2^i(s)\right\}_s$ to the household at dates 1 and 2. We assume that both the household and banks have a limited ability to commit to honoring the contract at dates 1 and 2.

In particular, at dates 1 and 2, the bank chooses whether to honor the contract and make payments $d_1^i(s)$ and $d_2^i(s)$ to the household. If the bank does not pay, it makes the household a take-it-or-leave-it offer regarding the date 1 and 2 payments. If the household refuses the offer, the bank is liquidated. In the event of liquidation, there is a kind of 'pecking order' among bank i's claimants: the household can seize the bank's net capital holdings (described below); any of the bank's interbank obligations must be paid out of the bank's remaining assets.[30]

[30] That the household is the first claimant on the bank's assets is not important for the results, but simplifies the

How much of a defaulting bank's assets can the household seize at date 1? Although the precise assumptions about this are not critical for our results, we assume the following: In the event of liquidation at date 1, the household can seize bank i's own capital holdings k_0^i and also the capital holdings that i lent to other banks j in the interbank market at date 0 $\sum_j \ell^{ij}$. However, the household cannot seize the capital that bank i borrowed from other banks j in the interbank market $\sum_j \ell^{ji}$.

In addition, we assume the household can seize a fraction $\Gamma < 1$ of the bank's profits date 2 profits, where Γ satisfies $\Gamma < \underline{q}$. Any profits not seized by the household is retained by the bank. (While bank profits eventually find their way to to the household in the form of dividends at date 2, this general equilibrium result is not internalized by the atomistic households).

Any assets that the household seizes can be converted to capital and invested in the date 1 project, after incurring the maintenance cost γ. Therefore, the value to the household of a liquidated bank i at date 1 is $(q(s)-\gamma)\left(k_0^i + \sum_j \left[\ell^{ij} - \ell^{ji}\right]\right)$, and at date 2 it is $\Gamma k_1^i(s)$. Then bank i never defaults in equilibrium if and only if the following conditions are met: in each period, the value of repayment does not exceed the liquidation value to the household of bank i.

$$d_1^i(s) + d_2^i(s) \leq (q(s)-\gamma)\left(k_0^i + \sum_j \left[\ell^{ij} - \ell^{ji}\right]\right) \tag{26}$$

$$d_2^i(s) \leq \Gamma k_1^i(s) \tag{27}$$

Similarly, the household can always walk away from the contract without consequence. Therefore, the household does not default in equilibrium if and only if two conditions hold.

$$0 \leq d_1^i(s) + d_2^i(s) \tag{28}$$

$$0 \leq d_2^i(s) \tag{29}$$

We can scale the contract by the value of i's net capital holdings at dates 0 and 1 in units of the numeraire, so that the contract is denoted $\left(d_0^i, \left\{b_1^i(s), b_2^i(s)\right\}\right)$ where $b_1^i(s)$ and $b_2^i(s)$ are given by $b_1^i(s) \equiv \frac{d_1^i(s)+d_2^i(s)}{k_0^i+\sum_j[\ell^{ij}-\ell^{ji}]}$ and $b_2^i(s) \equiv \frac{d_2^i(s)}{k_1^i(s)}$. (Using each bank's binding date 0 budget constraint, $b_1^i(s)$ can equivalently be expressed as $b_1^i(s) \equiv \frac{d_1^i(s)+d_2^i(s)}{n+d_0^i}$.) Then we can rewrite the no-default constraints (26)-(29) as

model, since the optimal household contract doesn't depend directly on the solution to the interbank contracting problems.

$$0 \leq b_1^i(s) \leq q(s) - \gamma \tag{30}$$

$$0 \leq b_2^i(s) \leq \Gamma. \tag{31}$$

To entice the household to accept the contract, bank i's contract must satisfy a participation constraint, which is the household's optimality condition (1).

Interaction between the contracting problems The above discussion implies that the two limited enforcement problems interact in two ways. First, the limited enforcement constraints between households and banks depend on banks' interbank exposures through ℓ^{ij}. (This can be seen from (26) and the definition of $b_1^i(s)$.) Second, interbank contracts must respect the 'pecking order' among a bank's claimants. In other words, in the event that a bank defaults on its obligations to the household at date 1 and its assets are seized, the bank's remaining assets must be sufficient to meet its interbank obligations. As a result, optimal interbank contracts are contingent not only on the state of world, but also on whether the bank defaults on the household at date 1 or not. This ensures that banks always have the resources to meet their interbank obligations in all states and contingencies.

In equilibrium, banks never default on their obligations to households. So for simplicity of exposition, in the paper we characterize the optimal interbank contracts only for the case in which banks do not default on the household (given in Lemma 3), and we omit the off-equilibrium case in which the banks default on the household and their assets are seized.

Externalities and the household contracting problem Note that both banks and their household creditors fully internalize that by choosing a risky portfolio of interbank claims, the banks are changing the state contingent payoff profile. In this sense, banks act in the best interest of the households that hold their claims. However, while individual, atomistic lenders do not internalize how the bank's investments affect the marginal utility of consumption in general equilibrium. In this sense, the externalities in the paper are all at the general equilibrium level, and do not derive directly from the contracting friction between the household and banks.

APPENDIX 3: Bank optimization problems

We can now put these elements together to solve each bank's optimization problem. At date 0, each bank i chooses the financial contract $\left(d_0^i, \left\{b_1^i(s), b_2^i(s)\right\}\right)$ with the household, the financial contract $\left\{\ell^{ji}, r^{ji}(s)\right\}_s$ with each other bank j, how much to lend to other banks $\left\{\ell^{ij}\right\}_j$, investment levels k_0^i,$k_1^i(s)$, and portfolio allocation ω^i across projects, to maximize the value of its investment bank. Here, $m_2(s)$ denotes the stochastic discount factor at date 2 given state s, and reflects the risk-aversion of the household.

$$\max \quad E_0\left[m_2(s)\left(1-b_2^i(s)\right)k_1^i(s)\right] \tag{32}$$

subject to budget constraints

$$k_0^i+\sum_j \ell^{ij} \leq n+d_0^i+\sum_j \ell^{ji} \tag{33}$$

$$q(s)k_1^i(s) \leq \theta_k^i\left(s,\omega^i,g^i\right)k_0^i+\sum_j \theta_\ell^i(s,j)\ell^{ij}-\sum_h\left(r^{hi}(s)-b_1^i(s)\right)\ell^{hi}+b_2^i(s)k_1^i(s) \tag{34}$$

no-default constraints for the household contract

$$0 \leq b_1^i(s) \leq q(s)-\gamma \tag{35}$$

$$0 \leq b_2^i(s) \leq \Gamma \tag{36}$$

the household participation constraint, where we have combined the household's optimality conditions (1) and (2)

$$u'(c_0)d_0^i \geq E\left[u'\left(c_1(s)\right)b_1^i(s)\right]\left(k_0^i-\sum_h \ell^{hi}+\sum_j \ell^{ij}\right) \tag{37}$$

and the other banks' participation constraints for each j

$$u^{ji}\left(\ell^{ji},\left\{r^{ji}(s)\right\}_s\right) \geq \bar{u}^j \tag{38}$$

and non-negativity constraints on capital holdings and inter-bank loans.

$$k_0^i,k_1^i(s),\ell^{ij} \geq 0 \quad \forall j \tag{39}$$

Let z_0^i, $z_1^i(s)$, $\bar{\lambda}^i(s)$, $\underline{\lambda}^i(s)$, $\bar{\mu}^i(s)$, $\underline{\mu}^i(s)$, and ν^{ji} denote Lagrange multipliers on the date 0 budget constraint (33), the date 1 budget constraint (34), the upper and lower bounds on $b_1^i(s)$, the upper and lower bounds on $b_2^i(s)$, and bank j's participation constraint (38) respectively. Also, let $g'(s,k_0^i,\omega^i)$ denote the derivative of the government transfer g^i to bank i with respect to ω^i, which represents how a marginal increase in ω^i affects the bailout that i receives conditional on i being bailed out. (Importantly, this may in general depend on not only the state of the world and i's investment, but also on the investment decisions ω^j of all other banks j.) Because the household has access to a riskless bond at date 1 with gross return 1, and all uncertainty is resolved in date 1, we will have in equilibrium

$$u'\left(c_2(s)\right) = u'\left(c_1(s)\right). \tag{40}$$

The optimality conditions are then given by

$$\frac{\partial L^i}{\partial k_0^i} \leq 0 \iff z_0^i \left(\frac{1}{u'(c_0)} E\left[u'\left(c_1(s)\right) b_1^i(s) \right] - 1 \right) + E\left[z_1^i(s) \theta_k^i\left(s, \omega^i, g^i\right) \right] \leq 0 \tag{41}$$

$$\frac{\partial L^i}{\partial k_1^i(s)} \leq 0 \iff m_2(s)\left(1 - b_2^i(s)\right) \leq z_1^i(s)\left(q(s) - b_2^i(s)\right) \tag{42}$$

$$\frac{\partial L^i}{\partial b_1^i(s)} \leq 0 \iff \left[\frac{u'\left(c_1(s)\right)}{u'(c_0)} z_0^i - z_1^i(s) \right] \left(k_0^i - \sum_h \ell^{hi} + \sum_j \ell^{ij} \right) \leq \lambda_1^i(s) - \lambda_0^i(s) \tag{43}$$

$$\frac{\partial L^i}{\partial b_2^i(s)} \leq 0 \iff \left[z_1^i(s) - m_2(s) \right] k_1^i(s) \leq \mu_1^i(s) - \mu_0^i(s) \tag{44}$$

$$\frac{\partial L^i}{\partial \omega^i} \leq 0 \iff E\left[z_1^i(s) k_0^i \frac{\partial \theta_k^i\left(s, \omega^i, g^i\right)}{\partial \omega^i} \right] \leq 0 \tag{45}$$

$$\frac{\partial L^i}{\partial \ell^{ij}} \leq 0 \iff E\left[z_1^i(s) \theta_\ell^i(s, j) \right] \leq z_0^i \left(1 - E\left[\frac{u'\left(c_1(s)\right)}{u'(c_0)} b_1^i(s) \right] \right) \tag{46}$$

$$\frac{\partial L^i}{\partial r^{ji}(s)} \leq 0 \iff -\nu^{ji} \frac{\partial u^{ji}\left(\ell^{ji}, \left\{ r^{ji}(s) \right\}_s\right)}{\partial r^{ji}(s)} \leq \pi(s) z_1^i(s) \ell^{ji} \tag{47}$$

APPENDIX 4: Optimal household contract

Notice from (42) and (44) that when the optimality condition for $k_1^i(s)$ holds, that for $b_2^i(s)$ cannot hold since $b_2^i(s) \leq \Gamma < 1$ and $q(s) \leq 1$. Therefore, given that in equilibrium the optimality condition for $k_1^i(s)$ holds, we have $z_1^i(s) \geq m_2(s)$. Although $b_2^i(s) \in [0, \Gamma]$ when $z_1^i(s) = m_2(s)$, we assume for simplicity it is at its upper bound in this situation. (This does not affect our main results.) Consequently, we always have a corner solution for $b_2^i(s)$ as it is set at its maximum.

$$b_2^i(s) = \Gamma \tag{48}$$

And since $m_2(s) > 0$ by the Inada condition of $u(\cdot)$, it follows that $z_1^i(s) > 0$, so that i's date 1 budget constraint always binds in equilibrium.

Notice from i's optimality condition for $b_1^i(s)$, the household's optimality condition for the bond and the definition of the stochastic discount factor $m_2(s) = \frac{u'(c_2(s))}{u'(c_0)}$, we can write (27) the optimality condition for $b_1^i(s)$ as

$$z_0^i m_2(s) \leq z_1^i(s) \tag{49}$$

Then $b_1^i(s)$ is set at its maximum $q(s) - \gamma$ (a corner solution) if and only if $z_0^i > \frac{z_1^i(s)}{m_2(s)} = \frac{1-\Gamma}{q(s)-\Gamma}$, at its minimum 0 (corner solution) if and only if $z_0^i < \frac{1-\Gamma}{q(s)-\Gamma}$, and is indeterminate if and only if $z_0^i = \frac{1-\Gamma}{q(s)-\Gamma}$. Lemma 1 characterizes the individually optimal financial contract in light of these conditions.

APPENDIX 5: Proof of Lemma 3

Proof: The proof relies on two results. First, perfect competition between atomistic banks implies that, in each state, the interbank contract issued from any i to h equates the return on the contract to h to the return on i's assets in each state, such that $\theta_\ell^h(s,i) = \theta^i(s)$. Second, Lemma 1 showed that each bank is always at a corner solution in its portfolio choice. This implies only one contract accepted: the contract with highest private valuation $E\left[z_1(s)\theta_\ell^h(s,i)\right]$. It follows that $\theta_\ell^h(s,i) = \theta_k^w(s,\omega^w,g^w)$, where $W \equiv \left\{w \mid E\left[z_1(s)\theta_k^w(s,\omega^w,g^w)\right] \geq E\left[z_1(s)\theta_k^i\left(s,\omega^i,g^i\right)\right] \; \forall\, i \in I\right\}$.

APPENDIX 6: Aggregate investment at date 1

In order to evaluate the date 1 spot market for capital, we first characterize aggregate net investment in capital at date 1. Consider net aggregate investment by all banks in state s at date 1, defined as the difference between aggregate capital holdings at date 1 and aggregate date 0 holdings of capital, $K_1(s) - K_0$, where we have defined $K_0 \equiv \sum_i k_0^i$ and $K_1(s) \equiv \sum_i k_1^i(s)$ to be the aggregate capital holdings of the banking sector at dates 0 and 1, respectively. We can write aggregate net investment in state s as

$$K_1(s) - K_0 = \sum_i \Delta^i\left(s,\omega^i,g^i\right) \tag{50}$$

where $\Delta^i\left(s,\omega^i,g^i\right) \equiv k_1^i(s) - \left[n + d_0^i\right] = k_1^i(s) - \left[k_0^i + \sum_h \ell^{ih} - \sum_h \ell^{hi}\right]$ denotes the difference between bank i's choice of date 1 capital $k_1^i(s)$ and its date 0 funds $n + d_0^i$ available for investment in any asset.[31]

[31] To see this, first note that we can re-write aggregate date 0 holdings of capital as $\sum_i k_0^i = \sum_i \left[n + d_0^i + \sum_j \left(\ell^{ji} - \ell^{ij}\right)\right] = \sum_i \left[n + d_0^i\right] + \sum_i \sum_j \left(\ell^{ji} - \ell^{ij}\right) = \sum_i \left[n + d_0^i\right] = \sum_i \left[k_0^i + \sum_h \ell^{ih} - \sum_h \ell^{hi}\right]$. Given our definition of $D^i\left(s,\omega^i,g^i\right)$, it follows that aggregate net investment can be written as $\sum_i k_1^i(s) - \sum_i k_0^i = \sum_i D^i\left(s,\omega^i,g^i\right)$.

This object can be derived from each bank i's date 1 budget constraint in state s, after imposing the partial equilibrium characterization of optimal interbank contracts given in Lemma 3 $\theta_\ell^i(s) = \theta_\ell^j(s) = \theta_k^w(s,\omega^w,g^w)$ for all i,j in the set of intermediary banks L.

$$K_1(s) - K_0 = \sum_i \Delta^i(s,\omega^w,g^w) = K_0\left[\frac{\theta_k^w(s,\omega^w,g^w)}{q(s)-\Gamma} - 1\right] \tag{51}$$

Equation (51) says that aggregate net investment in capital by the banking sector at date 1 is given by the aggregate rate of return on capital holdings at date 1, discounted by the cost of capital at date 1. At date 1, the aggregate rate of return on banks' date 0 capital holdings K_0 is given by the rate of return earned by bank w's assets $\theta_k^w(s,\omega^w,g^w)$. Since banks do not pay out dividends at date 1, this return is invested in capital at date 1. The cost of capital at date 1 is given by the spot price $q(s)$ net of the date 2 repayment to the household $b_2^i = \Gamma$. Therefore, the aggregate net investment in new capital by the banking sector at date 1 is given by (51).

APPENDIX 7: Proof of Lemma 4

Proof of Part (A)

Recall that the assumption that the consumption good can be costlessly converted into the capital good one-for-one, but not vice versa, implies $q(s) \leq 1$. This also implies that aggregate investment cannot be negative in equilibrium, i.e. $k_1^T(s) + \sum_i \left(k_1^i(s) - \chi(s)k_0^i\right) \geq 0$. If aggregate investment is strictly positive, then $q(s) = 1$ by arbitrage, and so equation (4) implies that $k_1^T(s) = 0$ since $1 = F'(0)$. If, on the other hand, aggregate investment is 0, then we have $k_1^T(s) = \sum_i \left(k_0^i - k_1^i(s)\right)$. These two cases imply that $q(s) = F'(k_1^T(s))$ and $k_1^T(s) = \max\left\{0, \sum_i k_0^i - k_1^i(s)\right\}$. Assumption 1 implies that $\gamma < \underline{q} < q(s)$. Therefore, in equilibrium, we have

$$q(s) = F'(k_1^T(s))$$

$$k_1^T(s) = \max\left\{0, K_1(s) - K_0\right\}.$$

Q.E.D.

Proof of Part (B)

Recall that the return to i's risky project is given by

$$R_A^i(s) = \rho^i R_A(s) - \mu^i \tag{52}$$

where $\mu^i = R_C\left(\rho^i - 1\right)$. First we show that we have misallocation if and only if $R^w(s) < b_1^w(s) + \gamma - \Gamma A$. (Recall that we have normalized $A = 1$.) Suppose $b_1^w(L) = 0$. Assumption 2 that $R_C \geq \gamma$

and $R_C + \Gamma A \geq 1$ implies that $R_C > \gamma - \Gamma A$. And for even the smallest ρ^i, we have $R_C - (\gamma - \Gamma A) < \rho^i (R_C - R_A(L))$. Then it follows that $R^w(L) < \gamma - \Gamma A$. It is also easy to see that misallocation does not hold for R_C, i.e. that $R_C \geq \gamma - \Gamma A$. This is true by Assumption 2. Since this holds for $R_A^w(L)$ but not R_C this condition holds for any equilibrium value of $b_1(s)$.

Now we show that there is no misallocation if and only if $R^w(s) \geq q(s) - \Gamma A$. This holds for R_C by Assumption 2 that $R_C + \Gamma A \geq 1$. This holds for $R_A^w(L)$ because of Assumption 2 and the assumption that $\underline{q} > \gamma$, which implies $q(s) \geq \underline{q} > \gamma$. Therefore, these conditions hold for any equilibrium value of $b_1(s)$. So, in equilibrium, $K_1(s) - K_0 < 0$ if and only if $s = L$ and $\omega^w = 0$. Q.E.D.

APPENDIX 8: Deriving the government's optimal bailout policy

Proof of Part (A):

At date 1, the government solves its problem taking date 0 variables as given. First substitute out of the household's date 1 budget constraint lump sum taxes $T_1 = K_0 g^w$ using the governments binding budget constraint.

$$c_1(s) + B_1(s) \leq e_1 + \sum_i f_0^i d_1^i(s) - K_0 g^w - q(s) k_1^T(s) \tag{53}$$

Recall that we ruled out counterfactual situations in which the government bails out banks outside of a crisis. Since the government takes agents' optimizing behavior as given, we impose the conditions for equilibrium at date 1. Below we characterize the bailout per unit of capital g^w, but this is equivalent to characterizing the total bailout $G = g^w K_0$, since the distribution of bailout funds across investing banks is allocatively irrelevant at date 1.

It turns out that, when the conditions for a misallocation of capital at date 1 are satisfied (namely, when $\omega^w = 0$ and $s = L$), we have $\frac{dK_1(L)}{dg^w} > 0$. From the government's date 1 budget constraint, we then have $\frac{d2c_1(s)}{dg^w} = K_1(s)\frac{dq(s)}{dg^w} + A(1-\Gamma)\frac{dK_1(s)}{dg^w} > 0$. So when $\omega^w = 0$ and $s = L$, household welfare is increasing in g^w when $k_1^T(s) > 0$. Hence, when there is a misallocation of capital at date 1, the government sets g at the minimum to ensure that capital is no misallocated to the traditional sector. This optimal choice of g^w therefore satisfies $k_1^T(s) = 0$ and is given by

$$g^i(s, \omega^w) = \begin{cases} q(s) - \Gamma A - R_A^i(s) & for\ i = w,\ s = L\ and\ \omega^i = 0 \\ 0 & otherwise \end{cases}$$

It follows that the total bailout is given by $K_0 g^i(s, \omega^w) = K_0\left(q(s) - \Gamma - R_A^w(s)\right)$. This proves part (A) of Lemma 5.

Proof of Part (B):

With regard to part (B), first recall that the government cannot verify the losses that a bank incurs on its interbank claims. As a result, the government does not bail out any intermediaries in equilibrium, and so $g^i(s, \omega^w) = 0$ for all $i \notin W$. How does the government prefer to distribute the bailout across investing banks? First note that any bailout that satisfies the conditions in part (A) will prevent a misallocation of capital ex post, regardless of how it is distributed across investing banks. This is because the aggregate investment of the banking sector, given in (12), is independent of the distribution of funds across banks due to banks' constant returns-to-scale technology. Therefore, any arbitrary distribution of bailout funds across banks which satisfies part (A) is optimal ex post. For ease of exposition, we therefore simply assume that the government bails out investing banks in proportion to their capital holdings.

In principle, however, how the bailout is distributed across banks may affect banks' ex ante incentives. Nevertheless, we show in Appendix 9B that our general equilibrium results are quite robust to alternative assumptions. This relies on the characterization of general equilibrium in Section 3.

APPENDIX 9: Discussion of government problem

9A. Discussion of the frictions faced by the government

An important assumption in the literature on collective moral hazard, and also in our model, is that bailouts cannot be perfectly targeted across banks (e.g. see Farhi and Tirole (2012)). If bailouts could be perfectly targeted to any bank in the financial system, the government could always design a transfer scheme which punishes SIFIs, thereby getting rid of the moral hazard problem (for example, by bailing out all banks except for the SIFIs). In practice, however, there are frictions which prevent the government from doing this, be it informational frictions, political constraints, etc. In the model, we impose a straightforward assumption which can capture this. While our results do not depend on the precise nature of this assumption, it is an empirically plausible and tractable way to generate imperfect targeting.

Our assumption is that it is difficult for the government to verify the losses that a bank incurs on its holdings of interbank claims. This assumption captures the fact that it is difficult for the government to identify banks' bilateral exposures during a crisis, due to the complexity of interbank markets and the fact that these markets are typically over-the-counter. Indeed, the losses that financial institutions incurred in 2008 from their (frequently off-balance-sheet) exposures to other banks on interbank markets were difficult to verify externally, and often these institutions did not themselves know the extent of these exposures in the midst of the crisis.

In the model, this assumption implies that, in general equilibrium, bailouts can be only imperfectly targeted to investing banks. Nevertheless, the results would hold under a broad class of alternative assumptions to the extent that bailouts cannot be perfectly targeted.

9B. Robustness to alternative bailout policies

In this section, we consider alternative policies for Part (B) of Lemma 5 and discuss their implications for our results. All the policies considered are ex post optimal (i.e. they conform with Lemma 5), and differ only in how the bailout is distributed across investing banks at date 1. Ex post, these policies lead to identical outcomes as those in the body of the paper, so below we analyze to what extent they alter banks' ex ante incentives and equilibria. Overall, our results are quite robust to these various alternatives, primarily because interbank risk sharing at date 0 ensures that the benefits of a bailout are widely shared across banks regardless of the government's policy.

1. Transfer of capital from SIFIs to non-SIFIs

One alternative policy would be for the government to simply transfer capital from SIFIs to other banks during a crisis, in a way which keeps production at the first best ex post and eliminates the risk taking incentive of SIFIs ex ante. It is important to note, however, that this would be isomorphic to a bailout of non-SIFI banks. To see why, suppose that, in a crisis, the government obtains the capital of the SIFIs (either through expropriation, or by purchasing the capital at some price) and grants it directly to non-SIFI banks. In a crisis, non-SIFI banks are also, in aggregate, facing losses. Therefore, these non-SIFI banks would be forced to liquidate these capital holdings to the traditional sector, and we would still end up with a misallocation of capital. This is because, in the bad state of the world, there are losses, incurred from risky investments, that need to be absorbed by some agents in the economy. In order to prevent a misallocation of capital, the government would need to cover losses of other banks via a transfer financed by taxing the household. This is effectively a bailout of non-SIFI banks.

However, recall from Section 2.9 that the government cannot bail out banks whose losses it cannot verify. Because the government cannot verify exposures from interbank claims, it would then be infeasible for the government to bail out non-SIFI banks, as these banks are facing losses only from their holdings of interbank claims. These frictions prevent the government from perfectly targeting bailouts to non-SIFI banks. Otherwise, the government could simply design a bailout of all banks except for the SIFIs, without ever having to directly reallocate capital across banks. As we discussed above in Part (A), this does not happen in practice for various reasons.

2. Randomized bailouts

We next consider randomized bailouts at date 1, similar to the policies analyzed in Nosal and Ordonez (2016). We consider two alternatives.

i) *Randomizing the occurrence of a bailout* In Nosal and Ordonez (2016), the government faces uncertainty about whether a crisis is systemic, and therefore delays intervention to attain more information. This forces banks to internalize the riskiness of their investments to some extent, mitigating the ex ante moral hazard problem. In our setting, there is no such uncertainty; the government knows with certainty whether there is a crisis, and so this mechanism is not at play. Moreover, given the inefficiencies associated with a crisis, it would be suboptimal (and therefore not credible) for the government not to intervene during a crisis with positive probability.

Nevertheless, in practice, a lack of confidence in the government's ability to carry out its optimal bailout policy could mitigate risk taking ex ante. We do not take up this issue in this paper.

ii) *Randomizing the bailout across investing banks* A government could conceivably choose to randomize *which* investing banks it bails out during a crisis. In our setting, however, risk sharing between banks in the interbank market always ensures that the benefits of bailouts are shared perfectly. As a result, randomization does not mitigate the collective moral hazard problem.

To see this, let $\pi^i \in (0,1]$ denote the probability (chosen by the government) that bank i receives a transfer in the event of a bailout, while $\tilde{g}^i \geq 0$ is the transfer to i per unit of capital, conditional on it receiving one. Consider a bailout policy which satisfies $\sum_{i\in W} \tilde{g}^i k_0^i = G(s, \omega^w)$ for all investing banks which are chosen to receive a bailout, so that the total size of the bailout is consistent with part (A) of Lemma 5 regardless of which banks receive the bailout.

First note that randomness of the bailout effectively adds an additional source of risk to risky assets which is uncorrelated to the aggregate shock: when an investing bank invests in a risky project, it bears not only the aggregate on the project's return, but also the risk that it is not bailed out during a crisis. Off equilibrium, this means that investing banks are no longer in a corner solution in their portfolio choice, but rather diversify this risk by lending to other investing banks in addition to investing in their own projects. The perfect risk sharing between banks facilitated interbank contracts means that each investing bank can fully diversify away the risk of not receiving bailout funds in a crisis. (Recall that the government will bailout at least one of them with probability 1.)

However, this cannot be sustained as part of an equilibrium. Consider two investing banks i and j such that $\rho^i < \rho^j$. Since perfect interbank risk sharing fully diversifies away the risk of not receiving a bailout during a crisis, all investing banks receive the same return R_C in the bad state. However, in the good state, bank j receives a higher return. Hence, from bank i's perspective, the risk-adjusted return to lending to bank j is larger than investing in its own project. As a result, each atomistic bank in i prefers to forgo its own investment and lend entirely to bank j. As a result, all other investing banks forgo their own investments in favor of lending to the riskiest investing bank. Hence, the riskiest investing banks become the only SIFIs and are bailed out during a crisis with probability 1. Thus, bailout policies of this type would yield identical equilibria to those analyzed in Section 3 of the paper.

3. Other bailout policies which alter ex ante incentives

We now consider other bailout policies which satisfy Lemma 5, but may lead to different ex ante incentives. Two examples of such policies are a credible commitment by the government to bail out only the least risky investing bank, or only the largest investing bank in a crisis. While these policies may alter the equilibria of the economy, we show that the implications for welfare and policy outlined in Sections 5 and 6 still apply under these alternatives.

Under either of these bailout policies, any equilibrium featuring risk taking features a single type of SIFI. To see why, suppose that the government bails out only the least risky bank, and that at date 0, there are two investing banks i and j such that $\rho^i < \rho^j$. In a crisis, only bank i will be bailed out by the government. Then atomistic banks in j prefer to deviate and forgo their investments in favor of lending to i. So this cannot be an equilibrium. Alternatively, suppose that only the largest investing bank is bailed out, and that i is larger than j. Again, atomistic banks in j prefer to deviate and forgo their investments in favor of lending to i. Hence, a risky equilibrium cannot feature more than one type of SIFI.

When only the least risky investing banks are bailed out, the unique SIFIs are the least risky banks. To see why, suppose that at date 0 bank j is the only investing bank. Bank j will be bailed out in equilibrium. However, atomistic banks in i have incentive to deviate and instead of lending to j, invest in their risky projects: then bank i will be bailed out, and atomistic banks in j will prefer to lend to i rather than invest in their risky projects, since they will not be bailed out. Hence, this bailout policy will imply that there is a unique risky equilibrium (in addition to the prudent equilibrium described in Section 3 of the paper) in which the unique SIFIs are the least risky banks.

When only the largest investing bank is bailed out, however, the identity of the SIFIs is not pinned down uniquely. To see why, suppose that bank i is the largest investing bank at date 0. Then bank i will be bailed out in a crisis. Consider bank j where $\rho^j \neq \rho^i$. Atomistic banks in j does not have incentive to deviate and start investing in its own asset, because bank i would remain the largest investing bank and therefore the only bank to be bailed out in a crisis. Similarly, atomistic banks in i have no incentive to deviate and start lending to j for the same reason. Therefore, we can have N risky equilibria (one for each bank type in the economy) in addition to the prudent equilibrium which feature a single type of SIFI.

Therefore, with bailout policies of this type, the risky equilibria may differ from those characterized in Section 3 of the paper. However, the welfare implications of each risky equilibrium remain the same. As a result, the scope for ex ante regulation to improve welfare, and the policy implications outlined in Sections 5 and 6 still apply even under these alternative bailout policies.

APPENDIX 10: Proof of Proposition 1

We prove Proposition 1 by backward induction. We have already characterized banks' optimal decisions at dates 1 and 2. Given these, we also characterized each investing bank's best response function for its date 0 portfolio choice. We now prove that, given these best response functions, there exist exactly two subgame perfect Nash equilibria.

Recall that, to complete the characterization of general equilibrium, it remains to determine the investment choices ω^w of investing banks, and to determine which banks are in the set W of investing banks in equilibrium. Once these are determined jointly, the investment choices ω^i of all other banks (i.e. banks in the set $L = I/W$, who simply invest in the liabilities of investing banks) are irrelevant for the allocation.

Proof: The proof is in three parts. In all cases, we make use of the best response functions

$$\omega^i\left(\{\omega^w\}_{w\in W}\right) = \begin{cases} 1 & if\ g^w(L,\omega^w) = 0 \\ 0 & otherwise \end{cases}.$$

Claim (i): $\{\omega^w = 1 \quad \forall w \in W\}$ is an equilibrium. This is the 'prudent' equilibrium, as all banks undertake the prudent investment.

Proof: We will show that, when all investing banks in set W choose $\omega^w = 1$, then bank $w \in W$ has no incentive to deviate from $\omega^w = 1$. Suppose that all investing banks choose $\omega^w = 1$. Recall from the government's optimal bailout policy that when all investing banks are exposed to risky projects, then there is never a bailout in the low state at date 1, i.e. $g^i(s,\omega^w) = 0$. The best response function for ω^w then implies that bank w finds it optimal to set $\omega^w = 1$.

Also, recall in that we showed in the partial equilibrium characterization of optimal interbank contracts that the set of investing banks J is given by $J = W \equiv \left\{w \mid w \ \equiv \max_{i\in M}\ E\left[z_1(s)\theta_k^i\left(s,\omega^i,g^i\right)\right]\right\}$. In this case when $\omega^w = 1 \quad \forall w \in W$, all banks are invested in only to prudent assets, so that that $E\left[z_1(s)\theta_k^i\left(s,\omega^i,g^i\right)\right]$ is the same for all banks i. Therefore, the structure of interbank lending in this equilibrium, and therefore the set of investing banks W, is indeterminate – in this prudent equilibrium, we can have any combination of banks investing in the prudent project on their own behalf, with rest of banks investing in their liabilities. W is non-empty, so that at least one bank invests in the prudent project in equilibrium.

Claim (ii): $\{\omega^w = 0 \quad \forall w \in W\}$ is also an equilibrium, where $w \in W \iff \rho^w = \bar{\rho}$. This is the 'risky' equilibrium, as all investing banks invest in the riskiest project available.

Proof: We will show that, when all banks set $\omega_C^j = 0$, then bank i has no incentive to deviate from $\omega_C^i = 0$. Suppose that all investing banks choose $\omega^w = 0$. Recall from the government's optimal bailout policy that when all investing banks are exposed to risky projects, then there is a bailout in the low state at date 1 given by $\hat{g}^i(s,\omega^w) = q(s) - \Gamma A - R_A^i(s)$. The best response function for ω^w then implies that bank w finds it optimal to set $\omega^w = 0$.

Again, we showed that interbank contracts in equilibrium are such that the set of investing banks J is given by $J = W \equiv \left\{ w \mid w \ \equiv max_{i\in M}\, E\left[z_1(s)\theta_k^i\left(s,\bar{\omega}^i,g^i\right)\right]\right\}$. Since $z_1(s) = m_1(s)$ is proportional to $u'\left(c_1(s)\right)$ and in this case $\theta_k^i\left(s,\omega^i,g^i\right) = R_A^i(s) + g^i(s,\omega^i) = \rho^i R_A(s) - \mu^i + g^i(s,\omega^i)$, it is easy to show that $E\left[z_1(s)\theta_k^i\left(s,\omega^i,g^i\right)\right]$ is monotonically increasing in ρ^i. This is because: (i) $u(\cdot)$ is strictly concave; (ii) the variance of $R_A^i(s)$ is increasing in ρ^i, while its mean is independent of ρ^i; and (iii) the government's optimal $g^i(s,\omega^i)$ bounds $\theta_k^i\left(s,\omega^i,g^i\right)$ from below by $1-\Gamma$. Therefore, $E\left[z_1(s)\theta_k^i\left(s,\omega^i,g^i\right)\right]$ is highest for the bank with the greatest potential exposure to the aggregate shock, $\rho^i = \bar{\rho}$. Hence, $W = \{w \in W \mid \rho^w = \bar{\rho}\}$, i.e. only banks with access to the riskiest projects invest in equilibrium, while the rest of banks invest in the liabilities of these risky banks.

Claim (iii): There are no other equilibria.

Proof: Suppose for the sake of contradiction that some $\{\omega^w\}_{w\in W}$ is an equilibrium, where $\{\omega^w\}_{w\in W} \neq \{\omega^w = 1 \quad \forall w \in W\}$ and $\{\omega^w\}_{w\in W} \neq \{\omega^w = 0 \quad \forall w \in W\}$. The government's optimal bailout policy implies that, in any equilibrium, either $g^w(L,\omega^w) = 1 - \Gamma A - R_A^i(L)$ for some $w \in W$ (i.e. a crisis and bailout occurs in the bad state) or $g^w(s) = 0$ for all s (i.e. a crisis and bailout never occur). Take the latter case in which we always have $g^w(s) = 0$. Then all investing bank w's best response functions favor investing only in the prudent project by setting $\omega^w = 1$. Moreover, this is consistent with having $g^w(s) = 0$. So we must have $\{\omega^w\}_{w\in W} = \{\omega^w = 1 \quad \forall w \in W\}$, which contradicts the premise that this equality does not hold. So this cannot be an equilibrium.

Now suppose that we have a bailout in the bad state. Then the best response function of each investing bank implies all investing banks invest only in the risky their risky projects by choosing $\omega^w = 0$, which is consistent with having a bailout in the bad state. So we must have $\{\omega^w\}_{w\in W} = \{\omega^w = 0 \quad \forall w \in W\}$, which contradicts the premise that this equality does not hold. So this cannot be an equilibrium either. Therefore, any equilibrium must be either the prudent equilibrium in which $\{\omega^w = 1 \quad \forall w \in W\}$, or the risky equilibrium in which $\{\omega^w = 0 \quad \forall w \in W\}$. Q.E.D.

Uniqueness of representative SIFI

Although the results above imply that, in the risky equilibrium, the SIFIs are always the riskiest banks (i.e. the banks with the highest ρ^i), it may be instructive to reiterate why this is necessarily the case. Suppose we have an equilibrium with risk taking in which bank j is the only investing bank, where $\rho^j < \rho^h$ for some h (i.e. bank j is not the riskiest bank). Can this be an equilibrium? Given that bank j is the only investing bank, it will be bailed out in the bad state. All other banks have incentive to lend their funds to bank j in order to benefit from the bailout in the bad state. Bank j in turn invests in its risky project. Indeed, other banks may not have incentive to deviate and lend to a different bank (since it may not be bailed out) or invest in its own project. (This would indeed be the case if the government announced in advance that it would bail out the least risky investing bank.) However, ex ante, bank h has incentive to deviate and invest its funds in its own risky project rather than lend to j. The reason for this is that, per the government bailout policy in Lemma 5, bank h will be bailed out in the bad state and therefore receive the same return from lending to j. But in the good state, the return on h's own project exceeds that paid on j's liability, since $\rho^j < \rho^h$. Therefore, this cannot be an equilibrium.

Now suppose that we have a situation with risk taking in which two banks j and h are both investing banks, where $\rho^j < \rho^h$ (i.e. bank h's project is riskier). Can this be an equilibrium? Recall that Lemma 5 implies that bank h will be bailed out in equilibrium, since it is an investing bank. Then ex ante, each atomistic bank in j has incentive to deviate and lend all of its funds to the riskiest bank h. This is because, giving the perfect risk sharing facilitated by interbank contracts, it would benefit from a higher upside in the good state (since $\rho^j < \rho^h$), and still benefit equally from the bailout in the bad state. Therefore, this cannot be an equilibrium.

Therefore, any equilibrium with risk taking features banks $w \in W$ (the riskiest banks) as the only investing banks. Appendix 9B discusses the robustness of this result to alternative assumptions about the government's bailout policy.

APPENDIX 11: Proof of Lemma 1

This follows from the linearity of the firm's portfolio allocation problem. Namely, the optimality conditions for the bank's portfolio allocation decisions for k_0^i,ℓ^{ij}, and ω^i do not depend on size of the firm's investment. Therefore, it immediately follows that, for each firm i, we have one of two cases. Either we are in case 1, in which there is a firm $j \neq i$ such that $E\left[z_1^i(s)\theta_\ell^i(s,j)\right] \geq E\left[z_1^i(s)\theta_\ell^i(s,h)\right]$ for all other firms h, and $E\left[z_1^i(s)\theta_\ell^i(s,j)\right] \geq E\left[z_1^i(s)\theta_k^i\left(s,\omega^i,g^i\right)\right]$ for any $\omega^i \in [0,1]$. In this case, the contract offered by firm j to firm i has a more favorable risk-return tradeoff that that offered to i by any other firm h. In addition, the return to lending to firm j is preferable to investing any amount in either the risky or prudent project on i's own behalf. In case 1, we have $k_0^i = 0$ and $\ell^{ij} > 0$, meaning the firm forgoes investing in its own projects in favor of lending to firm j.

The other possibility is that we are in case 2, in which there is a $\tilde{\omega}^i \in [0,1]$ such that $E\left[z_1^i(s)\theta_k^i\left(s,\tilde{\omega}^i,g^i\right)\right] \geq E\left[z_1^i(s)\theta_k^i\left(s,\omega^i,g^i\right)\right]$ for all $\omega^i \neq \tilde{\omega}^i$ and $E\left[z_1^i(s)\theta_k^i\left(s,\tilde{\omega}^i,g^i\right)\right] \geq E\left[z_1^i(s)\theta_\ell^i(s,h)\right]$ $\forall h$. This implies that at the optimal ω^i, the return to investing ω^i in the prudent project and $1-\omega^i$ of its capital has a more favorable risk-return profile than the returns offered by any firm's inter-firm contract. In case 2, we have $k_0^i > 0$ and $\ell^{ij} = 0$ for all j, meaning the firm does not lend to any other firm.

Furthermore, since the condition for ω^i does not depend on ω^i, firm i will always be at a corner solution in its choice of ω^i, so that the optimal ω^i satisfies $\tilde{\omega}^i \in \{0,1\}$. (This is partly due to the fact that, in the government's optimization problem, we will show that g^i will be zero for $\omega^i = 1$.) Q.E.D.

APPENDIX 12: Benchmark 2: Comparative static on degree of risk aversion

How does risk sharing between the SIFIs and non-SIFI banks generate excessive risk taking? In this benchmark variant of the model, we isolate the role of risk sharing *per se* in generating excessive risk taking by all banks by varying the degree of risk aversion of agents in the model.

In general, the interbank market plays two roles in the risky equilibrium. First, it directs funds at date 0 to the projects with the highest expected return. Second, as we showed in Section 3.2.3., the interbank market facilitates risk sharing between SIFIs and other banks by allowing other banks to benefit from the government guarantee indirectly, thereby reducing the variance of their portfolios. This second risk sharing motive of interbank lending arises because the stochastic discount factor reflects the household's risk aversion. To elucidate this point we modify the model in this section so that only the risk sharing role of the interbank market ultimately affects banks' portfolio choices. Then when capture how risk sharing incentivizes risk taking through a comparative static exercise by varying the degree of risk aversion of the household.

To do this, we modify the baseline model in three respects. First, for concreteness, we suppose that the representative household's utility features constant relative risk aversion so that, $u(c) = \frac{c^{1-\eta}-1}{1-\eta}$, where $0 \leq \eta \leq 1$. Second, rather than assuming that all risky projects are a mean-preserving spread of the prudent project, we now assume that $R_C > E\left[R_A^i(s)\right]$ for all i.[32] This implies that the risky projects are not only riskier than the prudent project, but also offer a lower expected return. Moreover, we assume that a stronger condition holds: $\pi(H)R_A^i(H) + \pi(L)(1-\Gamma) < R_C$. This assumption will ensure that the higher expected return on risky assets afforded by the government guarantee is not sufficient by itself to entice banks to invest in risky assets. For the purpose of this exercise, we also make an assumption to ensure that there is some threshold degree of risk aversion above which banks prefer to lend to SIFIs and below which they prefer hold prudent assets only.

Assumption OA.1: a) $\frac{(1-\pi(L))}{\pi(L)}\left(\frac{c_1(L)}{c_1(H)}\right)^{\eta} < \frac{(1-\theta_\ell(L,w))}{(\theta_\ell(H,w)-1)}$; b) $\frac{(1-\pi(L))}{\pi(L)}\left(\frac{c_1(L)}{c_1(H)}\right)^{\eta-1} > \frac{(1-\theta_\ell(L,w))}{(\theta_\ell(H,w)-1)}$; and c) $log\left[\frac{\pi(L)}{(1-\pi(L))}\frac{(R_C-\theta_\ell(L,w))}{(\theta_\ell(H,w)-R_C)}\right] < log\left(\frac{c_1(L)}{c_1(H)}\right)$ hold.

(Note that (b) and (c) can be assured by setting $\pi(L)$ sufficiently low. While these conditions depend in part on equilibrium variables, these can solved in closed form. For ease of exposition do not present that here.)

[32]For this to hold, we need to modify our assumption that $R_C \geq 1-\Gamma$ instead holds with strict inequality.

In this modified environment, the characterization of the date 1 spot market for capital and optimal interbank and household contracts all go through. Moreover, the government's optimal bailout policy is still characterized by Lemma 5. Therefore, to characterize the equilibrium in this version of the model, it remains to characterize banks' best response functions for their date 0 portfolio choices and interbank lending decisions. We characterize these best response functions for different degrees of the household's risk aversion η.

How does risk sharing affect portfolio choices, risk taking? Recall from Section 3.5.1. that the value to bank i of an interbank claim issued by a SIFI w promising a return $\theta_\ell^i(s,w)$ is given by the sum of the expected discounted return $E_A \equiv E\left[m_1(s)\right] E\left[\theta_\ell^i(s,w)\right]$ and a safety premium component given by $SP_A \equiv Cov\left(m_1(s), \theta_\ell^i(s,w)\right)$, where the total value V_A of the claim is given by the sum of the two. We already showed in Proposition 2 that the implicit guarantee lowers riskiness of SIFI's assets, and that the interbank market facilitates risk sharing between the SIFIs and non-SIFI banks whereby banks can benefit from safety of the SIFIs interbank claims. These results apply in this modified setting as well. We now vary the degree of risk aversion of the household to show how this interbank risk sharing actually exacerbates excessive risk taking, generating collective risk shifting problem.

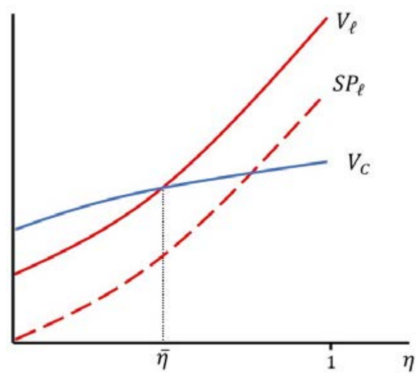
First suppose that $\eta = 0$, so that the household is risk neutral. In this case, the stochastic discount factor $m_1(s)$ is constant across states, and so the covariance term is 0. Agents do not value risk sharing - the variance of their portfolios is irrelevant for their portfolio choice and they care only about the expected return. Since the bailout policy $g^w(s)$ is given by Lemma 5, our assumption above $\pi(H)R_A^w(H) + \pi(L)(1-\Gamma) < R_C$ implies that $E[R_A^w(s) + g^w(s)] < R_C$. Therefore, the value of the investing in the claim issued by the SIFI V_A (which is backed by the SIFI's risky project) exceeds that of investing in the prudent project V_C, i.e. we have $V_A < V_C \equiv E\left[m_1(s)\right] R_C = m_1(s) R_C$. Banks never want to invest in interbank claims issued by SIFIs, because the government guarantee does not increase the expected return on these claims sufficiently to entice banks away from prudent assets. As a result, each bank i's best response function is to always invest in prudent assets. As a result, no bank ever undertakes a risky investment in equilibrium. This is summarized in the corollary below.

Corollary: *No excessive risk taking with risk neutrality*

Under Benchmark economy 2, when the household is risk neutral ($\eta = 0$), there is never excessive risk taking in equilibrium by any bank.

Now suppose that the household is risk averse, so that $\eta > 0$. As the household's risk aversion increases, banks care more about the covariance of their portfolio returns with the stochastic discount factor, and therefore the risk premium on an interbank claim issued by a SIFI w is lower, as captured by a higher safety premium $SP_A \equiv Cov\left(m_1(s), \theta_\ell^i(s,w)\right)$. In other words, the safety offered by the SIFI's interbank claim is valued more by non-SIFI banks. By Assumption OA.1, there is a threshold risk aversion value $\overline{\eta}$ above which the value of the SIFI's interbank claim exceeds the value of the prudent project. This is summarized in the figure below, which plots the value of investing in the prudent project $V_C \equiv E[m_1(s)] R_C$ together with the total value of the interbank claim issued by SIFI V_A and the safety premium component SP_A of this claim, each as a function of risk aversion parameter η. (The difference between V_A and SP_A is given by $E_A \equiv E[m_1(s)] E\left[\theta_\ell^i(s,w)\right]$.)

Figure 3:



How does this affect banks' portfolio choices? Recall that, for sufficiently high risk aversion $\eta > \overline{\eta}$, we have $E[m_1(s)(R_A(s)+g(s))] > E[m_1(s)R_C]$. As a result, non-SIFI banks choose to invest in claims issued by the SIFI for all $\eta > \overline{\eta}$: The insurance value of interbank claims issued by the SIFI (together with expected discounted return) is sufficiently high to entice banks to forgo their prudent projects in favor of buying financial claims issued by the SIFI. (At same time, the SIFI invests in its risky project.) As a result, the risk sharing facilitated by the interbank market incentivizes excessive risk taking.

Corollary: *Risk sharing generates excessive risk taking by all banks*

When the household is risk averse, the insurance value of interbank claims issued by SIFIs is sufficiently high to entice non-SIFI banks to forgo their prudent investments in favor of buying claims on the SIFIs' portfolio. As a result, in equilibrium, the SIFIs invests in their risky project and non-SIFIs invest in financial claims issued by SIFIs.

Takeaway These comparative static exercises show that, in Benchmark economy 2, risk sharing between the SIFIs and non-SIFI banks in the risky equilibrium is precisely what facilitates excessive risk taking in the first place. When the insurance value of interbank claims on the SIFIs are low, banks do not have incentive to invest in risky assets. Only when the insurance provided by these SIFI claims is sufficiently high do banks undertake excessive risks.

APPENDIX 13: Full planner problem

The planner's problem is to choose $c_t(s)$, f_0^i, $B_1(s)$, d_0^i, $b_1^i(s)$, $b_2^i(s)$, ℓ^{ji}, $r^{ji}(s)$, k_0^i, $k_1^i(s)$, ω^i, $T_1(s)$, and $g^i(s,\omega^i)$ for all banks i, j, all states s and all periods t to solve

$$\max \; E\left[u(c_0)+u(c_1(s))+u(c_2(s))\right]$$

s.t.

$$c_0+\Sigma_i f_0^i d_0^i \le e_0 - T_0 \tag{54}$$

$$c_1(s)+B_1(s) \le e_1 + \sum_i f_0^i d_1^i(s) - T_1(s) - q(s)k_1^T(s) \tag{55}$$

$$c_2(s) \le e_2 + B_1(s) + \sum_i f_0^i d_2^i(s) + \Pi_2(s) \tag{56}$$

Final dividend payout (including dividend from traditional firms)

$$\Pi_2(s) = \sum_i \left(A - b_2^i(s)\right) k_1^i(s) + F\left(k_1^T(s)\right)$$

budget constraints

$$k_0^i + \sum_j \ell^{ij} \le n + d_0^i + \sum_j \ell^{ji} \tag{57}$$

$$q(s)k_1^i(s) \le \theta_k^i\left(s,\omega^i,g^i\right)k_0^i + \sum_j \theta_\ell^i(s,j)\ell^{ij} - \sum_h \left(r^{hi}(s) - b_1^i(s)\right)\ell^{hi} + b_2^i(s)k_1^i(s)$$

no-default constraints for the household contract

$$0 \le b_1^i(s) \le q(s) - \gamma \tag{58}$$

$$0 \le b_2^i(s) \le \Gamma A \tag{59}$$

the other firms' participation constraints for each j

$$u^{ji}\left(\ell^{ji},\left\{r^{ji}(s)\right\}_s\right) \ge \bar{u}^j \tag{60}$$

and non-negativity constraints on capital holdings and interbank loans.

$$k_0^i, k_1^i(s), \ell^{ij} \ge 0 \quad \forall j \tag{61}$$

asset prices

$$q(s) = F'(k_1^T(s))$$

$$k_1^T(s) = \max\{0, K_1(s) - K_0\}.$$

the government's optimal bailout policy

$$k_0^w g^w(s, \omega^w) = \begin{cases} \left(q(s) - b_2^w(s)\right) \sum_i k_0^i - \sum_i \frac{q(s)-b_2^w(s)}{q(s)-b_2^i(s)} X & for\, s = L \\ 0 & otherwise \end{cases}$$

where

$$X \equiv \left(q(s) + \omega^{\mathbf{i}} \mathbf{R}^{\mathbf{i}}(s) - \gamma - b_1^i(s)\right) k_0^i + \sum_j \theta_\ell^i(s, j) \ell^{ij} - \sum_h \left(r^{hi}(s) - b_1^i(s)\right) \ell^{hi}$$

and the government budget constraint

$$\sum_j k_0^j g^j(s, \omega^j) + D\left(k_1^T(s)\right) = T_1(s). \tag{62}$$

Since the planner has the same limited commitment that the government does, the planner solves its problem recursively. The planner first solves the date 1 problem taking as given date 0 variables. The optimal government transfers at date 1 in the planner's solution will, by construction, coincide with the government's optimal bailout policy. Given this date 1 solution, the planner then solves the date 0 problem. The recursive nature of this problem is captured by including the optimal bailout policy as a constraint in the planner's date 0 problem above. Note, however, that this optimal bailout policy is a generalized version of that which appears in the competitive equilibrium, because we do not impose equilibrium conditions, such as full interbank risk sharing, in the planner's problem. Recall that the government's optimal bailout policy implies capital is never misallocated at date 1. Therefore, we have $q(s) = 1, k_1^T(s) = 0$. Imposing that the government budget constraint binds, replace date 1 taxes $T_1(s)$. We also replace $d_1^i(s)$ and $d_2^i(s)$ using the definitions of $b_1^i(s)$ and $b_2^i(s)$.

Notice that the planner takes the constraints of all banks i as constraints simultaneously in the Lagrangian. Hence, unlike in the competitive economy, the planner's first order conditions for ℓ^{ij} and $r^{ji}(s)$ will also capture how they affect the budget constraints of other banks j (i.e. k_0^j and $k_1^j(s)$). The planner's first order conditions are

$$\frac{\partial L'}{\partial f_0^i} \leq 0 \iff E\left[u'\left(c_2(s)\right)\right] b_2^i(s) k_1^i(s) + \ldots \tag{63}$$

$$\ldots + E\left[u'\left(c_1(s)\right)\left(\left[b_1^i(s)\left(k_0^i - \sum_h \ell^{hi} + \sum_j \ell^{ij}\right) - b_2^i(s) k_1^i(s)\right]\right)\right] - E\left[u'\left(c_0\right)\right] d_0^i \leq 0$$

$$\frac{\partial L'}{\partial B_1(s)} \leq 0 \iff E\left[u'\left(c_2(s)\right)\right] - E\left[u'\left(c_1(s)\right)\right] \leq 0 \tag{64}$$

$$\frac{\partial L'}{\partial d_0^i} \leq 0 \iff -u'\left(c_0\right) f_0^i + z_0^i \leq 0 \tag{65}$$

$$\frac{\partial L'}{\partial k_0^i} \leq 0 \iff E\left[u'\left(c_1(s)\right) f_0^i b_1^i(s)\right] - z_0^i + E\left[z_1^i(s) \theta_k^i\left(s, \omega^i, g^i\right)\right] - E\left[u'\left(c_1(s)\right) \frac{\partial T_1(s)}{\partial k_0^i}\right] \leq 0 \tag{66}$$

$$\frac{\partial L'}{\partial k_1^i(s)} \leq 0 \iff -u'\left(c_1(s)\right) f_0^i b_2^i(s) + u'\left(c_2(s)\right) f_0^i b_2^i(s) + \ldots \tag{67}$$

$$+ u'\left(c_2(s)\right)\left(A - b_2^i(s)\right) - z_1^i(s)\left(1 - b_2^i(s)\right) \leq 0$$

$$\frac{\partial L'}{\partial b_1^i(s)} \leq 0 \iff \left(u'\left(c_1(s)\right) f_0^i - z_1^i(s)\right)\left(k_0^i - \sum_h \ell^{hi} + \sum_j \ell^{ij}\right) - u'\left(c_1(s)\right) \frac{\partial T_1(s)}{\partial b_1^i(s)} \leq \bar{\lambda}_1^i(s) - \bar{\lambda}_0^i(s) \tag{68}$$

$$\frac{\partial L'}{\partial b_2^i(s)} \leq 0 \iff -u'\left(c_1(s)\right) f_0^i k_1^i(s) + u'\left(c_2(s)\right) f_0^i k_1^i(s) - u'\left(c_2(s)\right) k_1^i(s) + z_1^i(s) k_1^i(s) - \ldots \tag{69}$$

$$\ldots - u'\left(c_1(s)\right) \frac{\partial T_1(s)}{\partial b_2^i(s)} \leq \mu_1^i(s) - \mu_0^i(s)$$

$$\frac{\partial L'}{\partial \omega^i} \leq 0 \iff E\left[z_1^i(s) k_0^i \frac{\partial \theta_k^i\left(s, \omega^i, g^i\right)}{\partial \omega^i}\right] - E\left[u'\left(c_1(s)\right) \frac{\partial T_1(s)}{\partial \omega^i}\right] \leq 0 \tag{70}$$

$$\frac{\partial L'}{\partial \ell^{ij}} \leq 0 \iff E\left[u'\left(c_1(s)\right) f_0^i b_1^i(s)\right] - z_0^i + z_0^j + E\left[z_1^i(s) \theta_\ell^i(s, j)\right] - E\left[u'\left(c_1(s)\right) \frac{\partial T_1(s)}{\partial \ell^{ij}}\right] \leq 0 \tag{71}$$

$$\frac{\partial L'}{\partial r^{ji}(s)} \leq 0 \iff -z_1^i(s)\ell^{ji} + z_1^j(s)\ell^{ji} - \hat{v}^{ji}\frac{\partial u^{ji}\left(\ell^{ji}, \{r^{ji}(s)\}_s\right)}{\partial r^{ji}(s)} - u'\left(c_1(s)\right)\frac{\partial T_1(s)}{\partial r^{ji}(s)} \leq 0 \quad (72)$$

APPENDIX 14: Motivating empirical evidence

Here, we present a brief review of the empirical evidence on the structure of interbank markets, with a focus on three 'stylized facts'. The overall picture painted by these facts is one of a highly concentrated financial system in which a small number of large and interconnected institutions hold riskier assets, and benefit from an implicit government guarantee which lowers the cost of their liabilities.

The first stylized fact is that interbank financial markets typically exhibit a strong core-periphery structure, in which a few highly interconnected institutions at the core interact with the many sparsely connected institutions in the periphery. This has been shown for a wide range of markets including inter-dealer markets for corporate bonds, over-the-counter derivatives markets, interbank markets, and fed funds markets.[33]

The second fact is that these large and interconnected financial institutions often benefit from an implicit government guarantee of their assets or liabilities. Moreover, this guarantee lowers their costs of funding on deposit or wholesale funding markets, and lowers their cost of insurance via credit default swaps or put options on equity prices.[34]

The third fact is that these large and interconnected institutions often make riskier investments than those in the periphery. Afonso, Santos and Traina (2015), and several papers cited therein, show that the anticipation of government support is associated with increased risk taking. Moreover, Elliott, Georg and Hazell (2021) provide evidence that banks who are more interconnected also undertake more correlated risks.

Consistent with these three features of the data, our model will endogenously feature a core-periphery structure in the interbank market in which large, interconnected banks at the core benefit from an implicit government subsidy and undertake riskier investments. In addition, the liabilities of these SIFIs will command a lower risk premium, reflecting the insurance value provided by the

[33]For example, see Di Maggio, Kermani and Song (2017) for evidence of a core-periphery structure in the inter-dealer market for corporate bonds, Peltonen, Scheicher and Vuillemey (2014) and Vuillemey and Breton (2013) for over-the-counter derivatives markets, Boss et al. (2004), Chang et al. (2008), Craig and von Peter (2014), and van Lelyveld and in 't Veld (2014) for interbank markets, and Afonso and Lagos (2015), Allen and Saunders (1986), Bech and Atalay (2010) for the fed funds market.

[34]See Kelly, Lustig and Van Nieuwerburgh (2016) for evidence of the size of implicit government guarantees from out of the money put options, and Veronesi and Zingales (2010) from data on credit default swaps for the largest firms from 2008 Paulson plan, and Lucas and McDonald (2006) and Lucas (2019) for the size of guarantees government-sponsored enterprises. See also O'Hara and Shaw (1990), Baker and McArthur (2009), and Demirguc-Kunt and Huizinga (2013).

implicit government guarantee.

APPENDIX 15: Proof that selective bailouts can eliminate a TCTF equilibrium

We show that the Pigouvian taxes outlined in Lemma 6 are sufficient to eliminate all risk taking in equilibrium. By design, the Pigouvian taxes in Lemma 6 prevents SIFIs from arising in equilibrium by ensuring that no bank wants to lend to any bank with risky portfolios. However, it could still be the case that each bank invests in its own risky project, since these are perfectly correlated (a too-correlated-to-fail equilibrium). Therefore, it remains to show that there is a credible bailout policy which eliminates the too-correlated-to-fail equilibrium. This equivalent to considering an economy with the interbank market shut down, and show that there is a credible selective bailout policy which can eliminate all risk taking in equilibrium. A sufficient condition for the proof below is strengthening Assumption 2(a) to$R_C > 1-\Gamma$ instead of $R_C \geq 1-\Gamma$. (This amounts to assuming that the returns from prudent investments are high enough that banks can afford buying capital on net at $q(s) = 1$.)

Suppose that, in the event of a fire sale, the government is anticipated to bail out only some number $N-x \in (0,N)$ of banks which are suffering losses on their risky projects, and is anticipated to force the remainder $x \in (0,N)$ to absorb losses without a government transfer. Let S denote the set of banks who are anticipated to receive bailout funds in the event of a bailout ('selected' banks), and S^C the set of investing banks who are not ('non-selected' banks). (Note that S and S^C are both subsets of the set W of investing banks, since we've shut down the interbank market.) As we argued in section 2.9, the government can credibly commit to this policy as long as it it ensures that $K_1(s) \geq K_0$ so that all capital remains within the banking sector (and $q(s) = 1$). (Recall from section 2.8 that a fire sale occurs if and only if $K_1(s) < K_0$.)

The best response functions characterized in section 3.1 imply that each non-selected bank $i \in S^C$ invests only in prudent assets since it internalizes the riskiness of its projects, so that $\omega^i = 1$. Therefore, the portfolio return of these banks in the bad state is given by $\theta_k^i\left(s_L,\omega^i\right) = R_C + 1 - \gamma - b_1^i(s_L,\omega^i)$. Each selected bank $w \in S$ invests in the risky project, if and only if there is expected to be a bailout in the bad state, and their portfolio returns in the bad state are given by $\theta_k^w(s_L,\omega^w) = R_A^w(s_L) + 1 - \gamma - b_1^w(s_L,\omega^w)$.

Our goal is to show that the government can credibly choose a number $x \in (0,N)$ of selected banks such that, given banks' best response functions, there is never a fire-sale in the bad state even in the absence of a bailout. (This amounts to that the aggregate resources available to banking sector as a whole are sufficient to the bear losses in bad state without liquidating capital to the traditional sector, so that $K_1(s_L) \geq K_0$ even without government bailout.) If that is the case, even selected banks $w \in S$ would choose to invest in prudent assets only.

Under the government's selective bailout policy, a fire sale does not occur in the bad state if and only if the aggregate investment in capital at date 1 is weakly positive. Note that, while the portfolio returns of each non-selected bank are the same since they invest in the prudent project only, the portfolio returns of selected banks may differ since the returns on each of their risky projects may differ. Therefore, aggregate net investment in capital is comprised of the portfolio returns $\frac{\theta_k^i(s_L,\omega^i)}{1-\Gamma}-1$ of non-selected banks times the amount of capital $\frac{x}{N}K_0$ owned by these banks, plus the sum of the portfolio returns $\frac{\theta_k^w(s,\omega^w,g^w)}{1-\Gamma}-1$ of each selected bank w times the amount of capital owned by each of these banks $\sum_{w\in S}k_0^w$. This is a version of equation (12) in the bad state, where we have imposed $q(s_L)=1$ and $g^w(s)=0$.

$$\frac{x}{N}K_0\left[\frac{\theta_k^i\left(s_L,\omega^i\right)}{1-\Gamma}-1\right]+\sum_{w\in S}k_0^w\left[\frac{\theta_k^w\left(s_L,\omega^w\right)}{1-\Gamma}-1\right]\geq 0$$

Let $\tilde{x}$ be defined by the above condition holding with equality. Then $\tilde{x}$ is the threshold number of selected banks below which there is never a fire sale. Essentially, we want to show that $\tilde{x}\in(0,N)$, or $\frac{\tilde{x}}{N}\in(0,1)$. (This amounts to showing that there is a selective bailout policy which ensures that there is never a fire sale in equilibrium.) As we show below, it will be trivially true that $\frac{\tilde{x}}{N}>0$ (since, if all banks are bailed out in a crisis and they anticipate this, then all banks will make risky investments, so that there is a fire sale in the absence of a bailout). It is less obvious that $\frac{\tilde{x}}{N}<1$.

Because net aggregate investment is increasing in the returns $\theta_k^w\left(s_L,\omega^w\right)$ on each selected bank's portfolio in the bad state, it suffices to show that $\frac{\tilde{x}}{N}<1$ even when all selected banks receive the lowest return among them $\theta_k^v\left(s_L,\omega^v\right)$, where $v\in S$ and $R_A^v(s_L)\leq R_A^w(s_L)$ for all $w\in S/v$. (Put differently, in the absence of a bailout, if the resources of non-selected banks in bad state are sufficient to buy all capital sold by failing banks $w\in S$ even when all failing banks get the worst return, then these resources will also be sufficient if each failing bank gets the return on its risky project.) Suppose that all selected banks receive the lowest return $\theta_k^v\left(s_L,\omega^v\right)$ in the bad state. Then we could express the total capital holdings of selected banks as $\sum_{w\in S}k_0^w=\left(1-\frac{x}{N}\right)K_0$, and so net aggregate investment would be positive if and only if

$$\frac{x}{N}K_0\left[\frac{\theta_k^i\left(s_L,\omega^i\right)}{1-\Gamma}-1\right]+\left(1-\frac{x}{N}\right)K_0\left[\frac{\theta_k^v\left(s_L,\omega^v\right)}{1-\Gamma}-1\right]\geq 0.$$

So it suffices to show that $\frac{\hat{x}}{N}\in(0,1)$, where $\hat{x}$ is defined by the above condition holding with equality. Solving for $\frac{\hat{x}}{N}$ yields

$$\frac{\hat{x}}{N}=\frac{1-\Gamma-\theta_k^v\left(s_L,\omega^v\right)}{\theta_k^i\left(s_L,\omega^i\right)-\theta_k^v\left(s_L,\omega^v\right)} \tag{73}$$

(So there is no fire sale as long as x exceeds this threshold $\hat{x}$.) We want to show that $\frac{\hat{x}}{N}\in(0,1)$. Note that from section 2.5, we have that, in the absence of bailouts, $\theta_k^v\left(s_L,\omega^v\right)=1+R_A^v(s_L)-\gamma-b_1^i(s)$

and $\theta_k^i\left(s_L,\omega^i\right)=1+R_C-\gamma-b_1^i(s)$.

First we show that $\frac{\hat{x}}{N}>0$. Note that the denominator is always strictly positive. To see this, note that $\theta_k^i\left(s_L,\omega^i\right)>\theta_k^v\left(s_L,\omega^v\right)$ if and only if $R_C-R_A^v(s_L)>b_1^i(s,\omega^i)-b_1^v(s,\omega^v)$. Note also that, since $b_1^i(s)\in[0,1-\gamma]$, we have $b_1^i(s,\omega^i)-b_1^v(s,\omega^v)\leq 1-\gamma$. So it suffices to show that $R_C-R_A^w(s_L)>1-\gamma$. To see this, recall assumption 2a that $R_C-R_A(L)>R_C-\gamma+\Gamma$. Then it suffices to show that $R_C-\gamma+\Gamma>1-\gamma$, which holds based on our assumption above that $R_C>1-\Gamma$.

Turn to the numerator. If $b_1^v(s,\omega^v)=0$, then we have $1-\Gamma>\theta_k^v\left(s_L,\omega^v\right)=1-\gamma+R_A^v(s_L)$, by Assumption 2(a) that $\gamma-\Gamma-R_A^L>0$. So the numerator is strictly positive. If, on the other hand $b_1^v(s,\omega^v)=1-\gamma$, then we have $1-\Gamma>\theta_k^v\left(s_L,\omega^v\right)=1-\gamma-(1-\gamma)+R_A^v(s_L)$ which also holds by Assumption 2(a) and the assumption that $\gamma<1$. So the numerator is strictly positive in any case. Thus, we have $\frac{\hat{x}}{N}>0$.

Now we show that $\frac{\hat{x}}{N}<1$. This holds if and only if $1-\Gamma-\theta_k^v\left(s_L,\omega^v\right)<\theta_k^i\left(s_L,\omega^i\right)-\theta_k^v\left(s_L,\omega^v\right)$, i.e. if and only if $1-\Gamma<\theta_k^i\left(s_L,\omega^i\right)$. If $b_1^i(s,\omega^i)=0$, then this condition holds since $\gamma-\Gamma<R_C$, which holds because $R_C\geq\gamma$ and $\Gamma>0$. If, on the other hand, $b_1^i(s,\omega^i)=1-\gamma$, then this condition still holds since $1-\Gamma<R_C$ (which holds based on our assumption above). Thus, we always have that $\frac{\hat{x}}{N}<1$.

Thus, the government can design a selective bailout policy in which it concentrates a bailout on $N-x$ and leaves the remaining x investing banks to bear losses in the event of a crisis. As long as $x>\hat{x}$, this policy will ensure that there are no fire sales or bailouts in the bad state. As a result, even banks who would be selectively bailed out in the event of a fire sale choose to invest in prudent projects only, since they would never be bailed out in the bad state, and hence $\omega^w=1$ for all $w\in S$ as well. Therefore, any selective bailout policy in which $x>\hat{x}$, is optimal and credible ex ante. Q.E.D.