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A STRUCTURAL EMPIRICAL MODEL OF R&D INVESTMENT, FIRM HETEROGENEITY, AND INDUSTRY EVOLUTION

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ABSTRACT

This paper develops and estimates an industry equilibrium model of manufacturing plants in the Korean electric motor industry from 1991 to 1996. Plant-level decisions on R&D, physical capital investment, entry, and exit are integrated in a dynamic setting with knowledge spillovers. We use a simulated method of moments estimator and the novel approximation method of Weintraub, Benkard and Van Roy (2008) to estimate the R&D cost, magnitude of knowledge spillovers, adjustment costs of physical investment, and plant scrap value distribution. Knowledge spillovers are essential to explaining the firm-level productivity evolution and the equilibrium market configuration. A counterfactual experiment reveals that a 15% R&D subsidy maximizes industry output and is broadly consistent with a past policy initiative of the Korean government.

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1 Introduction

A large empirical literature has documented substantial and persistent heterogeneity in firm productivity even within narrowly defined industries.[1] Motivated by these facts, theoretical models of industry dynamics have been developed by Jovanovic (1982), Hopenhayn (1992), and Ericson and Pakes (1995) to explain individual firm size, success, and failure patterns observed in longitudinal micro-level data. These existing theoretical models share a common feature: a stochastic process that changes a firm's productivity (or belief on its productivity) over time. This process of productivity evolution is a key component that drives the growth and failure of individual firms and overall evolution of the industry structure.

In this paper, we study two important sources of productivity evolution: investment in R&D by individual firms and knowledge spillovers from their competitors. There exists strong empirical evidence that a firm's technological position does not just evolve exogenously. Using the knowledge production function framework pioneered by Zvi Griliches, numerous papers have investigated the linkages of firm R&D spending, knowledge spillovers, and productivity growth.[2] More recently, Aw et al. (2011) and Doraszelski and Jaumandreu (2013) extend this framework to explicitly model firm's optimal R&D decision. These papers utilize the insights from the modern production function estimation literature to jointly recover firm's production technology and the impact of R&D on firm productivity. Nevertheless, this line of empirical work has so far treated each firm as a single agent. This paper investigates firm R&D decisions and knowledge spillovers within a dynamic industry equilibrium model.

The idea of simultaneously investigating firm R&D, interfirm or intrafirm spillovers, and the industry structure dates back to classic theoretical papers like Dasgupta and Stiglitz (1980) and Spence (1984). Nonetheless, few empirical studies have attempted to estimate these forces in a dynamic equilibrium model. Some notable exceptions include Benkard (2004), Goettler and Gordon (2011), and Igami (2017). These papers focus on industries that are dominated by a few large firms and where strategic interactions in the product market play a central role in firms' innovation decisions.[3] Our paper instead

[1] See Bartelsman and Doms (2000) and Syverson (2011) for an excellent survey of the micro productivity literature.

[2] Griliches (2007) provides an extensive survey of the empirical literature in this tradition.

[3] For a related computational framework, see also Besanko et al. (2010).

focuses on a more typical manufacturing industry in an emerging economy: the Korean electric motor sector. Similarly to the settings examined in previous works, this industry features strong innovation activities and scope for knowledge spillovers; however, it also has a much larger number of producers. While these producers are highly heterogeneous, the overall size distribution is quite continuous, and there is no clear dominance by a small subset of firms. Firms still maximize their value of continuation given expectations about the evolution of their own and competitors' states (e.g, market structure) since these equilibrium objects impact the overall product market competition and, more importantly, the strength of knowledge spillovers. We tailor our empirical strategy to these features. In particular, we use the oblivious equilibrium concept proposed by Weintraub et al. (2008) to circumvent the well-known computational burden of the Markov perfect equilibrium (MPE) in the framework of Ericson and Pakes (1995). When there is a large number of firms within the industry, the oblivious equilibrium—where firms are assumed to ignore current information about competitors' states and condition their choices on their belief on the long-run average industry structure—closely approximates a Markov perfect equilibrium.

Equipped with this novel concept of equilibrium, we contribute to the existing empirical literature along several dimensions. First, it is widely observed that a large fraction of firms report no R&D activity, even in high-tech industries. We reconcile this observation with the commonly used first-order Markov productivity process in the firm dynamics literature (e.g., Hopenhayn, 1992),[4] by allowing firms to survive and grow by imitation. Within the context of our model, we propose an identification strategy to capture the effect of knowledge spillover. We discipline the spillover by matching the extent of "mean reversion" of measured firm productivity, i.e., the chance of backward firms catching up in the data.[5] Second, our equilibrium industry structure is also determined by market selection based on entry and exit. A high firm turnover rate is a salient feature of a broad range of manufacturing industries. We are able to recover the quantitative magnitude of the entry cost and scrap values and investigate how firm turnover interacts with the firm

[4]This assumption has also been one of the fundamental building blocks of the modern production function estimation literature, pioneered by Olley and Pakes (1996) and extended by Levinsohn and Petrin (2003) and Ackerberg et al. (2015).

[5]As we explain in detail later, the standard Ericson and Pakes (1995) step-by-step innovation setting implies a firm-level stochastic process closely approximated by a random walk and thus inconsistent with the empirical regularity of firm growth in our micro-data.

innovation and imitation mechanism. Finally, since most of the manufacturing sector is also capital intensive, we adapt the model of Ericson and Pakes (1995) to incorporate both physical capital and R&D investments. We allow a rich set of adjustment frictions in physical capital to match the observed interdependence of R&D and physical capital in our micro-data.

The model is estimated with micro-data from the Korean electric motor industry. In the first step, we utilize the model specification for static market competition to estimate the firm's revenue production function. We modify the approach of Ackerberg et al. (2015) such that a firm's productivity is influenced by its own R&D. Due to the presence of knowledge spillover, we treat the estimated process as only "reduced-form" moments that need to be matched in the subsequent estimation of the dynamic model. In the second step, we apply a simulated methodmof moments estimator to the industry equilibrium model and recover the cost of R&D, magnitude of the spillover, adjustment costs of investment, and distribution of plant scrap values. By accommodating imperfect competition, productivity heterogeneity, and investment in both physical and knowledge capital, the model is rich enough to reproduce the observed market structure and industry turnover patterns.

Our empirical results show, first, that each firm's own R&D effort improves its future productivity although this process is subject to substantial idiosyncratic uncertainty. The within-industry knowledge spillover is significant and helps to reconcile the observed producer R&D spending and productivity evolution patterns. Taking into account that the total knowledge pool is a public good, spillovers are quite important for less productive producers. For a firm located at the lower end of the industry productivity distribution and not engaged in R&D, the spillover that it receives increases the chance that its productivity improves by 12%. This accounts for the large amount of zero R&D behavior in our model and the data. Second, each producer also incurs substantial adjustment costs for physical capital investment. These costs prevent firms from instantaneously responding to positive R&D outcomes and, in turn, affect the dynamics of firm R&D investment incentives. Third, the mean random scrap value and entry cost equal 4.89 years and 12.33 years of average firm profit, respectively. The relatively narrow hysteresis band, defined as the difference between the entry cost and the mean scrap value, explains the high turnover rate observed in the industry data.

Using the point estimates of the parameters, we first conduct a counterfactual analysis that isolates the role of knowledge spillover in shaping firm R&D and productivity and the industry structure. When we increase the strength of knowledge spillover by 50% relative to our baseline, more firms have free-riding motives, and the total R&D effort of the industry is cut by around 24%. However, backward firms can catch up more easily. As a result, the industry has lower productivity dispersion. The free-riding motive is important in accounting for these responses: in an environment where firms stay with their baseline R&D policy function, aggregate productivity increases more, reflecting a better overall productivity distribution due to easier imitation.

Finally, inspired by Korea's S&T policy, we conduct a series of experiments that provide linear R&D subsidies. We assume that the government finances such subsidies with corporate income tax such that it is revenue neutral. It turns out that due to the knowledge spillover, such a policy could improve aggregate industry output and TFP. The optimal linear R&D subsidy is around 15%, which is quite close to that implemented by the Korean government. Industry output would increase by a modest 0.14% with the subsidy policy. We also show that—as standard theory indicates—such government policies would unambiguously reduce aggregate output if there were no spillovers.

This paper is related to three strands of literature. First, our focus on R&D and knowledge spillover is shared by a large number of papers in the productivity literature. Many of the earlier empirical works, such as that of Jaffe (1986), are summarized in Zvi Griliches's excellent book *R&D and Productivity: The Econometric Evidence* (Griliches, 2007). The literature has further modernized in terms of its identification strategies in the recent decade, for instance, with the work of Bloom et al. (2013). Our paper is consistent with this literature. However, while we construct our knowledge production process to incorporate both the firm's own and its rivals' R&D effort, our emphasis is more on modeling and estimating firm R&D decisions and industry evolution. Thus, we utilize an identification strategy for spillovers that depends more than the approaches in previous works on the model structure and ensure that the recovered spillover is consistent with various firm decisions and industry structure. Second, our paper is related to generations of industry dynamic models (e.g., Jovanovic (1982), Hopenhayn (1992), and, in particular, Ericson and Pakes (1995)). We modify the innovation process of firms and incorporate it into an otherwise standard Ericson and Pakes (1995) model. Our paper is one of the first to utilize the novel approximation of Weintraub et al. (2008) to facilitate the estimation and computation of such a model. This approach is necessary given the specificities of our empirical context.[6] Third, our paper is also related to the theoretical literature that emphasizes the role of knowledge diffusion as a source of economic growth. Our model's backward learning structure of knowledge spillover, where firms learn only from their more productive peers, is heavily motivated by the works of Eeckhout and Jovanovic (2002) and Jovanovic and MacDonald (1994). A particularly related recent paper is Benhabib et al. (2017), which incorporates both diffusion and innovation. Our theory is less technically challenging since we focus on idiosyncratic knowledge depreciation and a stationary environment. However, our paper adds to the scant empirical micro-level evidence in support of the core learning mechanisms in this line of work.

The rest of our paper is organized as follows. The second section summarizes the industry background and motivates our model elements with data. The third section describes the economic environment and the industry equilibrium. The fourth section estimates both the revenue production function and the full dynamic equilibrium model. Finally, the fifth section implements counterfactual simulations of a set of policy changes.

2 Industry Background and Data Descriptives

2.1 Korean Electric Motor Industry

This paper analyzes panel data of Korean manufacturers in the electric motor and generator industry for 1991 to 1996.[7] The data are part of the Korean Annual Mining and Manufacturing Survey of all establishments with more than 5 workers. These data cover both the large and small firms, which is important for our study of R&D and knowledge spillovers in an industry equilibrium framework.[8] The electric motor industry is an in-

[6] There has also been an important breakthrough allowing the empirical estimation of dynamic oligopoly models without directly solving them. Aguirregabiria et al. (2007), Bajari et al. (2007), and Pakes et al. (2007) laid the foundations for this line of work. However, since we observe one single industry over a relatively short period, the data are not sufficient to represent the industry state in its ergodic set. We thus have to rely on a full solution method.

[7] The Korean industry classification code SIC31101 and SIC31102 is equivalent to NAICS 335312 (motor and generator manufacturing) in the US census. These establishments primarily engage in manufacturing electric motors (except internal combustion engine starting motors), power generators (except battery charging alternators for internal combustion engines), motor generator sets (except turbine generator set units), and transformers.

[8] In contrast, the majority of previous studies on R&D investment and knowledge spillovers use data from Compustat or R&D surveys, which usually include only a limited number of firms spanning multiple

termediate input sector, where low cost and energy efficiency are the key indicators of a firm's technological position. Process innovation plays a predominant role in a firm's R&D and learning effort. Studying such a relatively mature industry in our model helps us abstract from the product innovation often emphasized in other high-tech industries. The Korean government started a strong science and technology initiative to promote knowledge-intensive industries after the Asian financial crisis in 1997. The electric motor industry is part of this initiative and a natural candidate as a setting in which to study how R&D policy impacts firm R&D, knowledge spillovers, and aggregate industry productivity.

2.2 Data Descriptives

The Korean Annual Manufacturing Survey reports detailed annual information on each manufacturing establishment's value added, physical capital, employment, physical investment, and, most importantly, R&D investment. On the cost side, we have information on total material expenditure and the total wage bill. Table 1 summarizes some of the key data patterns for the electric motor industry. The average R&D expenditure is 25.8 million won (33 thousand USD) per year.[9] However, only a small fraction of producers, accounting for 11% of the total observations during the sample years, report positive R&D expenditure.[10] The average R&D expenditure of performers is much higher, around 236.5 million won (300 thousand USD) per year. The major components of the reported R&D are wages for R&D workers and materials for R&D. Producers show large dispersion in their value added and wage expenditure, with the top firms (99th percentile) often a few hundred times larger than the bottom ones (1st percentile). This indicates significant revenue productivity differences across establishments within the industry. To rescale producers' R&D investment by their size, we define the R&D intensity as total R&D expenditures divided by value added. The R&D intensity has a mean value of 0.13 and a median of 0.06. Physical investment includes net capital expenditures (purchase minus sales) on buildings, machinery/equipment, and transport vehicles and averages 110 million won (140 thousand USD) per year. Similar to R&D

industries.

[9]The average exchange rate between the won and US dollar during the sample period is 786:1.

[10]Doraszelski and Jaumandreu (2013) reports similar patterns of R&D expenditure in Spanish Manufacturing Survey data.

investment, plant physical investment shows large differences: it ranges from -39 to $2,209$ million won (-49 thousand to 2.8 million USD) from the 1st to the 99th percentile.

Previous empirical literature has documented a zero or slightly negative correlation between R&D intensity and firm size (Klette and Kortum, 2004). A similar pattern holds in our data. Figure 1 illustrates the relation between log R&D intensity and log value added among R&D performers in our data. It shows that producers with larger value added tend to have lower R&D intensity. This suggests that larger producers in our data could have higher per-unit R&D cost; however, it could also be consistent with the fact that knowledge spillovers tend to decrease the private return to R&D for industry leaders.

Figure 2 reports the distribution of the investment-to-capital ratio (capital investment relative to the stock of physical capital) and shows that 46% of firms do not invest in a given year. In addition, reflecting a pattern often called "investment spikes", 24% of firms replace more than 20% of their capital stock in a given year. These lumpy investment patterns suggest that investment decisions are subject to nonconvex adjustment costs.[11] Figure 2 also shows that negative investment is relatively rare: only 4% of firms disinvest in a given year. This motivates us to consider an asymmetry in the purchase and resale prices of physical capital in our model.

Table 2 summarizes market concentration in this industry. It shows that in the Korean electric motor industry, on average, the market share of the top 4 firms is 29% and of the top 20 firms 59%. These numbers are comparable to those reported for manufacturing industries in the US by Autor et al. (2020). The average Herfindahl–Hirschman index (HHI) of this industry is 323, far below the Department of Justice's definition of a moderately concentrated market (one with an HHI between 1,500 and 2,500 points). The data pattern suggests that the Korean electric motor industry is neither perfectly competitive nor dominated by a very few large firms. Therefore, our empirical setting matches the economic environment described in Weintraub et al. (2008). Their paper develops the novel equilibrium concept of oblivious equilibrium, characterized by each firm making decisions based only on its own state and the long-run average industry state.

[11] By way of comparison, Chen et al. (2019) show that 49% of firms do not invest in a given year and 17% of firms replace more than 20% of their capital stock in China, and Zwick and Mahon (2017) reports that 34% of firms in the US replace less than 1% of their capital and that 16% of firms replace more than 20% each year.

Last, we show that this industry has a very high turnover rate. Table 3 reports that, on average, the entry rate is 15.7% and the exit rate 16.0%.[12] Entrants account for approximately 5% of the total market share each year, and exiting firms account for 11% of the total market share on average. Each entrant cohort's importance grows over time. For example, the market share of the cohort born in 1992 accounts for 5% of the market share, and this number increases to 10% in 1996. Given the high turnover rate and nontrivial market share accounted for by entries and exits, it is important to include them in our model.

We next outline an industry equilibrium model consistent with the documented data patterns for producer R&D, physical investment and turnover and the market structure.

3 A Dynamic Model of R&D Investment

3.1 Sequence of Actions

In this section, we extend the model of dynamic competition by Weintraub et al. (2008) to incorporate knowledge spillover and physical capital accumulation. Time is discrete and indexed by t. For each model period t, firm's state $\omega \in \Omega$ can be described by a pair of values representing its knowledge capital $x \in \mathbb{X}$ and physical capital $k \in \mathbb{K}$, where both $\mathbb{X}$ and $\mathbb{K}$ take discrete values. Accordingly, $\Omega \in \mathbb{X} \times \mathbb{K}$ takes all the possible combinations of knowledge and physical capital values. The industry state in each period t is denoted by s_t. We focus on a symmetric equilibrium such that each s_t's element $s_t(\omega)$ is the total mass of firms in state ω. The set of possible industry states is denoted by $\mathbb{S}$.

At the beginning of period t, all incumbent firms engage in competition in the product market and simultaneously set their prices. A firm with individual state (x_t, k_t) earns profit $\pi(x_t, k_t; s_t)$. Incumbent firms and potential entrants then make their exit and entry decisions. Each incumbent firm observes an idiosyncratic scrap value ϕ_t.[13] The incumbent firm decides whether to exit based on industry state s_t and its own state (x_t, k_t). If it decides to exit, it obtains the period profit plus the scrap value. The exit strategy is defined as an indicator function $\chi(x_t, k_t; s_t, \phi_t)$. If it decides to remain in the industry, it can choose to invest in knowledge capital, physical capital, or both.

[12] We are not able to report the exit rate for 1991 since that is the first year of our sample period.

[13] The scrap value ϕ_t is *i.i.d.* across different firms and time and has a well-defined density function with support $\mathbb{R}_+$.

There is a pool of ex ante identical potential entrants. An entrant needs to pay a fixed entry cost κ to enter the industry. Entrants draw their initial states from a distribution Φ^e. Potential entrants keep entering until the expected payoff from entry is zero. The resulting number of firms entering at industry state s_t is a Poisson random variable, with mean $M(s_t)$.[14]

3.2 Static Payoff

We first describe how firms interact in the product market each period. We assume that each firm within an industry has a standard Cobb-Douglas production function:

$$q_t = exp(x_t)(l_t)^{1-\alpha_k}(k_t)^{\alpha_k},$$

where q_t is the output of the individual firm. x_t captures the firm's production efficiency, k_t is physical capital input and l_t is labor input.

Each firm produces a differentiated product and faces a residual demand function:

$$q_t = Q_t(p_t/P_t)^\eta = \frac{I}{P_t}\left(\frac{p_t}{P_t}\right)^\eta,$$

where p_t is the price set by the firm while Q_t and P_t are the industry-level output and price index. I is defined as the industry market size. This demand function is consistent with the standard monopolistic competition model of Dixit and Stiglitz (1977). The parameter η governs the elasticity of substitution between different products.

Each period, a firm takes quasifixed factors (x_t, k_t), exogenous variable factor prices w, and aggregate market price P_t as given and chooses variable inputs l_t to maximize its profit:

$$\pi_t = p_t(q_t; I, P_t)q_t - wl_t.$$

We could rewrite this problem as

$$max_{l_t} P_t^{1+\frac{1}{\eta}} I^{-\frac{1}{\eta}} (exp(x_t)k_t^{\alpha_k})^{1+\frac{1}{\eta}} (l_t^{1-\alpha_k})^{1+\frac{1}{\eta}} - wl_t.$$

The optimal variable input decision is derived as

$$l_t^* = \left[\frac{wI^{\frac{1}{\eta}}}{(exp(x_t)k_t^{\alpha_k})^{1+\frac{1}{\eta}} P_t^{1+\frac{1}{\eta}}(1+\frac{1}{\eta})(1-\alpha_k)}\right]^{\frac{1}{(1+\frac{1}{\eta})(1-\alpha_k)-1}}.$$

[14] Weintraub et al. (2008) provides a game-theoretic microfoundation for this setting.

In equilibrium, the industry price index P_t is determined by the industry state s_t. Summarizing each firm's own state using a single index $\varphi_t = exp(x_t)k_t^{\alpha_k}$ and letting $s_t(\varphi)$ be the number of firms whose $\varphi_t = \varphi$, then

$$P_t = I^{\alpha_k} \left(\frac{w}{(1+\frac{1}{\eta})(1-\alpha_k)} \right)^{1-\alpha_k} \left(\sum_{\varphi} s_t(\varphi)\varphi^{\sigma} \right)^{-\frac{1}{\sigma}},$$

where $\sigma = \frac{1+\frac{1}{\eta}}{(1-(1+\frac{1}{\eta})(1-\alpha_k))}$. Finally, the equilibrium maximized profit for a firm with individual state φ_t is

$$\pi(\varphi_t, s_t) = I \left(1 - (1+\frac{1}{\eta})(1-\alpha_k) \right) \frac{\varphi_t^{\sigma}}{\sum_{\varphi} s_t(\varphi)\varphi^{\sigma}} \equiv (1+\frac{1}{\eta})\frac{I}{\sigma}\frac{\varphi_t^{\sigma}}{\sum_{\varphi} s_t(\varphi)\varphi^{\sigma}}. \tag{1}$$

In summary, the profit of each firm depends only on its knowledge capital x_t and physical capital k_t and the industry structure $s_t(\varphi)$.

3.3 Knowledge Production and Physical Capital Investment

Each period, a firm chooses to invest in knowledge capital, physical capital or both to improve its own state. There are major differences between the two types of investment. Consider a firm with state (x_t, k_t). The investment in physical capital has a deterministic outcome. It also involves an adjustment cost $c(k_t, k_{t+1})$. Thus, a firm could directly choose the level of physical capital k_{t+1} for the next period. In contrast, improvements to knowledge capital are uncertain and depend on the firm's own R&D and knowledge spillover.

More specifically, the input of knowledge production consists of two components. One part is the firm's own innovation effort $d_t = d(x_t, k_t; s_t)$. The cost of innovation is $c_d \cdot (d_t \cdot k_t^{d_k})$. Parameter c_d governs the overall cost level, and parameter d_k allows the per-unit innovation cost to vary by firm size. If d_k is larger than 0, the per-unit cost is higher for larger firms. The other part is the knowledge spillover from other producers competing in the same industry. Thus, the transition of knowledge capital from x_t to x_{t+1} is determined jointly by d_t, each firm's relative technological position x_t in the productivity distribution, and is subject to uncertainty. We focus on within-industry knowledge spillover. Firms receive knowledge spillovers by randomly meeting with a rival in each period t. If the rival happens to be less productive, then the firm does not benefit from the meeting. However, if the rival is more productive than the firm, the latter obtains a constant unit of knowledge spillover θ.[15] As a result, the average spillover for a firm with individual state (x_t, k_t) is defined as

$$\theta \sum_{x>x_t} \sum_{k} \frac{s_t(x,k)}{N_t} \equiv \theta P(x > x_t).$$

Recall that s_t is the industry state at time t and that N_t is defined as the total mass of incumbents at time t. The composite term entering the knowledge production takes the following form:

$$D_t = d(x_t, k_t; s_t) + \theta \sum_{x>x_t} \sum_{k} \frac{s_t(x,k)}{N_t}.$$

In Jovanovic and MacDonald (1994), learning also depends on the firm's state, actions, and the state of the industry, including the distribution of know-how in use.[16] However, unlike in Jovanovic and MacDonald (1994), in our model, the decision to invest in R&D does not preclude the opportunity to benefit from the knowledge externality.

As in Weintraub et al. (2008), there is an idiosyncratic exogenous depreciation shock that each firm suffers with probability δ. In reality, such a shock could capture the firm-level organizational forgetting documented by Benkard (2004), which causes the production process to be less efficient. With all the pieces that we have described so far, we can now introduce the knowledge production function. For $x_t = x^j \in \mathbb{X}$,

$$x_{t+1} = \begin{cases} x^{j+1}, & \text{with probability } \frac{(1-\delta)D_t}{1+D_t}; \\ x^{j-1}, & \text{with probability } \frac{\delta}{1+D_t}; \\ x^{j}, & \text{with probability } \frac{(1-\delta)+\delta D_t}{1+D_t} \end{cases} \tag{2}$$

The firm's physical investment decision is deterministic and follows conventional specifications in the literature. There are two types of investment frictions. First, the firm needs to pay an extra convex cost $c(k_t, k_{t+1})$ for adjusting its physical capital level from k_t to k_{t+1}. By normalizing the purchase price of capital $c_k = 1$, we specify the adjustment

[15] This specification of backward advantage in learning by random meeting is motivated by the approach of Benhabib et al. (2017). Using UK plant-level data, Griffith et al. (2005) shows that technology transfer plays an important role in productivity improvements in nonfrontier establishments.

[16] Jovanovic and MacDonald (1994) show that in a competitive industry, imitation by the firms that lag behind the frontier force some technology convergence among establishments as the industry matures.

cost of each establishment as:

$$c(k_t, k_{t+1}) = c_a(i_t/k_t)^2 k_t,$$

where $i_t = k_{t+1} - (1-\delta_c)k_t$ is the investment (disinvestment) and c_a is the parameter for the cost of adjustment. Second, the unit resale price of capital is $(1-\wp)$ of the purchase price, where parameter $\wp$ measures the degree of irreversibility of the installed capital.[17] This partial irreversibility is consistent with the data pattern of infrequent capital adjustment and, in particular, disinvestment.

3.4 Incumbent's and Entrant's Problem

Given the knowledge production function described in the last section, for a producer with (x_t, k_t), the value of continuation $V_c(x_t, k_t; s_t)$ is given by

$$\begin{aligned} V_c(x_t, k_t; s_t) = max_{d_t, k_{t+1}}\{ & -c_d d_t k_t^{d_k} - c_k(1 - \mathbb{1}_{\{i_t<0\}} \cdot \wp) \cdot i_t \\ & -c(k_t, k_{t+1}) + \beta E_{s_{t+1}}[V(x_{t+1}, k_{t+1}; s_{t+1})|x_t, d_t, s_t]\}, \end{aligned} \tag{3}$$

where $d_t(x_t, k_t; s_t)$ and $k_t(x_t, k_t; s_t)$ are associated policy functions. The perceived transition kernel of s_t is a key determinant of the value of continuation.

Let $V(x_t, k_t; s_t)$ be the establishment's value at the beginning of the current period. Firms can exit for both exogenous and endogenous reasons. We assume that a firm exits with probability ξ for reasons not directly related to firm profitability. This is useful to account for the exit of highly productive and large producers in our data. Whether the reason for exit is endogenous or exogenous, the firm obtains a random scrap value ϕ_t upon exit:

$$V(x_t, k_t; s_t) = \pi^*(x_t, k_t; s_t) + E_{\phi_t}\left[(1-\xi)max\{V_c(x_t, k_t; s_t), \phi_t\} + \xi\phi_t\right], \tag{4}$$

where the scrap value ϕ_t has an exponential distribution with parameter λ. The incumbent's decision rule $\chi(x_t, k_t; \phi_t, s_t) = 1$ if it decides to exit and $\chi(x_t, k_t; \phi_t, s_t) = 0$ otherwise.

Potential entrants are ex ante identical. Upon entry, they draw their initial endowment of x and k from a time-invariant distribution Φ^e. Each potential entrant incurs

[17] The presence of irreversibility is emphasized by Abel and Eberly (1996) and follow-up works.

entry cost κ. Potential entrants' decision $\epsilon = 1$ if

$$V_e(s_t) \equiv \beta E_{s_{t+1}}[\int V(x_e, k_e, s_{t+1}) d\Phi^e | s_t] \geq \kappa. \tag{5}$$

In a setting with a finite and large number of firms, Weintraub et al. (2008) show that the mass of entrants for time t is a Poisson random variable with mean $M(s_t)$.[18]

3.5 Oblivious Equilibrium

Following Ericson and Pakes (1995), the standard symmetric Markov perfect strategies can be defined by actions $a \in \mathbb{A}$ and entry decision $\epsilon \in \{0, 1\}$. In our application, $a = \{d, k, \chi\}$, where $d : \Omega \times \mathbb{S} \to \mathbb{R}_+$ is each firm's R&D investment strategy, $k : \Omega \times \mathbb{S} \to \mathbb{K}$ is its physical investment strategy, and $\chi : \Omega \times \mathbb{S} \to \{0, 1\}$ is its exit strategy. Similarly, we define the entry strategy for potential entrants as $\epsilon : \mathbb{S} \to \{0, 1\}$. Then, Markov perfect equilibrium strategies a and ϵ satisfy the following:

1. Each incumbent and potential entrant chooses optimal strategies given the *perceived transition kernel* of industry state s.

2. These perceptions are consistent with the behaviors of each agent's competitors.

Computing the Markovian transition kernel of industry state s is burdensome since it depends on the optimal incumbent strategy a and entrant strategy ϵ for each industry player. It becomes quickly infeasible for any manufacturing industry with more than a dozen firms. As a result, we rely on the concept of oblivious equilibrium, developed in Weintraub et al. (2008). Their paper establishes that when the number of firms is large, oblivious strategies, which ignore current information about competitors' states and are conditioned only on knowledge of the long-run average industry state, can closely approximate a Markov perfect equilibrium.

Let $\tilde{\mathbb{A}} \in \mathbb{A}$ be the set of oblivious strategies. Then, for oblivious strategies $a = (d, k, \chi) \in \tilde{\mathbb{A}}$ and $\epsilon \in \{0, 1\}$, the associated expected state of the industry in the long

[18] The Poisson random variable is justified by the following entry model: there are N potential entrants, and $v_N(i)$ is the expected present value for each entering firm if i firms enter simultaneously. Each potential entrant employs the same strategy, and the condition for a mixed-strategy Nash equilibrium is $\sum_{i=0}^{N-1} C^i_{N-1} p^i_N (1 - p_N)^{N-1-i} v_N(i + 1) = \kappa$. The equation has a unique solution $p^*_N \in (0, 1)$; the number of firms entering is a binomial random variable Y_N with parameters (N, p^*_N). As $N \to \infty$, Y_N converges to a Poisson random variable with mean M.

run is $S_{a,\epsilon}$. Define $\tilde{V}(x, k|a', S_{a,\epsilon})$ as the expected payoff of an incumbent under the assumption that the industry state will be equal to $S_{a,\epsilon}$ in all future periods. Then, given self-generated $S_{a,\epsilon}$, oblivious equilibrium strategies a and ϵ satisfy the following:

1. For an incumbent $\tilde{V}(x, k|a, S_{a,\epsilon}) \geq \tilde{V}(x, k|a', S_{a,\epsilon})$, $\forall a' \in \mathbb{A}$.

2. Entrants satisfy the zero-profit condition such that $\beta \int \tilde{V}(x, k|S_{a,\epsilon}) d\Phi^e \leq \kappa$ with equality if the mass of entrants $M > 0$.

Weintraub et al. (2008) prove that when the equilibrium incumbent strategies and entry rate function are oblivious, the industry state s_t is an irreducible, aperiodic and positive recurrent Markov chain. Their key insight is that when there is a large number of firms and the market tends not to be concentrated, individual firms do not benefit by unilaterally deviating to an optimal (nonoblivious) strategy by keeping track of the true industry state *averaged over the invariant distribution of industry states*. In other words, in any industry state that has a significant probability of occurrence, the oblivious strategy approximates the Markov perfect strategy.

Our procedure for calculating the equilibrium follows the above definition closely. Given a set of parameters, the steps to compute the equilibrium are as follows:

1. Initial guess of the mean number of entrants M.

2. Initial guess of the average long-run industry structure s_0.

3. Solve the incumbent's maximization problem and recover its optimal investment policy and exit policy: $a = (d, k, \chi)$ given s_0.

4. Construct the transition matrix $T_{x,k,\chi}$ with the optimal policies. The long-run average industry structure is calculated as $s_1 = M(I - T_{x,k,\chi})^{-1}\Phi^e$.

5. If $|s_0 - s_1|$ is not close enough, go back to step (2).

6. Check the free-entry condition of potential entrants. If it does not hold, go back to step (1).

4 Model Estimation

4.1 Revenue Function

The first stage of estimation focuses on the static part of the model. To be consistent with the theoretical model, we assume that labor is a short-run variable input while capital is quasi fixed. The empirical log *revenue* function for each establishment i at time t is then

$$\ln r_{it} = \alpha_{0t} + \tilde{\alpha}_k \ln k_{it} + \tilde{x}_{it} + u_{it}. \tag{6}$$

α_{0t} absorbs all common parameters and industry aggregates (I and P_t). $\tilde{\alpha}_k = \sigma\alpha_k$ is the capital coefficient in the static revenue equation with labor optimized out, and the scaled productivity $\tilde{x}_{it} = -(1+\eta)(1-\tilde{\alpha}_k)x_{it}$. u_{it} is an i.i.d. measurement error shock.

We then follow Ackerberg et al. (2015) in estimating parameter $\tilde{\alpha}_k$ in equation 6 and the productivity process. Assume that a firm's material input at time t enters final production together with value added in a Leontief fashion. This assumption then justifies the standard proxy function function $f_t(\cdot)$ where

$$m_{it} = f_t(\tilde{x}_{it}, k_{it}, l_{it}).$$

We then obtain

$$\ln r_{it} = \alpha_{0t} + \tilde{\alpha}_k \ln k_{it} + f_t^{-1}(m_{it}, k_{it}, l_{it}) + u_{it}.$$

Our model has a step-by-step innovation setup where the outcome can be influenced both by the firm's own R&D and by knowledge spillovers. Note that absent spillovers, each firm's productivity change is a random walk that depends only on the firm's own R&D (adjusted by its size). In our model, the existence of a backward advantage in obtaining spillovers rationalizes the mean reversion of the first-order Markov productivity process that is often assumed to be exogenous in the empirical literature. In this sense, despite the fact that the empirical productivity process does not map directly to our structural parameters, we use it to define the key moments to target for identification of the underlying model knowledge production parameters. Specifically, we approximate the productivity evolution using a first-order Markov process, where

$$\begin{aligned}\tilde{x}_{it} &= g(\tilde{x}_{it-1}, d_{it-1}, k_{it-1}) + \xi_{it} \\ &= m_0 + m_1\tilde{x}_{it-1} + m_2 \log(\frac{d_{it-1}}{k_{it-1}}) + \xi_{it}.\end{aligned} \tag{7}$$

Then, we rely on the following moment conditions to estimate the revenue production function parameters and the empirical productivity process.

$$E\left[\xi_{it}\begin{pmatrix} l_{it-1} \\ m_{it-1} \\ k_{it} \end{pmatrix}\right] = 0 \tag{8}$$

The estimation results are reported in Table 4. We find that the coefficient of capital in the revenue equation is 0.46. It is well known that one cannot identify the elasticity parameter η and the production technology parameter α_k separately with only revenue data. As a result, we calibrate the value of $\eta = -5$, following conventional findings in the trade literature.[19] Combined, these imply a $\sigma = -(1+\eta)(1-\tilde{\alpha}_k) = 2.14$ and a capital share coefficient $\alpha_k = 0.22$.

More importantly, we also find that the measured productivity process features mean reversion with $m_1 = 0.89$. The coefficient of the private R&D-to-capital ratio is also positive and significant with $m_2 = 0.007$. These estimates are generally in line with recent studies on R&D and productivity such as Aw et al. (2011) and Doraszelski and Jaumandreu (2013). However, unlike these earlier studies, we treat this measured productivity process as an auxiliary regression to target in our estimation of the dynamic parameters.

4.2 Parameterization and Estimation of Dynamic Parameters

We start by setting a few fixed parameters based on our empirical setting and data. First, we set the annual discounting rate β at 0.925 to reflect the relatively high interest rate during our sample period in Korea. Second, both the annual rate of depreciation $\delta_c = 0.15$ and the exogenous exit rate $\xi = 0.02$ are directly calculated from our data. Finally, the investment cost c_k is normalized to 1. The aggregate market size I is calculated as the annual average of industry total value added over the sample period $I = 562,025$ million won. We use the average number of entrants over the years from 1991 to 1996 to approximate an empirical estimate of the mass of entrants M in our model. The average number of entrants is $M = 78$.

Given the first-stage estimates of the production function coefficients $\tilde{\alpha}_0$ and $\tilde{\alpha}_k$ and the preset parameters $\beta, \delta_c, \xi, and \eta$, we estimate the set of dynamic parameters $\Theta_0 = [d_k, c_d, c_a, \lambda, \delta, \theta, \wp]$ in the second stage. Recall that d_k is the cost parameter of

[19]The implied markup in a monopolistic competitive setting is 1.25.

R&D with respect to capital size, which governs the effect of capital size on the R&D cost. c_d is the effectiveness of R&D inputs in improving plant knowledge capital. c_a is the physical capital adjustment cost, and $\wp$ captures the asymmetry in disinvestment of physical capital. λ is the parameter of the plant scrap value distribution. δ represents idiosyncratic uncertainty over the depreciation in plant-level knowledge capital. Most importantly, θ controls the size of knowledge spillover from frontier firms. Since the estimation involves solving a dynamic industry equilibrium with no closed-form solutions, we use the method of simulated moments (MSM), which minimizes a distance criterion between key moments from the actual and simulated data.

Recent empirical techniques have been proposed to estimate the dynamic industry equilibrium model without solving the equilibrium. Especially related to this study is the estimation procedure proposed by Bajari et al. (2007), which handles both continuous and discrete control variables. Their approach breaks the estimation into two stages. In the first stage, firm policy functions are recovered by regressing observed actions on the observed state variables. The probability distribution defining the evolution of the industry state is also recovered at this stage. In the second stage, the structural parameters that make these observed policies optimal are estimated. The major breakthrough of their approach is to avoid the computational burden of calculating the Markov perfect equilibrium, albeit with a trade-off with respect to the precise calculation of the agent's value function and policy function.

We have the plant-level data for only *one industry* over a six-year period, while the possible state space is very large. This would make the sampling error of estimating the policy functions a major concern if we were to adopt the strategy of Bajari et al. (2007). On the other hand, the large number of firms and low industry concentration make the weaker notion of equilibrium—oblivious equilibrium—especially attractive since it has been proved to be a good approximation of MPE in this case.

In the literature, there is often a concern over using the simulated method of moments at this stage in comparison with the approach of Bajari et al. (2007): the dynamic competition model that we use has not been proved to have a unique equilibrium in general. However, since we are focusing on only a single market, this problem is alleviated, at least in our estimation, by the matching of a full set of moments with policies and state transitions from observed and simulated data, which allow the data to confirm the correct equilibrium.

4.2.1 Moments

In this section, we describe the set of data moments utilized and their relevance for the identification of key parameters. The sample that we use to estimate the dynamic parameters is an unbalanced panel of incumbent plants in the electric motor industry in 1991 and their subsequent annual observations through 1996, including their exits. On the other hand, all entrants in subsequent years are used only to construct a frequency estimate of the initial state distribution Ψ^e and are excluded from the construction of moments.

Table 5 reports the moments that we use in our estimation. The first set of moments captures the key features of optimal plant R&D investment behavior in equilibrium. The R&D investment cost c_d affects a plant's R&D investment intensity. It is also the driving force behind the long-run productivity advantage of R&D performers. On the other hand, the idiosyncratic shock δ is shaped by the proportion of positive R&D investors. Given its own technological position and expectation on industry productivity evolution, a plant makes an optimal decision on whether to stick with the "corner solution" of investing nothing in R&D. Note here that all moments of the change in firm productivity over time are constructed conditional on survival. Thus, they are also affected by industry competition and turnover.

The second set of moments described in Table 5 relates to the plant's physical investment behavior. Following Cooper and Haltiwanger (2006), the cost parameter c_a helps capture the nonlinear relationship between plant-level investment and profitability. The level of the investment ratio and fraction of the plant's positive investment depend on the magnitude of this parameter. The fraction of disinvestment identifies parameter $\wp$, which controls the asymmetry of disinvestment, and the covariance between adjusted investment and the productivity level helps identify d_k, which determines the effect of capital on R&D expenditure. Since we model the firm's investment behavior within an industry equilibrium, the investment moments also indirectly influence other key model parameters such as technological spillover and R&D costs.

Third, the long-run exit pattern helps identify the scrap value distribution parameter.[20] Finally, we use the autocorrelation between productivity to identify parameter

[20] Obviously, the other moments are also affected by the exit pattern of incumbent firms in an industry equilibrium setting.

θ, which controls the R&D spillover effect: we simulate a sequence of $\{x_{it}\}$ from our structural model and calculate the autocorrelation based on the simulated sequence of productivity. More specifically, we run the second-stage estimation of the production function on the simulated sequence of productivity, where

$$\begin{aligned}\hat{x}_{it} &= g(\hat{x}_{it-1}, \hat{d}_{it-1}) + \xi_{it} \\ &= \tilde{m}_0 + \tilde{m}_1 x_{it-1} + \tilde{m}_2 \log(\frac{\hat{d}_{it-1}}{\hat{k}_{it-1}}) + \xi_{it}.\end{aligned} \tag{9}$$

Then, we match the estimated $\tilde{m}$ coefficients with the m coefficients estimated from the data by changing the spillover parameter θ.

4.2.2 Empirical Implementation and Computation Details

The estimation of the dynamic parameters Θ_0 is implemented according to the following procedures. First, denote the set of data moments in Table 5 as Γ^d, which is a 10-by-1 vector. Second, for a given set of parameters Θ, the industry equilibrium is solved, and optimal policy functions for R&D expenditure, physical investment and survival (d^*, k^*, χ^*) are generated. Third, we use the optimal policy functions to simulate the path for each plant in the oblivious equilibrium. We define the simulated moments as Γ^S. The MSM estimate $\hat{\Theta}$ minimizes the weighted distance between the data moments and the simulated moments:

$$L(\Theta) = min_{\Theta}[\Gamma^d - \Gamma^S(\Theta)]' W [\Gamma^d - \Gamma^S(\Theta)],$$

where W is a positive definite matrix. In our numerical analysis, $\hat{W}$ is calculated through a bootstrap procedure: we randomly resample the data and calculate the moments of interest for each sample; then, we obtain a variance–covariance matrix based on these bootstrap samples.

The range of $\mathbb{X}$ is determined by the standard deviation of productivity in the data. The space of capital $\mathbb{K} \equiv [k_{\min}, k_{\max}]$ is chosen to match $x_{\min}$ and $x_{\max}$ based on the firm's static profit maximization strategy, where $k_{\min} = \frac{\log(\frac{\sigma\alpha_K}{c_k+\delta_k})+\sigma x_{\min}}{1-\sigma\alpha_K}$ and $k_{\max} \approx \frac{\log(\frac{\sigma\alpha_K}{(1-\beta)(c_k+\delta_k)})+\sigma x_{\max}}{1-\sigma\alpha_K}$. The dynamic optimization problem is solved with a collocation method as in Midrigan and Xu (2014). In addition, we follow Andrews et al. (2017) and use finite differencing to calculate the standard errors of the estimated parameters.

4.2.3 Estimation Results

Table 6 reports the point estimates and their $5\% - 95\%$ confidence intervals. By solving the industry equilibrium with the reported point estimates Θ, we can also infer the fixed entry cost κ on the basis of the model's free entry condition, equation 5. To further evaluate the overall fit of the estimation, we also report the simulated moments at the point estimates in Table 7. The simulated data do a good job of replicating the pattern of R&D investment and productivity evolution, which are the core pieces of this model.

With the estimated set of parameters, the policy function derived from our simulation is shown in Figure 3. Panel (a) of Figure 3 shows that firms with higher productivity are more willing to invest in R&D but that a higher capital level lowers the R&D incentive because it increases the unit R&D investment cost. The kink in panel (a) is driven by the limit of maximum productivity that a firm can achieve in the model. Panel (b) shows the summation of individual R&D activity and knowledge spillover. From panel (b), we can see that backward firms enjoy significant spillover, which helps them move upward on the spectrum of the productivity distribution. Panel (c) shows the probability of exit of firms in the productivity–capital space: firms with both low productivity and low capital have the highest probability of exit, at approximately 0.17. The probability of exit decreases when either productivity or capital increases. After reaching a certain threshold in the productivity–capital space, firms suffer from possibility of exogenous exit only, the probability of which is 0.02 in our model. Panel (d) shows the investment-to-capital ratio. As expected, firms with high productivity and low capital have the highest incentive to invest.

Figure 4 plots the distribution of the industry structure. Panel (a) shows the distribution of entrants directly obtained from our data, which has the highest density in the middle but is smoothly distributed over the entire productivity–capital space. Panel (b) shows the distribution of the industry structure in equilibrium, from which we can observe that the industry is much more concentrated in states with medium to high levels of capital and medium levels of productivity. Panels (c) and (d) zoom in on the marginal distributions of capital and productivity in equilibrium, respectively. Overall, the visualization of the policy functions and the long-run market structure illustrate that our model is consistent with many of the salient facts on firm and industry dynamics.

The knowledge spillover effect varies across different firms. The lower a firm's own productivity is (i.e., the further it lags behind the technology frontier), the larger the knowledge spillover effect that it gains from the industry. The value of the knowledge spillover parameter θ of 4.5 can be interpreted as follows: among active R&D performers, on average, the knowledge spillover effect is 1.31 times the importance of an individual firm's R&D effort. The spillover effect is smaller for firms with higher productivity. For a firm with productivity at the 95th percentile in this industry, the spillover effect is only 0.29 times the importance of its own R&D spending. Among non-R&D performers, the spillover effect increases their probability of technology advancement by 10% on average. For a firm with the lowest productivity in this industry, the knowledge spillover effect increases its probability of technology advancement by 12%.

In terms of the quadratic adjustment cost, Cooper and Haltiwanger (2006) report a value of 0.225 in the absence of controls for fixed costs and 0.025 with controls for fixed costs. Bloom (2006) reports a quadratic adjustment coefficient of 4.743 on a monthly basis, which implies a yearly value of 0.39. Our estimate of c_a, which equals 0.04, indicates that it is less costly to acquire or sell physical capital in the Korean electric motor industry than in the settings examined in these prior works. However, our estimate of the disinvestment parameter $\wp$ is 0.50, which implies that disinvestment comes with a price discount of capital of approximately 50%.

The estimated scrap value implies an unconditional mean of $1,667$ million won, which is approximately five times the industry average of static profit. On the other hand, the entry cost implied from the free-entry condition is $4,200$ million won. These values results in quite a narrow hysteresis band, driven by the high turnover rate observed in the data.

5 Policy Simulation: Optimal R&D Subsidy Plans

We now use our estimated model to further understand the role of knowledge spillover and conduct policy analysis.

First, we examine by how much knowledge spillover affects aggregate R&D efforts and hence productivity dispersion among firms. With a higher spillover effect θ, firms have a lower incentive to invest in R&D because it is harder to pull away from other firms by performing R&D. Both the reduction in R&D effort among leading firms and the easier catch-up among laggards imply a smaller productivity dispersion. To disentangle the effect of θ on productivity dispersion and R&D incentives, we compare the benchmark case with two cases: in the first case, θ is increased by 50%, and the R&D policy of firms is allowed to change endogenously. Second, θ is increased by 50%, but R&D policy is exogenously *fixed* to be the same as in the benchmark case. Table 8 summarizes the aggregate R&D efforts and productivity dispersion. Comparing columns (1) and (2) of case one with our benchmark results in Table 8, we can see that a 50% increase in spillover parameter θ results in an 24% decrease in aggregate R&D efforts and a 0.6% increase in aggregate productivity. This is because firms have a lower incentive to invest in R&D when spillover to other firms is larger. On the other hand, when the spillover effect is larger, the variance in productivity is also smaller in this industry, as we can see by comparing column (3) of case 1 with the benchmark case.

To separate out the effect of increased spillover and lowered R&D efforts, in case two, we focus solely on the increased spillover effect by fixing firms' R&D policy to what it was in the benchmark case. The result shows that if firms' R&D policy remains unchanged when the spillover effect is larger, aggregate productivity increases by 2.4%, while aggregate R&D efforts stay basically the same in the new equilibrium, as we can see by comparing columns (1) and (2) of case two with the benchmark case. This additional aggregate productivity gain over that in case one highlights the importance of taking into account how R&D firms respond to free-riding on their spillovers in equilibrium.

Right after our sample period, Korea launched its S&T policy, geared toward the acquisition of core competences in strategic technology areas and development of an innovation system to enable the nation to successfully transition toward a knowledge–based economy. To achieve this policy goal, the Special Law for S&T Innovation was enacted in 1997. In accordance with the law, the Five–year Plan for S&T Innovation was launched the same year. The program contains specific action plans to achieve the policy goal:

1. A corporate tax deduction of 50% of the increase in R&D and human resource development (HRD) investments over the annual average investments of the past four years or 5% of the current expenditures for the same purposes (15% for SMEs).

2. A corporate tax deduction of 5% of the total investment in equipment and facilities for R&D and/or HRD and a direct R&D subsidy for SMEs of up to 100 million won or 75% of the total investment.

Motivated by the actual R&D subsidy plans in Korea, our main counterfactual results are targeted at finding the optimal subsidy plan with a revenue-equivalent tax. Because of the existence of the spillover effect, there is an externality from firm's own R&D on other firms' productivity in this industry, which is not endogenized in firm's own R&D decisions. Therefore, from the social planner's point of view, firm's own R&D effort is less than socially optimal. To increase this effort, we propose a subsidy plan whereby for every dollar of firm R&D spending, a certain fraction is subsidized by the government. The total subsidy expenditure is financed through a tax on firm's profits. In other words, the following balance condition of subsidy expenditures and tax revenues need to be satisfied:

$$\sum_{x,k} \tau \cdot \pi(x,k) \cdot s(x,k) = \sum_{x,k} \mathfrak{s} \cdot c_d dk^{d_k} \cdot s(x,k),$$

where τ is the uniform tax rate on firm's static profit $\pi(x,k)$ and $\mathfrak{s}$ is the rate of R&D subsidy. Our main counterfactual aims at finding the optimal subsidy plan $\mathfrak{s}$ that maximizes industry output.

Table 9 shows how the total industry quantity changes with respect to different subsidy plans. We see an inverted U-shape of the total industry quantity with respect to an increase in total subsidy expenditures. This arises because on the one hand, in the new equilibrium with the revenue-equivalent subsidy and tax plan, firms with a lower elasticity-weighted marginal cost are taxed to subsidize firms with a higher marginal cost. Therefore, the revenue-equivalent tax policy creates a misallocation of resources if we shut off knowledge spillover. On the other hand, when there is knowledge spillover, the R&D subsidy increases firms' incentive to conduct R&D and hence endogenize the knowledge spillover effect. From Table 9, we can see that the total industry quantity is maximized at a subsidy rate of about 15%, with a tax rate of 1.27%.

6 Conclusion

This paper develops and estimates a structural model of R&D investment and productivity evolution among manufacturing plants in the Korean electric motor industry from 1991 to 1996. Plant-level decisions on R&D investment, physical capital investment, entry, and exit are developed with an equilibrium industry evolution model. Plant productivity is affected by the plant's own R&D and by spillovers from the R&D of its competitors. The model provides a detailed set of pathways connecting the R&D investment, plant productivity, plant physical investment and industry turnover patterns observed in the data.

The structural parameter estimates show that a plant's own R&D expenditure has a positive effect on its future productivity. Among active R&D performers, on average, the knowledge spillover effect is 1.31 times the firm's individual R&D effort. Among non-R&D performers, the spillover effect increases their probability of technology advancement by 10% on average. The public externality from R&D is important given the large number of firms within the same industry. A narrow difference between the entry cost and the mean scrap value explains the high turnover rate in this industry. Finally, the industry equilibrium model provides a natural link from individual plant R&D decisions to aggregate industry productivity and output. This feature of the model provides us with a powerful tool to evaluate various industry or innovation policies. As our counterfactual experiments show, the optimal linear R&D subsidy rate is approximately 15%, which is quite close to that implemented by the Korean government for SMEs. Industry output would increase by a modest 0.14% with the subsidy policy.

There are quite a few possible extensions of the current framework. An interesting one would be to look at the interaction of firms' decision to export, R&D, and the overall industry evolution. Given the fact that trade and innovation policy are considered to be among the most important institutional arrangements in emerging economies such as Korea, it is important to provide a general framework for evaluating how these policies interact and affect long-run industry performance.

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Figure 1: Relation between R&D Intensity and Firm Size among R&D Performers

Note: Figure 1 illustrates the relation between log R&D intensity and log value added among R&D performers in our data. R&D intensity is defined as the ratio of R&D expenditure over value added. The results show that producers with larger value added tend to have lower R&D intensity.

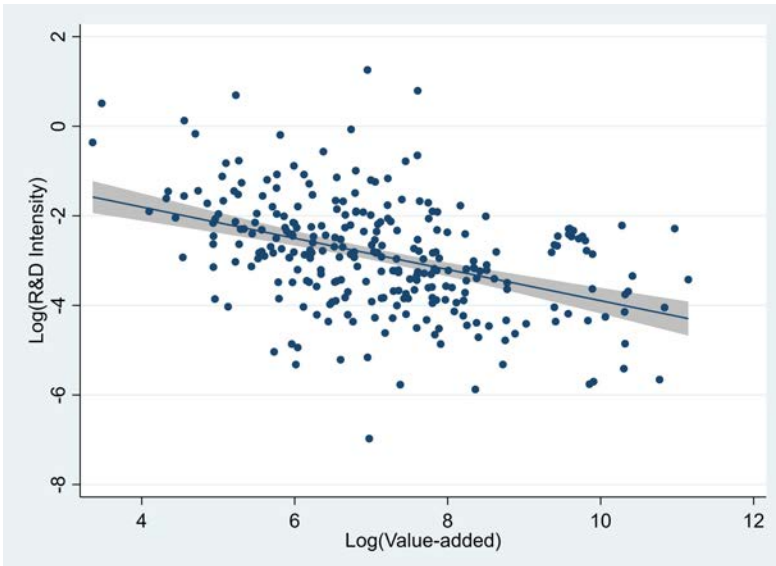


Figure 2: Investment Rate

Note: Figure 2 reports the distribution of the investment-to-capital ratio (capital investment relative to the stock of physical capital) in the Korean electric motor industry. The results show that 46% of firms do not invest in a given year. In addition, 24% of firms replace more than 20% of their capital stock in a given year (displaying a pattern often called "investment spike").

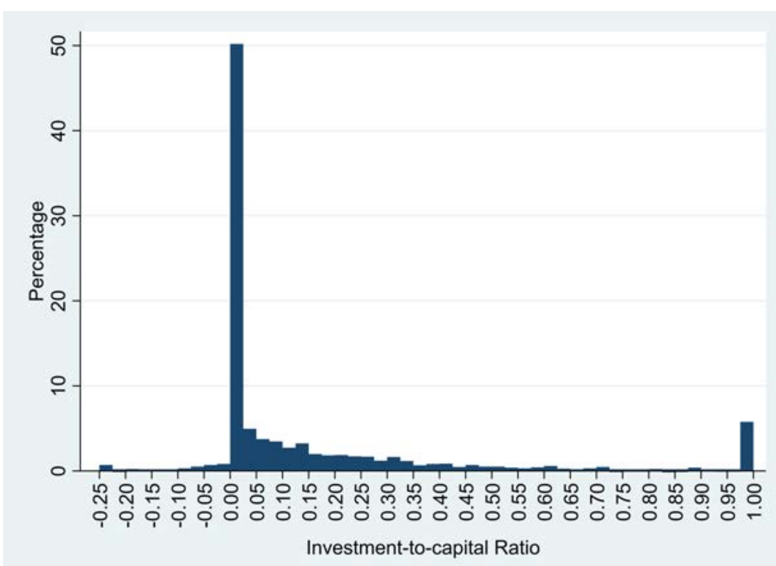
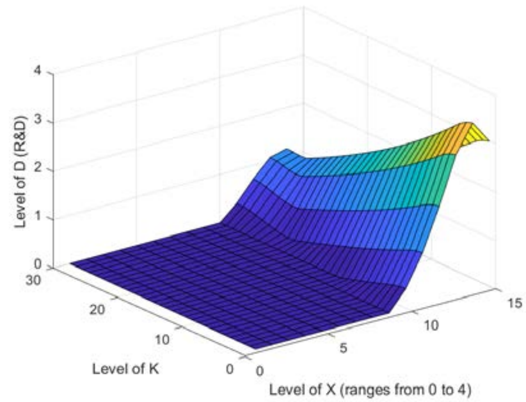
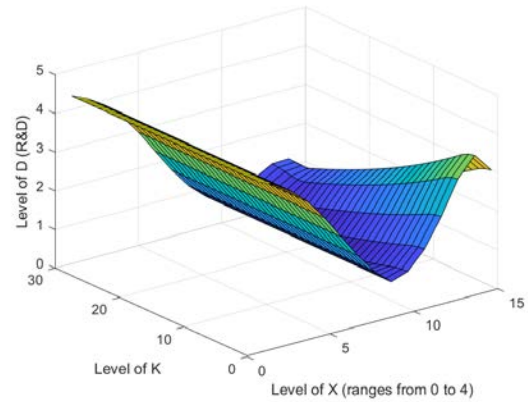


Figure 3: Plots of Policy Functions

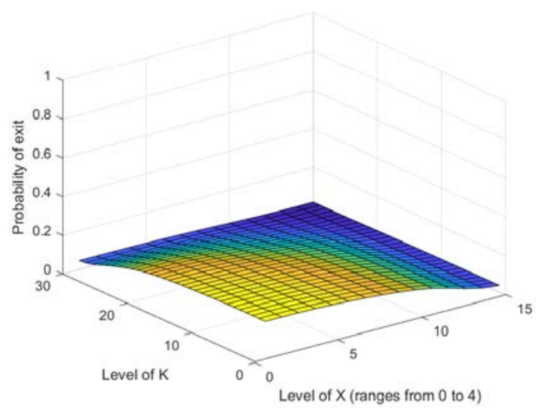
Figure 3 shows the equilibrium policy functions at the estimated parameter values. Panel (a) shows the R&D decision in the space of physical capital K and productivity X. Panel (b) shows the summation of individual R&D expenditures and knowledge spillover. Panel (c) shows firms' probability of exit. Panel (d) shows the investment-to-capital ratio.



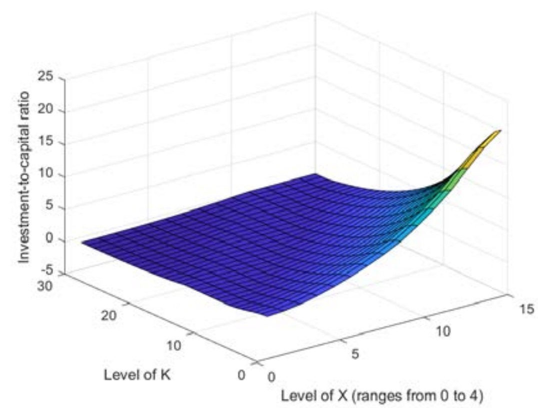
(a) R&D Expenditures



(b) R&D Expenditures Plus Knowledge Spillover



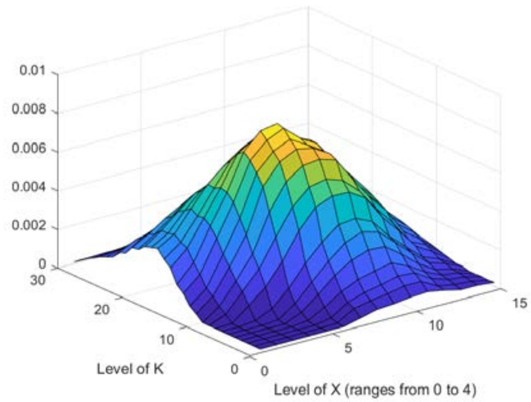
(c) Probability of Exit



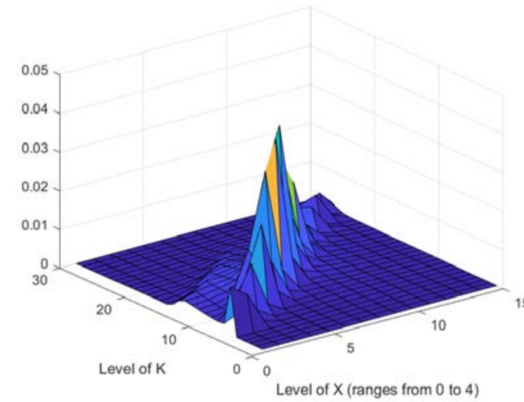
(d) Investment-to-Capital Ratio

Figure 4: Plots of Joint and Marginal Distributions

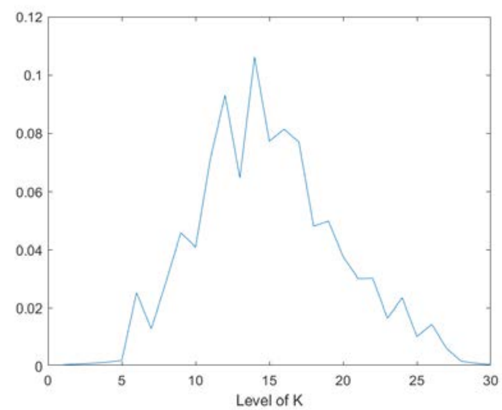
Figure 4 shows the distributions of the industry structure. Panel (a) shows the distribution of entrants, which is exogenously obtained from the data. Panel (b) shows the joint distribution of physical capital and productivity in the industry in equilibrium. Panels (c) and (d) show the marginal distribution of capital and productivity in equilibrium, respectively.



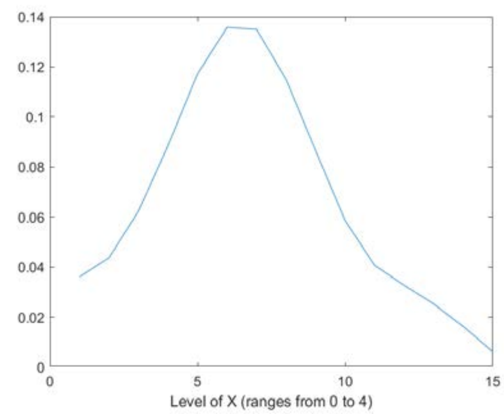
(a) Distribution of Entrants



(b) Joint Distribution



(c) Marginal Distribution of Physical Capital



(d) Marginal Distribution of Productivity

Table 1: Summary Statistics

Note: Table 1 summarizes the key data patterns of the Korean electric motor industry. The data are from the Korean Annual Manufacturing Survey, which reports detailed annual information on each manufacturing establishment's value added, physical capital, employment, physical investment, and R&D investment. On the cost side, we have information on total material expenditure and the total wage bill. All units are in million won. The average exchange rate between the won and US dollar during the sample period was 786:1.

	Mean	Std.	1%	25%	Med.	75%	99%
R&D expenditure	25.8	210.9	0.0	0.0	0.0	0.0	694.0
R&D intensity of performers	0.13	0.31	0.00	0.02	0.06	0.12	2.00
Physical capital	688.6	2977.2	4.7	36.4	90.0	296.4	15805.3
Physical investment	110.0	647.5	-39.2	0.0	0.0	25.5	2209.4
Investment-to-capital ratio	0.27	1.09	-0.18	0.00	0.00	0.19	4.57
Value added	1118.0	4257.7	42.1	133.8	262.8	634.8	19766.7
Wage bill	455.8	1675.9	24.0	67.1	121.2	268.1	7288.0

Unit of variables: million won

Table 2: Market Concentration

Note: Table 2 summarizes market concentration in the Korean electric motor industry. It shows the market share of the top 4 firms and the top 20 firms and the average HHI of this industry from 1991 to 1996. In the Korean electric motor industry, on average, the market share of the top 4 firms is 29% and of the top 20 firms 59%. The average HHI of this industry is 323, far below the Department of Justice's definition of a moderately concentrated market (one with an HHI between 1,500 and 2,500 points). The data pattern suggests that the Korean electric motor industry is neither perfectly competitive nor dominated by a very few large firms.

Year	Market Share of C4	Market Share of C20	HHI
91	27%	59%	280
92	27%	56%	266
93	31%	60%	342
94	30%	58%	355
95	32%	61%	353
96	31%	60%	341

Table 3: Turnover Rates

Note: Table 3 summarizes the entry and exit rate in the Korean electric motor industry. On average, the entry rate is 15.7%, and the exit rate is 16.0%. The exit rate for 1991 is not reported since that is the first year of our sample period. Entrants account for about 5% of the total market share each year, and exiting firms account for 11% of the total market share on average. Each entrant cohort's importance grows over time. For example, the market share of the cohort born in 1992 accounts for 5% of the market share, and this number increases to 10% in 1996.

Year	Entry Rate	Exit Rate	Market Share of Entrants	Market Share of Exits
91	19%	N/A	4%	N/A
92	17%	17%	5%	13%
93	17%	9%	4%	9%
94	16%	17%	5%	13%
95	15%	12%	6%	6%
96	11%	25%	3%	13%

Table 4: Production Function Parameters

Note: Table 4 shows the estimated values of the production function parameters. Standard errors are reported in parentheses.

	ACF
$\widetilde{\alpha}_k$	0.4655*
	(0.000)
m_1	0.8897*
	(0.000)
m_2	0.0071*
	(0.000)

*significant at the 1% level

Table 5: Key Data Moments

Note: Table 5 reports the moments that we use in our estimation. The first set of moments captures the key features of optimal plant R&D investment behavior in equilibrium. We use the fraction of R&D performers, R&D intensity of performers, and standard deviation of the relative productivity level. The second set of moments relates to the plant's physical investment behavior. We use the mean investment ratio, the fraction of positive investment, fraction of disinvestment, and covariance between capital investment (weighted by physical capital) and individual productivity. The third set of moments relates to the turnover rate in the Korean electric motor industry. We use the mean exit rate in this industry. The fourth set of moments relates to the autocorrelation between productivity to identify the parameter that affects the knowledge spillover effect. We simulate a sequence of productivity $\{x_{it}\}$ from our structural model and calculate the autocorrelation based on this sequence. Establishment productivity x_{it} evolves according to $\hat{x}_{it} = \tilde{m}_0 + \tilde{m}_1 x_{it-1} + \tilde{m}_2 \log(\frac{\hat{d}_{it-1}}{\hat{k}_{it-1}}) + \xi_{it}$, where d_{it-1} is the R&D expenditure of establishment i at time $t-1$ and k_{it-1} is physical capital. Arguments for identification can be found in section 4.2.1.

	Identification	Data
R&D Investment and Productivity Improvement		
fraction of R&D performers	pin down δ	11%
R&D intensity of performers (R&D/Value-added)	pin down c_d	0.13
std relative productivity level	pin down δ/c_d	0.31
Physical Capital Investment		
mean investment ratio $(\frac{i}{k}_{it})$	pin down c_a	.27
fraction of positive investment	pin down c_a	49.6%
fraction of disinvestment	pin down $\wp$	4%
Cov(i/k, x)	pin down d_k	0.08
Firm Turnover		
mean exit rate	pin down u_b	16%
Estimated Evolution Path of Productivity		
m_1 (coefficient of lagged productivity)	pin down θ	0.8897* (0.000)
m_2 (coefficient of $\frac{d_{it}}{k_{it}}$)	pin down d_k	0.007* (0.000)

*significant at the 1% level.

Table 6: Dynamic Parameter Estimates

Note: Table 6 reports the point estimates and their 95% confidence intervals. δ represents idiosyncratic uncertainty over the change in plant-level knowledge capital. c_d is the effectiveness of R&D inputs in improving plant knowledge capital. θ controls the size of the knowledge spillover effect. d_k is the cost parameter of R&D with respect to capital size, which captures the effect of capital size on the R&D cost. c_a is the physical capital adjustment cost. $\wp$ captures the asymmetry of disinvestment of physical capital. λ is the parameter of the plant scrap value distribution.

	Point Estimate	95% confidence interval
δ	0.85	$[0.79, 0.91]$
c_d	0.20	$[0.17, 0.23]$
θ	4.50	$[3.02, 5.98]$
d_k	0.30	$[0.24, 0.36]$
c_a	0.04	$[0.01, 0.06]$
$\wp$	0.50	$[0.45, 0.55]$
λ	0.61	$[0.46, 0.75]$

Table 7: Model Fit

Note: Table 7 reports the data moments and simulated moments at the point estimates in Table 6.

	Data	Simulation
R&D Investment and Productivity Improvement		
fraction of R&D performers	11%	13%
R&D intensity of performers (R&D/Value-added)	0.13	0.10
std relative productivity level	0.31	0.27
Physical Capital Investment		
mean investment ratio ($\frac{i}{k}_{it}$)	.27	.25
fraction of positive investment	49.6%	49.0%
fraction of disinvestment	4%	4%
Cov(i/k, x)	0.08	0.08
Firm Turnover		
mean exit rate	16%	13%
Estimated Evolution Path of Productivity		
m_1 (coefficient of lagged productivity)	0.8897*	0.8909*
	(0.000)	(0.000)
m_2 (coefficient of $\frac{d_{it}}{k_{it}}$)	0.007*	0.004*
	(0.000)	(0.000)

*significant at the 1% level.

Table 8: Effect of θ on R&D Efforts and Productivity Dispersion

Note: Table 8 shows the counterfactual results on how much knowledge spillover affects aggregate R&D efforts and hence productivity dispersion among firms. θ measures the level of knowledge spillover. We compare the benchmark case with two other cases: first, θ is increased by 50%, and the R&D policy of firms is allowed to change endogenously. Second, θ is increased by 50%, but R&D policy is exogenously fixed to be the same as in the benchmark case.

	(1) Aggregate productivity	(2) Aggregate R&D efforts	(3) Variance of productivity
Benchmark θ	0.3094	57.8003	0.0572
Case 1: $\tilde{\theta} = \theta * (1 + 50\%)$	0.3112	43.7572	0.0508
Case 2: $\tilde{\theta} = \theta * (1 + 50\%)$, but fix R&D policy at θ	0.3167	57.6590	0.0507

Table 9: Results of Policy Simulations

Note: Table 9 shows how the total industry quantity changes with respect to different subsidy plans. In our main counterfactual analysis, we propose a subsidy plan whereby for every dollar of firm R&D spending, a certain fraction is subsidized by the government. The total subsidy expenditure is financed through a tax on firm profits. In other words, the following balance condition of subsidy expenditures and tax revenues needs to be satisfied: $\sum_{x,k} \tau \cdot \pi(x,k) \cdot s(x,k) = \sum_{x,k} \mathfrak{s} \cdot c_d dk^{d_k} \cdot s(x,k)$, where τ is the uniform tax rate on firms' static profit $\pi(x,k)$ and $\mathfrak{s}$ is the rate of the R&D subsidy.

Subsidy Rate (Percentage of R&D Cost C_d)	Tax Rate	Industry Quantity	Total Expenditures of Subsidy
0.00%	0.00%	613.21	0.00
2.00%	0.13%	613.46	0.28
4.00%	0.27%	613.59	0.59
5.00%	0.33%	613.73	0.75
10.00%	0.78%	614.08	1.63
15.00%	**1.27%**	**614.10**	**2.67**
20.00%	1.85%	614.06	3.89
25.00%	2.58%	612.74	5.34
30.00%	3.45%	610.91	7.08
35.00%	4.37%	608.62	9.22
40.00%	5.59%	605.19	11.76
45.00%	6.87%	602.18	14.57

A Profit from Static Competition

The static competition of heterogeneous firms is built in a standard monopolistic competition setting. We assume that firm i within an industry has a standard Cobb-Douglas production function with constant returns to scale. We describe individual firm's problem, suppressing the notation i for convenience.

$$q_t = exp(x_t)(l_t^{1-\alpha}(k_t)^{\alpha}),$$

where q_t is the output of firm i. The firm's efficiency x_t captures each firm's knowledge capital. k_t is physical capital input, and l_t is labor input.

Each firm produces a differentiated product and faces a demand function such that

$$q_t = Q_t(p_t/P_t)^{\eta} = \frac{I}{P_t}(\frac{p_t}{P_t})^{\eta}, \tag{10}$$

where p_t is the price set by firm i while Q_t and P_t are the industry-level output and price index. Accordingly, I is defined as the industry market size. This demand function is from the widely used monopolistic competition model of Dixit and Stiglitz (1977). The parameter η captures the elasticity of substitution between different products.

Thus, in each period, a firm takes quasi-fixed factors (k_t, x_t), exogenous variable factor prices w_t, and the aggregate market price P_t and chooses variable inputs l_t to maximize its profit:

$$\pi_t = p(I, P_t, q_t)q_t - w_t l_t. \tag{11}$$

We could rewrite this problem as

$$max_{l_t} \underbrace{P_t^{1+\frac{1}{\eta}} I^{-\frac{1}{\eta}}}_{D_t} (\underbrace{exp(x_t)k_t^{\alpha}}_{\varphi_t})^{1+\frac{1}{\eta}} (l_t^{(1-\alpha)})^{1+\frac{1}{\eta}} - w_t l_t, \tag{12}$$

where the optimal labor decision is

$$l_t^* = \left[\frac{D_t(\varphi_t)^{\frac{1}{\eta}+1}(1+1/\eta)(1-\alpha)}{w_t}\right]^{\frac{1}{1-(1+1/\eta)(1-\alpha)}}. \tag{13}$$

Substitute the optimal labor decision into the individual price equation $p(I, P_t, q_t)$:

$$\begin{aligned} p(I, P_t, \varphi_t) &= D_t(\varphi_t)^{\frac{1}{\eta}} \left[\frac{D_t(\varphi_t)^{\frac{1}{\eta}+1}(1+1/\eta)(1-\alpha)}{w_t}\right]^{\frac{1}{1-(1+1/\eta)(1-\alpha)}\frac{(1-\alpha)}{\eta}} \\ &= \left[D_t^{\alpha}\left(\frac{(1+1/\eta)(1-\alpha)}{w}\right)^{\frac{1-\alpha}{\eta}} \varphi_t^{\frac{1}{\eta}}\right]^{\frac{\eta\sigma}{1+\eta}}, \end{aligned} \tag{14}$$

where $\sigma = \frac{1}{\eta/(1+\eta)-(1-\alpha)}$ and $\frac{\eta\sigma}{1+\eta} = \frac{1}{1-(1+1/\eta)(1-\alpha)}$.

Define $s_t(\varphi)$ as the number of firms whose $\varphi_t = \varphi$. In equilibrium, the industry price index P_t is determined by the industry state s_t.

$$P_t = [\sum_{\varphi} s_t(\varphi) p(I, P_t, \varphi)^{1+\eta}]^{\frac{1}{1+\eta}}$$

$$= \left[\sum_{\varphi} s_t(\varphi)\left(D_t^{\alpha}\left(\frac{(1+1/\eta)(1-\alpha)}{w}\right)^{\frac{1-\alpha}{\eta}} \varphi^{\frac{1}{\eta}}\right)^{\eta\sigma}\right]^{\frac{1}{1+\eta}} \longrightarrow$$

$$P_t^{1+\eta} = P_t^{(1+\eta)\alpha\sigma}\left[I^{-\alpha\sigma}\sum_{\varphi} s_t(\varphi)\left(\frac{(1+1/\eta)(1-\alpha)}{w}\right)^{(1-\alpha)\sigma}(\varphi)^{\sigma}\right]$$

$$P_t^{-\sigma} = \left[I^{-\alpha\sigma}\sum_{\varphi} s_t(\varphi)\left(\frac{(1+1/\eta)(1-\alpha)}{w}\right)^{(1-\alpha)\sigma}(\varphi)^{\sigma}\right], \tag{15}$$

where we use the fact that $(1+\eta)(1-\alpha\sigma) = -\sigma$. As a result,

$$P_t = I^{\alpha}\left(\frac{w}{(1+1/\eta)(1-\alpha)}\right)^{1-\alpha}\left(\sum_{\varphi} s_t(\varphi)\varphi^{\sigma}\right)^{-\frac{1}{\sigma}}. \tag{16}$$

Given the industry aggregate price P_t and market size I, we can also define total sales as

$$r(I, \varphi_t) = I\left(\frac{p(I, P_t, \varphi_t)}{P_t}\right)^{1+\eta} \equiv I\frac{\varphi_t^{\sigma}}{\sum_{\varphi} s_t(\varphi)\varphi^{\sigma}}. \tag{17}$$

Finally, we have the equilibrium maximized profit for the firm with individual state φ_t as

$$\pi(\varphi_t, s_t) = I(1-(1+\frac{1}{\eta})(1-\alpha))\frac{\varphi_t^{\sigma}}{\sum_{\varphi} s_t(\varphi)\varphi^{\sigma}}. \tag{18}$$