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ABSTRACT

A prominent challenge when drawing causal inference using observational data is the ubiquitous presence of endogenous regressors. The classical econometric method to handle regressor endogeneity requires IVs that must satisfy the stringent condition of exclusion restriction, making it infeasible to use in many settings. We propose a new IV-free method using copulas to address the endogeneity problem. Existing copula correction methods require nonnormal endogenous regressors: normally or nearly normally distributed endogenous regressors cause model non-identification or significant finite-sample bias. Furthermore, existing copula control function methods presume the independence of exogenous regressors and the copula control function. Our proposed two-stage copula endogeneity correction (2sCOPE) method simultaneously relaxes the two key identification requirements, and we prove that 2sCOPE yields consistent causal-effect estimates with correlated endogenous and exogenous regressors as well as normally distributed endogenous regressors. Besides relaxing identification requirements, 2sCOPE has superior finite-sample performance and addresses the significant finite-sample bias problem due to insufficient regressor nonnormality. 2sCOPE employs generated regressors derived from existing regressors to control for endogeneity, and is straightforward to use and broadly applicable. Overall, 2sCOPE can greatly increase the ease and broaden the applicability of using IV-free methods to handle regressor endogeneity. We further demonstrate the performance of 2sCOPE via simulation studies and an empirical application.

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Causal inference is central to many problems faced by academics and practitioners. It increasingly gains importance as rapidly-available observational data in this digital era promise to offer real-world evidence on cause-and-effect relationships for better decision makings. However, a prominent challenge faced by empirical researchers to draw valid causal inferences from these data is the presence of endogenous regressors that are correlated with the structural error in the population regression model representing the causal relationship of interest. For example, omitted variables such as ability would cause endogeneity of schooling when examining schooling's effect on wages (Angrist and Krueger 1991).

Regressor endogeneity poses great empirical challenges to researchers and demands special handling of the issue in order to draw valid causal inferences. One classical method to deal with the endogeneity issue is using instrumental variables (IV). The ideal IV has to meet two requirements: it is correlated with the endogenous regressor via an explainable and validated relationship (i.e., relevance restriction), yet is uncorrelated with the structural error and does not directly affect the outcome (i.e., exclusion restriction). Although the theory of IVs is well-developed, researchers often face the challenge of finding good IVs satisfying these two requirements. Potential IVs often suffer from either weak relevance or challenging justification for exclusion restriction, which hampers using IVs to correct for the underlying endogeneity concerns (Rossi 2014).

To address the lack of suitable IVs, there has been a growing interest in developing and applying IV-free endogeneity-correction methods (Ebbes, Wedel, and Böckenholt 2009). Park and Gupta (2012) propose an IV-free method that uses the copula model (Danaher 2007; Danaher and Smith 2011; Christopoulos, McAdam, and Tzavalis 2021) to directly model the regressor-error dependence.[1] In addition to requiring no IVs, their approach is straightforward to use: one can simply add the latent copula data for the endogenous regressors as control variables to correct for endogeneity. These features considerably increase

[1] In statistics, a copula is a multivariate cumulative distribution function where the marginal distribution of each variable is a uniform distribution on $[0, 1]$. Copulas permit modeling dependence without imposing assumptions on marginal distributions.

the feasibility of endogeneity correction, as evidenced by the rapidly increasing use of the copula correction method (see examples of recent applications in the next section on literature review). However, similar to other IV-free methods, the copula correction methods also require the distinctiveness between the distributions of the endogenous regressor and the structural error. This means that the endogenous regressor is required to have a nonnormal distribution for model identification with the commonly assumed normal structural error distribution (Park and Gupta 2012; Papies, Ebbes, and Van Heerde 2017; Becker, Proksch, and Ringle 2021; Haschka 2022; Eckert and Hohberger 2022; Qian, Xie, and Koschmann 2022). Furthermore, we show that the existing copula control function correction method implicitly requires all exogenous regressors to be uncorrelated with the linear combination of copula transformations of endogenous regressors (henceforth referred to as copula control function (CCF) used to control for endogeneity), and may yield significant bias when there are noticeable correlations between the CCF and exogenous regressors.

In practical applications, both requirements of sufficient regressor nonnormality and no correlations between CCF and exogenous regressors can be too strong, and pose significant challenges for applying the copula correction method. We often encounter endogenous regressors or include transformations of endogenous variables as regressors that have close-to-normal distributions. Examples of such regressors in economics and marketing management studies include stock market returns (Sorescu, Warren, and Ertekin 2017), corporate social responsibility (Eckert and Hohberger 2022), the organizational intelligent quotient (Mendelson 2000), and the logarithm of price (see Figure 4 in the Application). Theoretically, the endogenous regressor and the structural error can contain a common set of unobservables that collectively have a normal distribution, which can lead to a close-to-normal distribution of the endogenous regressor. In these situations, even if the model is identified asymptotically, close-to-normality of endogenous regressors can cause estimation bias even in moderate sample sizes and require a large sample size to mitigate the finite-sample bias (Becker, Proksch, and Ringle 2021). Correlations between the CCF and exogenous regressors are also quite common in practical applications, especially when the exogenous regressors are included to control for observed confounders. Examples of such exogenous control variables abound in marketing and management studies, such as customer-specific variables (age, household size, income, past purchase behaviors, etc.) when estimating the returns of consumer targeting strategies on product sales (Papies, Ebbes, and Van Heerde 2017) and firms' similarity when estimating the effect of competition on innovation (Aghion et al. 2005). Although insufficient regressor nonnormality leads to more severe identification issues, including model non-identifiability and poor finite-sample performance (Table 1), correlations between CCF and exogenous regressors may occur more frequently than close-to-normality of endogenous regressors. Thus, we consider the two requirements of sufficient regressor nonnormality and no correlations between CCF and exogenous regressors as being equally stringent, which call for more general and flexible copula correction methods that relax both requirements.

In this paper, we develop a generalized two-stage copula endogeneity correction method, denoted as 2sCOPE, that relaxes the above two requirements. Similar to the existing copula methods, 2sCOPE requires neither IVs nor the assumption of exclusion restriction. The 2sCOPE method corrects for endogeneity by adding residuals, obtained from regressing latent copula data for each endogenous regressor on the latent copula data for exogenous regressors, as generated regressors in the structural model. Unlike the original copula method (Park and Gupta 2012; Becker, Proksch, and Ringle 2021; Eckert and Hohberger 2022, henceforth denoted as $\text{Copula}_{\text{Origin}}$), 2sCOPE can account for the dependence between endogenous and exogenous regressors. Thus, $\text{Copula}_{\text{Origin}}$ is a special case of 2sCOPE. Under a Gaussian copula model for the endogenous regressors, correlated exogenous regressors and the structural error, we prove that 2sCOPE can identify causal effects under weaker assumptions than $\text{Copula}_{\text{Origin}}$ and overcome the above two key limitations of $\text{Copula}_{\text{Origin}}$ (Table 1).

The contributions of this work are threefold. *First*, to our knowledge, this work is among the first in the literature to provide formal proofs for theoretical properties of copula correction methods. These theoretical results are needed because model identifiability is central to addressing the endogeneity issue. Recent work notes the lack of rigorous proofs of required model identification conditions and estimation properties (consistency and efficiency) for copula correction as one major area requiring further research (Becker, Proksch, and Ringle 2021; Haschka 2022)[2]. The theoretical results presented here can fill in this important knowledge gap, and contribute to a better understanding of the properties of the copula correction methods and guiding their use.

Features	Park and Gupta (2012)	Haschka (2022)	2sCOPE
nonnormality of Endogenous Regressors[1]	Required	Required	Not Required[2]
No Correlated Exogenous Regressors[3]	Required	Not Required	Not Required
Intercept Included	YES	NO[4]	YES
Theoretical Proof	YES	NO	YES
Estimation Method	Control Function & MLE	MLE	Control Function
Structural Model	Linear Regression RCL Slope Endogeneity	LPM-FE	Linear Regression LPM-FE, LPM-RE, LPM-ME RCL, Slope Endogeneity

Table 1: A Comparison of Copula Correction Methods

Note: [1]: When required, normality of any endogenous regressor leads to non-identifiable models. Insufficient nonnormality of endogenous regressors can also cause poor finite-sample performance (finite-sample bias and large standard errors) and require extremely large sample sizes to perform well.
[2]: nonnormality of endogenous regressors is not required as long as at least one correlated exogenous regressor is not normally distributed.
[3]: In our paper, correlated exogenous regressors refer to those exogenous regressors correlated with the CCF (copula control function) used to control for endogeneity.
[4]: The approach cannot estimate the intercept term, which is removed from the panel model prior to estimation using first-difference or fix-effects transformation (Web Appendix A.8 of Haschka (2022)). Becker, Proksch, and Ringle (2021) shows the importance of including intercept in marketing applications.
LPM: Linear Panel Model; FE: Fixed Effects for individual-specific intercepts with common slope coefficients; RE: Random Effects; ME: Mixed-Effects (including both fixed-effects and random coefficients); RCL: Random Coefficient Logit

Two novel theoretical findings emerge from this study. First, we identify an implicit assumption required for $\text{Copula}_{\text{Origin}}$ to yield consistent estimation, and provide conditions to

[2]For instance, owing to the complex form of the estimation method, Haschka (2022) notes the lack of theoretical proofs of required model identification conditions and estimation consistency as one limitation of the copula correction method developed there, and thus has to rely solely on simulation studies to evaluate its empirical properties.

verify this implicit assumption. This helps improve the effectiveness of the rapidly adopted method for addressing the endogeneity issue. A useful result is that the existence of the correlations between endogenous and exogenous regressors alone does not automatically introduce bias to $\text{Copula}_{\text{Origin}}$. Instead, we show that the implicit assumption is the uncorrelatedness of the exogenous regressors with the CCF, the *linear combination* of copula transformations of endogenous regressors used to control for endogeneity. The difference between the implicit assumption and the condition of zero pairwise correlations between endogenous and exogenous regressors can be substantial, especially with multiple endogenous regressors.[3] We prove that the proposed 2sCOPE yields consistent causal effect estimates when the implicit assumption above is violated, which can cause biased causal effect estimates for $\text{Copula}_{\text{Origin}}$.

The second novel finding of our theoretical investigation is as follows. Although the exogenous regressors that are correlated with the CCF require special handling for consistent causal effect estimation, we prove that they can be leveraged efficiently by 2sCOPE to substantially improve the finite-sample performance of copula correction and to relax the model identification requirement of nonnormality of endogenous regressors. We prove that the structural model with normally distributed endogenous regressors can be identified using 2sCOPE as long as one of the exogenous regressors correlated with endogenous ones is nonnormal, which is considerably more feasible in many practical applications.

Second, the proposed 2sCOPE method is the first copula-correction method that simultaneously relaxes the nonnormality assumption of endogenous regressors and handles correlated endogenous and exogenous regressors (Table 1). Existing copula correction methods do not account for correlated endogenous and exogenous regressors. An exception is Haschka (2022), which generalizes Park and Gupta (2012) to fixed-effects linear panel models with correlated regressors by jointly modeling the structural error, endogenous and exogenous

[3] Although Haschka (2022) explains why correlated regressors can cause potential bias for $\text{Copula}_{\text{Origin}}$, no condition of when bias can occur is given. Specifically, it is possible that with multiple endogenous regressors, the CCF is uncorrelated with exogenous regressors when pairwise correlations between endogenous and exogenous regressors are non-zeros. Even if there is only one endogenous regressor and CCF reduces to be proportional to the copula transformation of the endogenous regressor, the correlation coefficient is not invariant to nonlinear transformations and thus changes after the copula transformation of the endogenous regressor (Danaher and Smith 2011).

regressors using copulas and maximum likelihood estimation (MLE). However, as noted in Haschka (2022), Haschka's approach still requires the nonnormality of endogenous regressors. Thus, all existing copula correction methods require sufficient nonnormality assumption of endogenous regressors for model identification; even when the model is identified, insufficient regressor nonnormality can cause significant finite-sample bias in a sample size of less than 2,000 (Haschka 2022; Becker, Proksch, and Ringle 2021; Eckert and Hohberger 2022). Becker, Proksch, and Ringle (2021) suggest a minimum absolute skewness of 2 for an endogenous regressor to ensure good performance of Gaussian copula correction methods in a sample size of less than 1000 (Figure 8 in Becker, Proksch, and Ringle 2021). These requirements can significantly limit the use of copula correction methods in practical applications.

Our proposed 2sCOPE method overcomes these important restrictions of existing copula correction methods. Consistent with our theoretical results, the evaluation in Case 2 and Case 3 of the simulation studies demonstrates the superior finite-sample performance of 2sCOPE and shows that 2sCOPE eliminates or substantially reduces the significant problem of finite-sample bias due to insufficient regressor nonnormality raised in Becker, Proksch, and Ringle (2021) and Eckert and Hohberger (2022). In fact, even when the endogenous regressor is normal or close-to-normal with skewness of 0, 2sCOPE is still capable of reducing substantial estimation bias to be negligible for a sample size as small as 200 as shown in Figure 1. We further conduct systematic simulation studies and provide an actionable guideline for using 2sCOPE in Figure 2. The guideline establishes sufficient conditions regarding exogenous regressors, verifiable using tests of nonnormality and relevance to endogenous regressors, for 2sCOPE to effectively handle endogenous regressors with insufficient nonnormality using data at hand. When these conditions are not satisfied, we develop a novel bootstrap re-sampling method (Algorithm 1) to detect and quantify the finite-sample bias due to insufficient regressor nonnormality. The bootstrap method directly informs the specific size of finite-sample bias and the applicability of 2sCOPE for the data at hand, and thus complements the rules of thumb using tests of normality and relevance. We illustrate the use of the guideline and the bootstrap algorithm to control for potential finite-sample bias caused by insufficient nonnormality of the endogenous regressor (logarithm of the price) in our empirical application. Overall, 2sCOPE can greatly broaden the applicability of the IV-free methods for handling endogeneity issues in practice.

Third, 2sCOPE is the first copula control function method using generated regressors to handle endogenous regressors with insufficient nonnormality or correlated with exogenous regressors. Despite that the vast majority of applications of the copula correction method have used the generated-regressor approach (Becker, Proksch, and Ringle 2021; Eckert and Hohberger 2022), no copula control function method exists that can handle endogenous regressors having insufficient nonnormality or correlated with exogenous regressors. The proposed 2sCOPE overcomes this hurdle and provides a versatile and feasible copula control function method to handle regressor endogeneity. By including generated regressors in the structural model to control for endogeneity, 2sCOPE enjoys a number of benefits associated with using the control function to address endogeneity as compared with the alternative MLE approach. These include but are not limited to incurring little extra computational and modeling burdens to be integrated with complex outcome models, broader applicability with weaker assumptions, and increased robustness to model mis-specifications.[4] We demonstrate that 2sCOPE retains and enhances these desirable properties of the control function approach for a range of commonly used models in marketing studies, as shown in Table 1.

In many of these models, the MLE approach becomes much more difficult or computationally infeasible, but 2sCOPE is straightforward. We present an example with Footnote 8 showing that extending the MLE approach of Haschka (2022) to random coefficient linear panel models (RC-LPMs) with correlated endogenous and exogenous regressors requires numerically evaluating potentially high-dimensional integrals of complicated functions containing the product of copula density functions, evaluated at repeated measurement occasions. Yet 2sCOPE involves none of these integrals and can be implemented using standard

[4]As shown in Becker, Proksch, and Ringle (2021), Gaussian copula control function approach is more robust against error term mis-specifications than the Gaussian copula MLE approach.

software programs for RC-LPMs, assuming all regressors are exogenous. Furthermore, although 2sCOPE assumes a normal error distribution, we show its robustness to symmetric nonnormal error distributions (Web Appendix E.4), in contrast to the sensitivity to such error mis-specifications in the existing copula methods (Becker, Proksch, and Ringle 2021). Thus, the 2sCOPE control function approach together with correlated exogenous regressors included in 2sCOPE can increase robustness to model mis-specifications. Last but not the least, the generated-regressor approach facilitates studying the theoretical properties of 2sCOPE.

The remainder of this paper unfolds as follows. It begins with a review of the related literature on methods for causal inference with endogenous regressors. We then propose 2sCOPE and prove the consistency of 2sCOPE with normally distributed and correlated regressors. Next, we evaluate the performance of 2sCOPE using simulation studies under different scenarios and provide a flowchart to guide the use of 2sCOPE in practical applications. We further apply 2sCOPE to estimate price elasticity using store purchase databases. The paper then concludes.

LITERATURE REVIEW

The marketing, economics, and statistics literature develops a rich set of methods to draw causal inferences. The gold standard to estimate causal effects is randomized assignments such as controlled lab experiments and field experiments (Johnson, Lewis, and Nubbemeyer 2017, Anderson and Simester 2004, Godes and Mayzlin 2009). When controlled experiments are not feasible, quasi-experimental designs such as regression discontinuity, difference-in-difference, and synthetic control are used to mimic randomized experiments and to enable the identification of causal effects with observational data (Hartmann, Nair, and Narayanan 2011, Narayanan and Kalyanam 2015, Athey and Imbens 2006, Shi et al. 2017, Kim, Lee, and Gupta 2020). However, these quasi-experimental designs have special data and design requirements, and are not aimed at coping with the general issue of endogenous regressors when estimating causal effects using observational data.

There is a large literature on various approaches to addressing endogenous regressors when inferring causal effects. Papies, Ebbes, and Van Heerde (2017), Rutz and Watson (2019), and Park and Gupta (2012) provide an overview of addressing endogeneity in marketing. Three broad classes of solutions are discussed, and the most commonly used solution is the IV approach (Angrist and Krueger 1991, Qian 2008, Novak and Stern 2009, Ataman, Van Heerde, and Mela 2010, Van Heerde et al. 2013, Li and Ansari 2014). Rossi (2014) surveyed 10 years of publications in *Marketing Science* and *Quantitative Marketing and Economics*, revealing that the most commonly used IVs are lagged variables, costs, fixed effects and Hausman-style variables from other markets. However, the survey found that the strength of the instruments is rarely measured or reported, which is needed to detect the weak instrument problem. Moreover, one generally cannot test the exclusion restriction condition and verify the validity of the instruments. The survey also found that most papers lack a discussion of why the instruments used are valid. In short, though the theory of IVs is well-developed, good instruments are difficult to find, making the IV approach hard to implement in practice.

The second class of solutions to mitigate endogeneity is to specify the economic structure that generates the observational data including endogenous regressors (e.g., a supply-side model for marketing-mix variables) (Chintagunta et al. 2006, Sudhir 2001, Yang, Chen, and Allenby 2003, Sun 2005, Dotson and Allenby 2010 and Otter, Gilbride, and Allenby 2011). The key concern with this approach is that incorrect assumptions or insufficient information on the supply side can lead to biased estimates (Chintagunta et al. 2006).

The third class of solutions in the domain of endogeneity correction is IV-free methods. This is a more recent stream of methodological development. Three extant IV-free approaches are discussed in Ebbes, Wedel, and Böckenholt (2009): the higher moments (HM) approach (Lewbel 1997), the identification through heteroscedasticity (IH) estimator (Rigobon 2003), the latent instrumental variables (LIV) method (Ebbes et al. 2005). Recently Wang and Blei (2019) proposed a deconfounder approach that has some flavor of the LIV approach. All these methods decompose an endogenous regressor into an exogenous part and an endogenous part. The assumption of the endogenous regressor containing an exogenous component not affecting the outcome directly is akin to the stringent condition of exclusion restriction for observed IVs, and thus can be difficult to justify.

Park and Gupta (2012) introduce another IV-free method that doesn't require the stringent condition of exclusion restriction. It directly models the association between the structural error and the endogenous regressor via copula. The copula method has been rapidly adopted by researchers to deal with the endogeneity problem because of its feasibility to use without requiring instruments (Becker, Proksch, and Ringle 2021; Haschka 2022; Eckert and Hohberger 2022; Qian, Xie, and Koschmann 2022; Datta, Foubert, and Van Heerde 2015; Heitmann et al. 2020; Atefi et al. 2018; Elshiewy and Boztug 2018). The 2sCOPE method contributes to the literature by overcoming important limitations of existing copula correction methods, as described upfront, and being applicable in more general settings with the capability to leverage exogenous regressors to improve model identification and estimation.

METHODS

In this section, we develop a copula-based IV-free method to handle endogenous regressors with insufficient nonnormality and correlation with exogenous regressors. We first review $\text{Copula}_{\text{Origin}}$ and show that $\text{Copula}_{\text{Origin}}$ implicitly assumes no correlations between exogenous regressors and the CCF, as well as the bias in the structural model parameter estimates that may arise from the violation of this assumption. Then we propose a new method to deal with the problem and the detailed estimation procedure. We also show how exogenous regressors correlated with endogenous regressors can sharpen structural model parameter estimates and enable the identification of the structural model containing normally distributed endogenous regressors, which are known to cause the model non-identifiability issue for $\text{Copula}_{\text{Origin}}$.

Assumptions of Existing Copula Endogeneity-Correction Method (Copula$_{Origin}$)

Consider the following linear structural regression model with an endogenous regressor and a vector of exogenous regressors [5]:

$$Y_t = \mu + P_t\alpha + W_t'\beta + \xi_t, \quad (1)$$

where $t = 1, 2, ..., T$ indexes either time or cross-sectional units, Y_t is a (1×1) dependent variable, P_t is a (1×1) continuous endogenous regressor, W_t is a (k×1) vector of exogenous regressors, ξ_t is the structural error term, and (μ, α, β) are model parameters. P_t is correlated with ξ_t, and this correlation generates the endogeneity problem. W_t is exogenous, which means it is not correlated with ξ_t, but can be correlated with the endogenous variable P_t.

The key idea of Copula$_{\text{Origin}}$ (Park and Gupta 2012) is to use a copula to jointly model the correlation between the endogenous regressor P_t and the error term ξ_t. The advantage of using copula is that marginals are not restricted by the joint distribution. Using information contained in the observed data, marginals of the endogenous regressor and the error term are first obtained respectively. Then the copula model enables researchers to construct a flexible multivariate joint distribution that captures the correlation among these variables.

Let $F(P, \xi)$ be the joint cumulative distribution function (CDF) of the endogenous regressor P_t and the structural error ξ_t with marginal CDFs $H(P)$ and $G(\xi)$, respectively. For notational simplicity, we may omit the index t in P_t and ξ_t below when appropriate. According to Sklar's theorem (Sklar 1959), there exists a copula function $C(\cdot, \cdot)$ such that

$$F(P, \xi) = C(H(P), G(\xi)) = C(U_p, U_\xi), \quad (2)$$

where $U_p = H(P)$ and $U_\xi = G(\xi)$, and they both follow uniform(0,1) distributions. Thus, the copula maps the marginal CDFs of the endogenous regressor and the structural error to their joint CDF, and makes it possible to separately model the marginals and correlations of these random variables. To capture the association between the endogenous regressor P and the error ξ, Park and Gupta (2012) uses the following Gaussian copula for its desirable

[5] As shown in Becker, Proksch, and Ringle (2021), it is important to include the intercept term when evaluating the copula correction method.

properties (Danaher 2007; Danaher and Smith 2011):

$$F(P,\xi) = C(U_p, U_\xi) = \Psi_\rho(\Phi^{-1}(U_p), \Phi^{-1}(U_\xi))$$
$$= \frac{1}{2\pi(1-\rho^2)^{1/2}} \int_{-\infty}^{\Phi^{-1}(U_p)} \int_{-\infty}^{\Phi^{-1}(U_\xi)} \exp\left[\frac{-(s^2 - 2\rho\cdot s\cdot t + t^2)}{2(1-\rho^2)}\right] dsdt, \quad (3)$$

where $\Phi(\cdot)$ denotes the univariate standard normal distribution function and $\Psi_\rho(\cdot,\cdot)$ denotes the bivariate standard normal distribution with the correlation coefficient ρ. With empirical marginal CDFs, the above Gaussian copula model depends on the rank order of raw data only, and is invariant to strictly monotonic transformations of variables in (P,ξ). Thus, the above Gaussian copula model is considered general and robust for most marketing applications (Danaher and Smith 2011). In the Gaussian copula model, ρ captures the endogeneity of the regressor P, and a non-zero value of ρ corresponds to P being endogenous.

Under the above copula model for (P_t, ξ_t) and the commonly-assumed normal distribution for the structural error ξ_t, Park and Gupta (2012) develop the following generated regressor procedure to correct for regressor endogeneity. Let $P_t^* = \Phi^{-1}(U_p)$ and $\xi_t^* = \Phi^{-1}(U_\xi)$, the above Gaussian copula assumes $[P_t^*, \xi_t^*]'$ follow the standard bivariate normal distribution with the correlation coefficient ρ as follows:

$$\begin{pmatrix} P_t^* \\ \xi_t^* \end{pmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}\right). \quad (4)$$

Under the assumption that the structural error ξ_t follows $N(0, \sigma_\xi^2)$, Park and Gupta (2012) show that the structural error can be split into two parts as follows:

$$\xi_t = \sigma_\xi \xi_t^* = \sigma_\xi \rho P_t^* + \sigma_\xi \sqrt{1-\rho^2}\omega_t, \quad (5)$$

where the first part $\sigma_\xi \rho P_t^*$ captures the correlation between ξ_t and the endogenous regressor, and the other part $\sigma_\xi \cdot \sqrt{1-\rho^2}\omega_t$ being an independent new error term. Equation (1) can then be rewritten as follows:

$$Y_t = \mu + P_t\alpha + W_t\beta + \sigma_\xi \cdot \rho \cdot P_t^* + \sigma_\xi \cdot \sqrt{1-\rho^2} \cdot \omega_t. \quad (6)$$

Based on the above representation, Park and Gupta (2012) suggest the following generated regressor approach to correcting for the endogeneity of P_t: the ordinary least squares (OLS) estimation of Equation (6) with $P_t^* = \Phi^{-1}(U_p)$ included as an additional regressor will yield consistent model estimates. Park and Gupta (2012) also point out that for the above approach to work, P_t needs to have a nonnormal distribution. Suppose P_t is normally distributed, $P_t = P_t^* \cdot \sigma_p$, resulting in perfect collinearity between P_t and P_t^* and violating the full rank assumption required for identifying the linear regression model in Equation (6). Thus, P_t should follow a different distribution from the normal error term so that the causal effect of P, which is independent of all other regressors, can be identified.

However, we show here that an implicit assumption for the above generated regressor approach to yield consistent model estimates is the uncorrelatedness between P_t^* and W_t. For the OLS estimation to yield consistent estimation, the error term ω_t in Equation (6) is required to be uncorrelated with all the regressors on the right-hand side of the equation: P_t, W_t, P_t^*. When W_t is not correlated with P_t^*, W_t is also uncorrelated with ω_t, which is determined by ξ_t and P_t^*, because of the exogenous feature of W_t. However, when W_t is correlated with P_t^*, W_t would become correlated with ω_t because (1) ω_t is a linear combination of ξ_t and P_t^* (Equation 5), and (2) W_t is uncorrelated with ξ_t. Thus, the correlation between the exogenous regressor W_t and the generated regressor P_t^* would cause biased OLS estimates of Equation (6) using $\text{Copula}_{\text{Origin}}$ because of the induced correlation between the error term ω_t and W_t. In short, W_t becomes endogenous in Equation (6) when W_t and P_t^* are correlated, which is formally proved in Theorem 1 below.

Theorem 1. *Assuming (1) $(1, P, W)$ is full rank and W is exogenous, (2) the error term is normal, (3) a Gaussian Copula for the structural error term and P_t, and (4) P_t is endogenous: $\rho \neq 0$, $Cov(\omega_t, W_t) = -\frac{\rho}{\sqrt{1-\rho^2}} Cov(W_t, P_t^*) \neq 0$ if P_t^* and W_t are correlated.*

Proof: See Web Appendix A.1, Proof of Theorem 1.

To summarize, the generated regressor procedure ($\text{Copula}_{\text{Origin}}$) based on Equation (6) makes the set of assumptions listed in Table 2. Assumption 5 has been discovered by Haschka (2022). However, as shown in Web Appendix A.2, Assumption 5 should be replaced with the more general Assumption 5(b) for the case of multiple endogenous regressors.[6] Assumptions

[6] For instance, in the 2-endogenous regressors case, Assumption 5(b) means

5 and 5(b) are verifiable and provide users with criteria to check whether $\text{Copula}_{\text{Origin}}$ would provide consistent estimation when there exist exogenous regressors that may be correlated with the CCF. With only one endogenous regressor, one can simply check the correlations between the copula transformation of this endogenous regressor with each exogenous regressor. For multiple endogenous regressors, one should check the correlations between the CCF (i.e., the linear combination of copula transformations of these endogenous regressors used to control for endogeneity) in $\text{Copula}_{\text{Origin}}$ with each exogenous regressor, using the Fisher's Z test described in Web Appendix E.7. If there exists at least one exogenous regressor in W_t that fails Assumption 5 or 5(b), $\text{Copula}_{\text{Origin}}$ yields biased estimates, and our proposed 2sCOPE can be used, which is derived in the next subsection.

Assumption 1. *Full rank* [1] *of all regressors and exogeneity of* W.
Assumption 2. *The structural error follows a normal distribution.*
Assumption 3. P_t *and the structural error follow a Gaussian copula.*
Assumption 4. *Nonnormality of the endogenous regressor* P_t.
Assumption 5. *For a scalar endogenous regressor* P, W_t *and* P_t^* *are uncorrelated.*
Assumption 5(b). *For multiple endogenous regressors,* W_t *and the CCF* [2] *are uncorrelated.*

[1]: Full rank means $\text{rank}(X'X) = k$, in which $X = (1, P, W)$ with column dimension of k;
[2]: CCF (copula control function) is the linear combination of P_t^* used to control for endogenous regressors.

Table 2: Assumptions in $\text{Copula}_{\text{Origin}}$

The full rank of all regressors and exogeneity of W_t in Assumption 1 of Table 2 are assumptions made in many other commonly used econometric methods to ensure estimation consistency, including OLS and IV methods. For Assumptions 2 to 4, Park and Gupta (2012) have shown reasonable robustness of their copula method to nonnormal distributions of the structural error (Assumption 2) and alternative forms of copula functions (Assumption 3), although it is not surprising to observe the sensitivity of $\text{Copula}_{\text{Origin}}$ to gross violations of these assumptions, such as highly skewed error distributions (Becker, Proksch, and Ringle 2021; Eckert and Hohberger 2022). By contrast, the assumption that the endogenous regressor P_t follows a nonnormal distribution (Assumption 4) is critical. An endogenous regressor

$Cov(W_t, \frac{\rho_{\xi 1} - \rho_{12}\rho_{\xi 2}}{1-\rho_{12}^2} \cdot P_{1,t}^* + \frac{\rho_{\xi 2} - \rho_{12}\rho_{\xi 1}}{1-\rho_{12}^2} \cdot P_{2,t}^*) = 0$ (Web Appendix A.2), which is not the same as either $Cov(W_t, P_{1,t}^*) = 0$, $Cov(W_t, P_{2,t}^*) = 0$ or $Cov(W_t, P_{1,t}) = 0, Cov(W_t, P_{2,t}) = 0$.

following a normal distribution violates the full-rank condition in Equation (6) and causes the model to be unidentifiable regardless of the sample size; a nearly normally distributed endogenous regressor may require a very large sample size for the method to perform well and may cause the method to have poor performance for a finite sample size. Moreover, we have shown above that for their method to work best, there should be no exogenous regressors that are correlated with the CCF (Assumption 5(b)). Both Assumptions 4 and 5(b) can be too strong and substantially limit the applicability of the IV-free copula method in practice.

Proposed Method: Two-stage Copula Endogeneity-correction (2sCOPE)

In this subsection, we propose a two-stage copula endogeneity-correction (2sCOPE) method and show that it can relax both the uncorrelatedness assumption between CCF and the exogenous regressors (Assumption 5(b)) and the key identification assumption of nonnormality on the endogenous regressors (Assumption 4). The 2sCOPE method jointly models the endogenous regressor, P_t, the correlated exogenous variable, W_t, and the structural error term, ξ_t, using the Gaussian copula model, which implies that $[P_t^*, W_t^*, \xi_t^*]$ follows the multivariate normal distribution:

$$\begin{pmatrix} P_t^* \\ W_t^* \\ \xi_t^* \end{pmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho_{pw} & \rho_{p\xi} \\ \rho_{pw} & 1 & 0 \\ \rho_{p\xi} & 0 & 1 \end{bmatrix} \right), \tag{7}$$

where $P_t^* = \Phi^{-1}(H(P_t))$, $W_t^* = \Phi^{-1}(L(W_t))$, and $\xi_t^* = \Phi^{-1}(G(\xi_t))$, and $H(\cdot)$, $L(\cdot)$ and $G(\cdot)$ are marginal CDFs of P_t, W_t and ξ_t, respectively.

Under the above Gaussian copula model in Equation (7), one can develop a direct extension of $\text{Copula}_{\text{Origin}}$, which adds generated regressors P_t^* and W_t^* into the structural regression model to correct for endogeneity bias. The resulting method, denoted as COPE, is shown to yield consistent causal effect estimates without requiring the exogeneity of W and Assumption 5 (or Assumption 5(b)) needed for $\text{Copula}_{\text{Origin}}$ (Web Appendix A.1). However, COPE requires endogenous regressors P_t and exogenous regressors W_t to both have sufficient nonnormality and yields substantial bias when regressors have insufficient nonnormality (see simulation results in Table 6 and Figure 1). Furthermore, adding many generated regressors for control variables W can cause severe multicollinearity issues and have significantly adverse impacts on causal effect estimation efficiency and stability (simulation results in Web Appendix E.3 showing COPE can require 5 times the sample size than our proposed method to achieve the same estimation precision). Overall, COPE suffers from the low face validity problem because it can add many more generated regressors than needed into the structural outcome model. To overcome these limitations of COPE, below we derive the 2sCOPE method that relaxes both Assumptions 4 and 5(b) of $\text{Copula}_{\text{Origin}}$.

Under the above Gaussian copula model, we have the following system of equations:

$$Y_t = \mu + P_t\alpha + W_t\beta + \xi_t \tag{8}$$

$$P_t^* = W_t^*\gamma + \epsilon_t. \tag{9}$$

Under the assumption that ξ_t follows a normal distribution, ϵ_t and ξ_t follow a bivariate normal distribution, since they are a linear combination of tri-normal variate (ξ_t^*, P_t^*, W_t^*) under the Gaussian copula assumption. Equation (9) expresses the copula transformation of the endogenous regressor, determined by the rank order of P_t, as a linear combination of observed and unobserved variables. The two error terms ϵ_t and ξ_t are correlated because of the endogeneity of P_t. For example, both ξ_t and ϵ_t may contain an additive component corresponding to a common omitted variable. The above model is then obtained when the omitted variable and regressors follow a Gaussian copula model.

The main idea of 2sCOPE is to make use of the fact that, by conditioning on ϵ_t, the structural error term ξ_t becomes independent of both P_t and W_t. That is, by conditioning on the component of P_t causing the endogeneity of P_t (i.e, ϵ_t here), the structural error is not correlated with both P_t and W_t, thereby ensuring the consistency of standard estimation methods. In this sense, ϵ_t serves as a (scaled) control function to address the endogeneity bias. To demonstrate this point, note that the Gaussian copula model in Equation (7) can be rewritten as follows:

$$\begin{pmatrix} P_t^* \\ W_t^* \\ \xi_t^* \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ \rho_{pw} & \sqrt{1-\rho_{pw}^2} & 0 \\ \rho_{p\xi} & \frac{-\rho_{pw}\rho p\xi}{\sqrt{1-\rho_{pw}^2}} & \sqrt{1-\rho_{p\xi}^2-\frac{\rho_{pw}^2\rho_{p\xi}^2}{1-\rho_{pw}^2}} \end{pmatrix} \cdot \begin{pmatrix} \omega_{1,t} \\ \omega_{2,t} \\ \omega_{3,t} \end{pmatrix},$$

$$\begin{pmatrix} \omega_{1,t} \\ \omega_{2,t} \\ \omega_{3,t} \end{pmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right). \tag{10}$$

Given the above joint normal distribution for (P_t^*, W_t^*, ξ_t^*) and $\xi_t = \sigma_\xi \xi_t^*$, we have

$$P_t^* \;=\; \rho_{pw}W_t^* + \sqrt{(1-\rho_{pw}^2)} \cdot \omega_{2,t} = \rho_{pw}W_t^* + \epsilon_t, \tag{11}$$

which shows γ in Equation (9) is ρ_{pw} and $\epsilon_t = \sqrt{(1-\rho_{pw}^2)} \cdot \omega_{2,t}$, and

$$\begin{aligned} Y_t &= \mu + P_t\alpha + W_t\beta + \frac{\sigma_\xi\rho_{p\xi}}{1-\rho_{pw}^2}P_t^* + \frac{-\sigma_\xi\rho_{pw}\rho_{p\xi}}{1-\rho_{pw}^2}W_t^* + \sigma_\xi\sqrt{1-\rho_{p\xi}^2-\frac{\rho_{pw}^2\rho_{p\xi}^2}{1-\rho_{pw}^2}} \cdot \omega_{3,t} \\ &= \mu + P_t\alpha + W_t\beta + \frac{\sigma_\xi\rho_{p\xi}}{1-\rho_{pw}^2}(P_t^* - \rho_{pw}W_t^*) + \sigma_\xi\sqrt{1-\rho_{p\xi}^2-\frac{\rho_{pw}^2\rho_{p\xi}^2}{1-\rho_{pw}^2}} \cdot \omega_{3,t}, \\ &= \mu + P_t\alpha + W_t\beta + \frac{\sigma_\xi\rho_{p\xi}}{1-\rho_{pw}^2}\epsilon_t + \sigma_\xi\sqrt{1-\rho_{p\xi}^2-\frac{\rho_{pw}^2\rho_{p\xi}^2}{1-\rho_{pw}^2}} \cdot \omega_{3,t}. \end{aligned} \tag{12}$$

Equation (12) suggests adding the estimate of the error term ϵ_t from the first stage regression as a generated regressor to the outcome regression instead of using P_t^* and W_t^*. As shown in Theorem 2 below, the linear model in Equation (12) satisfies both the full column rank condition of the regressor matrix and zero correlation between the new error term $\omega_{3,t}$ and each regressor in Equation (12), ensuring the consistency of OLS model estimates (Chpt. 4, Wooldridge 2010). This two-step procedure, named as 2sCOPE, adds the first-stage residual term $\widehat{\epsilon}_t$ to control for endogeneity and in this aspect is similar to the control function approach of Petrin and Train (2010). However, unlike Petrin and Train (2010), 2sCOPE requires no use of IVs.

Theorem 2. *Estimation Consistency.* *Assuming (1)* $(1, P, W)$ *is full rank and* W *is exogenous, (2) the error is normal, (3) the endogenous regressor* P_t *or correlated regressors*

W_t is nonnormal, and (4) a Gaussian Copula for (ξ_t, P_t, W_t), the regressor matrix $(1, P, W, \epsilon)$ is full rank and $Cov(\omega_{3,t}, W_t)$= $Cov(\omega_{3,t}, P_t) = Cov(\omega_{3,t}, \epsilon_t) = 0$ in Equation (12).

Proof: See Web Appendix B.1, Proof of Theorem 2.

According to Theorem 2, the proposed method 2sCOPE can yield consistent estimates when assumptions are met. Specifically, Assumption 5(b) is relaxed because 2sCOPE can handle the case when the model includes exogenous regressors correlated with the CCF. Theorem 3 further shows that 2sCOPE relaxes Assumption 4 (the nonnormality assumption on endogenous regressors), a critical model identification condition required in all other copula correction methods.

Theorem 3. ***Nonnormality Assumption Relaxed.*** *Assuming (1) $(1, P, W)$ is full rank and W is exogenous, (2) the error term is normal, (3) one of the correlated exogenous regressors W_t is nonnormal, and (4) a Gaussian Copula for the error term, P_t and W_t, 2sCOPE estimator is consistent when P_t follows a normal distribution.*

Proof: See Web Appendix B.2, Proof of Theorem 3.

Theorem 3 shows that as long as one exogenous regressor correlated with the endogenous regressor P_t is nonnormally distributed, 2sCOPE can correct for endogeneity for a normal regressor P_t while COPE cannot. Intuitively, when P_t (or W_t) is normal, P_t^* (or W_t^*) becomes a linear function of P_t (or W_t) under the Gaussian copula assumption, rendering COPE to fail the full rank assumption and become unidentified. Thus, COPE cannot deal with normal endogenous/exogenous regressors. For 2sCOPE in Equation (12), adding the first stage residual $\widehat{\epsilon}_t$ as the generated regressor improves model identification. As long as not all W_t are normal, $\widehat{\epsilon}_t$ would not be a linear function of P_t and W_t and thus the second stage model (Equation 12) in 2sCOPE would satisfy the full rank requirement for model identification. Thus, 2sCOPE can relax the nonnormality assumption on the endogenous regressor required in Park and Gupta (2012) as long as one of the W_t is nonnormally distributed.

To sum up, we have proven the consistency of 2sCOPE (Theorem 2). Theorem 3 and Proposition 1 (Web Appendix B.3) further establish that 2sCOPE outperforms COPE, the extended Copula$_{\text{Origin}}$, in terms of estimation efficiency gain and relaxation of the nonnormality assumption on endogenous regressors in Copula$_{\text{Origin}}$ by satisfying a looser condition.

Multiple Endogenous Regressors

In this subsection, we extend 2sCOPE to the general case of multiple endogenous regressors. Consider the following structural linear regression model with two endogenous regressors ($P_{1,t}$ and $P_{2,t}$) that are potentially correlated with the exogenous regressor W_t:

$$Y_t = \mu + P_{1,t} \cdot \alpha_1 + P_{2,t} \cdot \alpha_2 + W_t\beta + \xi_t. \tag{13}$$

Under the multivariate Gaussian distribution assumption on $(\xi_t, P^*_{1,t}, P^*_{2,t}, W^*_t)$, the system of equations for the 2sCOPE method in Equations (8, 9) are readily extended to the case with two endogenous regressors as

$$Y_t = \mu + P_{1,t}\alpha_1 + P_{2,t}\alpha_2 + W_t\beta + \xi_t, \tag{14}$$

$$P^*_{1,t} = \rho_{wp1}W^*_t + \epsilon_{1,t}, \tag{15}$$

$$P^*_{2,t} = \rho_{wp2}W^*_t + \epsilon_{2,t}, \tag{16}$$

where Equations (15) and (16) can be directly derived from the Gaussian copula assumption; $(\xi_t, \epsilon_{1,t}.\epsilon_{2,t})$ are linear transformations of $(\xi_t, P^*_{1,t}, P^*_{2,t}, W^*_t)$, and thus also follow a multivariate Gaussian distribution. As a result, we can decompose the structural error ξ_t as additive terms for $\epsilon_{1,t}$, $\epsilon_{2,t}$ and a remaining independent error term $\omega_{4,t}$ as follows

$$Y_t \;\; = \mu + P_{1,t}\alpha_1 + P_{2,t}\alpha_2 + W_t\beta + \eta_1\epsilon_{1,t} + \eta_2\epsilon_{2,t} + \sigma_\xi \cdot m \cdot \omega_{4,t}, \tag{17}$$

where $\epsilon_{1,t} = P^*_{1,t} - \rho_{wp1}W^*_t$ and $\epsilon_{2,t} = P^*_{2,t} - \rho_{wp2}W^*_t$, m is a constant depending only on the correlation coefficients in the Gaussian copula, η_1, η_2 and $\omega_{4,t}$ are defined in Equation (W11) in Web Appendix B.1, and the new (scaled) error term $\omega_{4,t}$ is independent of latent copula data $(P^*_{1,t}, P^*_{2,t}, W^*_t)$ as well as all functions of these latent data including $P_{1,t}, P_{2,t}, W_t, \epsilon_{1,t}, \epsilon_{2,t}$. Because $\omega_{4,t}$ is independent of all regressors on the right-hand side of Equation (17), the OLS estimation of Equation (17) yields consistent estimates of structural model parameters as long as the regressor matrix $(1, P_1, P_2, W, \epsilon_1, \epsilon_2)$ is of full column rank.

The proof for the estimation consistency, relaxation of the regressor-nonnormality assumption, and estimation efficiency gain for 2sCOPE can be found in Web Appendix B under the related Theorems 2, 3, and Proposition 1. Table 3 summarizes the assumptions

Copula$_{\text{Origin}}$	2sCOPE
• Full-rank condition and exogeneity of W_t(Asm. 1); • The structural error follows a normal distribution (Asm. 2); • P_t and the structural error follow a Gaussian copula (Asm. 3); • All regressors in P_t are nonnormally distributed (Asm. 4); • W_t is uncorrelated with the CCF (copula control function which is the linear combination of all P_t^* used to control for endogeneity) (Asm. 5, 5(b)).	• Full-rank condition and exogeneity of W_t(Asm. 1); • The structural error follows a normal distribution (Asm. 2); • P_t, W_t and the structural error follow a Gaussian copula; • P_t can be normally distributed as long as one of the related W_t is nonnormal.

Table 3: Summary of Assumptions for the 2sCOPE and Copula$_{\text{Origin}}$

for our proposed 2sCOPE method and the existing copula method Copula$_{\text{Origin}}$. Table 4 summarizes the estimation procedure of 2sCOPE.

Stage 1: • Obtain empirical CDFs for each regressor in P_t and W_t, $\widehat{H}(P_t)$ and $\widehat{L}(W_t)$; • Compute $P_t^* = \Phi^{-1}(\widehat{H}(P_t))$ and $W_t^* = \Phi^{-1}(\widehat{L}(W_t))$; • Regress each endogenous regressor in P_t^* separately on W_t^* and obtain residual $\widehat{\epsilon}_t$; Stage 2: • Add $\widehat{\epsilon}_t$ to the outcome structural regression model as generated regressors.
• Standard errors of parameter estimates are estimated using bootstrap (Web Appendix F).

Table 4: Estimation Procedure for 2sCOPE

2sCOPE for Random Coefficient Linear Panel Models

We consider the following random coefficient model for linear panel data

$$Y_{it}|\mu_i, \alpha_i, \beta_i \quad = \quad \bar{\mu} + \mu_i + P_{it}'\alpha_i + W_{it}'\beta_i + \xi_{it}, \tag{18}$$

where $i = 1, \cdots, N$ indexes cross-sectional units and $t = 1, \cdots, T$ indexes occasions. P_{it} (W_{it}) denotes a vector of endogenous (exogenous) regressors. P_{it} and W_{it} can be correlated. The error term $\xi_{it} \overset{iid}{\sim} N(0, \sigma_\xi^2)$, which is correlated with P_{it} due to the endogeneity of P_{it} but is uncorrelated with the exogenous regressors in W_{it}. The individual-specific intercept μ_i and individual-specific slope coefficients (α_i, β_i) permit heterogeneity in both intercepts and regressor effects across cross-sectional units. Extant marketing studies have shown the ubiquitous presence of heterogeneous consumers' responses to marketing mix variables (e.g., price sensitivity) and substantial bias associated with ignoring such heterogeneity in slope coefficients. Thus, it is important to allow individual-specific slope coefficients.

The linear panel data model as specified in Equation (18) is general and includes the linear panel model with only individual-specific intercepts considered in Haschka (2022) as a special case. Specifically, Haschka (2022) fixes (α_i, β_i) to be the same value (α, β) across all units, assuming all cross-sectional units have the same slope coefficients. In contrast, the model in Equation (18) relaxes this strong assumption and can generate unit-specific slope parameters, which can be used for targeting purposes.

A random coefficient model typically assumes $(\mu_i, \alpha_i, \beta_i)$ follows a multivariate normal distribution. When all regressors are exogenous, estimation algorithms for such random coefficient models are well-established and computationally feasible even for a high-dimensional vector of random effects $(\mu_i, \alpha_i, \beta_i)$: with the normal conditional distribution for $Y_{it}|(\mu_i, \alpha_i, \beta_i)$ in Equation (18) and the multivariate normal prior distribution for random effects $(\mu_i, \alpha_i, \beta_i)$, marginally Y_{it} follows a normal distribution with a closed-form expression containing no integrals with respect to random effects $(\mu_i, \alpha_i, \beta_i)$, leading to an easy-to-evaluate likelihood function (Greene 2003). For instance, R function `lme()` can be used to obtain MLEs of population parameters and empirical Bayes estimates of individual random effects. Alternatively, one can assume a mixed-effect model where μ_i is a fixed effect parameter with μ_i's allowed to be correlated with the regressors P_{it} and W_{it}. To avoid the potential incidental parameter problem, one often uses the first-difference or fixed-effects transformation to eliminate the incidental intercept parameters as follows

$$\tilde{y}_{it}|\alpha_i, \beta_i \quad = \quad \tilde{P}'_{it}\alpha_i + \tilde{W}'_{it}\beta_i + \tilde{\xi}_{it}, \tag{19}$$

where $\tilde{y}_{it}$, $\tilde{P}_{it}$, $\tilde{W}_{it}$ and $\tilde{\xi}_{it}$ denote new variables obtained from the first-difference or fixed-effect transformation. Haschka (2022) considered a special case of Equation (19) by fixing (α_i, β_i) to be constants.

It is straightforward to apply 2sCOPE to address regressor endogeneity in the general random coefficient model for linear panel data in Equation (18) and the transformed one without intercepts in Equation (19).[7] The 2sCOPE procedure adds the residuals obtained from regressing P^*_{it} on W^*_{it}. Thus, 2sCOPE can be implemented using standard software programs for random coefficient linear panel models assuming all regressors are exogenous (see Simulation Study Case 4 for an illustration using the R function `lme()`). By contrast, the MLE approach for copula correction in the random coefficients model accounting for correlated endogenous and exogenous regressors is not developed yet and would require constructing complicated joint likelihood on the error term, P_t and W_t, which involves newly appearing numerical integrals with respect to random effects and cannot be maximized by standard estimation algorithms for random coefficient models.[8] Finally, current applications applying $\text{Copula}_{\text{Origin}}$ do not consider the role of exogenous regressors. Our analysis shows that this may yield bias if any exogenous regressor is correlated with the CCF added to control endogeneity, for which 2sCOPE should be used to address regressor endogeneity.

2sCOPE for Slope Endogeneity and Random Coefficient Logit Model

In Web Appendices C and D, we derive the 2sCOPE method to tackle the slope endogeneity problem and address endogeneity bias in random coefficient logit models with correlated

[7] Similar to Haschka (2022), a GLS transformation can be applied to both sides of Equation (19), resulting in a pooled regression for which 2sCOPE can be directly applied.

[8] With endogenous regressors, the individual random effects parameters enter into both the density function for the outcome $Y_{it}|(\mu_i, \alpha_i, \beta_i)$ and the density of copula function $C(U_{\xi,it}, U_{P,it}, U_{W,it})$ via $U_{\xi,it}$, and thus cannot be integrated out in closed-form any more from the likelihood function even with the normal structural error term and normal random effects. Therefore, numerical integration is required for obtaining MLEs in random coefficient models with endogenous regressors, which cannot be performed with standard software programs for random coefficient model estimation.

and normally distributed regressors. In these two cases, we show how to apply 2sCOPE to correct for the endogeneity bias, which can avoid the potential bias of $\text{Copula}_{\text{Origin}}$ due to the potential correlations between the exogenous regressors and CCF, as well as make use of the correlated exogenous regressors to relax the nonnormality assumption of endogenous regressors, improve model identification and sharpen model estimates. As shown there, 2sCOPE can be implemented using standard estimation methods by adding generated regressors to control for endogenous regressors. By contrast, the maximum likelihood approach can require constructing a complicated joint likelihood that is not what the standard estimation method uses and thus requires separate development and significantly more computation involving numerical integration.

SIMULATION STUDY

In this section, we conduct Monte Carlo simulation studies for the following goals: (a) to assess the performance of the proposed method for correlated regressors, (b) to assess the performance of the proposed method under regressor normality and near normality, (c) to assess the performance of the proposed method under various types of structural models, and (d) to assess the robustness of the proposed method to violations of model assumptions. Following Park and Gupta (2012), we measure the estimation bias using t_{bias} calculated as the ratio of the absolute difference between the mean of the sampling distribution and the true parameter value to the standard error of the parameter estimate. As defined above, t_{bias} represents the size of bias relative to the sampling error.

Case 1: nonnormal Regressors

We first examine the case when P and W are correlated. The data-generating process (DGP) is summarized below:

$$\begin{pmatrix} P_t^* \\ W_t^* \\ \xi_t^* \end{pmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho_{pw} & \rho_{p\xi} \\ \rho_{pw} & 1 & 0 \\ \rho_{p\xi} & 0 & 1 \end{bmatrix} \right) = N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0.5 & 0.5 \\ 0.5 & 1 & 0 \\ 0.5 & 0 & 1 \end{bmatrix} \right), \tag{20}$$

$$\xi_t = G^{-1}(U_{\xi,t}) = G^{-1}(\Phi(\xi_t^*)) = \Phi^{-1}(\Phi(\xi^*)) = 1 \cdot \xi_t^*, \tag{21}$$

$$P_t = H^{-1}(U_{P,t}) = H^{-1}(\Phi(P_t^*)), \quad W_t = L^{-1}(U_{W,t}) = L^{-1}(\Phi(W_t^*)), \tag{22}$$

$$Y_t = \mu + \alpha \cdot P_t + \beta \cdot W_t + \xi_t = 1 + 1 \cdot P_t + (-1) \cdot W_t + \xi_t, \tag{23}$$

where ξ_t^* and P_t^* are correlated ($\rho_{p\xi} = 0.5$), generating the endogeneity problem; W_t^* is exogenous and uncorrelated with ξ_t^*; W_t^* and P_t^* are correlated ($\rho_{pw} = 0.5$), and thus W_t and P_t are correlated. We consider four different estimation methods: (1) OLS, (2) $\text{Copula}_{\text{Origin}}$ in the form of Equation (6), (3) the extended method COPE in the form of Equation (W2) in Web Appendix A.1, and (4) the proposed 2sCOPE in the form of Equation (12). We set the sample size $T = 1000$, and generate 1000 data sets as replicates using the DGP above. In the simulation, we use the gamma distribution $Gamma(1, 1)$ with shape and rate equal to 1 for P_t and the exponential distribution $Exp(1)$ with rate 1 for W_t. Models are estimated on all generated data sets, providing the empirical distributions of parameter estimates.

Table 5 reports estimation results. As expected, OLS estimates of both α and β are biased ($t_{bias} = 15.75/8.24$) due to the regressor endogeneity. $\text{Copula}_{\text{Origin}}$ reduces the bias, but still shows significant bias for the coefficient estimates of P_t and W_t. The bias of $\text{Copula}_{\text{Origin}}$ depends on the strength of the correlation between W and P. Stronger correlations between P^* and W^* can cause a larger bias of $\text{Copula}_{\text{Origin}}$ estimates. For example, when the correlation between W^* and P^* increases from 0.5 to 0.7, the bias of estimated α increases by around five times (from 0.055 to 0.260 in Table 5 under the column "$\text{Copula}_{\text{Origin}}$"). The bias confirms our derivation in the model section, demonstrating that using the existing copula method may not solve the endogeneity problem completely with correlated regressors.

ρ_{pw}	Parameters	True	OLS			Copula$_{\text{Origin}}$			COPE			2sCOPE		
			Mean	SE	t_{bias}	Mean	SE	t_{bias}	Mean	SE	t_{bias}	Mean	SE	t_{bias}
0.5	μ	1	0.689	0.045	6.964	1.231	0.081	2.849	1.012	0.093	0.129	1.009	0.059	0.157
	α	1	1.571	0.036	15.75	1.055	0.069	0.791	0.985	0.072	0.213	0.986	0.070	0.197
	β	-1	-1.259	0.031	8.236	-1.289	0.031	9.169	-0.997	0.067	0.038	-0.995	0.042	0.123
	$\rho_{p\xi}$	0.5	-	-	-	0.570	0.047	1.504	0.505	0.055	0.090	0.504	0.038	0.097
	σ_ξ	1	0.862	0.020	6.902	1.011	0.043	0.244	1.008	0.041	0.206	1.006	0.040	0.143
	D-error			-			-			0.002613			0.001614	
0.7	μ	1	0.730	0.041	6.629	1.307	0.076	4.037	1.011	0.085	0.124	1.005	0.053	0.088
	α	1	1.800	0.041	19.67	1.260	0.068	3.838	0.988	0.078	0.148	0.991	0.075	0.118
	β	-1	-1.529	0.037	14.21	-1.567	0.037	15.36	-0.997	0.071	0.041	-0.994	0.056	0.110
	$\rho_{p\xi}$	0.5	-	-	-	0.633	0.043	3.130	0.503	0.057	0.048	0.500	0.026	0.000
	σ_ξ	1	0.799	0.018	11.18	0.980	0.044	0.468	1.007	0.041	0.160	1.003	0.040	0.084
	D-error			-			-			0.002902			0.001760	

Table 5: Results of the Simulation Study Case 1: nonnormal Regressors

Note: Mean and SE denote the average and standard deviation of parameter estimates over all the 1,000 simulated samples.

The proposed 2sCOPE method provides consistent estimates without using instruments. The average estimate of $\rho_{p\xi}$ is close to the true value 0.5 and is significantly different from 0, implying regressor endogeneity detected correctly using 2sCOPE. Moreover, 2sCOPE shows greater estimation efficiency. The standard error of $\alpha(\beta)$ in 2sCOPE is 0.070 (0.042), which is 2.78% (37.31%) smaller than the corresponding standard errors using COPE. We further calculate the estimation precision of COPE and 2sCOPE using the D-error measure $|\Sigma|^{1/K}$ (Arora and Huber 2001, Qian and Xie 2022), where Σ is the covariance matrix of the regression coefficient estimates, and K is the number of explanatory variables in the structural model. A smaller D-error means greater estimation efficiency and improved estimation precision. When $\rho_{pw} = 0.5$, the D-error measure is 0.002613 for COPE and 0.001614 for 2sCOPE (Table 5), and thus 2sCOPE increases estimation precision by 38.2%, meaning that for 2sCOPE to achieve the same precision with COPE, the sample size can be reduced by 38.2%. A 39.3% efficiency gain for 2sCOPE is observed for $\rho_{pw} = 0.7$ (Table 5).

We perform a further simulation study for a small sample size. Specifically, we use the same DGP as described above to generate synthetic data, except with the sample size T=200. Web Appendix E Table W1 reports the results and shows that OLS estimates have endogeneity bias and $\text{Copula}_{\text{Origin}}$ reduces the endogeneity bias but significant bias remains. Our proposed method, 2sCOPE, performs well and has unbiased estimates for the small sample size T=200. The efficiency gain of 2sCOPE relative to COPE appears to be greater when the sample size becomes smaller. When the correlation between P^* and W^* is 0.5, the D-error measures are 0.0166 and 0.0091 for COPE and 2sCOPE (Web Appendix Table W1), respectively, meaning that 2sCOPE increases estimation precision by 1-0.0091/.0166=46% compared with COPE. Thus, the sample size can be reduced by almost half (∼50%) for 2sCOPE to achieve the same estimation precision as that achieved by COPE. A similar magnitude of efficiency gain for 2sCOPE relative to COPE (∼50%) is observed when the correlation between P^* and W^* is 0.7 (Web Appendix Table W1).

Case 2: Normal Regressors

Next, we examine the case when the endogenous regressor and (or) the correlated exogenous regressor are normally distributed. We pay special attention to this case because normality is not allowed for endogenous regressors in Park and Gupta (2012). We use the same DG as described in Equations (20) to (23) to generate the data, except that the marginal CDFs for regressors, $H(\cdot)$ and $L(\cdot)$, are chosen according to the distributions listed in the first two columns in Table 6.

Table 6 summarizes the estimation results. As expected, OLS estimates are biased. $\text{Copula}_{\text{Origin}}$ produces biased estimates whenever the endogenous regressor P follows a normal distribution. The estimates of $\text{Copula}_{\text{Origin}}$ are biased when P follows a gamma distribution (first row of Table 6) for a different reason: P and W are correlated. Similar to $\text{Copula}_{\text{Origin}}$, the COPE estimators are biased in all the three scenarios when either P_t or W_t is normal. When W_t is normal, β is 0.323 away from the true value -1; when P_t is normally distributed, α is 0.684 away from the true value; when both P_t and W_t are normal, α is 0.663 away from the true value 1 and β is 0.324 away from the true value -1. This is expected because COPE adds P_t^* and W_t^*, the copula transformation of regressors, as additional regressors, and will cause perfect co-linearity and model non-identification problem whenever at least one of these regressors is normally distributed.

Distribution				OLS			Copula$_{\text{Origin}}$			COPE			2sCOPE		
P	W	Parameters	True	Mean	SE	t_{bias}	Mean	SE	t_{bias}	Mean	SE	t_{bias}	Mean	SE	t_{bias}
Gamma	Normal	μ	1	0.431	0.045	12.63	1.018	0.078	0.227	1.017	0.080	0.217	1.015	0.077	0.190
		α	1	1.569	0.037	15.40	0.979	0.070	0.302	0.979	0.070	0.296	0.985	0.070	0.212
		β	-1	-1.259	0.030	8.619	-1.333	0.028	11.78	-1.323	0.433	0.746	-0.997	0.045	0.067
		$\rho_{p\xi}$	0.5	-	-	-	0.640	0.039	3.556	0.589	0.141	0.631	0.506	0.036	0.151
		σ_ξ	1	0.861	0.019	7.240	1.064	0.046	1.394	1.135	0.162	0.837	1.005	0.038	0.134
Normal	Exp	μ	1	1.286	0.042	6.777	1.286	0.045	6.374	0.994	0.073	0.081	1.023	0.070	0.334
		α	1	1.628	0.031	20.36	1.532	0.462	1.152	1.684	0.437	1.568	1.048	0.126	0.381
		β	-1	-1.286	0.032	8.956	-1.287	0.032	8.960	-0.992	0.066	0.127	-1.024	0.062	0.383
		$\rho_{p\xi}$	0.5	-	-	-	0.089	0.419	0.980	-0.167	0.384	1.738	0.465	0.074	0.473
		σ_ξ	1	0.829	0.018	9.492	0.940	0.151	0.394	0.981	0.151	0.129	0.980	0.063	0.318
Normal	Normal	μ	1	1.001	0.026	0.046	1.002	0.030	0.052	1.001	0.033	0.024	1.002	0.028	0.057
		α	1	1.668	0.030	22.38	1.663	0.450	1.474	1.663	0.460	1.441	1.655	0.395	1.657
		β	-1	-1.335	0.029	11.44	-1.335	0.029	11.42	-1.324	0.438	0.740	-1.328	0.197	1.668
		$\rho_{p\xi}$	0.5	-	-	-	0.006	0.412	1.198	0.001	0.412	2.426	0.010	0.303	1.616
		σ_ξ	1	0.816	0.019	9.687	0.917	0.155	0.534	1.003	0.211	0.016	0.879	0.092	1.317

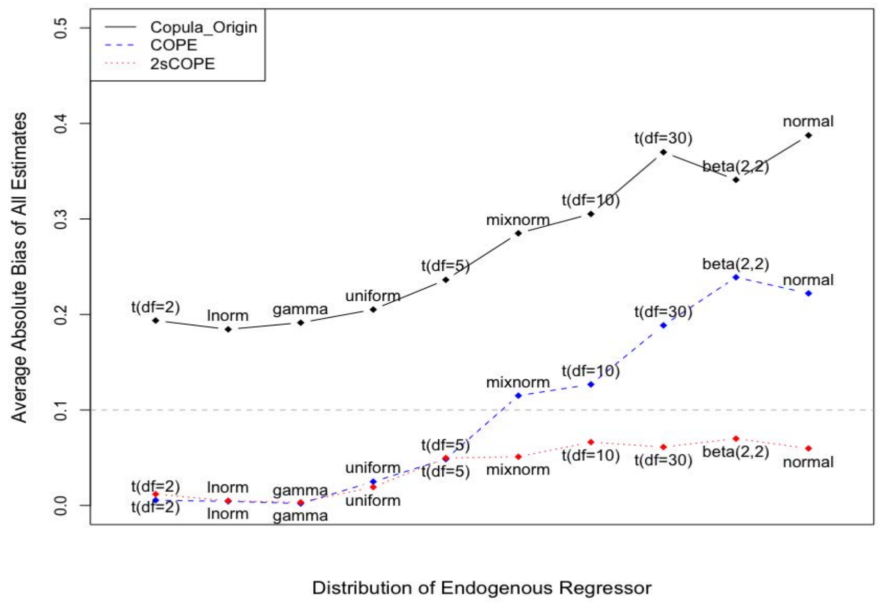
Table 6: Results of Case 2: Normal Regressors

By contrast, the proposed 2sCOPE method provides consistent estimates as long as P_t and W_t are not both normally distributed. Both α and β are tightly distributed near the true value whenever P_t or W_t is nonnormally distributed. Unlike Copula$_{\text{Origin}}$ and COPE, 2sCOPE adds the residual term obtained from regressing P_t^* on W_t^* as the generated regressor. Thus, as long as P_t and W_t are not both normally distributed, the residual term is not perfectly co-linear with the original regressors, permitting model identification. Only when both P_t and W_t are normally distributed (the last scenario in Table 6), the residual term added into the structural regression model becomes a linear combination of P_t and W_t, causing perfect co-linearity and model non-identification. Overall, this simulation study demonstrates the capability of the proposed 2sCOPE to relax the nonnormality assumption in $\text{Copula}_{\text{Origin}}$ as long as one of P_t and W_t is nonnormally distributed.

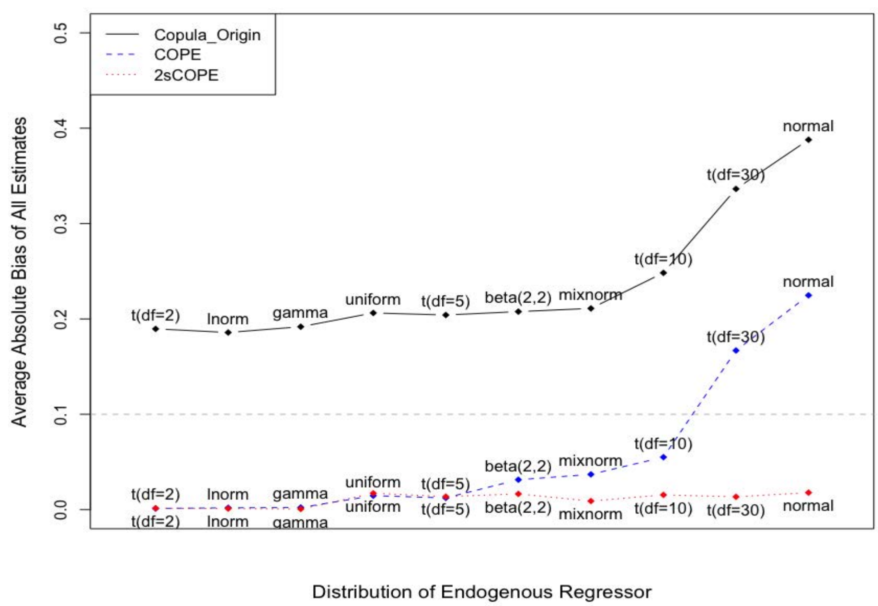
Case 3: Insufficient Nonnormality of Endogenous Regressors

The above case shows that the proposed 2sCOPE can deal with normal endogenous regressors, while $\text{Copula}_{\text{Origin}}$ and COPE cannot. In this case, we examine the performance of these methods in the more common situation of close-to-normal regressors. Although models are identified asymptotically (i.e., infinite sample size), appreciable finite-sample bias can occur with realistic sample sizes commonly seen in marketing studies, if the endogenous regressor is too close to a normal distribution (Becker, Proksch, and Ringle 2021; Haschka 2022; Eckert and Hohberger 2022). Becker, Proksch, and Ringle (2021) suggest a minimum absolute skewness of 2 for an endogenous regressor in order for $\text{Copula}_{\text{Origin}}$ to have good performance in sample sizes less than 1000. This requirement can significantly limit the use of copula correction methods in practical applications. Given that 2sCOPE can handle normal endogenous regressors, we expect that 2sCOPE can handle much better the finite-sample bias caused by insufficient regressor nonnormality than the existing copula correction methods. Thus, in this case, we examine the finite-sample performance of those methods when the distribution of the endogenous regressor has various closeness to normality. We use the DGP as described in Equations (20) to (23) to generate data, except that the marginal CDF for the endogenous regressor ($H(\cdot)$) is varied from some common distributions with varying closeness to normality. Specifically, we consider uniform, log normal, t, mixture normal, gamma, beta and normal distributions, and use the average absolute estimation bias of all the regression parameters (μ, α, β) in the structural model to measure the performance.

Figure 1 plots the estimation bias with different distributions of the endogenous regressor P. Results show estimates of $\text{Copula}_{\text{Origin}}$ are biased with correlated endogenous and exogenous regressors, consistent with our theoretical proof (Theorem 1). COPE performs well when P has sufficient nonnormality ($t(2)$, log normal, gamma) and has no bias even for a sample size as small as 200. However, COPE cannot handle a normal endogenous regres-



(a) Sample Size N=200



(b) Sample Size N=1000

Figure 1: Average absolute estimation bias of all the regression parameters (μ, α, β) in the structural model for different distributions of endogenous regressor.

Note: 'lnorm' is lognormal(0,1), 'mixnorm' is N(-1,1) with the probability 0.5 and N(1,1) with the probability 0.5, 'uniform' is U[0,1], and 'gamma' is $Gamma(1, 1)$.

sor and yields a large estimation bias that remains unchanged as the sample size increases, consistent with our theoretical proof in Theorem 3 (Web Appendix B.2) and the simulation result in Case 2. Furthermore, COPE suffers from finite-sample bias when the endogenous regressor P has distributions with insufficient nonnormality (e.g., beta(2,2), t(df = 30)). Moreover, the estimation bias of COPE is larger when the sample size is smaller or the distribution of the endogenous regressor P is closer to normal. For instance, t-distribution with a degree of freedom 30 is closer to normal than the t distribution with degrees of freedom 10, 5 and 2, resulting in a larger estimation bias. For $t(\text{df} = 30)$ which is very close to normal, increasing the sample size from T=200 to 1000 barely changes the size of the estimation bias. By contrast, our proposed 2sCOPE method yields consistent estimates for all normal and close-to-normal regressor distributions and has negligible finite-sample bias even for a sample size as small as 200 (bias $<$ 5% of parameter values).

Case 4: Random Coefficient Linear Panel Model

We investigate the performance of 2sCOPE in the random coefficient linear panel model. We use the copula function and marginal distributions of $[P_{it}, W_{it}, \xi_{it}]$ as specified in Case 1 (Equations 20-22). We assign $\rho_{pw} = 0.7$ as an example. We then generate the outcome Y_{it} using the following standard random coefficient linear panel model:

$$Y_{it} = \bar{\mu} + \mu_i + P_{it}(\bar{\alpha} + a_i) + W_{it}(\bar{\beta} + b_i) + \xi_{it} = 1 + \mu_i + P_{it}(1 + a_i) + W_{it}(-1 + b_i) + \xi_{it},$$

where $[\mu_i, a_i, b_i] \sim N(0, I_3)$, $t = 1, ..., 50$ indexes occasions for repeated measurements, and $i = 1, ..., 500$ indexes the individual units. The above random coefficients model permits individual units to have heterogeneous baseline preferences (μ_i) and heterogeneous responses to regressors (a_i, b_i). Such random coefficient models are frequently used in marketing studies to capture individual heterogeneity and to profile and target individuals. The correlation between ξ_{it} and P_{it} creates the regressor endogeneity problem, which can cause biased estimates for standard linear random coefficient estimation methods ignoring the regressor-error correlation. We generate individual-level panel data as described above 1000 times and use the data for estimation. Estimation results are in Table 7. LME is the standard estimation method for linear mixed models assuming all regressors are exogenous, as implemented in the R function `lme()`. LME and $\text{Copula}_{\text{Origin}}$ are biased because of endogeneity and correlated exogenous regressors, respectively. Our proposed method 2sCOPE provides unbiased estimates that are tightly distributed around the true values for all parameters.

		LME			$\text{Copula}_{\text{Origin}}$			2sCOPE		
Parameters	True	Mean	SE	t_{bias}	Mean	SE	t_{bias}	Mean	SE	t_{bias}
$\bar{\mu}$	1	0.722	0.046	6.052	1.314	0.049	6.399	1.004	0.048	0.091
$\bar{\alpha}$	1	1.853	0.045	18.83	1.293	0.045	6.469	1.000	0.046	0.008
$\bar{\beta}$	-1	-1.557	0.045	12.39	-1.598	0.044	13.56	-1.000	0.044	0.005
σ_μ	1	0.985	0.033	0.459	0.982	0.033	0.547	0.984	0.031	0.522
σ_α	1	0.988	0.036	0.326	0.987	0.034	0.397	0.989	0.035	0.316
σ_β	1	0.993	0.031	0.235	0.992	0.033	0.249	0.992	0.033	0.248
$\rho_{p\xi}$	0.5	-	-	-	0.646	0.009	16.33	0.507	0.005	1.365
σ_ξ	1	0.794	0.004	57.71	0.957	0.010	4.439	0.985	0.009	1.640

Table 7: Results of the Simulation Study Case 4: Random Coefficient Linear Panel Model
Note: $\sigma_\mu, \sigma_\alpha, \sigma_\beta$ are standard deviations of μ_i, a_i, b_i.

Additional Simulation Results and Robustness Checks

Web Appendix E provides additional simulation results on a small sample size (E.1), model estimation with multiple endogenous regressors (E.2), estimation with multiple exogenous control covariates including binary and close-to-normal control covariates (E.3), the robustness of 2sCOPE to mis-specifications of the structural error distribution (E.4), the robustness of 2sCOPE to mis-specifications of the copula dependence structure (E.5 & E.6), the experimental studies to obtain practical recommendations for using 2sCOPE (E.8), and the performance of 2sCOPE with one 'strongly-nonnormal' exogenous regressor vs. multiple 'weakly-nonnormal' exogenous regressors for handling an endogenous regressor with insufficient nonnormality (E.9). Overall, these results demonstrate 2sCOPE is robust to small sample sizes and reasonable violations of normal error and Gaussian copula assumptions, and provide guidance of using 2sCOPE to obtain good performance as summarized in the next section. Interestingly, results in Web Appendix E.9 show that a 'strongly-nonnormal' W is considerably more effective than multiple 'weakly-nonnormal' Ws in helping the identification of the causal effect for an endogenous regressor with insufficient nonnormality.

GUIDELINES FOR USING 2SCOPE

To summarize, we have established theoretical conditions that guarantee desirable large-sample properties of 2sCOPE when there exist correlated exogenous regressors (Theorem 2) and endogenous regressors have insufficient nonnormality (Theorem 3). As expected, simulation studies demonstrate the good performance of 2sCOPE when the sample size is sufficiently large. Meanwhile, simulation studies also reveal that, in finite samples, good performance of 2sCOPE may require sufficient nonnormality of regressors and sufficient relevance between P and W (e.g., Figure 1 (a)). To provide actionable guidelines for using 2sCOPE for data at hand, we conduct systematic simulation studies to establish the boundary conditions for using 2sCOPE. Specifically, the studies employ a factorial experimental design, which vary systematically distributions of P and W, sample sizes, the level of endogeneity, and the strength of correlation between P and W. We evaluate the performance of 2sCOPE using the relative bias of structural model parameters. Details of the experimental design and results are described in Web Appendix E.8.

Figure 2 shows the decision tree of when to use 2sCOPE based on the results from the simulation studies. The decision tree contains three steps in total. In step 1, we test Assumption 5 (or 5(b) for multiple endogenous regressors) to choose between 2sCOPE and $\text{Copula}_{\text{Origin}}$. When Assumption 5 (5(b)) is satisfied, $\text{Copula}_{\text{Origin}}$ is preferred over 2sCOPE because though both methods can provide consistent estimates, $\text{Copula}_{\text{Origin}}$ estimator is more efficient (Web Appendix E.7). In this case, Becker, Proksch, and Ringle (2021) provide a flowchart for the use of $\text{Copula}_{\text{Origin}}$. If Assumption 5 (5(b)) is violated, this means the presence of relevant exogenous regressors which can be leveraged by 2sCOPE to better han-

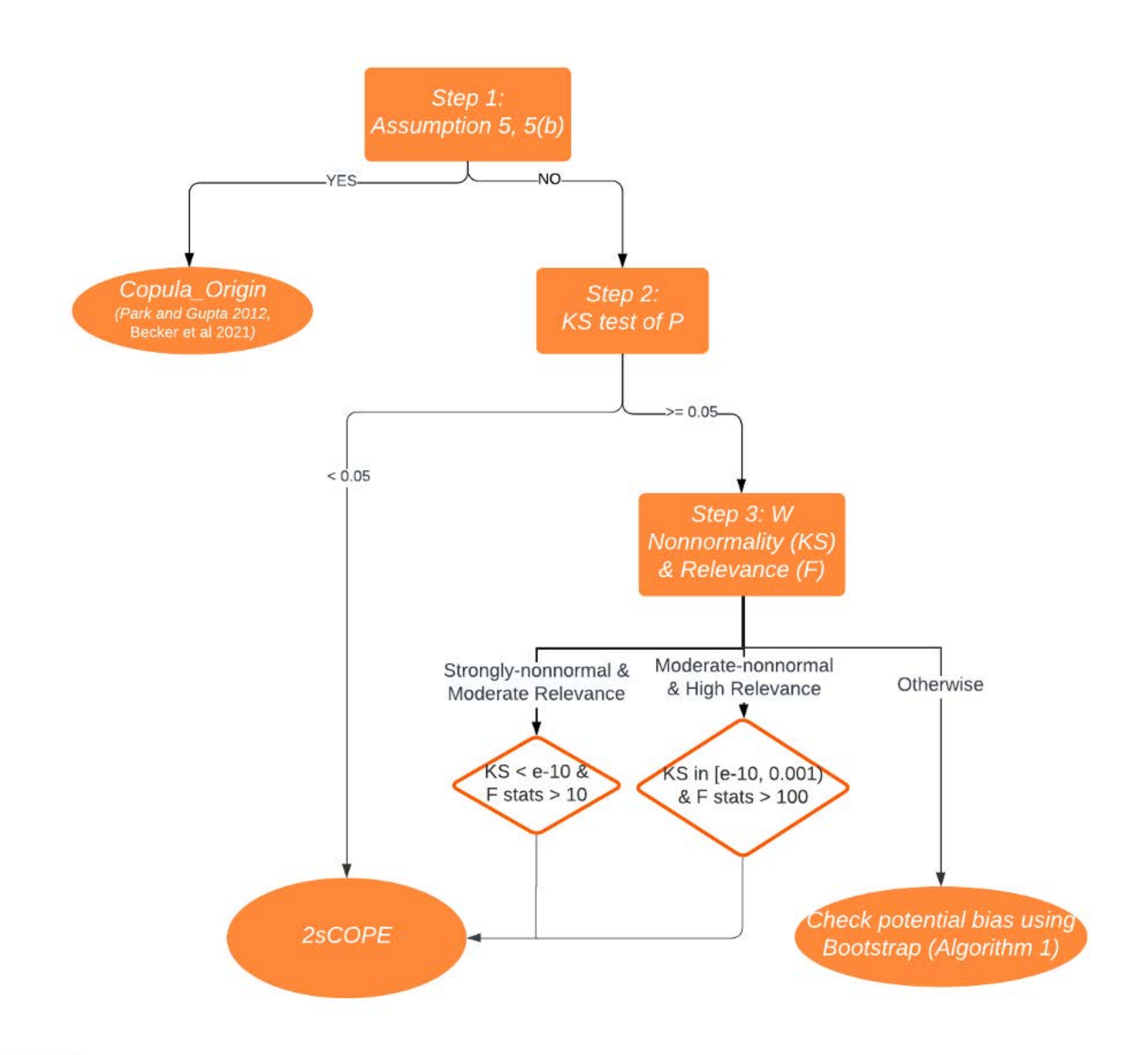


Figure 2: Decision Tree for Using 2sCOPE.
Note: P and W stand for endogenous and exogenous regressors, respectively.

dle endogeneity. In step 2, we test the nonnormality of the endogenous regressor P using the Kolmogorov -Smirnov (KS) test (see Web Appendix E.8 for the rationale of using the test of normality). If the KS test rejects the null at the 0.05 level, this means P possesses sufficient nonnormality and 2sCOPE has a high probability of success in correcting endogeneity bias based on the results in Web Appendix E.8. Otherwise, P has a close-to-normal distribution, which requires related exogenous regressors with sufficient nonnormality to help identification. Thus, in step 3, we check the nonnormality of W and its relevance to P. Results in

Web Appendix E.8 show: (1) If the p-value of the KS test of an exogenous regressor W is smaller than e^{-10} (i.e., strong nonnormality of W), only a moderate relevance between P and W (F statistic for the effect of W^* on $P^* > 10$ in the first-stage regression) is required for 2sCOPE to have a high probability of success; (2) If the p-value of the KS test of W is only smaller than 0.001 (i.e., moderate nonnormality of W), a relatively high relevance between P and W (F statistic for W^* on $P^* > 100$) is required for 2sCOPE to have a high probability of success.

We have provided the sufficient conditions of endogenous and exogenous regressors above in steps 2 and 3 for 2sCOPE to have good finite-sample performance. These are not necessary conditions but are conservative ones to be on the safe side. In particular, to obtain the sufficient conditions, we consider the extreme cases in which either the exogenous regressor in step 2 or endogenous regressor in step 3 follows the normal distribution. However, in practice, regressors are likely to have close-to-normal rather than exact normal distributions. The failure of the sufficient condition tests of W in practice does not mean 2sCOPE cannot be used. For instance, the estimation result of scenario 1 in Table W11 (P and W are close-to-normal and weakly nonnormal, respectively) demonstrates that 2sCOPE may still have acceptable finite-sample performance when the above (conservative) sufficient conditions are not satisfied. In this situation (the rightmost branch in Figure 2), one can rely on our proposed bootstrap resampling Algorithm 1 to evaluate the finite-sample performance of 2sCOPE on a case-by-case basis.

Bootstrap simulations can be used to evaluate the size of the bias in parameter estimates that may arise when sample size is small to moderate (Efron and Tibshirani 1994, Chpt. 10; Hooker and Mentch 2018)[9], even if the estimation performs well for large samples. Specifically, the proposed Algorithm 1 randomly draws the same number of observations from the underlying copula model and the structural model estimated using the original sample,[10]

[9]Note that this bootstrap simulation is different from and should not be confused with the bootstrap method mentioned in Table 4 and used to obtain the standard errors of 2sCOPE estimates.

[10]When $\rho_{P\xi}$ is set at zero (i.e., no endogeneity), 2sCOPE is expected to have no finite-sample bias since in this case 2sCOPE reduces to OLS which is unaffected by regressor normality. Thus, if $\hat{\rho}_{P\xi}$ is small and not significantly

Algorithm 1 A Bootstrap Algorithm for Evaluating Finite-sample Bias of 2sCOPE

Series Input: data Y, P, W, sample size N, $\widehat{\theta}(Y, P, W)-$ 2sCOPE estimates of the structural model parameters, $(\widehat{H}, \widehat{L})-$ empirical CDFs of P and W, and $\widehat{\Sigma}-$ Gaussian copula correlation structure estimate. If the $\hat{\rho}_{P\xi}$ is small and not significantly different from zero, set $\hat{\rho}_{P\xi} = \pm 0.5$ in $\widehat{\Sigma}$.

for $b = 1$ to B **do**

Simulate P_b^*, W_b^*, ξ_b^* from Gaussian Copula $\Psi_{\widehat{\Sigma}}(\Phi^{-1}(U_P), \Phi^{-1}(U_w), \Phi^{-1}(U_\xi))$, sample size=N;

Obtain $P_b = \widehat{H}^{-1}(\Phi(P_b^*)), W_b = \widehat{L}^{-1}(\Phi(W_b^*))$ and $\xi_b = \widehat{\sigma}_\xi \cdot \xi_b^*$, where $\widehat{\sigma}_\xi$ is the 2sCOPE estimate of the standard deviation of structural error term;

Obtain $Y_b = f(P_b, W_b, \xi_b, \widehat{\theta}(Y, P, W))$, where f is the linear regression in this setting;

Obtain the 2sCOPE estimate $\widehat{\theta}_b = \widehat{\theta}(Y_b, P_b, W_b)$ using the bth bootstrap sample.

end for

Calculate potential bias of the 2sCOPE estimator: $\frac{1}{B}\sum_{b=1}^{B}\widehat{\theta}_b - \widehat{\theta}(Y, P, W)$.

and then performs the 2sCOPE estimation on the bootstrap sample as done with the original sample. We repeat this simulation B times, and obtain a distribution for each model coefficient estimate. We then compare the mean of each coefficient estimate's distribution with the corresponding coefficient estimate using the original data, which is the true parameter value in our model-based bootstrap re-sampling. The small-sample bias of a coefficient estimate is the difference between the average coefficient estimate from bootstrap samples and the coefficient estimate from the original sample.

EMPIRICAL APPLICATION

In this section, we apply our method to a real marketing application. We illustrate the proposed method to address the price endogeneity issue using store-level sales data of the toothpaste category in Chicago over 373 weeks from 1989 to 1997 [11]. To control for product size, we select toothpastes with the most common size, which is 6.4 oz. Specifically, we estimate the following sales model:

$$\log(\text{Sales}_t) = \beta_0 + \log(\text{Retail Price}_t) \cdot \beta_1 + W_t'\beta_2 + \xi_t,$$

different from zero, we recommend setting $\widehat{\rho}_{P\xi}$ to a plausible non-zero value (e.g., ± 0.5 as suggested in Algorithm 1).

[11] We obtained the data from https://www.chicagobooth.edu/research/kilts/datasets/dominicks.

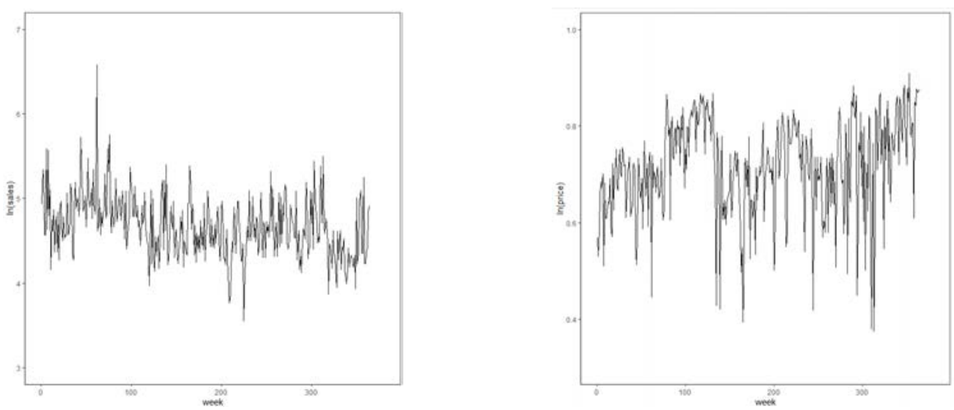
where $t = 1, 2, ..., T$ indexes week. Retail price is usually considered endogenous in the demand model. The endogeneity of retail price can come from unmeasured product characteristics or demand shocks that can influence both consumers' and retailers' decisions. Since these variables are unobserved by researchers, they are absorbed into the structural error, leading to the endogeneity problem. Prices of different stores are correlated and often used as an IV for each other. This allows us to test the performance of the proposed 2sCOPE method in an empirical setting where a good IV exists. Besides the endogenous price, two promotion-related variables, bonus promotion and direct price reduction, would also affect demand. Following Park and Gupta (2012), we treat the promotion variables as exogenous regressors. In general, the promotion decisions during the study period were made on a quarterly basis or even longer, plus a long lead time (e.g. several weeks) for implementation; thus, they were unlikely to be correlated with the weekly unobserved demand shock, and can be considered exogenous (Chintagunta 2002, Sriram, Balachander, and Kalwani 2007).[12] We focus on category sales in two large stores in Chicago (referred to as Stores 1 and 2). We convert retail price, in-store promotion and sales from UPC level to aggregate category level. They are computed as weekly market share-weighted averages of UPC-level variables.

The correlation between log retail price and bonus promotion in Store 1 (Store 2) is -0.30 (-0.15), and the correlation between log retail price and price reduction promotion in Store 1 (Store 2) is -0.23 (-0.35). The appreciable correlations between price and promotion variables actually provide a good setting for testing our method with correlated endogenous and exogenous regressors. The moderate sample size (T=373) also provides an opportunity to evaluate the finite-sample performance of the 2sCOPE method in the presence of potentially insufficient regressor nonnormality in real data. Summary statistics of key variables are summarized in Table 8. Figure 3 plots log sales and log retail prices of toothpaste at store 1 over time (store 2 is very similar). To control for the possible trend of retail price over

[12] We also checked the endogeneity of the bonus and price reduction promotion variables using Hausman test employing Hausman-style IVs (promotions in the other store). The p-values of the Hausman test are 0.30 for bonus promotion and 0.144 for price reduction in store 1 (store 2 is similar), which means there is no evidence that the two promotion variables are endogenous, consistent with prior literature.

	Store 1				Store 2			
Variables	Mean	SD	Max	Min	Mean	SD	Max	Min
Sales (Unit)	115	52.8	720	35	165.7	93.7	1334	26
Price ($)	2.06	0.20	2.48	1.46	2.10	0.21	2.48	1.47
Bonus	0.18	0.20	0.80	0.00	0.16	0.19	0.79	0.00
PriceRedu	0.10	0.19	0.72	0.00	0.10	0.19	0.73	0.00

Table 8: Summary Statistics



(a) Store 1 log sales **(b)** Store 1 log retail price

Figure 3: Log Sales and Log Retail Price of Toothpaste in Store 1.

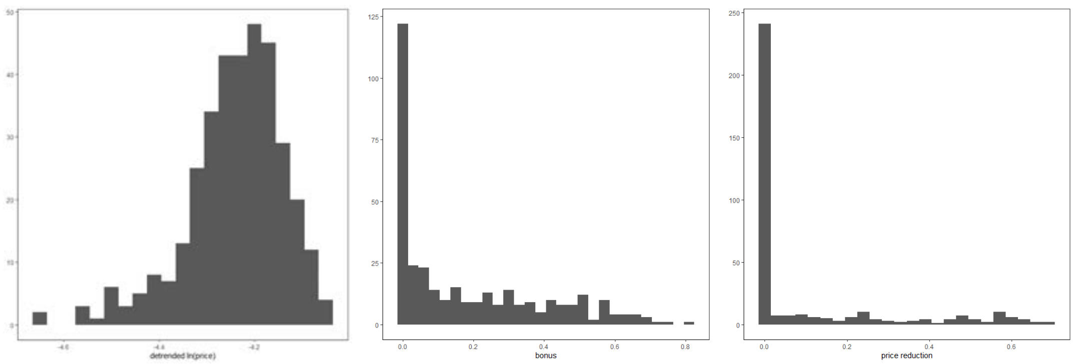
time, we use detrended log retail prices (and for IVs as well) for estimation below. Figure 4 shows the histograms of detrended log retail prices and the two promotion variables. All the three variables are continuous variables.

We follow the flowchart in Figure 2 to guide the use of 2sCOPE in the application. First, the correlations between $\log P^*$ and the exogenous regressors are -0.44[13] for bonus promotion and -0.26 for price reduction promotion, both of which are substantially different from zero with p-value $< 2.2 \times e^{-16}$ and $7.542 \times e^{-08}$ respectively, indicating a violation of Assumption 5 required for $\text{Copula}_{\text{Origin}}$ to yield consistent estimates. Next, we check the sufficient nonnormality of the endogenous regressor. The KS test of the endogenous price

[13]These correlations are different from the correlations between the original endogenous and exogenous regressors without copula transformation. For example, cor(logP,bonus) = -0.30.

yields a p-value of $0.063 > 0.05$, concluding insufficient nonnormality of the endogenous price which $\text{Copula}_{\text{Origin}}$ (or COPE) cannot handle appropriately. We then move to the next step to check the nonnormality of the exogenous regressors, and the relevance between the endogenous and exogenous regressors. Bonus variable is strongly nonnormal (p-value of KS test=$3.159 \times e^{-12}$), and is moderately relevant (F statistic $= 89.5 > 10$). Price reduction is also strongly nonnormal (p value of KS test $< 2.2 \times e^{-16}$), and is moderately relevant (F statistic $= 27.3 > 10$). Thus, according to Figure 2, this empirical data set is appropriate for using 2sCOPE to correct endogeneity, and 2sCOPE is expected to have a high probability to achieve good finite-sample performance. Thus, we proceed to estimate the demand model using the 2sCOPE method, in addition to the OLS and two-stage least-squares (TSLS).

We use the IV-based TSLS estimator as a benchmark to test the validity of our proposed method. Following Park and Gupta (2012), we use retail price at the other store as an instrument for price. This variable can be a valid instrument as it satisfies the two key requirements. First, retail prices across stores in a same market can be highly correlated because wholesale prices are usually offered the same (or very close). The Pearson correlation between the detrended log retail prices at Stores 1 and 2 is 0.79, providing strong explanatory power on the endogenous price. The correlation is comparable to that in Park and Gupta (2012). Second, some unmeasured product characteristics such as shelf-space allocation, shelf location and category location are determined by retailers and are usually not systematically related to wholesale prices (exclusion restriction). However, like any other IVs, the validity claim cannot be fully verified, and is debatable. We therefore perform both 2sCOPE and TSLS to cross-validate each other. Like TSLS, 2sCOPE includes (and makes use of) the existing exogenous regressors in the first-stage regression; however, unlike TSLS, no extra IVs are needed or included in 2sCOPE. Specifically, we first regress $\text{logP}^* = \Phi^{-1}(\widehat{H}(\text{logP}))$ on $\text{Bonus}^* = \Phi^{-1}(\widehat{L}_1(\text{Bonus}))$ and $\text{PriceRedu}^* = \Phi^{-1}(\widehat{L}_2(\text{PriceRedu}))$, and then add the residual as the only "generated regressor" to the outcome regression. $\widehat{H}(\cdot), \widehat{L}_1(\cdot), \widehat{L}_2(\cdot)$ are all estimated CDFs using the univariate empirical distribution for each regressor. Standard



(a) detrended log price **(b)** bonus **(c)** price reduction

Figure 4: Histogram of Log Retail Price, Bonus and Price Reduction in Store 1

errors of parameter estimates are obtained using bootstrap (Web Appendix F).

Table 9 reports the estimation results. Beginning with the results from Store 1, OLS estimates are significantly different from TSLS estimates, indicating that the price endogeneity issue occurs. Instrumenting for retail price changes the price coefficient estimate from -0.767 to -1.797, implying that there is a positive correlation between unobserved product characteristics and the price. The 2sCOPE estimate of ρ, representing the correlation between the endogenous regressor P_t and the error term, is 0.297 (t-value=3.34) and significantly positive, further confirming our previous conclusion. This direction of correlation is consistent with previous empirical findings (e.g., Villas-Boas and Winer 1999, Chintagunta, Dubé, and Goh 2005). The price elasticity estimate from the proposed method 2sCOPE is -2.014, which is close to the estimate of -1.797 from the TSLS method. We confirm in the literature that the TSLS and 2sCOPE estimates are reasonable because the price elasticity of toothpaste category is around -2.0 (Hoch et al. 1995, Mackiewicz and Falkowski 2015).

Unlike Store 1, the results from Store 2 indicate that the retail price is not endogenous. First, the estimate of ρ (the correlation between price and the error term) is not significantly different from 0 for 2sCOPE (t-value ≤ 1.96 under column "2sCOPE" for Store 2 in Table 9). Second, the estimated price coefficient of OLS is -1.982, which is very close to the estimates of TSLS and 2sCOPE in Store 1, further confirming no endogeneity of price in Store 2. Overall, the price elasticity estimates from TSLS and 2sCOPE method are close to each

		OLS			TSLS			2sCOPE		
Store	Parameters	Est	SE	t-value	Est	SE	t-value	Est	SE	t-value
Store 1	Constant	1.301	1.197	0.25	-2.993	1.646	1.82	-3.908	2.314	1.69
	Price	-0.767	0.288	2.66	-1.797	0.396	4.54	-2.014	0.555	3.63
	Bonus	0.371	0.122	3.31	0.104	0.141	0.74	0.064	0.171	0.37
	PriceRedu	0.498	0.115	4.33	0.285	0.125	2.28	0.275	0.143	1.92
	ρ	-	-	-	-	-	-	0.297	0.089	3.34
Store 2	Constant	-3.898	1.246	3.13	0.763	1.943	0.39	0.001	2.702	0.00
	Price	-1.982	0.300	6.61	-0.864	0.467	1.85	-1.048	0.648	1.62
	Bonus	0.062	0.116	0.53	0.286	0.148	1.93	0.239	0.151	1.58
	PriceRedu	0.283	0.111	2.55	0.540	0.137	3.94	0.467	0.152	3.07
	ρ	-	-	-	-	-	-	-0.188	0.109	1.72

Table 9: Estimation Results: Toothpaste Sales

other for Store 2, and the observed differences between them and the OLS estimate can be attributed to estimation variability incurred from using more complicated models instead of the presence of endogeneity.

Evaluating Finite-Sample Performance of Copula Correction Using Bootstrap In the above, the convergence of results between TSLS and the proposed 2sCOPE in both stores supports the validity of the proposed method in addressing the endogeneity issue. The flowchart in Figure 2 also suggests our empirical data satisfy the boundary conditions under which 2sCOPE is expected to have good finite-sample performance. Though, under this case, there is little need to empirically evaluate the finite-sample performance using the bootstrap resampling in Algorithm 1, we apply the algorithm to illustrate its usage in the empirical application. Specifically, we apply the bootstrap algorithm (Algorithm 1) to our empirical application with the true parameter values set to be the store 1's 2sCOPE estimates reported in Table 9 rounded to the first non-zero number when generating bootstrap samples. We also consider the case in which ρ is set at 0.5, somewhat larger than the estimated value of

ρ (=0.3), to assess the robustness of the bootstrap findings. The detailed steps to generate these bootstrap samples can be found in Web Appendix G.

Table 10 summarizes means and standard deviations of parameter estimates for OLS and 2sCOPE over the 1000 bootstrap samples, unlike the estimation result on one single observed data set reported in Table 9. The estimation results are broadly consistent with those in Table 9. In both cases ($\rho = 0.3$ and 0.5), the estimates of 2sCOPE are distributed closely to the true values, demonstrating that 2sCOPE corrects the bias of OLS estimates and performs well with little finite-sample bias in our empirical application.

		OLS			2sCOPE				OLS			2sCOPE		
Parameters	True	Est	SE	t_{bias}	Est	SE	t_{bias}	True	Est	SE	t_{bias}	Est	SE	t_{bias}
Constant	-4	1.514	0.777	7.098	-3.782	1.619	0.135	-4	5.256	0.635	14.57	-3.601	1.393	0.287
Price	-2	-0.678	0.186	7.099	-1.946	0.388	0.139	-2	0.220	0.152	14.59	-1.904	0.334	0.287
Bonus	0.1	0.458	0.088	4.046	0.113	0.128	0.103	0.1	0.706	0.073	8.290	0.126	0.112	0.236
PriceRedu	0.3	0.571	0.089	3.058	0.309	0.112	0.079	0.3	0.764	0.075	6.160	0.323	0.095	0.240
ρ	0.3	-	-	-	0.284	0.071	0.222	0.5	-	-	-	0.483	0.048	0.360

Table 10: Finite-Sample Performance of Copula Correction. "Est" and "SE" denote the mean and standard deviation of the estimates over 1000 bootstrap samples of Store 1 Data.

CONCLUSION

Causal inference lies at the center of social science research, and observational studies often beg rigorous study designs and methodologies to overcome endogeneity concerns. It is preferable to bring information via good instruments for identification, although this is not always possible. In this paper, we focus on the IV-free copula method to handle endogenous regressors. We propose a generalized two-stage copula endogeneity correction (2sCOPE) method that extends the existing copula correction methods (Park and Gupta 2012; Becker, Proksch, and Ringle 2021; Haschka 2022; Eckert and Hohberger 2022) to more general settings. Specifically, 2sCOPE allows exogenous regressors to be correlated with endogenous regressors and relaxes the nonnormality assumption on the endogenous regressors. Similar to the original copula correction method ($\text{Copula}_{\text{Origin}}$), 2sCOPE corrects endogeneity by adding "generated regressors" derived from the existing regressors and is straightforward to use. However, unlike $\text{Copula}_{\text{Origin}}$ that adds the latent copula transformations of endogenous regressors directly into the model, 2sCOPE has two stages. The first stage obtains the residuals from regressing latent copula data for the endogenous regressor on the latent copula data for the exogenous regressors. The second stage uses the first-stage residual as a "generated regressor" in the structural regression model. We theoretically prove that 2sCOPE yields consistent cause-effect estimates when exogenous regressors are correlated with the endogenous regressors. 2sCOPE can also relax the nonnormality assumption on endogenous regressors and substantially improve the finite-sample performance of copula correction.

We evaluate the performance of 2sCOPE via simulation studies and demonstrate its use in an empirical application. The simulation results show that 2sCOPE yields consistent estimates under relaxed assumptions. Moreover, 2sCOPE outperforms $\text{Copula}_{\text{Origin}}$ (and COPE) in terms of dealing with close-to-normal or normal endogenous regressors and improving estimation efficiency. Endogenous regressors are allowed to be close-to-normal or even normal distributions with the help of exogenous regressors (see conditions in Figure 2). The efficiency gain relative to COPE is substantial and can be up to $\sim$80% in simulation studies (Web Appendix E.3), implying that 2sCOPE can reduce the sample size by $\sim$80% needed to achieve the same estimation efficiency as compared with COPE that does not exploit the correlations between endogenous and exogenous regressors. Last but not the least, our robustness checks show that the proposed 2sCOPE is reasonably robust to the structural error distributional assumption and non-Gaussian copula correlation structure (Web Appendix E.4, E.5 & E.6). We further apply 2sCOPE to a public dataset in marketing. When dealing with endogenous price, we find that the estimated price coefficient using our proposed 2sCOPE is very close to the TSLS estimate and the price coefficient reported in the literature, while OLS estimator shows large biases. We further illustrate the use of a novel bootstrap simulation algorithm to evaluate and validate the finite-sample performance of 2sCOPE in the empirical application.

These findings have rich implications for guiding the practical use of the copula-based IV-free methods to handle endogeneity. A known critical assumption for $\text{Copula}_{\text{Origin}}$ is the nonnormality of endogenous regressors. The users of the method in the literature have all been practicing the check and verification of this assumption. However, our work shows that this is insufficient: one also needs to check Assumption 5 for the one-endogenous-regressor case, and Assumption 5(b) for the multiple-endogenous-regressors case. Note that neither assumption is the same as checking the pairwise correlations between the endogenous and exogenous regressors. Assumption 5 evaluates pairwise correlations involving copula transformation of the endogenous regressor, which, as shown in the literature (Danaher and Smith 2011) and in our specific empirical application, can be substantially different from the pairwise correlations using the original variables. Assumption 5(b) evaluates the correlations between exogenous regressors and the linear combination of generated regressors, which are even more different from checking pairwise correlations on the regressors themselves. When the above assumptions are satisfied, $\text{Copula}_{\text{Origin}}$ is preferred to our proposed 2sCOPE method (Step 1 in the flowchart depicted in Figure 2), since the simpler and valid model outperforms more general but more complex models.

If any endogenous regressor has insufficient nonnormality, or any exogenous regressor violates the Assumptions 5 or 5(b), our proposed 2sCOPE method can be used instead of $\text{Copula}_{\text{Origin}}$. The 2sCOPE is straightforward to extend to many other settings, and we have derived 2sCOPE for a range of commonly used marketing models, including linear regression models, linear panel models with mixed-effects, random coefficient logit models and slope endogeneity. The 2sCOPE method proposed here can be applied to these and many other cases not studied here, while accounting for correlations between exogenous and endogenous regressors and exploiting the correlations for model identification in the presence of insufficient nonnormality of endogenous regressors. When endogenous regressors all have sufficient nonnormality (p-value of KS test < 0.05), our evaluation shows that 2sCOPE is expected to perform well. If any endogenous regressor has insufficient nonnormality, 2sCOPE exploits exogenous regressors with sufficient levels of relevance and nonnormality (with detailed sufficient conditions shown in Figure 2) for satisfactory model identification in finite samples. One can empirically check and verify whether these conditions are satisfied for data at hand, using tests of normality and relevance. When these sufficient conditions are not satisfied, we also propose a bootstrap algorithm to directly gauge and validate the finite-sample performance of 2sCOPE in real applications on a case-by-case basis, complementing the above rules of thumb using tests of normality and relevance.

Unlike the two-stage least-squares method (TSLS), 2sCOPE does not require any IVs that satisfy exclusion restriction (ER). Compared with the exogeneity condition, ER is much more stringent in that the IV is not only exogenous but also does not appear in the outcome model, meaning that the IV cannot affect the outcome Y through any other way besides the endogenous regressor. It is typically impossible to test ER; one has to rely on institutional knowledge and theoretical arguments to establish the credibility of ER that is often the most challenging part in IV applications. By contrast, our approach eliminates the requirement of any variable satisfying the ER assumption, which is an important gain. Using 2sCOPE, one does not need to argue for ER.

Meanwhile, 2sCOPE is capable of leveraging relevant exogenous variables in W pre-existing in the outcome model (e.g., in Equation (8)) for model identification. Marketing models rarely contain only endogenous regressors. In fact, the vast majority of the outcome models estimated in marketing include exogenous variables for various reasons, such as the inclusion of exogenous regressors as control variables to mitigate the concern of endogeneity of the primary explanatory variables, to make the outcome models substantively complete and relevant, or to make the ER assumption of IVs more plausible. These exogenous regressors are not used for generating the copula control function in $\text{Copula}_{\text{Origin}}$. By contrast, 2sCOPE can leverage these exogenous variables W pre-existing in the OLS, IV or $\text{Copula}_{\text{Origin}}$ estimation of the outcome model, and requires no more arguments made to justify the exogeneity of W than these other methods.[14] Furthermore, exogeneity is considerably weaker than ER. Thus, 2sCOPE imposes little extra burden in finding relevant exogenous regressors, but can simply leverage the exogenous regressors that already exist in the model and have been used by alternative methods, such as OLS or TSLS. As mentioned above, 2sCOPE gains by not requiring any IVs satisfying the stringent ER condition. No theoretical arguments for the direction and intuition of correlation between W and P are needed. An empirical correlation is sufficient. Finally, when the endogenous regressor does not have sufficient nonnormality, 2sCOPE can leverage exogenous regressors with certain nonnormality and relevance levels (Figure 2), feasible in many applications, for identification.

Although 2sCOPE contributes to solving regressor endogeneity by relaxing key assumptions of the existing copula correction methods and extending them to more general settings, it is not without limitations. For 2sCOPE to work best, the distributions of the endogenous regressors need to contain adequate information. The condition is violated when the endogenous regressors follow Bernoulli distributions or discrete distributions with small support, as noted in Park and Gupta (2012). The proposed 2sCOPE method does not address this limitation. The simplicity of 2sCOPE hinges on the normal structural error and Gaussian copula dependence structure. Our evaluation shows 2sCOPE is robust to symmetric nonnormal error distributions, linear dependence among endogenous and exogenous regressors, and certain non-Gaussian copula structure (Web Appendix E.4, E.5 & E.6). Such robustness may not hold for asymmetric nonnormal error distributions or other forms of dependence structure. Future research is needed for more flexible methods to test and relax these assumptions. Despite these limitations, we expect that 2sCOPE will provide a useful alternative to a broad range of empirical problems when instruments are not available.

[14] These other methods (OLS, IV and $\text{Copula}_{\text{Origin}}$) all require the exogeneity of W as 2sCOPE does. For instance, for the model in Eqn (1), OLS estimate of $\hat{\alpha} = (P'P)^{-1}P'Y - (P'P)^{-1}P'[1, W][\hat{\mu}, \hat{\beta}']'$. The estimates of parameters depend on each other, and thus the inconsistency of $\hat{\beta}$ will make $\hat{\alpha}$ biased even if P is exogenous. In TSLS, only exogenous regressors and IVs can enter the first-stage regression in TSLS and so any endogenous regressors cannot be included in W for TSLS (Wooldridge 2010). The exogeneity of W means that certain types of variables that violate the exogeneity condition, such as colliders, should be excluded from W. Thus, for all these econometric methods, substantive knowledge about the causal mechanism helps inform appropriate exogenous regressors useful for model identification.

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Addressing Endogeneity using a Two-stage Copula Generated Regressor Approach

WEB APPENDIX

These materials have been supplied by the authors to aid in the understanding of their paper. The AMA is sharing these materials at the request of the authors.

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WEB APPENDIX A: PROOFS RELATED TO COPULA$_{\textbf{Origin}}$

Web Appendix A.1: Proof of Theorem 1

Under the Gaussian copula assumption for structural error ξ_t and the endogenous regressor P_t, and the normality assumption of ξ_t, the outcome regression becomes (Equation 6)

$$Y_t = \mu + P_t\alpha + W_t\beta + \sigma_\xi \cdot \rho \cdot P_t^* + \sigma_\xi \cdot \sqrt{1-\rho^2} \cdot \omega_t.$$

Because of the exogeneity assumption of W_t (Assumption 1 in Table 2), $Cov(W_t, \xi_t) = 0$,

$$\begin{aligned} Cov(W_t, \xi_t) &= Cov(W_t, \sigma_\xi \cdot \rho \cdot P_t^* + \sigma_\xi \cdot \sqrt{1-\rho^2} \cdot \omega_t) \\ &= \sigma_\xi \cdot \rho \cdot Cov(W_t, P_t^*) + \sigma_\xi \cdot \sqrt{1-\rho^2} \cdot Cov(W_t, \omega_t) = 0. \end{aligned}$$

Thus, whenever W_t and P_t^* is correlated, the covariance between W_t and P_t^* is

$$Cov(W_t, \omega_t) = -\frac{\rho}{\sqrt{1-\rho^2}} Cov(W_t, P_t^*) \neq 0,$$

and W_t would be correlated with the new error term ω_t. **Theorem proved.**

COPE Method: A Direct Extension of Copula$_{\textbf{Origin}}$

Under the Gaussian copula model for the endogenous regressor, P_t, the correlated exogenous regressor, W_t, and the structural error term, ξ_t in Equations (7,10), the structural error in the main model (Equation 1) can be rewritten as

$$\xi_t = \sigma_\xi \cdot \xi_t^* = \frac{\sigma_\xi \rho_{p\xi}}{1-\rho_{pw}^2} P_t^* + \frac{-\sigma_\xi \rho_{pw} \rho_{p\xi}}{1-\rho_{pw}^2} W_t^* + \sigma_\xi \sqrt{1-\rho_{p\xi}^2 - \frac{\rho_{pw}^2 \rho_{p\xi}^2}{1-\rho_{pw}^2}} \omega_{3,t}. \tag{W1}$$

In this way, the structural error term ξ_t is split into two parts: one part as a function of P_t^* and W_t^* that captures the endogeneity of P_t and the association of W_t with $\xi_t|P_t$ [15], and the other part as an independent new error term. Then, we substitute Equation (W1) into the main model in Equation (1), and obtain the following regression equation:

$$Y_t = \mu + P_t\alpha + W_t\beta + \frac{\sigma_\xi \rho_{p\xi}}{1-\rho_{pw}^2} P_t^* + \frac{-\sigma_\xi \rho_{pw}\rho_{p\xi}}{1-\rho_{pw}^2} W_t^* + \sigma_\xi \sqrt{1-\rho_{p\xi}^2 - \frac{\rho_{pw}^2\rho_{p\xi}^2}{1-\rho_{pw}^2}} \cdot \omega_{3,t}. \quad \text{(W2)}$$

Given P_t^* and W_t^* as additional regressors, $\omega_{3,t}$ is not correlated with all regressors on the right-hand side of Equation (W2) as proven in Theorem 2 in Web Appendix B.1, and thus we can consistently estimate the model using the least squares estimator. The regressors P_t^* and W_t^* can be generated from the nonparametric distribution of P_t and W_t as $P_t^* = \Phi^{-1}(\widehat{H}(P_t))$ and $W_t^* = \Phi^{-1}(\widehat{L}(W_t))$, where $\widehat{H}(P_t)$ and $\widehat{L}(W_t)$ are the empirical CDFs of P_t and W_t, respectively.

COPE method, the direct extension of Copula$_{\text{Origin}}$, does not require the uncorrelatedness between P_t^* and W_t for consistent model estimation, an assumption needed for Copula$_{\text{Origin}}$. However, similar to Copula$_{\text{Origin}}$, COPE requires the nonnormality of the endogenous regressor P_t to fulfill the full-rank identification assumption. In addition, COPE requires the nonnormality of all the exogenous regressor W_t to fulfill the full-rank identification assumption, while our proposed 2sCOPE method relaxes these assumptions.

[15] Although the exogenous regressor W_t and ξ_t are uncorrelated, W_t and $\xi_t|P_t$ (the error component in ξ_t remaining after removing the effect of the endogenous regressor P_t) can be correlated.

Web Appendix A.2: Assumption 5(b) in $\text{Copula}_{\text{Origin}}$

According to Park and Gupta (2012), under a Gaussian copula model for $(P_{1,t}, P_{2,t}, \xi_t)$, the structural model in Equation (13) with two endogenous regressors can be re-expressed as

$$\begin{aligned} Y_t =& \mu + P_{1,t}\alpha_1 + P_{2,t}\alpha_2 + W_t\beta + \sigma_\xi \frac{\rho_{\xi 1} - \rho_{12}\rho_{\xi 2}}{1-\rho_{12}^2} \cdot P_{1,t}^* + \sigma_\xi \frac{\rho_{\xi 2} - \rho_{12}\rho_{\xi 1}}{1-\rho_{12}^2} \cdot P_{2,t}^* \\ & + \sigma_\xi \cdot \sqrt{1 - \rho_{\xi 1}^2 - \frac{(\rho_{\xi 2} - \rho_{12}\rho_{\xi 1})^2}{1-\rho_{12}^2}} \cdot \omega_t. \end{aligned} \tag{W3}$$

where $P_{1,t}^* = \Phi^{-1}(H_1(P_{1,t}))$, $P_{2,t}^* = \Phi^{-1}(H_2(P_{2,t}))$, and $H_1(\cdot)$ and $H_2(\cdot)$ are CDFs of $P_{1,t}$ and $P_{1,t}$, respectively, ρ_{12} is the correlation between $P_{1,t}^*$ and $P_{2,t}^*$, $\rho_{\xi 1}$ is the correlation between ξ and $P_{1,t}^*$, $\rho_{\xi 2}$ is the correlation between ξ and $P_{2,t}^*$, and ω_t is a standard normal random variable that is independent of $P_{1,t}^*$ and $P_{2,t}^*$. For the OLS estimation of Equation (W3) to yield consistent estimates, W_t need also be uncorrelated with ω_t, which requires that $Cov(W_t, \sigma_\xi \cdot \sqrt{1 - \rho_{\xi 1}^2 - \frac{(\rho_{\xi 2} - \rho_{12}\rho_{\xi 1})^2}{1-\rho_{12}^2}} \cdot \omega_t) = -Cov(W_t, \frac{\rho_{\xi 1} - \rho_{12}\rho_{\xi 2}}{1-\rho_{12}^2} \cdot P_{1,t}^* + \frac{\rho_{\xi 2} - \rho_{12}\rho_{\xi 1}}{1-\rho_{12}^2} \cdot P_{2,t}^*) = 0$ (Assumption 5(b) in the main text) where $\frac{\rho_{\xi 1} - \rho_{12}\rho_{\xi 2}}{1-\rho_{12}^2} \cdot P_{1,t}^* + \frac{\rho_{\xi 2} - \rho_{12}\rho_{\xi 1}}{1-\rho_{12}^2} \cdot P_{2,t}^*$ is the CCF used to control for endogeneity in $\text{Copula}_{\text{Origin}}$.

WEB APPENDIX B: PROOFS FOR 2SCOPE

Web Appendix B.1: Proof of Theorem 2 Consistency of 2sCOPE

We have shown the derivation of 2sCOPE method in the main text. The system of equations used in the 2sCOPE method (Equations 8, 9) leads to the following equations

$$\begin{aligned} Y_t &= \mu + P_t\alpha + W_t\beta + \frac{\sigma_\xi \rho_{p\xi}}{1-\rho_{pw}^2}\epsilon_t + \sigma_\xi\sqrt{1-\rho_{p\xi}^2 - \frac{\rho_{pw}^2\rho_{p\xi}^2}{1-\rho_{pw}^2}} \cdot \omega_{3,t}, \\ P_t^* &= \rho_{pw}W_t^* + \epsilon_t. \end{aligned}$$

Since $\omega_{3,t}$ is independent of P_t^* and W_t^*, it would also be uncorrelated with any functional form of P_t^* and W_t^*, and thus $\omega_{3,t}$ is uncorrelated with P_t, W_t and ϵ_t, which satisfies the population orthogonality condition required for consistency of OLS (OLS.1 assumption in Wooldridge 2010). Once P_t or W_t is nonnormal, ϵ_t is not a linear function of P_t and W_t, satisfying the full rank condition required for model consistency of OLS (OLS.2 assumption in Wooldridge 2010). **Theorem proved**.

2sCOPE in Multiple Exogenous Regressors Case Next, we show that this result can be easily extended to the multi-dimension W_t case. We first derive the system of equations of the 2sCOPE method. Here we take 2-dimension W_t as an example. When there are one endogenous regressor P_t and two exogenous regressors W_t, the linear regression becomes:

$$Y_t = \beta_0 + \beta_1 P_t + \beta_2 W_{1,t} + \beta_3 W_{2,t} + \xi_t \tag{W4}$$

Under the Gaussian Copula assumption,

$$\begin{pmatrix} P_t^* \\ W_{1,t}^* \\ W_{2,t}^* \\ \xi_t^* \end{pmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho_1 & \rho_2 & \rho_\xi \\ \rho_1 & 1 & \rho_w & 0 \\ \rho_2 & \rho_w & 1 & 0 \\ \rho_\xi & 0 & 0 & 1 \end{bmatrix} \right) \tag{W5}$$

The multivariate normal distribution can be written as follows:

$$\begin{pmatrix} P_t^* \\ W_{1,t}^* \\ W_{2,t}^* \\ \xi_t^* \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ \rho_1 & \sqrt{1-\rho_1^2} & 0 & 0 \\ \rho_2 & \frac{\rho_w - \rho_1\rho_2}{\sqrt{1-\rho_1^2}} & \sqrt{1-\rho_2^2 - \frac{(\rho_w-\rho_1\rho_1)^2}{1-\rho_1^2}} & 0 \\ \rho_\xi & \frac{-\rho_1\rho_\xi}{\sqrt{1-\rho_1^2}} & \frac{\frac{(\rho_w-\rho_1\rho_2)\rho_1\rho_\xi}{1-\rho_1^2} - \rho_2\rho_\xi}{\sqrt{1-\rho_2^2-\frac{(\rho_w-\rho_1\rho_2)^2}{1-\rho_1^2}}} & \gamma \end{pmatrix} \cdot \begin{pmatrix} \omega_{1,t} \\ \omega_{2,t} \\ \omega_{3,t} \\ \omega_{4,t} \end{pmatrix},$$

where $\omega_{k,t} \sim N(0,1), k = 1,2,3,4,\ \gamma = \sqrt{1-\rho_\xi^2 - \frac{\rho_1^2\rho_\xi^2}{1-\rho_1^2} - \left(\frac{\frac{(\rho_w-\rho_1\rho_2)\rho_1\rho_\xi}{1-\rho_1^2} - \rho_2\rho_\xi}{\sqrt{1-\rho_2^2-\frac{(\rho_w-\rho_1\rho_2)^2}{1-\rho_1^2}}}\right)^2}$. Structural error ξ_t can then be written as a function of P_t^* and W_t^*,

$$\xi_t = \sigma_\xi \xi_t^* = \frac{\sigma_\xi\rho_\xi(1-\rho_w^2)}{1-\rho_1^2-\rho_2^2+2\rho_1\rho_2\rho_w+\rho_w^2}\left(P_t^* - \frac{\rho_1-\rho_2\rho_w}{1-\rho_w^2}W_{1,t}^* - \frac{\rho_2-\rho_1\rho_w}{1-\rho_w^2}W_{2,t}^*\right) + \sigma_\xi\gamma \cdot \omega_{4,t}. \tag{W6}$$

Then we derive the first-stage regression of 2sCOPE

$$\begin{aligned} P_t^* &= \frac{\rho_1-\rho_2\rho_w}{1-\rho_w^2}W_{1,t}^* + \frac{\rho_2-\rho_1\rho_w}{1-\rho_w^2}W_{2,t}^* + \sqrt{1-\rho_1^2-\frac{(\rho_2-\rho_1\rho_w)^2}{1-\rho_w^2}}\omega_{3,t} \\ &= \frac{\rho_1-\rho_2\rho_w}{1-\rho_w^2}W_{1,t}^* + \frac{\rho_2-\rho_1\rho_w}{1-\rho_w^2}W_{2,t}^* + \epsilon_{2,t} \\ &= \gamma_1 W_{1,t}^* + \gamma_2 W_{2,t}^* + \epsilon_{2,t}. \end{aligned} \tag{W7}$$

The structural error ξ_t in Equation (W4) and the first-stage error term $\epsilon_{2,t}$ are linear transformations of the Gaussian data $(\xi_t, P_t^*, W_{1,t}^*, W_{2_t}^*)$ and thus follow a bivariate normal distribution. Thus, ξ_t can be decomposed to a sum of one term containing $\epsilon_{2,t}$ and an independent new error term, resulting in the following regression equation:

$$Y_t = \beta_0 + \beta_1 P_t + \beta_2 W_{1,t} + \beta_3 W_{2,t} + \beta_4 \epsilon_{2,t} + \sigma_\xi \gamma \cdot \omega_{4,t}. \tag{W8}$$

where

$$\beta_4 = \frac{\sigma_\xi \rho_\xi (1 - \rho_w^2)}{1 - \rho_1^2 - \rho_2^2 + 2\rho_1 \rho_2 \rho_w + \rho_w^2}.$$

Since $\omega_{4,t}$ is independent of P_t^*, $W_{1,t}^*$ and $W_{2,t}^*$, it is uncorrelated with any functional form of P_t^*, $W_{1,t}^*$ and $W_{2,t}^*$, and thus $\omega_{4,t}$ is uncorrelated with P_t, $W_{1,t}$, $W_{2,t}$ and $\epsilon_{2,t}$ in Equation (W8). Thus, 2sCOPE that performs OLS regression of Equation (W8) yields consistent model estimates. Without loss of generality, the result can be extended to cases with any dimension of W_t.

2sCOPE in Multiple Endogenous Regressors Case

Under the Gaussian Copula assumption that $[P_{1,t}^*, P_{2,t}^*, W_t^*, \xi_t^*]$ follows a multivariate normal distribution:

$$\begin{pmatrix} P_{1,t}^* \\ P_{2,t}^* \\ W_t^* \\ \xi_t^* \end{pmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho_p & \rho_{wp1} & \rho_{\xi p1} \\ \rho_p & 1 & \rho_{wp2} & \rho_{\xi p2} \\ \rho_{wp1} & \rho_{wp2} & 1 & 0 \\ \rho_{\xi p1} & \rho_{\xi p2} & 0 & 1 \end{bmatrix} \right),$$

we have:

$$\begin{pmatrix} P^*_{1,t} \\ P^*_{2,t} \\ W^*_t \\ \xi^*_t \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ \rho_p & \sqrt{1-\rho_p^2} & 0 & 0 \\ \rho_{wp1} & \frac{\rho_{wp2}-\rho_p \rho wp1}{\sqrt{1-\rho_p^2}} & \sqrt{1-\rho_{wp1}^2 - \frac{(\rho_{wp2}-\rho_p\rho_{wp1})^2}{1-\rho_p^2}} & 0 \\ \rho_{\xi p1} & \frac{\rho_{\xi p2}-\rho_p\rho_{\xi p1}}{\sqrt{1-\rho_p^2}} & \frac{-\rho_{wp1}\rho_{\xi p1}-\frac{(\rho_{wp2}-\rho_p\rho_{wp1})(\rho_{\xi p2}-\rho_p\rho_{\xi p1})}{1-\rho_p^2}}{\sqrt{1-\rho_{wp1}^2-\frac{(\rho_{wp2}-\rho_p\rho_{wp1})^2}{1-\rho_p^2}}} & m \end{pmatrix} \cdot \begin{pmatrix} \omega_{1,t} \\ \omega_{2,t} \\ \omega_{3,t} \\ \omega_{4,t} \end{pmatrix},$$

$$\begin{pmatrix} \omega_{1,t} \\ \omega_{2,t} \\ \omega_{3,t} \\ \omega_{4,t} \end{pmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \right), \tag{W9}$$

where m is a function of all the ρs. Under the Gaussian Copula assumption above, we can derive ξ^*_t as a function of P_t and W_t. After simplification, the structural error in Equation (13) can be decomposed as

$$\xi_t = \sigma_\xi \xi^*_t = \eta_1 P^*_{1,t} + \eta_2 P^*_{2,t} - (\eta_1\rho_{wp1} + \eta_2\rho_{wp2})W^*_t + \sigma_\xi \cdot m \cdot \omega_{4,t}. \tag{W10}$$

where

$$\begin{aligned} \eta_1 &= \frac{\sigma_\xi\rho_{\xi p1}(1-\rho_{wp2}^2) - \sigma_\xi\rho_{\xi p2}(\rho_p - \rho_{wp1}\rho_{wp2})}{1-\rho_p^2-\rho_{wp1}^2-\rho_{wp2}^2+2\rho_p\rho_{wp1}\rho_{wp2}}, \\ \eta_2 &= \frac{\sigma_\xi(\rho_{wp1}\rho_{wp2}\rho_{\xi p1} + \rho_{\xi p2} - \rho_p\rho_{\xi p1} - \rho_{wp1}^2\rho_{\xi p2})}{1-\rho_p^2-\rho_{wp1}^2-\rho_{wp2}^2+2\rho_p\rho_{wp1}\rho_{wp2}}. \end{aligned} \tag{W11}$$

The 2sCOPE method with one endogenous regressor in Equation (12) is then extended to

$$
\begin{aligned}
Y_t &= \mu + P_{1,t}\alpha_1 + P_{2,t}\alpha_2 + W_t\beta + \eta_1\epsilon^*_{1,t} + \eta_2\epsilon^*_{2,t} + \sigma_\xi \cdot m \cdot \omega_{4,t}, \\
\epsilon_{1,t} &= P^*_{1,t} - \rho_{wp1}W^*_t, \\
\epsilon_{2,t} &= P^*_{2,t} - \rho_{wp2}W^*_t.
\end{aligned}
$$

The main model in the first equation above is the same as Equation (17). The new error term $\omega_{4,t}$ is uncorrelated with all the regressors on the right-hand side of Equation (17). Thus, the OLS estimation of Equation (17) provides consistent estimates of structural regression model parameters $(\mu, \alpha_1, \alpha_2, \beta)$.

Web Appendix B.2: Proof of Theorem 3 nonnormality Assumption Relaxed

In this section, we prove that our proposed 2sCOPE method can relax the nonnormality assumption on the endogenous regressors imposed in $\text{Copula}_{\text{Origin}}$, while COPE does not.

We first examine the COPE method in Equation (W2),

$$Y_t = \mu + P_t\alpha + W_t\beta + \frac{\sigma_\xi \rho_{p\xi}}{1-\rho_{pw}^2} P_t^* + \frac{-\sigma_\xi \rho_{pw}\rho_{p\xi}}{1-\rho_{pw}^2} W_t^* + \sigma_\xi \sqrt{1-\rho_{p\xi}^2 - \frac{\rho_{pw}^2 \rho_{p\xi}^2}{1-\rho_{pw}^2}} \cdot \omega_{3,t}.$$

If the endogenous regressor P_t is normally distributed, $P_t = \Phi_{\sigma_p}^{-1}(\Phi(P_t^*)) = \sigma_p P_t^*$ and thus P_t^* and P_t would be fully collinear, violating the full rank assumption and making the model unidentified.

We then examine the 2sCOPE method in Equation (12).

$$Y_t = \mu + P_t\alpha + W_t\beta + \frac{\sigma_\xi \rho_{p\xi}}{1-\rho_{pw}^2} \epsilon_t + \sigma_\xi \sqrt{1-\rho_{p\xi}^2 - \frac{\rho_{pw}^2 \rho_{p\xi}^2}{1-\rho_{pw}^2}} \cdot \omega_{3,t},$$

$$\epsilon_t = P_t^* - \rho_{pw} W_t^*.$$

When the endogenous regressor P_t is normally distributed, $P_t = \Phi_{\sigma_p}^{-1}(\Phi(P_t^*)) = \sigma_p P_t^*$. Since we add the residual ϵ_t from the first stage to the outcome regression instead of adding each P_t^* and W_t^*, ϵ_t would not be perfectly collinear with P_t and W_t as long as one of the Ws correlated with P_t is not normally distributed. **Theorem proved.**

Web Appendix B.3: Variance Reduction Proposition of 2sCOPE

Proposition 1. ***Variance Reduction.*** *Assuming (1) the error term is normal, (2) the endogenous variable P_t and correlated regressors W_t are nonnormal, and (3) a Gaussian Copula for the error term, P_t and W_t, $\mathbf{Var}(\widehat{\theta}_\mathbf{2}) \leq \mathbf{Var}(\widehat{\theta}_\mathbf{1})$, where $\widehat{\theta}_1$ and $\widehat{\theta}_2$ denote parameter estimates from COPE and 2sCOPE, respectively.*

According to the COPE method in Equation (W2),

$$Y_t = \mu + P_t\alpha + W_t\beta + \frac{\sigma_\xi \rho_{p\xi}}{1-\rho_{pw}^2} P_t^* + \frac{-\sigma_\xi \rho_{pw}\rho_{p\xi}}{1-\rho_{pw}^2} W_t^* + \sigma_\xi \sqrt{1-\rho_{p\xi}^2 - \frac{\rho_{pw}^2 \rho_{p\xi}^2}{1-\rho_{pw}^2}} \cdot \omega_{3,t}.$$

The coefficients of P_t^* and W_t^* follows a linear relationship. Denote δ_3 and δ_4 the coefficients of P_t^* and W_t^* respectively. Then,

$$\delta_4 + \rho_{pw}\delta_3 = 0.$$

With the two-stage estimation in 2sCOPE (Equation 12), ρ_{pw} is estimated in the first stage and is thus treated as a known parameter in the main regression. That is, 2sCOPE can be viewed as the COPE method with a linear restriction. The linear restriction is,

$$\delta_4 + \hat{\rho}_{pw}\delta_3 = 0. \tag{W12}$$

In this case, the two-stage copula method (2sCOPE) can be viewed as one kind of restricted least squares estimation based on COPE. We next prove that restricted least squares can achieve reductions in standard errors. Suppose we simplify the regression expression in Equation (W2) as

$$y = X\theta + \epsilon,$$

where $\epsilon \sim N(0, \sigma^2 I)$, $X \equiv (1, P_t, W_t, P_t^*, W_t^*)$, and $\theta = (\mu, \alpha, \beta, \delta_3, \delta_4)$. The restriction in Equation (W12) becomes

$$R\theta = 0, \text{where} R = (0, 0, 0, \hat{\rho}_{pw}, 1).$$

Thus, the 2sCOPE yields the least squares estimates $\hat{\theta}_2$ of Equation (W2) subject to the above restriction, whereas COPE yields the unrestricted least squares estimates, $\hat{\theta}_1$, as follows.

$$\hat{\theta}_1 \sim N(\theta, \sigma^2 (X'X)^{-1}),$$

$$\hat{\theta}_2 \sim N(\theta, \sigma^2 M(X'X)^{-1}M').$$

where according to restricted least squares theory, $M = I - (X'X)^{-1}R'(R(X'X)^{-1}R')^{-1}R$. Let us compare the variance of $\hat{\theta}_1$ and $\hat{\theta}_2$. Note that,

$$\begin{aligned} & M(X'X)^{-1}M' \\ =& (I - (X'X)^{-1}R'(R(X'X)^{-1}R')^{-1}R)(X'X)^{-1}(I - R'(R(X'X)^{-1}R')^{-1}R(X'X)^{-1}) \\ =& (X'X)^{-1} - (X'X)^{-1}R'(R(X'X)^{-1}R')^{-1}R(X'X)^{-1}. \end{aligned}$$

Therefore,

$$\begin{aligned} Var(\hat{\theta}_1) - Var(\hat{\theta}_2) &= \sigma^2 \{(X'X)^{-1} - M(X'X)^{-1}M'\} \\ &= \sigma^2 (X'X)^{-1}R'(R(X'X)^{-1}R')^{-1}R(X'X)^{-1} \geq 0. \end{aligned}$$

Since the matrix $Var(\hat{\theta}_1) - Var(\hat{\theta}_2)$ is positive semi-definite, all the diagonal elements should be greater than or equal to zero. Thus, the imposition of the linear restriction brings about a variance reduction. **Theorem proved.**

WEB APPENDIX C: 2SCOPE FOR SLOPE ENDOGENEITY

In this section, we describe the 2sCOPE approach to addressing slope endogeneity with correlated regressors in the following model:

$$Y_t \;=\; \mu + P_t\alpha_t + W_t'\beta_t + \eta_t, \qquad \text{where } \alpha_t = \bar{\alpha} + \xi_t, \tag{W13}$$

α_t, β_t are individual-specific regression coefficients and $\bar{\alpha}$ is the mean of α_i, $\xi_t \sim N(0, \sigma_\xi^2)$. The normal error term η_i is uncorrelated with the regressors P_t and W_t and thus causes no endogeneity concern. However, the random coefficient ξ_t can be correlated with the regressor P_t, causing the problem of "slope endogeneity". P_t and W_t can be correlated. Assuming that (P_t, W_t, α_t) follows a Gaussian copula model, the COPE approach to addressing the slope endogeneity problem is derived as follows.

$$\begin{aligned} Y_t &= \mu + P_t\Big(\bar{\alpha} + \frac{\sigma_\xi\rho_{p\xi}}{1-\rho_{pw}^2}P_t^* + \frac{-\sigma_\xi\rho_{pw}\rho_{p\xi}}{1-\rho_{pw}^2}W_t^* + \sigma_\xi\sqrt{1-\rho_{p\xi}^2 - \frac{\rho_{pw}^2\rho_{p\xi}^2}{1-\rho_{pw}^2}}\,\omega_{3,t}\Big) + W_t'\beta_t + \eta_t \\ &= \mu + P_t\bar{\alpha} + \frac{\sigma_\xi\rho_{p\xi}}{1-\rho_{pw}^2}P_t \times P_t^* + \frac{-\sigma_\xi\rho_{pw}\rho_{p\xi}}{1-\rho_{pw}^2}P_t \times W_t^* + W_t'\beta_t + \\ &\quad \sigma_\xi\sqrt{1-\rho_{p\xi}^2 - \frac{\rho_{pw}^2\rho_{p\xi}^2}{1-\rho_{pw}^2}}\,P_t \times \omega_{3,t} + \eta_t. \end{aligned} \tag{W14}$$

Given both $P_t \times P_t^*$ and $P_t \times W_t^*$ in Equation (W14), the unobserved variable $w_{3,t}$ is independent of all regressors (P_t, W_t, P_t^*, W_t^*) and uncorrelated with functions of these regressors. Thus, Equation (W14) can be estimated using standard methods for random-effects models with $\omega_{3,t}$ as the random effect and $(P_t \times P_t^*,\ P_t \times W_t^*)$ as generated regressors. The method of Park and Gupta (2012) adds only $P_t \times P_t^*$ as a generated regressor, and may fail to yield consistent estimates when P_t and W_t are correlated, resulting in the correlation between the random effect in their method and the regressor W_t.

The 2sCOPE for addressing the slope endogeneity problem with correlated regressors is derived as follows

$$
\begin{aligned}
Y_t &= \mu + P_t(\bar{\alpha} + \frac{\sigma_\xi \rho_{p\xi}}{1-\rho_{pw}^2}\epsilon_t + \sigma_\xi \sqrt{1-\rho_{p\xi}^2 - \frac{\rho_{pw}^2 \rho_{p\xi}^2}{1-\rho_{pw}^2}} \cdot \omega_{3,t}) + W_t' \beta_t + \eta_t \\
&= \mu + P_t \bar{\alpha} + \frac{\sigma_\xi \rho_{p\xi}}{1-\rho_{pw}^2} P_t^* \times \epsilon_t + W_t' \beta_t + \sigma_\xi \sqrt{1-\rho_{p\xi}^2 - \frac{\rho_{pw}^2 \rho_{p\xi}^2}{1-\rho_{pw}^2}} P_t \times \omega_{3,t} + \eta_t, \quad \text{(W15)}
\end{aligned}
$$

where only one generated regressor, $P_t^* \times \epsilon_t$, is needed, given which the random effect $\omega_{3,t}$ is independent of all regressors in Equation (W15).

The 2sCOPE estimation can be implemented using the standard methods for random effects models by simply adding generated regressors to control for endogenous regressors. By contrast, the maximum likelihood approach requires constructing a complicated joint likelihood of $(\xi_t, \eta_t, P_t^*, W_t^*)$, which is not what the standard random effects method uses and thus requires separate development and significantly more computation involving numerical integration.

WEB APPENDIX D: 2SCOPE FOR RANDOM COEFFICIENT LOGIT MODEL

We next consider endogeneity bias in the following random utility model with correlated endogenous and exogenous regressors:

$$
\begin{aligned}
u_{hjt} &= \psi_{hj} + P'_{jt}\alpha_h + W'_{jt}\beta_h + \xi_{jt} + \epsilon_{hjt}, \qquad j = 1, \cdots, J, \\
u_{h0t} &= \epsilon_{h0t}, \qquad j = 0 \text{ if no purchase,}
\end{aligned}
$$

where u_{hjt} denotes the utility for household $h = 1, \cdots, n_h$ at occasion $t = 1, \cdots, T$ with $j = 1, \cdots, J$ alternatives and $j = 0$ denotes the option of no purchase. In the utility function, ψ_{hj} is the individual-specific preference for choice j with ψ_{hJ} normalized to be zero for identification purpose, (P_{jt}, W_{jt}) include the choice characteristics, and (α_h, β_h) denote the individual-specific random coefficients. These individual-specific coefficients $(\psi_{hj}, \alpha_h, \beta_h)$ permit heterogeneity in both intercepts and regressor effects across cross-sectional units, such as consumers or households. In this model, the association between regressors in P_{jt} and the unobserved common shock ξ_{jt} causes endogeneity bias. We further allow P_{jt} and W_{jt} to be correlated. The term ϵ_{hjt} is the idiosyncratic error uncorrelated with all regressors. An individual at any occasion chose the alternative with the largest utility, i.e., $Y_{hjt} = 1$ iff $u_{hjt} > u_{hj't}\ \forall j' \neq j$. When ϵ_{hjt} follows an *i.i.d* Type I extreme value distribution, the choice probability follows the random-coefficient multinomial logit model.

The 2sCOPE approach can be used to address the endogeneity issue using the following two-step procedure. In the first step, we estimate the model

$$
u_{hjt} = \delta_{jt} + \widetilde{\psi}_{hj} + P'_{jt}a_h + W'_{jt}b_h + \epsilon_{hjt},
$$

where $\delta_{jt} = \mu_j + P'_{jt}\bar{\alpha} + W'_{jt}\bar{\beta} + \xi_{jt}$, $(\mu_j, \bar{\alpha}, \bar{\beta})$ is the mean of random effects $(\psi_{hj}, \alpha_h, \beta_h)$, $\widetilde{\psi}_{hj} = \psi_{hj} - \mu_j, a_h = \alpha_h - \bar{\alpha}$ and $b_h = \beta_h - \bar{\beta}$. δ_{jt} is treated as occasion- and choice-specific fixed-effect parameters in this model. Since the regressors are uncorrelated with the error term ϵ_{hij}, there is no endogeneity bias in the model. In the second step, we estimate the equation below.

$$\widehat{\delta}_{jt} = \mu_j + P'_{jt}\bar{\alpha} + W'_{jt}\bar{\beta} + \xi_{jt} + \eta_{jt}, \tag{W16}$$

where $\widehat{\delta}_{jt}$ denotes the estimate of the fix-effect δ_{jt}; η_{jt} denotes the estimation error of $\widehat{\delta}_{ij}$ and is approximately normally distributed. In the second-step model, the structural error is correlated with P_{jt}, leading to endogenous bias. We then apply 2sCOPE to correct for the endogenous bias, which can avoid the potential bias of $\text{Copula}_{\text{Origin}}$ due to the potential correlations between P and W, as well as make use of this correlation to relax the nonnormality assumption of P_{it}, improve model identification and sharpen model estimates. The above development is for individual-level data. Park and Gupta (2012) also derived their copula method for addressing endogeneity bias in random coefficient logit models using aggregate-level data. It is straightforward to extend the 2sCOPE to the setting with correlated regressors and (nearly) normal regressor distributions.

WEB APPENDIX E: ADDITIONAL SIMULATION RESULTS

Web Appendix E.1: Additional Results for Smaller Sample Size for Case 1

In the simulation study case 1, we use the sample size T=1000. Here we further check the robustness of results with respect to a smaller sample size. We simulate 1000 data sets, each of which has the sample size T=200, and use the same DGP as described in Case 1. Table W1 shows that 2sCOPE has unbiased estimates for a small sample size T=200. Hence, our proposed method is robust and can be applied to small sample sizes.

			OLS			$\text{Copula}_{\text{Origin}}$			COPE			2sCOPE		
ρ_{pw}	Parameters	True	Mean	SE	t_{bias}	Mean	SE	t_{bias}	Mean	SE	t_{bias}	Mean	SE	t_{bias}
0.5	μ	1	0.683	0.097	3.264	1.228	0.191	1.194	1.020	0.223	0.091	0.999	0.137	0.005
	α	1	1.583	0.079	7.388	1.048	0.178	0.271	0.990	0.184	0.056	0.996	0.175	0.023
	β	-1	-1.265	0.068	3.902	-1.291	0.068	4.293	-1.019	0.166	0.116	-1.004	0.101	0.044
	$\rho_{p\xi}$	0.5	-	-	-	0.559	0.122	0.489	0.493	0.139	0.048	0.489	0.097	0.109
	σ_ξ	1	0.857	0.044	3.224	1.016	0.107	0.148	1.018	0.100	0.176	1.001	0.094	0.013
	D-error			-			-			0.016598			0.009069	
0.7	μ	1	0.723	0.091	3.050	1.304	0.175	1.740	1.006	0.197	0.031	0.983	0.114	0.153
	α	1	1.817	0.095	8.583	1.255	0.161	1.584	1.032	0.182	0.175	1.044	0.174	0.253
	β	-1	-1.539	0.084	6.388	-1.574	0.086	6.686	-1.045	0.180	0.250	-1.033	0.131	0.251
	$\rho_{p\xi}$	0.5	-	-	-	0.624	0.103	1.200	0.490	0.135	0.077	0.480	0.067	0.297
	σ_ξ	1	0.796	0.039	5.156	0.988	0.105	0.116	0.999	0.096	0.011	0.982	0.090	0.205
	D-error			-			-			0.016245			0.008867	

Table W1: Results of the Simulation Study for Case 1 with Sample Size of 200

Web Appendix E.2: Multiple Endogenous Regressors

In this case, we examine the performance of our proposed 2sCOPE when the model has multiple endogenous regressors. Specifically, we use the DGP with two endogenous regressors and one exogenous regressor that is correlated with the endogenous regressors below:

$$\begin{pmatrix} P^*_{1,t} \\ P^*_{2,t} \\ W^*_t \\ \xi^*_t \end{pmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0.3 & 0.4 & 0.5 \\ 0.3 & 1 & 0.4 & 0.5 \\ 0.4 & 0.4 & 1 & 0 \\ 0.5 & 0.5 & 0 & 1 \end{bmatrix} \right), \tag{W17}$$

$$\xi_t = G^{-1}(U_{\xi,t}) = G^{-1}(\Phi(\xi^*_t)) = \Phi^{-1}(\Phi(\xi^*)) = 1 \cdot \xi^*_t, \tag{W18}$$

$$P_{1,t} = H_1^{-1}(U_{p1}) = H_1^{-1}(\Phi(P^*_{1,t})), \;\; P_{2,t} = H_2^{-1}(\Phi(P^*_{2,t})), \tag{W19}$$

$$W_t = L^{-1}(U_{W,t}) = L^{-1}(\Phi(W^*_t)), \tag{W20}$$

$$Y_t = \mu + \alpha \cdot P_t + \beta \cdot W_t + \xi_t = 1 + 1 \cdot P_{1,t} + 1 \cdot P_{2,t} + (-1) \cdot W_t + \xi_t, \tag{W21}$$

where $H_1^{-1}(\cdot)$, $H_2^{-1}(\cdot)$ and $L^{-1}(\cdot)$ are the inverse distribution functions of the Gamma(1,1), t(30) and Exp(1) distributions used to generate these regressors. We generate 1000 data sets, each of which has a sample size T=1000.

Table W2 shows the estimation results. First, the OLS estimates are biased. The COPE, the extended $\text{Copula}_{\text{Origin}}$, estimates are biased as well because of the close-to-normal endogenous regressor, t(30). However, our proposed 2sCOPE method provides unbiased estimates for all parameters, indicating that 2sCOPE performs well with multiple endogenous regressors, even for close-to-normal endogenous regressors. Moreover, 2sCOPE provides a much smaller d-error (0.002695) compared with COPE (0.006943), indicating that 2sCOPE can largely increase the estimation efficiency. The efficiency gain is 61.2% in this case.

		OLS			COPE			2sCOPE		
Parameters	True	Mean	SE	t_{bias}	Mean	SE	t_{bias}	Mean	SE	t_{bias}
μ	1	0.903	0.040	2.426	1.002	0.079	0.019	1.016	0.076	0.213
α_1	1	1.436	0.029	14.88	0.998	0.058	0.032	0.995	0.058	0.079
α_2	1	1.487	0.025	19.23	1.367	0.479	0.765	1.028	0.141	0.197
β	-1	-1.338	0.029	11.76	-1.000	0.055	0.007	-1.010	0.054	0.196
$\rho_{\xi p1}$	0.5	-	-	-	0.394	0.136	0.781	0.501	0.042	0.029
$\rho_{\xi p2}$	0.5	-	-	-	0.110	0.422	0.923	0.472	0.095	0.295
σ_ξ	1	0.742	0.017	15.51	0.992	0.166	0.050	0.993	0.073	0.093
D-error						0.006943			0.002695	

Table W2: Results of the Simulation Study: Multiple Endogenous Regressors.

Web Appendix E.3: Multiple Exogenous Control Covariates

We investigate the performance of our proposed method when there exist multiple exogenous regressors consisting of both continuous and discrete variables. We generate the data using the following DGP:

$$\begin{pmatrix} \xi_t^* \\ P_{1,t}^* \\ W_{1,t}^* \\ P_{2,t}^* \\ W_{2,t}^* \end{pmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho & 0 & \rho & 0 \\ \rho & 1 & q & 0 & 0 \\ 0 & q & 1 & 0 & 0 \\ \rho & 0 & 0 & 1 & q \\ 0 & 0 & 0 & q & 1 \end{bmatrix} \right), \tag{W22}$$

$$\xi_t = G^{-1}(\Phi(\xi_t^*)) = \Phi^{-1}(\Phi(\xi^*)) = 1 \cdot \xi_t^*, \tag{W23}$$

$$P_{1,t} = \chi^2(2)^{-1}(\Phi(P_{1,t}^*)), \quad P_{2,t} = \chi^2(2)^{-1}(\Phi(P_{2,t}^*)), \tag{W24}$$

$$W_{1,t} = \Phi^{-1}(\Phi(W_{1,t}^*)), \tag{W25}$$

$$W_{2,t} = \begin{cases} 1, & \text{if} \quad \Phi(W_{2,t}^*) \geq 0.5 \\ 0. & \text{if} \quad \Phi(W_{2,t}^*) < 0.5 \end{cases}, \tag{W26}$$

$$Y_t = \mu + \alpha_1 \cdot P_{1,t} + \alpha_2 \cdot P_{2,t} + \beta_1 \cdot W_{1,t} + \beta_2 \cdot W_{2,t} + \xi_t \tag{W27}$$

$$= 1 + P_{1,t} + P_{2,t} + (-1) \cdot W_{1,t} + (-1) \cdot W_{2,t} + \xi_t, \tag{W28}$$

where $W_{1,t}$ is normally distributed and $W_{2,t}$ is a binary variable that follows a Bernoulli distribution. ρ is set to 0.4, and q is set to two cases, $\{0.3, 0.6\}$. We set the sample size $T =$ 1000 and generate 1000 data sets to estimate parameters using OLS and copula methods. For binary W, we compute W^* in the same way as for the continuous case, $W^* = \Phi^{-1}(F(W_t))$, where F is the cdf function of W.

The estimation results for the multiple-exogenous-regressor case with both discrete and continuous ones are summarized in Table W3. The OLS and $\text{Copula}_{\text{Origin}}$ estimates are biased because of endogeneity and correlated exogenous regressors, respectively. The proposed 2sCOPE method performs well and provides consistent estimates for all parameters. This indicates that our proposed method performs well with multiple exogenous correlated regressors. Moreover, correcting for endogeneity using our proposed method does not require every exogenous correlated regressor to be informative (i.e., continuously distributed) and nonnormally distributed.

			OLS			$\text{Copula}_{\text{Origin}}$			COPE			2sCOPE		
q	Parameters	True	Mean	SE	t_{bias}	Mean	SE	t_{bias}	Mean	SE	t_{bias}	Mean	SE	t_{bias}
0.3	μ	1	0.312	0.055	12.54	1.100	0.098	1.012	1.100	0.101	0.983	1.000	0.091	0.001
	α_1	1	1.195	0.015	12.86	1.000	0.031	0.006	1.000	0.031	0.008	1.000	0.031	0.014
	α_2	1	1.191	0.015	12.57	1.000	0.032	0.002	1.000	0.033	0.002	1.000	0.032	0.001
	β_1	-1	-1.104	0.028	3.726	-1.130	0.027	4.897	-1.145	0.694	0.209	-0.999	0.036	0.015
	β_2	-1	-1.167	0.055	3.052	-1.206	0.053	3.882	-1.206	0.053	3.884	-1.003	0.070	0.042
	$\rho_{P_1,\xi}$	0.4	-	-	-	0.430	0.053	0.561	0.371	0.152	0.188	0.396	0.050	0.084
	$\rho_{P_2,\xi}$	0.4	-	-	-	0.417	0.055	0.307	0.364	0.078	0.468	0.398	0.052	0.042
0.6	μ	1	0.236	0.052	14.63	1.256	0.096	2.667	1.256	0.098	2.609	1.005	0.083	0.056
	α_1	1	1.256	0.017	14.68	0.999	0.032	0.037	0.999	0.032	0.033	1.000	0.031	0.016
	α_2	1	1.222	0.016	13.67	0.996	0.029	0.136	0.996	0.029	0.137	0.996	0.029	0.137
	β_1	-1	-1.277	0.032	8.620	-1.373	0.031	12.14	-1.367	0.629	0.584	-1.002	0.047	0.039
	β_2	-1	-1.379	0.057	6.621	-1.497	0.053	9.306	-1.497	0.053	9.314	-0.993	0.082	0.082
	$\rho_{P_1,\xi}$	0.4	-	-	-	0.566	0.045	3.659	0.487	0.232	0.376	0.396	0.036	0.110
	$\rho_{P_2,\xi}$	0.4	-	-	-	0.477	0.047	1.630	0.437	0.082	0.451	0.403	0.040	0.071

Table W3: Results of the Simulation Study: Multiple Exogenous Control Covariates.

We further show the advantage of using 2sCOPE, compared with the direct extension of $\text{Copula}_{\text{Origin}}$ (COPE), in estimation efficiency and consistency in high-dimensional W using simulation. In particular, we use some commonly used distributions for the exogenous regressors Ws. The data-generating process (DGP) is summarized below:

$$\begin{pmatrix} P_t^* \\ W_t^* \\ \xi_t^* \end{pmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0_{10} \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho_{pw} & \rho_{p\xi} \\ \rho_{pw} & \Sigma_w & 0 \\ \rho_{p\xi} & 0 & 1 \end{bmatrix} \right)$$

$$\xi_t = G^{-1}(U_{\xi,t}) = G^{-1}(\Phi(\xi_t^*)) = \Phi^{-1}(\Phi(\xi^*)) = 1 \cdot \xi_t^*,$$

$$P_t = H^{-1}(U_{P,t}) = H^{-1}(\Phi(P_t^*)), \quad W_t = L^{-1}(U_{W,t}) = L^{-1}(\Phi(W_t^*)),$$

$$Y_t = \mu + \alpha \cdot P_t + \beta \cdot W_t + \xi_t = 1 + 1 \cdot P_t + (-1) \cdot W_t + \xi_t,$$

where ξ_t^* and P_t^* are correlated ($\rho_{p\xi} = 0.5$), generating the endogeneity problem; W_t^* is a 8-dimentional exogenous regressors uncorrelated with ξ_t^*; Each exogenous regressor in W_t^* is correlated with P_t^* with $\rho_{pw} = 0.5$; Σ_w denotes the covariance matrix of W_t^* with all the diagonal items equal to one and all non-diagonal items equal to $\rho_w = 0.3$. We set the sample size $T = 1000$, and generate 1000 data sets as replicates using the DGP above. In the simulation, we use normal distribution $N(0,1)$ for P_t, and the eight distributions, Exp(1), t(2), binary, mixnorm, Gamma(1,1), truncated-normal, lognorm(0,1), Cauchy(0,0.5), in sequence for the 8-dimentional W_t.

Table W4 summarizes the estimation results, and confirms that 2sCOPE outperforms COPE in several dimensions. First, the estimated coefficient of the endogenous regressor for COPE is 2.360, which is far away from the true value, indicating that COPE cannot handle normally-distributed endogenous regressor, while 2sCOPE can provide unbiased estimate. Second, COPE estimates of some exogenous regressors with certain distributions are biased (31.9% bias for binary W, 23.5% bias for mix-normal W and 25.7% bias for truncated normal W), indicating that COPE is sensitive to the distributions of exogenous regressors included in the model, making all the estimates vulnerable. Third, the D-error of COPE and 2sCOPE estimates is 0.002531 and 0.000534 respectively, indicating that 2sCOPE is much more efficient than COPE and increases the efficiency by 78.9%. Adding too many generated regressors in the model, as COPE does, will significantly decrease the estimation efficiency. In this section, we illustrate using the exactly normally-distributed endogenous regressor as an example. Please refer to Table W11 for weakly-nonnormal P case.

			OLS			COPE			2sCOPE		
Distribution	Parameters	True	Mean	SE	t_{bias}	Mean	SE	t_{bias}	Mean	SE	t_{bias}
	μ	1	1.848	0.051	16.73	1.281	0.228	1.229	1.040	0.089	0.453
N(0,1)	α	1	2.170	0.036	32.75	2.360	0.496	2.741	1.057	0.101	0.565
Exp(1)	β_1	-1	-1.210	0.024	8.862	-0.999	0.044	0.029	-1.009	0.035	0.245
t(2)	β_2	-1	-1.066	0.025	2.608	-1.001	0.013	0.107	-1.004	0.012	0.313
binary	β_3	-1	-1.363	0.045	8.119	-1.319	0.039	8.193	-1.014	0.073	0.196
mix-norm	β_4	-1	-1.233	0.022	10.76	-1.235	0.432	0.544	-1.012	0.039	0.311
Gamma(1,1)	β_5	-1	-1.212	0.024	8.812	-1.001	0.043	0.017	-1.011	0.035	0.313
truncated-N(0,1)	β_6	-1	-1.261	0.024	10.81	-1.257	0.470	0.547	-1.012	0.043	0.270
lognorm(0,1)	β_7	-1	-1.078	0.017	4.425	-1.000	0.015	0.025	-1.004	0.014	0.286
cauchy(0,0.5)	β_8	-1	-1.005	0.006	0.883	-1.000	0.002	0.023	-1.000	0.002	0.146
	ρ	0.5	-	-	-	-0.392	0.394	2.262	0.487	0.022	0.580
	σ_ξ	1	0.644	0.016	22.33	1.191	0.341	0.559	0.968	0.055	0.580
	Bias		0.345			0.246			0.016		
	RMSE		0.347			0.327			0.047		
	D-error		0.000424			0.002531			0.000534		

Table W4: The Performance of 2sCOPE with large-dimension of W

Web Appendix E.4: Misspecification of ξ_t

Similar to Park and Gupta (2012), we assume the structural error ξ_t to be normally distributed, a reasonable and commonly used assumption in marketing and economics literature. However, the true distribution of ξ_t is often unknown. Thus, in this simulation study, we examine the robustness of 2sCOPE to the departures from the normality of ξ_t. We generate 1,000 data sets using the same multivariate normal distribution as in Equation (20). The rest of DGP is:

$$\xi_t = G^{-1}(U_{\xi,t}) \quad = \quad G^{-1}(\Phi(\xi_t^*)), \tag{W29}$$

$$P_t = H^{-1}(U_{p,t}) = H^{-1}(\Phi(P_t^*)), \qquad W_t = L^{-1}(U_{w,t}) = L^{-1}(\Phi(W_t^*)), \tag{W30}$$

$$Y_t = \mu + \alpha \cdot P_t + \beta \cdot W_t \quad + \quad \xi_t = 1 + 1 \cdot P_t + (-1) \cdot W_t + \xi_t, \tag{W31}$$

where we set $P_t \sim Gamma(1,1)$ and $W_t \sim Exp(1)$ in the simulation. We check the robustness of the structural error ξ_t using different distributions (e.g., a uniform distribution, beta distribution and t distribution) instead of a normal distribution. For estimation, we assume normality of ξ_t and use the OLS estimator, $\text{Copula}_{\text{Origin}}$ and the proposed 2sCOPE method.

Table W5 reports estimation results. As shown in Table W5, 2sCOPE can recover the true parameter values despite the misspecification of ξ_t, demonstrating the robustness of the proposed 2sCOPE method to the normal error assumption.

			OLS			2sCOPE		
Distribution of ξ_t	Parameters	True	Mean	SE	t_{bias}	Mean	SE	t_{bias}
U[-0.5,0.5]	μ	1	0.912	0.013	6.808	1.002	0.017	0.105
	α	1	1.160	0.010	16.41	0.996	0.017	0.233
	β	-1	-1.072	0.009	8.033	-0.998	0.011	0.147
	$\rho_{p\xi}$	0.5	-	-	-	0.495	0.035	0.155
	σ_ξ	0.289	0.251	0.004	9.018	0.290	0.008	0.197
Beta(0.5,0.5)	μ	1	0.896	0.016	6.461	1.003	0.020	0.145
	α	1	1.190	0.012	15.72	0.994	0.018	0.318
	β	-1	-1.086	0.011	7.763	-0.998	0.014	0.183
	$\rho_{p\xi}$	0.5	-	-	-	0.481	0.033	0.593
	σ_ξ	0.354	0.311	0.005	9.046	0.356	0.009	0.258
Beta(4,4)	μ	1	0.948	0.008	6.928	1.000	0.010	0.009
	α	1	1.095	0.006	16.61	1.000	0.010	0.044
	β	-1	-1.043	0.005	8.149	-1.000	0.007	0.030
	$\rho_{p\xi}$	0.5	-	-	-	0.499	0.037	0.025
	σ_ξ	0.167	0.144	0.003	7.969	0.167	0.006	0.011
t (df=3)	μ	1	0.504	0.082	6.071	0.983	0.127	0.135
	α	1	1.903	0.089	10.13	1.024	0.217	0.110
	β	-1	-1.410	0.064	6.448	-1.012	0.109	0.111
	$\rho_{p\xi}$	0.5	-	-	-	0.454	0.069	0.676
	σ_ξ	1.732	1.503	0.231	0.992	1.698	0.244	0.141
t (df=5)	μ	1	0.603	0.059	6.723	0.997	0.080	0.039
	α	1	1.727	0.053	13.65	1.006	0.113	0.057
	β	-1	-1.328	0.043	7.642	-1.002	0.067	0.037
	$\rho_{p\xi}$	0.5	-	-	-	0.486	0.047	0.292
	σ_ξ	1.291	1.118	0.049	3.506	1.289	0.070	0.032

Table W5: Results of the Simulation Study: Misspecification of ξ_t

We further examine the performance of 2sCOPE with misspecification of ξ under normally-distributed endogenous regressor case. The estimation result in Table W6 shows that 2sCOPE can even work for normal endogenous regressor case under misspecification of ξ.

			OLS			2sCOPE		
Distribution of ξ	Parameters	True	Mean	SE	t_{bias}	Mean	SE	t_{bias}
U[-0.5,0.5]	μ	1	1.080	0.012	6.778	1.002	0.020	0.086
	α	1	1.178	0.008	23.28	1.004	0.037	0.116
	β	-1	-1.080	0.009	9.070	-1.002	0.018	0.113
	$\rho_{p\xi}$	0.5	-	-	-	0.475	0.072	0.344
	σ_ξ	0.289	0.241	0.004	10.64	0.288	0.018	0.054
Beta(0.5,0.5)	μ	1	1.095	0.015	6.483	1.003	0.025	0.116
	α	1	1.211	0.009	22.85	1.007	0.045	0.157
	β	-1	-1.095	0.011	8.560	-1.003	0.022	0.130
	$\rho_{p\xi}$	0.5	-	-	-	0.457	0.075	0.573
	σ_ξ	0.354	0.299	0.005	10.51	0.352	0.021	0.088
Beta(4,4)	μ	1	1.047	0.007	6.962	1.002	0.012	0.146
	α	1	1.105	0.005	22.30	1.004	0.021	0.194
	β	-1	-1.047	0.005	9.219	-1.002	0.011	0.157
	$\rho_{p\xi}$	0.5	-	-	-	0.479	0.072	0.294
	σ_ξ	0.167	0.138	0.003	9.969	0.165	0.011	0.159
T(df=3)	μ	1	1.443	0.075	5.884	1.022	0.133	0.165
	α	1	1.988	0.077	12.82	1.052	0.260	0.201
	β	-1	-1.443	0.059	7.508	-1.021	0.125	0.168
	$\rho_{p\xi}$	0.5	-	-	-	0.438	0.089	0.695
	σ_ξ	1.732	1.461	0.221	1.226	1.692	0.247	0.161
T(df=5)	μ	1	1.358	0.052	6.932	1.012	0.090	0.135
	α	1	1.795	0.045	17.66	1.030	0.168	0.176
	β	-1	-1.359	0.041	8.795	-1.012	0.082	0.151
	$\rho_{p\xi}$	0.5	-	-	-	0.472	0.075	0.376
	σ_ξ	1.291	1.073	0.046	4.726	1.277	0.096	0.145

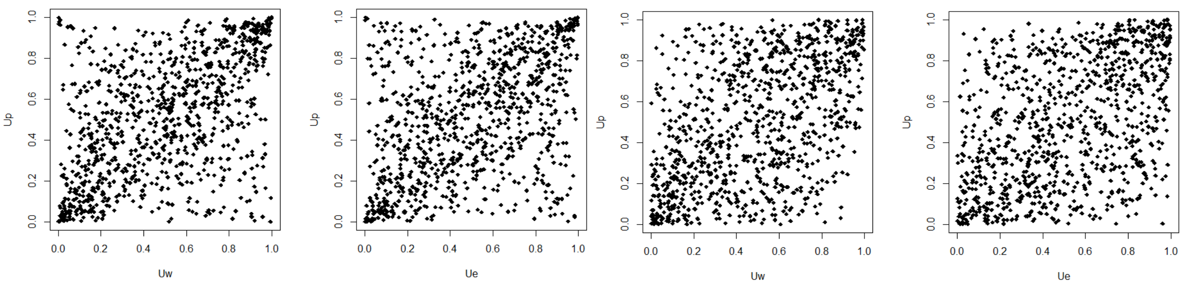
Table W6: Results of the Simulation Study: Misspecification of ξ with Normal Endogenous Regressor

Web Appendix E.5: Misspecification of Copula

In the proposed method, we use the Gaussian copula to capture the dependence structure among the regressors and error term (U_p, U_w and U_ξ). In practice, the dependence might come from an economic mechanism (such as marketing strategic decisions) and thus might be different from what the Gaussian copula generates. In this section, we examine the robustness of the Gaussian copula assumption in capturing the dependence among the endogenous regressors, exogenous regressors and the error term using simulated data. Specifically, we generate the dependence among U_p, U_w and U_ξ using copula models other than the Gaussian copula. Our simulation setting requires the availability of a random number generation routine from a tri-variate copula model other than Gaussian copula with non-homogeneous correlations among the three variables. Among copula models other than Gaussian copula, we find only T copula has this flexibility of providing flexible random number generation from arbitrary and heterogeneous correlation structures among more than two variables. We thus consider using the following T copula models in which

$$C(U_p, U_w, U_\xi) = \int_{-\infty}^{t_\nu^{-1}(U_p)} \int_{-\infty}^{t_\nu^{-1}(U_w)} \int_{-\infty}^{t_\nu^{-1}(U_\xi)} \frac{\Gamma(\frac{\nu+d}{2})}{\Gamma(\frac{\nu}{2})\sqrt{(\pi\nu)^d|\Sigma|}} \left(1 + \frac{x'\Sigma^{-1}x}{\nu}\right) dx, \qquad \text{(W32)}$$

where t_ν^{-1} denotes the quantile function of a standard univariate t_ν distribution. We set the degree of freedom ν=2, and the dimension of the copula d=3 in this example. Σ is covariance matrix capturing correlations among variables. The data-generating process (DGP) of t



(a) T Copula ($\rho = 0.5$) **(b)** Gaussian Copula ($\rho = 0.5$)

Figure W1: Scatter plots of Randomly Generated Pairs U_p, U_w (U_p,U_ξ) for Considered Copulas.

copula is summarized below:

$$\begin{pmatrix} P_t^* \\ W_t^* \\ \xi_t^* \end{pmatrix} \sim t_\nu^d \left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho_{pw} & \rho_{p\xi} \\ \rho_{pw} & 1 & 0 \\ \rho_{p\xi} & 0 & 1 \end{bmatrix} \right) = t_\nu^d \left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0.5 & 0.5 \\ 0.5 & 1 & 0 \\ 0.5 & 0 & 1 \end{bmatrix} \right). \tag{W33}$$

Figure W1 shows the scatter plots of randomly generated (U_p, U_w, U_ξ) pairs from the above copulas, as well as the Gaussian copula with the same correlation of 0.5. The figure shows disparate dependence structures between U_p and ξ_t (U_p and U_w) for these two copulas.

We then use the following process to generate P_t, W_t and ξ_t:

$$\xi_t = G^{-1}(U_\xi) = \Phi^{-1}(U_\xi), \tag{W34}$$

$$P_t = H^{-1}(U_p), W_t = L^{-1}(U_w), \tag{W35}$$

$$Y_t = 1 + 1 \cdot P_t + (-1) \cdot W_t + \xi_t. \tag{W36}$$

where $H(\cdot)$ is a gamma distribution and $L(\cdot)$ is an exponential distribution. We set $T = 1000$, generate 1000 data sets and estimate the parameters using the OLS estimator and the proposed 2sCOPE method.

Table W7 summarizes the estimation results. OLS and $\text{Copula}_{\text{Origin}}$ estimates are still biased for all parameters. By contrast, estimates from the proposed 2sCOPE method are centered closely around the true values. Therefore, the proposed method based on the Gaussian copula is reasonably robust to the mis-specifications of the copula dependence structure among the regressors and the structural error.

		OLS			2sCOPE		
Parameters	True	Mean	SE	t_{bias}	Mean	SE	t_{bias}
μ	1	0.710	0.530	5.463	0.988	0.077	0.156
α	1	1.580	0.044	13.13	1.029	0.116	0.250
β	-1	-1.289	0.047	6.142	-1.017	0.070	0.248
$\rho_{p\xi}$	0.5	-	-	-	0.458	0.067	0.622
σ_ξ	1	0.864	0.026	5.236	0.988	0.054	0.230

Table W7: Results of the Simulation Study: Misspecification of Copula

Web Appendix E.6: Linear Dependence Among Regressors

When using 2sCOPE with Gaussian Copula, the implied relation among regressors is restricted to a specific non-linear form. In this section, we further check the performance of the 2sCOPE estimator when the regressors are linearly related to each other. Specifically, we consider the following DGP.

$$\begin{pmatrix} X_t^* \\ \\ \xi_t^* \end{pmatrix} \sim N\left(\begin{bmatrix} 0 \\ \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho_{p\xi} \\ \\ \rho_{p\xi} & 1 \end{bmatrix}\right) = N\left(\begin{bmatrix} 0 \\ \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0.5 \\ \\ 0.5 & 1 \end{bmatrix}\right), \tag{W37}$$

$$\xi_t = G^{-1}(\Phi(\xi_t^*)) = \Phi^{-1}(\Phi(\xi^*)) = 1 \cdot \xi_t^*, \tag{W38}$$

$$W_t \sim N(0,1), \tag{W39}$$

$$P_t = \gamma W_t + F^{-1}_{\chi^2(1)}(\Phi(X_t^*)), \tag{W40}$$

$$Y_t = \mu + \alpha \cdot P_t + \beta \cdot W_t + \xi_t = 1 + 1 \cdot P_t + (-1) \cdot W_t + \xi_t \tag{W41}$$

Equation (W40) shows the linear dependence among P and W, which violates the assumption of 2sCOPE estimator. However, the estimation result in Table W8 shows that 2sCOPE estimation method can still get consistent estimates. Therefore, the results demonstrate the robustness of the proposed 2sCOPE method based on Gaussian copula, which implicitly requires a non-linear relationship among regressors, to the linear dependence among regressors.

			OLS			2sCOPE		
γ	Parameters	True	Mean	SE	t_{bias}	Mean	SE	t_{bias}
0.6	μ	1	0.545	0.041	11.07	0.967	0.084	0.389
	α	1	1.226	0.016	14.48	1.015	0.039	0.383
	β	-1	-1.136	0.030	4.607	-1.010	0.038	0.270
	σ_ξ	1	0.891	0.021	5.176	0.987	0.041	0.305
1.2	μ	1	0.546	0.042	10.74	0.969	0.104	0.294
	α	1	1.227	0.016	14.37	1.015	0.050	0.294
	β	-1	-1.273	0.036	7.653	-1.019	0.069	0.274
	σ_ξ	1	0.892	0.020	5.554	0.990	0.048	0.217

Table W8: Results of the Simulation Study: Linear Dependence Among Regressors

Web Appendix E.7: Test Assumption 5(b)

As shown in the METHODS section, when P contains only one endogenous regressor, Assumption 5 (W and P^* are uncorrelated) has to be satisfied for $\text{Copula}_{\text{Origin}}$ to yield consistent estimates. In multiple-endogenous-regressors case, W should be uncorrelated with the CCF term, the linear combination of P^*s (Assumption 5(b)). Assumption 5 for a single endogenous regressor is easy to check, while Assumption 5(b) is not that obvious. In this subsection, we describe how to test Assumption 5(b). Specifically, we consider two simulation scenarios: one satisfies Assumption 5(b) while the other doesn't. In addition, we will show that $\text{Copula}_{\text{Origin}}$ performs better if Assumption 5(b) is satisfied, and 2sCOPE is preferred if the assumption is violated. The data-generating process is summarized below.

$$\begin{pmatrix} P^*_{1,t} \\ P^*_{2,t} \\ W^*_{1,t} \\ W^*_{2,t} \\ \xi^*_t \end{pmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & p & q_1 & q_1 & \rho_1 \\ p & 1 & q_2 & q_2 & \rho_2 \\ q_1 & q_2 & 1 & q_{ww} & 0 \\ q_1 & q_2 & q_{ww} & 1 & 0 \\ \rho_1 & \rho_2 & 0 & 0 & 1 \end{bmatrix} \right), \tag{W42}$$

$$\xi_t = \Phi^{-1}(\Phi(\xi^*)) = 1 \cdot \xi^*_t, \quad W_t = L^{-1}(\Phi(W^*_t)), \tag{W43}$$

$$P_{1,t} = H^{-1}(\Phi(P^*_{1,t})), \quad P_{2,t} = H^{-1}(\Phi(P^*_{2,t})), \tag{W44}$$

$$Y_t = \mu + \alpha_1 \cdot P_{1,t} + \alpha_2 \cdot P_{2,t} + \beta \cdot W_t + \xi_t = 1 + P_{1,t} + P_{2,t} + (-1) \cdot W_t + \xi_t, \tag{W45}$$

where $P_t \sim Gamma(1,1)$ and $W_t \sim Exp(1)$ in both scenarios. The two scenarios differ in the covariance matrix in W42.

In Scenario 1, we set $p = 0$, $q_1 = q_2 = 0.4$, $q_{ww} = 0.2$, $\rho_1 = 0.5$ and $\rho_2 =$ -0.5;

In Scenario 2, we set $p = 0$, $q_1 = q_2 = 0.4$, $q_{ww} = 0.2$, $\rho_1 = 0.5$ and $\rho_2 = 0.5$.

We set $T = 1000$, generate 1000 data sets and estimate the parameters using the $\text{Copula}_{\text{Origin}}$ and 2sCOPE methods. To test the assumption 5(b) for $\text{Copula}_{\text{Origin}}$, we first estimate the coefficients of P_1^* and P_2^* ($\widehat{\gamma}_1$ and $\widehat{\gamma}_2$) using $\text{Copula}_{\text{Origin}}$, and obtain the CCF by calculating $\widehat{\gamma}_1 P_1^* + \widehat{\gamma}_2 P_2^*$. Then we check $\text{Cor}(W, \text{CCF})$, the correlation between W and the CCF=$\hat{\gamma}_1 \hat{P}_1^* + \hat{\gamma}_2 \hat{P}_2^*$ for each W. We use Fisher's Z test to test the null hypothesis of $\text{Cor}(W, \text{CCF}) = 0$. Assumption 5(b) is violated when the null hypothesis is rejected using the Fisher's Z test.

Table W9 summarizes the estimation results for the two scenarios with sample size N=1000. In Scenario 1, the average correlation between W_1 (W_2) and the CCF term across 1000 simulated data sets is -0.001316 (0.000369) with the average p-value of 0.493 (0.508), which means that the correlation is not significantly different from 0 and Assumption 5(b) holds. Correspondingly $\text{Copula}_{\text{Origin}}$ performs well with all the estimates center closely around the true values. By contrast, estimates from $\text{Copula}_{\text{Origin}}$ are biased in Scenario 2, and the average correlation between W_1 (W_2) and the CCF term across 1000 simulated data sets is 0.504 (0.503) with the average p-value $< 2.2e^{-16}$($< 2.2e^{-16}$), violating Assumption 5(b). In both scenarios, 2sCOPE provides unbiased estimates. However, when Assumption 5(b) holds, $\text{Copula}_{\text{Origin}}$ is more efficient than 2sCOPE with a smaller D-error ($0.001420 < 0.001611$) (Table W9), which means $\text{Copula}_{\text{Origin}}$ increases the estimation efficiency by 13.45%. When the dimension of W_t increases, we expect the efficiency gain of $\text{Copula}_{\text{Origin}}$ to be greater than that in this example.

To summarize, this subsection provides an example of how to test Assumption 5(b) with multiple endogenous regressors. When Assumption 5(b) holds, $\text{Copula}_{\text{Origin}}$ is preferred over

2sCOPE, as the simpler $\text{Copula}_{\text{Origin}}$ procedure yields more efficient estimates with smaller D-error. Otherwise, the proposed 2sCOPE method is preferred.

			OLS			$\text{Copula}_{\text{Origin}}$			2sCOPE		
Simulation	Parameters	True	Mean	SE	t_{bias}	Mean	SE	t_{bias}	Mean	SE	t_{bias}
Scenario 1	μ	1	0.999	0.050	0.014	0.999	0.089	0.017	0.998	0.063	0.038
	α_1	1	1.454	0.033	13.94	0.998	0.060	0.026	1.000	0.059	0.003
	α_2	1	0.546	0.032	14.19	1.004	0.058	0.074	1.005	0.057	0.087
	β_1	-1	-1.000	0.029	0.010	-1.000	0.027	0.008	-1.002	0.039	0.050
	β_2	-1	-1.001	0.029	0.036	-1.001	0.026	0.040	-1.001	0.038	0.021
	ρ_1	0.5	-	-	-	0.499	0.049	0.014	0.499	0.034	0.041
	ρ_2	-0.5	-	-	-	-0.500	0.046	0.010	-0.501	0.032	0.019
	σ_ξ	1	0.769	0.017	13.86	1.004	0.043	0.084	1.002	0.043	0.049
	D-error					0.001420			0.001611		
	Cor(W_1, CCF), p value					-0.001316, 0.493					
	Cor(W_2, CCF), p value					0.000369, 0.508					
Scenario 2	μ	1	0.384	0.040	15.32	1.753	0.078	9.699	0.995	0.060	0.085
	α_1	1	1.763	0.031	24.26	1.161	0.047	3.426	1.001	0.040	0.038
	α_2	1	1.764	0.034	22.32	1.164	0.044	3.699	1.003	0.041	0.072
	β_1	-1	-1.456	0.025	18.58	-1.542	0.023	23.54	-1.000	0.031	0.013
	β_2	-1	-1.456	0.024	19.30	-1.541	0.023	23.78	-1.000	0.032	0.010
	ρ_1	0.5	-	-	-	0.666	0.032	5.239	0.491	0.034	0.260
	ρ_2	0.5	-	-	-	0.665	0.033	5.047	0.490	0.035	0.274
	σ_ξ	1	0.569	0.017	25.01	1.099	0.044	2.237	1.001	0.033	0.043
	Cor(W_1, CCF), p value					0.504, <2.2e-16					
	Cor(W_2, CCF), p value					0.503, <2.2e-16					

Table W9: Results of the Simulation Study: Testing Assumption 5(b)

Web Appendix E.8: Simulation Experiments to Inform the Decision Tree of Using 2sCOPE

As noted in the main paper, as long as the sample size is sufficiently large and the assumptions of Theorems 2 and 3 are satisfied, 2sCOPE yields unbiased structural model estimates. However, in practical data sets with finite sample sizes, good performance of 2sCOPE may require sufficient nonnormality of regressors and sufficient relevance between P and W. Thus, in this section, we conduct systematic simulation studies to assess boundary conditions for using 2sCOPE in finite samples, and to inform the decision tree in Figure 2 of the main text.

To achieve this goal, in the simulation studies we systematically vary distributions of P and W, sample size, the endogeneity level, and the relevance level between P and W, to obtain empirically verifiable boundary conditions under which we can expect good performance of 2sCOPE with a high probability. In the simulation studies, we use the Kolmogorov-Smirnov (KS) test to evaluate the regressor nonnormality for the following reasons. The KS test statistic compares the empirical cumulative distribution of the standardized regressor with the CDF of the standard normal distribution, and is an overall and comprehensive measure to quantify nonnormality. Furthermore, because the performance of 2sCOPE improves with sample size when everything else is fixed, the measures for the sufficient nonnormality of regressors should also change with sample size: a minor departure from normality that is considered as insufficient nonnormality for a small sample can become sufficient for 2sCOPE to have good performance when the sample size is large. The p-value from the KS normality test satisfies this condition. Thus, we use the p-value from the KS test to inform sufficient nonnormality of regressors. Finally we consider cases in which Assumptions 5 and 5(b) are violated because otherwise copula$_{\text{Origin}}$ should be used instead of 2sCOPE (Web Appendix E.7).

We first consider the scenario when W is normally distributed, in which case the relevant W is expected to provide less help in identifying the causal effect of a close-to-normal endogenous regressor P than if W is sufficiently nonnormal. To identify the boundary condition of sufficient nonnormality of P for using 2sCOPE in this scenario, we conduct a factorial experiment using the data generating process from Equations (20-23) with a wide range of variations in sample size, endogeneity level, relevance, and distributions of regressors. Specifically, we simulate the endogenous regressor P using the nine nonnormal distributions in Figure 1, four levels of sample size {200, 500, 1000, 5000}, eight endogeneity level $\rho_{p\xi}$ $\{0.1, 0.2, ..., 0.8\}$, and eight relevance level between P^* and W^* $\{0.1, 0.2, ..., 0.8\}$. This results in a total of $9 \times 4 \times 8 \times 8 = 2304$ cases. We generate 1000 data sets for each case, estimate the parameters using 2sCOPE, and calculate the average relative bias of $[\mu, \alpha, \beta]$ to measure the performance in each of the 2304 cases. In the end, we obtain 2304 observations in total. By examining the performance of 2sCOPE across all these cases, we evaluate the boundary condition of the nonnormality level of P for good performance of 2sCOPE. Table W10 shows that in 1280 out of 2304 cases considered, the average p-value of the KS test of the normality of P over 1000 simulated data sets is less than 0.05 (i.e., rejecting the normality of P). As long as the p-value of the KS test of P is smaller than 0.05, 2sCOPE yields estimates with minor bias (relative bias less than 15%) with high probability ($\frac{1263}{1280}$ =98.7%) (Table W10). Among the 1263 cases with average relative bias less than 15%, the bias is small with mean relative bias of 1.6% and standard deviation (SD) of 2.7% over these 1263 cases (Table W10). Furthermore, among the 17 cases in which the average relative bias of the 2sCOPE estimates exceeds 15%, the bias is not large with mean relative bias of 22% and standard deviation of 6% (Table W10). Overall, we can conclude from this simulation experiment that 2sCOPE is expected to perform well in finite samples with high probabilities when the p-value from the KS test of the endogenous regressor P is less than 0.05.

KS Test P-value of Endogenous Regressor P	Number of Cases	Number of Cases Bias $\leq 15\%$	Number of Cases Bias $> 15\%$	Percentage of Good Performance
< 0.05	1280	1263 (mean=1.6%, SD=2.7%)*	17 (mean=22%, SD=6%)*	98.7%

Table W10: Condition for Sufficient Nonnormality of P to Use 2sCOPE in Step 2 of the Decision Tree.

Note: *: Mean and standard deviation of the relative bias across cases are reported in the parenthesis.

Next we consider the scenario when P fails the nonnormality test in step 2. Specifically, we consider the extreme case when P is normally distributed, and examine the sufficient condition of W for 2sCOPE to have good finite-sample performance. Similarly, to identify the sufficient condition of W, we simulate the exogenous regressor W using the nine nonnormal distributions in Figure 1, four levels of sample size {200, 500, 1000, 5000}, eight endogeneity level $\rho_{p\xi}$ $\{0.1, 0.2, ..., 0.8\}$ and eight relevance level between P^* and W^* $\{0.1, 0.2, ..., 0.8\}$. This results in a total of $9 \times 4 \times 8 \times 8 = 2304$ cases. We generate 1000 data sets for each case, estimate the parameters using 2sCOPE, and calculate the average relative bias of $[\mu, \alpha, \beta]$ to measure the performance. In the end, we obtain 2304 observations in total. By examining the performance of 2sCOPE in all those simulation studies, we evaluate the sufficient condition of W for 2sCOPE to have good performance when P is normally distributed. Figure W2 shows the aggregate simulation results. When W is strongly-nonnormal with the avearge p-value of KS test smaller than e^{-10} over 1000 simulated data sets, we only require a moderate relevance between P^* and W^* (average F stats > 10 for the effect of W^* on P^* in the first stage regression of 2sCOPE over 1000 simulated data sets) to have good performance of 2sCOPE (relative bias $\leq 15\%$) with a high probability ($\frac{457}{457+16} = 96.6\%$ in Figure W2): in 457 out of 473 (457+16) cases in which W satisfies the strongly-nonnormality and moderate relevance requirements above, 2sCOPE performs well with relative bias $\leq 15\%$. When W is moderately-nonnormal with the average p-value of the KS test smaller than 0.001 but greater than e^{-10} over 1000 simulated data sets, a relatively strong relevance between P^* and W^* (F stats > 100) is required for 2sCOPE to have good performance (relative bias $\leq 15\%$) with a high probability $\frac{276}{276+11} = 96.2\%$: in 276 out of 287 (276+11) cases in which W satisfies the moderately-nonnormality and strong relevance requirements above, 2sCOPE performs well with relative bias $\leq 15\%$. In other cases, we observe a considerably lower probability ($\frac{400}{1144+400}$ =25.9% [16]) of good finite-sample performance. The simulation experiment informs the sufficient condition of W for 2sCOPE to have good performance when the endogenous regressor P is normally distributed. Overall, this simulation experiment demonstrates that a combination of certain levels of nonnormality and relevance of W are needed to identify the normally distributed endogenous regressor with good finite-sample performance.

We have provided the sufficient conditions of endogenous and exogenous regressors above for 2sCOPE to have good finite-sample performance. These are not necessary conditions but are conservative ones to be on the safe side. In particular, to obtain sufficient conditions, we consider the extreme cases in which either the exogenous regressor in step 2 or the endogenous

[16] The 25.9% is the worst case in extreme scenario (P is set to be normally distributed). In practice when P is close-to-normal instead of exact normal, 2sCOPE can have a larger probability of achieving good finite-sample performance than 25.9%.

regressor in step 3 follows the normal distribution. However, in practice, regressors are likely to have close-to-normal rather than exact normal distributions. The failure of the sufficient condition tests of W in practice does not mean 2sCOPE cannot be used. For instance, the estimation result of scenario 1 in Table W11 (P and W are close-to-normal and weakly nonnormal, respectively) demonstrates that 2sCOPE may still have acceptable finite-sample performance when the above (conservative) sufficient conditions are not satisfied. In this situation, one can rely on our proposed bootstrap resampling Algorithm 1 to evaluate the finite-sample performance of 2sCOPE on a case-by-case basis.

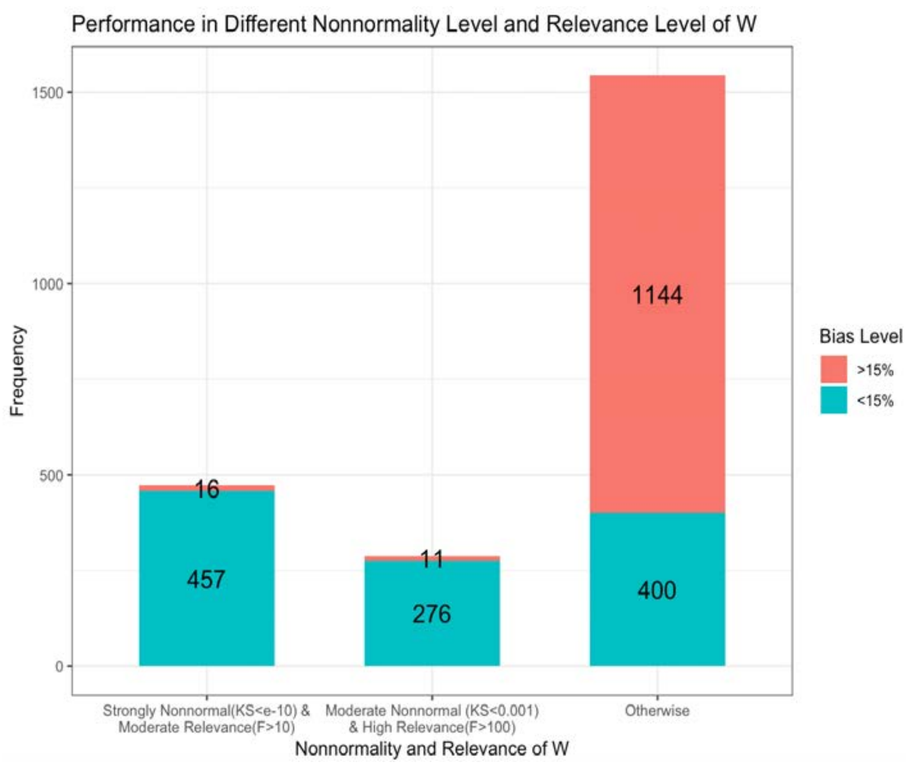


Figure W2: Sufficient Condition of W for Using 2sCOPE in Step 3 of the Decision Tree.

Web Appendix E.9: Multiple 'Weakly-nonnormal' Exogenous Covariates vs. One 'Strongly-nonnormal' Exogenous Covariate

According to the decision tree in Figure 2, 2sCOPE with one relevant exogenous regressor having sufficiently strong nonnormality can achieve good finite-sample performance when the endogenous regressor is normally distributed. In this section, we conduct further simulation studies to examine whether several 'weakly-nonnormal' exogenous covariates can add up to achieve the same performance as one 'strongly-nonnormal' exogenous covariate when the endogenous regressor is close to normal. We set the sample size $T = 1000$, and use the same data generating process as in Equations (20 - 23) except for the dimension of W and the distributions of W and P. Specifically, we set the distribution of P to a close-to-normal distribution, t(30), and use three different scenarios of W to examine the capability of W to help identify the causal effect of the endogenous regressor. In scenario 1, we have one W following the $t(4)$ distribution, with the average p-value of the KS test over 1000 simulated data sets being 0.0054$>$ 0.001 and thus is a 'weakly-nonnormal' exogenous covariate defined in Figure 2. In scenario 2, we increase the number of 'weakly-nonnormal' Ws from 1 to 3, with 0.2 correlation between different Ws. In scenario 3, we use one 'strongly-nonnormal' W following the $t(2)$ distribution, with the average p-value of KS test over 1000 data sets $1.88e^{-11} < e^{-10}$.

Table W11 shows the estimation results of the three scenarios. In both scenarios 1 (one weakly-nonnormal W) and 2 (three weakly-nonnormal Ws), the estimate of α of 2sCOPE have similar minor but noticeable finite-sample bias. Adding multiple 'weakly-nonnormal' exogenous regressors does not improve the estimation (the estimate of 1.137 for α in scenario

2 is farther away from the true value than 1.101 in scenario 1). However, the 2sCOPE's performance becomes better when using a 'strongly-nonnormal' W in scenario 3 (the estimate of 1.018 for α is closer to the true value than 1.101 in scenario 1 and 1.137 in scenario 2, Table W11). The D-error for 2sCOPE is also smallest when using a 'strongly-nonnormal' W in scenario 3 (Table W11). Thus, a 'strongly-nonnormal' W is better than multiple 'weakly-nonnormal' Ws in helping the identification for close-to-normal endogenous regressors. Adding multiple 'weakly-nonnormal' exogenous covariates will not help the identification of a normal (close-to-normal) endogenous regressor as effectively as one 'strongly-nonnormal' exogenous regressor. Moreover, the estimation results further confirm that our proposed 2sCOPE can largely improve the performance, compared with COPE (the extended $\text{Copula}_{\text{Origin}}$). For instance, in scenario 3, 2sCOPE has large improvement over COPE in both estimation consistency (the estimate of α is improved from 1.494 in COPE to 1.018 in 2sCOPE) and efficiency (the D-error is improved from 0.004830 in COPE to 0.001031 in 2sCOPE, increased by 78.7%), when both the endogenous and exogenous regressors are close-to-normally distributed.

			OLS			COPE			2sCOPE		
Scenario	Parameters	True	Mean	SE	t_{bias}	Mean	SE	t_{bias}	Mean	SE	t_{bias}
1 W, $t(4)$	μ	1	1.001	0.025	0.026	1.001	0.034	0.030	1.000	0.030	0.016
Weakly-nonnormal	α	1	1.634	0.030	21.25	1.516	0.556	0.928	1.101	0.209	0.486
	β	-1	-1.227	0.026	8.684	-1.010	0.086	0.119	-1.038	0.078	0.482
	ρ	0.5	-	-	-	-0.018	0.454	1.142	0.425	0.132	0.569
	σ_ξ	1	0.822	0.019	9.438	1.016	0.225	0.072	0.965	0.095	0.368
	D-error						0.013740			0.002724	
3 Ws, $t(4)$	μ	1	1.000	0.021	0.016	1.002	0.031	0.080	0.999	0.029	0.019
Weakly-nonnormal	α	1	1.994	0.034	29.40	1.814	0.467	1.742	1.137	0.146	0.941
	β_1	-1	-1.258	0.027	9.677	-1.018	0.071	0.252	-1.036	0.043	0.848
	β_2	-1	-1.259	0.025	10.38	-1.022	0.073	0.297	-1.037	0.042	0.892
	β_3	-1	-1.258	0.024	10.89	-1.020	0.073	0.279	-1.035	0.042	0.851
	ρ	0.5	-	-	-	-0.258	0.394	8.345	0.459	0.046	0.903
	σ_ξ	1	0.697	0.017	17.93	1.005	0.206	1.923	0.933	0.072	0.928
	D-error						0.007816			0.001095	
1 W, $t(2)$	μ	1	1.001	0.026	0.040	1.002	0.033	0.054	1.001	0.031	0.037
Strongly-nonnormal	α	1	1.574	0.040	14.36	1.494	0.577	0.855	1.018	0.094	0.188
	β	-1	-1.086	0.033	2.617	-1.002	0.018	0.110	-1.003	0.018	0.194
	ρ	0.5	-	-	-	-0.005	0.467	1.081	0.486	0.057	0.249
	σ_ξ	1	0.839	0.019	8.273	1.025	0.229	0.108	0.993	0.052	0.133
	D-error						0.004830			0.001031	

Table W11: Multiple 'Weakly-nonnormal' Exogenous W vs. One 'Strongly-nonnormal' W

WEB APPENDIX F: OBTAINING STANDARD ERRORS USING BOOTSTRAP

We generate B bootstrap data sets by randomly resampling the original data set with replacement, and re-estimate the structural model parameters using 2sCOPE for each data set. Then calculate the standard errors by calculating the standard deviation of the estimates obtained from these data sets. Algorithm 2 summarizes the detailed steps of how to obtain the standard errors of estimates using bootstrap.

Algorithm 2 Bootstrap Algorithm for Calculating Standard Error of 2sCOPE Estimates

Series Input: data Y, P, W, sample size N, and number of bootstrap B.
for $b = 1$ to B **do**
Randomly resample (Y_b, P_b, W_b) from the original data (Y, P, W) with replacement, sample size $= N$;
Obtain $P_b^* = \Phi^{-1}(\widehat{H}(P_b)), W_b^* = \Phi^{-1}(\widehat{L}(W_b))$, where $\hat{H}(\cdot)$ and $\hat{L}(\cdot)$ are estimated CDFs of P_b and W_b;
Obtain the 2sCOPE estimate $\widehat{\theta}_b = \widehat{\theta}(Y_b, P_b, W_b, P_b^*, W_b^*)$ using the bth bootstrap sample.
end for
Calculate standard error of the estimator: $\sqrt{\frac{\sum_{b=1}^{B}(\widehat{\theta}_b - \frac{1}{B}\sum_{b=1}^{B}\widehat{\theta}_b)^2}{B-1}}$.

WEB APPENDIX G: IMPLEMENTING THE BOOTSTRAP METHOD TO EVALUATE FINITE-SAMPLE BIAS IN EMPIRICAL APPLICATION

To gauge and validate the finite-sample performance of 2sCOPE, we apply the bootstrap algorithm described in Algorithm 1 to our empirical application and conduct a bootstrap re-sampling study by drawing repeated samples of the same size as the observed data from the underlying copula model and the structural model estimated from the original sample using data from store 1 in the application, and perform estimation on each bootstrap sample. Specifically, we generate data using the following DGP:

$$\begin{pmatrix} \text{Price}^* \\ \text{Bonus}^* \\ \text{PriceRedu}^* \\ \xi_t^* \end{pmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & -0.5 & -0.3 & 0.3 \\ -0.5 & 1 & -0.3 & 0 \\ -0.3 & -0.3 & 1 & 0 \\ 0.3 & 0 & 0 & 1 \end{bmatrix} \right), \tag{W46}$$

$$\xi_t = G^{-1}(\Phi(\xi_t^*)) = \Phi_\sigma^{-1}(\Phi(\xi^*)) = 0.4 \cdot \xi_t^*, \tag{W47}$$

$$\text{Price} = \hat{H}^{-1}(\Phi(\text{Price}^*)), \quad \text{Bonus} = \hat{L}_1^{-1}(\Phi(\text{Bonus}^*)), \tag{W48}$$

$$\text{PriceRedu} = \hat{L}_2^{-1}(\Phi(\text{PriceRedu}^*)), \tag{W49}$$

$$Y_t = -4 + (-2) \cdot \text{Price} + 0.1 \cdot \text{Bonus} + 0.3 \cdot \text{PriceRedu} + \xi_t, \tag{W50}$$

where $\hat{H}(\cdot), \hat{L}_1(\cdot), \hat{L}_2(\cdot)$ are all estimated CDFs using the univariate empirical distribution in the application for regressors Price, Bonus and PriceRedu, respectively. The correlation matrix of the copula transformation of variables (i.e., Price*, Bonus*, PriceRedu*, ξ^*) in Equation (W46) and the standard deviation of the error term (i.e., σ_ξ) are set according to the estimated parameter values using real data. After generating the regressors and the structural error, we set the coefficients using the 2sCOPE estimates of original data to generate Y in Equation (W50). We set the sample size $T = 373$, which is the same as the sample size in the application, and generate $B = 1000$ bootstrap data sets in each of which we estimate the structural model parameters using OLS and 2sCOPE.