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LIQUIDITY, LIQUIDITY EVERYWHERE, NOT A DROP TO USE -
WHY FLOODING BANKS WITH CENTRAL BANK RESERVES
MAY NOT EXPAND LIQUIDITY

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Working Paper 29680
<http://www.nber.org/papers/w29680>

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
January 2022

We are grateful to Richard Berner, Douglas Diamond, Will Diamond, Wenxin Du, Charles Goodhart, Robin Greenwood, Sam Hanson, Yunzhi Hu, Max Jager, Anil Kashyap, Yiming Ma, Stefan Nagel, Carolin Pflueger, Charles Plosser, Rafael Repullo, Bruce Tuckman, Alexi Savov, Philipp Schnabl, Andrei Sheleifer, Jeremy Stein, Adi Sunderam, Quentin Vanderweyer, Annette Vissing-Jorgensen, Olivier Wang, Zhengyang Jiang, and participants at seminars at CEMFI, New York University Stern School of Business, the Office of Financial Research (OFR), and the Federal Reserve Bank of Cleveland and OFR's 2021 Financial Stability Conference "Planning for Surprises, Learning from Crises" for helpful comments and discussions. We also benefited from excellent research assistance from Huan He and Stefano Pastore. Rajan thanks IGM and the Fama Miller Center at the Booth School as well as the Hoover Institution for research support. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

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NBER Working Paper No. 29680
January 2022
JEL No. E0,G0

ABSTRACT

Central bank balance sheet expansion is financed by commercial banks. It involves not just a substitution of liquid central bank reserves for other assets held by commercial banks, but also a counterpart alteration in commercial bank liabilities, such as in short-term deposits issued to finance reserves and in off-balance-sheet encumbrances pledged against reserves, which are also claims on liquidity. In ordinary times, when these claims are not called on, central bank balance sheet expansion will typically enhance the net availability of liquidity to the system. However, in times of stress when these offsetting claims on liquidity are exercised, the demand for liquidity can be significantly more. Furthermore, liquid commercial banks, desiring to maintain unimpeachable balance sheets, may provide only limited re-intermediation of liquidity and contribute significantly to liquidity shortages. Commercial banks do not fully internalize prospective stress ex ante or take sufficient steps to avoid it. Consequently, central bank balance sheet expansion need not eliminate episodes of stress; it may even exacerbate them. This may also attenuate any positive effects of central bank balance sheet expansion on economic activity.

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Despite a significant expansion in central bank balance sheets, some markets like the US money market have experienced increasing interest rate volatility, including significant spikes in the repo rate, notably in September 2019 (see Copeland, Duffie and Yang (2021), Correa, Du, and Liao (2021), D’Avernas and Vanderweyer (2021), and Yang (2021)). This apparent disruption in money markets that depend intimately on the availability of liquidity seems puzzling when the cash and central bank reserves held by the US private sector at the end of 2019 were around 4 times their holdings before the Global Financial Crisis in 2007. Greater liquid holdings do not seem to have made markets for liquidity more immune to liquidity shocks. Indeed, markets were disrupted yet again in March 2020 at the onset of the COVID-19 pandemic² and the banking system was found short in its ability to accommodate the demand for liquidity.³ In response, the Federal Reserve expanded its balance sheet yet more (see, for example, Kovner and Martin (2020)), buying financial assets from the private sector and placing large quantities of liquid reserves with it (or promising to do so). Where had all the prior liquidity gone? Our paper focuses on this question, not so much to explain the microstructure of interest rate spikes, for which there is an extensive literature now,⁴ but to analyze more general theoretical underpinnings of the consequences of central bank balance sheet expansion.

It seems natural that the liquid central bank reserves issued to finance central bank balance sheet expansion should enhance the supply of liquidity, bringing down illiquidity premia in the market, and reducing the cost to firms of financing. Yet this view neglects three key private sector responses. First, central banks effectively issue these reserves to commercial banks (henceforth “banks”), which have to finance them. For a variety of reasons, the best way for a bank to finance short-term assets is with short-term liabilities such as deposits. In times of liquidity stress, this offsetting liability could claim some of the liquidity created by the central bank.⁵

Second, the liquid reserves themselves get further encumbered. While a number of papers have focused on regulatory encumbrances such as capital requirements that inhibit banks’ use of reserves, encumbrances may also be endogenous. For example, banks with liquid assets sitting on their balance

² This was the case especially for corporate debt, but segments of the US Treasuries market also experienced significant illiquidity, see Duffie (2020), Fleming and Ruela (2020), He, Nagel and Song (2020), Liang and Parkinson (2020), Schrimp, Shin and Sushko (2020)), and Vissing-Jorgensen (2020).

³ Corporates drew down significantly on bank credit lines, see Acharya, Engle and Steffen (2021); and, dealer banks appear to have faced regulatory constraints in extending their balance-sheets for market-making, see Boyarchenko, Kovner and Shachar (2020), Breckenfelder and Ivashina (2021), Liang and Parkinson (2020), Kargar *et al.* (2021), and Vissing-Jorgensen (2020).

⁴ Copeland, Duffie, and Yang (2021) and IAWG Treasury Market Surveillance Report (2021) provide a discussion of the various channels.

⁵ Such financing could initially happen near-automatically – the central banks buys financial assets from non-banks, who deposit the proceeds in their banks, giving the commercial banks both reserves and offsetting deposits. Of course, banks and non-banks might eventually want to rebalance, but banks in aggregate have to hold the reserves.

sheets want to earn returns to that liquidity. So they sell it – by offering contingent lines of credit or guaranteeing margin calls on speculation (see, for example, Anderson, Du, and Schlusche (2021)), so long as liquidity is available when they need it. Unfortunately, not only do they sell too much relative to the social optimal, but bank (and depositor) behavior can lead to unintended outcomes.

Third, and perhaps most novel, in times of liquidity stress healthy banks may see a valuable *convenience yield* to liquid reserves – for instance, because it is dry powder in case conditions worsen. Consequently, healthy banks may hoard liquidity and maintain unimpeachable balance sheets, in order to be perceived as safe and attract more deposit flows, rather than lending it out to stressed banks.

Put differently, new sources of demand as well as constraints on supply may offset the new supply of liquidity. Consequently, it is possible that central bank balance sheet expansion under some circumstances does not reduce illiquidity premia, but may even enhance them. Instead of enhancing financial stability, it makes the financial system more fragile. Rather than expanding real activity, it disincentivizes it. While these are extreme possibilities, the norm may be that central bank balance sheet expansion contributes much less to liquidity, financial stability, and real activity than suggested by an analysis that abstracts from the considerations we lay out.

Let us elaborate on all this. We assume the central bank wants to expand its balance sheet, buying financial assets from the public markets with newly issued reserves. We take any direct effect of the asset purchases on economic activity as given, so as to focus on what happens to liquidity after that. We assume the reserves eventually find their way back to commercial bank balance sheets (so cash holdings with the public do not go up), and the banks optimally issue liabilities to finance them. Key in the analysis that follows is the mix of how banks finance these reserves. A number of authors (Calomiris and Kahn (1991), Dang, Gorton, and Holmstrom (2010), Flannery (1986), and Gorton and Pennacchi (1990), among others) have argued that banks have a comparative advantage in issuing short-term or demandable debt. Others (see, for example, Diamond and Dybvig (1984) or Stein (2012)) have attributed an implicit liquidity/money premium to short-term bank liabilities that makes them relatively attractive for investors, and Diamond and Rajan (2001) argue that one leads to the other. We are agnostic as to why longer-term financing (that is, capital) is costlier for banks, but assume functional forms that make it so. Naturally then, banks finance a large portion of the reserve expansion by issuing short-term liabilities.

Indeed, the evidence suggests this is the case (see Exhibit 1). The Federal Reserve bought financial assets between November 2010 and June 2011 (“QE II”), between September 2012 and October 2014 (“QE III”), and between March 2020 to the end of 2020 (the Pandemic Intervention which is still continuing). Exhibit 1, put together from the Flow of Funds data, suggests that commercial banks

increased their assets considerably over the same period – so central bank reserves did not simply substitute for existing bank assets. Furthermore, bank deposit issuance was a multiple of the increase in commercial bank holdings of reserve balances and repos in each case. Of course, banks may also have expanded their holdings of other liquid assets such as vault cash and securities over these periods, but the increase in deposits exceeds even these. Indeed, in both QEII and the Pandemic purchases, the increase in bank deposits exceeds the overall increase in bank assets, while in QE III, it is 80 percent of the increase (the period of QE III was also one when bank loans went up considerably, along with bank liquid assets). In both QE III and the Pandemic Intervention, uninsured deposits account for the majority of the deposit financing. In Exhibit 2, we plot the cumulative increase in outstanding reserves against the rise in uninsured deposits over the last two decades, which confirms this financing pattern cumulates across programs. These data inform our modeling choices.

We assume that after commercial banks get reserves, make loans, and set their capital structure to accord with these assets, there is a probability that the demand for liquidity in the real economy will increase significantly, and will be concentrated on some banks. Call these the *stressed* banks. We assume that their wholesale depositors, fearful of any loss, withdraw their cash in such states, increasing the stressed banks' need for funds. A fraction of healthy banks will choose to become *tainted* by lending their reserves to stressed banks, making money on the lending. The remaining fraction of healthy banks will abstain from lending so as to be seen as *safe*, thereby attracting the flight-to-safety deposits fleeing stressed banks. To start with, we take the fraction of safe and tainted banks among healthy banks as given and endogenize it later. Finally, we assume that not all the central bank reserves held on commercial bank balance sheets can be used to pay withdrawers – some reserves are encumbered because regulators demand set-asides or the banks create further contingent claims on reserves (we explain shortly).

The financing of partially encumbered reserves with short-term deposits, coupled with reserve hoarding by some of the healthy banks that are recipients of flight-to-safety deposits, sets up an interesting dynamic when liquidity is stressed: loan rates in the interbank market can shoot up as stressed banks try and attract liquidity from healthy banks (see, Acharya and Mora (2015) for empirical documentation of such a dynamic during 2007-08). Importantly, the extent of illiquidity, and therefore the premium paid on borrowing in this situation (which will also affect the fire-sale prices of illiquid financial assets), need not fall in the reserves the central bank issues ex ante. Indeed, under plausible circumstances, every additional dollar of reserves the central bank issues up front can increase the net demand for liquidity in situations of liquidity stress, and can increase the interbank borrowing premium.

A higher anticipated bank borrowing rate in the future then cascades up front into a higher rate for term loans made by banks (as in Diamond and Rajan (2011), Shleifer and Vishny (2010), or Stein

(2012)), lower investment by firms, and lower aggregate activity. Somewhat perversely, therefore, higher central bank reserve issuance can create more headwinds even to current activity by increasing future, and thus current, borrowing premia. Put differently, the expansion in available reserve assets may be outweighed by claims created on them, or more succinctly, *the ex-ante supply of reserves affects the ex-post demand for them*.

Importantly, individual banks take the expected rates in the interbank market as given, and do not take account of the effects of their financing or activity choices on those rates. Ex ante, if they financed their own reserve holdings with more long-term capital (as the social planner would desire), there would be less call on liquidity when the economy is liquidity stressed. However, such financing is not privately optimal (as suggested by Caballero and Krishnamurthy (2003), Lorenzoni (2008) and Stein (2012)), and not observed in practice.

Consider the second contributor to liquidity stress, the extent of reserve encumbrance. Why might only a fraction of a dollar set aside as reserves be available to pay out on a future date? Two possibilities are speculation and regulation. A bank holding highly liquid reserves, with the reserves being required only in situations of liquidity stress, will want to try and “sell” liquidity in all the states it does not need it, for instance, by backstopping the liquidity needs of firms or speculators for a fee. To the extent that such backstopping cannot be fine-tuned, it will spill over into the states where the economy is liquidity-stressed and there is a high value for liquidity.⁶ The amount of free liquidity in such states will shrink relative to the ex-ante size of the reserves. Indeed, defaults on such speculative trades (or even the price impact of unwinding trades because of increasing haircuts or margins, see Aramonte, Schrimpf, and Shin (2021)) may be a source of ex-post contagion in the financial sector.

To reduce its incidence and impact, regulation of centralized clearing may require dealer banks to encumber a portion of the liquid assets as guarantee funds for the settlement of defaulted trades; similarly, regulation may require non-centrally cleared positions of dealer banks to also be backed by liquid assets for prudential management of the ex-post risks they pose. Such liquidity regulations, in response to speculation, are another source of encumbrance. Shouldn’t the regulator suspend a liquidity requirement imposed ex ante in the face of ex-post stress so that more reserves are available to alleviate market

⁶ Yankov (2020) examines the changes in the liquidity management at banks and nonbank financial firms in the United States that occurred following the proposal of the liquidity coverage ratio (LCR) requirement in 2010 and its finalization in 2014. He concludes that “While banks increased their liquid assets to meet the new regulatory liquidity requirements, nonbank financial institutions—such as insurance companies, finance companies, real estate investment trusts, pension funds, asset managers, mutual funds, and others—decreased their liquid assets and increased their reliance on bank credit lines to manage their liquidity risks.” Thus banks effectively sold claims that would be collectively called upon in states where liquidity would likely be scarce.

illiquidity? Diamond and Kashyap (2016) explain why the regulator may not want to release banks from holding reserves for fear that localized stress morphs into a full-blown panic.⁷ Furthermore, ratchet effects whereby supervisors scrutinize reductions in reserves closely no matter what the prior level held (see Nelson (2019)) would also contribute to reserve hoarding.

Consider now the third influence on ex-post liquidity – reserve hoarding by profit-maximizing banks. We assume banks see a convenience yield in directly holding reserves in times of stress. Given this benefit to holding liquidity, healthy banks can choose between two options: to either remain perceptibly safe and attract flight-to-safety deposits (with their associated convenience yield); or to lend in the interbank market and realize the associated rents but in the process become *tainted* and attract none of the flight-to-safety reserve flows. The interbank rate will have to move up to convince banks to lend and forego the convenience yield that comes from attracting flight-to-safety deposits. Clearly, the higher the perceived convenience yield, the higher the interbank rate will have to be. Indeed, the interbank market may remain shut altogether because it is cheaper for stressed banks to raise capital than to pay the exorbitant required rate. The bottom line though is liquidity hoarding by healthy banks can contribute to the ex-post shortage even if aggregate liquidity is plentiful.

In sum, the key problem is that central bank reserves are placed with commercial banks. Commingled with those balance sheets, they influence other activities (lending, deposit financing, hoarding) that have the potential for significantly enhancing the net demand for liquidity. If, instead, central bank reserves were placed directly with households, or with financial intermediaries that did not issue claims on liquidity, the effects we hypothesize would be mitigated.

Given current practice, our paper suggests that under certain circumstances, there is a threshold size of the central bank balance sheet beyond which further expansion will increase the severity of future liquidity problems. Consequently, the balance sheet size that is optimal from a purely monetary perspective may be excessive from a financial stability perspective. More generally, even though the central bank has no direct cost of creating additional fiat money (Friedman (1969)), our paper proposes a social cost stemming from the reactions of market participants with consequences for financial stability.

Much of the related literature has focused on frictions such as market segmentation, capital regulation, and timing mismatches (from intraday payments and Treasury sales) to explain price spikes in

⁷ One function of a liquidity requirement is to prevent a panic by assuring depositors that the regulated bank has plenty of liquidity to meet both expected and unexpected needs. Depositors may hold off on running on a bank even when other banks are being run if it is convinced regulators will force the bank to hold on to liquidity under almost all circumstances. For this reason, regulations like the liquidity coverage ratio may limit the amount of its reserves a healthy bank is free to lend even in extremis, and these regulations may be hard to suspend even in the midst of an episode of liquidity stress.

usually liquid money markets. To alleviate these spikes, a number of commentators suggest that the central bank provide more liquidity in stressed times to a wider array of market participants (see the G30 Report on US Treasury Markets 2021), that it permanently expand the quantum of reserves (Copeland, Duffie, and Yang (2021)), or that it reduce or eliminate capital requirements against reserves (Liang and Parkinson (2020)). While these proposals will likely reduce stress ex post, we also need an ex-ante analysis of why the system is so fragile despite abundant reserves to understand the full consequences of the proposed policies.

For instance, reducing capital requirements on reserves could make them more available ex post, but it will increase the short-term financing of reserves, which contributes to the problem. Similarly, the central bank can certainly flood the market with reserves ex post. Such intervention is not without cost. Ex post, it crowds out lending by healthy banks, increasing the scale of the needed intervention. Ex ante, market participants are even more inclined to write future claims on liquidity and ever more reliant on the central bank backstop. Consequently, we should expect escalating and more frequent central bank interventions over time, with broader categories of assets accepted as collateral for the central bank intervention, and potential distortions creeping into asset prices as well as asset allocations (see Acharya, Shin and Yorulmazer (2011), Diamond and Rajan (2012) or Farhi and Tirole (2011)). In our analysis, growing liquidity dependency of the system on the central bank is a concerning possibility.

Most closely related to our paper is Diamond, Jiang, and Ma (2021), who ask how the reserve build-up by the Federal Reserve could affect bank lending. While they too emphasize the need to finance reserves, their focus is on the crowding-out effects of such reserve holdings on corporate loans. Using structural estimation methods, they conclude the \$2.7 trillion increase in Fed reserves from quantitative easing reduced bank lending by over \$500 billion. Our focus instead is on the effects of reserves on ex-post liquidity, and how that would impact corporate lending. Even though our papers are complementary given their focus, the implications, for example on whether capital requirements should be relaxed ex ante for reserve holdings, are different.

The rest of the paper is as follows. In section I, we lay out a simple model of central bank balance sheet expansion; in section II we analyze the benchmark model which takes as exogenous the fraction of healthy banks that choose to hoard reserves; in section III we examine the central bank or social planner's considerations; in section IV we endogenize the level of reserve hoarding among healthy banks, and in section V the encumbrances on reserves. In section VI, we examine robustness and extensions, and then conclude.

I. The Model

Consider an economy with three dates, 0, 1, and 2. Subscripts denote the date in what follows and Greek letters are parameters. There are four sets of agents in the economy: firms, banks, risk-averse savers, and risk-neutral savers (with the central bank playing a cameo role in determining reserves). The state of the economy \tilde{y} is revealed at date 1. It can be healthy ($y = 0$) or liquidity stressed ($y = 1$). Firms and banks maximize expected profits.

1.1. Firms

Each firm could be thought of as representing an entire sector of the real economy. The firm has access to an investment opportunity at date 0. The state of the firm \tilde{z} is revealed at date 1. It is always healthy ($z = 0$) when the economy is healthy. However, the firm can be stressed ($z = 1$) with probability θ when the economy is liquidity stressed, which occurs with probability $\frac{q}{\theta}$. The firm is otherwise healthy. So the date-0 probability of a firm getting stressed at date 1 is q . The time line for the state space of economic outcomes is in Figure 1 (we will explain shortly the bank-level outcomes illustrated therein).

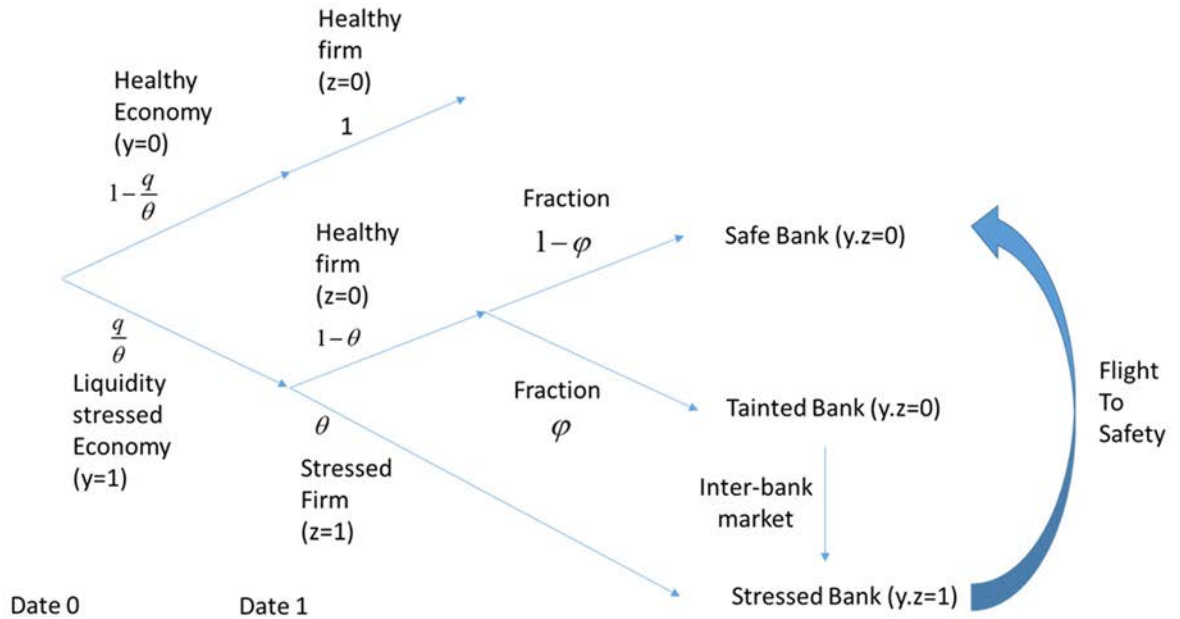


Figure 1: The state space of economic and bank-level outcomes

Liquidity stress in our model stems from real needs for spending, which in turn precipitate larger financial demands for liquidity. An investment of I_0 at date 0 produces $g_0(I_0)$ at date 2 if the firm is

healthy. If stressed, the firm produces nothing at date 2 from its original investment. However, it has the possibility of “rescuing” some of its earlier investment by investing an additional amount I_1 . The expected output from such investment is $g_1(I_1)$. This output of the rescue investment is high enough in expectation to allow the firm to repay the expected value of its loans, both for the initial investment and the rescue investment, but there is a non-zero probability that nothing is produced from the rescue investment and the entire sequence of investments is a write-off. Both g_0 and g_1 are increasing and concave, and obey Inada conditions. We focus on real investment but a model where losses on financial investment precipitate margin calls, which necessitate new funding to avoid distressed selling, would have similar effects.

The firm starts out with own funds of W_0^F , and will supplement it with L_0^F of long-term borrowing from the bank. Apart from the real investment at date 0, it can also place deposits of D_0^F in the bank. We can think of this as the firm’s precautionary liquidity holdings, and is isomorphic (up to the fees charged) to pre-contracted credit lines from the bank.

1.2. Banks

Each bank lends to a firm (or in the alternative interpretation, an entire sector). So a bank and a firm constitute a pair, and we will refer to a bank that has lent to a firm that has become stressed also as “stressed”. At date 0, the bank can make a two period loan of amount L_0^B (think of L_0^F as loan demand and L_0^B as loan supply, and in equilibrium, the two will be equal at L_0) at a cumulative gross interest rate of R_0^L . The bank incurs a cost of $\frac{1}{2} \lambda (L_0^B)^2$ in making the loan – the cost is increasing and convex because the bank has to manage, and lay off, an increasing amount of risk. At date 0, each bank also has to hold S_0 of reserves that the central bank has issued. For now, we assume it has no choice about the size of reserves it holds, these flow automatically from its (symmetric) share of financial activity, which is given.

1.3. Bank Financing

The entire analysis will be conducted on a per bank-firm pair basis. To finance its asset holdings, a bank can raise deposits at date 0 from the risk-averse saver, whose rate of time preference is 1. So if D_0 is the quantum of overall deposits it raises, then $(D_0 - D_0^F)$ is what it raises from the public, receiving the

rest from the firm. Implicit here is the assumption that there are only a limited number of risk-neutral savers in the economy so deposits cannot be financed by them.

The risk-averse saver has log utility over consumption at date 2. We assume that if the low probability event that the stressed firm repays nothing on the rescue loan materializes at date 2, the bank will have to default on deposits at date 2.⁸ Anticipating their deposits to be haircut, risk-averse depositors will run on the bank at date 2 to avoid being the one at the back of the line that gets nothing. In turn, anticipating a run at date 2 and thus possible zero consumption even with small probability, risk-averse depositors will ask for their money back from a stressed bank at date 1. Put differently, even though the bank is solvent at date 1, as in Stein (2012) it will have to repay its risk-averse depositors immediately if stressed. Think of risk-averse depositors as institutions such as companies, hedge funds, and pension funds where their CFO loses their job if they have inadvertently left low-yielding transaction deposits in a bank that is risky or fails – this induces extreme risk-aversion about transaction deposit accounts.⁹ The firm also withdraws some or all of its bank deposits to make the date-1 rescue investment.

The bank can raise long-term funding (that is, it can raise bank capital consisting of long-term bonds or equity) from the risk-neutral investor (an investor like Warren Buffet or a sovereign wealth fund), both at date 0 and date 1. The bank faces a repayment cost of $e_t + \frac{\alpha_t}{2} e_t^2$ at date 2 when it raises amount e_t at date t . The quadratic term could be composed of a variety of costs associated with long-term capital relative to short-term deposits, including higher illiquidity premia, higher term premia, higher borrower moral hazard, and due diligence costs. These costs could be significantly higher if the bank has to raise capital at date 1 (typically when the economy is liquidity stressed) rather than date 0.

1.4. Firm Financing in the Stressed State at Date 1

To make its date-1 rescue investment, the stressed firm can borrow l_1^F from its bank at date 1 to supplement the deposits it withdraws. The bank will have to do significant due diligence and monitoring, given the stressed state of the firm, so the interest rate charged will be $(1+r_1+\gamma)$ where γ is the bank's deadweight due-diligence and monitoring costs which are passed on to the firm. For simplicity, we

⁸ We make the reasonable assumption therefore that the bank's assets, including reserves, are not enough to pay deposits in full when its loans default in entirety.

⁹ Alternative depositor behaviors such as their perception that only perfectly safe assets have the requisite "moneyness" (Stein (2012)) or that depositors do not want to monitor the bank (Dang, Gorton, Holmstrom (2010)) would have similar consequences.

assume that all interest rates reflect expected values (so that face values are set to deliver that rate after accounting for any default risk). This reduces notation and lets us focus on liquidity.

1.5. Reserves Encumbrance and Interbank Market

At date 1, a stressed bank can use its reserves to meet depositor/firm needs. We assume that a fraction τ of the reserves a bank has at date 1 is encumbered, *i.e.*, it cannot be used or lent, either for regulatory reasons or because they have been pledged elsewhere – we will endogenize this fraction later as well as examine the consequences of having a fixed level of encumbrance even as reserves increase. For now, only $(1 - \tau)$ fraction of the initial reserves is available at date 1.

A stressed bank can also borrow in the interbank market, where healthy banks with surplus reserves can lend. The gross interest rate over the second period in the inter-bank market is 1 if there is an excess of loanable funds relative to demand. If not, the gross interest rate will rise to equalize the demand and supply for funds, and will be higher at $(1 + r_1)$; when this is the case, stressed banks and healthy banks that are active in the interbank market will find it attractive to issue some capital at date 1.

1.6. Flow of Reserves due to Deposit Flight and Capital Issuance at Date 1

Where do deposits that flee the distressed banks (as well as the incremental deposits that are created by the payments on the date-1 rescue investment) go? This is a critical issue and will influence important results in the paper. We assume these deposits get parked in *safe* banks. But what is safe? Any healthy bank that lends in the interbank market bears some risk of not being repaid. Institutional depositors will wonder how much total risk such a bank is taking. We therefore assume that to be seen as *safe*, a healthy bank should maintain unimpeachable balance sheet and in particular not lend to distressed banks in the date-1 interbank market. It will then attract a proportional share (with other safe banks) of the flight-to-safety deposits that flee the distressed banks. The healthy banks that choose to lend in the interbank market will be referred to as *tainted* (low-reserve-balance) banks and will not attract any flight-to-safety deposits if the economy is liquidity stressed. We assume that their existing deposits do not withdraw, though alternative assumptions are easily analyzed. We also assume that only a fraction ϕ of the healthy banks lend in the interbank market in the stressed state of the economy at date 1 and become tainted. At date 0, any bank will assume that conditional on being healthy, there is a probability ϕ it will become tainted. We take the fraction ϕ as exogenous for now. We will endogenize it in section IV.

We also assume that any bank capital issued at date 1 is bought by risk-neutral investors who first acquire deposits in safe banks (for instance, by selling risk averse depositors their treasury bills), and then

transfer the safe bank's reserves to the capital-issuing bank by writing the latter a check (alternative assumptions would worsen the date-1 illiquidity problem). Similarly, any payment by firms for the rescue investment such as the purchase of equipment or inventory goes as a check to goods or service providers, who then deposit the check in the safe banks.

These detailed assumptions about payments are necessary to track the flow of reserves through the banking system. Note that the banking system does not gain or lose reserves as a result of depositor movement or capital issuance; the latter moves reserves from safe banks to capital-issuing banks, whereas the former moves reserves to safe banks. Importantly, there is no shortage of payment media at date 1. However, safe banks hoard reserves, which leads to a tight interbank loan market, and thus affects date-1 and date-0 investment.

1.7. Central Bank

The central bank issues reserves S_0 per bank at date 0, which each bank has to hold at date 0. Further

- i) The banking system as a whole has to hold reserves issued by the central bank. With no ex-ante differentiation, banks assume they will be held symmetrically. Put differently, no bank can avoid holding reserves without refusing to accept legal tender as payment. We also do not initially allow for central bank reserves to be held directly by the non-bank sector. We study incentives to hold reserves later.
- ii) We net out the volume of deposit creation engendered by the issuance of high-powered reserves, looking only at final "reduced-form" balance sheets. The pyramiding of deposits via the money multiplier typically introduces complications as to how claims are run upon, netted and settled (see, for example, Kashyap (2020)) that would magnify the problems we examine.

II. Analysis

2.1. The Firm's Problem

To ease understanding of the calculations that follow, we present firm and bank balance sheets at date 0 and date 1 in Exhibit 3. With probability q the firm will be stressed at date 1, and it will be healthy with probability $(1-q)$. So its date-0 maximization problem and its date-1 maximization problem are as follows:

$$\text{Date 0: } \underset{L_0^F, D_0^F}{\text{Max}} (1-q) \left[g_0(I_0) + D_0^F \right] + q \left[g_1(I_1) - l_1^F (1 + \gamma + r_1) \right] - R_0^L L_0^F$$

$$\text{Date 1: } \underset{l_1^F}{\text{Max}} g_1(I_1) - l_1^F (1 + \gamma + r_1)$$

$$\text{s.t. } I_0 = L_0^F + W_0^F - D_0^F \text{ and } I_1 = l_1^F + D_0^F$$

The constraints are just budget constraints at each date. The firm's first order conditions (FOC's) then are

$$\text{w.r.t. } L_0^F : (1-q)g_0' - R_0^L = 0 \quad (1.1)$$

$$\text{w.r.t. } D_0^F : (1-q)(-g_0' + 1) + q(g_1') = 0 \quad (1.2)$$

$$\text{w.r.t. } l_1^F : g_1' - (1 + \gamma + r_1) = 0 \quad (1.3)$$

Substituting the value of g_1' from (1.3) into (1.2), we get $(1-q)g_0' = (1 + q\gamma + qr_1)$. Term the right-hand side of this expression R_0^{DF} : it is the expected opportunity return for the firm of holding an additional dollar of deposit, and thus avoiding borrowing from the bank at date 1 if stressed. Comparing with (1.1) where the firm's marginal expected return on date 0 investment is equal to the cost of long-term borrowing from the bank, we get $R_0^L = R_0^{DF}$. In words, the cost of long-term borrowing is equal to the opportunity return on holding an additional dollar of deposit. Let us now turn to the bank's problem.

2.2. The Bank's Problem

The bank maximizes profits given constraints, that is,

$$\begin{aligned} \text{Max}_{L_0^B, e_0, e_1} \quad & R_0^L L_0^B + S_0 - e_0 - \frac{\alpha_0}{2} e_0^2 - D_0 \\ & + \frac{q}{\theta} \theta \left[-e_1 - \frac{\alpha_1}{2} (e_1)^2 - (1+r_1)(b_1(y=1, z=1) - l_1^B) \right] \\ & + \frac{q}{\theta} (1-\theta) \varphi \left[-e_1 - \frac{\alpha_1}{2} (e_1)^2 - (1+r_1)b_1(y=1, z=0) \right] \end{aligned}$$

$$\text{s.t. } D_0 + e_0 = L_0^B + \frac{1}{2} \lambda (L_0^B)^2 + S_0 \quad (1.4)$$

$$b_1(y=1, z=1) = l_1^B + D_0 - S_0(1-\tau) - e_1 \quad (1.5)$$

$$b_1(y=1, z=0) = -S_0(1-\tau) - e_1 \quad (1.6)$$

$$l_1^B = l_1^F = I_1 - D_0^F \quad (1.7)$$

The first line of the maximization is the bank's profits if the economy does not become stressed. The second line is the expected return for a stressed bank, which comes from lending $l_1^B (=l_1^F)$ in equilibrium to the stressed firm less financing costs. The stressed bank's loan to the firm and the deposit outflows are funded by unencumbered reserves, capital raised, and interbank borrowing (of $b_1(y=1, z=1)$, see (1.5)). The third line of the maximization is the expected profits to a healthy bank from becoming tainted and lending $-b_1(y=1, z=0)$ into the interbank market when the economy is stressed; it makes the loan out of its unencumbered reserves and the capital it raises (see (1.6)). Note that both the stressed bank and the tainted bank will raise the same amount of capital in equilibrium because they will both see the same marginal return of using that capital in the interbank market (for the stressed bank it reduces borrowing, and for the tainted bank it increases loans).¹⁰ The constraint (1.4) simply reflects the sources and uses of funds at date 0 (the bank raises money from deposits and long-term capital, and invests in loans, the cost of making loans, as well as forced reserve holdings). Then, the first order conditions are

$$\text{w.r.t. } L_0^B : \quad R_0^L - (1 + \lambda L_0^B)(1 + qr_1) = 0$$

From the bank's perspective, the date-0 return from making another dollar of loan should equal the cost of funding that dollar (and the associated marginal cost of managing the risk of the additional loan, λL_0^B) via flighty deposits, which cost a net rate of r_1 per dollar if the bank gets stressed. Let us term as R_0^{DB} the expected cost of funding via deposits, which equals $(1 + qr_1)$. Hence, $R_0^L = (1 + \lambda L_0^B)R_0^{DB}$. Next, FOC

$$\text{w.r.t. } e_0 : \quad -(1 + \alpha_0 e_0) + (1 + qr_1) = 0$$

This implies the marginal cost of raising an additional dollar of long-term funding or capital at date 0

should equal the saving on funding via deposits. So $e_0 = \frac{(R_0^{DB} - 1)}{\alpha_0} = \frac{qr_1}{\alpha_0}$. In words, the bank raises more

capital at date 0 the higher the expected premium it will pay in the interbank market in the stressed state – note importantly that it takes the date-1 rate as given and does not see that its capital-raising will have an effect on that rate. Finally, FOC

¹⁰ Note that banks that remain safe and hoard reserves will receive a lump sum in flight-to-safety deposits that depends on choices of *other* banks, and therefore will not affect the date-0 optimization problem of a given bank. When we endogenize φ in Section IV, this lump sum will matter but still not affect the date-0 problem.

$$\text{w.r.t. } e_1 : \quad -(1 + \alpha_1 e_1) + (1 + r_1) = 0$$

So at the margin, the bank's cost of raising an additional dollar of capital at date 1 equals the cost of borrowing in the interbank market. Simplifying,

$$r_1 = \alpha_1 e_1 \quad (1.8)$$

Hence the prevailing premium in the interbank market drives capital-raising at date 1 by stressed and healthy tainted banks and vice versa. Importantly, the firm and bank's maximization decisions link the various interest rates to the date-1 premium in the interbank market r_1 . So

$$R_0^L = R_0^{DF} = (1 + q\gamma + qr_1) = (1 + \lambda L_0^B) R_0^{DB} \quad (1.9)$$

We know that the inter-bank premium is necessary in order to equalize the date-1 demand and supply of funds when the economy is liquidity stressed – essentially the premium draws forth more date-1 issuance in the capital market by stressed and tainted banks even while reducing rescue investment by stressed firms and the associated demand for funds that spills over into the interbank market. The net date-1 shortfall in the interbank market in the liquidity stressed economy is

$\theta [I_1 + (D_0 - D_0^F)] - [\varphi(1 - \theta) + \theta] S_0(1 - \tau)$. The first term in the first square bracket is the “rescue” investment by the stressed firms, and the second term is the expected withdrawal by the risk-averse depositors from stressed banks (which is redeposited in safe banks). The sum is the call on liquidity by the system, which is reduced by the available shrunken reserves with stressed and tainted healthy banks (the last term). This overall shortfall, when positive, exactly equals $[\varphi(1 - \theta) + \theta] e_1$, the date-1 capital raised by the tainted and stressed banks (note that safe banks do not raise date-1 capital because they have no profitable way to deploy it). So when r_1 is positive, we have from (1.8),

$$[\varphi(1 - \theta) + \theta] \alpha_1^{-1} r_1 = \theta [I_1 + (D_0 - D_0^F)] - [\varphi(1 - \theta) + \theta] S_0(1 - \tau)$$

or equivalently, $r_1 \equiv \alpha_1 f(r_1, S_0)$ (1.10)

$$\text{where } f(r_1, S_0) \equiv \frac{\theta}{[\varphi(1 - \theta) + \theta]} [I_1 + (D_0 - D_0^F)] - S_0(1 - \tau).$$

Of course, if there are sufficient reserves in the banking system to meet funding needs without capital issuance, then $r_1 = 0$. Throughout, we denote the equilibrium interbank rate premium as \bar{r}_1 .

2.3. Date-1 Interbank Rate

Let us now analyze the interbank rate and how it varies with reserves issued, S_0 .

Lemma 1: The date-1 equilibrium interest rate in the inter-bank market is unique.

Proof: See Appendix.

The proof essentially demonstrates that the net need for capital issuance, $f(r_1, S_0)$, falls in r_1 . Since the left hand side of (1.10) is increasing in r_1 and the right hand side is decreasing, there is a unique positive solution when the right hand side is positive at $r_1 = 0$, and it is 0 otherwise.

How does the possible positive equilibrium rate, \bar{r}_1 , implicitly determined by (1.10), vary with central bank reserves? Totally differentiating (1.10), we have $\frac{1}{\alpha_1} \frac{d\bar{r}_1}{dS_0} = \frac{\partial f}{\partial r_1} \cdot \frac{d\bar{r}_1}{dS_0} + \frac{\partial f}{\partial S_0}$. Therefore,

$$\left(\frac{1}{\alpha_1} - \frac{\partial f}{\partial r_1} \right) \frac{d\bar{r}_1}{dS_0} = \frac{\partial f}{\partial S_0} = \frac{\theta}{[\varphi(1-\theta) + \theta]} - (1-\tau).^{11} \text{ Since } \frac{\partial f}{\partial r_1} \text{ is negative as shown in the proof of}$$

Lemma 1, $\left(\frac{1}{\alpha_1} - \frac{\partial f}{\partial r_1} \right)$ is positive, and $\text{Sign} \left(\frac{d\bar{r}_1}{dS_0} \right) = \text{Sign} \left(\frac{\theta}{[\varphi(1-\theta) + \theta]} - (1-\tau) \right)$. Consequently,

Lemma 2: If

$$(i) \quad \theta > \frac{\varphi(1-\tau)}{\tau + \varphi(1-\tau)}, \frac{d\bar{r}_1}{dS_0} > 0; \quad (ii) \quad \theta < \frac{\varphi(1-\tau)}{\tau + \varphi(1-\tau)}, \frac{d\bar{r}_1}{dS_0} < 0; \quad (iii) \quad \theta = \frac{\varphi(1-\tau)}{\tau + \varphi(1-\tau)}, \frac{d\bar{r}_1}{dS_0} = 0 .$$

Essentially, Lemma 2 (i) suggests the stress in the interbank market, and the associated equilibrium rate for funds, can increase in the extent of reserves that the central bank injects into the system at date 0. Importantly, this will also reduce date-0 and date-1 real investments. At first pass, the result seems counterintuitive. How can more liquidity supply at date 0 increase liquidity stress at date 1? However, this result is counterintuitive only from a partial-equilibrium perspective. Recognize first that the marginal source of funding of the reserves is demand deposits, which potentially create their own demand for liquidity in the stressed state (in proportion to the fraction of stressed banks, θ). Moreover,

¹¹ This requires substituting in (1.10)

$(D_0 - D_0^F) = (S_0 + L_0^B + \frac{1}{2} \lambda (L_0^B)^2 - e_0) - (W_0^F + L_0^F - I_0) = I_0 - e_0 + (S_0 - W_0^F) + \frac{1}{2} \lambda (L_0^B)^2$, where the second equality uses $L_0^B = L_0^F$.

only a fraction $\varphi(1-\theta)$ of healthy banks use their unencumbered reserves to meet the liquidity demands of stressed banks, and only $(1-\tau)$ of each dollar of their reserves is available at date 1. Put differently,

$\frac{d\bar{r}_1}{dS_0} > 0$ whenever the marginal liquidity provided by each dollar of reserves, $(1-\tau) [\varphi(1-\theta) + \theta]$, is

lower than the marginal call on liquidity when demand deposits are withdrawn from stressed banks, θ .

Simplifying, the required condition for more date-0 reserves to constrain date-1 liquidity further and raise

the equilibrium interbank rate is $\theta > \frac{\varphi(1-\tau)}{\tau + \varphi(1-\tau)}$ as in lemma 2.

Note that a fraction $(1-\varphi)$ of healthy banks not only hoard *all* of their reserves, but also any flight-to-safety reserves they obtain. With full liquidity hoarding ($\varphi = 0$), the condition for Lemma 2 (i) is always met as long as $\tau > 0$; conversely, when the interbank market is fully open ($\varphi = 1$), then Lemma 2 (i) holds whenever $\theta > (1-\tau)$. Lemma 2 (ii) is, of course, the more traditional view that more reserves injected at date 0 will reduce the date-1 interbank premium.

2.4. Threshold Reserve Levels

Because $f(r_1, S_0)$ is decreasing in r_1 , it must be that \bar{r}_1 is positive iff $f(0, S_0) > 0$. We have

$$f(r_1, S_0) = \frac{\theta}{[\varphi(1-\theta) + \theta]} \left(g_1'^{-1}(1+\gamma+r_1) + g_0'^{-1} \left(\frac{1+q\gamma+qr_1}{(1-q)} \right) - \frac{qr_1}{\alpha_0} - W_0^F + \frac{1}{2} \frac{q^2}{\lambda} \left(\frac{\gamma}{1+qr_1} \right)^2 \right) + S_0 \left(\frac{\theta}{[\varphi(1-\theta) + \theta]} - (1-\tau) \right)$$

So, for $f(0, S_0) > 0$, it must be that

$$S_0 \left(\frac{\theta}{[\varphi(1-\theta) + \theta]} - (1-\tau) \right) > \frac{\theta}{[\varphi(1-\theta) + \theta]} \left(-g_1'^{-1}(1+\gamma) - g_0'^{-1} \left(\frac{1+q\gamma}{(1-q)} \right) + W_0^F - \frac{1}{2} \frac{q^2}{\lambda} (\gamma)^2 \right) \equiv NLS \quad (1.11)$$

The left hand side is the net liquidity demand created by reserves. The right hand side is the net liquidity supplied (NLS) by the corporate sector anticipating a date-1 interbank premium of zero (and adjusting for any cost to the bank of long term lending). NLS is high when the corporate sector has high levels of starting internal funds W_0^F and relatively low demand for funds for investment and loans. The

equilibrium interbank rate (and date-1 capital market rate) is positive if the net liquidity demand exceeds supply at a rate of zero. Since $f(r_1, S_0)$ decreases in r_1 , and changes in S_0 and τ only shift the term containing S_0 and not the slope of $f(r_1, S_0)$, using lemma 2 we can describe the level of date-0 central bank reserves \hat{S}_0 , at which the interbank rate turns positive. We can also describe how the interbank rate moves with reserves around that threshold. We have

Theorem 1:

- (i) If $\theta > \frac{\varphi(1-\tau)}{\tau + \varphi(1-\tau)}$, then $\bar{r}_1 > 0$ is the unique equilibrium for $S_0 > \hat{S}_0$ with \bar{r}_1 increasing in S_0 ; and, $\bar{r}_1 = 0$ for $S_0 \leq \hat{S}_0$. Note that $\hat{S}_0 \leq 0$ if $NLS \leq 0$.
- (ii) If $\theta \leq \frac{\varphi(1-\tau)}{\tau + \varphi(1-\tau)}$, then $\bar{r}_1 > 0$ is the unique equilibrium for $S_0 < \hat{S}_0$ with \bar{r}_1 decreasing in S_0 ; and, $\bar{r}_1 = 0$ for $S_0 \geq \hat{S}_0$. Note that $\hat{S}_0 \geq 0$ if $NLS \geq 0$.

Proof: Follows from the discussion above.

2.5. Discussion

Theorem 1 (ii) is the traditional view of reserves. An increase in reserves should alleviate future illiquidity, reduce the interbank rate, and increase current (and future) real investment. A preponderance of reserves, $S_0 \geq \hat{S}_0$, ensure that the date-1 interbank interest rate premium will be zero.

Let us plot in Figure 2A the threshold value of reserves at which the date 1 interbank rate is zero, \hat{S}_0 , for different values of θ (the fraction of the banking sector that becomes liquidity stressed) for the more plausible case that the corporate sector absorbs liquidity so $NLS < 0$ (In Figure 2B in the Appendix we analyze $NLS > 0$). Note that θ is on the horizontal axis and the size of reserves is on the vertical axis. When $\theta < \frac{\varphi(1-\tau)}{\tau + \varphi(1-\tau)}$ (to the left of the vertical axis), \hat{S}_0 is positive, and rises in θ .

Because higher ex-ante reserves loosen liquidity conditions, \bar{r}_1 falls in S_0 , and the unhatched region below the \hat{S}_0 curve is where \bar{r}_1 is positive.

Theorem 1 (i) is the alternative view our model also offers. When $\theta > \frac{\varphi(1-\tau)}{\tau + \varphi(1-\tau)}$ (to the right of the vertical axis), \hat{S}_0 is negative, and increases in θ . Because higher ex-ante reserves tighten liquidity in the stressed state, \bar{r}_1 increases in S_0 , and the unhatched region above the \hat{S}_0 curve is where \bar{r}_1 is positive. In this unconventional case, the central bank cannot provide the demanded date-1 liquidity via banks through asset purchases at date 0 – reserve issuance also tends to absorb liquidity on net.

To summarize, when the economy is healthy, the inter-bank premium is always zero. Each claimant’s idiosyncratic liquidity demand is likely to be diversified away across a large set of diverse claimants (see Kashyap, Rajan, and Stein (2002), for example). Central bank supplied liquidity is likely to be ample for such needs. We are focused on the net availability of liquidity in tail situations when liquidity demands become much more strongly positively correlated (for example, as witnessed by banks in the United States during 2007-08 and documented in Acharya and Mora (2015), or as witnessed by firms at onset of the pandemic in March 2020), or when interbank market is tight due to hoarding of liquidity by a significant proportion of surplus banks (as witnessed during the repo rate spike of September 2019, as shown by Copeland, Duffie and Yang, 2021). In that case, the way the reserve holdings are financed matters, and the net demand for liquidity could increase in the size of reserves being financed. There is then a threshold central bank reserve issuance level (and balance sheet size) beyond which liquidity conditions tighten.

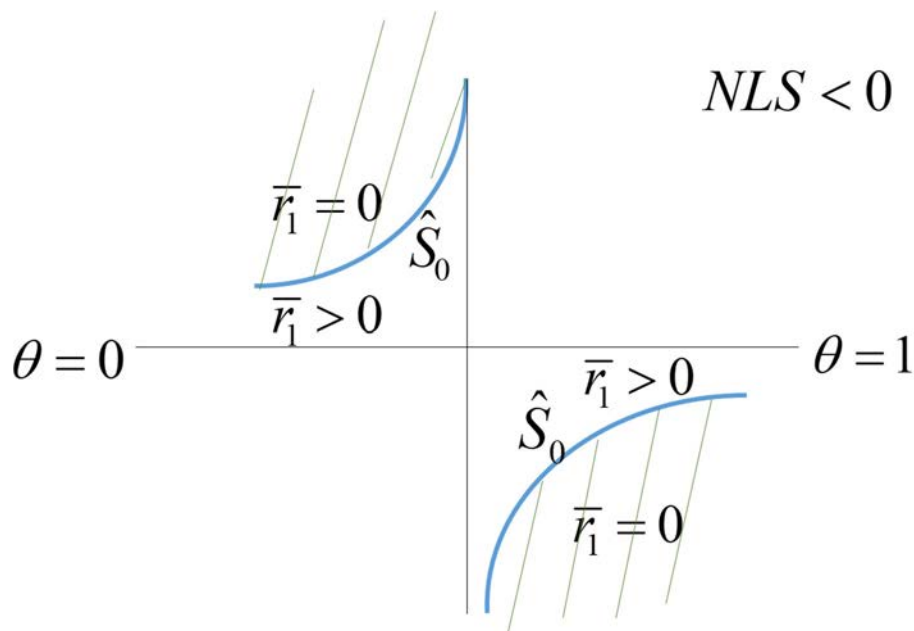


Figure 2A: Reserves are on the y axis, θ on x axis, with the axes intersecting at $\theta = \frac{\varphi(1-\tau)}{\tau + \varphi(1-\tau)}$

III. The Central Bank's Problem

We have taken the ex-ante level of reserves as given, and examined the consequences for the ex-post availability of liquidity as well as credit market rates and investment. What we have in mind thus far is the central bank may be setting reserves for monetary purposes, for instance to effect a target level of quantitative easing. What if the planner/central bank instead set reserves with the view of maximizing welfare in our framework? Throughout this section, we will continue to take the fraction φ of surplus banks that lend in interbank markets as exogenous in order to focus on the inefficiency arising from the manner in which banks finance their reserves holdings.

3.1. The Planner/Central Banker's Problem and Optimal Reserves

The planner/central bank wants to maximize output net of real costs, that is, maximize w.r.t. S_0

$$U \equiv ((1-q)g_0(I_0) - I_0) + q(g_1(I_1) - I_1 - (I_1 - D_0^F)\gamma) - \frac{1}{2}\alpha_0 e_0^2 - \frac{q}{\theta}[(1-\theta)\varphi + \theta](\frac{1}{2}\alpha_1 e_1^2) - \frac{1}{2}\lambda(L_0)^2 \quad (1.12)$$

where $(I_1 - D_0^F)$ is the firm's date-1 borrowing from the bank that is associated with a per unit

deadweight cost γ . It follows that $\frac{dU}{dS_0} = \frac{\partial U}{\partial \bar{r}_1} \cdot \frac{d\bar{r}_1}{dS_0}$ since $\frac{\partial U}{\partial S_0} = 0$ (the planner has no direct cost of

supplying reserves as suggested by Friedman (1969)). It is easily shown (see Appendix) that $\frac{\partial U}{\partial \bar{r}_1} < 0$.

Consequently, the planner wants to raise S_0 only if it brings down the date-1 interbank market rate

premium, i.e., $\frac{d\bar{r}_1}{dS_0} < 0$. Conversely, if $\frac{d\bar{r}_1}{dS_0} > 0$, the planner wants to reduce reserve issuance. In the

cases we have seen so far, the answer to the optimization is obvious: the planner will set the reserves at any level such that the anticipated interbank rate premium \bar{r}_1 is zero.

3.2. Negative \hat{S}_0

When the threshold level of reserves, \hat{S}_0 , is positive, it is clear that the planner/central bank will set reserves at any level at or above \hat{S}_0 when $\theta < \frac{\varphi(1-\tau)}{\tau + \varphi(1-\tau)}$ and at or below \hat{S}_0 when $\theta > \frac{\varphi(1-\tau)}{\tau + \varphi(1-\tau)}$.

When the threshold level of reserves, \hat{S}_0 , is negative, matters are equally easy when $\theta < \frac{\varphi(1-\tau)}{\tau + \varphi(1-\tau)}$.

Essentially, the corporate sector is in liquidity surplus, and the banking sector can utilize a lot of liquidity before the date-1 interbank market tightens enough to make the rate positive. If the intent is to set the interbank premium to zero, any positive level of reserves will also do since every dollar of reserves adds to date-1 liquidity.

The problem arises when $\theta > \frac{\varphi(1-\tau)}{\tau + \varphi(1-\tau)}$. Additional reserves will exacerbate the liquidity

shortage since every dollar of date-0 reserves subtracts from date-1 liquidity. While “negative” reserves may be required in theory, it is unclear how this can be implemented. Perhaps it is best to recognize that when reserves do not add to future liquidity, the central bank should find other instruments so that the banking sector can provide liquidity to the deficient corporate sector – for instance by making long-term loans to the banking sector to encourage the purchase of long-term corporate financial assets/loans, and then being prepared to lend against those assets in case the economy becomes liquidity stressed.

Parenthetically, this may resemble the European Central Bank’s Long-Term Refinancing Operation (LTRO) interventions.

3.3. Role for Capital Requirements

We have focused on the optimal level of reserves that the central bank will issue *given our framework*. This level can be considered as one that is based on *financial stability* considerations. In practice, the central bank typically sets the ex-ante level of reserves based on purely monetary considerations such as signaling through quantitative easing (see, for example, Krishnamurthy and Vissing-Jorgensen (2011)). The optimal reserve level from a financial stability standpoint can differ substantially from what the central bank might want to issue given purely monetary considerations. The problem then will be that the forces we have described could impede the effectiveness of monetary policy. For instance, if the central bank has to signal patience through a prolonged period of quantitative easing, while optimal reserves in our framework are negative and higher reserves increase loan premiums, then monetary and financial stability considerations are in contradiction. If the former triumphs, its effectiveness will be less than envisaged because of the latter.

In such cases, when the central bank cannot ensure that the interbank market rates will be zero in all states, it would benefit from having an additional instrument. The pecuniary externality it is trying to remedy is a well-known one (see Kehoe and Levine (1993), Lorenzoni (2008), and Stein (2012), among others); the individual banks take the date-1 interbank rate as given, and optimize their lending and capital

raising given that rate. However, the planner/central bank knows that when the date-1 interbank market is expected to be stressed, more bank capital-raising at date 0 would alleviate the stress all round and be socially beneficial.

Lemma 3: If $\bar{r}_1 > 0$, $\left. \frac{dU}{de_0} \right|_{e_0 = \frac{qr_1}{\alpha_0}} > 0$, so at the private bank's optimal financing choice, the central bank/planner wants the bank to finance reserve holdings with more capital.

Proof: See Appendix.

The bank raises too little long-term finance at date 0 relative to the social optimal. Intuitively, it equates the cost of raising capital at date 0 to the expected benefit of avoiding funding at date 1, ignoring the other system-wide benefits of higher date-0 capital on reducing \bar{r}_1 and thus increasing overall investment and term borrowing. Therefore, when the central bank cannot use reserves alone to bring the expected interbank rate premium, \bar{r}_1 , down to zero, it might use a mix of tools, including higher capital requirements. While additional capital-raising imposes costs on the bank, it can be socially beneficial, and in addition, could allow reserves to be set at a level required by monetary policy even while interbank premia are brought to zero.

3.4. Discussion

To see why this analysis may be useful, consider the literature that finds the liquidity premium paid for near-money assets such as T-bills (as measured by the spread they pay below illiquid assets of similar maturity and risk) falls with the quantum of issuance of such near-money assets (see, for example, Bansal, Coleman, and Lundblad (2010) and Krishnamurty and Vissing Jorgensen (2012)). Some economists (see, in particular, Stein (2012) and Greenwood, Hanson, and Stein (2016)) have argued that the large-scale expansion of central bank reserves can similarly reduce the money premium in bank deposits. This will discourage short-term bank financing. Essentially, the argument is that central bank reserves will compete with short-term bank deposits for place on private investor portfolios. Being a better source of liquidity, the former will displace the latter, and make the financial system safer (by avoiding the run risk associated with short-term bank financing).

However, Nagel (2016) questions the basic premise that an issuance of near-money assets (such as T-bills) reduces the implied money premium (by satiating some fixed demand for liquidity). He documents that the money premium is positively correlated with the level of interest rates, which in turn is positively correlated with the issuance of near-money assets like T-bills. When the level of interest

rates is introduced as an explanatory variable, the correlation between the money premium and T-bill issuance loses significance. Indeed, given a targeted interest rate, any fall in near-money T-bills will lead to an undersupply of liquidity and a potential rise in interest rates, which the Fed will offset by expanding reserves. Since banks finance reserves with deposits in his model, this could lead to a negative correlation between T-bills and deposits, without the money premium actually changing.

We make a somewhat different point – that the demand for liquidity is not unaffected by the issuance of reserves. Because of their very nature, banks will finance a central bank reserve expansion with short-term liabilities. Far from crowding out bank deposits, central bank reserve issuance may enhance them (as also pointed out by Nagel (2016)). Our focus, however is on stress situations. Because placing reserves with banks commingles them with other bank activities, there may be much less freely available liquidity in stress situations than suggested by the level of reserve issuance. Indeed, we argue more reserve issuance may even reduce ex-post liquidity and raise financial fragility.

Greenwood, Hanson, and Stein (2016) argue that the optimal way for the central bank to crowd out the money premium in deposits is to do reverse repo transactions directly with a broader set of non-bank investors (which most central banks do not do today). This is right, of course. If, for instance, households could hold reserves directly (that is, accounts at the central bank), it would crowd out their need to hold bank deposits. If, however, the final resting place of reserves is on the balance sheet of some non-bank finance company, it is essential that they see reserves as substitutes for bank deposits. If they have to hold reserves in addition to deposits, we will argue in the Appendix that the problems we have highlighted will not diminish significantly, since these intermediaries will also fund the incremental short-term assets with short-term liabilities so as to avoid asset-liability mismatches and reduce risk.

IV. Flight to Safety and Liquidity Hoarding

We assumed that an *exogenous* fraction φ of healthy banks lend in interbank markets, and the remaining fraction receives the flighty deposits of stressed banks. To endogenize the fraction φ , we will allow healthy banks to choose between lending in the interbank markets and consequently being *tainted* by the stress, or staying clear of profitable albeit risky interbank lending and instead attracting flight-to-safety deposits in stressed situations.

4.1. Convenience Yield on Reserves in Stressed State of the Economy

For this choice to be interesting, there should be some value to attracting flight-to-safety deposits and passing up the opportunity to earn a premium in interbank markets. To this end, we assume that when the economy is liquidity stressed, some banks want to accumulate super-safe and liquid reserves, and each

dollar of reserves that migrates between banks has a *convenience yield* δ to the final holder. This could be thought of as the precautionary value of reserves in case there is further un-modeled stress, their value in signaling a “fortress balance sheet” to other stakeholders looking for safety, or the franchise value of deposits associated with those reserves. Any interest on excess reserves that the central bank pays over and above the market rate would also add to δ . We assume this convenience yield accompanies the flow of reserves to the safe banks; in other words, it is a private wealth transfer that washes out in the aggregate.

An immediate question is why safe banks do not compete for flight-to-safety deposits by raising rates. Acharya and Mora (2015) show that safe banks did not raise deposit rates during the GFC, while distressed banks did. One explanation is that safe banks are trying to signal that they do not need funds in order to avoid the stigma associated with risky banks. Relatedly, the inflow from flight-to-safety depositors may be driven by convenience and a desire for principal protection rather than to exploit small differences in rates. For instance, depositors may go to the most proximate safe bank to their existing stressed bank. Finally, a bank will have to pay any higher rate to all its depositors. If the flight-to-safety deposits are substantial in magnitude, but are only a small fraction of a bank’s overall deposits, safe banks may be reluctant to compete for them. This is a similar effect to Drechsler, Savov and Schnabl (2017), who document that banks in concentrated banking areas are reluctant to pay depositors higher rates when the Fed raises rates, since they have to also pay captive depositors that rate. Given these considerations, we assume safe banks cannot (or will not) raise rates to attract more flight-to-safety deposits.

4.2. Bank Choices at Date 1

Let us denote the incremental value of a bank at date 1 as $V_1(y, z)$ where recall that $y = 1$ if the economy is liquidity stressed and zero otherwise, while $z = 1$ if the bank is stressed and zero otherwise. In the benchmark model of Section III, the interbank market was always open so that r_1 was both the interbank rate as well as the lending rate to stressed firms net of bank monitoring cost γ (i.e., firms borrow at $r_1 + \gamma$ but the bank earns the net rate r_1). Now, the interbank market may be shut, but stressed firms will still borrow from their banks; so we will denote as r_1 the bank lending rate to the firm net of the monitoring cost and it will always exist. As before, we will use the notation \bar{r}_1 for the equilibrium interbank lending rate when the interbank market is open (in which case it will equal r_1). For emphasis, when there is liquidity stress but the interbank market is closed, we will denote r_1 as the *autarky rate* r_1^A .

Now, in the liquidity stressed state, there are 3 cases to consider when δ is sufficiently large.¹² The first case, perhaps the least interesting economically, is one where the level of reserves is low enough that stressed banks do not need any liquidity from surplus banks to meet their needs; in the second case, the inter-bank market remains shut even though stressed banks are liquidity-deficient; and, in the third case, the inter-bank market opens up as some surplus banks are induced by the high inter-bank premium to provide liquidity to deficient banks.

Case 1: *Stressed banks have enough liquidity to meet the needs of deposit outflows and rescue investment without accessing the inter-bank market.*

Since reserves have a convenience yield δ , stressed banks will issue some capital e_1 to add to reserves even if they do not need to use it for loans or deposit outflows. Furthermore, no bank will loan out reserves without earning at least the convenience yield. Finally, since liquidity is in surplus, any competition to make bank loans would push the bank lending rate down to the convenience yield. The

bank solves $\max_{e_1} V_1(y=1, z=1) \equiv \left[r_1 (I_1(r_1) - D_0^F) - \delta (D_0 - D_0^F + I_1(r_1) - e_1) - \frac{\alpha_1}{2} e_1^2 \right]$ where the

first term of the maximization is the return on loans, the second term the cost of the reserve outflow reduced by the inflow of capital, while the last term is the incremental cost of raising capital over and above the gross cost of 1. Since $r_1 = \delta$, the stressed bank makes no profit from the rescue loan. Solving,

$e_1 = \frac{\delta}{\alpha_1} > 0$ even if the stressed bank has no need to use the funds to meet depositor outflows or loan

demand (details are in the Appendix but the analysis follows a similar structure to Section 3).

Safe banks will not issue capital since they know capital issuance will not alter their reserves on net – any investor in capital will first acquire reserves from the safe banks to buy the capital.¹³ Since there is no need for interbank loans, no healthy bank will become tainted. This means that the reserve outflows from the stressed banks are spread across all the healthy banks, and their incremental date-1 value is

$V_1(y=1, z=0) = \frac{\delta \theta (D_0 - D_0^F + I_1 - e_1)}{(1 - \theta)}$ where the numerator is the value of flight-to-safety deposit

¹² The condition is formally stated in Appendix, Proof of Theorems 2-3. When the convenience yield δ is small, it is possible that only Cases 1 and 3 arise since the “breakeven rate” at which some surplus banks find it advantageous to enter the inter-bank market may be lower than δ at the level of reserves that requires a switch out of Case 1.

¹³ Of course, safe banks may issue capital assuming it will come from reserve flows from other safe banks. If everyone does this, no one will have any additional reserves, but everyone will have issued capital commensurate with the size of the convenience yield and incurred the associated costs. Allowing for this adds little to the analysis.

outflows (plus new deposits created by purchases less capital issued) to the healthy banks, and the denominator is the measure of healthy banks.

Finally, this case arises when the stressed bank's reserves are enough to meet the demands on it, that is, $S_0(1-\tau) \geq (D_0 - D_0^F + I_1 - e_1)$. Substituting for the endogenous $(D_0 - D_0^F)$, we see this case

arises when $\tau S_0 \leq \left[\left(\frac{q\delta}{\alpha_0} + \frac{\delta}{\alpha_1} \right) + W_0^F - I_0 - I_1 - \frac{1}{2} \lambda L_0^2 \right]$ where I_1 is the optimized value

evaluated at $r_1 = \delta$, I_0 at $R_0^L = (1+q\gamma+q\delta)$ and $L_0 = \frac{q\gamma}{\lambda(1+q\delta)}$ (see Appendix for derivations). Note

that as S_0 increases, deposits increase dollar for dollar since no additional capital is issued. Since a fraction τ of the reserves will be encumbered, the distressed bank's net need for date-1 funds grows as S_0 grows. Eventually, it will exhaust available own funds at date 1, and have to issue more capital (compared to the amount that would be optimal considering only the convenience yield). This is when the economy moves into Case 2.

Case 2: *The liquidity needs of each stressed bank can be entirely met by its raising date-1 capital (beyond that warranted by the convenience yield).*

Now, the rate at which the stressed bank lends to the firm, r_1 , rises above δ to incentivize further capital-raising. However, the rate stays too low for any of the healthy banks to lend in the interbank market or compete for loans to stressed firms. Essentially, the stressed bank is in *autarky* and has to issue costly capital even though there is plentiful lending capacity in the system. This is effectively equivalent to setting $\theta = 1$ in our earlier analysis. Let the equilibrium bank lending rate in autarky be r_1^A .

The stressed bank maximizes

$$V_1(y=1, z=1) = \max_{e_1} \left[r_1^A (I_1(r_1^A) - D_0^F) - \delta S_0(1-\tau) - \frac{\alpha_1}{2} e_1^2 \right] \quad \text{such that}$$

$e_1 = (D_0 + I_1(r_1^A) - D_0^F - S_0(1-\tau))$. It follows that $e_1 = r_1^A / \alpha_1$ (and $e_0 = q r_1^A / \alpha_0$). As before, a rise in

the date-1 interest rate equilibrates the demand and supply of liquidity by decreasing the size of the rescue investment and increasing the capital raised. Expanding the constraint for the maximization, we get

$r_1^A / \alpha_1 = \left(I_0 + I_1 + \frac{1}{2} \lambda L_0^2 - W_0^F - \frac{q r_1^A}{\alpha_0} + \tau S_0 \right)$. Furthermore, because the stressed banks are on their

own, once again an increase in ex-ante reserves S_0 always raises r_1^A , regardless of the size of τ (so long as $\tau > 0$).¹⁴

Since the stressed banks just meet liquidity demand using all their unencumbered reserves, the healthy banks get all of it. So $V_1(y=1, z=0) = \frac{\delta\theta S_0(1-\tau)}{(1-\theta)}$.

Case 3: *The liquidity needs of the stressed banks are high enough that at the equilibrium autarkic interest rate, some of the healthy banks are willing to lend in the interbank market and become tainted. The equilibrium rate then is lower than the (now counterfactual) autarkic rate.*

To see this, let a share φ of the healthy banks choose to lend in the interbank market to stressed banks. They will lend all their unencumbered reserves as well as the capital raised at date 1 into the interbank market at rate r_1 . The date-1 profits from doing so are $V_1^\varphi(y=1, z=0) =$

$\left[(r_1 - \delta)S_0(1-\tau) + \frac{r_1^2}{2\alpha_1} \right]$, where the first term is the incremental value from lending out own

reserves, and the second term is the profit from raising capital ($e_1 = \frac{r_1}{\alpha_1}$) and lending the proceeds. The

reserve outflows from the stressed and now tainted banks amount to $S_0(1-\tau)(\theta + (1-\theta)\varphi)$ and these go to the $(1-\theta)(1-\varphi)$ banks that choose to be seen as safe. So the profit from being seen as safe and

attracting the flight-to-safety deposits is $V_1^{1-\varphi}(y=1, z=0) = \frac{\delta S_0(1-\tau)(\theta + (1-\theta)\varphi)}{(1-\theta)(1-\varphi)} =$

$\delta S_0(1-\tau) \left(\frac{1}{(1-\theta)(1-\varphi)} - 1 \right)$. In equilibrium, healthy banks should be indifferent between choosing to

become tainted or stay safe. So $V_1^\varphi = V_1^{1-\varphi}$, and rearranging terms, we get

$$(1-\varphi) = \frac{\delta S_0(1-\tau)}{(1-\theta) \left(r_1 S_0(1-\tau) + \frac{r_1^2}{2\alpha_1} \right)} \quad (1.11)$$

¹⁴ Since all the endogenous variables on the right hand side are decreasing in r_1^A while the left hand side is increasing in r_1^A , there is a unique equilibrium r_1^A , and S_0 shifts it up whenever $\tau > 0$.

Inspecting (1.11), it is clear that $\frac{\partial \varphi}{\partial S_0} < 0$, $\frac{\partial \varphi}{\partial \delta} < 0$, $\frac{\partial \varphi}{\partial r_1} > 0$. In words, the share of healthy banks lending in the interbank market falls in the ex-ante level of reserves (because the relative profits from raising and lending capital fall relative to attracting the flight-to-safety deposits as S_0 increases) as well as in the convenience yield, and increases in the available interbank rate.

As an aside, as $\delta \rightarrow 0$, we have $\varphi \rightarrow 1$. That is, as the convenience yield of reserves falls to zero, virtually all healthy banks choose to lend in the interbank market. Only a sliver of the healthy banks prefers being seen as safe, and these attract all the flight-to-safety reserves, which carry an infinitesimal convenience yield δ .

For Case 3 to occur, it must be that $\varphi > 0$, that is,
$$\frac{\delta S_0(1-\tau)}{(1-\theta)\left(r_1 S_0(1-\tau) + \frac{r_1^2}{2\alpha_1}\right)} < 1.$$

Rearranging, this requires $\left[\frac{r_1^2}{2\alpha_1} + r_1 S_0(1-\tau) - \frac{\delta S_0(1-\tau)}{(1-\theta)}\right] > 0$. Since the expression on the left hand

side of the inequality is increasing in r_1 , it must be that the threshold value or the “breakeven interbank rate” r_1^φ that induces banks to lend in the interbank market is the positive root of the quadratic equation

obtained by setting the expression to zero. So
$$r_1^\varphi = \alpha_1 S_0(1-\tau) \left[\sqrt{1 + \frac{2\delta}{\alpha_1(1-\theta)S_0(1-\tau)}} - 1 \right].$$
 Since this

increases in S_0 , we know that in Case 2, an increase in S_0 expands both the autarky rate r_1^A as well as the rate r_1^φ necessary for the system to move into Case 3. However, it is easily seen that r_1^φ increases at a decreasing rate while r_1^A does not, so at a high enough S_0 , $r_1^A > r_1^\varphi$ and the interbank market will open.

Finally, we show in the Appendix that a sufficient condition for the equilibrium r_1 to be increasing in S_0 is $\theta > \frac{\varphi(1-\tau)}{\tau + \varphi(1-\tau)}$ as in section II. It is only a sufficient condition, however, since with endogenous φ , the incentive to hoard also increases in S_0 , further increasing the equilibrium interest rate. Furthermore, since φ rises from zero at the threshold interest rate r_1^φ at which the interbank market

opens, there is always a region in Case 3 in which increases in S_0 will raise the ex-post interbank rate premium r_1 .

4.3. Summarizing the Equilibrium as S_0 changes

Formally, we have

Theorem 2: For $\delta > 0$ and $\tau > 0$, there exist critical thresholds for the level of reserves, S_0^* and S_0^{**} ,

where $S_0^{**} > S_0^* > 0$, such that the inter-bank market is open, *i.e.*, $\varphi > 0$, only for $S_0 > S_0^{**}$, and

(i) for $S_0 \leq S_0^*$, stressed banks are not liquidity-deficient (taking into account the capital raise dictated by the convenience yield), and the equilibrium lending rate to firms r_1 (net of monitoring cost) equals δ ;

(ii) for $S_0 \in (S_0^*, S_0^{**}]$, stressed banks are liquidity-deficient and raise more capital at date1 than dictated by the convenience yield, but the inter-bank market remains shut (autarky). Furthermore, the autarkic lending rate to firms r_1^A satisfies $r_1^A > \delta$, $\frac{dr_1^A}{dS_0} > 0$, and $r_1^A(S_0^{**}) = r_1^\varphi(S_0^{**}) > 0$; and,

(iii) for $S_0 > S_0^{**}$, stressed banks are liquidity-deficient and raise capital as well as borrow in the inter-bank market at date 1; the inter-bank rate satisfies $\bar{r}_1(S_0) \geq r_1^\varphi(S_0) > 0$, with $r_1^\varphi(S_0)$ increasing in S_0 .

4.4. Examples and Details

Figures 3A and 3B illustrate model outcomes for a specific parameterization where $\delta = 0.2, \tau = 0.2$. In 3A, $\theta = 0.8 = (1 - \tau)$ and in 3B, $\theta = 0.6 < (1 - \tau)$. Other parameters are $\lambda = 1, \gamma = 0.4, q = 0.1, \alpha_0 = \alpha_1 = 1, W_0^F = 2, g_0' = g_1' = 1/I$. The green line is the breakeven interbank rate $r_1^\varphi(S_0)$, the yellow line the autarkic bank lending rate $r_1^A(S_0)$ (with its hypothetical value extrapolated if the inter-bank market were to remain shut even for $S_0 > S_0^{**}$), and blue line is the equilibrium interbank rate $\bar{r}_1(S_0)$ when some healthy banks choose to enter the inter-bank market. While the entry of some healthy banks pulls the inter-bank rate down (blue line relative to the yellow line), it nevertheless remains above $r_1^\varphi(S_0)$ and is increasing in S_0 for both parameter sets.

As the convenience yield δ associated with the possession of reserves increases, the inter-bank market remains shut over a wider range of the level of reserves, and the level of the inter-bank rate increases with δ whenever the inter-bank market is open. Formally,

Theorem 3: (i) S_0^* and S_0^{**} are increasing in δ ; and, (ii) for $S_0 > S_0^{**}$, $\frac{d\bar{r}_1}{d\delta} > 0$.

We illustrate the effects of varying δ in Figures 4A and 4B, where θ is set at 0.6, less than $(1-\tau)$. In Figure 4C, δ ranges among values close to zero, whereas in Fig 4b, it ranges between significantly higher values (the range of equilibrium interest rates is commensurately higher in the latter case). In both cases, we see that as δ increases, the threshold level of reserves above which the inter-bank market opens up shifts to the right to a higher value though this shift is relatively modest at low values of δ ; we also observe that as δ increases, the inter-bank rate is higher whenever the inter-bank market is open. Finally, Figure 4C shows that as δ increases, the proportion φ of surplus banks that enter the inter-bank market decreases.

4.5. Discussion

As we have just seen, a key factor in exacerbating liquidity stress is the convenience yield δ associated with reserves. A higher convenience yield implies more liquidity stress. Arguably, δ is higher in environments where bank assets other than reserves are very illiquid, and where the incidence of systemic stresses are positively serially correlated.

This then leads to the possible desirability of ex-post central bank intervention. The central bank could try to bring down \bar{r}_1 by injecting reserves at date 1 if the economy becomes liquidity stressed. This may run up against similar frictions to the fear of taint we have documented. Banks may face “stigma” in interbank markets if they access central bank facilities (see Hu and Zhang, 2020, for example); tapping intraday into the central bank could be problematic if it prompts rumors of potential stress at the bank, which cause other banks to freeze lending and wholesale deposits to flee.

Assuming banks overcome stigma in extreme situations, there are still three important caveats here. First, the most effective way for the central bank to intervene ex post is for it to lend unsecured into the interbank market. However, this entails significant risk of losses. If it does lend against high-quality securities, though, the financial sector will have to hold those high-quality securities ex ante. As we will show in section 6.4, bank incentives to voluntarily hold reserves (or equivalently, other high-quality safe assets that could be used to borrow reserves at date 1) can be lower than the socially desirable level. Of course, the central bank can broaden the range of assets it will lend against (for example, lend against

corporate securities) even while increasing the size of the haircut it levies on collateral value. The larger the quantum of intervention, the more the central bank is likely to depart from alleviating just liquidity risk, instead taking on other risks such as credit risk.¹⁵

But this leads to the second concern. Central bank intervention at date 1 will reduce the interbank rate. But then fewer healthy banks will want to lend into that interbank market. So the act of intervention ex post will crowd out private lending and increase the ex-post quantum of needed central bank intervention. Of course, central bank intervention may also reduce the convenience yield on reserves (assuming the private sector, in response, does not build up illiquidity again – see below), especially if the central bank commits to lending freely in the foreseeable future without much concern for security, and this may elevate private interbank lending. The net quantum of ex-post central bank intervention depends on how the interbank rate moves relative to the convenience yield. Once again, though, the central bank will have to take on balance sheet risks to effect a rescue.

And finally, there are the better-known ex-ante consequences on banks taking on liquidity risk. If leveraged illiquid banks expect to receive central bank funds ex post, they may not reduce their illiquid assets in a timely manner by transferring assets to less leveraged, more liquid banks, taking on further liquidity risk in the process (Acharya and Tuckman, 2014). Similarly, the more the financial sector expects central bank intervention, the more it will increase the ex-ante issuance of claims on liquidity, effectively reducing liquidity holdings net of liquidity promises (see Acharya, Shin and Yorulmazer (2011), Diamond and Rajan (2012) or Farhi and Tirole (2011)), and necessitating intervention of yet greater magnitude.¹⁶

In sum, illiquidity premia can be brought down ex post through central bank intervention – indeed, some see this as the fundamental purpose of a central bank. Yet repeated central bank intervention is not without cost. The central bank can distort the pricing and quantum of liquidity in the market considerably by displacing the market – it will tend to overdo intervention and underprice it. Participants will become extremely dependent on a fallible and not always predictable central bank, and will even

¹⁵ The central bank could offer secured lending to the entire financial sector against high quality assets rather than just to the banking sector (see Liang and Parkinson (2020)), and the recent move to Standing Repurchase Facilities with a wider set of market participants. This will prevent the banking sector from becoming an impediment to the transfer of liquidity to stressed firms in the financial sector. It will not necessarily ensure that financial firms with surplus liquidity will recirculate it to stressed firms in the real economy or in the banking sector.

¹⁶ Indeed, some central banks recognize that their provision of liquidity on demand creates dependence for more. Nelson (2019) cites a Norges Bank statement in 2010 justifying its move to a deficit reserves position in the system thus: “*When Norges Bank keeps reserves relatively high for a period, it appears that banks gradually adjust to this level... With ever increasing reserves in the banking system, there is a risk that Norges Bank assumes functions that should be left to the market. It is not Norges Bank’s role to provide funding for banks... If a bank has a deficit of reserves towards the end of the day, banks must be able to deal with this by trading in the interbank market.*”

game it into intervening. That too has costs. For instance, the scale of central bank liquidity interventions has got only larger and larger over the past three decades (in 2020, the Fed rolled out many of the programs it had created during the Great Recession plus some new ones) and the normalization of resulting liquidity injections has been difficult. Such liquidity dependence, which we have argued can portend greater future liquidity stress, appears to be an important unintended consequence of central bank balance sheet expansion.

V. Encumbrance on Reserves

We have assumed an encumbrance τ on reserves. We propose speculation and regulation as two channels through which reserves encumbrance can be endogenized. Next, we show the robustness of our primary insights to assuming a fixed *level* of encumbrance on reserves (instead of assuming a fixed encumbrance *share* of reserves).

5.1. Endogenizing Encumbrance Share τ : Speculation

Reserves, as we argued in the introduction, have an optionality embedded in them. Ideally, banks would like to sell that option when they do not need it (when the economy is healthy), and retain it when the economy is liquidity stressed. Unfortunately, such selective sales of liquidity may be difficult.

Consider, for example, the prime brokerage services that banks offer. Let each bank serve one speculator. Let the speculator put on trades at date 0 of size x . In normal economic times, the bets pay off and return ηx to the speculator and fees of ρx to the bank. Conditional on the economy getting liquidity stressed (with probability $\frac{q}{\theta}$), the bank has to meet margin calls on the speculator, putting up reserves of κx . These calls have priority over all other claims on the bank (else it will have to default on exchanges, and see its brokerage business shut down). Alternatively, if the trades are centrally cleared to reduce any risk of contagion from such speculative positions, the clearinghouse would require the clearing members (banks) to over-collateralize their positions and contribute initial and variation margins and guarantee fund contributions. The resulting funds with clearinghouses are typically not allowed to be rehypothecated (or face significant limits on rehypothecation), and a large fraction of it is in the form of reserves deposited with the central bank, thereby being unavailable for further private use.¹⁷

Finally, assume each speculator's search costs of putting on a profitable trade is increasing in the size of a trade (that is, there are fewer remaining low hanging fruit as they trade more) and decreasing in

¹⁷ The need for such higher priority of clearinghouse claims on banks is underscored in their disclosures around liquidity risks: “*The major liquidity risks for derivatives CCPs result from the nature of their payment flows. To make timely payments to some clearing members, the CCPs rely upon timely collections from others.*” (OFR, 2017)

the unencumbered liquidity of the system. It is $\frac{\nu}{2} \frac{x^2}{(S_0 - \kappa \bar{x})}$, where ν is a parameter and \bar{x} is the

equilibrium level of trade per bank. This captures the notion that liquidity facilitates speculation, but speculators are aware that liquidity gets tied up as there is more speculative trade. Assume that $\eta > \rho$ which ensures that speculation is profitable net of fees. For simplicity, we focus on the model of Section 2 with an exogenously given share of surplus banks in the interbank market (in the sub-section with fixed encumbrance, we will examine endogenous shares). The speculator's maximization problem is then:

$$\text{Max}_x \quad \left(1 - \frac{q}{\theta}\right) [\eta - \rho] x - \frac{\nu}{2} \frac{x^2}{(S_0 - \kappa \bar{x})}$$

The first order condition is $\left(1 - \frac{q}{\theta}\right) [\eta - \rho] = \frac{\nu x}{(S_0 - \kappa \bar{x})}$. Recognizing that $x = \bar{x}$ in equilibrium, we have

$$\kappa \bar{x} = \frac{S_0 \kappa \left(1 - \frac{q}{\theta}\right) (\eta - \rho)}{\nu + \kappa \left(1 - \frac{q}{\theta}\right) (\eta - \rho)} = \tau S_0 \text{ where } \tau = \frac{\kappa \left(1 - \frac{q}{\theta}\right) (\eta - \rho)}{\nu + \kappa \left(1 - \frac{q}{\theta}\right) (\eta - \rho)}.$$

Assuming that the market for provision of prime-brokerage services to speculators is competitive among banks at date 0, the fee ρ per unit of speculative activity is set such that in expectation banks are compensated just adequately for the cost of providing the per-unit margin call κ . This zero-profit condition implies then that $\left(1 - \frac{q}{\theta}\right) \rho = \frac{q}{\theta} [\varphi(1 - \theta) + \theta] (1 + \bar{r}_1) \kappa$. Substituting above for the implied $\rho(\bar{r}_1)$, we obtain that the encumbrance per unit of reserves is a function of the date-1 interbank rate premium, i.e., $\tau(\bar{r}_1)$, such that $\tau'(\bar{r}_1) \leq 0$. This implies then that at low levels of the expected rate, there is greater speculation, and if liquidity needs in the stressed state rise, then speculative activity is tempered by the expectation of a rising interbank rate, creating an additional equilibrating force that clears the market for reserves. Using $\tau'(\bar{r}_1) \leq 0$ and logic analogous to the proof of Theorem 1, it follows that

Theorem 4:

- (i) If $\theta > \frac{\varphi(1-\tau(0))}{\tau(0)+\varphi(1-\tau(0))}$, then $\bar{r}_1 > 0$ is the unique equilibrium for $S_0 > \hat{S}_0$; \bar{r}_1 increases with S_0 over a range $[\hat{S}_0, S_0^*]$ till it reaches r_1^* where $\frac{\varphi(1-\tau(r_1^*))}{\tau(r_1^*)+\varphi(1-\tau(r_1^*))} = \theta$, after which \bar{r}_1 does not increase with further increases in S_0 . Also $\bar{r}_1 = 0$ for $S_0 \leq \hat{S}_0$. Note that $\hat{S}_0 \leq 0$ if $NLS \leq 0$.
- (ii) If $\theta \leq \frac{\varphi(1-\tau(0))}{\tau(0)+\varphi(1-\tau(0))}$, then $\bar{r}_1 > 0$ is the unique equilibrium for $S_0 < \hat{S}_0$; \bar{r}_1 increases as S_0 falls till it reaches r_1^{**} at $S_0 = S_0^{**}$ where $\frac{\varphi(1-\tau(r_1^{**}))}{\tau(r_1^{**})+\varphi(1-\tau(r_1^{**}))} = \theta$, after which \bar{r}_1 does not increase with further decreases in S_0 . Also $\bar{r}_1 = 0$ for $S_0 \geq \hat{S}_0$. Note that $\hat{S}_0 \leq 0$ if $NLS \geq 0$.

In essence, case (i) which formalizes our novel insight continues to hold with the endogenous modeling for reserves encumbrance. As long as additional reserves create a net demand for liquidity when interbank rate is zero, i.e., $\theta > \frac{\varphi(1-\tau(0))}{\tau(0)+\varphi(1-\tau(0))}$, increasing reserves leads to an interbank rate that is greater than zero, and which rises with reserves until the speculative encumbrance τ falls to the point that an incremental increase in reserves does not change (per reserve) net demand for liquidity, and in turn, \bar{r}_1 or $\tau(\bar{r}_1)$. Interestingly, therefore, $\tau(\bar{r}_1)$ never falls to zero; hence, our starting assumption that prime-brokerage fee is lower than the speculative return, $\eta > \rho$, always holds in equilibrium as long as speculation is attractive at a zero interbank rate, which holds whenever $\left(1 - \frac{q}{\theta}\right)\rho > \frac{q}{\theta}\kappa$, or in words, that margin requirement on speculative positions is not too high. Importantly, the lower is the average or expected margin requirement on speculative activity, the greater the ex-ante speculative activity (all else equal), and in turn, the range of parameters for which more reserves can tighten interbank markets. In a richer model with multiple states of liquidity stress, margins may rise in a state-contingent – procyclical – manner when the liquidity stress is most severe due to the attendant increase in counterparty risk (see Aramonte, Schrimpf, and Shin (2021)).¹⁸

5.2. Endogenizing Encumbrance Share τ : Regulation

¹⁸ Barth and Kahn (2021) provide evidence, for instance, that speculative hedge fund positions in relative value (cash-futures) trades in Treasury markets grew from under \$200 billion in 2013 to \$800 billion in January 2020, rising sharply during 2018 and 2019, with required margins on their futures position rising sharply with the rise of VIX in March 2020. Schnabel (2020) also observes that initial and variation margins collected by the four European central counterparties rose immediately around the outbreak of the pandemic, with variation margins often growing more than fivefold and exceeding pre-pandemic cash positions for several of the derivatives counterparties.

To offset speculation, regulators may place their own encumbrances on reserves. Farhi, Golosov and Tsyvinski (2009) suggest a floor on liquidity holdings to prevent a bank from free riding on other banks *a la* Jacklin (1987). Calomiris, Heider, and Hoerova (2014) suggest a minimum level of cash reserves to limit risk shifting. Such regulations are likely to be insufficiently contingent. The most obvious such regulation is a requirement that a certain fraction of assets have to be held at all times in the most liquid form (see, for example, Diamond and Kashyap (2016)) or a capital requirement that binds precisely when a bank ought to lend out its excess reserves (see, for example, Vanderweyer (2019)).

Why cannot such requirements be dropped in times of stress? As Goodhart (2008) emphasizes, a policy of having at least one taxi at the station is of little benefit to the late-arriving traveler if it cannot be used. Diamond and Kashyap (2016) argue, however, that it may make sense for the regulator to prevent a bank from using all its liquid reserves in stressed times if the anticipation that it would do so would prompt more concern about its health, and also cause the stress to spread to unaffected banks – the saver might get alarmed if she believes healthy banks have no mandated liquid assets and might lend them all to stressed banks. Instead of running on just the stressed banks, they may run on all banks. To avoid this, banks might be asked to hold on to a portion of their liquid funds under all circumstances so as to stave off any contagion. We show in the Appendix how such regulatory requirements can be embedded in our analysis.

Some bank actions in response to uncertain regulation could also amplify encumbrances. D’Avernas and Vanderweyer (2021) attribute enhanced volatility and fragility in repo markets to regulations on intra-day bank liquidity holdings. In answer to the question of why JP Morgan did not lend in the repo markets when spreads blew out, they cite Jamie Dimon, CEO of JP Morgan, who responded thus: “[...] *we have \$120 billion in our checking account at the Fed, and it goes down to \$60 billion and then back to \$120 billion during the average day. But we believe the requirement under CLAR and resolution and recovery is that we need enough in that account, so if there’s extreme stress during the course of the day, it doesn’t go below zero.*” In other words, regulations force JP Morgan to hold a portion of reserves back for really extreme market events – since no one really knows what these might be, some portion of the reserves might be permanently encumbered.

Furthermore, Nelson (2019) documents that in a Bank Policy Institute (BPI) survey conducted in January 2019, bank examiner expectations about liquidity holdings were mentioned overwhelmingly as “important” or “very important” reasons for reserve demand by banks. Indeed, Nelson points out that in times of abundant reserves, bank supervisors would scrutinize any drawdowns carefully, creating a ratchet effect (higher the held reserves, higher the reserves supervisor expect) limiting the ability of healthy banks to redeploy reserves when needed.

The constraint on bank lending of reserves could lie elsewhere such as with capital requirements. Diamond, Jiang, and Ma (2021) and Liang and Parkinson (2020) suggest the supplementary leverage ratio (requiring capital to be held against *all* assets including the relatively safe ones) should not apply to reserves. In times of stress, this seems very sensible for it frees up reserves to be used as needed. Yet ex ante, our model suggests that banks ought to finance reserves with more capital, so that the liquidity provided by reserves is not fully offset by deposit withdrawals. Our model would suggest a contingent lightening of regulations like the supplementary leverage ratio rather than a permanent abandonment of the ratio or a permanent exclusion of reserves from the leverage ratio calculations.

5.3. Fixed Encumbrance on Reserves ($\tau S_0 \equiv \bar{E}$)

Suppose that instead of a constant fraction, the required level of reserves to be maintained by each bank is a fixed amount \bar{E} , independent of reserves, S_0 . Our analysis carries over to this case.

To show this, consider the full model of Section 4 with the endogenized share of surplus banks in the interbank market. Case 1 in which each stressed bank is self-sufficient in liquidity at the convenience

yield δ arises whenever $\bar{E} \leq \bar{E}^* \equiv \left[\left(\frac{q\delta}{\alpha_0} + \frac{\delta}{\alpha_1} \right) + W_0^F - I_0 - I_1 - \frac{1}{2} \lambda L_0^2 \right]$, a condition that is

independent of the level of reserves; note that I_1 is the optimized value evaluated at $r_1 = \delta$, I_0 at

$R_0^L = (1 + q\gamma + q\delta)$ and $L_0 = \frac{q\gamma}{\lambda(1 + q\delta)}$. For $\bar{E} > \bar{E}^*$, the interbank market may be shut (autarky) or

open; when shut, the autarkic rate $r_1^A(\bar{E})$ satisfies $\bar{E} = \left[\left(\frac{qr_1^A}{\alpha_0} + \frac{r_1^A}{\alpha_1} \right) + W_0^F - I_0 - I_1 - \frac{1}{2} \lambda L_0^2 \right]$,

and is now a function of the fixed level of encumbrance and not of the level of reserves, with I_1 , I_0 and

L_0 set accordingly. Naturally, $r_1^A(\bar{E})$ is increasing in the encumbrance \bar{E} , and $r_1^A(\bar{E}) > \delta$ for $\bar{E} > \bar{E}^*$.

A key question then is when is the autarkic rate above or below the breakeven rate r_1^φ that induces banks to lend in the interbank market. With fixed encumbrance, this rate is given by

$$r_1^\varphi = \alpha_1 (S_0 - \bar{E}) \left[\sqrt{1 + \frac{2\delta}{\alpha_1(1-\theta)(S_0 - \bar{E})}} - 1 \right],$$

which as before is increasing and concave in S_0 . Note

also that the endogenous share φ of surplus banks that lend in the interbank market for a given rate r_1

satisfies $(1-\varphi) = \frac{\delta(S_0 - \bar{E})}{(1-\theta)\left(r_1(S_0 - \bar{E}) + \frac{r_1^2}{2\alpha_1}\right)}$ and the equilibrium interbank rate \bar{r}_1 is given by the

usual market-clearing condition adjusted for encumbrance being now at a fixed level:

$$\frac{r_1}{\alpha_1} = \frac{\theta}{[\varphi(1-\theta) + \theta]} \left[I_0 + I_1 - e_0 + \frac{1}{2} \lambda (L_0^B)^2 - W_0^F \right] - \frac{\varphi(1-\theta)}{[\varphi(1-\theta) + \theta]} S_0 + \bar{E}. \text{ It can then be shown that}$$

Theorem 5: For $S_0 > \bar{E} > \bar{E}^*$, there exists a critical threshold $\bar{S}_0 > \bar{E}$ such that

- (i) For $S_0 \in (\bar{E}, \bar{S}_0)$, the autarkic rate $r_1^A(\bar{E})$ exceeds the breakeven rate $r_1^\varphi(S_0)$, the interbank market is open ($\varphi > 0$), and the equilibrium interbank rate $\bar{r}_1 \in (r_1^\varphi(S_0), r_1^A(\bar{E}))$.
- (ii) For $S_0 \geq \bar{S}_0$, the autarkic rate $r_1^A(\bar{E})$ is at or below the breakeven rate $r_1^\varphi(S_0)$, the interbank market is shut ($\varphi = 0$), and the equilibrium interbank rate \bar{r}_1 equals the autarkic rate $r_1^A(\bar{E}) > \delta$.
- (iii) When fixed encumbrance \bar{E} is sufficiently small such that $r_1^A(\bar{E}) < r_1^\varphi(S_0 \rightarrow \infty) = \frac{\delta}{(1-\theta)}$, then both cases (i) and (ii) arise and \bar{r}_1 is strictly increasing in S_0 for at least some range of S_0 in $(\bar{E}, \bar{S}_0]$; otherwise, when $r_1^A(\bar{E}) \geq \frac{\delta}{(1-\theta)}$, only case (i) arises and $\bar{S}_0 \rightarrow \infty$.

This result is illustrated in Figures 5A and 5B, with same parameters as in Figures 3-4 ($\theta = 0.6$) and respectively \bar{E} being low (below 1), for which both cases of Theorem 5 arise, and \bar{E} being high (above 1), for which only case (i) arises as the autarkic rate always exceeds the breakeven rate and $\bar{S}_0 \rightarrow \infty$. In both parameterizations, \bar{r}_1 is strictly increasing in S_0 .¹⁹

It is therefore a robust feature of the equilibrium that the interbank market may remain shut and the interbank rate can increase in the level of reserves when the interbank market is open. However,

¹⁹ Note that in Figures 5A-B, the equilibrium interbank rate is rising in the level of reserves; in general, this holds when the systemic extent of liquidity shock θ is sufficiently small and the convenience yield δ is sufficiently high; when this is not the case, it can be shown that an increase in the supply of reserves can cause the rate to decrease as it starts out high, close to the autarkic rate, when reserves are close to the fixed encumbrance, and then decreases towards the breakeven rate.

whether the interbank market remains open or shut as a function of reserves is qualitatively different with fixed encumbrance compared to fixed share encumbrance (Theorem 3); in that case, the interbank market opens up for high enough level of reserves.

VI. Robustness

We now elaborate on some of the assumptions we have made so far and discuss their robustness.

6.1. Nature of Liquidity Shock

We have assumed the liquidity shock at date 1 is a shock to firm fundamentals – and thereby affects both sides of the funding bank’s balance sheet (in terms of loan demands and deposit withdrawals). Yet to the extent that liquidity shocks precipitate solvency concerns, any large-scale flow out of the financial system, such as a build-up of Treasury deposit balances at the Federal Reserve due to depositors making tax payments (or the seasonal shift in reserves across the US banking system stemming from agricultural needs in the late nineteenth century), would trigger similar effects, as would other forms of contagion such as firms drawing down credit lines (following the collapse of Lehman Brothers, see Ivashina and Scharfstein (2010), and at onset of the pandemic, see Acharya, Engle and Steffen (2021)).²⁰

An important simplification is that the share of stressed banks, θ , is invariant to the build-up in reserves. It might seem that the risks to commercial bank balance sheets should fall as the share of reserves composing those balance sheets increases. If so, our model would hold for only the range of reserve expansion where commercial bank credit risk is not swamped by reserve expansion. Yet this neglects three possible sources of risk. First, beyond a certain point, incremental reserves are entirely funded by demandable deposits (as implied by Exhibits 1 and 2). So absent commercial bank capital-raising, even small amounts of credit risk relative to the size of the commercial bank’s assets can have large consequences. Second, the monetary effects of central bank balance sheet expansion, if sizeable (though see below), should expand the size of corporate borrowing and increase the risk thereof (that is, it should increase I_0). Third, we have assumed the encumbrance on reserves from speculative activity to be risk-free. In practice, some of it will be risky (speculators will go bust). Our sketch in Section 5.1 of the speculative elements forcing the reserves encumbrance share τ to be higher at low interbank rates also suggests that a prolonged period of easy money could cause the encumbrance rate to rise (also because

²⁰ There is some evidence that credit line “promises” by banks have risen along with the supply of reserves. Undrawn credit lines issued by banks in the United States expanded by \$452,367 million during QE III (between September 2012 and October 2014) and \$457,539 million post the pandemic (between March 2020 and Dec 2020). We are grateful to Sascha Steffen for sharing with us these calculations based on FDIC Call Reports. See also Yankov (2020) who attributes growth in bank credit lines to non-banks to the proposal in 2010 that banks be subject to a liquidity coverage ratio (LCR) requirement.

the optimistic speculators get richer and put on bigger bets as in Geanakoplos (2008)). The same factors causing greater speculation could also cause θ to rise. Prolonged easy conditions may then switch ex-ante reserves from alleviating future liquidity stress to exacerbating it. A deeper analysis of the underpinnings of θ and τ , and their interconnected dynamics, offers an interesting avenue for future research.

6.2. Convenience Yield on Reserves under Liquidity Stress

We have taken the convenience yield δ on reserves as given. Clearly, its size depends on a variety of factors that are exogenous to the model (for example, the franchise value associated with prime-brokerage deposit accounts of hedge funds) or that could be endogenized (the possibility of panic continuing beyond date 1 and spreading, the likelihood of future fire sales where unencumbered funds will come in handy, the actions of the central bank in supplying liquidity as needed, etc.). Each of these is an interesting extension that we leave to future research. What appears plausible is that even endogenized, such convenience yield would rise in times of stress, strengthening ex-post liquidity hoarding.

6.3. Ex-ante Reserves and Activity

We have looked at economic activity after central-bank-issued reserves find their way to bank balance sheets. However, the act of issuing reserves may itself propel activity at date 0, setting up the monetary policy/financial stability tradeoff we referred to earlier. For instance, some kind of mandated reserve requirement could hold back bank deposit creation and thus lending if reserves are scarce (Stein (1998)). Suppose $D_0 \leq \zeta S_0$ so that deposits cannot be more than ζ (greater than one) times bank reserve holdings. On the one hand, a binding constraint on deposit issuance will limit ex-ante lending, as well as increase the use of capital in financing. On the other, it will limit the extent of liquidity stress ex post, and thus reduce the interbank premium, with attendant positive effects on ex-ante lending and deposit issuance. In general, the presence of a binding reserve requirement will enhance the positive ex-ante effects of reserve expansion on economic activity, and limit the negative ex-post effects. Many industrial country central banks have done away with such requirements, so they may be a historical curiosity (though our model suggests a reason to re-examine their value in today's context).

More recently, central banks have sought to expand reserves in order to implement unconventional monetary policy, where the effects range from signaling monetary policy stance (see Krishnamurthy and Vissing-Jorgensen (2011)) to recapitalizing banks through the back door or repairing markets (see Acharya, Eisert, Eufinger, and Hirsch (2019), Brunnermeier and Sannikov (2014), Foley-Fisher, Ramcharan, and Yu (2016), Grosse-Rueschkamp, Steffen, and Streit (2019), Rodnyansky and Darmouni (2017)). Unfortunately, it is hard to discern unambiguously the net macroeconomic effects of

these interventions, perhaps because so much else is going on over the term of the interventions (see Greenlaw et al. (2018)). Indeed, Fabo, Jancokova, Kempf, and Pastor (2021) find that the effect may depend on the identity of the investigator. Studies by central bank staff of the effect of central bank asset purchases on output seem to be uniformly positive, while only half the academic studies find it to be significant! Regardless of the reason why central banks expand their balance sheets, our analysis suggests possible offsetting effects on credit and liquidity, which may partly account for why the effects of unconventional monetary policy are hard to discern.

6.4 Other Extensions

Our simple model allows for many possible extensions and explorations. Two are worth sketching. First, what if banks were not forced to hold S_0 ? It turns out, not surprisingly, that banks will not have the same incentives as the central bank and will tend to optimally hold different levels in a variety of circumstances (see Appendix). This is all the more likely if the central bank has different motives to expand reserves than financial stability, such as signaling or committing to a monetary policy stance. Importantly, the prospect of greater liquidity shortages need not increase the commercial banks' private incentives to hold more reserves; indeed, recognizing that the source of the shortage is the financing of reserves, they may want to hold less.

What if the central bank issues reserves directly to the non-bank private sector? Here again (see Appendix), a desire to match the duration of liabilities with assets to reduce risk will result in non-banks financing with short-term liabilities. Many of the consequences we have documented will follow. In April 2021, the Federal Reserve reinstated the supplementary leverage ratio (SLR) for commercial banks. This is a regulatory capital requirement that was suspended in April 2020 in the wake of the pandemic (see Covas, 2021). Given the increased cost to banks of funding reserves with long-term capital, they released reserves. Interestingly, money market funds, themselves funded with short term liabilities, took on the reserves and redeposited them at the central bank via overnight reverse-repo facilities. This suggests the natural form of reserve funding is short-term even in the non-bank financial sector.

Conclusion

The significant expansion of central bank balance sheets in recent years should have reduced liquidity stress, and even perhaps increased real activity. We propose reasons why central bank balance sheet expansion may be less helpful in stress situations than one might think a priori. In particular, the financing of reserves on bank balance-sheets leads to the issuance of short-term liabilities that is excessive from a social standpoint and creates claims on future liquidity that offset the reserves; furthermore, the

encumbrance of reserves due to speculation and regulation, and reserves hoarding by healthy banks in times of liquidity stress, may prevent liquidity flowing to stressed banks. Ex ante, these effects may partly explain why central bank balance sheet expansion has less effect on real activity than one might anticipate. We have likely only scratched the surface in modeling and sketching out implications of the phenomenon that the ex-ante supply of reserves affects the ex-post demand for them. There is clearly more work to be done in understanding and mitigating liquidity stress implied by this phenomenon.

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Exhibit 1

Incremental Depository Institution balance sheets (obtained from Flow of Funds data Z1.111 – Level Data: U.S.-Chartered Depository Institutions)

All entries under Assets and Liabilities are increments, i.e., changes, for that entry in Millions of Dollars; all ratios are increment or change in the numerator divided by that for the denominator.

QE II (between November 2010 and June 2011)

<u>Assets</u>		<u>Liabilities</u>	
Cash	60	Bonds	-62,773
Debt Securities	94,351	Holding company investment	42,870
Loans	-103,791	Commercial paper	-46,539
Misc	18,748	Loans	-80,632
Repos	-11,639	Miscellaneous	-315,306
Reserves	194,070	Insured deposits	1,264,014
		Uninsured deposits	-534,919
	<u>191,799</u>		<u>266,715</u>
		Deposits/Total Liabilities	2.73361
		Deposits/(Cash+Securities+ Repos+ Reserves)	2.63361
		Deposits/(Repos+Reserves)	3.99655
		Uninsured deposits/(Repos+ Reserves)	-2.93217
		Uninsured deposits/(Uninsured+ insured deposits)	-0.73368

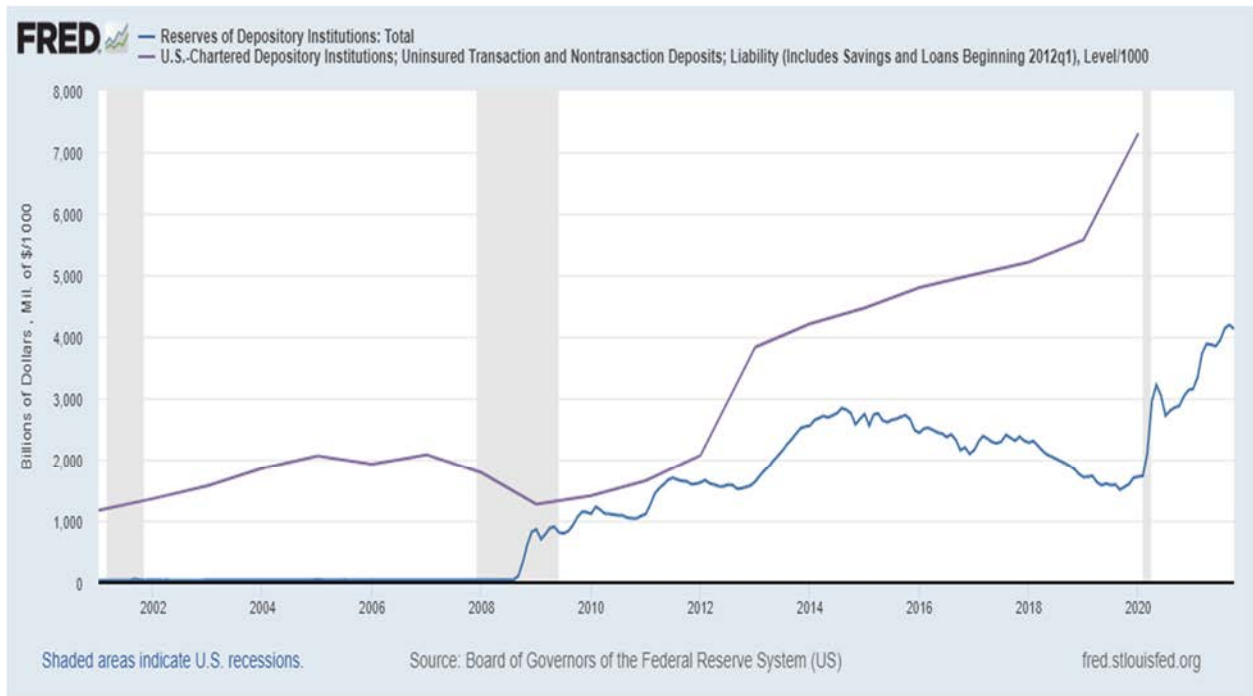
QE III (between September 2012 and October 2014)

<u>Assets</u>		<u>Liabilities</u>	
Cash	11,191	Bonds	-112,030
Debt Securities	504,642	Holding company investment	332,381
Loans	804,170	Commercial paper	-86,743
Misc	-64,076	Loans	108,019
Repos	-29,398	Miscellaneous	184,540
Reserves	713,351	Insured deposits	-810,496
		Uninsured deposits	2,528,429
	<u>1,939,880</u>		<u>2,144,100</u>
		Deposits/Total Liabilities	0.80124
		Deposits/(Cash+Securities+ Repos+ Reserves)	1.43187
		Deposits/(Repos+Reserves)	2.51177
		Uninsured deposits/(Repos+ Reserves)	3.69679
		Uninsured deposits/(Uninsured+ insured deposits)	1.47179

Pandemic (between March 2020 to end 2020)

<u>Assets</u>		<u>Liabilities</u>	
Cash	15,843	Bonds	26,083
Debt Securities	1,041,056	Holding company investment	202,606
Loans	289,404	Commercial paper	26,651
misc	272,661	Loans	-227,272
Repos	179,821	Miscellaneous	-125,790
Reserves	1,282,417	Insured deposits	1,317,938
		Uninsured deposits	1,719,650
	<u>3,081,202</u>		<u>2,939,866</u>
		Deposits/Total Liabilities	1.03324
		Deposits/(Cash+Securities+ Repos+ Reserves)	1.20581
		Deposits/(Repos+Reserves)	2.07736
		Uninsured deposits/(Repos+ Reserves)	1.17604
		Uninsured deposits/(Uninsured+ insured deposits)	0.56612

Exhibit 2



Reserves of depository institutions and uninsured deposit liabilities. The data are from the Federal Reserve Bank of St Louis database (FRED).

Exhibit 3: Bank and Firm Balance Sheets

Firm Balance Sheet at Date 0	
<u>Assets</u>	<u>Liabilities</u>
I_0	$L_0^F (= L_0^B)$
D_0^F	W_0^F
	Net worth

Firm Balance Sheet at Date 1 if stressed	
<u>Assets</u>	<u>Liabilities</u>
I_1	l_1^F
	L_0^F
	Net worth

Firm Balance Sheet at Date 1 if healthy	
<u>Assets</u>	<u>Liabilities</u>
I_0	L_0^F
D_0^F	W_0^F
	Net worth

Bank Balance Sheet at Date 0	
<u>Assets</u>	<u>Liabilities</u>
$L_0^B + \frac{1}{2}\lambda(L_0^B)^2$	D_0
S_0	e_0
	Net worth

Bank Balance Sheet at Date 1 if bank stressed	
<u>Assets</u>	<u>Liabilities</u>
$L_0^B + \frac{1}{2}\lambda(L_0^B)^2$	Possible interbank borrowing = b_1
τS_0	e_1
$l_1^B (= l_1^F)$	e_0
	Net worth

Bank Balance Sheet at Date 1 if economy stressed, bank healthy but "tainted" (makes interbank loans)	
<u>Assets</u>	<u>Liabilities</u>
$L_0^B + \frac{1}{2}\lambda(L_0^B)^2$	D_0
Interbank loans of up to $e_1 + (1-\tau)S_0$	e_1
Reserves of $(S_0 + e_1 - \text{interbank loans})$	e_0
	Net worth

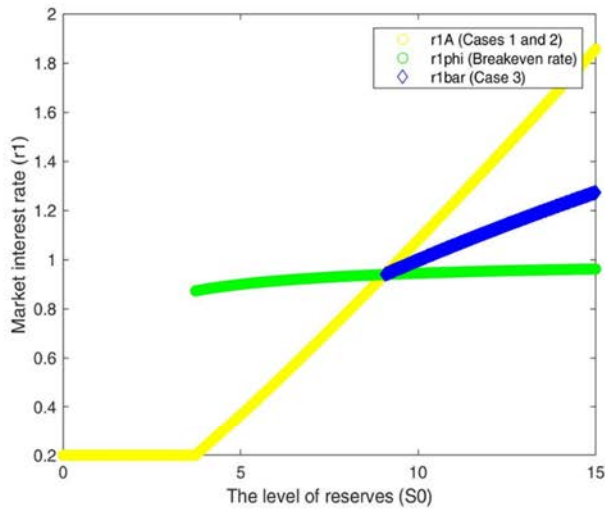


Figure 3A: Market Rate and Reserves

$$\delta = 0.2, \tau = 0.2, \theta = 0.8$$

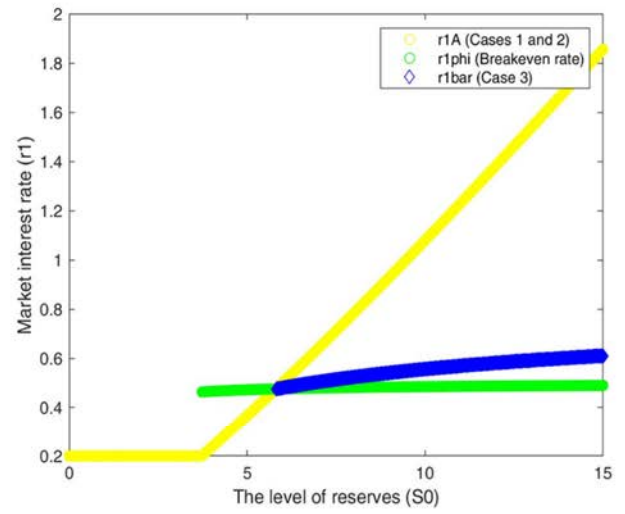


Figure 3B: Market Rate and Reserves

$$\delta = 0.2, \tau = 0.2, \theta = 0.6$$

Figure 3: Numerical example for the extended model of section IV with endogenous interbank entry: The effect of varying the size of the shock θ

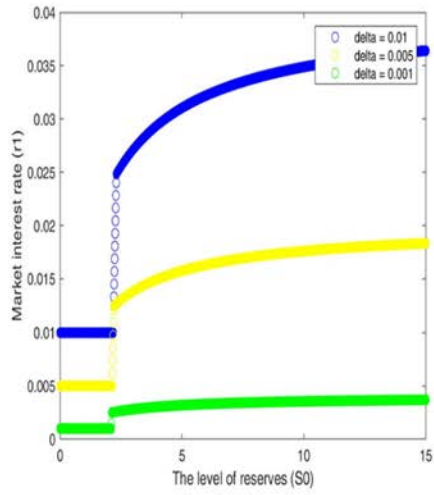


Figure 4A: Market Rate and Reserves for low δ

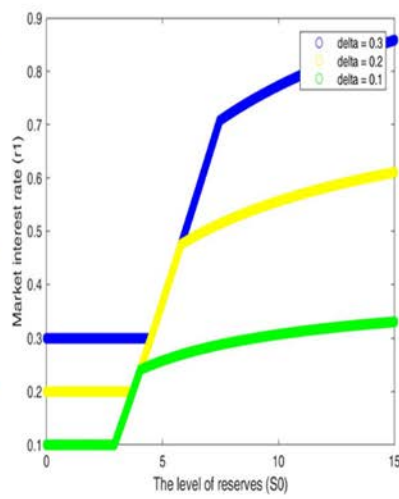


Figure 4B: Market Rate and Reserves for high δ

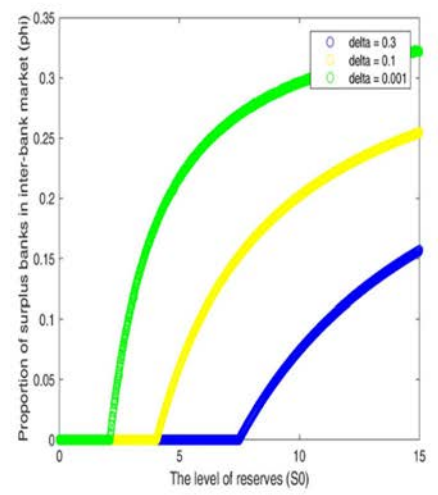


Figure 4C: ϕ and reserves for various δ

Figure 4: Numerical example for the extended model of section IV with endogenous interbank entry: The effect of varying the convenience yield on reserves δ

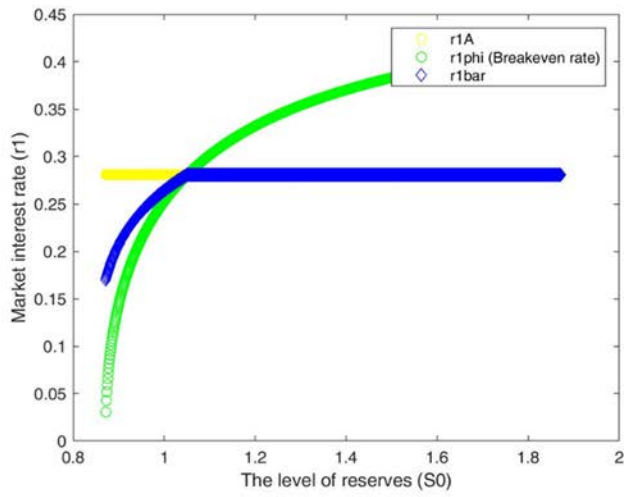


Figure 5A: Market Rate and Reserves

$$\bar{E} = 0.87, \theta = 0.6$$

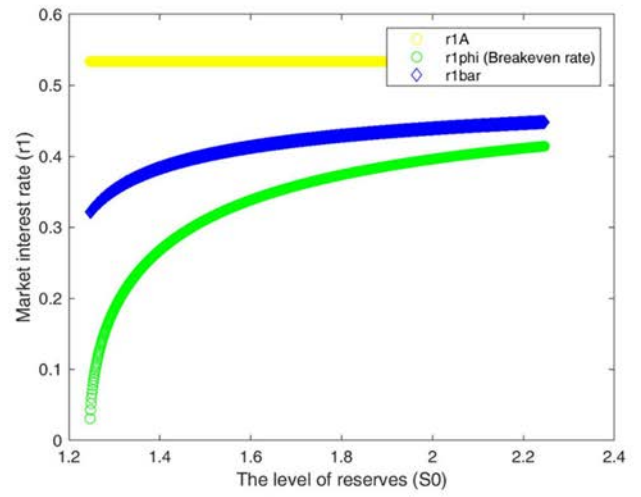


Figure 5B: Market Rate and Reserves

$$\bar{E} = 1.25, \theta = 0.6$$

Figure 5: Numerical example for the extended model of section 5.3 with endogenous interbank entry: The effect of varying the fixed encumbrance on reserves \bar{E}

Appendix

Proof of Lemma 1:

The right hand side of (1.10) is decreasing in r_1 (we will see this shortly). The left hand side is obviously increasing in r_1 . If so, if the right hand side of (1.10) is positive when $r_1 = 0$ then there is excess demand for funds in the inter-bank market when the premium is zero, and hence there is an unique positive crossing point, the equilibrium r_1 . If the right hand side is non-positive when $r_1 = 0$, there is (weakly) excess supply, and r_1 is zero.

So it remains to show the right hand side of (1.10) is decreasing in r_1 . From (1.3),

$$I_1 = g_1'^{-1}(1 + \gamma + r_1), \text{ which is decreasing in } r_1.$$

Turn next to the second term in the square brackets, $(D_0 - D_0^F)$. This equals

$$\left[I_0 - e_0 + (S_0 - W_0^F) + \frac{1}{2}\lambda(L_0^B)^2 \right]. \text{ We know } I_0 = g_0'^{-1}\left(\frac{1 + q\gamma + qr_1}{(1 - q)}\right) \text{ which is decreasing in } r_1. \text{ Also,}$$

$$-e_0 = -\frac{qr_1}{\alpha_0}, \text{ which is decreasing in } r_1. \text{ The next term, } (S_0 - W_0^F), \text{ is a constant. That leaves the last}$$

$$\text{term, in the expression for } (D_0 - D_0^F), \frac{1}{2}\lambda(L_0^B)^2. \text{ From (1.9), } L_0^B = \frac{1}{\lambda}\left(\frac{R_0^{DF} - R_0^{DB}}{R_0^{DB}}\right) = \frac{q}{\lambda}\left(\frac{\gamma}{1 + qr_1}\right),$$

which decreases in r_1 whence given L_0^B is positive, $\frac{1}{2}\lambda(L_0^B)^2$ also decreases in r_1 . So we have

$$(D_0 - D_0^F), \text{ the deposits the bank raises from the public, decreasing in } r_1.$$

Finally, the last term on the right hand side of (1.10), $-\alpha_1 S_0(1 - \tau)$, is a constant. So the right hand side of (1.10) is decreasing in r_1 and the equilibrium r_1 is unique. Q.E.D.

Threshold level of reserves when $NLS > 0$:

Recall that NLS is the net liquidity supplied by the corporate sector anticipating a date-1 interbank premium of zero (and adjusting for any cost to lending). When $NLS > 0$ and the risk of

liquidity stress in the economy is high, *i.e.*, $\theta > \frac{\varphi(1 - \tau)}{\tau + \varphi(1 - \tau)}$ (this is the region to the right of the vertical

axis in Figure 2B), \hat{S}_0 is positive, and falls in θ . Intuitively, because higher ex-ante reserves tighten liquidity in the stressed state, and a higher θ consumes more liquidity per dollar of reserves, the reserve threshold at which the net liquidity supplied by the corporate sector is fully consumed is positive and falls in θ . Furthermore, \bar{r}_1 increases in S_0 , and the unhatched region above the \hat{S}_0 curve is where \bar{r}_1 is positive. When $NLS > 0$ and the risk of liquidity stress in the economy is low, *i.e.*, $\theta < \frac{\varphi(1-\tau)}{\tau + \varphi(1-\tau)}$ (that is, in the region to the left of the vertical axis in Figure 2B), \hat{S}_0 is negative, and falls in θ . Because higher ex-ante reserves loosen liquidity conditions, \bar{r}_1 falls in S_0 , and the unhatched region below the \hat{S}_0 curve is where \bar{r}_1 is positive. The hatched area is where \bar{r}_1 is zero.

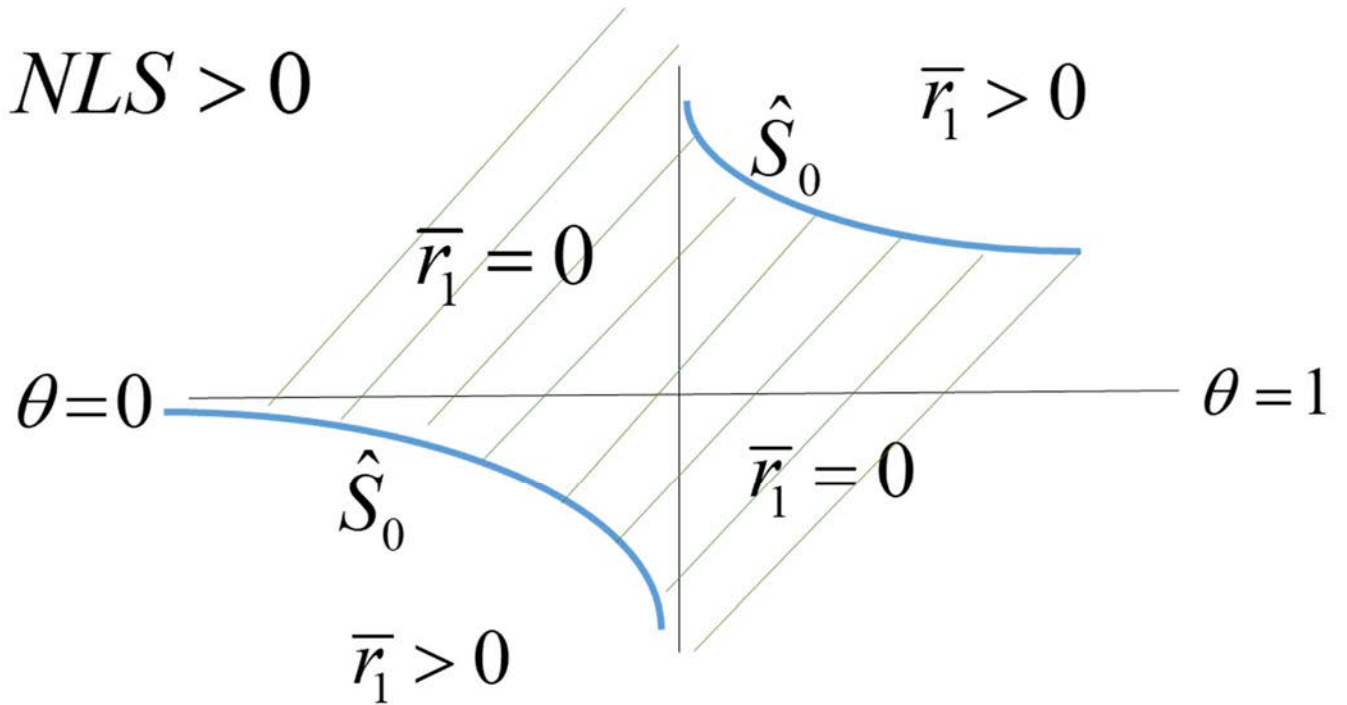


Figure 2B: Reserves are on the y axis, θ on x axis, with the two axes intersecting at $\theta = \frac{\varphi(1-\tau)}{\tau + \varphi(1-\tau)}$

Proof that $\frac{\partial U}{\partial \bar{r}_1} < 0$ in section III:

Since $D_0^F = (L_0 + W_0^F - I_0)$, substituting and rewriting, the planner maximizes

$$((1-q)g_0(I_0) - I_0(1+q\gamma)) + q(g_1(I_1) - I_1(1+\gamma)) - \frac{1}{2}\alpha_0 e_0^2 - \frac{q}{\theta}[\varphi(1-\theta) + \theta]\left(\frac{1}{2}\alpha_1 e_1^2\right) - \frac{1}{2}\lambda(L_0)^2 + q(L_0 + W_0^F)\gamma$$

Differentiating U w.r.t r_1 , we get $\frac{\partial U}{\partial r_1} =$

$$\left((1-q)g_0' - (1+q\gamma)\right)\frac{dI_0}{dr_1} + q\left(g_1' - (1+\gamma)\right)\frac{dI_1}{dr_1} - \alpha_0 e_0 \frac{de_0}{dr_1} - \frac{q}{\theta}[\varphi(1-\theta) + \theta]\alpha_1 e_1 \frac{de_1}{dr_1} + (q\gamma - \lambda L_0)\frac{dL_0}{dr_1}$$

(1.12)

On inspection, the first 4 elements are all negative, so $\frac{\partial U}{\partial r_1} < 0$ if $(q\gamma - \lambda L_0)\frac{dL_0}{dr_1} \leq 0$. But

$$L_0 = \frac{q}{\lambda} \left(\frac{\gamma}{1+qr_1} \right). \text{ So } (q\gamma - \lambda L_0) \geq 0. \text{ Since } \frac{dL_0}{dr_1} < 0, \frac{\partial U}{\partial r_1} < 0. \text{ Q.E.D.}$$

Proof of Lemma 3:

Note that for the social-planning central bank

$$\frac{dU}{de_0} = \frac{\delta U}{\delta e_0} + \frac{\delta U}{\delta r_1} \frac{dr_1}{de_0} \quad (1.13)$$

Differentiating (1.12), we get $\frac{\delta U}{\delta e_0} = -\alpha_0 e_0$. Differentiating (1.10) after substituting in for $(D_0 - D_0^F)$, we

get $\frac{dr_1}{de_0} = -\alpha_1 \frac{\theta}{[\varphi(1-\theta) + \theta]}$. From (1.12), $\frac{\delta U}{\delta r_1} = -\frac{q}{\theta}[\varphi(1-\theta) + \theta]\alpha_1 e_1 \frac{de_1}{dr_1} + \text{four negative terms}$.

Substituting $\frac{de_1}{dr_1} = \frac{1}{\alpha_1}$, and all terms back into (1.13), we get

$$\frac{dU}{de_0} = -\alpha_0 e_0 + \left(-\frac{q}{\theta}[\varphi(1-\theta) + \theta]e_1 + \text{four negative terms} \right) \left(-\alpha_1 \frac{\theta}{[\varphi(1-\theta) + \theta]} \right). \text{ At the private}$$

optimal, $e_0 = \frac{qr_1}{\alpha_0}$ and $e_1 = \frac{r_1}{\alpha_1}$. Substituting, we get

$$\frac{dU}{de_0} \Big|_{e_0 = \frac{qr_1}{\alpha_0}} = (\text{four negative terms}) \left(-\alpha_1 \frac{\theta}{[\varphi(1-\theta) + \theta]} \right) > 0. \text{ Q.E.D.}$$

Detailing the maximization in section IV:

The firm's maximization problem remains unchanged. The bank's maximization problem is

$$\text{Max}_{L_0^B, e_0} R_0^L L_0^B + S_0 - e_0 - \frac{\alpha_0}{2} e_0^2 - D_0 + E_0 \left[V(y, z) \mid L_0^B, e_0 \right]$$

$$\text{s.t. } D_0 + e_0 = L_0^B + \frac{1}{2} \lambda (L_0^B)^2 + S_0$$

Case 1: The convenience yield associated with reserves in the stressed state, δ , is an opportunity cost for stressed banks, and they pass it on while lending to their firm at date 1. So they lend at rate $(1 + \gamma + \delta)$ where γ is their monitoring cost. Therefore,

$$V(y=1, z=1) = \text{Max}_{e_1} \left[-\delta(D_0 - e_1) - \frac{\alpha_1}{2} e_1^2 \right]$$

Therefore, $e_1 = \frac{\delta}{\alpha_1}$, $e_0 = \frac{q\delta}{\alpha_0}$. The $(1 - \theta)$ healthy banks divide the deposit outflows from the stressed banks so

$$V(y=1, z=0) = \frac{\theta\delta(D_0 - D_0^F + I_1 - \frac{\delta}{\alpha_1})}{(1 - \theta)}. \text{ Note that in making decisions at date 0, the inflows that}$$

come into the bank if it were healthy at date 1 are unrelated to any decision it takes at date 0 – it stems from decisions (on the size of loans, capital raise, and deposit funding) taken by other banks.²¹ So, maximizing at date 0 w.r.t. L_0^B , we get $R_0^L = (1 + q\delta)(1 + \lambda L_0^B)$. From the firm's maximization, we

know $R_0^D = (1 + q\gamma + q\delta) = R_0^L$, so $L_0^B = \frac{q\gamma}{\lambda(1 + q\delta)}$. We have worked out the condition under which

this case arises in the text, that is, $(D_0 - D_0^F + I_1 - e_1) < S_0(1 - \tau)$. Since

$(D_0 - D_0^F) = (S_0 + L_0^B + \frac{1}{2} \lambda (L_0^B)^2 - e_0) - (W_0^F + L_0^F - I_0) = I_0 - e_0 + (S_0 - W_0^F) + \frac{1}{2} \lambda (L_0^B)^2$, where the second equality uses $L_0^B = L_0^F$, the condition simplifies to

$$\tau S_0 \leq \left[\left(\frac{q\delta}{\alpha_0} + \frac{\delta}{\alpha_1} \right) + W_0^F - I_0 - I_1 - \frac{1}{2} \lambda L_0^2 \right].$$

Case 2: Here the opportunity cost of lending at date 1 is r_1 (since this is the marginal cost of raising capital, the source of incremental funding at date 1), and it replaces δ in the bank's maximization in Case 1. The stressed bank sees

²¹ Put differently, all the variables in this expression should have a superscript O to signify they are decisions made by other banks. In the symmetric equilibrium, however, they will be equal to the values chosen by the bank whose maximization decisions we are studying.

$V(y=1, z=1) = \underset{e_1}{\text{Max}} \left[-r_1(D_0 - e_1) - \frac{\alpha_1}{2} e_1^2 \right]$. The healthy banks receive

$V(y=1, z=0) = \frac{\theta \delta S_0 (1-\tau)}{(1-\theta)}$. Furthermore, $\tau S_0 = \left[\left(q r_1 / \alpha_0 + r_1 / \alpha_1 \right) + W_0^F - I_0 - I_1 - \frac{1}{2} \lambda L_0^2 \right]$ for

liquidity demand to equal liquidity supply. Since the right hand side increases in r_1 , a higher S_0 always induces a higher r_1 , whatever the level of τ so long as it is positive. In other words, since the stressed bank is in autarky, it is as if $\theta = 1$, and the condition $\theta > (1-\tau)$ is always satisfied for any positive τ .

Case 3: For the bank, the date-0 maximization is similar to the one in Case 2. In this case, if healthy, the bank may use its reserves to lend at date 1. However, this will not enter its maximization since it takes the reserves as given. The bank's maximization problem at date 0, and the stressed bank's problem at date 1 then is as in case 2, where it takes r_1 as given.

Proof of Theorems 2 and 3:

From the text, S_0^* is the level of reserves at which the stressed bank can just meet liquidity needs with the (shadow) rate δ and reserves having a convenience yield δ . That is,

$$S_0^* = \frac{1}{\tau} \left[\left(q \delta / \alpha_0 + \delta / \alpha_1 \right) + W_0^F - I_0 - I_1 - \frac{1}{2} \lambda L_0^2 \right] \text{ where } g_0'(I_0) = \frac{1+q(\gamma+\delta)}{1-q},$$

$g_1'(I_1) = (1+\gamma+\delta)$, and $L_0 = \frac{q\gamma}{\lambda(1+q\delta)}$. Note the right hand side is increasing in δ so S_0^* is

increasing in δ . Furthermore, the net rate the stressed banks charge firms is $(\gamma+\delta)$ for $S_0 < S_0^*$.

When S_0 rises from S_0^* , the (shadow autarky) rate r_1^A rises from δ . It solves

$$\tau S_0 = \left[\left(q r_1 / \alpha_0 + r_1 / \alpha_1 \right) + W_0^F - I_0 - I_1 - \frac{1}{2} \lambda L_0^2 \right] \text{ where } g_0'(I_0) = \frac{1+q(\gamma+r_1)}{1-q},$$

$g_1'(I_1) = (1+\gamma+r_1)$, and $L_0 = \frac{q\gamma}{\lambda(1+q r_1)}$. Once again, since the right hand side increases in r_1 , r_1^A is

increasing in S_0 . If we further assume g_0''' , g_1''' are both positive, then it is convex.

Also $r_1^\phi = \alpha_1 S_0 (1-\tau) \left[\sqrt{1 + \frac{2\delta}{\alpha_1(1-\theta)S_0(1-\tau)}} - 1 \right]$. So $r_1^\phi > 0$ for $S_0 > 0$. Furthermore, it is

straightforward to show that r_1^ϕ is increasing in S_0 and it is concave. Assume for now, and we will revisit

later, that at S_0^* , $r_1^\phi > r_1^A$. Since both rates are increasing in S_0 , and r_1^A is convex in S_0 while r_1^ϕ is

concave, they can intersect only once at S_0^{**} . So the (shadow) rate is r_1^A as S_0 increases from S_0^* to S_0^{**}

after which it becomes the rate dictated by the interbank market. Finally, r_1^ϕ increases in δ (as does r_1^A ,

see above). So S_0^{**} increases in δ . Finally, since the equilibrium φ falls in δ , the required equilibrating interbank rate also increases in δ . Now, it can be shown using the 2nd order Taylor-series expansion of $\sqrt{(1+x)}$ in r_1^φ , that at S_0^* , $r_1^\varphi - r_1^A > \frac{\delta\theta}{(1-\theta)} - \frac{\delta^2}{2\alpha_1(1-\theta)^2 S_0^*(1-\tau)} > 0$, if $\delta < 2\alpha_1\theta(1-\theta)(1-\tau)S_0^*$. Substituting for S_0^* , and doing some algebra, it can be shown that a sufficient condition for this in turn is that $\theta(1-\theta) > \frac{\tau}{2(1-\tau)}$ and $\delta > \delta^*$ where δ^* satisfies

$$\frac{\tau}{2\alpha_1\theta(1-\theta)(1-\tau)}\delta^* = \left[\left(q\delta^*/\alpha_0 + \delta^*/\alpha_1 \right) + W_0^F - I_0(\delta^*) - I_1(\delta^*) - \frac{1}{2}\lambda[L_0(\delta^*)]^2 \right]. \text{ Q.E.D.}$$

Condition for r_1 to be increasing in S_0 in Case 3 (section IV):

Recognize that in this region, r_1 is determined by equating the demand by stressed banks for loans in the inter-bank market to the supply by tainted banks of those loans. So

$$\theta[D_0 - D_0^F + I_1 - S_0(1-\tau) - e_1] = \varphi(1-\theta)[S_0(1-\tau) + e_1]. \text{ Substituting}$$

$$(D_0 - D_0^F) = [I_0 - e_0 + (S_0 - W_0^F) + \frac{1}{2}\lambda(L_0^B)^2] \text{ and } e_1 = \frac{r_1}{\alpha_1} \text{ and rearranging, we get}$$

$$\frac{r_1}{\alpha_1} = \frac{\theta}{[\varphi(1-\theta) + \theta]} [I_0 + I_1 - e_0 + \frac{1}{2}\lambda(L_0^B)^2 - W_0^F] + \frac{[\theta\tau - \varphi(1-\theta)(1-\tau)]}{[\varphi(1-\theta) + \theta]} S_0. \text{ Let the right hand side}$$

of this equality be denoted by F . Then totally differentiating, we get $\left[\frac{1}{\alpha_1} - \frac{\partial F}{\partial r_1} \right] \frac{dr_1}{dS_0} = \frac{\partial F}{\partial \varphi} \frac{\partial \varphi}{\partial S_0} + \frac{\partial F}{\partial S_0}$.

Since $\frac{\partial F}{\partial r_1} < 0$, $\frac{dr_1}{dS_0} > 0$ if $\frac{\partial F}{\partial \varphi} \frac{\partial \varphi}{\partial S_0} + \frac{\partial F}{\partial S_0} > 0$. But $\frac{\partial F}{\partial \varphi} < 0$ by inspection, and we argued in the text that

$\frac{\partial \varphi}{\partial S_0} < 0$. So $\frac{\partial F}{\partial \varphi} \frac{\partial \varphi}{\partial S_0} > 0$ and a sufficient condition for $\frac{dr_1}{dS_0} > 0$ is that $\frac{\partial F}{\partial S_0} > 0$. This then requires

$$[\theta\tau - \varphi(1-\theta)(1-\tau)] > 0, \text{ which on simplifying requires } \theta > \frac{\varphi(1-\tau)}{\tau + \varphi(1-\tau)}. \text{ Note that this is only}$$

sufficient, since even if it does not hold, it may still be that $\frac{dr_1}{dS_0} > 0$. Intuitively, there is now a new

channel through which a higher S_0 leads to a higher r_1 : a higher S_0 leads to a lower φ ceteris paribus, since healthy banks have more reason to stay on the sideline given the larger flight to safety flows, which in turn leads to a greater net need for liquidity from capital-raising, and hence a higher r_1 .

Embedding Liquidity Regulations (section 5.2)

In the context of our framework, suppose that after reserves are set and speculation is under way, regulators can affect overall $\tau (= \tau^{Spec} + \tau^{Reg})$ by setting τ^{Reg} . Let the fraction of banks that suffer withdrawals at date 1 be $K(\tau^{Reg})\theta$ instead of θ , with $K' < 0$, $K'' > 0$ and $K(0) = 1$. This means the share of banks that are stressed falls in mandatory regulatory reserve holdings (in part because that also curbs the effects of speculation). However, this also hampers the liquidity available from healthy banks in times of liquidity stress. Hence, if regulators are narrowly focused on maximizing overall liquidity available per dollar of reserves ex post, given the central bank has set reserves, they would maximize $(1-\tau) - K(\tau^{Reg})\theta$. So they would optimally choose $\tau^{Reg*} = K'^{-1}(-\frac{1}{\theta})$. On inspection, and bearing in mind that risk reduction has diminishing returns so that $K'' > 0$, the higher is θ the greater will be the regulatory encumbrance τ^{Reg*} . Depending on functional forms, that is, how effectively a higher τ^{Reg} reduces the share of banks that are stressed, it can be shown that all the cases we have discussed earlier could still be possible with optimal regulation.

Our model easily allows for an analysis of alternative formulations of the regulatory requirement. For instance, if banks are required to maintain τD_0 of deposits as reserves at all times (that is, a traditional reserve requirement), we can show easily that once again $\tau^{Reg*} = K'^{-1}(-\frac{1}{\theta})$ since deposit issuance moves one for one with reserves.

Proof of Theorem 5:

Following earlier derivations, but with a fixed encumbrance, we know

$$r_1^\varphi = \alpha_1 (S_0 - \bar{E}) \left[\sqrt{1 + \frac{2\delta}{\alpha_1(1-\theta)(S_0 - \bar{E})}} - 1 \right]. \text{ Clearly } r_1^\varphi \rightarrow 0 \text{ as } S_0 \rightarrow \bar{E}. \text{ Also, } r_1^\varphi \rightarrow \frac{\delta}{(1-\theta)} \text{ as } S_0 \rightarrow \infty.$$

Finally, r_1^φ is increasing in S_0 . We also know that r_1^A is the value of r_1 that solves

$$\bar{E} = \left[\left(\frac{qr_1}{\alpha_0} + \frac{r_1}{\alpha_1} \right) + W_0^F - I_0 - I_1 - \frac{1}{2} \lambda L_0^2 \right], \text{ where } I_0, I_1, L_0 \text{ depend on } r_1 \text{ in the usual manner.}$$

Since none of the elements on the right hand side change with S_0 , r_1^A does not change with S_0 .

Therefore, if $r_1^A < \frac{\delta}{(1-\theta)}$ because \bar{E} is small, there is an $\bar{S}_0 > \bar{E}$ such that $r_1^\varphi = r_1^A$ at $S_0 = \bar{S}_0$, and

$r_1^\varphi > r_1^A$ for $S_0 > \bar{S}_0$. So the equilibrium interbank rate $\bar{r}_1 \in (r_1^\varphi(S_0), r_1^A(\bar{E}))$ for $\bar{S}_0 > S_0 > \bar{E} > \bar{E}^*$ and

$\bar{r}_1 = r_1^A(\bar{E})$ for $S_0 \geq \bar{S}_0$. If, however, $r_1^A \geq \frac{\delta}{(1-\theta)}$, then $r_1^\varphi < r_1^A$ for all finite S_0 , and the equilibrium interbank rate $\bar{r}_1 \in (r_1^\varphi(S_0), r_1^A(\bar{E}))$ for all finite S_0 . Q.E.D.

Private Incentives to Hold Reserves (section 6.4)

What if banks were not forced to hold S_0 ? To make the problem relevant, we assume there is a carrying cost or capital cost of holding excess reserves of $C(S_0)$ such that $C'(S_0) > 0$, $C''(S_0) > 0$. Focusing only on the bank's optimal choice of reserve holdings, and assuming that $\delta \rightarrow 0$ so that $\varphi \rightarrow 1$ (all surplus banks lend in the interbank market in the stressed state of the economy), this choice boils down to

$$\max_{S_0} S_0 - C(S_0) + E_0 \left[-(1+r_1(y))(yzD_0 - yS_0(1-\tau)) - (1-yz)D_0 \right]$$

The FOC w.r.t. S_0 is $(1-C') - (1+qr_1) + \frac{q}{\theta}(1+r_1)(1-\tau) = 0$. Essentially, the bank reaps the direct return from reserve holdings $(1-C')$, has to refinance the deposit used to finance it at date 1 in the interbank market if stressed, and can lend the unencumbered portion of reserves in the interbank market (or equivalently, reduce its borrowing) if the economy is liquidity stressed. Simplifying

$$S_0^{opt} = C'^{-1} \left[q \left(1 - (1+r_1) \left(1 - \frac{(1-\tau)}{\theta} \right) \right) \right]$$

There is no guarantee that the level of (per bank) reserves that the central bank wants to issue (given its other concerns such as conducting unconventional monetary policy) are at, or below, the level that commercial banks want to optimally hold. Suppose the central bank wants to initially place higher level of reserves than S_0^{opt} . If $\theta > (1-\tau)$, an increase in anticipated interest rates reduces the S_0^{opt} that the commercial bank would optimally like to hold. Since under this same condition, we know that anticipated interbank rates are rising in S_0 , the divergence, between what commercial banks are willing to hold and what the central bank has to place, grows with S_0 . Put differently, the prospect of greater liquidity shortages need not increase the commercial banks' private incentives to hold more reserves; indeed, recognizing that the source of the shortage is the financing of reserves, they may want to hold less.

Risk Management and Maturity Matching or Short-term Financing of Reserves by Shadow Banks (section 6.4)

We have assumed that the reserves end up on bank balance sheets. What if the central bank departs from normal practice and allows non-bank financial firms to hold reserves directly? Unless the central bank buys money-like assets from the non-bank private sector, we may not get significantly different outcomes; if the central bank buys long-term financial assets and pays with reserves, for standard risk management reasons the non-bank private sector may match the maturity of their liability structure to their shorter-maturity asset holdings.

To see this, let us focus on the healthy state (that is, assume $q = 0$), and assume that economy-wide date-1 short-term (gross) interest rates in the healthy state are $(1 + r)$ with probability p and $(1 - r)$ with probability $(1 - p)$. The net rates $(+r$ and $-r)$ represent the state-contingent cost of rolling over each bank's liquidity shortfall given by $(D_0 - S_0)$. Further, assume the financial firm holding reserves wants to finance it so as to minimize costs, but it also dislikes the variability of its date-2 profits given by the variance of profits, $p(1-p)4r^2(D_0 - S_0)^2$, with aversion parameter $\Psi / 2$. Finally, the cost of capital issuance at date 0 is $R_0^E = [p(1+r) + (1-p)(1-r) + \Delta_0^E]$, where Δ_0^E is a capital risk premium. So ignoring the other activities of the financial firm, its objective function for choosing the maturity structure of its liabilities, given the need to finance reserve holdings, is as follows (where variables have their earlier connotation):

$$\begin{aligned} \text{Max}_{D_0} & \left[-R_0^E e_0 - p(1+r)(D_0 - S_0) - (1-p)(1-r)(D_0 - S_0) - \frac{\Psi}{2} p(1-p)4r^2(D_0 - S_0)^2 \right] \\ \text{s.t.} & \quad e_0 = S_0 - D_0 \end{aligned}$$

It is straightforward from the maximization that $D_0 = \left[S_0 + \frac{\Delta_0^E}{\Psi p(1-p)4r^2} \right]$. So deposits increase one for one with reserves and also increase with the capital premium – the point is that longer term financing for reserves can increase the variability of profits by locking in financing costs while leaving returns on reserves variable. Financial firms will match maturity to avoid this variability. Put differently, so long as central-bank-issued reserves have to be financed somewhere in the economy, there will be some offsetting short-term liabilities.