MINIMUM WAGES, EFFICIENCY AND WELFARE

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ABSTRACT

It has long been argued that a minimum wage could alleviate efficiency losses from monopsony power. In a general equilibrium framework that quantitatively replicates results from recent empirical studies, we find higher minimum wages can improve welfare, but most welfare gains stem from redistribution rather than efficiency. Our model features oligopsonistic labor markets with heterogeneous workers and firms and yields analytical expressions that characterize the mechanisms by which minimum wages can improve efficiency, and how these deteriorate at higher minimum wages. We provide a method to separate welfare gains into two channels: efficiency and redistribution. Under both channels and Utilitarian social welfare weights the optimal minimum wage is $15, but alternative weights can rationalize anything from $0 to $31. Under only the efficiency channel, the optimal minimum wage is narrowly around $8, robust to social welfare weights, and generates small welfare gains that recover only 2 percent of the efficiency losses from monopsony power.

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1   Introduction

The U.S. federal minimum wage has been roughly constant in real terms since the 1980s, and fallen almost 40 percent from its peak in the late 1960s (Figure 1A). As a result, the 2019 *Raise the Wage Act* proposed raising the Federal minimum wage to \$15 an hour. If enacted, the minimum wage would exceed current wages for 41 percent of workers without a college education, 11 percent of college educated workers and 29 percent of workers overall (Figure 1B).[1] In this paper, we develop a tractable, general equilibrium oligopsony model of the labor market with heterogeneous workers and firms that is quantitatively consistent with the empirical minimum wage literature and use it to answer the following questions: *Is there a rationale for a non-zero minimum wage?*, if so, *why?* and *how high should it be?*, and finally, *what would be the labor market consequences of raising minimum wages to a much higher level?*

There are two key rationales for a positive minimum wage: efficiency and redistribution. First, if firms have market power in the labor market, wages are generically less than the marginal product of labor, and employment at each firm is inefficiently low. Even before the introduction of the federal minimum wage in 1938, it was known that a well-targeted minimum wage could help alleviate the efficiency losses from monopsony power by inducing firms to hire more workers.[2] Second, a higher minimum wage has the potential to benefit low income workers and reduce profits that tend to accrue to business owners and high income workers, redistributing economic output. While both channels are potentially important, our analysis returns to the beginning of the minimum wage debate and focuses on the ability of a national minimal wage to address inefficiencies due to labor market power.[3]

Our model, which includes both firm and worker heterogeneity, captures the three empirically documented channels that could lead to higher efficiency. First, there is a *direct effect* by which firms with market power increase their wages and expand employment when faced with a binding minimum wage. Evidence on such direct effects come from increases in employment and wages following small minimum wage increases (for example Clemens and Strain, 2021). Second, an *spillover effect* by which firms, whose competitors start paying higher wages, increase their wage in response and in doing so expand beyond their inefficiently low levels employment. Evidence on such spillovers comes from competitors' response to Amazon increasing its wage (Derenoncourt, Noelke, Weil, and Taska, 2021) and hospitals raising their wages in response to competitors (Staiger, Spetz, and Phibbs, 2010). Third, there is a *reallocation effect* by which a minimum wage may destroy jobs at unproductive firms, with labor reallocated to more productive firms. Recent evidence on reallocation effects come from Germany (Dustmann, Lind-

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[1] Raise the Wage Act (S. 53) was introduced to the Senate on January 26, 2021, after being introduced a H.R. 582 in the House of Representatives in March 2019. The Congressional Budget Office reports on both bills can be found here for S. 53 and H.R. 582.

[2] *The Economics of Imperfect Competition* (1933), Joan Robinson pp 294. *"The amount of employment given by the monopsonist organization will be restricted to the amount at which the marginal cost of labour to the whole group is equal to its demand price for each particular type of organization. The wage will be equal to the supply price of labour, and this, in each case, will be less than the value of the marginal physical product of labour. Thus exploitation will occur. Monopsonistic exploitation of this type can be removed by the imposition of a minimum wage."*

[3] Furthermore, a focus on the redistributive properties of the minimum wage has the undesirable feature in that the answer depends significantly on what the social welfare weights are and, holding social welfare weights fixed, the availability and efficiency of other fiscal instruments.
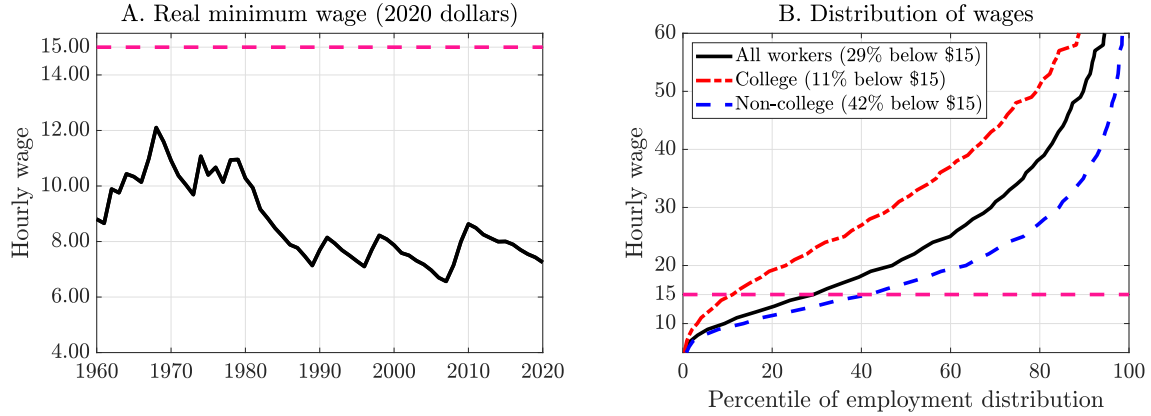
Figure 1: Minimum wages in the US, 1960-2020

Notes: **Panel A.** Real wages computed by deflating by the CPI-U. **Panel B.** CPS data constructed using MORG from 2019, and weighted using `hwtfinl`. Wages are computed as weekly earnings (`earnweek`) divided by usual weekly hours worked (`uhrsworkt`). We follow the Federal Reserve Bank of Atlanta Wage Growth Tracker, and remove individuals whose hourly pay is below the current federal minimum wage for tip-based workers ($2.13). We drop individuals who are coded as *hours vary* (`uhrsworkt=997`). We keep all other workers aged between 16 and 65.

ner, Schoenberg, Umkehrer, and vom Berge, 2021).

Our model extends the textbook model of monopsony under a minimum wage to incorporate firm heterogeneity in productivity, worker heterogeneity in wealth and productivity, and a well-defined notion of labor market power: firms compete strategically in many concentrated labor markets. This general equilibrium framework captures the relevant trade-offs necessary to evaluate a higher minimum wage. On the one hand, the presence of monopsony power means that a higher minimum wage can raise wages with little disemployment effects, while the presence of oligopsony power allows for empirically plausible responses to competitors' wage policies. Together, this creates the possibility that a higher minimum wage may be welfare improving. On the other hand, when the minimum wage exceeds a firms' competitive wage, employment and profits fall. Moreover, as the wage differences between firms becomes compressed, some workers will choose to move from the high paying, high productivity firms to newly higher paying but less productive firms. This leads to misallocation. These two forces imply that above a certain threshold, a higher minimum wage will lower welfare.

To quantify these forces we calibrate our model to the U.S. economy. The calibrated model reproduces the empirical distribution of markets in terms of the number of firms in each market, average firm employment and payroll, relative wages across worker types, distribution of consumption and nonwage income across worker types, average market concentration, labor and capital share, and the observed relationship between labor market share and wage and employment responses to shocks (Berger, Herkenhoff, and Mongey, 2021). We then quantitatively replicate the above studies, which discipline the channels through which minimum wages may improve efficiency. We adapt our environment to the empirical setting studied in each paper, repeat the associated natural experiment and reproduce the authors' empirical analysis. Hence, both theoretically and empirically, the model captures the key margins through which minimum wages can lead to expanded output in the economy. Our model also

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reproduces recent evidence (i) that the employment effects of minimum wages may be positive in concentrated markets and negative in less concentrated markets (Azar, Huet-Vaughn, Marinescu, Taska, and von Wachter, 2019), (ii) on the spillover effects of minimum wages across workers (Autor, Manning, and Smith (2016), in the US, and Engbom and Moser (2021) and Haanwickel (2020) in Brazil). Hence we make a separate contribution in providing a unifying quantitative framework for many existing empirical results.

We then use our calibrated and validated model of the economy to answer the following questions: what is the minimum wage that maximizes efficiency, what are the welfare gains, and through which channels do these accrue? This is complicated by the fact that, as noted, a minimum wage has both *efficiency* and *redistributive* implications. A methodology for disentangling these is required for two reasons. First, we do not know what the correct social welfare weights are and this choice matters quantitatively. In particular, we show that under Utilitarian weights a planner would choose a minimum wage of $15.12, while if it cared only about college educated workers it would choose $31.53, or down to $6.97 if it used the social welfare weights that rationalize the observed competitive equilibrium. Second, governments have access to additional tools for redistribution via the tax and transfer system, which may be changed along with the minimum wage. We cannot model all tools of the tax and transfer system, or compute how each tool should be reoptimized under each level of a minimum wage considered.

Our methodology addresses both issues: we ask *what is the optimal minimum wage in the presence of budget-neutral, unrestricted lump-sum transfers across households?* This addresses the first issue, as taking labor market imperfections as given, the government can reoptimize transfers to meet the redistribution objectives encoded in any arbitrary social welfare weights. It addresses the second issue in that lump sum transfers encapsulate all possible tax and transfer schemes. With redistribution taken care of, the minimum wage can be used entirely for efficiency. To this end we provide aggregation theorems that allow us to compute optimal lump-sum taxes under arbitrary social welfare weights. Now, rather than a range of $0 to $31 per hour—depending on social welfare weights, and keeping fiscal policies fixed— the answer narrows to $7.50 to $10 per hour—invariant to social welfare weights, and with flexible redistributive policy. Moreover, we find that the associated efficiency gains are small: equivalent to a 0.1 percent increase in TFP. We benchmark these gains by showing that they shift the economy only 2 percent of the way toward an economy with no labor market power. Our exercise therefore concludes that higher minimum wages would not be justified not on efficiency grounds.

There are three reasons why the efficiency gains from a minimum wage increase are limited. First, the direct efficiency gains come from firms for whom the minimum wage is binding but still below the firm's competitive wage (the wage they pay with no markdown). These are lower productivity firms and the efficiency gains that can be squeezed from them are small because (i) they would have endogenously narrow markdowns absent a minimum wage, limiting the scope of efficiency gains, (ii) they face highly elastic labor supply, so only a small increase in their wage is required to shift them to their efficient employment level. Second, in our calibration firms have a relatively flat marginal revenue product of labor schedule which implies that once firms start rationing employment, they do so very quickly. Third, the biggest efficiency gains would come from narrowing markdowns at highly productive, un-

constrained firms who have the largest markdowns in each market. However, because their low wage competitors that pay the minimum wage have small market shares, large firms largely ignore them and hence spillover effects are small.

These results do not rule out the minimum wage as a tool for improving welfare. The potency of minimum wages to redistribute has been well documented (Derenoncourt and Montialoux, 2021; Cengiz, Dube, Lindner, and Zipperer, 2019), and we show our model generates spillovers consistent with empirical evidence. Instead our exercises quantify the extent to which redistribution is the *primary channel* through which minimum wages improve welfare.[4] Our key result is that under the $15.12 per hour minimum wage that maximizes social welfare under utilitarian weights, less than 5 percent of the welfare gains come from improved efficiency, whereas more than 95 percent come from redistribution. To understand this result we first deliver aggregation theorems that allow us to compute the social welfare weights that would lead a planner to choose the observed competitive equilibrium allocation of consumption and labor. The weights we back out load disproportionately on college educated households. Importantly, their weight is higher than their population share, which is the weight assigned by a utilitarian planner.[5] Hence, a minimum wage policy that maximizes a utilitarian objective will place a large emphasis on redistribution, which is exactly what we find.

These results also do not rule out the minimum wage as a tool for reducing income inequality or increasing labor's share of income, which are common empirical proxies for inequality and worker power, respectively. Indeed, we show that under a higher minimum wage, income inequality falls within and across worker types, and labor's share of income increases.[6] Both indicators are *monotonic* as the minimum wage increases, despite the clear hump-shape of the welfare effects of the minimum wage. This result warns that observations such as a higher labor share of income or lower wage inequality caused by a higher minimum wage can be compatible with *declining* welfare.

Robustness. We provide a number of robustness checks of our main results. First, we split our economy into low, medium and high income regions, calibrated separately to data on the distribution of workers and average wages in groups of US states. We find the welfare gains from three region specific minimum wages are only marginally greater than the welfare gains from a federal minimum wage.

Second, we analyze the optimal minimum wage in Mississippi—the state with the lowest per capita

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[4]This provides a quantitative answer to the first set of questions left open in Cahuc and Zylberberg (2004). The authors conclude their section on minimum wages by questioning whether a minimum wage primarily acts through efficiency or redistribution. The further set of questions, taken up by Hurst, Kehoe, Pastorino, and Winberry (2021), relates to whether alternative fiscal policies can do better at redistribution than a minimum wage.

[5]This is a common result in normative applications of quantitative heterogeneous agent models. For example, part of the exercise in Heathcote and Tsujiyama (2021) is to ask *"For what set of social welfare weights would the observed schedule of labor income taxes be the optimal schedule of labor income taxes?"*. They find that the answer is a set of welfare weights tilted toward high income households.

[6]Recently, Deb, Eeckhout, Patell, and Warren (2020) extends Berger, Herkenhoff, and Mongey (2021) to study inequality between skill types, but do not study minimum wages. In their model, built for positive analysis, both skill types live in the same household, share the same budget constraint, and consumption is joint. This removes the role of exposure to profits and wealth effects on labor supply from the analysis. Huneeus, Kroft, and Lim (2021) study a minimum wage policy in an environment where workers are all similarly exposed to profits, like in Deb, Eeckhout, Patell, and Warren (2020), but firms are monopsonistic and so do not respond to competitors' responses to the minimum wage.

income and hence *a priori* the state with the most to lose from a high federal minimum wage. Under Utilitarian weights, we find that Mississippi would benefit from a $15 minimum wage, and the Mississippi-specific optimal minimum wage is $14.89, very close to the US level. While Mississippi has low average income, which pushes toward a lower minimum wage, it has a relatively larger share of high school educated workers, who support higher minimum wages.

Third, we compute the optimal minimum wage as the Frisch elasticity of labor supply varies. Varying the Frisch elasticity $\pm 50\%$ has little effect on the optimal minimum wage, and preserves our main result that more than 95 percent of the gains from the optimal minimum wage derive from redistribution, not efficiency.

Fourth, our main exercise is inherently long-run, so we consider a short-run exercise where, within each firm, capital is fixed type-by-type and the minimum wage then increased. This is a simplified version of the exercises in the putty-clay models of Aaronson, French, Sorkin, and To (2018) and Sorkin (2015) that study short versus long run elasticities. We theoretically characterize the effect of fixing capital on the equilibrium in our model: the fixity of capital has the largest negative impact on the smallest, least productive firms. Quantitatively, this leads the optimum to decreases by only around one dollar.

Literature. Our paper makes a set of theoretical contributions related to analyzing concentrated markets with strategic interactions in the presence of price controls. This problem of price controls has been studied in stylized cases with symmetric firms in the case of an oligopoly setting (Molho, 1995; Reynolds and Rietzke, 2018), while others have studied capacity constraints and rationing but no price setting (e.g. de Palma, Picard, and Waddell (2007) and Ching, Hayashi, and Wang (2015)). Our characterization is new, and handles firms that are heterogeneous in their productivity. We show that the equilibrium can be stated in terms of *shadow wages* which are *shadow markdowns* relative to marginal revenue products. At the micro level, shadow wages encode multipliers on firm-specific constraints that ration equilibrium labor under a minimum wage. At a minimum wage of $\underline{w} = \$50$, a store hiring only $n = 1$ worker would equate the firm's marginal product of labor to the wage ($mrpl = \underline{w}$), while many more workers would want to work at the store. There exists a shadow wage $\widetilde{w} \ll \$50$ such that only 1 worker would want to work at the firm. And while the firm's markdown ($\mu = w/mrpl$) is equal to one, its shadow markdown ($\widetilde{\mu} = \widetilde{w}/mrpl$), is much less than one. At the macro level, we provide a characterization of how firm shadow markdowns aggregate to wedges that encode the deviations of the entire economy from an efficient benchmark in which there is no labor market power. The first, which we denote $\widetilde{\mu}$, reflects an economically meaningful measure of an aggregate shadow wage markdown, which first narrows as minimum wages increase and then widens, reducing welfare. The second, which we denote $\widetilde{\omega}$, reflects misallocation of factors, which improves for small value of the minimum wage and then dramatically worsens at higher values, reducing welfare. We quantify how each component contributes to a hump-shaped profile of efficiency gains with respect to the minimum wage.

The most closely related to our exercise of constructing a general equilibrium model with a minimum wage is complementary work by Hurst, Kehoe, Pastorino, and Winberry (2021). The emphasis of

Hurst, Kehoe, Pastorino, and Winberry (2021) is redistribution and the positive implications of minimum wages, rather than the normative exercises regarding efficiency that we consider. In particular, Hurst, Kehoe, Pastorino, and Winberry (2021) have an expanded role for worker heterogeneity and homogeneous firms, which allows a richer discussion of redistribution, while our setting has an expanded role for firm heterogeneity as is required of a discussion of efficiency via the three channels discussed earlier. We focus on replicating studies in the empirical minimum wage literature that pertain to these channels. Hurst, Kehoe, Pastorino, and Winberry (2021) compare a $15 minimum wage to other existing transfer policies, and find that policies such as the EITC dominate the minimum wage for the majority of non-college workers. They also consider dynamic capital accumulation in a putty-clay setting, whereas we consider a simple short- and long-run exercise with fixed- and flexible- capital in order to demonstrate the robustness of our results.

That monopsony can rationalize small, and positive, employment responses to minimum wages is in part responsible for the theory's historical development (Card and Krueger, 1994; Boal and Ransom, 1997; Manning, 2003). Whether minimum wages have positive or negative employment effects is a contentious topic. On the one hand, a lengthy review by Neumark and Wascher (2006) concludes that the balance of the empirical literature demonstrates negative employment effects. On the other hand, a summary by Allegretto, Dube, Reich, and Zipperer (2017) concludes that employment effects are small and positive. Our model provides a tent for all parties, by demonstrating circumstances under which employment effects are negative, and under which they are positive. A common theme in our results is that non-linearities warn against extrapolating from small increases in minimum wages to the large changes.

Our paper studies a neoclassical labor market, similar to Cahuc and Laroque (2014), Lee and Saez (2012) among others, while the minimum wage has often been studied in frictional settings. Flinn (2010) and Flinn (2006) document the economic forces that shape the optimal minimum wage in a frictional setting. Flinn and Mullins (2021) study the choice of firms' optimal wage setting strategy in this environment, finding that higher minimum wages lead more firms to prefer renegotiation to wage-posting. Engbom and Moser (2021) equip a Burdett and Mortensen (1998) model for quantitative analysis and study the effects of a large increase in the minimum wage in Brazil on wage inequality, but do not compute or delineate the forces that would shape an optimal minimum wage in that framework.

To provide a starting point for addressing heterogeneity, concentration, strategic interaction and minimum wages in market- and general-equilibrium we abstract from two effects of minimum wages that have been empirically documented. First, recent work has shown that higher minimum wages can be passed through to prices (for example, Renkin, Siegenthaler, and Montialoux, 2021). In our model, a decreasing marginal revenue product of labor is generated by decreasing returns in production, but could be replaced by downward sloping demand under monopolistic competition. In such an environment, the pass-through of minimum wages to prices is one for firms that ration employment. Second, Harasztosi and Lindner (2019) document that firms substitute away from labor and toward capital, increasing purchases of computers and other capital goods. Our model features a unit elasticity of substitution between labor and capital. In general both channels will weaken redistribution toward low wage workers,

and since redistribution is not our focus here, we leave adding these mechanisms to future work. Researchers or policy makers may also wish to compute optimal minimum wages as they vary by market concentration or some other observable. On the latter, some countries feature occupation specific minimum wages (e.g. Australia). Satisfactorily including such policies would require modelling occupational choice for different types of workers, which is beyond the scope of this paper.

Overview. The rest of the paper proceeds as follows. Sections 2 lays out the model environment. Section 3 first simplifies the model to one worker type and provides our characterization of the efficiency effects of a minimum wage in terms of partial equilibrium firm optimality and market Nash equilibrium, and then extends these results to general equilibrium. In Section 4 we calibrate the model. Section 5 validates the model by replicating the design and estimates of empirical studies that point to the channels through which minimum wages can increase efficiency: Derenoncourt, Noelke, Weil, and Taska (2021) and Dustmann, Lindner, Schoenberg, Umkehrer, and vom Berge (2021). Section 6 further replicates studies on the employment effects of minimum wages by labor market concentration and spillovers across the distribution of workers. Section 7 documents the positive implications of the minimum wage for allocations, inequality, the labor share and concentration. Section 8 separates out efficiency from redistribution and in doing so computes the optimal minimum wage. Section 9 provides our robustness exercises. Section 10 concludes.

2 Model

2.1 Environment

Agents. The economy consists of K households and a continuum of firms. Firms are heterogeneous in two dimensions. First, firms inhabit a continuum of local labor markets $j \in [0,1]$, within which there exists an exogenously given finite number of firms indexed $i \in \{1, 2, ... M_j\}$.[7] Second, firms differ in their total factor productivity $z_{ij} \in (0, \infty)$. The *only ex-ante difference* between markets is the number of firms $M_j \in \{1, \ldots, \infty\}$. Households are heterogeneous in four dimensions: their measure π_k, disutility of labor supply $\overline{\varphi}_k$, factor-augmenting productivity ξ_k and share of capital and profit income κ_k.

Goods and technology. Each firm produces one good. These goods are perfect substitutes, so trade in a perfectly competitive market at a price P which we normalize to one. Goods are used for consumption and investment. Firms operate a *value-added* production function that uses labor of each type n_{ijk}.[8] Let $\overline{Z}$ be a common component of productivity across firms. A firm produces y_{ij} units of net-output according to the production function:

$$y_{ij} \;=\; \overline{Z} z_{ij} \sum_{k=1}^{K} \left(\left[\xi_k n_{ijk} \right]^{\gamma} k_{ijk}^{1-\gamma} \right)^{\alpha} \;\;,\;\; \gamma \in (0,1] \;\;,\;\; \alpha > 0$$

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[7]In our short-run exercise, we allow this quantity responds to changes in the minimum wage.
[8]Since aggregating firm-level value-added yields aggregate output (GDP), we abuse terminology and refer to the output of this production function interchangeably in terms of goods and value-added.

This production function has a unit elasticity of substitution between capital and labor for each type, with decreasing returns to scale at the type level if $\alpha < 1$, and is additively separable across types.

Preferences. Each household has a unit measure of workers and preferences over per-capita consumption and labor:

$$\mathcal{U}_k = \sum_{t=0}^{\infty} \beta^t u^k \left(\frac{c_{kt}}{\pi_{kt}}, n_{kt} \right) = \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(c_{kt}/\pi_k \right)^{1-\sigma}}{1-\sigma} - \frac{1}{\widetilde{\varphi}_k^{1/\varphi}} \frac{n_{kt}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right]. \tag{1}$$

The parameter $\widetilde{\varphi}_k$ expresses the disutility of labor supply on a per capita basis which we normalize by an aggregate measure $\overline{\varphi}$: $\widetilde{\varphi}_k = \left(\overline{\varphi}_k / \overline{\varphi} \right) \pi_k^{1+\varphi}$.[9] Labor supply disutility is defined by a nested *CES* expression as in Berger, Herkenhoff, and Mongey (2021) and depends on the allocation of labor across and within labor markets, while consumption goods are perfect substitutes:

$$c_{kt} := \int \sum_{i=1}^{M_j} c_{ijt} dj \quad , \quad n_{kt} := \left[\int_0^1 n_{jkt}^{\frac{\theta+1}{\theta}} dj \right]^{\frac{\theta}{\theta+1}} \quad , \quad n_{jkt} := \left[\sum_{i=1}^{M_j} n_{ijt}^{\frac{\eta+1}{\eta}} \right]^{\frac{\eta}{\eta+1}} \quad , \quad \eta \geq \theta. \tag{2}$$

The elasticities of substitution η and θ are such that the household finds jobs within a market to be closer substitutes than across markets. This implies labor supply to firms is more elastic within than across markets. Our formulation nests (i) perfect competition, where firms within markets face perfectly elastic labor supply curves ($\eta \to \infty$), and (ii) complete monopsony when there is one firm in a labor market and labor supply to the market is perfectly inelastic ($\theta \to 0$).[10]

Budget constraints and endowments. Each household has its own budget constraint. It receives income from supplying labor to all firms in all markets in the economy, capital income and profits, and chooses how much to consume and invest. That is, within household risk associated with labor being rationed due to the minimum wage is insured, but across household risk is not. We discuss this further in Section 9.4. The initial distribution of capital in the economy is a free-parameter of the competitive equilibrium. We denote each households share of the initial capital stock by κ_k and assume they earn an equivalent share of profits:

$$P_t c_{kt} + k_{kt+1} = \int \sum_{i=1}^{M_j} w_{ijkt} n_{ijkt} \, dj + R_t k_{kt} + (1-\delta) k_{kt} + \kappa_k \Pi_t \quad , \quad k_{k0} = \kappa_k K_0. \tag{3}$$

Minimum wages and rationing constraints. Absent a minimum wage, we have the necessary ingredients to specify a competitive equilibrium. To define an equilibrium of the minimum wage economy we

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[9]Our preferences over consumption and labor supply for heterogeneous households with measure π_k is similar to that used in Dyrda, Kaplan, and Rios-Rull (2019), who consider an economy with different ages of households.

[10]In Berger, Herkenhoff, and Mongey (2021) we show how the labor supply curves that obtain under these preferences can also be obtained by individuals making discrete labor supply decisions (i) across an employment / non-employment margin, (ii) across markets, (iii) across firms within markets. If preferences across these three are drawn from a correlated Gumbel distribution, then the parameter φ maps into overall variance of draws, θ into the conditional variance across markets, and η into the conditional variance within markets. This is a straight-forward extension of techniques from the demand system literature: Anderson, De Palma, and Thisse (1987) (single nested) and Verboven (1996) (double nested).

introduce a novel feature to the household information set: a set of *rationing constraints*. This is a set of numbers that the household takes as given and will be determined in equilibrium.[11]

At each firm in each market, the rationing constraint $\bar{n}_{ijkt}$ tells the household the maximum amount of labor it may supply to the firm:

$$n_{ijkt} < \bar{n}_{ijkt}. \tag{4}$$

From the firm side, the wage w_{ijkt} is bounded below by a minimum wage $\underline{w} \geq 0$.

Markets and competition. Households behave competitively: they take rationing constraints and prices as given. Since they produce a homogeneous good and there are infinitely many firms in the economy, competition in the goods market is perfectly competitive. Competition is also competitive in the rental market for capital. Because there are only a finite number of firms in each local labor market, firms behave strategically, competing under Cournot, where they choose the quantity of employment given the quantities chosen by their competitors. Since each labor market is infinitesimal with respect to all other labor markets in the economy, firms take quantities and wages outside of their labor market as given.

2.2 Equilibrium

We present the solution to the full model. Appendix D derives all conditions in a simplified version of the economy, which may be pedagogically useful.[12]

Household problem. Given its initial endowment of capital, each household maximizes its utility (1) subject to its budget constraint (3), and rationing constraints (4). Let the multiplier on the household's budget constraint be μ_{kt}. Let ν_{ijkt} be the multiplier on the firm ij rationing constraint. The rationing constraint binds at high minimum wages, where labor supply to the firm would exceed its labor demand. We scale ν_{ijkt}, and write it as $\nu_{ijkt} = \mu_{kt} w_{ijkt}(1 - p_{ijkt})$. The first two terms are a normalization; the second re-writes the multiplier in terms of p_{ijkt}, such that the multiplier is slack if $p_{ijkt} = 1$ and binding when $p_{ijkt} < 1$. In the household's Lagrangian, the period t terms in the budget constraint and rationing constraint appear as:

$$\cdots + \mu_{kt} \int \sum_{i \in j} w_{ijkt} n_{ijkt} dj + \int \sum_{i \in j} \underbrace{\mu_{kt} w_{ijkt} \left(1 - p_{ijkt}\right)}_{\text{Multiplier: } \nu_{ijkt}} \underbrace{\left[\bar{n}_{ijkt} - n_{ijkt}\right]}_{\text{Rationing constraint}} dj$$

which can be rearranged:
$$\cdots + \mu_{kt} \int \sum_i \underbrace{\left(p_{ijkt} w_{ijkt}\right)}_{:=\widetilde{w}_{ijkt}} n_{ijkt} dj + \mu_{kt} \int \underbrace{\sum_{i \in j} w_{ijkt} \left(1 - p_{ijkt}\right) \bar{n}_{ijkt} dj}_{\text{Terms that do not involve } n_{ijkt}}.$$

The second line provides an intuitive interpretation of the constraint. The household makes decisions *as if* it faces a wage $\widetilde{w}_{ijkt} = p_{ijkt} w_{ijkt}$. When labor is rationed $\widetilde{w}_{ijkt}$ is less than w_{ijkt} by a factor p_{ijkt}. We

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[11]In equilibrium, this constraint will bind when labor supply would exceed labor demand for a particular firm.
[12]The simplified model features one type of labor, GHH preferences and no capital.

call $\widetilde{w}_{ijt}$ the *firm shadow wage*. It encodes the wage the firm pays and the bindingness of the rationing constraint, and is allocative in terms of employment.

With this formulation of the constraint, first order conditions for household k consumption and labor supply to firm-ij imply the following. We also define market- and type- shadow wages:

$$\widetilde{w}_{ijkt} = \left(\frac{n_{ijkt}}{n_{jkt}}\right)^{\frac{1}{\eta}} \left(\frac{n_{jkt}}{n_{kt}}\right)^{\frac{1}{\theta}} \left(-\frac{u_n^k(c_{kt}/\pi_k, n_{kt})}{u_c^k(c_{kt}/\pi_k, n_{kt})}\right) \quad , \quad \widetilde{w}_{jkt} := \left[\sum_{i=1}^{M_j} \widetilde{w}_{ijkt}^{1+\eta}\right]^{\frac{1}{1+\eta}} \quad , \quad \widetilde{w}_{kt} := \left[\int_0^1 \widetilde{w}_{jkt}^{1+\theta} dj\right]^{\frac{1}{1+\theta}}$$

Using only these expressions, we can write the labor supply curve to the firm, inverse labor supply curve and type supply curve as:

$$\underbrace{n_{ijkt} = \left(\frac{\widetilde{w}_{ijkt}}{\widetilde{w}_{jkt}}\right)^{\eta} \left(\frac{\widetilde{w}_{jkt}}{\widetilde{w}_{kt}}\right)^{\theta} n_{kt}}_{\text{Labor supply curve}} \quad , \quad \underbrace{\widetilde{w}_{ijkt} = \left(\frac{n_{ijkt}}{n_{jkt}}\right)^{\frac{1}{\eta}} \left(\frac{n_{jkt}}{n_{kt}}\right)^{\frac{1}{\theta}} \widetilde{w}_{kt}}_{\text{Inverse labor supply curve}} \quad , \quad \underbrace{n_{kt} = \pi_k \widetilde{\varphi}_k \widetilde{w}_{kt}^{\varphi} \left(\frac{c_{kt}}{\pi_{kt}}\right)^{-\sigma\varphi}}_{\text{Household labor supply curve}} \tag{5}$$

Combining these results, we can show that shadow wages aggregate in the following sense:

$$\widetilde{w}_{jkt} n_{jkt} = \sum_{i \in j} \widetilde{w}_{ijkt} n_{ijkt} \quad , \quad \widetilde{w}_{kt} n_{kt} = \int \widetilde{w}_{jkt} n_{jkt} dj. \tag{6}$$

Lastly, if we define the *shadow share* of a firm as the share of total shadow payroll of the market, then:

$$\underbrace{\widetilde{s}_{ijkt} := \frac{\widetilde{w}_{ijkt} n_{ijkt}}{\sum_{i \in j} \widetilde{w}_{ijkt} n_{ijkt}}}_{\text{Shadow share definition}} \quad \underbrace{= \frac{\widetilde{w}_{ijkt} n_{ijkt}}{\widetilde{w}_{jkt} n_{jkt}}}_{\text{From aggregation (6)}} \quad \underbrace{= \left(\frac{\widetilde{w}_{ijkt}}{\widetilde{w}_{jkt}}\right)^{1+\eta} = \left(\frac{n_{ijkt}}{n_{jkt}}\right)^{\frac{1+\eta}{\eta}}}_{\text{Using labor supply system (5)}} \quad \underbrace{= \frac{\partial \log n_{jkt}}{\partial \log n_{ijkt}}}_{\text{Differentiating preferences (2)}}$$

These results show that when we aggregate, labor is allocated across firms according to equilibrium shadow wages, where the market- and type-level shadow wages encode the full distribution of multipliers on rationing constraints that exist due to the minimum wage. We use the final equality below.

This relationship is key to the efficiency properties of the minimum wage. When the minimum wage increases, labor will be rationed at some firms. How unconstrained competitors respond in equilibrium will determine the magnitude of spillover effects. Competitors' responses depend on their residual labor supply curves, and since shadow wages are allocative, the position and elasticity of a firm's labor supply curve is determined by its competitors' shadow wages.

Firm problem. Each firm solves a static optimization problem in which it hires all types of workers and rents capital to maximize profits. It takes as given the rental rate of capital R, type-level shadow wages $\widetilde{w}_{kt}$ and type-level labor supply n_{kt}, which are determined outside of its market. It also takes as given its direct competitors' employment decisions n_{-ijkt}, and understands how its decisions impact market-level labor n_{jkt}:

$$\pi_{ijt} = \max_{n_{ijkt}, k_{ijkt}} \overline{Z} z_{ij} \sum_{k=1}^{K} \left(\left[\xi_k n_{ijkt}\right]^{\gamma} k_{ijkt}^{1-\gamma}\right)^{\alpha} - R \sum_{k=1}^{K} k_{ijkt} - \sum_{k=1}^{K} w_{ijkt} n_{ijkt}$$

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subject to the constraint that it pays at least the minimum wage to each type of worker:

$$w_{ijkt} = \max \left\{ \left(\frac{n_{ijkt}}{n_{jkt}}\right)^{\frac{1}{\eta}} \left(\frac{n_{jkt}}{n_{kt}}\right)^{\frac{1}{\theta}} \widetilde{w}_{kt}, \underline{w} \right\} \quad , \text{ where } \quad n_{jkt} = \left[n_{ijkt}^{\frac{\eta}{1+\eta}} + \sum_{l \neq i} n_{ljkt}^{\frac{\eta}{1+\eta}} \right]^{\frac{1+\eta}{\eta}} \quad \text{for each } k$$

The firm can treat this problem separately type by type. Given the optimal choice of k_{ijkt} for each type, it then chooses n_{ijkt}. This gives the type-k problem:

$$\pi_{ijkt} = \max_{n_{ijkt}} \widetilde{Z} \widetilde{\xi}_k \widetilde{z}_{ij} n_{ijkt}^{\widetilde{\alpha}} - w_{ijkt} n_{ijkt}$$

subject to the same constraints as above, where 'tilde' objects transform the underlying parameters.[13] Firms may be either unconstrained by the minimum wage, or constrained.

Unconstrained firm. Consider a firm that is unconstrained. That is, the solution to the above problem ignoring the constraint, delivers a quantity n_{ijkt} that implies a wage $w_{ijkt} > \underline{w}$. The firm's optimality conditions can be expressed as a wage w_{ijkt} that is a markdown μ_{ijkt} relative to marginal cost:

$$w_{ijkt} = \mu_{ijkt} \times mrpl_{ijkt} \quad , \quad mrpl_{ijkt} = \widetilde{\alpha} \widetilde{Z} \widetilde{\xi}_k \widetilde{z}_{ijkt} n_{ijkt}^{\widetilde{\alpha}-1}.$$

Differentiating the inverse labor supply curve implies that the markdown depends on the firms' inverse labor supply elasticity. With a minimum wage, that labor supply elasticity depends on the firm's *shadow share* of *shadow payrolls* in the market:

$$\mu_{ijkt} = \frac{\varepsilon_{ijkt}}{\varepsilon_{ijkt}+1} \quad , \quad \frac{1}{\varepsilon_{ijkt}} = \frac{\partial \log w_{ijkt}}{\partial \log n_{ijkt}}\bigg|_{\overline{n}_{-ijkt}} = \frac{1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right) \frac{\partial \log n_{jkt}}{\partial \log n_{ijkt}} = \frac{1}{\eta}\widetilde{s}_{ijkt} + \frac{1}{\theta}\left(1 - \widetilde{s}_{ijkt}\right). \tag{7}$$

In the presence of a minimum wage and oligopsonistic competition, even an unconstrained firm's optimal markdown is affected by the minimum wage. Why? Competitors may be paying a high minimum wage, but such competitors' rationing constraints may be severely binding leading to much lower shadow wages. Such competitors employ a high payroll share of the market, but a low shadow share. This implies a higher shadow share for the unconstrained firm, a consequently lower labor supply elasticity, inducing it to choose a wider markdown.

Constrained firms. Constrained firms come in two varieties. Consider the above problem subject to paying the minimum wage, and allow for the fact that firms can always choose to employ less labor than the household supplies:

$$\pi_{ijkt} = \max_{n_{ijkt}} \widetilde{Z} \widetilde{\xi}_k \widetilde{z}_{ij} n_{ijkt}^{\widetilde{\alpha}} - \underline{w} n_{ijkt} \quad , \quad \text{subject to} \quad n_{ijkt} \leq \underbrace{\left(\frac{\underline{w}}{\widetilde{w}_{jkt}}\right)^{\eta} \left(\frac{\widetilde{w}_{jkt}}{\widetilde{w}_{kt}}\right)^{\theta} n_{kt}}_{:=\widehat{n}_{ijkt}}$$

---

[13]In particular, the exponent is $\widetilde{\alpha} = \frac{\gamma\alpha}{1-(1-\gamma)\alpha}$, firm productivity is $\widetilde{z}_{ij} = \left[1 - (1-\gamma)\alpha\right] \left(\frac{(1-\gamma)\alpha}{R}\right)^{\frac{(1-\gamma)\alpha}{1-(1-\gamma)\alpha}}$, aggregate productivity is $\widetilde{Z} = \overline{Z}^{\frac{1}{1-(1-\gamma)\alpha}}$, and worker type productivity is $\widetilde{\xi}_k = \xi_k^{\widetilde{\alpha}}$. We also define output net of capital expenditures $\widetilde{y}_{ijkt} := \widetilde{Z}\widetilde{\xi}_k\widetilde{z}_{ij}n_{ijkt}^{\widetilde{\alpha}}$, and associated aggregate $\widetilde{Y} = \sum_k \int \sum_i \widetilde{y}_{ijkt} dj$. This implies that aggregate output is $Y = \widetilde{Y}/(1-(1-\gamma)\alpha)$.

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Marginal cost for all constrained firms is the minimum wage $\underline{w}$. If absorbing the $\widehat{n}_{ijkt}$ workers that household k would supply at $\underline{w}$ results in a marginal product of labor that exceeds marginal cost, the firm will hire all $\widehat{n}_{ijkt}$ workers. If doing so would reduce marginal product below marginal cost, it chooses $n_{ijkt} < \widehat{n}_{ijkt}$ to equate the two:

$$
n_{ijkt} = \begin{cases} \left(\frac{\underline{w}}{\widetilde{w}_{jkt}}\right)^{\eta} \left(\frac{\widetilde{w}_{jkt}}{\widetilde{w}_{kt}}\right)^{\theta} n_{kt} & , \quad \text{if} \quad \underline{w} \leq \widetilde{\alpha} \widetilde{Z} \widetilde{\xi}_{k} \widetilde{z}_{ijkt} \widehat{n}_{ijkt}^{\widetilde{\alpha}-1} \quad \left[\text{On labor supply curve, } mrpl_{ijkt} > \underline{w}\right] \\ \left(\frac{\widetilde{\alpha} \widetilde{Z} \widetilde{\xi}_{k} \widetilde{z}_{ijkt}}{\underline{w}}\right)^{\frac{1}{1-\widetilde{\alpha}}} & , \quad \text{if} \quad \underline{w} > \widetilde{\alpha} \widetilde{Z} \widetilde{\xi}_{k} \widetilde{z}_{ijkt} \widehat{n}_{ijkt}^{\widetilde{\alpha}-1} \quad \left[\text{On labor demand curve, } mrpl_{ijkt} = \underline{w}\right] \end{cases} \tag{8}
$$

This delivers both the labor demand curves of the firm and the equilibrium rationing constraints that the household takes as given:

$$
\overline{n}_{ijkt} = \left(\frac{\widetilde{\alpha} \widetilde{Z} \widetilde{\xi}_{k} \widetilde{z}_{ijkt}}{\underline{w}}\right)^{\frac{1}{1-\widetilde{\alpha}}}.
$$

Note that $\overline{n}_{ijkt}$ is defined for all firms whether they are constrained or not. It also does not depend on household decisions, only on $\underline{w}$ and technology parameters. Hence it is valid to include these objects as constraints in the household problem.

Equilibrium. Throughout we focus our analysis on an economy that begins in steady-state, such that initial aggregate capital K_0—which is endogenous—is at its steady-state value. At the end of the following section we derive aggregation conditions that deliver a parsimonious set of equilibrium conditions. At this point we leave the definition terse: a *competitive equilibrium* is an allocation of employment n_{ijk}, wages w_{ijk}, shadow wages $\widetilde{w}_{ijk}$, rationing constraints $\overline{n}_{ijk}$, for all types at all firms in all markets, such that labor supply and shadow wages are consistent with household optimality under rationing constraints, and market-by-market firm employment constitutes a Nash equilibrium.

3 A shadow wage characterization of a minimum wage economy

We characterize the comparative statics of the economy as the minimum wage increases from zero, highlighting the efficiency role of minimum wages. We proceed in three steps: partial equilibrium, market equilibrium, and general equilibrium. Since our focus is efficiency we consider a single type of labor and remove capital—which is competitively traded—such that firm output is simply $y_{ij} = z_{ij} n_{ij}^{\alpha}$. We come back to redistribution when we turn to general equilibrium.

3.1 Partial equilibrium

First, consider a single firm with an isoelastic labor supply curve with fixed elasticity ε_i: $n(w_i) = w_i^{\varepsilon_i}$. Depending on the level of $\underline{w}$, the firm can be in one of three regions: Region I (unconstrained), Region II (constrained, on labor supply curve), Region III (constrained, on labor demand curve).
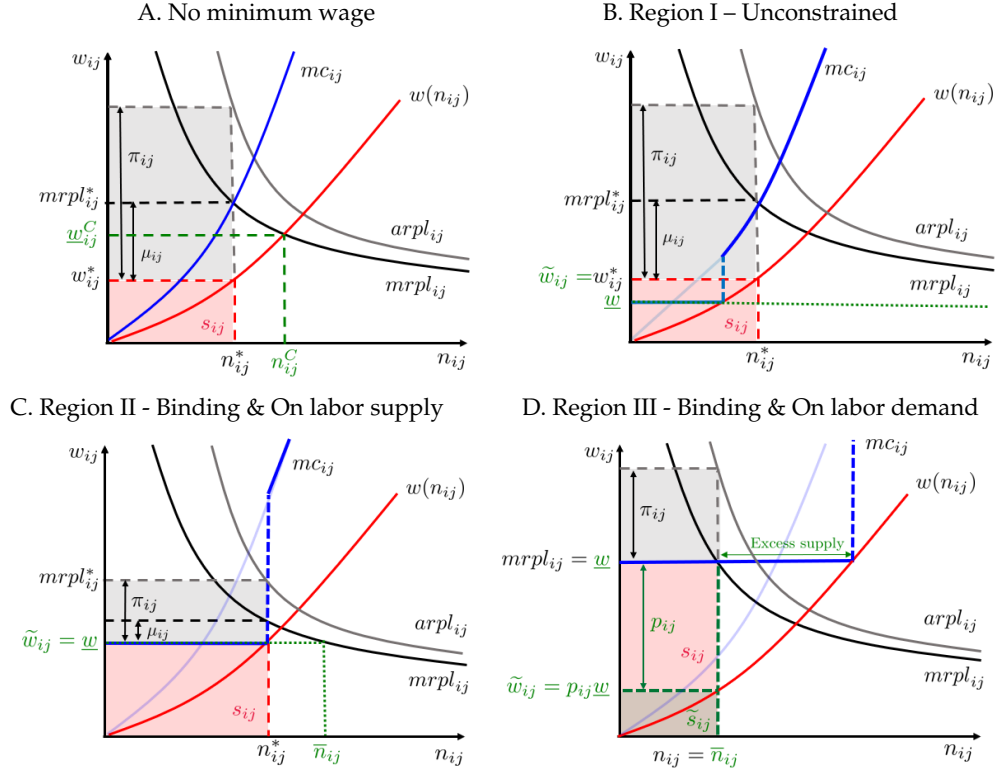
Figure 2: Comparative static increase in the minimum wage: Partial equilibrium

We now define an additional object, which we call the *shadow markdown*:

$$
\widetilde{\mu}_i := \frac{\widetilde{w}_i}{mrpl_i} = \begin{cases} \frac{w_i^*}{\alpha z_i n(w_i^*)^{\alpha-1}} = \frac{\varepsilon_i}{\varepsilon_i+1} & , \quad \text{if in Region I} \\[2ex] \frac{\underline{w}}{\alpha z_i n(\underline{w})^{\alpha-1}} = \frac{\underline{w}^{1+\varepsilon_i(1-\alpha)}}{\alpha z_i} & , \quad \text{if in Region II} \\[2ex] \frac{p_i \underline{w}}{\alpha z_i n(\underline{w})^{\alpha-1}} = p_i & , \quad \text{if in Region III} \end{cases}
$$

If unconstrained, the shadow markdown is given by the standard markdown formula. If constrained in Region II, the shadow markdown narrows as the minimum wage increases and supply of labor to the firm increases, while its marginal product falls. In Region III, the shadow markdown is given exactly by the multiplier on the rationing constraint, p_i.

We can use this new object to characterize efficiency. From these expressions it is clear that at the border of Region II and Region III, the shadow markdown $\widetilde{\mu}_i = 1$ as the constraint only binds weakly. At this point the marginal revenue product and wage are equated, which delivers the efficient allocation of employment to firm i. This observations leads to a useful benchmark to keep in mind as we proceed. With heterogeneous firms a firm-specific minimum wage $\underline{w}_i$ set at this point for each firm, delivers the efficient allocation of employment. Outside of this benchmark, we describe the trade-offs that shape an optimal minimum wage.

We characterize efficiency properties via partial equilibrium comparative statics, illustrated in Figure 2. Starting in Panel B, we study the firms' problem when the minimum wage is not binding. With

decreasing returns the firm faces a downward sloping marginal revenue product of labor ($mrpl_i$) which is below its average revenue product ($arpl_i$). Monopsony power implies that the marginal cost curve mc_i exceeds the wage w_i (read off of the labor supply curve). Increasing labor by one more unit increases costs by w_i plus the monopsonist internalizes the increase in the wage of existing workers $w'(n_i)n_i$. Optimal n_i^* equates marginal revenue and marginal cost resulting in a wage w_i^* that is a markdown μ_i on $mrpl_i$. The firm earns profits due to the markdown and decreasing returns. If the downward sloping marginal revenue product of labor reflected diminishing marginal revenue—as would be the case for a monopolistically competitive producer—this second component of profits would be due to a price markup.

In Panel C, the higher minimum wage has pushed the firm into Region II: the minimum wage now binds, but employment is pinned down by household labor supply. As the minimum wage increases, the firm's shadow markdown shrinks with an elasticity of $1 + \varepsilon_i(1 - \alpha)$. This improves efficiency, as employment is shifted in the direction of equating the marginal product and the wage. Because labor supplied at this wage is less than labor demanded, the rationing constraint $\overline{n}_i$—determined by the intersection of $mrpl_i$ and the minimum wage—is not binding. Since the constraint is slack, $p_i = 1$, and the firms' shadow wage $\widetilde{w}_i$ coincides with the minimum wage $\underline{w}$. Relative to Panel A, profits have shrunk on both margins, with the losses born by the firm.[14]

Increasing the minimum wage further pushes the firm into *Region III* (Panel D). Here the minimum wage is above the competitive wage so, absent the rationing constraint, labor supply would exceed labor demand.[15] Since the rationing constraint binds, $n_i = \overline{n}_i$, employment falls, reducing efficiency. On the other hand, as opposed to an aggregate labor supply-demand framework in which this would be labelled 'unemployment', in our framework less employment at an unproductive firm may lead to reallocation to more productive firms which increases efficiency. Our aggregation results below make clear these trade-offs. Thus, in response to minimum wage increases the dynamics of employment are the opposite of those in Region II. In Region III firms contract in response to higher minimum wages. Their shadow markdown $\widetilde{\mu}_i = p_i$ also widens.

In summary, at the microeconomic level of the firm in partial equilibrium, the introduction of rationing constraints delivers a clear picture of the wages and shadow wages that rationalize equilibrium employment. Shadow markdowns capture inefficiencies due to (i) market power in Region I, (ii) diminished market power in Region II, and (iii) binding rationing constraints in Region III. We now show how these objects characterize the efficiency effects of minimum wages at the market level.

---

[14]In Region II, the marginal cost curve is different from the benchmark economy. The new marginal cost curve is horizontal and equal to $\underline{w}$ until it reaches the labor supply curve. Up to this point workers are paid $\underline{w}$. Marginal cost then jumps, as above the minimum wage, additional hiring requires increasing pay for existing workers. Since marginal cost jumps above the marginal revenue product of labor, profit maximizing employment is on the labor supply curve at $\underline{w}$.

[15]Note that *Region III* does not exist with constant returns to scale. With constant returns to scale the competitive wage is equal to $mrpl_i$ which is a constant. Therefore as $\underline{w}$ increases past the competitive wage, the firm exits.

3.2   Market equilibrium

In partial equilibrium, the only channel through which minimum wages improve efficiency is via moving firms toward their competitive wage. We now consider the same comparative static in *market equilibrium* which delivers two additional channels: spillovers and reallocation.

Comparative static.   Figure 3 considers the same comparative static but in a market equilibrium with three firms. All aggregates are held fixed. On the *x*-axis we plot the minimum wage relative to the unconstrained optimal wage of the low productivity firm: $\underline{w}/w_L^*$. Panel A plots how the low productivity firm transits through the three regions described in Figure 2. Its wage increases one-for-one across Region II and Region III as the minimum wage increases (Panel C), but its shadow wage and shadow share decline as it moves back along its labor demand curve in Region III (Panel B). As such it expands (Panel D), doubling in size, before it shrinks as the minimum wage increases further.

   The behavior of the medium and high productivity firms reflect the Nash equilibrium at the market level. Absent a minimum wage, these firms are larger, and pay higher wages. Since they have higher shares of the market, they face less elastic labor supply and hence these wages represent wider markdowns on their marginal product of labor.

Spillovers.   As the low productivity firm's wage increases in Region II, its share increases, which lowers the share of the unconstrained firms. With lower market shares due to stiffer competition, the unconstrained firms' equilibrium markdowns narrow which increases their wages. This has positive implications for efficiency. Not only is the markdown of the constrained firm narrowing in Region II, but the equilibrium markdowns of its competitors in Region I are also narrowing, expanding each toward their efficient level of employment. The elasticity of competitors' response to other firms' wages is therefore a key determinant of the efficiency properties of minimum wages. We replicate observed spillovers in Derenoncourt, Noelke, Weil, and Taska (2021) and Staiger, Spetz, and Phibbs (2010) in order to discipline this channel.

Reallocation.   As the low productivity firm enters Region II the fact that competitors respond less than one-for-one to its wage implies that employment is reallocated in its direction. This reduces efficiency, as more labor is employed at a less productive firm. However, as the firm enters Region III its workers are reallocated to the medium productivity firm, which leads to positive reallocation effects. The reallocation of employment from lower to higher productivity firms is therefore also a key determinant of the efficiency properties of minimum wages. We replicate observed reallocation in Dustmann, Lindner, Schoenberg, Umkehrer, and vom Berge (2021) in order to discipline this channel.

Aggregation.   Aggregating equilibrium conditions across firms, delivers a precise representation of how all three channels affect efficiency at the market level. We have two sets of conditions involving quantities and shadow wages:
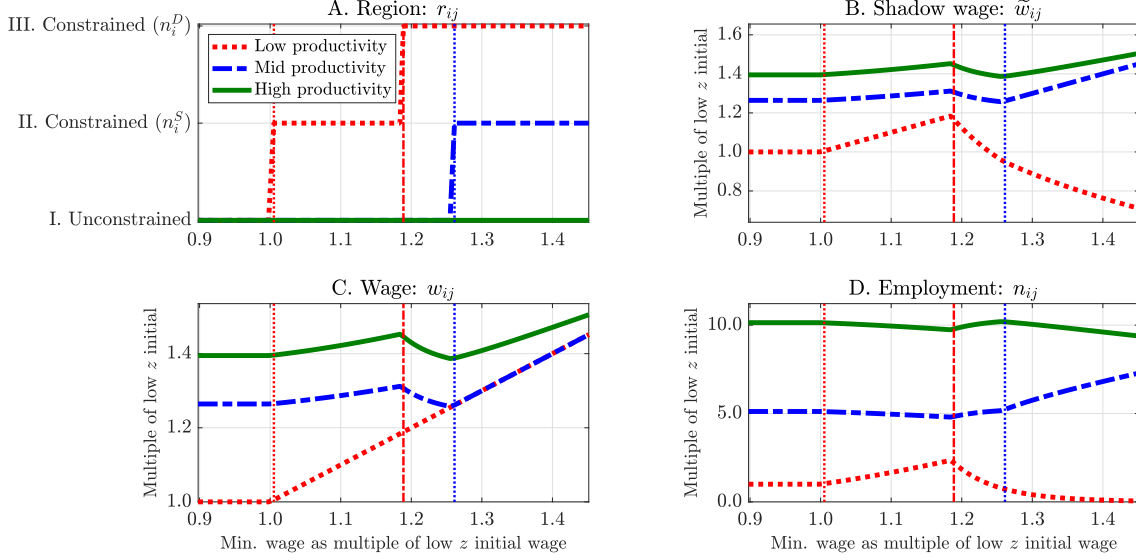
Figure 3: Comparative static increase in the minimum wage: Market equilibrium

<u>Notes:</u> All aggregates are held fixed and we plot outcomes for a market with three firms as the minimum wage is increased. The x-axis plots the minimum wage relative to unconstrained optimal wage of the low productivity firm: $\underline{w}/w_L^*$. We increase the minimum wage from 10 percent below to 50 percent above this wage. Panel *A* plots the regions corresponding to Figure 2. Panels *B* and *C* plots the shadow wage $\widetilde{w}_{ij} = p_{ij}w_{ij}$, and actual wage w_{ij}. Panel *D* plots employment relative to unconstrained employment at the low productivity firm.

1. **Firm conditions.** We have the following sets of conditions:

$$y_{ij} = z_{ij}n_{ij}^\alpha \quad , \quad n_{ij} = \left(\frac{\widetilde{w}_{ij}}{\widetilde{w}_j}\right)n_j \quad , \quad \widetilde{w}_{ij} = \widetilde{\mu}_{ij}mrpl_{ij} \quad , \quad mrpl_{ij} = \alpha z_{ij}n_{ij}^{\alpha-1}$$

2. **Market conditions.** Let y_j denote market-level output, then we have

$$\widetilde{w}_j = \left[\sum_{i \in j} \widetilde{w}_{ij}^{1+\eta}\right]^{\frac{1}{1+\eta}} \quad , \quad y_j = \sum_{i \in j} y_{ij}$$

Combining these obtains the following characterization of market-level output, wages and employment:

Lemma 1 - Market equilibrium *Market-level output y_j, employment disutility n_j and shadow wage $\widetilde{w}_j$ satisfy:*

$$\underbrace{y_j = \omega_j z_j n_j^\alpha}_{\text{1. Output}} \quad , \quad \underbrace{\widetilde{w}_j = \widetilde{\mu}_j \alpha z_j n_j^{\alpha-1}}_{\text{2. Shadow wage}} \quad , \quad \underbrace{\widetilde{n}_j = \left(\frac{\widetilde{w}_j}{\widetilde{w}}\right)^\theta n}_{\text{3. Labor supply}}$$

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where the variables z_j, $\widetilde{\mu}_j$ and ω_j are defined by the following expressions:

$$\underbrace{z_j := \left[\sum_{i \in j} z_{ij}^{\frac{1+\eta}{1+\eta(1-\alpha)}}\right]^{\frac{1+\eta(1-\alpha)}{1+\eta}}}_{\text{1. Market productivity}} \quad , \quad \underbrace{\widetilde{\mu}_j := \left[\sum_{i \in j} \left(\frac{z_{ij}}{z_j}\right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} \widetilde{\mu}_{ij}^{\frac{1+\eta}{1+\eta(1-\alpha)}}\right]^{\frac{1+\eta(1-\alpha)}{1+\eta}}}_{\text{2. Market shadow markdown}} \quad , \quad \underbrace{\omega_j := \sum_{i \in j} \left(\frac{z_{ij}}{z_j}\right)^{\frac{1+\eta}{1+\eta(1-\alpha)}} \left(\frac{\widetilde{\mu}_{ij}}{\widetilde{\mu}_j}\right)^{\frac{\eta\alpha}{1+\eta(1-\alpha)}}}_{\text{3. Market misallocation}}.$$

Given aggregate labor supply conditions $(n, \widetilde{w})$, market-level labor supply solves the labor demand and supply conditions on the first line, which then determines output.

The two wedges $(\widetilde{\mu}_j, \omega_j)$ encode the efficiency effects of minimum wages via the shadow wages of firms. In an efficient economy $\widetilde{\mu}_{ij} = 1$, so $(\widetilde{\mu}_j, \omega_j) = (1, 1)$. The wedges $\widetilde{\mu}_j$ and ω_j are productivity weighted averages of firm level outcomes. The first, aggregates shadow markdowns and, when narrower ($\uparrow \widetilde{\mu}_j$), expands market labor demand. In a minimum wage economy this occurs for two reasons: markdowns of unconstrained firms narrow in response to Region I competitors, or are directly narrowed by the minimum wage in Region II. The aggregate shadow markdown widens ($\downarrow \widetilde{\mu}_j$), contracting market labor demand for two reasons: unconstrained competitors gain more market power and widen markdowns, or rationing constraints increasingly bind on firms in Region III which widen shadow markdowns.

The second represents misallocation. Given n_j, a worse allocation of employment across firms generates a lower ω_j which represents a direct output loss. If all shadow markdowns were identical, this term would equal 1, but with heterogeneity this term is lower when z_{ij} is negatively correlated with $\widetilde{\mu}_{ij}$. This is the case across firms absent a minimum wage: higher productivity firms have wider markdowns, which reduces their employment, leading to misallocation. With a minimum wage, however, *low productivity firms* in Region III may have *low shadow markdowns*, which can lead to improvements in misallocation and an increase in ω_j.

Summary. The utility of Lemma 1 is it allows us to focus on the role of the minimum wage in shaping ω_j and $\widetilde{\mu}_j$, which fully account for the distribution of binding rationing constraints across firms in a market. Figure 4 shows how these change in our example market from Figure 3. There are two key take-aways. First, productivity weighting implies that the market shadow-markdown is shaped by the endogenous response of unconstrained firms (Panel A). The model has a strong role for spillovers in shaping efficiency. Second, misallocation has ambiguous effects (Panel B). Misallocation worsens with the expansion of the low productivity firm as its wage increases, improves as it slowly gets pushed out of business, and then worsens again as employment is reallocated from the high to the medium productivity firm. For a single minimum wage, our general equilibrium framework appropriately aggregates markets that are distributed across this spectrum.

3.3 General equilibrium

General equilibrium requires that we add back in capital and multiple types of workers. With capital the above characterization of the market equilibrium holds under a production function $\widetilde{y}_{ijk} = \widetilde{Z}\widetilde{\xi}_k \widetilde{z}_{ij} n_{ij}^{\alpha}$,
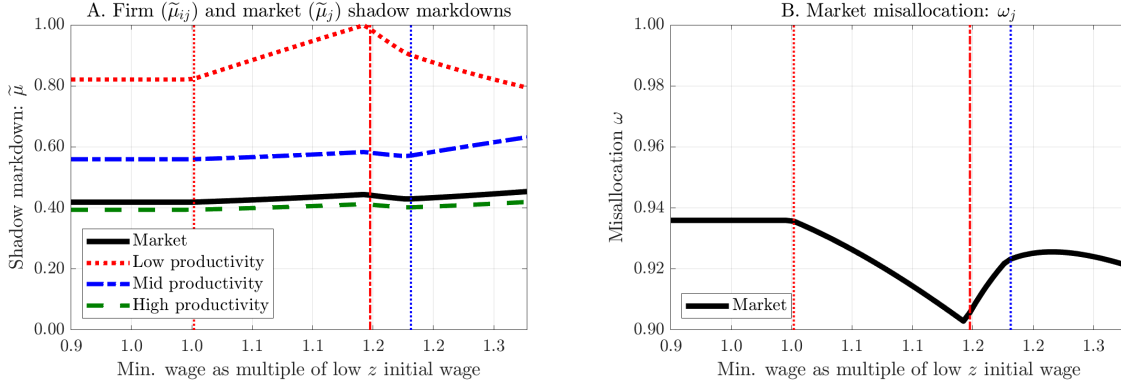
Figure 4: Market level outcomes: Shadow markdown and misallocation

<u>Notes:</u> All aggregates are held fixed and we plot outcomes for a market with three firms as the minimum wage is increased. The x-axis plots the minimum wage relative to unconstrained optimal wage of the low productivity firm: $\underline{w}/w_L^*$. We increase the minimum wage from 10 percent below to 50 percent above this wage. Panel A plots the market markdown $\widetilde{\mu}_j$ from Lemma 1. Panel B plots the market markdown $\widetilde{\omega}_j$ from Lemma 1. Moving from left to right, the vertical dotted lines correspond to the low and then medium productivity firms move from Region I to Region II. The vertical dashed line corresponds to the low productivity firm moving from Region II to Region III.

where the 'tilde' objects, defined earlier, account for the optimal choice of capital type-by-type. We can then aggregate the market-level expressions from Lemma 1 across markets to type-level expressions. These are important, as they will later allow us to decompose the efficiency channels of minimum wages—encoded in shadow markdowns and misallocation terms—in our calibrated model. In Appendix D we provide a full derivation of all mathematical expressions in the text.

1. **Macro to micro** - Suppose the following are determined by market equilibria for all types of workers k and in all markets j, where firms take aggregate quantities $\{C_k, N_k, Y_k, K_k\}_{k=1}^K$ and prices $\{\widetilde{W}_k\}_{k=1}^K$, R as given

$$\underbrace{\widetilde{z}_k := \left[\int \widetilde{z}_{jk}^{\frac{1+\theta}{1+\theta(1-\widetilde{\alpha})}} dj \right]^{\frac{1+\theta(1-\widetilde{\alpha})}{1+\theta}}}_{\text{1. Type productivity}}, \underbrace{\widetilde{\mu}_k := \left[\int \left(\frac{\widetilde{z}_{jk}}{\widetilde{z}_k} \right)^{\frac{1+\theta}{1+\theta(1-\widetilde{\alpha})}} \widetilde{\mu}_{jk}^{\frac{1+\theta}{1+\theta(1-\widetilde{\alpha})}} \right]^{\frac{1+\eta(1-\alpha)}{1+\eta}}}_{\text{2. Type shadow markdown}}, \underbrace{\omega_k := \int \left(\frac{\widetilde{z}_{jk}}{\widetilde{z}_k} \right)^{\frac{1+\theta}{1+\theta(1-\widetilde{\alpha})}} \left(\frac{\widetilde{\mu}_{jk}}{\widetilde{\mu}_k} \right)^{\frac{\theta\alpha}{1+\theta(1-\widetilde{\alpha})}} \omega_{jk}}_{\text{3. Type misallocation}}.$$

2. **Micro to macro** - For each k, under $\{\widetilde{z}_k, \widetilde{\mu}_k, \omega_k\}_{k=1}^K$, aggregate quantities $\{C_k, N_k, Y_k, K_k\}_{k=1}^K$ and prices $\{\widetilde{W}_k\}_{k=1}^K$, R satisfy:

$$\textit{Output:} \quad \widetilde{Y}_k = \omega_k \widetilde{Z} \widetilde{\xi}_k \widetilde{z}_k N_k^{\widetilde{\alpha}} \quad , \quad Y_k = \frac{1}{1 - (1-\gamma)\alpha} \widetilde{Y}_k$$

$$\textit{Capital supply and demand:} \quad 1 = \beta(R + (1-\delta)) \quad , \quad R = \alpha(1-\gamma) \frac{Y_k}{K_k} \quad , \quad K_k = \kappa_k K$$

$$\textit{Labor supply and demand:} \quad N_k = \pi_k \widetilde{\varphi}_k \left(\frac{\widetilde{W}_k}{P} \right)^{\varphi} C_k^{-\sigma\varphi} \quad , \quad \widetilde{W}_k = \widetilde{\mu}_k \widetilde{\alpha} \widetilde{Z} \widetilde{\xi}_k \widetilde{z}_k N_k^{\widetilde{\alpha}-1}$$

$$\textit{Budget constraint:} \quad C_k + \delta K_k = \int \sum_{i \in k} w_{ijk} n_{ijk} \, dj + R K_k + \kappa_k \Pi$$

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where aggregate profits are consistent:

$$\Pi = \sum_k Y_k - \int \sum_{i \in k} w_{ijk} n_{ijk} \, dj - R \sum_k K_k$$

These conditions yield three results. First, they show that the market-level lesson of focusing on the shadow markdown and misallocation carries over to the aggregate economy, when these wedges are appropriately aggregated. Second, they provide an algorithm to solve the competitive equilibrium, given $\{z_k, \widetilde{\mu}_k, \omega_k\}_{k=1}^K$, which later will allow us understand the role of different wedges in aggregate welfare. Third, they show how the shadow wages that we have constructed are allocative for quantities. Household labor supply N_k is pinned down by the shadow wage $\widetilde{W}_k$ that the household faces.

Having laid out the channels through which minimum wages impact efficiency, we now turn to the problem that the government solves in choosing an optimal minimum wage, both with and without lump sum taxes.

3.4 Government problem

To separate out the redistribution and efficiency effects of a minimum wage, we consider the problem of a government with social welfare weights $\{\psi_k\}_{k=1}^k$. The government faces prices determined by the imperfectly competitive labor market where firms are subject to the minimum wage. The government is given access to lump-sum taxes $\{T_k\}_{k=1}^K$, with the restriction that total lump sum taxes add to zero. We take the standard approach of solving for the optimal allocation, then the transfers that implement it.

Problem. The government chooses allocations of consumption and labor to maximize

$$\mathcal{U} = \sum_k \psi_k \sum_{t=0}^{\infty} u^k \left(\frac{C_{kt}}{\pi_k}, N_{kt} \right) = \sum_k \psi_k \sum_{t=0}^{\infty} \left[\frac{C_{kt}^{1-\sigma}}{1-\sigma} - \frac{1}{\widetilde{\varphi}_k^{1/\varphi}} \frac{N_{kt}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right].$$

We can define the following aggregate consumption and labor indices, and use these to write social welfare as follows:

$$\mathcal{U} = \sum_{t=0}^{\infty} \beta^t \left[\frac{\mathcal{C}_t^{1-\sigma}}{1-\sigma} - \frac{1}{\widetilde{\varphi}^{1/\varphi}} \frac{\mathcal{N}_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right] \quad , \quad \mathcal{C}_t := \left[\sum_k \psi_k \left(\frac{C_{kt}}{\pi_k} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \quad , \quad \mathcal{N}_t := \left[\sum_k \frac{\psi_k}{\widetilde{\varphi}_k^{1/\varphi}} N_{kt}^{\frac{1+\varphi}{\varphi}} \right]^{\frac{\varphi}{1+\varphi}}$$

This problem can be solved subject to an aggregate budget constraint, and then implemented using lump sum taxes. The government also takes labor rationing constraints into account. Under this approach, the government is endowed with K_0 units of capital, and maximizes social welfare subject to:

$$\sum_k C_{kt} + K_{t+1} = \sum_k \int \sum_{i \in j} w_{ijkt} n_{ijkt} \, dj + R_t K_t + (1-\delta) K_t + \Pi_t \quad , \quad n_{ijkt} \geq \bar{n}_{ijkt} \tag{9}$$

Optimization delivers an identical set of labor supply conditions to firms as the competitive equilibrium given in (5). These involve $\{\widetilde{W}_k, N_k, C_k\}$, which were determined by each household's labor supply

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curve and budget constraint. In the government's allocation problem these are instead determined by the government's optimality conditions:

$$C_{kt} = \pi_k \left(\frac{\psi_k}{\pi_k}\right)^{\frac{1}{\sigma}} \left(\frac{1}{\mathcal{P}_t}\right)^{-\frac{1}{\sigma}} \mathcal{C}_t \quad , \quad N_{kt} = \pi_k \widetilde{\varphi}_k \left(\frac{\psi_k}{\pi_k}\right)^{-\varphi} \left(\frac{\widetilde{W}_{kt}}{\widetilde{\mathcal{W}}_t}\right)^{\varphi} \mathcal{N}_t \quad , \quad \mathcal{N}_t = \overline{\varphi} \left(\frac{\widetilde{\mathcal{W}}_t}{\mathcal{P}_t}\right)^{\varphi} \mathcal{C}_t^{-\sigma\varphi}. \tag{10}$$

Higher social welfare weights relative to population shares entail a higher share of consumption and less labor supply, where the latter is offset if relative wages of the type are higher, or disutility of work is lower (higher $\widetilde{\varphi}_k$). The aggregate shadow price $\mathcal{P}_t$—which is such that $\mathcal{P}_t \mathcal{C}_t = \sum_k C_{kt}$—and aggregate shadow wage $\widetilde{\mathcal{W}}_t$ indexes are given by

$$\mathcal{P}_t = \left[\sum_k \psi_k^{\frac{1}{\sigma}} \pi_k^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \quad , \quad \widetilde{\mathcal{W}}_t = \left[\sum_k \widetilde{\varphi}_k \pi_k \left(\frac{\psi_k}{\pi_k}\right)^{-\varphi} \widetilde{W}_{kt}^{1+\varphi}\right]^{\frac{1}{1+\varphi}}.$$

The derivation of all of these results are given in detail in Appendix D.

Implementation. The planner can implement this allocation in a competitive economy in which households are endowed with shares of capital and profits, by choosing lump sum transfers T_k. These can be read off of each household's budget constraint under equilibrium prices and the government's desired allocation:

$$T_k = \int \sum_{i \in j} w_{ijk} n_{ijk} \, dj + (R + \delta) \kappa_k K + \kappa_k \Pi - C_k. \tag{11}$$

To see that this implements the government's solution, observe that combining conditions in (10) yields the decentralized household labor supply curves, and that the government's steady-state Euler equation coincides with each household's in the competitive equilibrium. Since taxes are lump-sum, their presence does not distort these conditions.[16] Finally, summing budget constraints (11) obtains the planner's budget constraint (9), and hence transfers sum to zero.

Aggregates. To solve the government's problem still requires the determination of aggregates $\mathcal{C}$, $\widetilde{\mathcal{W}}$, and $\mathcal{N}$. Under a given set of social welfare weights, market equilibria can be aggregated to obtain shadow markdowns for all types: $\{\widetilde{\mu}_k, \omega_k\}_{k=1}^K$. These can be further aggregated, where $\phi_k = \pi_k^{1+\varphi} \widetilde{\varphi}_k \psi_k^{-\varphi}$:

$$\widetilde{z} = \left[\sum_k (\varsigma_k \widetilde{z}_k)^{\frac{1+\varphi}{1+\varphi(1-\alpha)}} \phi_k^{\frac{\alpha}{1+\varphi(1-\alpha)}}\right]^{\frac{1+\varphi(1-\alpha)}{1+\varphi}} , \quad \widetilde{\mu} = \left[\sum_k \left(\frac{\varsigma_k \widetilde{z}_k}{\widetilde{z}}\right)^{\frac{1+\varphi}{1+\varphi(1-\alpha)}} \phi_k^{\frac{\alpha}{1+\varphi(1-\alpha)}} \widetilde{\mu}_k^{\frac{1+\varphi}{1+\varphi(1-\alpha)}}\right]^{\frac{1+\varphi(1-\alpha)}{1+\varphi}} , \quad \omega = \sum_k \left(\frac{\varsigma_k \widetilde{z}_k}{\widetilde{z}}\right)^{\frac{1+\varphi}{1+\varphi(1-\alpha)}} \phi_k^{\frac{\alpha}{1+\varphi(1-\alpha)}} \left(\frac{\widetilde{\mu}_k}{\widetilde{\mu}}\right)^{\frac{\varphi\alpha}{1+\varphi(1-\alpha)}} \omega_k$$

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[16]As is standard, comparing households' and the planner's first order conditions for consumption reveal that the social welfare weights map into multipliers on households budget constraints, which are constant in steady-state. Denote these multipliers ν_k. Normalize $\psi_1 = 1$, then $\psi_k = \nu_1/\nu_k$. Hence, starting with some social welfare weights, the implied allocation can be decentralized by budget-neutral lump-sum taxes. Lump sum transfers tighten and loosen budget constraints, so can be chosen to align multipliers with the planner's social welfare weights.

Weights now account for productivity shifters ξ_k, household measures π_k, labor disutility $\widetilde{\varphi}_k$ and social welfare weights ψ_k. Given $\{\mathcal{P}, \widetilde{z}, \widetilde{\mu}, \omega\}$, the following can be solved for $\mathcal{C}, \widetilde{\mathcal{W}}$, and $\mathcal{N}$ in closed form:

$$\underbrace{Y = \frac{\omega \widetilde{Z} \widetilde{z} \mathcal{N}^{\widetilde{\alpha}}}{1 - (1 - \gamma)\alpha}}_{\text{Output}} \quad , \quad \underbrace{\mathcal{P}\mathcal{C} = Y - \delta K}_{\text{Resource constraint}} \quad , \quad \underbrace{\widetilde{\mathcal{W}} = \widetilde{\mu}\widetilde{\alpha} \widetilde{Z} \widetilde{z} \mathcal{N}^{\widetilde{\alpha}-1}}_{\text{Labor demand}} \quad , \quad \underbrace{\mathcal{N} = \overline{\varphi}\left(\frac{\widetilde{\mathcal{W}}}{\mathcal{P}}\right)^{\varphi} \mathcal{C}^{-\varphi\sigma}}_{\text{Labor supply}} \quad (12)$$

To allocate these aggregates across households, we use the government's first order conditions (10).

Negishi weights. Our baseline calibration of the model is a competitive equilibrium with zero lump sum taxes. This yields an allocation of labor, consumption and capital. Note that there exists a vector of social welfare weights $\{\psi_k^*\}_{k=1}^K$ such that a government with these weights would choose the same allocation, also with zero lump sum taxes. As is standard, we refer to this vector of social welfare weights as the *Negishi weights*. Computing the Negishi weights associated with the benchmark competitive equilibrium is a key step in our welfare exercise. Optimal policy under this benchmark can be compared to optimal policy under alternative weights, such as Utilitarian weights. Incidentally, we also exploit the associated *Negishi algorithm* to make feasible the computation of the competitive equilibrium with K types.[17]

4 Calibration

We now calibrate the economy to US data, using a combination of the Census Longitudinal Business Database (LBD, using moments released by Berger, Herkenhoff, and Mongey, 2021), BLS Current Population Survey (CPS), and Survey of Consumer Finances (SCF). Our LBD data is from 2014, which is the latest data available to us in the Census. We use pre-Covid 2019 data from the CPS and SCF. Parameters and moments are summarized in Tables 1 and 2. There are three sets of parameters we describe below.

Households. We consider four households types, $K = 4$, and note that our approach could be extended to much richer heterogeneity given suitable data. First, we split households by education, which is available in the CPS and SCF. The first two are workers that have not completed high school (NHS) and workers that have a high school diploma but not completed college (HS). The remainder represent workers that have completed college. Second, we use the SCF to split college households into two groups: those that receive the majority of their income from labor income (which we term *Workers*), and those that earn the majority of their income from capital income (*Owners*). We measure capital income

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[17]In particular, we can guess a set of Negishi weights, normalizing $\psi_1^* = 1$. First, we solve market equilibria, to obtain $\{\widetilde{\mu}_k, \omega_k\}_{k=1}^K$. Using the guessed Negishi weights we can compute $\widetilde{z}, \widetilde{\mu}, \omega, \mathcal{P}$ from the above expressions, and then use these to solve for $Y, \mathcal{W}, \mathcal{C}, \mathcal{N}$ using equations (12). Using the planner's first order conditions (10), we can allocate $\mathcal{C}$ among households, and hence compute implied household consumption C_k. We can also compute firm wages and employment. Following the tradition of the Negishi algorithm, we then compute the implied residual in the household's budget constraint—under $T_k = 0$— and update our guess of $\{\psi_k^*\}_{k\neq 1}$, until this residual is zero. We lower ψ_k^* for households with a deficit, and increase ψ_k^* for households with a surplus.

as interest and dividend income, business and farm income, and realized capital gains.[18] By this metric, we allocate less than a fifth of college households to owners. When aggregated, non-college workers' capital income is not zero, but it is small, and hence our assumption that only college households are owners is reasonable.[19] Table 2B reports the implied population shares $\{\pi_k\}_{k=1}^K$; notably only 6 percent of households are owners.

4.1 Externally calibrated

The first set of parameters are those we externally calibrate (Table 1A). The discount rate β implies a risk free rate of 4 percent annually. The depreciation rate δ is 10 percent. We set the preference parameters governing curvature in marginal utility of consumption σ to 1.05, so approximately log, and the Frisch elasticity of aggregate labor supply φ to 0.62. In Appendix C, we contribute a simple method that combines recent evidence to infer a data-consistent φ for any σ.[20] In Section 9.1 we repeat our main minimum wage counterfactuals under alternative values of these parameters, recalibrating all remaining parameters in each case.

The distribution of firms across markets matches LBD data. We treat markets as in Berger, Herkenhoff, and Mongey (2021), and define a market as a combination of a NAICS 3-digit industry and a commuting zone. We define a firm in the data as the collection of all establishments with the same *firmid* in the commuting zone and compute total employment and average worker wage across these establishments. The distribution of firms across markets $G(M_j)$ is comprised of a mass point of 0.09 at $M_j = 1$ and a generalized Pareto distribution for $M_j > 1$. The tail, shape and location parameters chosen to best match the mean (113.10), standard deviation (619.0) and skewness (26.1) of the empirical distribution of M_j, which we measure in the LBD. We solve the model with $J = 5,000$ markets.

Preference parameters (θ, η) are taken from Berger, Herkenhoff, and Mongey (2021). With $M_j < \infty$, firms exercise market power in their local labor markets. If $\eta > \theta$, then labor supply is more elastic within- rather than across- markets, and firms with a larger market share will be less responsive to shocks. BHM uses the relative response of large and small market share firms to changes in state corporate taxes—which by distorting capital decisions, are shocks to the marginal revenue product of labor

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[18]We also consider an alternative approach, where we determine capital income as a residual in the household budget constraint. By this approach capital income is defined as total income minus labor income and transfers. This yields a very similar split of households.

[19]Two different cuts of the data support this. First, of the households that earn more than half of their income from capital income, 70 percent are college households, 25 percent are high-school, and only 5 percent are non-high school. Second, the share of college households that earn more than half of their income from capital income is 17.3 percent, while this is true for only 6.7 percent for high-school households and 3.7 percent for non-high-school households. As a robustness, we ran a calibration where we increased the number of business owners by 30% (pooling all business owners together), assuming for simplicity that these additional business owners come the college worker households so we do not need to add additional household types. This gives an upper bound to the possible effects for the optimal minimum wage (Section 8) because we are moving mass from the group with the highest desired minimum wage (college workers) to the those with the lowest (business owners). Doing lowers the optimal minimum wage under utilitarian social welfare weights by less than 1% from $15.12 to $14.99. Thus quantitatively, including these additional business owners makes little difference.

[20]In Appendix C we show how one can fix σ and then use recent evidence to infer φ by combining (i) estimates on marginal propensities to consume and earn from Golosov, Graber, Mogstad, and Novgorodsky (2021), (ii) data on the average propensity to consume from the BLS, and (iii) estimates of the progressivity of labor income taxes from Heathcote, Storesletten, and Violante (2020).

Parameters		Value	Moment and source	Value
A. External				
Discount rate	β	0.962	Risk free rate	0.04
Depreciation rate	δ	0.10		
Coefficient of risk aversion	σ	1.05	Fixed, approximately log	
Aggregate Frisch elasticity	φ	0.62	Consistent with recent evidence given σ (see Appendix C)	
Number of markets	J	5,000	Normalization	
Distribution of number of firms	$G(M_j)$		Mean, variance, skewness of distribution of M_j (LBD)	
Pareto with mass point at $M_j = 1$			9 percent of markets have 1 firm	
Across market substitutability	θ	0.42	Estimate from BHM (2021)	
Within market substitutability	η	10.85	Estimate from BHM (2021)	
B. Internally estimated				
Productivity dispersion	$\text{Std}[\log z_{ij}]$	0.268	Payroll weighted $\mathbb{E}[HHI^{wn}]$ (LBD)	0.11
Decreasing returns in production	α	0.940	Labor share	0.57
Labor exponent in production	γ	0.808	Capital share	0.18

Table 1: Calibration of common parameters

A. Aggregate parameters						
Parameters		**Values**		**Moments**		**Values**
Labor disutility shifter	$\overline{\varphi}$	2.61×10^6		Average firm size (LBD)		22.8*
Productivity shifter	$\widetilde{Z}$	17.63		Binding at \$15 (CPS, %)		29.3*

B. Household parameters										
Parameters		**Non-HS**	**HS**	**Coll**	**Own**	**Moments**	**Non-HS**	**HS**	**Coll**	**Own**
Relative population (%)	$\pi_k / \sum \pi_k$	12.4	52.8	28.8	6.0	Population shares (CPS, %)	12.4*	52.8*	28.8*	6.0*
Relative disutility labor supply	$(1/\overline{\varphi}_k)^\varphi$	3.09	0.71	1	0.45	Share of agg. labor income (CPS, %)	2.2*	55.3*	35.1*	7.4*
Relative productivity	ξ_k	0.25	0.55	1	0.85	Relative ave. earnings per hour (CPS, %)*	41.7*	60.1*	100	
Capital income share (%)	κ_k	0.09	3.88	4.14	91.89	Ratio of h'hold capital/labor inc. (SCF)	0.022*	0.037*	0.062*	6.568

C. Additional statistics										
Statistics		**Non-HS**	**HS**	**Coll**	**Own**	**Statistics**	**Non-HS**	**HS**	**Coll**	**Own**
Implied Negishi weights (%)	$\psi_k^* / \sum \psi_k^*$	1.3	36.0	23.7	39.0	Ratio of h'hold capital/labor inc. (model)	0.022	0.037	0.062	7.121
Binding at \$15 (model, %)		82.7	35.6	9.1		Consumption share (model, %)	1.5	37.6	61.0	
Binding at \$15 (CPS, %)		68.7	38.1	11.1		Consumption share (BLS, %)	2.7	38.2	59.1	

Table 2: Calibration of constants and additional statistics

Notes: Data with an ∗ indicates that the model matches the data exactly, by a direct inversion of data to model parameters. In both the CPS and BLS consumption data, we cannot split college households into owners and non-owners as we do in the SCF. Hence for *Binding at \$15*, *Consumption share*, *Relative average earnings* we consider all college educated households.

of only *C-corp* firms—to identify θ and η. The generalization to multiple types of workers does not interfere with this exercise and hence we use the same parameter values: $(\theta, \eta) = (0.42, 10.85)$.

4.2 Shifters

The second group of parameters comprise a large set of constants. Common parameters $\widetilde{Z}$ and $\overline{\varphi}$ are identified by average firm size and any arbitrary moment of the wage distribution. In the LBD we compute an average size of a firm at the commuting zone level of 22.83, and in the CPS, 29 percent of workers earn below \$15 per hour. Given any other parameters, these moments pin down $\widetilde{Z}$ and $\overline{\varphi}$ exactly.

Parameters that are heterogeneous across households are relative shifters in productivity and labor supply disutility $\{\xi_k, \widetilde{\varphi}_k\}_{k=1}^K$, and shares of aggregate profits and capital income $\{\kappa_k\}_{k=1}^K$. We normalize $\xi_k = \widetilde{\varphi}_k = 1$ for college worker households. For any $\{\kappa_k\}_{k=1}^K$, the remaining productivity and labor disutility parameters can be inverted from data on average earnings per hour and each household's
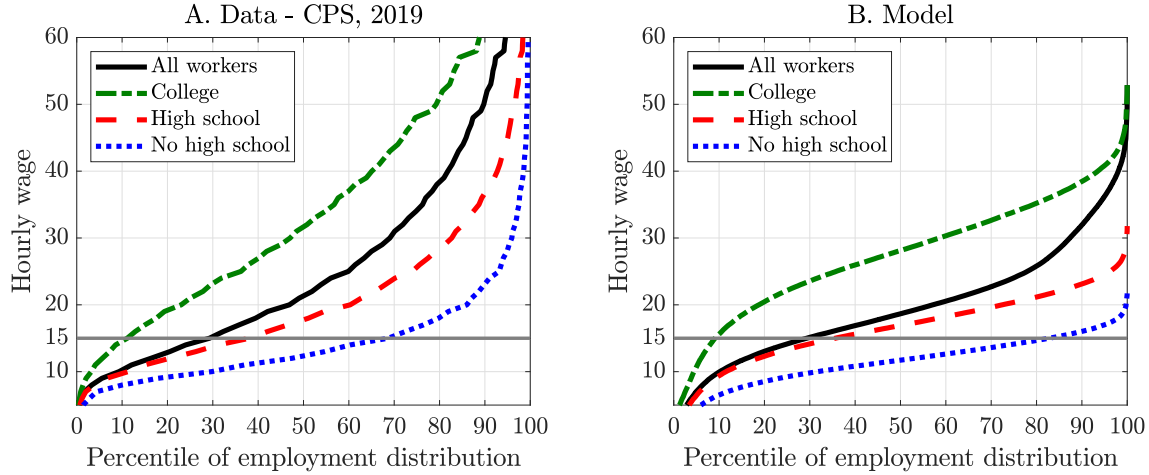
Figure 5: Distribution of wages by worker type and $15 minimum wage

<u>Notes:</u> CPS data constructed using combined MORG and March survey from 2019, using `hwtfinl`. Wages are computed as weekly earnings (`earnweek`) divided by usual weekly hours worked (`uhrsworkt`). We follow the Federal Reserve Bank of Atlanta Wage Growth Tracker, and remove individuals whose hourly pay is below the current federal minimum wage for tip-based workers ($2.13). We drop individuals who are coded as *hours vary* (`uhrsworkt=997`). We keep all other workers aged between 16 and 65.

share of aggregate labor income, which we compute in the CPS. We assign college worker and owner households the same wage.[21] The average wage of non-high-school (high-school) workers is 42 percent (60 percent) of the average college wage. Productivity shifters are in line with these data.[22] Shares of aggregate labor income exactly pin down relative disutilities of labor supply.

Using the SCF we compute the ratio of total capital income to total labor income for each group of households, and choose κ_k for each of the three non-owner households to match these ratios exactly. In the data, this ratio is below 0.10 for all non-owner households, and more than 6 for owners. This provides further support for our approach of including owners as a separate group. Since the shares must sum to one, the share of owners is determined as a residual. As an over-identifying test, we verify that the ratio for owners is consistent with the data (Table 2C).

4.3 Internally calibrated

The final set of parameters are internally calibrated. Productivity dispersion σ and decreasing returns α are identified by the average level of concentration in labor markets, and the labor share. The argument is as follows. First, more productivity dispersion increases the market power of the most productive firms. This increases concentration and decreases the labor share. Second, more linear technology also makes the most productive firms larger, but reduces profits. This increases concentration and increases the labor share. We infer a level of productivity dispersion of 0.268, which is consistent with direct empirical estimates (see Decker, Haltiwanger, Jarmin, and Miranda, 2020), and moderate decreasing returns to

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[21]This allows us to combine SCF and CPS data since we do not observe assets in the CPS. In the SCF, labor earnings are similar across the two college household types.

[22]Heterogeneity in capital income requires a different ξ_k for owners to match the same wage as college workers.

scale: $\alpha = 0.940$. The value of α implies a relatively elastic marginal revenue product of labor, hence firms will shrink relatively quickly in Region III. Given all other parameters, γ can be chosen to match the capital share, which we set to 0.18.

4.4 Implied distribution of wages, consumption and Negishi weights

Wages.   Figure 5 plots the distribution of wages in the benchmark economy and in the 2019 CPS data used to calibrate the model. Details on computation of wages and sample selection are given in the figure footnote. By construction, the calibration matches 29 percent of workers earning wages less than $15. The model also does well on the non-targeted fraction of college workers below $15 (11% in data vs. 9% in model) and high school workers (38% in data vs. 36% in model).

There are two features of the data the model misses. First, it slightly *overstates* non-high-school workers below $15 (69% in data vs. 83% in model). To address this, Appendix A.3 provides results for an alternative calibration where we target the average level of wages of all types of workers, ignoring the fraction below $15. Under this calibration the fraction of non-high-school and high school workers below $15 is 51% and 16%, respectively, which both *understate* the data. We show that our main results are not substantially effected. Second, the model misses the fat tails of the wage distribution. Additional worker heterogeneity would be needed to capture these, however the tails are less consequential for the analysis of minimum wages.

Consumption.   Table 2 reports the share of consumption of each group. Data for consumption by household is available from the BLS, where household education is the education of the highest earner. We therefore do not use this data in calibration. Nonetheless, the model accurately replicates this data, with non-high-school households accounting for around 2 percent of consumption despite being 12 percent of workers, and college households accounting for around 60 percent of consumption despite being only 35 percent of households.

Negishi weights.   The inferred Negishi weights motivate our approach to separating out the efficiency and redistribution components of minimum wage policy. The competitive equilibrium is consistent with the allocation chosen by a planner with a combined weight of 62 percent on college households and a combined weight of 38 percent on non-college households, while these groups make up 35 and 65 percent of the population, respectively. Suppose the government has a *utilitarian* objective, which applies population weights. Given the asymmetry in Negishi weights and population weights, such a planner would primarily want to use the minimum wage to redistribute. Recognizing this, we will give our government access to lump sum taxes that can be used to meet these redistribution objectives. Any benefit due to a positive minimum wage will then be entirely due to efficiency gains.

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5   Validation

Section 3 described two of the channels through which minimum wages may improve efficiency: (i) real-location of employment to more productive firms, (ii) increasing wages via strategic interactions—which undo markdown distortions—at unconstrained firms. Two recent empirical papers speak directly to these channels. Dustmann, Lindner, Schoenberg, Umkehrer, and vom Berge (2021) provides evidence on reallocation of employment to higher productivity firms following a moderate minimum wage increase in Germany. Derenoncourt, Noelke, Weil, and Taska (2021) provides evidence on the size of employer responses to competitor's *voluntary* minimum wages. We show that the model can replicate both studies in sign and magnitude, which gives us confidence in assessing the efficiency implications of minimum wages.

5.1   Reallocation Effects of Minimum Wages

Dustmann, Lindner, Schoenberg, Umkehrer, and vom Berge (2021) (DLSUB) *"Reallocation Effects of the Minimum Wage,"* studies the effect of the introduction of a minimum wage in Germany and its impact on the cross-section of workers and firms. A national minimum wage of 8.50 euros per hour was introduced in January 2015, into an environment with no pre-existing minimum wage. Moreover, the minimum wage introduced in Germany was large. Pre-reform, 15 percent of workers earned below 8.50, which was 48 percent of the median wage. The key findings are significant employment reallocation effects in which firms exit, and larger more productive firms expand, increasing average firm size.

Empirical setting.   DLSUB consider a number of empirical approaches. The one we focus on computes the elasticity of firm characteristics with respect to minimum wage exposure. The authors compute a measure they call the minimum wage *Gap*: the percent increase in total earnings required to satisfy the new minimum wage, holding employment and hours fixed at their pre-reform level. In the data, let workers be indexed by $\ell \in \{1, \dots, n\}$. DLSUB define *Gap* using workers pre-reform hours h_ℓ and wages w_ℓ:

$$Gap := \frac{\sum_\ell \max\{\underline{w} - w_\ell, 0\} h_\ell}{\sum_\ell w_\ell h_\ell}$$

The authors group firms by geographic regions r, and regress changes in region outcomes around the policy on Gap_r. Since Gap_r is in percentage changes, these regressions yield elasticities. We focus on the following moments reported in their paper: (i) total employment $n = \sum_\ell \mathbf{1}_{[h_\ell > 0]}$, (ii) average wage: $\overline{w} = \sum_\ell w_\ell h_\ell / \sum_\ell h_\ell$, (iii) total number of operating firms, and (iv) average firm size. Their results are reported in Table 7, page 54.

Model replication.   To an economy with no minimum wage, we introduce a minimum wage of \$8.95/hr which equals 48 percent of the pre-reform median wage. Since the empirical setting is a national reform, we solve the pre- and post-reform economy in general equilibrium. The regions considered in DLSUB, comprise all industries in multiple commuting zones and rural areas. These are much larger than the markets j in our model. We therefore treat our whole economy as one region, which generates

a single *Gap* measure that is directly comparable to that of DLSUB:

$$Gap = \frac{\sum_k \int \sum_i \max\{\underline{w} - w_{ijk}, 0\} n_{ijk} \, dj}{\sum_k \int \sum_i w_{ijk} n_{ijk} \, dj} \tag{13}$$

We then compute the elasticity of variable x with respect to the minimum wage exposure *Gap*, by dividing the ratio of economy-wide $\Delta \log x$ by *Gap*.

Results. Figure 6 gives the results. There are two sets of the authors' results: 'Data 1' and 'Data 2'. The former features controls that account for observable regional differences (e.g. average age) and region specific trends in the moments, the latter additionally interacts these trends with year fixed effects. We plot results for a range of minimum wages, indexed by the ratio of $\underline{w}$ to the pre-reform median wage, and mark the case corresponding to the German case with a vertical line.

First, consistent with other empirical studies of minimum wage effects, Figure 6A shows that there are no disemployment effects, with employment increasing marginally. Employment increases in Region II, dominate reductions in Region III. At much higher minimum wages, however, this flips, and the effect becomes negative. Market wage increases significantly in response to the minimum wage change (Figure 6B). Through the lens of the model, both constrained and unconstrained firms pay higher wages, regardless if they cut or expand employment.

Second, consistent with the new reallocation facts in DLSUB, Panel C shows that small firms exit and Panel D shows that reallocation causes average firm size to grow. In the model all firms still operate due to decreasing returns and the fact that n_{ijk} is continuous and can go below one (recall Figure 2D). To compare our model to DLSUB, we classify a firm as 'operating' when their employment is above one worker. We find that the elasticity of the number of operating firms with respect to minimum wage exposure is negative and thus correctly signed, but moderately less responsive compared to the data. The model's elasticity of firms size with respect to minimum wage exposure is positive and moderately higher than the data, and consistent with the German data for slightly larger minimum wage increases. The increase in firm size is moderated at larger minimum wage increases due to firms shrinking in Region III.

Interpretation. One of the key take-aways of DLSUB is that minimum wage increases have heterogeneous effects across firms. Low productivity firms exit, but their workers do not move out of the labor market. Jobs which existed due to the small amount of market power at these low productivity firms are destroyed, but workers are reallocated to larger, more productive firms. This can improve allocative efficiency, and our model generates dynamics consistent with these observations.

5.2 Derrenoncourt et al (2021) - Firms' responses to competitor's minimum wages

Derenoncourt, Noelke, Weil, and Taska (2021) (DNWT) *"Spillover effects from voluntary employer minimum wages"*, studies how *voluntary minimum wages* as part of large firms' policies affect the wages and employment of firms within the large firms' market. In the context of a \$15/hr minimum wage instituted nationally by Amazon in 2018, which increases Amazon wages on average by 18.1 percent, the authors
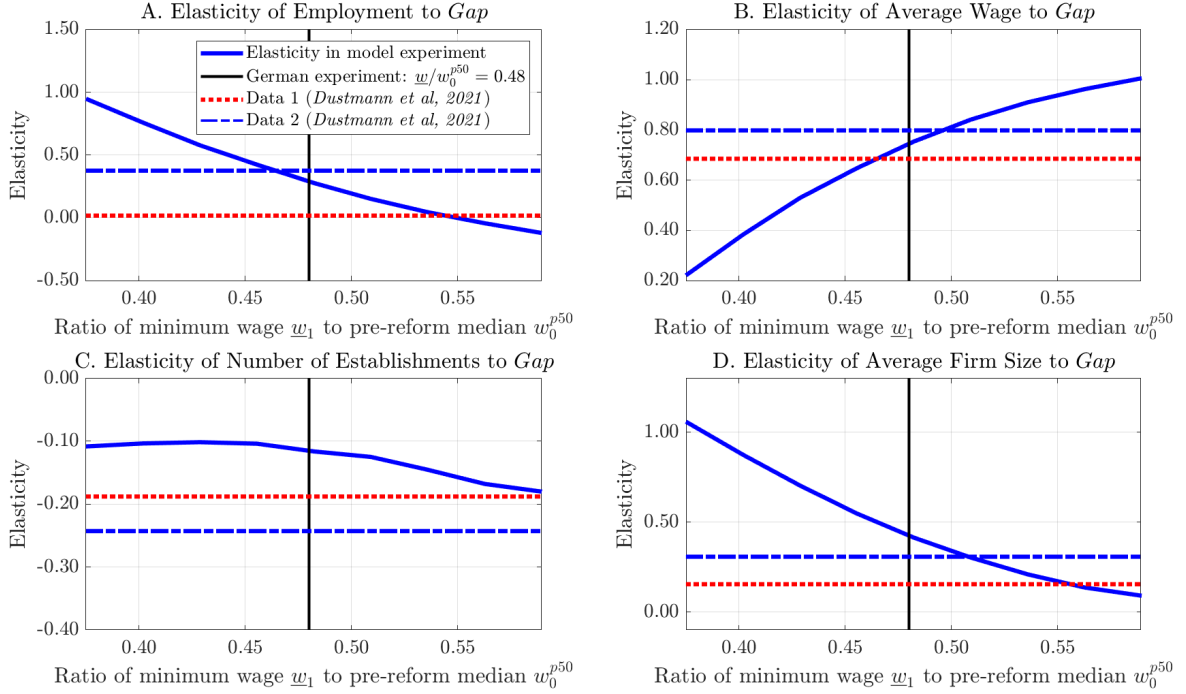
Figure 6: Replication of DLSUB (2021) - Reallocation effects of minimum wages

find that competitors increase their wages by 4.7 percent. The authors' headline empirical result is that this constitutes a *cross-employer wage elasticity* of 0.26.[23]

Replication. To replicate the exercise, we need to identify firms in markets that we can call 'Amazon', and institute a policy that increases their observed wage by 18.1 percent. We do this by exogenously narrowing markdowns. First we solve the baseline model, and then take a firm i in market j and set its new markdown for all types to:

$$\mu'_{ijk} = (1 - \zeta) \times \mu_{ijk} + \zeta \times 1 \quad , \quad \zeta \in (0, 1).$$

That is, we narrow the firms' markdown a fraction ζ toward the efficient markdown $\mu^*_{ijk} = 1$. We refer to firm i as the *focal firm*. We then solve the Nash equilibrium among the remaining firms in each market. We run this experiment in every market. We keep aggregates fixed, since this is a partial equilibrium

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[23]These results are summarized on page 2 of DNWT (2021): *"In the case of Amazon, we estimate an increase in average hourly wages [of competitors] as a result of the policy of 4.7%, controlling for unrelated trends in wages at the occupation and commuting zone level. Given the size of the increase for Amazon's wages, roughly 20%, our results imply a cross-employer wage elasticity of 0.26.* Note that 4.7%/20% would imply a cross employer wage elasticity of 0.235, not 0.26. The authors refer to the 0.26 and 0.047 numbers, while only here mention the "roughly 20%" Amazon wage increase. Therefore we target an Amazon wage increase of 0.181 such that $0.047/0.181 = 0.26$.

exercise.

In order to proceed we need to choose a focal firm among the firms in each market that corresponds to 'Amazon'. DNWT do not provide summary statistics on (i) the average size of Amazon relative to competitors, or (ii) the number of competitors that Amazon faces in each market. Absent (i) we consider two cases, one where the focal firm is the most productive firm in each market, and one where it is the second most productive firm. Absent (ii) we conduct our experiment for firms in all markets, and then consider dropping markets based on a cut-off for the number of firms operating in a market, starting at $\underline{M} = 2$ and going up to $\underline{M} = 30$. If ζ were left fixed, we would find that the average change in the focal firm's wage is smaller when $\underline{M}$ is larger, due to tighter competition, and larger when the focal firm is smaller relative to the market. We therefore recalibrate ζ to keep the average wage change of the focal firm constant with the data. As we increase $\underline{M}$, the required value of ζ increases from around 0.40 to 0.60 when the focal firm is the leader. The interpretation of this result is that leaders can increase wages by 20 percent while still maintaining markdowns that are around half as wide.

Results. Figure 7 gives the results of this exercise. Panel A shows that our strategy for recalibrating ζ generates data consistent increases in focal firm wages as we vary $\underline{M}$ and in our two specifications of the identity of the focal firm. Qualitatively, as in the data, Panel B shows that the model generates an increase in competitors' wages. The increase in the leaders' wage increases its market share and reduces that of its competitors. This tightening of competition leads competitors' markdowns narrow and their wages to increase. Quantitatively, the effect is in the range of the data.[24] In the cases where Amazon is the largest firm and in markets with at least 36 firms, or is the second largest firm and in markets with at least 12 firms, the outcome is exactly as estimated by DNWT.

Apart from replicating the empirical evidence, these results suggest an additional margin of cross-sectional variation that empirical research following DNWT may wish to explore. In markets that are more competitive, or when the focal firm is relatively smaller, the cross-employer elasticity is lower. This provides an early warning that it may be challenging to extrapolate from this evidence to the effects of minimum wages more broadly. Within a market, minimum wages will first affect small firms, and most employment is in competitive markets. Both of these facts suggests that spillovers within markets may be small, the second fact suggests these might not matter when aggregated.

Section summary. We have shown that the model successfully replicates and gives a natural interpretation to key papers in the empirical literature on the reallocative effects of minimum wages and the strategic nature of wage setting. These are necessary features of the data for a model to replicate. We therefore view this section as positioning the model well for the main quantitative contribution which is to compute the optimal minimum wage.

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[24]As in DNWT we compute the average log change in competitors' wages market by market, and then take an unweighted average across markets.
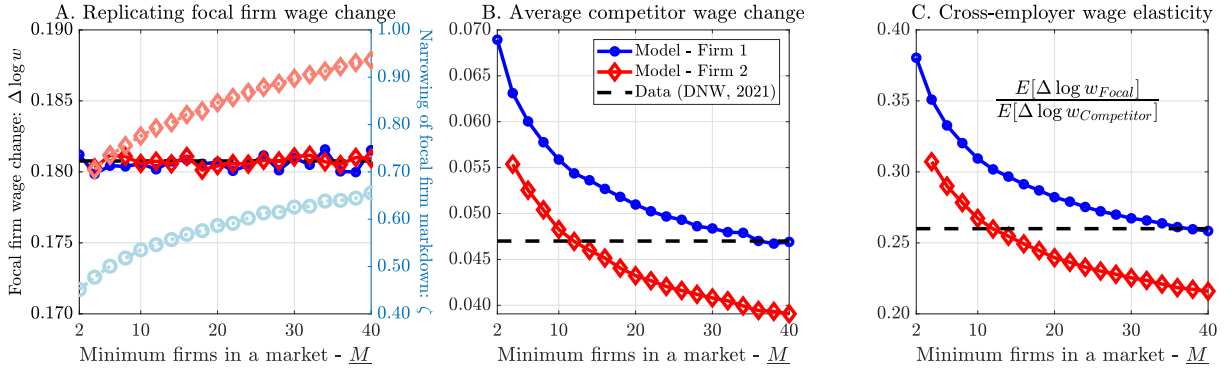
Figure 7: Replication of DNWT (2021) - Competitor responses to voluntary minimum wages

<u>Notes:</u> **Panel A.** Faint, increasing, lines are plotted on the righthandside vertical axis and give the value of ζ required to deliver the observed average increase in focal firm wages. Solid lines are plotted on the lefthandside vertical axis and give the model implied average increase in focal firm wages. These are compared to the data, with a value of $0.181 (= 0.047/0.26)$ (page 2 of DNWT). **Panel B.** Plots the average non-focal firm wage change. Blue-circle line corresponds to the case where the focal firm is the most productive firm in each market. Red-diamond line corresponds to the case where the focal firm is the second most productive firm in each market. Data value is 0.047. **Panel C.** Plots the *cross-employer wage elasticity*, which is the ratio of panel B to panel A. Data value is 0.26.

6 Additional validation

We have shown that spillovers of wages across firms are consistent with the data. We now document that our model generates empirically reasonable measures of spillovers from the perspective of workers. Since our paper contributes a model of minimum wages in concentrated markets, we also show that the model generates heterogeneity in employment responses to the minimum wage by market concentration consistent with recent evidence.

6.1 Spillovers

Empirical setting. Rather than consider evidence from short-run employment responses to small minimum wage changes in the US (Autor, Manning, and Smith, 2016), where spillovers are observed up to around the 20th percentile, we consider recent leading empirical studies of the large minimum wage increase over two decades in Brazil: Engbom and Moser (2021), Haanwickel (2020).[25] Both papers follow the procedure of Autor, Manning, and Smith (2016). We focus on Engbom and Moser (2021) since the paper contains additional summary statistics that aid our replication. They compute that in 1996 the minimum wage was 34 percent of the median wage, and then increased by 119 percent between 1996 and 2012 (Engbom and Moser, 2021, page 11). To replicate this experience we solve our economy under a minimum wage of $6.34, which is 34.9 percent of the median wage, and then increase it to $14.23 which is a 119 percent increase. We denote these period zero and period one.

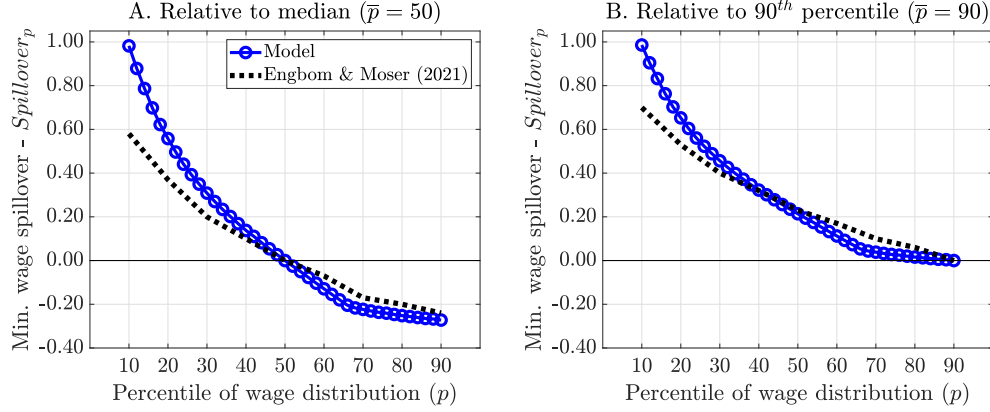[25]See the former, Figure 4, and the latter Figure 8.

Figure 8: Common measure of spillovers from a large increase in the minimum wage

<u>Notes:</u> Consistent with the minimum wage in Brazil in 1996, the initial minimum wage is 34 percent of the median wage. Consistent with the minimum wage increase in Brazil from 1996 to 2012, the minimum wage increases by 119 percent. These statistics are reported in Engbom and Moser (2021, page 12). We compare model results to those of Engbom and Moser (2021), Figure 4, under the State Fixed Effects plus IV specification.

Statistic. Let $\overline{p}$ be a reference percentile of the wage distribution, and let $w_{p,t}$ be the percentile p wage in period t. We compute spillovers at p by

$$Spillover_p = \frac{\log(w_{p,1}/w_{\overline{p},1}) - \log(w_{p,0}/w_{\overline{p},0})}{\log(\underline{w}_1/w_{\overline{p},1}) - \log(\underline{w}_0/w_{\overline{p},0})} \tag{14}$$

By construction $Spillover_{\overline{p}} = 0$. If wages below $\overline{p}$ compress upward, then $Spillover_p > 0$. If wages above $\overline{p}$ compress upward, then $Spillover_p < 0$. Engbom and Moser (2021) use a regression framework to obtain estimates of $Spillover_p$, whereas we simply compute $Spillover_p$ non-parameterically via (14).[26] We compute results for non-High school workers. As shown by Engbom and Moser (2021, Figure A2), as far up as at the 70th percentile of the earnings distribution more than 80 percent of workers have not completed high school in Brazil.[27]

Results. Figure 8 plots $Spillover_p$ for $p \in [10, 12, \ldots, 90]$ and compares estimates to those from Engbom and Moser (2021, Figure 4) for the case where reference percentiles are $\overline{p} = 50$ (panel A) and $\overline{p} = 90$ (panel B).[28] We find very similar patterns of spillovers, with compression far up into the wage distribution. Again, we find that the model is consistent with key empirical facts that arise in the discussion of minimum wages.

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[26]Note that if one wanted to estimate $Spillover_p$ at percentile p via regression, its a regression of $\Delta Gap_{p,t}$, where $Gap_{p,t} = \log w_{p,t} - \log w_{\overline{p},t}$, on the commonly named 'Kaitz' index $\Delta Kaitz_{p,t}$, where $Kaitz_{p,t} = \log \underline{w}_t - \log w_{\overline{p},t}$, which measures the 'bite' of the minimum wage. Estimating this p-by-p would be excessively demanding of the data, so in practice this is implemented parameterically, with instruments for $Kaitz_{p,t}$. For details see Engbom and Moser (2021, Section 3.2).

[27]To clarify: take all workers in the 70th percentile of the earnings distribution. Of these workers, more than 80 percent had not completed high school. Another statistic that reflect this is as follows: at the 90th percentile of the earnings distribution, around 90 percent of workers do not have a college degree.

[28]We compare our results to their IV specification that controls for state-level trends and state fixed effects. This delivers similar results to their specification with state-level fixed effects only.

6.2 Heterogeneity of employment effects by market concentration

Empirical setting. Azar, Huet-Vaughn, Marinescu, Taska, and von Wachter (2019) compute the response of employment in low wage occupations to changes in state minimum wages, but stratify responses by the concentration of the labor market for each occupation. They estimate statistically significant positive effects in markets in the upper tercile of concentration, and statistically significant negative effects in markets in the lower tercile of concentration. We show that the same results hold in our economy.[29]

Statistic. Holding aggregates fixed, we increase the minimum wage by fifty cents, compute the increase in employment in each market j, and regress the change in market employment $\Delta \log n_j$ on the change in the minimum wage $\Delta \log \underline{w}$, interacted with dummies for the tercile of market concentration $\overline{hhi}_j^n$:[30]

$$\Delta \log n_j = \psi_1 \Delta \log \underline{w} + \sum_{k=2}^{3} \psi_k \mathbf{1}[\overline{hhi}_j^n \in \text{Tercile}_k] \times \Delta \log \underline{w}.$$

This is equivalent to the main specification with market fixed effects estimated in the paper, but where we have computed concentration in employment as opposed to in job postings, which is observed in their study. In their sample, the average pre- and post-policy minimum wages are \$7.43 and \$7.83.[31] Given the focus of Azar, Huet-Vaughn, Marinescu, Taska, and von Wachter (2019) on low wage jobs (Stock Clerks, Retail Sales, and Cashiers), we compute market hhi_j^n and n_j using non-Highschool and Highschool workers. To understand potential heterogeneity by the level of the initial minimum wage, we repeat this exercise for initial minimum wages $\underline{w}_0$ between \$2 and \$10 per hour.

Results. Figure 9 plots the estimated coefficients for low $(\widehat{\psi}_1)$ and high $(\widehat{\psi}_1 + \widehat{\psi}_3)$ concentration markets, holding the increase in the minimum wage constant (50c), but varying the initial minimum wage $\underline{w}_0$ on the horizontal axis. For initial minimum wages consistent with the settings the paper studies, i.e. less than \$8.00 per hour, the model is consistent with its key empirical findings. High concentration markets see large, positive, employment effects, and low concentration markets see small negative employment effects. Low productivity firms in more concentrated markets have more market power, wider markdowns, and hence have larger positive employment gains available in Region II before shrinking in Region III. In less concentrated markets these firms have initially narrow markdowns and move quickly into Region III, incurring employment losses.

Section summary. Results in Section 5 related directly to efficiency. Section 6 has shown that on two additional dimensions our model captures key statistics estimated in empirical studies, and from a range of countries. Apart from lending credibility to our quantitative results, these exercises show that these facts are mutually consistent *with one another*, through the lens of our theory.

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[29] We are unable to replicate their study directly and conduct a quantitative comparison due to concentration being computed using Burning Glass data on job openings.

[30] As in their paper, the measure of concentration we use to determine terciles is the average of concentration pre- and post-policy change

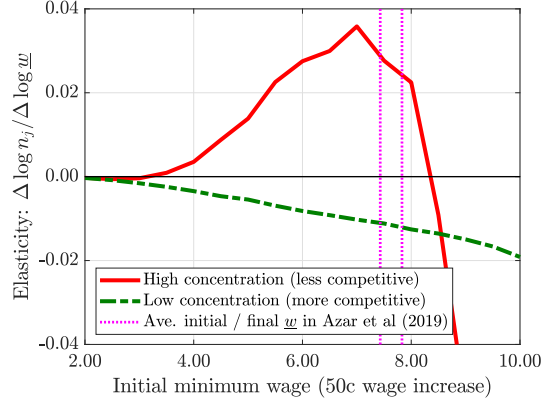[31] We thank the authors for sharing these two moments with us.

Figure 9: Effect of a minimum wage increase on employment, by concentration of labor market

<u>Notes:</u> Horizontal axis gives the initial minimum wage $\underline{w}_0$. The minimum wage is then increased by 50 cents. Red solid line plots estimated elasticity in high concentration markets $(\widehat{\psi}_1 + \widehat{\psi}_3)$. Green dashed line plots estimated elasticity in low concentration markets $(\widehat{\psi}_1)$.

7 Positive implications of the minimum wage

Before turning to our normative exercises we describe the positive implications of the minimum wage for (i) aggregate and cross-sectional outcomes, and (ii) inequality metrics usually tied to welfare absent an explicit welfare criterion: wage inequality and the labor share.

Aggregates. Qualitatively, a main result is the non-linear effects of minimum wages on many aggregate variables. The top row of Figure 10 shows the increasing and then decreasing effect of the minimum wage on consumption, capital, output (Panel A), and employment (Panel C). Consumption, output and employment increase due to a more efficient allocation of resources across firms, with small firms shrinking and employment and capital reallocated to more productive firms. These aggregates rapidly deteriorate at higher minimum wages as misallocation via more productive firms entering Region III sets in (Appendix A.2 plots the share of jobs and employment in each region for each type of worker). We return to this in the next section. Panel B shows that despite monotonically increasing wages, the aggregate shadow wage is also hump-shaped. Initially, narrowing shadow markdowns in Region II increase the wage and shadow wage in tandem, but as rationing constraints bind the shadow wage declines. Panel C shows that the shadow wage is indeed allocative, determining aggregate employment rather than the increasing average wage. Profits monotonically decrease, reallocating payments from owners to non-owners.

Employment non-linearities. Quantitatively, the minimum wage has small positive effects on aggregates, with less than one percent increases in output, consumption, capital and employment. Importantly, Panel C shows the model can rationalize either positive or negative employment effects of the minimum wage. This is consistent with the recent, broad study of state minimum wage increases by Clemens and Strain (2021). They find that positive minimum wage effects are found following relatively
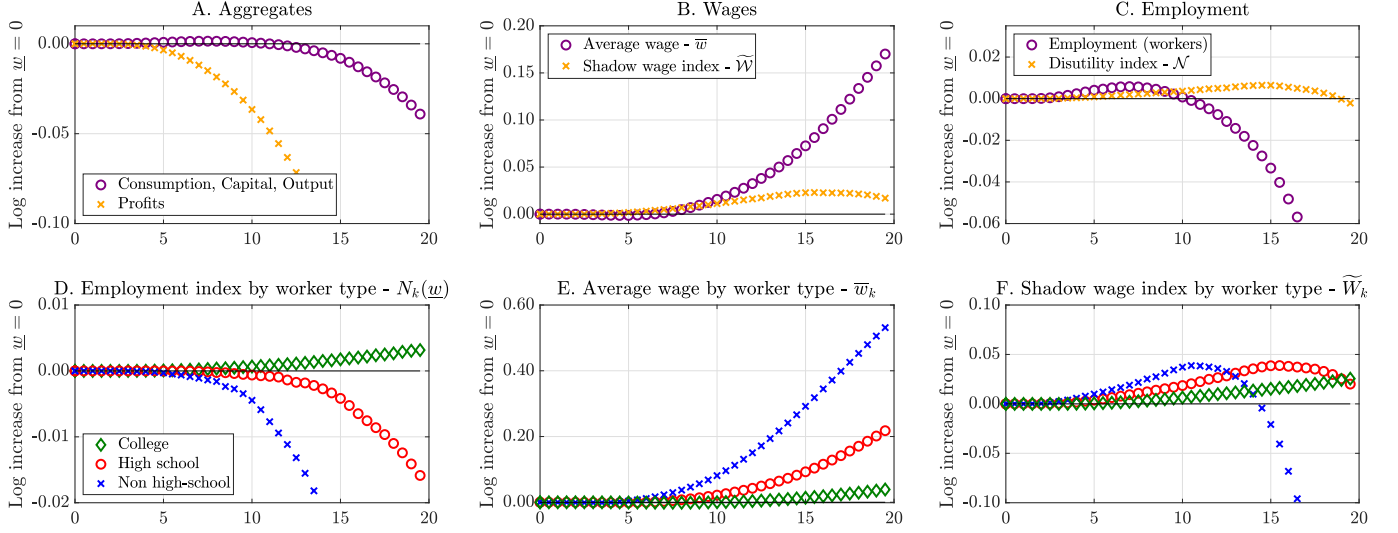
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Figure 10: Aggregate and worker type outcomes

smaller minimum wage increases from relatively lower initial minimum wages.[32] Crucially, our model implies that while a higher minimum wage can increase employment levels, once the minimum wage is greater than $10, aggregate employment effects become quickly negative, with job losses concentrated at workers with less education (Panel D). Thus, the effect of a higher minimum wage is highly non-linear and caution must be used if extrapolating based on evidence developed from minimum wage increases at lower minimum wage levels.

7.1 Aggregates and worker outcomes

Cross-sectional outcomes. The bottom row of Figure 10 plots employment, wage and shadow wages for each worker type. For sake of exposition, we suppress owners from the figures. As a summary, each panel is consistent with negative effects emerging more swiftly for non-high school workers. Again, the allocative wage for each type of worker is the hump-shaped shadow wage (Panel C), rather than the sharply increasing average wage (Panel B).

Figure 10 shows that accounting properly for heterogeneity in income from labor and profits is necessary to understanding employment effects of minimum wages in general equilibrium. Declining profits have positive wealth effects on employment. However the skewed distribution of capital and profit income in the population implies that these effects are siloed and quantitatively small. Owners earn 92 percent of dividends, but only have a 7 percent share of labor income (Table 2B). If households had equal shares of profits, wealth effects would dampen the negative employment effects in Panel D.

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[32]Appendix A.1 replicates this finding in the context of Card and Krueger (1994). In the 2019 context a minimum wage increase consistent with their 1992 data would be from $7.74 to $9.20. We plot the employment change for non-college workers and show that in our model calibrated to 2019 data, the employment effect would be small and negative, with positive wage effects. Importantly, for slightly smaller initial minimum wages a similar wage increase can deliver employment increases.
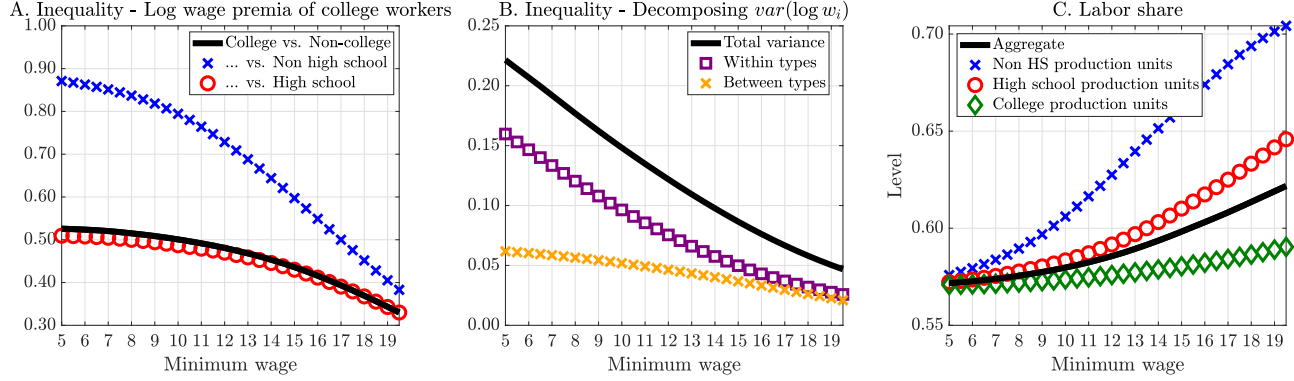
Figure 11: Minimum wages and commonly used measures of welfare

7.2 Empirical proxies for welfare

Often used empirical proxies for welfare are monotonically increasing in the minimum wage. Below we show that well posed measures of aggregate welfare inherit the non-monotonicities found in Figure 10. These proxies are therefore misleading. Figure 11 plots wage inequality and the labor share as the minimum wage increases. Panel A shows that the log wage premia between college and non-college workers declines by one fifth (0.53 to 0.43) as the minimum wage is increased from $7.50 to $15. Panel B presents an alternative measure of wage inequality: the cross-sectional variance of log wages. The total variance of wages declines by nearly a half over the same range, driven equally by declining in within- and between-type inequality. As profits decline (Figure 10A), the aggregate share of payments to labor increases by about 3 ppt over this range. The share of output created in non-high school jobs paid to non-high school workers increases by 7 ppt (Panel C). Hence measures of (i) income inequality and (ii) worker power in the form of the labor share, both suggest ever higher minimum wages. This is inconsistent with our following results: a positive minimum wage that maximizes welfare.

8 Results - Social welfare and efficiency

We (i) describe our measurements of welfare, (ii) compare welfare maximizing minimum wages under alternative social welfare weights, (iii) use our partial planner with flexible lump sum transfers to separate out efficiency and redistribution, and (iv) explain the quantitative mechanisms behind our results. For convenience, we benchmark welfare gains relative to an economy with a zero minimum wage. This choice is easy to alter in future versions of the paper, and has little implications for our results (see also Appendix A.4, referenced below).

8.1 Measurement

For each household, under a particular minimum wage $\underline{w}$, the *consumption equivalent welfare gain relative to a no minimum wage economy* (henceforth, *welfare gains*) is the proportional increase in consumption

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$\lambda_k(\underline{w})$ that delivers the same utility as the minimum wage economy. The aggregate welfare gain, $\Lambda(\underline{w})$, is defined similarly. It requires taking a stand on *social welfare weights* $\{\psi_k\}_{k=1}^K$:

$$\underbrace{u^k\left((1+\lambda_k(\underline{w}))\frac{c_k(0)}{\pi_k}, n_k(0)\right) = u^k\left(\frac{c_k(\underline{w})}{\pi_k}, n_k(\underline{w})\right)}_{\text{Worker type welfare gains, } \lambda_k(\underline{w})} \quad , \quad \underbrace{\sum_k \psi_k u^k\left((1+\Lambda(\underline{w}))\frac{c_k(0)}{\pi_k}, n_k(0)\right) = \sum_k \psi_k u^k\left(\frac{c_k(\underline{w})}{\pi_k}, n_k(\underline{w})\right)}_{\text{Aggregate welfare gains, } \Lambda(\underline{w})}.$$

With separable, power utility, aggregate welfare gains are a weighted harmonic mean of gains type-by-type:

$$1 + \Lambda(\underline{w}) = \left[\sum_k \widetilde{\psi}_k(0)(1+\lambda_k(\underline{w}))^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \quad , \quad \widetilde{\psi}_k(0) := \frac{\psi_k\left(\frac{c_k(0)}{\pi_k}\right)^{1-\sigma}}{\sum_k \psi_k\left(\frac{c_k(0)}{\pi_k}\right)^{1-\sigma}}. \tag{15}$$

We provide results for two sets of social welfare weights: (i) Utilitarian weights, under which $\psi_k = \pi_k$, (ii) Negishi weights associated with a zero minimum wage economy $\psi_k(0) = \psi_k^*(0)$.[33]

8.2 Social welfare maximizing minimum wage, absent transfers

Figure 12 depicts our first set of welfare results. Panel A plots consumption equivalent welfare gains for each type of worker under minimum wages in the range of zero to \$20.00 per hour. Consistent with the positive effects of minimum wages described in the previous section, welfare gains are shortest lived for the least productive groups of workers, and owners of capital face welfare losses due to the erosion of profits.

Panel B plots aggregate welfare gains. As per equation (15), aggregate welfare gains are averages over the worker level gains, where the weights reflect social welfare weights. Under Utilitarian weights, the optimal minimum wage is \$15.12, at which point welfare gains of non-college workers are high. Welfare of college workers increases at higher minimum wages, but by this point welfare losses among non-college workers are steep. Under Negishi weights, the optimal minimum wage is much lower at \$6.97. Recall from Table 2, that the Negishi weights put a weight of about 65 percent on college workers and owners, whereas they represent only 35 percent of the population. In this sense Negishi weights are roughly the inverse of the Utilitarian weights, and with more weight on college workers and owners, imply a much lower optimal minimum wage.

These results make clear that the optimal minimum wage, holding other fiscal instruments fixed, is in the eye of the beholder in the sense that it depends largely on social welfare weights. This is because the minimum wage redistributes significantly. This leads us, in the following section, to separate out the efficiency and redistributive components.

The magnitude of the welfare gains also depend on social welfare weights, with large gains under Utilitarian weights and relatively small gains under Negishi weights. In both cases we compute that

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[33]Under the Negishi weights, using the governments's optimality conditions (10) in (15) imply that $\widetilde{\psi}_k(0)$ are equal to consumption shares: $\widetilde{\psi}_k(0) = c_k(0)/C(0)$. If we benchmark welfare gains to an alternative minimum wage $\underline{w}_0$, then the effective weights $\widetilde{\psi}_k(\underline{w}_0)$ will be different. However Appendix A.4 shows that under both Negishi and Utilitarian weights, the associated effective weights $\psi_k^*(\underline{w}_0)$ vary very little in $\underline{w}_0$.
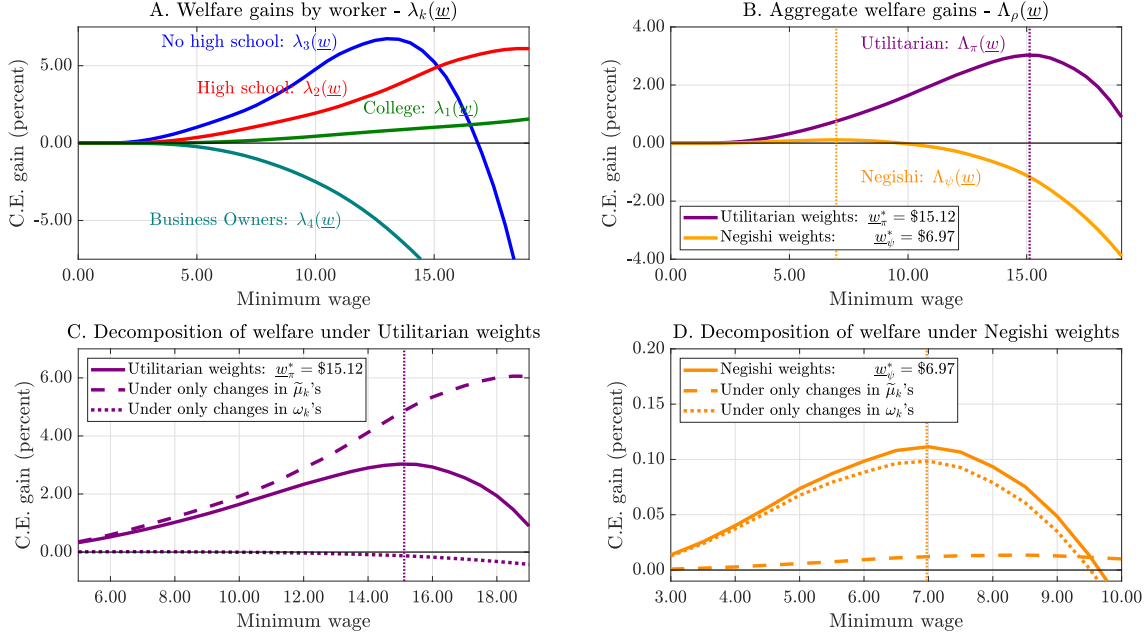
Figure 12: Minimum wages and welfare

Notes: In all cases we plot objects from the equilibrium under various values of the minimum wage $\underline{w}$, on the horizontal axis. In all cases the vertical axis plots consumption equivalent welfare gains relative to an economy with a zero minimum wage. **Panel A.** Plots the consumption equivalent welfare gains of each household: $\lambda_k(\underline{w})$. **Panel B.** Plots the aggregate consumption equivalent welfare gains, under alternative sets of social welfare weights ψ_k. **Panel C.** Plots consumption equivalent welfare gains under utilitarian welfare weights under changes in shadow markdown wedges only, $\widetilde{\mu}_k(\underline{w})$, and under misallocation wedges only, $\omega_k(\underline{w})$. In each case a competitive equilibrium is solved under only the specified wedges changing, while the remaining wedges are kept at their value under $\underline{w} = 0$. **Panel D.** Repeats panel C, but with welfare evaluated under Negishi weights.

these gains are small with respect to the potential welfare gains available in the economy. Consider the efficient allocation which obtains when all firms' markdowns are one, and hence wages are equal to marginal revenue products at all firms. In the case of Utilitarian weights, the welfare gains from the efficient allocation are 28.61 percent, and the optimal minimum wage gains deliver about one ninth of this (3.04 percent).

The decomposition of welfare gains also depends on the perspective of the analysts' social welfare weights. Leveraging our construction of the general equilibrium, Panels C and D decompose welfare gains into those due to changes in shadow markdowns, and those due to changes in misallocation. At $\underline{w}^*$ under Utilitarian weights, misallocation is worse than in the zero minimum wage economy, but the planner is happy to trade this off against large gains from narrowing markdowns in Region II, which redistribute income to low wage workers. The presence of labor market power provides the possibility of these gains from redistribution. At $\underline{w}^*$ under Negishi weights, more than 80 percent of the welfare gains are instead driven by improved reallocation. As we showed earlier, these gains stem from reallocating employment from low z to medium z firms. Quantitatively, gains from misallocation are limited, as reallocation begins to occur from high z to medium z firms. We return to these channels in Section 8.4.

In summary, redistribution generated by the minimum wage implies that the optimal minimum wage

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Policy	Weights ψ_k	Min. wage $\underline{w}^*$	Welfare gain $\Lambda(\underline{w}^*)$ vs. Λ^{Eff}	Welfare gains by type, $\lambda_k(\underline{w}^*)$				Transfers (T_k/GDP, %)			
(1)	(2)	(3)	(4)	Non-HS (5)	HS (6)	Coll. (7)	Own (8)	Non-HS (9)	HS (10)	Coll. (11).	Own (12)
A. No transfers	Utilitarian	$15.12	3.04% vs. 28.61%	5.04%	4.91%	1.03%	-8.60%	-	-	-	-
	Negishi	$6.97	0.11% vs. -2.43%	2.10%	0.90%	0.14%	-0.79%	-	-	-	-
	97% HS	$18.32	5.69% vs. 34.37%	-7.14%	6.09%	1.43%	-14.54%	-	-	-	-
B. Transfers	Utilitarian	$8.27	0.17% vs. 15.26%	0.31%	0.03%	0.34%	0.19%	10.4%	18.2%	5.4%	-34.0%
	Negishi	$7.76	0.16% vs. 15.26%	-0.81%	-0.11%	0.37%	0.35%	-0.0%	-0.4%	0.0%	0.4%
	97% HS	$9.95	0.20% vs. 15.26%	-0.56%	0.35%	-8.66%	-2.68%	0.4%	76.7%	-39.9%	-37.2%

Table 3: Optimal minimum wage

Notes: 'Utilitarian' corresponds to population share weighted consumption equivalent welfare. 'Negishi' corresponds to the weights that rationalize current U.S. data. '97% HS' places weights of 0.01 on all other types, and 0.97 on high-school grad-uates. Panel A restricts transfers to be zero across household types, consistent with Figure 12. Panel B allows for optimal lump sum transfers across household types. Column (3) reports the optimal minimum wage. Column (4) reports the aggregate consumption equivalent welfare gain $\Lambda(\underline{w}^*)$ and compares this to the aggregate consumption equivalent welfare gain associ-ated with the efficient allocation Λ^{Eff} in which firms have no market power. Columns (5) through (8) report the type specific consumption equivalent welfare gains. Columns (9) through (12) report the transfers relevant for exercises in Panel B.

depends crucially on the social welfare weights of the policy maker. The focus of our second set of results, therefore, will be efficiency, which our model—and the associated empirical exercises that we have validated it against—are well purposed to discuss.

8.3 Efficiency maximizing minimum wage

We now allow the planner to choose the minimum wage and unrestricted, balanced-budget, lump sum transfers $\{T_k\}_{k=0}^K$, in order to maximize welfare. If the government considers conducting policy under different sets of social welfare weights, then lump sum transfers may adjust to soak up the different re-distributive motives that come from the different weights. With redistribution taken care of, the optimal minimum wage now reflects efficiency. We call this the *efficiency maximizing minimum wage*.

Table 3 provides our second set of results. There are four key results, found in Panel B. First, as anticipated, with flexible lump sum transfers, the efficiency maximizing minimum wage is robust to social welfare weights, and in the range of $7.50 to $8.50. Under Utilitarian weights, the government can meet its redistributive objectives by transfers (columns 9 to 12), leading to a much lower optimal minimum wage. Second, the welfare gains associated with the efficiency maximizing minimum wage are small. Column 4 shows that the welfare gains are robustly around 0.16 percent, and represent only one percent of the welfare gains associated with the efficient allocation with no labor market power.[34] Third, a back of the envelope calculation suggests that under Utilitarian weights, around 94 percent of the welfare gains come from redistribution (1- 0.17%/3.04%). Fourth, our method is robust to even extremely skewed social welfare weights, such as a 97 percent weight on high school workers.

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[34]The welfare gains associated with the efficient benchmark are computed with optimal lump sum transfers in the efficient economy, and hence are constant with respect to social welfare weights.

Removing heterogeneity.   To highlight that the efficiency implications of minimum wages are largely independent of the distribution of households, we conduct an additional exercise in Appendix A.5. We recalibrate the model with no household heterogeneity, and find that the optimal minimum wage is $7.74. This is very close to the values with optimal lump sum transfers in Table 3B. In this sense, heterogeneity can be added or removed, but the efficiency maximizing minimum wage is robust.

Summary.   We find that the efficiency gains from minimum wages are small, and—when recalibrated—robust to the amount of heterogeneity in the economy. In Section 9 we show that this is robust to (i) alternative preference parameters, (ii) region specific minimum wages, (iii) short- versus long-run effects. This is despite the fact that the model matches key empirical evidence disciplining the channels through which efficiency improvements under a minimum wage could occur: direct effects, spillovers, and reallocation.

8.4   Mechanisms

We conclude this section by providing some understanding as to why these channels have quantitatively small effects.

Decomposition.   Section 3.3 provided a characterization of the equilibrium of the economy that hinged on a set of shadow markdown and misallocation wedges for each type of worker. We therefore can understand the efficiency implications of minimum wages via these wedges. Figure 13 panels A and B plots the wedges for each type of worker household (for clarity, we suppress owners). First, labor market power implies that as the minimum wage increases shadow markdowns narrow, generating welfare gains. However, these gains are limited and gradual on the way up, and then sharply decline. This nonlinearity owes to the quick erosion of employment in Region III, and associated rapid tightening of the rationing constraint. We use an example to expound on this below. Second, the gains from misallocation are even more swiftly undone, as firms in Region III contribute toward misallocation.

   To study this in more detail, Panels C and D restrict attention to high school workers and add counterfactuals under which all markdowns are kept at their level under $\underline{w} = 0$, apart from firms in a particular region. If firms in Region I responded strongly to the binding minimum wages at their competitors in Region II, then the purple dashed line would steeply increase. However its increase is quantitatively small. Despite matching empirical evidence on the spillovers across firms, an increase in the minimum wage has quantitatively negligible spillovers on the markdowns of unconstrained firms. There are two reasons. First, the firms in Region II are small, and so increases in their employment does not substantially shift the elasticity of unconstrained firms' residual supply curves. Second, firms in Region III shrink quickly, with declining shadow wages, which delivers more market power to firms in Region I, making their supply curves more inelastic which supports wider markdowns. The aggregate markdown is instead shaped by the narrowing markdowns of firms in Region II, up to about $15, and then by the widening shadow markdowns—capturing binding rationing constraints—in Region III.
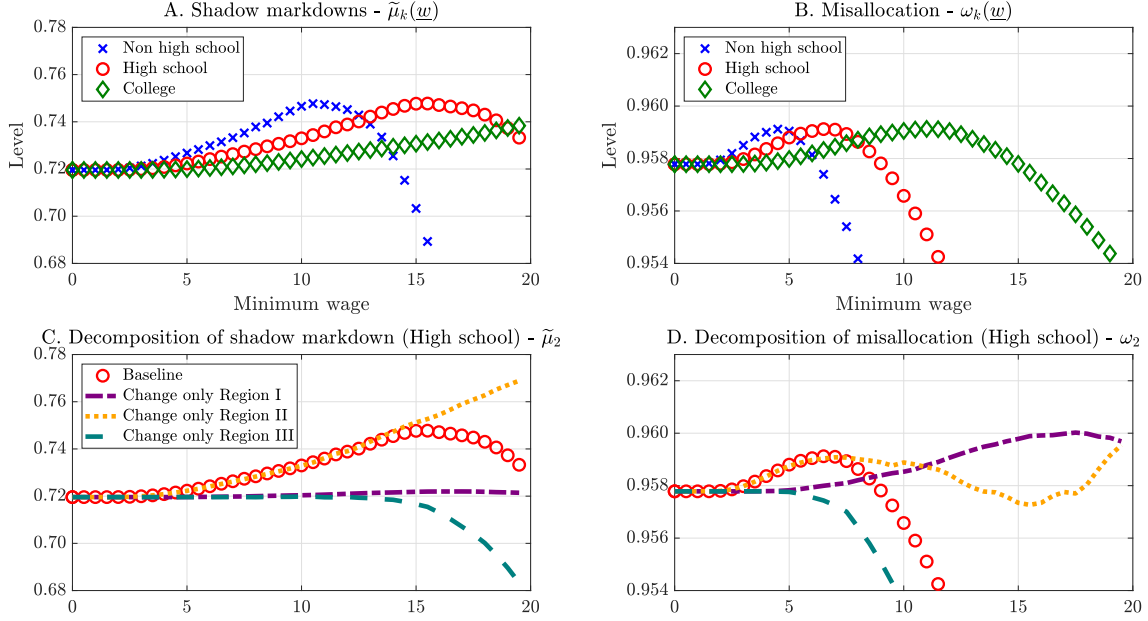
Figure 13: Decomposing efficiency wedges across the distribution of workers and firms

<u>Notes:</u> Panel A plots shadow markdowns faced by household types. Panel B plots misallocation by household types. Panel C decomposes the shadow markdown movements for high school grads into those due to each Region (I-III). Panel D repeats this exercise for misallocation.

Example market. To expand on the forces described above, Figure 14 provides an example of a market under a \$15 minimum wage. Each marker is a different firm, with most firms being in Region III under $\underline{w} = \$15$ (panel A). We use this figure to make three points.

First, the range of productivity for which firms are in Region II is small (orange diamonds). This limits the direct efficiency gains. Low productivity firms in Region II have relatively small market shares, and hence face a relatively elastic labor supply curve (equation 7). Therefore, the range over which the minimum wage situates them in Region II is relatively small, as small increases in the minimum wage quickly increase their employment toward the competitive level.

Second, once in Region III, the relatively flat marginal revenue product of labor schedule implies that firms quickly shrink (Panel C). In Panel C we compare actual employment to the firm's partial equilibrium employment n_{ij}^*, if it were to behave competitively, absent a minimum wage. Holding market aggregates fixed, n_{ij}^* equates firm labor supply and demand under a markdown of one:

$$w_{ij}^* = \widetilde{\alpha}\widetilde{z}_{ij}n_{ij}^{*\,\widetilde{\alpha}-1} \quad , \quad n_{ij}^* = \left(\frac{w_{ij}^*}{w_j}\right)n_j.$$

The largest firms in Region I have wider markdowns, so their size is relatively more distorted away from the competitive level. However firms in Region III, are even further away from their competitive size, due to rationing. This asymmetry, leads to large efficiency losses from firms in Region III. Figures in Appendix A.2 plot the fraction of jobs and employment in each region for each worker type.

Third, Panels E and F plot the contribution of each firm to the market-level shadow markdown and misallocation, using the proudctivity weighted formulas from Section 3.3. The largest contributions are
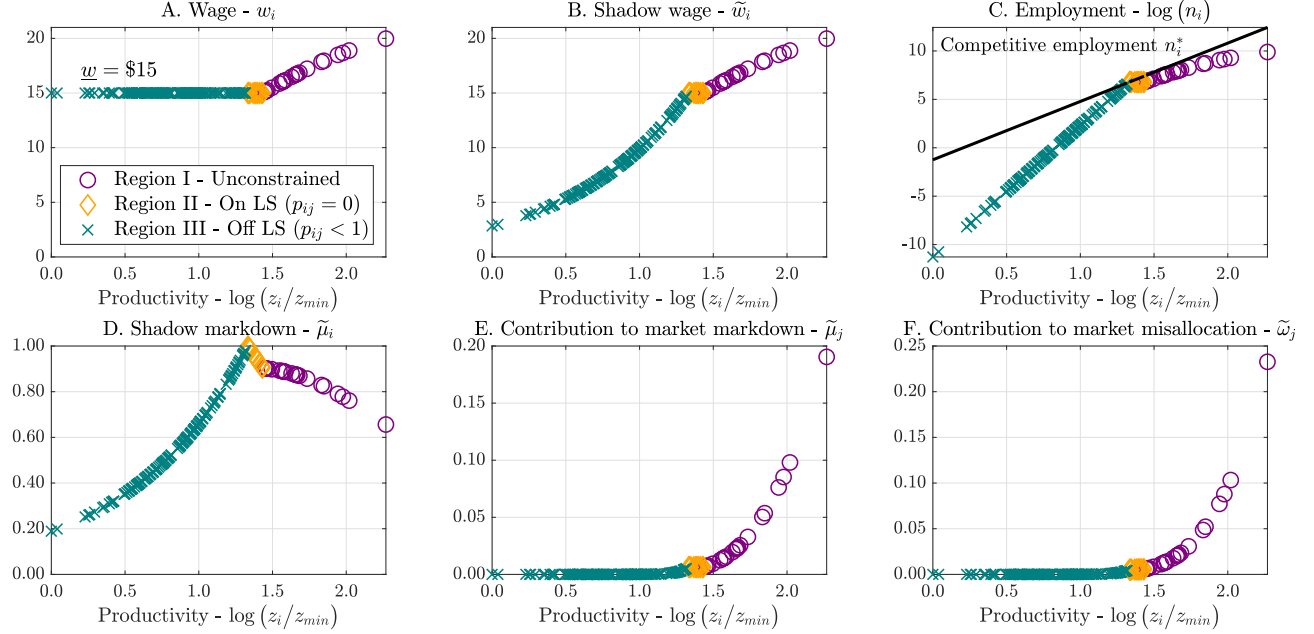
40

Figure 14: Example market - 200 firms, \$15 minimum wage and Non-high school workers

<u>Notes:</u> The pictured market has 200 firms and was drawn at random from markets with more than 150 firms.

from firms that have unconstrained wages that are far away from the minimum wage. As discussed above, large firms respond little to the increase in wages of their low wage competitors, as their low wage competitors have small market shares. Panels E and F show that had these responses been large, then they would have large effects on the market. The empirical evidence in Derenoncourt, Noelke, Weil, and Taska (2021) concerns competitors responding to wage changes at a large firm (Amazon), whereas the key question for the minimum wage is the large firm responses to small wage competitors, which we need a model to compute.

9 Robustness

First, we provide bounds on optimal minimum wages and welfare gains under different configurations of the aggregate elasticity of labor supply φ. Our key result that efficiency gains are relatively small holds across parameterizations. Second, we extend the model to multiple regions, calibrated to high, medium and low income US states. Quantitatively, we find very little regional heterogeneity in region-specific minimum wages. Third, we consider short-run effects of minimum wages by keeping capital fixed, as opposed to the previous results which can be viewed as long-run effects. We characterize the theory of the short-run and quantitatively find that optimal minimum wages fall by only around one dollar.

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Weights		Frisch		Optimal minimum wage		Welfare gain		Frac. due to
ψ_k		φ		No transfers	Transfers	No transfers	Transfers	redistribution
(1)		(2)		(5)	(6)	(7)	(8)	(9)
A. Utilitarian	Baseline	0.62		$15.12	$10.11	3.04%	0.20%	93.6%
	Low	0.30		$15.10	$7.67	3.01%	0.14%	95.4%
	High	0.86		$15.17	$8.87	3.05%	0.18%	94.0%
B. Negishi	Baseline	0.62		$6.97	$8.30	0.11%	0.17%	-
	Low	0.30		$6.87	$7.43	0.10%	0.14%	-
	High	0.86		$7.05	$7.97	0.11%	0.18%	-

Table 4: Robustness exercise - Varying the elasticity of labor supply φ

<u>Notes:</u> In the case of Utilitarian social welfare weights, column (9) gives gains due to efficiency and reports one minus the welfare gains with transfers (column 8) divided by the welfare gains with no transfers (column 7).

9.1 Sensitivity to the elasticity of labor supply

We assess the role of the aggregate elasticity of labor supply in the efficiency implications of the minimum wage. Recall the aggregate labor supply curve for each type of worker:

$$N_k = \pi_k \widetilde{\varphi}_k \left(\frac{\widetilde{W}_k}{P} \right)^{\varphi} C_k^{-\sigma\varphi}.$$

A higher Frisch elasticity of labor supply φ, increases the positive employment effects of the minimum wage when shadow wages are increasing, and increases the negative disemployment effects of the minimum wage when shadow wages are falling.

Approach. To assess this, we consider two values of φ either side of the baseline value of 0.62. These values are informed by our simple exercise in Appendix C using data from Golosov, Graber, Mogstad, and Novgorodsky (2021). Their results imply larger φ for high income households (lower MPC, higher MPE) than low income households (higher MPC, lower MPE). We consider the values for both groups: $\varphi \in \{0.30, 0.86\}$.[35] For each value we recalibrate all other shifters in Table 2 to match the same data as our baseline calibration.

Results. Table 4 provides the main result: levels of φ have essentially zero effect on our calculations. We conclude that our main results are robust to the Frisch elasticity of labor supply.

9.2 Sensitivity to heterogeneity in income across states

If average wages in a region are lower, or non-high school or high-school workers are a larger share of the population, this may have implications for the optimal minimum wage. To understand the scope of these potential differences across regions, we ask how our answers for the optimal minimum wage and its decomposition into efficiency and redistributive elements might depend on the level and distribution of wages.

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[35]This range subsumes the range used by the Congressional Budget Office when modeling policy, which is around 0.30 to 0.53. See the following (link).

Weights	Regions	Average	Fraction of workers				Optimal minimum wage		Welfare gain		Frac. due to
ψ_k (1)	by income (2)	wage (3)	< \$15 (4)	Non-HS (5)	HS (6)	Coll. (7)	No transfers (8)	Transfers (9)	No transfers (10)	Transfers (11)	redistribution (12)
A. Utilitarian	Baseline	\$19.69	29.3%	12.4%	52.8%	34.8%	\$15.12	\$10.11	3.04%	0.20%	93.6%
	Low	\$18.20	33.0%	12.4%	56.2%	31.4%	\$14.91	\$7.71	3.07%	0.18%	94.1%
	Medium	\$19.46	29.8%	12.0%	53.5%	31.4%	\$15.04	\$8.25	3.01%	0.17%	94.4%
	High	\$21.64	26.0%	12.9%	49.2%	31.4%	\$14.90	\$10.03	3.10%	0.16%	94.7%
	Mississippi	\$16.93	41.3%	15.8%	63.1%	21.2%	\$14.89	\$7.66	4.01%	0.18%	95.6%
B. Negishi	Baseline						\$6.97	\$8.30	0.11%	0.17%	-
	Low						\$6.67	\$7.50	0.12%	0.18%	-
	Medium		Same as above				\$6.92	\$7.75	0.11%	0.17%	-
	High						\$7.36	\$8.17	0.09%	0.14%	-
	Mississippi						\$6.22	\$6.88	0.12%	0.17%	-

Table 5: Robustness exercise - Optimal minimum wages by US region

<u>Notes:</u> In the case of Utilitarian social welfare weights, column (11) gives gains due to efficiency and reports one minus the welfare gains with transfers (column 10) divided by the welfare gains with no transfers (column 9). Baseline refers to the single region benchmark calibration from Tables 1 and 2. See footnote 37 for states included in Low, Medium and High income regions.

Approach. We split our economy into three separate regions, which we denote *r* and consider a separate household type for each region. We make the simplifying assumption that labor is immobile across regions.[36] We calibrate each region to data from three sets of US states, grouped by median household income, such that each region contains approximately one third of the civilian labor force.[37] Across regions, we keep some preference and technology parameters the same, as well as the distribution of number of firms in a market: $\{\beta, \theta, \eta, \delta, \alpha, \gamma, G(M_j)\}$. We calibrate region-specific shifters in labor disutility and productivity $(\overline{\varphi}_r, \overline{Z}_r)$, corresponding type-parameters $\{\overline{\varphi}_{kr}, \xi_{kr}\}_{k=1,r=1}^{K,R}$ and measures $\{\pi_{kr}\}_{k=1,r=1}^{K,R}$, to match CPS data from each region: the fraction of workers earning less than \$15 an hour, distribution of worker types, their relative average earnings per hour, and their share of region total labor income. Since the SCF does not identify an individual's state, we impose two further restrictions across regions. First, we keep the target moments for the ratio of household capital to labor income constant. Nonetheless, since other parameters change, we recalibrate the share parameters in each region $\{\kappa_{kr}\}_{k=1,r=1}^{K,R}$ to match the benchmark targets. Second, we keep constant across states the total fraction of *all households* that are owners.[38] We also assume average firm size is constant across regions. Columns 3 to 6 of Table 5 describe some of these moments. Relative to High income states, in Low income states the average wage is 16 percent lower, 7 percent more of the workforce has a wage below \$15 per hour, and 6.6 percent fewer of the workers have a college degree, with relatively similar proportion of workers that do not complete high school

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[36]In this economy, capital and consumption goods are traded at the same rental rate and price across all regions.

[37]States are allocated to regions as followed, ordered by 2019 median household income within each region: Low income states: MS, LA, NM, WV, AR, KY, AL, TN, GA, FL, OK, MT, MS, NC, SC, MI, SD. Medium income states: OH, WY, ID, IA, ME, IN, WI, TX, ND, RI, PA, AZ, NV, NY, CO, NE, KS, DE, VT. High income states: IL, OR, CA, AK, VA, MN, WA, UT, NH, CT, MA, NJ, HI, DC, MD.

[38]For example, if Group A has 37% of workers have a college degree, and Group B has 29%, then in both Group A and Group B we maintain that 6% of households are *college-owners* (Table 2) and set the share of households that are *college-workers* to 31% in Group A and 23% in Group B.

Results.   Table 5 provides the results of this exercise. In the absence of transfers, the optimal minimum wage varies very little, but follows the distribution of workers. In the case of utilitarian weights, $\underline{w}_\pi^*$ is slightly lower in regions with more non-high-school workers, whose utility declines most at higher minimum wages (Figure 12). Negishi weights prioritize college workers and owners, hence it may seem odd that $\underline{w}_\psi^*$ is higher in high income regions which have more of these types. However, with fewer workers on low wages, profits that accrue to college workers and owners decline less steeply as $\underline{w}$ increases.

There is larger and more systematic variation in the optimal minimum wage *with transfers*. The optimal minimum wage is monotonic in income, increasing a third between low and high income regions. With redistribution looked after, the region-government can push the minimum wage higher in high income states.[39] Despite this, the welfare gains are similar across states, and consistent with our baseline results: the efficiency gains—measured as welfare gains under optimal lump sum transfers—are similar and robust to social welfare weights.

A conclusion of this exercise is that *absent additional redistributive policy*, and conditional on a set of welfare weights, the welfare losses from a national minimum wage are minimal relative to a set of region-specific minimum wages. Appendix Section A.6 describes welfare by worker type in each region $\{\lambda_{kr}(\underline{w})\}_{k=1,r=1}^{K,R}$, replicating Figure 12. Appendix Section A.7 describes region-level welfare $\{\Lambda_r(\underline{w})\}_{r=1}^{R}$ and national welfare $\Lambda(\underline{w})$, which is a weighted harmonic mean across regions. We compare aggregate welfare under R optimal region minimum wages to aggregate welfare under one optimal national minimum wage. Under Utilitarian and Negishi weights, welfare losses from a national minimum wage are small.

Mississippi.   Mississippi (MS) has the lowest income per capita among the 50 U.S. states and a \$15 minimum wage would bind for 41.3 percent of residents. There currently is no state-level minimum wage in Mississippi. *A priori*, one would expect Mississippi to be the most likely state to experience a welfare loss from a federal \$15 minimum wage. However, we find that a federal minimum wage of \$15 yields welfare gains in Mississippi, even after we recalibrate our model economy to match the fraction of workers earning less than \$15 an hour (MS 41.3% vs. US 29.4%), distribution of worker types, their relative average earnings per hour, and each household types' share of total labor income.

Two off-setting forces lead to this result. Average wages are lower in MS, which would push toward a lower optimal minimum wage. However, despite the fact that only 21.2% of those in Mississippi have a college degree (versus 24.9% in the U.S.), there are significantly more individuals who have a high-school degree (63% in MS vs. 52.8% in U.S.). High-school graduates prefer a higher optimal minimum wage of roughly \$17 (see Figure 12). These effects wash out and the optimal minimum wage is similar to that of the US as a whole.

These examples shed light on the relative stability of the optimal minimum wage. In states with fewer college educated, high-wage workers there are typically more high-school workers who actually prefer higher minimum wages. These offsetting compositional forces generate stable minimum wages

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[39] We assume redistribution by lump sum taxes within, but not across, regions. This is equivalent to a balanced budget requirement at the region-level.

across disparate regions.[40]

9.3 Sensitivity to short vs. long run

In comparing steady-states we are implicitly studying the long-run effects of the minimum wage. Our theory suggests a smaller optimal minimum wage in the short-run if the cost of labor increases but the level and distribution of capital across workers in each firm is slow to adjust. Even assuming maximal stickiness in reallocation of capital, we find these effects are quantitatively small.

Approach. To compare short- and long-run effects we vary the minimum wage, but keep capital at the firm fixed for each type at the allocation $\bar{k}_{ijk}$ under a zero minimum wage. We think of these as firm-worker specific installations of capital. Firm profits from each type are as follows, with three main implications:

$$\pi_{ijkt} = \overline{Z}\xi_k \left(z_{ij}\bar{k}_{ijk}^{(1-\gamma)\alpha} \right) n_{ijk}^{\gamma\alpha} - w_{ijk}n_{ijk} - R\bar{k}_{ijk}.$$

First, with fixed capital, the production function has sharper decreasing returns in labor: $\gamma\alpha < \tilde{\alpha}$. Second, firms face overheard costs of pre-installed capital, $R\bar{k}_{ijk}$, which will cause firms to shut down non-profitable jobs at high minimum wages. This requires adding an endogenous margin of operation into the solution of the model.[41] Third, the aggregate equilibrium conditions are the same as in Lemma 3, minus the capital demand condition. Capital supply remains infinitely elastic at $R = 1/\beta + (1 - \delta)$, but capital demand is pinned down at $\overline{K}(\underline{w}) = \sum_k \int \sum_i \chi_{ijk}(\underline{w})\bar{k}_{ijk}\,dj$, where $\chi_{ijk}(\underline{w}) \in \{0,1\}$ indicates whether the firm operates worker-type-k capital in equilibrium under minimum wage $\underline{w}$.

Theory. Figure 15 characterizes the mechanism behind a lower optimal minimum wage in this environment. Panel A considers a firm in an economy without a minimum wage, where capital is fixed at the allocation consistent with long-run employment n_{ij}^*. Short-run marginal and average products coincide with long-run values at this point. Away from n_{ij}^*, short-run $mrpl_{ij}^{SR}$ is steeper due to sharper decreasing returns with fixed capital: if $n_{ij} > n_{ij}^*$, then $mrpl_{ij}^{SR} < mrpl_{ij}^{LR}$. With fixed overhead capital, the $arpl_{ij}^{LR}$ goes to zero as n_{ij} goes to zero since overhead per worker explodes. The peak in $arpl_{ij}^{SR}$ intersects $mrpl_{ij}^{SR}$ and gives the maximum minimum wage the firm could afford and still operate type-k capital: $\underline{w}_{ij}^{Max}$. At a higher minimum wage, equating $\underline{w} = mrpl_{ij}^{SR}$ would imply $arpl_{ij}^{SR} < \underline{w}$ and shutdown is optimal.

Panels B and C show how these differences constrain the positive efficiency gains from narrowing $\tilde{\mu}_k$. Take the firm in Panel A, in the long run, at the minimum wage pictured in Panel B, the firm is in

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[40] As a proof of concept that the model can generate lower optimal minimum wages, Appendix Figure A11 shows that *counterfactually* large differences across regions in the moments used to calibrate the model can have significant effects on the optimal minimum wage. If we calibrate the model such that *65 percent of workers* have a wage less than $15, then the optimal minimum wage declines by about $3 under either Utilitarian or Negishi weights. This is counterfactual in that Mississippi has only 41.3 percent of workers at less than $15.

[41] Market-by-market we first assume that all firms enter, and then solve the Nash equilibrium of the market and general equilibrium of the economy. We then compute firm-type profits π_{ijkt}, which account for fixed capital costs. If any firm has profits $\pi_{ijkt} < 0$, we drop the lowest productivity firm in the market and then solve the market equilibrium again. With fewer firms, labor market power of the remaining firms increases, which increases profits, hence the need to remove only one firm at a time. We continue in this way until we reach a Cournot Nash equilibrium: no firm with shut-down jobs wishes to re-open them given competitor's operation and intensive margin labor decisions. This general equilibrium algorithm is similar to that in Loecker, Eeckhout, and Mongey (2021).
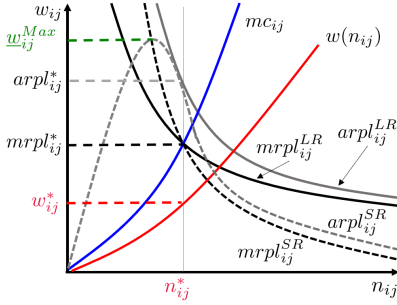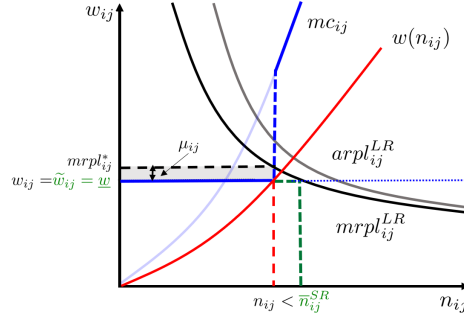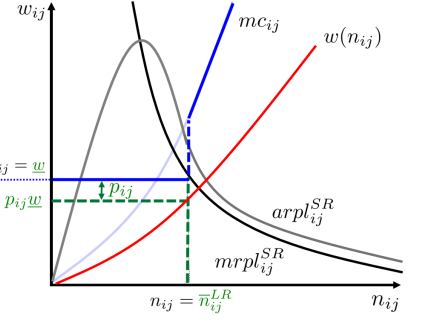
Figure 15: Partial equilibrium theory of minimum wage with capital fixed in the short-run
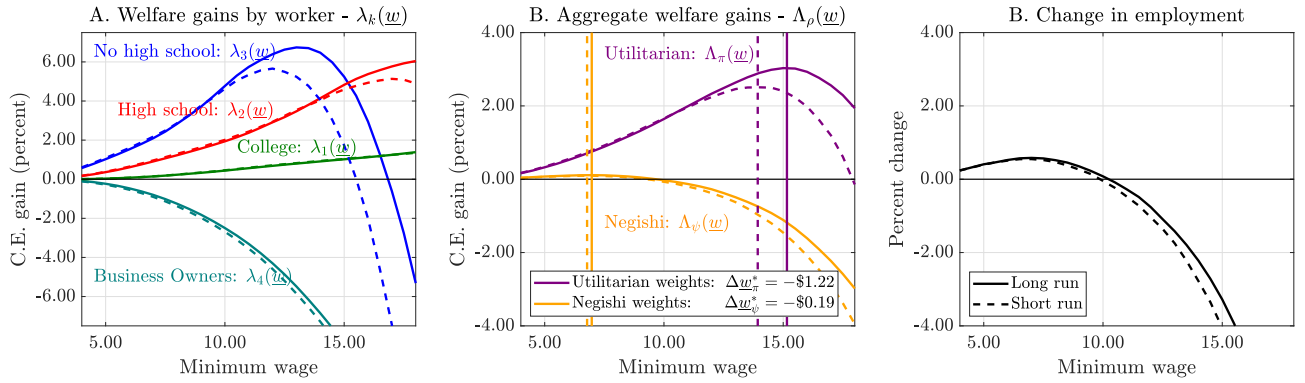

Figure 16: Short- and long-run effects of minimum wages

<u>Notes:</u> Long-run results, which are identical to Figure 12, in solid lines, short-run results in dashed lines. Legend in Panel B gives the decline in the optimal minimum wage when comparing short-run to long-run.

Region II: employment is non-rationed ($n_{ij} < \overline{n}_{ij}^{SR}$), and wages are a narrower markdown on $mrpl_{ij}^{LR}$. A small increase in the minimum wage *increases employment* and *narrows shadow markdowns*. Panel C considers the short run, at the same minimum wage. The lower $mrpl_{ij}^{SR}$ places the firm in Region III, where employment is rationed. A small increase in the minimum wage now *decreases employment* and *widens shadow markdowns*. In the short run, the range of $\underline{w}$ over which firms are in Region II—where $\widetilde{\mu}_{ij}$ is narrowing in $\underline{w}$—is smaller. This constrains the efficiency gains from improvements in $\widetilde{\mu}_k$ for non-college workers.

Results. Figure 16 plots the results. Panel B shows that the short-run optimal minimum wage under Utilitarian weights declines by about one dollar. Consistent with the theory, this is driven by a sharp decline in the welfare gains to non-college households. With capital being unable to adjust, the welfare losses to business owners are slightly larger. Under the Negishi weights the optimal minimum wage declines by only 20 cents. Panel C shows that the positive employment effects of the minimum wage are slightly more limited. This exercise delivers the additional result that, quantitatively, short- and long-run elasticities in our model are similar which is reassuring for our earlier interpretations of empirical studies of short-run changes.

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9.4 Incomplete markets

In our baseline model with limited household heterogeneity, the optimal federal minimum wage with transfers ranges from $7 to $10, very close to the current federal minimum wage of $7.25. While our model only features market incompleteness *across* types, our framework can readily incorporate more types given the data necessary to discipline the additional parameters. In previous iterations of this paper (available upon request), we treated high-school graduates and non-high school graduates as one single household type which could fully insure against employment losses *within* the household. We found a very similar efficiency maximizing minimum wage. As discussed above, we find that the efficiency maximizing minimum wage in an economy with heterogeneity is very similar to the efficiency maximizing minimum wage in an economy with no heterogeneity, where social welfare weights play no role (Appendix A.5). Adding more household heterogeneity alters redistributive motives provides limited pressure on the efficiency maximizing minimum wage.

With respect to the optimal minimum wage under Utilitarian weights and without transfers, we also argue that further disaggregating households will have little effect. Suppose non-high school workers were split into equally sized groups A and B, where Group A suffers greater employment losses. The non-linearity of employment losses in Group A—sharp declines in utility past the group-specific optimum (Figure 12)—would lead to a reduction in the optimal minimum wage. However if Group A is smaller than Group B, Group B's preference for a higher minimum wage would lead to a higher optimal minimum wage. For a Utilitarian government, non-linear employment losses of Group A but a larger size of Group B net out. We anticipate that splitting non-high-school workers into a small group that is highly affected by the minimum wage, and a larger group that is less affected would yield such a result. Indeed this is what we found when moving from three workers types Non-college, College, Owners, to four worker types Non-high-school, High-school, College, Owners. Non-high school workers are more sensitive to the minimum wage, but there are more than 4 high school graduates for every non high school graduate (Table 2B).

10   Conclusion

In this paper we have provided a theoretical framework for studying the effect of minimum wages on welfare and the allocation of employment across firms in the economy. The framework has three key features. First, each market features strategic interaction between firms, which we have shown to be important for (i) quantifying the reallocating effects of minimum wage policies, (ii) interpreting empirical evidence documenting such reallocation, (iii) interpreting empirical evidence on employers' responses to competitors' minimum wages. Second, workers are of heterogeneous types, which allows us to decompose the heterogeneous impacts of minimum wages on employment, labor and capital income. Third, we provide a parsimonious nesting of this market model into a general equilibrium economy and show how the economy aggregates, allowing for a succinct representation of the efficiency effects of minimum wages via two wedges: the *shadow markdown* $\widetilde{\mu}$, and misallocation ω. We showed that, when calibrated

to US data, this model is consistent with a wide body of empirical research on the effects of minimum wage changes.

We have shown that in such an economy an optimal minimum wage exists, and that this trades-off positive effects on markdowns against negative misallocation effects. Quantitatively, we find that the efficiency maximizing minimum wage is around $7 per hour, consistent with the current US Federal minimum wage, but that higher minimum wages can be justified through redistribution. Under Utilitarian social welfare weights, and ignoring alternative fiscal instruments for redistribution, we find an optimal minimum wage of around $15 an hour. Under such a policy, 95 percent of welfare gains come from redistribution and only 5 percent from improved efficiency, with welfare gains equivalent to a 3 percent increase in TFP.

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This Appendix is organized as follows. Section A provides additional tables and figures referenced in the text. Section B details the algorithm for solving the minimum wage economy. Section C contains derivations associated with our calibration of preference parameters. Section D contains mathematical derivations of all equations in the main text and derivations associated with the solution of the government problem and its implementation via lump sum transfers.

A Additional tables and figures

A.1 Card and Krueger (1994) in a model calibrated to 2019 data

To show it is feasible to get positive or negative employment effects from a minimum wage increase, we consider an approximate replication of Card and Krueger (1994). This seminal paper studies an 80c minimum wage increase from $4.25 to $5.05 in 1992. In 2019 dollars, this corresponds to a $1.46 minimum wage increase from $7.74 to $9.20. Figure A1 plots the log change in employment and average wages following a $1.46 increase in the minimum wage in the model. We plot these for various initial values of the minimum wage, and mark the $7.74 initial $\underline{w}$ with a vertical line. Running this experiment in our model calibrated to 2019 data would generate a negative employment effects for all workers or when we restrict attention to workers of the two non-college groups. However it is clear that at lower initial levels of the minimum wage the estimated effects may be positive. The experiment is also consistent with findings in Clemens and Strain (2021), that employment effects tend to be positive for low initial minimum wages and small minimum wage increases.
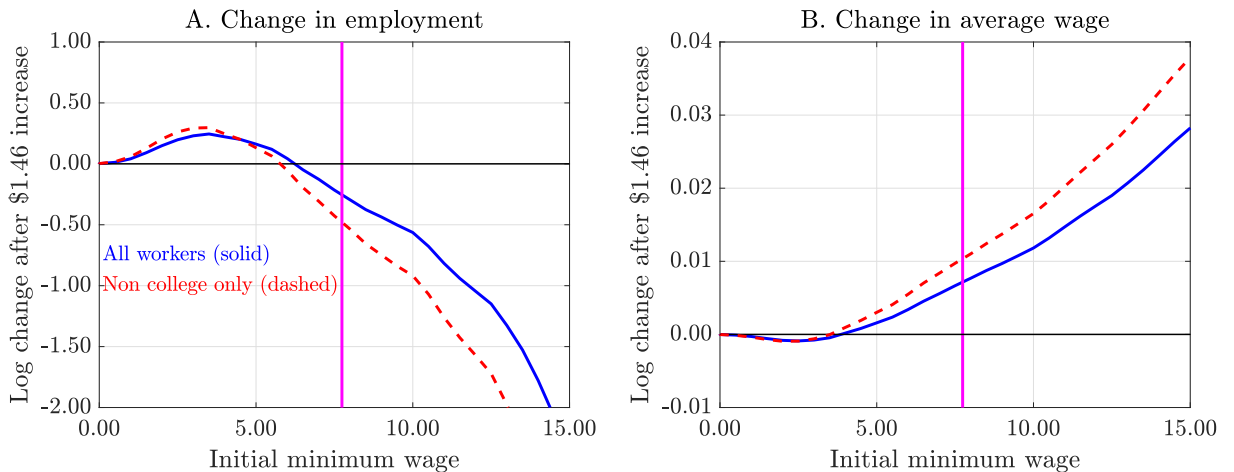

Figure A1: Replication of Card and Krueger (1994)

Notes: Vertical line corresponds to an initial minimum wage of $7.74. This is in 2019 dollars. In 1992 dollars, this corresponds to $4.25, which is the initial minimum wage in the Card and Krueger (1994) study.

A.2 Distribution of activity across regions

Figure A2 plots the fraction of production units (e.g. a firm-worker-type pair), and fraction of employment in each of the three Regions described in Section 2.
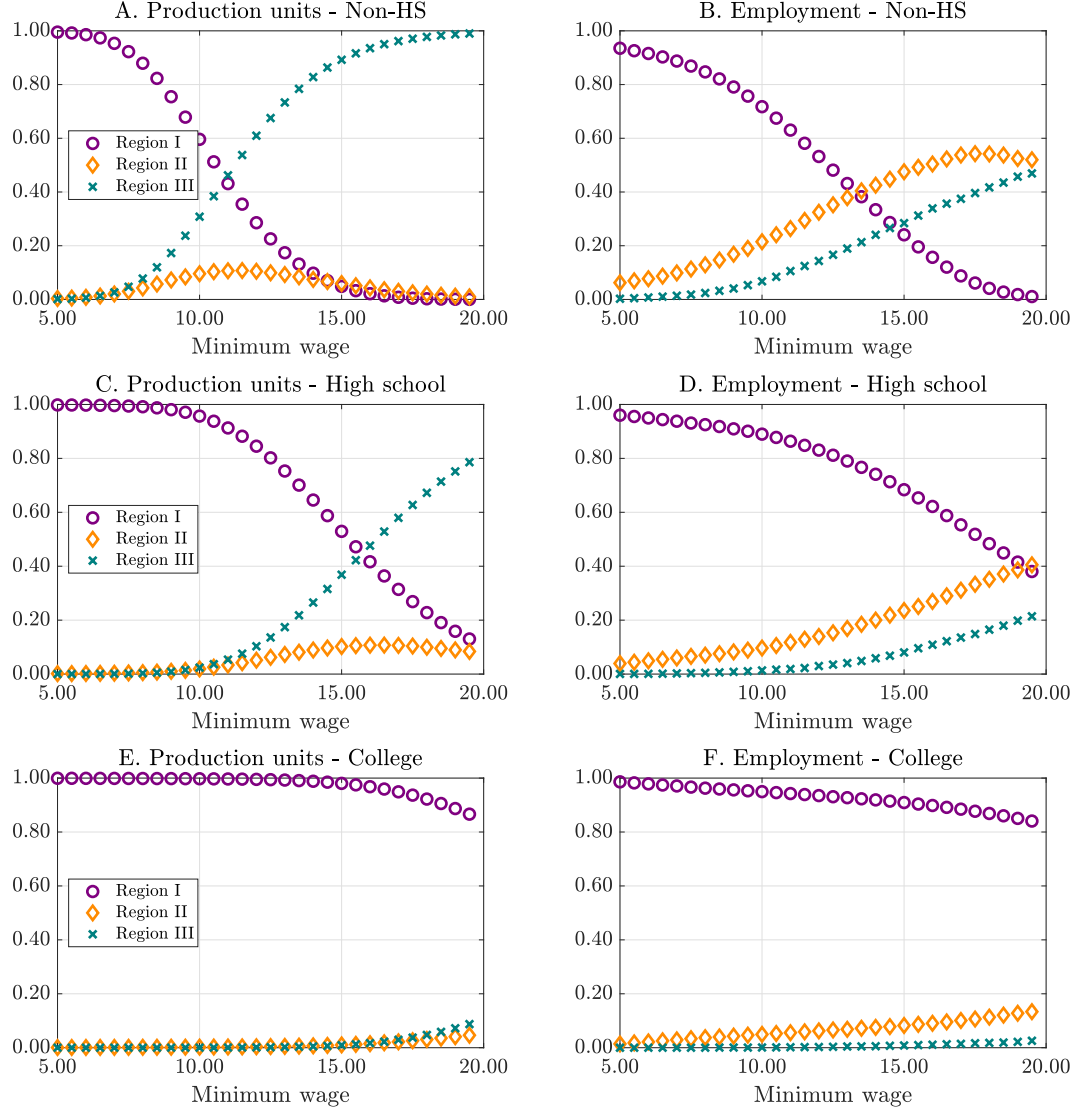
Figure A2: Distribution of positions and employment across regions - High school workers

Notes: A production unit is a firm-worker-type pair. For example, firm i in market j employs all four types of labor, and for each type of labor it may either be unconstrained by the minimum wage (Region I), constrained by the minimum wage but on its labor supply curve (Region II), constrained by the minimum wage but on its labor demand curve (Region III).

A.3 Alternative calibration of worker-type productivity parameters $\{\tilde{\xi}_k\}_{k=1}^K$

Figures A3 and A4 give results for an alternative calibration of the model in which the the three values of ξ_k are chosen to match the *average wage of each worker type*, and we normalize $\overline{Z} = 1$. Figures A3B compares the distribution of wages under this calibration to the baseline (thin lines). For all workers (black) and for non-high school workers (blue) the fraction of workers with a wage less than \$15 shifts from being higher than what is observed in the data (under the baseline), to less than what is observed in the data (under the alternative calibration). Nonetheless, Figure A4 shows similar magnitudes of welfare gains from the optimal minimum wage (compare to Figure 12). Moreover, the welfare gains associated with efficiency—which we have shown are consistent with welfare gains under the Negishi weights with no lump sum transfers—remain small and the optimal minimum wage reflecting efficiency is less than \$9.
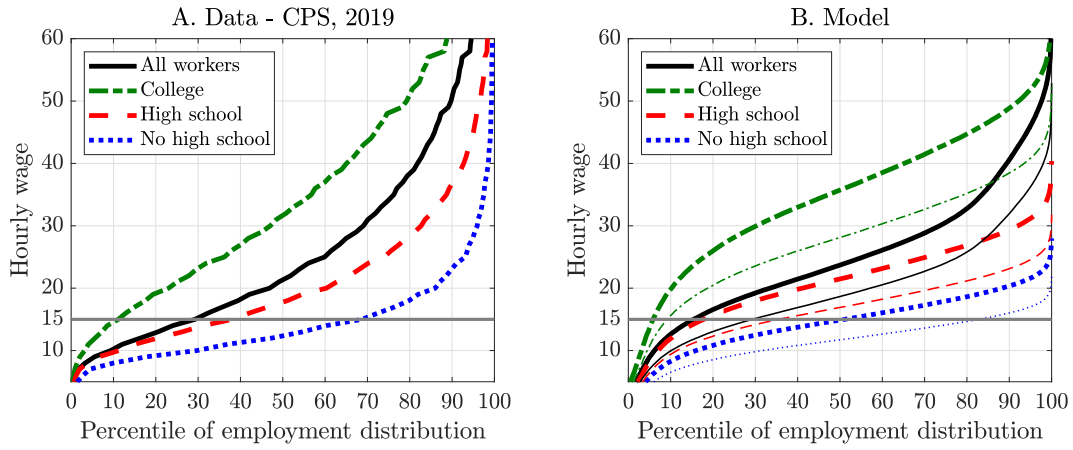
Figure A3: Distribution of wages by worker type and \$15 minimum wage - Alternative calibration

Notes: For Panel A see footnotes to Figure 5. Thick lines in Panel B refer to the alternative calibration of the model (see text). Thin lines in Panel B refer to the baseline calibration in the text, in which the two values of ξ_k are chosen to match the ratio of average college-to-non-high-school and college-to-high-school worker wages and ξ_k for college workers is normalized to one, and $\overline{Z}$ is chosen to match the fraction of workers for which a \$15 minimum wage would bind.
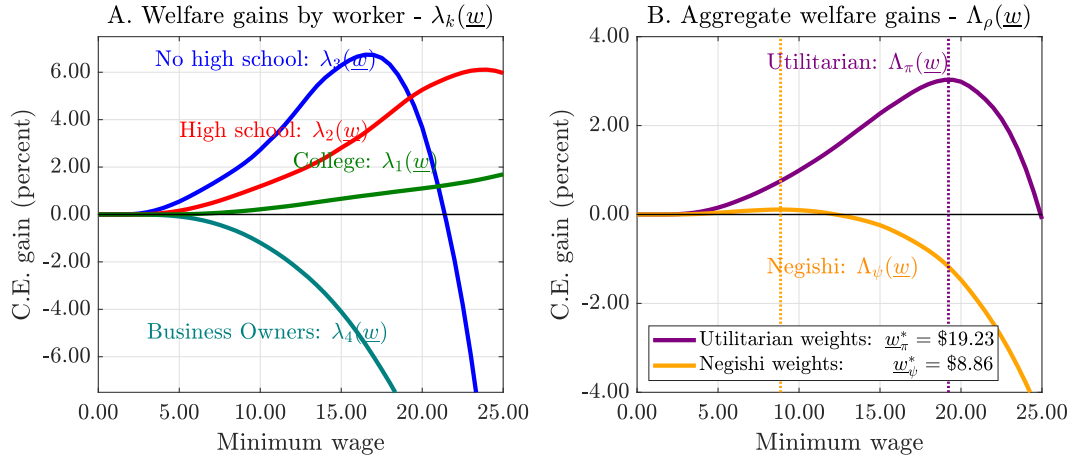

Figure A4: Minimum wages and welfare - Alternative calibration

Notes: For details see footnote to Figure 12.

A.4 Effective social welfare weights for alternative minimum wage benchmarks

In the main text we compute the welfare gains relative to a zero minimum wage economy. This implies that the social welfare weights ψ_k are benchmarked to a zero minimum wage economy. For example, when using *Negishi weights*, then the welfare weights can be shown to be equal to consumption shares in the benchmark (i.e. zero minimum wage economy): $\psi_k = \psi_k^*(0)$. Figure A5 shows that our findings are robust to alternative benchmarks since, quantitatively, the effective weights move very little with the minimum wage. Panel A plots the implied Negishi weights $\psi_k^*(\underline{w})$, and shows that they vary little as the minimum wage changes, while in Panel B, Utilitarian weights are constant. Panels C and D show how these map into the *effective weights*, described below, which are also relatively flat.

Recall the main formula from the text, but now written to reflect that the welfare gains in an economy with minimum wage $\underline{w}_1$ are being computed with respect to a benchmark economy indexed by a minimum wage $\underline{w}_0$:

$$
1 + \Lambda(\underline{w}_1) = \left[\sum_k \widetilde{\psi}_k(\underline{w}_0) \left(1 + \lambda_k(\underline{w}_1)\right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \quad , \quad \widetilde{\psi}_k(\underline{w}_0) := \frac{\psi_k(\underline{w}_0) \left(\frac{c_k(\underline{w}_0)}{\pi_k}\right)^{1-\sigma}}{\sum_k \psi_k(\underline{w}_0) \left(\frac{c_k(\underline{w}_0)}{\pi_k}\right)^{1-\sigma}}. \tag{A1}
$$

For different $\underline{w}_0$, Figure A5 plots in panels A and B the underlying weights $\psi_k(\underline{w}_0)$ for A. Negishi weights, and B. Utilitarian weights. Panels C and D plot the *effective weights* $\widetilde{\psi}_k(\underline{w}_0)$ for C. Negishi weights, and D. Utilitarian weights. The main conclusion is that $\widetilde{\psi}_k(0)$, which are the effective weights used as the benchmark in the text, are very similar to $\widetilde{\psi}_k(\underline{w}_0)$ for alternative benchmark minimum wages $\underline{w}_0 \in [0, 20]$.
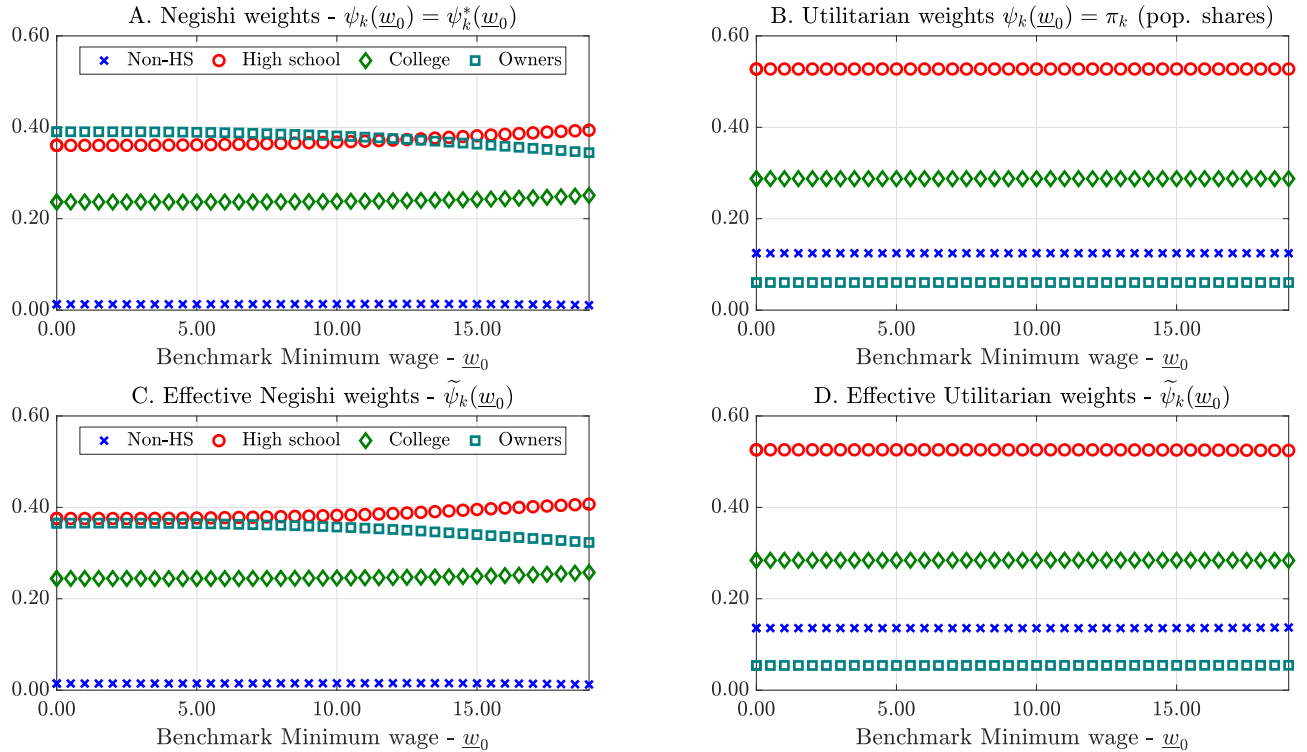

Figure A5: Negishi and Utilitarian weights as the minimum wage varies

A.5 Alternative calibration of heterogeneity - Homogeneous worker calibration

Our baseline model has four types of workers, here we show results for an alternative specification with only one type of worker. The parameters $\overline{Z}$, $\overline{\varphi}$ are still calibrated to match the same baseline targets as the benchmark model: 29 percent of workers earn less than \$15 in the initial economy, and average firm employment is 22.8 workers. We compare aggregate welfare and the decomposition of welfare for this homogeneous worker economy to the benchmark economy under the implied Negishi weights.

Figure A6 plots the results. The orange lines are those for the benchmark economy with heterogeneous workers, and are identical to the orange lines in Figure 12B and 12D. The grey lines are those of the homogeneous worker economy. The main result is that the welfare gains from the minimum wage are similar in shape and size in the homogeneous worker economy and the heterogeneous economy under the implied Negishi weights. They also imply optimal minimum wages that are within one dollar of each other. This further reflects the extent to which minimum wages that are more than \$10 are welfare improving precisely due to their interaction with the heterogeneity in the economy via redistribution, rather than through efficiency. We view this as a further check on our efficiency results.
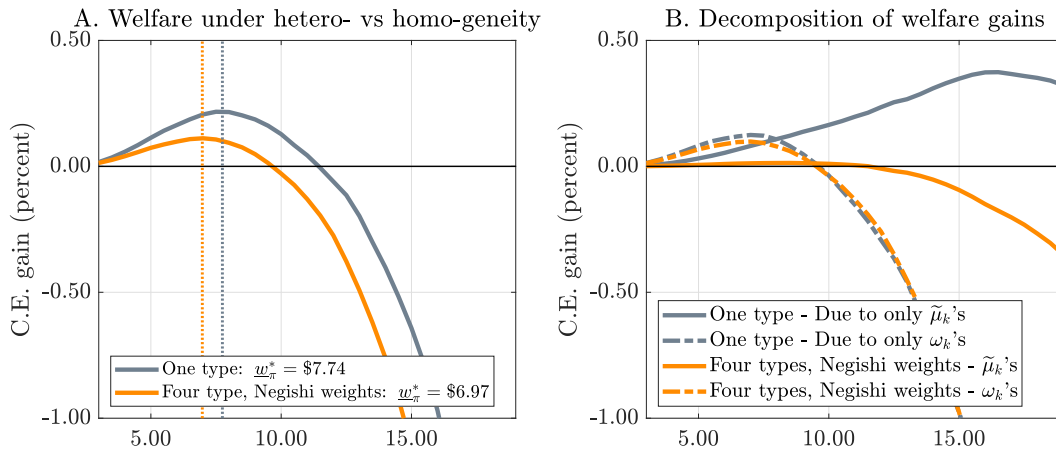

Figure A6: Minimum wages and welfare - Homogeneous vs. Heterogeneous households

Notes: For details see text.

A.6 Within region welfare results for heterogeneous state calibration

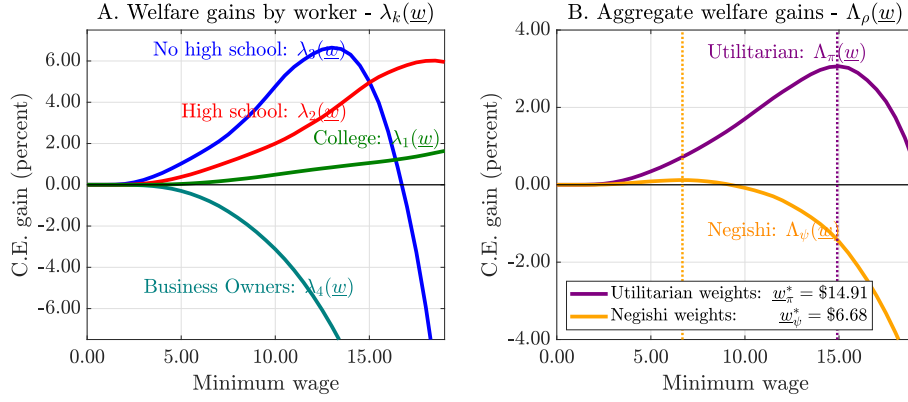Figure A7 to A9 plot within-region welfare comparative statics and region-specific optimal minimum wages.

Figure A7: Low income states
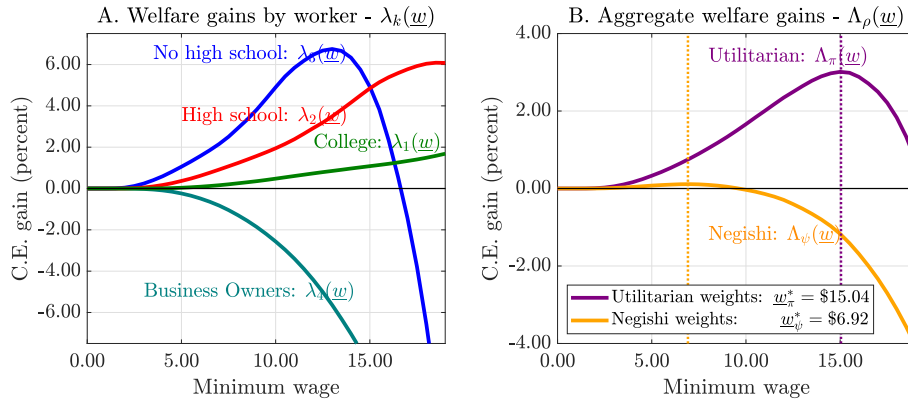
Figure A8: Medium income states
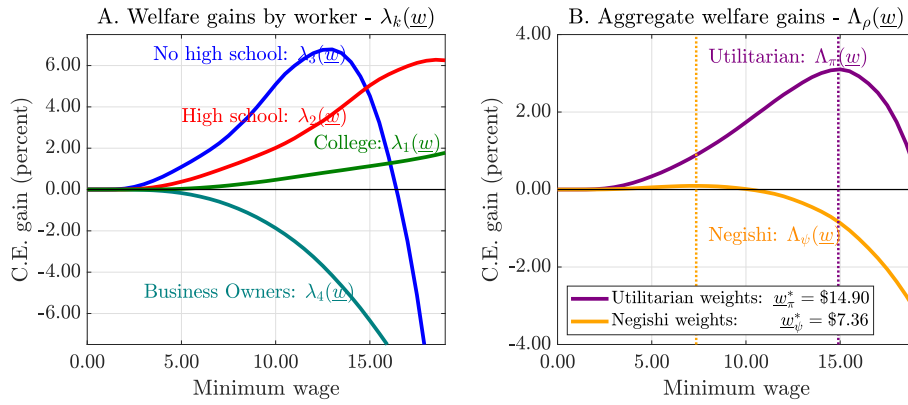
Figure A9: High income states

A.7 Welfare gains under region-specific and national minimum wage

In Section 9 we computed optimal minimum wages in a set of three regions. Figure A10 plots the welfare gains in each region, along with aggregate welfare gains. Aggregate welfare gains are simply the harmonic mean of region welfare gains, since each region accounts for one third of the total population. The red crosses in each panel denote the *average of region-specific optimal minimum wages*, these are the minimum wages that attain the maximum of welfare in each state, and the aggregate welfare associated with the region-optimal minimum wages. The black line gives aggregate welfare under a *national minimum wage*, and the black circle gives the associated welfare maximizing minimum wage and welfare. In both cases the aggregate welfare gains associated with region specific optimal minimum wages are two orders of magnitude less than the level. Under utilitarian weights, the welfare gains from a national minimum wage are 3.059 percent, and 3.060 percent under region-specific minimum wages. Under Negishi weights, the welfare gains from a national minimum wage are 0.108 percent, and 0.109 percent under region-specific minimum wages.

Figure A10: Minimum wages and welfare in a multiregion calibration

Notes: For details see text.

Figure A11: Counterfactual calibration such that 65 percent of workers initially earn less than $15 per hour

Notes: For details see text.

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B Algorithm for the minimum wage economy

The aim of this section is to clearly lay out the algorithm for solving the minimum wage equilibrium, and to present a full solultion of a simplified model, which may be pedagogically useful relative to the extensive derivations in Ap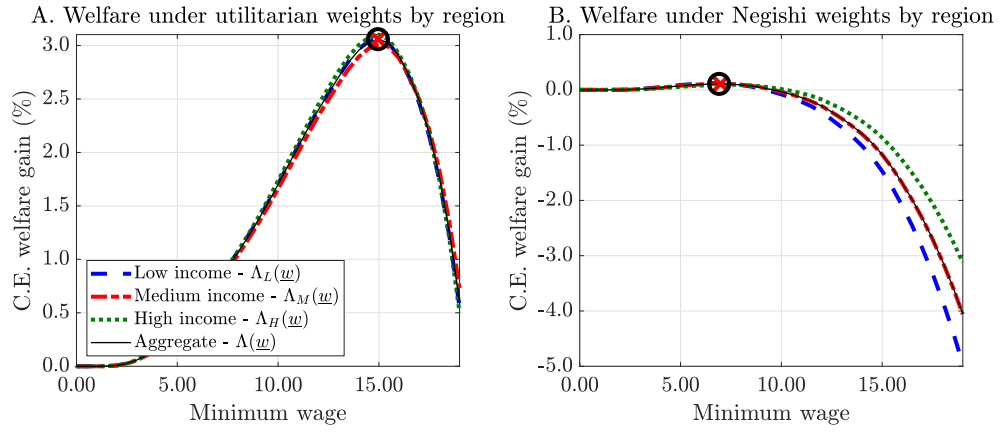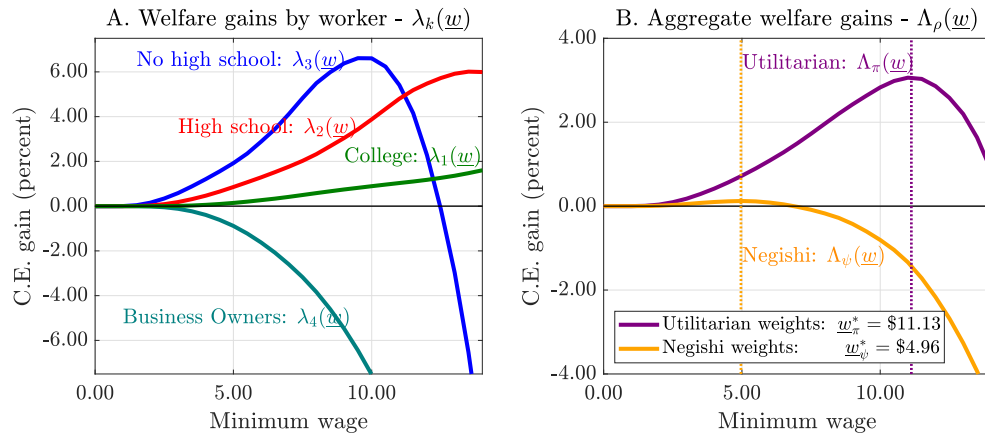pendix D. The algorithm for the minimum wage equilibrium is nested in the broader solution to the equilibrium of the model described in Appendix D.

For ease of exposition, we lay out the minimum wage problem (i) ignoring capital, (ii) consider an economy with a single type of household, (iii) to simplify exposition we also consider GHH preferences, which are not used in the main text, (iv) as well as a static environment, (v) set the coefficient on labor in utility $\overline{\varphi} = 1$. We derive conditions for this simplified economy and then present the algorithm.

B.1 Model

- Consider the household problem with the rationing constraint $n_{ij} \leq \overline{n}_{ij}$. For ease of interpretation we attach multiplier $\zeta_{ij} = \lambda w_{ij} \left(1 - p_{ij} \right)$ to the rationing constraint, normalized by the household budget multiplier λ:

$$U_0 = \max_{\{n_{ij}, c_{ij}\}} u \left(C - \frac{N^{1+\frac{1}{\varphi}}}{1 + \frac{1}{\varphi}} \right)$$

$$
\begin{aligned}
C &= \int \sum_{i \in j} w_{ij} n_{ij} \, dj + \Pi \qquad [\lambda] \\
n_{ij} &\leq \overline{n}_{ij} \qquad [\lambda w_{ij} \left(1 - p_{ij} \right)] \\
C &= \int \sum_{i \in j} c_{ij} dj \\
N &= \left[\int n_j^{\frac{\theta+1}{\theta}} \, dj \right]^{\frac{\theta}{\theta+1}} \\
n_j &= \left[\sum_{i \in j} n_j^{\frac{\eta+1}{\eta}} \, dj \right]^{\frac{\eta}{\eta+1}}
\end{aligned}
$$

- The first order condition for n_{ij} yields

$$\lambda w_{ij} - \lambda w_{ij} \left(1 - p_{ij} \right) = u' \left(\cdot \right) \left(\frac{\partial n_j}{\partial n_{ij}} \right) \left(\frac{\partial N}{\partial n_j} \right) N^{\frac{1}{\varphi}}$$

$$\lambda w_{ij} p_{ij} = u' \left(\cdot \right) \left(\frac{\partial n_j}{\partial n_{ij}} \right) \left(\frac{\partial N}{\partial n_j} \right) N^{\frac{1}{\varphi}}$$

- The first order condition for consumption yields $u' \left(\cdot \right) = \lambda$.

- Define the *shadow wage* $\widetilde{w}_{ij} = p_{ij} w_{ij}$, use the first order condition for consumption $u' \left(\cdot \right) = \lambda$, and

use the derivatives of N and n_j:

$$\widetilde{w}_{ij} = \left(\frac{n_{ij}}{n_j}\right)^{\frac{1}{\eta}} \left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}} \qquad (*)$$

- Now define the *shadow wage indexes*

$$\widetilde{w}_j = \left[\sum_{i \in j} \widetilde{w}_{ij}^{1+\eta}\right]^{\frac{1}{1+\eta}} \quad , \quad \widetilde{W} = \left[\int \widetilde{w}_j^{1+\theta} dj\right]^{\frac{1}{1+\theta}}.$$

- Using these definitions in $(*)$ along with the definition of n_j:

$$\sum_{i \in j} \widetilde{w}_{ij}^{1+\eta} = \left[\left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}\right]^{1+\eta} \sum_{i \in j} \left(\frac{n_{ij}}{n_j}\right)^{\frac{1+\eta}{\eta}}$$

$$\widetilde{w}_j = \left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}$$

- Using this along with the definition of N:

$$\int \widetilde{w}_j^{1+\theta} dj = \left[N^{\frac{1}{\varphi}}\right]^{1+\theta} \int \left(\frac{n_j}{N}\right)^{\frac{1+\theta}{\theta}} dj$$

$$\widetilde{W} = N^{\frac{1}{\varphi}}$$

- Note that $\widetilde{W}N \neq \int \sum_{i \in j} w_{ij} n_{ij} dj$, however the aggregate labor supply $N = \widetilde{W}^{\varphi}$ is *as if*, the household had maximized

$$U_0 = \max_{C,N} u\left(C - \frac{N^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}\right) \quad \text{subject to} \quad C = \widetilde{W}N + \Pi.$$

This makes clear the extent to which the shadow wage index $\widetilde{W}$ captures the full distribution of binding minimum wages.

- Note that shadow wages aggregate:

$$\widetilde{w}_{ij} n_{ij} = n_{ij}^{\frac{1+\eta}{\eta}} \left(\frac{1}{n_j}\right)^{\frac{1}{\eta}} \left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}$$

$$\sum_{i \in j} \widetilde{w}_{ij} n_{ij} = \left[\sum_{i \in j} n_{ij}^{\frac{1+\eta}{\eta}}\right] \left(\frac{1}{n_j}\right)^{\frac{1}{\eta}} \left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}$$

$$\sum_{i \in j} \widetilde{w}_{ij} n_{ij} = n_j \widetilde{w}_j$$

- **Shadow shares** - We can define the shadow share $\widetilde{s}_{ij}$ as

$$\widetilde{s}_{ij} := \frac{\widetilde{w}_{ij} n_{ij}}{\sum_{i \in j} \widetilde{w}_{ij} n_{ij}}.$$

Substituting in the labor supply system $(*)$ for $\widetilde{w}_{ij}$

$$\widetilde{s}_{ij} := \frac{n_{ij}^{\frac{1+\eta}{\eta}}}{\sum_{i \in j} n_{ij}^{\frac{1+\eta}{\eta}}} = \left(\frac{n_{ij}}{n_j}\right)^{\frac{1+\eta}{\eta}} = \left(\frac{\widetilde{w}_{ij}}{\widetilde{w}_j}\right)^{1+\eta}$$

- The firm's problem is

$$\pi_{ij} = \max_{n_{ij}} z_{ij} n_{ij}^{\alpha} - w_{ij} n_{ij}$$

subject to

$$n_{ij} = \left(\frac{\widetilde{w}_{ij}}{\widetilde{w}_j}\right)^{\eta} \left(\frac{\widetilde{w}_j}{\widetilde{W}}\right)^{\theta} N$$

$$w_{ij} \geq \underline{w}$$

- Let $r_{ij} \in \{1, 2, 3\}$ denote the region that the firm is in.

- **Region I -** If the firm is in Region I, then its wage is the optimal markdown on the marginal revenue product of labor

$$w_{ij} = \mu_{ij} \alpha z_{ij} n_{ij}^{\alpha-1}$$
$$p_{ij} = 1$$
$$\widetilde{w}_{ij} = w_{ij}$$
$$n_{ij} = \left(\frac{w_{ij}}{\widetilde{w}_j}\right)^{\eta} \left(\frac{\widetilde{w}_j}{\widetilde{W}}\right)^{\theta} \widetilde{W}^{\varphi}$$

where the markdown depends on its shadow share of the labor market. That is, $\mu_{ij} = \mu\left(\widetilde{s}_{ij}\right)$, where $\mu\left(\widetilde{s}_{ij}\right) = \frac{\varepsilon\left(\widetilde{s}_{ij}\right)}{\varepsilon\left(\widetilde{s}_{ij}\right)+1}$. We have shown that

$$\widetilde{s}_{ij} = \left(\frac{\widetilde{w}_{ij}}{\widetilde{w}_j}\right)^{1+\eta} \quad \implies \quad \widetilde{w}_j = \widetilde{w}_{ij} \widetilde{s}_{ij}^{-\frac{1}{1+\eta}}$$

Using these, we can write:

$$w_{ij} = \left[\mu\left(\widetilde{s}_{ij}\right) \alpha z_{ij} \widetilde{s}_{ij}^{-\frac{(1-\alpha)(\eta-\theta)}{1+\eta}} \widetilde{W}^{(1-\alpha)(\theta-\varphi)}\right]^{\frac{1}{1+\theta(1-\alpha)}}$$

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- **Region II** - In Region II, then

$$w_{ij} = \underline{w}$$

$$p_{ij} = 1$$

$$\widetilde{w}_{ij} = \underline{w}$$

$$n_{ij} = \left(\frac{\underline{w}}{\widetilde{w}_j}\right)^{\eta} \left(\frac{\widetilde{w}_j}{\widetilde{W}}\right)^{\theta} N$$

- **Region III** - In Region III, then

$$w_{ij} = \alpha z_{ij} n_{ij}^{\alpha - 1}$$

$$p_{ij} < 1$$

$$\widetilde{w}_{ij} = p_{ij}\underline{w}$$

$$n_{ij} = \left(\frac{p_{ij}\underline{w}}{\widetilde{w}_j}\right)^{\eta} \left(\frac{\widetilde{w}_j}{\widetilde{W}}\right)^{\theta} N$$

B.2 Minimum wage solution algorithm

We implement the following solution algorithm. We denote the *Region* that a firm is in by $r_{ijt} \in \{I, II, III\}$. Initialize the algorithm by (i) guessing a value for $\widetilde{W}^{(0)}$, (ii) assuming all firms are in *Region I*, $r_{ij}^{(0)} = I$, which implies guessing $p_{ij}^{(0)} = 0$. These will all be updated in the algorithm.

1. **Solve all market equilibria in shadow shares**

    (a) Guess shadow shares $\widetilde{s}_{ij}^{(0)}$.

    (b) *Region I* - Using the above optimality condition

    $$w_{ij} = \left[\mu\left(\widetilde{s}_{ij}\right) \alpha z_{ij} \widetilde{s}_{ij}^{(0) - \frac{(1-\alpha)(\eta-\theta)}{1+\eta}} \widetilde{W}^{(0)(1-\alpha)(\theta-\varphi)}\right]^{\frac{1}{1+\theta(1-\alpha)}}$$

    (c) *Regions II, III* - Here the minimum wage is binding so set $w_{ij} = \underline{w}$.

    (d) Given the guess $p_{ij}^{(k)}$ and w_{ij}, compute the shadow wage: $\widetilde{w}_{ij} = p_{ij} w_{ij}$.

    (e) With all shadow wages in hand, update shadow shares using $\widetilde{w}_{ijt}$:

    $$\widetilde{s}_{ij}^{(l+1)} = \frac{\widetilde{w}_{ij}^{1+\eta}}{\sum_{i \in j} \widetilde{w}_{ij}^{1+\eta}}.$$

    (f) Iterate over (b)-(e) until shadow shares converge: $\widetilde{s}_{ij}^{(l+1)} = \widetilde{s}_{ij}^{(l)}$.

2. **Recover employment** - Here we use the wages from the previous step plus the current guess of each firms' region. First aggregate $\widetilde{w}_{ij}$ to compute $\widetilde{w}_j$ and $\widetilde{W}$. Then by region $r_{ijt}^{(k)}$:

(a) *Region I - Firm is unconstrained:*

$$n_{ij} = \left(\frac{w_{ij}}{\widetilde{w}_j}\right)^{\eta} \left(\frac{\widetilde{w}_j}{\widetilde{W}}\right)^{\theta} \widetilde{W}^{\varphi}$$

(b) *Region II - Firm is constrained and n_{ij} determined by household labor supply curve at $\underline{w}$:*

$$n_{ij} = \left(\frac{\underline{w}}{\widetilde{w}_j}\right)^{\eta} \left(\frac{\widetilde{w}_j}{\widetilde{W}}\right)^{\theta} \widetilde{W}^{\varphi}$$

(c) *Region III - Firm is constrained and n_{ijt} determined by firm labor demand curve at $\underline{w}$:*

$$\underline{w} = \alpha z_{ij} n_{ij}^{\alpha-1} \quad \implies \quad n_{ij} = \left(\frac{\alpha z_{ij}}{\underline{w}}\right)^{\frac{1}{1-\alpha}}.$$

3. Update the multipliers: $p_{ij}^{(k)}$

    (a) Aggregate n_{ij} to compute n_j and N.

    (b) Update p_{ij} from the *household's* first order conditions: $\widetilde{w}_{ij} = p_{ij} w_{ij}$

$$p_{ij}^{(k+1)} = \frac{\left(\frac{n_{ij}}{n_j}\right)^{\frac{1}{\eta}} \left(\frac{n_j}{N}\right)^{\frac{1}{\theta}} N^{\frac{1}{\varphi}}}{w_{ij}}$$

4. Update $\widetilde{W}^{(k)}$:

    (a) Compute $\widetilde{w}_{ij} = p_{ij}^{(k+1)} w_{ij}$

    (b) Use $\widetilde{w}_{ij}$ to update the aggregate shadow wage index to $\widetilde{W}^{(k+1)}$.

5. Update firm regions. For each region:

    (a) Compute the marginal product of labor of all firms $mrpl_{ij} = \alpha z_{ij} n_{ij}^{\alpha-1}$.

    (b) If in market j there exists a firm in *Region I* with $w_{ij} < \underline{w}$, then move the firm with the lowest wage into *Region II*.

    (c) If in market j there exists a firm that was initially in *Region II* and has a marginal product of labor that is less than marginal cost ($\underline{w}$), move that firm into *Region III*.

6. Iterate over (1) to (5) until $p_{ij}^{(k+1)} = p_{ij}^{(k)}$ and $\widetilde{W}^{(k+1)} = \widetilde{W}^{(k)}$ and $r_{ij}^{(k+1)} = r_{ij}^{(k)}$.

C Disciplining preference parameters

This Section details how we use recent evidence from Golosov, Graber, Mogstad, and Novgorodsky (2021) to discipline preference parameters σ and φ.

Background. Consider a budget constraint, where b_i is unearned income and $\mathcal{T}$ gives taxes and transfers which depend on pre-tax labor income y_i:

$$c_i = y_i - \mathcal{T}(y_i) + b_i$$

Totally differentiating with respect to b_i:

$$\frac{dc_i}{db_i} = \frac{dy_i}{db_i} - \frac{d\mathcal{T}_i}{db_i} + 1$$

$$MPC_i = MPE_i - MPT_i + 1$$

Table 4.1 of Golosov, Graber, Mogstad, and Novgorodsky (2021, henceforth GGMN) gives estimates of the marginal propensity to consume (MPC) and marginal propensity to earn (MPE) for different income groups, where lottery winnings are used as an instrument for the endogenous variable b_i. For example, results are of the type: *An extra dollar in unearned income leads to a $MPE = -0.52$ cent reduction in labor earnings.* We show how their results can be used to discipline preference parameters (φ, σ) in a simple labor supply setting that is consistent with our model.

Derivation. Consider the following individual problem, where preferences are as in the main text, and $y = wn$, where w is taken as given:

$$u(c,n) = \frac{c^{1-\sigma}}{1-\sigma} - \frac{1}{\overline{\varphi}^{1/\varphi}} \frac{n^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \tag{C1}$$

$$c = wn + \mathcal{T}(wn) + b$$

Optimality conditions for c and n give labor supply, which can be expressed in terms of earnings:

$$y = \overline{\varphi} c^{-\varphi\sigma} w^{\varphi+1} \left(1 - \mathcal{T}'(y)\right)^{\varphi}$$

Totally differeniating with respect to b

$$\frac{dy}{db} = -\varphi\sigma \frac{dc}{db}\left(\frac{y}{c}\right) - \varphi\left(\frac{\mathcal{T}''(y)y}{1-\mathcal{T}'(y)}\right)\frac{dy}{db}.$$

Now suppose that post-tax labor earnings were of the form used in Heathcote, Storesletten, and Violante (2020, henceforth HSV): $y - \mathcal{T}(y) = \lambda y^{1-\tau}$. In this case, the elasticity term is simply the progressivity of taxes, τ.

$$\frac{dy}{db} = -\varphi\sigma \frac{dc}{db}\left(\frac{y}{c}\right) - \varphi\tau\frac{dy}{db}.$$

Using the definitons of MPC, MPE, the average propensity to consume $APC = c/y$, and after rearranging, we have a closed-form relationship between σ and φ, given data on $\{MPC, MPE, APC, \tau\}$:

$$\varphi = -\frac{1}{\sigma\frac{MPC}{MPE}\frac{1}{APC} - \tau}. \tag{C2}$$

If we let $\sigma = 1$ and $\tau = 0$, it is straightforward to observe that a lower MPC and higher MPE in absolute terms (as will be the case for richer households), requires a higher φ.

$$\varphi = \frac{|MPE|}{MPC} APC.$$

Data. We use BLS data to compute APC for non-high-school, high-school, and college completion households. We map these into the four quartiles of income groups in GGMN Table 4.1 as given in the following table. We take a value of $\tau = 0.086$ from HSV (JEEA, 2020).[42] This uses *pre-government-transfer* income for y, that is y only considers labor income earnings.

	All	Group		
BLS category		Non-High School	High school	Completed college
GGMN category		Q1	Q2-Q3	Q4
APC (BLS)	0.69	0.73	0.71	0.67
MPE (GGMN)	-0.5227	-0.3080	-0.5549	-0.6735
MPC (GGMN)	0.5836	0.7315	0.5429	0.4990

Table C1: Data used in calibrating preference parameters
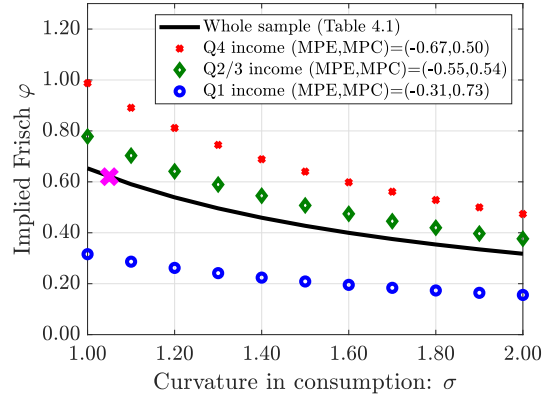
Figure C1: Implied parameters

<u>Notes</u>: Given a value for the coefficient of relative risk aversion σ, this figure plots the Frisch elasticity of labor supply φ required for the optimality conditions of the simple labor supply model C1 to be consistent with (i) empirical measures of the marginal propensity to earn and marginal propensity to consume following changes in unearned income from Golosov, Graber, Mogstad, and Novgorodsky (2021), (ii) estimates of the average propensity to consume from the BLS, (iii) estimates of the progressivity of post-tax labor income to pre-tax-and-transfer income from Heathcote, Storesletten, and Violante (2020).

Results. Using equation (C2), we can then determine φ given σ. Figure C1 plots $\varphi(\sigma)$ for $\sigma \in [1, 2]$. As a benchmark, with log preferences, and when calibrated to the whole sample values, $\varphi(1) = 0.65$. For low income (Q1) households $\varphi(1) = 0.32$, for high income households $\varphi(1) = 0.987$. High income (Q4) households have higher MPE's, and their MPC is lower, reducing $|MPC/MPE|$, and requiring a higher φ. The pink cross corresponds to $(\sigma, \varphi) = (1.05, 0.62)$, which are the values used in the baseline calibration of our model (see Table 1).

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[42]See: http://violante.mycpanel.princeton.edu/Workingpapers/JEEA_final.pdf, Table 1, Row 5

D Mathematical details

- We first derive results for the competitive equilibrium, then the government's allocation problem. We then use results from the competitive equilibrium to prove that the solution to the government's allocation problem can be decentralized in a competitive equilibrium with revenue neutral lump sum taxes

D.1 Competitive equilibrium

D.1.1 Household problem - Labor supply system, shadow wages

- In the competitive equilibrium, household k solves the following problem:

$$\max_{c_{kt}, h_{kt}} \sum_{t=0}^{\infty} \beta^t \left[\frac{(c_{kt}/\pi_k)^{1-\sigma}}{1-\sigma} - \frac{1}{\widetilde{\varphi}_k^{1/\varphi}} \frac{n_{kt}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right]$$

where $\widetilde{\varphi}_k = \overline{\varphi}_k \pi_k^{1+\varphi}$ is adjusted for the measure of workers of the household,

$$n_{kt} = \left[\int n_{jkt}^{\frac{\theta+1}{\theta}} \, dj \right]^{\frac{\theta}{\theta+1}} \quad , \quad n_{jkt} = \left[\sum_{i \in j} n_{ijkt}^{\frac{\eta+1}{\eta}} \right]^{\frac{\eta}{\eta+1}}$$

subject to the budget constraint

$$c_{kt} + k_{kt+1} = \int \sum_{i \in j} w_{ijkt} n_{ijkt} dj + R_t k_{kt} + (1-\delta) k_{kt} + \kappa_k \Pi_t.$$

with the initial condition $k_{k0} = \kappa_k K_0$.

- Since we focus on steady-state we normalize the price of consumption to one.

- In the text we refer to these preferences as $u^k \left(\frac{c_{kt}}{\pi_k}, n_{kt} \right)$:

$$u^k \left(\frac{c_{kt}}{\pi_k}, n_{kt} \right) = \frac{(c_{kt}/\pi_k)^{1-\sigma}}{1-\sigma} - \frac{1}{\widetilde{\varphi}_k^{1/\varphi}} \frac{n_{kt}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}$$

- The household is also subject to the firm by firm rationing constraints: $n_{ijkt} \leq \overline{n}_{ijkt}$.

- Let $\beta^t v_{kt}$ be the multiplier on the household's budget constraint and write the multiplier on the ratioining constraint as $\zeta_{ijkt} = \beta^t v_{kt} w_{ijkt} \left(1 - p_{ijkt} \right)$.

- The the household's Lagrangean features the following terms in n_{ijkt}

$$\mathcal{L} = \cdots + \beta^t u^k \left(\frac{c_{kt}}{\pi_k}, n_{kt} \right) + \cdots + \beta^t v_{kt} w_{ijkt} n_{ijkt} + \beta^t v_{kt} w_{ijkt} \left(1 - p_{ijkt} \right) \left[\overline{n}_{ijkt} - n_{ijkt} \right] + \ldots$$

$$\mathcal{L} = \cdots + u^k \left(\frac{c_{kt}}{\pi_k}, n_{kt} \right) + \cdots + \beta^t v_{kt} \left\{ w_{ijkt} p_{ijkt} \right\} n_{ijkt} + \beta^t v_{kt} w_{ijkt} \left(1 - p_{ijkt} \right) \overline{n}_{ijkt} + \ldots$$

- The first order condition for consumption is

$$u_c^k \left(\frac{c_{kt}}{\pi_k}, n_{kt} \right) = \nu_{kt}$$

- The first order condition for labor supply is

$$\nu_{kt} w_{ijkt} p_{ijkt} = -u_n^k \left(\frac{c_{kt}}{\pi_k}, n_{kt} \right) \frac{\partial n_{kt}}{\partial n_{jkt}} \frac{\partial n_{jkt}}{\partial n_{ijkt}}$$

$$w_{ijkt} p_{ijkt} = -\frac{u_n^k \left(\frac{c_{kt}}{\pi_k}, n_{kt} \right)}{u_c^k \left(\frac{c_{kt}}{\pi_k}, n_{kt} \right)} \left(\frac{n_{jkt}}{n_{kt}} \right)^{\frac{1}{\theta}} \left(\frac{n_{ijkt}}{n_{jkt}} \right)^{\frac{1}{\eta}}$$

- Define the *shadow wage* by $\widetilde{w}_{ijkt} := w_{ijkt} p_{ijkt}$.

- Then

$$\widetilde{w}_{ijkt} = -\frac{u_n^k \left(\frac{c_{kt}}{\pi_k}, n_{kt} \right)}{u_c^k \left(\frac{c_{kt}}{\pi_k}, n_{kt} \right)} \left(\frac{n_{jkt}}{n_{kt}} \right)^{\frac{1}{\theta}} \left(\frac{n_{ijkt}}{n_{jkt}} \right)^{\frac{1}{\eta}}.$$

- Now define the following *shadow wage indexes*:

$$\widetilde{w}_{jkt} = \left[\sum_{i \in j} \widetilde{w}_{ijkt}^{1+\eta} \right]^{\frac{1}{1+\eta}} \quad , \quad \widetilde{w}_{kt} = \left[\int \widetilde{w}_{jkt}^{1+\theta} dj \right]^{\frac{1}{1+\eta}}$$

- Using this

$$\sum_{i \in j} \widetilde{w}_{ijkt}^{1+\eta} = \left[-\frac{u_n^k \left(\frac{c_{kt}}{\pi_k}, n_{kt} \right)}{u_c^k \left(\frac{c_{kt}}{\pi_k}, n_{kt} \right)} \left(\frac{n_{jkt}}{n_{kt}} \right)^{\frac{1}{\theta}} \right]^{1+\eta} \sum_{i \in j} \left(\frac{n_{ijkt}}{n_{jkt}} \right)^{\frac{1+\eta}{\eta}}$$

$$\widetilde{w}_{jkt} = -\frac{u_n^k \left(\frac{c_{kt}}{\pi_k}, n_{kt} \right)}{u_c^k \left(\frac{c_{kt}}{\pi_k}, n_{kt} \right)} \left(\frac{n_{jkt}}{n_{kt}} \right)^{\frac{1}{\theta}}$$

$$\widetilde{w}_{jkt}^{1+\theta} = \left[-\frac{u_n^k \left(\frac{c_{kt}}{\pi_k}, n_{kt} \right)}{u_c^k \left(\frac{c_{kt}}{\pi_k}, n_{kt} \right)} \right] \left(\frac{n_{jkt}}{n_{kt}} \right)^{\frac{1+\theta}{\theta}}$$

$$\int \widetilde{w}_{jkt}^{1+\theta} dj = \left[-\frac{u_n^k \left(\frac{c_{kt}}{\pi_k}, n_{kt} \right)}{u_c^k \left(\frac{c_{kt}}{\pi_k}, n_{kt} \right)} \right] \int \left(\frac{n_{jkt}}{n_{kt}} \right)^{\frac{1+\theta}{\theta}} dj$$

$$\widetilde{w}_{kt} = -\frac{u_n^k \left(\frac{c_{kt}}{\pi_k}, n_{kt} \right)}{u_c^k \left(\frac{c_{kt}}{\pi_k}, n_{kt} \right)}$$

- Using our form of preferences, this gives the household k labor supply curve:

$$n_{kt} = \overline{\varphi}_k \pi_k \widetilde{w}_{kt}^\varphi \left(\frac{c_{kt}}{\pi_k} \right)^{-\varphi\sigma}$$

- Using this we can show that shadow wages aggregate, as claimed in the text,

- First across markets:

$$\widetilde{w}_{ijkt} = \widetilde{w}_{kt} \left(\frac{n_{jkt}}{n_{kt}} \right)^{\frac{1}{\theta}} \left(\frac{n_{ijkt}}{n_{jkt}} \right)^{\frac{1}{\eta}}.$$

$$\widetilde{w}_{ijkt}^{1+\eta} = \left[\widetilde{w}_{kt} \left(\frac{n_{jkt}}{n_{kt}} \right)^{\frac{1}{\theta}} \right]^{1+\eta} \left(\frac{n_{ijkt}}{n_{jkt}} \right)^{\frac{1+\eta}{\eta}}$$

$$\left[\sum_{i \in j} \widetilde{w}_{ijkt}^{1+\eta} \right]^{\frac{1}{1+\eta}} = \widetilde{w}_{kt} \left(\frac{n_{jkt}}{n_{kt}} \right)^{\frac{1}{\theta}}$$

$$\widetilde{w}_{jkt} = \widetilde{w}_{kt} \left(\frac{n_{jkt}}{n_{kt}} \right)^{\frac{1}{\theta}}$$

$$\widetilde{w}_{jkt} n_{jkt} = \widetilde{w}_{kt} n_{jkt} \times \left(\frac{n_{jkt}}{n_{kt}} \right)^{\frac{1+\theta}{\theta}}$$

$$\int \widetilde{w}_{jkt} n_{jkt} dj = \widetilde{w}_{kt} n_{jkt}$$

- Then using these results, across firms within a market:

$$\widetilde{w}_{ijkt} = \widetilde{w}_{kt} \left(\frac{n_{jkt}}{n_{kt}} \right)^{\frac{1}{\theta}} \left(\frac{n_{ijkt}}{n_{jkt}} \right)^{\frac{1}{\eta}}$$

$$\widetilde{w}_{ijkt} = \widetilde{w}_{jkt} \left(\frac{n_{ijkt}}{n_{jkt}} \right)^{\frac{1}{\eta}}$$

$$\widetilde{w}_{ijkt} n_{ijkt} = \widetilde{w}_{jkt} n_{jkt} \times \left(\frac{n_{ijkt}}{n_{jkt}} \right)^{\frac{1+\eta}{\eta}}$$

$$\sum_{i \in j} \widetilde{w}_{ijkt} n_{ijkt} = \widetilde{w}_{jkt} n_{jkt}$$

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- Summarizing results so far, we have:

$$\widetilde{w}_{ijkt} = \left(\frac{n_{ijkt}}{n_{jkt}}\right)^{\frac{1}{\eta}} \widetilde{w}_{jkt}$$

$$\widetilde{w}_{jkt} = \left(\frac{n_{jkt}}{n_{kt}}\right)^{\frac{1}{\theta}} \widetilde{w}_{kt}$$

$$\widetilde{w}_{kt} n_{jkt} = \int \widetilde{w}_{jkt} n_{jkt} dj$$

$$\widetilde{w}_{jkt} n_{jkt} = \sum_{i \in j} \widetilde{w}_{ijkt} n_{ijkt}$$

- Note that these can be combined to give the entire labor supply system of household k in shadow wages:

$$n_{ijkt} = \left(\frac{\widetilde{w}_{ijkt}}{\widetilde{w}_{jkt}}\right)^{\eta} \left(\frac{\widetilde{w}_{jkt}}{\widetilde{w}_{kt}}\right)^{\theta} n_{kt}$$

$$n_{kt} = \overline{\varphi}_k \pi_k^{1+\varphi\sigma} \widetilde{w}_{kt}^{\varphi} c_{kt}^{-\varphi\sigma}$$

- A key result, used below, is that if the household received lump sum transfers T_k, then the same labor supply system would be obtained.

- Now consider our results regarding shadow shares. We define the *shadow share* as

$$\widetilde{s}_{ijkt} := \frac{\widetilde{w}_{ijkt} n_{ijkt}}{\sum_{i \in j} \widetilde{w}_{ijkt} n_{ijkt}}.$$

- Using the above aggregation results, labor supply system, and definition of the aggregator n_{jkt}:

$$\widetilde{s}_{ijkt} = \frac{\widetilde{w}_{ijkt} n_{ijkt}}{\widetilde{w}_{jkt} n_{jkt}} = \left(\frac{\widetilde{w}_{ijkt}}{\widetilde{w}_{jkt}}\right)^{1+\eta} = \left(\frac{n_{ijkt}}{n_{jkt}}\right)^{\frac{1+\eta}{\eta}} = \frac{\partial \log n_{ijkt}}{\partial \log n_{jkt}}$$

which we use below in the firm optimality conditions.

D.1.2 Firm optimality

- **Simplifying the firm problem -** First we simplify the firm problem by separating it out across types and optimizing out capital for each type of worker:

- Consider the maximization problem of the firm in the text:

$$\pi_{ij} = \max_{\{n_{ijk}, k_{ijk}\}_{k=1}^{K}} \overline{Z} z_{ij} \sum_{k=1}^{K} \left(\left[\xi_k n_{ijk}\right]^{\gamma} k_{ijk}^{1-\gamma} \right)^{\alpha} - R \sum_{k=1}^{K} k_{ijk} - \sum_{k=1}^{K} w_{ijk} n_{ijk}$$

subject to the labor supply system and minimum wage constraints.

- First observe that this can separated out by type of worker k.

- The problem for type *k* labor at the firm is

$$\pi_{ijk} = \max_{n_{ijk}, k_{ijk}} \overline{Z} z_{ij} \left(\left[\xi_k n_{ijk} \right]^\gamma k_{ijk}^{1-\gamma} \right)^\alpha - R k_{ijk} - w_{ijk} n_{ijk}$$

- We first optimize out capital. This yields the objective function

$$\pi_{ijk} = \max_{n_{ijk}} \widetilde{Z} \widetilde{z}_{ij} \widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}} - w_{ijk} n_{ijk}$$

  where

$$\widetilde{Z} = \overline{Z}^{\frac{1}{1-(1-\gamma)\alpha}}$$

$$\widetilde{z}_{ij} = [1 - (1-\gamma)\alpha] \left(\frac{(1-\gamma)\alpha}{R} \right)^{\frac{(1-\gamma)\alpha}{1-(1-\gamma)\alpha}} z_{ij}^{\frac{1}{1-(1-\gamma)\alpha}}$$

$$\widetilde{\xi}_k = \xi_k^{\widetilde{\alpha}}$$

$$\widetilde{\alpha} = \frac{\gamma\alpha}{1-(1-\gamma)\alpha}$$

- We denote output net of capital expenses as $\widetilde{y}_{ijk} := \widetilde{Z} \widetilde{z}_{ij} \widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}}$.
- We can also define a market-level aggregate $\widetilde{y}_{jk} = \sum_{i \in j} \widetilde{y}_{ijk}$, and a type-level aggregate $\widetilde{y}_k = \int \widetilde{y}_{jk} dj$.
- Note that

$$y_{ijk} = \frac{\widetilde{y}_{ijk}}{1-(1-\gamma)\alpha} \quad , \quad y_{jk} = \frac{\widetilde{y}_{jk}}{1-(1-\gamma)\alpha} \quad , \quad y_k = \frac{\widetilde{y}_k}{1-(1-\gamma)\alpha}.$$

- Using the simplified problem we now consider optimality of the firm in each of the three regions described in the text.

- **Region I - Unconstrained**

  – Consider an unconstrained firm. Its problem is

$$\pi_{ijk} = \max_{n_{ijk}} \widetilde{Z} \widetilde{z}_{ij} \widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}} - w_{ijk} n_{ijk}$$

  subject to its wage being given by the above labor supply system:

$$w\left(n_{ijkt} \right) = \left(\frac{n_{ijkt}}{n_{jkt}} \right)^{\frac{1}{\eta}} \left(\frac{n_{jkt}}{n_{kt}} \right)^{\frac{1}{\theta}} \widetilde{w}_{kt}.$$

- The first order condition is

$$w_{ijk} + w'\left(n_{ijk}\right)n_{ijk} = \widetilde{\alpha}\widetilde{Z}\widetilde{z}_{ij}\widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}-1}$$

$$w_{ijk}\left(1 + \frac{w'\left(n_{ijk}\right)n_{ijk}}{w_{ijk}}\right) = \widetilde{\alpha}\widetilde{Z}\widetilde{z}_{ij}\widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}-1}$$

$$w_{ijk}\left(1 + \frac{1}{\varepsilon_{ijk}}\right) = \widetilde{\alpha}\widetilde{Z}\widetilde{z}_{ij}\widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}-1}$$

$$w_{ijk} = \frac{\varepsilon_{ijk}}{1+\varepsilon_{ijk}}\widetilde{\alpha}\widetilde{Z}\widetilde{z}_{ij}\widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}-1}$$

where using the inverse labor supply curve gives

$$\frac{1}{\varepsilon_{ijk}} := \frac{w'\left(n_{ijk}\right)n_{ijk}}{w_{ijk}} = \frac{\partial \log w_{ijk}}{\partial \log n_{ijk}} = \frac{1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right)\frac{\partial \log n_{jk}}{\partial \log n_{ijk}} = \frac{1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right)\widetilde{s}_{ijk}$$

$$\varepsilon_{ijk} = \left[\frac{1}{\eta} + \left(\frac{1}{\theta} - \frac{1}{\eta}\right)\widetilde{s}_{ijk}\right]^{-1}.$$

- Therefore

$$w_{ijk} = \mu_{ijk}\widetilde{\alpha}\widetilde{Z}\widetilde{z}_{ij}\widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}-1}$$

where the markdown depends on the firms' elasticity of labor supply.

- Note that since $p_{ijk} = 1$ since the firm is unconstrained, then $\widetilde{w}_{ijk} = p_{ijk}w_{ijk} = w_{ijk}$, so

$$\widetilde{w}_{ijk} = \mu_{ijk} \times \widetilde{\alpha}\widetilde{Z}\widetilde{z}_{ij}\widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}-1}$$

- **Region III - Constrained, on labor demand curve**

  - Now consider a constrained firm in Region III, this firm's problem is

  $$\pi_{ijk} = \max_{n_{ijk}} \widetilde{Z}\widetilde{z}_{ij}\widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}} - \underline{w}n_{ijk}$$

  - The solution to this problem is to choose employment to equate the marginal revenue product of labor to the minimum wage:

  $$\underline{w} = \widetilde{\alpha}\widetilde{Z}\widetilde{z}_{ij}\widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}-1}$$

  - For convenience when aggregating, we can express this in terms of shadow wages by multiplying through by the equilibrium multiplier on the rationing constraint

  $$\underline{w}p_{ijk} = p_{ijk}\widetilde{\alpha}\widetilde{Z}\widetilde{z}_{ij}\widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}-1}$$
  $$\widetilde{w}_{ijk} = p_{ijk} \times \widetilde{\alpha}\widetilde{Z}\widetilde{z}_{ij}\widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}-1}$$

- **Region II - Constrained, on labor supply curve**

  - Now consider a constrained firm in Region II, this firm simply has labor determined by the labor

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supply curve, but since the rationing constraint is slack, $\widetilde{w}_{ijk} = p_{ijk} w_{ijk} = \underline{w}$.

$$n_{ijk} = \left(\frac{\underline{w}}{\widetilde{w}_{jk}} \right)^{\eta} \left(\frac{\widetilde{w}_{jk}}{\widetilde{W}_k} \right)^{\theta} n_k.$$

– Nonetheless, we can express the shadow wage of the firm as

$$\widetilde{w}_{ijk} = \widetilde{\mu}_{ijk} \widetilde{\alpha} \widetilde{Z} \widetilde{z}_{ij} \widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}-1}$$

$$\widetilde{\mu}_{ijk} = \frac{\underline{w}}{\widetilde{\alpha} \widetilde{Z} \widetilde{z}_{ij} \widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}-1}} \quad , \quad n_{ijk} = \left(\frac{\underline{w}}{\widetilde{w}_{jk}} \right)^{\eta} \left(\frac{\widetilde{w}_{jk}}{\widetilde{W}_k} \right)^{\theta} n_k.$$

– Therefore, in all three regions, we can express the *shadow wage* as a *shadow markdown* on the marginal revenue product of labor:

$$\widetilde{w}_{ijk} = \widetilde{\mu}_{ijk} \widetilde{\alpha} \widetilde{Z} \widetilde{z}_{ij} \widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}-1}.$$

D.1.3   Aggregation of output and labor demand conditions

- Using the above results for firm optimality and the household's labor supply system we can aggregate the optimality conditions of agents. This is a key step in solving the government problem and optimal transfers, which we describe below.

- **Aggregation - Firm-Type to Market-Type**

  – From the above we have the following set of five conditions at the firm and market level:

  – **Firm level**:

$$\widetilde{y}_{ijk} = \widetilde{Z} \widetilde{z}_{ij} \widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}}$$
$$\widetilde{w}_{ijk} = \widetilde{\mu}_{ijk} \widetilde{\alpha} \widetilde{Z} \widetilde{z}_{ij} \widetilde{\xi}_k n_{ijk}^{\widetilde{\alpha}-1}.$$
$$n_{ijk} = \left(\frac{\widetilde{w}_{ijk}}{\widetilde{w}_{jk}} \right)^{\eta} n_{jk}$$

  – **Aggregates**:

$$\widetilde{y}_{jk} = \sum_{i \in j} \widetilde{y}_{ijk}$$
$$\widetilde{w}_{jk} = \left[\sum_{i \in j} \widetilde{w}_{ijk}^{1+\eta} \right]^{\frac{1}{1+\eta}}$$

– Following steps from Berger, Herkenhoff, Mongey (2022), these can be combined to yield:

$$\widetilde{y}_{jk} = \omega_{jk}\widetilde{Z}\widetilde{\xi}_k\widetilde{z}_j n_{jk}^{\widetilde{\alpha}}$$

$$\widetilde{w}_{jk} = \widetilde{\mu}_{jk}\widetilde{\alpha}\widetilde{Z}\widetilde{z}_j\widetilde{\xi}_k n_{jk}^{\widetilde{\alpha}-1}$$

$$n_{jk} = \left(\frac{\widetilde{w}_{jk}}{\widetilde{w}_k}\right)^{\theta} n_k$$

where the three wedges $\left\{\widetilde{z}_j, \widetilde{\mu}_{jk}, \omega_{jk}\right\}$ are given by

$$\widetilde{z}_j = \left[\sum_{i\in j}\widetilde{z}_{ij}^{\frac{1+\eta}{1+\eta(1-\widetilde{\alpha})}}\right]^{\frac{1+\eta(1-\widetilde{\alpha})}{1+\eta}}$$

$$\widetilde{\mu}_{jk} = \left[\sum_{i\in j}\left(\frac{\widetilde{z}_{ij}}{\widetilde{z}_j}\right)^{\frac{1+\eta}{1+\eta(1-\widetilde{\alpha})}}\widetilde{\mu}_{ijk}^{\frac{1+\eta}{1+\eta(1-\widetilde{\alpha})}}\right]^{\frac{1+\eta(1-\widetilde{\alpha})}{1+\eta}}$$

$$\omega_{jk} = \left[\sum_{i\in j}\left(\frac{\widetilde{z}_{ij}}{\widetilde{z}_j}\right)^{\frac{1+\eta}{1+\eta(1-\widetilde{\alpha})}}\left(\frac{\widetilde{\mu}_{ijk}}{\widetilde{\mu}_{jk}}\right)^{\frac{1+\eta}{1+\eta(1-\widetilde{\alpha})}}\right]^{\frac{1+\eta(1-\widetilde{\alpha})}{1+\eta}}$$

– Note that this implies that if $\left\{\widetilde{z}_j, \widetilde{\mu}_{jk}, \widetilde{\omega}_{jk}\right\}$ are known, then $\left\{n_{jk}, \widetilde{w}_{jk}, \widetilde{y}_{jk}\right\}$ can be determined.

• **Aggregation - Market-Type to Type**

– The same approach can be followed to aggregate to the household level, which delivers:

$$\widetilde{y}_k = \omega_k\widetilde{Z}\widetilde{\xi}_k\widetilde{z}_k n_k^{\widetilde{\alpha}}$$

$$\widetilde{w}_k = \widetilde{\mu}_k\widetilde{\alpha}\widetilde{Z}\widetilde{z}_k\widetilde{\xi}_k n_k^{\widetilde{\alpha}-1}$$

where

$$\widetilde{z}_k = \left[\int \widetilde{z}_j^{\frac{1+\theta}{1+\theta(1-\widetilde{\alpha})}}dj\right]^{\frac{1+\theta(1-\widetilde{\alpha})}{1+\theta}}$$

$$\widetilde{\mu}_k = \left[\int\left(\frac{\widetilde{z}_{ij}}{\widetilde{z}_j}\right)^{\frac{1+\theta}{1+\theta(1-\widetilde{\alpha})}}\widetilde{\mu}_{jk}^{\frac{1+\theta}{1+\theta(1-\widetilde{\alpha})}}dj\right]^{\frac{1+\theta(1-\widetilde{\alpha})}{1+\theta}}$$

$$\omega_k = \left[\int\left(\frac{\widetilde{z}_{ij}}{\widetilde{z}_j}\right)^{\frac{1+\theta}{1+\theta(1-\widetilde{\alpha})}}\left(\frac{\widetilde{\mu}_{jk}}{\widetilde{\mu}_k}\right)^{\frac{1+\theta}{1+\theta(1-\widetilde{\alpha})}}\omega_{jk}\right]^{\frac{1+\theta(1-\widetilde{\alpha})}{1+\theta}}$$

• The conditions derived thus far all hold in a competitive equilibrium with lump sum transfers.

• In a competitive equilibrium, the above conditions are satisfied and budget constraints clear for each household.

D.2 Government problem

- We consider the government primal problem where it chooses an allocation of (i) labor from each household to all firms (ii) consumption of all households, (iii) investment. We then show that the government can decentralize this allocation in a competitive equilibrium by choosing appropriate lump sum transfers.

- This has the flavor of a 'partial' planning problem. 'Partial' in the sense that the government takes as given the prices of firms in the economy, and firms' rationing constraints, where these are due to the market power of firms. The government therefore faces a *budget constraint* rather than a resource constraint.

D.2.1 Allocation problem

- The government is endowed with K_0, takes prices $\left\{ w_{ijkt} \right\}$, profits $\{\Pi_t\}$, rationing constraints $\left\{ \overline{n}_{ijkt} \right\}$ as given and chooses directly $\left\{ c_{kt}, n_{ijkt}, K_{t+1} \right\}$ to maximize

$$U_0 = \sum_{t=0}^{\infty} \beta^t \sum_k \psi_k \left[\frac{(c_{kt}/\pi_k)^{1-\sigma}}{1-\sigma} - \frac{1}{\widetilde{\varphi}_k^{1/\varphi}} \frac{n_{kt}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right]$$

where

$$n_{kt} = \left[\int n_{jkt}^{\frac{\theta+1}{\theta}} dj \right]^{\frac{\theta}{\theta+1}}$$

$$n_{jkt} = \left[\sum_{i \in j} n_{ijkt}^{\frac{\eta+1}{\eta}} \right]^{\frac{\eta}{\eta+1}}$$

subject to its budget constraint

$$\sum_k c_{kt} + K_{t+1} = \sum_k \int \sum_{i \in j} w_{ijkt} n_{ijkt} dj + R_t K_t + (1-\delta) K_t + \Pi_t$$

rationing constraints

$$n_{ijk} \leq \overline{n}_{ijk}$$

- Here $\widetilde{\varphi}_k = \overline{\varphi}_k \pi_k^{\varphi+1}$ is adjusted for the measure of workers of the household.

- We can rewrite the objective function as

$$U_0 = \sum_{t=0}^{\infty} \beta^t \left[\frac{\mathcal{C}_t^{1-\sigma}}{1-\sigma} - \frac{1}{\widehat{\varphi}^{1/\varphi}} \frac{\mathcal{N}_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right]$$

where

$$\mathcal{C}_t = \left[\sum_k \psi_k \left(\frac{c_{kt}}{\pi_k} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$$

$$\mathcal{N}_t = \left[\sum_k \left(\frac{\psi_k}{\widehat{\varphi}_k^{1/\varphi}} \right) n_{kt}^{\frac{\varphi+1}{\varphi}} \right]^{\frac{\varphi}{\varphi+1}}$$

and $\widehat{\varphi}_k = \widetilde{\varphi}_k / \widehat{\varphi}$.

- Let $\beta^t \Lambda_t$ be the multiplier on the government's budget constraint.

D.2.2 Allocation problem - Consumption

- The first order condition for c_{kt} gives the following:

$$\psi_k \left(\frac{c_{kt}}{\pi_k} \right)^{1-\sigma} = \Lambda_t p_{kt} c_{kt}$$

$$c_{kt} = \pi_k \left(\frac{\Lambda_t p_{kt} \pi_k}{\psi_k} \right)^{-\frac{1}{\sigma}}$$

- Suppose there exists some $\mathcal{P}_t$ such that aggregate consumption $\mathcal{C}_t = \sum_k c_{kt} = \mathcal{P}_t \mathcal{C}_t$.

- Using the first order condition we can obtain:

$$\Lambda_t = \frac{\mathcal{C}_t^{-\sigma}}{\mathcal{P}_t}$$

$$c_{kt} = \pi_k \left(\frac{\psi_k}{\pi_k} \right)^{\frac{1}{\sigma}} \left(\frac{1}{\mathcal{P}_t} \right)^{-\frac{1}{\sigma}} \mathcal{C}_t$$

$$\mathcal{P}_t = \left[\sum_k \psi_k^{\frac{1}{\sigma}} \pi_k^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{1-\sigma}}$$

- We can substitute $\sum_k c_{kt} = \mathcal{P}_t \mathcal{C}_t$ into the planner's problem to obtain the following problem, where the distribution of $\mathcal{C}_t$ among households is determined by

$$c_{kt} = \pi_k \left(\frac{\psi_k}{\pi_k} \right)^{\frac{1}{\sigma}} \left(\frac{1}{\mathcal{P}_t} \right)^{-\frac{1}{\sigma}} \mathcal{C}_t$$

- The government's reduced problem is therefore to choose $\left\{\mathcal{C}_t, n_{ijkt}\right\}$ to maximize

$$U_0 = \sum_{t=0}^{\infty} \beta^t \left[\frac{\mathcal{C}_t^{1-\sigma}}{1-\sigma} - \frac{1}{\widehat{\varphi}^{1/\varphi}} \frac{\mathcal{N}_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right]$$

$$\mathcal{N}_t = \left[\sum_k \left(\frac{\psi_k}{\widehat{\varphi}_k^{1/\varphi}} \right) n_{kt}^{\frac{\varphi+1}{\varphi}} \right]^{\frac{\varphi}{\varphi+1}}$$

$$n_{kt} = \left[\int n_{jkt}^{\frac{\theta+1}{\theta}} dj \right]^{\frac{\theta}{\theta+1}}$$

$$n_{jkt} = \left[\sum_{i \in j} n_{ijkt}^{\frac{\eta+1}{\eta}} \right]^{\frac{\eta}{\eta+1}}$$

subject to

$$\mathcal{P}_t \mathcal{C}_t + K_{t+1} = \sum_k \int \sum_{i \in j} w_{ijkt} n_{ijkt} dj + R_t K_t + (1-\delta) K_t + \Pi_t$$

and rationing constraints

$$n_{ijk} \leq \overline{n}_{ijk}$$

- The planner's first order condition for $\mathcal{C}_t$ is then

$$U_{\mathcal{C}} \left(\mathcal{C}_t, \mathcal{N}_t \right) = \Lambda_t \mathcal{P}_t$$

where

$$U \left(\mathcal{C}_t, \mathcal{N}_t \right) = \frac{\mathcal{C}_t^{1-\sigma}}{1-\sigma} - \frac{1}{\widehat{\varphi}^{1/\varphi}} \frac{\mathcal{N}_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}.$$

D.2.3 Allocation problem - Labor

- Consider the terms in the government's Lagrangean that feature n_{ijkt}

- Write the multiplier on the rationing constraint as $\zeta_{ijkt} = \beta^t \Lambda_t w_{ijkt} \left(1 - p_{ijkt} \right)$

- These terms are

$$\mathcal{L} = \cdots + \beta^t U \left(\mathcal{C}_t, \mathcal{N}_t \right) + \ldots + \beta^t \Lambda_t w_{ijkt} n_{ijkt} + \beta^t \Lambda_t w_{ijkt} \left(1 - p_{ijkt} \right) \left[\overline{n}_{ijkt} - n_{ijkt} \right] + \ldots$$

$$\mathcal{L} = \cdots + \beta^t U \left(\mathcal{C}_t, \mathcal{N}_t \right) + \ldots + \beta^t \Lambda_t w_{ijkt} p_{ijkt} n_{ijkt} + \beta^t \Lambda_t w_{ijkt} \left(1 - p_{ijkt} \right) \overline{n}_{ijkt} + \ldots$$

- The first order condition for consumption is as above:

$$\Lambda_t = U_{\mathcal{C}} \left(\mathcal{C}_t, \mathcal{N}_t \right) / \mathcal{P}_t$$

- The first order condition for n_{ijkt} is

$$w_{ijkt} p_{ijkt} = -\frac{U_{\mathcal{N}} \left(\mathcal{C}_t, \mathcal{N}_t \right)}{U_{\mathcal{C}} \left(\mathcal{C}_t, \mathcal{N}_t \right) / \mathcal{P}_t} \left(\frac{\partial \mathcal{N}_t}{\partial n_{kt}} \right) \left(\frac{\partial n_{kt}}{\partial n_{jkt}} \right) \left(\frac{\partial n_{jkt}}{\partial n_{ijkt}} \right)$$

- Using the definitions of aggregators $\mathcal{N}_t$, n_{kt}, n_{jkt}:

$$w_{ijkt} p_{ijkt} = -\frac{U_{\mathcal{N}}\left(\mathcal{C}_t, \mathcal{N}_t\right)}{U_{\mathcal{C}}\left(\mathcal{C}_t, \mathcal{N}_t\right)/\mathcal{P}_t} \left(\frac{\psi_k}{\widehat{\varphi}_k^{1/\varphi}}\right) \left(\frac{n_{kt}}{\mathcal{N}_t}\right)^{\frac{1}{\varphi}} \left(\frac{n_{jkt}}{n_{kt}}\right)^{\frac{1}{\theta}} \left(\frac{n_{ijkt}}{n_{jkt}}\right)^{\frac{1}{\eta}}$$

- Define the *shadow wate* $\widetilde{w}_{ijkt} := w_{ijkt} p_{ijkt}$.

- Using this definition:

$$\widetilde{w}_{ijkt} = -\frac{U_{\mathcal{N}}\left(\mathcal{C}_t, \mathcal{N}_t\right)}{U_{\mathcal{C}}\left(\mathcal{C}_t, \mathcal{N}_t\right)/\mathcal{P}_t} \left(\frac{\psi_k}{\widehat{\varphi}_k^{1/\varphi}}\right) \left(\frac{n_{kt}}{\mathcal{N}_t}\right)^{\frac{1}{\varphi}} \left(\frac{n_{jkt}}{n_{kt}}\right)^{\frac{1}{\theta}} \left(\frac{n_{ijkt}}{n_{jkt}}\right)^{\frac{1}{\eta}} \qquad (*)$$

- We now define the following *shadow wage indexes* at the market, type and aggregate level:

$$\widetilde{w}_{jkt} := \left[\sum_{i \in j} \widetilde{w}_{ijkt}^{1+\eta}\right]^{\frac{1}{1+\eta}}$$

$$\widetilde{w}_{kt} := \left[\int \widetilde{w}_{jkt}^{1+\theta} dj\right]^{\frac{1}{1+\theta}}$$

$$\widetilde{\mathcal{W}}_t := \left[\sum_k \left(\frac{\widehat{\varphi}_k}{\psi_k^{\varphi}}\right) \widetilde{w}_{kt}^{1+\varphi}\right]^{\frac{1}{1+\varphi}}$$

- Using the definition of $\widetilde{w}_{jkt}$ and $n_{jkt} = \left[\sum_{i \in j} n_{ijkt}^{\frac{\eta+1}{\eta}}\right]^{\frac{\eta}{\eta+1}}$ in $(*)$ we have

$$\widetilde{w}_{jkt} = -\frac{U_{\mathcal{N}}\left(\mathcal{C}_t, \mathcal{N}_t\right)}{U_{\mathcal{C}}\left(\mathcal{C}_t, \mathcal{N}_t\right)/\mathcal{P}_t} \left(\frac{\psi_k}{\widehat{\varphi}_k^{1/\varphi}}\right) \left(\frac{n_{kt}}{\mathcal{N}_t}\right)^{\frac{1}{\varphi}} \left(\frac{n_{jkt}}{n_{kt}}\right)^{\frac{1}{\theta}}$$

- Then using the definition of $\widetilde{w}_{kt}$ and $n_{kt} = \left[\int n_{jkt}^{\frac{\theta+1}{\theta}} dj\right]^{\frac{\theta}{\theta+1}}$:

$$\widetilde{w}_{kt} = -\frac{U_{\mathcal{N}}\left(\mathcal{C}_t, \mathcal{N}_t\right)}{U_{\mathcal{C}}\left(\mathcal{C}_t, \mathcal{N}_t\right)/\mathcal{P}_t} \left(\frac{\psi_k}{\widehat{\varphi}_k^{1/\varphi}}\right) \left(\frac{n_{kt}}{\mathcal{N}_t}\right)^{\frac{1}{\varphi}}$$

- Then using the definition of $\widetilde{\mathcal{W}}_t$ and $\mathcal{N}_t = \left[\sum_k \left(\frac{\psi_k}{\widehat{\varphi}_k^{1/\varphi}}\right) n_{kt}^{\frac{\varphi+1}{\varphi}}\right]^{\frac{\varphi}{\varphi+1}}$:

$$\widetilde{\mathcal{W}}_t = -\frac{U_{\mathcal{N}}\left(\mathcal{C}_t, \mathcal{N}_t\right)}{U_{\mathcal{C}}\left(\mathcal{C}_t, \mathcal{N}_t\right)/\mathcal{P}_t}$$

- Using $U\left(\mathcal{C}_t, \mathcal{N}_t\right)$ we can obtain what we will refer to as the *aggregate labor supply curve*

$$\widetilde{\mathcal{W}}_t = \widehat{\varphi}^{-\frac{1}{\varphi}} \mathcal{P}_t \mathcal{C}_t^\sigma \mathcal{N}_t^{\frac{1}{\varphi}}$$

$$\mathcal{N}_t = \widehat{\varphi} \left(\frac{\widetilde{\mathcal{W}}_t}{\mathcal{P}_t}\right)^\varphi \mathcal{C}_t^{-\varphi\sigma}$$

- A key result is that this is the labor supply curve that would obtain from a government that maximizes $U\left(\mathcal{C}_t, \mathcal{N}_t\right)$ subject to a budget constraint

$$\mathcal{P}_t \mathcal{C}_t + K_{t+1} = \widetilde{\mathcal{W}}_t \mathcal{N}_t + R_t K_t + \left(1 - \delta\right) K_t + \Pi_t$$

and faced no rationing constraints. However such a budget constraint is incorrect, in that $\widetilde{\mathcal{W}}_t \mathcal{N}_t \neq \sum_k \int \sum_{i \in j} w_{ijkt} n_{ijkt} dj$. Nonetheless, the interpretation of the aggregate labor supply curves holds, and shows exactly the extent to which the economy supplies labor *as if* it faced a wage $\widetilde{\mathcal{W}}_t$.

D.2.4 Implied labor supply system to firms

- Using the above results we can refine the labor supply system.

- Using the aggregate labor supply curve in the type-level expression above, we have

$$\widetilde{w}_{kt} = \left(\frac{\psi_k}{\widehat{\varphi}_k^{1/\varphi}}\right) \left(\frac{n_{kt}}{\mathcal{N}_t}\right)^{\frac{1}{\varphi}} \widetilde{\mathcal{W}}_t$$

$$n_{kt} = \left(\frac{\widehat{\varphi}_k}{\psi_k^\varphi}\right) \left(\frac{\widetilde{w}_{kt}}{\widetilde{\mathcal{W}}_t}\right)^\varphi \mathcal{N}_t$$

- Using this in the market-type-level expression above:

$$\widetilde{w}_{jkt} = \left(\frac{n_{jkt}}{n_{kt}}\right)^{\frac{1}{\theta}} \widetilde{w}_{kt}$$

$$\implies n_{jkt} = \left(\frac{\widetilde{w}_{jkt}}{\widetilde{w}_{kt}}\right)^\theta n_{kt}$$

- Using this in the firm-market-type-level expression $(*)$ above, we then recover the same labor supply system as the competitive equilibrium:

$$\widetilde{w}_{ijkt} = \left(\frac{n_{jkt}}{n_{kt}}\right)^{\frac{1}{\theta}} \left(\frac{n_{ijkt}}{n_{jkt}}\right)^{\frac{1}{\eta}} \widetilde{w}_{kt}$$

which can then be written:

$$n_{ijkt} = \left(\frac{\widetilde{w}_{ijkt}}{\widetilde{w}_{jkt}}\right)^{\eta} \left(\frac{\widetilde{w}_{jkt}}{\widetilde{w}_{kt}}\right)^{\theta} n_{kt}$$

$$\widetilde{w}_{jkt} = \left[\sum_{i \in j} \widetilde{w}_{ijkt}^{1+\eta}\right]^{\frac{1}{1+\eta}}$$

$$\widetilde{w}_{kt} = \left[\int \widetilde{w}_{jkt}^{1+\theta} dj\right]^{\frac{1}{1+\theta}}$$

- **Result A** - *This corresponds to the labor supply system from type-k household optimality in a competitive equilibrium with lump sum transfers T_k.*

  - This follows immediately from our derivation in the competitive equilibrium, and the fact that the presence of lump sum transfers in household k's budget constraint do not affect any such derivations.

D.2.5 Implied household labor supply curves

- Combining the planner's allocation of consumption

$$c_{kt} = \pi_k \left(\frac{\psi_k}{\pi_k}\right)^{\frac{1}{\sigma}} \left(\frac{1}{\mathcal{P}_t}\right)^{-\frac{1}{\sigma}} \mathcal{C}_t$$

the aggregate labor supply curve

$$\mathcal{N}_t = \widehat{\varphi} \left(\frac{\widetilde{\mathcal{W}}_t}{\mathcal{P}_t}\right)^{\varphi} \mathcal{C}_t^{-\varphi\sigma}$$

and the planner's allocation of labor

$$n_{kt} = \left(\frac{\widehat{\varphi}_k}{\psi_k^{\varphi}}\right) \left(\frac{\widetilde{w}_{kt}}{\widetilde{\mathcal{W}}_t}\right)^{\varphi} \mathcal{N}_t$$

by substituting out the planner's social welfare weight ψ_k, obtains

$$n_{kt} = \pi_k^{\varphi(\sigma-1)} \widehat{\varphi} \widehat{\varphi}_k \widetilde{w}_{kt}^{\varphi} c_{kt}^{-\varphi\sigma}$$

- Using definition of $\widehat{\varphi}_k = \widetilde{\varphi}_k / \widehat{\varphi}$

$$n_{kt} = \pi_k^{\varphi(\sigma-1)} \widetilde{\varphi}_k \widetilde{w}_{kt}^{\varphi} c_{kt}^{-\varphi\sigma}$$

- Using the definition of $\widetilde{\varphi}_k = \overline{\varphi}_k \pi_k^{\varphi+1}$:

$$n_{kt} = \pi_k \overline{\varphi}_k \widetilde{w}_{kt}^{\varphi} \left(\frac{c_{kt}}{\pi_k}\right)^{-\varphi\sigma}$$

- **Result B** - *This corresponds to the household labor supply curve from type-k household optimality in a competitive equilibrium with lump sum transfers T_k.*

- This follows immediately from our derivation in the competitive equilibrium, and the fact that the presence of lump sum transfers in household k's budget constraint do not affect any such derivations.

D.2.6 Further conditions

- From the above we have obtained the aggregate supply curve. We also have the aggregate resource constraint, which in steady-state is:

$$Y_t = \sum_k c_{kt} + \delta K_t.$$

- Using the consumption results from above, this can be written

$$Y_t = \mathcal{P}_t \mathcal{C}_t + \delta K_t.$$

- Recall also, that we have the aggregation of output

$$Y_t = \frac{1}{1 - \gamma(1 - \alpha)} \widetilde{Y}_t$$

$$\widetilde{Y}_t = \sum_k \widetilde{y}_{kt}$$

- The steady-state Euler equation of the government is

$$1 = \beta[R + (1 - \delta)]$$

- **Result C** - *This corresponds to the household Euler equation from type-k household optimality in a competitive equilibrium with lump sum transfers T_k.*

  - This follows immediately from our derivation in the competitive equilibrium, and the fact that the presence of lump sum transfers in household k's budget constraint do not affect any such derivations.

D.2.7 Aggregating labor demand and output

- From **Result A** above, the labor supply system for type k labor from the solution to the government's primal (allocation) problem corresponds to the labor supply system in the competitive equilibrium.

- Firm optimality conditions will therefore be the same as in the competitive equilibrium, and the aggregation results derived earlier hold up to the type-k level.

- Recall that these results yielded the following. For type-k, output, the shadow wage index, labor supply are as follows, where the third line is the new solution to the government's supply of type-k labor

$$\widetilde{y}_k = \omega_k \widetilde{Z} \widetilde{\xi}_k \widetilde{z}_k n_k^{\widetilde{\alpha}}$$

$$\widetilde{w}_k = \widetilde{\mu}_k \widetilde{\alpha} \widetilde{Z} \widetilde{z}_k \widetilde{\xi}_k n_k^{\widetilde{\alpha}-1}$$

$$n_k = \phi_k \left(\frac{\widetilde{w}_k}{\widetilde{\widetilde{\mathcal{W}}}}\right)^\varphi \mathcal{N}$$

where $\phi_k = \pi_k^{1+\varphi} \left(\frac{\overline{\varphi}_k}{\overline{\varphi}}\right) \psi_k^{-\varphi}$, where $\{\omega_k, \widetilde{\mu}_k, \widetilde{z}_k\}$ are as in the competitive equilibrium, derived above.

- We also have two aggregation conditions, where we have substituted in

$$\widetilde{Y} = \sum_k \widetilde{y}_k$$

$$\widetilde{\mathcal{W}} = \left[\sum_k \phi_k \widetilde{w}_{kt}^{1+\varphi} \right]^{\frac{1}{1+\varphi}}$$

- In the same way as we used the sets of 5 conditions to aggregate output and labor demand in the competitive equilibrium we can also use the same approach on this set of conditions.

- The result is that aggregate output and the aggregate shadow wage can be expressed using aggregate shadow markdown, productivity and misallocation wedges:

$$\widetilde{Y} = \omega \widetilde{Z} \widetilde{z} \mathcal{N}^\alpha$$

$$\widetilde{\mathcal{W}} = \widetilde{\mu} \alpha \widetilde{Z} \widetilde{z} \mathcal{N}^{\alpha-1}$$

where

$$\widetilde{z} = \left[\sum_k \left(\xi_k \widetilde{z}_k \right)^{\frac{1+\varphi}{1+\varphi(1-\alpha)}} \phi_k^{\frac{\alpha}{1+\varphi(1-\alpha)}} \right]^{\frac{1+\varphi(1-\alpha)}{1+\varphi}}$$

$$\widetilde{\mu} = \left[\sum_k \left(\frac{\xi_k \widetilde{z}_k}{\widetilde{z}} \right)^{\frac{1+\varphi}{1+\varphi(1-\alpha)}} \phi_k^{\frac{\alpha}{1+\varphi(1-\alpha)}} \widetilde{\mu}_k^{\frac{1+\varphi}{1+\varphi(1-\alpha)}} \right]^{\frac{1+\varphi(1-\alpha)}{1+\varphi}}$$

$$\omega = \sum_k \left(\frac{\xi_k \widetilde{z}_k}{\widetilde{z}} \right)^{\frac{1+\varphi}{1+\varphi(1-\alpha)}} \phi_k^{\frac{\alpha}{1+\varphi(1-\alpha)}} \left(\frac{\widetilde{\mu}_k}{\widetilde{\mu}} \right)^{\frac{\varphi\alpha}{1+\varphi(1-\alpha)}} \omega_k$$

- Capital demand is as in the competitive equilibrium:

$$Rk_{ijk} = \alpha \left(1 - \gamma \right) y_{ijk}$$

which when aggregated yeilds

$$RK = \alpha \left(1 - \gamma \right) Y$$

D.2.8 Full set of conditions for the solution of government allocation problem and competitive equilibrium

- Equilibrium under the government allocation problem, can therefore be summarized in the following conditions

$$\widetilde{Y} = \omega \widetilde{Z} \widetilde{z} \mathcal{N}^{\alpha}$$

$$\mathcal{W} = \widetilde{\mu} \alpha \widetilde{Z} \widetilde{z} \mathcal{N}^{\alpha-1}$$

$$\mathcal{N} = \widehat{\varphi} \left(\frac{\widetilde{\mathcal{W}}}{\mathcal{P}} \right)^{\varphi} C^{-\varphi\sigma}$$

$$Y = \mathcal{P}C + \delta K$$

$$Y = \frac{1}{1 - \gamma (1 - \alpha)} \widetilde{Y}$$

$$\mathcal{P} = \left[\sum_k \psi_k^{\frac{1}{\sigma}} \pi_k^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{1-\sigma}}$$

$$1 = \beta \left[R + (1 - \delta) \right]$$

$$RK = \alpha (1 - \gamma) Y$$

- This system of 8 equations in 8 unknowns $\left\{ \widetilde{Y}, Y, \mathcal{N}, C, \mathcal{P}, R, K, \mathcal{W} \right\}$ can be solved in closed form.

- Once solved, household type variables can be determined from the government's first order conditions:

$$n_k = \phi_k \left(\frac{\widetilde{w}_k}{\widetilde{\mathcal{W}}} \right)^{\varphi} \mathcal{N}$$

$$c_k = \pi_k \left(\frac{\psi_k}{\pi_k} \right)^{\frac{1}{\sigma}} \left(\frac{1}{\mathcal{P}} \right)^{-\frac{1}{\sigma}} C$$

- The allocation of labor n_k to firms is then determined by the labor supply system:

$$n_{ijk} = \left(\frac{\widetilde{w}_{ijk}}{\widetilde{w}_{jk}} \right)^{\eta} \left(\frac{\widetilde{w}_{jk}}{\widetilde{w}_k} \right)^{\theta} n_k$$

D.2.9 Implementation with lump-sum taxes

- **Results A, B, C** above imply that the government's optimality conditions of its allocation problem coincide with those of households in a competitive equilibrium with lump sum transfers T_k.

- Therefore the government can choose arbitrary lump sum transfers and yield the same set of optimality conditions.

- The only thing that is left is to determine the lump sum transfers themselves.

- These can simply be read off of the household's budget constraints, which in steady state are:

$$c_k = \int \sum_{i \in j} w_{ijk} n_{ijk} \, dj + \kappa_k \left[(R - \delta) K + \Pi \right] + T_k$$

$$T_k = c_k - \int \sum_{i \in j} w_{ijk} n_{ijk} \, dj - \kappa_k \left[(R - \delta) K + \Pi \right]$$

- Transfers clearly sum to zero since summing the household budget constraint yeilds the government budget constraint if and only if $\sum_k T_k = 0$.

D.3   Leveraging the government solution to solve the competitive equilibrium

- In practice we leverage the government problem described above to solve the competitive equilibrium of the economy.

- We do this in the trandition of the *Negishi algorithm.*

- The above section described how we can first *fix social welfare weights*, then solve the government problem, then determine the required lump-sum transfers.

- The competitive equilibrium can be solved under *guessing of social welfare weights*, then solve the government problem, then determine the required lump-sum transfers, and then *iterating on the guess of social welfare weights,* until the implied lump sum transfers are all equal to zero.

- Under the social welfare weights that deliver zero lump sum transfers, the competitive equilibrium budget constraints of all households hold by construction:

$$c_k = \int \sum_{i \in j} w_{ijk} n_{ijk} \, dj + \kappa_k \left[(R - \delta) K + \Pi \right]$$

and all remaining competitive equilibrium conditions also hold (i.e. each household's Euler equation, labor supply system to firms, household labor supply curve, resource constraint, capital and labor demand).

- The solution of the government problem, which can be achieved largely in closed form, is therefore a key part of our computational strategy.