NBER WORKING PAPER SERIES

INNOVATION NETWORKS AND INNOVATION POLICY

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Working Paper 29607
http://www.nber.org/papers/w29607

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
December 2021

We thank Daron Acemoglu, John Grigsby, Benny Kleinman, Atif Mian, Ezra Oberfield, Alessandra Peter, Swapnika Rachapalli, Stephen Redding, Peter Schott, Aleh Tsyvinski, Sifan Xue, and Yucheng Yang for useful feedback. We also thank seminar participants at Princeton University. Xugan Chen provided excellent research assistance. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

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NBER Working Paper No. 29607
December 2021
JEL No. F43,O33,O38

ABSTRACT

We study the optimal allocation of R&D resources in an endogenous growth model with an innovation network, through which one sector's past innovations may benefit other sectors' future innovations. First, we provide closed-form sufficient statistics for the optimal path of R&D resource allocation, and we show that planners valuing long-term growth should allocate more R&D toward key sectors that are upstream in the innovation network. Second, we extend to an open-economy setting and illustrate an incentive for countries to free-ride on fundamental technologies: an economy more reliant on foreign knowledge spillovers has less incentive to direct resources toward innovation-upstream sectors, leading to cross-country differences in unilaterally optimal R&D allocations across sectors. Third, we build the global innovation network based on over 30 million global patents and establish its empirical importance for knowledge spillovers. Fourth, we apply the model to evaluate R&D allocations across countries and time. Adopting optimal R&D allocations can generate substantial welfare improvements across the globe. For the United States, R&D misallocation accounts for about 0.68 percentage points of missing annual growth since the 2000s.

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1 Introduction

Innovation is the source of long-run growth, and how to design policy to foster innovation has long been a central question in economics. Innovation activities require researchers and scientists to build on previous discoveries, often from outside their own fields or sectors. Innovators in decentralized markets may not internalize the future spillovers they generate, leading to inefficient allocations of innovation investment. This is particularly true for more fundamental technologies for which the social value from long-run innovation spillovers dwarfs their private value.

How should innovation resources be allocated *across sectors,* in the presence of an inter-sector innovation network through which knowledge spillovers occur? For example, how many resources should the economy devote to R&D in semiconductors relative to consumer electronics, or chemistry relative to pharmaceutics? How does the optimal allocation depend on the structure of the innovation network? How should R&D allocations differ across countries? How much does innovation resource allocation matter for economic growth?

We answer these questions theoretically and quantitatively. The key novelty of our approach is that we introduce a network perspective into modeling the spillover structure of innovation. Specifically, we embed an innovation network into an otherwise canonical multisector, quality-ladder endogenous growth model. A finite amount of resources (i.e., scientists) may be deployed across sectors to innovate and improve product quality. The innovation network is defined by the structure of cross-sector knowledge spillovers, as one sector's past innovations may subsequently benefit other sectors' future innovation activities by helping scientists in those sectors generate new ideas more productively. The state variables of the economy are sectoral knowledge stocks, which reflect the accumulation of past innovations in each sector. Through dynamic spillovers across the network, the state variables form a dynamical system, in which the evolution of the knowledge stock *in each sector* depends endogenously on the entire time path of resource allocation *across all sectors* of the economy. The key decision of interest is how to efficiently allocate R&D resources across sectors in the network as the economy grows.

We begin by modeling a closed economy. Despite the complexity of dynamic network spillovers, we are able to explicitly solve for the optimal path of R&D resource allocation and express the closed-form solution as sufficient statistics in terms of consumer preferences across sectoral products and the structure of the innovation network. These sufficient statistics are qualitatively intuitive: they account for the direct and indirect network effects of R&D on sectoral output, discounting benefits that occur far into the future. A key parameter for the optimal cross-sector allocation of R&D resources is the society's discount rate. A more patient society valuing long-term growth (i.e., with a low discount rate) should optimally allocate more resources toward fundamental sectors that are upstream in the innovation network, such as semiconductors. These are sectors that can generate widespread and long-lasting knowledge spillovers to many other sectors, directly or indirectly. By contrast, an impatient, short-termist society should allocate more R&D resources toward innovation-downstream sectors such as consumer goods. We show that the optimal R&D allocation can be implemented by imposing simple R&D subsidies to the market equilibrium, and we also provide closed-form solutions for the welfare impact of adopting the optimal R&D allocation starting away from the balanced growth path and taking into account entire transitional dynamics.

A key object of theoretical and quantitative importance is what we call the *innovation centrality* of each sector. The vector of innovation centrality is the dominant eigenvector of the innovation network and, intuitively, captures the extent to which a sector's R&D activities contribute to economic growth, taking into account the network effects. Formally, the innovation centrality vector is a sufficient statistic for evaluating the growth impact of R&D allocations: the economic growth rate along a balanced growth path is affine in the inner product between the innovation centrality vector and the vector of log-R&D allocation shares. Consequently, the innovation centrality vector also coincides with the growth-maximizing R&D allocation along a balanced growth path, and sectors with higher innovation centrality are therefore more *fundamental* in the innovation network. We show the optimal R&D allocation chosen by a benevolent planner can be written as a weighted average between the innovation centrality vector and the vector of consumer expenditure shares. The former represents the planner's incentives to take advantage of knowledge spillovers for future growth, and the latter represents the planner's incentives to expand knowledge in ways directly benefiting the consumer. A more patient planner places a higher weight on the former.

Not every economy features a self-contained domestic innovation network, and many benefit from foreign knowledge spillovers. Intuitively, global knowledge spillovers would lead to cross-country differences in the optimal cross-sector R&D allocations. To formalize this, we extend our model to an open-economy setting with international knowledge spillovers and trade. We provide sufficient statistics for the R&D allocations that maximize each country's domestic welfare—what we call *unilaterally optimal* allocations—along a balanced growth path. Our analysis highlights an incentive for countries to free-ride on fundamental technologies: holding the total level of R&D constant, an economy more reliant on foreign knowledge spillovers has less incentive to direct resources toward fundamental or innovation-central sectors—and more toward consumer goods—as if the planners in these economies are more impatient. This leads to cross-country differences in unilaterally optimal R&D allocations. Economies with well-developed domestic innovation networks, such as the United States and Japan, should conduct more R&D in innovation-central sectors; by contrast, economies that heavily rely on foreign knowledge spillovers should direct more R&D toward consumer goods.

A main advantage of our sufficient statistics for optimal R&D allocations is that we can easily compute them using data on sectoral production and the innovation network. By comparing real-world R&D resource allocations with optimal ones, we can quantitatively evaluate the importance of R&D misallocation on both economic growth and welfare.

To leverage our theory and evaluate R&D allocations in the data, we construct a novel, global innovation network from over 36 million patents and their citations, collected from over 40 main patent authorities around the world and compiled by Google Patents. The data contain patent-level information, covering innovations that took place in most economies between 1976 and 2020. We construct the innovation network based on citation shares across countries and sectors. Innovation centrality is highly skewed across 645 international technological classes (IPCs). A handful of IPCs—such as digital data processing, semiconductors, medical diagnoses, and digital communications—should be allocated disproportionately large shares of R&D resources in order to maximize growth. Countries vary widely regarding reliance on foreign spillovers: 80% of citations from U.S. patents are toward other U.S. patents, but most other economies—including China, South Korea, Germany, and, in early periods of the sample, Japan—are foreign-reliant with domestic citation shares well below 50%.

As a model validation exercise, we empirically test the key mechanism behind our theory, namely that a sector's innovation activities benefit from past innovation in upstream sectors linked through the innovation network. We show the mechanism holds in both the U.S. domestic innovation network and the global innovation network and across a variety of innovation output metrics, including patent counts, future citations, and patent value measured by stock market reactions upon patent approval. We show knowledge spillovers are directional: each sector's innovation output responds only to past upstream innovations and does not respond to past downstream innovations. We find that the innovation network is only weakly correlated with the input-output production network, such that there is substantial independent variation in both network structures. Relative to input-output linkages, the innovation network is a significantly stronger channel through which knowledge spillovers take place.

Our main empirical application uses the model to evaluate cross-sector R&D allocations in the real world. For each country and year, we compute the unilaterally optimal R&D allocation using our sufficient statistics and data on both production and the innovation network. We compare optimal R&D allocations against sectoral R&D expenditures and patent output in the data. In economies generally perceived to be more innovative, such as the United States, Germany, and Japan, the data indicate that sectors that should optimally receive more resources do receive more resources. This positive relationship also holds in later sample periods for South Korea and China, during phases of rapid growth in these economies. By contrast, for many other economies, such as Brazil, India, Indonesia, Mexico, and Russia, real-world R&D allocations deviate significantly from the optimal allocation throughout our sample.

What accounts for the R&D allocations in economies such as the United States, Japan, and Germany, and why are innovation-central sectors receiving more resources in those countries? As one potential explanation, we provide some evidence that a significant share of innovation activities in these countries take place in innovation hubs that operate and hold intellectual property rights across a wide range of technological classes. Such hubs include IBM, Sony, and Siemens, whose R&D activities build heavily on internal knowledge within the firm. These innovation hub's self-reliance suggests they internalize some of the network effects and thereby allocate R&D resources more like a planner would.

Finally, we evaluate each country's potential growth and welfare gains when adopting its optimal R&D allocation. We show that, around the globe, substantial welfare gains are left on the table. Even for the United States, where real-world R&D resource allocation correlates strongly with the theoretical optimal, there is still substantial misallocation. Through the lens of our model, R&D misallocation accounts for about 0.68 percentage points of missing annual growth in the United States since the 2000s. The welfare losses are even larger: once transitional dynamics are considered, welfare losses are equivalent to about 2.5 percentage points of missing annual economic growth.

This study relates to several strands of existing work. First, our study contributes to a long line of research on knowledge spillovers and innovation policy (Aghion, Bloom, Blundell, Griffith and Howitt, 2005, Bloom, Schankerman and Van Reenen, 2013, Lucking, Bloom and Van Reenen, 2018, Bloom, Van Reenen and Williams, 2019, Jones and Summers, forthcoming), particularly in the context of endogenous economic growth (Jones and Williams, 1998, Acemoglu, Akcigit, Bloom and Kerr, 2018b, Akcigit and Kerr, 2018, Atkeson and Burstein, 2019, Garcia-Macia, Hsieh and Klenow, 2019, Bloom, Jones, Van Reenen and Webb, 2020, Akcigit, Hanley and Serrano-Velarde, 2021, Cai and Tian, 2021, Koenig, Song, Storesletten and Zilibotti, forthcoming). We contribute to this literature by tackling a key open question: how to optimally allocate R&D resources across sectors in the presence of an innovation network with cross-sector knowledge spillovers. There are studies that, like ours, investigate cross-sector knowledge linkages, including Acemoglu, Akcigit and Kerr (2016), Cai and Li (2019), Huang and Zenou (2020), and Yang and Zhu (2020), and, in an open economy setting, Cai, Li and Santacreu (forthcoming); unlike ours, these studies do not analyze optimal resource allocation and policy interventions. There is also a large literature on cross-country knowledge diffusion (Caballero and Jaffe, 1993, Jaffe, Trajtenberg and Henderson, 1993, Eaton and Kortum, 1999, 2006, Coe and Helpman, 1995, Coe, Helpman and Hoffmaister, 2009, Santacreu, 2015, Buera and Oberfield, 2020); see Keller (2004) and Melitz and Redding (2021) for surveys. Relative to this literature, which focuses on the country-level implications of foreign spillovers, the novelty of our open-economy analysis is to show how sectoral-level foreign dependence interacts with the innovation network and shapes the unilaterally optimal R&D allocation across sectors.

Second, we contribute to the fast-growing literature on networks in macroeconomics and trade (Carvalho, 2010, Gabaix, 2011, Acemoglu, Carvalho, Ozdaglar and Tahbaz-Salehi, 2012, Jones, 2011, 2013, Grassi, 2017, Acemoglu, Akcigit and Kerr, 2015, Baqaee, 2018, Lim, 2018, Oberfield, 2018, Liu, 2019, Baqaee and Farhi, 2019, 2020, Chaney, 2018, Taschereau-Dumouchel, 2020, Kleinman, Liu and Redding, 2021b). Particularly related are recent papers that introduce methodologies for dynamic network analysis (Kleinman, Liu and Redding, 2021a, Liu and Tsyvinski, 2021) and studies on policy interventions targeting specific sectors in production and strategic networks (King, Tarbush and Teytelboym, 2019, Liu, 2019, Galeotti, Golub and Goyal, 2020). Relative to this literature, our contribution is to embed an innovation network into a growth model and study innovation policy.

Third, we contribute to the large literature on resource misallocation (Restuccia and Rogerson, 2008, Hsieh and Klenow, 2009, Jones, 2013, David and Venkateswaran, 2019, Hsieh, Hurst, Jones and Klenow, 2019, Baqaee and Farhi, 2020). While such literature mostly focuses on the misallocation of production resources, we instead study the misallocation of innovation resources, and our analysis is therefore inherently dynamic in nature.

2 Optimal Innovation Policy in a Closed Economy

We study the optimal allocation of R&D resources in a multisector, quality-ladder growth model with an innovation network. This section studies a closed economy. After setting up the model, we first analyze the efficient allocation of R&D resources across sectors (Section 2.2), before discussing potential inefficiencies in a decentralized equilibrium and how to implement the efficient allocation using R&D taxes and subsidies (Section 2.3). We analyze both the long-run impact of optimal R&D allocation on the balanced growth path (Section 2.4) and the welfare impact of adopting the optimal R&D allocation, taking into account the transitional dynamics (Section 2.5).

2.1 Setup

Preferences and Production Technology There is a representative consumer with log flow utility and exponential discounting at rate ρ:

$$V_t = \int_t^\infty e^{-\rho(s-t)} \ln c_s \, \mathrm{d}s. \tag{1}$$

The instantaneous consumption aggregator c_t at each time t is a Cobb-Douglas combination over sectoral composite goods c_i, $i = 1, ..., K$:

$$c_t = \prod_{i=1}^{K} c_{it}^{\beta_i}, \quad \sum_{i=1}^{K} \beta_i = 1. \tag{2}$$

We refer to β_i as the consumption share of sector i.

Each sectoral composite good i is a Cobb-Douglas aggregator over a continuum of varieties of intermediate products. Each intermediate product can be potentially supplied in a countably infinite number of qualities. Let $q_{it}(\nu)$ denote the highest quality of variety ν available in sector i. The sectoral composite good aggregator c_{it} is given by

$$\ln c_{it} = \int_0^1 \ln \left(q_{it}(\nu)\, x_{it}(\nu|q) \right) d\nu, \tag{3}$$

where $x_{it}(\nu|q)$ is the quantity of the variety ν of quality q in the production process. The sectoral aggregator (3) implicitly imposes that only the highest quality variety will be used in production. The intermediate varieties are produced linearly, one-for-one from production workers:

$$x_{it}(\nu|q) = \ell_{it}(\nu) \text{ for all } i, t, \nu, q. \tag{4}$$

The Innovation Process R&D can improve product quality. Let q_{it} denote the average quality of the intermediate varieties used for production in sector i at time t:

$$\ln q_{it} \equiv \int_0^1 \ln q_{it}(\nu) \, \mathrm{d}\nu.$$

We also refer to q_{it} as sector i's *knowledge stock* at time t. The collection of cross-sector knowledge stocks $\{q_{it}\}_{i=1}^{K}$ are the state variables of the economy.

At each time t, mass s_i of scientists employed in sector i generate new ideas n_{it}:

$$n_{it} = s_{it} \eta_i \chi_{it}, \qquad \chi_{it} \equiv \prod_{j=1}^{K} q_{jt}^{\omega_{ij}}, \qquad \sum_{j=1}^{K} \omega_{ij} \equiv 1. \tag{5}$$

η_i is the exogenous component of innovation productivity, and χ_{it} is the endogenous component. Specifically, χ_{it} is a Cobb-Douglas combination of knowledge stock across all sectors. The aggregator χ_{it} implies that a larger knowledge stock q_j in sector j facilitates new idea generation in sector i with elasticity ω_{ij}, thereby making scientists in sector i conduct R&D more productively. Our formulation thus captures the notion that scientists stand on the shoulders of giants spread across all sectors of the economy. We impose the assumption that χ_{it} has constant returns to scale ($\sum_{j=1}^{K} \omega_{ij} = 1$) to ensure sustained and nonexplosive growth. Absent knowledge spillovers, $\omega_{ij} = 1$ if $i = j$ and is zero otherwise.

New ideas stochastically translate into innovation, thereby improving product quality. Specifically, we assume innovation of each variety ν in sector i occurs following a Poisson process with arrival rate $\ln(n_{it}/q_{it})$. Upon innovation, a new vintage of the improved variety is discovered, with proportional quality improvement e^{λ}, $\lambda > 0$. The new vintage thus has quality $e^{\lambda} q_{it}(\nu)$.

Even though innovation is stochastic at the variety level, the law of motion for quality is deterministic at the sector level. Sector i's knowledge stock evolves according to:

$$\dot{q}_{it}/q_{it} = \lambda \ln \left(n_{it}/q_{it}\right). \tag{6}$$

The arrival rate $\ln \left(n_{it}/q_{it}\right)$ decreases in existing knowledge stock q_{it}, capturing the notion that innovation is harder to find as the knowledge stock in sector i expands (Bloom et al. (2020)).

Throughout the rest of the paper, we use boldface variables to denote column vectors (lowercase) and matrices (uppercase). Let $\boldsymbol{q}_t$ denote the column vector whose i-th entry is q_{it}; $\boldsymbol{q}_t$ captures the state variables of the economy.

Definition 1. (**Innovation Network**) The innovation network $\boldsymbol{\Omega} \equiv [\omega_{ij}]$ is the $K \times K$ matrix whose ij-th entry is ω_{ij}.

The $\boldsymbol{\Omega}$ matrix is a central object of this study. Absent cross-sector knowledge spillovers, $\boldsymbol{\Omega} = \boldsymbol{I}$ is the identity matrix. Elements of the $\boldsymbol{\Omega}$ matrix ω_{ij} capture the degree to which sector i's idea generation relies on sector j's existing knowledge; we refer to sector j as *upstream* to sector i and, conversely, i as *downstream* to j; this terminology captures the notion that knowledge flows from upstream sector j to downstream sector i.

Resources We close the model with resource constraints. The economy is endowed with two exogenous stocks of resources: production workers of mass $\bar{\ell}$, and research scientists of mass $\bar{s}$. Workers are employed to produce intermediate goods as in (4). Scientists are employed to conduct R&D and improve the quality of intermediate products. Let ℓ_{it} denote the total mass of workers employed in sector i; the market clearing conditions for production workers and scientists are:

$$\textstyle\sum_{i=1}^{K} \ell_{it} = \bar{\ell}, \qquad \ell_{it} \equiv \int_0^1 \ell_{it}\left(\nu\right) d\nu; \qquad \sum_{i=1}^{K} s_i = \bar{s}. \tag{7}$$

Remarks on Model Features

Remark 1. *Testable Implications of the Innovation Process.* The law of motion (6) for each sector's knowledge stock implies that the (log-) knowledge stock at a given time can be written as the discounted sum of (log-) past ideas:

$$\ln q_{it} = \lambda \textstyle\int_0^\infty e^{-\lambda s} \ln n_{i,t-s} \, \mathrm{d}s. \tag{8}$$

Taking logs of the ideas production function (5) and substituting (8) for q_{it}, we obtain a log-linear relationship between new ideas in sector i, the amount of R&D resources employed in the sector, and past ideas from other sectors:

$$\ln n_{it} = \ln \eta_i + \ln s_{it} + \lambda \textstyle\sum_{j=1}^{K} \omega_{ij} \left(\int_0^\infty e^{-\lambda s} \ln n_{j,t-s} \, \mathrm{d}s \right). \tag{9}$$

Equation (9) is empirically testable. We assume new ideas are patented. Equation (9) thus implies that, controlling for R&D expenditures, a sector tends to create more patents at times when its upstream sectors had more patents in the past, and the effect weakens over longer time lags. In Section 5.2 we test and show this relationship holds empirically.

Remark 2. *Input-Output (I-O) Linkages.* The baseline model features an innovation network $\mathbf{\Omega}$ in the form of cross-sector knowledge spillovers, without a production network of I-O linkages. In the Appendix, we generalize our results to a setting that features both production and innovation networks, and we show, with straightforward modifications, our characterizations extend to that setting. Furthermore, we later empirically test equation (9) and show knowledge spillovers that occur through the innovation network dominate the potential spillovers through I-O linkages; for this reason, we abstract away from the production network in the baseline model.

Remark 3. *Separate R&D and Production Resources.* In the baseline model, we specify that R&D and production require two distinct resource types: scientists $\bar{s}$ and production workers $\bar{\ell}$. This separation implies the total endowment of R&D resources ($\bar{s}$) is exogenously fixed and is thus not a potential source of inefficiency. We choose this specification for simplicity; as we show below, our results characterize the cross-sector allocation shares of R&D resources ($s_{it}/\bar{s}$), and our characterization is invariant to the level of R&D resources $\bar{s}$. Hence, our analysis of cross-sector allocation shares holds even in a richer model in which a single worker type can move between R&D and production.

Remark 4. *Constant Returns.* We have assumed that the knowledge aggregator χ_{it} in the ideas production function (5) is constant returns to scale, that is, the innovation network $\mathbf{\Omega}$ is a row-stochastic matrix with a spectral radius of one. When this assumption is violated and if the spectral radius of $\mathbf{\Omega}$ is strictly below one, our characterization of the efficient allocation and transitional dynamics continues to hold, but the economy no longer features sustained growth unless R&D resources grow exogenously, as in semi-endogenous growth models (Jones (1995)).

2.2 Efficient Allocation of R&D Resources

In this section we study the efficient allocation of R&D resources in the economy. We postpone discussing the decentralized equilibrium and inefficiencies therein until after we characterize the efficient allocation.

Consider a benevolent social planner who chooses the entire time sequence of worker and scientist allocations across sectors to maximize consumer utility. We can write the planner's problem as

$$V\left(\{q_{i0}\}\right) \equiv \max_{\{\ell_{it}(\nu), s_{it}\}} \int_0^\infty e^{-\rho t} \sum_{i=1}^K \beta_i \ln c_{it} \, \mathrm{d}t,$$

subject to the sectoral aggregator (3) for c_{it}, production function of ideas (5), the law of motion for sectoral knowledge (6), and the resource constraints (7).

Lemma 1. *The planner allocates production workers in proportion to the consumption share vector* $\boldsymbol{\beta}$*: for all* t, $\ell_{it}(\nu) = \ell_{it} = \beta_i \bar{\ell}$ *for each sector* i *and variety* ν.

We use Lemma 1 to simplify the planner's problem into choosing how to allocate scientists only. Recall $\boldsymbol{\Omega} \equiv [\omega_{ij}]$ is the matrix that encodes the innovation network, and $\ln \boldsymbol{q}_t \equiv [\ln q_{it}]_{i=1}^{K}$ is the vector of log-knowledge stock at time t. Let $\gamma_{it} \equiv s_{it}/\bar{s}$ denote the share of scientists allocated to sector i at time t, and let $\boldsymbol{\gamma}_t$ denote the vector $[\gamma_{it}]_{i=1}^{K}$ which sums to one. Using equations (3) and (4) to express consumption in terms of production worker allocation and then applying Lemma 1, we rewrite the planner's problem in vector form as

$$\max_{\{\gamma_t\} \text{ s.t. } \gamma_t{}'\mathbf{1}=1 \forall t} \int_0^\infty e^{-\rho t} \boldsymbol{\beta}' \ln \boldsymbol{q}_t \, \mathrm{d}t \tag{10}$$

$$\text{s.t. } \mathrm{d} \ln \boldsymbol{q}_t \big/ \mathrm{d}t = \lambda \cdot \left(\ln \boldsymbol{\eta} + \ln \bar{s} + \ln \boldsymbol{\gamma}_t + (\boldsymbol{\Omega} - \boldsymbol{I}) \ln \boldsymbol{q}_t \right), \tag{11}$$

where we obtain (11) by substituting the ideas production function (5) into $\boldsymbol{q}_t$'s law of motion (6).

The planner's problem may seem intractable: the economy features an entire vector of state variables (sectoral knowledge stocks), and the law of motion involves dynamic network spillovers across sectors. Our formulation, however, is especially tractable: both the planner's objective function (10) and the law of motion (11) are log-linear in the state variables $\boldsymbol{q}_t$. Such tractability enables us to characterize the solution—the entire time path of optimal R&D allocation—in closed form.

Proposition 1. *Starting from any vector of initial knowledge stock* $\boldsymbol{q}_0$*, the optimal R&D allocation is time-invariant and follows, along the entire time path,*

$$\boldsymbol{\gamma}' = \frac{\rho}{\rho + \lambda} \boldsymbol{\beta}' \left(\boldsymbol{I} - \frac{\boldsymbol{\Omega}}{1 + \rho/\lambda} \right)^{-1}. \tag{12}$$

Proposition 1 shows that the optimal cross-sector R&D allocation is time-invariant and follows $\boldsymbol{\gamma}' \propto \boldsymbol{\beta}' \left(\boldsymbol{I} - \frac{\boldsymbol{\Omega}}{1+\rho/\lambda} \right)^{-1}$; the proportionality constant, $\frac{\rho}{\rho+\lambda}$, ensures that the scientist allocation shares sum to one ($\sum_i \gamma_i = 1$). To understand the intuition for the result, note that another way to write the optimal allocation vector of R&D resources $\boldsymbol{\gamma}'$ is:

$$\boldsymbol{\gamma}' \;\; \propto \;\; \boldsymbol{\beta}' \sum_{m=0}^{\infty} \left(\frac{\boldsymbol{\Omega}}{1 + \rho/\lambda} \right)^m = \boldsymbol{\beta}' \left(\boldsymbol{I} + \frac{\boldsymbol{\Omega}}{1 + \rho/\lambda} + \left(\frac{\boldsymbol{\Omega}}{1 + \rho/\lambda} \right)^2 + \cdots \right).$$

That is, the Leontief inverse $\left(\boldsymbol{I} - \frac{\boldsymbol{\Omega}}{1+\rho/\lambda} \right)^{-1}$ in (12) can be written as a power series of $\frac{\boldsymbol{\Omega}}{1+\rho/\lambda}$. The first term in the infinite summation, $\boldsymbol{\beta}'\boldsymbol{I} = \boldsymbol{\beta}'$, captures the direct impact of each sector's knowledge stock on consumer welfare, through new product varieties created directly by new knowledge. This term coincides with the allocation of production workers. Subsequent terms in the power series capture the indirect effect of knowledge creation on consumer welfare, through future innovations. Innovations in sector j benefit sector i by raising the endogenous efficiency of subsequent R&D in sector i, captured by the innovation aggregator χ_{it} in equation (5) with elasticity ω_{ij}, which is the ij-th entry of the innovation network matrix $\mathbf{\Omega}$. Improved innovation efficiency in sector i further generates additional knock-on effects, as new knowledge in sector i facilitates future innovations in all sectors that benefit from sector i's knowledge stock; the higher-powered terms in the infinite summation capture these indirect effects.

Because the network spillovers affect innovative efficiency in the future, the importance of network effects in the optimal R&D allocation is modulated by the discount rate ρ relative to the innovation step size λ: the former (ρ) captures discounting of the future, and the latter (λ) captures how quickly those future benefits materialize. When ρ/λ is high, the planner discounts the future benefits heavily, and the network effects play a smaller role. In the limit as $\rho/\lambda \to \infty$, the planner becomes myopic, and the optimal R&D allocation coincides with the allocation of workers. Conversely, a more patient (low ρ/λ) planner allocates more R&D resources to sectors that benefit more sectors in the future, directly or indirectly. Proposition 1 implies that a patient planner directs R&D into basic science; an impatient planner directs R&D into consumer goods that are more downstream in the innovation network, such as textiles and food products.

In section B.1 of the Online Appendix, we provide an example with three sectors, and we analytically express the optimal allocation based on the network structure and parameters ρ/λ.

2.3 Decentralized Equilibrium

In an innovation network, knowledge creation benefits subsequent R&D in other sectors and all future periods. In a decentralized economy, R&D decisions are made by firms in pursuit of profits; to the extent that the knowledge spillovers are not fully monetized, decentralized markets may allocate R&D resources sub-optimally.

To demonstrate potential inefficiency as clearly as possible, in this section we construct a decentralized equilibrium with a stark market structure in which each intermediate good is produced by an atomistic monopolist that produces only that variety, and potential entrants conduct R&D only in pursuit of product market profits, disregarding any beneficial spillovers their R&D activities may provide for other firms. In Section B.4 of the Online Appendix, we extend this analysis to incorporate granular, multisector firms that are innovation hubs. We show innovation hubs may partially internalize knowledge spillovers, resulting in R&D allocations somewhere in between the optimal and those chosen by atomistic firms. Innovation hubs play an important role in real-world R&D allocations, as we show empirically in Section 6.

Market Structure with Atomistic, Single-Sector Firms Every vintage of each intermediate variety can be produced by a distinct atomistic monopolist. Different vintages of the same intermediate variety are perfect substitutes. Because the most recent vintage's quality is e^{λ} proportionally higher than the next-best vintage, the monopolist who holds production rights to the highest-quality vintage conducts limit pricing under Bertrand competition and charges a markup e^{λ} over the marginal cost of production. No vintages with dominated quality are produced in equilibrium.

In each sector, innovation is carried out by a continuum of potential entrants, who hire scientists to conduct R&D and generate new ideas according to (5). Ideas lead to quality improvements of a random variety within sector i at Poisson rate $\ln(n_{it}/q_{it})$. All ideas are patented, but only quality improvements bring profits: once a variety improves, the innovating firm obtains a patented production right and becomes the producing monopolist of that variety. We assume patents expire at rate δ. For simplicity, we assume that once a patent expires, a new, random firm becomes the monopolistic producer of that variety, until the variety is improved upon. This simplification ensures that firms charge the same markup across all varieties, allowing us to abstract away from production inefficiencies due to markup dispersions and focus instead on cross-sector knowledge spillover as the only source of inefficiency. Also for simplicity, we assume that potential entrants conduct R&D only for unpatented varieties. Because all entrants in a sector choose to hire the same measure of scientists and there is no sector-level innovation uncertainty, the law of motion for sectoral knowledge stock coincides with (6).

The representative consumer receives all workers' and scientists' income and firm profits. Given the initial state variables $\{q_{i0}\}_{i=1}^{K}$, a decentralized equilibrium is the sequence of prices, quantities, and knowledge stocks such that production firms set prices to maximize profits, the consumer chooses bundles of goods to consume to maximize utility, and potential entrants hire scientists for R&D to maximize expected profits.

We now solve for the decentralized equilibrium. We normalize the consumer price index to one for all times t. The consumer at each time t spends a constant fraction β_i of its income on sectoral composite good i, with

$$p_{it}c_{it} = \beta_i c_t \quad \text{for all } i, t. \tag{13}$$

The sectoral composite aggregator (3) further implies that the total revenue of each variety ν is also equal to $\beta_i c_t$, and, using the fact that each monopolist sets a markup e^{λ}, we derive the profits in each sector i as

$$\pi_{it}(\nu) = \left(1 - e^{-\lambda}\right)\beta_i c_t \quad \text{for all } i, t, \nu. \tag{14}$$

Because all varieties have identical markups, the worker allocation is identical across varieties, and the total workers in each sector i is also proportional to the consumption shares β_i:

$$\ell_{it}(\nu) = \ell_{it} = \beta_i \bar{\ell} \quad \text{for all } i, t, \nu. \tag{15}$$

Entrants conduct R&D to obtain patented production rights, with present discounted value

$$v_{it} \equiv \int_0^\infty e^{-(r_s+\delta-g_s)(s-t)} \pi_{is} \, \mathrm{d}s,$$

where r_s is the interest rate at time s and g_s is the growth rate of aggregate consumption c_s. Note we have suppressed the index for variety since all varieties have the same profits and thus the same value within each sector. Because sectoral profits are always proportional to the consumption shares at all times, we have

$$v_{it}/v_{jt} = \beta_i/\beta_j \quad \text{for all } i, j, t. \tag{16}$$

Entrants hire scientists to conduct research, and the marginal value from an additional scientist ($v_{it} \times \partial \ln(n_{it}/q_{it})/\partial s_{it}$) must be equalized across sectors. Substitute n_{it} using the ideas production function (5) and v_{it}/v_{jt} using equation (16), we obtain that scientist allocation must also follow the consumption share, that is, $s_{it}/\bar{s} = \beta_i$ for all t.

Proposition 2. *In the decentralized equilibrium, the allocations of R&D and production resources both follow the consumption shares at all times, that is,* $\ell_{it}(\nu) = \ell_{it} = \beta_i \bar{\ell}$ *and* $s_{it} = \beta_i \bar{s}$.

Intuitively, β_i is proportional to each sector's revenue, and since markups are constant, β_i is also proportional to the production inputs as well as profits in each sector. Profits in turn pin down the cross-sector allocation of R&D resources (scientists).

Policy Implementation of the Optimal R&D Allocation The differences between the socially optimal and the decentralized R&D allocations originate from the fact that the decentralized R&D allocation is driven by profits, as firms do not fully internalize subsequent knowledge spillovers from their own innovative activities. Given a broad set of tax instruments, the planner may have many equivalent ways to implement the optimal R&D allocation. The most direct implementation is through R&D subsidies and taxes. Intuitively, the planner should tax R&D activities in sectors with high β_i (which encodes market incentives) relative to γ_i (which encodes social incentives) and subsidize R&D activities in sectors with low β_i relative to γ_i. Formally, suppose the planner has access to R&D tax instruments such that firms pay $(1+\tau_i)$ times the wage rate of each scientist in sector i. Suppose the planner has access to lump-sum taxes on the consumer in order to balance its budget.

Proposition 3. *The planner can decentralize the optimal allocation by setting R&D taxes to be* $1+\tau_i \propto \frac{\beta_i}{\gamma_i}$*, with the appropriate lump-sum tax levied on the consumer to balance the budget.*

2.4 Economic Growth Along the Balanced Growth Path

In this section we characterize how R&D allocations affect the economic growth rate along a balanced growth path (BGP), that is, a steady-state equilibrium in which the knowledge stock in every sector grows at the same constant rate.

Definition 2. The vector of sectoral *innovation centrality*, $\boldsymbol{a} \equiv [a_i]_{i=1}^{K}$, is the dominant left-eigenvector of $\boldsymbol{\Omega}$ with an associated eigenvalue of one, satisfying $\boldsymbol{a}' = \boldsymbol{a}'\boldsymbol{\Omega}$ and $\sum_{i=1}^{K} a_i = 1$.

The innovation centrality vector $\boldsymbol{a}$ exists and is unique by the Perron-Frobenius theorem. Our next Proposition shows that $\boldsymbol{a}$ is a key determinant of the BGP growth rate.

Let $\boldsymbol{b}$ denote a generic vector of allocation shares with nonnegative entries and $\sum b_i = 1$.

Proposition 4. *Consider a BGP in which R&D allocation shares follow the vector $\boldsymbol{b}$ ($s_i = \bar{s}b_i$). Then the aggregate consumption and the stock of knowledge in every sector grow at the same rate $g(\boldsymbol{b})$:*

$$g(\boldsymbol{b}) = \mathit{const} + \lambda \cdot \boldsymbol{a}' \ln \boldsymbol{b}, \tag{17}$$

where the exogenous constant on the right-hand side is equal to $\lambda \cdot (\ln \bar{s} + \boldsymbol{a}' \ln \boldsymbol{\eta})$. The vector of knowledge stock $\boldsymbol{q}_t$ satisfies the fixed point equation

$$\ln \boldsymbol{q}_t = \lambda \cdot [\ln \bar{s} - g(\boldsymbol{b}) + \ln \boldsymbol{\eta} + \ln \boldsymbol{b}] + \boldsymbol{\Omega} \ln \boldsymbol{q}_t. \tag{18}$$

The first part of Proposition 4 analytically expresses the growth rate of knowledge stock along a BGP as a function of the R&D allocation, $\boldsymbol{b}$. The endogenous component of the growth rate is innovation step size λ times the inner product between the innovation centrality $\boldsymbol{a}$ and the vector of log-R&D allocation shares, $\ln \boldsymbol{b}$. The exogenous component on the right-hand side of (17) shows that the growth rate is higher when there are more scientists $\bar{s}$, when scientists are more productive at generating new ideas (higher $\boldsymbol{\eta}$), and when the step-size λ of quality improvements is larger. The second part of Proposition 4 characterizes the knowledge stock along a BGP. Equation (18) expresses $\ln \boldsymbol{q}_t$ as a fixed point, rather than in levels, because the knowledge stock grows at a constant rate along a BGP and the levels are therefore not time-invariant.[1]

Corollary 1. *(i) The difference in growth rate between the BGP where R&D allocation follows $\boldsymbol{b}$ and the BGP where R&D allocation follows $\tilde{\boldsymbol{b}}$ is $g\left(\tilde{\boldsymbol{b}}\right) - g(\boldsymbol{b}) = \lambda \cdot \boldsymbol{a}' \left(\ln \tilde{\boldsymbol{b}} - \ln \boldsymbol{b}\right)$.*

(ii) The R&D allocation that maximizes the BGP growth rate coincides with the innovation centrality $\boldsymbol{a}$, as it is the solution to the following problem: $\boldsymbol{a} = \arg\max_{\boldsymbol{b}} \boldsymbol{a}' \ln \boldsymbol{b}, \quad \text{s.t. } \boldsymbol{b} \geq 0, \mathbf{1}'\boldsymbol{b} = 1$.

This corollary highlights that the innovation centrality serves as a sufficient statistic for growth evaluation of policy counterfactuals and that innovation centrality coincides with the

[1] Note that if $\ln \boldsymbol{q}_t$ is a solution to equation (18), $2 \times \ln \boldsymbol{q}_t$ is also a solution.

R&D allocation vector that leads to the highest BGP growth rate. Intuitively, innovation centrality captures the extent to which a sector's R&D activities contribute to economic growth, taking into account the network effects. Sectors with higher innovation centrality are more fundamental in the innovation network.

Qualitatively, the corollary also demonstrates that the social planner does not necessarily choose R&D allocations to maximize the economy's growth rate. Unlike the socially optimal allocation $\boldsymbol{\gamma}$, which depends on the discount rate relative to step-size of innovation ρ/λ, the growth-maximizing allocation is equal to the innovation centrality and is independent of these parameters. Intuitively, the social planner maximizes the welfare of the consumer, who may prefer to have better-quality products in the near future from consumption-intensive sectors (e.g., consumer goods such as textiles and food products), and knowledge in these sectors may not generate much knowledge spillovers for future innovations.

One can rewrite the optimal R&D allocation vector $\boldsymbol{\gamma}' = \frac{\rho}{\rho+\lambda}\boldsymbol{\beta}'\left(\boldsymbol{I} - \frac{\boldsymbol{\Omega}}{1+\rho/\lambda}\right)^{-1}$ as the solution to the following fixed point equation, which usefully shows how $\boldsymbol{\gamma}'$ varies with ρ/λ:

$$\boldsymbol{\gamma}'\left(\boldsymbol{I} - \boldsymbol{\Omega}\right) + \frac{\rho}{\lambda}\left(\boldsymbol{\gamma}' - \boldsymbol{\beta}'\right) = \boldsymbol{0}'. \tag{19}$$

The two terms on the left-hand side represent the innovation centrality $\boldsymbol{a}$ and consumer preferences $\boldsymbol{\beta}$ as two determinants of the efficient R&D allocation vector $\boldsymbol{\gamma}$. When the first term is equal to the zero vector ($\boldsymbol{\gamma}'\left(\boldsymbol{I} - \boldsymbol{\Omega}\right) = \boldsymbol{0}'$), it must be the case that $\boldsymbol{\gamma}$ coincides with the innovation centrality $\boldsymbol{a}$. When the second term is the zero vector ($\frac{\rho}{\lambda}\left(\boldsymbol{\gamma}' - \boldsymbol{\beta}'\right) = \boldsymbol{0}'$), it must be the case that $\boldsymbol{\gamma} = \boldsymbol{\beta}$. Under the optimal R&D allocation, the sum of the two terms must be equal to the zero vector, and ρ/λ modulates the relative importance of these two terms. When ρ/λ is close to zero—a patient social planner—the first term dominates ($\lim_{\rho/\lambda\to 0}\boldsymbol{\gamma} = \boldsymbol{a}$). When ρ/λ is large—an impatient planner—consumer preferences dominate, and the efficient R&D allocation is closer to the decentralized R&D allocation ($\lim_{\rho/\lambda\to\infty}\boldsymbol{\gamma} = \boldsymbol{\beta}$).

Corollary 2. *As the planner becomes infinitely patient ($\rho/\lambda \to 0$), the optimal R&D allocation converges to the innovation centrality, and the economic growth rate is maximized:* $\lim_{\rho/\lambda\to 0}\boldsymbol{\gamma} = \boldsymbol{a}$*. As the planner becomes infinitely impatient ($\rho/\lambda \to \infty$), the optimal R&D allocation converges to the decentralized allocation vector:* $\lim_{\rho/\lambda\to\infty}\boldsymbol{\gamma} = \boldsymbol{\beta}$*.*

2.5 Transitional Dynamics and Welfare Impact of Optimal R&D

We now analytically characterize the transitional dynamics under the optimal R&D allocation, starting from arbitrary levels of the initial state variables $\boldsymbol{q}_t$. We undertake a spectral analysis and show that the eigenvalues and eigenvectors of the innovation network $\boldsymbol{\Omega}$ determine the rate of convergence toward the BGP. We provide the closed-form solution for the welfare impact of adopting the optimal R&D allocation starting from arbitrary levels of the initial knowledge stock.

Along any BGP, each sector's knowledge stock grows at the same rate, and the relative knowledge stock between any two sectors is constant. Let $\ln \bar{q}_t \equiv \boldsymbol{a}' \ln \boldsymbol{q}_t$ denote the $\boldsymbol{a}$-weighted average log-knowledge stock at time t, and let $\widetilde{\boldsymbol{q}}_t \equiv [\ln \boldsymbol{q}_{jt} - \ln \bar{q}_t]_{j=1}^K$ encode the log knowledge stock across sectors relative to the average $\ln \bar{q}_t$. We refer to $\widetilde{\boldsymbol{q}}_t$ as the vector of *relative knowledge stock* at time t. Let $\widetilde{\boldsymbol{q}}^*$ denote relative knowledge stock along the optimal BGP. We now provide results for how $\widetilde{\boldsymbol{q}}_t$ converges to $\widetilde{\boldsymbol{q}}^*$ along the transition under the optimal R&D allocation.

Proposition 5. *Starting from an arbitrary initial relative knowledge stock $\widetilde{\boldsymbol{q}}_0$, the law of motion for the relative knowledge stock $\widetilde{\boldsymbol{q}}_t$ under the optimal R&D allocation is*

$$\widetilde{\boldsymbol{q}}_t - \widetilde{\boldsymbol{q}}^* = e^{-\lambda(\boldsymbol{I}-\boldsymbol{\Omega})t} \left(\widetilde{\boldsymbol{q}}_0 - \widetilde{\boldsymbol{q}}^* \right),$$

where $\widetilde{\boldsymbol{q}}^* = \left((\boldsymbol{I} - \boldsymbol{\Omega} + \boldsymbol{1}\boldsymbol{a}')^{-1} - \boldsymbol{1}\boldsymbol{a}' \right) [\ln \boldsymbol{\eta} + \ln \boldsymbol{\gamma}]$ *and* $e^{-(\boldsymbol{I}-\boldsymbol{\Omega})t} \equiv \sum_{k=0}^{\infty} \frac{(-(\boldsymbol{I}-\boldsymbol{\Omega})t)^k}{k!}$.

Proposition 5 provides the closed-form solution for the evolution of knowledge stock $\widetilde{\boldsymbol{q}}_t$ under the optimal R&D allocation, starting from an arbitrary initial knowledge stock $\widetilde{\boldsymbol{q}}_0$.

Spectral Analysis and the Welfare Impact of the Optimal R&D Allocation Recent developments in the literature on network dynamics (Liu and Tsyvinski, 2021, Kleinman et al., 2021a) indicate spectral analysis can be used to analyze the speed of convergence toward BGP, an approach applicable to the closed-form transitional dynamics in Proposition 5. Intuitively, the rate at which the economy converges to the BGP depends on the initial relative knowledge stock $\widetilde{\boldsymbol{q}}_0$. The economy should converge quickly if its underdeveloped sectors are downstream in the innovation network; conversely, the economy may converge slowly if the upstream and central sectors—whose knowledge benefits many others—are underdeveloped. Spectral analysis formalizes these intuitions and can be taken to data quantitatively.

We contribute to this literature with a new result: we show spectral analysis can be used to provide closed-form solutions for the welfare impact of adopting the optimal R&D allocation, starting from arbitrary levels of initial knowledge stock.

Specifically, consider an eigendecomposition of the innovation network, $\boldsymbol{\Omega} = \boldsymbol{U}\boldsymbol{\Psi}\boldsymbol{V}$, where $\boldsymbol{\Psi}$ is a diagonal matrix of eigenvalues arranged in decreasing order by absolute value. For each eigenvalue ψ_k, the k-th column of $\boldsymbol{U}$ (denoted as $\boldsymbol{u}_k$) is the corresponding right-eigenvector of $\boldsymbol{\Omega}$, and the k-th row of $\boldsymbol{V}$ (denoted as $\boldsymbol{v}_k'$) is the corresponding left-eigenvector.

$$\psi_k \boldsymbol{u}_k = \boldsymbol{\Omega} \boldsymbol{u}_k, \qquad \psi_k \boldsymbol{v}_k' = \boldsymbol{v}_k' \boldsymbol{\Omega}.$$

Without loss of generality, we normalize the 1-norm of these eigenvectors to one.

To see why spectral analysis reveals convergence rates, note that if the initial relative knowledge stock coincides with a right-eigenvector in terms of deviation from the BGP (i.e., $\widetilde{\boldsymbol{q}}_0 - \widetilde{\boldsymbol{q}}^* =$

$\boldsymbol{u}_k$), then the economy converges at a constant rate $\lambda(1-\psi_k)$ governed by the corresponding eigenvalue:

$$\widetilde{\boldsymbol{q}}_t - \widetilde{\boldsymbol{q}}^* \Big|_{\widetilde{\boldsymbol{q}}_0 - \widetilde{\boldsymbol{q}}^* = \boldsymbol{u}_k} = e^{-\lambda(1-\psi_k)t} \left(\widetilde{\boldsymbol{q}}_0 - \widetilde{\boldsymbol{q}}^*\right), \qquad \text{half-life} = \frac{\ln 2}{\lambda(1-\psi_k)}.$$

The smaller the eigenvalue, the faster the convergence rate. Studying the sectoral loadings of each right-eigenvector $\boldsymbol{u}_k$ (i.e., the value on each coordinate) therefore enables one to quantitatively examine which underdeveloped sectors lead to slow convergence. Moreover, any initial deviation from BGP can generically be written as a linear combination of the right-eigenvectors by projecting $(\widetilde{\boldsymbol{q}}_0 - \widetilde{\boldsymbol{q}}^*)$ onto the eigenbasis; hence, spectral analysis fully reveals the dynamical system's convergence properties.

Our next result leverages spectral analysis to derive the welfare impact of R&D allocations. For completeness, we also provide closed-form solution for the impact on the economic growth rate along the entire transition.

Proposition 6. *Given any initial knowledge stock at time 0, consider two time-invariant R&D allocation plans $\boldsymbol{b}$ and $\widetilde{\boldsymbol{b}}$ while holding production worker allocation constant at arbitrary levels.*

(i) The difference in consumer welfare between the two R&D plans is

$$\begin{aligned} V(\boldsymbol{b}) - V\left(\widetilde{\boldsymbol{b}}\right) &\equiv \int_0^\infty e^{-\rho t}\left(\ln c_t(\boldsymbol{b}) - \ln c_t\left(\widetilde{\boldsymbol{b}}\right)\right) \mathrm{d}t \\ &= \boldsymbol{\beta}' \sum_{k=1}^{K} \boldsymbol{u}_k \boldsymbol{v}_k' \frac{1}{\rho\left(\rho + \lambda(1-\psi_k)\right)} \times \lambda\left(\ln \boldsymbol{b} - \ln \widetilde{\boldsymbol{b}}\right). \end{aligned} \tag{20}$$

(ii) The difference in the path of consumption growth rates between the two R&D plans is

$$\begin{aligned} g(t;\boldsymbol{b}) - g(t;\boldsymbol{b}') &= \boldsymbol{\beta}' e^{-\lambda(\boldsymbol{I}-\boldsymbol{\Omega})t} \times \lambda\left(\ln \boldsymbol{b} - \ln \widetilde{\boldsymbol{b}}\right) \\ &= \boldsymbol{\beta}' \sum_{k=1}^{K} \boldsymbol{u}_k \boldsymbol{v}_k' e^{-\lambda(1-\psi_k)t} \times \lambda\left(\ln \boldsymbol{b} - \ln \widetilde{\boldsymbol{b}}\right). \end{aligned} \tag{21}$$

The first part of Proposition 6 is a closed-form formula for the welfare difference between two time-invariant R&D allocation plans, starting from any initial knowledge stock and taking into account transitional dynamics. The formula can be used for quantitative analysis to calculate, for instance, the welfare impact of replacing the real-world R&D allocation ($\widetilde{\boldsymbol{b}}$) with the optimal (setting $\boldsymbol{b} = \boldsymbol{\gamma}$). The second part of the proposition provides the difference in the path of consumption growth rates between two R&D allocation plans.

Intuitively, the summations in (20) and (21) enumerate the welfare and growth impact of R&D allocations along each of the K eigencomponents. All components $k \geq 2$ represent temporary effects along the transition, with each component having a constant rate of decay for its dynamic effects. A faster decay—lower eigenvalue ψ_k—implies the growth effect dissipates more rapidly, and the welfare effect along this component k is smaller.

The dominant eigencomponent $k = 1$ has the largest eigenvalue $\psi_1 = 1$, with $\boldsymbol{u}_1$ being the constant vector that sums to one (thus $\boldsymbol{\beta}'\boldsymbol{u}_1 = 1$), and $\boldsymbol{v}_1' = \boldsymbol{a}'$ is the innovation centrality. This component represents the long-run impact due to differential BGP growth rates. To see this, note that the $k = 1$ component on the right-hand side of (21) simplifies to $\lambda\boldsymbol{a}'\left(\ln\boldsymbol{b} - \ln\widetilde{\boldsymbol{b}}\right)$, the BGP growth differential under the two R&D plans (Corollary 1). The $k = 1$ component in (20) simplifies to $\lambda\rho^{-2}\boldsymbol{a}'\left(\ln\boldsymbol{b} - \ln\widetilde{\boldsymbol{b}}\right)$, which is precisely the welfare impact of differential consumption growth rates under log-utility and exponential discounting.[2]

2.6 Extensions

As noted above, we consider a number of extensions in the Online Appendix. In Section B.2, we allow a single worker type to be mobile between production and R&D, and we show the optimal share of R&D allocated to each sector continues to be characterized by Proposition 1. In Section B.3, we incorporate a production network to the model and show the term $\boldsymbol{\beta}'$ in the formula for the optimal R&D allocation should be replaced by sectoral value-added as a share of GDP, and otherwise our characterization continues to hold. In Section B.4, we formulate a decentralized equilibrium with granular, multi-sector firms that are innovation hubs. We show innovation hubs partially internalize knowledge spillovers, resulting in R&D allocations somewhere in between the optimal ones and those chosen by atomistic firms.

3 International Knowledge Spillovers and Policy

This section studies innovation policy in a setting with international knowledge spillovers and trade. We analyze the problem of a country's planner choosing R&D allocations to maximize welfare for its citizens, taking the sequence of foreign knowledge stocks as given. We derive unilaterally optimal innovation policy in terms of sufficient statistics that can be measured from data on production, trade, and cross-country patent citations.

The open-economy version of the model highlights a key externality across countries: the incentive to free-ride on fundamental technologies. Even when sector j creates knowledge spillovers to other sectors, each country-specific planner may not have sufficient incentives to direct R&D resources into sector j and may instead choose to free ride on other countries' R&D efforts. This incentive to free ride is stronger if a country's domestic R&D productivity depends significantly

[2]Specifically, consider utility function $V(g; c_0) = \int_0^\infty e^{-\rho t} \ln c_t \, \mathrm{d}t$ when $c_t = e^{gt}c_0$, in which case the impact of changing growth rate g on welfare is $V(g_2; c_0) - V(g_1; c_0) = (g_2 - g_1)/\rho^2$.

on foreign knowledge stock in sector j. Reliance on foreign knowledge is a key determinant of the unilaterally optimal R&D allocation, and self-reliant economies should optimally direct resources toward more innovation-upstream or central sectors.

3.1 Setup

Consider a collection $\mathcal{M}$ of countries. We exposit the model from the perspective of a generic country $m \in \mathcal{M}$, which we refer to as *domestic*. Country m is endowed with $\bar{s}_m$ units of scientists and $\bar{\ell}_m$ units of production workers.

Country m is an open economy in two ways. First, it may benefit from foreign knowledge spillovers. We generalize the closed-economy ideas production function (5) and posit that scientists in country m may benefit from foreign knowledge $\left\{q^f_{mjt}\right\}$ across sectors j at time t:

$$n_{mit} = \eta_{mi} s_{mit} \chi_{mit}, \qquad \text{where } \chi_{mit} = \prod_{j=1}^{K} \left(q_{mjt} + q^f_{mjt}\right)^{\omega_{ij}}. \tag{22}$$

When $q^f_{mjt} = 0$ for all j, the open-economy ideas production function (22) coincides with the closed-economy counterpart (5) in Section 2. All agents in country m, including the social planner, treat the time path of $\left\{q^f_{mjt}\right\}$ as given when making decisions, although $\left\{q^f_{mjt}\right\}$ may arise endogenously as an equilibrium outcome. This is a flexible, reduced-form formulation and nests many realistic cases. For instance, q^f_{mjt} may represent the following aggregator

$$q^f_{mjt} = \int_{n \in \mathcal{M}, n \neq m} \phi_{mnjt} q_{njt} \, \mathrm{d}n, \tag{23}$$

where q_{njt} is the knowledge stock in sector j of country n, and $\phi_{mnjt} \leq 1$ captures the fraction of knowledge from country n, sector j that is available for idea generation in country m at time t. Variation in ϕ_{mnjt} may arise from cultural and political ties between countries m and n, intellectual property protections in country n, and the adaptability of sector j's technology. In any case, we posit that agents in country m treat $\left\{q^f_{mjt}\right\}$ as given when making decisions, and we provide sufficient statistics for q^f_{mjt} without needing to specify the functional form it represents.

Second, country m may also be open to trade. The representative consumer values foreign varieties in all sectors. The final consumption aggregator is

$$c_{mt} = \prod_{i=1}^{K} \left(c_{mit}^{\frac{\sigma-1}{\sigma}} + \left(c^f_{mit}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\beta_i \frac{\sigma}{\sigma-1}}, \quad \sigma > 1,$$

where c_{mit} continues to be the aggregator of domestic varieties in sector i (c.f. equation 3), and c^f_{mit} is the bundle of foreign varieties imported into country m for consumption. We assume domestic and foreign bundles are gross substitutes ($\sigma > 1$), as is standard in the trade literature.

Country m can export domestic sectoral bundles and import foreign bundles. We provide a reduced-form formulation of the trade block in the main text, treating both the supply of foreign goods and the foreign demand for domestic goods as exogenous. Specifically, let $\boldsymbol{p}_{mt} \equiv [p_{mit}]_{i=1}^{K}$ and $\boldsymbol{d}_{mt} \equiv [d_{mit}]_{i=1}^{K}$ respectively denote the price vector of m's sectoral composite goods and the export quantity of these goods; let $\boldsymbol{p}_{mt}^{f} \equiv \left[p_{mit}^{f}\right]_{i=1}^{K}$ and $\boldsymbol{c}_{mt}^{f} \equiv \left[c_{mit}^{f}\right]_{i=1}^{K}$ be vectors that respectively capture m's import prices and quantities of foreign bundles. We assume the import prices of foreign goods are reduced-form functions of country m's imports and exports: $\boldsymbol{p}_{mt}^{f} \equiv \boldsymbol{p}_{m}^{f}\left(\boldsymbol{c}_{mt}^{f}, \boldsymbol{d}_{mt}; \boldsymbol{\theta}_{t}\right)$, where $\boldsymbol{\theta}_{t}$ is the vector that parametrizes international demand and supply at time t. We refer to $\boldsymbol{\theta}_{t}$ as "international trade conditions." For simplicity, we assume balanced trade at every t:

$$\boldsymbol{p}_{mt}' \cdot \boldsymbol{d}_{mt} = \boldsymbol{p}_{m}^{f}\left(\boldsymbol{c}_{mt}^{f}, \boldsymbol{d}_{mt}; \boldsymbol{\theta}_{t}\right)' \cdot \boldsymbol{c}_{mt}^{f}, \tag{24}$$

where "$\cdot$" represents the inner product. This reduced-form formulation of the trade block is quite general, as the import price functions $\boldsymbol{p}_{m}^{f}\left(\boldsymbol{c}_{mt}^{f}, \boldsymbol{d}_{mt}; \boldsymbol{\theta}_{t}\right)$ can incorporate general equilibrium forces and rich geography with importer-exporter-industry specific trade costs embedded in the vector of parameters $\boldsymbol{\theta}_{t}$. Because $\boldsymbol{\theta}_{t}$ enters the pricing function freely, it is without loss of generality to normalize $\boldsymbol{p}_{m}^{f}(\cdot)$ to be homogeneous of degree zero: when $\boldsymbol{c}_{mt}^{f}$, $\boldsymbol{d}_{mt}$, and $\boldsymbol{\theta}_{t}$ all double, the vector of import prices stays constant. This normalization simplifies our exposition and is meant to capture the notion that import prices stay invariant when foreign technology ($\boldsymbol{\theta}_{t}$), domestic imports ($\boldsymbol{c}_{mt}^{f}$), and exports ($\boldsymbol{d}_{mt}$) all change proportionally.

In the Appendix, we microfound the price functions in several ways, including a small monopolistic economy setup similar to Gali and Monacelli (2005), as well as a full-fledged multicountry, multisector general equilibrium model with a constant trade elasticity, similar to Costinot, Donaldson and Komunjer (2012). This reduced-form formulation also nests trade autarky as a special case when the import prices go to infinity ($p_{mi}^{f} \to \infty$) for any trade quantities. The main reason we adopt this reduced-form formulation of the trade block is for expositional brevity—we do not have anything conceptually new to say about international trade, and in the data, as we discuss later, different treatments of the trade block do not generate quantitatively meaningful differences in how each country should allocate R&D resources.

We study country m's planner problem of allocating R&D resources to maximize domestic welfare, while taking the time path of foreign knowledge $\left\{q_{mjt}^{f}\right\}$ and trade conditions $\{\boldsymbol{\theta}_{t}\}$ as given and decentralizing both production worker allocation and international trade. By decentralizing production and trade decisions, we are able to focus solely on innovation policies while abstracting away from trade policies, which are well-studied in the literature, and industrial policies in open economies, which are the topic of an emerging literature (see, e.g., Chen, Liu and Song (2021)).

Definition 3. (Instantaneous Equilibrium.) At any time t, given the state variables $\boldsymbol{q}_{mt}, \boldsymbol{q}_{mt}^{f}$ and the import price function $\boldsymbol{p}_{m}^{f}(\cdot; \boldsymbol{\theta}_{t})$, an *instantaneous equilibrium* in country m is the collection of prices—wages w_{mt} of workers, prices of intermediate varieties $\{p_{mit}(\nu)\}$ and sectoral composite goods $\left\{p_{mit}, p^f_{mit}\right\}$—and quantities—worker allocation $\{\ell_{mit}(\nu)\}$, production output of intermediate varieties $\{x_{mit}(\nu)\}$ and sectoral bundles consumed $\{c_{mit}\}$, exported $\{d_{mit}\}$, and imported $\left\{c^f_{mit}\right\}$—such that the representative consumer chooses domestic and foreign bundles to maximize flow utility given income, the monopolistic producer of every variety sets markup to maximize profits, production workers are fully employed, trade is balanced, and the supply of composite goods is equal to the sum of domestic and foreign demand:

$$c_{mit} + d_{mit} = y_{mit}, \qquad \ln y_{mit} \equiv \int_0^1 \ln\left(q_{mit}(\nu)\, x_{mit}(\nu)\right) \mathrm{d}\nu \quad \text{for all } i.$$

Let $c^*_m\left(\boldsymbol{q}_{mt}, \boldsymbol{q}^f_{mt}, \boldsymbol{\theta}_t\right)$ denote the aggregate consumption in the instantaneous equilibrium at time t as a function of state variables $\boldsymbol{q}_{mt}, \boldsymbol{q}^f_{mt}$, and parameters $\boldsymbol{\theta}_t$ of the import price function. The planner's problem is to allocate R&D to maximize consumer welfare:

$$V\left(\boldsymbol{q}_{m0}, \left\{\boldsymbol{q}^f_{mt}, \boldsymbol{\theta}_t\right\}_{t=0}^{\infty}\right) \equiv \max_{\{s_{it}\} \text{ s.t. } \sum_i s_{it} = \bar{s}} \int_0^\infty e^{-\rho t} \ln c^*_m\left(\boldsymbol{q}_{mt}, \boldsymbol{q}^f_{mt}, \boldsymbol{\theta}_t\right) \mathrm{d}t,$$

subject to the open-economy ideas production function (22) and the law of motion for domestic knowledge (6), while taking the time path of foreign knowledge and trade conditions as given.

Our next result characterizes the unilaterally optimal R&D allocation of country m along an international balanced growth path (IBGP), a steady-state equilibrium in which domestic and foreign knowledge, exports, and imports in every sector, as well as aggregate consumption, all grow at the same rate, with time-invariant relative prices of imports and exports. We express the optimal R&D allocation in terms of sufficient statistics. Let $\boldsymbol{x}_{mt}$ denote the $K \times 1$ vector whose i-th entry is $x_{mit} \equiv \frac{q_{mit}}{q_{mit} + q^f_{mit}}$, the domestic share of useful knowledge in sector i that benefits idea generation within country m. Let $\boldsymbol{\Theta}_{mt} \equiv \mathbf{1}\boldsymbol{x}'_{mt} \circ \boldsymbol{\Omega}$ denote the $K \times K$ matrix whose ij-th entry is the elasticity of innovation efficiency in sector i with respect to the domestic knowledge stock in sector j: $[\boldsymbol{\Theta}_{mt}]_{ij} \equiv \frac{\partial \ln \chi_{mit}}{\partial \ln q_{mit}} = \omega_{ij} x_{mjt}$, where $\mathbf{1}$ is the $K \times 1$ vector of one's and $\circ$ denotes the Hadamard product. Finally, let $\boldsymbol{\phi}_{mt}$ denote the $K \times 1$ vector whose i-th entry is the elasticity of domestic aggregate consumption with respect to the knowledge stock in sector i: $\phi_{mit} \equiv \frac{\mathrm{d} \ln c^*_m\left(\boldsymbol{q}_{mt}, \boldsymbol{q}^f_{mt}, \boldsymbol{p}^f_{mt}\right)}{\mathrm{d} \ln q_{mit}}$. Along the IBGP, $\boldsymbol{x}_{mt}$, $\boldsymbol{\Theta}_{mt}$, and $\boldsymbol{\phi}_{mt}$ are all time-invariant; thus, we drop the time subscript in the proposition.

Proposition 7. *Along an international balanced growth path (IBGP) where $\boldsymbol{q}_{mt}$, $\boldsymbol{q}^f_{mt}$, and $\boldsymbol{\theta}_t$ all grow at the same rate, the unilaterally optimal R&D allocation in country m follows $s_{mi}/\bar{s}_m = \gamma_{mi}$, where*

$$\boldsymbol{\gamma}'_m \propto \boldsymbol{\phi}'_m \left(\boldsymbol{I} - \frac{\boldsymbol{\Theta}_m}{1 + \rho/\lambda}\right)^{-1}, \tag{25}$$

and where the proportionality constant is chosen so that $\sum_i \gamma_{mi} = 1$.

Intuitively, ϕ_{mi} captures the extent to which domestic consumers directly benefit from knowledge expansion in sector i. As is well known, in quality-ladder growth models, changes in sectoral knowledge q_{it} have the same aggregate implication as changes in sector i's productivity. Hence, the vector $\boldsymbol{\phi}_m$ can be interpreted as the elasticity of domestic consumption with respect to a sectoral productivity shock, taking into account such shock's effect on both domestic production and, through international trade, on the relative prices of imports.

The ij-th entry of the Leontief inverse $\left(\boldsymbol{I} - \frac{\boldsymbol{\Theta}_m}{1+\rho/\lambda}\right)^{-1}$ captures the infinite rounds of how indirect network spillovers of additional *domestic knowledge* from sector j impact subsequent *domestic innovation* in sector i. The incentive to free-ride on fundamental technologies manifests through the fact that $\boldsymbol{\Theta}_m \neq \boldsymbol{\Omega}$. That is, when deliberating how much R&D to allocate to sector j, country m's planner does not internalize the full effect (ω_{ij}) new knowledge in sector j will have on future R&D efficiency in each sector i, since its own R&D will only partially affect the knowledge stock in sector j that is useful for subsequent innovation in sector i.

Both $\boldsymbol{\phi}_m$ and $\boldsymbol{\Theta}_m$ are endogenous equilibrium objects. Under trade autarky, $\boldsymbol{\phi}_m$ is the consumption share vector $\boldsymbol{\beta}'$ and is proportional to sectoral value-added, as in our closed-economy model characterized by Proposition 1. With cross-country trade in goods, $\boldsymbol{\phi}_m$ also incorporates terms-of-trade considerations and is co-determined by international comparative advantage. Similarly, $\boldsymbol{\Theta}_m$ is determined by the distribution of innovation efficiency across countries and sectors, the total R&D resources in each country, equilibrium R&D intensity, and the ease with which country m benefits from foreign knowledge. Relation (25) therefore describes an endogenous relationship that holds along the BGP for a country m that adopts the unilaterally optimal R&D allocation.

Each country's unilaterally optimal R&D allocation can be measured in the data: the object $\boldsymbol{\Theta}_m$ can be measured from the innovation network $\boldsymbol{\Omega}$ and country m's reliance on domestic knowledge x_{mi}, and, given a microfoundation for the import price function $\boldsymbol{p}_t(\cdot; \boldsymbol{\theta}_t)$—such as the small monopolistic economy setting in Gali and Monacelli (2005)—$\boldsymbol{\phi}_m$ can be measured from data on production and trade. As we will show, a good empirical approximation of $\boldsymbol{\phi}_{mi}$ is simply the value-added in sector i as a share of GDP in country m, because changes in sectoral knowledge stock have little real-world effects on the terms of trade. Most cross-country differences in unilaterally optimal R&D allocations originate from variations in R&D's degree of self-reliance ($\boldsymbol{x}_{mt}$).

Cross-Country Implications The incentive to free-ride on fundamental technologies highlighted in (25) implies that countries with more self-contained innovation networks (often large economies at the knowledge frontier across many sectors) should allocate R&D in more fundamental sectors, that is, those with higher innovation centrality, that tend to generate knowledge spillovers to other sectors. Conversely, countries that are more reliant on foreign knowledge spillovers have incentives to direct R&D toward sectors that account for greater domestic value-added. In other words, using our intuition from the close-economy Proposition 1, it is as if economies with self-contained innovation networks have patient planners while economies reliant on foreign knowledge have impatient planners.

To see this, let us compare a country m in which 90% of useful knowledge for idea creation is domestic across all sectors ($x_{mj} = 0.9$ for all j), with another country n in which only 10% of useful knowledge is domestic ($x_{nj} = 0.1$ for all j). The Leontief inverse in (25) simplifies to $\left(\boldsymbol{I} - 0.9 \times \frac{\boldsymbol{\Omega}}{1+\rho/\lambda}\right)^{-1}$ for country m and to $\left(\boldsymbol{I} - 0.1 \times \frac{\boldsymbol{\Omega}}{1+\rho/\lambda}\right)^{-1}$ for country n. Greater reliance on domestic knowledge (higher x_m) is therefore isomorphic to a lower discount rate ρ; that is, when a country's future innovations build more heavily on its own knowledge stock, the planner allocates R&D resources as if it were more patient. Even though in the real world foreign reliance could be sector-specific—x_{mj}'s are not necessarily constant across sectors j—the intuition continues to hold: economies with self-contained innovation networks—such the United States and Japan, in which idea generation relies more on domestic rather than on foreign knowledge—should optimally allocate more R&D resources into sectors that create more network externalities; by contrast, countries reliant on foreign knowledge spillovers should optimally allocate R&D resources more myopically, focusing disproportionately on short-term benefits.

It is worth emphasizing again that Proposition 7 characterizes the unilaterally optimal R&D allocation from the perspective of self-serving planners and does not characterize the Pareto-optimal allocation from a global perspective. The Pareto-optimal allocation depends on a broader set of model primitives, such as the cross-country distribution of innovation efficiency, which our formulation does not specify. We have decided to exclude this analysis from the current study.

4 Data

This section describes the data for our empirical analyses. One part of our analysis deals with the United States as a closed economy, and the other part is a cross-country analysis based on the global economy. Both parts need sectoral data on production, consumption, and innovation; we also need patent citation data across sectors and countries to construct the corresponding U.S. and global innovation networks. For the cross-country analysis, we also require data on international trade. We now briefly describe how we construct and harmonize these data; we provide more details in Section D of the Online Appendix.

4.1 Patents Data, Citations, and Sectoral Innovation

U.S. Patents U.S. patent data are obtained from the United States Patent and Trademark Office (USPTO). Their database provides detailed patent-level records on nearly seven million patents granted by the USPTO between 1976 and 2020. The data include, for each patent, the application and grant years, the technology classifications based on the International Patent Classification (IPC) system, and the geographic locations of the patent assignee and patent inventors (the former holds legal ownership rights to the patent while the latter may not). Central to our network analysis, we observe each patent's citations of prior patents as well as the citations it receives from subsequent patents up to the year 2020.

Global Patents To capture global innovation, we use Google Patents' global patent data, which contain information on more than 36 million patents from over 40 main patent authorities around the world, including those from the United States, the European Union, Japan, and China, among others, during the period 1976–2020. For each patent, Google Patents provides similar information similar to the USPTO data described above.

We construct the global innovation network from global patent citations. To our knowledge, we are the first team to use this data to construct the global innovation network. The main challenge while working with these data is multi-filing: to protect intellectual properties, it is common practice for innovators to file the same innovation with multiple patent authorities in different countries. For our analysis, we trace each innovation's original location using available geographic information for the patent assignees and inventors, and we unwind multi-filings so that we count each innovation only once even if it was filed with multiple authorities. Our unwinding procedure uses information such as the patent family ID assigned by Google Patents, self-reported multi-filing status, and the unique identifier for patents filed under the Patent Cooperation Treaty, which is an international law treaty aimed at protecting innovations across countries. In Section D of the Online Appendix, we provide details of these tracing and unwinding procedures.

Measuring Sectoral Innovation We build a few measures to capture innovation output at sector-year levels for U.S.-based analysis and country-sector-year levels for our global studies. We measure raw patent counts—the number of patents produced in the (country)-sector-year—and with quality adjustments using total citations received by each patent. For U.S. patents, we also measure patents' monetary value based on the algorithm of Kogan, Papanikolaou, Seru and Stoffman (2017) that calculates the value using stock market reactions upon patent approval. To capture innovation's timing, we use the year a patent was filed rather than granted, to abstract from bureaucratic delays that are orthogonal to innovative activities.

4.2 Sectoral Data on Production, Consumption, Trade, and R&D

In our cross-country analysis, for each country and sector, we use the World Input-Output Database (WIOD, Timmer, Dietzenbacher, Los, Stehrer and de Vries (2015)) to obtain data on value-added, employment, revenue, intermediate inputs, value used for consumption, imports, and exports. The data cover the years 2000–2014 and 43 major economies, which altogether represent more than 85% of global GDP. WIOD's sectoral categorization follows the two-digit International Standard Classification (ISIC) revision 4, with a total of 56 sectors covering the entire production spectrum, including primary, manufacturing, and service sectors. We obtain each country's sector-level R&D expense data from the Analytical Business Enterprise Research and Development (ANBERD) database, which is available for more than 30 countries in the WIOD data from 1987 onward.

For the United States, we obtain sectoral production, consumption, and import-export data from the national accounts of the Bureau of Labor Statistics (BLS), comprising 181 sectors from 1990 to 2019. We obtain sectoral R&D expenditures based on public firms in COMPUSTAT.

4.3 Concordances

Both U.S. and international patents are classified according to the IPC system, which is based on the concept of technology class and is distinct from the classifications in our sectoral data. We build concordance between these two data types by leveraging publicly traded firms' sectoral classifications and innovation output across IPCs. For the United States, we concord patents' IPC classification with BLS sectors. We first map IPCs into the North American Industry Classification System (NAICS) codes using the bridge files provided by NBER, Kogan et al. (2017), and Ma (2020, 2021) to cover our entire sample period. We then map NAICS codes to the BLS sectors using the crosswalk file provided by the BLS. For the global analysis, we provide a novel mapping from IPCs to the 56 ISIC sectors in WIOD using global firms from the Worldscope and Datastream databases accessed through the Wharton Research Data Services (WRDS). The data cover more than 109,000 global firms located in 160 countries, and we use fuzzy matching based on firm-level observables to link these firms' sectoral codes to their patent output across IPCs. We provide details of these matching procedures in Section D of our Online Appendix.

5 Innovation Network and Knowledge Spillovers

In this section, we first construct the innovation network Ω and then empirically validate of our theory's key law of motion (9), that there are knowledge spillovers through the innovation network.

5.1 Innovation Network

Constructing the Innovation Network We construct the innovation network from patent citations. Let $Cites_{ijt}$ denote the total number of times that patents in sector i cite patents in sector j, among all patents filed in year t in our global sample. We define ω_{ijt} as the share of total citations made by patents in sector i to sector j in year t:

$$\omega_{ijt} \equiv \frac{Cites_{ijt}}{\sum_{k=1}^{K} Cites_{ikt}}. \tag{26}$$

The object ω_{ijt} measures the extent to which upstream sector j's prior knowledge benefits innovation in sector i. The matrix $\mathbf{\Omega}_t$, whose ij-th entry is ω_{ijt}, captures the network of knowledge flows and is what we refer to as the *innovation network*.

There are several degrees of freedom when constructing the innovation network: we can construct a country-specific network using patents from each country; we can also include patents from a time window broader than one year. Empirically, the innovation network is highly persistent over time and also highly correlated across countries. Table A.2 of the Online Appendix shows that the serial correlation of the entries in Ω_t is near-perfect when a decade apart and remains above 0.8 even when three decades apart. Table A.3 of the Online Appendix shows that the innovation network constructed by pooling patents from all countries is almost perfectly correlated with the U.S.-specific network (correlation 0.97) and highly correlated (correlation≈0.8) with the country-specific innovation networks from Japan, China, Germany, Canada, the United Kingdom, and France. Such high correlations mean that decisions about country and time specificities of the innovation network do not affect our results; hence, for expositional simplicity, we adopt the time-specific, location-invariant measure in (26) as our baseline notion of the innovation network.

The Innovation Network is Weakly Correlated with Input-Output Networks The innovation network $\mathbf{\Omega}$ encodes cross-sector linkages via knowledge spillovers. Another prominent type of cross-sector linkage occurs through input-output relations, as sectors purchase intermediate inputs from one another during production. Table 1 shows that the innovation and production networks are very distinct; the two network relations indeed capture different connections across sectors. Specifically, for each of the top ten countries ranked by total patent output, we compute the industry-by-industry, input-output expenditure share matrix, which is a row stochastic matrix (as is $\mathbf{\Omega}$) commonly used to represent input-output relationships. Table 1 presents the correlation between entries in $\mathbf{\Omega}$ and those in the input-output matrix. The correlation is weak (<0.35) in all economies.

Table 1. Correlations Between the Innovation Network and Country-Level Production Networks

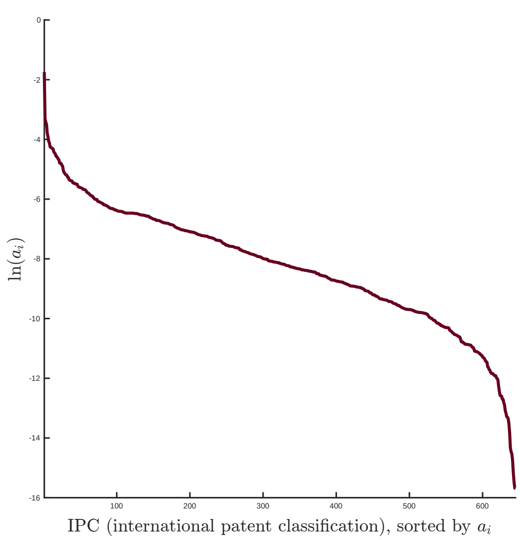
USA	JPN	CHN	KOR	DEU	CAN	GBR	FRA	RUS	SWE
0.32	0.31	0.35	0.24	0.28	0.25	0.28	0.32	0.15	0.11

Notes. This table presents the correlations between the innovation network matrix and the country-level input-output expenditure share matrix for the top ten countries ranked by total patent counts during our full sample period.

Innovation Centrality Across Sectors We now describe some properties of the innovation network. We first compute the innovation centrality $\boldsymbol{a}'$, which is the dominant left eigenvector of the innovation network $\boldsymbol{\Omega}$. Recall that $\boldsymbol{a}$ is also the R&D allocation vector that maximizes the growth rate of a closed economy (Corollary 1) and is an important determinant of the optimal R&D allocation in a closed economy (equation 19). The left panel of Figure 1 plots $\log(a_i)$ across 645 IPC sectors, where sectors are ordered along the x-axis in descending a_i. The figure shows innovation centrality is significantly skewed across sectors. To maximize economic growth, the highest innovation centrality sector should be allocated about five times as many R&D resources as the second sector ranked by a_i, about 12 times as many as the tenth sector, and about 100 times as many as the one-hundredth sector. The right panel of Figure 1 identifies the top ten IPC sectors; these include several technological classes related to digital data processing, semiconductors, telecommunications, and healthcare and pharmaceuticals technologies, among others.

Figure 1. Innovation Centrality and Key Sectors

(a) Innovation Centrality Across IPCs



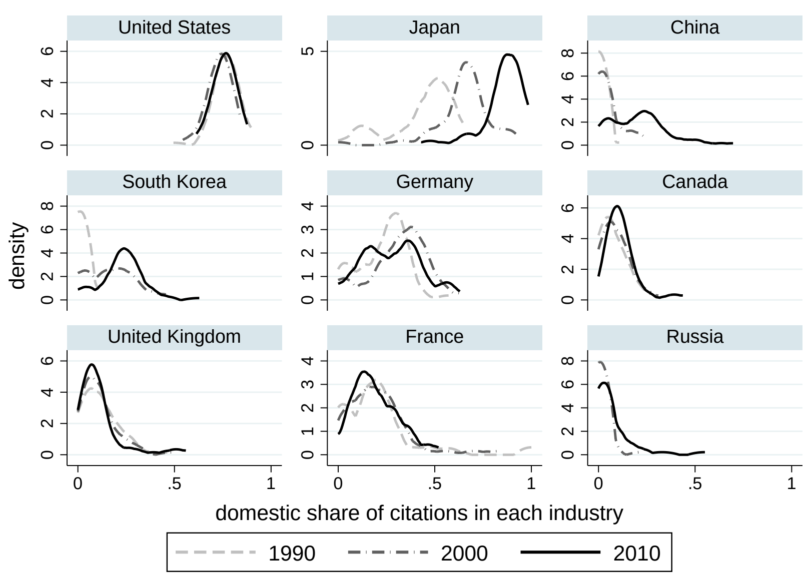
(b) Top Ten IPCs By Innovation Centrality a_i

1	electrical digital data processing
2	semiconductor devices; electric solid state devices
3	diagnosis; surgery; identification (analyzing biological material)
4	transmission of digital information; telegraphic communication
5	data processing systems or methods, spec. adapted for administrative, commercial, financial, managerial, supervisory, forecasting purposes
6	devices or methods specially adapted for bringing pharmaceutical products into particular physical or administratering forms
7	pictorial communication (e.g., television)
8	investigating or analyzing materials by determining their chemical or physical properties
9	containers for storage or transport of articles or materials
10	earth or rock drilling (mining, quarrying); obtaining oil, gas, water, soluble or meltable materials

Notes. This figure presents the innovation centrality of different technology classes categorized using IPCs. Panel (a) plots $\log(a_i)$, and the sectors are ranked in descending order based on a_i. Panel (b) lists the top ten IPCs by their innovation centrality.

Cross-Country Linkages in the Innovation Network The innovation network $\mathbf{\Omega}$ is constructed using patent citations across sectors, yet many patents are cited across countries. To the extent that the innovation network captures knowledge spillovers, how much do countries benefit from foreign knowledge? To answer this, for each country m, sector i, and year t, we compute the share of all citations mit made to patents created domestically in country m. Figure 2 shows the distribution of the domestic citation shares across all sectors for the nine economies with the highest patent counts in our sample, for years $t \in \{1990, 2000, 2010\}$. The United States relies relatively sparingly on foreign knowledge: consistently across these three decades, about 80% of citations made by U.S. patents are to other U.S. patents, and, across industries, these domestic citation shares are quite narrowly distributed around the mean. In contrast, citations to foreign patents account for the vast majority of citations made by all other economies except Japan, suggesting that these economies benefit significantly from foreign knowledge, most notably from the United States. The Japanese self-citation shares averaged to 50% in 1990 and increased over time, reaching close to 90% in 2010. A similar decline in foreign knowledge dependence over time is also observed for China and South Korea, the two other fast-growing Asian economies shown in Figure 2.

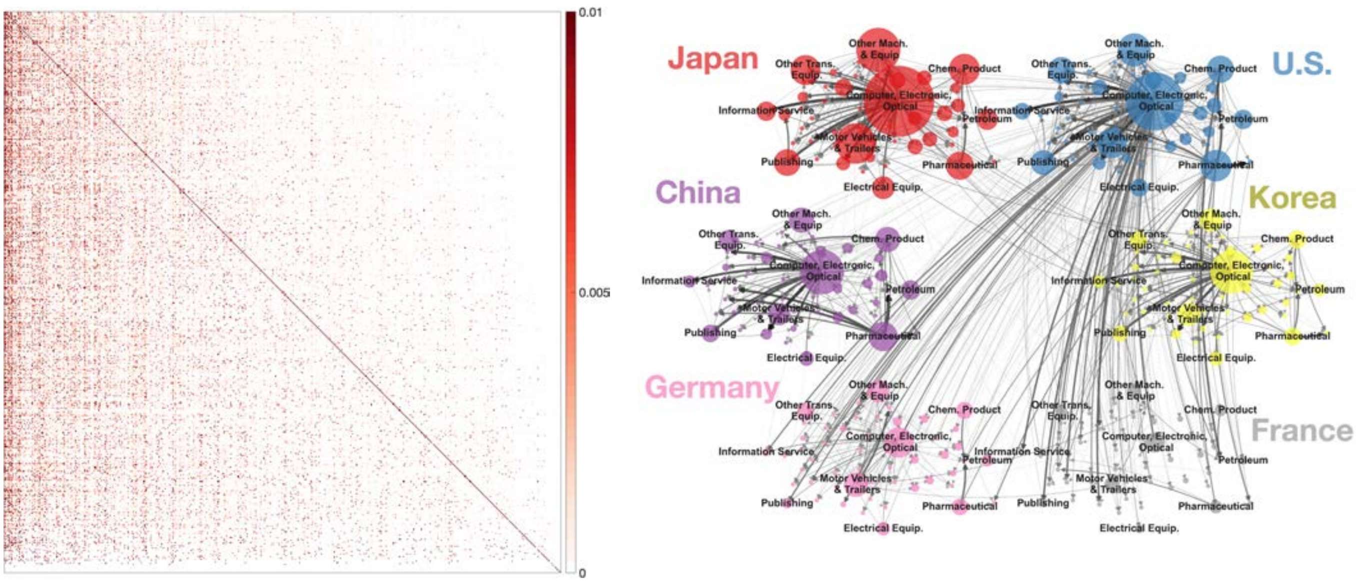
Figure 2. Cross-Sector Distribution of Domestic Citation Shares by Country



Notes. This figure presents the cross-sector distribution of domestic citation shares for each country, showing the distribution using 1990, 2000, and 2010 data. Sector definitions follow WIOD categorizations. Domestic citation share is defined as the number of citations made to domestic patents as a share of total citations made by new patents invented in each country-sector.

Figure 3. Visualizing the Innovation Network

(a) IPC-to-IPC (645×645) network $\mathbf{\Omega}$ **(b)** The global innovation network across country-sectors



Notes. The left panel visualizes the IPC-to-IPC network $\mathbf{\Omega}$ as a heatmap, with darker colors representing larger matrix entries; sectors are ordered according to their innovation centrality. The right panel visualizes the global innovation network. Each node is a country-sector, with size drawn in proportion to patent output. Arrows represent knowledge flows, with width drawn in proportion to citation shares.

Visualizing the Innovation Network Figure 3, Panel (a) visualizes the innovation network by plotting the matrix $\mathbf{\Omega}$ as a heatmap. Each row and each column is an IPC sector, where the color in the i-th row and j-th column correspond to ω_{ij} using the colormap listed to the right of the figure. Sectors are sorted by decreasing innovation centrality. A key feature is that IPC sectors follow a "hierarchical" structure: the innovation network is highly asymmetric, and there is a "pecking order" across sectors. Innovation-central sectors account for a disproportionate share of citations from all other sectors (columns are dense on the left but become progressively sparser to the right), yet these innovation-central sectors do not significantly cite noncentral sectors (rows are sparse on the top but become progressively denser towards the bottom).

Figure 3, Panel (b) visualizes the global innovation network by plotting each country-sector as a node, with size drawn in proportion to the total patent counts in our sample. An arrow from country m sector j to country n sector i indicates knowledge flow from mj to ni, with arrow width drawn in proportion to the share of ni's citations that are to mj. For visual clarity, only the largest countries and sectors are shown. Several patterns emerge from this figure. First, Japan and the United States produce the most patents in our sample. Second, the United States receives significantly more foreign citations than any other economy in our sample; it is a major knowledge exporter and only a minor knowledge importer, corroborating Figure 2.

5.2 Knowledge Spillovers Through the Innovation Network

5.2.1 Evidence Based on the United States

As a validation exercise, we now test the key mechanism behind our theory, namely that knowledge spillovers occur through the innovation network. We first test the mechanism using U.S. patents, treating the United States as a closed economy. Specifically, our law of motion (6) implies that the (log-)knowledge stock in each sector is a discounted sum of (log-)patents as in (8), and the ideas production function (5) further implies a log-linear relationship (equation 9, reproduced below) between sector i's new patents, sectoral R&D, and past patents from other sectors:

$$\ln n_{it} = \ln \eta_i + \ln s_{it} + \underbrace{\lambda \sum_{j=1}^{K} \omega_{ij} \left(\int_0^\infty e^{-\lambda s} \ln n_{j,t-s} \, \mathrm{d}s \right)}_{\substack{\equiv \ln \chi_{it}, \text{ the knowledge stock that benefits} \\ \text{idea generation in sector } i \text{ at time } t}} . \tag{9}$$

Equation (9) is empirically testable. It implies that, after controlling for sectoral R&D expenditures, past patents $\ln n_{j,t-s}$ in sector j predict new patent output in sector i through the innovation network ω_{ij}, and the effect decays over longer time lags with rate λ. Equation (9) also implies that the effect of knowledge spillovers is not only weighted but also directed: the knowledge flow from sector j to sector i operates through ω_{ij} and not ω_{ji}.

We test the discrete-time analogue of (9) by constructing the knowledge aggregator χ_{it} from past patents. Specifically, for each sector i, we enumerate over all sectors j from which knowledge flows to i, aggregating j's log patent counts $\ln n_{j,t-\tau}$ in the past ten years ($1 \leq \tau \leq 10$), weighted by $\omega_{ij,t-\tau}$, the strength of the knowledge connection from j to i in the corresponding year:

$$\text{Knowledge}_{it}^{Up} \equiv \textstyle\sum_{j \neq i} \sum_{\tau=1}^{10} \omega_{ij,t-\tau} \ln n_{jt-\tau}. \tag{27}$$

$\text{Knowledge}_{it}^{Up}$ captures the stock of past knowledge "upstream" of sector i, meaning it is the stock of knowledge ($\ln \chi_{it}$) that can benefit sector i's subsequent idea generation. We then perform the following regression:

$$\ln n_{it} = \beta_1 \times \text{Knowledge}_{it}^{Up} + \beta_2 \times \ln R\&D_{i,t-1} + \xi_i + \xi_t + \text{controls}_{it} + \epsilon_{it}, \tag{28}$$

where n_{it} is the number of patents filed in sector i year t and $R\&D_{i,t-1}$ is the R&D expenditure, with a one-year lag to reflect the delayed nature of patent filing (our results are robust to controlling for concurrent R&D expenditures). We control for sector fixed effects ξ_i, to purge time-invariant sectoral factors that affect patent output, as well as for year fixed effects ξ_t, to purge time-varying shocks common across all sectors.

We discuss two details before showing the results. First, when constructing the upstream knowledge aggregator (27) for each sector i, we exclude the lagged patent output from sector i itself; doing so ensures that the coefficient β_1 in regression (28) is not driven by serially correlated shocks to sectoral patent output. Second, theoretically the knowledge aggregator in (9) features exponential decay of past patents' effects, yet our empirical construction (27) features a discrete cutoff window for $\tau \leq 10$ years. We make this choice to be agnostic about the parameter λ; later we also nonparametrically estimate the effect at different time lags.

Table 2. Directed Nature of Knowledge Flow

$Y =$	ln(Patents)			ln(Cites)			ln(Patent Value)		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
$\text{Knowledge}_{it}^{Up}$	0.678***	0.705***	0.939***	0.883***	0.914***	1.054***	1.083***	1.088***	1.126***
	(0.188)	(0.214)	(0.236)	(0.195)	(0.205)	(0.328)	(0.325)	(0.335)	(0.412)
$\ln(R\&D)_{i,t-1}$	0.233***	0.232***	0.177**	0.228**	0.228**	0.198*	0.263***	0.263***	0.137
	(0.072)	(0.071)	(0.071)	(0.091)	(0.090)	(0.107)	(0.104)	(0.104)	(0.092)
$\text{Knowledge}_{it}^{Down}$		-0.058			-0.067			-0.011	
		(0.188)			(0.107)			(0.143)	
$\text{Knowledge}_{it}^{Up,IO}$			0.301			0.282			0.160
			(0.187)			(0.244)			(0.244)
No. of Sectors		95			95			95	
No. of Obs		1,900			1,900			1,900	
Fixed Effects		Sector, Year			Sector, Year			Sector, Year	

Notes. This table tests the relation between innovation in a focal sector and past innovation in sectors connected through the innovation network, using the U.S. data over BLS sectors. We restrict the sample to sectors that have at least 100 patents over the full sample period. To measure innovation production (Y), we use the number of patents, the number of future citations per patent through the end of our sample, and the commercial value estimated using stock market reactions upon patent approval (Kogan et al., 2017). The key variable of interest, $\text{Knowledge}_{it}^{Up}$, is the knowledge from upstream, defined in (27). Lagged sectoral R&D expenses and sector and year fixed effects are included as controls. Columns (2), (5), and (8) include downstream knowledge as a control. Columns (3), (6), and (9) include knowledge accumulated from upstream sectors in the production network as a control. Standard errors in parentheses are clustered at the sector level. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

Table 2, column (1) presents the results of regression (28). Sectoral R&D expenditure significantly predicts the number of new patents filed in a given year, with an elasticity of 0.233. The knowledge stock upstream of each sector—$\text{Knowledge}_{it}^{Up}$, or $\ln \chi_{it}$—also significantly predicts patent output, with an elasticity of 0.62. Columns (4) and (7) show that both variables also predict patent quality: sectors with greater R&D and greater upstream knowledge stock tend to produce patents with more future citation counts and greater commercial value, as measured using the stock market reaction upon patent approval (Kogan et al. (2017)).

These regressions paint a picture consistent with our proposed mechanism, that past knowledge in sectors upstream of i benefits the subsequent patent production in the focal sector i. An alternative story relates to common shocks: a group of sectors that is connected to each other via citation linkages may face similar demand, supply, and investment opportunities, leading to co-movements of innovation activities. Such common shocks would lead to a positive coefficient β_1 in regression (28) even without cross-sector knowledge spillovers. This is a classic version of the "reflection problem" documented in Manski (1993) and, more relevant to our setting, in Bloom et al. (2013).

To address the "common shock" concern, we construct the aggregator of knowledge stock for sectors *downstream* of i:

$$\text{Knowledge}_{it}^{Down} \equiv \sum_{k\neq i}\sum_{\tau=1}^{10}\omega_{ki,t-\tau}\ln n_{kt-\tau}.$$

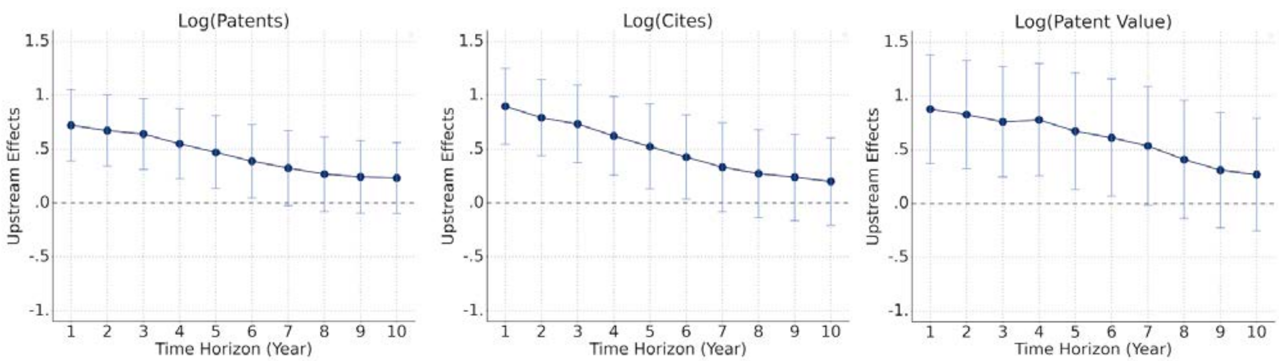
That is, $\text{Knowledge}_{it}^{Down}$ aggregates the (log-)patent output in all sectors $k \neq i$, weighted by the extent to which patents in sector k cite those in sector i. It is therefore a measure of the knowledge stock *downstream* of sector i. Because knowledge flow is directional, our theory implies the following asymmetry: while the upstream aggregator $\text{Knowledge}_{it}^{Up}$ should positively predict subsequent patent output in sector i, the downstream aggregator $\text{Knowledge}_{it}^{Down}$ should not. Yet any common shocks hitting this network should generate symmetric correlations in innovation output for focal sector i and both its upstream and downstream sectors.

Columns (2), (5), and (8) of Table 2 add $\text{Knowledge}_{it}^{Down}$ as a control variable to our baseline regressions. We make two observations. First, adding $\text{Knowledge}_{it}^{Down}$ as a control does not meaningfully affect the economic or statistical significance of our two baseline variables. This suggests our baseline regressions are not simply picking up correlated shocks to local technology clusters. Second, the coefficient on $\text{Knowledge}_{it}^{Down}$ is precisely zero, confirming our key model mechanism and that knowledge flow along the innovation network is directional in nature—it goes only from upstream to downstream, and not the other way around.

Another related concern is that common shocks operate not through technological linkages but through input-output (IO) linkages. To address this, we construct the aggregator $\text{Knowledge}_{it}^{Up,IO}$ similarly to $\text{Knowledge}_{it}^{Up}$, but patents from other sectors are weighted not by the innovation network, as in (27), but instead by sector i's expenditure share on inputs from sector j. Columns (3), (6), and (9) of Table 2 show the regression results when including $\text{Knowledge}_{it}^{Up,IO}$ as a control variable. Knowledge from sectors that are innovation-upstream of sector i remains an economically and statistically significant predictor of subsequent innovation in sector i, measured by patent counts, citations per patent, and patent value. By contrast, $\text{Knowledge}_{it}^{Up,IO}$ is not a significant predictor of sector i's innovation in these specifications. We find that the coefficient on $\text{Knowledge}_{it}^{Up,IO}$ is significant when we omit the main variable $\text{Knowledge}_{it}^{Up}$ from the regressions (not reported in Table 2), but the effects seem to be dwarfed by the spillover effects through the innovation network. These results, along with the fact that the innovation network is only weakly correlated with the IO network (see Table 1), imply that the innovation network provides valuable incremental information that is particularly powerful for understanding knowledge spillovers across industries.

Finally, we revisit the dynamic prediction of our key law of motion (9), that upstream knowledge from the more distant past has less effect on patent output. To explore this, we perform our baseline regression (28) using lagged versions of the upstream knowledge aggregator on the right-hand side. We plot the coefficients and confidence intervals in Figure (4), where the one-year lag corresponds to the baseline estimates in columns (1), (4), and (7) in Table 2. The figure shows an obsolescence-like pattern (Ma, 2021) in which the effect of past upstream knowledge on subsequent innovation weakens over time, precisely as our theory predicts. The half-life of these effects is about four years. Because our theory features an exponential decay of rate λ, the half-life of four years translates into $\lambda = -\left(\ln \frac{1}{2}\right)/4 \approx 0.173$.

Figure 4. Dynamic Responses of Innovation Output to Upstream Knowledge



Notes. This figure presents the dynamic responses of the focal sector's innovations to past innovations from upstream sectors in the innovation network. The coefficients are from regressions of focal sector's innovations at times $t+1$ through $t+10$ on upstream knowledge measured at time-t. We control for log R&D with time-1 lag as well as sector and year fixed effects.

5.2.2 Evidence Based on the Global Innovation Network

We now test international knowledge spillovers in our global sample. We construct an analogous measure of upstream knowledge stock: for each focal country m, sector i in year t, we enumerate over all countries c and sectors j in our sample, aggregating the (log-)patent output in cj over the past ten years, weighted by the share of mi's citations that are to cj in the corresponding year:

$$\text{Knowledge}^{Up}_{mit} \equiv \sum_{cj \neq mi} \sum_{\tau=1}^{10} \frac{Cites_{mi \to cj, t-\tau}}{\sum_{c'=1}^{N} \sum_{k=1}^{K} Cites_{mi \to c'k,, t-\tau}} \ln n_{aj,t-\tau}. \tag{29}$$

We then adapt our closed-economy test of knowledge spillovers to perform on the global innovation network. In this case, the unit of observation is at the country-industry-year level:

$$\ln n_{mit} = \beta_1 \times \text{Knowledge}^{Up}_{mit} + \beta_2 \times \ln R\&D_{mi,t-1} + \xi_{mi} + \xi_{mt} + \xi_{it} + \text{controls}_{mit} + \varepsilon_{ict}. \tag{30}$$

We include a saturated set of fixed effects. The country-industry fixed effect ξ_{mi} accounts for any time-invariant unobserved heterogeneity in patent output (e.g., IT industries in the United States and France have different patent productivity); the country-year fixed effects ξ_{mt} control for time-varying country-level shocks (e.g., patent productivity; business cycles) that are common across industries; and the industry-year fixed effects ξ_{it} account for time-varying global shocks to patent output that are common within industries and across countries.

Table 3 shows the results: knowledge stock upstream of each country-industry significantly predicts subsequent patent output (column 1) and average citations per patent (column 5) even in the global setting. The coefficients are lower than those estimated for the United States, suggesting that knowledge spillovers are stronger across sectors within the United States than they are across countries. The data for sectoral R&D across countries, which are from ANBERD, have regrettably spotty coverages: they only cover 16 of the 56 WIOD sectors in earlier years, and the data exist for only 30 out of 43 countries. Hence, regressions where we control for sectoral R&D (columns 2 and 6) have substantially smaller sample sizes. Nevertheless, we find the coefficient on the upstream knowledge aggregator to be stable and quantitatively unchanged with the control.

To rule out common shocks to technological and input-output clusters, we again—similar to our closed-economy tests—construct and control for aggregators $\text{Knowledge}_{mit}^{Down}$ to capture knowledge from downstream and $\text{Knowledge}_{mit}^{Up,IO}$ to capture potential knowledge spillovers through the input-output network. Columns (3), (4), (7), and (8) show that the coefficients on these controls are insignificant, and our coefficients on $\text{Knowledge}_{mit}^{Up}$ do not materially change when adding these controls. Overall, these results validate our mechanism of knowledge spillovers through the international innovation network.

6 Application: R&D Resource Allocations in the Data

In this section we use our model to evaluate cross-sector allocations of innovation resources in the data. We compute the unilaterally optimal allocation of R&D resources across sectors for each country and year in our sample. We show that optimal allocations do predict sectoral patent output for countries and time periods generally perceived to be more innovative, such as the United States, Germany, and, more recently, Japan and South Korea, but the relationship does not hold for many other economies such as India, Mexico, and Russia. As we demonstrate, what distinguishes the first group of countries is that a small number of multisector innovation hubs account for the vast majority of those countries' patent output. These innovation hubs have significant internal knowledge stock across many sectors, and because they partially internalize the cross-sector knowledge spillovers, they allocate resources more like a planner would. Finally, we conduct policy counterfactuals and show that replacing the real-world R&D allocations with the unilaterally optimal ones can generate substantial welfare improvement for countries around the globe.

Table 3. Evidence of the Global Innovation Network for Knowledge Spillovers

$Y=$	ln(Patents)				ln(Cites)			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\text{Knowledge}_{mit}^{Up}$	0.084***	0.058**	0.055**	0.057**	0.113***	0.098***	0.098***	0.097***
	(0.019)	(0.024)	(0.023)	(0.028)	(0.022)	(0.031)	(0.031)	(0.033)
$\ln{(R\&D)}_{i,t-1}$		0.000	0.000	0.002		-0.010*	-0.009*	-0.008
		(0.004)	(0.004)	(0.005)		(0.005)	(0.005)	(0.06)
$\text{Knowledge}_{mit}^{Down}$			0.008				0.001	
			(0.009)				(0.012)	
$\text{Knowledge}_{mit}^{Up,IO}$				0.008				-0.003
				(0.042)				(0.060)
R^2	0.949	0.964	0.964	0.969	0.921	0.943	0.943	0.948
No. of Obs	19,027	12,693	12,693	9,897	19,027	12,693	12,693	9,897
No. of Country×Sectors	998	895	895	875	998	895	895	875
Fixed Effects	Country×Sector, Country×Year, Sector×Year							

Notes. This table tests the relation between innovation in a focal sector and past innovation in connected sectors through the innovation network, in an international setting. We restrict the sample to country-sectors that have at least 10 patents over the full sample period. To measure innovation production (Y), we use the number of patents and total number of citations. The key variable of interest, $\text{Knowledge}_{it}^{Up}$, is the knowledge from upstream, defined in (29). Fixed effects at the country-sector, country-year, and sector-year levels are included as controls. Columns (3) and (7) include downstream knowledge as a control. Columns (4) and (8) include knowledge accumulated from upstream sectors in the production network as a control. Standard errors in parenthesis are clustered at the country-sector level. *, **, and *** indicate significance at the 10%, 5%, and 1% levels respectively.

6.1 Optimal R&D Allocations and Reliance on Foreign Knowledge

For each country m and year t, we calculate the unilaterally optimal cross-sector allocation of R&D resources $\boldsymbol{\gamma}_m$ using Proposition 7:

$$\boldsymbol{\gamma}_{mt}' \propto \boldsymbol{\phi}_{mt}' \left(\boldsymbol{I} - \frac{\boldsymbol{\Theta}_{mt}}{1+\rho/\lambda}\right)^{-1}, \tag{31}$$

where the proportionality constant is chosen to ensure the vector $\boldsymbol{\gamma}_{mt}$ sums to one. Recall that $\boldsymbol{\Theta}_{mt} \equiv \boldsymbol{1}\boldsymbol{x}_{mt}' \circ \boldsymbol{\Omega}_t$ is the matrix obtained by multiplying each column j of the innovation network $\boldsymbol{\Omega}_t$ with $x_{mjt} \equiv \frac{q_{mjt}}{q_{mjt}+q_{mjt}^f}$, the share of domestic knowledge in sector j used for subsequent idea generation in other sectors. Empirically, we measure x_{mjt} as the share of citations to sector j that are from domestic patents. As Figure 2 shows, x_{mjt} averages to above 80% across sectors for the United States but is significantly lower for all other countries except Japan in recent years.

We compute the object $\boldsymbol{\phi}_{mt}$, which is the vector encoding the elasticity of domestic aggregate consumption to sectoral productivity shocks, using import pricing function $\boldsymbol{p}_{mt}\left(\cdot;\boldsymbol{\theta}_t\right)$ microfounded from a trade model in which country m is a small open economy with intermediate varieties produced monopolistically, similar to the trade block of Gali and Monacelli (2005).

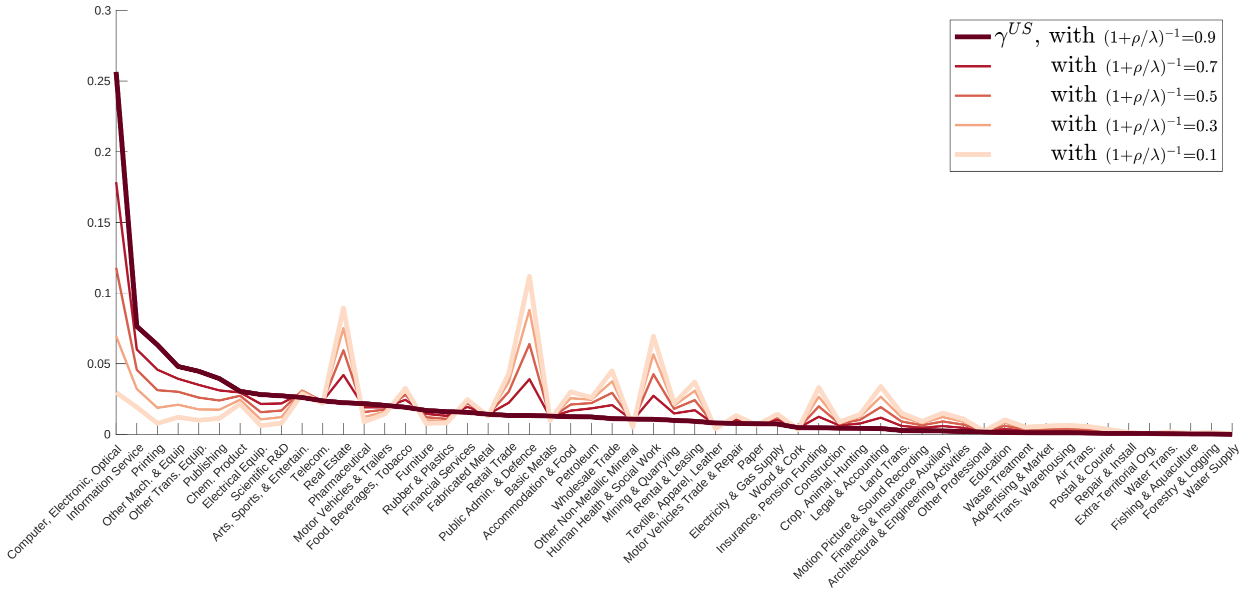
This microfoundation provides the mapping from data on country m's employment shares, imports, and sectoral imports and exports to $\boldsymbol{\phi}_{mt}$. The microfoundation details appear in our Online Appendix; here we simply note that, empirically, $\boldsymbol{\phi}_{mt}$ correlates extremely highly with the consumption share vector $\boldsymbol{\beta}_{mt}$ (correlations average to 0.95) and the employment share vector (correlations average to 0.88) of each country and year. Intuitively, in a closed economy, $\boldsymbol{\phi}_{mt}$ coincides with the consumption share and employment share vectors, and in open economies, $\boldsymbol{\phi}_{mt}$ deviates from these other vectors due to terms-of-trade considerations: because of imperfect substitutability between domestic and foreign bundles, improving the domestic knowledge stock in a sector affects domestic demand for imports and hence their prices. In practice, the effect of terms-of-trade changes on domestic consumption is very small; in fact, the results we present below are largely unchanged even if we simply replace $\boldsymbol{\phi}_{mt}$ with the consumption share vector $\boldsymbol{\beta}_{mt}$ when calculating the unilaterally optimal R&D allocation using (31).

To implement formula (31), we need to specify the discount rate relative to the step size of innovation, ρ/λ. As a baseline, we choose the discount rate to be $\rho = 2\%$, and we use the step size $\lambda = 0.173$ as implied by the half-life in Figure 4, which is consistent with calibrations of λ in a number of recent studies based on quality-ladder models.[3] Qualitatively, our cross-country analysis is not sensitive to these parameter values. As discussed previously and evident in formula (31), an increase in ρ/λ has the same implication for the optimal R&D allocation as an increase in a country's reliance on foreign knowledge (i.e., a uniform decrease in the domestic citation shares x_{mit} across all sectors). As shown in Figure 2, empirical cross-country variation in domestic citation shares is an order of magnitude larger than the variation in $(1 + \rho/\lambda)^{-1}$ within reasonable ranges of the discount rate and step size. For instance, as the discount rate quadruples from $\rho = 2\%$ to 8%, $(1 + \rho/\lambda)^{-1}$ experiences a decline from 0.90 to 0.63; however, the average domestic citation shares vary across countries by as much as 400 percent in proportion, for instance when comparing the United Kingdom (20% domestic citations) to the United States (>80% domestic citations). Hence, the qualitative cross-country differences in unilaterally optimal R&D allocations are not sensitive to our calibration of ρ/λ. For our quantitative exercises, we report alternative values of ρ/λ in the Online Appendix as sensitivity checks.

To illustrate how foreign reliance can affect optimal R&D allocations, Figure 5 plots the share of R&D resources to be optimally allocated across sectors in the United States in the year 2010, for varying levels of $(1 + \rho/\lambda)^{-1}$, noting again that a decline in $(1 + \rho/\lambda)^{-1}$ is isomorphic to a uniform decline in domestic citation shares. The x-axis represents WIOD sectors, and the level of optimal R&D resources γ_{it} is shown on the y-axis. The darkest curve reflects our baseline calibration where $(1 + \rho/\lambda)^{-1} = 0.9$, with sectors sorted in descending order by γ_{it}. The other

[3] Acemoglu, Akcigit, Bloom and Kerr (2018a) set $\lambda = 0.13$; Aghion, Bergeaud, Boppart, Klenow and Li (2021) set $\lambda = \ln(1.249) = 0.22$; Liu, Mian and Sufi (forthcoming) set $\lambda = \ln(1.21) = 0.19$.

Figure 5. Optimal R&D Allocations for Varying Levels of Discounting/Foreign Reliance



Notes. This figure shows the optimal R&D allocation $\boldsymbol{\gamma}$ across WIOD sectors for varying levels of ρ/λ. Sectors are sorted by $\boldsymbol{\gamma}$ when $(1+\rho/\lambda)^{-1} = 0.9$, our baseline calibration. Values of $\boldsymbol{\gamma}$ in the baseline calibration are shown in the darkest red curve. The curves with progressively lighter shades correspond with lower values of $(1+\rho/\lambda)^{-1}$.

curves with progressively lighter colors represent economies with higher ρ/λ—or, equivalently, that rely more on foreign knowledge.

Two patterns emerge. First, for the United States, WIOD sectors that obtain the most R&D resources under the optimal allocation are related to computers, electronics, optical apparatus, and information services; these few sectors should account for over a third of total U.S. R&D resources. These are also sectors with the highest innovation centrality. In fact, the correlation between the optimal R&D allocation $\boldsymbol{\gamma}$ and the innovation centrality $\boldsymbol{a}$ is well above 0.9 in most years. This is because the United States is a large economy with a self-contained innovation network; hence, its planner should internalize most of the knowledge spillovers, so that the optimal allocation ($\boldsymbol{\gamma}$) is not too different from the growth-maximizing ones ($\boldsymbol{a}$), and consumer preferences and international trade conditions (both are encoded in $\boldsymbol{\phi}_{US}$) play a relatively small role in determining $\boldsymbol{\gamma}$. Second, as ρ increases—equivalently, as countries become more reliant on foreign knowledge and have lower domestic citation shares—sectors that are important in the consumption bundle, such as real estate and retail trade, require progressively more resources in the optimal R&D allocation. Note that we show these patterns using the WIOD sectors for visual clarity; they also hold for the 645 IPC technological classes, but are more visually cluttered.

Qualitatively, these results suggest that, from each country's self-serving perspective, economies with self-reliant innovation networks, such as the United States and Japan, should optimally allocate more resources to innovation-central sectors; by contrast, economies that depend on foreign knowledge spillovers should optimally allocate more resources to sectors that are more important in domestic production, either due to domestic consumer demand or exports.

Table 4 shows the empirical correlation in optimal R&D allocations between country pairs, with the lower triangular panel showing the Pearson correlation and the upper triangular panel showing the Spearman rank correlation, which is equal to the Pearson correlation of the rank values. Unlike the strong correlation in the country-specific innovation networks (see Table A.3 of the Online Appendix), unilaterally optimal R&D allocations differ significantly across countries. This variation is primarily driven by cross-country heterogeneity in domestic citation shares ($\boldsymbol{x}_{mt}$), though heterogeneity in production structures ($\boldsymbol{\phi}_{mt}$) plays a role as well.

Table 4. Unilaterally Optimal R&D Allocations Differ Significantly Across Countries

Correlations of $\boldsymbol{\gamma}_m$ across countries (for the year 2010)

Countries	US	Japan	China	Korea	Germany	Canada	UK	France	Russia	Sweden
US		0.80	0.51	0.65	0.71	0.58	0.50	0.53	0.53	0.51
Japan	0.93		0.64	0.66	0.63	0.45	0.30	0.41	0.57	0.43
China	0.49	0.58		0.73	0.48	0.46	0.30	0.40	0.75	0.43
Korea	0.60	0.64	0.74		0.71	0.58	0.56	0.62	0.70	0.63
Germany	0.45	0.39	0.29	0.62		0.69	0.78	0.84	0.62	0.82
Canada	0.34	0.21	0.24	0.47	0.75		0.83	0.74	0.71	0.81
UK	0.22	0.11	0.09	0.42	0.83	0.84		0.89	0.55	0.82
France	0.23	0.14	0.20	0.50	0.87	0.82	0.93		0.63	0.84
Russia	0.21	0.18	0.59	0.54	0.57	0.65	0.55	0.67		0.64
Sweden	0.39	0.30	0.21	0.50	0.85	0.80	0.88	0.87	0.51	

Notes. This table shows the pair-wise correlations of optimal R&D allocations across countries. The lower triangular panel shows the Pearson correlation coefficients; the upper triangular panel shows Spearman's rank correlation.

6.2 Innovation Allocations in the Data

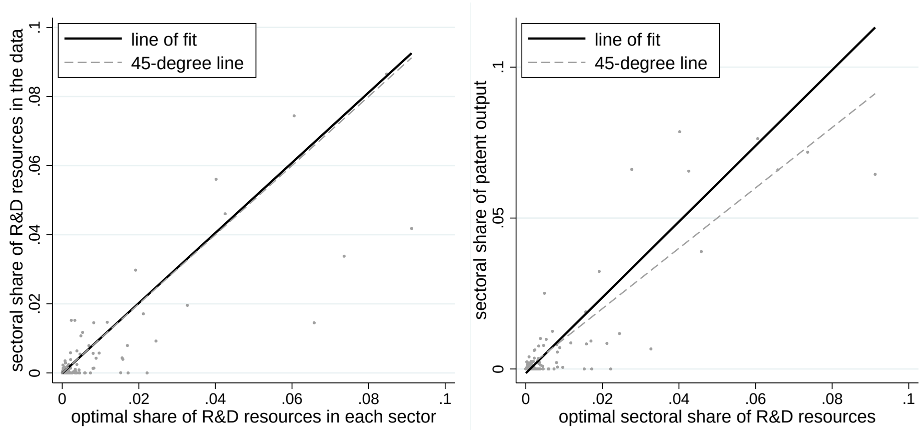
We now use optimal R&D allocations $\boldsymbol{\gamma}_{mt}$ to assess innovation activities around the globe. Ideally, for each country m and year t, we would like to observe sectoral R&D expenditures, which, according to our theory, should align with $\boldsymbol{\gamma}_{mt}$ if resources are allocated optimally. By contrast, any misalignment between $\boldsymbol{\gamma}_{mt}$ and R&D expenditures indicates resource misallocation for country m in year t.

We start with analysis based on the United States over BLS sectors. The left panel of Figure 6 shows the scatter plot of sectoral R&D expenditure (as a share of total R&D) against the optimal R&D expenditure share $\boldsymbol{\gamma}_{US}$ for the year 2010. The linear fit (solid line) is visually indistinguishable from the 45-degree line (dashed) and has a slope of 1.02. The figure thus demonstrates that that on average, sectors that should have received more R&D resources do indeed receive more R&D resources. In the right panel of Figure 6, we change the y-axis to sectoral patent output (as a share of total patent output); again, sectoral patent output aligns very well with $\boldsymbol{\gamma}_{US}$, with a significantly positive slope of 1.26.

One important concern about how we interpret the striking pattern in Figure 6 is whether we are picking up something mechanical and circular. In particular, it may be possible that sectors receiving more resources simply end up with more patents and citations; because the United States is a large economy, these sectors could appear to be more important in the global innovation network $\mathbf{\Omega}$, leading our theory to suggest that these sectors should be allocated more resources.

To rule out this possibility, we construct an alternative innovation network that is independent from sectoral R&D in the United States. Specifically, we construct a network based only on Japanese patent citations to non-U.S. patents. Although this alternative innovation network $\hat{\mathbf{\Omega}}_{JP}$ is independent of U.S. R&D expenditures, we nevertheless find $\hat{\mathbf{\Omega}}_{JP}$ to be nearly perfectly correlated with $\mathbf{\Omega}$, and all our findings continue to hold. This robustness test suggests that the innovation centrality $\boldsymbol{a}$—which correlates strongly with the optimal R&D allocation $\boldsymbol{\gamma}_{US}$ of the United States—is indeed picking up sectoral importance in the innovation network and is independent of sectoral R&D expenditures.

Figure 6. U.S. Sectoral R&D and Patent Output Align Well With $\boldsymbol{\gamma}_{US}$ in 2010



Notes. This figure shows scatter plots of the real-world R&D expenditure shares (left panel) and patent output (right panel) against the optimal R&D allocation shares, for the U.S. in 2010. The solid line is the linear fit; the dashed line is the 45-degree line.

To be clear, the on-average alignment between real-world and optimal R&D allocations, as in Figure 6, does not imply the United States is allocating R&D optimally: there is substantial residual variation in R&D allocations as they disperse around the 45-degree line. The vertical distance between each observation and the 45-degree line measures the amount of R&D resources that need to be reallocated to achieve the optimal allocation. We quantitatively assess the importance of such dispersion in Section 6.3.

We now analyze the correlation between optimal and real-world innovation allocations across countries and time in our global sample. As discussed previously, the data on sectoral R&D spendings are regrettably spotty and cover only a subset of our sample; the coverage is especially poor for less-developed economies and in early years of the sample. Hence, we examine both R&D expenditures and sectoral patent outputs, using the latter as an imperfect proxy for R&D expenditures in order to cover more countries and time periods. For the subset of data that include both R&D expenditure and patent output, the two variables have a correlation of 0.66.

Table (5) shows the results. Columns (1)–(3) use (log-) R&D expenditures for each country-sector-year as the outcome variable, with a different set of fixed effects across columns. Columns (4)–(6) use (log-) patent counts as the outcome variable, and columns (7)–(9) use (log-) total future citation counts, from the filing year through the end of our sample. We use saturated fixed effects to control for unobserved heterogeneity, including sector-year fixed effects, which control for time-varying sectoral shocks (e.g., to innovation productivity) common across countries; country-year fixed effects, which control for time-varying, country-level shocks common across sectors; and country-sector fixed effects, which control for time-invariant unobserved heterogeneity that varies across countries and sectors. The coefficients on the main dependent variable γ_{mit} are significant at the 5% level across all specifications except for column (3), which uses R&D as the outcome variable with a limited sample size; nevertheless, this coefficient has a p-value of 0.058 and is therefore still marginally significant. These results suggest that, on average and from the perspectives of self-serving social planners in our global sample, sectors that should obtain greater innovation inputs do receive more R&D resources and have greater innovation output.

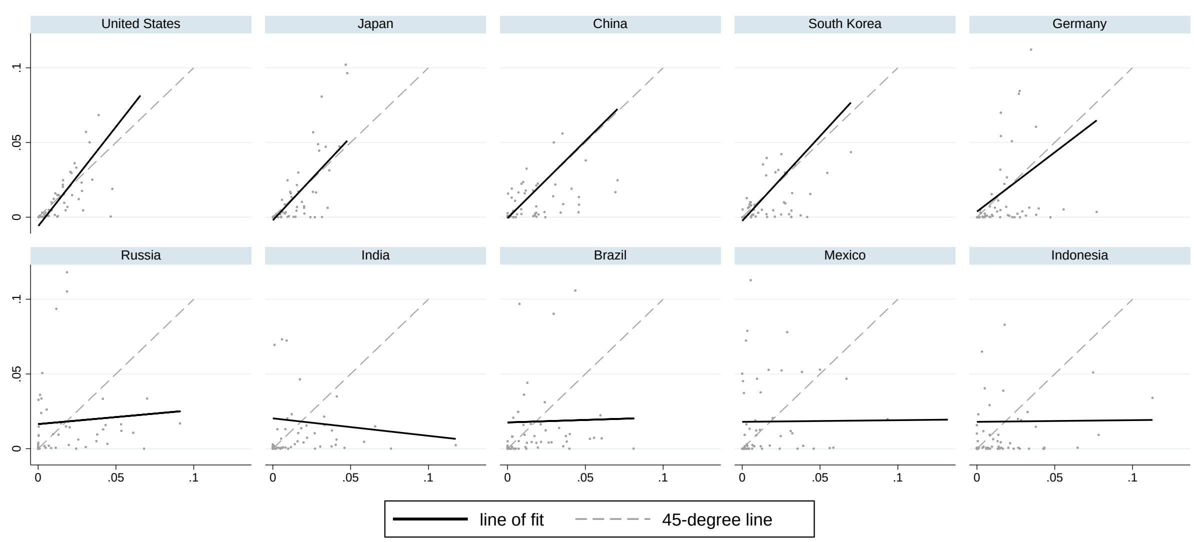
Table 5. Across Countries, Sectors that Should Optimally Have Greater Innovation Inputs Do Generate Greater Innovation Output

	$\ln(\text{R\&D expenditure}_{mit})$			$\ln(\#\text{Patents}_{mit})$			$\ln(\text{citations}_{mit})$		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
γ_{mit}	22.47***	23.75***	6.935*	10.41***	10.41***	14.41**	10.82**	10.82**	13.56**
	(3.237)	(3.068)	(3.432)	(3.047)	(3.046)	(3.666)	(3.306)	(3.305)	(3.767)
Fixed Effects									
Country	Yes			Yes			Yes		
Year	Yes			Yes			Yes		
IPC	Yes	Yes		Yes	Yes		Yes	Yes	
Country×Year		Yes	Yes		Yes	Yes		Yes	Yes
IPC×Year			Yes			Yes			Yes
Country×IPC			Yes			Yes			Yes
R^2	0.863	0.896	0.969	0.613	0.677	0.910	0.606	0.665	0.884
No. of Obs	26041	26033	25891	141900	141900	141900	141900	141900	141900
No. of Countries	34	34	34	42	42	42	42	42	42

Notes. This table presents the relation between the unilaterally optimal innovation allocations and measures of sectoral innovation, including (log-) R&D expenditure, patent output, and total future citation counts through the end of our sample. The regressions pool all countries, sectors, and years in our sample. Standard errors in parentheses are clustered at the country level. *, **, and *** indicate significance at the 10%, 5%, and 1% levels respectively.

There is substantial cross-country heterogeneity in R&D resource allocations. Figure 7 shows scatter plots of sectoral patent output against unilaterally optimal R&D allocations, for ten selected countries in the year 2010. Sectoral patent output—as a proxy for R&D inputs—correlate strongly with optimal R&D allocations for the five countries shown in the top row (the United States,[4] Japan, China, South Korea, and Germany). On the other hand, there does not seem to be a positive relationship between patent output and unilaterally optimal R&D allocations for those five economies at the bottom (Russia, India, Brazil, Mexico, and Indonesia). As we have noted above, a positive line-of-fit does not imply optimal resource allocation, but on average, observations are vertically closer to the 45-degree line in the top row than in the bottom, suggesting that, in the latter group of countries, more resources need to be reallocated in order to achieve optimal R&D allocations. Note that we use patent output to proxy R&D inputs in this figure because patent output is available for more countries, particularly the five less-developed economies in the bottom row; Figure A.1 in the Online Appendix uses sectoral R&D expenditure as the y-axis shows similar findings for the set of economies in the top row.

Figure 7. Patent Output Aligns Well With γ_m for Some Countries But Poorly for Others in 2010



Notes. This figure shows scatter plots of sectoral patent output (as a fraction of total patent output in each country) against the optimal sectoral share of R&D allocation for ten selected countries and the year 2010. The solid line is the linear fit; the dashed line is the 45-degree line. For visual clarity, outliers—sectors that account for >12% of national patent output—are not shown in the scatter plots, but all sectors are used when constructing the linear fit.

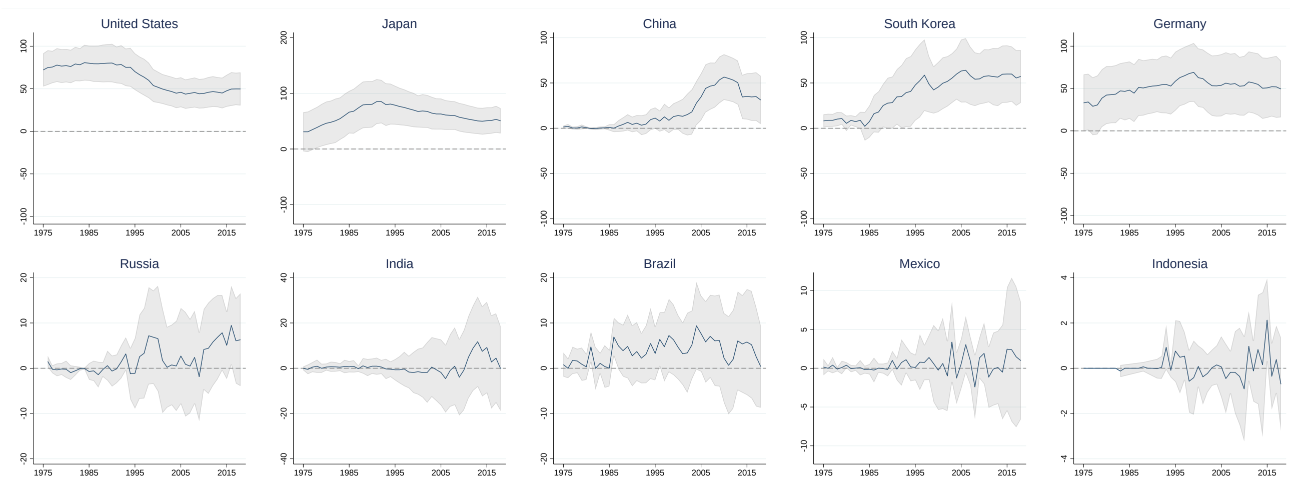
In Figure 8, we further demonstrate over-time heterogeneity in R&D allocations within each country. Specifically, for each country m and year t, we perform a bivariate regression of (log-) patent count in each sector i on γ_{mit}, and we show the slope coefficient and the confidence interval for ten selected countries over our entire sample period. The slope coefficients for the United States and Germany are consistently positive throughout our sample period. The slope

[4] We include the United States in Figure 7, which is based on WIOD sectors, as a comparison to Figure 6, which is based on BLS sectors.

coefficients for India, Mexico, and Russia are consistently indistinguishable from zero. Most interestingly, for the three Asian economies, the slope coefficients were zero at the beginning of our sample and only became positive during these economies' rapid growth periods.

Taking stock, our test suggests that during time periods generally perceived to be more innovative countries do seem to allocate R&D resources in ways that correlate with unilaterally optimal allocations, while other economies perceived to be less innovative—perhaps due to poor institutions and lack of incentives—seem to misallocate R&D resources.

Figure 8. Alignment Between Patent Output and Optimal Allocations, Across Countries and Over Time

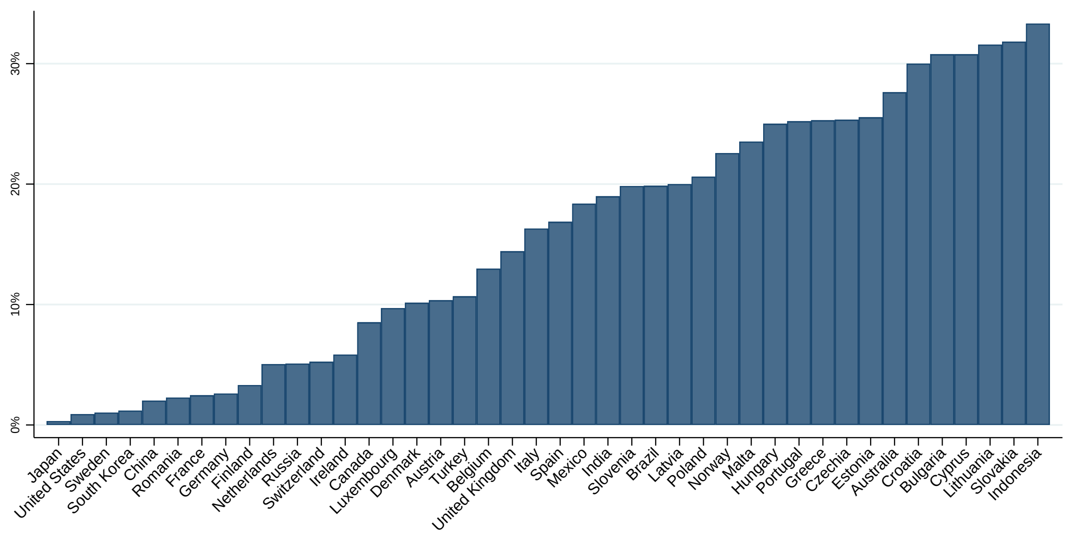


Notes. For each country and year, we perform a bilateral regression of (log-) sectoral patent output on the optimal R&D allocation shares. This figure shows the time-series of the regression coefficients with confidence intervals for ten selected countries in our sample.

Innovation Hubs Even in innovative economies such as the United States, Germany, and modern Japan and South Korea, most R&D activities occur in the private sector. How do these economies decentralize the cross-sector allocation of R&D resources? We argue this could be through multisector innovation hubs: the IBMs, Siemens, Sonys, and Samsungs of the world. These firms operate and hold intellectual property rights across a wide range of technological classes, and their R&D activities build heavily on past knowledge created internally within each firm. The self-reliance on within-firm knowledge spillovers implies that these firms internalize some of the network effects, so that they allocate R&D resources not purely driven by myopic profits but instead in ways closer to the planner's solutions. In Section B.4 of the Online Appendix, we formalize this intuition by extending our closed-economy model to include granular, multi-sector innovation hubs. Here we provide descriptive evidence about the presence of innovation hubs.

To demonstrate that the majority of innovations in the most innovative economies take place in a small collection of firms, Figure 9 shows the minimum fraction of firms in each country that is needed to account for 50% of patent output in the year 2010. In economies such as Japan, the United States, and South Korea, a tiny fraction of firms (0.3%, 0.9%, and 1.2% respectively, or 64, 315, and 290 firms) account for half of the entire country's total innovation output; these firms are the innovation hubs.

Figure 9. Fraction of Innovating Firms that Can Account for 50% of Each Country's Patent Output in the Year 2010

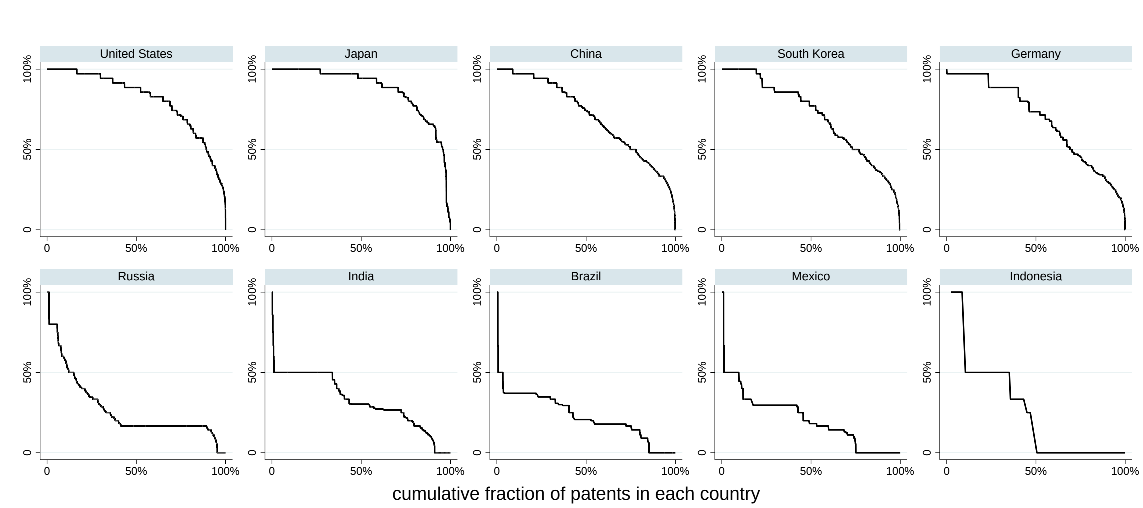


Notes. This figure shows the minimum fraction of innovating firms that can account for 50% of patent output for each country in the year 2010.

Figure 10 demonstrates the self-reliant nature of these innovation hubs' R&D activities. Specifically, each time a patent cites a sector, we ask whether the patent assignee holds other patents in that cited sector. We aggregate this information at the firm level into a measure of what we call *internal knowledge coverage ratio*, that is, the fraction of each firm's citations that are to sectors in which the firm holds intellectual property rights. The measure indicates how much the firm relies on internal knowledge when conducting innovations; a firm with patents in every sector has an internal knowledge coverage ratio of 100%.

In Figure 10, we show the distribution of patents with the corresponding firm's internal knowledge coverage ratio for each country in the year 2010. Specifically, for each country, a point (x, y) on the curve should be read as "at least x% of patents are produced by firms with an internal knowledge coverage ratio of over y%." If a single firm accounts for the entire patent output in the economy—thus all citations are internal—the figure would show a flat line at $y = 100\%$, and if every firm holds only a single patent—thus all citations must be external—the figure would show a flat line at $y = 0\%$. For every real-world economy, the curve declines from $y = 100\%$ on the left ($x = 0$) to $y = 0$ on the right ($x = 100\%$); a faster decline in the y-value implies the economy's innovations rely less on each innovator's internal knowledge. The figure shows a slow decline for the five countries in the top row and a fast decline for those the bottom, indicating a significant degree of knowledge self-reliance at the firm-level for the United States, Japan, China, South Korea, and Germany, highlighting the importance of innovation hubs in these economies.

Figure 10. The Cumulative Distribution of Patents by the Innovator's Internal Knowledge Coverage Ratio



Notes. This figure shows the distribution of patents with the corresponding firm's internal knowledge coverage ratio for ten selected countries in 2010. The knowledge coverage ratio of a firm is defined as the fraction of its patent citations that are to sectors in which the firm holds patents. For each country, a point (x, y) on the curve should be read as "at least $x\%$ of patents are produced by firms with internal knowledge coverage ratio of over $y\%$."

6.3 Growth and Welfare Gains from Adopting Optimal Allocations

Even though our analysis in Section 6.2 suggests that in innovative economies like the United States, sectors that should have more R&D resources under unilaterally optimal allocations do tend to get more resources, this does not mean these countries optimally allocate resources. That is, while Figure 7 shows that sectoral R&D allocations correlate positively with γ_{mt} for the economies in the top row, there is still substantial residual variation in R&D allocations as they disperse around the 45-degree line. Our theory suggests that by allocating R&D exactly along the 45-degree line, these economies would experience positive welfare gains.

We now use our model to evaluate the growth and welfare gains for countries switching from observed to optimal R&D allocations. First, we model the United States as a closed economy. We apply Proposition 4 and its corollary to evaluate the BGP growth differentials between optimal and real-world R&D allocations, and we apply Proposition 6 to evaluate the potential welfare gains, taking into account the transitional dynamics under optimal allocation. Second, we provide an open-economy version of Proposition 6 and evaluate the potential welfare gains across the globe.

For the United States, the dashed line in Figure 11 shows the time series of the potential gain in the BGP economic growth rate when optimal R&D allocations replace the observed ones, applying Corollary 1. R&D misallocation has been stable since the 2000s, accounting for about 0.68 percentage points of missing annual growth on average.

We apply Proposition 6 to derive the potential gain in welfare when replacing the real-world R&D allocations with the optimal ones, taking into account the transitional dynamics. For ease of interpretation, we express welfare gains in growth-equivalent terms.

Definition 4. Consider two consumption paths $\{c_t\}_{t\geq 0}$ and $\{\hat{c}_t\}_{t\geq 0}$. The *growth-equivalent welfare gain* of replacing $\{c_t\}$ with $\{\hat{c}_t\}$ is the level of g such that the consumer is indifferent between the consumption path $\{c_t e^{gt}\}_{t\geq 0}$ and $\{\hat{c}_t\}_{t\geq 0}$.

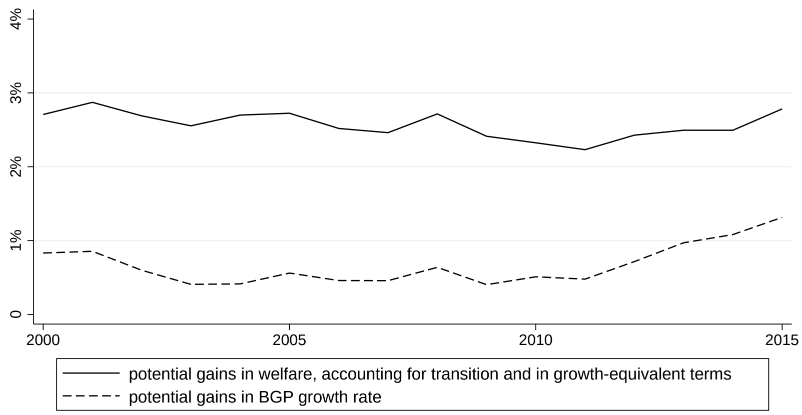
The growth-equivalent welfare gain expresses the consumer value change from switching to a different consumption path in terms of a constant growth factor that, once applied to the original consumption path, makes the consumer indifferent between switching and not. Deriving the growth-equivalent welfare gain of counterfactual R&D allocations is straightforward: Proposition 6 provides the formula for the welfare change under two R&D allocations; to convert to growth-equivalent terms, we simply multiply the welfare change by ρ^2. This is because under log-utility and exponential discounting, the welfare difference between $\{c_t\}$ and $\{c_t e^{gt}\}$ is precisely g/ρ^2:

$$\int_0^\infty e^{-\rho t} \ln\left(c_t e^{gt}\right)\, \mathrm{d}t - \int_0^\infty e^{-\rho t} \ln c_t \, \mathrm{d}t = \int_0^\infty e^{-\rho t} g t \, \mathrm{d}t = g/\rho^2.$$

The solid line in Figure 11 shows the time series of the growth-equivalent welfare gains when replacing the real-world R&D allocations with the unilaterally optimal ones. While the solid line tracks the dashed line very well—periods when optimal R&D allocations lead to greater gains in BGP growth rates also see greater welfare gains—the growth-equivalent welfare gains are always higher than the BGP growth rate gains throughout our sample period. Transitional dynamics explain this. While the long run welfare effects of adopting optimal R&D allocations are captured by the differences in the BGP growth rate, Figure 11 shows there are additional, short run welfare gains, which correspond to the even larger but temporary boost in growth along the transition path, as the cross-sector distribution of knowledge stocks converges to the BGP levels. Overall, R&D reallocation can bring very substantial welfare gains to the United States, equivalent to gaining about 2.5 percentage points in perpetual economic growth.

We now turn to open-economy analysis based on our global sample. Because of our reduced-form formulation of international knowledge spillovers and trade, analysis of how country-level

Figure 11. Growth and Welfare Gains from Adopting Optimal R&D Allocations in the United States



Notes. This figure shows the time series of growth gains (dashed line) and welfare gains (solid line) from adopting optimal R&D allocations in the United States. The growth gains are calculated as the difference in the implied BGP growth rate between the real-world R&D allocations and the optimal ones, and the welfare gains are calculated in growth-equivalent terms (Definition 4), taking into account the transitional dynamics.

R&D allocations affect global BGP is beyond the scope of this paper. Nevertheless, we provide an open-economy version of Proposition 6.

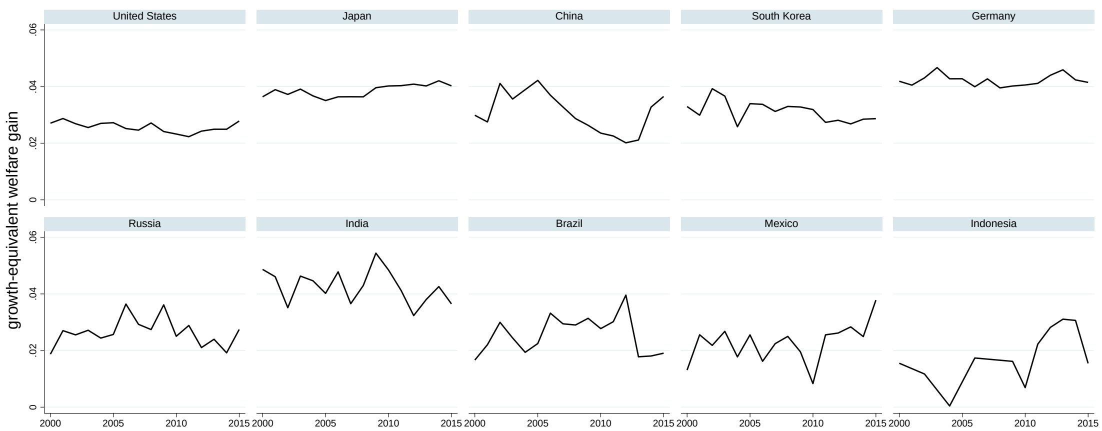
Proposition 8. *Consider an open economy m with arbitrary initial knowledge stock $\boldsymbol{q}_{m0}$ at time 0. Consider two time-invariant R&D allocation plans $\boldsymbol{b}$ and $\widetilde{\boldsymbol{b}}$ while holding the innovation network $\boldsymbol{\Theta}_m$, the sequence of trade conditions $\{\boldsymbol{\theta}_t\}$ and production worker allocation constant. The difference in consumer welfare between the two R&D plans is*

$$
\begin{aligned}
V(\boldsymbol{b}) - V\left(\widetilde{\boldsymbol{b}}\right) &\equiv \int_0^\infty e^{-\rho t}\left(\ln c_t(\boldsymbol{b}) - \ln c_t\left(\widetilde{\boldsymbol{b}}\right)\right) \mathrm{d}t \\
&= \boldsymbol{\phi}_m' \sum_{k=1}^{K} \boldsymbol{u}_k^{\Theta} \boldsymbol{v}_k^{\Theta\prime} \frac{1}{\rho\left(\rho + \lambda\left(1-\psi_k^{\Theta}\right)\right)} \times \lambda\left(\ln \boldsymbol{b} - \ln \widetilde{\boldsymbol{b}}\right),
\end{aligned} \tag{32}
$$

where $\boldsymbol{u}_k^{\Theta}$, $\boldsymbol{v}_k^{\Theta\prime}$, and ψ_k^{Θ} are respectively the k-th right eigenvector, the k-th left eigenvector, and the corresponding eigenvalue of Θ_m.

Proposition 8 provides the formula for welfare changes in small open economies under different R&D allocations. We implement the formula using our global sample. Figure 12 shows the 2000–2015 time series for selected countries' growth-equivalent welfare gains, with the U.S. result on the top left as a comparison. Two features stand out. First, R&D misallocation leaves substantial welfare gains on the table: as much as 6 percentage points in growth-equivalent terms for Mexico, and close to 4 percentage points for some of the other economies. Second, despite the better alignment between actual and optimal R&D allocations for countries in the top row, the misallocation welfare losses are not necessarily smaller in these economies than those in the bottom row. That is because, on average, the economies in the bottom row rely more on foreign knowledge, and foreign reliance limits the scope of welfare gain from domestic R&D policies. In terms of the formula in (32), greater foreign reliance translates into lower eigenvalues ψ_k^{Θ} across the entire spectrum $k = 1, \cdots, K$. Intuitively, an important channel through which domestic R&D policies can improve welfare is domestic knowledge spillovers that facilitate subsequent idea production, and this channel only operates if domestic knowledge is used for subsequent idea production.

Figure 12. Country-Level Welfare Gains from Adopting the Unilaterally Optimal R&D Allocations



Notes. This figure shows welfare gains from adopting unilaterally optimal R&D allocations for ten selected countries between 2000 and 2015. Welfare gains on the y-axis are expressed in growth-equivalent terms (Definition 4).

7 Conclusion

We study the optimal allocation of R&D resources in an endogenous growth model with an innovation network. We provide closed-form solutions for the optimal dynamic path of R&D resource allocations, and we show that planners valuing long-term growth (i.e., with low discount rates) should allocate more R&D toward key sectors that are upstream in the innovation network. We show innovation centrality, i.e., the dominant eigenvector of the innovation network, is a sufficient statistic for evaluating the growth impact of R&D allocations. In an open-economy setting, we derive the unilaterally optimal R&D allocations as a function of each country's dependence on foreign knowledge spillovers, and we demonstrate the incentive for countries to free-ride on fundamental technologies: an economy more reliant on foreign knowledge spillovers has less incentive to direct resources toward fundamental sectors, leading to cross-country differences in unilaterally optimal R&D allocations.

The main advantage of our sufficient statistics for optimal R&D allocations is that we can easily compute them using data on sectoral production and innovation network. By comparing the real-world R&D allocations with the optimal ones, we are able to quantitatively evaluate the importance of R&D misallocations on economic growth and welfare.

To leverage our theory and evaluate R&D allocations in the data, we build the global innovation network based on over 30 million global patents and establish its empirical importance for knowledge spillovers. We use our sufficient statistics to evaluate R&D allocations across countries and time. We find that in economies generally perceived as innovative, such as the United States, Germany, and Japan, cross-sector R&D allocations and patent production correlate with our sufficient statistics, but the relationship does not hold for many other economies. Adopting optimal R&D allocations can generate substantial welfare improvements across the globe; for the United States, R&D misallocation accounts for about 0.68 percentage points of missing annual growth since the 2000s.

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