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CHANGING INCOME RISK ACROSS THE US SKILL DISTRIBUTION:
EVIDENCE FROM A GENERALIZED KALMAN FILTER

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Changing Income Risk across the US Skill Distribution: Evidence from a Generalized Kalman Filter
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ABSTRACT

For whom has earnings risk changed, and why? To answer these questions, we develop a filtering method that estimates parameters of an income process and recovers persistent and temporary earnings for every individual at every point in time. Our estimation flexibly allows for first and second moments of shocks to depend upon observables as well as spells of zero earnings (i.e., unemployment) and easily integrates into theoretical models. We apply our filter to a unique linkage of 23.5m SSA-CPS records. We first demonstrate that our earnings-based filter successfully captures observable shocks in the SSA-CPS data, such as job switching and layoffs. We then show that despite a decline in overall earnings risk since the 1980s, persistent earnings risk has risen for both employed and unemployed workers, while temporary earnings risk declined. Furthermore, the size of persistent earnings losses associated with full year unemployment has increased by 50%. Using geography, education, and occupation information in the SSA-CPS records, we refute hypotheses related to declining employment prospects among routine and low-skill workers as well as spatial theories related to the decline of the Rust-Belt. We show that rising persistent earnings risk is concentrated among high-skill workers and related to technology adoption. Lastly, we find that rising persistent earnings risk while employed (unemployed) leads to welfare losses equivalent to 1.8% (0.7%) of lifetime consumption, and larger persistent earnings losses while unemployed lead to a 3.3% welfare loss.

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For whom has persistent and temporary earnings risk changed over time, and why? Prior work has shown that persistent earnings shocks are often not well insured (e.g., Blundell, Pistaferri, and Preston (2008)), and hence understanding how and why the dispersion in persistent and temporary shocks (i.e., persistent and temporary *risk*) has evolved over time is critical for individual welfare and policy design. To answer these questions, we develop a filtering method that estimates parameters of an income process and recovers persistent and temporary earnings for every individual at every point in time. Using our method on a linked sample of earnings records from the Social Security Administration (SSA) and the Current Population Survey's Annual Social and Economic Supplement (CPS), we find that since the 1980s, persistent earnings risk has increased while temporary earnings risk has declined. Exploiting the demographic information from our CPS sample, we find that the increase in persistent earnings risk is concentrated among high-skill workers and related to technology adoption. Finally, we find that there have been large welfare declines due to the increase in persistent earnings risk.

This paper makes three contributions. First, we show how the Kalman filter and an Expectation-Maximization (EM) algorithm can be used to estimate the parameters of a flexible, but easily interpretable model of income dynamics. As is consistent with much of the income process literature, we write down a low-dimensional representation of individual earnings as the sum of latent persistent and temporary components.[1] Using the EM algorithm, we derive updating equations for the parameters of the income process, which resemble generalized least squares regressions. These closed form updating equations allow the model to easily handle income process parameters that depend on a potentially large number of observables (e.g., age, employment status, education, occupation, and firm characteristics). We show that the parameters from our estimated income process are simple to integrate into quantitative models. Finally, using the Kalman Filter, we are able to recover estimates of persistent and temporary earnings for each individual in every period. By directly studying these filtered estimates, we can conduct further explorations of drivers of income dynamics.

An additional benefit of our Kalman filter and EM algorithm approach is that it naturally allows for the inclusion of individuals with very low or zero earnings. Motivated by economic theories of human capital depreciation during unemployment (e.g., Ljungqvist and Sargent, 1998, among others), we posit a law of motion for persistent earnings when individuals have zero earnings (with a slight abuse of convention, we use 'zero earnings' and 'unemployment' interchangeably). During unemployment spells, individuals receive shocks to persistent earn-

[1]While we work with the canonical example in which persistent earnings follow a persistent AR(1) process throughout, our approach can naturally extend to incorporate additional linear dynamics—including individual fixed effects in earnings levels and growth rates, as well as moving average components—while remaining tractable.

ings; these shocks have a different mean and variance than those received during periods of employment. Despite individuals' lack of earnings information during unemployment spells, the law of motion for persistent earnings is identified via earnings upon re-entry to work.

We estimate our filter on a linked sample of SSA-CPS earnings records from 1982 to 2016. The estimated parameters reveal that earnings are very persistent (persistent earnings has an annual AR(1) parameter of 0.94) and that the unemployed (i.e., those with very low or zero earnings) face substantial earnings risk.[2] We estimate that the standard deviation of shocks to persistent earnings to the unemployed is nearly double that for the employed and that the unemployed face persistent earnings losses of nearly 15% per year of unemployment (compared with a 0.4% gain for the employed). Finally, our estimation also yields a panel of persistent and temporary earnings shocks.

We then compare observed CPS events with our recovered individual-level shocks in order to validate our econometric specification of earnings. Specifically, we plot the distribution of estimated shocks to temporary and persistent earnings in response to job loss and job switching in the SSA-CPS data. We show that compared with staying in a job, job switching and unemployment are associated with greater dispersion in temporary and persistent shocks, as is consistent with job ladder models of the labor market. In particular, layoffs associated with recalls (return to same employer) display much less persistent downside risk than non-recall (switch employers) layoffs. Likewise, job switchers are associated with much more dispersed persistent earnings shocks than job stayers, and we find that characteristics of the destination firms help to explain this variation. Compared with workers who move down the ladder, workers who move up (switch to higher-paying firms) face considerably less persistent downside risk.

Our second contribution is to examine how earnings risk has varied over time and then use our linked survey data to shed light on a potential mechanism. We extend our filter to allow for age- and time-dependent variances of persistent and temporary earnings shocks. We additionally allow for the mean of persistent earnings shocks to vary over time. We document an upward trend in persistent earnings risk since the 1980s. Among the employed, the standard deviation of persistent earnings shocks rose by nearly 10%. Conversely, there has been an offsetting downward trend in temporary earnings risk over the same time period. Through the combination of these two factors, overall earnings risk among the employed has a moderate downward trend (statistically insignificant), indicating that examining only overall earnings risk can mask heterogeneous trends in the underlying temporary and persistent components.

One unique feature of the way we filter the data is our ability to measure persistent earnings

[2] In our baseline estimation, we define an individual to be unemployed if his or her annual earnings are below the equivalent of working full-time at the real federal minimum wage for 2 quarters of the year (approximately $8*k* in 2019 dollars).

trends among the unemployed. We find that persistent earnings losses have been accelerating for unemployed individuals. A year of unemployment translates to a -11% decline in persistent earnings in 1985, but by 2013, this rate of loss accelerates by over 50%, reaching -17% per annum. On top of this more rapid negative drift in persistent earnings, the magnitude of persistent earnings risk has increased among the unemployed by 15%. We then show these time trend results are robust to a number of different specifications and samples. Notably, we find similar results in a random sample of individuals from the Longitudinal Employer-Household Dynamics database.

By linking our administrative earnings database to survey responses in the CPS, we test a number of explanations of rising persistent earnings risk. Motivated by the job polarization literature, we consider three potential hypotheses that relate the rise in persistent earnings risk to (H1) declining employment prospects of low-skill workers, (H2) the decline of the rust belt (e.g. declining union protection, manufacturing employment, etc.), and (H3) reduced employment and wages in routine occupations. Our results provide strong evidence against H1, H2, and H3.[3] We begin by documenting that the rise in persistent earnings risk for both the employed and the unemployed is particularly pronounced among *college educated* workers. Likewise, the acceleration of earnings losses among the unemployed is also pronounced among college educated workers. These facts provide strong evidence against theories related to the declining employment prospects of low-skill workers (e.g., H1). We then show that the rise in persistent earnings risk since the 1980s is pervasive and fairly uniform across the vast majority of U.S. states. In particular, the well-documented deterioration of labor market conditions in the Rust Belt is not driving the trends we document, allowing us to rule out hypotheses related to declining union protectionism, and the decline of manufacturing (e.g., H2). Lastly, we use the CPS occupation information to show that the rise in persistent earnings risk is uncorrelated with an occupation's routine task content. This finding suggests declining that labor demand for workers in routine occupations is not driving the trends in persistent earnings risk.

We argue instead that the increase in persistent earnings risk is a phenomenon affecting high skill workers (H4). To test H4, we consider three measures of the degree to which an occupation is high skill: (1) the degree of non-routine cognitive task content as measured in Acemoglu and Autor (2011), (2) average years of completed education, and (3) average (log) earnings. All three measures show that, since the 1980s, workers employed in high skill occupations have faced a larger increase in persistent earnings risk both while employed and while unemployed, as well as larger declines in persistent earnings during spells of unemployment.

[3] To clarify the interpretation of this result, note that while a substantial literature has documented trends related to H1, H2, and H3 that explain nontrivial differences in average earnings *between* groups, we do not find that the increase in earnings risk is particularly pronounced *within* these groups.

One potential mechanism for why high skill workers are facing greater persistent risk is that they face greater exposure to the introduction of new, skill-biased technologies (e.g., Krueger (1993) and Deming and Noray (2020)). While existing work (e.g., Krusell, Ohanian, Ríos-Rull, and Violante (2000)) suggests that these new technologies are complementary to skilled labor, they also create a risk of skill displacement. For instance, workers may be unable to easily acquire the skills required to adapt to the new technology, or they may find that previously valuable/scarce expertise is no longer required to produce with the new technology. Hence, new vintages of technology can create winners and losers, where the biggest gains and losses occur among skilled workers.[4] Since it is costly to acquire new skills and changes in the technological frontier are permanent, such a phenomenon naturally generates substantial and persistent variation in earnings across workers. To empirically test this mechanism, we link Burning Glass vacancy data to our SSA-CPS data in order to measure which occupations introduced intensive computer and software use in the workplace as a proxy for the introduction of new skill-biased technologies.[5] We find that occupations with high computer use in 2010 were the occupations that saw the largest increases in persistent earnings risk.

Our third contribution in quantitative. We examine the welfare and macroeconomic effects of changing earnings risk over time. We show that our income process can be easily discretized and incorporated into a Bewley-Huggett-Aiyagari model with incomplete markets. In the model, agents differ by a permanent type that corresponds to their level of education (e.g., no college degree, college graduate, etc.) and receive shocks to labor income based upon our estimated income process. We use the model to measure the welfare losses from each component of the income process: (i) persistent and temporary earnings risk while employed, (ii) persistent earnings risk while unemployed, and (iii) downward drift in earnings while unemployed. We find that the increase in persistent earnings risk while employed (unemployed) generates a welfare losses equivalent to 1.8% (0.7%) of lifetime consumption. The acceleration of earnings losses while unemployed causes significant welfare losses equivalent to 3.3% of lifetime consumption. The welfare losses are largest among the most highly educated, as these individuals have seen the largest increase in persistent earnings risk and the greatest acceleration of persistent earnings losses while unemployed.

[4]Quantitative papers with this mechanism include Chari and Hopenhayn (1991), and Violante (2002). For direct empirical evidence and related theory for how high skilled workers see larger increases in risk following innovations, see Kogan, Papanikolaou, Schmidt, and Seegmiller (2021) and Kogan, Papanikolaou, Schmidt, and Song (2020). See also Goldin and Katz (2010), Akerman, Gaarder, and Mogstad (2015), Atack, Margo, and Rhode (2019), and Feigenbaum and Gross (2020).

[5]Burning Glass collects detailed information on the skills listed in vacancies.We follow recent work by Hershbein and Kahn (2018) and Atalay, Phongthiengtham, Sotelo, and Tannenbaum (2020), which argues that the skill requirements in vacancy postings are informative on the technology of the firm posting the vacancy.

Literature. This paper contributes to recent work that has examined how earnings risk has changed over time. While Sabelhaus and Song (2010), and Bloom, Guvenen, Pistaferri, Sabelhaus, Salgado, and Song (2017) find that earnings volatility has declined over time, Moffitt (2020) and the papers summarized therein find relatively flat long term trends in male earnings volatility over the past 30 years.[6] We in part reconcile these different findings by showing that the value of the minimum earnings criterion (the minimum value of earnings for a person to be included in the sample) plays a large role in shaping the evolution of earnings risk over time. In our baseline estimation, we find a minimal to slightly declining trend in earnings volatility, as is consistent with the work summarized by Moffitt (2020). We then show that if we lower the value of the minimum earnings criterion to the one set in Bloom et al. (2017), we find a significant decline in earnings risk over time. Further, we show that lowering the minimum earning threshold primarily impacts the trend in temporary earnings over time, with lower thresholds implying larger declines in temporary earnings risk and thus lower overall earnings risk. Lowering the minimum earnings threshold leaves the trend in persistent earnings risk largely unchanged, increasing by approximately 10% over the sample period as in our baseline estimation.

Relative to these papers, ours make several contributions. First, we decompose earnings risk over time into its temporary and persistent components.[7] This decomposition reveals that trends in overall earnings risk can mask heterogeneous trends in persistent and temporary earnings. This is because temporary earnings risk has declined since the 1980s, while persistent risk has increased. Second, we allow for arbitrary spells of unemployment (e.g., low/zero earnings) in the analysis, and allow the mean and variance of persistent shocks to differ during unemployment spells.[8] We then show that earnings losses of the unemployed have accelerated since the 1980s and that persistent risk to this group has increased. Finally, we measure the welfare effects of changes in earnings risk and show that despite the flat trend (or declining trend, depending on the minimum cutoff), there are large welfare implications due to the increase in persistent risk.

This paper also contributes to the literature that has attempted to estimate persistent earn-

[6]The papers summarized by Moffitt (2020) are part of a coordinated effort to use common sampling structures and definitions across datasets to document changes in male earnings volatility over time. The papers in this series include: Carr, Moffitt, and Wiemers (2020), McKinney and Abowd (2020), Moffitt and Zhang (2020), and Ziliak, Hokayem, and Bollinger (2020).

[7]See Moffitt and Zhang (2018) for a summary of prior work estimating trends in persistent and temporary earnings risk over time.

[8]Prior work has incorporated spells of zero earnings using arc-percent changes. However, arc changes require positive earnings in at least one year. Our method allows for more general and arbitrary spells of zero earnings. Relatedly, Daly et al. (2016) argue that adjusting for partial years of employment at the start and end of earnings histories reconciles earnings estimation in levels and differences.

ings at the individual level. Although they do not explicitly estimate the income process, there are influential studies that assume that the persistent component of income is well approximated by a moving average of income and other simple moments. This allows the authors to recover, individual-by-individual, a persistent and temporary component of income. An early example is Gottschalk and Moffitt (1994), who measure the persistent component of income using 7-year moving averages of earnings while labeling the residual as temporary earnings. Commault (2021) uses income expectations from the Survey of Consumer Expectations along with a structural model to identify individual-specific persistent earnings shocks. Guvenen, Ozkan, and Song (2014), and Guvenen, Karahan, Ozkan, and Song (2021) use 1-year and 5-year changes in earnings to approximate temporary and persistent earnings risk, respectively.[9] We complement their work by decomposing changes in earnings risk into temporary and persistent components using the additional structure of our model.

Our paper also contributes to the literature that estimates statistical representations of individual earnings.[10] There are three ways prior studies have recovered income processes: (1) fitting statistical models of the earnings processes using GMM (e.g., Storesletten et al. (2004), Karahan and Ozkan (2013), Guvenen et al. (2014), and Guvenen et al. (2015)), (2) fitting structural models of earnings processes using simulated method of moments (e.g., Blundell et al. (2008), Guvenen and Smith (2014), and Madera (2016)), and (3) using Bayesian methods to estimate individual specific earnings processes (Geweke and Keane (2000), Jensen and Shore (2011), Nakata and Tonetti (2015), Gu and Koenker (2017), Borella, De Nardi, and Yang (2019) and Chatterjee, Morley, and Singh (2021)).[11] The first two methods do not allow researchers to recover realized shocks person-by-person, while the Bayesian methods, which encompass the filter presented in this paper, do.[12] The paper most closely related to ours is Chatterjee et al. (2021), who use Bayesian methods to estimate pass-through from income shocks to consumption in the PSID. We build on their work by positing a law of motion for persistent earnings among the unemployed, estimating our earnings process using a combination of the Kalman filter and the EM algorithm, and finally, we use our filtered estimates to understand the drivers

[9] Another approach to identify individual specific shocks is to use event studies, e.g. tax rebates or mass layoff episodes. Recent work by Baker and Yannelis (2015) and Gelman, Kariv, Shapiro, Silverman, and Tadelis (2015) identify temporary shocks from the 2013 government shutdown. Others have studied tax rebates (e.g. Kaplan and Violante (2014)) and mass layoffs (e.g. Saporta-Eksten (2013)) to isolate specific persistent and temporary income shocks. See Commault (2017) for a recent summary.

[10] See Meghir and Pistaferri (2011) for a detailed discussion of the literature.

[11] See also Browning et al. (2010) who estimate rich parametric models of individual income process heterogeneity using simulated minimum distance methods. Meghir and Pistaferri (2004) estimate models of persistent and transitory variances via GMM which also allow for ARCH effects.

[12] For detailed discussion of prior work, we refer readers to Gu and Koenker (2017). Additionally, see Nakata and Tonetti (2015) for a review of the literature on income process estimation and a discussion of the small sample properties of Bayesian estimation techniques for earnings processes.

of persistent and temporary earnings risk in both the cross-section and over time.

Our paper also contributes to recent work that uses EM algorithms to estimate income processes. Arellano and Bonhomme (2016) and Arellano, Blundell, and Bonhomme (2017) also use an EM approach to estimate models of earnings dynamics using fully nonparametric methods.[13] Since we focus on a model of first and second moments only, our approach is more restrictive than theirs in terms of providing information about higher moments. However, this additional restriction allows us to use closed form expressions, rather than numerical approximations, to compute posterior distributions of latent state variables. It confers two substantial advantages. First, our approach can be quickly implemented, even on very large datasets, and does not require specification of additional hyperparameters (e.g., on Census servers, estimating the parameters and recovering the full panel of persistent and temporary earnings for 23.5m observations takes roughly 3 hours). Second, since our updating formulas resemble GLS regressions, we can quite easily incorporate information from many sources of observed data (i.e., allow for fairly high dimensional parameter vectors) without suffering from the curse of dimensionality. This allows us to incorporate non-linearities that depend on observable events, such as periods of zero earnings (unemployment) or job switching. In other words, our approach can generate non-linearities and still yield very attractive scaling properties by imposing additional structure.

1 Empirical framework

In this section, we describe our econometric framework for modeling income. We then discuss how our framework can be estimated to recover estimates of persistent and temporary earnings at the individual level as well as the parameters that govern the income process.

1.1 General setup

We begin with a panel dataset of income, $Y_{i,t}$, where $i \in \{1, \ldots, N\}$ indexes individuals and $t \in \{1, \ldots, T\}$ indexes years.[14] We are interested in understanding the evolution of earnings net of predictable lifecycle components. Let $\widehat{y}_{i,t}$ characterize how observable components (e.g. age and birth cohort) influence log earnings. Then define residual log earnings, denoted $y_{i,t}$, as $y_{i,t} = \log(Y_{i,t}) - \widehat{y}_{i,t}$. In the remainder of the paper, we focus on the factors that influence changes in residual log earnings $y_{i,t}$, which we hereafter refer to as income.

[13] See also Bonhomme and Robin (2010) and Bonhomme and Weidner (2021) for related approaches.

[14] For ease of notation, we assume here that the panel is balanced, but the extension to an unbalanced panel setting is immediate.

The income process we define below depends upon whether an individual is employed in a given year. Let $l_{i,t} = [l_{E,i,t} \;\; l_{U,i,t}]'$ be a vector that identifies an individual's labor market status. Element $l_{E,i,t}$ is an indicator variable that equals one when individual i is employed in year t and zero otherwise. Likewise, $l_{U,i,t}$ equals one when individual i is unemployed, and zero otherwise. We define an individual to be employed when they have labor income above a minimum earnings criterion $\bar{y}$ (i.e., $Y_{i,t} > \bar{y}$) and unemployed otherwise.

For employed individuals, we model the process for income $y_{i,t}$ as the sum of persistent and temporary earnings. The persistent and temporary components of income $y_{i,t}$ are not observed. Let $z_{i,t}$ denote the unobserved persistent component of income, and let $\omega_{i,t}$ denote the temporary shock. When an individual is unemployed we set their income ($y_{i,t}$) to missing. Temporary shocks to an individual's earnings ($\omega_{i,t}$) are drawn from a normal distribution with mean zero and variance $R(l_{i,t}; X_{i,t})$, where the variance depends upon the individual's labor market status as well as other observable variables $X_{i,t}$. We assume that observed earnings satisfy[15]

$$
\begin{aligned}
y_{i,t} &= \begin{cases} z_{i,t} + \omega_{i,t} & \text{if } l_{E,i,t} = 1 \\ \cdot & \text{if } l_{E,i,t} = 0 \end{cases} \qquad (1) \\
\mathbb{V}(\omega_{i,t} \mid l_{i,t}, X_{it}) &= R(l_{i,t}; X_{i,t}),
\end{aligned}
$$

where $\omega_{i,t}$ is independent of $z_{i,t}$ conditional on $l_{i,t}$ and $X_{i,t}$

We next discuss the law of motion for persistent earnings. We model the process for persistent earnings $z_{i,t}$ as an autoregressive process subject to normally distributed innovations. Let $F(X_{i,t+1})$ denote the persistence of $z_{i,t}$. Let $B(l_{i,t}; X_{i,t})$, denote the drift of an individual's persistent income in period t. Observe that the drift of persistent earnings varies by employment status $l_{i,t}$. Hence the mean of the shock to persistent earnings varies by an individual's labor market status. Finally, let $\nu_{i,t}$ be an i.i.d shock to persistent income. The draw of $\nu_{i,t}$ is from a normal distribution with mean zero and variance $Q(l_{i,t}; X_{i,t})$, where the variance depends upon the individual's labor market status and other observables. An individual's persistent income evolves according to the following equation:

$$
\begin{aligned}
z_{i,t+1} &= F(X_{i,t+1})\, z_{i,t} + B(l_{i,t+1}; X_{i,t+1}) + \nu_{i,t+1} \qquad (2) \\
\mathbb{V}(\nu_{i,t+1} \mid l_{i,t+1}, X_{i,t+1}) &= Q(l_{i,t+1}; X_{i,t+1}).
\end{aligned}
$$

Observe that in this income process, persistent income continues to evolve during spells of

[15] Since $y_{i,t}$ is set to missing when earnings are below the cutoff, temporary shocks to the unemployed ($R(U; X_{i,t})$) are arbitrary and do not inform any parameter estimates. We then normalize $R(U; X_{i,t}) = 1$.

unemployment. Additionally, the mean (drift) and variance of the shocks to persistent earnings differ by an individual's employment status. This process for persistent earnings resembles a Ljungqvist and Sargent (1998) style human capital process, in which human capital is subject to different shocks during spells of employment and unemployment. Since income is set to missing during unemployment, contemporaneous observations of income during unemployment contain no information about $z_{i,t}$. However, the mean and standard deviation of income at re-employment will inform the parameters governing the law of motion for persistent earnings $z_{i,t}$ during unemployment.

We assume that the dynamics of the system evolve as follows. The individual first draws an observation of latent initial persistent income ($z_{i,0}$) from a normal distribution with mean zero and variance $u_{z0}(X_{i,0})$. Next, conditional on the values of $z_{i,0}$ and $X_{i,0}$, nature draws a realization of $l_{E,i,1}$ and $X_{i,1}$ with probabilities which potentially depend on $z_{i,0}$ and $X_{i,0}$. Moving forward, we draw independent permanent and transitory innovations ($\nu_{i,t}$ and $\omega_{i,t}$) from distributions that depend on $l_{E,i,t}$ and $X_{i,t}$. We then draw $l_{E,i,t}$ and $X_{i,t}$ with probabilities that potentially depend on $z_{i,0}$, $X_{i,0}$ and $\{l_{E,i,\tau}, z_{i,\tau}, X_{i,\tau}\}_{\tau=1}^{t-1}$. This timing assumption yields *sequential exogeneity*. We formally discuss the implications of this timing assumption in Section 1.3.

1.2 Simple Setup: Canonical income process with unemployment

The majority of papers in the income process estimation literature often assume that latent persistent income $z_{i,t}$ evolves as an AR(1) process with iid, mean zero innovations. Consider a minimal deviation from the existing literature that maintains the AR(1) structure while allowing for unemployment. The observation equation (1) remains unchanged, and we simplify equation (2) to have scalar persistence (i.e. $F(X_{i,t}) = F$) and drift that only depends on employment (i.e. $B(l_{i,t+1}; X_{i,t+1}) = B(l_{i,t+1})$):

$$z_{i,t+1} = Fz_{i,t} + B(l_{i,t+1}) + \nu_{i,t+1}, \tag{3}$$

where $B(l_{i,t+1}) = B_E$ for the employed ($l_{E,i,t} = 1$) and $B(l_{i,t+1}) = B_U$ for the unemployed ($l_{U,i,t} = 1$), and $\mathbb{V}(\nu_{i,t+1}) = Q(l_{i,t+1})$, where Q_E denotes the variance of persistent shocks to the employed and Q_U denotes the variance of persistent shocks to the unemployed. Let us further assume that the variance of temporary earnings shocks is constant when the agent is employed, i.e., $\mathbb{V}(\omega_{i,t}|l_{E,i,t} = 1) = R_E$. Since we only observe earnings during periods in which the worker is employed, so we can normalize $\mathbb{V}(\omega_{i,t}|l_{E,i,t} = 0) = 1$ without loss of generality. We maintain the prior timing assumptions regarding realization of shocks and employment status.

We argue more formally in Appendix H that such a law of motion for earnings follows from a directed search model with on-the-job human capital accumulation and skill depreciation during unemployment. In the model, $z_{i,t}$ is proportional to persistent human capital, and ω_{it} is proportional to transitory human capital. Total earnings is proportional to both components of human capital and only observed during periods of employment. Moreover, the law of motion for $z_{i,t}$ has both a drift and volatility which depend on an individual's employment status. Such a model naturally predicts that expected persistent earnings growth rates are higher for employed relative to unemployed workers ($B_E > B_U$). If, in addition, there is more uncertainty about skill loss during unemployment than there is about skill acquisition while employed, one would naturally expect $Q_E < Q_U$. Both properties are consistent with our estimates below.

1.3 Discussion of model assumptions

Before discussing how we estimate the parameters and recover latent shocks for the income process described above, we pause to discuss the key assumptions embedded in our specification outlined in the previous section.

1.3.1 The role of observables and timing of shocks

We assume that first uncertainty about employment status is resolved, followed by uncertainty about persistent and temporary earnings.

Formally, in writing the conditional likelihood function which we maximize in our estimation, we assume that contemporaneous innovations to persistent and temporary earnings ($\omega_{i,t}$ and $\nu_{i,t}$) have mean zero conditional on employment status $l_{i,t}$, other observable variables in $X_{i,t}$, and $z_{i,0}$, $X_{i,0}$, and $\{l_{i,\tau}, X_{i,\tau}, z_{i,\tau}\}_{\tau=1}^{t-1}$. In other words, we assume that there is a compound lottery in which we first draw the conditioning variables, then draw income shocks conditional on these observed variables $l_{i,t}$ and $X_{i,t}$ in a second stage. As above, we do not need to restrict the dependence between $(l_{i,t}, X_{i,t})$ and $z_{i,0}$, $X_{i,0}$, and $\{l_{i,\tau}, X_{i,\tau}, z_{i,\tau}\}_{\tau=1}^{t-1}$; e.g., there can be a correlation between $l_{i,t}$ and $\omega_{i,\tau}$ and $\nu_{i,\tau}$ for $\tau = 1, \ldots, t-1$.

This *sequential exogeneity* assumption rules out certain classes of Tobit models with selection on *unobservables*.[16] However, the filter easily handles rich selection on *observables*. The shocks $\omega_{i,t}$ and $\nu_{i,t}$ can be modeled as flexible functions of observables that are correlated with employment ($l_{i,t}$), including occupation, firm characteristics, prior income $y_{i,t-1}$, age and other demographics.

The main benefit from this timing assumption is that it lets us condition on the observables $l_{i,t}$ and $X_{i,t}$ when writing down the full information log likelihood given earnings and $z_{i,0}$, $X_{i,0}$, and $\{l_{i,\tau}, X_{i,\tau}, z_{i,\tau}\}_{\tau=1}^{T}$, yielding closed form expressions for posterior means and variances of

[16] Incorporating selection on unobservables would break the tractability and linearity of the filtering problem.

latent persistent earnings given the data.[17] These analytical expressions are critical for tractably updating the parameters of the income process. Thus, the timing assumption lets us tractably estimate rich stochastic processes for income dynamics *conditional on the path of observables*.

Note that in many scenarios one might also want to model the uncertainty associated with the conditioning variables in order to obtain a more complete characterization of total income risk. For example, when we embed our income process into a Bewley-Huggett-Aiyagari model in Section 5, we must specify a law of motion for employment status. In practice, we find that an individual's prior persistent earnings ($z_{i,t-1}$) and employment status $(l_{i,t-1})$ are strong predictors of an individual's current employment status ($l_{i,t}$).[18] We estimate a flexible process for employment status based on these variables as well as other observables ($X_{i,t}$) which are easily nested into the quantitative model. Further, as we will demonstrate below, the fact that we recover estimates of the posterior income changes experienced by each person and in each time period in our sample, it is easy to integrate out the empirical distribution of $l_{i,t}$ and/or $X_{i,t}$ and thereby obtain measures of income risk which take this "first stage lottery" into account.

Simple examples: conditioning on job switching/firm characteristics To give a sense for some of the potential applications which can be explored via a framework like ours, note that, in most administrative earnings datasets, it is often possible to match workers to firms and also to identify whether or not the identity of a worker's main employer has changed. Moreover, there is substantial evidence that first and especially second moments of income growth rates differ for job switchers versus job stayers. Since these switching indicators are always available in these datasets, it is easy to allow income process parameters to depend on whether or not workers switch employers, e.g., by having an additional dummy capturing whether an employed worker has recently switched jobs. We could also allow for B and Q to differ for the first versus subsequent years in a nonemployment spell. Likewise, a number of variables can often be computed at the employer or industry level (such as size, average pay per worker, employment/payroll growth rates) which could allow for a richer characterization of income dynamics than is typically done with extant permanent-transitory models. For example, we estimate an income process with parameters that depend on job switching status in section 3.3.

[17]A closely related statement is that, given our linearity and sequential exogeneity assumptions, the vector of means and the variance-covariance matrix of $y_{i1}, \ldots, y_{iT}$ given the observed $l_{i1}, \ldots, l_{i,T}, X_{i,0}, \ldots, X_{iT}$ is exactly the same, regardless of the law of motion for $l_{i,t}, X_{i,t} \, | z_{i0}, X_{i0}, \{z_{i\tau}, X_{i\tau}, l_{i,\tau}\}_{\tau=1}^{t-1}$.

[18]We find that individuals with lower prior persistent earnings and individuals unemployed in the prior period are more likely to be unemployed in the current period.

1.3.2 Tractable functional forms for B, Q, R, and F

The specification in Section 1.1 allows for rich models of earnings dynamics that are easily estimated using the Kalman filter and EM algorithm. For example, in Section 4 we are able to tractably estimate measures of persistent and temporary risk over time by geography, occupation, and education. Our flexible time-varying parameter model can incorporate a considerable amount of heterogeneity by allowing the specific parameters of the income process (e.g., B, R, Q, F, z_{i0}) to depend on linear combinations of observable variables (e.g. time, age, education, occupation, etc.). More concretely, we assume that these parameters satisfy a linear-in-parameters structure–that is,

$$B(l_{i,t}; X_{i,t}) = g_B(l_{i,t}; X_{i,t})'\Lambda_B \qquad \text{and} \qquad F(X_{i,t}) = g_F(X_{i,t})'\Lambda_F,$$

where $g_B(\cdot)$ and $g_F(\cdot)$ are *known* functions mapping the observed data to a set of basis functions and Λ_B and Λ_F are an unknown set of parameters. Note that $g_B(l_{i,t}; X_{i,t}) = l_{i,t}$ and $g_F(l_{i,t}; X_{i,t}) = 1$ in our simple example in section 1.2. We impose an analogous linear-in-parameters structure for $F(\cdot)$. Via such a specification, we have the ability to characterize rich interactions between observed variables and the dynamics of latent persistent income. We make an analogous assumption for the *logarithm* of the relevant variances; i.e., $Q(l_{i,t}; X_{i,t}) = \exp\left[g_Q(l_{i,t}; X_{i,t})'\Lambda_Q\right]$, with similar expressions for $R(\cdot)$ and $u_{z0}(\cdot)$.[19] Under these assumptions, we obtain closed-form expressions to update parameters within the EM algorithm (described next), which makes our analysis highly tractable even with very large numbers of parameters.

1.4 Income process estimation with the Kalman filter and EM algorithm

In this section, we discuss how we use the Kalman filter and an EM algorithm to estimate the income process outlined in equations (1) and (2). We separately discuss the Kalman filtering and EM algorithm steps, then conclude with an overview of the estimation procedure. The estimation returns the parameters of the income process as well as a panel of estimates of temporary and persistent earnings at the individual level.

[19] Assuming that the log variance rather than variance itself is affine in observables has the advantage of guaranteeing that variances are positive (and guarantees a well-defined log-likelihood). Further, Schmidt and Zhu (2021) show that such a multiplicative separability assumption for variances is implied by a location-scale version of the generalization of differences in differences proposed by Athey and Imbens (2006). See Appendix C for details.

1.4.1 Kalman Filter

We first discuss the use of the Kalman filter in the estimation of the income process to recover estimates of persistent and temporary earnings at the individual-observation level. To do this, we recast the income process from Section 1.1 into state-space form. We treat persistent earnings $z_{i,t}$ as the state variable, and we treat the law of motion for persistent earnings in equation (2) as the *state equation*. Note that conditional on the observable sequence of $\{l_{i,t}, X_{i,t}\}_{t=0}^{T}$, the state equation is linear, albeit with parameters that vary with observables. Since we observe income $y_{i,t}$ and employment status $l_{i,t}$, we can treat the income process specified in equation (1) as the *measurement equation*, which is also linear but with parameters that potentially vary with observables. When an agent is unemployed ($l_{E,i,t} = 0$), the value of the observation $y_{i,t}$ provides no additional signal about latent earnings other than what can be inferred from other observables, so the Kalman filter will not directly use $y_{i,t}$ to update its guess about $z_{i,t}$.

With the income process modeled in state space form (equations (2) and (1)), we can use the Kalman Smoother to recover an estimate of persistent earnings each period $\{\{\hat{z}_{i,t}\}_{t=1}^{T}\}_{i=1}^{N}$, given the observed data.[20] We refer to our estimated mean persistent earnings ($\hat{z}_{i,t}$) simply as *persistent earnings*, and we define the differenced persistent earnings ($\hat{z}_{i,t} - F(X_{i,t})\hat{z}_{i,t-1}$) as the *persistent earnings shock*. Our estimated *temporary earnings shock* is given by $\hat{\omega}_{i,t} = y_{i,t} - \hat{z}_{i,t}$. When we recover subpopulation earnings volatility from our estimates of $\hat{z}_{i,t}$, we adjust for uncertainty of the mean persistent shock $\hat{z}_{i,t}$ using the law of total variance.[21] In Appendix B we present the further details of our Kalman Filtering algorithm.

The Kalman filtering step in our estimation provides us with an estimate of temporary and persistent earnings for *every* individual in *every* period. These individual level estimates of persistent and temporary earnings depend upon the parameters of the income process in equations (1) and (2). In the next section, we discuss how we use the EM algorithm to estimate the parameters of the income process.

1.4.2 EM Algorithm

In this section, we discuss how we use the EM algorithm (Dempster, Laird, and Rubin (1977)) to estimate the parameters of the income process.

The EM algorithm begins with the full-information log likelihood, which is the likelihood

[20] Hamilton (1994b) shows that the Kalman filter recovers an estimate of the unobserved state variable – in out context, persistent earnings $\hat{z}_{i,t}$ – with the minimum mean squared error even in cases where the shocks are non-normal.

[21] In practice, we do this by adding normal noise to the estimates from the Kalman smoother, where the amount of noise added is governed by the amount of "filtering uncertainty" as estimated by the Kalman filter. See Appendix B.1 for details.

function if the state variable (i.e., persistent earnings, $z_{i,t}$) is observed. By taking expectations, which is the equivalent of integrating out the unobserved state variable, we obtain a likelihood that is a function of posterior means and variances of the unobserved state variable as well as data ($y_{i,t}$, $X_{i,t}$). From the Kalman filtering step, we have an estimate of the unobserved state variable, i.e. persistent earnings $\hat{z}_{i,t}$. Taking first-order conditions of the expected likelihood, we arrive at a series of expressions to update the parameters of the income process. In Appendix C we go through the derivation of these expressions and present the expressions that are used to update the parameters of the income process.

Under the functional forms proposed in section 1.3.2, the formulas for updating the parameters resemble generalized least squares regressions formulas. For example, the persistence parameter (F) is updated by regressing lagged persistent earnings $(\hat{z}_{i,t-1})$ onto current persistent earnings $(\hat{z}_{i,t})$, and it is then adjusted to take into account the covariance of persistent earnings with its lag, as well as the variance of lagged persistent earnings. In practice, these closed form expressions for updating the parameters of the income process make it very tractable to update the parameters and allow us to scale up the income process to consider risk in very fine partitions of the data (e.g., detailed occupation codes).

Our overall estimation procedure proceeds as follows. The first step of the estimation is to guess the parameters of the income process. Given parameters, the second step is to estimate the posterior distribution of the unobserved state variable (i.e. persistent earnings, $\hat{z}_{i,t}$) using the Kalman filter. The third step uses the estimates of persistent earnings ($\hat{z}_{i,t}$) along with the data to update the parameters using closed form expressions from the EM algorithm. We then repeat steps two and three until the likelihood has been maximized.[22]

1.4.3 Consistency under non-normality

We conclude this section by discussing the role of distributional assumptions for our analysis. In deriving the likelihood function we maximize to estimate parameters and compute posteriors, we assume that the shocks to temporary and persistent earnings (for both the employed and unemployed) are normally distributed; however, each step of our estimation procedure yields consistent estimates even if shocks are non-normal. The key step of the Kalman filter infers the unobserved state variable (persistent earnings) using the linear projection updating formula (detailed in Chapter 4.5 and Chapter 13.2 of Hamilton (1994a)). The linear projection

[22] Dempster et al. (1977) prove that the EM algorithm is guaranteed to increase the likelihood function at each iteration for general MLE problems. While poor choices of starting values could in principle lead to convergence to a local maximum, we have found our results to be generally quite insensitive to these choices in our applications and convergence to be quite rapid. In Appendix B, equation (12) we show the likelihood function that is maximized as part of the estimation.

updating formula infers the unobserved state variable by minimizing the mean squared error of the forecast. As long as the underlying state-space model is linear, the linear projection updating formula yields consistent estimates of the unobserved state under non-normality (see the longer discussion in the handbook chapter Hamilton (1994b)).[23] Additionally, the EM algorithm produces consistent estimates of the income process parameters even in cases when the shocks are not normally distributed (see Chapter 13 of Hamilton (1994a)).[24] The intuition for the EM result is that the formulas to update the parameters of the income process resemble GLS-style regression formulas. Hence, the Gauss-Markov theorem applies, and we obtain the best linearly unbiased estimator (BLUE) for the parameters.

Recent work has emphasized that log income changes are non-Gaussian, and exhibit negative skewness as well as excess kurtosis (e.g., Guvenen et al. (2021)). While the shocks to temporary and persistent earnings in our income process are drawn from normal distributions, our income process produces skewness and kurtosis in log earnings changes by incorporating unemployment spells as well as making the shocks functions of other observables. By conditioning on these observables, we naturally estimate mixture distributions; therefore, integrating out these observables yields non-Gaussian shock distributions even if shocks were Gaussian conditional on l_{it} and $X_{i,t}$.

In Figure 1, we show how the estimated shocks to persistent (Panel (a)) and temporary (Panel (b)) earnings deviate from a normal distribution. In the figure, the black solid line represents the distribution of shocks to persistent and temporary earnings from our baseline income process in Section 3. The red dashed line is a normal distribution with the same standard deviation as the shock to persistent (Panel (a)) and temporary earnings (Panel (b)). The figure shows that our baseline income process exhibits negative skewness as well as excess kurtosis. In Section 3.3, we incorporate job switching (blue dashed-dotted line), which generates persistent (temporary) skewness of -0.5 (-1.15) and kurtosis of 7.2 (11.2).[25] Hence, by incorporating rich sets of observables, our method tractably generates non-Gaussian shocks to temporary and persistent earnings.[26]

[23]In Section 2.5 Hamilton (1994b) writes, "Thus, while the Kalman filter forecasts need no longer be optimal for systems that are not normal, no other forecast based on a linear function of $[z_t]$ will have a smaller mean squared error [see Anderson and Moore (1979, pp. 92298) or Hamilton (1994, Section 13.2)]. These results parallel the Gauss-Markov theorem for ordinary least squares regression."

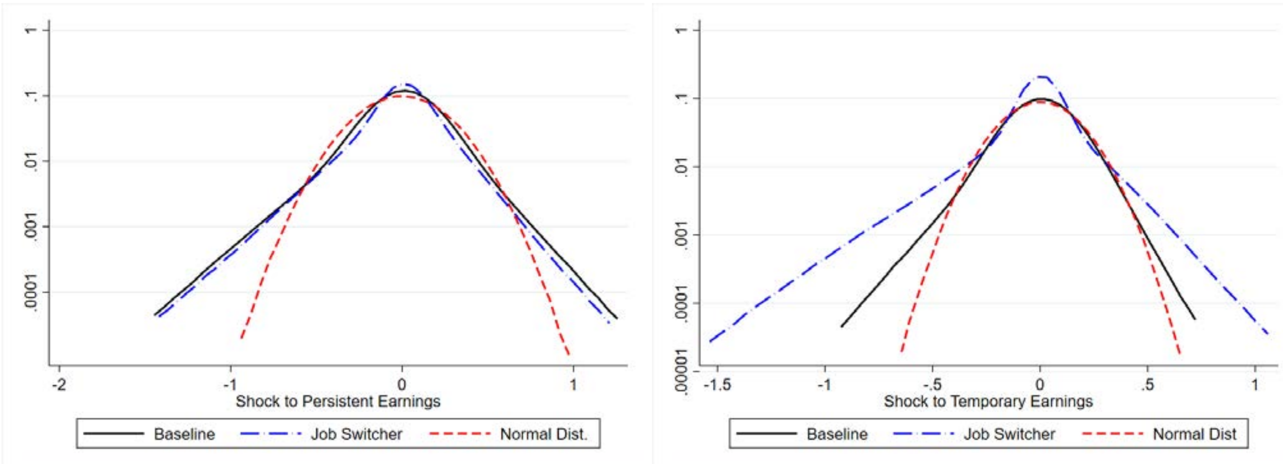
[24]See p.388 of Hamilton (1994a) entitled "Quasi-Maximum Likelihood Estimation." He writes, that even with non-normal shocks, the likelihood function can be interpreted as a quasi-maximum likelihood function and estimation "will still yield consistent estimates of the elements of F, Q, A, H, and R"

[25]In Appendix G.7 we include additional information on the higher order moments of the estimated shocks to temporary and persistent earnings. Section 3 contains additional information on the income processes used in creating the histogram of shocks shown in Figure 1.

[26]Analogously, a version of this model with means and variances which depend on job switching status is able to replicate the patterns of countercyclical tail risk in persistent income shocks, consistent with the evidence in

Figure 1: Histogram of persistent and temporary shocks

(a) Persistent shocks (b) Temporary shocks



Note: Figure shows the histogram of shocks to persistent (Panel (a)) and temporary (Panel (b)) earnings from alternative estimations of the model. The black solid line is our baseline estimation from Section 3, and the blue dashed-dotted line is from the estimation that takes into account job switching from section 3.3. The red dashed line is a normal distribution with the same standard deviation as the baseline model. For ease of interpretation, the y-axis is given in log scale and represents the share of the population in each bin.
Source: 1973, 1991, 1994, 1996-2016 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1982 to 2016.

2 Drivers of earnings risk

In this section, we first discuss our source of linked survey and administrative earnings records. Second, we present the benchmark parameter estimates of our income process. Third, we examine the observable labor market events associated with persistent and temporary income shocks identified by our filter.

To facilitate comparison with existing income process estimates, throughout this section, our analysis is based on the simplified income process in Section 1.2. The law of motion for observed income is assumed to follow equation (1) and the law of motion for persistent earnings is assumed to follow equation (3).

2.1 Data

To estimate the parameters governing the income process, we use annual labor earnings from administrative earnings records that have been linked to survey information. Our source of administrative earnings records is the Social Security Administration's Detailed Earnings Records

Guvenen et al. (2014). These results are available upon request.

(DER). The DER is a database of job-level W-2 earnings from 1978 to 2016. We supplement the DER with survey responses from the Annual Social and Economic Supplement (ASEC) to the Current Population Survey (CPS). The ASEC asks a series of questions about labor income, receipt of transfer income, occupation, and industry; it also provides detailed demographic information. Using scrambled Social Security numbers (called Protected Identification Keys), the Census Bureau links individuals from the CPS ASEC to their earnings information in the DER.[27]

Our sample includes individuals who were in the ASEC in the years 1973, 1991, 1994, and 1996-2016. Earnings records from the DER are included in all years in which the individual is observed and not just the years for which an individual is in the ASEC. We use earnings from the DER from 1982 through 2016, owing to concerns about data quality before 1982.[28] For the majority of individuals in our sample, we have 2 years of detailed information on demographics, income (labor and non-labor income), labor market information (e.g., weeks worked, occupation, etc.), and a full time series of an individual's labor income over their career from the DER.[29]

This ability to link administrative earnings with individual ASEC responses provides two primary benefits to our analysis. First, the information on labor market events (e.g., layoffs) allows us to examine the events that are associated with changes in persistent and temporary earnings. We view these comparisons as a means to validate our individual-level estimates of temporary and persistent earnings. Second, the information on education and occupation allows us to create finely partitioned groups to examine for whom earnings risk has changed over time. We can therefore examine these changes in a manner that would be impossible if we were using U.S. administrative data sources alone.

To study earnings dynamics, we focus on a sample of individuals with a minimum degree of labor force attachment. To be included in our estimation sample, an individual must: (1) satisfy a minimum earnings requirement in at least 5 (non-consecutive) years, (2) satisfy the minimum earnings criterion in at least 50% of years (inclusive) between the first and last year they satisfy the minimum earnings criterion, (3) be between the ages of 25 and 60, and (4) enter the sample by 2010. For conditions (1) and (2), we impose a minimum earnings criterion equal to the the real federal minimum wage for 40 hours a week for 26 weeks (on average, approximately $7,900

[27] See Wagner and Layne (2014) for more information on the assignment of PIKs to survey and administrative data. Note also that going forward we interchangeably use "the CPS", "March CPS", and "CPS ASEC" to denote the CPS ASEC.

[28] See Song, Price, Guvenen, Bloom, and Von Wachter (2018) and Guvenen, Kaplan, and Song (2020) for additional details. Song et al. (2018) and Guvenen et al. (2020) start their analyses in 1981, we start in 1982 because of concerns about the 1981 data. Our results are not sensitive to starting in 1982 rather than 1981 or 1978.

[29] In our estimation, we use an individual's sampling weight from the ASEC.

in 2019 dollars). These criteria allow for extremely long spells of zero earnings, potentially equal to half of the individual's panel of earnings. Condition (4) is included so that entrants to the sample in the final year are not selected towards individuals with the strongest labor force attachment (i.e. individuals with earnings above the minimum threshold for 5 consecutive years). Finally, to focus on labor market risk for workers, we additionally remove from the sample individuals who have self-employment income that exceeds 50% of their total income (labor income plus self-employment income) in at least 5 years. These sampling criteria result in a sample of 1,157,000 individuals.[30]

We use earnings information from the DER to study income risk. Our measure of income is the sum of Box 1 (total wages, tips, and bonuses) and Box 12 (earnings deferred to a 401(k) type account) earnings across all jobs the individual held during the year. We report earnings in 2019 dollars, where earnings are deflated by the CPI price index. To remove the impact of outliers, we winsorize real earnings at the 99.9th percentile in each year. Table 1 provides summary statistics for the individuals in our sample.[31]

Table 1: Summary statistics

Variable	**Mean**
Real Annual Earnings	$55,650
Age	40.78
Share Unemployed	6.9%
Share Less than College Degree	62.3%
Share College Degree Plus	29.4%
Share Education Not Reported	8.3%
Share Male	52.1%
Observations	23,500,000
Individuals	1,157,000

Note: Sample selection criteria in Section 2.1. Real annual earnings are measured in 2019 dollars. The variable "share unemployed" is the share of individuals whose average earnings in a given year do not satisfy the minimum earnings criterion.
Source: 1973, 1991, 1994, 1996-2016 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1982 to 2016.

[30]Owing to Census Bureau disclosure rules, the number of individuals is rounded to the nearest thousand.

[31]Note that we do not use an individuals reported education if they are less than 25 when in the ASEC. We do so to avoid mis-classifying individuals who have not yet completed their education. In Table 1 these individuals are classified as *Education Not Reported*.

2.2 Estimated income process

In this section, we discuss the estimated parameters governing the income process presented in Section 1.2. As a first step, we remove the predictable lifecycle component of log earnings, as in Guvenen et al. (2014) (see Appendix A.1 for details). When removing the predictable lifecycle component of log earnings, we only use earnings that satisfy our minimum earnings requirement. Our income process allows for the type of shock an individual receives to depend upon their employment status (i.e., employed or unemployed). For the filtering exercise, we classify individuals with annual earnings below the minimum earnings requirement as unemployed, and individuals with annual earnings above the requirement are classified as employed. To recover the parameters of the income process, we use the EM algorithm presented in Appendix C.4.

Table 2 presents the parameter estimates from estimating our income process, which incorporates only employment information $(l_{i,t})$ and omits all other observable variables $(X_{i,t})$. The parameter estimates reveal that persistent earnings shocks are very persistent ($F = 0.94$). Existing estimates of persistence of persistent earnings in mixture models range from .953 to .999 (see, e.g., Guvenen et al. (2014)). However, we make two departures from the literature, which make comparison difficult. First, the mean (drift) of the shock to persistent earnings depends upon an individual's employment status. Second, the prior literature drops observations with zero earnings (i.e., earnings below a minimum earnings criteria). Since our approach produces an estimate of persistent earnings even when an individual has zero labor earnings, these observations are taken into account when we estimate F. The estimation reveals that compared with employed individuals, unemployed individuals experience persistent earnings shocks with both a different mean (captured by the drift) and a different standard deviation. In particular, when an individual is unemployed, shocks to persistent earnings have a standard deviation that is nearly double the size of the standard deviation of shocks for employed individuals ($Q_U^{1/2} = .4171$, $Q_E^{1/2} = .2261$). Additionally, an unemployed individual's persistent earnings decline on average by nearly 15% per year, while an employed individual's persistent earnings increase by .4% per year. Hence unemployed individuals draw persistent earnings shocks from a distribution with a significantly lower mean and greater dispersion relative to that of employed individuals.

In addition to recovering income process parameters, the main benefit of our approach is that we obtain estimates of persistent earnings for each individual in each year. Next, we compare these individual-level shocks with observable labor market events in order to validate the filter.

Table 2: Parameter estimates

Description	Parameter	Value
Persistence of Perm. Earnings	F	0.9401
		(0.0002)
Std. Dev. of Shocks to Perm. Earnings (Emp.)	$Q_E^{1/2}$	0.2261
		(0.0002)
Std. Dev. of Shocks to Perm. Earnings (Unemp.)	$Q_U^{1/2}$	0.4171
		(0.0008)
Std. Dev. of Shocks to Temp. Earnings	$R^{1/2}$	0.1604
		(0.0002)
Drift of Perm Earnings (Emp.)	B_E	0.0038
		(0.0001)
Drift of Perm Earnings (Unemp.)	B_U	-0.1472
		(0.0006)
Std. Dev. of Initial Draw of Perm Earnings	$z_0^{1/2}$	0.7002
		(0.0008)

Note: Table presents parameter estimates from estimating income process in Section 1. Bootstrapped SE in parenthesis.
Source: 1973, 1991, 1994, 1996-2016 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1982 to 2016.

2.3 What causes persistent and temporary income shocks?

In this section, we examine how our filtered estimates of temporary and persistent earnings shocks align with job switches, self-reported layoffs in the CPS, and self-reported recalls in the CPS. For each labor market event, we report the joint density of persistent and temporary shocks. We illustrate these joint densities as heatmaps whose colors correspond to the mass of individuals with a given combination of persistent and temporary shocks.

2.3.1 Job Switching

We first plot heatmaps of the shocks to persistent and temporary earnings for individuals who remain at the same primary employer (EIN) across two consecutive years (Panel (a) of Figure 2) and for individuals who switch their primary employer (Panel (b) of Figure 2).[32] Panel (a) shows that among job stayers, the majority of individuals have small shocks to temporary and persistent earnings. These shocks likely reflect changes in hours and weeks worked, as well as promotions, and raises, etc. Conversely, Panel (b) shows that among job switchers, the mass

[32]An individual's primary employer in a given year is the defined as the EIN where the individual had the largest share of earnings in that year.

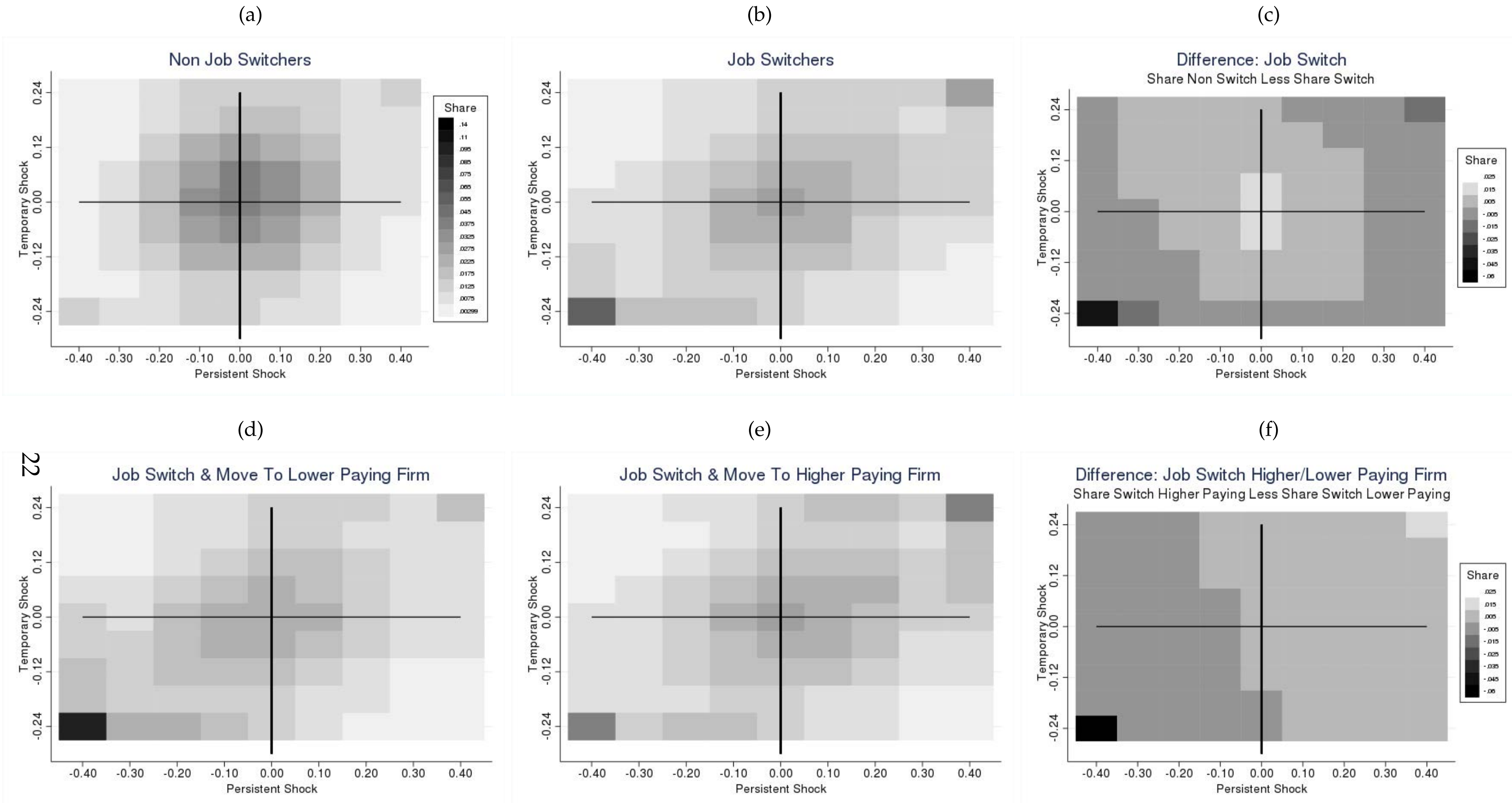
of individuals spreads out of the middle of the distribution towards more extreme persistent and temporary shocks (either positive or negative). To facilitate comparison, Panel (c) of Figure 2 subtracts the joint density in Panel (a) from Panel (b). The resulting difference in densities more clearly illustrates that job switching is associated with larger shocks (both positive and negative) to persistent and temporary earnings. Among non-switchers, roughly 2.5% have the most extreme earnings outcomes (lowest or highest persistent and temporary shocks). Among switchers, approximately 10 percent have the most extreme earnings outcomes, representing a fourfold increase.

We further split job switchers by the type of job switch. Using data from the Longitudinal Business Database (LBD), we measure average earnings per employer.[33] We separate job switchers into those who move to an employer with average earnings per worker that are 25% lower (higher) than their previous employer. Panel (d) (Panel (e)) of Figure 2 shows that when an individual moves to a lower (higher) paying employer, they become more likely to experience a large negative (positive) shock to persistent earnings. To facilitate comparison, Panel (f) of Figure 2 subtracts the joint density in Panel (d) from Panel (e). Panel (f) demonstrates that moving to a higher-paying firm is associated with positive shocks, especially to persistent earnings. We find that the mass of individuals with a persistent earnings loss (i.e. all mass left of the origin) is 33% higher among those who switch to firms that pay 25% less rather than 25% more.

The results of this section demonstrate that the estimates of temporary and persistent shocks align with observable labor market events. In particular, the estimates align with job ladder models of the labor market in which job switching is associated with shocks to temporary and persistent earnings that are larger than those associated with remaining at the same employer. Further, the direction of the job switch (i.e., moving to a higher or lower paying employer) aligns with the notion of climbing up and falling down the job ladder.

[33] See Jarmin and Miranda (2002) for details on the construction of the LBD.

Figure 2: Shocks to temporary and persistent earnings around job switching



Note: Figure plots a heatmap of temporary and persistent shocks by observable labor market event. Higher (lower) paying firms are identified by moving to an employer with average earnings that are 25% above (below) an individuals current employer. The heatmaps in panels (a), (b), (d) and (e) all use the same scale that is presented in panel (a).
Source: 1973, 1991, 1994, 1996-2016 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1982 to 2016.

2.3.2 Layoff

In this section, we examine temporary and persistent earnings shocks around layoff. While many papers have studied the average response of earnings to layoffs, we examine the heterogeneous behavior of temporary and persistent earnings following layoff. We document substantial heterogeneity in earnings following layoff and how it correlates with observable features of the layoff.

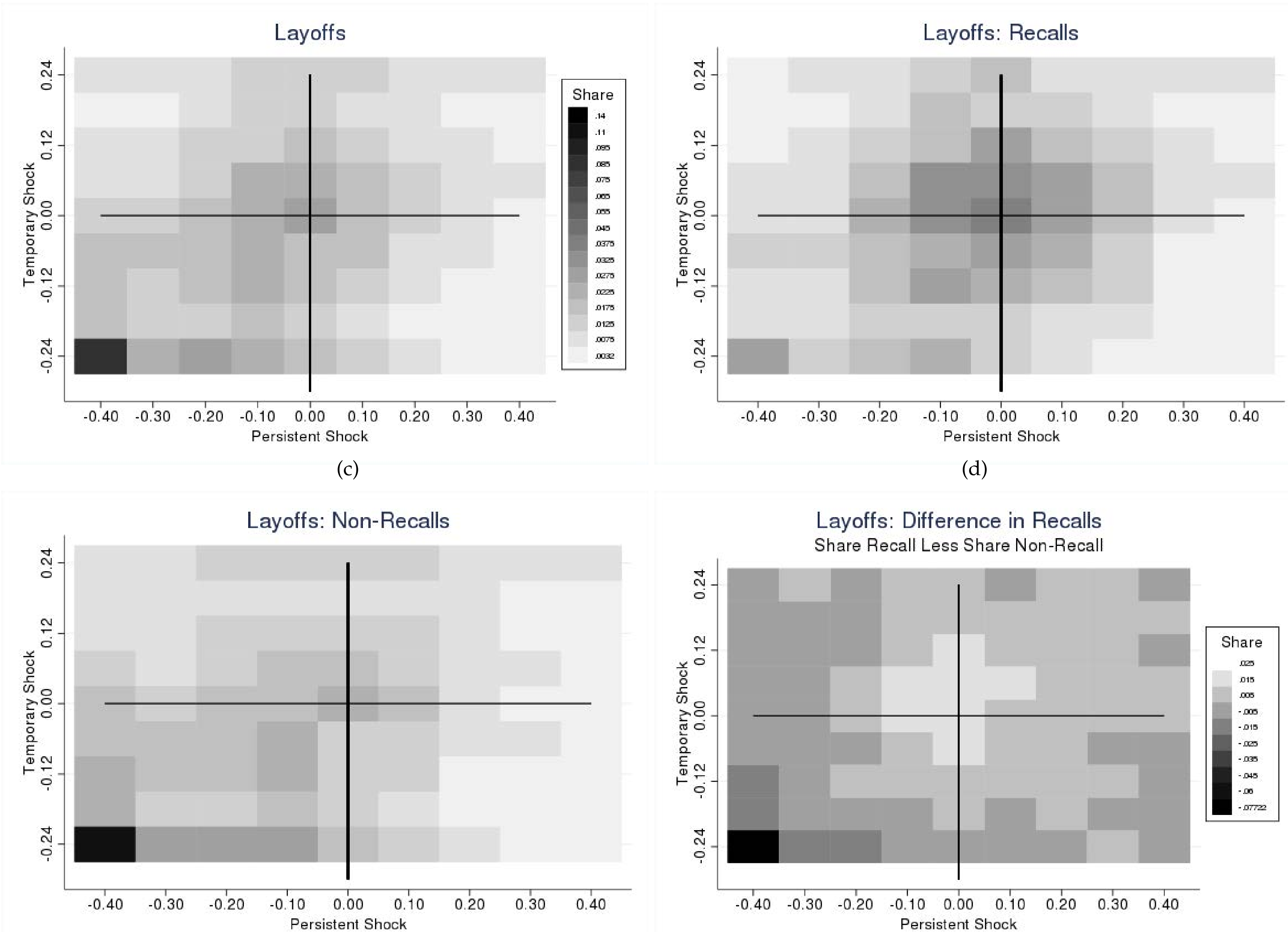
We identify layoffs using an individual's self-reported CPS responses. In particular, we define an individual to have been laid off in year t if they report having positive weeks on layoff in year t and zero weeks on layoff in year $t-1$. We impose the requirement that an individual have zero weeks on layoff in year $t-1$ so that we are able to accurately measure the inflow of individuals into unemployment. In Panel (a) of Figure 3, we plot the heatmap of persistent and temporary earnings shocks in year t for individuals we identify as laid off in the CPS. The figure shows that there is a large mass of individuals in the bottom left hand corner of the heatmap, which indicates that a sizeable mass of laid off individuals has large negative persistent and temporary shocks. Roughly 29.6% of laid off individuals have negative persistent losses that exceed their temporary losses. Interestingly, there is also a large mass of individuals with small shocks, and there are even some individuals with positive shocks.

We investigate the heterogeneity in shocks around layoffs by distinguishing *recalls* from *non-recalls*. We define an individual to be recalled if their primary employer in the year before layoff is also their primary employer in the year after layoff.[34] We define an individual to be non-recalled if they have different primary employers in the years before and after layoff. Panel (b) of Figure 3 plots the heatmap of persistent and temporary shocks among recalled individuals, while Panel (c) of Figure 3 plots the heatmap for non-recalled individuals, and Panel (d) illustrates the difference. Comparing Panels (b) and (c) shows that relative to individuals who are not recalled after layoff, individuals who are recalled exhibit much smaller negative shocks to temporary and persistent earnings, and they are more likely to have a positive shock. Among those who are recalled, 19.9% have negative persistent losses that exceed their temporary losses. In contrast, among those who are not recalled, their earnings losses are much more persistent: 33.8% have negative persistent losses that exceed their temporary losses.

[34] As in section 2.3.1, we define an individual's primary employer in a given year as the EIN where they have the largest labor earnings.

Figure 3: Shocks to temporary and persistent earnings around layoff

(a) (b)



Note: Figure plots a heatmap of temporary and persistent shocks around layoff. Layoffs are identified using the CPS. Individuals are defined as "recalled" if their primary employer in the year after layoff is the same as the year before layoff.
Source: 1973, 1991, 1994, 1996-2016 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1982 to 2016.

The results of this section showed that our estimates of temporary and persistent earnings align with observable shocks that individuals face in the labor market. As the filter is unaware of the shocks individuals face, we view these results as a validation of the method. We next utilize this method to examine how and why persistent and temporary earnings risk has changed over time.

3 The changing nature of earnings risk

Having validated our filter, we now turn to our main exercise, which is to measure time trends in persistent and temporary earnings risk. We show that since the 1980s, persistent earnings risk has risen, while temporary earnings risk has declined. We further show that the decline in persistent earnings during spells of unemployment has worsened. In the following section, we exploit the rich demographic, geographic, and occupation information in the linked SSA-CPS data to characterize for whom earnings risk has changed.

To measure time trends in earnings risk, we incorporate the following features into our income process: (1) the standard deviation of shocks to temporary and persistent earnings are a function of year fixed effects and a quadratic in age, which vary separately for both the employed and unemployed; (2) the standard deviation of initial draws of persistent earnings are also a function of year fixed effects and a quadratic in age; (3) the drift in persistent earnings is also a function of year fixed effects that vary separately for both the employed and unemployed. We include the age quadratic in order to control for changes in the age composition of the sample over time.[35] We relegate a full exposition of this augmented income process to Appendix D. We use data from 1982 to 2016 to estimate our model; however, the year fixed effects at the start and end of the sample are not well identified. For this reason, we bin together the first and last three years into single year fixed effects at the start and end of our sample (i.e., 1982-1984, and 2014-2016.). In the graphs that follow, we omit these grouped year fixed effects, and we present the individual year fixed effects that cover the period 1985-2013.[36]

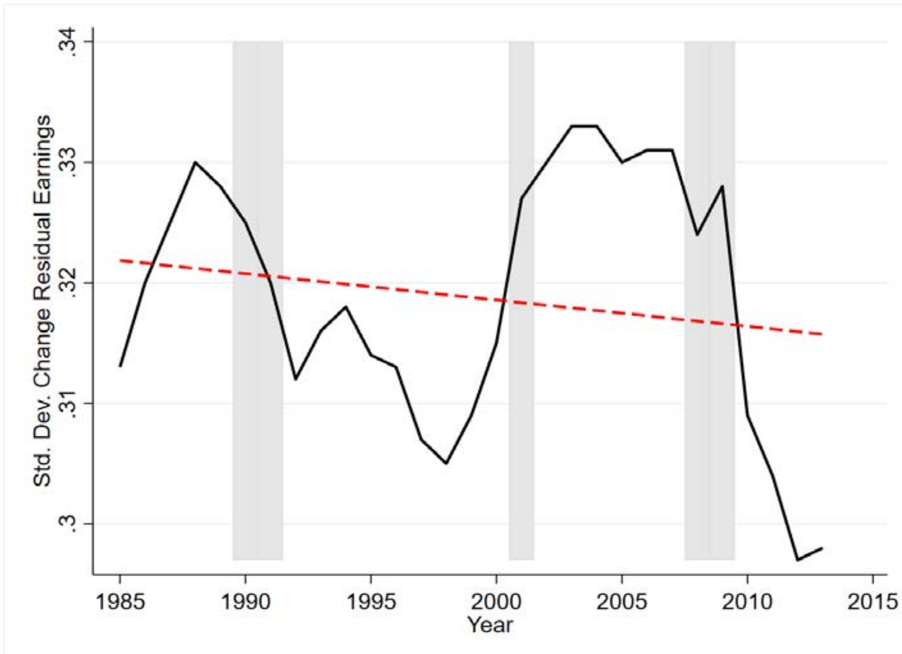
Estimation Results. We first illustrate earnings risk among the employed, ignoring the decomposition between persistent and temporary shocks. Figure 4 shows that earnings risk, as

[35]From the age quadratic, the filter produces estimates of earnings risk over the lifecycle, complementing the work of Karahan and Ozkan (2013) and Blundell, Graber, and Mogstad (2015). Since lifecycle earnings are not the focus of this paper, we relegate these results to Figure 18 in Appendix G.2.

[36]Our results are robust to changing the number of years that are included in the "grouped" fixed effects at the start and end of the sample. The results are also robust to including the 3-year grouped fixed effects at the start and end of the sample.

measured by the standard deviation of residual log earnings changes, exhibits a mild downward trend.[37] However, we show below that simply looking at trends of earnings changes masks offsetting trends in persistent and temporary earnings risk, as well a major acceleration of persistent earnings losses among the unemployed.

Figure 4: Earnings risk over time



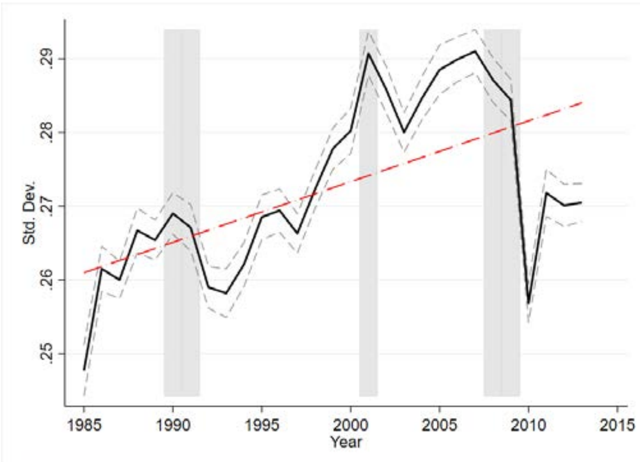
Note: Figure presents the standard deviation of residualized log earnings over time (black, solid line) and a linear trend line (red, dashed line).
Source: 1973, 1991, 1994, 1996-2016 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1982 to 2016.

We next decompose changes in earnings risk into its persistent and temporary components. Figure 5 illustrates how persistent earnings risk and temporary earnings risk have changed over time.[38] Panel (a) shows that the standard deviation of persistent earnings among the employed rose by 2 percentage points (or 9%) from 0.25 to 0.27 between 1985 and 2013. While persistent earnings risk rose, temporary earnings risk declined. Panel (b) shows that the standard deviation of temporary earnings risk declined by over 1.5 percentage points (or nearly 9%) over the sample period.

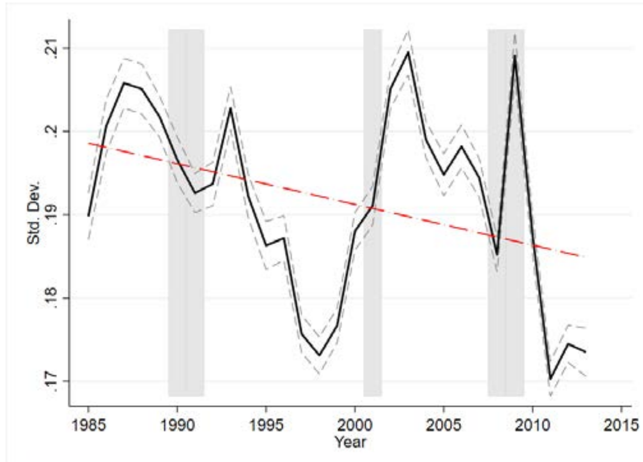
[37]In Section 3.1, we discuss how our estimate of earnings risk over time relates to the existing literature.
[38]Note that in Figure 5 we present the standard deviation of shocks to income over time, holding the age component fixed at the value for individuals who are 25.

Figure 5: Persistent and temporary earnings risk over time

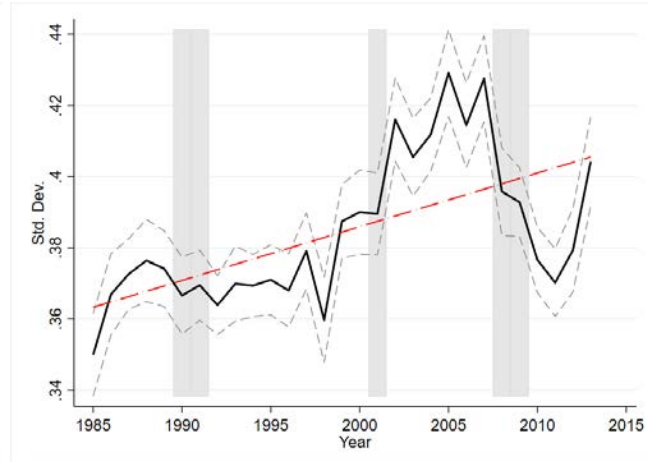
(a) Std. dev. persistent earnings (Emp.)



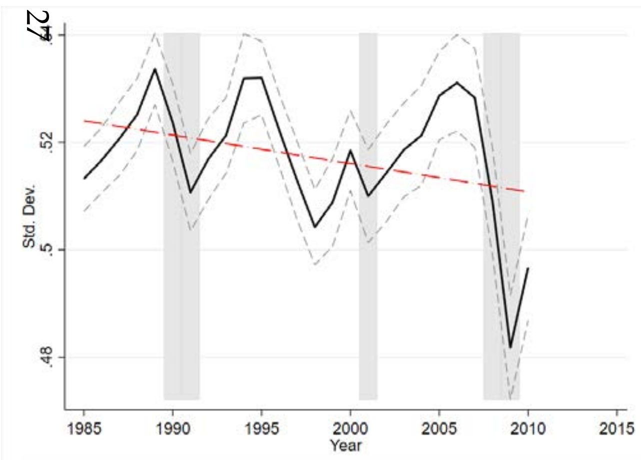
(b) Std. dev. temporary earnings



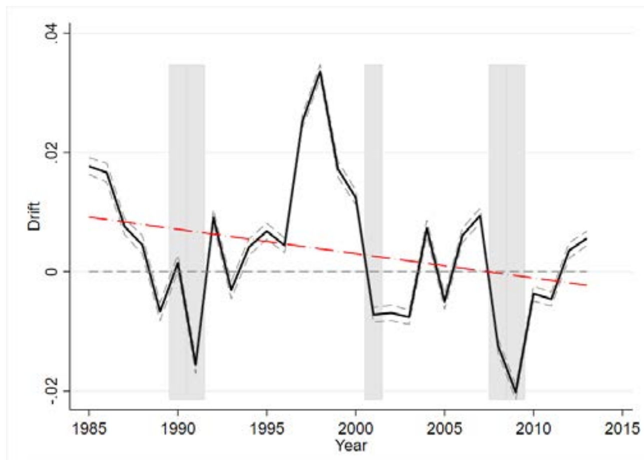
(c) Std. dev. persistent earnings (Unemp.)



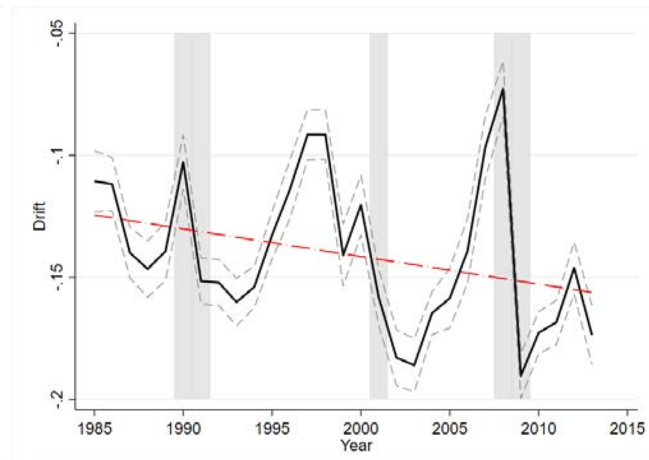
(d) Std. dev. initial draw persistent earnings



(e) Drift persistent earnings (Emp.)



(f) Drift persistent earnings (Unemp.)



Note: Figure presents parameter estimates of the shocks to earnings over time (black, solid line) along with a linear trend line (red, dashed line). Dashed gray lines denote a 95% confidence interval.
Source: 1973, 1991, 1994, 1996-2016 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1982 to 2016.

This result highlights that simply looking at the standard deviation of log earnings changes by year can mask heterogeneous trends in temporary and persistent earnings risk. As shown in Figure 4, there was a minimal trend in overall earnings risk, but this masks the fact that persistent earnings risk increased while temporary earnings risk declined.

Our income process also produces an estimate of the standard deviation of shocks to persistent earnings among the unemployed. Panel (c) shows that the standard deviation of shocks to persistent earnings among the unemployed also increased over time. Among the unemployed, shocks to persistent earnings increased by over 15%. Hence, the unemployed faced greater dispersion in their earnings upon re-entering employment in the 2010s than they did in the 1980s.

Panel (d) of Figure 5 plots the standard deviation of initial persistent earnings. The figure shows there is no significant time trend over the sample period. However, the time series exhibits a strong pro-cyclical pattern, with initial persistent earnings compressed during downturns.

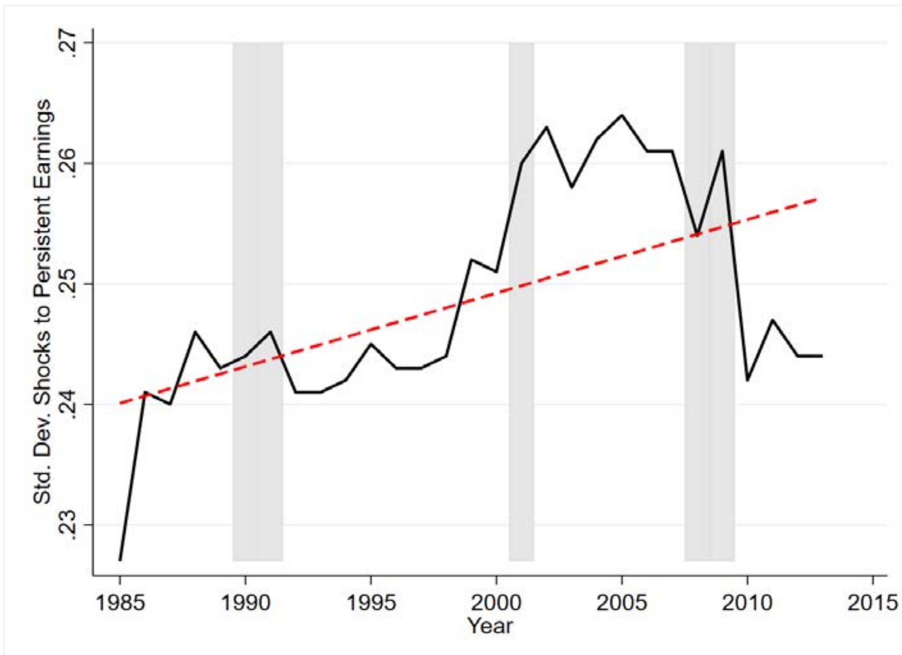
Finally, our income process also estimates trends in the mean (drift) of shocks to persistent earnings for both the employed and unemployed. Panel (e) of Figure 5 presents the trend in the mean shock to persistent earnings among the employed. The figure shows that the drift among the employed is highly cyclical, with large declines in recessions and increases in expansions. Panel (f) shows the trend in the drift of persistent earnings among the unemployed. The drift of persistent earnings during spells of unemployment is also highly cyclical and becomes more negative during recessions. Additionally, the drift in persistent earnings during spells of unemployment exhibits a strong negative trend over the sample period. At the start of the sample, a year of unemployment is associated with an -11% decline in persistent earnings, and by the end of the sample, this decline in persistent earnings has accelerated to over -17%. This estimate reveals that the "scarring" effect of unemployment on persistent earnings has become more severe over time.

These results highlight that workers have become exposed to greater persistent earnings risk over time. A novel feature of our method is that it allows us to estimate persistent earnings risk over both employment and unemployment spells. Using our individual level estimates of persistent earnings shocks, we can create a measure of *combined* persistent earnings risk that takes into account persistent earnings risk among both the employed and unemployed.[39] This measure of persistent earnings risk naturally takes into account unemployment risk, which is

[39]To estimate combined persistent earnings risk we simply take the standard deviation of shocks to persistent earnings at the individual level by year from the Kalman filtering step in our estimation. Since this measure is from the individual-level estimates, it takes into account changes in the age composition of the sample. Incorporating changes in age structure is why the level of combined persistent risk is lower than the estimates in Figure 5, which are based upon the parameters of the income process and are presented for a 25-year-old.

missing from estimates of earnings risk that simply look at log earnings changes.[40] Figure 6 presents the standard deviation of combined earnings risk over time, which has exhibited a strongly increasing trend over the sample period. In particular, combined persistent earnings risk has increased by just under 10% over the sample period.

Figure 6: Persistent earnings risk over time



Note: Figure presents the standard deviation of combined persistent earnings risk over time.
Source: 1973, 1991, 1994, 1996-2016 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1982 to 2016.

The results of this section showed that individuals face greater persistent earnings risk than they did in the 1980s. Additionally, workers going through spells of unemployment are subjected to larger declines in their persistent earnings. In the following subsections, we relate our measures of earnings risk to other estimates in the literature and discuss how we identify the trends in persistent and temporary earnings risk over time. We then examine for whom earnings risk has changed over time.

3.1 Relation to prior measures of earnings risk over time.

In this section, we briefly compare our estimates of earnings risk with existing estimates in the literature. Recently, there has been a debate on the extent to which earnings volatility has

[40]Relative to measures that allow for one period of missing earnings, such as arc-changes, our combined measure of persistent earnings risk allows for arbitrary spells of zero/low earnings.

changed over time, with some papers finding a significant decline in earnings risk over time (e.g., Bloom et al. (2017)) and others reporting no trend (e.g., Moffitt (2020) and papers summarized therein). We find that the value of the minimum earnings cutoff significantly influences the trend in the standard deviation of residual log earnings changes (hereafter referred to as "earnings risk"); however, it does not impact the interpretation of the trends in persistent and temporary earnings risk over time.

In this section, we lower the minimum earnings cutoff to the equivalent of working part-time (20 hours per week) at the real federal minimum wage for one quarter, which corresponds to approximately $2k per year in 2019 dollars.[41] This value of the minimum earnings criterion follows from Bloom et al. (2017). In this extension, we maintain our prior sample requirements that an individual must have earnings above the alternative minimum earnings criterion in at least 5 (non-consecutive) years and 50% of the years between the first and last year that the individual satisfied the minimum criterion.

We first examine how the lower minimum earnings cutoff impacts earnings risk over time by plotting the standard deviation of residual log earnings changes by year. Panel (a) of Figure 7 plots the standard deviation of residual log earnings changes by year, using our baseline minimum earnings cutoff (black, solid line), along with the standard deviation of residual log earnings changes using the lower threshold (black, solid line with black circles).[42] With the lower minimum earnings cutoff, there is a significant trend decline in earnings risk over time. In particular, with the lower minimum earnings cutoff, the standard deviation of changes in residual log earnings decreases by 10% over the sample period, whereas it decreases by 5% with the baseline minimum earnings cutoff.

We next examine how the minimum earnings cutoff affects the trends in persistent and temporary earnings risk.[43] Panel (b) of Figure 7 plots the standard deviation of temporary and persistent shocks over time with the baseline minimum earnings cutoff (blue dashed and red dashed-dotted lines, respectively), along with the corresponding time series with the lower minimum earnings criteria (blue dashed and red dashed-dotted lines with blue and red circles, respectively). First, we observe a significantly larger decline in temporary earnings risk with the lower minimum earnings threshold, compared with the decline under the baseline estimate.

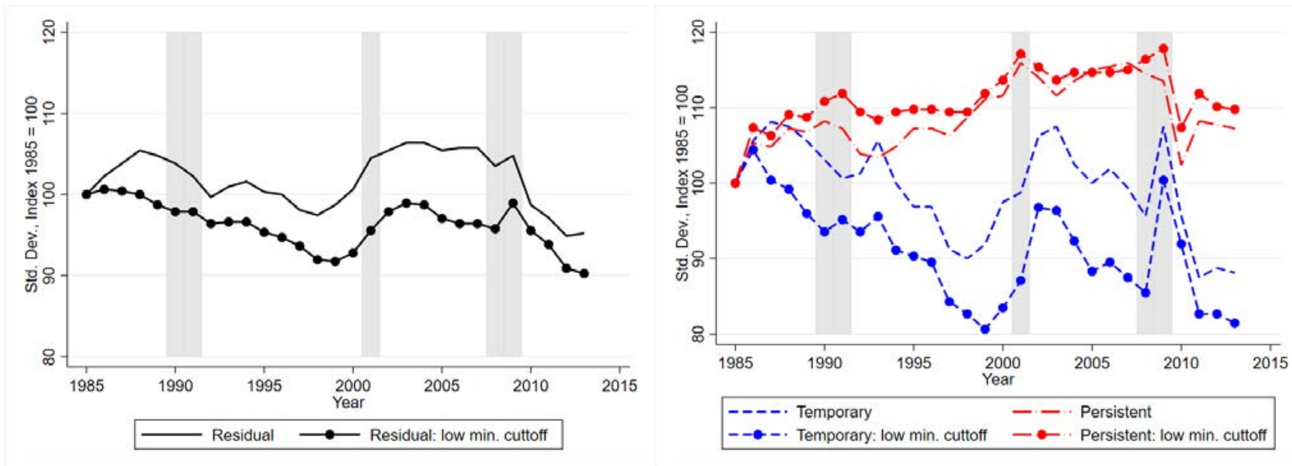
[41]In our baseline estimation, the minimum earnings criterion is the equivalent of working full-time at the real federal minimum wage for two quarters.

[42]For ease of interpretation, we have normalized the values in 1985 to be equal to 100. In Appendix G.3 we plot the time series of the level of the standard deviation of log earnings changes with the lower minimum earnings cutoff. With the lower minimum earnings cutoff, the level of earnings risk is substantially higher (by approximately 40%) than in the baseline.

[43]In Appendix G.3, we present all parameter estimates from the estimation with the lower minimum earnings criterion.

Figure 7: Earnings risk over time by minimum earnings cutoff

(a) Residual log earnings changes (b) Persistent and temporary earnings risk



Note: Figure examine how the trends in earnings risk have changed over time with the baseline minimum earnings cutoff and a lower minimum earnings cutoff. The baseline (lower) minimum earnings cutoff is set to the equivalent of working 40 (20) hours per week for 2 quarters (1 quarter) at the real federal minimum wage.
Source: 1973, 1991, 1994, 1996-2016 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1982 to 2016.

In particular, the standard deviation of temporary earnings shocks declines by nearly 20% over the sample period with the lower minimum earnings threshold, whereas it declines by 10% in the baseline. Second, the trends for persistent earnings risk over time are largely unchanged by the minimum earnings cutoff. Under both estimations, there is an increase of nearly 10% in persistent earnings risk.

Putting the results of this section together, we find that through its impact on temporary earnings, the value of the minimum earnings criterion plays a large role in the trend in earnings risk. With a lower minimum earnings threshold, there is a larger decline in temporary earnings risk over time, which generates a decline in overall earnings risk. However, across both estimations, we find a common rise in persistent earnings risk. In the next section, we further decompose the trend in earnings risk into its temporary and persistent components, and we discuss identification.

3.2 Understanding trends in persistent and temporary earnings risk

In this section, we discuss intuition and provide validation for the trends in persistent and temporary earnings risk. We first discuss the trends in persistent risk for the employed, and then consider the trends for the unemployed.

3.2.1 Trends in temporary and persistent earnings among employed

In this section, we discuss the intuition for how the trends in temporary and persistent income are identified among the *employed*.[44] We show below that the variance of shocks to persistent and temporary earnings can be identified from the variance in log earnings changes over different horizons. We can then identify trends in temporary and persistent risk by examining how these variances evolve over time. Below, we illustrate this identification using 1-year and 5-year changes in residual log earnings. It is important to note that the baseline estimation in Section 1 uses the full path of earnings (and hence their changes) for identification.

For ease of exposition, we abstract from incorporating periods of zero earnings, remove the drift parameters, and assume that persistent earnings follow a unit root process. With these assumptions the income process is given by

$$
\begin{aligned}
y_{i,t} &= z_{i,t} + \omega_{i,t} \\
z_{i,t} &= z_{i,t-1} + \eta_{i,t},
\end{aligned}
$$

where the temporary earnings shock ($\omega_{i,t}$) has a variance R, and the persistent shock ($\nu_{i,t}$) has variance Q. In this pedagogical example, we treat the variances as constant over time to simplify exposition, and we note that this is not important for our results (Appendix G.6 relaxes this assumption). With the income process specified in this manner, we can write the variance of earnings changes over a 1-year horizon and a 5-year horizon as follows:

$$
var(y_{i,t} - y_{i,t-1}) = Q + 2R \tag{4}
$$

$$
var(y_{i,t} - y_{i,t-5}) = 5Q + 2R. \tag{5}
$$

Hence, with an estimate of the variance of earnings changes over 1 year and 5 years, we can identify the variance of temporary and persistent shocks. Then, by examining these variances at different points in time, we can identify how the variance of temporary and persistent earnings has changed over time. Let $V_{1,t} = var(y_{i,t} - y_{i,t-1})$ and $V_{5,t} = var(y_{i,t} - y_{i,t-5})$. Then, solving the system of equations in (4) and (5), we have

$$
Q(V_{1,t}, V_{5,t}) = \frac{V_{5,t} - V_{1,t}}{4} \tag{6}
$$

$$
R(V_{1,t}, V_{5,t}) = \frac{5V_{1,t} - V_{5,t}}{8}. \tag{7}
$$

[44] In Appendix G.6 we show how changes in the variance of log earnings changes can be decomposed into changes due to the temporary and persistent component of earnings.

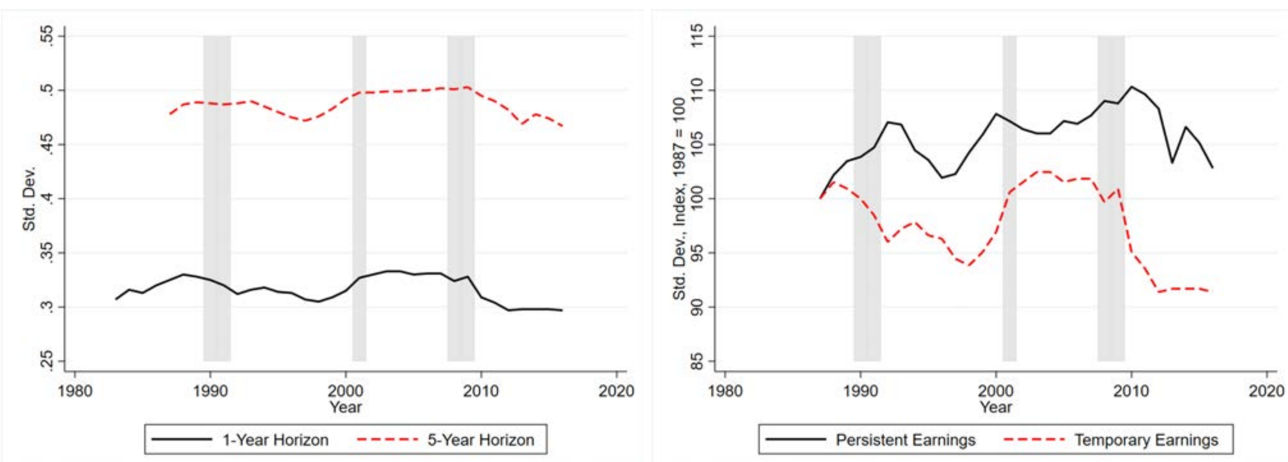
In panel (a) of Figure 8, we plot the standard deviation of log earnings changes over a 1-year horizon (black, solid line) and 5-year horizon (red, dashed line). Next, in panel (b) of Figure 8, we use $V_{5,t}$ and $V_{1,t}$ for each year to calculate the implied standard deviation of shocks to persistent earnings (black, solid line) and temporary earnings (red, dashed line), using equations (6) and (7). To ease the illustration of the trends, we normalize the initial values to be equal to 100.

The time series in Panel (b) of Figure 8 show that the path of persistent and temporary earnings implied by the 1-year and 5-year variances aligns with our baseline estimates. In particular, we find an increase in persistent risk and a decline in temporary risk over the sample period. This result highlights that simply looking at the standard deviation of 1-year or 5-year log changes in earnings is not sufficient to characterize temporary or persistent risk. In fact, the standard deviation of 5-year earnings changes can have no trend or can even decline, while persistent earnings risk can rise. Identifying persistent and temporary risk requires examining the joint evolution of earnings changes at different horizons. A benefit of our filtering exercise is that it examines the full path of an individual's earnings (and changes in earnings) to inform the estimates of temporary and persistent earnings risk.

Figure 8: Identifying trends persistent and temporary risk

(a) Std. residual log earnings changes

(b) Implied persistent and temporary earnings risk



Note: Panel (a) of the figure plots the standard deviation of residual log earnings changes over 1-year (black solid line) and 5-year (red, dashed line) horizons. Panel (b) plots the time series of persistent and temporary earnings implied by equations (6) and (7).
Source: 1973, 1991, 1994, 1996-2016 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1982 to 2016.

3.2.2 Trends in persistent risk among unemployed

In this section, we further discuss the trends in persistent earnings among the unemployed. In Section 3, we showed that the standard deviation of persistent earnings shocks to the unemployed has increased over time, and there are larger declines in persistent earnings during spells of unemployment. One potential concern is that our time series patterns may be attributable to individuals with no observable labor market shocks (i.e., no 'true' labor market risk), and thus the time trends reflect misspecification and/or life events that are unrelated to labor market risk. In this section, we validate our finding of rising persistent earnings risk among the unemployed by showing that those who self-reported job loss in the CPS exhibit rising persistent earnings risk.

In what follows, we define an individual to be *CPS unemployed* in year t if they report being on layoff for at least one week in year t or report not working in year t. With this definition, we examine how the standard deviation of shocks changed over time for individuals we classify as CPS unemployed, and how these trends compare with our full sample of individuals. We note that these individuals need not fall below the minimum earnings threshold defined in Section 2.1.

In Figure 9, we compare the time series of earnings risk among our full sample of individuals to the individuals we classify as CPS unemployed. Panel (a) of Figure 9 compares the time series of the standard deviation of shocks to persistent earnings among individuals in the full sample whom we classify as unemployed (black, solid line) to individuals whom we classify as CPS unemployed (red, dashed line).[45] The CPS unemployed have a standard deviation of shocks to persistent earnings that is smaller than that of the full sample of individuals; however, the time series reveals a steadily increasing trend from the early 1990s, similar to the trend we observe in the full sample of unemployed individuals (correlation = 0.887). Hence, under the significantly less stringent definition of CPS unemployment, we see a similar trend increase in the standard deviation of shocks to persistent earnings among the unemployed.

Next, we examine the mean change in persistent earnings among the full sample of unemployed individuals and among individuals who report being unemployed in the CPS. Panel (b) of Figure 9 compares the average shock to persistent earnings among individuals in the full sample whom we classify as unemployed (black, solid line) to individuals whom we classify as CPS unemployed (red, dashed line). The figure shows that those who self-report CPS unemployment have experienced a similar acceleration of persistent earnings losses during unemployment (correlation = 0.794).

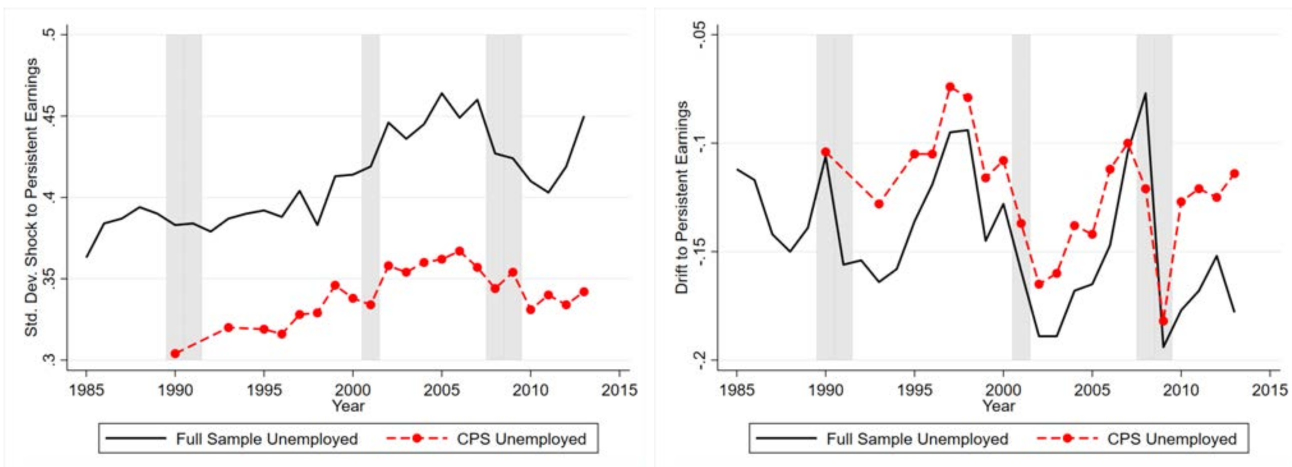
[45] An individual is defined as full sample unemployed in year t if they have earnings below the minimum earnings cutoff in year t.

In summary, we show that individuals with observable labor market risk in the CPS (i.e., those who self-report positive weeks on layoff) are precisely those who have rising persistent earnings risk implied by our filter. We view this as a demonstration of our filter's ability to capture economically meaningful labor market risk not just cross-sectionally but also over time. We next examine the sensitivity of our results to alternative estimations of the income process and samples.

Figure 9: Comparison to shocks among CPS Unemployed

(a) Std. dev. shock to persistent earnings

(b) Drift to persistent earnings



Note: Figure compares the time series of the standard deviation and mean of filtered shocks to persistent and temporary earnings among individuals in the full estimation sample (black, solid line) and individuals whom we classify as CPS unemployed. See Section 3.2.2 for definitions of CPS Unemployed as well as Full Sample Unemployed.

Source: 1973, 1991, 1994, 1996-2016 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1982 to 2016.

3.3 Sensitivity of trends in persistent and temporary earnings risk

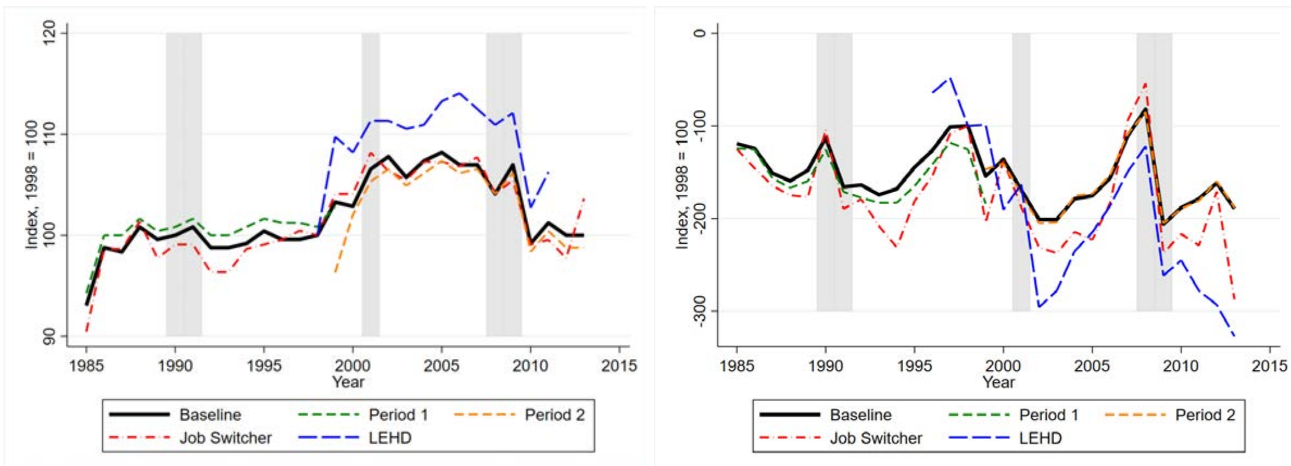
In this section, we exploit our estimation procedure's ability to tractably estimate rich models of the income process to assess the sensitivity of our results to alternative specifications and samples. We show that the increase in persistent earnings risk since the 1980s, as well as the larger decline in persistent earnings during spells of unemployment, is robust to alternative specifications and samples.

Figure 10 presents the results of this sensitivity analysis. In Figure 10, for ease of comparison, we normalize the time series to be equal to 100 (-100) in the year 1998. We normalize the time series to their 1998 value, as that is the first year for which we have estimates for our

Longitudinal Employer Household Dynamics (LEHD) sample. For ease of presentation, we discuss only the trend in the standard deviation of combined persistent earnings changes for the employed and unemployed as well as the drift in persistent earnings for the unemployed. In Appendix G.8, we show results for the remaining components of the income process.

Figure 10: Sensitivity of earnings risk over time

(a) Shocks to persistent earnings (comb.) over time (b) Drift to persistent earnings (unemp.) over time



Note: Figure compares the estimates of combined persistent earnings risk (Panel (a)) and the drift in persistent earnings among the unemployed (Panel (b)) over time for alternative income proccesses and estimations of the income process over different samples. See Section 3.3 for details on the different estimations and samples.
Source: 1973, 1991, 1994, 1996-2016 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1982 to 2016. Longitudinal Employer-Household Dynamics (LEHD) database for 1995 to 2014.

Our first sensitivity exercise splits our estimation sample into two distinct periods and estimates the income process parameters separately across the two periods. The two time periods are 1985-1999 (*Period 1*) and 1999-2013 (*Period 2*). A key feature of this estimation is that the parameter that governs the persistence of shocks (F) is allowed to differ over sample periods. The green and orange dashed lines in Figure 10 present the results of estimating the income process over separate time periods. The results show that the separately estimated trends closely track the baseline estimates over the full sample period (black, solid line). Thus we conclude that increasing persistent earnings risk is not being driven simply by changes in the persistence parameter (F) over time.[46]

Our next sensitivity exercise allows for the shocks to persistent and temporary earnings that an individual receives to be conditioned on whether or not they remain employed at the

[46]We find that the persistent parameter F has increased moderately over these two time periods. In Period 1 we estimate $F = 0.9374$ (se 0.0003) and in Period 2 we estimate $F = 0.9465$ (se 0.0002).

same primary employer across years. In particular, this estimation includes two shocks to both persistent and temporary earnings for the employed: one for job switchers and one for job stayers.[47] We allow the standard deviations of shocks to temporary and persistent earnings to depend upon job switching, and we allow the drift parameters for persistent earnings to differ by job switching status. The red, dash-dotted line in Figure 10 plots the trends in earnings risk for the estimation of the income process that accounts for job switching. Allowing for the shocks to persistent earnings to depend upon job switching status does not materially affect our estimate of the trend in persistent earnings risk or the decline in persistent earnings losses during spells of unemployment.[48]

Finally, we examine the sensitivity of our results to using an alternative sample of workers. The blue, long-dashed line in Figure 10 presents the results of estimating our baseline income process from Section 3 on earnings records from the Longitudinal Employer-Household Dynamics (LEHD) database. The LEHD is a matched employee-employer database of earnings that covers over 95% of private sector workers. In Appendix G.8.1 we provide additional detail on the LEHD and our sample of workers from the LEHD. The figure shows that we obtain similar trends for persistent earnings risk, as well as the drift in persistent earnings risk among workers from the LEHD. This result indicates that the trends we are uncovering are not driven by our SSA-CPS sampling frame of workers.

The results of this section document that since the 1980s, workers have faced an increase in persistent earnings risk. Additionally, spells of unemployment have become associated with larger declines in persistent earnings. In the following section, we exploit the additional information from our SSA-CPS data to examine for whom earnings risk is changing.

4 For whom is earnings risk changing?

We have shown that persistent earnings risk has increased, and the decline in persistent earnings during spells of unemployment has worsened. In this section, we exploit the demographic, geographic, and occupation information in the linked SSA-CPS data to characterize for whom these changes are occurring. We frame this discussion by testing several hypotheses for rising persistent earnings risk. We rule out hypotheses related to declining employment prospects of low skill workers and regional theories of persistent earnings losses, including the decline of the Rust-Belt. We additionally rule out theories based upon routine employment using infor-

[47] An individual is defined as a job stayer in year t if they have the same primary employer (EIN) in year $t-1$, t and $t+1$. Individuals who are not job stayers are job switchers.

[48] As shown in Figure 1 incorporating job switching into the estimation increases the skewness and kurtosis of the shocks to temporary and persistent earnings.

mation on the skill content in occupations (e.g. Acemoglu and Autor (2011)). Instead, we show that the rise in persistent earnings risk is a high skill worker phenomenon and provide supportive evidence that the rise in persistent risk among high skill workers is due to exposure to the introduction of new technologies. We organize this section around tests of four hypotheses:

Hypothesis 1: Low human capital workers have experienced declining employment prospects, which have caused a rise in persistent earnings risk.

Hypothesis 2: The declining manufacturing sector along with declining union protection has led to a geographically concentrated rise in persistent earnings risk in the Rust Belt.

Hypothesis 3: Declining wages and employment in routine occupations have caused rising persistent earnings risk.

Hypothesis 4: High skill workers have experienced the largest increase in persistent earnings risk due to greater exposure to the introduction of new, skill-biased technologies.

To rule out concerns regarding parameter restrictions among these disparate subgroups, we re-estimate our income process for each subgroup (i.e., by education, occupation, gender, state) in all results shown in this section.

4.1 Education

We start by testing the first hypothesis: rising persistent earnings risk is being driven by workers with low levels of human capital. We partition individuals into two groups based upon their first recorded level of education in the CPS: those who do not have a college degree, and those who have a college degree or higher. We use an individual's reported level of education only if they are over the age of 25 at the time of the CPS survey. We then estimate the parameters of our income process separately for each group.[49]

In Figure 11, we present evidence on the trends in persistent earnings risk for the two education groups. For ease of comparison, we normalize the time series for each group to be equal to 100 (or -100 in panel (b)) in the year 1985. Panel (a) shows that the the increase in combined persistent earnings risk is more pronounced among college graduates than non-college-graduates. Among college graduates, there has been an increase of over 10% in persistent earnings risk. Conversely, among non-college-graduates, the increase in persistent earnings risk has been closer to 5%.

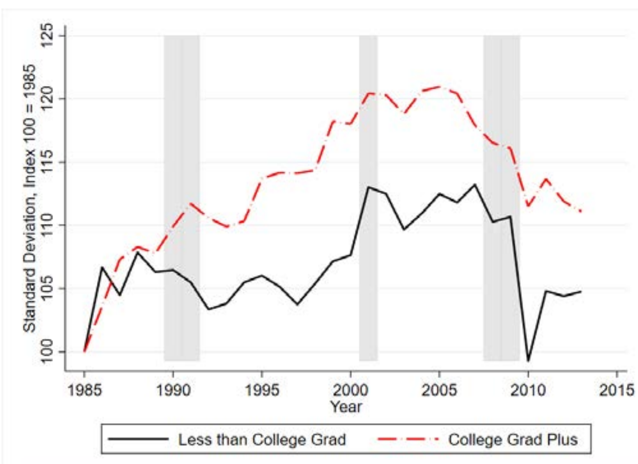
[49]Note that we residualize earnings separately for each group. In Appendix G.3 we present the parameter estimates by education level.

Panel (b) shows a similar phenomenon for the drift in persistent earnings during spells of unemployment. For college graduates, the decline in persistent earnings during spells of unemployment has nearly doubled over the sample period. Alternatively, for non-college-graduates there are larger declines in persistent earnings during unemployment spells, but the size of the increase is smaller than for college graduates.

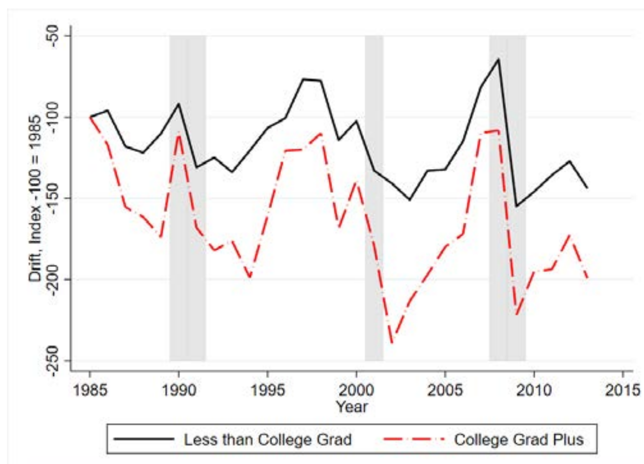
Taken together, the results presented in Figure 11 provide evidence that the increase in persistent earnings risk and greater scarring during unemployment is not a low skill phenomenon. We find that individuals with a college degree or higher have a more pronounced increase in persistent earnings risk and have seen a larger increase in persistent earnings declines during unemployment spells. We therefore rule out our first hypothesis that rising persistent earnings risk is attributable to low-skill, non-college-educated workers.

Figure 11: Changes in persistent earnings risk by education

(a) Shocks to persistent earnings (comb.) over time



(b) Drift of persistent earnings (unemp.) over time



Note: Figure presents parameter estimates from estimating the income process by education level. Panel (a) compares the shock to persistent earnings (combined for employed and unemployed) over time, while panel (b) shows the drift of persistent earnings while unemployed over time. The black, solid line represents individuals without a college degree, while the red, dashed line represents individuals with a college degree or higher.
Source: 1973, 1991, 1994, 1996-2016 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1982 to 2016.

We further explore differences across genders. In Figure 21 in Appendix G.3, we show that the trends in earnings risk documented above have occurred for both men and women. Therefore, our results are unlikely to be driven by declining employment prospects of low-skill men (e.g. Binder and Bound (2019)).

4.2 Geography

Next, we explore whether the rise in persistent earnings risk varies geographically. Our analysis tests theories of regional persistent earnings risk, such as those centered on declining manufacturing employment and weakened unions in the Rust Belt. Since individuals that initially live in hard-hit communities may move states, we split individuals across states using the earliest self-reported state of residence in the CPS survey. We estimate our income process separately across states in order to alleviate concerns regarding parameter restrictions. Our time period is from 1985 to 2013. It therefore includes 29 years of observations (a prime number), so there is no way to evenly divide the time period. To resolve this issue, we split the time period into 7-year windows (1985-1991, 1992-1998, 1999-2005, and 2006-2013), with the extra year going in the final time interval. We show results that compare the first time window, 1985-1991, with the last time window, 2006-2013. Our results are not sensitive to this choice of time windows.

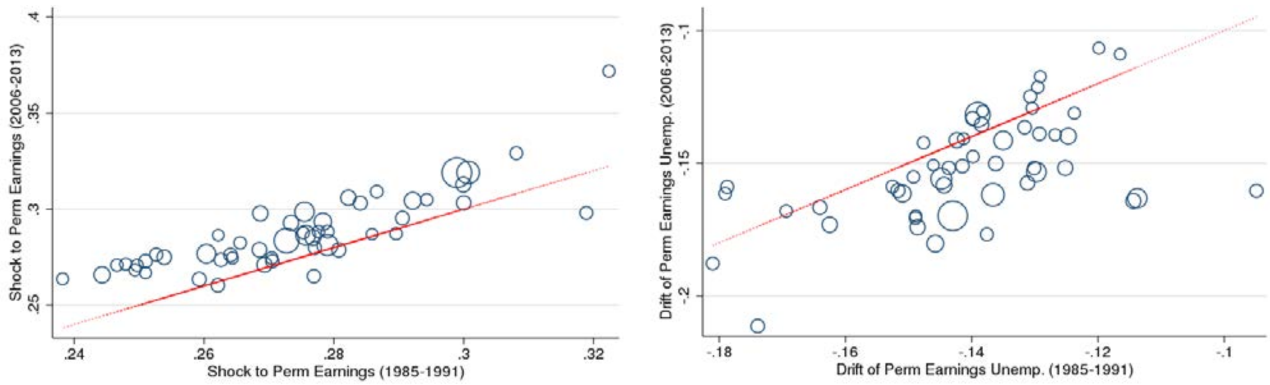
Figure 12 plots the change in our estimated earnings process parameters by state between 1985-1991 (x-axis) and 2006-2013 (y-axis), along with a 45-degree line (red, dashed line). Each circle represents a state, and the size of the circle corresponds to the relative population of the state in our sample. Panel (a) plots the change in persistent earnings risk, combined among both the employed and unemployed. In the vast majority of states, we estimate increasing earnings risk over time. Panel (b) plots the change in the drift of persistent earnings among the unemployed. There is considerably more variation across states, but fewer than half of the states had slower persistent earnings deterioration among the unemployed between 2006 and 2013 than they did between 1985 and 1991. These findings suggest that regional factors are unlikely to explain rising persistent earnings risk. All states followed a fairly similar path of rising persistent earnings risk between 1985 and 2013, with no clear outliers emerging in our analysis. More formally, in Appendix G.4, we show that changes in persistent earnings risk are largely uncorrelated with changes in state level union coverage and manufacturing employment. Thus, state-level theories of rising persistent earnings risk, such as declining manufacturing employment and/or declining rates of unionization in the Rust Belt, are unlikely to rationalize our results.

4.3 Occupations

In this section, we test our third and fourth hypotheses that rising persistent earnings risk is related to the skill content of occupations. Using an individual's earliest reported occupation in the CPS, we classify individuals into one of 334 time-consistent occupation codes developed

Figure 12: Changes in persistent earnings risk by U.S. state

(a) Shocks to persistent earnings (comb.) over time (b) Drift of persistent earnings (unemp.) over time



Note: Figure presents parameter estimates from estimating the income process by state. The x-axis shows the parameter estimate for the 1985-1991 time period, while the y-axis shows the parameter estimate for the 2006-2013 time period. Panel (a) compares the shock to persistent earnings (combined for employed and unemployed) over time, while panel (b) shows the drift of persistent earnings while unemployed over time.
Source: 1973, 1991, 1994, 1996-2016 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1982 to 2016.

by Autor and Dorn (2013).[50] We then estimate the parameters of our income process separately for each occupation.

In Figure 13, we plot how the volatility of earnings has changed between 1985-1991 (x-axis) and 2006-2013 (y-axis) for each occupation, along with a 45-degree line (red, dashed line). Each circle represents an occupation, and the size of the circle represents an occupation's relative employment share in our sample. Panel (a) shows that the vast majority of occupations experienced rising persistent earnings risk. However, there remains considerable variation across occupations. Similarly, Panel (b) shows that persistent earnings losses among the unemployed accelerated in the majority of occupations, but a significant share of occupations exhibited slower persistent earnings deterioration.

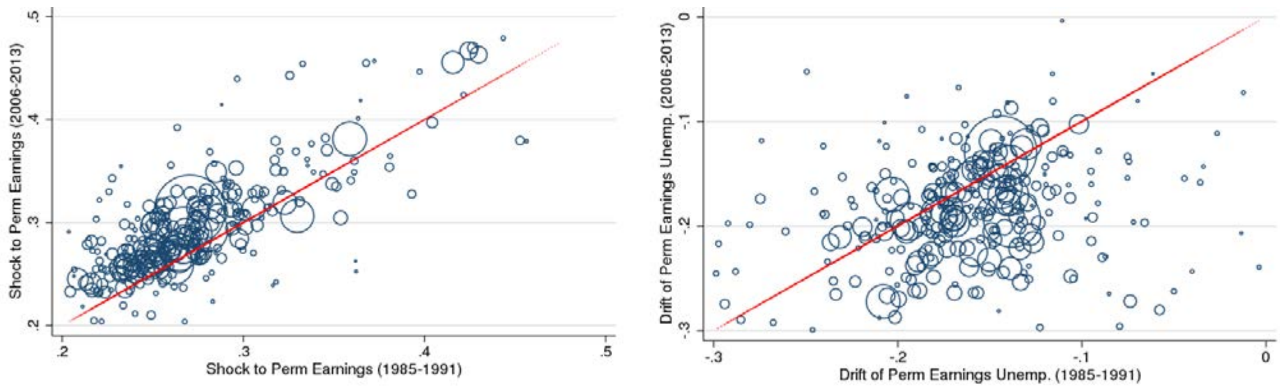
We next exploit variation across occupations to test the third and fourth hypotheses that rising persistent earnings risk is related to the skill-content of occupations. Let X_o be a measure of the skill-content of occupation o (e.g. measure of routine task content, non-routine cognitive task content, etc.). Let $\Delta Y_{o,j} = Y_{o,j} - Y_{o,(1985-1991)}$ denote the change in parameter Y (e.g., the standard deviations of shocks to persistent earnings among employed etc.) for occupation o between time period j and $1985-1991$.[51] Let γ_j denote a set of 7-year window fixed effects.

[50] We thank Bryan Seegmiller for creating the mapping from CPS occupation codes to the Autor and Dorn (2013) occupation codes.

[51] As in Section 4.2, we split the time period into 7-year windows (1985-1991, 1992-1998, 1999-2005, and 2006-

Figure 13: Changes in persistent earnings risk by occupation

(a) Shocks to persistent earnings (comb.) over time (b) Drift of persistent earnings (unemp.) over time



Note: Figure presents parameter estimates from estimating the income process by occupation. The x-axis shows the parameter estimate for the 1985-1991 time period, while the y-axis shows the parameter estimate for the 2006-2013 time period. Panel (a) compares the shock to persistent earnings (combined for employed and unemployed) over time, while panel (b) shows the drift of persistent earnings while unemployed over time.
Source: 1973, 1991, 1994, 1996-2016 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1982 to 2016.

The specification we use is of the form

$$\Delta Y_{o,j} = \alpha + \eta X_o + \gamma_j + \epsilon_{o,j}. \tag{8}$$

The parameter of interest is η which measures whether occupations with a greater task requirement X experienced a larger increase in parameter Y over the sample period. Hence, if $\eta > 0$ then an occupation with a greater amount of skill content X experienced a larger change in earnings risk over the sample period.

Routine Occupations. We first split occupations by their degree of routine skill content as measured by Acemoglu and Autor (2011) using O*NET data.[52] Table 3 presents the results of estimation equation (8) where the independent variable is the measure of the routine task content of an occupation.[53] Column (1) shows that higher routine-skill content in an occupa-

2013).

[52]Acemoglu and Autor (2011) provide a measure of the routine manual and routine cognitive task content of an occupation. We combine their estimates into a single measure of the routine task content of an occupation by averaging the two measures. As in Acemoglu and Autor (2011), we normalize the index to be mean zero and have unit variance. Results with routine manual and routine cognitive skills as the main independent variable are available upon request.

[53]Table 9 in Appendix G.5 shows the results of estimating equation (8) for different measures of earnings risk (e.g. standard deviation of shocks to persistent earnings among employed/unemployed, etc.).

Table 3: Routine Occupations and Changes in Earnings Risk

	(1) Chg. Std. Dev. Pers Comb.	(2) Chg. Mean Pers. Unemp.
Routine	-0.000611	0.00418**
	(0.000981)	(0.00172)
Year Fixed Effects	Yes	Yes
R-Sq.	0.123	0.109
No. Obs. (Occ.)	1000	1000

Notes: Table presents results from estimating equation (8) where the independent variable is the measure of routine task content of an occupation from Acemoglu and Autor (2011). The measure of routine task content is normalized to have mean zero and unit variance. Clustered SE in parenthesis, where the clustering is performed at the occupation level. ***$p < 0.01$, **$p < 0.05$,*$p < 0.1$. *Source: 1973, 1991, 1994, 1996-2016 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1982 to 2016.*

tion is not correlated with the change in the standard deviation of combined shocks to persistent earnings. Additionally, Column (2) shows that compared with non-routine occupations, occupations with greater routine task content have experienced *smaller* declines in persistent earnings during unemployment spells over time. Putting these results together, we conclude that workers in routine occupations are not driving the increase in persistent earnings risk over time. This is consistent with theories in which routine workers are highly substitutable and non-specialized (e.g., Edmond and Mongey (2019)), implying that they face low mean wages but very little earnings risk.

High Skill Workers. In this section, we examine the hypothesis that the rise in persistent earnings risk has occurred among high skill workers. We test this high skill workers hypotheses by using three measures an occupation's skill intensity. We first split occupations by their degree of "Non-Routine Cognitive Analysis" skills (henceforth, *non-routine cognitive skills*) as measured by Acemoglu and Autor (2011) using O*NET data.[54] We additionally measure the degree to which an occupation is high-skill by using its mean years of completed education and mean (log) earnings.[55] To ease the comparison across measures, we normalize each measure to have mean zero and unit standard deviation.

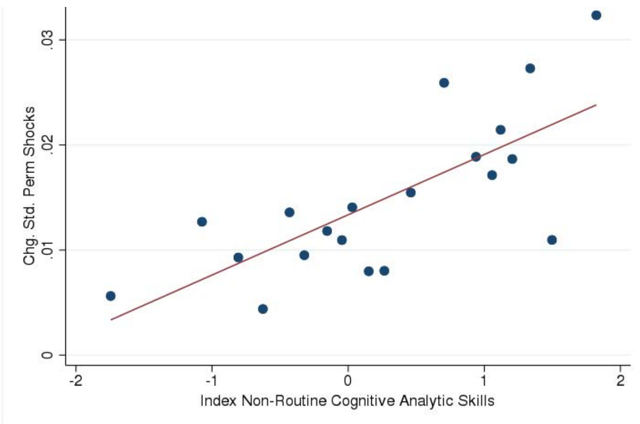
Figure 14 graphically represents the regression in equation (8) where the independent vari-

[54]This measure is created from the O*NET tasks measures on the importance of: (1) analyzing data/information, (2) thinking creatively, and (3) interpreting information for others. The index is constructed to be mean zero and have unit variance. The measure is derived from O*NET vintage 14.0, which contains data collected between 2002 and 2009.

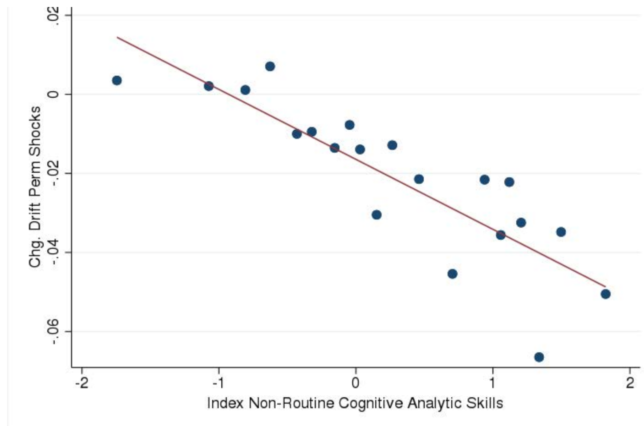
[55]We measure average years of completed education and average (log) earnings in the years 1985-1991.

Figure 14: Changes in earnings risk by non-routine cognitive skills

(a) Chg. std. dev. persistent earnings (Comb.)



(b) Chg. drift persistent earnings (Unemp.)



Note: Figure presents a graphical representation of the regression in equation (8), where the measure of task content is non-routine congitive analytic skills as measured in Acemoglu and Autor (2011).
Source: 1973, 1991, 1994, 1996-2016 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1982 to 2016.

able is the non-routine cognitive skill content of an occupation. Panel (a) plots the change in combined persistent earnings risk among the employed between 1985-1991 and 2006-2013 on the y-axis, and the workers' ventile of cognitive skill requirements on the x-axis. The figure shows that occupations with a greater amount of non-routine cognitive skill content have seen a larger increase in combined persistent earnings risk. Over the time interval, the standard deviation of combined persistent earnings risk among those in the bottom ventile of cognitive skill requirements increased by 0.5 percentage points between 1985-1991 and 2006-2013. Persistent earnings risk for those in the top ventile increased by 3 percentage points over the same time period, a factor of 6 larger than the bottom ventile.

In addition to greater combined persistent earnings risk, higher skill occupations have seen a larger decline in persistent earnings during spells of unemployment. Panel (b) shows that the acceleration of persistent earnings losses among the unemployed is particularly pronounced among those with the highest cognitive skill requirements. Between 1985-1991 and 2006-2013, those in the bottom ventile of cognitive skills had no change in their persistent earnings drift while unemployed. However, persistent earnings among those in the top ventile of cognitive skills deteriorated 6 percentage points per annum faster in the 2006-2013 time period.

In Table 10 in Appendix G.5, we present formal regression results for Figure 14. Consistent with the graphical evidence, occupations with greater non-routine cognitive skill content have experienced (1) a larger increase in the combined standard deviation of persistent earnings shocks, as well as (2) a larger decline in persistent earnings while unemployed. In Appendix G.5 we show that we obtain similar results when using average (log) earnings (Table 12) as well as years of completed education (Table 13) as our measure of the high skill content of an occupation. Putting these results together, we conclude that the increase in persistent earnings risk since the 1980s has been a high skill worker phenomenon.

Finally, we examine a potential mechanism for why high skill workers experience a larger increase in persistent earnings risk. Previous work has shown that high skill workers are more exposed to the introduction of new, skill-biased technologies (e.g., Krueger (1993) and Deming and Noray (2020)).[56] New technologies allow workers to increase their output, but they also require workers to have new skills to perform their job. Hence, for workers with the sufficient skill to use the new technology their output increases, which increases their wages. For workers who do not have the skills to use the new technology, the demand for their services declines in their original occupation (or the worker has to move to another occupation where their skills are still employable), lowering their wages.[57]

We test this mechanism by exploiting variation in the introduction of new technologies across occupations. In particular, we measure changes in persistent earnings risk among occupations that adopted greater computer and software skill requirements by 2010. Since computers were not prevalent in the workplace during the 1980s, individuals in these occupations faced a greater degree of new technology introduction.[58] We use detailed skill requirements from online vacancies collected by Burning Glass Technologies to measure computer use in an occupation.[59] As in Hershbein and Kahn (2018) and Braxton and Taska (2020), we measure the degree of computer use in an occupation by measuring the share of vacancies that list a computer or software requirement.[60] To facilitate comparison to our other measures of occupation skill content, we normalize the measure of computer skills to have mean zero and standard deviation equal to one.

[56]Krueger (1993) provides evidence that the computer revolution of the workplace was more pronounced for high skill workers. Recently, Deming and Noray (2020) showed that there are greater changes in the skill requirements (a proxy for technologies used by firms) of jobs over time for workers with technology intensive college majors, e.g. science, technology, and business.

[57]Consistent with this mechanism Braxton and Taska (2020) shows that workers displaced from occupations that have experienced a greater increase in computer and software requirements suffer larger earnings losses. Additionally, Kogan, Papanikolaou, Schmidt, and Song (2020) find that within industry increases in the rate of innovation are associated with substantial increases in earnings risk, particularly for high income workers.

[58]From Card and DiNardo (2002), "... many observers date the beginning of the *computer revolution* to the introduction of the IBM-PC in 1981. This was followed by the IBM-XT (the first PC with built-in disk storage) in 1982, and the IBM-AT in 1984."

[59]We follow recent papers by Hershbein and Kahn (2018) and Atalay et al. (2020), which argue that the skill requirements in vacancies are informative on the technology of the firm posting the vacancy.

[60]See Hershbein and Kahn (2018) and Braxton and Taska (2020) for more details on the Burning Glass database.

Table 4: Computer skills and changes in earnings risk

	(1) Chg. Std. Dev. Perm Comb.	(2) Chg. Mean Perm. Unemp.
Computer Skills	0.00717***	-0.0145***
	(0.00132)	(0.00296)
Year Fixed Effects	Yes	Yes
R-Sq.	0.229	0.176
No. Obs. (Occ.)	1000	1000

Note: Table presents results from estimating equation (8) where the independent variable is computer skills as measured in Braxton and Taska (2020). Computer skills are measured for each occupation in 2010 and have been normalized to have mean zero and unit variance. Clustered standard errors in parenthesis where the clustering is performed at the occupation level. $***p < 0.01, **p < 0.05, *p < 0.1$

Source: 1973, 1991, 1994, 1996-2016 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1982 to 2016.

Table 11 presents the results of estimating equation (8) where the independent variable is computer skills in 2010. We find that computer use is a strong predictor of the increase in persistent earnings risk. In particular, Table 11 shows that occupations with a greater amount of computer skill requirements have experienced (1) a larger increase in combined persistent earnings risk (Column (1)), and (2) larger persistent earnings losses while unemployed (Column (2)).[61]

We have shown that since the 1980s, there has been a steady increase in persistent earnings risk. Additionally, the average size of persistent earnings losses while unemployed has increased by nearly 50%. The results of this section show that this increase in persistent earnings risk has been a high skill worker phenomenon. In the next section, we quantify the welfare implications of the observed changes in earnings risk over time.

5 Welfare effects of changing earnings risk

In this section, we use a finite lifecycle Bewley-Huggett-Aiyagari model to examine the welfare effects of changes in earnings volatility between 1985 and 2013.

[61] In Appendix G.5, Table 11, we show the results for estimating equation (8) for all measures of earnings risk where the independent variable is computer skills in 2010.

5.1 Steady State Model

In this section, we introduce a steady state version of a finite lifecycle Bewley-Huggett-Aiyagari model. We assume there are $T \geq 2$ overlapping generations of agents, and let $t \in \{1, ..., T\}$ denote the age of an agent. Agents exit the model exogenously at age T, and there is no retirement. Let i denote an agent's type, and in the estimation that follows, we assume types map to educational attainment. An individual's type is fixed indefinitely, and let $\pi_i \in [0, 1]$ denote the share of agents that are type i.

Agents are heterogeneous along several dimensions. Let $e \in \{E, U\}$ denote the employment status of an agent, where $e = E$ denotes employed and $e = U$ denotes unemployed. Let $b \in \mathbb{R}$ denote the net asset position of an agent. When $b > 0$, the agent is saving, and when $b < 0$, the agent is borrowing. The agent's asset choice is constrained by a borrowing limit $\underline{b}$. Agents save and borrow at the risk-free rate, denoted r_f. Let $z \in \mathbb{R}$ denote an agent's persistent earnings. Let $\epsilon \in \mathbb{R}$ denote an agent's temporary shock to earnings.

At the start of each period, agents observe their employment status, as well as the shocks to persistent and temporary earnings. Let $\delta_i(z, e) \in [0, 1]$ denote the probability that an agent becomes unemployed. The probability that an agent becomes unemployed depends upon their persistent earnings and employment status from the prior period. In Section 5.2 we discuss how we estimate the function $\delta_i(z, e)$ using the filtered estimates of persistent earnings $\hat{z}_{i,t-1}$ and realizations of earnings that fall below the minimum earnings threshold (i.e., unemployment).

Let $w_{i,t}(z, \epsilon, e)$ be a function that maps an individual's (i) type, (ii) age, (iii) persistent earnings, (iv) temporary shock, and (v) employment status into a wage. We define the wage $w_{i,t}(z, \epsilon, e)$ such that

$$w_{i,t}(z, \epsilon, e) = \begin{cases} \exp(\kappa_{i,t} + z + \epsilon) & \text{if } e = E \\ \gamma \exp(\kappa_{i,t}) & \text{if } e = U, \end{cases}$$

where $\kappa_{i,t}$ is a deterministic age profile of log earnings. $\gamma \in [0, 1]$ can be thought of as a replacement rate of persistent income for the unemployed (alternatively, one can model home production as proportional to z, consistent with our empirical interpretation of z as capturing "income risk" among the unemployed – see Appendix H for such a model). Wages are subject to labor income taxation. Let $\widetilde{w}_{i,t}(z, \epsilon, e)$ denote the after tax income for an age t agent with persistent earnings z, temporary shock ϵ and employment status e. We model taxes following Heathcote, Storesletten, and Violante (2017), where after-tax income is given by

$$\widetilde{w}_{i,t}(z, \epsilon, e) = \lambda w_{i,t}(z, \epsilon, e)^{1-\alpha}.$$

The parameter $\alpha > 0$ governs the degree of tax progressivity.

Finally, when an agent enters into the labor market they start as an employed agent and they draw their persistent earnings from a normal distribution with mean zero and variance $u_{z0,i}$.

Value Functions. We next define the value function for agents in the model. We write the value function for agents after the shocks to employment status as well as those to temporary and persistent earnings have been realized. Let $V_{i,t}(b, z, \epsilon, e)$ denote the value of being an age t, type i agent with employment status e, persistent earnings z and temporary earnings e.[62] The agent makes a consumption savings decision in the current period, taking into account the set of potential shocks to income in the next period. The value function for an age t agent is given by,

$$V_{i,t}(b, z, \epsilon, e) = \max_{c, b^{'} \geq \underline{b}} u(c) + \beta \mathbb{E}_{z^{'}, \epsilon^{'}, e^{'}} \left[V_{i,t+1}(b^{'}, z^{'}, \epsilon^{'}, e^{'}) \right] \quad \forall t \leq T$$

$$V_{i,T+1}(b, z, \epsilon, e) = 0,$$

subject to the budget constraint,

$$c + b^{'} \leq b(1 + r_f) + \widetilde{w}_{i,t}(z, \epsilon, e);$$

the law of motion for employment status,

$$e^{'} = \begin{cases} E & \text{w. prob } 1 - \delta_i(z, e) \\ U & \text{w. prob } \delta_i(z, e); \end{cases}$$

and the law of motion for persistent earnings,

$$z^{'} = \begin{cases} F_i z + \nu_{E,i,t+1} & \text{w. prob } 1 - \delta_i(z, e) \\ F_i z + \nu_{U,i,t+1} & \text{w. prob } \delta_i(z, e), \end{cases}$$

where $\nu_{e,i,t+1} \sim N(B_{e,i}, Q_{e,i,t+1})$. In addition to type, the mean of the shock depends on an individual's employment status (employed vs. unemployed), and the variance to the shock depends on the agent's employment status and age.

Finally, the law of motion for temporary earnings is given by,

[62] Note for unemployed the value of temporary earnings ϵ is irrelevant.

$$\epsilon' = \begin{cases} \epsilon_{i,t+1} & \text{w. prob } 1 - \delta_i(z,e) \\ 0 & \text{w. prob } \delta_i(z,e), \end{cases}$$

where $\epsilon_{i,t+1} \sim N(0, R_{i,t+1})$. The variance of the shock to temporary earnings depends on the agent's age and type.

5.2 Estimation

We next discuss the estimation of the model.[63] Some parameters are assigned using estimates from the literature, while others are calibrated to be consistent with the U.S. labor market in 1985.

Demographics and preferences. To align with the sample in Section 2, agents enter the model at age 25 ($t = 1$) , and work until age 60 ($T = 36$). When agents enter the model, they begin with zero assets and are employed. Upon entering the model, agents learn their type, and types are fixed indefinitely. There are three types of agents in the economy: (1) agents without a college degree, (2) agents with a college degree or higher, and (3) agents with unknown education.[64] We set the shares of each type (π_i) to be consistent with each group's share in the sample in 1985. The first row of Panel (b) in Table 5 presents the share of agents by type in the initial steady state.

Agents receive utility from consumption, with preferences given by

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}.$$

We set the risk aversion parameter to a standard value, $\sigma = 2$. Agents discount the future at rate $\beta = 0.977$. The parameter β is calibrated to match the median ratio of net worth to income. In the SCF, we measure this ratio to be 1.82.

Income process. Agents receive wages that are a function of their age, persistent earnings, and temporary earnings. The fixed age component of earnings is estimated as part of the residualization process in Section 4.1. Panel (a) of Appendix Figure 15 plots the deterministic path of earnings that is used in the model.

[63] In Appendix E, we discuss how we solve the model numerically.

[64] In the data, we only classify an individuals level of education if they are over the age of 25 when in the CPS. The third group is comprised of individuals who were in the CPS before the age of 25, for whom we do not classify their level of education. We include them in the model to align aggregate statistics in the model economy with our empirical sample.

When agents are unemployed they receive (pre-tax and transfers) a fraction $\gamma \in [0,1]$ of their persistent earnings. The parameter γ can be thought as the replacement rate of unemployment insurance. We set $\gamma = 0.4$, as in Shimer (2005).

We next discuss the estimation of the stochastic process that governs how earnings evolve in the model.

Shocks to income. The model's initial steady state is consistent with the 1985 values of the income process estimated in Section 4.1, where the income process is estimated separately for individuals by their level of education. The income process allows for the standard deviation of shocks to temporary and persistent income to be a function of age and time. In Panels (b) –(d) of Appendix Figure 15, we plot the shocks to persistent earnings while employed and unemployed, as well as shocks to temporary earnings over the lifecycle that are fed into the model. The figures show that agents of all types face (1) a U-shaped age profile of persistent earnings risk while employed (Panel (b)), (2) a decreasing age profile of temporary earnings risk (Panel (c)), and (3) an increasing profile of persistent earnings risk while unemployed (Panel (d)). The income process additionally allows for the drift of persistent earnings to depend upon employment status and type. The third and fourth rows of Panel (b) in Table 5 present the drift parameters while employed (row 3) and unemployed (row 4). For each type of agent, there is a modest average increase in persistent earnings while employed and a large average decline in persistent earnings while unemployed.

Probability of unemployment. We next discuss the estimation of the unemployment probability function $\delta_i(z,e)$. We use a functional form that allows an agent's unemployment probability to flexibly depend on their education, prior persistent earnings, and employment status. The specification we use is of the form

$$\delta_i(z,e) = \begin{cases} \mathbb{I}\{z \geq 0\} \left[\sum_{k=0}^{2} \alpha_{i,k,E}^{+} z^k\right] + \mathbb{I}\{z < 0\} \left[\sum_{k=0}^{2} \alpha_{i,k,E}^{-} z^k\right] & e = E \\ \alpha_{i,U} & e = U, \end{cases} \tag{9}$$

The functional form in equation (9) allows for the unemployment probability of the employed to be a quadratic function of prior persistent earnings (z) estimated separately for positive prior persistent earnings or negative prior persistent earnings. The persistence of unemployment is a education-specific constant.[65] We estimate equation (9) separately for each education group,

[65]When we estimated the quadratic function in equation (9) for the unemployed, we obtained results that were consistent with simply using a constant function by education.

using our filtered estimates of persistent earnings and individual realizations of being below the minimum earnings criteria.[66] Panel (e) of Appendix Figure 15 presents the implied probabilities of becoming unemployed by type and prior persistent earnings for individuals who are employed in the prior period. The figure shows that individuals with lower prior persistent earnings face significantly higher probabilities of becoming unemployed. Additionally, relative to non-college graduates, college graduates face a significantly lower probability of becoming unemployed. The second row of Panel (b) in Table 5 contains the parameter estimates that govern the probability of remaining unemployed by education level ($\alpha_{i,U}$).

Taxes. We model taxes as in Heathcote et al. (2017). As in Heathcote et al. (2017), we set the tax progressivity (α) parameters to be equal to 0.181. In addition to financing the UI system, we model the government as having exogenous expenditures G that are equal to share $g \in [0, 1]$ of before tax labor income. Using NIPA data on personal income and government consumption expenditure and investment, we set $g = 0.264$. We set the level parameter (λ) so that government revenue from taxes is equated to government spending on transfers and the exogenous government spending. Panel (f) of Appendix Figure 15 presents the implied tax function in the model economy. Agents with pre-tax incomes below approximately \$10K receive transfers from the government, while individuals with pre-tax incomes greater than \$10K pay labor income taxes.

Asset Markets. Agents are able to save and borrow at the risk-free rate of 4%. We set the borrowing limit $\underline{b}$ to the "natural borrowing limit," which requires that individuals exit the model with zero debt. Setting the borrowing limit at the natural borrowing limit represents an upper bound to the extent to which agents can use borrowing to smooth shocks to income.

Table 5 and Figure 15 in Appendix E.3 present the parameters that govern model economy. In the next section, we conduct the welfare experiment of adjusting labor income risk as documented in Section 3.

5.3 Welfare implications of changing earnings risk

In Section 3, we documented four facts about the changing nature of earnings risk in the U.S. over the past 30 years. We documented that (1) the standard deviation of persistent shocks

[66] In the data we use the lag of an indiviudals estimated persistent earnings ($\hat{z}^k_{i,t-1}$), and the realization of an individuals earnings above or below the minimum earnings cutoff to estimate equation (9). To maximize statistical power, we estimate equation (9) on all sample years, but include year fixed effects in the estimation to obtain a set of parameters for our initial steady state.

Table 5: Parameters

Variable	Type 1	Type 2	Type 3	Description
Panel A: Parameters fixed across types				
Variable	**Value**			**Description**
β	0.977			Discount factor
r_f	4%			Risk-free interest rate
σ	2			Coefficient of relative risk-aversion
α	0.181			Progessivity of tax function
γ	0.4			Replacement Rate UI
g	0.264			Ratio of government expenditure to pre-tax income
T	36			Number of years in labor market
Panel B: Income parameters by type				
Variable	**Type 1**	**Type 2**	**Type 3**	**Description**
π_i	0.687	0.287	0.026	Shares of agents
F_i	0.925	0.937	0.930	Persistence
$\delta_{i,U}$	0.389	0.413	0.379	Unemp. prob., unemp. in prior year
$B_{E,i}$	0.020	0.022	0.016	Drift while employed
$B_{U,i}$	-0.123	-0.110	-0.142	Drift while unemployed

Note: Table presents model parameters. Panel (A) shows parameters that do not depend upon type, and Panel (b) shows parameters of the income process that vary by type. In Panel (B), Type 1 refers to agents who have less than a college degree, Type 2 refers to individuals with a college degree or higher, and Type 3 refers to individuals for whom we do not classify a level of education.

while employed has increased; (2) the standard deviation of temporary shocks has declined; (3) the standard deviation of persistent shocks while unemployed has increased; (4) the decline in persistent earnings while unemployed has worsened. In this section, we use the calibrated model to assess the welfare implications of these changes in earnings risk across steady states of the model by updating each of these components of income risk to their 2013 value.

Changes in standard deviation of shocks. We start by discussing the welfare effects of changes in the standard deviation of shocks (i.e., Q_E, Q_U, and R). In order to isolate the role of increased risk when we counterfactually change the standard deviation of shocks, we must adjust the age earnings profiles for each type of agent so that mean earnings are equated between the counterfactual economy and the baseline economy.

We first increase the standard deviation of shocks to persistent earnings of the employed (Q_E) from its 1985 value to its 2013 value.[67] Column (2) of Table 6 presents the predictions of

[67] Moving from the 1985 to 2013 level of earnings alters the path of earnings risk over the life cycle as shown in Figure 16. Changes in earnings risk is specific to level of education and is determined from the year fixed effects recovered as part of the estimation in Section 4.1.

the calibrated model from increasing the standard deviation of shocks to persistent earnings for the employed. Across all agents in the economy, the standard deviation of shocks to persistent earnings while employed increases from 0.21 to 0.225. This increase in persistent earnings risk causes an average welfare decline of over 1.75% of lifetime consumption. These welfare losses occur in part as agents accumulate greater precautionary savings to insure against future income risk. From the increase in persistent earnings risk, the median asset-to-income ratio in the economy increases by over 5 percent, from 1.82 to 1.91. Additionally, the welfare loss is heterogeneous across types (i.e., across levels of education). Agents with a college degree or higher (type 2) experience the largest welfare losses of nearly −3.3% of lifetime consumption, while individuals without a college degree have earnings losses of −1.1% of lifetime consumption.

Table 6: Welfare experiment: changes in earnings risk

	(1) Baseline	(2) Q_E	(3) Q_U	(4) R	(5) Q_E, Q_U, R	(6) B_U
Mean Welfare		−1.773%	−0.691%	0.033%	−2.401%	−3.33%
Mean Welfare Type 1		−1.116%	−0.266%	0.018%	−1.353%	−2.837%
Mean Welfare Type 2		−3.262%	−1.745%	0.059%	−4.895%	−4.615%
Mean Welfare Type 3		−3.009%	−0.474%	0.171%	−3.069%	−2.459%
Std. Dev. Persistent (Emp.)	0.21	0.225	0.21	0.21	0.225	0.21
Std. Dev. Persistent (Unemp.)	0.407	0.407	0.463	0.407	0.463	0.407
Std. Dev. Temporary	0.161	0.161	0.161	0.149	0.149	0.161
Drift Persistent Employed	0.020	0.020	0.020	0.020	0.020	0.020
Drift Persistent Unemployed	-0.122	-0.121	-0.121	-0.122	-0.121	-0.187
Avg. Earnings Type 1	39800	39800	39800	39800	39800	39100
Avg. Earnings Type 2	71300	71300	71300	71300	71300	68700
Avg. Earnings Type 3	49100	49100	49100	49100	49100	48100
Assets/Income	1.816	1.912	1.868	1.811	1.955	2.005

Note: Table presents the results of the steady state welfare experiment. Column (1) presents the baseline estimation of the model where the parameters of the income process are set at their 1985 value. Column (2) updates the standard deviation of shocks to persistent earnings among the employed (Q_E) to its 2013 values, holding all other parameters fixed. Column (3) updates the standard deviation of shocks to persistent earnings among the unemployed (Q_U) to its 2013 value, holding all other parameters fixed. Column (4) updates the standard deviation of shocks to temporary earnings among the unemployed (R) to its 2013 value, holding all other parameters fixed. Column (5) updates the parameters Q_E, Q_U and Q_R to their 2013 values. Column (6) updates the drift of persistent earnings among the unemployed (B_U) to its 2013 value. Type 1 corresponds to agents without a college degree, type 2 corresponds to a college degree or higher, and type 3 corresponds to agents whom we do not classify with a level of education. In Appendix F we discuss our measure of welfare.

In Column (3) of Table 6, we examine the welfare effects of increases in the standard deviation of shocks to persistent earnings among the unemployed. Increasing the standard deviation of shocks to persistent earnings among the unemployed generates an average welfare loss of -0.7% of lifetime consumption. The welfare losses of increased persistent earnings risk are smaller because only a fraction of the population is unemployed. However, the greater risk associated with becoming unemployed creates a motive for accumulating more precautionary savings. The results of columns (2) and (3) show that increased persistent earnings risk has generated a substantial welfare decline. Conversely, in Column (4), we decrease shocks to temporary earnings, and we find that there is a small welfare gain of only 0.03% of lifetime consumption.

Finally, we introduce all three changes in earnings risk into the simulated economy, shown in Column (5). Increased persistent earnings risk for both the employed and unemployed, along with the decline in the standard deviation of shocks to temporary earnings, generates an average welfare decline of 2.4%. Since increased persistent earnings risk is concentrated among the college educated (Section 4.1), we find the largest welfare losses among these agents, with a welfare loss of nearly -4.9% of lifetime consumption. Therefore, rising persistent earnings risk has generated substantial welfare losses that are not mitigated by declining temporary earnings risk. We next examine how changes in the means of persistent earnings shocks impact welfare.

Changes in mean of shocks. In this section, we examine the welfare implications of changes in the mean (i.e., drifts) of shocks to persistent earnings among the unemployed. We decrease the drift of persistent earnings to the unemployed (B_U) from its 1985 value to its 2013 value. Averaging across all agents in the economy, persistent earnings drift of the unemployed decreases from -0.121 to -0.187. Column (6) of Table 6 shows that accelerating persistent earnings losses among the unemployed causes a welfare loss of 3.3% of lifetime consumption. These welfare losses occur in part because of an increase in precautionary savings, as the median asset to income ratio increases by approximately 10% from 1.82 to 2.

In summary, changes in both the standard deviation and mean of persistent earnings shocks over the past 30 years have generated significant welfare losses across all education groups. But rising persistent earnings risk for college-educated workers and faster persistent earnings losses among the unemployed contribute disproportionately to these welfare losses.

6 Conclusion

How have temporary and persistent earnings risk changed over time and why? By answering these questions our paper makes several contributions. First, we present a method that estimates parameters of the income process while also recovering temporary and persistent earnings at the individual level. We flexibly allow for arbitrary spells of zero earnings, allowing us to include estimates of persistent earnings risk for individuals who would otherwise be omitted using alternative methods, as well as a potentially high dimensional set of observable variables. We provide a simple persistent-temporary income process that allows for zeros – thus incorporating skewness and kurtosis – and can easily be integrated into heterogeneous agent models. As a practical contribution, our method of estimating persistent and temporary earnings risk on 23.5 million records can be completed in 3 hours on the Census servers.

Second, we estimate our income process on administrative earnings records that have been linked to survey responses from the CPS ASEC to examine how and why persistent and temporary earnings risk have changed over time. Our analysis of the time series yields four facts: (1) the standard deviation of persistent shocks while employed has increased; (2) the standard deviation of temporary shocks has declined; (3) the standard deviation of persistent shocks while unemployed has increased; (4) the decline in persistent earnings while unemployed has worsened. Of particular note, between 1985 and 2013, the downward drift of persistent earnings among the unemployed fell by 6 percentage points (i.e. it became 57% more negative).

We then evaluate various hypotheses for the rise in persistent earnings risk between 1985 and 2013. We rule out hypotheses related to low skill workers' declining employment prospects and regional theories of persistent earnings losses, including the decline of the Rust Belt. Using information on the skill content of occupations (e.g., Acemoglu and Autor (2011)), we additionally rule out theories related to declining employment and wages in routine occupations. Instead, we show that the increase in persistent earnings risk is a high skill worker phenomenon.

Our final contribution is to examine the welfare and macroeconomic effects of changing earnings risk over time. We show that the parameter estimates from our filtering method are easily incorporated into a Bewley-Huggett-Aiyagari model of incomplete markets. While all sources of rising persistent earnings risk generate welfare losses, we find that the acceleration of persistent earnings losses while unemployed generated the largest losses.

This paper is the first step of a research agenda that aims to open the black box of earnings dynamics. By recovering shocks to persistent and temporary earnings for *every* person in *every* period, the method presented in this paper can be used to further understand the factors that shape earnings at the individual level as well as how individuals respond to temporary and persistent shocks. Additionally, by allowing the parameters of our income process to be fairly arbitrary linear combinations of functions of observed data, our framework offers a practical approach to studying the relationship between income risk and other firm/industry/regional-level outcomes. For instance, it is straightforward to extend our approach to allow for firm-level shocks to pass through to workers' expected level of permanent income (e.g., Guiso et al., 2005; Lamadon et al., 2019; Chan et al., 2020) and income risk (Kogan et al., 2020). Likewise, consistent with our analysis in Section 4, one could consider the role of various shocks to local labor markets (e.g., Autor, Dorn, Hanson, and Song (2014)) in affecting workers' persistent and transitory income dynamics.

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A Data and Estimation Details

A.1 Residualzing Earnings

We remove the common age component of earnings (residualizing) as in Guvenen et al. (2014). Using all earnings observations from the base sample, we run a pooled regression of earnings on age and cohort dummies without a constant. This regression recovers the age profile of log earnings. We then scale the age dummies so as to match the average log earnings of 25-year-olds used in the regression. We subtract the age dummies from earnings to recover residualized earnings.

B Kalman Filter

In this Appendix, we present the Kalman Filter, which we use to recover estimates of temporary and persistent earnings at the individual level. For now assume that the parameters which govern the income process are all known. In Section C we discuss how we use an EM algorithm to recover the parameters that govern the income process.

In practice we make the state variable the current realization of persistent earnings as well as its lag. Let $\zeta_{i,t}$ denote an individual i's unobserved state in period t:

$$\zeta_{i,t} = \begin{bmatrix} z_{i,t} \\ z_{i,t-1} \end{bmatrix}$$

where $z_{i,t}$ is persistent earnings of individual i in period t. Based off of the law of motion in equation (2), the state vector $\zeta_{i,t}$ evolves according to the following law of motion (*state equation*),

$$\zeta_{i,t} = \begin{bmatrix} z_{i,t} \\ z_{i,t-1} \end{bmatrix} = \begin{bmatrix} B(l_{i,t}) \\ 0 \end{bmatrix} + \underbrace{\begin{bmatrix} F & 0 \\ 1 & 0 \end{bmatrix}}_{\hat{F}} \zeta_{i,t-1} + \begin{bmatrix} \nu_{i,t} \\ 0 \end{bmatrix} \tag{10}$$

Using the definition of the state vector $\zeta_{i,t}$ and the income process specified in equation (1), labor income evolves according to the following equation while employed (*measurement equation*),

$$y_{i,t} = H(l_{i,t})\zeta_{i,t} + l_{E,i,t}\, \omega_{i,t} \tag{11}$$

where $H(l_{i,t}) = \begin{bmatrix} l_{E,i,t} & 0 \end{bmatrix}$ governs the relationship between the state vector $(\zeta_{i,t})$ and earnings

$y_{i,t}$ among individuals who are employed ($l_{E,i,t} = 1$).[68]

For now assume that F, Q_E, Q_U, B_E, B_U, H_E, H_U, R_E and R_U are all known. Starting with estimates $\hat{\zeta}_{i,1|0}$ and $M_{i,1|0}$, which we will define below, we obtain our desired time series for $\zeta_{i,t}$ as follows:

1. Estimate the "Kalman Gain":

$$K_{i,t} = M_{i,t|t-1}H^{'}(l_{i,t})\left[H(l_{i,t})M_{i,t|t-1}H^{'}(l_{i,t}) + R(l_{i,t})\right]^{-1}$$

2. Update the state vector:

$$\hat{\zeta}_{i,t|t} = \hat{\zeta}_{i,t|t-1} + K_{i,t}\left(y_{i,t} - H(l_t)\hat{\zeta}_{i,t|t-1}\right)$$
$$\hat{\zeta}_{i,t+1|t} = \underbrace{\begin{bmatrix} F & 0 \\ 1 & 0 \end{bmatrix}}_{\hat{F}}\hat{\zeta}_{i,t|t} + \begin{bmatrix} B(l_{i,t}) \\ 0 \end{bmatrix}$$

3. Update the MSE matrix:

$$\begin{aligned} M_{i,t|t} &= M_{i,t|t-1} - K_{i,t}H(l_t)M_{i,t|t-1} \\ M_{i,t+1|t} &= \hat{F}M_{i,t|t}\hat{F}' + Q(l_{t+1})e_1^2 e_1^{2'} \end{aligned}$$

where $e_1^2 = [1,0]'$. Repeat steps 1-3 for $t = 2, ...T$, and for each individual $i \in \{1, .., N\}$.

Setting initial value. To run the Kalman Filter we need an initial estimate of the mean of the state-vector and the variance-covariance matrix. We set the initial mean of the state vector to zero. The initial variance of the state vector is given by:

$$M_{i,1|0} = var\left[\zeta_0|X\right] = \begin{bmatrix} var(z_{i,0}) & 0 \\ 0 & 0 \end{bmatrix}$$

where:

$$var(z_{i0}) = var(u_{z,i0})$$

[68]When an agent is unemployed ($l_{E,i,t} = 0$), the value of the observation $y_{i,t}$ provides no additional signal about latent earnings other than what can be inferred from other observables, so the Kalman filter will not directly use $y_{i,t}$ to update its guess about $z_{i,t}$.

Smoothed Kalman Filter. Hamilton (1994b) comments that when the value of the state vector is of interest in its own right, as in our application, we can improve the inference about the historical values of the state vector took in the middle of the sample by using the smoothed filter. The goal of this section will be to take the sequences we found from the Kalman filter above and estimate the smoothed sequence, which we denote $\{\{\hat{\zeta}_{i,t|T}\}_{t=1}^{T}\}_{i=1}^{N}$.

The steps for the smoothed Kalman filter are:

1. Run the Kalman Filter as presented above storing the sequences $\{M_{i,t|t-1}\}_{t=1}^{T}$ and $\{M_{i,t|t}\}_{t=1}^{T}$ as well as $\{\hat{\zeta}_{i,t|t-1}\}_{t=1}^{T}$ and $\{\hat{\zeta}_{i,t|t}\}_{t=1}^{T}$.

    (a) Notice that we will not use the estimates $\hat{\zeta}_{i,T+1|T}$ and $M_{i,T+1|T}$.

2. Store the element $\hat{\zeta}_{i,T|T}$ from $\{\hat{\zeta}_{i,t|t}\}_{t=1}^{T}$.

3. Calculate the sequence of smoothed estimations $\{\hat{\zeta}_{i,t|T}\}_{t=1}^{T-1}$ in reverse order by iterating on:

$$\hat{\zeta}_{i,t|T} = \hat{\zeta}_{i,t|t} + J_{i,t}(\hat{\zeta}_{i,t+1|T} - \hat{\zeta}_{i,t+1|t})$$

    for $t = T-1, T-2, ..., 1$, where $J_{i,t} = M_{i,t|t}\hat{F}'M_{i,t+1|t}^{-1}$.

4. Update the sequence of MSE by iterating on:

$$M_{i,t|T} = M_{i,t|t} + J_{i,t}(M_{i,t+1|T} - M_{i,t+1|t})J_{i,t}'$$

Likelihood Function. Finally, we present the likelihood function that is maximized as part of our estimation algorithm.[69] The likelihood for individual i in period t is given by,

$$\begin{aligned} LL_{i,t}(y_{i,t}|l_{i,t}, \{y_{i,j}, l_{i,j}\}_{j=1}^{t-1}) = & (2\pi)^{-n/2}|H(l_{i,t})'M_{t|t-1}H(l_{i,t}) + R(l_{i,t})|^{-1/2} \\ & \times \exp\{-\frac{1}{2}(y_t - H(l_{i,t})'\zeta_{t|t-1})'(H(l_{i,t})'M_{t|t-1}H(l_{i,t}) + R(l_{i,t}))^{-1}\} \\ & \times (y_t - H(l_{i,t})'\zeta_{t|t-1}) \end{aligned} \tag{12}$$

for $t = 1, 2, ..., T$.

B.1 Accounting for filtering uncertainty

In this Appendix, we discuss how we add normal noise to our estimates from the Kalman filter to account for filtering uncertainty.

[69] Note the full information log likelihood presented in Appendix C.1 is used the derive the updating equations for the EM algorithm. The likelihood presented here is the one maximized in the estimation.

The Kalman smoother presented above returns an estimate of persistent earnings for individual i in period t and the lag of persistent earnings in period $t-1$, i.e., $\hat{\zeta}_{i,t|T} = \begin{bmatrix} \hat{z}_{i,t|T} & \hat{z}_{i,t-1|T} \end{bmatrix}'$. The Kalman filter also produces an estimate about the uncertainty of this estimate, which is given by the MSE matrix $M_{i,t|T}$. To arrive at our estimate of persistent earnings for individual i in period t, denoted $\hat{z}_{i,t}$, we draw normal noise, denoted $\xi_{i,t}$, from a bi-variate normal distribution with mean zero and variance-covariance matrix $M_{i,t|T}$. Let $\xi_{1,i,t}$ denote the first element of $\xi_{i,t}$. We can then define our estimate of persistent earnings for individual i in period t as $\hat{z}_{i,t} = \hat{z}_{it|T} + \xi_{1,i,t}$. With the estimate of persistent earnings for individual i in period t ($\hat{z}_{i,t}$) we can then recover the temporary shock for individual i in period t, denoted $\hat{\omega}_{i,t}$, using $\hat{\omega}_{i,t} = y_{i,t} - \hat{z}_{i,t}$.

C EM Algorithm

In this Appendix, we outline the EM algorithm we use to estimate the parameters of the income process presented in Section 1.

The EM algorithm is an iterative algorithm to update the parameters that govern the income process. To start the algorithm we make an initial guess of the parameters of the income process, and using these parameters create an estimate of the state vector using the Kalman Filter presented in Appendix B. The next step in the EM algorithm is to use the estimates of the state vector along with the data to update the estimates of the parameters. The parameters are updated using a series of equations that we drive below. The algorithm then repeats by using the new parameters to update the estimate of the state vector, and then using the estimated state vector and data to update the parameters. This process continues until the log likelihood has been maximized. In the subsections below we derive the equations that will allow for closed form updating of the parameters. Finally, we provide a detailed description of the EM algorithm.

C.1 Log Likelihood

The EM algorithm uses a set of closed form updating equations to uncover parameters which allow the log likelihood function to be maximized. To derive these formulas we start with the full-information log likelihood, which is the likelihood function *if* the state-variables are observed. For an individual i, the full information log likelihood appears as:[70]

[70] Note the full information log likelihood is used to derive the equations which update the parameters of the income process via the EM algorithm. The likelihood that is maximized as part of the estimation is given by (12).

$$
\begin{aligned}
LL_i(\{y_{i,t}\}_{t=0}^{T}, \{z_{i,t}\}_{t=0}^{T} \mid \{l_{i,t}\}_{t=1}^{T}, \theta_0) = & -\frac{T+1}{2}\log(2\pi) \\
& - \frac{1}{2}\log(var(u_{z0})) - \frac{1}{2}\frac{(z_{i0})^2}{var(u_{z0})} \\
& - \frac{1}{2}\sum_{t=1}^{T}\log(Q(l_{i,t})) - \frac{1}{2}\sum_{t=1}^{T}\frac{(z_{i,t} - Fz_{i,t-1} - B(l_{i,t}))^2}{Q_t(l_{i,t})} \\
& - \frac{1}{2}\sum_{t=1}^{T}\log(R(l_{i,t})) - \frac{1}{2}\sum_{t=1}^{T}\frac{(l_{E,i,t})(y_{i,t} - z_{i,t})^2}{R_{i,t}(l_{i,t})}
\end{aligned}
$$

C.2 Updating Means

In this section, we derive the expressions that are used to update the mean parameters of the income process (e.g. the persistence of persistent earnings, and drifts of persistent earnings when employed/unemployed). Before deriving the formulas we present a series of useful expressions that will ease the derivations of the updating equations. Additionally, note the following notation. Define $E_T\left[z_{it}|\{x_{it}, y_{it}, l_{it}\}\right] = \mu_{it|T}$, that is the expected value of individual i's persistent earnings in period t (given the data) is denoted by $\mu_{it|T}$, which corresponds to the output of the smoothed Kalman filter. Define $\Sigma_{i0|T}(1,1)$ to be the estimated the variance of initial persistent earnings. Define $\Sigma_{it|T}(1,2)$ to be the estimated covariance between $z_{it|T}$ and $z_{it-1|T}$.[71]

For simplicity and ease of notation, we will first discuss in detail how to update parameters for the simple income process outlined in section 1.2. Then, we discuss how things extend to the more general case in which F and B are both assumed to be linear in a set of unknown parameters in section C.2.3 below.

C.2.1 Useful Expressions

In this section, we derive a series of useful expressions that will aid in the derivation of the updating equations in the following subsections.

[71]Note this covariance term is the $(1,2)$ element of the matrix $M_{i,t|T}$.

First, we show that $E_T\left[z_{i0}^2|\{x_{it},y_{it},l_{it}\}\right]=\Sigma_{i0|T}(1,1)+\mu_{i|T}^2$,

$$\begin{aligned}
E_T\left[z_{i0}^2|\{y_{it},x_{it},l_{it}\}\right] &= E_T\left[\left(z_{i0}-\mu_{i0|T}+\mu_{i0|T}\right)^2|\{y_{it},x_{it},l_{it}\}\right]\\
&= E_T\left[\left(z_{i0}-\mu_{i0|T}\right)^2+\mu_{i0|T}^2+2\left(z_{i0}-\mu_{i0|T}\right)\mu_{i0|T}|\{y_{it},x_{it},l_{it}\}\right]\\
&= E_T\left[\left(z_{i0}-\mu_{i0|T}\right)^2+\mu_{i0|T}^2|\{y_{it},x_{it},l_{it}\}\right]\\
&= \Sigma_{i0|T}(1,1)+\mu_{i0|T}^2 \qquad (13)
\end{aligned}$$

where in the third equality we used the fact that $E_T\left[z_{i0}-\mu_{i0|T}|\{y_{it},x_{it},l_{it}\}\right]=0$.

Next, we show that $E_T\left[z_{it}z_{i,t-1}|\{y_{it},x_{it},l_{it}\}\right]=\Sigma_{it|T}(1,2)+\mu_{it|T}\mu_{it-1|T}$,

$$\begin{aligned}
E_T\left[z_{it}z_{i,t-1}|\{y_{it},x_{it},l_{it}\}\right] &= E_T\left[\left(z_{it}-\mu_{it|T}+\mu_{it|T}\right)\left(z_{i,t-1}-\mu_{it-1|T}+\mu_{it-1|T}\right)|\{y_{it},x_{it},l_{it}\}\right]\\
&= E_T\left[\left(z_{it}-\mu_{it|T}\right)\left(z_{i,t-1}-\mu_{it-1|T}\right)+\mu_{it|T}\mu_{it-1|T}|\{y_{it},x_{it},l_{it}\}\right]\\
&= \Sigma_{it|T}(1,2)+\mu_{it|T}\mu_{it-1|T} \qquad (14)
\end{aligned}$$

where in the second equality we have used the fact that $E_T\left[\left(z_{it}-\mu_{it|T}\right)|\{y_{it},x_{it},l_{it}\}\right]=0$ and $E_T\left[\left(z_{it-1}-\mu_{it-1|T}\right)|\{y_{it},x_{it},l_{it}\}\right]=0$.

C.2.2 Updating F, B_E, B_U

In this section, we derive the expression we will use to update the parameters $\{F,B_E,B_U\}$. The relevant part of the log likelihood for updating the parameters $\{F,B_E,B_U\}$ is given by:

$$\frac{1}{Q_t(l_{i,t})}\sum_{t=1}^{T}\left(z_{i,t}-Fz_{i,t-1}-B(l_{i,t})\right)^2$$

The expected value can be written as:

$$\frac{1}{Q_t(l_{i,t})}E_T\left[\left(z_{i,t}-Fz_{i,t-1}-B(l_{i,t})\right)^2|\{y_{it},x_{it},l_{it}\}\right]$$

Completing the square we obtain the following expression:

$$
\begin{aligned}
\frac{1}{Q_t(l_{i,t})} E_T \Big[z_{i,t}^2 - z_{i,t} F z_{i,t-1} - z_{i,t} B(l_{i,t}) \\
+ F^2 z_{i,t-1}^2 - F z_{i,t-1} z_{i,t} + F z_{i,t-1} B(l_{i,t}) \\
+ B(l_{i,t})^2 - B(l_{i,t}) z_{i,t} + F B(l_{i,t}) z_{i,t-1} | \{y_{it}, x_{it}, l_{it}\} \Big]
\end{aligned}
$$

Combining terms we have:

$$
\begin{aligned}
\frac{1}{Q_t(l_{i,t})} E_T \Big[z_{i,t}^2 - 2F z_{i,t} z_{i,t-1} - 2 z_{i,t} B(l_{i,t}) \\
+ F^2 z_{i,t-1}^2 + 2F z_{i,t-1} B(l_{i,t}) \\
+ B(l_{i,t})^2 | \{y_{it}, x_{it}, l_{it}\} \Big]
\end{aligned} \tag{15}
$$

We will next use expressions from Section C.2.1 to simplify equation (15). First using equation (13) (adjusted for period t, and period $t+1$), we have:

$$
\begin{aligned}
\frac{1}{Q_t(l_{i,t})} \left(\Sigma_{it|T}(1,1) + \mu_{it|T}^2 + F^2 \left[\Sigma_{it-1|T}(1,1) + \mu_{it-1|T}^2 \right] \right) \\
+ \frac{1}{Q_t(l_{i,t})} E_T \big[-2F z_{i,t} z_{i,t-1} - 2 z_{i,t} B(l_{i,t}) + 2F z_{i,t-1} B(l_{i,t}) \\
+ B(l_{i,t})^2 | \{y_{it}, x_{it}, l_{it}\} \big]
\end{aligned}
$$

Next using equation (14), we have:

$$
\begin{aligned}
\frac{1}{Q_t(l_{i,t})} \left(\Sigma_{it|T}(1,1) + \mu_{it|T}^2 + F^2 \left[\Sigma_{it-1|T}(1,1) + \mu_{it-1|T}^2 \right] \right) \\
+ \frac{1}{Q_t(l_{i,t})} \left(-2F \left[\Sigma_{it|T}(1,2) + \mu_{it|T} \mu_{it-1|T} \right] \right) \\
+ \frac{1}{Q_t(l_{i,t})} E_T \left[-2 z_{i,t} B(l_{i,t}) + 2F z_{i,t-1} B(l_{i,t}) + B(l_{i,t})^2 | \{y_{it}, x_{it}, l_{it}\} \right]
\end{aligned}
$$

Then taking the expectation over the remaining terms we have:

$$
\begin{aligned}
\frac{1}{Q_t(l_{i,t})} \left(\Sigma_{it|T}(1,1) + \mu^2_{it|T} + F^2 \left[\Sigma_{it-1|T}(1,1) + \mu^2_{it-1|T} \right] \right) \\
\frac{1}{Q_t(l_{i,t})} \left(-2F \left[\Sigma_{it|T}(1,2) + \mu_{it|T}\mu_{it-1|T} \right] \right) \\
+ \frac{1}{Q_t(l_{i,t})} \left[\left(-2\mu_{it|T} B(l_{i,t}) + 2F\mu_{it-1|T} B(l_{i,t}) + B(l_{i,t})^2 \right) \right]
\end{aligned}
\tag{16}
$$

We want to optimize equation (16) with respect to F, B_E and B_U. For ease of exposition, we drop the terms in equation (16) that do not include F, B_E and B_U, which returns:

$$
\begin{aligned}
\frac{1}{Q_t(l_{i,t})} \left(F^2 \left[\Sigma_{it-1|T}(1,1) + \mu^2_{it-1|T} \right] \right) \\
\frac{1}{Q_t(l_{i,t})} \left(-2F \left[\Sigma_{it|T}(1,2) + \mu_{it|T}\mu_{it-1|T} \right] \right) \\
+ \frac{1}{Q_t(l_{i,t})} \left[\left(-2\mu_{it|T} B(l_{i,t}) + 2F\mu_{it-1|T} B(l_{i,t}) + B(l_{i,t})^2 \right) \right]
\end{aligned}
\tag{17}
$$

The expression in (17) gives the expected contribution to the likelihood for individual i in period t. We want to maximize the likelihood across all individuals and time periods. To perform this optimization it will be convenient to define the following vectors and matrices. Define:

$$
X_C \equiv \begin{bmatrix} \mu_{1,0|T} & l_{E,1,1} & l_{U,1,1} \\ \mu_{1,1|T} & l_{E,1,2} & l_{U,1,2} \\ \vdots & \vdots & \vdots \\ \mu_{1,T-2|T} & l_{E,1,T-1} & l_{U,1,T-1} \\ \mu_{1,T-1|T} & l_{E,1,T} & l_{U,1,T} \\ \mu_{2,0|T} & l_{E,2,1} & l_{U,2,1} \\ \vdots & \vdots & \vdots \\ \mu_{N,T-1|T} & l_{E,N,T} & l_{U,N,T} \end{bmatrix}_{NT\times 3} \qquad C \equiv \begin{bmatrix} F \\ B_E \\ B_U \end{bmatrix}_{3\times 1} \qquad Y_C \equiv \begin{bmatrix} \mu_{1,1|T} \\ \vdots \\ \mu_{1,T|T} \\ \vdots \\ \mu_{N,T|T} \end{bmatrix}_{NT\times 1}
\tag{18}
$$

We can rewrite terms in matrix notation as follows:

$$\begin{aligned}
C^{'}X_C^{'}X_C C &= \sum_{i=1}^{N}\sum_{t=1}^{T}\left(F\mu_{it-1|T} + B_E l_{Eit} + B_U l_{Uit}\right)^2 \\
&= \sum_{i=1}^{N}\sum_{t=1}^{T}\left(F^2\mu_{it-1|T}^2 + 2FB_E\mu_{it-1|T}l_{Eit} + 2FB_U\mu_{it-1|T}l_{Uit} + (B_E l_{Eit})^2 + (B_U l_{Uit})^2\right) \\
&= \sum_{i=1}^{N}\sum_{t=1}^{T}\left(F^2\mu_{it-1|T}^2 + 2F\mu_{it-1|T}B(l_{i,t}) + B(l_{i,t})^2\right) \qquad \text{using } B(l_{it}) \text{ def. from above.}
\end{aligned}$$

We also have,

$$\begin{aligned}
Y_C^{'}X_C C &= F\sum_{i=1}^{N}\sum_{t=1}^{T}\mu_{it|T}\mu_{it-1|T} + B_E\sum_{i=1}^{N}\sum_{t=1}^{T}\mu_{it|T}l_{Eit} + B_U\sum_{i=1}^{N}\sum_{t=1}^{T}\mu_{it|T}l_{Uit} \\
&= F\sum_{i=1}^{N}\sum_{t=1}^{T}\mu_{it|T}\mu_{it-1|T} + \sum_{i=1}^{N}\sum_{t=1}^{T}\mu_{it|T}B(l_{i,t}) \qquad \text{using } B(l_{it}) \text{ def. from above.}
\end{aligned}$$

To complete writing the sum of the log likelihood across individuals it will be convenient to define the following vectors:

$$\vec{\sigma}_{t-1}(1,1) \equiv \begin{bmatrix} \Sigma_{1,0|T}(1,1) \\ \Sigma_{1,1|T}(1,1) \\ \vdots \\ \\ \Sigma_{N,T-1|T}(1,1) \end{bmatrix}_{NT\times 1} \qquad \vec{\sigma}_{t}(1,2) \equiv \begin{bmatrix} \Sigma_{1,2|T}(1,1) \\ \Sigma_{1,2|T}(1,2) \\ \vdots \\ \\ \Sigma_{N,T|T}(1,2) \end{bmatrix}_{NT\times 1} \qquad e_1^3 \equiv \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad e^{NT} \equiv \underbrace{\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}}_{(NT\times 1)}$$

We can make further progress with matrix notation by noting,

$$\sum_{i=1}^{N}\sum_{t=1}^{T}\left(F^2\Sigma_{it-1|T}(1,1)\right) = C^{'}e_1^3 e_1^{3'} C\vec{\sigma}_{t-1}^{'}(1,1)e^{NT} \tag{19}$$

and

$$\sum_{i=1}^{N}\sum_{t=1}^{T}\left(F\Sigma_{it|T}(1,2)\right) = e^{NT^{'}}\vec{\sigma}_t(1,2)e_1^{3'}C. \tag{20}$$

Using (17) and the definitions above we have the following:

$$\sum_{i=1}^{N}\sum_{t=1}^{T} E_T\left[(z_{i,t}-Fz_{i,t-1}-B(l_{i,t}))^2\,|\{y_{it},x_{it},l_{it}\}\right] = C^{'}X_C^{'}X_C C - 2Y_C^{'}X_C C$$
$$- 2e^{NT^{'}}\vec{\sigma}_t(1,2)e_1^{3'}C + C^{'}e_1^3 e_1^{3'}C\vec{\sigma}^{'}_{t-1}(1,1)e^{NT}$$

Finally, define:

$$Q^{-1} \equiv \begin{bmatrix} \frac{1}{Q_1(l_{1,1})} & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & \frac{1}{Q_2(l_{1,2})} & 0 & \vdots & \vdots & \vdots & \vdots \\ \vdots & 0 & \ddots & 0 & \vdots & \vdots & \vdots \\ \vdots & \vdots & 0 & \frac{1}{Q_T(l_{1,T})} & 0 & \vdots & \vdots \\ \vdots & \vdots & \vdots & 0 & \frac{1}{Q_1(l_{2,1})} & 0 & \vdots \\ \vdots & \vdots & \vdots & \vdots & 0 & \ddots & 0 \\ 0 & \cdots & \cdots & \cdots & \cdots & 0 & \frac{1}{Q_T(l_{N,T})} \end{bmatrix}$$

Using (17) and the definitions above we have the following:

$$\sum_{i=1}^{N}\sum_{t=1}^{T} \frac{1}{Q_t(l_{i,t})} E_T\left[(z_{i,t}-Fz_{i,t-1}-B(l_{i,t}))^2\,|\{y_{it},x_{it},l_{it}\}\right] = C^{'}X_C^{'}Q^{-1}X_C C - 2Y_C^{'}Q^{-1}X_C C$$
$$- 2e^{NT^{'}}Q^{-1}\vec{\sigma}_t(1,2)e_1^{3'}C$$
$$+ C^{'}e_1^3 e_1^{3'}C\vec{\sigma}^{'}_{t-1}(1,1)Q^{-1}e^{NT}$$

Taking the FOC with respect to C returns:

$$0 = 2C^{'}X_C^{'}Q^{-1}X_C - 2Y_C^{'}Q^{-1}X_C - 2e^{NT^{'}}Q^{-1}\vec{\sigma}_t(1,2)e_1^{3'} + 2C^{'}e_1^3 e_1^{3'}\vec{\sigma}^{'}_{t-1}(1,1)Q^{-1}e^{NT} \tag{21}$$

Rearranging equation (21) returns:

$$C^{'}\left[X_C^{'}Q^{-1}X_C + e_1^3 e_1^{3'}\vec{\sigma}^{'}_{t-1}(1,1)Q^{-1}e^{NT}\right] = Y_C^{'}Q^{-1}X_C + e^{NT^{'}}Q^{-1}\vec{\sigma}_t(1,2)e_1^{3'} \tag{22}$$

Taking the transpose of both sides of equation (22) returns:

$$\left[X_C^{'}Q^{-1}X_C + e_1^3 e_1^{3'}\vec{\sigma}^{'}_{t-1}(1,1)Q^{-1}e^{NT}\right]C = X_C^{'}Q^{-1}Y_C + e_1^3\vec{\sigma}^{'}_t(1,2)Q^{-1}e^{NT} \tag{23}$$

where we have exploited the fact that the matrices on the LHS of equation (22) are symmetric.

Equation (23) gives us a closed form equation for updating the parameters $\{F, B_E, B_U\}$.

Intuition Equation (23) shows that the parameters $C = [F, B_E, B_U]$ are updated by using a GLS style regression equation. The persistence of persistent earnings (F) is updated by regressing lagged persistent earnings (the first column of X_C) onto current persistent earnings (Y_C), and is then adjusted to take into account the covariance of persistent earnings with its lag as well as the variance of lagged persistent earnings. The drift of persistent earnings when employed is updated by regressing a dummy variable for being employed (the second column of X_C) onto current persistent earnings (Y_C). Similarly, the drift of persistent earnings when unemployed is updated by regressing a dummy variable for being unemployed (the third column of X_C) onto current persistent earnings (Y_C). The GLS regression formula in equation (23) shows that these parameters are identified by running regressions that are informative about the evolution of persistent earnings over time, as well as during employment and unemployment spells. Note that we would obtain the same formula in the limit if we were instead to simulate a large number of draws from the posterior distributions of $z_{it|T}, z_{i,t-1}|T$ for each individual in our sample and run a standard OLS regression of z_{it} on $z_{i,t-1}$, $l_{E,i,t}$, and $l_{U,i,t}$ to update the unknown parameters.

C.2.3 Extension to the general case

Above, we assumed that the income process was quite simple. In particular, one could write $F_{it} = [0, 0, 1]C = e_1^{3'}C = F$ and $B_{it} = [0, l_{E,i,t}, l_{U,i,t}]C$. It turns out that it is straightforward to extend to a much more flexible setting in which

$$F(l_{it}; X_{it}) \equiv F_{it} = [x^F_{i,t}]'C \tag{24}$$

$$B(l_{it}; X_{it}) \equiv B_{it} = [x^B_{i,t}]'C, \tag{25}$$

where $x^F_{i,t}$ and x^B_{it} are *known* functions of l_{it} and X_{it}. Like in the example above, C is the set of unknown parameters which captures information which is relevant for the conditional mean in the state equation, which involves both the AR(1) coefficient on lagged permanent income as well as the drift in the state equation. Here, we allow for considerably more flexibility, but simply require that both F_{it} and B_{it} are linear in these parameters. In the vast majority of applications, one would tend to expect that things are partitioned so that the j^{th} element of x^F_{it} is always zero if the j^{th} element of x^B_{it} is nonzero with positive probability, but we don't need to require this per se.[72]

Let us define X_F as the design matrix constructed by concatenating the $[x^F_{it}]'$ vectors vertically, and X_B be the analogous object constructed by concatenating the $[x^B_{it}]'$ vectors vertically. Then, let us redefine

$$X_C \equiv \text{diag}(\vec{\mu}_{t-1})X_F + X_B,$$

where $\vec{\mu}_{t-1}$ is the first column of the definition of X_C in equation ((18))–i.e., the vector of lagged posterior means. In the special case in which F is constant, x^F_{it} has a 1 in its first element and a zero otherwise, and $x^B_{i,t}$ has a zero in its first element, we can use this extended X_C matrix in place of the one defined above, and the updating formulas defined in ((23)) apply without modification.

If F_{it} is not constant, we also need to make a minor modification to the additional terms which appear in the likelihood function which involve filtering uncertainty about current and lagged z_{it}. In the more general case, the expressions in equations ((19)-(20)) simplify to

$$\sum_{i=1}^{N}\sum_{t=1}^{T}\left(F^2_{it}\Sigma_{it-1|T}(1,1)\right) = \sum_{i=1}^{N}\sum_{t=1}^{T}\left(C'x^F_{it}\Sigma_{it-1|T}(1,1)[x^F_{i,t}]'C\right) = C'X'_F\text{diag}(\vec{\sigma}_{t-1}(1,1))X_F C$$

and

$$\sum_{i=1}^{N}\sum_{t=1}^{T}\left(F_{it}\Sigma_{it|T}(1,2)\right) = \sum_{i=1}^{N}\sum_{t=1}^{T}\left(\Sigma_{it|T}(1,2)[x^F_{i,t}]'C\right) = \vec{\sigma}_t(1,2)'X_F C.$$

If we use the alternative formulation which allows for F to vary as a linear function of x^F_{it}, we obtain the closely related expression to equation ((23)):

$$\left[X'_C Q^{-1} X_C + X'_F Q^{-1}\text{diag}(\vec{\sigma}_{t-1}(1,1))X_F\right] C = X'_C Q^{-1} Y_C + X'_F Q^{-1}\vec{\sigma}_t(1,2), \tag{26}$$

which still resembles a GLS regression equation. Clearly, this will not work for completely arbitrary X_F and X_B; we will need to be able to impose restrictions which ensure that the matrix $\left[X'_C Q^{-1} X_C + X'_F Q^{-1}\text{diag}(\vec{\sigma}_{t-1}(1,1))X_F\right]$ is invertible.

C.3 Updating Variances

In this Appendix, we derive the expressions that will be used to update the variance parameters. As above, we will economize on notation by restricting attention to the notation of the model in section 1.2 above. However, the extension will be immediate. Notice that, below, we already

[72]For example, in our base case above, F was assumed to be the first element of C and the remaining two parameters captured the unknown parameters which governed $B(l_{it})$.

already assume that log variances are linear in unknown sets of parameters. As such, allowing for a more flexible linear-in-parameters structure simply requires reinterpreting l_{it} as a broader set of observables than just employment/unemployment dummies.[73]

C.3.1 Shocks to persistent Earnings When Employed and Unemployed (Q_E and Q_U)

In this section we discuss how we update the variance of persistent earnings for the employed and unemployed. We can write the variance of persistent earnings as:

$$Q(l_{it}) \quad = \quad \exp(l_{it}'\phi_Q)$$

where $\phi_Q = [\phi_{Q,E}, \phi_{Q,U}]$. The relevant part of the negative log likelihood which depends on ϕ_Q is:

$$\Theta(\phi_Q;\beta,\omega) \equiv \sum_{i=1}^{N}\sum_{t=1}^{T}\log(Q(l_{it})) + \sum_{i=1}^{N}\sum_{t=1}^{T}\frac{(z_{i,t} - Fz_{i,t-1} - B(l_{i,t}))^2}{Q(l_{it})} \tag{27}$$

To arrive at an updating formula for the variance of persistent earnings, we will take the conditional expectation using the posterior distribution of the latent states given all of the missing data, and then take FOC with respect to ϕ_Q. Taking the conditional expectation using the posterior distribution of latent states (given all of the missing data) returns:

$$\Theta(\phi_Q;\beta,F) \equiv \sum_{i=1}^{N}\sum_{t=1}^{T}\log(Q(l_{it})) + \sum_{i=1}^{N}\sum_{t=1}^{T}\frac{E_T\left[(z_{i,t} - Fz_{i,t-1} - B(l_{i,t}))^2|x_{it}, y_{it}, l_{it}\right]}{Q(l_{it})} \tag{28}$$

Observe that this function is convex in ϕ_Q. Therefore, if we take a second order approximation of the objective, we obtain the following:

$$\Theta(\phi_Q;\beta,F) - \Theta(\phi_{Q,0};\beta,\omega) \equiv (\phi_Q - \phi_{Q,0})'\nabla\Theta + \tfrac{1}{2}(\phi_Q - \phi_{Q,0})'\,\nabla^2\Theta(\phi_Q - \phi_{Q,0}),$$

where the Jacobian matrix is defined as

$$\nabla\Theta(\phi_{Q,0};\beta,F) \equiv \sum_{i=1}^{N}\sum_{t=1}^{T}\left[1 - \frac{E\left[(z_{i,t} - Fz_{i,t-1} - B(l_{i,t}))^2|x_{it}, y_{it}, l_{it}\right]}{Q(l_{it})}\right] l_{i,t}$$

[73] Also, in expressions below, we would need to replace F with F_{it} and $B(l_{it})$ with B_{it} where appropriate.

and the Hessian matrix $\nabla^2\Theta$ is defined as

$$\nabla^2\Theta(\phi_{Q,0};\beta,F) \equiv \sum_{i=1}^{N}\sum_{t=1}^{T}\left[\frac{E\left[(z_{i,t}-Fz_{i,t-1}-B(l_{i,t}))^2|x_{it},y_{it},l_{it}\right]}{Q(l_{it})}\right] l_{i,t}l_{i,t}^{\prime}$$

Taking first order conditions, we obtain the familiar expressions for Newton's method:

$$\nabla^2\Theta\phi_Q = \nabla^2\Theta\phi_{Q,0} - \nabla\Theta \qquad \iff \qquad \phi_Q = \phi_{Q,0} - \left[\nabla^2\Theta\right]^{-1}\nabla\Theta, \tag{29}$$

which gives us a simple way of updating ϕ_Q.

Implementation Note that we can write the conditional expectation term as:

$$\begin{aligned} E\left[(z_{i,t}-Fz_{i,t-1}-B(l_{i,t}))^2|x_{it},y_{it},l_{it}\right] &= E\left[z_{i,t}-Fz_{i,t-1}-B(l_{i,t})|x_{it},y_{it},l_{it}\right]^2 \\ &\quad + var(z_{i,t}-Fz_{i,t-1}-B(l_{i,t})|x_{it},y_{it},l_{it}) \end{aligned} \tag{30}$$

Let $A_{it} = z_{it} - Fz_{i,t-1}$, then we can write the conditional variance expression as follows:

$$\begin{aligned} var(z_{i,t}-Fz_{i,t-1}-B(l_{i,t})|x_{it},y_{it},l_{it}) &= var(A_{it}-B(l_{i,t})|x_{it},y_{it},l_{it}) \\ &= var(A_{it}|x_{it},y_{it},l_{it}) + var(B(l_{i,t})|x_{it},y_{it},l_{it}) \\ &\quad - 2cov(A_{it},B(l_{i,t})|x_{it},y_{it},l_{it}) \\ &= var(A_{it}|x_{it},y_{it},l_{it}) \end{aligned}$$

where in the final equality we are using the fact that we are conditioning on l_{it}. Then using the definition of A_{it}, we have:

$$\begin{aligned} var(z_{it}-Fz_{it-1}-B(l_{i,t})|x_{it},y_{it},l_{it}) &= var(z_{it}|x_{it},y_{it},l_{it}) + F^2 var(z_{i,t-1}|x_{it},y_{it},l_{it}) \\ &\quad - 2Fcov(z_{it},z_{i,t-1}|x_{it},y_{it},l_{it}) \end{aligned} \tag{31}$$

Combining equations (30) and (31), we have the following expression for the conditional expectations terms.

$$\begin{aligned} E\left[(z_{it}-Fz_{it-1}-B(l_{i,t}))^2|x_{it},y_{it},l_{it}\right] &= E\left[(z_{it}-Fz_{it-1}-B(l_{i,t}))|x_{it},y_{it},l_{it}\right]^2 \\ &\quad + var(z_{it}|x_{it},y_{it},l_{it}) + F^2 var(z_{i,t-1}|x_{it},y_{it},l_{it}) \\ &\quad - 2Fcov(z_{it},z_{i,t-1}|x_{it},y_{it},l_{it}) \end{aligned}$$

C.3.2 Updating Variance of Temporary Earnings (R)

In this section we discuss how we update the variance of temporary earnings. We can write the variance of persistent earnings as:

$$R(l_{E,i,t}) \quad = \quad \exp(l_{E,i,t}\phi_R)$$

where $l_{E,i,t}$ is a dummy variable denoting whether an individual i is employed in period t.

The relevant part of the negative log likelihood which depends on ϕ_R is:

$$\Theta(\phi_R) \equiv \sum_{i=1}^{N}\sum_{t=1}^{T}\log(R(l_{E,i,t})) + \sum_{i=1}^{N}\sum_{t=1}^{T}\frac{H(l_{i,t})(y_{i,t}-z_{i,t})^2}{R_{i,t}(l_{E,i,t})} \tag{32}$$

Taking the conditional expectation using the posterior distribution of latent states (given all of the missing data) returns:

$$\Theta(\phi_R) \equiv \sum_{i=1}^{N}\sum_{t=1}^{T}\log(R(l_{E,i,t})) + \sum_{i=1}^{N}\sum_{t=1}^{T}H(l_{i,t})\frac{E\left[(y_{i,t}-z_{it})^2|x_{it},y_{it},l_{it}\right]}{R_{i,t}(l_{E,i,t})} \tag{33}$$

Similar to above, observe that this function is convex in ϕ_R. Therefore, if we take a second order approximation of the objective, we obtain the following:

$$\Theta(\phi_R) - \Theta(\phi_{R,0}) \equiv (\phi_R - \phi_{R,0})'\nabla\Theta + \tfrac{1}{2}(\phi_R - \phi_{R,0})'\,\nabla^2\Theta(\phi_R - \phi_{R,0}),$$

where the Jacobian matrix is defined as

$$\nabla\Theta(\phi_{R,0}) \equiv \sum_{i=1}^{N}\sum_{t=1}^{T}\left[1 - \frac{E\left[(y_{i,t}-z_{it})^2|x_{it},y_{it},l_{it}\right]}{R(l_{E,i,t})}\right]l_{E,i,t}$$

and the Hessian matrix $\nabla^2\Theta$ is defined as

$$\nabla^2\Theta(\phi_{R,0}) \equiv \sum_{i=1}^{N}\sum_{t=1}^{T}\left[\frac{E\left[(y_{i,t}-z_{it})^2|x_{it},y_{it},l_{it}\right]}{R(l_{E,i,t})}\right]l_{E,i,t}l_{E,i,t}'$$

Taking first order conditions, we obtain the familiar expressions for Newton's method:

$$\nabla^2\Theta\phi_R = \nabla^2\Theta\phi_{R,0} - \nabla\Theta \qquad \iff \qquad \phi_R = \phi_{R,0} - \left[\nabla^2\Theta\right]^{-1}\nabla\Theta, \tag{34}$$

Implementation Note that we can write the conditional expectations term as:

$$E\left[(y_{i,t}-z_{it})^2|x_{it},y_{it},l_{it}\right]=E\left[y_{i,t}-z_{it}|x_{it},y_{it},l_{it}\right]^2+var(y_{i,t}-z_{it}|x_{it},y_{it},l_{it})$$

Since we condition on $y_{i,t}$, the conditional variance term can be written as:

$$var(y_{i,t}-z_{it}|x_{it},y_{it},l_{it})=var(z_{it}|x_{it},y_{it},l_{it})$$

Then we have that:

$$E\left[(y_{i,t}-z_{it})^2|x_{it},y_{it},l_{it}\right]=E\left[y_{i,t}-z_{it}|x_{it},y_{it},l_{it}\right]^2+var(z_{it})$$

C.3.3 Updating Variance of Initial persistent Earnings Draw$(var(u_{z0}))$

In this section we discuss how we update the variance of the initial draw of persistent earnings. We can write the variance of initial persistent earnings as:

$$var(u_{z0})=exp(l^0_{E,i,t}\phi_{u_{z0}})$$

where $l^0_{E,i,t}$ is a dummy variable that is equal to 1 if individual i is employed E for the first time in the sample in period t.

The relevant part of the negative log likelihood which depends on $\phi_{u_{z0}}$ is:

$$\Theta(\phi_{u_{z0}})\equiv\sum_{i=1}^{N}\log(var(u_{z0}))+\sum_{i=1}^{N}\frac{(z_{i0})^2}{var(u_{z0})}$$

Taking the conditional expectation using the posterior distribution of latent states (given all of the missing data) returns:

$$\Theta(\phi_{u_{z0}})\equiv\sum_{i=1}^{N}\log(var(u_{z0}))+\sum_{i=1}^{N}\frac{E\left[(z_{i0})^2|x_{it},y_{it},l_{it}\right]}{var(u_{z0})}$$

Similar to above, observe that this function is convex in $\phi_{u_{z0}}$. Therefore, if we take a second order approximation of the objective, we obtain the following:

$$\Theta(\phi_{u_{z0}})-\Theta(\phi_{u_{z0},0})\equiv(\phi_{u_{z0}}-\phi_{u_{z0},0})'\nabla\Theta+\tfrac{1}{2}(\phi_{u_{z0}}-\phi_{u_{z0},0})'\,\nabla^2\Theta(\phi_{u_{z0}}-\phi_{u_{z0},0}),$$

where the Jacobian matrix is defined as

$$\nabla\Theta(\phi_{u_{z0},0}) \equiv \sum_{i=1}^{N}\left[1-\frac{E\left[(z_{i0})^2|x_{it},y_{it},l_{it}\right]}{var(u_{z0})}\right] l_{E,i,t}^{0}$$

and the Hessian matrix $\nabla^2\Theta$ is defined as

$$\nabla^2\Theta(\phi_{u_{z0},0}) \equiv \sum_{i=1}^{N}\left[\frac{E\left[(z_{i0})^2|x_{it},y_{it},l_{it}\right]}{var(u_{z0})}\right] l_{E,i,t}^{0} l_{E,i,t}^{0'}$$

Taking first order conditions, we obtain the familiar expressions for Newton's method:

$$\nabla^2\Theta\phi_{u_{z0}} = \nabla^2\Theta\phi_{u_{z0},0} - \nabla\Theta \qquad \Longleftrightarrow \qquad \phi_{u_{z0}} = \phi_{u_{z0},0} - \left[\nabla^2\Theta\right]^{-1}\nabla\Theta, \tag{35}$$

Implementation Note that we can write the conditional expectations term as:

$$E\left[(z_{i0})^2|x_{it},y_{it},l_{it}\right] = E\left[z_{i0}|x_{it},y_{it},l_{it}\right]^2 + var\left[z_{i0}|x_{it},y_{it},l_{it}\right]$$

C.4 Algorithm

In this section, we present the EM algorithm we use to recover the estimate of persistent earnings as well as the parameters which govern the income process.

1. Guess an initial set of parameters $\theta_0 = [F, Q_E, Q_U, B_E, B_U, R]'$.
2. Using the parameter guess θ_0 use the Kalman Filter for the state-space system in equations (2) and (1) to obtain an estimate of $\{\{z_{i,t}\}_{i=1}^{N}\}_{t=0}^{T}$, and estimate the log likelihood.
3. Using estimated persistent earnings $\{\{z_{i,t}\}_{t=0}^{T}\}_{i=1}^{N}$ and data $\{\{y_{i,t}\}_{t=0}^{T}, \{l_{i,t}\}_{t=0}^{T}, \{x_{i,t}\}_{t=0}^{T}\}_{i=1}^{N}$, update the parameter vector as follows:
   (a) Update F, B_U, B_E using equation (23).
   (b) Update the shocks to persistent earnings by iterating on equation (29).
   (c) Update the shocks to temporary earnings by iterating on equation (34).
   (d) Update the initial draw of persistent earnings by solving (35).
4. Repeat steps (2) and (3) until the log likelihood is maximized.

D Extended Model

In this appendix, we present the income process that we use in Section 3. The income process, we estimate in Section 3 includes:

1. The standard deviation of shocks to temporary and persistent earnings are a function of year fixed effects and a quadratic in age which vary separately for both the employed and unemployed.

2. The standard deviation of initial draws of persistent earnings are also a function of year fixed effects and a quadratic in age.

3. The drift in persistent earnings is also a function of year fixed effects which vary separately for both the employed and unemployed.

Standard deviation of shocks Let $\widetilde{Q}_{E,t,j}$ denote the log variance of shocks to the employed in year t for an individual of age j.[74] We model $\widetilde{Q}_{E,t,j}$ as follows:

$$\widetilde{Q}_{E,t,j} = \widetilde{Q}_E + \widetilde{Q}^Y_{E,t} + (j-25)\widetilde{Q}^A_{E,1} + (j-25)^2\widetilde{Q}^A_{E,2} \tag{36}$$

where $\widetilde{Q}_E$ denotes the log variance of the shock to persistent earnings in the initial year of the sample for an age 25 individual. The parameter $\widetilde{Q}^Y_{E,t}$ denotes the year fixed effect for the log variance of the shock to persistent earnings among the employed in year t. The parameters $\widetilde{Q}^A_{E,1}$ and $\widetilde{Q}^A_{E,2}$ govern the age quadratic for the shock to persistent earnings among the employed. Suppose that there are T periods. Then, the estimation recovers a set of parameters $\{\widetilde{Q}_E, \widetilde{Q}^Y_{E,1}, ..., \widetilde{Q}^Y_{E,T}, \widetilde{Q}^A_{E,1}, \widetilde{Q}^A_{E,2}\}$, which govern the variance of shocks to persistent earnings among the employed.

We similarly define $\widetilde{Q}_{U,t,j}$ as the log variance of shocks to persistent earnings for the unemployed in year t for an individual of age j, and proceed as above to estimate a set of parameters $\{\widetilde{Q}_U, \widetilde{Q}^Y_{U,1}, ..., \widetilde{Q}^Y_{U,T}, \widetilde{Q}^A_{U,1}, \widetilde{Q}^A_{U,2}\}$, which govern the log variance of shocks to persistent earnings among the unemployed. Additionally, we define $\widetilde{R}_{t,j}$ as the log variance of temporary shocks in year t for an individual of age j, and proceed as above to estimate a set of parameters $\{\widetilde{R}, \widetilde{R}^Y_1, ..., \widetilde{R}^Y_T, \widetilde{R}^A_1, \widetilde{R}^A_2\}$, which govern the log variance of shocks to temporary earnings.

[74] Assuming that the logarithm of variances rather than variances themselves are affine in observables has the advantage of guaranteeing that variances are positive (and guarantees a well-defined log-likelihood).

Finally, let $\widetilde{z}_{0,t,j}$ denote the log variance of the initial draw of persistent earnings for an individual who enter the estimation sample in period t when they are age j. We model $\widetilde{z}_{0,t,j}$ as follows:

$$\widetilde{z}_{0,t,j} = \widetilde{z}_0 + \widetilde{z}^Y_{0,t} + \mathbb{I}\{t = t_0\}\left\{(j-25)\widetilde{z}^A_{0,1} + (j-25)^2\widetilde{z}^A_{0,2}\right\} + \mathbb{I}\{t > t_0\}\left\{(j-25)\widetilde{z}^A_{0,3} + (j-25)^2\widetilde{z}^A_{0,4}\right\} \tag{37}$$

where $\widetilde{z}_0$ denotes the log variance of the initial draw of persistent earnings in the initial year of the sample for an age 25 individual. The parameter $\widetilde{z}^Y_{o,t}$ denotes the year fixed effect for the log variance of the initial draw of persistent earnings in year t. The parameters that govern the age quadratic depend upon when the individuals enters the sample. We include this distinction, because individuals of all ages enter the sample in the first year ($t = t_0$). When individuals enter the sample after the age of 25 in years after the first year, there is information in their later entry to the labor market, e..g additional schooling, difficulty finding work etc. The parameters $\widetilde{z}^A_{0,1}$ and $\widetilde{z}^A_{0,2}$ govern the age quadratic for individuals who enter the sample in the first year, while the parameters $\widetilde{z}^A_{0,3}$ and $\widetilde{z}^A_{0,4}$ govern the age quadratic for individuals who enter the sample after the first year. The estimation recovers a set of parameters $\{\widetilde{z}_0, \widetilde{z}^Y_{0,1}, ..., \widetilde{z}^Y_{0,T}, \widetilde{z}^A_{0,1}, \widetilde{z}^A_{0,2}\widetilde{z}^A_{0,3}, \widetilde{z}^A_{0,4}\}$, which govern the standard deviation of shocks to temporary earnings.

Drift of persistent shocks Let $B_{E,t}$ denote the drift of the persistent earnings shock to the employed in year t. We model the $B_{E,t}$ as follows:

$$B_{E,t} = B_E + B^Y_{E,t} \tag{38}$$

where B_E denotes the drift of the shock to persistent earnings in the initial year of the sample. The parameter $B^Y_{E,t}$ denotes the year fixed effect for the drift to the shock of persistent earnings among the employed in year t. Suppose that there are T periods. Then, the estimation recovers a set of parameters $\{B_E, B^Y_{E,1}, ..., B^Y_{E,T}\}$, which govern the drift of the shocks to persistent earnings among the employed.

We similarly define $B_{U,t}$ as the drift of shocks to persistent earnings for the unemployed in year t, and we proceed as above to estimate a set of parameters $\{B_E, B^Y_{E,1}, ..., B^Y_{E,T}\}$.

D.1 Additional Extensions.

In Section 4 we allow the parameters of the income process to depend upon: education, gender, geography as well as occupation. To recover these parameters, we use the approach from above and simply allow for each parameter to depend upon the appropriate conditioning variable (e.g. education, gender, geography, occupation, etc.).

E Model Estimation

In this Appendix, we discuss how we solve the lifecycle Bewley model. We solve the model using value function iteration on grids. Below we outline the algorithm for solving the model and discuss the process for discretizing income shocks.

E.1 Discretization Process (persistent Earnings)

In this section, we outline our process for discretizing shocks to persistent earnings where agents recieve different shocks when employed versus unemployed.

At the start of the period an agent draws whether or not they will be employed for the period. Let $\lambda_U \in [0,1]$ denote the probability that an agent is classified as unemployed. Recall that the process for persistent earnings is given by:

$$z' = \rho z + \mu_e + \eta_e$$

where $e \in \{E, U\}$ denotes employment status, μ_e denotes the drift of persistent earnings while in employment status e, and η_e is the shock to persistent earnings while in employment status e. We assume that the drifts to persistent earnings and the variance of the shocks to persistent earnings differ by employment status. That is $\eta_U \sim N(0, \sigma^2_{\eta,U})$, and $\eta_E \sim N(0, \sigma^2_{\eta,E})$.

Define a transition matrix for agents classified as employed, denoted π^E, and a transition matrix for agents classified as unemployed, denoted π^U. The elements of π^e_{jk} defines the probability that an agent with employment status e, moves from state j **today** to state k **tomorrow**.

Assume for now that we have specified a grid of values for z with N grid points, which are given by $[z_1, z_2, ..., z_N]$. Let the points be evenly spaced, with distance between grid points denoted by d.[75] The transition probability of going from state j **today** to state k **tomorrow** for

[75]In practice, we define the endpoints of the grid using $z_N = m \left(\frac{\sigma^2_{\eta,U}}{1-\rho}\right)^{\frac{1}{2}}$, setting $m = 3$, and $z_1 = -z_N$.

an individual with employment status e is given by

$$\begin{aligned}\pi^e_{jk} &= P(\widetilde{z}_t = z_k | \widetilde{z}_{t-1} = z_j | e) \\ &= P(z_k - \frac{d}{2} < \rho z_j + \mu_e + \eta_e < z_k + \frac{d}{2}) \\ &= P(z_k - \frac{d}{2} - \rho z_j - \mu_e < \eta_e < z_k + \frac{d}{2} - \rho z_j - \mu_e)\end{aligned} \tag{39}$$

For an interior point on the grid, the probability in equation (39) is given by:

$$\pi^e_{jk} = F(\frac{z_k + \frac{d}{2} - \rho z_j - \mu_e}{\sigma_{\eta,e}}) - F(\frac{z_k - \frac{d}{2} - \rho z_j - \mu_e}{\sigma_{\eta,e}})$$

where $F(\cdot)$ is the standard normal distribution. For the end points of the grid, define the probabilities using:

$$\begin{aligned}\pi^e_{j1} &= F(\frac{z_1 + \frac{d}{2} - \rho z_j - \mu_e}{\sigma_{\eta,e}}) \\ \pi^e_{jN} &= F(\frac{z_N - \frac{d}{2} - \rho z_j - \mu_e}{\sigma_{\eta,e}})\end{aligned}$$

E.2 Discretization Process (Temporary Earnings)

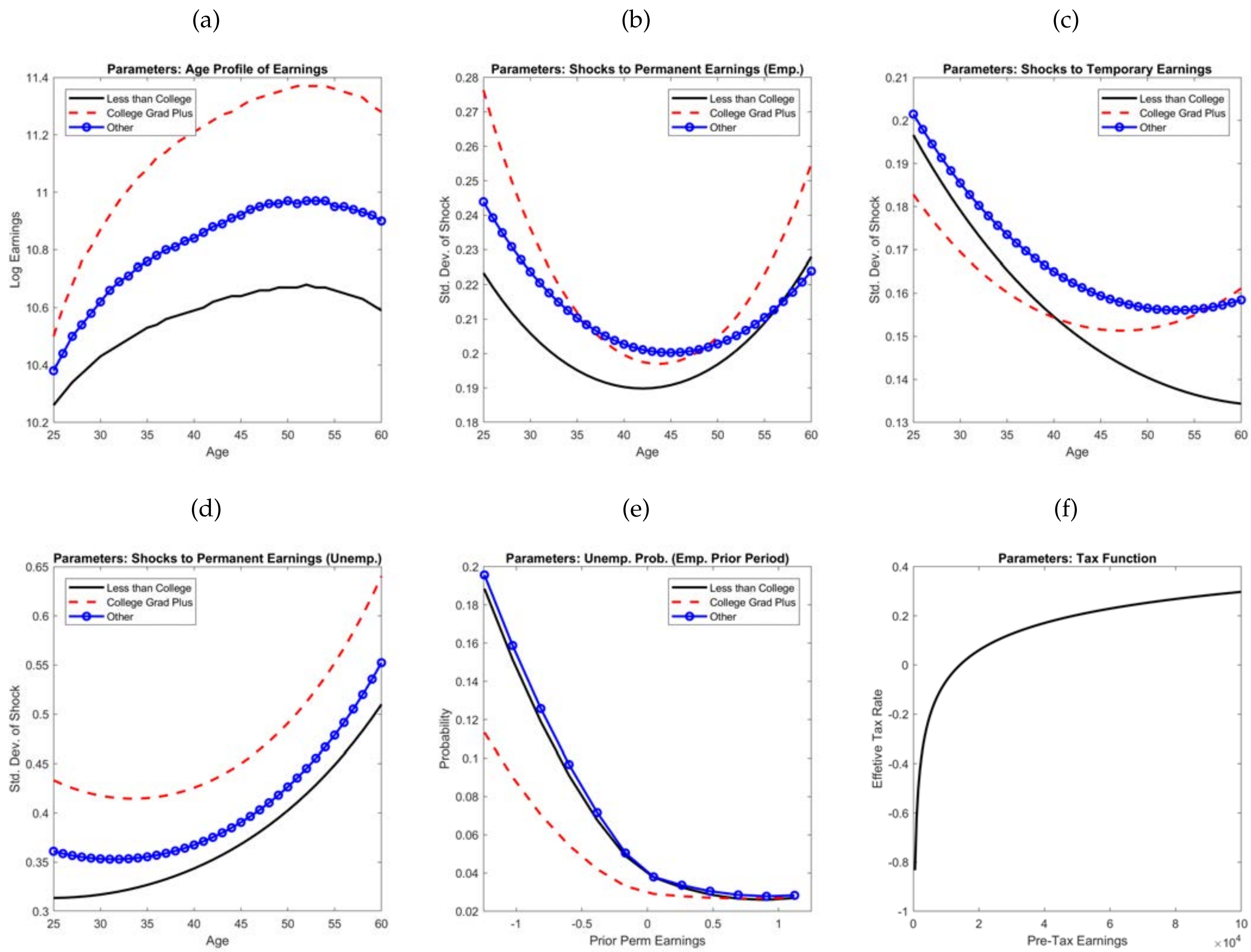
To discretize the process for temporary earnings, we use Tauchen's method with the persistence of the shock set to zero.

E.3 Additional Model Parameters

In this Appendix, we present additional parameters which govern the quantitative model and welfare experiment. Figure 15 presents the parameters that govern income risk in the model economy.

Figure 16 displays the parameters which govern income risk in the welfare experiment of Section 5.3. In each figure the solid lines correspond to the income process at the start of our sample in 1985. The lines with markers correspond to the income process at the end of our sample in 2013. To obtain the different paths of earnings over the life cycle in the two time periods, we estimate the income process detailed in Appendix D which includes year fixed effects and a quadratic in age where the parameters of the income process differ by level of education. The figure shows the path of earnings risk implied by the 1985 year fixed effect and age quadratic (solid lines) as well as the path implied by the 2013 year fixed effect and age quadratic (lines with markers).

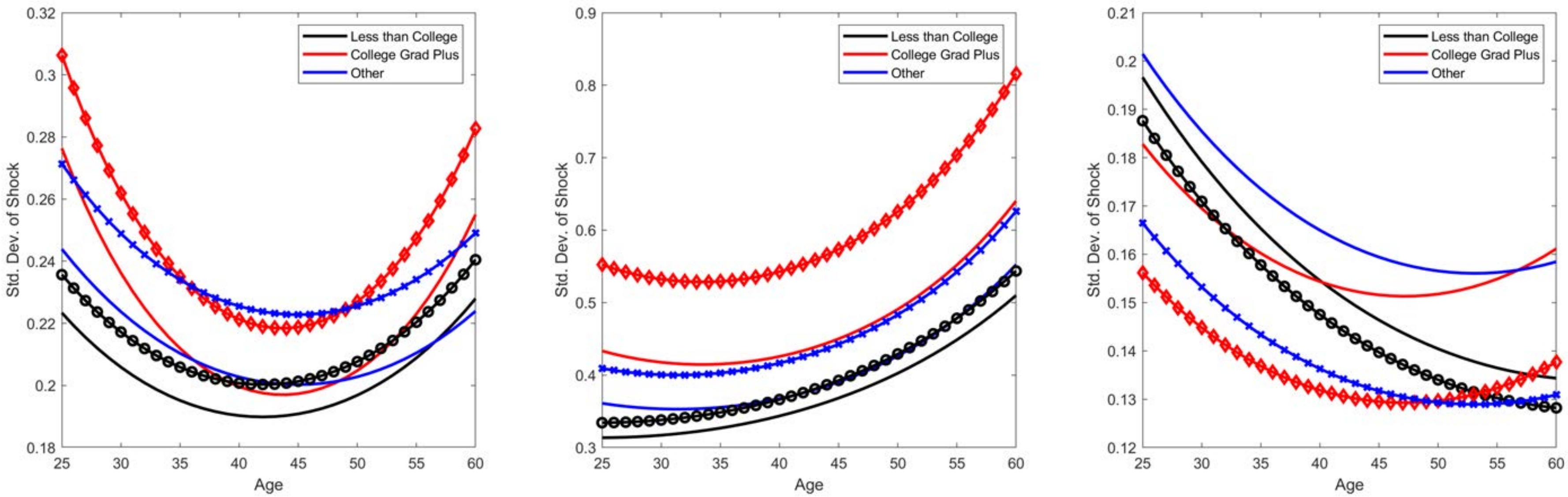
Figure 15: Parameters governing income process



Note: Figure presents parameters that govern the income process for agents in the model that are a function of age and type (Panels (a)-(e)). Panel (f) presents the tax function for the model economy.
Source: 1973, 1991, 1994, 1996-2016 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1982 to 2016.

Figure 16: Welfare Experiment: Income process parameters

(a) Std. Pers. Earnings Emp. (b) Std. Pers. Earnings Unemp. (c) Std. Temp. Earnings



Note: Figure presents parameters that govern the income process for agents in the model that are a function of age and type in the welfare experiment. Solid lines correspond to the 1985 steady state. The lines with markers correspond to the 2013 steady state.
Source: 1973, 1991, 1994, 1996-2016 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1982 to 2016.

F Welfare Calculation

In this section, we describe our process for performing the welfare calculation. Let $(\{c_t^j\}_{t=1}^T)$ be the consumption policy function for an individual j over their lifetime with baseline income risk. Let $(\{\tilde{c}_t^j\}_{t=1}^T)$ be the consumption policy function for an individual j under an alternative amount of income risk. We will perform welfare calculations by estimating the share of lifetime consumption an individual would be willing to forgo (or must receive) to leave the baseline economy and move to an economy with an alternative amount of income risk. Formally, we estimate the scaling factor for consumption λ_j that makes individual j indifferent between living under either amount of income risk:

$$\sum_{t=1}^{T} \beta^t \left(\frac{\left(\lambda_j c_t^j\right)^{1-\sigma} - 1}{1-\sigma} \right) = \sum_{t=1}^{T} \beta^t \left(\frac{\left(\tilde{c}_t^j\right)^{1-\sigma} - 1}{1-\sigma} \right) \tag{40}$$

Solving equation (40) for λ_j returns:

$$\lambda_j = \left[\frac{\sum_{t=1}^{T} \beta^t \left(\frac{\left(\tilde{c}_t^j\right)^{1-\sigma}}{1-\sigma} \right)}{\sum_{t=1}^{T} \beta_i^t \left(\frac{\left(c_t^j\right)^{1-\sigma}}{1-\sigma} \right)} \right]^{\frac{1}{1-\sigma}} \tag{41}$$

We use the model to simulate a large mass of individuals under a series of alternative amounts of labor income risk. We then compute utilitarian welfare (equal weights on λ_j):

$$Welfare = \frac{1}{N} \sum_{j=1}^{N} \lambda_j$$

G Additional Results

G.1 Unemployment Probability

In this Appendix, we presents results on the probability that an agent has earnings below the minimum earnings threshold (e.g. is classified as unemployed). The income process that we define in Section 1 is agnostic on the process for which individuals transition between having earnings above and below the minimum earnings threshold. We then use our estimates of filtered persistent earnings to examine the probability of being unemployed.

Let $\delta(z, e) \in [0, 1]$ denote the probability that an agent becomes unemployed. The probability that an agent becomes unemployed depends upon their persistent earnings and employment status from the prior period. In particular, we model the probability that an individual becomes unemployed using the following functional form:

$$\delta_i(z, e) = \begin{cases} \mathbb{I}\{z \geq 0\} \left[\sum_{k=0}^{2} \alpha_{i,k,E}^{+} z^k\right] + \mathbb{I}\{z < 0\} \left[\sum_{k=0}^{2} \alpha_{i,k,E}^{-} z^k\right] & e = E \\ \alpha_{i,U} & e = U \end{cases} \tag{42}$$

The functional form in equation (42) allows for the unemployment probability of the employed to be a quadratic function of prior persistent earnings ($\hat{z}_{i,t-1}$) estimated separately for positive prior persistent earnings or negative prior persistent earnings. We define the unemployment probability for the unemployed to be a constant.[76] We estimate equation (42) using our filtered estimates of persistent earnings and individuals realizations of being below the minimum earnings criteria.

To gauge the ability of the functional form in equation (42) to capture the dynamics of becoming unemployed in the data, we assign individuals to ventiles based upon their prior persistent earnings and measure the share of individuals by ventile who become unemployed in the next year. Figure 17 compares the observed share of unemployed individuals by ventile of prior persistent earnings (black, solid line) to the predicted value based upon the results of estimating equation (42) (red, dashed line). The figure shows that the functional form in equation (42) is able to accurately capture the dynamics of who becomes unemployed as a function of their prior persistent earnings.

G.2 Earnings risk over the lifecycle

Figure 18 shows parameter estimates of the shocks to earnings by age.[77]

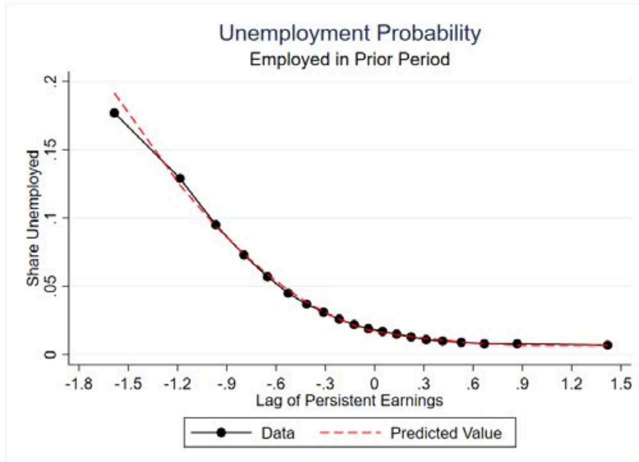
G.3 Alternative Estimations of the Income process

In this appendix we discuss the results from alternative estimations of our income process.

[76] When we estimated the quadratic function in equation (42) for the unemployed, we obtained results that were consistent with simply using a constant function by education.

[77] Note that in Figure 18 we present the standard deviation of shocks to income by age, holding the year component fixed at the initial value.

Figure 17: Probability of becoming unemployed



Note: This figure shows the predicted predicted probability of unemployment as estimated from equation (42) plotted against the actual observed probability of unemployment.
Source: 1973, 1991, 1994, 1996-2016 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1982 to 2016.

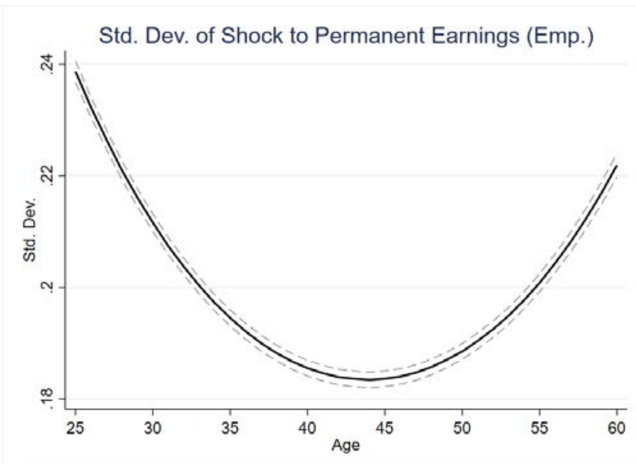
Lower minimum eanrings threshold In this extensions, we present the results from estimating the income process from Section 1 when the minimum earnings criteria is set to the equivalent of working part-time (20 hours per week) for 1-quarter at the real federal minimum wage, which corresponds to approximately \$2*k* in 2019 dollars. Figure 19 plots the standard deviation of residual log earnings changes by year. The figure shows that with the lower minimum earnings threshold there is a significant decline in earnings risk over the sample period. Figure 20 plots the parameter estimates from the estimation of the income process in Section 1 with the lower minimum earnings threshold. With the lower minimum earnings threshold, the trend in persistent earnings is nearly identical to our baseline estimation. However, we find a significantly larger decline in temporary earnings risk.

Changes in earnings risk over time by gender In this extension, we estimate the parameters of our income process separately by gender. Figure 21 shows parameter estimates of the shocks to earnings by gender over time. We find similar trends in persistent and temporary earnings risk by gender.

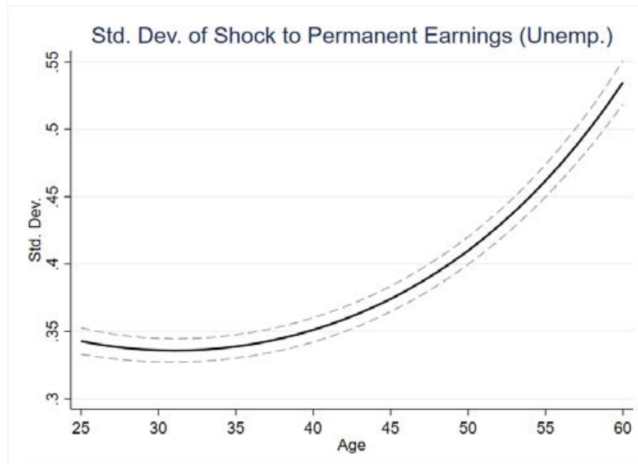
Changes in earnings risk over time by education level In this extension, we estimate the parameters of our income process by education level. We separate individuals into those with a college degree or higher and individuals with less than a college degree. Figure 22 shows parameter estimates of the shocks to earnings by education level over time. The figure shows

Figure 18: Earnings risk over the lifecycle

(a)

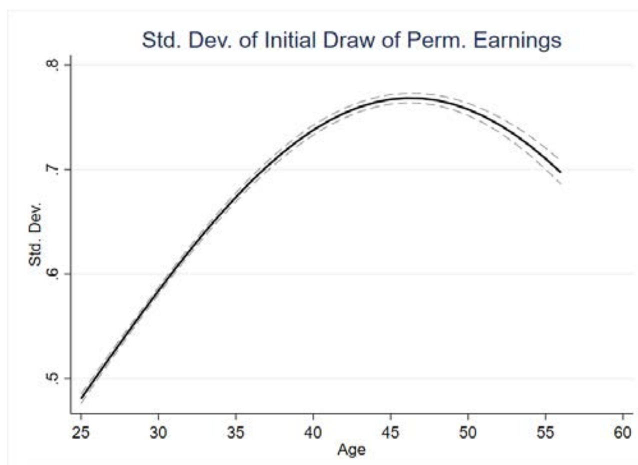


(b)



(c)

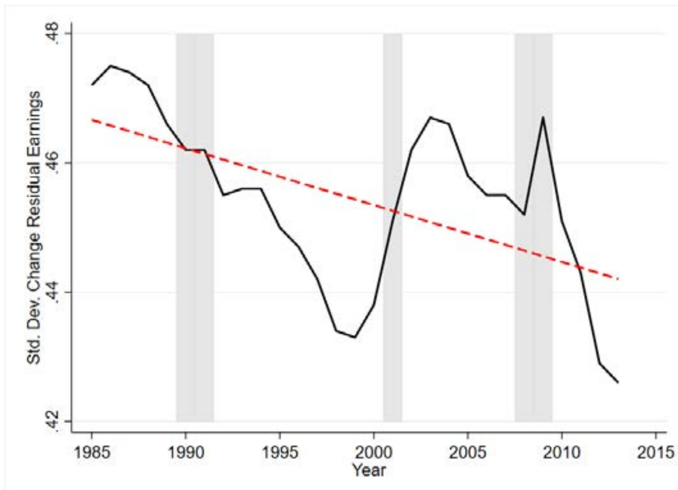
(d)



Note: Figure presents estimates for the standard deviation of : (1) the shock to persistent earnings among the employed (Panel (a)), (2) the shock to persistent earnings among the unemployed (Panel (b)), (3) the shock to temporary earnings (Panel (c)), and (4) the initial draw of persistent earnings, as a function of age based upon the estimation from Section 3. Dashed line represent a 95% confidence interval.
Source: 1973, 1991, 1994, 1996-2016 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1982 to 2016.

that we find a more pronounced increase in persistent earnings risk among individuals with a college degree or higher. We also find a larger decline in temporary earnings risk among individuals with a college degree or higher.

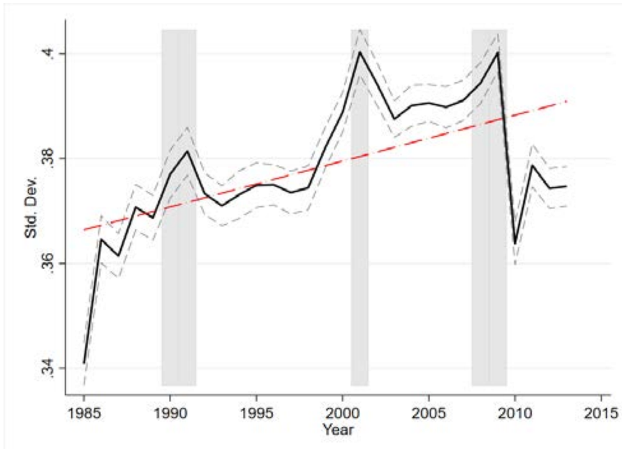
Figure 19: Earnings risk over time with lower minimum earnings cutoff



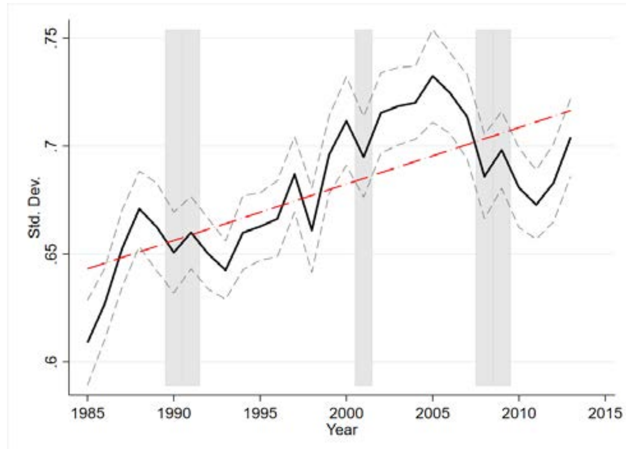
Note: Figure presents the standard deviation of residualized log earnings over time (black, solid line) and a linear trend line (red, dashed line) with the lower minimum earnings cutoff.
Source: 1973, 1991, 1994, 1996-2016 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1982 to 2016.

Figure 20: Changes in earnings risk over time with lower minimum earnings cutoff

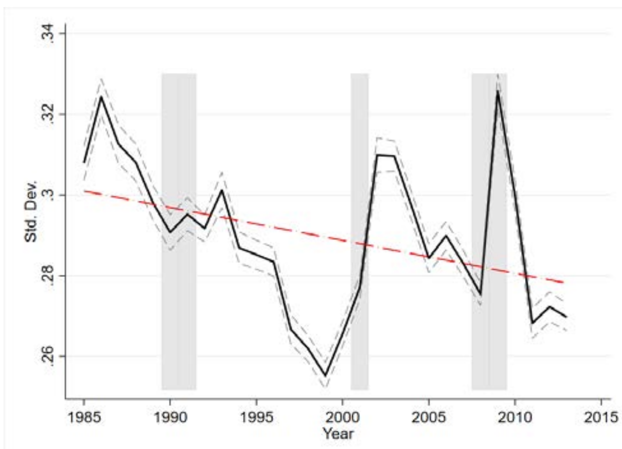
(a) Std. dev. persistent earnings (Emp.)



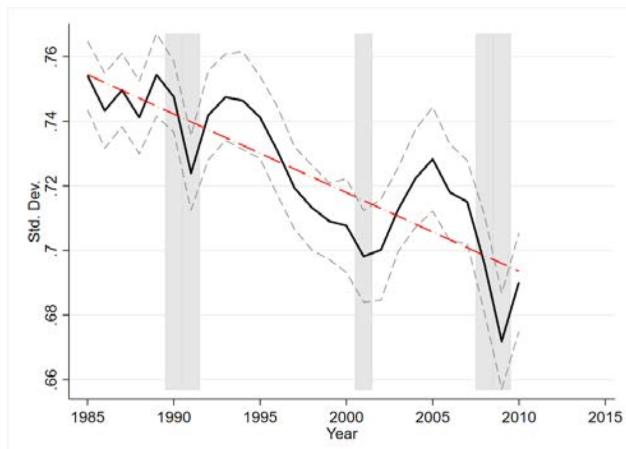
(b) Std. dev. persistent earnings (Unemp.)



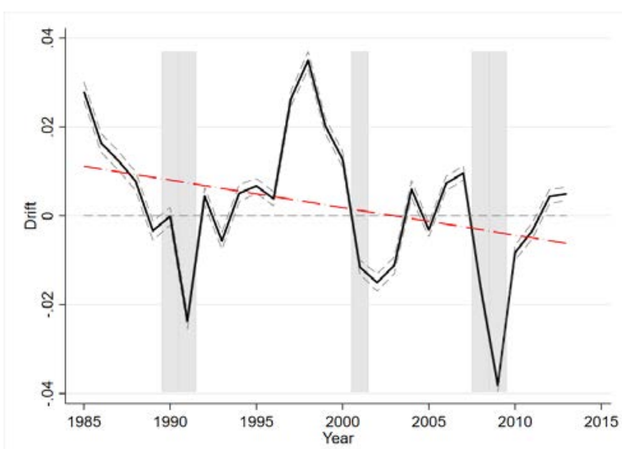
(c) Std. dev. temporary earnings



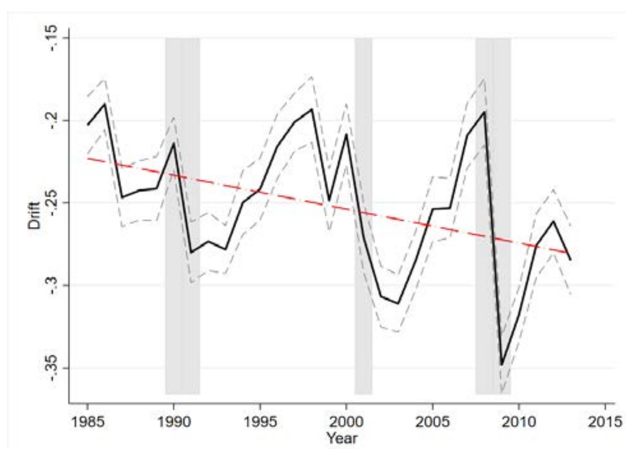
(d) Initial draw persistent earnings



(e) Drift persistent earnings (Emp.)



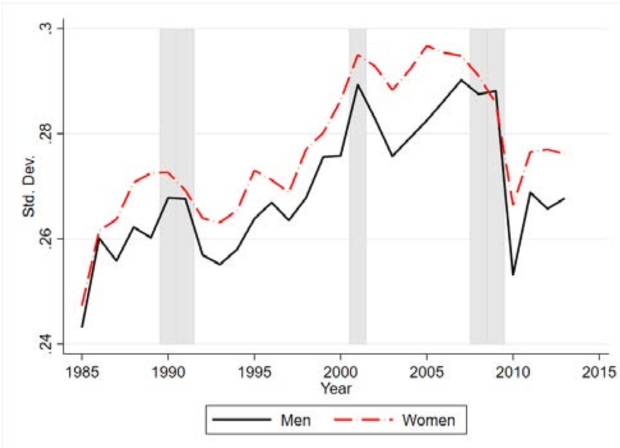
(f) Drift persistent earnings (Unemp.)



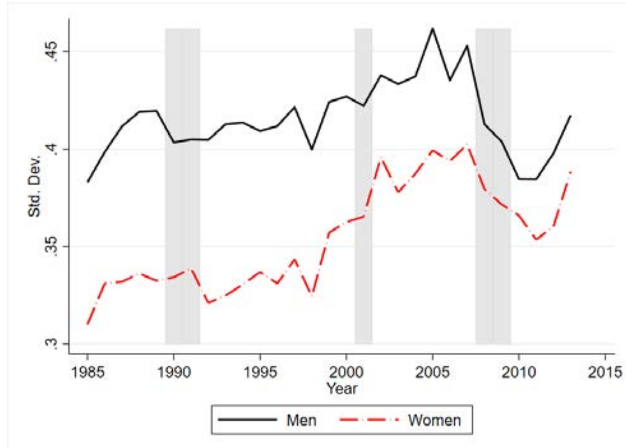
Note: Figure presents parameter estimates of the shocks to earnings over time with the lower minimum earnings cutoff.
Source: 1973, 1991, 1994, 1996-2016 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1982 to 2016.

Figure 21: Changes in earnings risk over time by gender

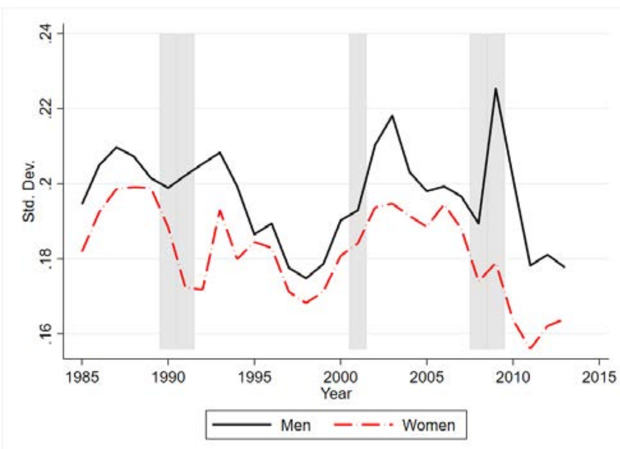
(a) Std. dev. persistent earnings (Emp.)



(b) Std. dev. persistent earnings (Unemp.)

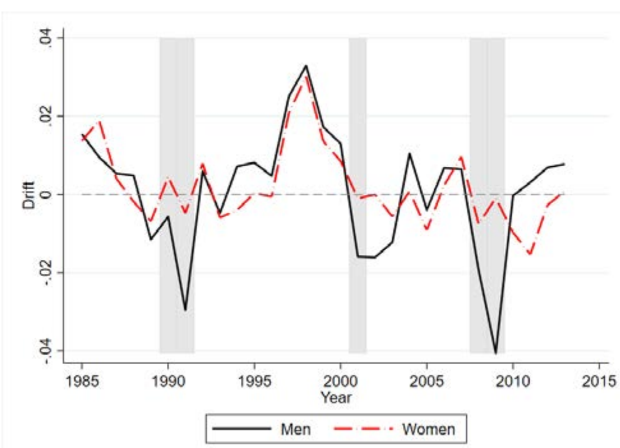


(c) Std. dev. temporary earnings

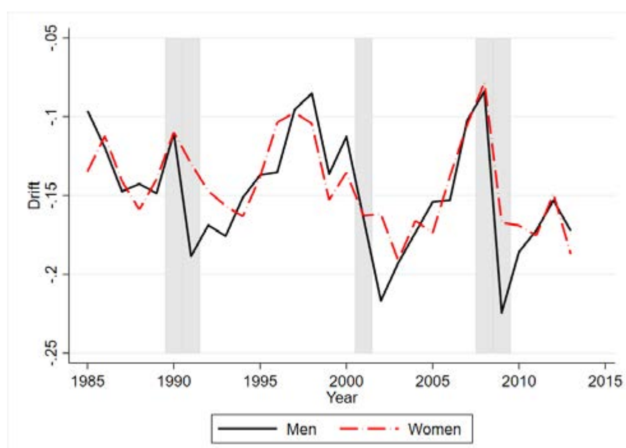


(d) Initial draw persistent earnings

(e) Drift persistent earnings (Emp.)

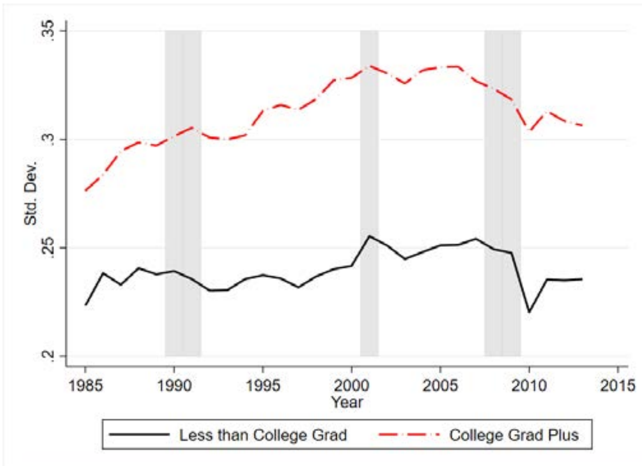


(f) Drift persistent earnings (Unemp.)

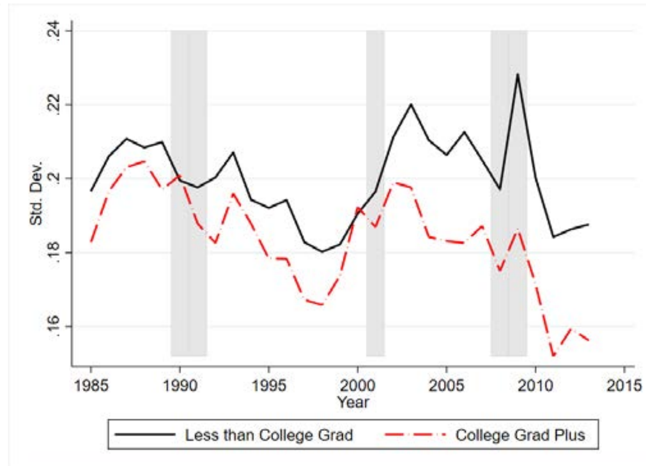


Note: Figure presents parameter estimates of the shocks to earnings over time by gender. The black solid line denotes men, and the red dashed line represents women.
Source: 1973, 1991, 1994, 1996-2016 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1982 to 2016.

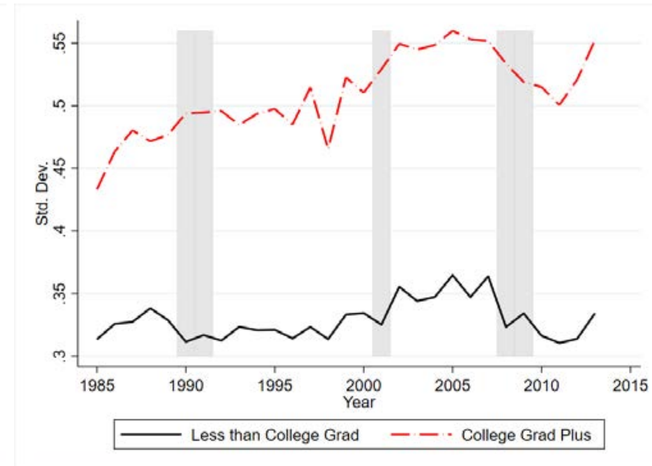
Figure 22: Changes in earnings risk over time by education



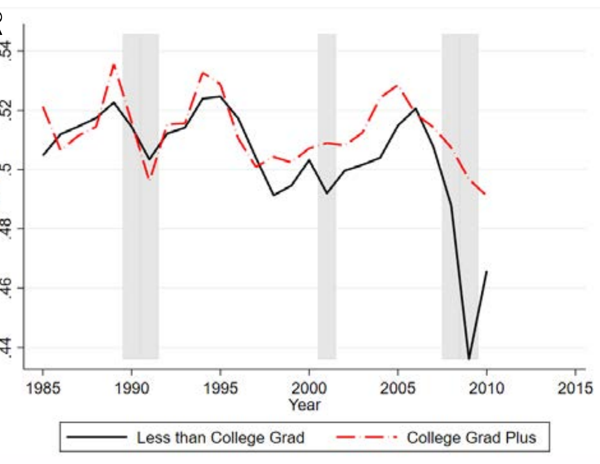
(a) Std. dev. persistent earnings (Emp.)



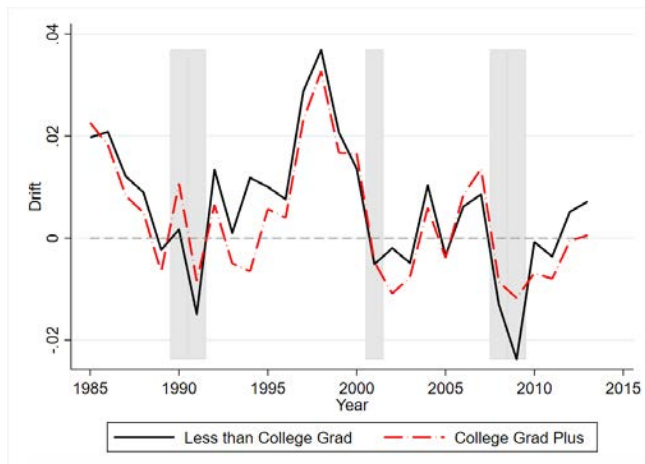
(b) Std. dev. temporary earnings



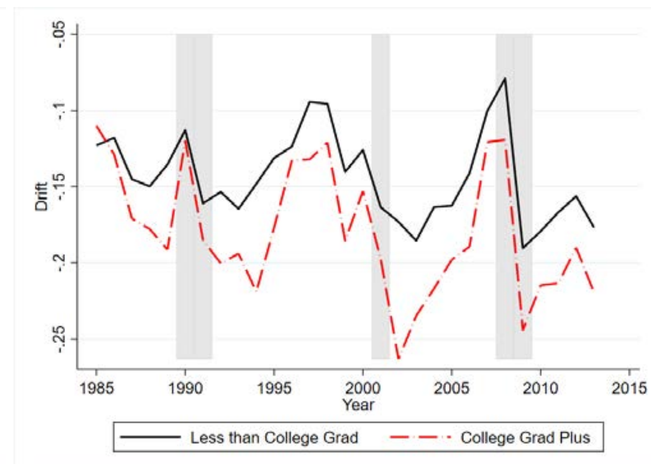
(c) Std. dev. persistent earnings (Unemp.)



(d) Std. dev. initial draw persistent earnings



(e) Drift persistent earnings (Emp.)



(f) Drift persistent earnings (Unemp.)

Note: Figure presents parameter estimates of the shocks to earnings over time by level of the education as reported in the CPS. The black solid line denotes individuals with less than a college degree, and the red dashed line represents individuals with a college degree or higher.
Source: 1973, 1991, 1994, 1996-2016 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1982 to 2016.

G.4 Additional Results: Geographic Variation

In this Appendix, we present additional results where we estimate the income process by state.

We formally test the second hypothesis that rising persistent earnings risk is related to the declines in manufacturing employment and union membership. Let X_s be the change in union membership (manufacturing employment) in state s between 1985-1991 and 2006-2013. The share of employed workers that are members of a union is measured in the CPS, while manufacturing employment is based upon Fort and Klimek (2016) industry classifications in the SSA data. Let $\Delta Y_{s,j} = Y_{s,j} - Y_{s,(1985-1991)}$ denote the change in parameter Y (e.g. the standard deviations of shocks to persistent earnings among employed etc.) for state s between time period j and $1985 - 1991$.[78] Let γ_j denote a set of year window fixed effects. The specification we use is of the form:

$$\Delta Y_{s,j} = \alpha + \eta X_s + \gamma_j + \epsilon_{s,j} \tag{43}$$

The parameter of interest is η which reports the correlation between the change in union membership (manufacturing employment) in a state and measures of earnings risk in that state. If $\eta < 0$, then we have evidence that in states with larger declines in union coverage (manufacturing employment) there have been larger increases in earnings risk.

Tables 7 and 8 present the results of estimating equation (43) for changes in union coverage and manufacturing employment, respectively. The tables show that changes in union coverage and manufacturing employment are largely uncorrelated with changes in earnings risk.

[78]Note the time-periods are 1992-1998, 1999-2005 and 2006-2013.

Table 7: Changes in union membership and earnings risk

	Chg. Std. Dev. Pers. Emp.	Chg. Std. Dev. Pers. Unemp.	Chg. Std. Dev. Pers. Comb.	Chg. Std. Dev. Temp.	Chg. Mean Pers. Emp.	Chg. Mean Pers. Unemp.
Chg. Union Membership	-0.00168	-0.00106	-0.00168	-0.00171	0.00250*	0.00480*
	(0.00135)	(0.00317)	(0.00143)	(0.00181)	(0.00125)	(0.00247)
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
R-Sq.	0.284	0.391	0.296	0.190	0.330	0.391
No. Obs (States, rounded)	200	200	200	200	200	200

*Notes: Table presents results from estimating equation (43) where the independent variable is the change in union membership by state. The change in union membership is measured between 1985-1991 and 2006-2013 for each state, and have been normalized to have mean zero and unit variance. Clustered SE in parenthesis, where the clustering is performed at the state level.****$p < 0.01$, **$p < 0.05$,*$p < 0.1$.

Source: 1973, 1991, 1994, 1996-2016 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1982 to 2016.

Table 8: Changes in manufacturing employment and earnings risk

	Chg. Std. Dev. Pers. Emp.	Chg. Std. Dev. Pers. Unemp.	Chg. Std. Dev. Pers. Comb.	Chg. Std. Dev. Temp.	Chg. Mean Pers. Emp.	Chg. Mean Pers. Unemp.
Chg. Manufacturing Share	-0.00348*	0.00598*	-0.00355*	-0.00587**	0.00591***	0.00285
	(0.00175)	(0.00329)	(0.00183)	(0.00254)	(0.00141)	(0.00291)
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
R-Sq.	0.315	0.417	0.329	0.271	0.467	0.369
No. Obs (States, rounded)	200	200	200	200	200	200

*Notes: Table presents results from estimating equation (43) where the independent variable is the change in manufacturing employment share by state. The change in manufacturing employment share is measured between 1985-1991 and 2006-2013 for each state, and have been normalized to have mean zero and unit variance. Clustered SE in parenthesis, where the clustering is performed at the state level.****$p < 0.01$, **$p < 0.05$,*$p < 0.1$.

Source: 1973, 1991, 1994, 1996-2016 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1982 to 2016.

G.5 Additional Results: Occupations

In this Appendix, we present addition results where we estimate the income process by occupation. We first present results for routine occupations, and then present results for alternative measures of high skill occupations.

G.5.1 Routine Occupations

In this section, we split occupations by their routine task content as measured in Acemoglu and Autor (2011). Acemoglu and Autor (2011) provide a measure of the routine manual as well as routine cognitive task content of an occupation. We combine their estimates into a single measure of the routine task content of an occupation by averaging the two measures. As in Acemoglu and Autor (2011) we normalize the index to be mean zero and have unit variance. Table 9 presents the results of estimating equation (8) where the independent variable is the routine task content of an occupation. The results presented in Table 9 shows that the degree of non-routine task content is not correlated with changes in the persistent earnings risk among the employed or unemployed. Additionally, the table shows that occupations with greater routine task content have seen an *increase* in temporary earnings risk.

G.5.2 High Skill Occupations

We next present results for our measures of high skill occupations.

Non-Routine Cognitive Skills In this section, we split occupation by their degree of "Non-Routine Cognitive Analysis" skills (henceforth, *non-routine cognitive skills*) as measured by Acemoglu and Autor (2011) using O*NET data. Figure 23 presents a graphical representation of estimating equation (8) for all measures of earnings risk. Table 10 presents the coefficient estimates underlying the graphical evidence in Figure 23. The results shows that occupations with a greater degree of non-routine cognitive skill content (e.g. high skill occupations) have experienced: (1) a larger increase in persistent earnings risk while employed and unemployed, (2) a larger decline in temporary earnigns risk, and (3) larger declines in persistent earnings during spells of unemployment.

Mean Earnings. In this section, we split occupations by their mean log earnings for the $1985 - 1991$ time period. To ease the interpretation we normalize the statistic to have mean zero and standard deviation equal to one. Table 12 presents the results of estimating equation (8) where the independent variable is mean log earnings in the occupation between 1985 and 1991. The results in Table 12 show that workers in higher paying occupations have experienced: (1) a larger increase in persistent earnings risk while employed and unemployed, and (2) larger declines in persistent earnings during spells of unemployment.

Years of Education. In this section, we split occupations by their average years of education in the $1985-1991$ time period. To ease the interpretation we normalize the statistic to have mean zero and standard deviation equal to one. Table 13 presents the results of estimating equation (8) where the independent variable is the mean years of completed education in the occupation between 1985 and 1991. The results in Table 13 show that workers in higher paying occupations have experienced: (1) a larger increase in persistent earnings risk while employed and unemployed, (2) a larger decline in temporary earnigns risk, and (3) larger declines in persistent earnings during spells of unemployment.

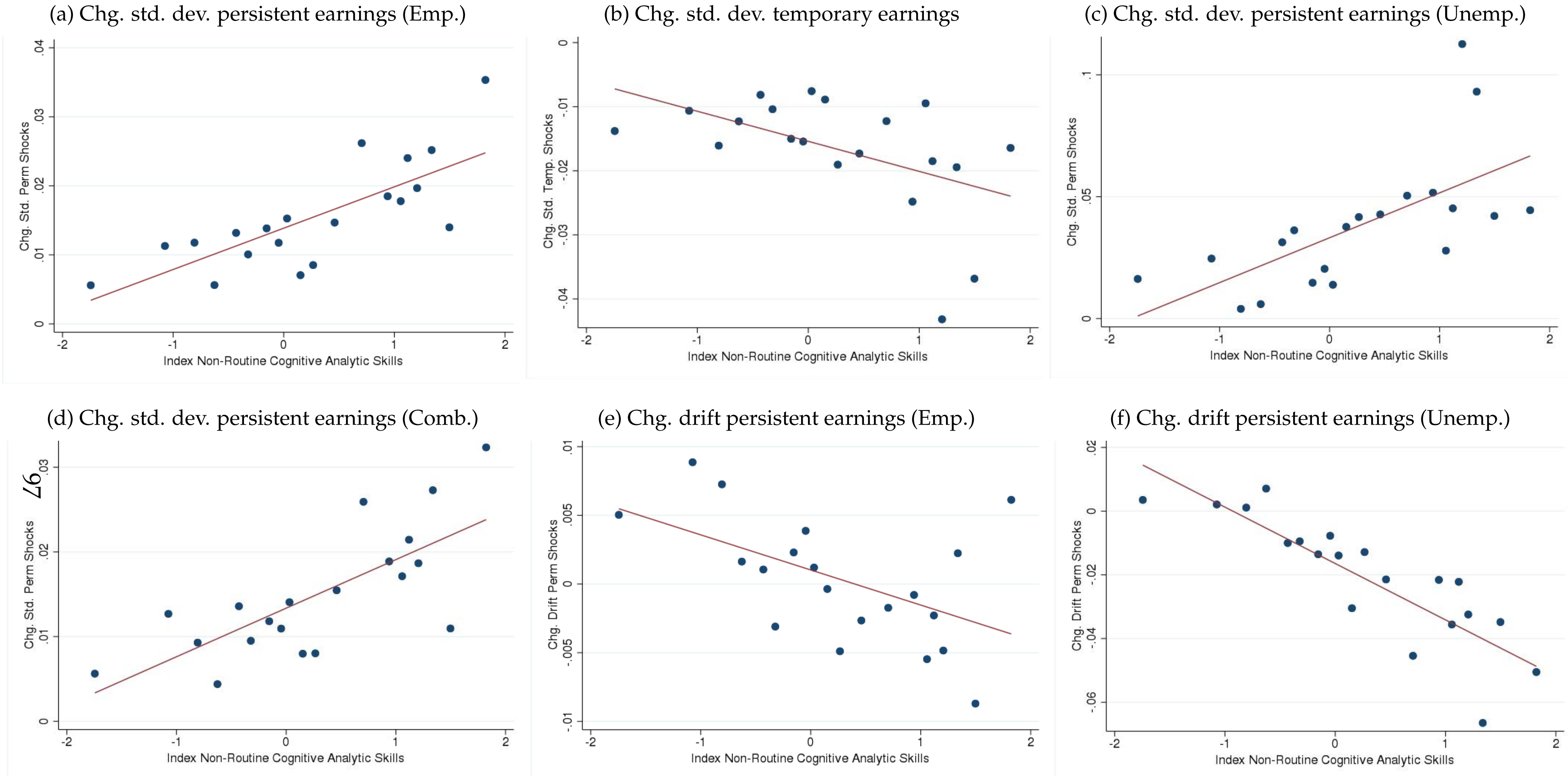
Computer Skills. In this section, we estimate an occupations exposure to technological change over time by measuring exposure to computer and software requirements. We measure exposure to computer and software requirements using data on the skill content of vacancies provided by Burning Glass Technologies. As in Braxton and Taska (2020) we measure exposure to computer and software requirements by measuring the share of vacancies in an occupation that list a computer or software requirements. We measure this share using data from vacancies posted in 2010. Table 11 presents the results of estimating equation (8) where the independent variable is exposure to computer and software requirements. The results in Table 11 show that workers in higher paying occupations have experienced: (1) a larger increase in persistent earnings risk while employed and unemployed, and (2) larger declines in persistent earnings during spells of unemployment.

Table 9: Routine Occupations and Changes in Earnings Risk

	(1) Chg. Std. Dev. Pers. Emp.	(2) Chg. Std. Dev. Pers. Unemp.	(3) Chg. Std. Dev. Pers Comb.	(4) Chg. Std. Dev. Temp.	(5) Chg. Mean Pers. Emp.	(6) Chg. Mean Pers. Unemp.
Routine	-0.000800	-0.00165	-0.000611	0.00314***	0.000919*	0.00418**
	(0.000975)	(0.00225)	(0.000981)	(0.00109)	(0.000506)	(0.00172)
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
R-Sq.	0.124	0.054	0.123	0.101	0.212	0.109
No. Obs. (Occ.)	1000	1000	1000	1000	1000	1000

*Notes: Table presents results from estimating equation (8) where the independent variable is the measure of routine task content of an occupation from Acemoglu and Autor (2011). The measure of routine task content is normalized to have mean zero and unit variance. Clustered SE in parenthesis, where the clustering is performed at the occupation level. ***$p < 0.01$, **$p < 0.05$,*$p < 0.1$. Source: 1973, 1991, 1994, 1996-2016 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1982 to 2016.*

Figure 23: Changes in earnings risk by non-routine cognitive analytic skills



Note: Figure presents a graphical representation of the regression in equation (8), where the measure of task content is non-routine congitive analytic skills as measured in Acemoglu and Autor (2011).
Source: 1973, 1991, 1994, 1996-2016 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1982 to 2016.

Table 10: Non-routine cognitive skills and changes in earnings risk

	Chg. Std. Dev. Pers. Emp.	Chg. Std. Dev. Pers. Unemp.	Chg. Std. Dev. Pers. Comb.	Chg. Std. Dev. Temp.	Chg. Mean Pers. Emp.	Chg. Mean Pers. Unemp.
Non-Routine Cognitive.	0.00605***	0.0175***	0.00574***	-0.00469**	-0.00255**	-0.0174***
	(0.00163)	(0.00412)	(0.00166)	(0.00189)	(0.00109)	(0.00254)
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
R-Sq.	0.186	0.097	0.180	0.095	0.238	0.194
No. Obs. (Occ.)	1000	1000	1000	1000	1000	1000

Note: Table presents results from estimating equation (8) where the independent variable is Non-Routine Cognitive Analysis as measured in Acemoglu and Autor (2011). The variable Non-Routine Cognitive Analysis is constructed to have mean zero and unit variance. Clustered standard errors in parenthesis where the clustering is performed at the occupation level. $***p < 0.01, **p < 0.05, *p < 0.1$.
Source: 1973, 1991, 1994, 1996-2016 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1982 to 2016.

Table 11: Computer skills and changes in earnings risk

	(1) Chg. Std. Dev. Pers. Emp.	(2) Chg. Std. Dev. Pers. Unemp.	(3) Chg. Std. Dev. Pers. Comb.	(4) Chg. Std. Dev. Temp.	(5) Chg. Mean Pers. Emp.	(6) Chg. Mean Pers. Unemp.
Computer Skills	0.00685***	0.00945**	0.00717***	0.00425***	0.00154	-0.0145***
	(0.00137)	(0.00382)	(0.00132)	(0.00140)	(0.00118)	(0.00296)
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
R-Sq.	0.222	0.068	0.229	0.094	0.219	0.176
No. Obs. (Occ.)	1000	1000	1000	1000	1000	1000

Note: Table presents results from estimating equation (8) where the independent variable is computer skills as measured in Braxton and Taska (2020). Computer skills are measured for each occupation in 2010, and have been normalized to have mean zero and unit variance. Clustered standard errors in parenthesis where the clustering is performed at the occupation level. $***p < 0.01, **p < 0.05, *p < 0.1$
Source: 1973, 1991, 1994, 1996-2016 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1982 to 2016.

Table 12: Mean Earnings and Changes in Earnings Risk

	(1) Chg. Std. Dev. Perm. Emp.	(2) Chg. Std. Dev. Perm. Unemp.	(3) Chg. Std. Dev. Perm Comb.	(4) Chg. Std. Dev. Temp.	(5) Chg. Mean Perm. Emp.	(6) Chg. Mean Perm. Unemp.
Mean Log Earnings	0.00588***	0.00981**	0.00592***	-0.000721	-0.000921	-0.0177***
	(0.00118)	(0.00400)	(0.00120)	(0.00212)	(0.00102)	(0.00262)
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
R-Sq.	0.196	0.069	0.195	0.064	0.208	0.216
No. Obs. (Occ.)	1000	1000	1000	1000	1000	1000

Notes: Table presents results from estimating equation (8) where the independent variable is mean log earnings in an occupation. Mean log earnings are measured in 1985-1991 for each occupation, and have been normalized to have mean zero and unit variance. Clustered standard errors in parenthesis where the clustering is performed at the occupation level. $***p < 0.01, **p < 0.05, *p < 0.1$

Source: 1973, 1991, 1994, 1996-2016 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1982 to 2016.

Table 13: Mean Years of Education and Changes in Earnings Risk

	(1) Chg. Std. Dev. Perm. Emp.	(2) Chg. Std. Dev. Perm. Unemp.	(3) Chg. Std. Dev. Perm Comb.	(4) Chg. Std. Dev. Temp.	(5) Chg. Mean Perm. Emp.	(6) Chg. Mean Perm. Unemp.
Mean Years Edu.	0.00416**	0.0142***	0.00370**	-0.00812***	-0.00346***	-0.0125***
	(0.00187)	(0.00355)	(0.00184)	(0.00180)	(0.000801)	(0.00307)
Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
R-Sq.	0.159	0.086	0.151	0.174	0.288	0.155
No. Obs. (Occ.)	1000	1000	1000	1000	1000	1000

Notes: Table presents results from estimating equation (8) where the independent variable is mean years of education in an occupation. Mean years of education are measured in 1985-1991 for each occupation, and have been normalized to have mean zero and unit variance. Clustered standard errors in parenthesis where the clustering is performed at the occupation level. $***p < 0.01, **p < 0.05, *p < 0.1$

Source: 1973, 1991, 1994, 1996-2016 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1982 to 2016.

G.6 Decomposing changes in earnings risk over time

In this section, we use the structure of the income process to further decompose changes in earnings risk over time into persistent and temporary components. Given the income process from Section 1, the variance of residual log income changes can be written as a function of persistent and temporary shocks as well as the variance of permanent earnings using the following formula,

$$var(y_t - y_{t-1}) = (F-1)^2\, var(z_{t-1}) + Q_{E,t} + R_t + R_{t-1} \tag{44}$$

Using equation (44), we next perform a counterfactual exercise of estimating the standard deviation of residual log earnings changes assuming that there was no change in either persistent or temporary income shocks since 1985.[79] Panel (a) of Figure 24 plots the standard deviation of residual log income changes (black, solid line) along with a counterfactual estimate of the standard deviation of residual log income changes where persistent earnings shocks ($Q_{E,t}$) have been fixed at their 1985 values (red, dashed line). For ease of interpretation we have normalized the timeseries to 100 in the year 1985. The figure shows that without the increase in persistent earnings risk, the standard deviation of residual log income changes would have declined by 7.5 percent, slightly more than the nearly 5 percent decline observed in the data.

We next examine the counterfactual path when temporary earnings risk is held fixed. Panel (b) of Figure 24 plots the counterfactual estimate of the standard deviation of residual log income changes where temporary earnings shocks (R_t and R_{t-1}) are fixed at their 1985 values (blue, dashed line). Without the decline in temporary earnings risk, residual log earnings changes would have increased by over 3 percent during the sample period, significantly higher from the nearly 5 percent decline observed in the data.

The results of this exercise demonstrate that variation in temporary earnings risk plays a larger role than changes in persistent earnings risk in shaping the evolution of overall earnings risk (e.g. the standard deviation of residual log earnings) over time. The rationale for the importance of temporary earnings risk is evident in equation (44), where there are two temporary earnings risk terms but only a single term for persistent earnings risk. Hence, declines in temporary earnings risk hold greater weight in equation (44) and lead to declines in overall earnings risk despite the increasing trend in persistent earnings risk.[80] We next examine trends

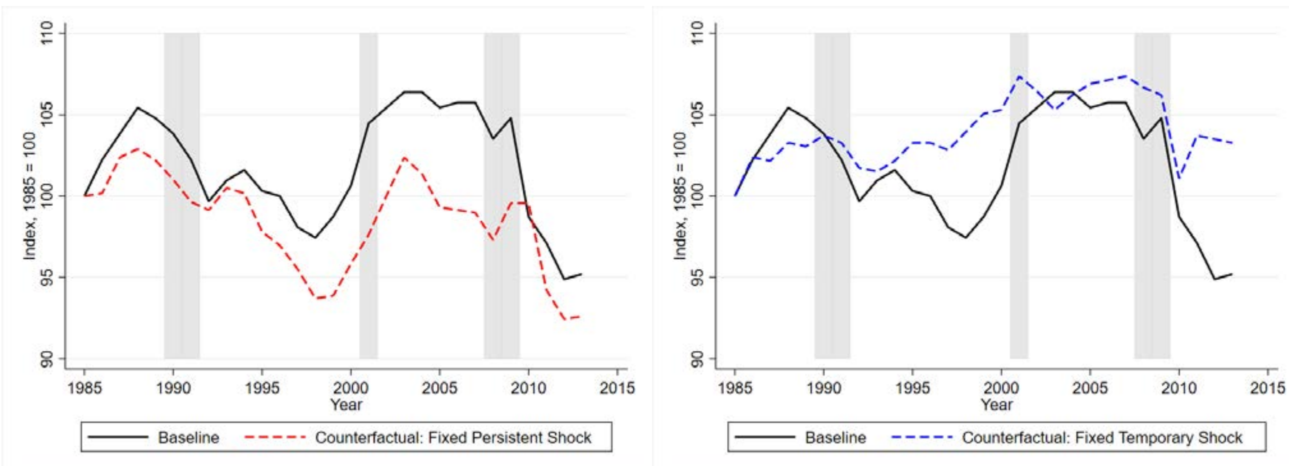
[79] For this decomposition, we include individuals whose earnings are above the minimum earnings criteria in both year t and year $t-1$. We additionally, use the individual level estimates of temporary and persistent risk when performing the decomposition exercise. Additionally note that since $F = 0.9424$, the first term in equation (44) plays a minor role in changes in the variance of log income.

[80] In Section 3.2.1, we show how the variance of earnings changes across different horizons can be used to identify the trends in temporary and persistent earnings risk.

in earnings risk among individuals in the CPS with observable shocks, and then explore the the drivers of these time series changes using the labor market, demographic, geographic, and occupation data available in the CPS.

Figure 24: Decomposing changes in earnings risk

(a) Fixing persistent earnings shocks (b) Fixing temporary earnings shocks



Note: Figure presents the standard deviation of residual log earnings changes over time (black, solid line), along with a counterfactual estimate where persistent earnings shocks are held fixed at their 1985 values (panel (a), red dashed line) and a counterfactual estimate where temporary earnings shocks are held fixed at their 1985 values (panel (b), blue dashed line). The counterfacutal estimates are generated using equation (44).
Source: 1973, 1991, 1994, 1996-2016 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1982 to 2016.

G.7 Additional Results: Higher order moments

In this Appendix, we provide summary statistics on the higher order moments generating by our estimation procedure for shocks to persistent and temporary earnings. Table 14 summarizes the higher order moments from our baseline estimation in Section 3 as well as our estimation which takes into account job switching (Section 3.3). The table shows that in our baseline estimation the shocks to persistent and temporary earnings exhibit negative skewness and excess kurtosis relative to a normal distribution, which as skewnewss equal to 0 and kurtosis of 3. By incorporating additional observables into the analysis we are able to obtain greater deviations from a normal distribution. In particular, in Section 3.3 we condition persistent and temporary shocks on job switching/job staying status. We refer to this estimation of the model as the "Job Switch" model. Table 14 shows that this addition noticeably increases the degree of negative skewness as well as excess kurtotis, especially for temporary shocks.

Table 14: Higher order moments to persistent and temporary shocks

Shock	Estimation	Std. Dev.	Skewnewss	Kurtosis
Persistent Earnings	Baseline	0.226	-0.347	5.89
Persistent Earnings	Job Switcher	0.204	-0.516	7.239
Temporary Earnings	Baseline	0.157	-0.282	4.654
Temporary Earnings	Job Switcher	0.176	-1.149	11.19

Notes: Table presents summary statistics on the higher order moments for persistent and temporary shocks. Source: 1973, 1991, 1994, 1996-2016 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1982 to 2016.

G.8 Additional Results: Sensitivity of trends in earnings risk

In this Appendix, we include additional details on our sensitivity analysis from Section 3.3. We first provide additional details on our sample of workers from the LEHD. We then provide time series for additional parameters of the income process across alternative estimations.

G.8.1 LEHD Sample

In this section we perform our filtering exercise using the data from the Longitudinal Employer-Household Dynamics (LEHD) database. The LEHD is a matched employee-employer dataset containing information on worker earnings and characteristics of the firm where the worker is employed.[81]

Our sample from the LEHD spans 1995-2014, and we consider an 1% random sample of workers from the states that have entered the LEHD program by the first quarter of 1995.[82] Using this sample of workers we impose the same sampling criteria as in the Section 2. Table 15 presents summary statistics for our LEHD sample and compares the sample to our SSA-CPS sample of workers.

G.8.2 Additional measures of income risk

In this Appendix, we present additional moments from the income process for the sensitivity analysis in Section 3.3. Figure 25 presents the results. In the figure the black solid line corresponds to our baseline estimates from Section 3. The figure shows that across alternative specfications and samples we find similar results for the: (1) trend increase in persistent earning risk among the employed (Panel (a)), (2) the trend decrease in temporary earnings risk (Panel

[81] See Abowd et al. (2009) for details on the construction of the LEHD.

[82] The states we use are: MD, AK, CO, ID, IL, IN, KS, LA, MO, WA, WI, NC, OR, PA, CA, AZ, WY, FL, MT, GA, SD, MN, NY, RI, TX. These states represent over 67% of employment as recorded in the QCEW in the first quarter of 2012. See McKinney and Abowd (2020) for details on when each state entered the LEHD program.

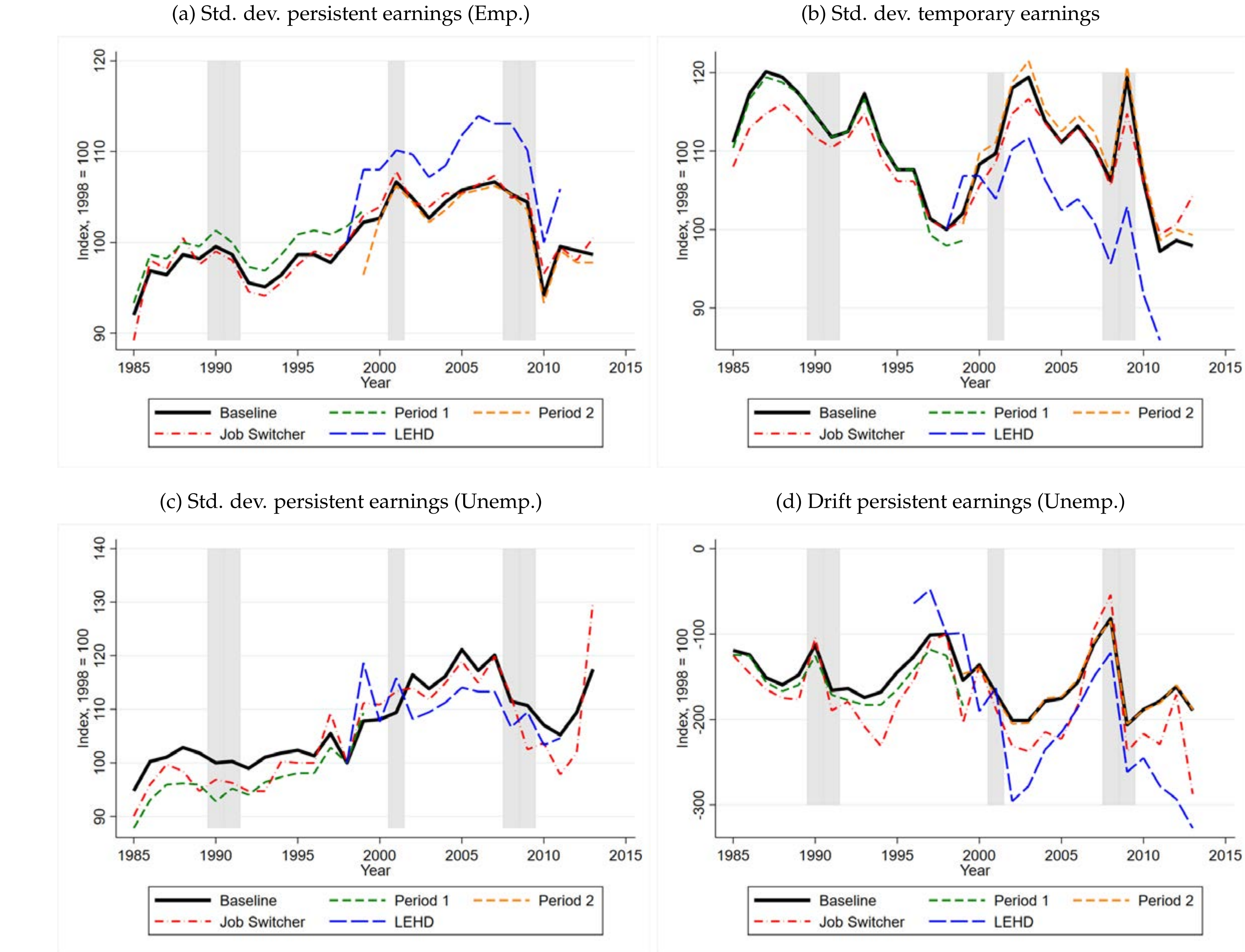
Table 15: Summary statistics

	(1)	(2)
Variable	Mean (LEHD)	Mean (SSA-CPS)
Real Annual Earnings	$58,890	$55,650
Age	40.93	40.78
Share Unemployed	7.30%	6.90%
Observations	18,020,000	235,000,000
Individuals	1,296,000	1,157,000

Note: Sample selection criteria in Section 2.1. Real Annual earnings is measured in 2019 dollars. The variable "share unemployed" is the share of individuals whose average earnings in a given year do not satisfy the minimum earnings criteria.
Source: 1973, 1991, 1994, 1996-2016 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1982 to 2016. Longitudinal Employer-Household Dynamics (LEHD) database for 1995 to 2014.

(b)), (3) the trend increase in persistent earnings risk among the unemployed (Panel (c)), and (4) the greater decline in persistent earnings during spells of unemployed (Panel (d)).

Figure 25: Persistent and temporary earnings risk over time in LEHD



Note: Figure compares the estimates of persistent and temporary earnings risk over time for alternative income processes and estimations of the income process over different samples. See Section 3.3 for details on the different estimations and samples.
Source: 1973, 1991, 1994, 1996-2016 Current Population Survey Annual Social and Economic Supplement linked to the Detailed Earnings Record for 1982 to 2016. Longitudinal Employer-Household Dynamics (LEHD) database for 1995 to 2014.

H Directed search model

Consider a search environment analogous to Braxton et al. (2020). Time is discrete and runs forever. There is a unit measure of individuals who live indefinitely. Individuals search for jobs in a labor market that is segmented by a worker's persistent human capital which we detail below. When a worker matches with a firm, they are paid a *piece-rate* $\omega \in \mathcal{W} \subseteq [0,1]$, where ω determines the share of production paid to the worker.[83] Let $e \in \{W, U\}$ denote employment status, where $e = W$ is employed and $e = U$ is unemployed. Individuals have two components of human capital: (1) persistent human capital, h^p, which follows an AR(1) and (2) transitory human capital, h^t, which is iid.

We assume that persistent and transitory human capital evolve as follows:

$$\ln h^{p'} = \rho \ln h^p + h_e + \epsilon, \qquad \epsilon \sim N(0, \sigma^p)$$

$$\ln h^t = u, \qquad u \sim N(0, \sigma^t)$$

where $h_e > 0$ for $e = W$ and $h_u < 0$ for $e = U$ capture on-the-job learning while employed and human capital depreciation while unemployed, respectively.

We assume the production function is multiplicative in human capital, i.e., $f(h^p, h^t) = h^p h^t$. Since workers are paid a piece-rate, worker income is $y = \omega f(h^p, h^t) = \omega h^p h^t$. Among workers, observed log income is given by $\ln y = \ln \omega + \ln h^p + \ln h^t$. Observed log income therefore inherits the properties of h^p, and is itself an AR(1). We assume the unemployed receive home production $b(h^p) = \gamma h^p$ which is proportional to their persistent human capital h^p and unobserved to the econometrician.

The model is closed with a free entry condition for firms. Firms post vacancies at cost κ, yielding equilibrium job finding probabilities $p(h^p)$.[84]

The timing of events from the start to end of a period is: (1) workers produce and consume, (2) unemployed workers search for a job/employed workers are laid off at rate $\delta(h^p)$, and lastly (3) at the end of the period, human capital transitions are realized; thus, job status is a function of lagged human capital realizations, and future human capital is a function of job status, but contemporaneous shocks to transitory and persistent human capital do not determine employment status. In terms of our econometric model, this timing assumption is equivalent to *sequential exogeneity* discussed in Section 1.3.

[83] The assumption of a single piece rate closely resembles per-period Nash-Bargaining over intra-period production, a common modeling device used by Huckfeldt (2014) and Kaplan and Menzio (2013).

[84] Allowing vacancy costs to vary by the persistent human capital of the worker allow the model to match the heterogeneity in transitioning from unemployment to employment seen in the data.

Formally, an unemployed worker continuation value is given by

$$U(h^p, h^t) = u(\gamma h^p) + \beta\{p(h^p)E_{h^{p'},h^{t'}|h^p}W(h^{p'}, h^{t'}) + (1 - p(h^p))E_{h^{p'}|h^p}U(h^{p'})\}.$$

An employed worker's continuation value is similarly defined, where we assume there is an exogenous job-loss probability $\delta(h^p)$ each period,

$$W(h^p, h^t) = u(\omega h^p h^t) + \beta(1 - \delta(h^p))E_{h^{p'},h^{t'}|h^p}W(h^{p'}, h^{t'}) + \delta(h^p)E_{h^{p'}|h^p}U(h^{p'}).$$

Income is given by $\ln y = \ln \omega + \ln h^p + \ln h^t$. Recall in our estimation we remove the predictable component of income, which in the above equation for income is given by $\ln \omega$. Hence, this fairly standard directed search environment yields an identical income process to the one estimated in Section 1.1. Other period-by-period bargaining protocols in the presence of random search (e.g. Diamond-Mortensen-Pissarides models), as well as McCall models with exogenous piece-rate wage distributions, yield analogous income processes that inherit the properties of h^p and h^t.