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**ABSTRACT**

We provide a model to analyze charter school educational practices. Students differ in cognitive ability, motivation, and household income. Student achievement depends on ability, match of their school's curriculum to their ability, and effort. Charter schools choose curriculum to maximize achievement gains, optimally setting curriculum to attract lower ability students. Achievement gains are modest, consistent with empirical evidence. We also investigate "no excuses" charter schools. These charters enforce an effort minimum that attracts highly motivated students. We find, consistent with the evidence, that these charters are highly effective in increasing achievement, with the largest gains accruing to lower ability students.

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A is available at http://www.nber.org/data-appendix/w29529

## 1. Introduction

Charter schools are playing an increasingly important role in US education. The first charter school opened in Minnesota in 1992. Charter schools now operate in 44 states. From fall 2009 to fall 2018, the percentage of public school students attending charter schools increased from 3 to 7 percent.[1] The charter share is much higher in many, especially urban, areas, exceeding 15 percent in 20 cities in the 2010-11 school year.[2] Moreover, more than 10% of public school students attend charter schools in 6 states. Charters are tuition-free public schools and are not permitted to impose selective criteria for admission. If oversubscribed, a charter must admit by lottery from its applicant pool. Charter schools are, however, permitted to adopt curriculum and educational practices different from those of traditional public schools. Much research has been devoted to comparing the performance of charter and public schools and to documenting educational approaches of charter schools. This research has established that large differences across charter schools exist in the extent to which they improve educational outcomes relative to the public schools their students would otherwise attend.

In contrast to this impressive body of empirical research, there has been little formal modeling of the decisions about curriculum and educational practices of a prospective charter school entrant, how those decisions vary with the educational environment the charter would enter, or the impacts on students of differing capabilities and socioeconomic background. There is, to the best of our knowledge, no existing framework that might be employed to address these fundamental issues.

In this paper, we develop such a framework, and apply it to analyze decisions about curriculum and educational practices made by a charter entrant seeking to maximize the educational gains of the students it attracts. We investigate how such choices vary with the characteristics of the student population from which the charter school draws its students. This in turn requires modeling the school choice decisions of students of differing capabilities and socioeconomic backgrounds. It also requires comparison of student outcomes in the equilibrium with charter entry relative to student outcomes in the equilibrium without charter entry. While charter schools cannot impose selective criteria for admission, and so must randomly admit students among their applicants if oversubscribed, their curriculum choice maximizes the achievement gains of their students, both by targeting their capabilities and by affecting student self-selection into the school. We find higher gains in schools that target lower ability students. Those students benefit more from a curriculum adapted to their abilities than do higher ability students with curriculum adapted to their abilities.

In addition to considering traditional charter schools, a key part of our analysis models "no-excuses charter schools." Such schools have been quite successful in increasing student achievement.[3] In addition to their curriculum choice, these charters enforce student work standards with parental support. We find these charters attract highly motivated students with a wider range of abilities.

[1] https://nces.ed.gov/programs/coe/indicator/cgb
[2] See Figure 4, panel C of Epple, Romano, and Zimmer (2016).
[3] The evidence is discussed in Section 2.

Our model features some elements that, while being central to the educational process, have not received much attention in the theoretical literature. Students differ by cognitive and non-cognitive skills, which can be interpreted as, respectively, academic ability and motivation. Students also differ by income of their household. A student's achievement depends on the quality of the school attended, the student's academic ability, and the (costly) effort the student exerts. Costs of effort in turn depend on motivation. Three key educational variables shape a school's educational process: academic standards, school curriculum and, in the case of a no-excuses charter school, the required effort minimum. Standards, set by the educational authority, determine how demanding it is for a student to graduate. Higher standards may push some students to exert greater effort than they would otherwise choose, but may discourage others from exerting any effort at all. A school's curriculum establishes the academic ability the school targets with its teaching, and thereby affects the quality of education received by every student. The model thereby captures an important feature of the educational process; the quality of the educational services provided by a school varies across students and depends on the alignment between a student's ability and the school's curriculum.

We follow with a computational counterpart to our theoretical model. For calibration, we utilize recent econometric evidence regarding the roles of different types of skills in determining educational outcomes. We couple this with information about the distributions of student ability and student achievement. We compute the gains in student achievement from different charter school designs and for different student populations. We compare the results to magnitudes estimated in evaluations of charter school impacts. There is a close congruence between the magnitudes from our computational model and those found in the empirical literature. Our calibration does not use the latter estimates. Hence, these comparisons provide supportive evidence for the validity of our model.

Our paper proceeds as follows. Section 2 reviews related literature and discusses further our contribution. Sections 3 and 4 present, respectively, our theoretical framework and results. Our computational model and its findings follow in Section 5, including comparison to empirical findings. Section 6 concludes. Some technical analysis is in the appendix.

**2. Literature Discussion.**

The relative newness and on-going growth of charter schools has inspired an extensive body of research. The focus has been on their performance in increasing student achievement relative to traditional public schools, with the primary identification challenge being student selection into charter schools on unobservables.[4] Here we provide a brief summary and refer the reader to Epple, Romano, and Zimmer (2016) for a comprehensive review of this research up through about 2015, and

[4] Empirical research on charter schools also investigates the impact on educational attainment and non-educational outcomes, where charter schools locate and what students they serve, and competitive effects on neighboring traditional public schools. See Epple, Romano, and Zimmer (2016) for citations and discussion.

to Chabrier, Cohodes, and Oreopoulos (2016) for review of lottery studies of charter schools.[5] The thrust of the literature is that: (i) Overall, charter schools have mixed performance in increasing achievement, but with an upward trend and average positive gains in urban areas (see especially National Charter School Study, CREDO 2015[6]). (ii) Charter schools that follow the "No-Excuses" approach dramatically increase student achievement, especially for students with lower prior scores and for black students.[7] For example, Abdulkadiroglu, et.al (2011), exploiting the lottery setting, find local-average-treatment-effect estimates of attending a no-excuses school equal to .198σ and .359σ in middle school ELA and math and .265σ and .364σ in high school ELA and math[8].

We next discuss the research most closely related to our theoretical modelling of charter schools and student choices. In Section 5.3, we discuss the predictions of our model in light of the empirical research. We present a model of the micro-foundations of charter school differentiation based on curriculum focus, with an explicit charter objective. Students choose between the charter and traditional public school. Preference across schools varies with student ability, motivation, and socioeconomic background. Glomm, Harris, and Lo (2005) appeal to horizontal charter school differentiation, generally defined, to explain why, in Michigan and California, charter schools entered districts that are characterized by more varied parental and more racially diverse populations. Exogenous differentiation of charter schools plays a role in several empirical papers on charter schools. Singleton (2019) estimates a structural model of charter school entry allowing costs of educating students to vary with their demographics and across profit, not-for-profit, and no-excuses charter schools. He provides evidence that charter schools are more likely to enter localities where they attract student types with lower educational costs. Lower cost students are then selected by the locality of entry, this related to our model's emphasis on student selection, but by educational practices.[9] Gilraine, Petronijevic, and Singleton (2019) investigate how achievement gains vary among charter schools whose applications specify that "learning is experiential or project-based as opposed to focused on core skills through traditional instruction." Bau (2019), though studying private schools in Pakistan, has schools specialize to serve rich or poor students, this related to our

[5] Lottery studies exploit over-subscribed charter schools that are required to admit students randomly, essentially comparing the educational outcomes of lottery winners and losers, thus controlling for selection (though then able to use only over-subscribed charter schools).
[6] https://credo.stanford.edu/publications/national-charter-school-study. Another very recent national study finding average gains and a positive trend is Shakeel and Peterson (2021).
[7] See, for example, Abdulkadiroglu et al. (2011), Angrist et al. (2012), Dobbie and Fryer (2013), Angrist, Pathak, and Walters (2013), and Walters (2018).
[8] These LATE estimates are per year spent in the charter school and control for demographics and baseline scores. While dramatic, these are typical among studies of no-excuses schools.
[9] Other evidence that charter schools select easier-to-educate students exists. Bergman and McFarlin (2020) conduct an experiment showing that charter schools may discourage attendance of high-cost students by being less responsive to (fictitious) emails inquiring about the school that signal special needs, as well as low achievement and poor behavior. Charter schools in states that reimburse for a large share of special-needs costs did not so behave.

curriculum specialization to student abilities.[10] Geographic locational choice is central to several empirical models of charter school entry, these taking residences of racially and socio-economically diverse populations as given, with students facing travel costs to attend schools. This includes Singleton (2019), Ferreyra and Kosenok (2018), and Mehta (2017). Ferreyra and Kosenok (2018) also have schools differentiated by whether they follow a core, language-oriented, arts, vocational, or "other" curriculum. Mehta (2017) has schools dictate student effort, which is similar to what we do in our model of no-excuses schools.[11] No-excuses charters differ from other schools by their educational practices. Those practices, which vary somewhat among such schools, include: (i) a longer school day and year; (ii) frequent testing; (iii) an ethos of comportment and a strong student work ethic;[12] (iv) extensive tutoring; (v) emphasis on core math and reading; and (vi) selective teacher hiring and professional development. Dobbie and Fryer (2013) construct an index of teacher feedback, use of data to guide instruction, tutoring, instructional time, and high expectations of students that explains 45 percent of the variation in charter school effectiveness. Angrist, Pathak, and Walters (2013) identify emphasis on discipline, school uniforms, cold-calling, adherence to school-wide standards, and employment of Teach-for-America teachers as the five practices that best explain effectiveness of no-excuses schools. As noted, we discuss further empirical findings below as related to our model's predictions. Our contribution is to provide a theoretical and computational model of key charter school practices, chosen endogenously to maximize achievement gains of students strategically drawn from a given educational market. This contrasts with the focus in the structural models on charter choice of location choice that is likely to make a charter school viable. Central to our model is the dichotomous nature of student capability shown by Cunha, Heckman, and Schennach (2010),

[10] It bears noting in the international context that publicly funded, privately run schools, akin to charter schools, are present in many countries, including Chile, England, the Netherlands, Spain, and Sweden. There is variation across these countries in private school autonomy (see Figure 1 in Eyles, Hupkau and Machin, 2016, and the discussion therein), practices that are afforded autonomy, funding models, and market shares. For instance, Chile only partially subsidizes private schools (called *escuelas particulares subvencionadas*), with parents contributing the rest of the school's funding. In Spain, the network of (mostly Catholic) *escuelas concertadas* educate about a quarter of the population of students. Like US charter schools, these schools are not permitted to charge fees or to select students but, unlike in the US, they cannot modify the curriculum, and there is controversy as to whether or not, contrary to the law, some do charge fees (see, for instance, El País, 6th October 2021). In Sweden, the so-called free schools (*friskolor*), are not allowed to charge fees, select students, or modify the curriculum. Closest schools to American charter schools are, perhaps, English *academy schools*: these cannot charge fees or be selective and have autonomy to modify the curriculum and other aspects of their educational model. Some English academies adopt similar practices to "no excuses" US charter schools. There are differences, however: *some* English academies are religious and can select pupils by their faith (this not allowed in the US). That is also the case in the Netherlands; publicly funded private schools can be religious. There, as in our model, the Dutch government sets the graduation standards for a set of compulsory subjects (so-called attainment targets), while schools choose how to reach those targets when they design the curriculum.

[11] An interesting element of Mehta's model is that, when charter schools require more student effort, school cost rises, because teachers then have more work. We do not consider this. On the other hand, we allow students to choose more than the required effort in our no excuses schools, while this is not considered by Mehta.

[12] In Knowledge is Power Program (KIPP) charter schools, parents and students sign contracts including a pledge of timely attendance, hard work, and completion of assignments. KIPP is the largest charter management organization in the nation, with 255 schools at the time of this writing.

Heckman and Kautz (2012), Heckman Stixrud, and Urzua (2006), Knudsen, Heckman, Cameron, and Shonkoff (2006), and Borghans, Golsteyn, Heckman, and Humphries (2016) to be important to predict achievement. These foundational contributions play a central role both in the formulation of our theoretical model and in the counterpart quantitative model. We are aware of no prior theoretical modelling of this skill dichotomy and its relation to student effort and schooling strategy.[13] While different focuses of schools are elements of some of the structural models, as noted above, our model stresses curriculum targeting as an explicit choice. Regarding this element, Duflo, Dupas, and Kremer (2011) similarly examine curriculum targeting in their modelling and experimental evaluation of tracking in Kenya. They provide evidence of targeting teaching to the abilities of students (linked to teacher rewards), supporting this component of the model we adopt. While student effort is not considered in their model, they state: "Student effort might also respond endogenously to teacher effort and the target teaching level. In such a model, ultimate outcomes will be a composite function of teacher effort, teacher focus level, and student effort (p. 1747)." Our model includes the interaction between student effort choice and the target. Finally, the model takes account of potential divergence between parental and student preferences, and we find conflict in equilibrium in the case of no excuses charter schools. Students do not like to study as much as their parents prefer, this consistent with some evidence on no excuses charter schools (Golann, 2015).

## 3. Model and Preliminary Results

### 3.1 The Model

Households, Education Production, and Achievement. We study a continuum of households with mass normalized to 1. Each household consists of one school-aged child and a parent or parents. Our model reflects and synthesizes the latest available evidence on skill formation and the determinants of school achievement in the literature cited just above. Households differ along three dimensions: parental income (y), the child's intellectual or cognitive ability (b), and motivation (m). The characteristics (b,m) of the child can be more broadly conceptualized as cognitive and non-cognitive skills, respectively. Students of greater cognitive ability have higher educational achievement for given school characteristics and effort (e) in school. Students who are more motivated are more efficient at or more inclined to study, which we capture by assuming they face lower costs of exerting schooling effort. For expositional ease, we refer to b as just "ability." We assume parents know their own child's ability and motivation.

[13] To study the impact of immigration on the school system, Albornoz et al. (2018) develop a model where parents (as well as schools) differ by the strength of their motivation to educate their child (or their pupils), and where adult labour market outcomes depend on learning effort exerted by the child at school. In contrast to our model, their model has students who lack an intrinsic motivation to learn. Instead, their model has parents and schools provide costly incentives that reward students' learning efforts.

The probability density function of student types in the population is denoted with f(b,m,y), and is positive on its support $[b_m, b_x]\times[m_m, m_x]\times[y_m, y_x]$. Let $\bar{b}$, $\bar{m}$ and $\bar{y}$ denote the population mean ability, motivation, and income, and $b_{med}$, $m_{med}$, $y_{med}$ denote their respective median values. In our computational analysis, we study cases where the household population is representative of the US population and, alternatively, a poverty population. This is to investigate charter school practices if entering a school district with a representative population (e.g., a typical town) or if entering a poor district that arises from Tiebout sorting (e.g., an inner-city district).

Education production combines the student's study effort with ability and school quality to produce achievement (a).[14] School quality varies among students depending on the match of the student's ability to the level of ability targeted in the school's instruction or curriculum (B). Specifically, the school's quality to a student of ability b equals $Q\Gamma(b,B)$, consisting of two elements: (i) a common quality component Q, a function of spending per pupil, and (ii) a student-specific component, which depends on the distance between the school's curriculum target and the student's ability. The function $\Gamma(b,B)$ captures the latter:

$$(1)\qquad \Gamma(b,B) \equiv \Gamma_m Z(b,B) \equiv \Gamma_m\left[\alpha + (1-\alpha)\exp\left\{-\frac{(b-B)^2}{\tau^2}\right\}\right],\ \alpha\in[0,1).$$

Note that $\Gamma$ reaches its global maximum at b = B with value $\alpha\Gamma^m$, and declines as the distance from b to B increases.[15] A lower $\alpha$ implies a stronger curriculum effect and a lower $\tau^2$ that a student's achievement declines more steeply as the curriculum ability target moves away from her ability.

Achievement is given by: $a(b,e,B,Q) = b^{\beta}Q\Gamma(b,B)e.$ To ensure that achievement increases with ability even if the distance to the school's curriculum target increases with ability (i.e., for b > B), we assume:

**Assumption 1 (A1):** $b^{\beta}\Gamma(b,B)$ increases with b.

The main analysis will have one (traditional) public school and one charter school, with limited capacity in the charter school denoted by κ.[16] Mehta (2017) indicates a charter school competing with only one public school is supported by the (North Carolina) data, but we realize this is not always the case. We assume the charter needs to fill to capacity to cover its fixed costs. Parents choose the school to maximize their child's achievement, or the school the student prefers if either school would

[14] De Fraja et al. (2010) constitutes an elegant piece of evidence supporting the notion that effort affects education outcomes.

[15] This is straightforward to verify: $\frac{\partial\Gamma(b,B)}{\partial(b-B)^2} = \frac{-\Gamma_m(1-\alpha)}{\tau^2}\exp\left\{-\frac{(b-B)^2}{\tau^2}\right\} < 0.$

[16] While charter schools are publicly funded and legally public schools, we refer to the traditional public school as the public school.

lead to the same achievement. Parental and child school preferences can diverge.[17] Parents apply to the charter school if attending would strictly increase achievement, or lead to the same achievement and is preferred by the child. If applications exceed capacity, the charter conducts a lottery. We assume that only a fraction $\mu \in (0,1]$ will accept an offer and that the matriculation probability ($\mu$) is constant among students. This allows parents to change their minds about sending their child to the charter and is empirically motivated. We discuss heterogeneity of this probability in the Conclusion.

While parents choose the school, the child chooses effort. Children care about achievement, but also about their effort costs (c). We assume effort costs are given by $c(e, m) = e^2/2m$, satisfying $\partial c/\partial e > 0$, $\partial c^2/\partial e^2 > 0$ and $\partial c/\partial m < 0$. Children choose effort to maximize their utility:

(2) $$U = a - c.$$

The child then may not prefer the school that maximizes achievement and, if both lead to the same achievement, prefers the school requiring less effort. A constraint on the child's effort choice arises in a "no-excuses charter school," which is able to enforce an effort minimum with parental help.

Schools. In addition to adopting a curriculum, a school has a "standard" S, the minimum achievement necessary for the student to pass or to "graduate." We normalize the achievement of non-graduates to zero. The constancy of achievement for non-graduates conforms to assuming utility does not vary with academic performance if failing, e.g., such individuals work in jobs that only require physical labor. The utility a student of type (b,m) derives from attending school j is therefore given by:

(3) $$U\left(b,m,e,B^j,Q^j,S^j\right)=\left\{\begin{array}{ll} a\left(b,e,B^j,Q^j\right)-c\left(e,m\right), & \text{if } a\left(b,e,B^j,Q^j\right)\geq S^j \\ -c\left(e,m\right), & \text{if } a\left(b,e,B^j,Q^j\right)< S^j \end{array}\right\}$$ where the

superscripts denote values for school j. Conditional on not graduating, students optimally choose to exert zero effort and "drop out."[18] The utility a dropout receives is therefore equal to zero. We consider three types of schools: a public school (P), a traditional charter school (TCS) and a no-excuses charter school (NEC). All schools must adopt the same standard, which we assume the state sets.[19] Generally, states set minimum academic skill requirements for passing grades and graduating, with 43 states having adopted the Common Core Standards.[20] We also assume all schools receive the same funding per student, which is becoming more common. Thus, Q is identical across schools. The public school targets its curriculum at the population median ability.[21] Local school boards

[17] Gibbons and Silva (2011) provide evidence from survey data in England that "school value added dominates school unhappiness [of students] in explaining parents' school satisfaction (p. 323)."
[18] The student may just fail, rather than actually drop out. However, we refer to these students as dropouts. No students will apply to a charter school where they would drop out.
[19] An extension would consider charter schools that adopt a different standard, perhaps at least as high as is the public school's.
[20] Information on the Common Core Standards is found at: www.corestandards.org/standards-in-your-state .
[21] We first consider targeting the median ability of the nationally representative population and then specialized to the poverty population. Tracking by public schools in discussed in the Conclusion.

generally have responsibility for setting curriculum, these boards usually having 3 to 9 elected members.[22] Targeting the median ability is a majority choice equilibrium among parents, thus also of a representative board.[23] The public school is not capacity constrained and accepts all students that do not apply to the charter, or that apply but do not attend either because they lose the lottery or have a change of mind. The TCS chooses its curriculum target to maximize *achievement gains* relative to what its students would achieve in the public school. Aggregate and average achievement gains from the charter school are denoted with $\Delta$ and $\overline{\Delta}$, respectively. A charter is subject to a size and viability constraint: it must attract enough students to fill its capacity $\kappa$ but cannot exceed it.[24] We assume the charter capacity is not large. Charter schools must accept all applicants if they have enough seats, or run a lottery if over-subscribed as discussed above. The NEC school is identical to the TCS except that, besides choosing the ability its curriculum will target (to maximize $\Delta$), it also establishes an enforceable minimum effort requirement that students must satisfy to attend the school. We denote this minimum effort with $e^m$. Schools know the distribution of (b,m) in their district but do not observe individual values. We use a j superscript to identify non-common values over schools, with $j \in \{P, T, N\}$ indicating, respectively, the public school, a traditional charter, and a no-excuses charter.

Timing. The public policies ($B^P$,S) are given. The charter school chooses and announces its curriculum ability target $B^j$, $j \in \{T, N\}$, and, in the case of the NEC school, its minimum effort requirement $e^m$. Parents then decide whether to apply to the charter school, knowing the school policies and their child's characteristics (b,m). If oversubscribed, the charter school runs a fair lottery, and each winner decides whether to matriculate or not. Finally, once assigned to schools, children decide how much effort to exert and payoffs result. Note that parents anticipate their child's effort choices when deciding to apply to the charter school or not.

### 3.2 Preliminary Results: Single Public School

As a baseline and to clarify the model, we begin with the case of only a public school. The school's curriculum $B^P$ targets the median student ability, while we will later calibrate the graduation standard S and the common component of quality Q to match key variables of US demographics and its school marketplace. We drop the superscript identifying the school in the remainder of this subsection.

[22] See "School Boards" at: https://education.stateuniversity.com/pages/2391/School-Boards.html .

[23] Parental utility (i.e., achievement) is single peaked in the ability target, with peak at their student's ability, though flat at targets distant enough that the student would drop out. Given indifference over flat ranges, multiple majority choice equilibria arise if there are dropouts. We select the "natural" median-target equilibrium, as if parents of dropouts voted for ability targeted to their children's abilities.

[24] Having a simple capacity constraint for school viability avoids introducing a cost function and strikes us as a reasonable approximation.

Student Effort Choice. We start by defining and identifying three effort levels of a student: (i) the optimal (unconstrained) effort choice ($e^*$); (ii) the effort necessary to reach the school's standards ($e^S$); and (iii) the effort cutoff above which the student would rather drop out ($e^0$).

The unconstrained utility-maximization problem is:

(4) $$\underset{e}{\text{Max}}\ U(b,m,e,B,Q) = b^{\beta}Q\Gamma(b,B)e - \frac{1}{2m}e^2,$$

From the FOC, we obtain:

(5) $$e^*(b,m,B,Q) = b^{\beta}Q\Gamma(b,B)m$$

One can see that the unconstrained optimal effort level increases with motivation (more motivated students face lower effort costs), with the common component of school quality (higher quality implies greater productivity of effort) and, for the same reason, with the student-specific element of quality. Thus, effort increases as the curriculum target gets closer to the ability of the student. Moreover, under A1, optimal effort goes up with ability, even when greater ability implies a greater distance to the school's curriculum target (i.e. for students whose ability is larger than the curriculum target).[25] Let $a^*$ denote the utility-maximizing achievement assuming it exceeds the standard:

(6) $$a^*(b,m,B,Q) \equiv a\left(b, e^*(b,m,B,Q), B, Q\right)$$

The graduation standard constrains the behavior of those students whose most-preferred level of effort is not enough to reach it. The minimum effort a student (b,m) requires to attain the school's standard S satisfies $b^{\beta}Q\Gamma(b,B)e_S \equiv S$. Solving for $e_S$:

(7) $$e_S(b,B,Q,S) \equiv \frac{S}{b^{\beta}Q\Gamma(b,B)}.$$

$e^S$ falls with the quality of education received by the student and, under A1, also with her academic ability. It increases with the school's graduation standard and is independent of student motivation.

The maximum effort the student can bear without dropping out satisfies

$b^{\beta}Q\Gamma(b,B)e_0 - \frac{1}{2m}(e_0)^2 \equiv 0$. Solving for $e_0$:

(8) $$e_0(b,m,B,Q) \equiv 2b^{\beta}Q\Gamma(b,B)m.$$

Notice that $e_0(b,m,B,Q) = 2e^*(b,m,B,Q)$. These three effort values ($e^*$, $e_S$, $e_0$) allow us to classify the students of a school into three groups: high achievers (those with $e^* > e_S$), effort-constrained students (those with $e^* \le e_S \le e_0$), and dropouts (those with $e^* < e_0 < e_S$). The following two loci partition the (b,m) plane into three regions: one with the school's high achievers, another with its effort-constrained students, and a third one with the dropouts. The first locus, denoted with $\hat{m}_S$,

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[25] $\frac{\partial e^*}{\partial b} = Qm\frac{\partial[b^{\beta}\Gamma(b,B)]}{\partial b} > 0$, by A1.

separates high-achievers from effort-constrained students and is defined by $e_S(b,B,Q,S) \equiv e^*(b,\hat{m}^S,B,Q)$. Solving for m:

$$\text{(9)} \qquad \hat{m}_S(b,B,Q,S) \equiv \frac{S}{\left[b^{\beta}Q\Gamma(b,B)\right]^2}$$

It is straightforward to show that $\hat{m}_S$ is negatively sloped in the (b,m) plane using A1. The second locus, denoted with $\hat{m}_0$, separates effort-constrained students from dropouts and is defined by $e_S(b,B,Q,S) \equiv e_0(b,\hat{m}^0,B,Q)$. Solving for m again yields:

$$\text{(10)} \qquad \hat{m}_0(b,B,Q,S) \equiv \frac{\hat{m}_S(b,B,Q,S)}{2}.$$

Lemma 1 examines the characteristics and effort choices of each of these groups.

**Lemma 1**. Given the school curriculum target B, graduation standard S and common quality component Q:

(i) *High achievers* are students who achieve above the graduation standards: $a^*(b,m,B,Q) > S$. They satisfy: $e^*(b,m,B,Q) > e^S(b,B,Q,S)$ and have types (b,m) such that $m > \hat{m}_S(b,B,Q,S)$.

(ii) *Effort-constrained students* are students who exert the minimum effort they require to graduate and so obtain achievement equal to S. They satisfy: $e^*(b,m,B,Q) \le e_S(b,B,Q,S) \le e_0(b,m,B,Q)$ and have types (b,m) such that $m \in \left[\hat{m}_0(b,B,Q,S), \hat{m}_S(b,B,Q,S)\right]$.

(iii) *Dropouts* are students who drop out of school and exert no effort. They satisfy: $e_S(b,B,Q,S) > e_0(b,m,B,Q) > e^*(b,m,B,Q)$ and have types (b,m) such that $m < \hat{m}_0(b,B,Q,S)$.

**Proof.** The proof is simple and in the online appendix.

We close this section by defining the *induced achievement function*. This is denoted by $\tilde{a}$ and equals the achievement of students of type (b, m) conditional on the school inputs (B, S, Q) and on utility-maximizing effort choices:

$$\text{(11)} \qquad \tilde{a}(b,m,B,S,Q) = \begin{cases} \left(b^{\beta}Q\Gamma(b,B)\right)^2 m; & \text{if } m > \hat{m}_S(b,B,S,Q) \\ S; & \text{if } m \in \left[\hat{m}_0(b,B,S,Q), \hat{m}_S(b,B,S,Q)\right] \\ 0; & \text{if } m < \hat{m}_0(b,B,S,Q) \end{cases}$$

Figure 1 illustrates the student partition and implied achievement levels.[26]

## 4. Charter Schools

[26] The numerical values of m and b in Figures 1 – 7 are from a simple example developed to illustrate student partitions. Later we present a seriously calibrated model.

4.1. Traditional Charter School. Here we study competition between the public school and a traditional charter school. The analysis proceeds by backward induction. First, we establish which households apply to the charter school as a function of its curriculum target. Second, we determine the school's optimal curriculum choice. The TCS selects its ability target to maximize the achievement gains of its students, subject to the viability constraint and given public school policies ($Q^P$, $S^P$, $B^P$).

School Choice. The educational quality a student enjoys at a particular school is higher the closer is the student's ability to the school's curriculum target. Given that the student-specific component of school quality is a function of $(b-B)^2$, the level of cognitive ability equidistant to the schools' targets, $B^T$ and $B^P$, separates students who receive higher quality from one school and the other. Let $\tilde{b}(B^T, B^P) \equiv (B^T + B^P)/2$ define this cutoff ability.

The next lemma orders the loci separating high-achievers, effort-constrained students and dropouts in the two schools to study household preferences for schools. Refer to Figure 2. Recall that the public and charter schools receive the same level of funding per student, so $Q^T = Q^P = Q$. Also, note that policy requires the same graduation standards, so $S^T = S^P = S$. Let $\hat{m}_S^j(b, B^j, S, Q)$, j = P, T, denote, respectively for the public and the TCS, the locus separating high-achievers from effort-constrained students; and $\hat{m}_0^j(b, B^j, S, Q)$, j = P, T, denote the locus separating the latter group from the dropouts for the relevant school:

**Lemma 2**.[27] Suppose that $B^T < B^P$. Then:

(i) $e_S\left(b, B^T, S, Q\right) \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} e_S\left(b, B^P, S, Q\right) \ \forall b \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} \tilde{b},$

(ii) $\hat{m}_0^T\left(b, B^T, S, Q\right) \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} \hat{m}_0^P\left(b, B^P, S, Q\right) \ \forall b \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} \tilde{b},$ and

(iii) $\hat{m}_S^T\left(b, B^T, S, Q\right) \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} \hat{m}_S^P\left(b, B^P, S, Q\right) \ \forall b \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} \tilde{b}.$

**Proof**. The proof follows easily from the definition of $\Gamma(b, B)$, which implies:

$|b - B^T| < (>)(=) |b - B^P|$ and $\Gamma(b, B^T) > (<)(=) \Gamma(b, B^P)$ for $b < (>)(=) \tilde{b}$. ■

Taking the example of students with ability below $\tilde{b}$, when $B^T < B^P$, they require less effort to graduate from the charter school than from the public one (part i), and so they need less motivation to graduate (part ii) or to achieve above the graduation standards (part iii). (See Figure 2).

[27] An analogous result holds with $B^T > B^P$: simply flip the latter set of inequalities in every part of the lemma.

The following lemma characterizes the set of students whose parents apply to the TCS. We denote such a set with $A^T(B^T, B^P)$. Figure 2 shows the applicant set.

**Lemma 3.** (Applicant set.) Suppose the TCS enters the market with curriculum target $B^T \in [b_m, b_x]$. Then, every student with ability closer to $B^T$ than to $B^P$ and sufficiently motivated to not drop out of the TCS applies. Formally:

(12) $$A^T(B^T, B^P) = \{(b,m) \,\|\, |b - B^T| < |b - B^P|,\ m > \hat{m}_0^T(b, B^T, \cdot)\}.$$

The proof is in the appendix. We provide intuition here. Observe in Figure 2 that the ability of TCS applicants is better suited to its curriculum target, $B^T$, than to the public school's one, $B^P$, all these students having ability closer to $B^T$. In addition, their motivation is strong enough to not drop out of the charter school. Several groups of students make up this applicant set. Those with $m > m_s^T$ are not effort constrained in the charter and have higher achievement there. Those with $m \in [\hat{m}_o^T, \hat{m}_s^T]$ just pass at the charter and either do the same at the public school or drop out. The former gain utility by attending the charter since they exert less effort, while the latter gain achievement and utility.

**Remark 1.** By lemma 3, $B^T = B^P \Leftrightarrow |b - B^T| = |b - B^P| \ \forall\, b \Leftrightarrow A^T(B^P, B^P) = \varnothing$. That is, the applicant set, $A^T(B^T, B^P)$, is empty whenever $B^T = B^P$. *Therefore, to be viable, the TCS must differentiate its curriculum from the public school's one.*

**Remark 2.** *Parental and student preferences between the public school and the TCS coincide, though student preferences include their effort costs.* On the one hand, for student types whose achievement differs among schools, the school that maximizes achievement also maximizes student utility since it offers them greater quality. On the other hand, for those whose achievement is identical in the two schools, recall that parents prefer the school the child prefers.

Curriculum Choice. The charter school's objective is to maximize the aggregate achievement *gains* relative to attending the public school. Given random admissions if over-subscribed, this is tantamount to maximizing the aggregate achievement gains of the anticipated applicant set. Moreover, since the charter school has fixed capacity, maximizing average achievement gains is also equivalent. We then write the curriculum choice problem as follows:[28]

(13)

$$\underset{B^T}{\text{Max}}\ \bar{\Delta} = \left[\begin{array}{l} \displaystyle\int_{y_m}^{y_x}\int_{b_m}^{\tilde{b}(B^T)}\int_{\hat{m}_0^T(b,B^T,\cdot)}^{\hat{m}_0^P(b,\cdot)} Sf(b,m,y)dbdmdy + \int_{y_m}^{y_x}\int_{b_m}^{\tilde{b}(B^T)}\int_{\hat{m}_S^T(b,B^T,\cdot)}^{\hat{m}_S^P(b,\cdot)} \left[a^*(b,m,B^T,\cdot) - S\right] f(b,m,y)dmdbdy + \\ \displaystyle\int_{y_m}^{y_x}\int_{b_m}^{\tilde{b}(B^T)}\int_{\hat{m}_S^P(b,\cdot)}^{m_x} \left[a^*(b,m,B^T,\cdot) - a^*(b,m,B^P,\cdot)\right] f(b,m,y)dmdbdy \end{array}\right] / N^T,$$

[28] Note that problem (13) drops the functional arguments that remain fixed throughout the analysis ($B^P$, Q, S).

$$\text{s.t.} \qquad \mu N^T \equiv \mu \int_{y_m}^{y_x} \int_{b_m}^{\tilde{b}(B^T)} \int_{\hat{m}_0^T(b,B^T,\cdot)}^{m_x} f(b,m,y)\,db\,dm\,dy \geq \kappa,$$

where $N^T$ defines the mass of students that apply to the charter school (lemma 3).[29] The constraint is the viability or size constraint. To streamline the presentation, we focus on cases where $B^T \leq B^P$ while noting that analog results arise if $B^T \geq B^P$ as further explained below.

To have a well-behaved objective function, we assume:

**Assumption 2 (A2):** $N^T$ is increasing with $B^T$ on $[b_x, B^P)$.[30]

The assumption is that increasing the curriculum ability target toward the public school's one will always increase the size of the applicant pool. With such a curriculum change, more students have ability levels closer to the charter school's target, increasing demand. Moreover, it entails a better match to the ability of some students, further increasing demand. However, the reverse holds for lower ability students. The assumption is that demand enhancing effects dominate the latter effect.

To gain insight into the TCS objective function, refer again to Figure 2. Applicants to the school have ability closer to its curriculum target than to the public school's, $b < \tilde{b}$, and sufficient motivation to not drop out, $m > \hat{m}_0^C(b, B^T, \cdot)$. Each of three of the colored areas of the graph corresponds to one addend in the numerator of (13). The first addend aggregates the achievement gains of students in the lower marked area, those with motivation $m \in [\hat{m}_0^T(b, B^T, \cdot), \hat{m}_0^P(b, B^P, \cdot)]$. These students would graduate from the TCS but would drop out of the public alternative. Thus, if they matriculate at the TCS, they experience gains given by the graduation standard, S. The second addend corresponds to students in the upper intermediate marked area, those with motivation $m \in [\hat{m}_S^T(b, B^T, \cdot), \hat{m}_S^P(b, B^P, \cdot)]$. These students would be high-achievers in the TCS but just graduate from the public school. Therefore, if they matriculate at the charter, they realize achievement gains equal to $a^*(b, m, B^T, \cdot) - S$. The third addend captures the achievement gains of students in the upper marked area of the graph, those sufficiently motivated to be high-achievers in both schools (i.e. with $m > \hat{m}_S^P(b, B^P, \cdot)$). They apply and, if they matriculate, derive gains equal to $a^*(b, m, B^T, \cdot) - a^*(b, m, B^P, \cdot)$. The remaining students among those with ability closer to $B^T$ than to $B^P$ would either dropout of the charter school and so do not apply (those with $m < \hat{m}_0^c(b, B^T, \cdot)$) or would just graduate from both the charter and public school, those with $m \in [\hat{m}_0^P(b, B^P, \cdot), \hat{m}_S^T(b, B^T, \cdot)]$. Students in the latter group still apply to the charter school because they can exert less effort there. They experience no achievement gains and vanish from the numerator of the objective but count as part of the denominator.

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[29] Not every one of those students will attend the TCS in general, either because not every student who would gain matriculates or because there is excess demand and the charter school conducts a lottery to assign its seats. Still, $\overline{\Delta}$ in (13) yields average achievement gains because we assume that the probability of not matriculating is constant across types and that lotteries are fair.

[30] We make the analogous assumption when considering ability targets above $B^P$.

The TCS may target high or low abilities. Indeed, its optimization problem has two local solutions, one with entry at the high-end of the ability distribution – that is, where the curriculum targets high ability students, $B^T > B^P$ – and another with entry at the low-end, $B^T < B^P$. Targeting low ability students is globally optimal in our computational examples. However, we will also report properties of the local maximum if high ability students are targeted.

A student's income plays no direct role in achievement. However, we examine different income populations that have different conditional distributions on (b,m). We then specify the objective using the joint distribution on (b,m,y).

Let $B_m^T$ $(B_x^T)$ denote the minimum (maximum) value of $B^T <$ (>) $B^P$ that would generate just enough demand to attend the TCS for it to be viable:

$$\mu \int_{y_m}^{y_x} \int_{b_m}^{\tilde{b}(B_m^T)} \int_{\hat{m}_0^T(b,B_m^T,\cdot)}^{m_x} f(b,m,y)dmdbdy \equiv \kappa. \tag{14}$$

$$\mu \int_{y_m}^{y_x} \int_{\tilde{b}(B_x^T)}^{b_x} \int_{\hat{m}_0^T(b,B_x^T,\cdot)}^{m_x} f(b,m,y)dmdbdy \equiv \kappa. \tag{15}$$

Assumption (A2) or its analogue for $B^T > B^P$ imply unique solutions to (14) and (15). A value of $B^T \in (B_m^T, B^P)$ or $B^T \in (B^P, B_x^T)$ would attract more than κ students and so would imply an excess demand and a need for a lottery. On the other hand, a value of $B^T < B_m^T$ or $B^T > B_x^T$ cannot be optimal since the capacity constraint would not be met and the school would not be viable. Proposition 2 characterizes the solution to the curriculum choice problem.

**Proposition 2.** (TCS Curriculum Choice.) The traditional charter school problem has at least two (local) maxima, at least one with entry at the low end, $B^{T*} < B^P$, and at least one with entry at the high end, $B^{T**} > B^P$. To avoid tedium, we assume a unique local maximum on both sides of $B^P$.[31] We have:

1. Low-end entry: Assuming $B^T \le B^P$, $B^{T*} = B_m^T$ if and only if

$$\left[\begin{array}{l} \displaystyle\int_{y_m}^{y_x} \int_{b_m}^{\tilde{b}(B_m^T)} \left\{ (\bar{\Delta} - S) \frac{\partial \hat{m}_0^T(b,B_m^T)}{\partial B^T} f(b, \hat{m}_0^T(b,B_m^T), y) db \right\} dy \\ \displaystyle + \int_{y_m}^{y_x} \int_{b_m}^{\tilde{b}(B_m^T)} \int_{\hat{m}_S^T(b,B_m^T)}^{m_x} \frac{\partial a^*(b,m,B_m^T)}{\partial B^T} f(b,m,y)dmdbdy - \bar{\Delta} \cdot \frac{\partial \tilde{b}}{\partial B^T} \int_{y_m}^{y_x} \int_{\hat{m}_S^T(\tilde{b},B_m^T)}^{m_x} f(\tilde{b},m,y)dmdy \end{array}\right] \le 0. \tag{16}$$

Otherwise, the (interior) local maximum is $B^{T*} = B_I^T \in (B_m^T, B^p)$ such that

[31] We find this in our computational model below. Notice that we are also assuming that κ is small relative to the student population so that viable entry at the high and low ends is feasible.

$$(16') \quad \left[\begin{array}{l} \int_{y_m}^{y_x}\int_{b_m}^{\tilde{b}(B_I^T)}\left\{(\bar{\Delta}-S)\dfrac{\partial \hat{m}_0^T(b,B_I^T)}{\partial B^T}f(b,\hat{m}_0^T(b,B_I^T),y)db\right\}dy \\ \\ +\int_{y_m}^{y_x}\int_{b_m}^{\tilde{b}(B_I^T)}\int_{\hat{m}_S^T(b,B_I^T)}^{m_x}\dfrac{\partial a^*(b,m,B_I^T)}{\partial B^T}f(b,m,y)dbdmdy-\bar{\Delta}\cdot\dfrac{\partial\tilde{b}}{\partial B^T}\int_{y_m}^{y_x}\int_{\hat{m}_S^T(\tilde{b},B_I^T)}^{m_x}f(\tilde{b},m,y)dmdy \end{array}\right]=0.$$

2. High-end entry: Assuming $B^T \geq B^P$, $B^{T**}=B_x^T$ if and only if

$$(17) \quad \left[\begin{array}{l} \int_{y_m}^{y_x}\int_{\tilde{b}(B_x^T)}^{b_x}\left\{(\bar{\Delta}-S)\dfrac{\partial \hat{m}_0^T(b,B_x^T)}{\partial B^T}f(b,\hat{m}_0^T(b,B_x^T),y)db\right\}dy \\ \\ +\int_{y_m}^{y_x}\int_{\tilde{b}(B_x^T)}^{b_x}\int_{\hat{m}_s^T(b,B_x^T)}^{m_x}\dfrac{\partial a^*(b,m,B_x^T)}{\partial B^T}f(b,m,y)dmdbdy-\bar{\Delta}\cdot\dfrac{\partial\tilde{b}}{\partial B^T}\int_{y_m}^{y_x}\int_{\hat{m}_s^T(\tilde{b},B_x^T)}^{m_x}f(\tilde{b},m,y)dmdy \end{array}\right]\leq 0.$$

Otherwise, the (interior) high-end local optimum is at $B^{T**}=B_{II}^c \in (B^p, B_x^T)$ such that

$$(17') \quad \left[\begin{array}{l} \int_{y_m}^{y_x}\int_{\tilde{b}(B_{II}^T)}^{b_x}\left\{(\bar{\Delta}-S)\dfrac{\partial \hat{m}_0^T(b,B_{II}^T)}{\partial B^T}f(b,\hat{m}_0^T(b,B_{II}^T),y)db\right\}dy \\ \\ +\int_{y_m}^{y_x}\int_{\tilde{b}(B_{II}^T)}^{b_x}\int_{\hat{m}_s^T(b,B_{II}^T)}^{m_x}\dfrac{\partial a^*(b,m,B_{II}^T)}{\partial B^T}f(b,m,y)dmdbdy-\bar{\Delta}\cdot\dfrac{\partial\tilde{b}}{\partial B^T}\int_{y_m}^{y_x}\int_{\hat{m}_S^T(\tilde{b},B_{II}^T)}^{m_x}f(\tilde{b},m,y)dmdy \end{array}\right]=0.$$

The complete proof is in the appendix. Taking the case of $B^T < B^P$, the proof shows in part that the sign of the LHS of (16) (or (16)') coincides with $\text{Sign}\left[\dfrac{\partial\bar{\Delta}}{\partial B^T}\right]$. The expression in (16) contains the effects of changing $B^T$ on the charter objective. To interpret it, consider an increase in $B^T$ and refer to Figure 3. There are three effects on average achievement gains. The increase in $B^T$ shifts marginally to the right the vertical lines at $B^T$ and $\tilde{b}$, i.e., the latter value also increases. The third term in the expression is the *loss* of bringing in students in the vicinity of $\tilde{b}$. These students add essentially 0 to the average achievement gain, while taking away seats, then reducing the average achievement gain $(\bar{\Delta})$ by an amount proportional to the average achievement gain (multiplied by the magnitude of their increased numbers). The second term corresponds to the effect on all students who are unconstrained in the charter, those who have achievement equal to $a^*(b,m,B^T)$. These are students in the charter with $b < \tilde{b}$ and $m > \hat{m}_s^T(b,B^T)$. Among them, those with $b >$ (<) $B^c$ have reduced (increased) achievement and so an achievement gain (loss) results from the increase in $B^T$. Thus, the second term in (16) is of ambiguous sign. The first term captures the last effect on achievement gains. This stems from a steepening of $\hat{m}_0^T$ at (approximately) point B in Figure 3. (The locus also shifts marginally, which is not shown, this having second-order effects.) Some of those with $b < B^T$ drop out, resulting in reduced achievement gain of S. The reverse is true for some with $b > B^T$. The change in $\overline{\Delta}$ is the gain or loss of S relative to $\overline{\Delta}$. It is likely that $S > \overline{\Delta}$. Therefore, students in this group attracted to the school increase the average gain, while dropouts decrease it. Thus, the first term is of ambiguous sign too. We note that $\hat{m}_s^T$ will also steepen as shown at (approximately) point A. But this has no first-order effect on $\overline{\Delta}$, since these students have $a^*(b,m,B^T) \approx S$.

To summarize, the first two terms on the LHS of (16) are of ambiguous sign, while the third is negative. In our computational analysis, we find that the net effect at $B^T = B_m^T$ is negative, implying the corner solution where a minimum number of applicants is optimal.

4.2. No-Excuses Charter School.

Now we examine competition between an NEC school and the public school. In addition to selecting its curriculum, denoted $B^N$, the NEC can enforce a minimum effort of its students, denoted $e^m$. NEC schools require significant parental involvement, and so we assume they can successfully enlist parental help in enforcing the minimum effort. Characterizing NEC's as enforcing an effort minimum is a tractable way to capture arguably several of the documented practices of these schools discussed in Section 2, examples being a longer school day, frequent testing, and having an ethos of comportment and a strong work ethic. Beyond the effort minimum, the NEC school is modeled like the TCS. It maximizes achievement gains of its students, its students must meet the same graduation standard, it faces the same size-viability constraint, and it must admit students randomly if over-subscribed. In short, the NEC is the same as the TCS but can enforce a minimum effort requirement.

Effort Choice. An NEC school student chooses the largest of three effort levels: the unconstrained optimal effort, $e^*$, the effort needed to graduate, $e_S$, or the minimum effort demanded by the school, $e^m$. This is provided such effort level is no greater than the maximum the student is willing to exert, $e_0$. If it is, the student would rather drop out and exert zero effort. Formally, let $e^N(b,m,B^N,Q,S)$ denote the effort exerted by students of type (b,m) at the NEC school. Using that $e_0 > e^*$, we have:

$$(18) \qquad e^N(b,m,B^N,\cdot) = \begin{cases} \text{Max}\left[e^*(b,m,B^N,\cdot), e_s(b,B^N,\cdot), e^m\right] & \text{if } e_0(b,m,B^N,\cdot) \geq \text{Max}[e_S(b,B^N,\cdot), e^m] \\ 0 & \text{if } e_0(b,m,B^N,\cdot) < \text{Max}[e_S(b,B^N,\cdot), e^m]. \end{cases}$$

The minimum-effort policy of the NEC school would constrain the choices of student types (b,m) for whom $e^m > \text{Max}[e^*(b,m,B^N,\cdot), e_S(b,B^N,\cdot)]$. Among them, those that satisfy $e^m \leq e_0(b,m,B^N,\cdot)$ choose to exert the minimum effort, while the rest would experience negative utility if they did, then prefer to drop out and would not apply to the charter school.

Applicant Set Given Choice of the Curriculum. An interesting difference from the TCS is that the NEC will adopt the same curriculum as the public school at its optimum. To show this, we must, of course, examine the applicant sets for both $B^N = B^P$ and for $B^N \neq B^P$. An analogous argument proves that the school prefers $B^N = B^P$ over any $B^N > B^P$ or any $B^N < B^P$. We then focus on the alternative with $B^N > B^P$ in developing the argument. We will prove that $B^N = B^P$ with optimal minimal effort choice is a local optimum. We assume:

**Assumption 3 (A3):** The NEC's local maximum at $B^N = B^P$ is the global optimum.

We do not have general conditions for this assumption, but we do verify it holds in our computational analysis. We will also show that the NEC does adopt an effort minimum that strictly binds students, in fact, all its students!

Turning to the analysis, while we provide the math, it is notable that one can understand well the several findings with the help of a few graphs. Refer to Figure 4. Let $\hat{m}_0^N(b, B^N, \cdot)$ and $\hat{m}_S^N(b, B^N, \cdot)$ denote, respectively, the loci in the (b,m)-plane separating students that would or would not drop out from the NEC, and those that would just meet the standard or exceed it *assuming effort unconstrained by the NEC effort minimum*. The definition of these loci is analogous to that given for the public and TCS schools above with $B = B^N$. In addition, Figure 4 depicts two iso-effort loci that identify students who would choose a given effort if attending the NEC school unconstrained by the effort minimum. The values of $b_i$, i = 1,2, satisfy $e_i = S / [b_i^\beta Q \Gamma(b_i, B^N)]$, where $e_i$ is the constant effort defining the locus. The vertical portion of an iso-effort locus above $b_i$ has students choose $e_i$ to just meet the standard. The downward-sloping section of each locus contains students for whom $e^* = e_i$, who optimally choose the same effort level but with achievement that exceeds the standard. Letting $\hat{m}_e^N(b, B^N, e_i, Q)$ denote the value of m along the latter part of an iso-effort locus, it is straightforward to confirm (using (5)) that it is given by:

$$(19) \qquad \hat{m}_e^N(b, B^N, e_i, Q) \equiv \frac{e_i}{b^\beta Q \Gamma(b, B^N)}.$$

It is also straightforward to confirm that the latter locus slopes down and lies above $\hat{m}_S^N(b, B^N, \cdot)$ for b > $b_i$. Note that effort *decreases* as the iso-effort loci shift rightward.

Consider a case with $B^N > B^P$ and with the NEC setting an effort minimum that would *not* constrain the $\tilde{b}$ type, the ability level that separates those who receive higher and lower quality at the charter relative to the public school.[32] Figure 5 depicts the applicant set. The assumed effort minimum is that along the iso-effort locus with the vertical portion above ability denoted $b_N$. Another locus

[32] Figure 5 does not show the values of ($B^P$, $B^N$).

$\hat{m}_0^e = \dfrac{\hat{m}_e^N}{2}$ separates students that choose not to attend the NEC because the effort minimum exceeds the effort that would give 0 utility ($e_0$).[33] As identified in Figure 5, three groups apply to the NEC school. One consists of students not constrained by the effort minimum who achieve the standard. Another has students neither constrained by the effort minimum nor by the need to meet the standard. These two groups are exactly as for a TCS with $B = B^N$. The third group are those effort constrained by $e^m$. For them, the effort minimum increases achievement, but does not induce them to drop out. Those "driven out" of the NEC due to the effort constraint may or may not attend the public school (depending on whether they are above or below the $\hat{m}_0^P$ locus, which Figure 5 does not depict).

If $B^N > B^P$ and the NEC school selects an effort minimum that would constrain the $\tilde{b}$ type, the applicant set changes. Figure 6 illustrates this case, with $b_N$ now less than $\tilde{b}$, again the ability of a student who would choose effort equal to the NEC minimum and just graduate (over a range of values of m). Some students with ability b below $\tilde{b}$ - hence offered a less efficient curriculum by the charter school - will still attend the NEC because the effort minimum increases their achievement. Here, the $\hat{m}_s^P$ locus and yet another one, denoted $\hat{m}_a^N$, come into play. Proposition 3 presents details, but it is possible to provide an intuitive explanation using Figure 6. Three groups of applicants arise. For those with $b > \tilde{b}$, one group consists of types with relatively lower m-values constrained by the effort minimum. The second group optimally chooses effort exceeding the minimum. The latter two groups have their analogues in the previous case with $b_N > \tilde{b}$ (see Figure 5). The third group have $b < \tilde{b}$, but higher achievement in the NEC school because they are effort constrained there. These students have ability $b \in [b_N, \tilde{b})$. A subset of them are relatively low m-types who would achieve the standard in the public school but above it in the NEC alternative due to its effort constraint (these have $m \in [\hat{m}_0^e, \hat{m}_s^P]$.) The other subset would exceed the standard in the public school, but the effort minimum imposed by the NEC school still induces higher achievement (these have $m \in [\hat{m}_s^P, \hat{m}_a^N]$.)

Two interesting observations about the latter applicant set are the following: (i) all applicants exceed the standard if matriculating to the NEC school; and (ii) unlike in the case with a TCS, it contains students who disagree with their parents about which school to attend. For example, all students with $b \in [b_N, \tilde{b})$ would prefer to attend the public school since it offers them a better curriculum match and does not push them to exert more effort than they would optimally choose.

Last, consider the case with $B^N = B^P$, which is simpler. *Only students that are effort constrained in the NEC would apply to it.* With the same curricula, the latter must hold for

[33] One can find this locus by setting $e_i = e^m = e_0$ and using (8).

achievement to increase. *In turn, this implies choosing a binding effort constraint is optimal for the charter school if $B^N = B^P$.* Otherwise, the NEC has nothing to offer and would attract no students.

It is easy to see that the applicant set is as shown in Figure 7, keeping in mind all students that would not be effort constrained in the NEC do not gain by attending the charter school and would not apply. *All applicants exceed the standard if they matriculate.* The nature of the applicant set is independent of how $b_N$ compares to $B_N$.

Proposition 3 collects the main results, paying more attention to the $B^N = B^P$ case, which we show arises in equilibrium below.

**Proposition 3.** Let $A^N(B^N,e^m,B^P)$ denote the applicant set to the NEC.

1. Suppose the NEC school enters the market with curriculum target $B^N = B^P$. Then, $A^N(B^N,e^m,B^P) = \{(b,m) | b > b_N(B^N,e^m,Q,S),\ m \in [\hat{m}_0^N(b,B^N,e^m,Q), \hat{m}_e^N(b,B^N,e^m,Q)]\}$. Every student that matriculates chooses $e = e^m$ and exceeds the graduation standard.

2. Suppose the NEC school enters the market with curriculum target $B^N > B^P$ and minimum effort $e^m$ such that $\tilde{b}(B^N,B^P) \le b_N(B^N,e^m,Q,S)$. Then:
$A^N(B^N,e^m,B^P) = \{(b,m) | b > \tilde{b}(B^N,B^P),\ m > \mathrm{Max}[\hat{m}_0^N(b,B^N,e^m,Q), \hat{m}_0^e(b,B^N,Q,S)]\}$.

3. Suppose the NEC school enters the market with curriculum target $B^N > B^P$ and minimum effort $e^m$ such that $b_N(B^N,e^m,Q,S) < \tilde{b}(B^N,B^P)$. For $b \in [b_N,\tilde{b}]$, define the locus

$$\hat{m}_a^N(b,B^N,e^m,B^P,Q) \equiv \frac{\Gamma(b,B^N)e^m}{b^\beta Q\Gamma(b,B^P)^2},$$ which satisfies $\hat{m}_a^N(\tilde{b},B^N,e^m,B^P,Q) = \hat{m}_S^N(\tilde{b},B^N,Q,S)$ and

$m_a^N(b_N,B^N,e^m,B^P,Q) = \hat{m}_S^P(b_N,B^N,Q,S)$. Then:

$$A^N(B^N,e^m,B^P) = \{(b,m) | b > \tilde{b}(B^N,B^P),\ m > \hat{m}_0^e(b,B^N,Q,S)]\} \cup$$
$$\{(b,m) | b \in (b_N,\tilde{b}(B^N,B^P)],\ m \in [\hat{m}_0^e(b,B^N,e^m,Q), \hat{m}_a^N(b,B^N,e^m,B^P,Q)]\}.$$

**Proof of Proposition 3**: See the appendix.

<u>Choice of Curriculum and Effort Minimum</u>. As we have already indicated, we show that setting the same curriculum as the public school and with $e^m$ at its optimal value, denoted $e^{m*}$, yields a local maximum in $(B^N,e^m)$. The results stated are:

**Proposition 4:**

1. Choice by the NEC of $B^N = B^P$ and $e^m = e^{m*}$ is a local maximum.
2. Under (A3), it is the global maximum.

**Proof**. See the appendix.

The results can be understood intuitively given the application spaces and effort choices for alternative values of $B^N$ and $e^m$ shown above. Obviously, if $B^N = B^P$, $e^m = e^{m*}$. We also know that this choice of $e^{m*}$ would constrain every applicant and thus matriculant, implying strict achievement gains within the boundaries of the applicant space (review Figure 7). Proposition 4-2 is also immediate. We next explain why $B^N = B^P$ and $e^m = e^{m*}$ is a local maximum.

Fix $e^m = e^{m*}$, and compare the applicant sets if $B^N = B^P$ to the alternative with $B^N = B^P + \delta$, for $\delta$ positive but vanishingly small. Also, assume that $e^{m*}$ is such that $b_N < \tilde{b}$ when $B^N = B^P + \delta$, recall this being a case where some students apply to the NEC though the curriculum match is worse than in the public school. Graphically, the applicant sets match Figures 6 and 7, though the left (just-described) marked group in Figure 6 for the case $B^N = B^P + \delta$ would be vanishingly small and can then be ignored (i.e., $b_n \approx \tilde{b}$). The substantive difference in the applicant sets is that the set of students not effort constrained in Figure 6 would not apply to the NEC with $B^N = B^P$, *and these students achievement gains when attending the NEC with* $B^N = B^P + \delta$ *are virtually zero. By choosing the same curriculum as the public school, the NEC avoids applicants who would otherwise gain slightly in achievement, such students diluting aggregate achievement gains generated by the charter school.*

Thus, aggregate achievement gains generated by the NEC are discretely higher if setting the curriculum target equal to the public school's as compared to it being locally higher. That achievement gains with $B^N = B^P + \delta$ are continuous in $e^m$ completes the argument of a local maximum for the case considered (positive $\delta$ and $e^{m*}$ such that $b_N < \tilde{b}$), with the formal proof showing the remaining cases to be analogous.

4.3 Efficiency.

Efficiency implications of charter schools are discussed here. We measure efficiency by parental utility or achievement. Proposition 5 summarizes.

**Proposition 5.**

1. Entry of a TCS or NEC implies a Pareto Improvement.
2. The TCS could increase aggregate achievement weakly if the charter school were able to directly admit students on their ability b. The NEC could increase aggregate achievement strictly if the charter school were able to directly admit students on their characteristics (b,m).

These results are immediate or at least very intuitive, so we simply discuss their logic here, and provide more formal analysis in the appendix. Proposition 5-1 holds simply because the charter school makes choices to maximize achievement gains of their students, who select in only when their gains are strictly positive (with no change in achievement for some students in the case of the TCS). With no spillovers on the public school, the result is immediate for both types of the charter school.[34]

[34] Student utility, which takes into account their study costs, also increases for all charter attendees in the case of the TCS. For the intuition, see Remark 2 to Lemma 3 above. For the NEC, student utility is lower for all attending students! This follows from the fact that the curriculum is the same as the public school's but all the attending students are required to put in more effort than they would like. Our focus on parental utility or

Proposition 5-2 indicates that requiring an open enrollment policy reduces efficiency generally. The main intuition is that charter schools bias their curriculum choice relative to that which maximizes aggregate achievement to select students. Consider the TCS and the case of an interior solution under open enrollment (i.e., not at the boundary of demand to meet the size constraint), taking the example of the charter school that chooses a curriculum target below the public school's. Allowing admission on ability, the charter could increase achievement of its students by increasing the curriculum target but not admitting the highest ability students that would be drawn in and for whom achievement gains are the lowest.[35] The inability of the open enrollment charter to limit admissions of marginal students is what induces the bias down in the curriculum. A curriculum adjustment along these lines might lead to gains for the TCS that is at the bound of its size constraint. However, it could be that the size constraint leads the charter to bias up its target choice to attract enough students, which it would still need to do allowing freedom to admit. In the latter case, no efficiency gains are feasible with selective admissions.

Consider now the NEC charter school, which has three margins for improvement if able to admit students. Recalling the NEC chooses the same curriculum target as the public school so as not to draw in highly motivated students that would not be constrained by the effort minimum, this curriculum choice then "extremely" biased. Being able to admit students on their characteristics eliminates any incentive to adopt the same curriculum as the public school, implying gains from curriculum adjustment better aligned to its students. Moreover, the NEC has an incentive to not admit the lowest motivated students it attracts for whom achievement gains are small, but who would then attend the public school. This margin is not present for the TCS, since the lowest motivated students that attend would drop out of school completely if not admitted. Finally, the NEC can adjust its minimum effort requirement as it varies the curriculum target and admission set. Given the additional leeway of the NEC that is permitted to admit students, we can show gains generally. A final point is that the gains identified by Proposition 5-2 regard aggregate achievement; allowing selective admissions would result in some losers.

The efficiency gains summarized in Proposition 5 have nothing to do with differences in charter productivity or potential competitive effects on the public school. The analysis assumes equally productive schools with fixed technologies. The charter hones its methods to particular students. A charter that is less productive could still attract students, while one that is more productive could do so more readily, these alternatives having obvious welfare implications relative to the case of equal productivities. To examine seriously potential competitive effects would require a model extension that entails distortions underlying inefficiencies. Without pursuing this, one thing interesting to observe that could play a role is the nature of "cream skimming" by the charter. We find below that the traditional charter school that enters targets lower ability students, then with

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achievement takes a paternalistic perspective that children having to study a lot is not costly for them in the long run. How to evaluate student study costs in welfare analysis is an interesting topic that we avoid.

[35] The TCS that is not at the boundary of the size constraint under open enrollment could also gain by not admitting the highest ability students that choose to attend for whom achievement gains are vanishingly small.

"negative cream skimming." Scores in the public school would increase, absent competitive effects. In contrast, the NEC charter does draw off from the public school high achievers, though superstar students are not drawn into the NEC charter school. The "cream skimming" by the NEC is not the standard case of drawing off the highest ability students (see Figure 7). The NEC is then more likely to spur improvement of the public school. Another potential competitive effect on the public school, though, is loss of students if per student funding exceeds marginal cost for both types of charter.

**5 Computational Model and Analysis.**

5.1 Calibration of the Computational Model. The building blocks of our model are the joint distribution of income, cognitive and non-cognitive skills, the achievement production function, and the cost function for student effort. Education policy establishes curriculum and sets standards for graduation. These policies influence students' effort decisions which, coupled with the distribution of skills and the achievement function, determine the realized distribution of achievement and the realized dropout rate. Our approach to the calibration is to combine "direct" matching of some parameters to empirical counterparts when straightforward and choice of other parameters such that equilibrium predictions conform to empirical findings. The model we fit is of a small nationally representative area with one public high school and no charter school. The area has household distribution of income corresponding to that in the US, and with a student population for which the joint distribution of cognitive and non-cognitive skills is representative of the student population in the US and for whom attending the public high school yields an achievement distribution representative of the 12th grade achievement distribution in the US.

The parameters of our model are the following:[36]

$\mu_y, \sigma_y$ : mean and standard deviation of the distribution of the ln(y).

$\mu_b, \sigma_b$ : mean and standard deviation of the distribution of the ln(b).

$\mu_m, \sigma_m$ : mean and standard deviation of the distribution of ln(m).

$\rho_{by}$ : correlation of ln(b) and ln(y).

$\rho_{bm}$ : correlation of ln(b) and ln(m).

$\beta$: exponent of b in the achievement function, Equation (20) below.

$\Omega$: product of $\Gamma_m$ and $Q^p$ in the achievement function, Equation (20) below.

$\alpha, \tau$: parameters of the curriculum function, Z(b,B), Equation (1).

$B^P$, $S^P$: ability target in curriculum of a standard in the public school.

In addition to the dropout rate and an $R^2$ ratio discussed below, three targeted values are:

$\mu_a, \sigma_a$ : mean and standard deviation of the distribution of ln(a).

$\omega$: effect in standard deviations of the logarithm of achievement of being at distance ($b_{.975} - B^P$) from the curriculum target.

[36] For simplicity, we have assumed the correlation of m and y equals 0.

We estimate the mean and standard deviation of the distribution of the logarithm of income as follows. The US income distribution is well approximated by a lognormal distribution. Let y denote income. Then $\ln(y) \sim N(\mu_y,\sigma_y)$. Note that we let $\mu_y$ denote the mean of ln(y) and $\sigma_y$ denote the standard deviation of ln(y). We use this same subscripting convention throughout. Mean and median household income in 2014 were, respectively \$72,641 and \$53,657.[37] Matching these empirical values implies $\mu_y=\log(53{,}657)=10.89$ and $\sigma_y=.78$. In 2014, 14.8% of households were below the poverty line.[38] Hence, when we analyze equilibrium with a poverty population, as when charter schools commonly enter in a central city, we take that population to be households below the 14.8th percentile of the income distribution.

We take the joint distribution of the logarithms of ability and income to be normally distributed. Solon (1992) and Zimmerman (1992) estimate the correlation between father's income and son's income and both find .4. Hence, in modeling the relationship of income and ability, we take the correlation of ln(y) and ln(b), $\rho_{by}$, to be .4. This in turn implies that a household with ability one-tenth of a standard deviation above the median will have earnings 3.5% higher than a household at the median. This corresponds closely to the 3.3% estimate of the gain in annual income at age 50 from a one-tenth standard deviation increase in cognitive ability in Table 3 of Lin, Lutter, and Ruhm (2018).

Findings of Cunha, Heckman, and Schennach (CHS) (2010) and Borghans, Golsteyn, Heckman, and Humphries (BGHH) (2016) play a central role in our calibration of non-cognitive skills, the relationship of non-cognitive to cognitive skills, and the roles of the two types of skills in determining achievement. From CHS, we obtain $\rho_{bm}=.2165$ and $\sigma_m/\sigma_b=\sqrt{.75}$. The details are presented in the accompanying appendix.

BGHH Appendix Table 7.8 columns (1) and (2) provide regressions of achievement on IQ and achievement on non-cognitive skills respectively. The ratio of the $R^2$ statistic in column (2) to that in column (1) is .173/.489 = .35. Our calibration requires the counterpart regressions in our simulated sample of 20,000 public-school students in equilibrium to produce the same $R^2$ ratio. In particular, let $R^2_{am}$ be the $R^2$ obtained in our model from regressing ln(a) on ln(m) and let $R^2_{ab}$ be the $R^2$ obtained from regressing ln(a) on ln(b). Our calibration requires that, in equilibrium, $(R^2_{am}/R^2_{ab})=.35$.

In their regression including both cognitive and non-cognitive skills, (Table 7.8, column 3), BGHH obtain an $R^2 = .53$. As they note, measurement errors and/or unmeasured factors account for the unexplained variance. In our calibration, we assume the $R^2$ would be .75 if all relevant elements on the right-hand-side were measured without error. This implies that the variance of achievement, $\sigma_a^2$, can be obtained from the variance of measured achievement, $\sigma_x^2$, as follows: $\sigma_a^2=.75\sigma_x^2$.

[37] https://en.wikipedia.org/wiki/Household_income_in_the_United_States#Mean_household_income
[38] https://www.census.gov/library/publications/2015/demo/p60-252.html

Our distribution of the logarithm of ability is as follows. As noted above, we take ability to be lognormally distributed, and we take the logarithm of ability to be IQ. The standard measure of IQ has mean 100 and standard deviation 15. The ratio of the latter to the former is .15. Hence, in our model, the ratio of the standard deviation of log ability to the mean of log ability is $\sigma_b / \mu_b = .15$.[39]

Values targeted in the calibration are the mean $(\mu_a)$ and standard deviation of the logarithm of achievement $(\sigma_a)$, To obtain empirical counterparts, we utilize data for the distribution of 12th grade scores from the National Assessment of Educational Progress, NAEP.[40] Median NAEP scores and standard deviations are reported for mathematics, reading, and writing. As detailed in the appendix, using this data and $\sigma_a^2 = .75\sigma_x^2$, we calibrate $\mu_a = 2.89$ and $\sigma_a = .48$.

Another targeted empirical value is the high school dropout rate. From the National Center for Education Statistics[41], we obtain the high school dropout rate of 5.4%.

The following equations will be used in detailing the calibration of the curriculum function in our computational model. Our achievement function can be expressed as:

(20) $$a(b,e,B,Q) = b^{\beta}Q\Gamma(b,B)e = b^{\beta}Q\Gamma_m Z(b,B)e = b^{\beta}\Omega Z(b,B)e.$$

Recall that $Z(b,B) = \left[\alpha + (1-\alpha)\exp\left\{-\frac{(b-B)^2}{\tau^2}\right\}\right]$. All else constant, the impact on the logarithm of achievement for a student with ability at the curriculum target, ln(B), relative to a student with logarithm of ability ln(B)+δ is the following:

(21) $$\begin{aligned} &\ln\left[Z(e^{\ln(B)+\delta}, e^{\ln(B)})\right] - \ln\left[Z(e^{\ln(B)}, e^{\ln(B)})\right] = \ln\left[Z(e^{\ln(B)+\delta}, e^{\ln(B)})\right] - \ln\left[Z(1,1)\right] \\ &= \ln\left[Z(e^{\ln(B)+\delta}, e^{\ln(B)})\right] \end{aligned}$$

We assume the public school targets students of median ability. Hence, $B^P = e^{\mu_b}$. Consider a student at the 97.5th percentile of the logarithm of ability, $b_{.975}$. We choose parameters $\alpha$ and $\tau$ of the Z(b,B) function so that the effect of being at the 97.5th percentile of ability relative to median ability is .04 standard deviations of the logarithm of achievement. That is, we choose $\alpha$ and $\tau$ with the objective of having ω in the equation below be .04. We have not found direct empirical evidence to calibrate this measure. Hence, this choice of ω is largely based on our judgment.

(22) $$\omega\sigma_a = \ln[Z(b_{.975}, B^P)].$$

The latter pins down one of the two parameters that characterize the Z(b,B) function. Recall that B denotes the ability level to which the curriculum is targeted. Students drop out if their achievement falls below the standard, $S^P$. We set α=.85. Hence, other things constant, a student at the extreme

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[39] Quantiles of a normal distribution are invariant to the choice of mean as long as the ratio of the standard deviation to the mean is preserved. For example, IQ can be measured with a mean of 1 and standard deviation of .15, a mean of 2 and standard deviation .3, etc. We exploit this feature at several points in our calibration.

[40] We obtained this NAEP information from: https://www.nationsreportcard.gov/ndecore/xplore/NDE

[41] This was the dropout rate in 2017 reported in Table 2.1 of https://nces.ed.gov/pubs2020/2020117.pdf

distance from the target, $(B-b)^2 \to \infty$, suffers a decrement of .15 standard deviations of achievement relative to the achievement obtained if the curriculum were targeted to that student's ability.

For calibrating $\beta$ in the achievement function (25), it is useful to consider a student (b,m) not constrained by the standard. Equation (5) shows that optimal effort implies:

$e^*(b,m,B,Q) = b^{\beta}\Omega Z(b,B)m.$

Hence, achievement for an unconstrained student is: $a^* = b^{\beta}\Omega Z(b,B)e^* = [b^{\beta}\Omega Z(b,B)]^2 m.$

Taking logs, we obtain the following[42].

$$(23) \qquad \ln(a^*) = 2\beta\ln(b) + 2\ln(\Omega) + 2[\ln(Z(b,B))] + \ln(m)$$

Taking expectations then yields:

$$\begin{aligned} E\ln(a^*) &= 2\beta E\ln(b) + 2\ln(\Omega) + 2E[\ln(Z(b,B))] + E\ln(m) \\ &= 2\beta\mu_b + 2\ln(\Omega) + 2E[\ln(Z(b,B))] + \mu_m \end{aligned}$$

As the preceding relationship shows, $\beta\mu_b$ scales the relationship between ln(b) and ln(a*). Thus, $\beta\mu_b$ can be treated as a single parameter. Hence, we set $\beta$=1 while preserving in our calibration the requirement derived above that specifies $\sigma_b / \mu_b$ =.15.

To complete our calibration, we employ an inner-loop, outer-loop approach with the inner loop solving for equilibrium in a simulated sample of 20,000 students and the outer-loop varying the parameters to be calibrated. Assembling the items above, in addition to fixing the already calibrated values of the joint income-ability distribution, our calibration objective is to have the following ten conditions be satisfied in equilibrium:

$\rho_{bm}$ =.2165, $\sigma_m = \sigma_b\sqrt{.75}$, $\sigma_b / \mu_b$ =.15, $\mu_a$ = 2.89, $\ln(B^P) = \mu_b$, $\alpha$ = .85, $\sigma_a / \mu_a$ =.167, ($R^2_{am}$/ $R^2_{ab}$)=.35, dropout rate=5.4%, and $\omega_{.975}$=.04. We proceed as follows. We impose the first six of the latter conditions. We then search for values of the remaining parameters targeting the last four conditions in equilibrium, i.e., $\sigma_a / \mu_a$ =.167, ($R^2_{am}$/ $R^2_{ab}$)=.35, dropout rate=5.4%, and $\omega_{.975}$=.04. In particular, we search for values of $\Omega$, $\mu_b$, $\mu_m$, $\tau$, and $S^P$. The chosen parameters must be consistent with the equilibrium requirement that all students choose their optimal effort given their type (b,m), the

[42] Earlier, we chose the mean and standard deviation of the logarithm of achievement, $\mu_a$ and $\sigma_a$, by benchmarking those parameters to the NAEP. In doing so, we did not impose the requirement that the logarithm of achievement in our model be normally distributed (see the appendix for the detail). As the preceding equation demonstrates, the logarithm of achievement is not normally distributed. While both ln(b) and ln(m) are normally distributed, ln(Z(b,B)) is not. The above equation is for students not constrained by the curriculum standard. There are also students who are constrained by the standard, including some who drop out. These factors also gives rise to a departure of the achievement distribution from lognormality. We emphasize that these departures from lognormality are not of concern. It is inevitable that a realistic model with curriculum standards and dropouts will have an achievement distribution that is not lognormal. Indeed, the NAEP itself is administered to students who are in school, and hence does not measure achievement of those who dropout.

curriculum $B^P$, and the graduation standard $S^P$. The equilibrium provides a good fit to the targets. A fine-grained grid search yields the following parameter values: $\Omega = .625;\ \mu_b = 1.8; \mu_m = .023;$

$\tau = 11.25; S^P = 10.6; \sigma_b = .27; \sigma_m = .0234; \rho_{bm} = .2165; \alpha = .85;$ and $B^P = e^{\mu_b} = 6.05.$

Measures of fit are in Table 1. The comparison of targets to outcomes illustrates that the calibration satisfies these conditions quite well.[43]

**Table 1: Model Fit**

| Measure | Target | Model |
|---|---|---|
| $R^2_{am}/ R^2_{ab}$ | .35 | .326 |
| Dropout Rate | .054 | .055 |
| $\sigma_a / \mu_a$ | .167 | .163 |
| $\omega_{.975}$ | .4 | .413 |

5.2 Computational Findings.

| | **Table 2: Computational Results** | | | | |
|---|---|---|---|---|---|
| | **(1)** | **(2)** | **(3)** | **(4)** | **(5)** |
| | **TCS Income Nat Rep** $B^C \leq B^P$ | **TCS Income Nat Rep** $B^C \geq B^P$ | **TCS Income Pov** $B^C \leq B^P$ | **TCS Income Pov** $B^C \geq B^P$ | **NEC Income Pov** |
| $\mathbf{S^P = S^C}$ | 10.6 | 10.6 | 10.6 | 10.6 | 10.6 |
| $\mathbf{B^C}$, **ln($\mathbf{B^C}$)** | 4.08, 1.41 | 9.10, 2.21 | 3.13, 1.14 | 6.63, 1.89 | 6.05, 1.80 |
| $\mathbf{B^C}$ **Percentile** | .072 | .935 | .007 | .632 | .5 |
| $\mathbf{B^P}$ **Percentile*** | .5 | .5 | .5 | .5 | .5 |
| **Prop. Attend Ch** | .1 | .1 | .1 | .1 | .1 |
| **Prop. Apply Ch** | .2 | .2 | .2 | .2 | .2 |
| **Med Ch Inc./Med Pop Inc.** | .72 | 1.60 | .97 | 1.08 | 1.06 |
| **Achieve Gain**** | $.091\sigma_N$ | $.022\sigma_N$ | $.122\sigma_P$ | $.003\sigma_P$ | $.516\sigma_P$ |
| **Prop Effort Constrained Charter** | NA | NA | NA | NA | 100% |
| **Prop Charter Exceed Standard** | 14.4% | 100% | 9.1% | 99.5% | 100% |
| **Prop Dropouts if Charter** | 5.41% | 5.50% | 13.76% | 13.87% | 13.87% |
| **Prop Dropouts if No charter** | 5.50% | 5.50% | 13.87% | 13.87% | 13.87% |

* Note that this is for nationally representative population across row.

** Standard deviations of public school achievement: National population, $\sigma_N$=.786; Poverty population, $\sigma_P$=.956. These standard deviations include dropouts, who put in zero effort. We include dropouts since most of the evidence to which we compare our predictions is from middle and grade schools, so that failing students would not yet actually drop out and would take tests. If we instead assume dropouts are not counted, the standard deviations of achievement in the public school would decrease substantially, by a factor of .589 for the nationally representative population and .360 for the poverty population.

In our computational analysis, we investigate five alternative cases of charter schools. Results are presented in Table 2. Columns (1) and (2) report results for traditional charter schools serving a

[43] Recall that we targeted $\omega_{.975}$=.04. With the above calibration, a student at the 2.5th percentile would then experience a $.015\sigma_a$ decrement relative to a curriculum targeted to the student's ability, $\omega_{.025}$=.015. The asymmetry in the magnitude of the two $\omega$ values arises because achievement is lognormally distributed. Hence, a student at the 2.5th percentile is closer to median ability than a student at the 97.5th percentile of ability.

population representative of the US population. Results with optimal curriculum target below the public school curriculum are in Column (1) while results with optimal curriculum target above the public school's are in Column (2). Recall that two local optima exist, one with curriculum target above and one with curriculum target below that at the public school. Columns (3) and (4) report results for traditional charter schools serving a population representative of a poverty population, with curriculum target below, Column (3), and above, Column (4), that of the public school. In both student populations, the global optimum is that with curriculum target below the public school's target. Column (5) presents results for a no-excuses charter school serving a poverty population.

5.2.1 Traditional Charter Schools. Column (1) of Table 2 is for a TCS in a neighborhood with population representative of the national population and with curriculum optimally chosen to serve low-ability students in that population. Looking down the column, we see that the TCS curriculum target is to a student at the 7.2$^{th}$ percentile of ability. Recall that the public school targets the student of 50$^{th}$ percentile ability. The charter school has capacity to serve 10% of the population in the school zone and 20% percent of students apply. The median income of charter school students is roughly three-fourths the median income of the neighborhood population. The mean achievement gain of students attending the charter school is almost one-tenth of a standard deviation of all students' achievement (if they attended the public school). Much of this gain comes from students that attend the charter school if admitted, but would drop out of the public school. The dropout rate with only public schooling is 5.50%. With the charter school, the population dropout rate decreases to 5.41%. This reduction in dropout rate is achieved by a charter school serving just 10% of the population. Most charter school students just meet the standard for graduation and most of them would do the same if attending the public school. However, 14.4% exceed the standard. The attendance spaces for this equilibrium are shown in Figure 8a. The ability level targeted by the curriculum is shown by the left vertical line. This vertical line intersects the horizontal axis the value of $\ln(B^C)=1.41$. This target, and the curriculum targets for the other cases, are shown in Table 2. Charter school applicants occupy the wedge in the upper-left region of Figure 8a, with right border midway between the public and charter curriculum targets.[44] This is comparable to Figure 2 from the theoretical development, though Figure 2 uses hypothetical parameters.[45] The widening of the wedge as one moves vertically arises because highly motivated low-ability students are willing to make the effort to meet the achievement standard required to be retained (graduate) in school. Students of lower motivation below and to the left of the wedge prefer to drop out and do not attend the charter school. Figure 8b shows detail for the charter school applicants, with additional detail for the charter attendees (lottery winners). Charter attendees in the upper left triangle in the figure are those who exceed the standard, comprising, as

[44] Here and in the remaining discussion, we use charter school applicants to refer to both those who win and those who lose the charter school lottery.
[45] Figure 2 also presents attendance choices in the (b,m) plane, rather than the (ln b,ln m) plane.

noted above, 14.4% of charter school attendees. All remaining charter attendees perform at the minimum effort required to just meet the standard.

Column (2) of Table 2 reports the results for a TCS targeting high ability students. The optimally chosen target is the 95.5$^{th}$ of ability. The charter school attracts a clientele with household income substantially above that of the population; median income of the TCS students is 1.6 times that of the population. Charter school students gain .022 standard deviations in achievement relative to those who applied and lost the charter lottery. This is a modest but non-negligible gain. This TCS attracts students who would not drop out whether in public or charter school. Hence, entry of this TCS does not affect the overall dropout rate, a key reason gains are small. As shown in Figure 9a, the charter school attracts high-ability students. Here, the left boundary of the space of charter school applicants lies halfway between the public school curriculum target and the charter school curriculum target. Figure 9b shows the charter school applicant space in more detail. All charter school applicants choose effort well above that required to meet the retention standard. Hence, in contrast to the TCS serving low ability students, no students in the high-ability charter are constrained by the standard.

Column (3) of Table 2 presents results for a charter school targeting low-ability students in a neighborhood with a poverty population. The average achievement gain of lottery winners relative to losers is .12 of a standard deviation of achievement of all poverty students if instead attending the public school. As shown in Figure 10a, the curriculum target for the charter school is set at a quite low level, far below the public school curriculum target, which we maintain at the median ability of the nationally representative population. By setting a low target, the charter differentiates itself enough from the public school to attract the lowest ability students and maximize achievement gains. The effort of almost all charter students, 91.9%, is constrained by the requirement to meet the achievement standard in order to not fail. Figure 10b shows the small number of students in the upper right corner of the charter school admission space who choose effort higher than that required to be retained in school. Entry of this TCS lowers the dropout rate somewhat because the targeting of low ability students enables some of them to learn material that they would not be able to learn with the public school curriculum target, these students having the largest achievement gains.

Column (4) of Table 2 reports results for a TCS serving a poverty population with curriculum target above that for the public school. The median income in this TCS is about 8 percent higher than the median income in the poverty neighborhood. The mean achievement gain of TCS students is negligible, .0003 standard deviations. Because the poverty population has a higher concentration of lower ability students than the nationally representative population, the charter curriculum target is not too far above the public school's target, even though optimally attracting higher ability students from the population. Almost all charter students, 99.5%, achieve above the standard required to pass. This selection is displayed in Figure 11a. Figure 11b shows the space of charter applicants in more detail. As the figure shows, all perform above the standard except the tiny fraction in the lower left corner who must exert effort above their preferred level in order to meet the standard for retention. Note that the dropout (or fail) rate is unchanged, this one reason for the small gains.

5.2.2 No Excuses Charter School. Column (5) of Table 2 displays results for a No-Excuses charter school serving a poverty neighborhood. We examine only the poverty population because NEC schools are usually urban. Recall that the optimum has $B^N = B^P$, which we verify to be the global optimum. Students attending this NEC obtain very large achievement gains, .516$\sigma_P$, relative to those who applied to the school but lost the lottery. Comparison of Figure 12a to Figure 11a shows that this NEC attracts a larger proportion of highly motivated students than the TCS despite the latter having a higher curriculum target, while the NEC does not attract students who have relatively high ability but low motivation. Students that attend the NEC must be sufficiently motivated to increase their effort to the required level.

5.2.3 Public School Curriculum Targeted to Poverty Population

The computational analysis to this point has maintained the public school's curriculum target at the 50th percentile of ability of the nationally representative population. A public school serving a poverty population might alternatively target the (lower) 50th percentile of that population. Thus, we have examined the effects of charter schools entering such an environment. The analogue of Table 2 for the cases of the poverty population in Columns 3, 4, and 5 is presented in the appendix. If there were no charter school, the lower ability target of the public school slightly decreases the dropout proportion of poverty students from 13.87% to 13.6%. The main results about achievement gains of charter school students are similar to those predicted in Table 2. Regarding the traditional charter school, the global optimum again has entry targeting lower ability students ($B^T < B^P$ is optimal), but still with smaller average achievement gains equal to .032$\sigma_P$. Because the public school has a lower ability target, the scope for gains by targeting lower ability students is reduced. Again, however, the no excuses charter school dramatically increases average achievement gain of charter students, of close to the same magnitude as in Table 2, here .507$\sigma_P$.

5.2.4 Model Predictions and the Literature

Our theoretical model predicts that traditional charter schools will differentiate their curriculum to maximize the achievement gains of the students they attract. Whether the student population is poor or representative, the curriculum choice that maximizes achievement gains targets lower ability students. Our computational model quantifies these predictions. As summarized above, we find, for a nationally representative neighborhood, that an entering TCS charter serving lower ability students population yields achievement gains of .091$\sigma_N$ whereas a TCS entrant serving higher ability students can induce only very small gains. For students representative of a poverty neighborhood, we find the same, but with larger gains of the .122$\sigma_P$ with the lower ability students targeted. This gain drops to .032$\sigma_P$ if the competing public school targets the median ability of the poverty population. Epple, Romano, and Zimmer (2016) provide an extensive review of charter school research. The evidence from this body of research is that traditional charter schools typically yield modest achievement gains, if any, relative to traditional public schools. The findings of our quantitative model about gains are reasonably consistent with this evidence. Regarding the predicted optimality of TCS's targeting lower ability students in the local educational market, the best evidence we can find here is from Booker, Zimmer, and Buddin (2005) who show that in Texas the students that switch to charter schools are lower performing than students in the public schools they exit.[46] While NEC charter school students are included in the data, a substantial majority of charter school students in Texas attend TCS's.

By contrast, the evidence is that students in no-excuses charter schools have very large achievement gains. While the NEC in our model can vary the ability target of the curriculum relative to the public school curriculum target, our model predicts that it will not. Rather, the NEC focuses on getting students to work hard. Dobbie and Fryer (2013) investigate practices of no excuses charter schools in New York City. One finding is: "Surprisingly, lesson plans at high-achieving charter schools are *not* (italics added) more likely to be at or above grade level and do not have higher Bloom's Taxonomy scores (p. 38)."[47] Thus, in choosing curriculum, the NEC does not appear to target brighter or weaker students. This is not to suggest that they have the same learning expectations as does the public school. To the contrary, the high expectations ethos, longer contact times, frequent testing, and parental engagement of NEC schools are consistent with the focus on student effort our model predicts (see the discussion and references in Section 2).

Our model predicts much larger achievement gains from no excuses charter schools than from traditional charter schools. As shown in Column (5) of Table 2, our model predicts that a no-excuses charter school serving a neighborhood with a poverty population increases achievement by .52 standard deviations (or .51$\sigma_P$ if the public school targets the median poverty ability). Walters (2018) provides empirical estimates that no-excuses charter schools serving poverty population increase achievement for mathematics and reading by .71 and .52 standard deviations respectively.

In addition, the predicted achievement gain in the no excuses charter school is higher for the lower scoring students. This is also consistent with estimates (see Abdulkadiroglu, et.al., 2011 and Angrist, et.al., 2012, and Walters (2018)). In his analysis, Walters (2018) finds that the gains in achievement in no-excuses charter schools are greatest for students who were previously relatively low achievers. For a random sample of 400 charter lottery applicants from our model, we simulated their charter school scores if they won the charter lottery and their public school scores if they had lost the lottery. This sample size is comparable to that in Walters (2018). The regression in (29) below using this simulated data is the counterpart to the regressions presented by Walters in Table 7. As in

[46] Overwhelming evidence shows charter schools usually locate in disadvantaged neighborhoods, but the prediction of interest is about the relative ability of students attracted from the competing public school(s).
[47] Bloom's Taxonomy identifies six levels of thinking that are used to "organize objectives and create lesson plans with appropriate content and instruction …," https://tophat.com/blog/blooms-taxonomy/.

Walters empirical estimates, we find that that the gains to charter school attendees are greatest for those who would otherwise have had the lowest achievement scores if they were in public school. In this simulated sample, these coefficients are both highly significant ($p<.001$). Our coefficient of -.46 is larger in absolute value than the estimates obtained by Walters. In practice, of course, an achievement test measures scores with error, which would tend to attenuate the magnitude of the estimated coefficient. In addition, Walters includes, quite appropriately, indicator variables for demographic groups. This may also reduce the magnitude of the coefficient estimate. Thus, while differences in magnitude are to be expected, our model delivers the kind of inverse relationship between achievement gains and public school achievement scores that is found by Walters in his empirical analysis.[48] This is another important respect in which the implications of our no-excuses model are supported by empirical evidence.

(29) $\ln(\text{Achievement Gain}) = 1.94 - .46 \cdot \ln(\text{Achievement in Public School})$

Finally, in our model, the students the no excuses charter school attracts from a poverty population have profile as found in urban no excuses charter schools. They are not as poor and have higher prior scores within the population of poverty students, with, again, the predicted score in the public school equated to prior scores. This is found e.g., by Abdulkadiroglu, et.al. (2011), Angrist, et.al. (2012), and Angrist, Pathak, and Walters (2013), and Walters (2018).[49] Thus, the key predictions of our no-excuses model conform nicely to the empirical evidence.

**6. Concluding Remarks**

We have developed a model to analyze charter school educational practices and entry. Charter schools aim to maximize achievement gains of their students, drawn from a student population differentiated by cognitive ability, motivation (or non-cognitive ability), and income. Students must put forth study effort to achieve, with achievement dependent on their cognitive ability, how well their school's curriculum matches their capability, and how hard they study. More motivated students are more inclined to study. We capture the autonomy of charter schools by allowing differentiation in the curriculum match to student ability. An entering charter school selects students by choosing the level of difficulty of its curriculum with the objective of maximizing student achievement gains subject to the requirement that it draw enough students to be viable. Two local optima exist, one with a charter school that enters with a curriculum targeted to higher ability students than the public school's

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[48] Walters (2018) also makes the point that the finding that marginal students that select into no excuses charter schools gain the most implies potential for higher gains: "charter expansion is likely to be most effective when targeted to students who are currently unlikely to apply (p. 2182)." In our model, students just below the margin of application to the charter are unwilling to put in the required effort and would then fail at the charter school. To induce them to select in and succeed (i.e., to target them) would require a lower effort minimum, this adversely affecting the performance of more motivated students. An interesting question is whether several charter schools with different effort minimum requirements would improve average achievement gains in light of student selection.

[49] Another well-established characteristic of no excuses charter students is that they have higher proportion black students, though race is not an element of our model.

curriculum, and another with curriculum targeted to lower ability students. In a calibrated computational counterpart model, the global optimum by a charter school targets lower ability students. Achievement gains are modest, consistent with most of the empirical evidence.

Motivated by the practices of the no-excuses segment of charter schools, we then also investigate charter schools that enforce an effort minimum. Interestingly, a no excuses charter chooses not to differentiate its curriculum from the public school curriculum, and instead employs the effort minimum to induce selection into the charter by highly motivated students. No excuses charter school students must be willing to study a lot. Consistent with the evidence on such charter schools, we find in our computational model that the no excuses approach is highly effective in increasing student achievement. For some students, this occurs at the expense of student utility, which triggers a disagreement in the preferences over schools of those children and their parents. We also find that the largest achievement gains accrue to the lower achieving students among those that attend the charter school. This is also consistent with the evidence.

Several extensions of the model for potential future research might be pursued. Our model has a graduation or retention standard for achievement, which we assume is dictated by the educational authority and must be adhered to by both the public and charter school. Varying this standard, if allowed, is another potential dimension on which a charter school might pursue its objective. Our model assumes households choose schools to maximize achievement, but with a proportion failing to do so and with that proportion independent of household-student characteristics. Hastings, Kane, and Staiger (2006) provide evidence that lower income households are less likely to choose schools to maximize achievement and Hastings and Weinstein (2008) provide evidence that this is associated with access to information. Considering heterogeneity in efficacy of school choice is well motivated with potentially interesting effects on "biases" in charter school curriculum targeting. Public schools do not practice tracking in our model, consistent with reported practice of a majority of public schools in the U.S., though tracking is not unusual.[50] Investigating public school tracking and whether public schools might seek to ward off competition from charter schools by adoption of tracking is also a well motivated topic. Relatedly, our model considers competition between just one charter school and one public school. Consideration of equilibrium with more schools is of interest.

Finally, we have argued (see footnote 3) that U.S. charter schools are quite close to (secular) English academies and Dutch publicly funded private schools. We thus believe that our model also does a good job at capturing essential elements of education provision in these two school systems, and that it could be readily adapted to study English and Dutch school markets.

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[50] See Pre-COVID Ability Grouping in U.S. Public School Classrooms, NCES, 2021.

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Figure 1. Student Partition in Public School

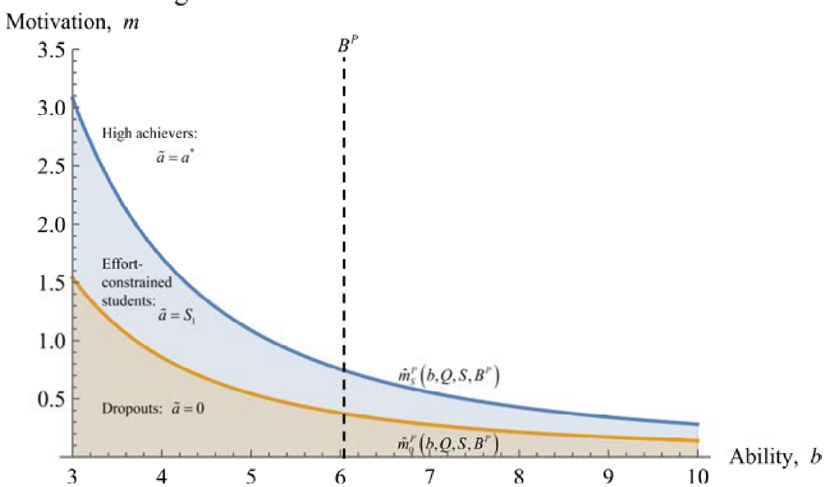


Figure 2. TCS Achievement Gains

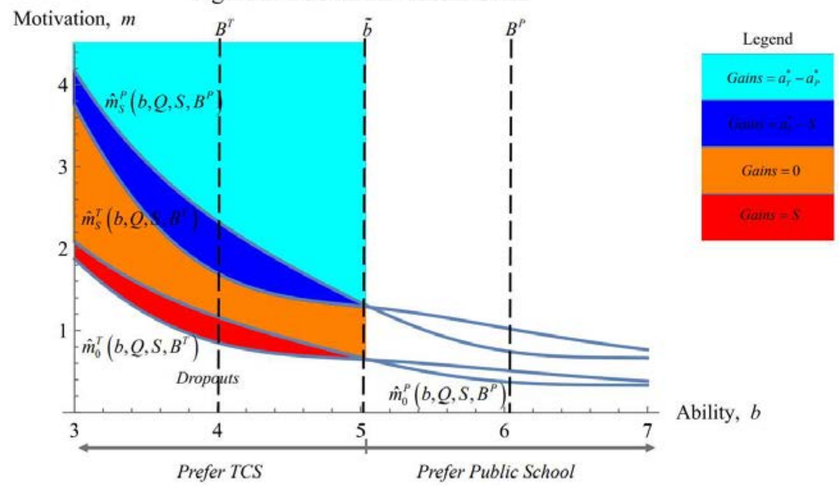


Figure 3. TCS Achievement Gains: Changes of $B^T$ Increase

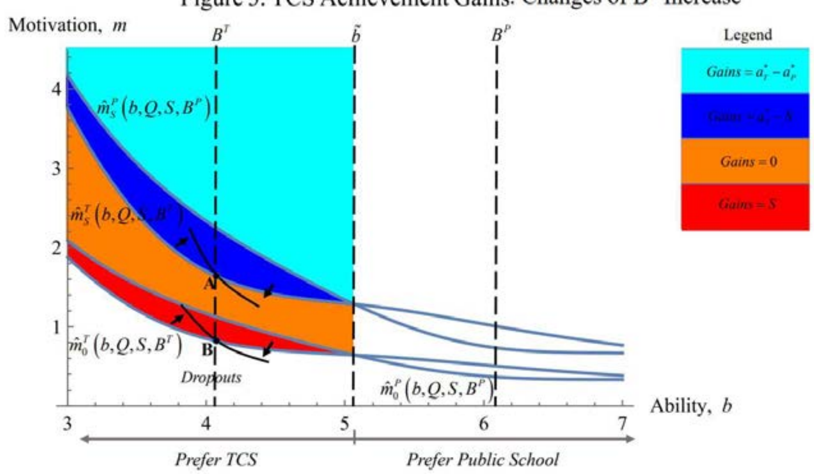


Figure 4. Iso – Effort Loci

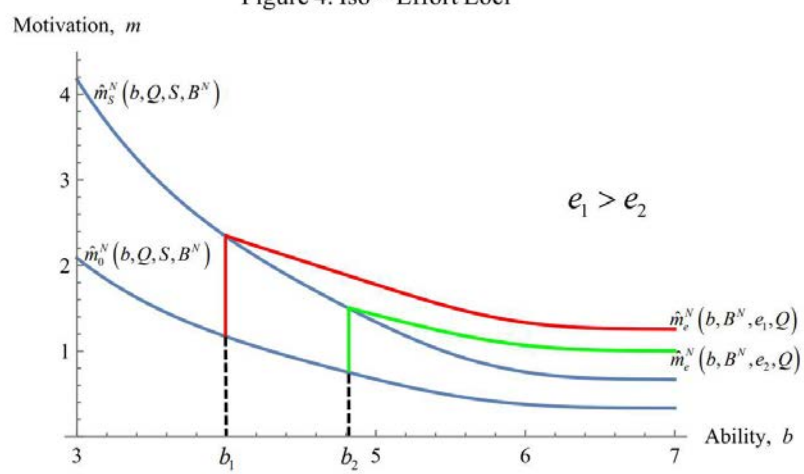

Figure 5. Applicants Set NEC $(B^N > B^P,\ b_N > \tilde{b})$

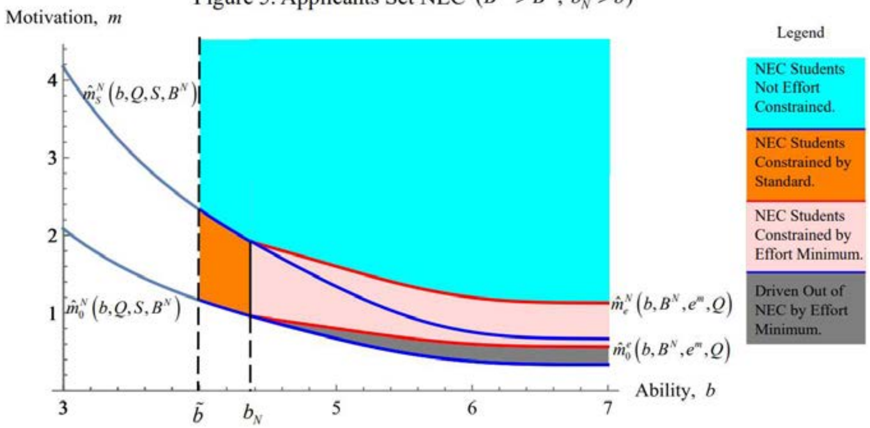


Figure 6. Applicants Set NEC $(B^N > B^P,\ b_N < \tilde{b})$

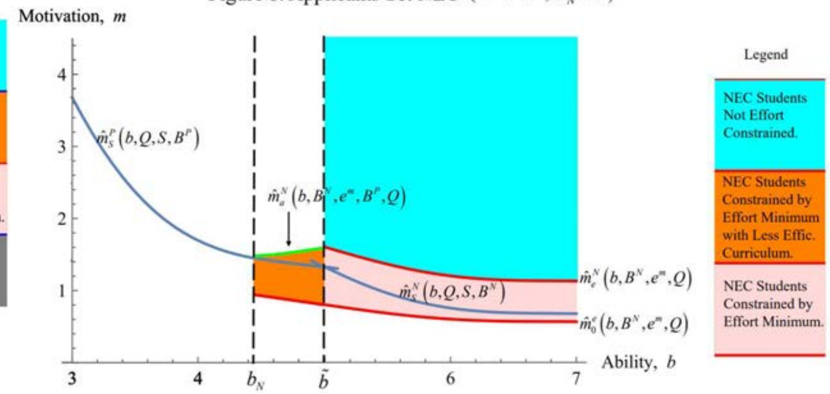


Figure 7. Applicants Set NEC $(B^N = B^P)$

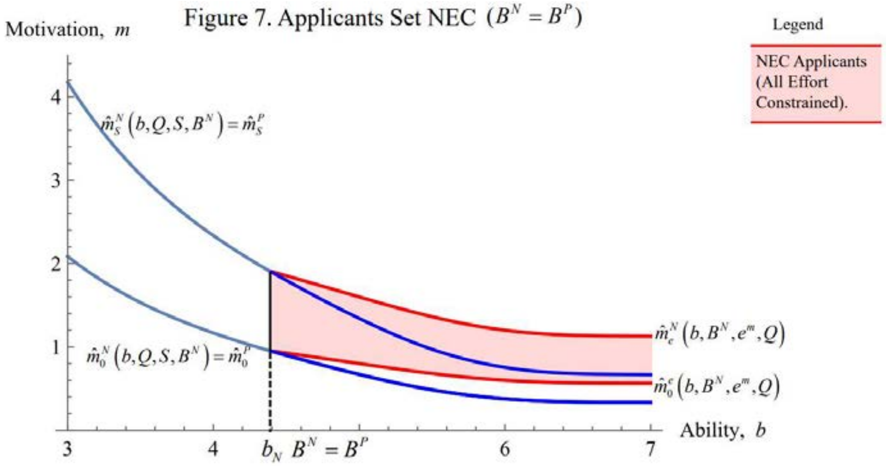

Figure 8a

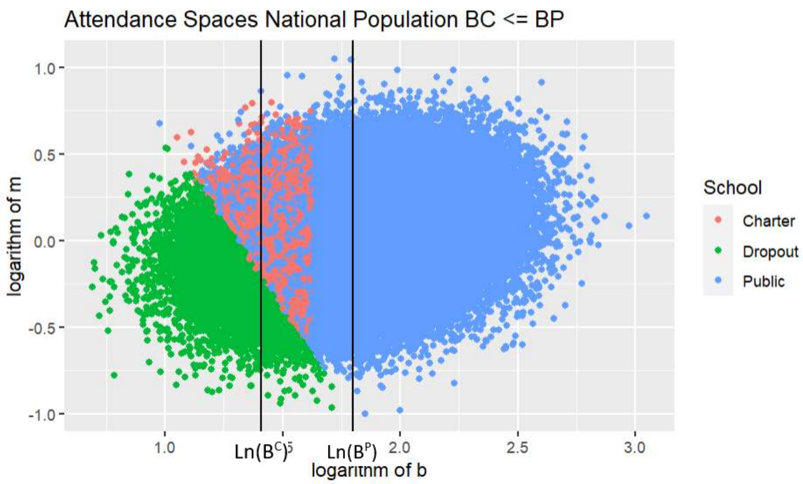


Figure 8b

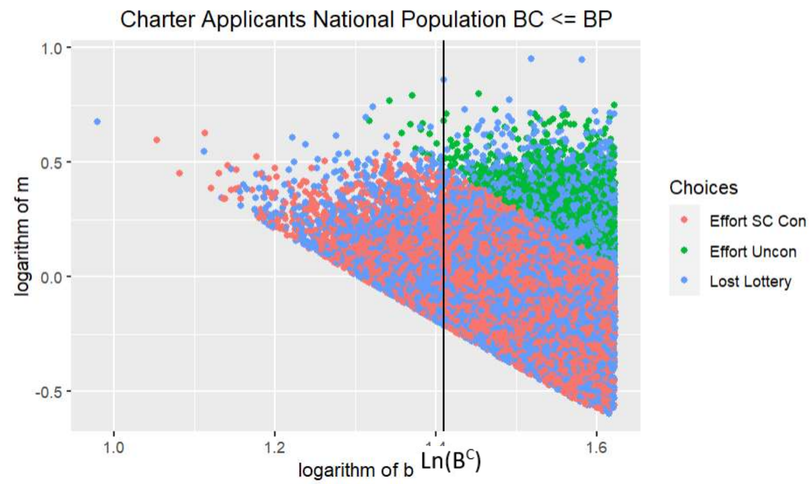


Figure 9a

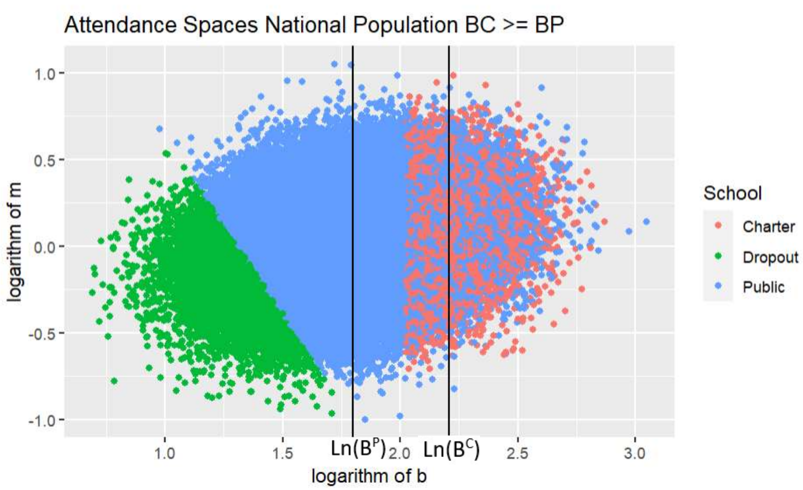


Figure 9b

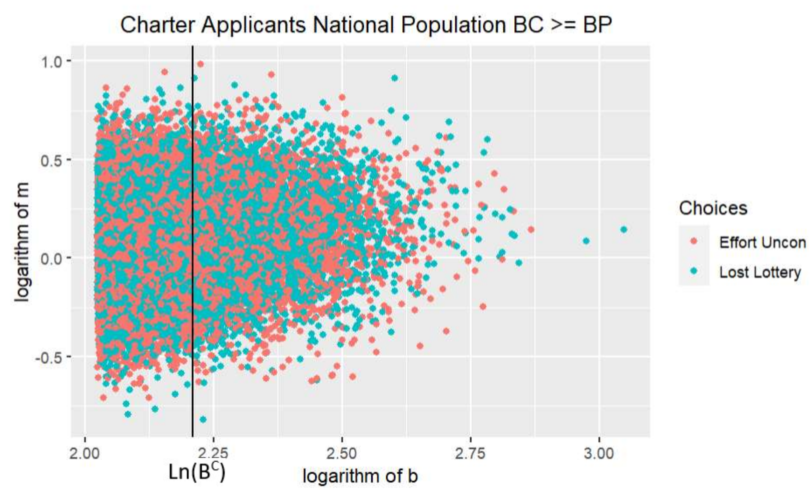

Figure 10a

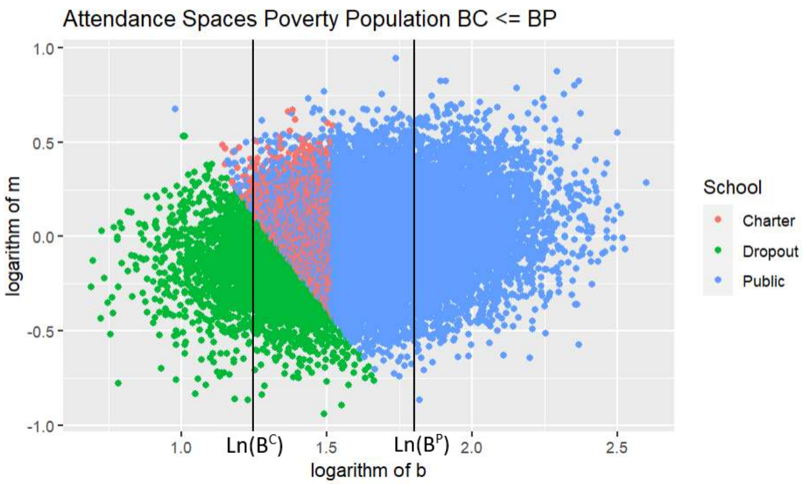


Figure 10b

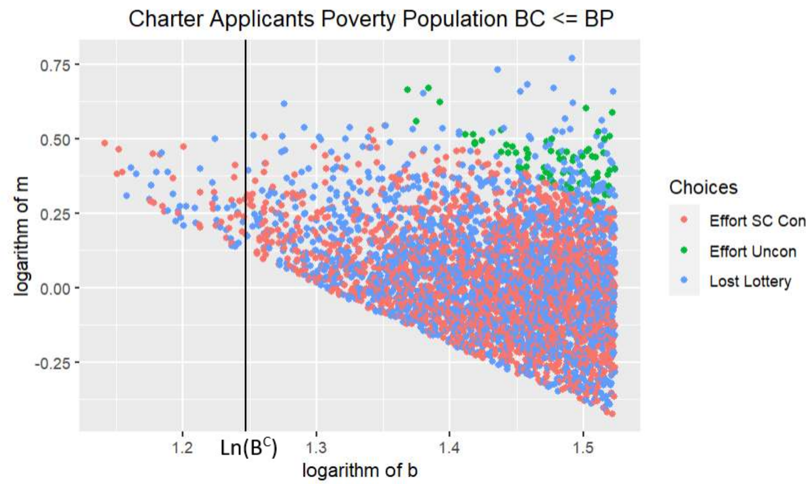


Figure 11a

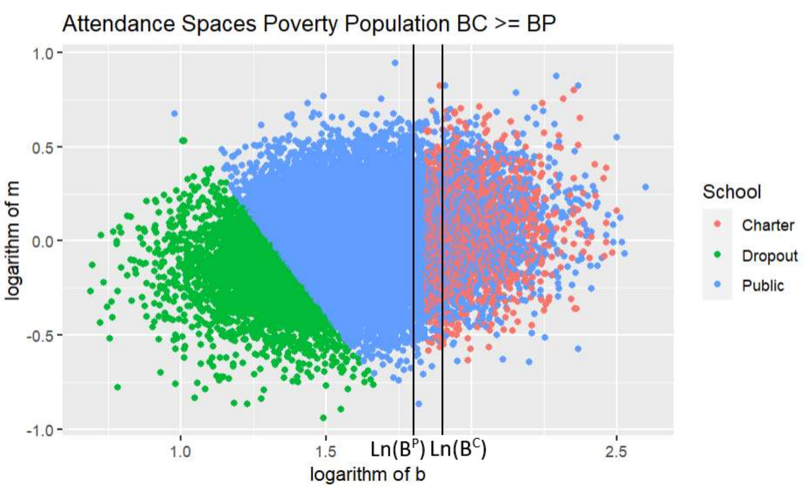


Figure 11b

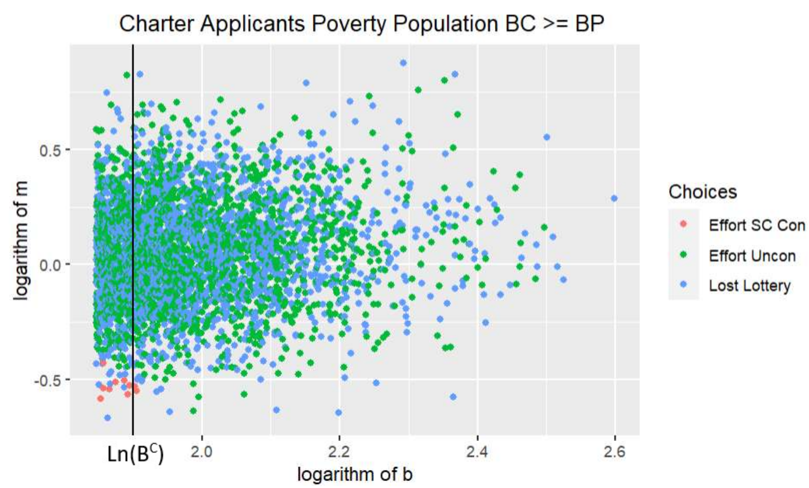

Figure 12a

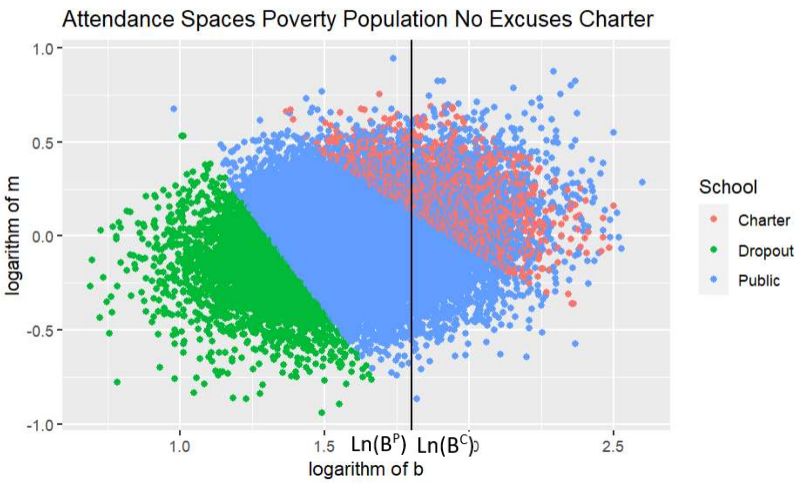


Figure 12b