NBER WORKING PAPER SERIES

SUPPLY CHAIN RESILIENCE:
SHOULD POLICY PROMOTE DIVERSIFICATION OR RESHORING?

Gene M. Grossman
Elhanan Helpman
Hugo Lhuillier

Working Paper 29330
http://www.nber.org/papers/w29330

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
October 2021

We are grateful to Maxim Alekseev and Alejandro Sabal for excellent research assistance. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

Supply Chain Resilience: Should Policy Promote Diversification or Reshoring?
Gene M. Grossman, Elhanan Helpman, and Hugo Lhuillier
NBER Working Paper No. 29330
October 2021
JEL No. F12,F13

ABSTRACT

Supply chain disruptions, which have become commonplace, are often associated with globalization and trade. Little is known about optimal policy in the face of insecure supply chains. Should governments promote resilience by subsidizing backup sources of input supply? Should they encourage firms to source from closer and presumably safer domestic suppliers? We address these questions in a very simple model of production with a single critical input and with exogenous risks of relationship-specific and country-wide supply disturbances. We follow Matsuyama and Ushchev (2020) in positing a class of preferences that are homothetic with a single aggregator and that obey Marshall's Second Law of Demand. The familiar case of CES preferences is a member of the class, but it imposes restrictions that are important for policy conclusions.We find that, in the CES case, a subsidy for diversification achieves the constrained social optimum and dominates a policy that promotes reshoring or offshoring. When the demand elasticity rises with price, two policy instruments generally are needed to achieve efficient supply chains, private investments in resilience may be socially excessive, and policy that alter incentives to invest at home versus abroad may achieve greater welfare than ones that encourage or discourage diversification.

Gene M. Grossman
International Economics Section
Department of Economics
Princeton University
Princeton, NJ 08544
and NBER
grossman@princeton.edu

Elhanan Helpman
Department of Economics
Harvard University
1875 Cambridge Street
Cambridge, MA 02138
and CEPR
and also NBER
ehelpman@harvard.edu

Hugo Lhuillier
Department of Economics
Princeton University
hugo.lhuillier@princeton.edu

1 Introduction

> The United States needs resilient, diverse, and secure supply chains to ensure our economic prosperity and national security. Pandemics and other biological threats, cyber-attacks, climate shocks and extreme weather events, terrorist attacks, geopolitical and economic competition, and other conditions can reduce critical manufacturing capacity and the availability and integrity of critical goods, products, and services. Resilient American supply chains will revitalize and rebuild domestic manufacturing capacity, maintain America's competitive edge in research and development, and create well-paying jobs.
>
> Joseph R. Biden, Jr., Executive Order on America's Supply Chains, February 24, 2021

Supply chain disruptions have become the new normal. The Great East Japan Earthquake of 2011, and the massive tsunami that it triggered, brought such occurrences to the attention of economists. Since then, hardly a month passes without news of a fresh disturbance. The pace of disruptions has quickened with the advent of the COVID-19 pandemic, and now we hear regularly of supply chain breakdowns in industries as disparate as automobiles, dishwashers, plastics, copper wire, lumber, pork, and toilet paper.

Disruptions have a myriad of causes. They result from natural disasters, geopolitical disputes, transportation failures, cyber-attacks, fires, power outages, labor shortages, human error and, of course, pandemics. McKinsey Global Institute (2020), which recently conducted a series of interviews with supply chain experts, reports that disruptions lasting one to two weeks happen to a given company on average every second year, while those lasting one to two months occur every 3.7 years. They find that an industry's exposure to shocks reflects its geographic footprint and its production technology, with greatest exposure in medical devices, wooden products, and fabricated metal products and least exposure in communications equipment, apparel, and petroleum products (see Exhibit E2 on p.6). The disruptions impose significant costs, presenting firms with expected losses of between 24% of a year's earnings before interest, taxes, depreciation and amortization (EBITDA) in pharmaceuticals to 67% in aerospace, over a ten-year period. Across the thirteen industries that McKinsey examined, the expected losses per decade amounted to about 42% of annual EBITDA (see Exhibit E5 on p.12).

Carvalho et al. (2021) provide a more rigorous quantitative assessment of one major disruption, namely the aforementioned Japanese earthquake of 2011. They focus on the role of input-output linkages as a mechanism for propagation and amplification of shocks, exploiting the exogenous nature of the event, its enormous impact on a subset of firms, and its localized incidence. Beyond the direct effects, they find that firms whose suppliers were hit hard by the natural disaster suffered substantial sale losses compared to others, as did firms whose downstream customers were hit. Using a calibrated general-equilibrium model of production networks, they conclude that the disaster imposed a 0.47 percentage point reduction in Japan's aggregate real GDP growth.

Many commentators associate the increasing frequency and severity of supply chain disruptions with the recent history of rapid globalization.[1] This perceived connection, in turn, has sparked soul searching amongst policy makers and a call to action in the broader public. If costly shocks reflect concentration of input supplies in a few regions or countries, wouldn't it be sensible for governments to encourage firms to diversify their sourcing? And if distance from suppliers intensifies the risk of disruption, wouldn't it be better to bring some parts of the supply chains back home? The preamble to President Biden's February 24th 2021 Executive Order suggests that "*diverse* and secure supply chains" are a prerequisite for economic prosperity and that "resilient American supply chains" will rest on "rebuil[t] *domestic* manufacturing capacity [emphasis added]."

Little is known about the efficacy of policies aimed at supply chain management in an environment with recurring disturbances. Disruptions generate input shortages that can give rise to price spikes or even outright unavailability of downstream products. Consumers suffer from their hampered ability to purchase the products they covet. To the extent that households forfeit consumer surplus in the face of supply chain disruptions, governments may have reason to enact policies that curtail their occurrence.

But the policy calculus is not so simple. Production impediments impact not only consumers' surplus, but also firms' bottom lines. The question for governments is not whether shortages adversely affect households, but whether firms' private incentives to avoid such shortages fall short of (or exceed) what is socially desirable. Firms may have inadequate incentives to invest in supply-chain resilience, because they do not capture all of the surplus from offering their products to the market. But they may also have an excessive taste for resilience arising from their desire to be in a position to capitalize on extraordinary profit opportunities when their rivals are hit with supply problems of their own.

In this paper, we propose a bare-bones framework that can aid with evaluating policy that influences the organization of supply chains. Our framework puts supply shortages front and center. We abstract from all complexity in the production process by assuming that each firm manufactures a differentiated variety of some good using a single, critical input. If the supply chain operates smoothly, the firm produces one unit of its variety from one unit of the customized input. But exogenous shocks may disrupt supplier-buyer relationships. We allow for two types of shocks, those that idiosyncratically sever a single chain and those that impede all supply from a particular source country. Each firm may establish a relationship with a potential supplier in a low-cost but riskier foreign country, in a higher-cost but safer home location, or in both. To form a relationship with some supplier, the firm incurs a fixed cost. A firm can invest in resilience by avoiding the riskier foreign supply or, even more so, by diversifying its supply base by establishing relationships in both countries. In equilibrium, there are four possible states of the aggregate economy: supply chains may be operative only with home suppliers, only with foreign suppliers, with neither, or with both. The fixed mass of final producers chooses among four strategies: invest in a single supply relationship domestically, invest in a single relationship abroad, diversify, or exit. Their collective choices determine equilibrium prices, equilibrium variety, profits, surplus and (with fiscal policies

[1] See, for example, Shih (2020a, 2020b) and Iakovou and White III (2020).

in place) government revenues in each state of the world. Using these state-contingent aggregate outcomes, we can calculate expected welfare under different policy regimes.

We consider three alternative policy options, separately and in combination. The government might promote or discourage diversification, which we operationalize as a subsidy or tax on a firm's formation of a second supply relationship. Also, the government might promote or discourage "reshoring" with a subsidy or tax for establishing a supply relationship specifically in the home country. Finally, the government might promote or discourage offshoring with a subsidy or tax for establishing a supply relationship abroad. These policies alter the availability of products in the various states of the world. We examine which types of policies increase welfare and how the optimal second and third-best policies compare in their efficacy. In order to elucidate the role that policy plays in each environment, we sometimes allow for an optimal consumption subsidy alongside the policies that directly affect the incentives for supply-chain organization. It is well-known that markup pricing generates undersupply in settings of imperfect competition (see, for example, Dhingra and Morrow, 2019, and Campolmi et al., 2021), so considering supply-chain policy together with consumption subsidies clarifies whether the former just substitutes (imperfectly) for the latter, or whether it can play some additional role in promoting continued availability in the face of adverse supply shocks.

Inasmuch as the social cost of supply-chain disruptions stems from loss of consumer surplus, the form of consumer preferences plays a crucial role in any policy analysis. It has become commonplace to use CES preferences in trade models with endogenous availability of differentiated products, but the very special properties of these preferences have been recognized since the seminal work by Dixit and Stiglitz (1977). The market undersupplies variety, because firms do not capture all of the surplus from their investments. But it oversupplies variety, because some of a firm's profits come at the expense of rival firms. In many contexts with CES preferences, these opposing forces happen exactly to offset one another. These considerations apply as much to investments in "resilience" as they do to investments in entry, so it is important for understanding the dictates of optimal policy addressing the organization of supply chains to allow for more flexible forms of demand. To this end, we follow Matsuyama and Ushchev (2020) in adopting a broader class of preferences that are *Homothetic with a Single Aggregator* (HSA). With HSA preferences, the demand for any variety per unit of income depends on the price of the variety relative to a (common) aggregator of all prices. The CES utility function is a member of this class wherein the ideal price index plays a dual role of inversely measuring welfare and capturing the competitive pressure from rival varieties. More generally, HSA preferences allow the demand elasticity for a variety to vary along the demand curve and the aggregator that enters demand to differ from the one that measures welfare.

Needless to say, there are many ways that our analysis could be enriched beyond allowing for a broader class of demands than CES. For example, we could introduce a richer technology with potential substitution between manufactured inputs and primary factors of production. We could entertain more complex supply chains, with multiple inputs and with a sequencing of them such that some inputs enter the production process upstream from others. We could allow for dynamics, which would render inventories an additional tool for firms to invest in resilience and give governments additional policy instruments such as stockpiling supplies or allowing accelerated depreciation of inventory costs. We could introduce political-economy considerations that might drive a wedge between the parameters that capture the risk aversion of managers versus that of policy makers. We see all of these potential extensions as worthwhile and germane to the ultimate policy assessment. Our simpler setting suggests a way to pose the question and our analysis provides a "proof of concept."

Our paper fits into an earlier literature on trade disruptions in a neoclassical setting. Much of this previous work addressed optimal policy responses to potential trade embargoes. Mayer (1977) showed that production subsidies are an optimal response to threats of trade interruption in the presence of costly adjustment. Bhagwati and Srinivasan (1976) made the likelihood of a disruption a function of the volume of trade and elucidated an efficiency role for tariffs to give agents an incentive to internalize the externality arising from their effect on the probability of a trade restriction. Arad and Hillman (1979) extended these earlier papers to allow for learning-by-doing in the production of a good that might later be subject to an embargo. Bergström et al. (1985) developed an infinite-horizon model to study the potential role of inventories to mitigate the threat of embargo. Perhaps the most sophisticated of these early studies was that by Cheng (1989), who considered recurrent embargo threats as a two-state stationary Markov process that plays out with constraints on the speed of intersectoral reallocation.

The main difference between our work and this earlier literature stems from our treatment of the endogenous availability of differentiated products. With perfect competition and homogeneous goods, aggregate quantities matter for welfare but the availability of a particular firm's offering does not. If a disruption causes some import good to be unavailable, there is no harm to consumers beyond the higher price of the domestic (perfect) substitutes. The number of viable producers plays no role in neoclassical welfare analysis and is not even well defined. Of course, higher sticker prices play a role also in a world with differentiated products, but there is also a direct harm to consumers from a particular variety not being available for purchase. For this reason, we believe that endogenous determination of the set of available products should feature prominently when evaluating policy toward supply-chain security.

The remainder of the paper is organized as follows. In Section 2 we develop a very simple model of risky supply chains in which final goods are produced with a single, critical input that may be subject to relationship-specific and country-wide supply shocks. Section 3 builds intuition by examining the case in which the home and foreign countries offer similar costs and pose similar risks. In this symmetric case, we are able to derive a number of analytical results concerning the relative ranking of policies that affect the incentives for diversification versus ones that encourage or discourage relationships in one country or the other. We also show that it is optimal for the government to encourage greater supply-chain resilience when demand elasticities are constant, but not necessarily so if demand for a brand becomes more elastic as its price rises. In Section 4, we address the more interesting but more difficult case in which suppliers in the home country are safer but more expensive than those abroad. Here too, the case of CES preferences admits an analytical approach and we find again that the competitive equilibrium provides too little resilience and that diversification policies dominate others that affect the incentives firms have to source in one country or the other. For more general HSA preferences we resort to numerical methods. Although a range outcomes is theoretically possible, the dominance of diversification policies over reshoring and offshoring policies seems reasonably robust when the elasticity of demand for differentiated products is sufficiently above one.

2 A Simple Model of Risky Supply Chains

2.1 Supply Relationships

The home economy produces a homogeneous, numeraire good and potentially a unit measure of nontraded differentiated final products. Firm ω in the latter industry converts a single, customized critical input into the final good ω using the linear production technology,

$$x(\omega) = m(\omega) \ ,$$

where $x(\omega)$ is output of good ω and $m(\omega)$ is the quantity of the customized input. If the firm has established a supplier relationship in country i and if that supply chain is operative, then the firm can procure the customized inputs at unit cost q_i, $i = H, F$, where the subscripts denote "home" and "foreign," respectively, and we assume $q_F \leq q_H$. This is, of course, the simplest imaginable production function; in future work we plan to allow for additional factors of production and more complex supply chains. But the linear production technology provides a good starting point.

To form a supply relationship anywhere, a firm must bear a sunk investment cost, k. This cost captures the up-front outlays associated with searching for a partner, negotiating a contract, and designing a suitable input. Once a supply relationship has been established, it is subject to two possible "disruption shocks." With probability $1 - \rho$, any particular supply chain breaks down for exogenous and idiosyncratic reasons, which might be a failure of the prototype input, a strike in the supplier factory, a localized weather event in the location where the input would be produced, or anything else that happens independently of all other supply relationships. In any of these circumstances, the downstream firm loses the ability to purchase its input from the particular supplier for the length of the period captured by our model. In the complementary event, with probability ρ, no idiosyncratic supply disruption occurs and the firm can buy as much as it wants from the particular supplier provided that the latter is located in a country that is "open for business." However, with probability $1 - \gamma_i$ a country-wide shock disrupts all chains with suppliers in country i. These shocks, which we assume to be independent across countries (to simplify the expressions, but with no substantive importance), represent events such as earthquakes, hurricanes, epidemics, political conflicts between national governments, or failures of the national transportation system. The relative safety of the home country is captured by the assumption that

$\gamma_H \geq \gamma_F$. It follows from all this that a particular relationship with a supplier located in country i survives with probability $\gamma_i \rho$.

After the realization of the supply shocks, firms produce their varieties if they can. A firm that has invested in a single supply relationship in country i operates with probability $\gamma_i \rho$ at unit cost q_i, whereas its product becomes unavailable in the face of any disruption, which happens with probability $1 - \gamma_i \rho$. A firm that instead pursues a strategy of diversification can produce if at least one of its supply chains remains viable. It prefers to produce at the lower unit cost q_F, which it can do with probability $\gamma_F \rho$. Should its offshore relationship be disturbed, it can turn to its home supplier with probability $\gamma_H \rho$. Therefore, the unconditional probability that it produces at cost q_H is $\gamma_H \rho (1 - \gamma_F \rho)$. Finally, with probability $(1 - \gamma_F \rho)(1 - \gamma_H \rho)$ it will find both of its potential supply chains disrupted and it will be unable to serve the market.[2]

2.2 Preferences and Demand

There is a unit mass of identical consumers in the home country. The representative consumer holds quasi-linear preferences over consumption of the homogeneous good, Y, and consumption of differentiated products, indexed by X, so that total utility is given by

$$V(X, Y) = Y + U(X), \tag{1}$$

where $U(\cdot)$ has a constant elasticity $\varepsilon \geq 1$; i.e.,

$$U(X) = \begin{cases} \frac{\varepsilon}{\varepsilon - 1}\left(X^{\frac{\varepsilon-1}{\varepsilon}} - 1\right) & \text{for } \varepsilon > 1 \\ \log X & \text{for } \varepsilon = 1 \end{cases}.$$

These preferences give rise to a constant-elasticity demand for differentiated products,

$$X = P^{-\varepsilon}, \tag{2}$$

where P is the real price index dual to U.

Following Matsuyama and Ushchev (2020), we assume that preferences for the bundle of differentiated products belong to a class they term *Homothetic with a Single Aggregator.* Homotheticity implies that the consumption index X is a linearly homogenous function of consumption of the individual varieties $\{x(\omega)\}_{\omega \in \Omega}$, where Ω is the set of varieties available in the relevant state of the world (i.e., in view of the realization of the various supply shocks). A single aggregator, A, which is a linearly homogenous function of the set of prices $\{p(\omega)\}_{\omega \in \Omega}$, guides the substitution between a particular variety ω and all other varieties. More formally, HSA preferences require the existence

[2] In principle, a firm that diversifies may choose to invest in multiple supply relationships in the same country. To avoid a taxonomy, we do not consider this possibility here; it will not be an attractive option for ρ close to one.

of a market-share function $s(z)$ that is non-negative for all z such that

$$\frac{d\log P}{d\log p(\omega)} = s[z(\omega)] \tag{3}$$

and

$$\int_{\omega\in\Omega} s[z(\omega)]\, d\omega = 1 \ , \tag{4}$$

where $z(\omega) \equiv p(\omega)/A$ represents the price of variety ω *relative to the aggregator*. Equation (3) expresses the demand for any variety ω in implicit form; the substantive assumption is that this demand depends only on the relative price p/A. Equation (4) stipulates that the market shares sum to one.

We place some mild restrictions on the market-share function, $s(z)$. First, we impose

Assumption 1 *The market-share function $s(z)$ is strictly decreasing when positive, with $lim_{z\to 0}s(z) = \infty$ and $lim_{z\to\bar{z}}s(z) = 0$, for $\bar{z} \equiv \inf\{z > 0 | \, s(z) = 0\}$.*

This assumption ensures that all varieties in X are gross substitutes. It admits both the case when $\bar{z} < \infty$, so that demand "chokes off" at some finite relative price, and the case $\bar{z} = \infty$, when positive quantities are demanded at any finite price.

Equation (3) implies that the elasticity of substitution between any two goods with equal prices is a function of the common price, and is given by

$$\sigma(z) = 1 - \frac{zs'(z)}{s(z)} > 1.$$

Second, we adopt

Assumption 2 *For all $z \in (0, \bar{z})$, $\sigma(z) > \varepsilon$ and $\sigma'(z) \geq 0$.*

The first part of Assumption 2 ensures that the demand for any variety ω increases when the aggregate price of competitor brands rises. The second part of Assumption 2 is known as *Marshall's Second Law of Demand (MSLD)*, namely that the demand for a good becomes more elastic as its price rises.[3] Melitz (2018) argues that violations of MSLD would contradict evidence on firms' pricing behavior.

Before proceeding, we highlight for future reference two familiar examples of HSA preferences. The first, of course, is the *Symmetric CES* formulation, wherein $s(z) = z^{1-\sigma}$ and $\sigma(z) \equiv \sigma$ is constant and independent of z. In this case, as is well known, the aggregator A is proportional to the price index P. The second example is the *Symmetric Translog,* developed by Feenstra (2003), drawing on Diewert (1974). It has $s(z) = -\theta \log z$, $z \in (0, 1)$, $\theta > 0$. Then $\sigma(z) = 1 - 1/\log z$,

[3] Zhelobodko et al. (2012) describe this assumption as increasing "relative love of variety" while Mrázová and Neary (2017) refer to it as the case of "sub-convex" demand.

which obeys MSLD. For this specification of preferences,

$$A = \frac{1}{\theta n} + \frac{1}{n} \int_{\omega \in \Omega} \log p(\omega)\, d\omega \ ,$$

where $n \equiv \int_{\omega \in \Omega} d\omega$ is the measure of products available on the market. Here, the aggregator A that enters demands differs from the price index P that enters the indirect utility function. More generally, Matsuyama and Ushchev (2020) prove that the alternative price aggregates are related by

$$\log P = C_P + \log A - \int_{\omega \in \Omega} \int_{p(\omega)/A}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta d\omega, \tag{5}$$

where C_P is a constant.

2.3 Profit Maximization

The firm producing variety ω maximizes profits by marking up price over its marginal cost $c(\omega)$, where $c(\omega) = q_H$ or $c(\omega) = q_F$ according to whether it sources its inputs domestically or offshore. Notice that the price of any variety might vary across states of the world. The markup reflects the elasticity of demand, as usual, but the latter need not be constant or independent of the state. Specifically, the firm solves

$$p(\omega) = \arg\max_{\mathsf{p}} P^{1-\varepsilon} s\left(\frac{\mathsf{p}}{A}\right) \mathsf{p}^{-1} \left[\mathsf{p} - c(\omega)\right],$$

taking the state-contingent price index P and the state-contingent aggregator A as given. Profit maximization requires

$$p(\omega) = \frac{\sigma\left(\frac{p(\omega)}{A}\right)}{\sigma\left(\frac{p(\omega)}{A}\right) - 1} c(\omega) \tag{6}$$

and yields operating profits

$$\pi(\omega) = \frac{s\left(\frac{p(\omega)}{A}\right)}{\sigma\left(\frac{p(\omega)}{A}\right)} P^{1-\varepsilon}. \tag{7}$$

2.4 Supply Chain Management

We allow firms to choose among three modes of organization (plus exit). Strategy h entails investment in a single supply relationship in the home country in the hope of "onshoring." Strategy f entails investment in a single relationship in the foreign country in the hope of "offshoring." Strategy b (for "both") involves diversification, i.e., an investment in a supply relationship in both places with the intention of sourcing from the low-cost foreign supplier if that is possible, and from the higher-cost domestic supplier if that is possible and the low-cost foreign option is not available.

Firms organize their supply chains to maximize expected profits.[4]

Firms calculate expected profits with rational expectations about prices, sales, and costs in each state of the world in view of the fraction of their competitors that pursue each strategy in equilibrium. Let μ_j be the fraction of firms that opt for strategy j, $j \in \{h, f, b\}$, with $\sum_j \mu_j \leq 1$.

In state H, supply chains in the foreign country are disrupted and only firms that chose strategy h or strategy b might be active in the market. Among these, only a fraction ρ escape the idiosyncratic demise of their relationships. Thus, in state H an active firm faces competition from $\rho(\mu_h + \mu_b)$ others, all of which have a unit cost of q_H. If the firm itself chooses strategy h or strategy b, it also faces a unit cost of q_H in this state, a state that occurs with probability $\gamma_H(1-\gamma_F)$. Otherwise, it earns no profits in state H.

In state F, supply chains in the home country are inoperative and only firms that pursued strategy f or strategy b might produce. Again, only a fraction ρ can do so, because the others suffer relationship-specific supply shocks. It follows that in state F, an active firm competes with $\rho(\mu_f + \mu_b)$ others, each of which has a unit cost of q_F. The firm in question also faces a cost of q_F if it adopts either strategy f or strategy b, but will be unable to produce if it chooses strategy h. State F arises with probability $\gamma_F(1-\gamma_H)$.

A firm's expected profit calculation for state B in which supply chains in both countries are active is slightly more complicated. In this state, the firm anticipates competition from $\rho(\mu_f + \mu_b)$ firms with costs of q_F inasmuch as the diversified firms that can do so will opt to purchase inputs from their low-cost, offshore source. In addition, a fraction $\rho(1-\rho)$ of the μ_b firms that are diversified will be unlucky with their low-cost suppliers but will be able to source from their backup source in the home country at a cost of q_H. Competition from firms with cost q_H includes as well the fraction $\rho\mu_h$ firms that choose strategy h and that avert an idiosyncratic shock. The firm itself anticipates a cost of q_F with probability ρ in state B if it chooses either strategy f or strategy b. It faces a cost of q_H with probability ρ if it opts for strategy h and with probability $\rho(1-\rho)$ if it opts for strategy b. Finally, we note that state B occurs with probability $\gamma_H\gamma_F$.

In the appendix, we tally the expected profits associated with each strategy as a function of the fractions of others in the industry that make the various choices. We denote these expected profit opportunities by Π_h, Π_f, and Π_b for strategies h, f and b, respectively. In equilibrium, if there is one dominant strategy j, all firms will make that choice, and so $\mu_j \leq 1$, while $\mu_\ell = 0$ for $\ell \neq j$. If two strategies yield equally high expected profits and higher than the third, then these two can have any positive fractions in equilibrium, while the third will find no takers. The fractions will be such as to generate indifference. Finally, if there exist $\mu_h > 0$, $\mu_f > 0$ and $\mu_b > 0$ such that $\Pi_h = \Pi_f = \Pi_b$, then the equilibrium will exhibit a positive number of firms pursuing each of the available strategies.

[4] An alternative would be to allow for risk aversion by firms. This would be hard to justify with a continuum of small firms, unless managers are distinct from shareholders and the former do not act fully in the interest of the latter. What would matter is the difference between the risk aversion of firms and the risk aversion of policy makers, whom we will also take here to be risk neutral.

2.5 Welfare

We adopt expected indirect utility as our welfare metric, weighting utility in each aggregate state by the likelihood of that state. Indirect utility comprises profits, tax revenues (if any) and consumer surplus.

Expected welfare reflects the fractions of firms that choose each organizational mode, outcomes that can be influenced by government policy. Let us consider the government's direct-control problem. When μ_h firms adopt strategy h, μ_f firms adopt strategy f and μ_b firms opt for diversified supply chains, aggregate expected profits amount to $\mu_h \Pi_h(\mu) + \mu_f \Pi_f(\mu) + \mu_b \Pi_b(\mu)$, where μ is the vector, (μ_h, μ_f, μ_b). Consumer surplus in state j is given by $\frac{1}{\varepsilon-1} P^J(\mu)^{1-\varepsilon}$ for $J \in \{H, F, B\}$. Therefore,

$$W(\mu) = \mu_h \Pi_h(\mu) + \mu_f \Pi_f(\mu) + \mu_b \Pi_b(\mu) + \frac{1}{\varepsilon - 1} \sum_{J=H,F,B} \delta^J P^J(\mu)^{1-\varepsilon} \ , \tag{8}$$

where δ^J is the *ex ante* probability of state J, i.e., $\delta^H = \gamma_H(1-\gamma_F)$, $\delta^F = \gamma_F(1-\gamma_H)$, and $\delta^B = \gamma_H \gamma_F$. To achieve a global maximum, the government must dictate not only the organizational choices μ_h, μ_f, and μ_b, but also the prices of the various differentiated products in each state. Supply-chain policy—that leaves firms free to set their own prices—can achieve only a constrained optimum. In general, achieving the second-best (constrained optimization of $W(\cdot)$ given markup pricing) requires two policy instruments inasmuch as there are two decision margins for firms, namely whether to diversify and where to invest.

3 The Symmetric Case

We begin the policy analysis with a limiting case that is most readily understood and that connects most directly with the existing literature: Suppose the home and foreign countries offer similar unit costs and pose similar risks, while differing only in the realizations of their supply shocks. Formally, we take $\gamma_H = \gamma_F = \gamma$ and $q_H = q_F = q$.

Consider the free-market outcome in this symmetric case. First, firms choose their organizational strategies, h, f, or b. Then, the state of nature is realized, H, F, or B. The shocks determine which firms can operate; all that do so produce with a common unit cost of q. Given this cost and the aggregator A, equation (6) dictates pricing in a given state. At the same time, the collection of optimal pricing decisions determines the aggregator A for that state. Then (7) delivers operating profits in a given state for any active firm. Using expected profits conditional on the various states, we can calculate a firm's overall expected profits from an entry strategy, noting that $\delta^H = \delta^F = \gamma(1-\gamma)$ and $\delta^B = \gamma^2$. Finally, equilibrium requires that the selected strategies are not dominated by any others.

The equilibrium outcomes are intuitive; more details can be found in the appendix. First, note that with $q_H = q_F = q$, the operating profits for any firm active in state J, $J = H, F, B$, depends only on the total number of other firms that are able to produce in that state. Among the fraction μ_h of firms that form a single supply relationship in the home country, a fraction ρ are active in state H or state B, but none is active in state F. Similarly, among the fraction μ_f of firms that form a single supply relationship in the foreign country, a fraction ρ are active in state F or state B, but none is active in state H. Finally, a fraction ρ of the μ_b diversified firms can produce in state H or state F, whereas a larger fraction $\rho + (1-\rho)\rho = \rho(2-\rho)$ can produce in state B, thanks to the backup options they have arranged. With this, we can readily calculate the total number of active firms in each state, so that $n^H(\mu) = (\mu_h + \mu_b)\rho$, $n^F(\mu) = (\mu_f + \mu_b)\rho$, and $n^B(\mu) = [\mu_h + \mu_f + \mu_b(2-\rho)]\rho$.

Equation (4) links the number of firms active in state J to the price p^J relative to the aggregator A^J for that state, namely

$$1 = n^J(\mu)\, s\left(z^J\right) \ , \ J \in \{H, F, B\} \ , \tag{9}$$

where $z^J = p^J/A^J$. Since $s(\cdot)$ is an decreasing function, the relative price, z^J, for any firm that operates in state J is an increasing function of the number of active firms, n^J. Also, the price index P^J is a function of z^J; see (5). It follows from (7) that operating profits for any firm that is able to produce in state J are a function of z^J,

$$\pi^J = \pi\left(z^J\right) = \frac{s\left(z^J\right)}{\sigma\left(z^J\right)} P^J\left(z^J\right)^{1-\varepsilon}. \tag{10}$$

We show in the appendix that $\pi(z)$ is a declining function for all ε despite the fact that $P^J(\cdot)$ is itself a declining function, which means that operating profits for every active firm in any state are lower, the greater is the number of competitors it faces in that state.

Now consider a firm's decision about where to source, conditional on opting for a single supplier. Total expected profits from strategy h are

$$\Pi_h = \delta^H \pi^H \rho + \delta^B \pi^B \rho - k. \tag{11}$$

Similarly,

$$\Pi_f = \delta^F \pi^F \rho + \delta^B \pi^B \rho - k. \tag{12}$$

With $\delta^H = \delta^F$, $\Pi_h > \Pi_f$ if and only if $\pi^H > \pi^F$, which in turn requires $n^H < n^F$ and thus $\mu_h < \mu_f$. Similarly, if $\mu_f < \mu_h$, $\Pi_f > \Pi_h$. Finally, $\Pi_f = \Pi_h$ requires $\mu_h = \mu_f$; i.e., equiprofitability of the alternative sourcing options requires equal fractions of each type of firm. We conclude that $\mu_h = \mu_f$ in any equilibrium in which both strategies are used by some firms. With symmetry, firms that opt for a single supplier choose the location that is less popular, which tends to equalize the numbers of suppliers in each country.

Firms also must decide between engaging a single supplier and investing in resilience. The total expected profits from forming two supply relationships amount to

$$\Pi_b = \delta^H \pi^H \rho + \delta^F \pi^F \rho + \delta^B \pi^B \rho(2-\rho) - 2k. \tag{13}$$

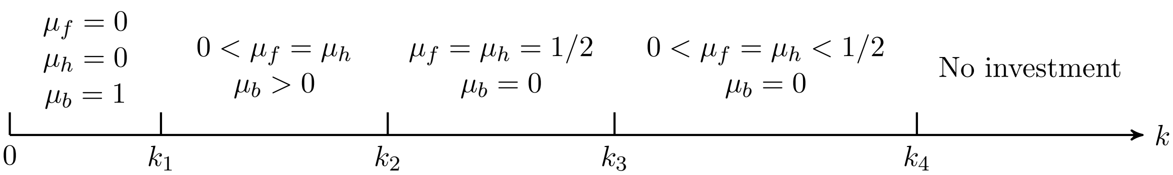


Figure 1:
Supply Chain Outcomes: Symmetric Case

Firms that diversify earn profits in both states H and F, whereas those that maintain a single relationship earn profits only in one of these states. Those that diversify also stand a better chance of surviving in state B, when relationship-specific shocks might disrupt their individual supply chains. Evidently, diversified firms enjoy higher expected operating profits. But they also pay an added fixed cost. The choice of whether to diversify hinges on the size of the fixed cost; for example, $\Pi_b > \Pi_h$ if and only if $k < \delta^F \pi^F \rho + \delta^B \pi^B \rho (1-\rho)$.

Figure 1 depicts the outcome as a function of k. For low enough k, i.e., $k < k_1$, firms view the insurance against supply disruptions as well worth the extra cost and all firms diversify. For the next range of fixed costs between k_1 and k_2, all available strategies are used in equilibrium by positive numbers of firms, with $\mu_h = \mu_f = \mu$ rising in k, and $\mu_b = 1 - 2\mu$ declining in k. All firms in the industry invest in at least one supply relationships. At fixed costs above k_2, it is no longer profitable for any firm to invest in resilience, because the potential profits in the event of a disruption do not justify the extra fixed cost of a second relationship. All firms continue to pursue either strategy h or strategy f (with equal number of each) for $k \in (k_2, k_3)$, whereas the total number of firms that form a relationship declines with k when $k \in (k_3, k_4)$.[5] Finally, at k_4 and above, fixed costs are so high as to render entry unprofitable for all modes of organization.[6]

In Figure 2, we illustrate an equilibrium at E for a typical (symmetric) case with $k \in (k_1, k_2)$. The figure shows μ_h and μ_f on the vertical and horizontal axes, respectively. Since k is in the range where all firms invest in at least one supply relationship, Figure 2 depicts the plane in (μ_f, μ_h, μ_b) space along which $\mu_b = 1 - \mu_f - \mu_h$. We have already argued that, for any value of μ_b, equiprofitability of the onshore and offshore options requires $\mu_h = \mu_f$. Therefore, the 45-degree line shows combinations of μ_h and μ_f such that single relationships at home and abroad are equally profitable.

The downward sloping curve labelled $\Pi_h = \Pi_b$ shows combinations of μ_h and μ_f (with $\mu_b = 1 - \mu_f - \mu_h$) for which investing in a single relationship at home yields the same expected profits as a strategy of diversification. Note that, with $\delta^H = \delta^F$, (11) and (13) yield the equation for this curve,

$$\gamma (1-\gamma) \pi^F \rho + \gamma^2 \pi^B \rho (1-\rho) = k. \tag{14}$$

The downward slope of the curve can be understood as follows. Starting from a point on the

[5] For $k \in (k_2, k_3)$, $\Pi_h = \Pi_f > 0 > \Pi_b$, whereas for $k \in (k_3, k_4)$, $\Pi_h = \Pi_f = 0 > \Pi_b$.

[6] If profits go to infinity as $\boldsymbol{\mu} \to (0,0,0)$, then $k_3 \to \infty$ as well. Entry always is profitable for finite k, for example, if $U(\cdot)$ takes the CES form.

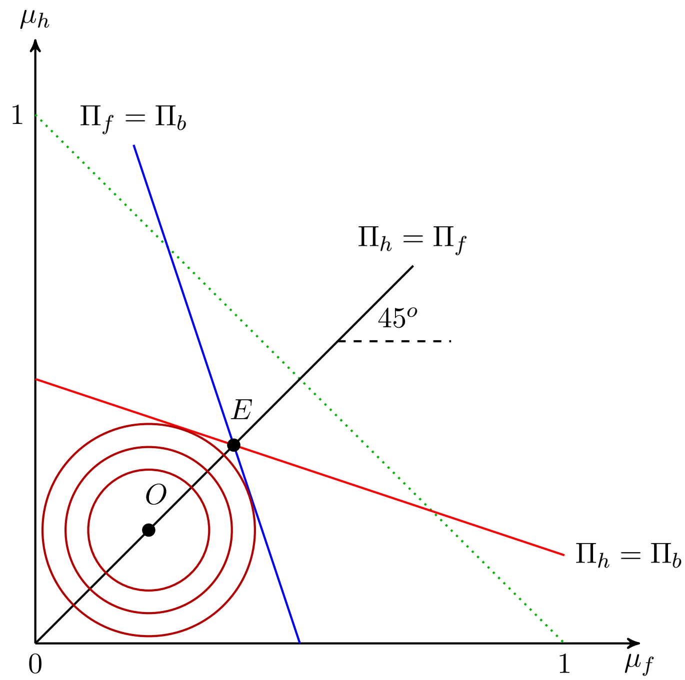


Figure 2:
Equilibrium and Constrained Optimum for Symmetric Case

curve, say E, suppose we move vertically upward. This shift corresponds to a rise in μ_h and a decline in μ_b of similar magnitudes, with μ_f held constant. The change in composition of strategies does not affect the total number of firms active in state H, but it decreases the numbers of firms active in states F and B. Thus, π^F and π^B rise, leaving a (positive) gap between Π_b and Π_h for points vertically above E.[7] Now consider a fall in μ_f accompanied by an offsetting rise in μ_b at a given μ_h, i.e., a horizontal movement to the left. This change in the composition of firms intensifies competition in state H, which contributes to lower expected profits for both h firms and b firms. Since both types earn operating profits of π^H with the same probability ρ in state H, the fall in π^H does not figure in the comparison between the two. Meanwhile, in state F, the offsetting changes in μ_f and μ_b leave the intensity of competition untouched and with it the operating profits π^F for any firm that is active in this state. Finally, in state B, the number of active firms rises, because a given firm is more likely to produce in this state if it is diversified than if it has only a single potential supplier. The intensification of competition in state B reduces π^B, which depresses expected profits more for diversified firms than for those that invest only in a domestic supplier, because the former firms are more likely to survive in this state. Thus, a decrease in μ_f offset by an increase in μ_b reduces Π_b relative to Π_h. It follows that a decrease in μ_f is needed to offset the effects of an increase in μ_h if strategies h and b are to be equally profitable. We note further that

[7]The fall in competition in state F serves to increase expected profits of diversified firms, but does not affect the expected profits of firms that can only source at home. The fall in competition in state B raises the profitability of an active h firm and an active b firm by similar amounts, but the diversified firms have a better chance of avoiding supply disruptions, so they reap a bigger boost to expected profits in this state as well.

the $\Pi_h = \Pi_b$ curve must have a slope less than one in absolute value.[8]

The curve labelled $\Pi_f = \Pi_b$ shows combinations of μ_h and μ_f for which investing in a single relationship abroad yields the same expected profits as a strategy of diversification. It too slopes downward, for analogous reasons, and the curve must have a slope greater than one in absolute value. Finally, the three curves intersect at E, where all three strategies are equally profitable, as befits an equilibrium in which positive numbers of firms select each option.

The figure also illustrates a constrained optimum at O. The constrained optimum does not achieve maximal expected welfare for the home country, because it maximizes W over the choice of μ in the presence of monopoly pricing of differentiated products. In the appendix, we show that the first-order conditions for a constrained maximum are satisfied with an appropriate choice of $\mu_h = \mu_f$ for any HSA preferences. We also show that the welfare function $W(\mu)$ is globally concave when $U(X)$ takes a CES form and that the constrained optimum must have $\mu_h = \mu_f$ when preferences take the symmetric translog form. Evaluating the second-order conditions for more general HSA preferences is challenging, but it seems compelling that the planner would want equal numbers of firms with single relationships at home and abroad. The figure depicts some iso-welfare loci for successively lower levels of expected welfare as we move away from O. Notice that they are symmetric about the 45-degree line, thanks to the symmetry across countries.

3.1 Should the government subsidize diversification?

In the symmetric case, a diversification policy alone can be used to achieve the constrained optimum. Suppose the government provides a subsidy of s_d to firms that add a second supplier (with $s_d < 0$ representing a tax). Such a subsidy has no effect on the profitability of engaging a single supplier, be it at home or abroad. Therefore, the curve that represents equiprofitability of the alternative locations for single suppliers continues to be the 45-degree line from the origin. With a subsidy (or tax) in place, indifference between investing in a single relationship and investing in resilience requires $\Pi_j = \Pi_b + s_d$ for $j = h, f$. Clearly, a subsidy of this sort shifts the curve representing indifference between h and b and that representing indifference between f and b downward, while a tax shifts these curves in the opposite direction. If point E lies above point O on the 45-degree line (as depicted in Figure 2), then a diversification subsidy can be used to achieve the constrained optimum. If point O lies above point E, the government needs to discourage diversification in order to achieve the constrained optimum. In general, either outcome is possible.

In the special case of CES preferences, however, point E always lies above point O, so the optimal policy that achieves the second best is a *subsidy* for diversification. By promoting resilience, the government augments the availability of products in all three states. In states H and F, the number of active firms increases by $(\rho/2)\,\Delta\mu_b > 0$, because $\mu_h + \mu_f + \mu_b = 1$ and $\mu_h = \mu_f = \mu$ imply $\Delta\mu_h = \Delta\mu_f = -\Delta\mu_b/2$. In state B, the number of active firms increases by $\rho(1-\rho)\,\Delta\mu_b > 0$,

[8]The effect on expected profits of a diversified firm relative to a home-only firm conditional on state B are equal and opposite for a given increase in μ_h and comparable decrease in μ_f. But the increase in μ_h (and accompanying decrease in μ_b) gives an added boost to the relative profitability of diversification, because it raises the expected profits for a b firm if state F arises. See the appendix for more details.

because $n^B = (\mu_h + \mu_f + \mu_b)\,\rho + \rho\,(1-\rho)\,\mu_b$ and $\mu_h + \mu_f + \mu_b = 1$. The increase in available products in each state boosts aggregate welfare, because the market undersupplies diversity in the CES case with an outside good.[9]

The gains from promoting resilience in the CES case can best be understood by considering a combination policy of a tax or subsidy on diversification and a hypothetical subsidy to consumption of differentiated products. The latter policy (if feasible) could serve to counteract the distortion caused by monopoly pricing, so that consumers face the marginal cost, q. We show in the appendix that, once the consumption distortion has been eliminated in this way, there is no further need for a policy to promote diversification; indeed, the first-best outcome is achieved by a consumption subsidy alone, with *zero tax or subsidy on supply-chain formation.* Only when a consumption subsidy is infeasible does it become beneficial to promote resilience. With constant markup pricing in one sector and competitive pricing in the other, the consumer surplus of an additional product exceeds the cost of bringing it to market. A diversification subsidy generates second-best gains by creating greater product availability in every state of nature.

What if the elasticity of substitution is not constant but instead rises with price, per Marshall's Second Law of Demand? In the appendix, we prove that the first best can be achieved with a subsidy to consumption and a *tax* on diversification. This finding echoes Matsuyama and Ushchev (2020), who show that, in a setting of monopolistic competition with a single sector, the market oversupplies diversity with HSA preferences when the elasticity of substitution increases with price. The explanation they give is that, with $\sigma'(z) > 0$, the business-stealing effect dominates the consumer-surplus effect, i.e., the fact that firms do not take account of the effect of their entry on the profits of others overrides the fact that they do not take account of the surplus they generate for consumers. The excessive incentive for entry that they describe translates into an excessive incentive for firms to diversify their supply chains in order to boost the likelihood that they can operate in the face of supply disruptions.

In the setting studied by Matsuyama and Ushchev (2020), markup pricing by homogeneous firms does not create any consumption distortion, because relative prices are not affected by firms' uniform markup pricing. Therefore, the powerful incentives for business stealing under MSLD generate socially excessive entry in the monopolistically competitive equilibrium. Had they allowed for a second sector with competitive pricing, their conclusion would have been more nuanced, much as ours is here. Excessive incentives for business stealing on its own suggests too much investment in resilience, but distorted relative prices across industries imply the opposite. Accordingly, the government might need either to promote or to discourage diversification in order to achieve the constrained optimum.

Investigation of the case of symmetric translog preferences affords some additional insight. Recall that, with these preferences, $s(z) = -\theta \log z$. In the appendix we show that, in the translog

[9] See the proof in the appendix. For an analogous result in a world without supply shocks, see, for example, Campolmi, Fadinger and Forlati (2021).

case, point E must lie above point O if

$$\varepsilon > \frac{\theta n^B (1+\theta n^B)}{1+3\theta n^B}, \tag{15}$$

where n^B is the number of active firms in state B in the competitive equilibrium. In such circumstances, the market undersupplies resilience. Since the right-hand side of (15) is increasing in n^B, which in turn is decreasing in k, it takes its maximum value for $k \in [k_1, k_2]$ at $k = k_1$ where $\mu = 0$ and $n^B = \rho\,(2-\rho)$. Accordingly, a diversification *subsidy* is indicated when the elasticity of demand for differentiated products and the risk of idiosyncratic supply disturbances are relatively large.[10] We also show, however, that point E must lie below point O when

$$\varepsilon < \frac{\theta n(1+\theta n)}{1+3\theta n}\ , \tag{16}$$

where n is the number of active firms in state H or in state F of the competitive equilibrium. This number also decreases with k, so the right-hand side takes its minimum value at $k = k_2$, where $\mu = 1/2$ and $n = \rho/2$. Therefore, a *tax* on diversification is optimal when the demand elasticity and the risk of relationship-specific supply shocks are relatively small.[11,12]

The central importance of the demand elasticity ε should be clear. When demand is elastic, the consumption distortion arising from markup pricing looms large, and the market's tendency to undersupply products in the face of supply disruptions weighs heavily compared to the excessive incentives for investment in resilience from the business-stealing effect. Then the optimal diversification policy with symmetric translog preferences is a subsidy, much as with CES preferences. In contrast, when demand for differentiated products is inelastic, the welfare effects of the consumption distortion are muted and firms' eagerness to steal profits from others in disrupted states of the world leads to excessive investment in resilience.

3.2 Should the government subsidize reshoring or offshoring?

Let s_r denote a subsidy (or tax, if negative) payable to any firm that invests in a supply relationship at home. Similarly, let s_o denote a subsidy payable to a firm that invests in a supply relationship abroad. In the presence of these policies that influence sourcing decisions, indifference between the available strategies requires $\Pi_h + s_r = \Pi_f + s_o$, $\Pi_h = \Pi_b + s_o$, and $\Pi_f = \Pi_b + s_r$.

Consider first a reshoring subsidy on its own. With $s_o = 0$, the policy does not affect the relative attractiveness of a single supplier at home versus diversification; the equilibrium continues to fall

[10] Proposition 1 in the appendix provides a sufficient condition for the constrained optimality of a diversification subsidy with translog preferences, namely (i) $\nu < 1+\sqrt{2}$ or (ii) $\nu > 1+\sqrt{2}$ and $\varepsilon > \theta\nu\,(1+\nu)\,/\,(1+3\nu)$, where $\nu \equiv \theta\rho\,(2-\rho)$.

[11] Proposition 1 in the appendix provides a sufficient condition for the constrained optimality of a diversification tax with translog preferences, namely $\varepsilon < \rho\theta(2+\rho\theta)/\,(4+6\rho\theta)$.

[12] In the two panels of Figure 7 in the appendix, we depict the equilibrium and the constrained optimum for alternative parameter values that satisfy (15) and (16), respectively, illustrating thereby that a diversifciation subsidy or a diversification tax might be needed to achieve the constrained optimum.

along the original $\Pi_h = \Pi_b$ schedule. Meanwhile, a subsidy favors both inshoring and diversification relative to forming a single relationship abroad. A subsidy shifts both of the remaining equiprofitability curves so that they intersect at a lower point than E along $\Pi_h = \Pi_b$. A tax shifts the curves in the opposite direction. The optimal reshoring policy is the one that induces a tangency between the iso-welfare curve and the $\Pi_h = \Pi_b$ schedule.

Now recall that $\Pi_h = \Pi_b$ has a slope less than minus one in absolute value, whereas the iso-welfare curves around O are symmetric and cross the 45-degree line with a slope of minus one. It follows that a subsidy to reshoring on its own raises expected welfare whenever point E lies above point O and a tax on reshoring raises expected welfare when point O lies above point E. Evidently, the sign of the optimal reshoring policy mirrors that of the optimal diversification policy.

The optimal reshoring policy cannot, however, be used to achieve the constrained optimum at O. Rather, such a policy distorts the allocation of supply chains between the (equally costly and equally risky) home and foreign countries. This policy must, therefore, be inferior to the second-best diversification tax or subsidy, which avoids such a distortion.

Notice that a subsidy or tax for offshoring offers exactly the same potential efficiency gains as does a policy for reshoring. A subsidy for offshoring—like one for reshoring—raises the expected number of firms that are active conditional on the occurrence of a single, country-wide supply disturbance. A subsidy also expands the number of firms that operate in state B. The increase in the expected number of active firms creates surplus gains for consumers that must be weighed against the profit losses for firms and the cost of the subsidy. If the gains in consumer surplus exceed the loss of profits and revenue, a subsidy to either offshore supply relationships or onshore supply relations offers third-best welfare improvements; if the losses exceed the gains, then a tax on these relationships is indicated. Clearly, it is not the location of the supplier that is germane for the welfare gain or loss, but rather the fact that the policies enhance or diminish the incentives for investing in resilience. Indeed, a combination of a reshoring subsidy and an offshoring subsidy at equal rates can duplicate the beneficial effects of a subsidy for diversification and thereby achieve the constrained optimum.

Again, the CES is a special case in which the equilibrium E lies above and to the right of the constrained optimum at O. In this case, a subsidy for domestic supply relationships raises welfare, as does a subsidy for offshore relationships. A combination of the two does better than either alone.

4 The Asymmetric Case

Now we turn to an environment in which costs and risks are not the same in the two countries. We examine the seemingly pertinent case with $\gamma_H > \gamma_F$ and $q_H > q_F$; i.e., the foreign country offers lower costs but poses greater risks of supply disruptions.

In the asymmetric case, diversified firms elect to source offshore whenever they can do so, while relying on their higher-cost backup option only when they must. Thus, these firms produce at a cost of q_H in state H with probability ρ and at cost q_F in state F also with probability ρ, much as do firms with sole suppliers in country H or in country F, respectively. In state B, the diversified firms produce at cost q_F with probability ρ and at cost q_H with probability $\rho(1-\rho)$.

We denote by $z^{B,i}$ the relative price charged by a firm in state B that sources from country i, $i = H, F$. Market shares must sum to one in every state, which gives (9) as before for $J \in \{H, F\}$. In state B, we have

$$1 = n^{B,H}(\mu)\, s\left(z^{B,H}\right) + n^{B,F}(\mu)\, s\left(z^{B,F}\right) \ , \tag{17}$$

where $n^{B,i}$ is the number of active firms in state B that source from country i and so $n^{B,F} = (1-\mu_h)\rho$ and $n^{B,H} = \mu_h\rho + \mu_b\rho(1-\rho) = (1-\mu_f)\rho(1-\rho) + \mu_h\rho^2$. The pricing equation (6) implies

$$\frac{z^{B,H}}{z^{B,F}} = \frac{\frac{\sigma\left(z^{B,H}\right)}{\sigma\left(z^{B,H}\right)-1} q_H}{\frac{\sigma\left(z^{B,F}\right)}{\sigma\left(z^{B,F}\right)-1} q_F} \ . \tag{18}$$

Now we can use (9) for $J = H, F$ together with (17) and (18) to solve for the four relative-price terms as functions of μ.

Operating profits for firms that do not suffer supply disruptions are given by

$$\pi\left(z^J\right) = \frac{s\left(z^J\right)}{\sigma\left(z^J\right)} P\left(z^J\right)^{1-\varepsilon}, \text{ for } J = H, F,$$

and

$$\pi^{B,i} = \pi^{B,i}\left(z^{B,i}; \boldsymbol{n^B}, \boldsymbol{z^B}\right) = \frac{s\left(z^{B,i}\right)}{\sigma\left(z^{B,i}\right)} P\left(\boldsymbol{n^B}, \boldsymbol{z^B}\right)^{1-\varepsilon}, \ i = H, F,$$

where $\boldsymbol{n^B} = \left(n^{B,H}, n^{B,F}\right)$ and $\boldsymbol{z^B} = \left(z^{B,H}, z^{B,F}\right)$. Finally, we calculate the expected profits associated with each strategy,

$$\Pi_h = \delta^H \pi\left(z^H\right)\rho + \delta^B \pi^{B,H}\left(z^{B,H}\right)\rho - k \ ,$$
$$\Pi_f = \delta^F \pi\left(z^F\right)\rho + \delta^B \pi^{B,F}\left(z^{B,F}\right)\rho - k \ ,$$

and

$$\Pi_b = \delta^H \pi\left(z^H\right)\rho + \delta^F \pi\left(z^F\right)\rho + \delta^B \pi^{B,F}\left(z^{B,F}\right)\rho + \delta^B \pi^{B,H}\left(z^{B,H}\right)\rho(1-\rho) - 2k \ ,$$

with $\delta^H = \gamma_H(1-\gamma_F)$, $\delta^F = \gamma_F(1-\gamma_H)$, and $\delta^B = \gamma_H\gamma_F$. These expected profit levels are all functions of $\boldsymbol{\mu}$.

Figure 3 shows the fraction of firms that choose each organizational form for different values of k. For $k < k_1$, all firms find it worthwhile to invest in resilience, so $\mu_b = 1$ and $\mu_h = \mu_f = 0$. Next comes a range of fixed cost for which some firms are diversified, and others form relationships in a single country, with all sole-source suppliers located either in country H or in country F. The figure depicts a case with $q_H \approx q_F$ and $\gamma_H > \gamma_F$. In such circumstances, there exists a k_2 such that for $k \in (k_1, k_2)$, $\mu_h = 1 - \mu_b > 0$ and $\mu_f = 0$. Alternatively if $q_F < q_H$ and $\gamma_H \approx \gamma_F$, then there

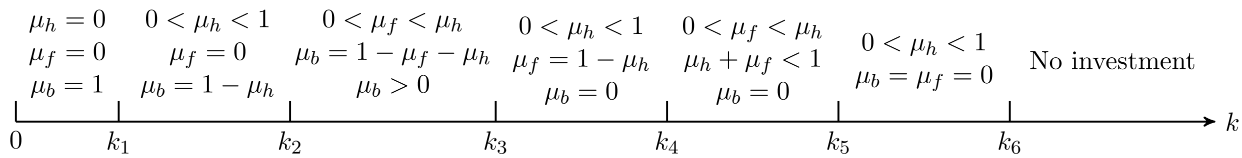


Figure 3:
Supply Chain Outcomes: Asymmetric Case

exists a k_2 such that for $k \in (k_1, k_2)$, $\mu_f = 1 - \mu_b > 0$ and $\mu_h = 0$.[13] Next, there exists a range of fixed costs $k \in (k_2, k_3)$ for which each of the three strategies is deployed in equilibrium by some positive number of firms. Then, as k rises further, diversification becomes unprofitable for all firms ($\mu_b = 0$), then sole sourcing becomes unprofitable in one of the two countries, and finally entry of any sort becomes unprofitable.[14]

Figure 4a illustrates an equilibrium for some $k \in (k_2, k_3)$, i.e., fixed costs in an intermediate range such that all three organizational forms are present in equilibrium. As before, the curve $\Pi_h = \Pi_f$ depicts combinations of μ_h and μ_f (with $\mu_b = 1 - \mu_h - \mu_f$) that deliver equal expected profits to firms that form supply chains only in H or only in F. The curves labeled $\Pi_h = \Pi_b$ and $\Pi_f = \Pi_b$, respectively, show combinations of μ_h and μ_f such that investing in a single relationship in H or in F yields the same expected profits as investing in both. With all strategies in use, the equilibrium E lies at the intersection of the three curves.

Aggregate welfare comprises the sum of expected profits and expected consumer surplus across the three states of nature, as before. The expression for $W(\mu)$ remains as in (8). Figure 4a depicts some iso-welfare curves around the constrained optimum at O.

We observe first that, for general HSA preferences, the constrained optimum, O, need not fall on any of the three equiprofitability curves. This means that, generically, the government requires two policy instruments to achieve the second best. Again, a combination of a diversification policy, s_d, and a reshoring policy, s_r, are sufficient to achieve the constrained optimum, because when these two policy instruments are available, the offshoring policy s_o becomes redundant.[15]

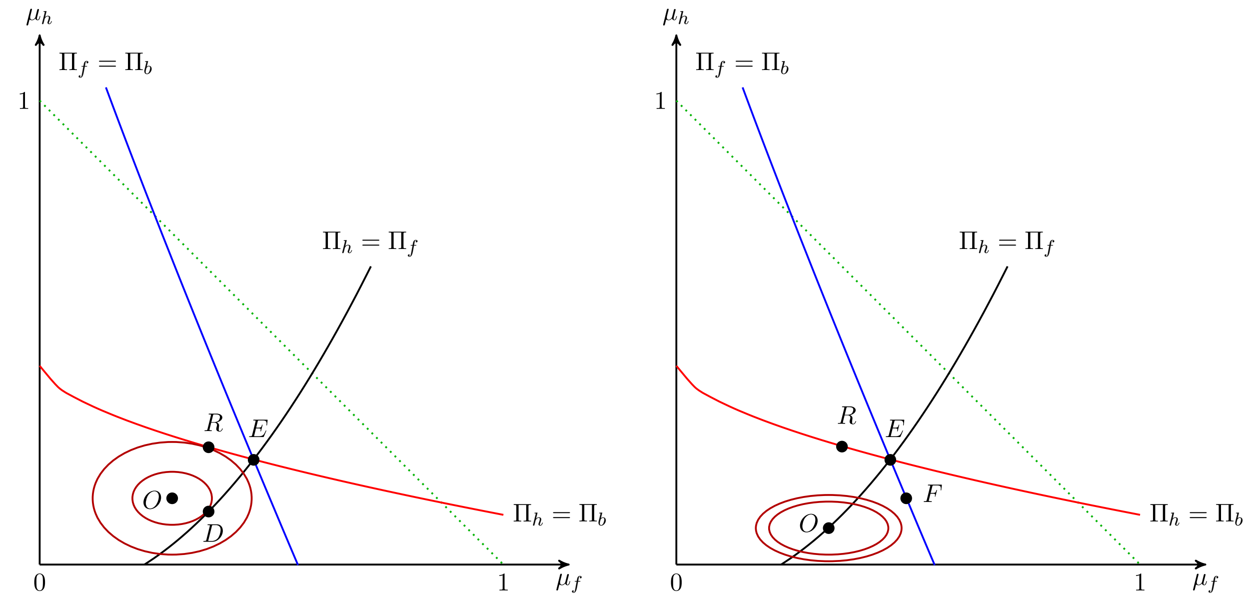
For the case depicted in Figure 4a, the constrained optimum lies to the southwest of point E. A subsidy to diversification shifts the $\Pi_h = \Pi_b$ and $\Pi_f = \Pi_b$ curves toward the origin such that the intersection remains on the original $\Pi_h = \Pi_f$ curve. At best, this can achieve the outcome at D, where $\Pi_h = \Pi_f$ is tangent to an iso-welfare locus. Further gains can be attained by a reshoring policy that shifts the $\Pi_h = \Pi_b$ so that it passes through O.

In the figure, we also label the point R, which is the best that can be achieved with a reshoring policy on its own. For the case illustrated, it is beneficial for the government to promote reshoring (with a subsidy) if it cannot enact a diversification policy. In the case illustrated, the optimal

[13]If $q_F < q_H$ and $\gamma_H > \gamma_F$, then we might have a range of fixed costs for which $\mu_H > 0 = \mu_F$ or with $\mu_F > 0 = \mu_H$, depending on whether the prospective cost savings in the foreign country outweigh the relatively greater reliability of the home suppliers.

[14]For $k \in (k_3, k_4)$, all firms form a single relationship either in H or in F, so that $\mu_h + \mu_f = 1$, whereas for $k \in (k_4, k_5)$ some firms choose not to invest at all, so that $\mu_h + \mu_f < 1$.

[15]As before, a policy equilibrium has $\Pi_h + s_r = \Pi_f + s_o$, $\Pi_h = \Pi_b + s_d + s_o$, and $\Pi_f = \Pi_b + s_d + s_r$.



(a) General HSA Preferences

(b) CES Preferences

Figure 4:
Equilibrium and Constrained Optimum for Asymmetric Case

reshoring policy does not yield as high a level of expected welfare as the optimal diversification policy.

Let us turn to the special case of CES preferences, as depicted in Figure 4b. In this case, consumer surplus in any state of nature J is proportional to the price index, P^J, raised to the power of $1-\varepsilon$. Meanwhile, profits in state J of any active firm are proportional to the same term, $\left(P^J\right)^{1-\varepsilon}$. We show in the appendix that—due to this exceptional feature of the CES, which reflects the dual role of the price index as both single aggregator and welfare metric—the constrained optimum satisfies $\Pi_h = \Pi_f$. In other words, the point O must lie on the $\Pi_h = \Pi_f$ curve, much like the equilibrium point E.

The fact that the constrained optimum falls on the $\Pi_h = \Pi_f$ curve means that the second best can be achieved with a single policy instrument, namely a tax or subsidy for diversification. A subsidy shifts the equilibrium downward from E, whereas a tax shifts the equilibrium upward. However, as we also show in the appendix, point O always lies *below* point E on the $\Pi_h = \Pi_f$ curve, so, with CES preferences, it is always desirable for the government to promote resiliency with a subsidy for all values of the cost and risk parameters. The explanation is the same as in the symmetric case; in the CES case, the market underprovides resiliency, because the consumer surplus plus profit gains from greater product availability in a given state exceeds the fixed cost of another link in the supply chain. This conclusion reflects the fixed and positive wedge that exists with constant-markup pricing between consumer prices and marginal costs.

In Figure 4b we also see that either a subsidy for supply relationships at home or a subsidy for supply relationships abroad can be used to raise expected welfare relative to the equilibrium at E, although neither of these policies can achieve the second-best level of welfare that is attainable with the optimal diversification subsidy.[16] The optimal reshoring subsidy shifts the equilibrium to point R, whereas the optimal offshoring subsidy shifts the equilibrium to point F. These outcomes cannot be uniquely ranked without further information about relative costs and risks.

Moving beyond CES, some results from the symmetric case generalize. For example, if consumers have symmetric translog preferences and the costs in the two countries do not differ by too much, then the optimal diversification policy promotes resilience when ε is large but discourages resilience for ε is small. But, with asymmetric costs and risks, HSA preferences admit a richer set of possibilities. For example, any of the three policy instruments (diversification tax or subsidy, reshoring tax or subsidy or offshoring tax or subsidy) might offer the greatest welfare improvement if the government can only implement one such policy. General results are difficult to come by, so we rely instead on numerical analysis to explore some of the possibilities. We highlight in particular how policy recommendations might reflect differences in the likelihood of supply chain disruptions at home and abroad.

Figure 5 depicts a reasonably typical case with symmetric translog preferences.[17] We take the translog parameter $\theta = 3.5$, the relationship-specific probability of supply disruption $1 - \rho = 0.1$, the elasticity of demand for differentiated products $\varepsilon = 1.4$, and the respective unit costs of the input $q_H = 0.1$ and $q_F = 0.08$, so that the cost advantage of the foreign country is 20%. The other parameters were chosen to ensure that all three strategies are used by some firms in equilibrium.[18]

The left-hand panel in the figure shows the fractions of firms that adopt each strategy (solid curves) and the constrained-optimal fractions (dashed curves) for each mode of operation for various degrees of risk asymmetry. The figure shows that too many firms establish a single relationship in the safer home market compared to the constrained optimum. This finding is very common in our numerical analysis. The excessive home sourcing often (but not always) comes at the expense of too little diversification, with single supplier relationships abroad also above the efficient level.

Not surprisingly, as the risk of foreign supply disruptions falls, both the equilibrium and optimal number of firms that invest in a single relationship at home declines. Meanwhile, the number of firms that invest in some manner in a supply relationship abroad grows. These new relationships might take the form of more single relationships abroad (μ_f increases) or more diversified firms (μ_b increases), or both. The figure depicts a case in which both μ_f and μ_b rise with γ_F, but it is not difficult to find reasonable parameter values for which one or the other falls.

The middle panel shows the size of the optimal subsidies for diversification, reshoring, and offshoring, as functions of relative risk. With $\varepsilon = 1.4$, the market accepts too much risk of disruption compared to the constrained optimum. Therefore, all three optimal subsidies are positive. The

[16]The iso-welfare curves have a slope of minus one where they cross the $\Pi_h = \Pi_f$ curves, just as in the symmetric case. And the $\Pi_h = \Pi_b$ and $\Pi_f = \Pi_b$ have slopes at less than and greater than one in absolute value, respectively, where they cross $\Pi_h = \Pi_f$, as drawn; see the appendix for the proof. This implies that a subsidy to reshoring or offshoring raises welfare, because they move the equilbrium to the left along the original $\Pi_h = \Pi_b$ curve or to the right along the original $\Pi_f = \Pi_b$ curve, respetively, in each case shifting onto an iso-welfare contour representing a higher level of welfare.

[17]A range of other numerical examples are provided in Figures 8-11 in the appendix.

[18]More specifically, the figure assumes $\gamma_H = 0.95, k = 0.34$ and $C_P = 0$.

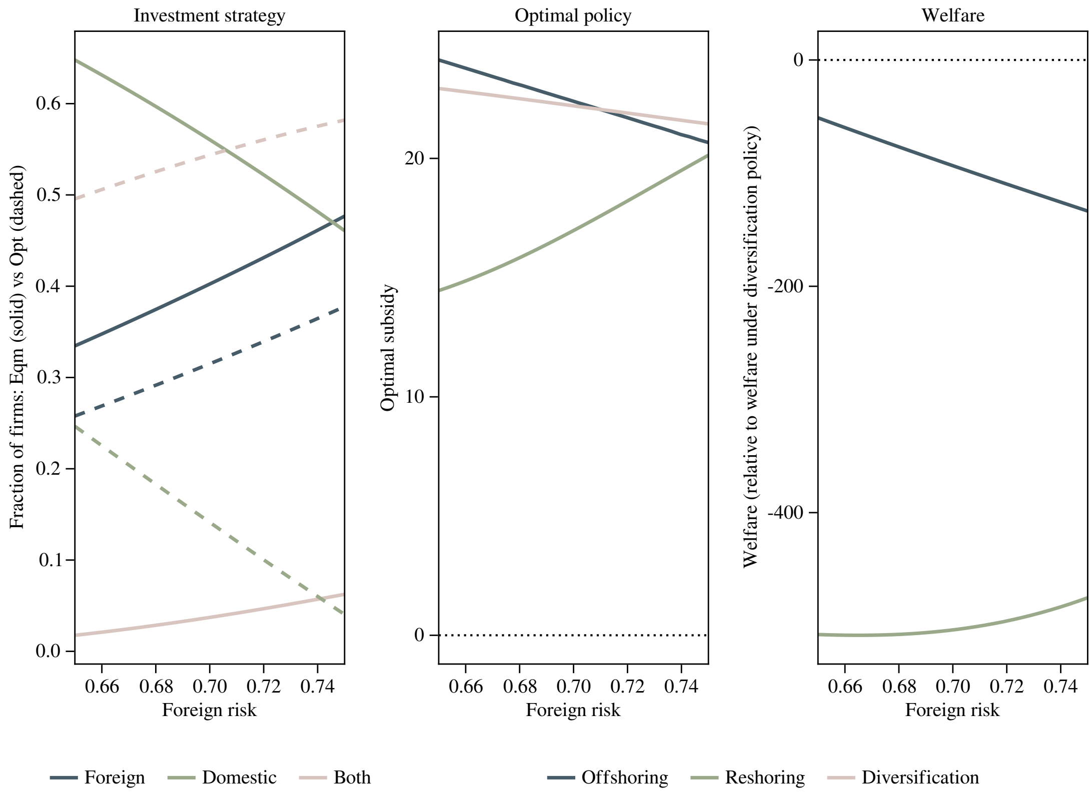


Figure 5:
Optimal Policy and Comparative Statics with respect to Risk

Note: the parameter values are $\varepsilon = 1.4$, $q_f = 0.08$, $q_h = 0.1$, $\gamma_h = 0.95$, $\rho = 0.9$, $\theta = 3.5$, $k = 0.34$ *and* $C_P = 0$.

optimal diversification subsidy rises as the foreign country becomes safer, as does the optimal offshoring subsidy.[19]

Finally, the right-hand panel shows the welfare that can be achieved by the optimal reshoring policy and the optimal offshoring policy, compared to what can be achieved by the optimal diversification policy. When the demand for differentiated products is reasonably elastic, we typically find that a diversification policy dominates any policy directed at supply chains in one country or the other. Moreover, we find that it is optimal for the government to encourage resilience with a subsidy for diversification. For the parameter values that underlie Figure 5, the optimal diversification subsidy yields at least 50 basis points higher welfare than the optimal subsidy for offshore investment, and more than 450 basis points higher welfare than the optimal subsidy for onshore investment. For other parameter values, this advantage can be even larger.

Figure 6 depicts a situation with a much smaller difference in costs (only 1%) and a much smaller elasticity of demand for differentiated products ($\varepsilon = 1.01$).[20] Here, the number of firms

[19] However, the optimal offshoring falls as the difference in riskiness rises when the cost difference between home and foreign is only 10%; see Figure 8 in the appendix.

[20] The parameters that underlie this figure are: $q_F = 0.999, q_H = 0.1$, $\gamma_H = 0.9$, $\rho = 0.9$, $\theta = 3.5.$, $k = 0.05$ and $C_P = 0$.

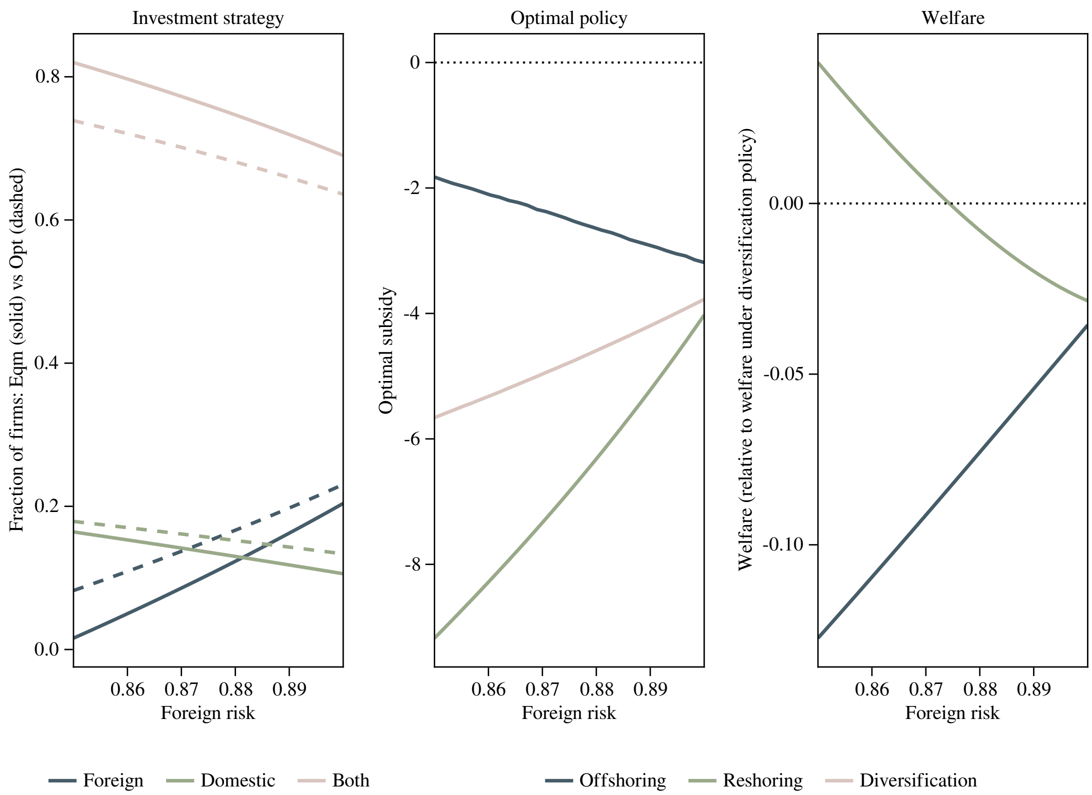


Figure 6:
Optimal Policy and Comparative Statics with respect to Risk: Small Cost Difference and $\varepsilon = 1.01$

Note: the parameter values are $\varepsilon = 1.01$, $q_f = 0.099$, $q_h = 0.1$, $\gamma_h = 0.9$, $\rho = 0.9$, $\theta = 3.5$, $k = 0.05$ *and* $C_P = 0$.

that diversify in equilibrium exceeds the socially optimal level, at the expense of firms that form single relationships both at home and abroad. Overall, the equilibrium provides excessive resilience, so it is optimal to tax diversification (see middle panel) or, if that is not possible, to discourage supply chain formation at home or abroad. The right-hand panel shows that a diversification tax dominates a tax on offshoring, whereas the relative ranking of the optimal diversification tax and the optimal reshoring tax depends on the size of the risk gap. However, for all values of γ_F, the difference in welfare is quite narrow, amounting to only a small fraction of one basis point. The tiny welfare effects are quite typical when unit costs are similar and ε differs little from one.

5 Conclusions

Supply chain disruptions are increasingly salient and often quite costly. Many commentators have been quick to conclude that governments ought to be doing something to promote greater market resilience. But the welfare-theoretic calculus around government intervention is rather subtle. Private actors have a clear self-interest in taking measures to avoid disruptions to their production processes. Only when the private incentives for resilience fall short of the social benefits will government encouragement be warranted. Pointing in that direction is the observation that consumers capture part of the surplus created by the ongoing availability of firms' products. But firms also have an incentive to be in a position to reap extra profits when their rivals are suffering. The temptation for "business stealing" suggests that excess resilience is also a possible market outcome.

Surprisingly little research has addressed the desirability of government policy to promote resilience or to encourage sourcing from safer locations. In this paper, we have taken a first step. We have proposed a simple framework in which the supply of any product requires the availability of a critical input. Exogenous shocks can disrupt the firms' relationships with their suppliers. We allow for idiosyncratic shocks that affect a single relationship and broader shocks that impinge on all sourcing from a particular country or region. Firms face the choice of where to develop a relationship and whether to protect their operation with backup sources of supply. We study the simplest case of two potential supply sources, one at home and one abroad and focus on a situation where domestic sourcing is (weakly) costlier than sourcing abroad, but also (weakly) less risky. This setting presents firms with three options: invest in a single supply relationship at home (onshoring), invest in a single relationship abroad (offshoring), or invest in supply relationships in both locations (diversification).

Since consumer gains from product availability reflect their preferences, the form of demand plays a critical role in the policy calculus. The CES demand system is popular and tractable for analysis such as ours. But it also introduces restrictions that may color the findings. We allow for a CES utility function, but also for a broader class of preferences that Matsuyama and Ushchev (2020) have developed and termed Homothetic with a Single Aggregator. The more general preferences admit non-constant markups and, in particular, application of Marshall's Second Law of Demand.

Our analysis yields several broad lessons. First, the government generally needs at least two policy instruments to achieve efficient sourcing, once allowance is made for the existence of markup pricing. One instrument regulates the margin between sourcing from one location or two. The other guides the choice between sourcing at home and abroad. For example, the government might subsidize or tax supply-chain diversification, while subsidizing or taxing onshoring. Or it might subsidize or tax diversification, while subsidizing or taxing offshoring. However, when preferences take the CES form, one instrument suffices to achieve the constrained optimum, namely a policy to promote diversification. In the CES case, even when the home and foreign sources bear different costs and risks, the private and social incentives to source at home and abroad coincide once the correct incentives for diversification have been created.

Second, when the government is limited to use only one policy instrument, either a policy that encourages or discourages diversification or one that alters the incentives for relationships in one country or the other might achieve a superior outcome. And the second best policy might be a subsidy or a tax. In the CES case, a diversification policy dominates a reshoring policy or an offshoring policy, and a subsidy (as opposed to a tax) for diversification is indicated. More generally, a high elasticity of demand for differentiated products tends to favor diversification subsidies whereas a low elasticity of demand for this group of products opens the possibility that a tax will be optimal. This finding reflects the fact that markup pricing creates an undersupply of product diversity in the market equilibrium that is especially pronounced when demand for differentiated products is highly elastic. In contrast, when demand is not so elastic, the private gains from potential business stealing may lead firms to overinvest in resilience.

Our paper opens the door to further research. For example, we have considered only the simplest possible production process whereby each firm produces a final product from a single critical input. Our framework could be extended to allow for more complex supply chains, including multiple purchased inputs from various sources that might also be combined with primary factors of production. We could also examine a sequential production process whereby some inputs enter into production upstream from others. Then we could ask whether the need for resilience or for safe sources of supply is more important for upstream inputs or downstream inputs and how private and social incentives differ at various stages. Further, we could introduce possibilities for storage in a multi-period model, so that firms might invest in resilience by stockpiling inputs and the government might promote availability by holding reserves of consumer goods. Issues of supply chain disruption are bound to be with us for a while and many important policy questions remain to be addressed.

References

[1] Arad, Ruth W. and Arye L. Hillman, 1979. "Embargo Threat, Learning and Departure from Comparative Advantage," *Journal of International Economics 9*, 265-275.

[2] Bergström, Clas, Loury, Glenn C. and Mats Persson, 1985. "Embargo Threats and the Management of Emergency Reserves, *Journal of Political Economy 93(1)*, 26-42.

[3] Bhagwati, Jagdish N. and T.N. Srinivasan, 1976. "Optimal Trade Policy and Compensation under Endogenous Uncertainty: The Phenomenon of Market Disruption," *Journal of International Economics 6*, 317-336.

[4] Campolmi, Alessia, Fadinger, Harald, and Chiata Forlati, 2021. "Trade and Domestic Policies in Models with Monopolistic Competition," http://fadinger.vwl.uni-mannheim.de/Research_files/CFF_20210722.pdf.

[5] Carvalho, Vasco M., Nirei, Makoto, Saito, Yukiko U. and Alifrez Tahbaz-Salehi, 2021. "Supply Chain Disruptions: Evidence from the Great East Japan Earthquake," *Quarterly Journal of Economics 136(2)*: 1255-1321.

[6] Cheng, Leonard K., 1989. "Intermittent Trade Disruptions and Optimal Production," *International Economic Review 30(4)*: 753-74.

[7] Dhingra, Swati and John Morrow, 2019. "Monopolistic Competition and Optimum Product Diversity under Firm Heterogeneity," *Journal of Political Economy 127(1)*, 196-232.

[8] Diewert, W.E., 1974. "Applications of Duality Theory," in M. Intriligator and D. Kendrick (eds.), *Frontiers of Quantitative Economics, vol. II*, Amsterdam: North Holland.

[9] Dixit, Avinash K. and Joseph E. Stiglitz, 1977. "Monopolistic Competition and Optimum Product Diversity," *American Economic Review 67(3)*: 297-308.

[10] Feenstra, Robert C., 2003. "A Homothetic Utility Function for Monopolistic Competition Models, without Constant Price Elasticity," *Economic Letters 78(1)*: 79-86.

[11] Iakovou, Eleftherios and Chelsea C. White III, 2020. "How to Build More Secure, Resilient, Next-Gen U.S. Supply Chains," *TechStream,* Brookings Institution.

[12] Matsuyama, Kiminori and Philip Ushchev, 2020. "When Does Procompetitive Entry Imply Excessive Entry," CEPR Discussion Paper No. 14991.

[13] Mayer, Wolfgang, 1977. "The National Defense Tariff Argument Reconsidered," *Journal of International Economics 7*, 363-77.

[14] McKinsey Global Institute, 2020. *Risk, Resilience and Rebalancing in Global Value Chains.*

[15] Melitz, Marc J., 2018. "Competitive Effects of Trade: Theory and Measurement," *Review of World Economics 154,* 1-13.

[16] Mrázová, Monika and J. Peter Neary, 2017. "Not So Demanding: Demand Structure and Firm Behavior," *American Economic Review 107(12)*: 3835-74.

[17] Shih, Willy C., 2020a. "Global Supply Chains in a Post-Pandemic World," *Harvard Business Review 98(5)*: 82-9.

[18] Shih, Willy C., 2020b. "Is it Time to Rethink Globalized Supply Chains," *MIT Sloan Management Review 61(4)*: 16-18.

[19] Zhelobodko, Evgeny. Kokovin, Sergey, Parenti, Mathieu and Jacques-François Thisse, 2012. "Monopolistic Competition: Beyond the Elasticity of Substitution," *Econometrica 80(6)*: 2765-2784.

Appendix

A Derivations

A.1 A Simple Model of Risky Supply Chains

We begin by deriving expected profits $\Pi_j(\boldsymbol{\mu},\boldsymbol{q})$, $j=f,h,b$, where $\boldsymbol{\mu}:=(\mu_h,\mu_f,\mu_b)$ and $\boldsymbol{q}:=(q_H,q_F)$. To this end recall the profit function (7),

$$\pi(\omega)=\frac{s(p(\omega)/A)}{\sigma(p(\omega)/A)}P^{1-\varepsilon}.$$

For a firm that adopts strategy j these profits depend on the state J, $J\in\{H,F,B\}$, the costs q_H and q_F, and the fraction of firms that adopt each one of the three available strategies, i.e., the vector $\boldsymbol{\mu}$. Here is a characterization of these relationships.

First consider a state $J\in\{H,F\}$, in which supply chains from exactly one country are not disrupted. In this state, only firms that invested in either supply chains from country $j=J$ only, μ_j, or supply chains from both countries, μ_b, can produce, as long as their bilateral relations do not break down, and every one of these firms pay q_J for the input. In this case (4) implies

$$1=n^J(\boldsymbol{\mu})\,s[z^J(\boldsymbol{\mu})],\qquad J\in\{H,F\}\ , \tag{19}$$

where

$$n^J(\boldsymbol{\mu})=(\mu_j+\mu_b)\,\rho, \tag{20}$$

and $z^J=p^J/A^J$. These equations yield relative prices z^J as functions of $\boldsymbol{\mu}$, $z^J(\boldsymbol{\mu})$.

Next, note that, in this state, the price index (5) can be expressed as

$$\log P^J=C_P+\log\frac{p^J}{z^J}-n^J\int_{z^J}^{\bar{z}}\frac{s(\zeta)}{\zeta}d\zeta,$$

where from (6),

$$p^J=\frac{\sigma\left(z^J\right)}{\sigma\left(z^J\right)-1}q_J.$$

Using the function $z^J(\boldsymbol{\mu})$, we can now express the price index P^J as a function of $z^J(\boldsymbol{\mu})$ and q_J,

$$\log P\left[z^J(\boldsymbol{\mu}),q_J\right]:=C_P+\log\frac{\sigma\left[z^J(\boldsymbol{\mu})\right]}{\sigma\left[z^J(\boldsymbol{\mu})\right]-1}+\log\frac{q_J}{z^J(\boldsymbol{\mu})}-\frac{1}{s\left[z^J(\boldsymbol{\mu})\right]}\int_{z^J(\boldsymbol{\mu})}^{\bar{z}}\frac{s(\zeta)}{\zeta}d\zeta. \tag{21}$$

This function, together with $z^J(\boldsymbol{\mu})$, can be used to express the profits of an active firm in state J

as

$$\pi\left[z^J(\boldsymbol{\mu}),q_J\right]:=\frac{s\left[z^J(\boldsymbol{\mu})\right]}{\sigma\left[z^J(\boldsymbol{\mu})\right]}P\left[z^J(\boldsymbol{\mu}),q_J\right]^{1-\varepsilon},\quad J\in\{H,F\}. \tag{22}$$

The functions $P(z,q)$ and $\pi(z,q)$, defined in (21) and (22), are decreasing in z. These are useful properties for future reference. The proof proceeds as follows. Differentiating $\log P(z,q)$ with respect to z yields[21]

$$\frac{P_z(z,q)}{P(z,q)}=-\frac{\sigma'(z)}{\sigma(z)\left[\sigma(z)-1\right]}+\frac{s'(z)}{s(z)^2}\int_z^{\bar z}\frac{s(\zeta)}{\zeta}d\zeta<0,$$

where $P_z(z,q)$ is the derivative of $P(z,q)$ with respect to z. Next, differentiating $\log\pi(z,q)$ with respect to z retuns

$$\frac{\pi_z(z,q)}{\pi(z,q)}=-\frac{\sigma'(z)}{\sigma(z)}\frac{\sigma(z)-\varepsilon}{\sigma(z)-1}+\frac{s'(z)}{s(z)}\left[1-\frac{\varepsilon-1}{s(z)}\int_z^{\bar z}\frac{s(\zeta)}{\zeta}d\zeta\right].$$

For $\varepsilon<1$, this directly implies $\pi_z(z,q)<0$. Next consider $\varepsilon>1$. Equation (3) in the main text implies (see Matsuyama and Ushchev (2020)):

$$\frac{s(\zeta)}{\zeta}=\frac{s'(\zeta)}{1-\sigma(\zeta)}.$$

Therefore

$$\int_z^{\bar z}\frac{s(\zeta)}{\zeta}d\zeta=\int_z^{\bar z}\frac{-s'(\zeta)}{\sigma(\zeta)-1}d\zeta<\int_z^{\bar z}\frac{-s'(\zeta)}{\sigma(z)-1}d\zeta=\frac{s(z)-s(\bar z)}{\sigma(z)-1}=\frac{s(z)}{\sigma(z)-1}. \tag{23}$$

Using this inequality we therefore obtain

$$\frac{\pi_z(z,q)}{\pi(z,q)}<\left[\frac{s'(z)}{s(z)}-\frac{\sigma'(z)}{\sigma(z)}\right]\frac{\sigma(z)-\varepsilon}{\sigma(z)-1}<0.$$

which we summarize in the following lemma.

Lemma 1 *The functions $P(z,q)$ and $\pi(z,q)$ are declining in z.*

In state B, where supply chains from both countries are available, every firm wants to source from the cheaper country F whenever possible (recall that $q_F<q_H$). In this case, the number of firms that source from F and pay q_F for inputs is

$$n^{B,F}(\boldsymbol{\mu})=(\mu_f+\mu_b)\,\rho. \tag{24}$$

These are firms that either invested in single supply chains in country F, or in two supply chains and do not suffer a breakdown of bilateral relations in F. At the same time, the number of firms

[21]Recall that $\sigma(z)>\varepsilon$, $\sigma'(z)\geq 0$ and $s'(z)<0$.

that source from country H and pay q_H for inputs is

$$n^{B,H}(\boldsymbol{\mu}) = \mu_h \rho + \mu_b \rho (1-\rho). \tag{25}$$

These are firms that either invested in single supply chains in H and did not suffer breakdowns in bilateral relations there, or invested in two supply chains and suffered breakdowns of bilateral relations in country F but not in H. In this case (4) implies

$$1 = n^{B,H}(\boldsymbol{\mu})\, s[z^{B,H}(\boldsymbol{\mu})] + n^{B,F}(\boldsymbol{\mu}) s[z^{B,F}(\boldsymbol{\mu})] , \tag{26}$$

which is equation (17) in the main text. The pricing equation (6) implies

$$\frac{z^{B,H}(\boldsymbol{\mu},\boldsymbol{q})}{z^{B,F}(\boldsymbol{\mu},\boldsymbol{q})} = \left(\frac{\sigma[z^{B,H}(\boldsymbol{\mu},\boldsymbol{q})]}{\sigma[z^{B,H}(\boldsymbol{\mu},\boldsymbol{q})]-1}\right) \Big/ \left(\frac{\sigma[z^{B,F}(\boldsymbol{\mu},\boldsymbol{q})]}{\sigma[z^{B,F}(\boldsymbol{\mu},\boldsymbol{q})]-1}\right) \frac{q_H}{q_F} ,$$

which is equation (18) in the main text. From here we obtain solutions to the relative prices $z^{B,i}$, $i = H, F$, as functions of the vectors $\boldsymbol{\mu}$ and $\boldsymbol{q}$, $z^{B,i}(\boldsymbol{\mu},\boldsymbol{q})$.

To derive the price index (5) for state B, first note that the pricing equation (6) implies

$$\frac{1}{A^B(\boldsymbol{\mu},\boldsymbol{q})} = \frac{z^{B,J}(\boldsymbol{\mu},\boldsymbol{q}) \left(\sigma[z^{B,J}(\boldsymbol{\mu},\boldsymbol{q})]-1\right)}{q_J \sigma[z^{B,J}(\boldsymbol{\mu},\boldsymbol{q})]}, \quad J \in \{H, F\}.$$

Together with (26), it can be expressed as

$$\log A^B(\boldsymbol{\mu},\boldsymbol{q}) = \sum_{J=H,F} n^{B,J}(\boldsymbol{\mu}) s\left[z^{B,J}(\boldsymbol{\mu},\boldsymbol{q})\right] \log\left\{\frac{q_J}{z^{B,J}(\boldsymbol{\mu},\boldsymbol{q})} \frac{\sigma\left[z^{B,J}(\boldsymbol{\mu},\boldsymbol{q})\right]}{\sigma\left[z^{B,J}(\boldsymbol{\mu},\boldsymbol{q})\right]-1}\right\}.$$

Using this equation, the price index (5) can be expressed as a function of the vectors $\boldsymbol{\mu}$ and $\boldsymbol{q}$:

$$\log P^B(\boldsymbol{\mu},\boldsymbol{q}) := \sum_{J=H,F} n^{B,J}(\boldsymbol{\mu})\, s\left[z^{B,J}(\boldsymbol{\mu},\boldsymbol{q})\right] \log P\left[z^{B,J}(\boldsymbol{\mu},\boldsymbol{q}), q_J\right], \tag{27}$$

where the function $\log P(z,q)$ is defined in (21). Using this result, profits of a firm that sources in state B from country J equal

$$\pi^{B,J}(\boldsymbol{\mu},\boldsymbol{q}) = \frac{s\left[z^{B,J}(\boldsymbol{\mu},\boldsymbol{q})\right]}{\sigma\left[z^{B,J}(\boldsymbol{\mu},\boldsymbol{q})\right]} P^B(\boldsymbol{\mu},\boldsymbol{q})^{1-\varepsilon}, \quad j \in \{H, F\}. \tag{28}$$

Now consider expected profits from strategy j, Π_j, $j = h, f, b$. For a firm that invests in a single-country supply chain the expected profits are

$$\Pi_j = \delta^j \pi^j \rho + \delta^B \pi^{B,j} \rho - k, \quad j \in \{H, F\},$$

where δ^j is the probability that only supply chains from country j will be available, $j = H, F$, and δ^B is the probability that supply chains from both countries will be available. These probabilities are $\delta^H = \gamma_H(1-\gamma_F)$, $\delta^F = \gamma_F(1-\gamma_H)$ and $\delta^B = \gamma_F\gamma_H$. Using the profit functions (22) and (28), this yields

$$\Pi_h = \Pi_h(\boldsymbol{\mu}, \boldsymbol{q}) := \delta^H \frac{s\left[z^H(\boldsymbol{\mu})\right]}{\sigma\left[z^H(\boldsymbol{\mu})\right]} P\left[z^H(\boldsymbol{\mu}), q_H\right]^{1-\varepsilon}\rho + \delta^B \frac{s\left[z^{B,H}(\boldsymbol{\mu}, \boldsymbol{q})\right]}{\sigma\left[z^{B,H}(\boldsymbol{\mu}, \boldsymbol{q})\right]} P^B(\boldsymbol{\mu}, \boldsymbol{q})^{1-\varepsilon}\rho - k, \quad (29)$$

$$\Pi_f = \Pi_f(\boldsymbol{\mu}, \boldsymbol{q}) := \delta^F \frac{s\left[z^F(\boldsymbol{\mu})\right]}{\sigma\left[z^F(\boldsymbol{\mu})\right]} P\left[z^F(\boldsymbol{\mu}), q_F\right]^{1-\varepsilon}\rho + \delta^B \frac{s\left[z^{B,F}(\boldsymbol{\mu}, \boldsymbol{q})\right]}{\sigma\left[z^{B,F}(\boldsymbol{\mu}, \boldsymbol{q})\right]} P^B(\boldsymbol{\mu}, \boldsymbol{q})^{1-\varepsilon}\rho - k. \quad (30)$$

For a firm that invests in supply chains from both countries, the expected profits are

$$\Pi_b = \sum_{J=H,F} \delta^J \pi^J \rho + \delta^B \left[\pi^{B,F}\rho + \pi^{B,H}(1-\rho)\rho\right] - 2k.$$

A firm that uses this strategy expects profits π^F if the supply chains from F do not break down but supply chains from H do break down, as long as its bilateral relation in F survives. It expects profits π^H if the supply chains from H do not break down but supply chains from F do break down, as long as its bilateral relation in H survives. In case supply chains from none of the countries break down, the firm expects profits $\pi^{B,F}$ if its bilateral relation survives in country F and profits $\pi^{B,H}$ if its bilateral relation in F does not survive but it survives in H. Using (22) and (28), this yields

$$\begin{aligned} \Pi_b = \Pi_b(\boldsymbol{\mu}, \boldsymbol{q}) := & \sum_{J=H,F} \delta^J \frac{s\left[z^J(\boldsymbol{\mu})\right]}{\sigma\left[z^J(\boldsymbol{\mu})\right]} P\left[z^J(\boldsymbol{\mu}), q_J\right]^{1-\varepsilon}\rho \\ & + \delta^B \left\{ \frac{s\left[z^{B,F}(\boldsymbol{\mu}, \boldsymbol{q})\right]}{\sigma\left[z^{B,F}(\boldsymbol{\mu}, \boldsymbol{q})\right]} + \frac{s\left[z^{B,H}(\boldsymbol{\mu}, \boldsymbol{q})\right]}{\sigma\left[z^{B,H}(\boldsymbol{\mu}, \boldsymbol{q})\right]}(1-\rho) \right\} P^B(\boldsymbol{\mu}, \boldsymbol{q})^{1-\varepsilon}\rho - 2k. \end{aligned} \quad (31)$$

Using these functions together with

$$P^J(\boldsymbol{\mu}, \boldsymbol{q}) := P\left[z^J(\boldsymbol{\mu},), q_J\right], \quad J \in \{H, F\},$$

we obtain the welfare function (8) in the main text. There, the dependence of the functions on the parameters $\boldsymbol{q}$ has been suppressed in order to focus on the endogenous vector $\boldsymbol{\mu}$.

A.2 The Symmetric Case

In the symmetric case $q_F = q_H = q$ and $\delta^F = \delta^H = \delta$. Firms are indifferent between sourcing from country F or H when both options are available. For notational simplicity, we omit the dependence of the equilibrium variables on q. As explained in the main text, in this case (see (9))

$$1 = n^J(\boldsymbol{\mu})\, s[z^J(\boldsymbol{\mu})], \quad J \in \{H, F, B\}, \quad (32)$$

where

$$n^B(\boldsymbol{\mu}) = [\mu_h + \mu_f + \mu_b(2-\rho)]\,\rho. \tag{33}$$

These equations yield the functions $z^J(\boldsymbol{\mu})$ and the price indexes $P\left[z^J(\boldsymbol{\mu})\right]$ for $J \in \{H, F, B\}$, where the function $\log P(z,q)$ is defined in (21). The resulting profits in state J are

$$\pi\left[z^J(\boldsymbol{\mu})\right] = \frac{s\left[z^J(\boldsymbol{\mu})\right]}{\sigma\left[z^J(\boldsymbol{\mu})\right]} P\left[z^J(\boldsymbol{\mu})\right]^{1-\varepsilon}, \quad J \in \{H, F, B\}. \tag{34}$$

In this case the expected profit functions (29)-(31) can be expressed as

$$\Pi_h(\boldsymbol{\mu}) = \delta\pi\left[z^H(\boldsymbol{\mu})\right]\rho + \delta^B\pi\left[z^B(\boldsymbol{\mu})\right]\rho - k, \tag{35}$$

$$\Pi_f(\boldsymbol{\mu}) = \delta\pi\left[z^F(\boldsymbol{\mu})\right]\rho + \delta^B\pi\left[z^B(\boldsymbol{\mu})\right]\rho - k, \tag{36}$$

$$\Pi_b(\boldsymbol{\mu}) = \delta\sum_{j=H,F}\delta\pi\left[z^j(\boldsymbol{\mu})\right]\rho + \delta^B\pi\left[z^B(\boldsymbol{\mu})\right](2-\rho)\rho - 2k. \tag{37}$$

The first thing to note from these functions is that $\Pi_h(\boldsymbol{\mu}) > \Pi_f(\boldsymbol{\mu})$ if and only if $z^H(\boldsymbol{\mu}) < z^F(\boldsymbol{\mu})$. This is a direct implication of Lemma 1. But $z^H(\boldsymbol{\mu}) < z^F(\boldsymbol{\mu})$ if and only if $\mu_f > \mu_h$, due to the fact that $s(z)$ is a declining function (see (19) and (20)). Since $\mu_f > \mu_h \geq 0$ implies that some firms use strategy f when h yields higher welfare, $\Pi_h(\boldsymbol{\mu}) > \Pi_f(\boldsymbol{\mu})$ is not an equilibrium outcome. For similar reasons $\Pi_h(\boldsymbol{\mu}) < \Pi_f(\boldsymbol{\mu})$ is not an equilibrium outcome. Therefore in equilibrium $\Pi_h(\boldsymbol{\mu}) = \Pi_f(\boldsymbol{\mu})$, $\mu_f = \mu_h = \mu$ and $z^H(\boldsymbol{\mu}) = z^F(\boldsymbol{\mu})$. We summarize this in

Lemma 2 *Suppose that $q_F = q_H = q$ and $\delta^F = \delta^H = \delta$. Then, in equilibrium, $\mu_f = \mu_h = \mu$ and $z^H(\boldsymbol{\mu}) = z^F(\boldsymbol{\mu})$.*

In a symmetric equilibrium, the same fraction of firms invest in single supply chains from countries H and F, because expected profits are higher in country i if a smaller fraction of firms invest in this country. As a result, relative prices are the same in both countries.

Now examine when it is more profitable to invest in two supply chains rather than one, under the constraint that $\mu_f = \mu_h = \mu$. For this purpose we consider the difference $\Pi_b(\mu,\mu,\mu_b) - \Pi_h(\mu,\mu,\mu_b)$, where $\Pi_h(\mu,\mu,\mu_b) \equiv \Pi_f(\mu,\mu,\mu_b)$.[22] Let $z(\mu,\mu_b) := z^H(\mu,\mu,\mu_b) \equiv z^F(\mu,\mu,\mu_b)$ and similarly $z^B(\mu,\mu_b) := z^B(\mu,\mu,\mu_b)$. Then

$$\Pi_b(\mu,\mu,\mu_b) - \Pi_h(\mu,\mu,\mu_b) = \delta\pi\left[z(\mu,\mu_b)\right]\rho + \delta^B\pi\left[z^B(\mu,\mu_b)\right](1-\rho)\rho - k.$$

It is evident from this equation that for all feasible vectors $\boldsymbol{\mu} = (\mu,\mu,\mu_b)$, i.e., that satisfy $\mu \geq 0$, $\mu_b \geq 0$ and $2\mu + \mu_b \leq 1$, $\Pi_b(\mu,\mu,\mu_b) > \Pi_h(\mu,\mu,\mu_b)$ for $k \to 0$. Therefore, for low values of investment costs, the equilibrium entails $\mu_h = \mu_f = 0$ and $\mu_b = 1$. As k increases, there exists a cutoff that makes firm indifferent between not investing in a single-country supply chain and

[22] We suppose that C_P is small enough so that no firms want to exit before $k < k_3$.

investing in diversification. We denote this cutoff k_1, defined as

$$k_1 = \delta\pi \left[z\left(0,1\right)\right]\rho + \delta^B \pi \left[z^B\left(0,1\right)\right]\left(1-\rho\right)\rho.$$

For larger values of k it is profitable to invest in both strategies. Consider the function

$$\kappa\left(\mu\right) := \delta\pi\left[z\left(\mu, 1-2\mu\right)\right]\rho + \delta^B\pi\left[z^B\left(\mu, 1-2\mu\right)\right]\left(1-\rho\right)\rho.$$

Conditions (20), (32) and (33) imply that both functions $\mu \to z\left(\mu, 1-2\mu\right)$ and $\mu \to z^B\left(\mu, 1-2\mu\right)$ are decreasing in μ for $\mu \in (0, 1/2)$. Therefore $\kappa\left(\mu\right)$ is increasing in μ for $\mu \in (0, 1/2)$. Let $k_2 = \kappa\left(1/2\right)$. It then follows that, for $k \in (k_1, k_2)$, the equilibrium consists of $\mu > 0$ and $\mu_b = 1-2\mu > 0$, where μ satisfies $\kappa\left(\mu\right) = k$. In this range of investment costs, μ is increasing in k and μ_b is declining.

For still higher investment costs, no firm invests in two supply chains, and $\mu_b = 0$ while $\mu_h = \mu_f = 1/2$. Profits from investing in a single-country supply chain are

$$\Pi_h = \Pi_f = \delta\pi\left[z\left(1/2, 0\right)\right]\rho + \delta^B\pi\left[z^B\left(1/2, 0\right)\right]\rho - k.$$

which is clearly decreasing in k. Hence, there exists k_3 so that $\Pi_h = \Pi_f = 0 > \Pi_b$. For $k > k_3$, only a fraction of the firms decide to invest in any supply chain. Profits from investing in a single-country supply chain are

$$\Pi_h = \Pi_f = \delta\pi\left[z\left(\mu, 0\right)\right]\rho + \delta^B\pi\left[z^B\left(\mu, 0\right)\right]\rho - k,$$

and μ is given by the condition $\Pi_h = \Pi_f = 0$. For $k > k_3$, these profits are declining in μ, because $z^J\left(\mu, \mu, 0\right)$ is increasing in μ for $J \in \{H, F, B\}$. Therefore, μ is declining in k for $k > k_3$ until $\mu = 0$, which obtains at

$$k_4 := \lim_{\mu \searrow 0} \delta\pi\left[z\left(\mu, 0\right)\right]\rho + \delta^B\pi\left[z^B\left(\mu, 0\right)\right]\rho.$$

This completes the proof of the pattern depicted in Figure 1.

In order to understand Figure 2, consider investment costs that satisfy $k \in (k_1, k_2)$. For these cost levels we know that the equilibrium satisfies

$$\mu_h = \mu_f > 0 \text{ and } \mu_b = 1 - \mu_h - \mu_f > 0.$$

All three investment strategies are equally profitable. Allowing for $\mu_h \neq \mu_f$, we express the equal-profit conditions for vectors $\boldsymbol{\mu} = \left(\mu_h, \mu_f, 1-\mu_h-\mu_f\right)$ as follows:

$$\begin{aligned}
\Pi_h\left(\mu_h, \mu_f, 1-\mu_h-\mu_f\right) &= \Pi_f\left(\mu_h, \mu_f, 1-\mu_h-\mu_f\right), \\
\Pi_h\left(\mu_h, \mu_f, 1-\mu_h-\mu_f\right) &= \Pi_b\left(\mu_h, \mu_f, 1-\mu_h-\mu_f\right), \\
\Pi_f\left(\mu_h, \mu_f, 1-\mu_h-\mu_f\right) &= \Pi_b\left(\mu_h, \mu_f, 1-\mu_h-\mu_f\right).
\end{aligned}$$

The first condition is satisfied along the 45 degree line in the figure, because it requires $z^H = z^F$ and therefore $\mu_h = \mu_f$. The second condition is satisfied if and only if

$$\delta\pi\left[z^F\left(\mu_h,\mu_f,1-\mu_h-\mu_f\right)\right]\rho+\delta^B\pi\left[z^B\left(\mu_h,\mu_f,1-\mu_h-\mu_f\right)\right](1-\rho)\,\rho=k.$$

Recall that the function z^F is obtained from (32), $s[z^F(\boldsymbol{\mu})]\rho(1-\mu_h)=1$, so that z^F only depends on μ_h. Similarly, the function z^H only depends on μ_f. Finally, as long as $k\in(k_1,k_2)$ so that $\mu_b=1-\mu_h-\mu_f$, condition (33) rewrites $s[z^B(\boldsymbol{\mu})]\rho[1+(1-\rho)(1-\mu_f-\mu_h)]=1$ and $z^B(\boldsymbol{\mu})$ is a function of $\mu_{h+f}:=\mu_h+\mu_f$. With this in mind, totally differentiating the second condition and re-arranging yields

$$\left.\frac{d\mu_h}{d\mu_f}\right|_{\Pi_h=\Pi_b}=-\frac{\delta^B d\pi\left[z^B\left(\mu_h,\mu_f,1-\mu_{h+f}\right)\right](1-\rho)\,\rho}{\delta d\pi\left[z^F\left(\mu_h,\mu_f,1-\mu_{h+f}\right)\right]\rho+\delta^B d\pi\left[z^B\left(\mu_h,\mu_f,1-\mu_{h+f}\right)\right](1-\rho)\,\rho}\in(-1,0),$$

where $d\pi[z^F(\mu_h,\mu_f,1-\mu_{h+f})]=\pi'(z^F)\cdot\partial_{\mu_h}z^F(\mu_h,\mu_f,1-\mu_{h+f})>0$ denote the total derivative of $\pi(z^F)$ and similarly for $d\pi[z^B(\mu_h,\mu_f,1-\mu_{h+f})]$. This proves that the slope of $\Pi_h=\Pi_b$ is negative and smaller than one in absolute value. Proceeding similarly with the third condition, $\Pi_f=\Pi_b$, we have

$$\left.\frac{d\mu_h}{d\mu_f}\right|_{\Pi_h=\Pi_f}=-\frac{\delta\rho d\pi[z^H(\mu_h,\mu_f,1-\mu_{h+f})]+\delta^B d\pi^B(z^B(\mu_h,\mu_f,1-\mu_{h+f})](1-\rho)\,\rho}{\delta^B d\pi^B[z^B(\mu_h,\mu_f,1-\mu_{h+f})](1-\rho)\rho}<-1.$$

This proves that the slope of $\Pi_h=\Pi_f$ is negative and greater than one in absolute value.

Now consider the welfare function (8) in the main text. We focus the analysis on $k\in(k_1,k_2)$, i.e., the range of investment costs in which all the investment strategies are used in equilibrium. Using (21) and (35)-(37), in the symmetric case this function can be expressed as

$$\begin{aligned}W\left(\mu_h,\mu_f\right)&:=\sum_{j=h,f,b}\mu_j\Pi_j+\frac{1}{\varepsilon-1}\sum_{J=H,F,B}\delta^J P\left[z^J\left(\mu_h,\mu_f,1-\mu_{h+f}\right),q\right]^{1-\varepsilon}\\&=\sum_{j=H,F}\delta\left(n^j(\mu_{-j})\pi[z^j(\mu_{-j})]+\frac{1}{\varepsilon-1}P[z^j(\mu_{-j})]^{1-\varepsilon}\right)+\\&\qquad\delta^B\left(n^B(\mu_{h+f})\pi[z^B(\mu_{h+f})]+\frac{1}{\varepsilon-1}P[z^B(\mu_{h+f})]^{1-\varepsilon}\right)-k(2-\mu_{h+f}).\end{aligned}$$

We dropped the dependence in the price functions on the μ's that are irrelevant, so that $z^j(\mu_h,\mu_f,1-\mu_{h+f})=z^j(\mu_{-j})$ for $j\in\{H,F\}$ and $z^B(\mu_h,\mu_f,1-\mu_{h+f})=z^B(\mu_{h+f})$. Welfare is therefore the sum of realized profits in each state, $n^j\pi^j$, and consumer surplus, $(P^j)^{1-\varepsilon}/(\varepsilon-1)$, weighted by the probability of each state, δ^j. Plugging (22) and (28) in the expression for welfare and using the market clearing conditions, $n^j(\mu)s[z^j(\mu)] = 1$, yields

$$W(\mu_h, \mu_f) = \sum_{j=H,F} \delta \left\{ \frac{1}{\sigma[z^j(\mu_{-j})]} + \frac{1}{\varepsilon - 1} \right\} P[z^j(\mu_{-j})]^{1-\varepsilon} + \\ \delta^B \left\{ \frac{1}{\sigma[z^B(\mu_{h+f})]} + \frac{1}{\varepsilon - 1} \right\} P[z^B(\mu_{h+f})]^{1-\varepsilon} - k(2 - \mu_{h+f}), \tag{38}$$

as long as $k \in (k_1, k_2)$. The constrained optimum (second-best) is obtained from maximizing this function over (μ_h, μ_f). We denote by $\boldsymbol{\mu} = (\mu_h, \mu_f, \mu_b)$ the vector of μ's in the decentralized equilibrium, and by $\boldsymbol{\mu}^o = (\mu_h^o, \mu_f^o, \mu_b^o)$ the vector of μ's in the constrained optimum.

We first show that the first-order conditions are satisfied for $\mu_h^o = \mu_f^o$, which is a straightforward implication of the symmetrical structure. The first order conditions are

$$-\frac{\partial W(\mu_h^o, \mu_f^o)}{\partial \mu_{-j}} = \delta P[z^j(\mu_{-j}^o)]^{1-\varepsilon} \left[\frac{\sigma'[z^j(\mu_{-j}^o)]}{\sigma[z^j(\mu_{-j}^o)]^2} + \frac{P'[z^j(\mu_{-j}^o)]}{P[z^j(\mu_{-j}^o)]} \left\{ \frac{\varepsilon - 1}{\sigma[z^j(\mu_{-j}^o)]} + 1 \right\} \right] \frac{\partial z^j(\mu_{-j}^o)}{\partial \mu_{-j}} + \\ \delta^B P[z^B(\mu_{h+f}^o)]^{1-\varepsilon} \left[\frac{\sigma'[z^B(\mu_{h+f}^o)]}{\sigma[z^B(\mu_{h+f}^o)]^2} + \frac{P'[z^B(\mu_{h+f}^o)]}{P[z^B(\mu_{h+f}^o)]} \left\{ \frac{\varepsilon - 1}{\sigma[z^B(\mu_{h+f}^o)]} + 1 \right\} \right] \frac{\partial z_b(\mu_{h+f}^o)}{\partial \mu_{-j}} - k = 0,$$

for $j \in \{H, F\}$. These two conditions are necessary in the constrained optimum as long as μ^o is interior, which imply

$$P[z^H(\mu_f^o)]^{1-\varepsilon} \left[\frac{\sigma'[z^H(\mu_f^o)]}{\sigma[z^H(\mu_f^o)]^2} + \frac{P'[z^H(\mu_f^o)]}{P[z^H(\mu_f^o)]} \left\{ \frac{\varepsilon - 1}{\sigma[z^H(\mu_f^o)]} + 1 \right\} \right] \frac{\partial z^H(\mu_f^o)}{\partial \mu_f^o} = \\ P[z^F(\mu_h^o)]^{1-\varepsilon} \left[\frac{\sigma'[z^F(\mu_h^o)]}{\sigma[z^F(\mu_h^o)]^2} + \frac{P'[z^F(\mu_h^o)]}{P[z^F(\mu_h^o)]} \left\{ \frac{\varepsilon - 1}{\sigma[z^F(\mu_h^o)]} + 1 \right\} \right] \frac{\partial z^F(\mu_h^o)}{\partial \mu_h^o}.$$

This condition is satisfied for $\mu_h^o = \mu_f^o$. The first order conditions are sufficient if W is globally concave.

Proving the global concavity of W for general HSA preferences turns out to be a tricky task. In Section A.4, we prove it for CES preferences. Additionally, we now show that in the case of symmetric translog preferences, $\mu_h^o = \mu_f^o$ is globally optimal. For that, we show that increasing μ_f is welfare-improving if and only if $\mu_f < \mu_h$. Specifically, we consider the variation $d\mu = (d\mu_h, d\mu_f, 0)$ along $d\mu_h = -d\mu_f$. Totally differentiating the welfare function (38) and imposing $d\mu_h = -d\mu_f$ returns

$$\frac{dW(\mu_h, \mu_f)}{d\mu_f} \propto d\Phi(\mu_h) - d\Phi(\mu_f),$$

where

$$\Phi(\mu) := -\left(n(\mu)\pi(z(\mu)) + \frac{1}{\varepsilon - 1} P(z(\mu))^{1-\varepsilon} \right),$$

and $n(\mu) = \rho(1 - \mu)$. Hence, $dW(\mu_h, \mu_f)/d\mu_f > 0 \iff d\Phi(\mu_h) > d\Phi(\mu_f)$. Under the assumption that preferences are symmetric translog, so that the market share function is $s(z) = -\theta \log(z)$ for $z \in (0,1)$, the function Φ can be rewritten as[23]

$$\Phi(\mu) \propto -\left\{ \left(\frac{1}{1+\theta\rho(1-\mu)} + \frac{1}{\varepsilon - 1} \right) \left(\frac{1+\theta\rho(1-\mu)}{\theta\rho(1-\mu)} \right)^{1-\varepsilon} \exp\left(\frac{1-\varepsilon}{2\theta\rho(1-\mu)} \right) \right\}.$$

The function $\mu \to \Phi(\mu)$ is convex as long as $\sigma(\mu) = 1 + \theta\rho(1-\mu) > \varepsilon$ – which holds by assumption. Hence, $d\Phi(\mu)$ is increasing, and $dW(\boldsymbol{\mu})/d\mu_f > 0 \iff d\Phi(\mu_h) > d\Phi(\mu_f) \iff \mu_h > \mu_f$, as claimed.

A.2.1 Should the Government Subsidize Diversification?

We have already argued that $\mu_h^o = \mu_f^o$ in the constrained optimum, so that the constrained optimum is located on the $\Pi_h = \Pi_f$ line. It automatically follows that, in the symmetric case, a diversification subsidy (or tax) can attain the second-best, while this is not true for a reshoring or offshoring subsidy.

Whether the diversification policy consists of a tax or a subsidy depends on the relative location of the constrained optimum (O) relative to the decentralized equilibrium (E) along the $\Pi_h = \Pi_f$ line. If O is located below E, so that $\mu^o < \mu$ and the constrained optimum features more diversification, then the governments wants to subsidize investment in a second supply chain. Suppose indeed that the government offers a subsidy s_d if firms invest in both supply chains. (35) and (36) remain unchanged, while (37) becomes

$$\Pi_b = \sum_{j=H,F} \delta\pi \left[z^j(\mu), q \right] \rho + \delta^B \pi \left[z^B(\mu), q \right] (2-\rho)\rho - 2k + s_d.$$

Since the expected profits of strategies h and f are unaffected by the subsidy, Lemma 2 remains valid and $\mu_h = \mu_f$ in equilibrium. Firms decide to invest in both supply chains as long as $\Pi_b(\mu, \mu, \mu_b) \geq \Pi_h(\mu, \mu, \mu_b)$, or

$$\Pi_b(\mu, \mu, \mu_b, q) - \Pi_h(\mu, \mu, \mu_b, q) = \delta\pi \left[z(\mu, \mu_b), q \right] \rho + \delta^B \pi \left[z^B(\mu, \mu_b), q \right] (1-\rho)\rho - k + s_d \geq 0.$$

Evidently, while μ is increasing in k, it is decreasing in s_d, and therefore a subsidy moves the equilibrium in the direction of O.

The relative location of E versus O depends on preferences and technological parameters. In Section A.4, we show that, with CES preferences, O is always located below E. However, this prediction is not robust to general HSA preferences. To see this, limiting ourselves to $k \in (k_1, k_2)$, consider a variation $(d\mu, d\mu, -2d\mu)$ around the decentralized equilibrium $(\mu, \mu, 1-2\mu)$. Totally differentiating (38), imposing $d\boldsymbol{\mu} = (d\mu, d\mu, -2d\mu)$, and re-arranging, the change in welfare associated

[23] Under symmetric translog, the elasticity of substitution is $1 - \sigma(z) = 1/\log(z)$. The market clearing condition implies $z(\mu) = \exp[-1/(\theta\rho(1-\mu))]$. Finally, $\int_z^1 s(\zeta)/\zeta \, d\zeta \cdot 1/s(z) = -\log(z)/2$.

with this variation is

$$-\frac{1}{2}\frac{dW(\mu,\mu)}{d\mu} = \delta\left(\left[\frac{\varepsilon-1}{\sigma[z(\mu)]}+1\right]\frac{P'[z(\mu)]}{P[z(\mu)]}+\frac{\sigma'[z(\mu)]}{\sigma[z(\mu)]^2}\right)P[z(\mu)]^{1-\varepsilon}\frac{\partial z(\mu)}{\partial\mu}+ \\ \delta^B\left(\left[\frac{\varepsilon-1}{\sigma[z^B(\mu)]}+1\right]\frac{P'[z^B(\mu)]}{P[z^B(\mu)]}+\frac{\sigma'[z^B(\mu)]}{\sigma[z^B(\mu)]^2}\right)P[z^B(\mu)]^{1-\varepsilon}\frac{\partial z^B(\mu)}{\partial\mu}-k, \tag{39}$$

where, as before, $z(\mu) := z^J(\mu,\mu,1-2\mu)$ for $J\in\{H,F\}$ and $z^B(\mu) := z^B(\mu,\mu,1-2\mu)$. Characterizing the sign of dW for general HSA preferences is complicated; however, this is feasible under symmetric translog preferences. Assuming $\sigma(z) = 1-1/\log(z)$ for $z\in(0,1)$, the change in welfare becomes

$$-\frac{1}{2}\frac{dW(\mu,\mu)}{d\mu} = \left(\frac{1}{2}+\frac{\varepsilon(1+3\theta n(\mu))}{2\theta n(\mu)(1+\theta n(\mu))}\right)\frac{\delta}{1+\theta n(\mu)}\frac{\rho\theta}{\theta n}P[z(\mu)]^{1-\varepsilon}+ \\ \left(\frac{1}{2}+\frac{\varepsilon(1+3\theta n^B(\mu))}{2\theta n^B(\mu)(1+\theta n^B(\mu))}\right)\frac{\delta^B}{1+\theta n^B(\mu)}\frac{\rho(1-\rho)\theta}{\theta n^B(\mu)}P[z^B(\mu)]^{1-\varepsilon}-k.$$

Since $k\in(k_1,k_2)$, we have $\mu\in(0,1/2)$, and the condition $\Pi(\mu,\mu,1-2\mu)=\Pi_b(\mu,\mu,1-2\mu)$ holds in the decentralized equilibrium. Solving for k in $\Pi(\mu,\mu,1-2\mu)=\Pi_b(\mu,\mu,1-2\mu)$ and plugging its value in the above equation yields

$$\frac{dW(\mu,\mu)}{d\mu} = \left(1-\frac{\varepsilon(1+3\theta n(\mu))}{\theta n(\mu)(1+\theta n(\mu))}\right)\frac{\delta}{1+\theta n(\mu)}\frac{\rho\theta}{\theta n(\mu)}P[z(\mu)]^{1-\varepsilon}+ \\ \left(1-\frac{\varepsilon(1+3\theta n^B(\mu))}{\theta n^B(\mu)(1+\theta n^B(\mu))}\right)\frac{\delta^B}{1+\theta n^B(\mu)}\frac{\rho(1-\rho)\theta}{\theta n^B(\mu)}P[z^B(\mu)]^{1-\varepsilon}.$$

The terms in parenthesis are increasing in $n(\mu)$ and $n^B(\mu)$ respectively. Furthermore, $n(\mu) = \rho(1-\mu) < \rho(1+(1-\rho)(1-2\mu)) = n^B(\mu)$. Therefore, if the first term is positive, so is the second, and it follows that

$$\frac{\theta n(\mu)(1+\theta n(\mu))}{1+3\theta n(\mu)} > \varepsilon \quad \Rightarrow \quad \left.\frac{dW(\mu,\mu)}{d\mu}\right|_{\mu=\mu^o} > 0.$$

Alternatively, if the second term is negative, so is the first, and

$$\frac{\theta n^B(\mu)(1+\theta n^B(\mu))}{1+3\theta n^B(\mu)} < \varepsilon \quad \Rightarrow \quad \left.\frac{dW(\mu,\mu)}{d\mu}\right|_{\mu=\mu^o} < 0.$$

We summarize this findings in the following lemma.

Lemma 3 *Suppose countries are symmetric, $\gamma_F=\gamma_H$ and $q_F=q_H$, and preferences are symmetric translog. Then, E lies below O if*

$$\eta(\mu) := \frac{\theta n(\mu)(1+\theta n(\mu))}{1+3\theta n(\mu)} > \varepsilon, \tag{40}$$

while E lies above O if

$$\eta^B(\mu) := \frac{\theta n^B(\mu)(1+\theta n^B(\mu))}{1+3\theta n^B(\mu)} < \varepsilon. \tag{41}$$

While the values of $n(\mu)$ and $n^B(\mu)$ are endogenous and functions of the underlying parameters, it is possible to derive parametric restrictions to guarantee that either (40) or (41) holds. The functions η and η^B are increasing in $n(\mu)$ and $n^B(\mu)$ respectively, thus decreasing in μ. Hence, if (40) is satisfied for $\min_\mu n(\mu) = n(1/2) = \rho/2$, it is satisfied for all equilibria with $\mu \in (0, 1/2)$. Similarly, if (41) is satisfied for $\max_\mu n^B(\mu) = n^B(0) = \rho(2-\rho)$, then it is satisfied for all equilibria with $\mu \in (0, 1/2)$. To these we need to add regularity conditions to ensure that $\sigma(\mu) = 1 + \theta n(\mu) > \varepsilon$ in equilibrium.[24] Clearly, $\eta(\mu) > \varepsilon$ implies $\sigma(\mu) > \varepsilon$, so that (40) is sufficient. Regarding (41), a sufficient additional condition is $1 + \theta n(1/2) > \varepsilon$, which guarantees that $1 + \theta n(\mu) > \varepsilon$ for all $\mu \in (0, 1/2)$ since $n(\mu)$ is decreasing in μ. Altogether, these remarks yield the following propositions:

Proposition 1 *Suppose that countries are symmetric, $\gamma_F = \gamma_H$ and $q_F = q_H$, and preferences are symmetric translog. Then,*

1. *if ε is relatively small, so that $\varepsilon < \eta(1/2)$, then O lies above E;*
2. *if ε is relatively large, such that $\varepsilon \in (\eta^B(0), 1 + \theta n(1/2))$, then O lies below E.*

Why does the planner want to promote or mitigate investments in a second supply chain? In particular, is it to counteract the distortion caused by monopoly pricing? To answer these questions, we consider a consumption subsidy so that consumers purchase the final goods at their marginal cost of production.

Let $1 - \nu$ be the subsidy rate provided to consumers, so that they pay only a fraction ν of the price. Then the consumer price of the final product is νp, where p is the producer price. Consumers and firms treat this subsidy as given, but the government chooses it so as to ensure $\nu p = q$. Firms now solve

$$p = \arg\max_{\breve{p}} P^{1-\varepsilon} s\left(\frac{\nu \breve{p}}{A}\right) (v\breve{p})^{-1} (\breve{p} - q).$$

Let a c subscript denote the equilibrium variables when the consumption subsidy is active. Prices remain given by

$$p_c^J = \frac{\sigma\left(z_c^J\right)}{\sigma\left(z_c^J\right) - 1} q,$$

for $J \in \{H, F, B\}$, which implies that the state-contingent consumption subsidy is

$$v\left(z_c^J\right) = \frac{\sigma\left(z_c^J\right) - 1}{\sigma\left(z_c^J\right)}.$$

[24] With symmetric translog preferences, $\sigma(z) = 1 - 1/\log(z)$ while $z^J(\mu) = \exp(-1/[\theta n^J(\mu)])$ for $J \in \{H, F, B\}$, so that $\sigma^J(\mu) = 1 + \theta n^J(\mu)$. Finally, $n^B(\mu) > n(\mu)$, so that $\sigma(\mu) > \varepsilon$ implies $\sigma^B(\mu) > \varepsilon$.

Regardless of the presence of a consumption subsidy, the market clearing conditions remain identical, $n(\boldsymbol{\mu})s[z_c^J(\boldsymbol{\mu})] = 1$ for $j \in \{H, F, B\}$, and therefore $z_c^J(\mu) = z^J(\boldsymbol{\mu})$. However, the price index and the profit per state expressions are changed. Specifically, the price index becomes

$$\log P_c[z^J(\boldsymbol{\mu})] = C_P + \log \frac{q}{z^J(\boldsymbol{\mu})} - \frac{1}{s[z^J(\boldsymbol{\mu})]} \int_{z^J(\boldsymbol{\mu})}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta, \tag{42}$$

and profits per state update to

$$\pi\left[z^J(\boldsymbol{\mu})\right] := \frac{s[z^J(\boldsymbol{\mu})]}{\sigma[z^J(\boldsymbol{\mu})] - 1} P_c[z^J(\boldsymbol{\mu})]^{1-\varepsilon}. \tag{43}$$

The expressions for expected profits remain qualitatively as before. Hence, there still exists $k_{1,c}$ and $k_{2,c}$ so that $\mu_h, \mu_f, \mu_b \in (0,1)$ and $\mu_b = 1 - \mu_h - \mu_f$ for $k \in (k_{1,c}, k_{2,c})$. As before, we focus on this interior case. The expression for welfare needs now to account for the fact that the government pays out consumption subsidies in the amount $\frac{1-\nu}{\nu} P_c^{1-\varepsilon}$.[25] Since $v(z) = \frac{\sigma(z)-1}{\sigma(z)}$, the consumption subsidy bill equals $\frac{1}{\sigma(z)-1} P_c^{1-\varepsilon}$, and welfare – under the assumption that $\mu_h + \mu_f \in (0,1)$ – reads

$$W(\mu_h, \mu_f) = \sum_{J=H,F,B} \delta^J n^J(\boldsymbol{\mu}) \pi^J[z^J(\boldsymbol{\mu})] + \sum_{J=H,F,B} \delta^J \left(\frac{1}{\varepsilon - 1} - \frac{1}{\sigma[z^J(\boldsymbol{\mu})] - 1} \right) P_c[z^J(\mu)]^{1-\varepsilon} - k(2 - \mu_{h+f}).$$

Plugging (43) into the welfare expression gives

$$W(\mu_h, \mu_f) = \frac{1}{\varepsilon - 1} \sum_{J=H,F,B} \delta^J P_c[z^J(\boldsymbol{\mu})]^{1-\varepsilon} - k(2 - \mu_{h+f}).$$

The first-best allocation is obtained by maximizing the above function over (μ_h, μ_f). As in the case without consumption subsidy, it is evident that $\mu_h = \mu_f =: \mu$ satisfies the first-order conditions due to the symmetry. We therefore focus on the case with $\mu_h = \mu_f$. Starting from the decentralized equilibrium, $\boldsymbol{\mu}$, consider a deviation $(d\mu, d\mu, -2d\mu)$. The change in welfare associated with this variation is

$$-\frac{1}{2} \frac{dW(\mu, \mu)}{d\mu} = \delta \rho \int_{z(\mu)} \frac{s(\zeta)}{\zeta} d\zeta P[z_c(\mu)]^{1-\varepsilon} + \delta^B \rho (1 - \rho) \int_{z^B(\mu)} \frac{s(\zeta)}{\zeta} d\zeta P[z_c^B(\mu)]^{1-\varepsilon} - k.$$

Considering only interior equilibrium, it must be that $\Pi = \Pi_b$. Solving for k and plugging in the

[25] Total consumer spending is νpxn while the firms collect pxn. Therefore the government's subsidy bill is

$$(1 - \nu) pxn = \frac{1 - \nu}{\nu} vpxn = \frac{1 - \nu}{\nu} P^{1-\varepsilon}.$$

above equation becomes

$$\frac{1}{2}\frac{dW(\mu,\mu)}{d\mu} = \delta\rho\left(\frac{s[z(\mu)]}{\sigma[z(\mu)]-1} - \int_{z(\mu)}\frac{s(\zeta)}{\zeta}d\zeta\right)P[z_c(\mu)]^{1-\varepsilon} +$$
$$\delta^B\rho(1-\rho)\left(\frac{s[z^B(\mu)]}{\sigma[z^B(\mu)]-1} - \int_{z^B(\mu)}\frac{s(\zeta)}{\zeta}d\zeta\right)P[z_c^B(\mu)]^{1-\varepsilon}.$$

Evidently, $dW(\mu,\mu) > 0$ if and only if

$$\frac{s(z)}{\sigma(z)-1} > \int_z^{\bar{z}}\frac{s(\zeta)}{\zeta}d\zeta,$$

for $z \in \{z(\mu), z^B(\mu)\}$, which is necessarily the case when the elasticity $\sigma(z)$ is increasing in z; i.e., Marshall's Second Law of Demand is satisfied. In the CES case, in which this elasticity is constant, $dW(\mu^\star,\mu^\star) = 0$. We therefore conclude this section with the following proposition.

Proposition 2 *Suppose that countries are symmetric, $\gamma_F = \gamma_H$ and $q_F = q_H$. Suppose, in addition, that consumption subsidies equate prices to the marginal cost of production. Then, if $\sigma'(z) > 0$, the first-best features less investment in a second supply chain. If $\sigma'(z) = 0$ (CES), the first-best is attained with consumption subsidies alone.*

A.2.2 Should the Government Subsidize Reshoring or Offshoring?

How does the sign of a reshoring (or offshoring) subsidy relate to that of the diversification subsidy? To answer, this question, let s_r denote the subsidy on the home supply chain. Intuitively, a subsidy on the home supply chain increases the fraction of firms that invest in country H, that is it increases μ_h and μ_b, while it decreases the fraction of firm that invests in the foreign country alone, μ_f. To see this, note that the three equations that pin down an interior equilibrium are now $\Pi_b(\boldsymbol{\mu}) = \Pi_h(\boldsymbol{\mu})$, $\Pi_h(\boldsymbol{\mu}) + s_r = \Pi_f(\boldsymbol{\mu})$ and $\mu_b = 1 - \mu_{h+f}$, or using (35-37)

$$\delta\rho\pi[z^F(\mu_h)] + \delta^B\rho(1-\rho)\pi[z^B(\mu_{h+f})] - k = 0,$$
$$\delta\rho\pi[z^F(\mu_h)] - \delta\rho\pi[z^H(\mu_f)] - s_r = 0.$$

Using the implicit function theorem,

$$\begin{pmatrix}\partial\mu_h/\partial s_r \\ \partial\mu_f/\partial s_r\end{pmatrix} = \frac{1}{\Delta}\begin{pmatrix} -\delta^B\rho(1-\rho)d\pi[z^B(\mu_{h+f}]) \\ \delta\rho d\pi[z^F(\mu_h)] + \delta^B\rho(1-\rho)d\pi[z^B(\mu_{h+f}]) \end{pmatrix},$$

where

$$\Delta := -\rho^2\delta d\pi[z^H(\mu_f)]\cdot\left\{\delta d\pi[z^F(\mu_h)] + \delta^B(1-\rho)d\pi[z^B(\mu_{h+f})]\right\} -$$
$$\rho^2\delta^B(1-\rho)d\pi[z^B(\mu_{h+f})]\cdot\delta d\pi[z^F(\mu_h)] < 0.$$

Using $d\mu_b = -d\mu_{h+f}$, we conclude that

$$\begin{aligned}
\frac{\partial \mu_h}{\partial s_r} &= -\frac{\rho(1-\rho)}{\Delta}\delta^B \frac{d\pi[z^B(\mu_{h+f})]}{d\mu_{h+f}} > 0, \\
\frac{\partial \mu_b}{\partial s_r} &= -\frac{\rho}{\Delta}\delta \frac{d\pi[z^F(\mu_h)]}{d\mu_h} > 0, \\
\frac{\partial \mu_f}{\partial s_r} &= -\left(\frac{\partial \mu_b}{\partial s_r} + \frac{\partial \mu_h}{\partial s_r}\right) < 0.
\end{aligned}$$

With this in mind, we can now proceed to show that a reshoring subsidy is welfare-improving if and only if the second-best equilibrium features a larger degree of diversification; that is, if and only if O is below E. When the government offers a subsidy to the home supply chain, it pays out $s_r(\mu_h + \mu_b) = s_r(1 - \mu_f)$ to firms and collect taxes on households of the same amount. Hence, the welfare expression (38) remains unchanged. Suppose now that, starting from $s_r = 0$, the government increases the subsidy on the home supply chain by $ds_r > 0$. The change in welfare associated with this variation reads

$$\begin{aligned}
\left.\frac{dW(\mu_h, \mu_f)}{ds_r}\right|_{s_r=0} \Big/ \frac{\partial \mu_b}{\partial s_r} = \delta \left(\left[\frac{\varepsilon - 1}{\sigma[z(\mu)]} + 1\right] \frac{P'[z(\mu)]}{P[z(\mu)]} + \frac{\sigma'[z(\mu)]}{\sigma[z(\mu)]^2}\right) P[z(\mu)]^{1-\varepsilon} \frac{\partial z(\mu)}{\partial \mu} + \\
+ \delta^B \left(\left[\frac{\varepsilon - 1}{\sigma[z^B(\mu)]} + 1\right] \frac{P'[z^B(\mu)]}{P[z^B(\mu)]} + \frac{\sigma'[z^B(\mu)]}{\sigma[z^B(\mu)]^2}\right) P[z^B(\mu)]^{1-\varepsilon} \frac{\partial z^B(\mu)}{\partial \mu} - k,
\end{aligned}$$

where we used the fact that, when $s_r = 0$, $\mu_h = \mu_f = \mu$ and $\mu_b = 1 - 2\mu$, as well as $d\mu_f + d\mu_h = -d\mu_b$. Comparing with (39), we immediately see that

$$\left.\frac{dW(\mu_h, \mu_f)}{ds_r}\right|_{s_r=0} \Big/ \frac{\partial \mu_b}{\partial s_r} = -\frac{1}{2}\frac{dW(\mu, \mu)}{d\mu},$$

and therefore, since μ_b is increasing in s_r,

$$\left.\frac{dW(\mu_h, \mu_f)}{ds_r}\right|_{s_r=0} > 0 \quad \Longleftrightarrow \quad \frac{dW(\mu, \mu)}{d\mu}.$$

That is, a subsidy on the home supply chain increases welfare only if the constrained optimum features more diversification, as claimed.

A.3 The Asymmetric Case

We now return to the general model described in Section A.1. When $q_F < q_H$, Lemma 1 no longer applies in state B since firms are not indifferent where to source from. We therefore start by extending Lemma 1 to functions $\pi^{B,H}(\boldsymbol{\mu})$ and $\pi^{B,F}(\boldsymbol{\mu})$. In terms of notation, we omit the dependence of the variables on $\boldsymbol{q}$ since we keep those costs constant.

We first show that, since $q_F < q_H$, prices of the goods produced with inputs from country F must be strictly cheaper in state B than goods produced with inputs from country H. Suppose not, so that $p^{B,F} \geq p^{B,H}$. From the pricing equation (6), we have

$$p^{B,H} = \frac{\sigma(p^{B,H}/A^B)}{\sigma(p^{B,H}/A^B)-1} q_H \leq \frac{\sigma(p^{B,F}/A^B)}{\sigma(p^{B,F}/A^B)-1} q_F = p^{B,F} < \frac{\sigma(p^{B,F}/A^B)}{\sigma(p^{B,F}/A^B)-1} q_H.$$

However, the mark-up $\sigma/(\sigma-1)$ is (weakly) increasing in p, contradicting the strict inequality above. Hence, for any allocation $\boldsymbol{\mu}$, we have $z^{B,H}(\boldsymbol{\mu}) > z^{B,F}(\boldsymbol{\mu})$, and firms supplying from the low-cost country make higher profits than firms supplying from the high-cost country in state B.[26]

Lemma 4 *If $q_F < q_H$, then $z^{B,F}(\boldsymbol{\mu}) < z^{B,H}(\boldsymbol{\mu})$ and $\pi^{B,F}(\boldsymbol{\mu}) > \pi^{B,H}(\boldsymbol{\mu})$ for all $\boldsymbol{\mu}$.*

To show that $\pi^{B,i}(\boldsymbol{\mu})$ is increasing in (μ_h, μ_f), we start by studying the response of profits to changes in the number of products available, $n^{B,H}$ and $n^{B,F}$. To keep the notation succinct, we omit the dependence of the variables on $\boldsymbol{\mu}$. In equilibrium, relative prices and the price aggregator are jointly determined by

$$\begin{aligned} p^{B,J} &= \frac{\sigma\left(p^{B,J}/A^B\right)}{\sigma\left(p^{B,J}/A^B\right)-1} q_J, \quad J \in \{H,F\} \\ 1 &= \sum_{J=H,F} n^{B,J} s\left(p^{B,J}/A^B\right). \end{aligned}$$

For the rest of the proof, define $\sigma^{B,J} := \sigma(p^{B,J}/A^B)$ and similarly for the other variables. For a proportional variation $(\hat{n}^{B,H}, \hat{n}^{B,F})$ in the number of active supply chains in country H and F respectively, the proportional change in prices read[27]

$$\hat{p}^{B,J} = -\frac{\alpha_J}{\Delta} \sum_{\ell=H,F} n^{B,\ell} s^{B,\ell} \hat{n}^{B,\ell} < 0,$$

for $J \in \{H, F\}$, and the proportional change in the price aggregator is

$$\hat{A}^B = -\frac{1}{\Delta} \sum_{J=H,F} n^{B,J} s^{B,J} \hat{n}^{B,J} < 0,$$

where we defined

$$\alpha_J := \frac{\eta_\sigma^{B,J}}{\sigma^{B,J} - 1 + \eta_\sigma^{B,J}} \in (0,1) \quad ; \quad \Delta := \sum_{J=H,F}^{2} (1-\alpha_J)\, n^{B,J} s^{B,J} \left(\sigma^{B,J} - 1\right) > 0, \tag{44}$$

for $\eta_\sigma := \sigma' \cdot z/\sigma$ the elasticity of σ at z. These imply that raising the number of firms reduces

[26] Recall that $\pi^{B,i}(\mu) = s(z^{B,i})/\sigma(z^{B,i}) \cdot (P^B)^{1-\varepsilon}$. Since $z \to s(z)/\sigma(z)$ is increasing and P^B is constant across i's, it automatically follows that $\pi^{B,F} > \pi^{B,H}$.

[27] $\hat{x} := dx/x$ denote the proportional change in x.

prices and reduces the aggregator. Furthermore,

$$\hat{z}_J := \hat{p}_J - \hat{A}^B = \frac{1-\alpha_J}{\Delta} \sum_{\ell=H,F} n^{B,\ell} s^{B,\ell} \hat{n}^{B,\ell} > 0, \quad J \in \{H, F\}.$$

It follows that the market share of every firm declines in the number of active firms and its elasticity of demand rises, thereby reducing markups. Moving on to the price index, we start by noting that the price index elasticity is negative,[28]

$$\frac{\partial \log P^B}{\partial \log n^{B,I}} = -\frac{n^{B,I} s^{B,I}}{\Delta} \sum_{\ell=H,F} n^{B,\ell} s^{B,\ell} \alpha_\ell - n^{B,I} \int_{p^{B,I}/A}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta < 0,$$

for $I \in \{H, F\}$. Therefore, the profit elasticity reads

$$\frac{\Delta}{n^{B,I} s^{B,I} (\varepsilon - 1)} \frac{\partial \log \pi^{B,J}}{\partial \log n^{B,I}} = -\left(\frac{\sigma^{B,J} - 1}{\varepsilon - 1}\right) + \sum_{\ell=H,F} n^{B,\ell} s^{B,\ell} \alpha_\ell + \frac{\Delta}{s^{B,I}} \int_{p^{B,I}/A}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta, \tag{45}$$

for $I, J \in \{H, F\}$. Clearly, if $\varepsilon < 1$, then the elasticity is negative for both I and J. Otherwise, recall that

$$\frac{1}{s(z)} \int_z^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta < \frac{1}{\sigma(z) - 1},$$

such that, using the definition of Δ, we have

$$\frac{\Delta}{n^{B,I} s^{B,I} (\varepsilon - 1)} \frac{\partial \log \pi^{B,J}}{\partial \log n^{B,I}} \leq -\left(\frac{\sigma^{B,J} - 1}{\varepsilon - 1}\right) + \sum_{\ell=H,F} n^{B,\ell} s^{B,\ell} \left(\alpha_\ell + (1 - \alpha_l) \frac{\sigma^{B,\ell} - 1}{\sigma^{B,I} - 1}\right).$$

Since $z^{B,H} > z^{B,F}$, we have $\sigma^{B,H} \geq \sigma^{B,F} > \varepsilon$, and therefore

$$\frac{\sigma^{B,H} - 1}{\varepsilon - 1} \geq \frac{\sigma^{B,\ell} - 1}{\varepsilon - 1} > \frac{\sigma^{B,\ell} - 1}{\sigma^{B,i} - 1},$$

for $\ell, i \in \{H, L\}$. It follows that the elasticity of $\pi^{B,H}$ is decreasing with both $n^{B,H}$ and $n^{B,F}$. Similarly,

$$\frac{\sigma^{B,F} - 1}{\varepsilon - 1} > 1 \geq \frac{\sigma^{B,\ell} - 1}{\sigma^{B,H} - 1},$$

and $\pi^{B,F}$ is decreasing with $n^{B,H}$. It is harder to sign the elasticity of $\pi^{B,F}$ with respect to $n^{B,F}$.

[28]Recall that the price index reads

$$\log P^B = C_P + \log A^B - \sum_{i=H,F} n^{B,i} \int_{p^{B,i}/A^B}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta$$

However, note that when $q_F \to q_H$, we have $\sigma^{B,F} \to \sigma^{B,H}$, and therefore

$$\frac{\Delta}{n^{B,I} s^{B,I}(\varepsilon-1)} \frac{\partial \log \pi^{B,J}}{\partial \log n^{B,I}} \leq -\frac{\sigma^B - \varepsilon}{\varepsilon - 1} < 0.$$

for $I, J \in \{H, F\}$. Furthermore, in the CES case, we have $\alpha_J = 0$ for both J, and therefore

$$\frac{\Delta}{n^{B,I} s^{B,I}(\varepsilon-1)} \frac{\partial \log \pi^{B,J}}{\partial \log n^{B,I}} = -\left(\frac{\sigma - \varepsilon}{\varepsilon - 1}\right) < 0.$$

We summarize these results in the following lemma.

Lemma 5 *Suppose that $q_F < q_H$. Then, $\pi^{B,H}$ is decreasing in both $n^{B,F}$ and $n^{B,H}$. Similarly, $\pi^{B,F}$ is decreasing in $n^{B,H}$. Finally, if $q_F \approx q_H$ or preferences are CES, then $\pi^{B,F}$ is also decreasing in $n^{B,F}$.*

For the differential costs used in the numerical simulations, $q_F/q_H \in [0.8, 1]$, we found Lemma 5 to always hold. From Lemma 5, it is possible to draw conclusions about the relationship between firm investment and profits. From (24) and (25), it is immediate that $\pi^{B,J}$ is increasing in μ_f for $J \in \{H, F\}$.

Corollary 1 *Suppose that $q_F < q_H$ but $\gamma_F = \gamma_H$. Then, $\pi^{B,F}$ and $\pi^{B,H}$ are increasing in μ_f.*

Regarding the impact of μ_h on profits, we have

$$\frac{\partial \log \pi^{B,J}}{\partial \mu_h} = \rho \left(\frac{\partial \log \pi^{B,J}}{\partial n^{B,H}} \rho - \frac{\partial \log \pi^{B,J}}{\partial n^{B,F}} \right).$$

Using (45),

$$\begin{aligned} \frac{\partial \log \pi^{B,J}}{\partial \mu_h} \propto \left(s^{B,F} - \rho s^{B,H}\right) \left\{ \left(\frac{\sigma^{B,J} - 1}{\varepsilon - 1} \right) - \sum_{\ell=H,F} n^{B,\ell} s^{B,\ell} \alpha_\ell \right\} - \\ \Delta \left(\int_{p^{B,F}/A}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta - \rho \int_{p^{B,H}/A}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta \right), \end{aligned} \tag{46}$$

for $J \in \{H, F\}$, where the constant of proportionality is identical across the J's. The elasticity of $\pi^{B,H}$ can be signed. Since $z^{B,H} > z^{B,F}$, we have $\sigma^{B,H} > \sigma^{B,F}$, and therefore

$$\begin{aligned} \left[\int_{z^{B,F}} \frac{s(\zeta)}{\zeta} d\zeta - \rho \int_{z^{B,H}} \frac{s(\zeta)}{\zeta} d\zeta \right] \Delta &< \left(\int_{z^{B,F}}^{z^{B,H}} \frac{s(\zeta)}{\zeta} d\zeta + (1-\rho) \int_{z^{B,H}} \frac{s(\zeta)}{\zeta} d\zeta \right) \left(\sigma^{B,H} - 1\right) \left(1 - \sum_{\ell=H,F} n^{B,\ell} s^{B,\ell} \alpha_\ell \right) \\ &< \left(\frac{s^{B,F} - s^{B,H}}{\sigma^{B,F} - 1} + (1-\rho) \frac{s^{B,H}}{\sigma^{B,H} - 1} \right) \left(\sigma^{B,H} - 1\right) \left(1 - \sum_{\ell=H,F} n^{B,\ell} s^{B,\ell} \alpha_\ell \right) \\ &< \left(s^{B,F} - \rho s^{B,H}\right) \frac{\sigma^{B,H} - 1}{\sigma^{B,F} - 1} \left(1 - \sum_{\ell=H,F} n^{B,\ell} s^{B,\ell} \alpha_\ell \right), \end{aligned}$$

where the first inequality follows from the definition of Δ (44), $\sigma^{B,H} > \sigma^{B,F}$ and the market clearing condition $\sum_{\ell=H,F} n^{B,\ell} s^{B,\ell} = 1$, the second from (23), and the last inequality from $\sigma^{B,H} > \sigma^{B,F}$ again. Combined with $(\sigma^{B,H}-1)/(\sigma^{B,F}-1) > (\sigma^{B,H}-1)/(\varepsilon-1)$, this is sufficient to show that $\pi^{B,H}$ is increasing with μ_h. Signing the elasticity of $\pi^{B,F}$ is trickier. However, when $q_F \to q_H$, the elasticity simplifies to

$$\frac{\partial \log \pi^{B,J}}{\partial \mu_h} \propto s^B \left(\frac{\sigma^B - 1}{\varepsilon - 1} - \alpha\right) - (1-\alpha)(\sigma^B - 1) \int_{p^B/A}^{\bar{z}} \frac{s(\zeta)}{\zeta} d\zeta \geq s^B \left(\frac{\sigma^B - \varepsilon}{\varepsilon - 1}\right) > 0,$$

for both $J \in \{H, F\}$ since firms become indifferent between the location of their supply chain. Alternatively, when preferences are CES,

$$\frac{\partial \log \pi^{B,J}}{\partial \mu_h} \propto \left(s^{B,F} - \rho s^{B,H}\right) \left(\frac{\sigma - \varepsilon}{\varepsilon - 1}\right) \geq 0,$$

where we used $\int_z s(\zeta)/\zeta d\zeta = s(z)/(\sigma - 1)$, $\alpha_J = 0$ and $\Delta = \sigma - 1$ under CES preferences.

Corollary 2 *Suppose that $q_F < q_H$ but $\gamma_F = \gamma_H$. Then, $\pi^{B,H}$ is increasing in μ_h. If $q_F \approx q_H$ or preferences are CES, then $\pi^{B,F}$ is also increasing in μ_h.*

The profit semi-elasticity (46) is also useful to show that the following assumption – which we need to establish the configuration depicted in Figure 3 when $q_F < q_H$ – is met under $q_F \approx q_H$.

Assumption 3 *The function $\Pi_{fh}(\mu_h, \mu_f) := \Pi_f(\mu_h, \mu_f, 1 - \mu_h - \mu_f) - \Pi_h(\mu_h, \mu_f, 1 - \mu_h - \mu_f)$ satisfies*

$$\frac{\partial \Pi_{fh}(\mu_h, \mu_f)}{\partial \mu_f} < 0 \quad ; \quad \frac{\partial \Pi_{fh}(\mu_h, \mu_f)}{\partial \mu_h} > 0.$$

We proceed to show that this assumption is met when $q_F \approx q_H$. Using the expressions for profits per state, the function of interest is $\Pi_{fh}(\boldsymbol{\mu})/\rho = \delta^F \pi[z^F(\boldsymbol{\mu}), q_F] - \delta^H \pi[z^H(\boldsymbol{\mu}), q_H] + \delta^B[\pi^{B,F}(\boldsymbol{\mu}) - \pi^{B,H}(\boldsymbol{\mu})]$. Since z^F is solely a function of μ_h, we have

$$\frac{\partial \Pi_{fh}(\mu_h, \mu_f)}{\partial \mu_f} \propto -\delta^H \frac{d\pi[z^H(\boldsymbol{\mu}), q_H]}{d\mu_f} - \delta^B \left(\frac{\partial \pi^{B,F}(\boldsymbol{\mu})}{\partial \mu_f} - \frac{\partial \pi^{B,H}(\boldsymbol{\mu})}{\partial \mu_f}\right).$$

From (45), it is easy to see that $\partial(\pi^{B,F} - \pi^{B,H})/\partial n^{B,H} \to 0$ as $q_F \to q_H$. Since the first term is strictly negative, this guarantees that Π_{fh} is decreasing in μ_f for $q_F \approx q_H$. Similarly, since z^H is independent of μ_h, we have

$$\frac{\partial \Pi_{fh}(\mu_h, \mu_f)}{\partial \mu_h} \propto \delta^F \frac{d\pi[z^F(\boldsymbol{\mu}), q_F]}{d\mu_h} + \delta^B \left(\frac{\partial \pi^{B,F}(\boldsymbol{\mu})}{\partial \mu_h} - \frac{\partial \pi^{B,H}(\boldsymbol{\mu})}{\partial \mu_h}\right).$$

From the expression for $\partial \log \pi^{B,i}/\partial \mu_h$, it is also easy to see that $\partial(\pi^{B,F} - \pi^{B,H})/\partial \mu_h \to 0$ for $q_F \to q_H$, and therefore Π_{fh} is increasing in μ_h.

We can now move to proving Figure 3.[29] For that, we proceed in two ways. First, we analyze an economy with $\gamma_F < \gamma_H$ but $q_F = q_H$. When that is the case, firms remain indifferent between the two countries in state B and there is a unique price, z^B, allowing us to re-use most of the results of Section A.2. The proof is by construction, and does not require Assumption 3.

Since country H is safer and offers the same marginal cost, relatively more firms want to settle a supply chain there than in country F. To see this, suppose the contrary: firms invest relatively more in the risky country, $\mu_f > 0$ and $\mu_f \geq \mu_h$. From the market clearing conditions, we then have

$$s[z^F(\boldsymbol{\mu})] = \frac{1}{\rho(\mu_f + \mu_b)} \leq \frac{1}{\rho(\mu_h + \mu_b)} = s[z^H(\boldsymbol{\mu})].$$

Since $z \to s(z)$ is decreasing, it must be that $z^H(\boldsymbol{\mu}) \leq z^F(\boldsymbol{\mu})$. But $z \to \pi(z)$ is decreasing, so that $\delta^F\pi[z^F(\boldsymbol{\mu})) \leq \delta^F\pi[z^H(\boldsymbol{\mu})] < \delta^H\pi[z^H(\boldsymbol{\mu})]$. However, this in turn implies that the expected profits of the foreign strategy are lower, $\Pi_f < \Pi_h$, and therefore $\mu_f = 0$, a contradiction. Hence, in equilibrium, it must either be that no firms invest in the risky country, $\mu_f = 0$, or if some firms do, relatively more firms need to invest in the safe country, $\mu_f > 0$ and $\mu_h > \mu_f$. Accordingly, it must also be that the expected profits of the safer strategy are (weakly) higher than the expected profits of the less safe strategy, $\Pi_h \geq \Pi_f$.

Given that the expected profits of the home supply chain are weakly higher than those with a supplier in the foreign country, firms' investments are dictated by two comparisons: home versus foreign supply chains, $\Pi_h(\mu_h, \mu_f, \mu_b) - \Pi_f(\mu_h, \mu_f, \mu_b)$, and single supply chain at home versus diversification, $\Pi_h(\mu_h, \mu_f, \mu_b) - \Pi_b(\mu_h, \mu_f, \mu_b)$.[30] Using the expressions for expected profits, these two optimality conditions respectively read

$$\Pi_h(\mu_h, \mu_f, \mu_b) \geq \Pi_f(\mu_h, \mu_f, \mu_b) \quad \iff \quad \delta^H\pi[z^H(\mu_h, \mu_f, \mu_b)] \geq \delta^F\pi[z^F(\mu_h, \mu_f, \mu_b)],$$

and

$$\Pi_b(\mu_h, \mu_f, \mu_b) \geq \Pi_h(\mu_h, \mu_f, \mu_b) \quad \iff \quad \delta^F\pi[z^F(\mu_h, \mu_f, \mu_b)] + \delta^B\rho(1-\rho)\pi[z^B(\mu_h, \mu_f, \mu_b)] \geq k.$$

In addition, profits must be positive, $\Pi_j(\mu_h, \mu_f, \mu_b) \geq 0$ for $j \in \{h, f, b\}$.

For $k \searrow 0$, investing in resilience is clearly the most profitable option, $\Pi_b > \Pi_h > \Pi_f$, where the second inequality follows from $\delta^H\pi[z^H(0,0,1)] > \delta^F\pi[z^F(0,0,1)]$. Hence, for low k, $\mu_h = 0 = \mu_f$ and $\mu_b = 1$. As the fixed cost increases, the gap between $\Pi_h(0,0,1) - \Pi_b(0,0,1)$ shrinks to the point where the two strategies yield the same expected profits. This occurs at k_1, defined by

$$k_1 := \delta^F\rho\pi[z^F(0,0,1)] + \delta^B\rho(1-\rho)\pi[z^B(0,0,1)].$$

At $k = k_1$, we thus have $\Pi_b(0,0,1) = \Pi_h(0,0,1) > \Pi_f(0,0,1)$. For $\mu_h \in [0,1]$, there always exists

[29] For both proofs we assume that C_P is small enough so that we do not have to check $\Pi_j < 0$ until $k > k_4$.

[30] Without $\Pi_h \geq \Pi_f$, we would also need to compare $\Pi_f - \Pi_b$.

a $k > k_1$ that ensures $\Pi_b(\mu_h, 0, 1-\mu_h) = \Pi_h(\mu_h, 0, 1-\mu_h)$, defined by[31]

$$\kappa^2(\mu_h) := \delta^F \rho\pi[z^F(\mu_h, 0, 1-\mu_h)] + \delta^B \rho(1-\rho)\pi[z^B(\mu_h, 0, 1-\mu_h)],$$

with $\kappa^2(0) = k_1$. As in Section A.2, $\pi[z^F(\boldsymbol{\mu})]$ and $\pi[z^B(\boldsymbol{\mu})]$ are increasing in μ_h, and therefore so is $\kappa^2(\mu_h)$.[32] Let $\mu_h^2(k)$ denote the inverse of $\kappa^2(\mu_h)$. The allocation $(\mu_h^2(k), 0, 1-\mu_h^2(k))$ is an equilibrium if and only if $\Pi_h(\mu_h(k), 0, 1-\mu_h(k)) > \Pi_f(\mu_h(k), 0, 1-\mu_h(k))$. The difference between the two expected profits can be rewritten as

$$\Pi_h(\mu_h, 0, 1-\mu_h) - \Pi_f(\mu_h, 0, 1-\mu_h) = \delta^H \rho\pi[z^H(\mu_h, 0, 1-\mu_h)] - \delta^F \rho\pi[z^F(\mu_h, 0, 1-\mu_h)].$$

When $\mu_f = 0$, z^H is obtained from $\rho s[z^H(\boldsymbol{\mu})] = 1$, so that z^H is constant. Hence, $\Pi_h - \Pi_f$ is monotonically decreasing in μ_h. There are therefore two cases. If $\Pi_h(1,0,0) \geq \Pi_f(1,0,0)$, then that is also the case for all $\mu_h \in [0,1]$, and the allocation $(\mu_h(k), 0, 1-\mu_h(k))$ is an equilibrium for all $k \in (k_1, k_2)$, where $k_2 := \kappa^2(1)$. On the contrary, if $\Pi_h(1,0,0) < \Pi_f(1,0,0)$, then there exists a $\bar{\mu}_h \in (0,1)$ for which $\Pi_h(\bar{\mu}_h, 0, 1-\bar{\mu}_h) = \Pi_f(\bar{\mu}_h, 0, 1-\bar{\mu}_h)$ and $\Pi_h(\mu_h, 0, 1-\mu_h) > \Pi_f(\mu_h, 0, 1-\mu_h)$ for $\mu_h < \bar{\mu}_h$. In that case, $(\mu_h(k), 0, 1-\mu_h(k))$ is an equilibrium for all $k \in (k_1, k_2)$, where $k_2 := \kappa^2(\bar{\mu}_h)$, but not for $k > k_2$. In both cases, μ_h is increasing with k.

Focusing on the second case, at $k = k_2$ the three strategies yield the same expected profits. For $\mu > \mu_h(k_2)$, define the function $\mu_f(\mu_h)$ that ensures equiprofitability between the home and foreign supply; specifically, let the function $\mu_f(\mu_h)$ satisfy

$$\delta^F \pi[z^F(\mu_h, \mu_f(\mu_h), 1-\mu_h-\mu_f(\mu_h))] = \delta^H \pi[z^H(\mu_h, \mu_f(\mu_h), 1-\mu_h-\mu_f(\mu_h))]. \tag{47}$$

The monotonicity of $\mu \to \pi[z(\mu)]$ ensures that $\mu_h \to \mu_f(\mu_h)$ is well-defined for all μ_h so that $\mu_h + \mu_f(\mu_h) \leq 1$ and is increasing. Abusing the notation, let $\bar{\mu}_h$ be such that $\bar{\mu}_h + \mu_f(\bar{\mu}_h) = 1$. Meanwhile, to guarantee that $(\mu_h, \mu_f(\mu_h), 1-\mu_h-\mu_f(\mu_h))$ is an equilibrium, we also need to ensure indifference between the home supply chain and the diversification strategy. This is guaranteed by appropriately adjusting the fixed cost; specifically, define $\kappa^3(\mu_h)$ on $\mu_h \in (\mu_h(k_2), \bar{\mu}_h)$ by

$$\kappa^3(\mu_h) := \delta^F \rho\pi[z^F(\mu_h, \mu_f(\mu_h), 1-\mu_h-\mu_f(\mu_h))] + \delta^B \rho(1-\rho)\pi[z^B(\mu_h, \mu_f(\mu_h), 1-\mu_h-\mu_f(\mu_h))].$$

Both $\pi(z^F)$ and $\pi(z^B)$ are increasing in μ_h, and therefore so is $\kappa^3(\mu_h)$.[33] Let $k_3 := \kappa^3(\bar{\mu}_h)$ and $\mu_h^3(k)$ denote the inverse function of $\kappa^3(\mu_h)$. By construction, the allocation $(\mu_h(k), \mu_f[\mu_h(k)], 1 - \mu_h(k) - \mu_f[\mu_h(k)])$ is thus an equilibrium for all k in (k_2, k_3), with both μ_h and μ_f increasing with k.

[31] For any $\mu_h \in [0,1]$, we have $\pi[z^j(\mu_h, 0, 1-\mu_h)] < \infty$. Hence, $\Pi_h - \Pi_b < 0$ for $k \searrow 0$, $\Pi_h - \Pi_b > 0$ for $k \nearrow 0$, and $\Pi_h - \Pi_b$ is monotonically increasing in k.

[32] The market clearing conditions are $(1-\mu_h)\rho s[z^F(\boldsymbol{\mu})] = 1$ and $\rho[1 + (1-\rho)(1-\mu_h)]s[z^B(\boldsymbol{\mu})] = 1$, from which it follows that z^F and z^B are decreasing in μ_h. The monotonicity of $\mu \to \pi[z(\mu)]$ then follows from Lemma 1.

[33] This again follows from the market clearing conditions and Lemma 1, which together ensures that $\pi[z^F(\boldsymbol{\mu})]$ is an increasing function of $\mu_f + \mu_h$.

At $k = k_3$, firms are indifferent between the three strategies, and $\Pi_h(\bar{\mu}_h, 1-\bar{\mu}_h, 0) = \Pi_f(\bar{\mu}_h, 1-\bar{\mu}_h, 0) = \Pi_b(\bar{\mu}_h, 1-\bar{\mu}_h, 0)$. For any $k > k_3$, investing in both supply chains is no longer the most profitable strategy, so that $\Pi_h(\bar{\mu}_h, 1-\bar{\mu}_h, 0) = \Pi_f(\bar{\mu}_h, 1-\bar{\mu}_h, 0) > \Pi_b(\bar{\mu}_h, 1-\bar{\mu}_h, 0)$. The allocation $(\bar{\mu}_h, 1-\bar{\mu}_h, 0)$ therefore constitutes an equilibrium for $k \geq k_3$ as long as $\Pi_h(\bar{\mu}_h, 1-\bar{\mu}_h, 0) = \Pi_f(\bar{\mu}_h, 1-\bar{\mu}_h, 0) \geq 0$. Let k_4 denote the level of fixed cost that make firms indifferent between investing in either supply chain or exiting,

$$k_4 := \delta^H \rho\pi[z^H(\bar{\mu}_h, 1-\bar{\mu}_h, 0)] + \delta^B \rho\pi[z^B(\bar{\mu}_h, 1-\bar{\mu}_h, 0)]$$

Clearly, for any $k > k_4$, $\Pi_h(\bar{\mu}_h, 1-\bar{\mu}_h, 0) = \Pi_f(\bar{\mu}_h, 1-\bar{\mu}_h, 0) < 0$, and therefore $(\bar{\mu}_h, 1-\bar{\mu}_h, 0)$ is no longer an equilibrium. Instead, with a slight abuse of notation, let $\mu_f(\mu_h)$ denote the function that guarantees the equiprofitability between the home and foreign supply chain when $\mu_b = 0$,[34]

$$\delta^F \pi[z^F(\mu_h, \mu_f(\mu_h), 0)] = \delta^H \pi[z^H(\mu_h, \mu_f(\mu_h), 0)].$$

The functions z^H and z^F are obtained from the market clearing conditions $\rho\mu_h s[z^H(\boldsymbol{\mu})] = 1$ and $\rho\mu_f s[z^F(\boldsymbol{\mu})] = 1$ respectively, so that z^H is increasing in μ_h and z^F is increasing in μ_f. Jointly with Lemma 1, this guarantees that the function $\mu_f(\mu_h)$ is well defined as long as $\mu_f(\mu_h) \geq 0$ and μ_f is increasing in μ_h. If $\lim_{\mu\to 0} \pi[z(\mu, \mu, 0)] = \infty$, then μ_f is well-defined on $(0, \bar{\mu}_h)$. Otherwise, since $\delta^F \pi[z^F(0,0,0)] < \delta^H \pi[z^H(0,0,0)]$, it must be that $\delta^F \pi[z^F(\underline{\mu}_h, 0, 0)] = \delta^H \pi[z^H(\underline{\mu}_h, 0, 0)]$ for some $\underline{\mu}_h > 0$.

Given that, by construction, firms are indifferent between the home and foreign supply chain, $(\mu_h, \mu_f(\mu_h), 0)$ constitutes an equilibrium for $k > k_4$ if and only if (1) profits are weakly positive, and (2) investing in both places is not more profitable. To satisfy the first condition, adjust the fixed cost so as to ensure $\Pi_h = 0$[35]

$$\kappa^5(\mu_h) = \delta^H \rho\pi[z^H(\mu_h, \mu_f(\mu_h), 0)] + \delta^B \rho\pi[z^B(\mu_h, \mu_f(\mu_h), 0)],$$

for all $\mu_h \in (\underline{\mu}_h, \bar{\mu}_h)$. We already know that $\pi[z^H(\boldsymbol{\mu})]$ is decreasing in μ_h. Meanwhile, the function z^B is given by $\rho(\mu_f + \mu_h)s[z^B(\boldsymbol{\mu})]$, so that z^B is increasing in μ_h, and $\pi[z^B(\mu_h, \mu_f(\mu_h), 0)]$ is decreasing in μ_h. Together, these imply that κ^5 is decreasing in μ_h on $(\underline{\mu}_h, \bar{\mu}_h)$, or said differently, μ_h is increasing in k for $k \in (k_4, k_5)$, where $k_5 := \kappa^5(\underline{\mu}_h)$. Regarding the second condition, the expected profits of the diversification strategy can be written as $\Pi_b = \Pi_h + \Pi_f - \delta^B \rho^2 \pi(z^B)$, such that $\Pi_h = \Pi_f = 0$ implies $\Pi_b < 0$ and no firm wants to invest in both supply chains. As before, let $\mu_h^5(k)$ denote the inverse of $k(\mu_h)$. For $k \in (k_4, k_5)$, the allocation $(\mu_h(k), \mu_f[\mu_f(k)], 0)$ is therefore an equilibrium, and both μ_h and μ_f are decreasing in k.[36]

[34]Note that this function is different from the one defined in (47) precisely because $\mu_b = 0$.

[35]Note that $\Pi_h(\mu_h, \mu_f(\mu_h), 0) = \delta^H \rho\pi[z^H(\mu_h, \mu_f(\mu_h), 0)] + \delta^B \rho(1-\rho)\pi[z^B(\mu_h, \mu_f(\mu_h), 0)] - k$ is decreasing in μ_h. Since $\Pi_h(\bar{\mu}_h, 1-\bar{\mu}_h, 0) = 0$ at k_4, it simply cannot be that, in equilibrium, $\Pi_h > 0$ for $k > k_4$.

[36]Note that, if $\underline{\mu}_h = 0$, then $k_5 = \infty$.

Finally, for $k > k_5$, the equilibrium consists of $(\mu_h, 0, 0)$, where μ_h ensures $\Pi_h = 0$, specifically

$$\delta^H \rho\pi[z^H(\mu_h, 0, 0)] + \delta^B \rho\pi[z^B(\mu_h, 0, 0)] = k.$$

The functions $\pi[z^H(\mu_h, 0, 0)]$ and $\pi[z^B(\mu_h, 0, 0)]$ remain decreasing in μ_h, so that μ_h decreases with k. As for $k \in (k_4, k_5)$, $\Pi_h = 0$ ensures that $\Pi_b < 0$. Furthermore, $\mu_h(k) < \underline{\mu}_h$, which guarantees that $\Pi_f < \Pi_h$. If $\pi[z(0, 0, 0)] < \infty$, there exists a final threshold k_6 above which no firms want to invest in any supply relationship,

$$k_6 := \lim_{\mu_h \searrow 0} \delta^H \rho\pi[z^H(\mu_h, 0, 0)] + \delta^B \rho\pi[z^B(\mu_h, 0, 0)].$$

We now move to the second economy of interest with asymmetric marginal costs but homogeneous risk, $q_F < q_H$ and $\gamma_F = \gamma_H$. The proof here is also by construction, but it requires Assumption 3. Since the proof is similar to the proof with asymmetric risk, some steps are omitted.

As always, when the fixed cost is small, it is strictly more profitable for firms to diversify their supply chains. For larger k, firms become indifferent between diversifying their supply chains and investing in the low-cost country only. Specifically, $k_1 > 0$ equates $\Pi_b(0, 0, 1) = \Pi_f(0, 0, 1) > \Pi_h(0, 0, 1)$.[37]

For slightly higher fixed costs, firms remain indifferent between investing in both supply chains or in the low-cost country only, while the home-only investment remains dominated. To see this, note that for any $\mu_f \in (0, 1)$, there exists a fixed cost $k > k_1$ that renders firms indifferent between strategy b and f. This fixed cost is

$$\kappa^2(\mu_f) := \delta\rho\pi[z^H(0, \mu_f, 1 - \mu_f), q_H] + \delta^B \rho(1 - \rho)\pi^{B,H}(0, \mu_f, 1 - \mu_f).$$

As in the case with symmetric marginal cost, $\pi(z^H)$ is increasing with μ_f. Combined with Corollary 1, this implies that κ^2 is an increasing function. Let $k \to \mu_f^2(k)$ denote the inverse of $\mu_f \to \kappa^2(\mu_f)$, so that μ_f^2 is increasing with k. The allocation $(0, \mu_f^2(k), 1 - \mu_f^2(k))$ is an equilibrium if and only if $\Pi_{fh}(0, \mu_f^2(k), 1 - \mu_f^2(k)) > 0$. We already know that $\Pi_{fh}(0, 0, 1) > 0$. Furthermore, by Assumption 3, $\mu_f \to \Pi_{fh}(0, \mu_f, 1 - \mu_f)$ is increasing in μ_f. Hence, if $\Pi_{fh}(0, 1, 0) < 0$, there exists a $\mu_f^\star < 1$ so that $\Pi_{fh}(0, \mu_f^\star, 1 - \mu_f^\star) = 0$. Letting $k_2 := \kappa^2(\mu_f^\star)$, it follows that $(0, \mu_f^2(k), 1 - \mu_f^2(k))$ is an equilibrium for $k \in (k_1, k_2)$.[38]

Assuming that $\Pi_{fh}(0, 1, 0) < 0$ indeed holds, at $k = k_2$, firms are indifferent between all three strategies, which remains valid for higher fixed cost of investment. To see this, let $\bar{\mu}_f$ be the (unique) solution to $\Pi_{fh}(1 - \bar{\mu}_f, \bar{\mu}_f, 0) = 0$.[39] Then, define the function $\mu_f \to \mu_h(\mu_f)$ to ensure equiprofitability between the foreign-only and home-only supply chain, $\Pi_{fh}(\mu_h(\mu_f), \mu_f, 1 - \mu_h(\mu_f) - \mu_f) = 0$, for

[37] $\Pi_f(0, 0, 1) > \Pi_h(0, 0, 1)$ follows from $\pi[z^F(0, 0, 1), q_F] > \pi[z^H(0, 0, 1), q_H]$ and $\pi^{B,F}(\mu) > \pi^{B,H}(\mu)$ for all μ.

[38] As in the asymmetric risk economy, $\Pi_{fh}(0, 1, 0) < 0$ is not necessarily satisfied. If $\Pi_{fh}(0, 1, 0) > 0$, then $(0, \mu_f^2(k), 1 - \mu_f^2(k))$ is an equilibrium for $k \in (k_1, k_2(1))$, and $\mu_h = 0$ for all k.

[39] This solution is unique because $\Pi_{fh}(1, 0, 0) > 0$ and $\Pi_{fh}(0, 1, 0) < 0$ while $\mu_f \to \Pi_{fh}(1 - \mu_f, \mu_f, 0)$ is decreasing.

$\mu_f \in (\mu_f(k_2), \bar{\mu}_f)$. This function is well-defined and increasing. Indeed, $\Pi_{fh}(0, \mu_f(k_2), 1-\mu_f(k_2)) = 0$ implies that $\Pi_{fh}(0, \mu_f, 1-\mu_f) < 0$ for all $\mu_f > \mu_f(k_2)$ per Assumption 3. Furthermore, $\Pi_{fh}(1-\mu_f, \mu_f, 0) \geq 0$ for all $\mu_f \leq \bar{\mu}_f$ since $\mu_f \to \Pi_{fh}(1-\mu_f, \mu_f, 0) \geq 0$ is decreasing. Finally, $\mu_h \to \Pi_{fh}(\mu_h, \mu_f, 1-\mu_h-\mu_f)$ is increasing (Assumption 3), which ensures the uniqueness and monotonicity of $\mu_f \to \mu_h(\mu_f)$.

The allocation $(\mu_h(\mu_f), \mu_f, 1-\mu_h(\mu_f)-\mu_f)$ is then an equilibrium if and only if firms are indifferent between investing only in the low-cost country or diversifying their supply chains. This is achieved by appropriately setting k. Specifically, define

$$\kappa^3(\mu_f) := \delta\rho\pi[z^H(\mu_h(\mu_f), \mu_f, 1-\mu_f-\mu_h(\mu_f)), q_H] + \delta^B\rho(1-\rho)\pi^{B,H}(\mu_h(\mu_f), \mu_f, 1-\mu_f-\mu_h(\mu_f)),$$

so as to ensure $\Pi_f = \Pi_b$ along $(\mu_h(\mu_f), \mu_f, 1-\mu_f-\mu_h(\mu_f))$. Both $\pi(z^H)$ and $\pi^{B,H}$ are increasing in μ_f, and therefore so is κ^3. Let $k \to \mu_f^3(k)$ denote the inverse function of $\mu_f \to \kappa^3(\mu_f)$ and $k_3 := \kappa^3(\bar{\mu}_f)$. Then, $(\mu_h[\mu_f(k)], \mu_f(k), 1-\mu_h[\mu_f(k)]-\mu_f(k))$ is an equilibrium for $k \in (k_2, k_3)$, and both μ_h and μ_f are increasing with k.

For $k > k_3$, it automatically follows that $\Pi_j(1-\bar{\mu}_f, \bar{\mu}_f, 0) = \Pi_f(1-\bar{\mu}_f, \bar{\mu}_f, 0) > \Pi_b(1-\bar{\mu}_f, \bar{\mu}_f, 0)$, and investing in strategy b is no longer the most profitable option. Let k_4 be the fixed cost that renders firms indifferent between existing and investing in a single-country supply chain,

$$k_4 := \delta\pi[z^F(1-\bar{\mu}_f, \bar{\mu}_f, 0)] + \delta^B\rho\pi^{B,F}(1-\bar{\mu}_f, \bar{\mu}_f, 0).$$

The allocation $(1-\bar{\mu}_f, \bar{\mu}_f, 0)$ therefore constitutes an equilibrium for $k \in (k_3, k_4)$. For $k > k_4$, $\Pi_j(1-\bar{\mu}_f, \bar{\mu}_f, 0) < 0$ for $j \in \{h, f\}$ and some firms want to exit. Since the focus of the policy analysis is for $k \in (k_2, k_3)$ and the remainder of the proof for $k > k_3$ is similar to the case with $\gamma_F < \gamma_H$ but $q_F = q_H$, we omit the details and move instead directly to the slopes of the iso-welfare curves.

For the rest of this section, we focus on values of $k \in (k_2, k_3)$ such that $\mu_h > 0$, $\mu_f > 0$ and $\mu_b = 1-\mu_h-\mu_f > 0$. For these fixed costs, firms are indifferent among all three strategies. The equilibrium is located at the intersection between the three functions $\mu_f \to \mu_h(\mu_f)$ defined implicitly by $\Pi_h(\boldsymbol{\mu}) = \Pi_f(\boldsymbol{\mu})$, $\Pi_h(\boldsymbol{\mu}) = \Pi_b(\boldsymbol{\mu})$ and $\Pi_f(\boldsymbol{\mu}) = \Pi_b(\boldsymbol{\mu})$. We now proceed to show that the curves defined by these indifference conditions have the qualitative properties shown in Figure **??**. Starting from the indifference condition between the home-only and foreign-only supply chain, $\Pi_f - \Pi_h = 0$ implies

$$\left.\frac{d\mu_h}{d\mu_f}\right|_{\Pi_f=\Pi_h} = -\frac{\partial\Pi_{fh}}{\partial\mu_f} \Big/ \frac{\partial\Pi_{fh}}{\partial\mu_h} > 0,$$

where the inequality follows from Assumption 3. Turning to the indifference between the foreign-only supply chain and the diversification strategy, we have

$$\left.\frac{d\mu_h}{d\mu_f}\right|_{\Pi_f=\Pi_b} = -\left(\frac{\delta^H}{1-\rho}\frac{d\pi[z^H(\mu_f), q_H]}{d\mu_f} + \frac{\partial \pi^{B,H}(\boldsymbol{\mu})}{\partial \mu_f}\right) \Big/ \frac{\partial \pi^{B,H}(\boldsymbol{\mu})}{\partial \mu_h} < 0,$$

where the inequality follows from Corollary 1 and 2. Finally, the equiprofitability of the home-only and resilience strategy implies

$$\left.\frac{d\mu_h}{d\mu_f}\right|_{\Pi_h=\Pi_b} \left[\delta^F \frac{d\pi[z^F(\mu_h), q_F]}{d\mu_h} + \delta^B\left(\frac{\partial \pi^{B,F}(\boldsymbol{\mu})}{\partial \mu_h} - \rho\frac{\partial \pi^{B,H}(\boldsymbol{\mu})}{\partial \mu_h}\right)\right] = -\delta^B\left(\frac{\partial \pi^{B,F}(\mu)}{\partial \mu_f} - \rho\frac{\partial \pi^{B,H}(\boldsymbol{\mu})}{\partial \mu_f}\right).$$

From Assumption 3, the left hand side is positive.[40] Furthermore, the right-hand side is negative if and only if $\Pi_h - \Pi_b = -\delta^F \pi[z^F(\mu_h), q_F] - \delta^B \rho[\pi^{B,F}(\boldsymbol{\mu}) - \rho\pi^{B,H}(\boldsymbol{\mu})] + k$ is decreasing in μ_f, which we now assume.

Assumption 4 *The function* $\Pi_{hb}(\mu_h, \mu_f) := \Pi_h(\mu_h, \mu_f, 1-\mu_h-\mu_f) - \Pi_b(\mu_h, \mu_f, 1-\mu_h-\mu_f)$ *satisfies*

$$\frac{\partial \Pi_{hb}(\mu_h, \mu_f)}{\partial \mu_f} < 0.$$

As with Assumption 3, this is satisfied for $q_F \approx q_H$.[41]

Lastly, we want to characterize optimal policies in the world with asymmetric costs and risks. Welfare as a function of (μ_h^o, μ_f^o) reads

$$W(\mu_h, \mu_f) = \sum_{j=H,F,B} \delta^j \Pi_j(\mu_h, \mu_f) + \frac{1}{\varepsilon - 1}\left(\sum_{j=H,F} \delta^j P[z^j(\mu_{-j})]^{1-\varepsilon} + \delta^B P^B(\boldsymbol{\mu})^{1-\varepsilon}\right).$$

Analyzing this function is complicated for general HSA preferences. However, it is possible to extend Lemma 3 to the case where $\gamma_F < \gamma_H$ but $q_F \approx q_H$. Under symmetric costs, firms are indifferent between the two countries in state B, and therefore (39) remains valid except that $\mu_h > \mu_f$ when $\gamma_F < \gamma_H$. In particular, under symmetric translog preferences, the change in welfare

[40] Indeed, $\partial \Pi_{fh}/\partial \mu_h > 0$ writes

$$\delta^F \frac{d\pi[z^F(\mu_h), q_F]}{d\mu_h} + \delta^B\left(\frac{\partial \pi^{B,F}}{\partial \mu_h} - \frac{\partial \pi^{B,H}}{\partial \mu_h}\right) > 0,$$

and $\rho \leq 1$.

[41] We have

$$\frac{\partial \Pi_{hb}}{\partial \mu_f} = -\delta^B \rho\left(\frac{\partial \pi^{B,F}(\boldsymbol{\mu})}{\partial \mu_f} - \rho\frac{\partial \pi^{B,H}(\boldsymbol{\mu})}{\partial \mu_f}\right) \to -\delta^B \rho \frac{\partial \pi^B(\boldsymbol{\mu})}{\partial \mu_f}(1-\rho) < 0.$$

caused by a variation $d\boldsymbol{\mu} = (d\mu_f, d\mu_h)$ is given by

$$-dW(\boldsymbol{\mu}) = \sum_{i=H,F}\left(\frac{1}{2}+\frac{\varepsilon(1+3\theta n^i(\boldsymbol{\mu}))}{2\theta n^i(\boldsymbol{\mu})(1+\theta n^i(\boldsymbol{\mu}))}\right)\frac{\delta^i}{1+\theta n^i(\boldsymbol{\mu})}\frac{\rho\theta}{\theta n^i(\boldsymbol{\mu})}P[z^i(\boldsymbol{\mu})]^{1-\varepsilon}d\mu_{-i}+$$
$$\left(\frac{1}{2}+\frac{\varepsilon(1+3\theta n^B(\boldsymbol{\mu}))}{2\theta n^B(\boldsymbol{\mu})(1+\theta n^B(\boldsymbol{\mu}))}\right)\frac{\delta^B}{1+\theta n^B(\boldsymbol{\mu})}\frac{\rho(1-\rho)\theta}{\theta n^B(\boldsymbol{\mu})}P[z^B(\boldsymbol{\mu})]^{1-\varepsilon}\sum_i d\mu_i - k(d\mu_h+d\mu_f),$$

while the indifference conditions for an interior equilibrium, $\Pi_f = \Pi_b$ and $\Pi_h = \Pi_b$, are

$$\frac{\rho\delta^H\theta}{\theta n^H(\boldsymbol{\mu})(1+\theta n^H(\boldsymbol{\mu}))}P[z^H(\boldsymbol{\mu})]^{1-\varepsilon}+\frac{\delta^B\rho(1-\rho)\theta}{\theta n^B(\boldsymbol{\mu})(1+\theta n^B(\boldsymbol{\mu}))}P[z^B(\boldsymbol{\mu})]^{1-\varepsilon}=k,$$
$$\frac{\rho\delta^F\theta}{\theta n^F(\boldsymbol{\mu})(1+\theta n^F(\boldsymbol{\mu}))}P[z^F(\boldsymbol{\mu})]^{1-\varepsilon}+\frac{\delta^B\rho(1-\rho)\theta}{\theta n^B(\boldsymbol{\mu})(1+\theta n^B(\boldsymbol{\mu}))}P[z^B(\boldsymbol{\mu})]^{1-\varepsilon}=k.$$

Combining the two,

$$2dW(\boldsymbol{\mu}) = \sum_i\left(1-\frac{\varepsilon(1+3\theta n^i(\boldsymbol{\mu}))}{\theta n^i(\boldsymbol{\mu})(1+\theta n^i(\boldsymbol{\mu}))}\right)\frac{\delta^i}{1+\theta n^i(\boldsymbol{\mu})}\frac{\rho\theta}{\theta n^i(\mu)}P[z^i(\boldsymbol{\mu})]^{1-\varepsilon}d\mu_{-i}+$$
$$\left(1-\frac{\varepsilon(1+3\theta n^B(\boldsymbol{\mu}))}{\theta n^B(\boldsymbol{\mu})(1+\theta n^B(\boldsymbol{\mu}))}\right)\frac{\delta^B}{1+\theta n^B(\boldsymbol{\mu})}\frac{\rho(1-\rho)\theta}{\theta n^B(\boldsymbol{\mu})}P[z^B(\boldsymbol{\mu})]^{1-\varepsilon}\sum_i d\mu_i.$$

As before, the terms in parenthesis are increasing in $n^i(\boldsymbol{\mu})$ for $i \in \{H, F, B\}$ respectively. Furthermore, $n^i(\boldsymbol{\mu}) < n^B(\boldsymbol{\mu})$ for $i \in \{H, F\}$. Finally, $\mu_h > \mu_f$ implies $n^H(\boldsymbol{\mu}) > n^F(\boldsymbol{\mu})$. It follows that if the expression in parenthesis is positive for $i = F$, then it is positive for $i \in \{H, F, B\}$. Hence, in this case, $dW(\boldsymbol{\mu}) > 0$, and increasing μ_f and μ_h – that is, reducing diversification – is welfare improving. On the contrary, if the expression is negative for $i = B$, then it is negative for all J, and increasing diversification raises W. This allows us to extend Lemma 3.

Lemma 6 *Suppose that $q_F = q_H$ but $\gamma_F < \gamma_H$ and preferences are symmetric translog. Then, O is above E if*

$$\frac{\theta n^F(\boldsymbol{\mu})(1+\theta n^F(\boldsymbol{\mu}))}{1+3\theta n^F(\boldsymbol{\mu})} > \varepsilon,$$

while O is below E if

$$\frac{\theta n^B(\boldsymbol{\mu})(1+\theta n^B(\boldsymbol{\mu}))}{1+3\theta n^B(\boldsymbol{\mu})} < \varepsilon.$$

A.4 A Special Case: CES Preferences

In this section, we focus on the limit case where the elasticity of substitution between varieties is constant, $\sigma'(z) = 0$. The corresponding market share function is $s(z) = z^{1-\sigma}$, where σ is the (constant) elasticity of substitution, $\sigma(z) = \sigma > \varepsilon > 0$. The pricing equation yields an explicit expression for prices,

$$p^J = \left(\frac{\sigma}{\sigma-1}\right) q_J,$$

for $J \in \{H, F\}$. In particular, prices become independent of the number of active firms and only depend on the location where the input is produced.

In states $J \in \{H, F\}$, the market clearing condition (4) implies $z^J(\boldsymbol{\mu}) = n^J(\boldsymbol{\mu})^{1/(\sigma-1)}$. Since $n^H(\boldsymbol{\mu}) = \rho(1-\mu_F)$, $z^H(\boldsymbol{\mu})$ is only a function of μ_F. Similarly, $z^F(\boldsymbol{\mu})$ is only a function of μ_h. Imposing $C_P = 1/(\sigma-1) - \log[\sigma/(\sigma-1)]$ for notational simplicity, the price index (21) simplifies to[42]

$$P[z^J(\boldsymbol{\mu}), q_J] = \frac{q_J}{z^J(\boldsymbol{\mu})} = n^J(\boldsymbol{\mu})^{\frac{1}{1-\sigma}} \cdot q_J, \tag{48}$$

for $J \in \{H, F\}$. As in standard trade models, the price index is decreasing in the number of available varieties in state J, $n^J(\boldsymbol{\mu})$. Plugging the functional form assumption and the expression for the price index in (22), the profit of an active firm in state $J \in \{H, F\}$ becomes

$$\pi[z^J(\boldsymbol{\mu}), q_J] = \left(\frac{q_J^{1-\varepsilon}}{\sigma}\right) n^J(\boldsymbol{\mu})^{\frac{\varepsilon-\sigma}{\sigma-1}}.$$

In state B, the price index (27) becomes a weighted geometric average of the marginal cost where the weights are the number of active firms in each country,[43]

$$P^B(\boldsymbol{\mu}) = \left(\sum_{J=H,F} n^{B,J}(\boldsymbol{\mu}) q_J^{1-\sigma}\right)^{\frac{1}{1-\sigma}}. \tag{49}$$

Profits of a firm that sources in state B from country $J \in \{H, F\}$ then equal

$$\pi^{B,J}(\boldsymbol{\mu}) = \left(\frac{q_J^{1-\sigma}}{\sigma}\right) \left(\sum_{\ell=H,F} n^{B,\ell}(\boldsymbol{\mu}) q_\ell^{1-\sigma}\right)^{\frac{\varepsilon-\sigma}{\sigma-1}}.$$

Adapting (29) and (30), the expected profits of a firm that invests in a single-country supply

[42] From the market clearing condition and the pricing equation, we have $A[z^J(\boldsymbol{\mu}), q_J] = p^J n(\boldsymbol{\mu})^{1/(1-\sigma)}$. Furthermore, from the price index (21), we also know that $\log P[z^J(\boldsymbol{\mu}), q_J] - \log A[z^J(\boldsymbol{\mu}), q_J] = C_P - 1/(\sigma-1)$ when preferences are CES. Assembling the two together and imposing the normalization on C_P yields the desired expression.

[43] The market clearing condition implies $A^B(\boldsymbol{\mu}) = (\sum n_j(\boldsymbol{\mu}) p_j^{1-\sigma})^{1/(1-\sigma)}$, while it remains the case under CES preferences that $\log P^B(\boldsymbol{\mu}) - \log A^B(\boldsymbol{\mu}) = C_P - 1/(\sigma-1)$. Using $C_P = 1/(\sigma-1) - \log[\sigma/(\sigma-1)]$ then returns (49)

chain in country $j \in \{h, f\}$ becomes

$$\Pi_j = \delta^j \left(\frac{q_j^{1-\varepsilon}}{\sigma}\right) n^j(\boldsymbol{\mu})^{\frac{\varepsilon-\sigma}{\sigma-1}} \rho + \delta^B \left(\frac{q_j^{1-\sigma}}{\sigma}\right) \left(\sum_{i=H,F} n^{B,i}(\boldsymbol{\mu}) q_i^{1-\sigma}\right)^{\frac{\varepsilon-\sigma}{\sigma-1}} \rho - k. \tag{50}$$

Similarly, the expected profits of a firm that invest in both countries are

$$\Pi_b = \sum_{j=H,F} \delta^j \left(\frac{q_j^{1-\varepsilon}}{\sigma}\right) n^j(\boldsymbol{\mu})^{\frac{\varepsilon-\sigma}{\sigma-1}} \rho + \delta^B \left[\left(\frac{q_F^{1-\sigma}}{\sigma}\right) + (1-\rho)\left(\frac{q_H^{1-\sigma}}{\sigma}\right)\right] \left(\sum_{j=H,F} n^{B,j}(\boldsymbol{\mu}) q_j^{1-\sigma}\right)^{\frac{\varepsilon-\sigma}{\sigma-1}} \rho - 2k.$$

We start by showing that, when preferences are CES, the welfare function is globally concave so that the first-order conditions are sufficient. We focus on the case where the equilibrium is interior, $k \in (k_2, k_3)$. From (8), using the expressions for expected profits just derived, the market clearing conditions $s[z^J(\boldsymbol{\mu})]n^J(\boldsymbol{\mu}) = 1$ and $\sum_{J=H,F} s[z^{B,J}(\boldsymbol{\mu})]n^{B,J}(\boldsymbol{\mu}) = 1$, and the special feature of CES that $\sigma(z) = \sigma$ for all z, the welfare function simplifies to

$$W(\mu_h, \mu_f) = c \sum_{J=H,F,B} \delta^J P^J(\boldsymbol{\mu})^{1-\varepsilon} - k(2 - \mu_h - \mu_f),$$

where $c := 1/\sigma + 1/(\varepsilon - 1)$. Plugging in the expression for the price indices, (48) and (49), we have

$$W(\mu_h, \mu_f) = c \left[\sum_{J=H,F} \delta^J n^J(\boldsymbol{\mu})^{\frac{1-\varepsilon}{1-\sigma}} q_J^{1-\varepsilon} + \delta^B \left(\sum_{\ell=H,F} n^{B,\ell}(\boldsymbol{\mu}) q_\ell^{1-\sigma}\right)^{\frac{1-\varepsilon}{1-\sigma}}\right] - k(2 - \mu_h - \mu_f).$$

As in the general HSA preferences case, the constrained optimum is obtained by maximizing this function over (μ_h, μ_f); let (μ_h^o, μ_f^o) denote the constrained optimum. If the allocation is interior, the constrained optimum satisfies the problem's first-order conditions, which are

$$c\left(\frac{1-\varepsilon}{1-\sigma}\right) \rho \left(\delta^H n^H(\boldsymbol{\mu}^o)^{\frac{\varepsilon-\sigma}{\sigma-1}} q_H^{1-\varepsilon} + \delta^B (1-\rho) q_H^{1-\sigma} P^B(\boldsymbol{\mu}^o)^{\sigma-\varepsilon}\right) = k, \tag{51}$$

with respect to μ_f^o, and

$$c\left(\frac{1-\varepsilon}{1-\sigma}\right) \rho \left(\delta^F n^F(\boldsymbol{\mu}^o)^{\frac{\varepsilon-\sigma}{\sigma-1}} q_F^{1-\varepsilon} + \delta^B \left[q_F^{1-\sigma} - \rho q_H^{1-\sigma}\right] P^B(\boldsymbol{\mu}^o)^{\sigma-\varepsilon}\right) = k, \tag{52}$$

with respect to μ_h^o, where we used that z^F is independent of μ_f and z^H is independent of μ_h.

Furthermore, the elements of the Hessian matrix are

$$\frac{\partial^2 W(\mu_h,\mu_f)}{\partial(\mu_f)^2} \propto -\left(\delta^H n^H(\boldsymbol{\mu})^{\frac{\varepsilon-\sigma}{\sigma-1}-1} q_H^{1-\varepsilon} + \delta^B(1-\rho)^2 q_H^{2(1-\sigma)} P^B(\boldsymbol{\mu})^{2\sigma-1-\varepsilon}\right) < 0,$$
$$\frac{\partial^2 W(\mu_h,\mu_f)}{\partial(\mu_h)^2} \propto -\left(\delta^F n^F(\boldsymbol{\mu})^{\frac{\varepsilon-\sigma}{\sigma-1}-1} q_F^{1-\varepsilon} + \delta^B \left[q_F^{1-\sigma} - \rho q_H^{1-\sigma}\right]^2 P^B(\boldsymbol{\mu})^{2\sigma-1-\varepsilon}\right) < 0,$$
$$\frac{\partial^2 W(\mu_h,\mu_f)}{\partial\mu_f\partial\mu_h} \propto -\delta^B(1-\rho) q_H^{1-\sigma}\left[q_F^{1-\sigma} - \rho q_H^{1-\sigma}\right] P^B(\boldsymbol{\mu})^{2\sigma-1-\varepsilon} < 0,$$

where the constant of proportionality is the same. Inspection of the Hessian matrix shows that W is globally concave.

We now turn to proving that, when preferences are CES and the equilibrium is interior, then the constrained optimum lies on $\Pi_h = \Pi_f$. Using (50), the indifference condition $\Pi_h = \Pi_f$ becomes

$$\delta^H q_H^{1-\varepsilon} n^H(\boldsymbol{\mu})^{\frac{\varepsilon-\sigma}{\sigma-1}} - \delta^F q_F^{1-\varepsilon} n^F(\boldsymbol{\mu})^{\frac{\varepsilon-\sigma}{\sigma-1}} = \delta^B \left[q_F^{1-\sigma} - q_H^{1-\sigma}\right] P^B(\boldsymbol{\mu})^{\sigma-\varepsilon}.$$

Meanwhile, equating (51) and (52) returns

$$\delta^H q_H^{1-\varepsilon} n^H(\boldsymbol{\mu}^o)^{\frac{\varepsilon-\sigma}{\sigma-1}} - \delta^F q_F^{1-\varepsilon} n^F(\boldsymbol{\mu}^o)^{\frac{\varepsilon-\sigma}{\sigma-1}} = \delta^B \left[q_F^{1-\sigma} - q_H^{1-\sigma}\right] P^B(\boldsymbol{\mu}^o)^{\sigma-\varepsilon},$$

and therefore $\Pi_h(\mu_h^o,\mu_f^o) = \Pi_f(\mu_h^o,\mu_f^o)$. Clearly, if $q_H = q_F$ and $\gamma_H = \gamma_F$, then $\mu_h^o = \mu_f^o$. Furthermore, since a diversification subsidy moves the equilibrium along $\Pi_h = \Pi_f$ while a reshoring (offshoring) subsidy moves the equilibrium along $\Pi_h = \Pi_b$ ($\Pi_f = \Pi_b$), it is possible to rank policy instruments under CES preferences.

Proposition 3 *Suppose that preferences are CES with an elasticity of substitution $\sigma > \varepsilon$. Then, the constrained optimum (μ_h^o,μ_f^o) can always be achieved with a diversification tax or subsidy, but not with a reshoring or offshoring tax or subsidy.*

To conclude the analysis, it only remains to characterize whether the diversification policy consists of a subsidy or a tax. Specifically, we show that the constrained optimum always features more diversification. For that, consider the variation $(d\mu_h, d\mu_f)$. The associated change in welfare can be expressed as

$$dW(\mu_h,\mu_f) = \rho\left[\frac{\varepsilon+\sigma-1}{\sigma(1-\sigma)}\right]\Bigg[\sum_j \delta^j n^j(\boldsymbol{\mu})^{\frac{\varepsilon-\sigma}{\sigma-1}} q_j^{1-\varepsilon} d\mu_{-j} +$$
$$\delta^B P^B(\boldsymbol{\mu})^{\sigma-\varepsilon}\left(q_H^{1-\sigma}(1-\rho)d\mu_f + (q_F^{1-\sigma} - \rho q_H^{1-\sigma})d\mu_h\right)\Bigg] + k(d\mu_h + d\mu_f),$$

where again we use that z^F is independent of μ_f and z^H is independent of μ_h. If this variation is taken around the decentralized equilibrium, and that equilibrium is interior, then the indifference conditions $\Pi_f = \Pi_b$ and $\Pi_h = \Pi_b$ must hold. Plugging in the expressions for the expected profits, these conditions imply

$$\delta^H \rho \left(\frac{q_H^{1-\varepsilon}}{\sigma} \right) n^H(\boldsymbol{\mu})^{\frac{\varepsilon-\sigma}{\sigma-1}} + \delta^B \rho(1-\rho) \left(\frac{q_H^{1-\sigma}}{\sigma} \right) P^B(\boldsymbol{\mu})^{\sigma-\varepsilon} = k,$$

and

$$\delta^F \rho \left(\frac{q_F^{1-\varepsilon}}{\sigma} \right) n^F(\boldsymbol{\mu})^{\frac{\varepsilon-\sigma}{\sigma-1}} + \delta^B \rho \left[\left(\frac{q_F^{1-\sigma}}{\sigma} \right) - \rho \left(\frac{q_H^{1-\sigma}}{\sigma} \right) \right] P^B(\boldsymbol{\mu})^{\sigma-\varepsilon} = k.$$

respectively. Inserting these in $dW(\mu_h, \mu_f)$ returns

$$dW(\mu_h, \mu_f) = \rho \left[\frac{\varepsilon}{\sigma(1-\sigma)} \right] \left[\left\{ \delta^F q_F^{1-\varepsilon} n^F(\boldsymbol{\mu})^{\frac{\varepsilon-\sigma}{\sigma-1}} + \delta^B P^B(\boldsymbol{\mu})^{\sigma-\varepsilon}(q_F^{1-\sigma} - \rho q_H^{1-\sigma}) \right\} d\mu_h + \right.$$
$$\left. \left\{ \delta^H q_H^{1-\varepsilon} n^H(\boldsymbol{\mu})^{\frac{\varepsilon-\sigma}{\sigma-1}} + \delta^B P^B(\boldsymbol{\mu})^{\sigma-\varepsilon} q_H^{1-\sigma}(1-\rho) \right\} d\mu_f \right].$$

Since $q_F < q_H$ and $\rho \in (0,1)$, both expressions in the curly brackets are positive, from which it follows that $d\mu_f < 0$ and $d\mu_h < 0$ increases welfare.

Proposition 4 *Suppose that preferences are CES with an elasticity of substitution $\sigma > \varepsilon$. Then, the constrained optimum (μ_h^o, μ_f^o) always feature more diversification than the equilibrium.*

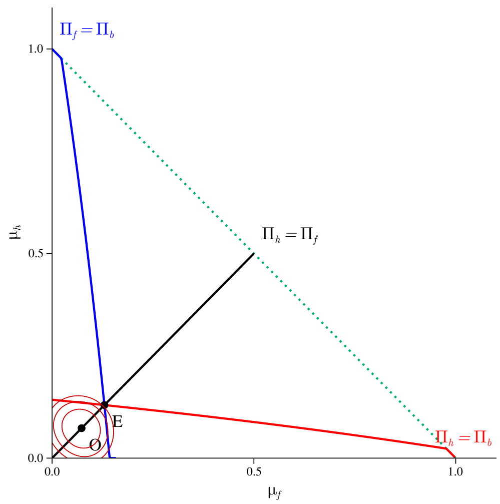
B Additional Figures

Figure 7 illustrates the equilibrium and the constrained optimum for an economy with symmetric costs and risks of disturbances. The left panel shows a case of relatively high ε; i.e., $\varepsilon = 1.4$. In these circumstances the constrained optimum at O lies below and to the left of the equilibrium at E, so the government optimally should promote diversification with a subsidy. The right panel shows a case of relatively low ε; i.e., $\varepsilon = 1.05$. Here, the constrained optimum at O lies above and to the right of the equilibrium at E, so a tax on diversification is indicated to discourage diversification.

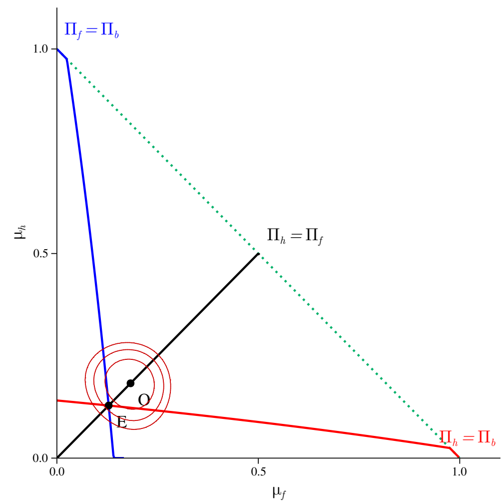
Figures 8-11 provide additional simulations for the asymmetric case, supplementing Figures 5 and 6. Specifically, in Figures 8 and 9, we take the parameterization of Figure 5 and analyze how the optimal policy differs when we consider a smaller cross-country difference in marginal cost (Figure 8) or a smaller idiosyncratic risk of supply chain disruption (Figure 9). In both cases, when ε is large, there is under-investment in diversification and over-investment in single-country supply chains. Hence, *subsidizing* investment in any of the three strategies is welfare-improving, and subsidizing diversification remains the best policy among the three instruments. Here, and for a range of other parameter values with ε relatively large, the welfare differences between the policies are substantial.

When the elasticity of demand for differentiated products is small, the conclusions are different. Figures 10 and 11 show, respectively, how larger cross-country marginal cost differences and higher idiosyncratic risks affect the findings depicted in Figure 6. Depending on the cross-country differences in marginal cost, there could be either under-investment or over-investment in single-country supply chains in equilibrium. Accordingly, the optimal policies could either be to promote or discourage such investments. Finally, by varying cross-country differences in aggregate risk, it is possible to generate any ranking of policy instruments. However, regardless of the ranking of policies that applies, it remains the case that when ε is small, there are small differences in the prospective welfare gains that can be achieved with the three policies.

Figure 7: Equilibrium and Optimum with Translog Preferences



(a) High ε

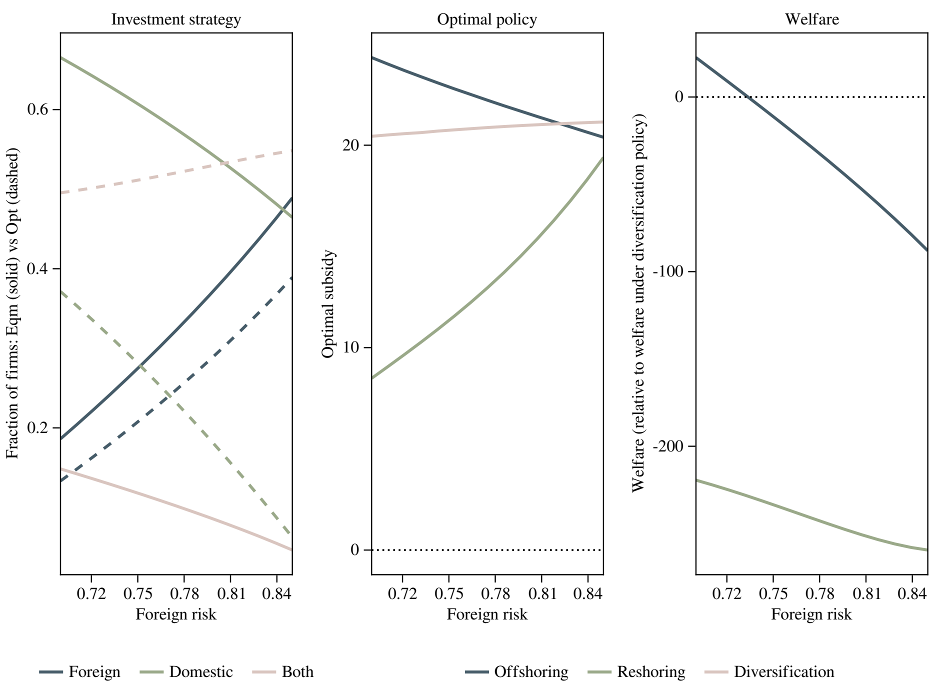


(b) Low ε

Note: for both panels, the parameter values are $q = 0.1$, $\gamma = 0.95$, $\rho = 0.9$, $\theta = 3.5$ *and* $C_P = 0$. *On the left,* $\varepsilon = 1.4$ *and* $k = 0.071$. *On the right,* $\varepsilon = 1.05$ *and* $k = 0.037$. *For both calibrations,* $n(\boldsymbol{\mu}) = 0.78$ *and* $n^B(\boldsymbol{\mu}) = 0.97$.

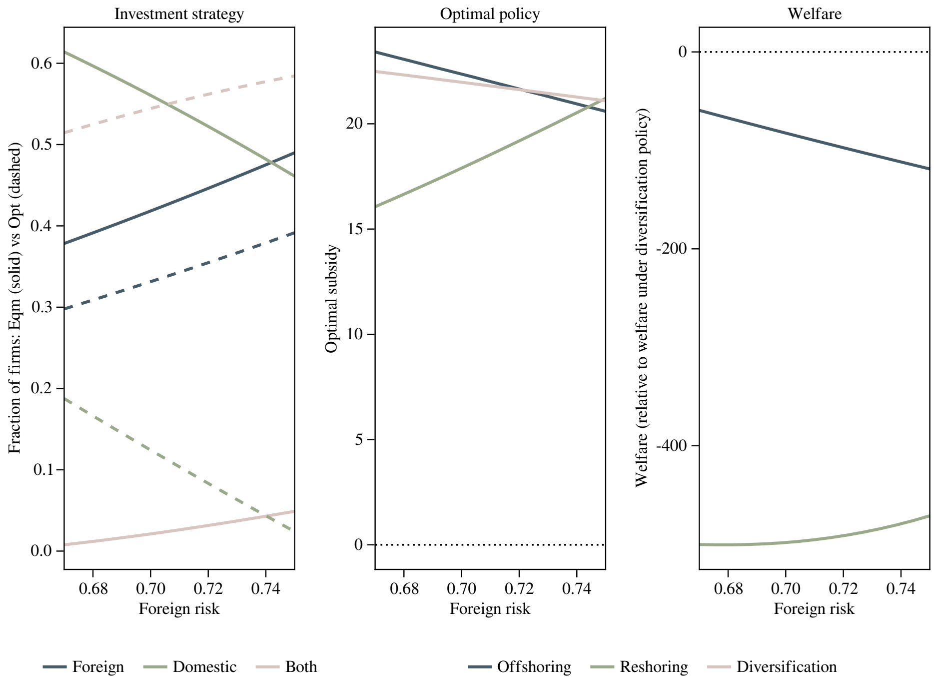
Figure 8: Optimal Policy Robustness

Large Epsilon and Smaller Cost Difference



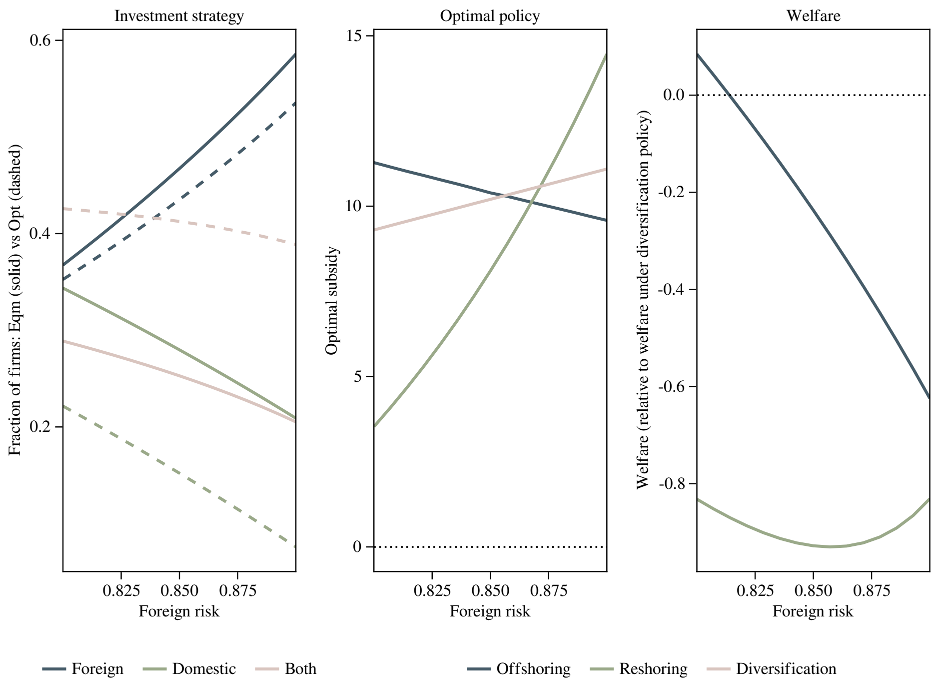
Note: the parameter values are $\varepsilon = 1.4$, $q_F = 0.09$, $q_H = 0.1$, $\gamma_H = 0.95$, $\rho = 0.9$, $\theta = 3.5$, $k = 0.25$ *and* $C_P = 0$.

Figure 9: Optimal Policy Robustness
Large Epsilon and Lower Idiosyncratic Risk



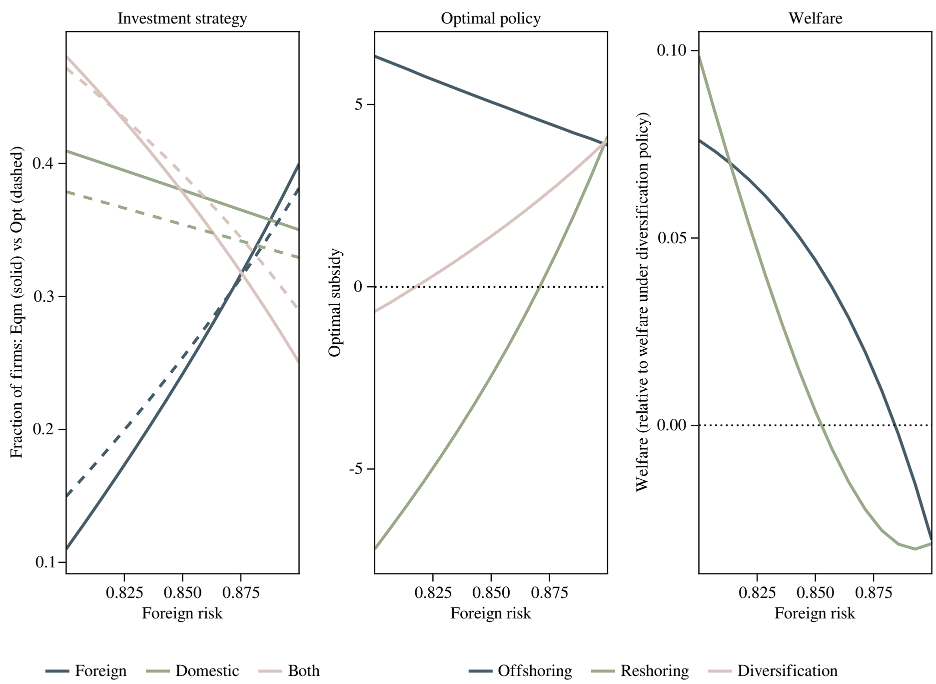
Note: the parameter values are $\varepsilon = 1.4$, $q_F = 0.08$, $q_H = 0.1$, $\gamma_H = 0.95$, $\rho = 0.95$, $\theta = 3.5$, $k = 0.25$ *and* $C_P = 0$.

Figure 10: Optimal Policy Robustness
Small Epsilon and Larger Cost Difference



Note: the parameter values are $\varepsilon = 1.01$, $q_F = 0.09$, $q_H = 0.1$, $\gamma_H = 0.9$, $\rho = 0.9$, $\theta = 3.5$, $k = 0.11$ *and* $C_P = 0$.

Figure 11: Optimal Policy Robustness
Small Epsilon and Higher Idiosyncratic Risk



Note: the parameter values are $\varepsilon = 1.01$, $q_F = 0.099$, $q_H = 0.1$, $\gamma_H = 0.99$, $\rho = 0.95$, $\theta = 3.5$, $k = 0.06$ *and* $C_P = 0$.