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EMPATHY AND THE EFFICIENT PROVISION OF PUBLIC GOODS

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ABSTRACT

I consider the effect of empathy towards others on the internalization of interpersonal externalities and on private contributions to the provision of public goods. I show that if preferences are empathetic in the sense of depending on the well-being of others, then in an extreme case external effects are fully internalized, and private contributions to the provision of a public good will be sufficient for it to be provided at an efficient level. Furthermore I show that an increase in the level of empathy shown by any agent will lead to an increase in the level of provision of the public good, and that as empathy levels increase towards their upper bound, the level of provision of the public good converges to the efficient level. Under certain conditions an increase in empathy is Pareto improving. As it is well-documented that people display some degree of empathy, it is arguable that our failure to provide public goods at efficient levels is attributable to lack of empathy as well as to the free rider problem.

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1. Introduction

Empathy, according to psychologists and neurologists, is an important determinant of human behavior (Iacoboni [17], Batson et al. [6], Decety et al. [12]). Discussions of it are nevertheless scarce in the economics literature. This paper takes some preliminary steps in investigating its implications for two classic problems in public and environmental economics: the management of external effects and the provision of public goods. In doing this I draw on two strands of literature: behavioral economics, and work on altruism and the private provision of public goods. Behavioral economists have imported ideas from psychology to improve our understanding of individual behavior (see Camerer [11] and Thaler [26]). The literature on public goods, altruism and warm glow (Bergstrom [9], Andreoni [2]) looks at how individual contributions to the cost of a public good vary with factors such as altruism and impure altruism or warm glow. My interest here is in how empathy affects contributions to the cost of a public good, or behavior that generates external costs and benefits for others.

Empathy is the ability to feel what others feel, to share their feelings and mental experiences, their pain and their happiness.[1] Mirror neurons are thought to be a mechanism - perhaps the main one - through which this happens. When we observe someone undergoing an intense experience, neurons in our brain behave as if we were ourselves undergoing the same experiences: these are referred to as mirror neurons. According to Kilner and Lemon [18], 'Mirror neurons were originally defined as neurons which "discharged both during monkey's active movements and when the monkey observed meaningful hand movements made by the experimenter"' ([14]). Kilner and Lemon [18] comment that

> "the key characteristics of mirror neurons are that their activity is modulated both by action execution and action observation, and that this activity shows a degree of action specificity."

Acharya and Shoukla [1] emphasize the connection between mirror neurons and empathy:

> "Studies have shown that people who are more empathic according to self-report questionnaires have stronger activations both in the mirror system for hand actions and the mirror system for emotions, providing more direct support for the idea that the mirror system is linked to empathy."

A famous non-human example concerns rats, which contrary to their public image show empathy for their fellows. In an experiment, rats were given a choice between releasing another rat from a cage or eating chocolate: the great majority proceeded to release the caged rat and then share the chocolate with it (for a summary see Wein [27] and for full details Bartal et al. [5]).

The importance of these ideas from an economic perspective is that empathy creates connections between individual welfares and may lead us to at least partially internalize any external effects we impose on others, provided we can see or imagine the impacts that these have on them. If something I do will have a negative effect

[1] The Oxford English Dictionary defines empathy as "the ability to understand and share the feelings of another." It goes on to say that people often confuse empathy and sympathy, whereas sympathy means "feelings of pity and sorrow for someone else's misfortune." The OED also says that empathy was first used in the early 20th century.

on my neighbor, and I am aware of this and can empathize with her, then I may be reluctant to carry our this act: my mirror neurons lead me to feel her response to my action. Similarly interest in the welfare of others may affect willingness to pay for public goods. It is the consequences of this that I investigate below.

In section 2 I define empathetic preferences, which extend regular preferences over consumption vectors (both own consumption and that of others) by including as arguments the welfare levels of others. In this case, each person's welfare depends on that of all others. As in Hori [16], no one's welfare can be specified independently of that of others.[2] I then show in section 3.1 that in the presence of interpersonal externalities, if people behave according to what I call maximum empathy, then a competitive equilibrium is Pareto efficient. If preferences are empathetic but weakly so, only a fraction of the externalities is internalized and the outcome is inefficient. In section 3.2 I consider a simple model of the provision of a public good: each person has to choose personal consumption levels in the light of the amount of a public bad produced in the making of the consumption good (think of the greenhouse gases emitted by traveling). A similar result holds in this case: with individuals choosing according to maximally empathetic preferences, the outcome is Pareto efficient, but a weaker form of empathy leads to an inefficient outcome. In section 3.3 I consider a general model of the production of a public good, in which each person has to choose between allocating their budget between private consumption and the financing of a public good. Again a competitive equilibrium is Pareto efficient and meets the Bowen-Lindahl-Samuelson (BLS) conditions in the presence of maximum empathy. With a lower level of empathy the BLS conditions are only partly satisfied and the public good is under-provided. I then show in subsection 4.1 that with log-linear preferences and a linear production technology for the public good, more empathetic preferences lead to more of the public good at equilibrium. Finally in section 4.2 I study the comparative statics of changes in the degree of empathy in a general model, and establish three results. One is that an increase in the degree of empathy on the part of any agent leads to an increase in the level at which the public good is provided, the second is that the equilibrium allocation is a continuous function of the agents' levels of empathy, and as these converge to maximum empathy the allocation of resources converges to a Pareto efficient outcome, and finally this convergence is Pareto improving under certain conditions.

It appears from these results that empathetic preferences may be a possible explanation for the finding that people make much larger contributions to the provision of pubic goods than conventional theory might suggest. More generally, empathetic preferences seem to give an additional explanation for apparently altruistic behavior, which economists have often explained by the "warm glow" effect (Andreoni [2] and [4]: Andreoni [3] looks at altruism as a possible explanation of behavior in public goods experiments).

The connection between empathy and altruism is complex. The two give rise to very similar models from an analytical perspective - the preference formulation that I use is identical to that developed by Hori [16] to study altruism in the context of social choice. Social psychologists see empathy as the basis for altruism: we show concern for others, which is interpreted as altruistic behavior, because of empathy

[2]For a similar modeling of interdependent preferences in an intertemporal context see Millner [21]

for others (see for example Batson et al. [6] and [8]). However, in the model that I develop below empathy implies that my wellbeing depends on that of others, so that by improving their welfare I am actually contributing to my own. My aid to them is strictly self-interested, and so not consistent with the Oxford English dictionary's definition of altruism, which is "disinterested and selfless concern for the well-being of others." The model is strictly one of individual welfare maximization in the best neoclassical tradition, albeit moderated by a recognition of the realities of psychological and social interdependence. Humans are, we are told by scientists, social animals, and have evolved to function in communities: empathy is one of the mechanisms via which we function communally. It is important to capture its implications for economics.

The idea of empathy in economics is not new: the founder of our discipline, Adam Smith, devotes a whole chapter of his *The Theory of Moral Sentiments* to sympathy,[3] and commented that

> "How selfish soever man may be supposed, there are evidently some principles in his nature, which interest him in the fortune of others, and render their happiness necessary to him, though he derives nothing from it except the pleasure of seeing it. Of this kind is pity or compassion, the emotion we feel for the misery of others, when we either see it, or are made to conceive it in a very lively manner. That we often derive sorrow from the sorrow of others, is a matter of fact too obvious to require any instances to prove it; for this sentiment, like all the other original passions of human nature, is by no means confined to the virtuous and humane, though they perhaps may feel it with the most exquisite sensibility. The greatest ruffian, the most hardened violator of the laws of society, is not altogether without it." (Adam Smith, Ii1 [24])

Pinker, in his very influential book *The Better Angels of Our Nature*[4] [22], distinguishes sharply between empathy and sympathy. Empathy, the ability to feel what others feel, he argues, gives rise to sympathy and it is this that leads to altruistic behavior. Batson and various co-authors argue that it is empathy itself that leads to altruism ([7], [6]). Neither Smith nor any other writers on this topic connected empathy with externalities and their internalization, but to a contemporary economist it is a natural step.

Although I emphasize cases in which empathy is uniform across all others, in reality it is likely that someone will display empathy for some but not for others. For small communities, for example the extended families in which our early ancestors lived, there may be empathy by all for all, leading to internalization of small-scale interpersonal externalities and efficient provision of local public goods. Pinker [22] makes the important point that in contemporary societies this can lead to nepotism and corruption. But for large groups, or the entire world as is relevant for climate change or biodiversity loss, such uniform empathy is most unlikely. In fact there are scientific studies that establish relevant results: the following is from Decety et al.([12])

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[3]The word "empathy" did not exist in Smith's time: it is a twentieth century invention. "Sympathy" and "compassion" were used in its stead.

[4]Which, according to its index, uses the word "empathy" 187 times.

> "Nonhuman animals show preference toward in-group members in detection and reaction to the distress of others. ... In one study, a female mouse moving toward a dyad member in physical pain led to a decrease in the physical symptoms of pain (less writhing) in the dyad member only when the mouse was a cage mate of the mouse in pain, not when they were strangers (Langford et al. 2010). Similarly, female mice exhibit higher fear responses when exposed to the pain of a close relative than when exposed to the pain of a more distant relative (Jeon et al. 2010). Importantly, it is not necessarily genetic affiliation that solely facilitates assistive behaviors. Rats fostered from birth with another strain have been shown to help strangers of the fostering strain but not rats of their own strain (Ben-Ami Bartal et al. 2014). Thus, strain familiarity, even to one's own strain, seems required for the expression of prosocial behavior in rodents."

The same authors go on to comment that

> "A priori, implicit, value-based attitudes toward conspecifics also modulate the response. For example, study participants were significantly more sensitive to the pain of individuals who had contracted AIDS as the result of a blood transfusion as compared to individuals who had contracted AIDS as the result of their illicit drug addiction .... Another fMRI study found modulation of empathic neural responses by racial group membership (Xu et al. 2009). Notably, the response in the ACC to viewing others in pain decreased remarkably when participants viewed faces of racial out-group members relative to racial in-group members. This effect was comparable in Caucasian and Chinese subjects and suggests that modulations of empathic neural responses by racial group membership are similar in different ethnic groups."

They also add that

> "Several aspects of empathy, such as accuracy and concern for others, as well as generosity and other-oriented behavior, are influenced by social status. Social class seems to shape not only people's values and behavior but also their affective responses that relate to sensitivity to the welfare of others. Research shows that lower class individuals, relative to their upper class counterparts, score higher on a measure of empathic accuracy, and judge the emotions of a stranger more accurately (Kraus, Cote, and Keltner 2010). Another set of studies indicates that relative to upper class people, lower class individuals exhibited more generosity, more support for charity, more trust behavior toward a stranger, and more helping behavior toward a person in distress (Piff et al. 2010). Despite their reduced resources and subordinate rank, lower class individuals are more willing than their upper class counterparts to increase another's welfare, even when doing so is costly to the self."

The conclusion to emerge from the psychology literature is that empathy is an important motivator of behavior towards others, but is generally not uniform and indeed may reinforce prejudices. In the next section I formalize the concept of empathetic preferences and investigate the consequences of uniform and non-uniform empathy.

2. Empathetic Preferences

We consider an economy in which individuals are indexed by j, $j = 1, 2, .., J$ and firms by i, $i = 1, 2, .., I$. Consumption vectors are $C_j \in R^N$, and $C_{-j} \in R^{N(J-1)}$ is the vector of consumption goods for all people other than person j. Firm i produces the $i - th$ good according to the formula $Y_i = f_i (K_i)$ where K_i is the capital stock of the $i - th$ firm and Y_i the total output of good i and the function f_i is strictly concave.

I make a distinction between a person's utility U_j and their welfare W_j: utility depends only on consumption, hers and possibly that of others via external effects, $U_j (C_j, C_{-j})$. In some models utility may depend on consumption and on the level of provision of a public good g, $U_j (C_j, g)$. In either case the utility function is strictly concave. As in Hori [16], welfare depends on utility U_j - on consumption - and also on the welfare levels of others W_k, $k \neq j$. Welfare is therefore a more general concept than utility, and depends on factors over and above consumption. The dependence a person's welfare on the welfares of others captures the idea of empathy: the more important the welfares of others are, the more empathetic a person is. Others have thought of this formulation as modeling altruism - see Bergstrom [9] and Hori [16].

In the first model considered, we suppose externalities to be generated by other peoples' consumption, according to the utility-of-consumption function $U_j (C_j, C_{-j})$. The inclusion of C_{-j} as an argument of U_j denotes a conventional externality: the consumption of people other than individual j affects $j's$ wellbeing via noise or pollution or some other mechanism. Empathy is then represented by assuming that $j's$ overall welfare depends not only on consumption vectors C_j and C_{-j} but also on the welfare levels of others. We assume $j's$ welfare W_j to be separable and linear in the welfares of others:

$$W_j = U_j (C_j, C_{-j}) + \sum_{k \neq j} W_k \alpha_{j,k} \tag{2.1}$$

where of course $W_k = U_k (C_k, C_{-k}) + \sum_{l \neq k} W_l$ and $\alpha_{j,k} \geq 0$ is the weight that each agent places on the welfare of others. Note that in this formulation individual j is accepting individual $k's$ specification of her welfare. In particular j may have no empathy for m whereas k may place a lot of value on $m's$ welfare: by accepting $k's$ own definition of her welfare, j is indirectly placing value on m. Hori [16] calls this nonpaternalisitic altruism: Millner [21] in the context of intertemporal preferences uses a similar specification and calls the resulting preferences non-dogmatic. This specification implies that changes in other people's utility levels do not change $j's$ ordering over the space of (C_j, C_{-j}), but do change the utility level associated with any point in the consumption space: they make people better or worse off without changing their underlying preferences over consumption. Another way of describing this is that changes in others' welfares renumber indifference surfaces. With this definition, each agent's welfare depends on that of all others: they are all simultaneously determined, and we cannot tell one person's welfare unless we know that of all others. We can cut through the interdependence the definition (2.1) gives rise to by using a system of simultaneous equations:

$$W_1 = U_1 + \alpha_{1,2}W_2 + \alpha_{1,3}W_3 + ... + \alpha_{1,J}W_J$$

$$W_2 = \alpha_{2,1}W_1 + U_2 + \alpha_{2,3}W_3 + ... + \alpha_{2,J}W_J$$

$$W_3 = \alpha_{3,1}W_1 + \alpha_{3,2}W_2 + U_3 + ... + \alpha_{3,J}W_J$$

....

$$W_J = \alpha_{J,1}W_1 + \alpha_{J,2}W_2 + ... + \alpha_{J,J-1}W_{J-1} + U_j$$

This system can be rewritten

$$AW = U \tag{2.2}$$

where A is the JxJ **empathy matrix**

$$\begin{bmatrix} 1 & -\alpha_{1,2} & ... & -\alpha_{1,J-2} & -\alpha_{1,J-1} & -\alpha_{1,J} \\ -\alpha_{2,1} & 1 & ... & -\alpha_{2,J-2} & -\alpha_{2,J-1} & -\alpha_{3,J} \\ -\alpha_{3,1} & -\alpha_{3,2} & ... & -\alpha_{3,J-2} & -\alpha_{3,J-1} & -\alpha_{3,J} \\ ... & ... & ... & ... & ... & ... \\ -\alpha_{J-1,1} & -\alpha_{J-1,2} & ... & -\alpha_{J-1,J-2} & 1 & -\alpha_{J-1,J} \\ -\alpha_{J,1} & -\alpha_{J,2} & ... & -\alpha_{J,J-2} & -\alpha_{J,J-1} & 1 \end{bmatrix}$$

and $W = W_1, W_2, W_3, ...W_J$ and $U = U_1, U_2, U_3, ...U_J$ (This is very similar to the formulation used by Hori [16]). If A is non-singular and possesses an inverse we can write

$$W = A^{-1}U \tag{2.3}$$

By way of example, consider the case of three agents: then the inverse of the matrix A is

$$\begin{bmatrix} (1-\alpha_{2,3}\alpha_{3,2})/D & (-\alpha_{1,2} + \alpha_{1,3}\alpha_{3,2})/D & (-\alpha_{1,3} + \alpha_{1,2}\alpha_{2,3})/D \\ (-\alpha_{2,1} + \alpha_{2,3}\alpha_{3,1})/D & (1-\alpha_{1,3}\alpha_{3,1})/D & (\alpha_{1,3}\alpha_{31} - \alpha_{2,3})/D \\ (-\alpha_{3,1} + \alpha_{2,1}\alpha_{3,2})/D & (\alpha_{1,2}\alpha_{3,1} - \alpha_{3,2})/D & (1-\alpha_{1,2}\alpha_{2,1})/D \end{bmatrix}$$

where D is the determinant of A, and

$$D = \{1 - \alpha_{1,2}\alpha_{2,1} - \alpha_{1,3}\alpha_{3,1} + \alpha_{1,2}\alpha_{2,3}\alpha_{3,1} + \alpha_{1,3}\alpha_{2,1}\alpha_{3,2} - \alpha_{2,3}\alpha_{3,2}\}$$

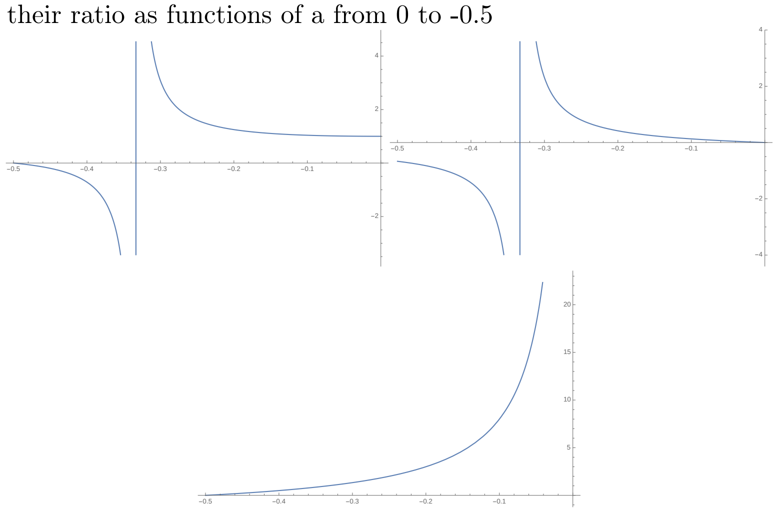
Using this solution, we can express the W_js as functions of the U_js :

$$W_j = \sum_k U_k (C_k, C_{-k}) \beta_{j,k} \tag{2.4}$$

where $\beta_{j,k}$ is the $j,k-th$ element of $B = A^{-1}$. So we can express each person's **welfare** as a weighted sum of everyone's **utilities**. If this result is to make economic sense, we need the weights $\beta_{j,k} \geq 0$: in fact we shall see below that we sometimes need them to be strictly positive. So we need the inverse of the matrix A to be non-negative or possibly positive: below I give necessary and sufficient conditions for this to be true. Intuitively, it makes sense that in a situation where everyone has empathy for everyone else (all $\alpha_{i,k} > 0$), the weights they implicitly place on others' utilities should be positive.

I will consider a simple special case below: when $\alpha_{i,j} = \alpha > 0 \, \forall i,j$, so that everyone puts the same weights on the welfares of others, implying that the off-diagonal terms in the matrix A are all the same, and this is a symmetric matrix

Figure 2.1. On-diagonal and off-diagonal elements of inverse and their ratio as functions of a from 0 to -0.5



with ones on the diagonal and $-\alpha s$ everywhere else. In this case its inverse can easily be shown to have the same form, with all diagonal elements equal and all off-diagonal elements equal. For example, the 4x4 matrix

$$\left\{\begin{matrix} 1 & a & a & a \\ a & 1 & a & a \\ a & a & 1 & a \\ a & a & a & 1 \end{matrix}\right\}$$

has as its inverse the matrix whose diagonal elements are all equal to $(1-3a^2+2a^3)/(1-6a^2+8a^3-3a^4)$ and whose off-diagonal elements are all $(-a+2a^2-a^3)/(1-6a^2+8a^3-3a^4)$.

I will be interested below in a special case which I call **maximum empathy**, in which equal weight is put by each agent on all utilities, their own included, so that $\beta_{ij} = \beta_i \; \forall j$, in which case the matrix A^{-1} would be singular. There is an intuitive explanation for this singularity: if each person gives the same weight to the utilities of all others, then all have the same welfare function up to a positive multiplier. (They have the same welfare function but not of course the same utility functions.) Hori's [16] theorems 1 and 2 address this issue in the context of social choice procedures. Effectively there is only one person and in maximizing her welfare she maximizes the sum of all utilities, which as we will see below is equivalent to finding a Pareto efficient allocation of resources.

If β_{jk} is the same for all j and so B is singular, this means that in the system of equations $W = BU$ all equations but one are redundant. Hence W must be on the diagonal of R^J - all welfare levels must be the same - and there is a set of possible solutions which is the intersection of a $J-1$ dimensional subspace with the positive orthant of R^J. This makes intuitive sense: if all preferences are the same then all welfare levels will be the same, and clearly there are many utility vectors U that are consistent with the weighted sum being W.

Figure 2.1 shows for the 4x4 matrix above the values of the on- and off-diagonal elements as a varies from 0 to -0.5, and also the ratio of these terms. Note that this ratio is one - the terms are equal - at $a = -1/3$ at which point the matrix is singular. As we approach the singularity of the A matrix, the weights being put on all utility functions by the inverse matrix tend to equality.

2.1. **Uneven Empathy.** The scientific results cited above show that an individual's feelings of empathy are directed at groups with which she feels close or for which she feels approval, but not necessarily at other groups for which she has no such feelings. This raises an interesting question: suppose A has empathy towards B but not C, and B has empathy towards C. Because A feels for B and B for C, does A effectively feel for C, because of her concerns for B and those of B for C? In other words, is empathy transitive? To investigate this we need to go back to the analysis of empathetic preferences: This example can be expressed as

$$W_1 = U_1 + \alpha_{1,2}W_2 + 0W_3$$
$$W_2 = 0W_1 + U_2 + \alpha_{2,3}W_3$$
$$W_3 = \alpha_{3,1}W_1 + 0W_2 + U_3$$

in which case the inverse of the matrix A is

$$\begin{bmatrix} 1/D & -\alpha_{1,2}/D & \alpha_{1,2}\alpha_{2,3}/D \\ \alpha_{2,3}\alpha_{3,1}/D & 1/D & -\alpha_{2,3}/D \\ -\alpha_{3,1}/D & \alpha_{1,2}\alpha_{3,1}/D & 1/D \end{bmatrix}$$

where $D = 1 - \alpha_{1,2}\alpha_{2,3}\alpha_{3,1}$. As $\alpha_{i,j} < 0$, the terms in this matrix are all positive. In this case each individual welfare level depends on the utilities of all others:

$$W_1 = U_1/D - \alpha_{1,2}U_2/D + \alpha_{1,2}\alpha_{2,3}U_3/D$$
$$W_2 = U_1\alpha_{2,3}\alpha_{3,1}/D + U_2/D - U_3\alpha_{2,3}/D$$
$$W_3 = -U_1\alpha_{3,1}/D + U_2\alpha_{1,2}\alpha_{3,1}/D + U_3/D$$

and empathy is effectively transitive. Note that agent 1 places no weight on the welfare of agent 3, but does place a weight on her utility proportional to $\alpha_{12}\alpha_{23}$ which is the product of the weight she places on 2's welfare and the weight 2 places on 3's. Likewise agent 3 places no weight on agent 2's welfare but places a weight on her utility proportional to $\alpha_{12}\alpha_{31}$. If however we change the problem so that $\alpha_{3,1} = 0$ then the inverse matrix becomes

$$\begin{bmatrix} 1 & \alpha_{1,2} & \alpha_{12}\alpha_{23} \\ 0 & 1 & \alpha_{2,3} \\ 0 & 0 & 1 \end{bmatrix}$$

so that two agents are insensitive to the utility of at least one other. This observation raises the question of conditions under which the elements of the inverse matrix A^{-1} are all positive. If all elements of A^{-1} are strictly positive then every agent places weight on the utility of every other agent: everyone cares about everyone. I call this **universal empathy**, and as these examples show this can arise because of the transitivity of empathy even if some agents place no weight on the welfares of some others, so that there are zeros in the A matrix. The next proposition formalizes this idea, but first we need some definitions.

The **digraph** (directed graph) of an nxn matrix A is constructed as follows: draw a set of n vertices in the plane, and connect vertices i and j with a directed segment from i to j if and only if $a_{ij} \neq 0$. The digraph is **strongly connected** if it is possible to move from any vertex to any other following the directed segments of the graph.

Note that the matrix A in (2.2) can be written as $A = I - G$ where if g_{ij} is a typical element of G, then $g_{ii} = 0$ and $g_{ij} = \alpha_{ij} \geq 0$.

Proposition 1. *If the modulus of the largest eigenvalue of A is less than one, then A^{-1} shows universal empathy (has strictly positive elements) if and only if the digraph of A is strongly connected.*

Proof. An nxn matrix A is **reducible** if there exists a permutation matrix P such that $PAP^T = \begin{bmatrix} A_{11} & A_{22} \\ 0 & A_{33} \end{bmatrix}$ where P^T is the transpose of P and the A_{jj} and 0 are sub-matrices. Otherwise it is irreducible. The digraph of a matrix M is strongly connected if and only if M is irreducible.[5] A Z-matrix is a matrix for which $a_{ij} \leq 0$ for $i \neq j$. An M-matrix is a matrix that can be expressed as $M = \tau I - G$, where G is non- negative and $\tau \geq \rho(G)$ where $\rho(G)$ is the spectral radius of G, that is the modulus of the largest eigenvalue. Note that the matrix A has ones on the diagonal and is non-positive off the diagonal. It is thus A Z-matrix. As noted above it can be written

$$A = I - G \tag{2.5}$$

where I is the identity matrix and G is a non-negative matrix with zeros on the diagonal: $g_{ij} = -a_{ij} \forall i, j$. As we have assumed the modulus of the largest eigenvalue of A to be less than one, it follows that A is an M-matrix as well as Z-matrix. We can now use the following theorem (Meyer and Stadelmaier ([20]): *(1) If A is a non-singular M-matrix then A is inverse positive. (2) If A is a Z-matrix then the converse of (1) is also true. (3) If A is an irreducible non-singular M-matrix then A is strictly inverse positive. (4) If A is a Z-matrix then the converse of (3) is also true.* Proposition 4 now follows: the digraph of A being strongly connected means that it is irreducible, and it is a Z-matrix and an M-matrix so we can invoke the Theorem cited by Meyer and Stadelmaier to asset that A is strictly inverse positive and so displays universal empathy. [6] □

There is an intuitive basis for this result: the directed graph being strongly connected means that everyone directly or indirectly cares about everyone else, and this is reflected in the positivity of the inverse matrix. The condition that the modulus of the largest eigenvalue of G be less than one essentially means that G is a contraction mapping that shrinks vectors on which it operates. We have some information about the largest eigenvalue of G : we know that the sum of the eigenvalues of a matrix equals the trace, which in this case is zero, and we know that the largest eigenvalue is bounded below by the lowest row sum of P and above by the greatest row sum.[7] A further intuitive understanding of this result comes from the standard result from input-output analysis that $A^{-1} = [I - G]^{-1} = \sum_{t=1}^{\infty} G^t$ so that the inverse of A is the sum of the powers of G, provided that this series

[5] This is a standard result - see for example https://www.stat.berkeley.edu/~mmahoney/s15-stat260-cs294/Lectures/lecture03-29jan15.pdf

[6] There is an alternative proof of Proposition 4 which is simpler but perhaps less intuitive. Berman and Plemmons [10] prove the following: *Let an nxn matrix A be irreducible. Then each of the following conditions is equivalent to the statement "A is a non-singular matrix." (1) $A^{-1} >> 0$ and (2) $Ax > 0$ for some $x >> 0$.* We assume A to be irreducible and know from the definition of A (2.2) that $AW = U$ where W, U are non-negative vectors in R^J. As a strictly positive vector U must exist we know that A satisfies the second condition here and so $A^{-1} >> 0$.

[7] This is part of the Frobenius-Perron theorem - see Heal Hughes and Tarling page 120 [15].

converges, which it does if and only if the maximum root of G is less than one. The i, j elements of the $t-th$ power of G gives the weight placed on $j's$ welfare by i via paths of length t in the digraph of G. So the i, jth element of G is just the weight i places directly on $j's$ welfare, and the same element of G^2 is the weight i places on the welfares of people who place weight directly on j, and so on.

Next we consider the implications of empathetic preferences for three models, one of a system with consumption externalities, one with a public good or bad produced as a side effect of the production of consumption goods, and finally a classical case of public goods.

3. Externalities and Public Goods with Maximum Empathy

In this section I show that interpersonal externalities are internalized, and pubic goods efficiently provided, if everyone has maximum empathy. This is a very special case, but is interesting for two reasons. One is that as levels of empathy increase, the system converges to this special case, which therefore provides a benchmark for evaluating other states. Another is that, as I argue in section 6, this is a case that is privileged by many important ethical systems. In the next on comparative statics I show that as empathy increases from levels below the maximum, allocations move towards efficiency, with the level of public good provision increasing monotonically with the levels of empathy. Under certain conditions this process is Pareto improving: an increase in anyone's empathy makes everyone better off.

3.1. **Consumption Externalities.** As mentioned above, I assume that firm i produces the $i-th$ good according to the formula $Y_i = f_i(K_i)$ where K_i is the capital stock of the $i-th$ firm and Y_i the total output of good i, and there are externalities in consumption, so that utility is $U_j(C_j, C_{-j})$. To characterize Pareto efficiency we need to solve the following optimization problem which maximizes the weighted sum of welfares subject to a resource constraint:

(3.1)
$$Max \sum_j w_j W_j = \sum_j w_j \sum_k \beta_{jk} U_k(C_k, C_{-k}), \ \sum_j C_{ij} = f_i(K_i), \ \sum_i K_i = K$$

where $w_j > 0$ is the weight assigned to agent $j's$ welfare: variations in these weights trace out the utility possibility or Pareto frontier. Rearranging the double sum, and letting $\gamma_j = \sum_k [w_k \beta_{kj}]$, this problem gives the Lagrangian

$$L = \sum_j \gamma_j U_j(C_j, C_{-j}) - \sum_i \lambda_i \left\{ \sum_j C_{ij} - f_i(K_i) \right\} - \theta \left\{ \sum_i K_i - K \right\} \tag{3.2}$$

and the corresponding first order conditions (FOCs) are

$$\gamma_j \frac{\partial U_j}{\partial C_{i,j}} + \sum_{k \neq j} \gamma_k \frac{\partial U_k}{\partial C_{i,j}} = \lambda_i \tag{3.3}$$

$$\lambda_i f_i^{'} = \theta \tag{3.4}$$

or equivalently

$$\frac{\partial U_j}{\partial C_{i,j}} = \theta / f_i^{'} \gamma_j - \sum_{k \neq j} \frac{\partial U_k}{\partial C_{i,j}} \left[\frac{\gamma_k}{\gamma_j} \right] \tag{3.5}$$

Clearly we can think of the second term on the RHS here as a Pigouvian tax on j's consumption of good i. It reflects the impact of this consumption on the welfare levels of others.

Next we consider the individual's choice with empathetic preferences. If the individual faces prices $p \in R^N$ and has income Y_j, then the individual optimization problem is then

$$Max\, \beta_{j,j} U_j\left(C_j, C_{-j}\right) + \sum_{k \neq j} \beta_{jk} U_k\left(C_k, C_{-k}\right)\ s.t.\ p.C_j = Y_j \tag{3.6}$$

giving as FOCs

$$\beta_{jj} \frac{\partial U_j}{\partial C_{i,j}} + \sum_{k \neq j} \frac{\partial U_k}{\partial C_{i,j}} \beta_{j,k} = \mu_j p_i \tag{3.7}$$

where μ_j is the Lagrange multiplier on individual $j's$ budget constraint. Equivalently

$$\frac{\partial U_j}{\partial C_{i,j}} = p_i \mu_j - \sum_{k \neq j} \frac{\partial U_k}{\partial C_{i,j}} \left[\frac{\beta_{jk}}{\beta_{jj}}\right] \tag{3.8}$$

Again we can think of the second term on the RHS as a Pigouvian tax on consumption.

Compare (3.5) with (3.8): these have the same general form, with the marginal utility of the $i-th$ good equal to its social marginal cost or price, minus a function of the external costs that it imposes on others (which will be negative if these are costs rather than benefits). But in the two equations the functions of external costs are different so that we cannot in general say that the private and social FOCs are aligned. However in some special cases we can say more than this.

Assume that in the matrix B all elements are the same, i.e. $\beta_{jk} = \beta_{ik} = \beta\, \forall i, j$. So all agents give the same weight to all other agents' utilities - **maximum empathy**. Then using the definition of γ_j above we see that $\gamma_j = \gamma_k\, \forall j, k$

Hence in the social FOC (3.5) we see that $\gamma_k / \gamma_j = \beta_{jk} / \beta_{jj}$ so in this case the two sets of FOCs are the same and hence the privately optimal outcome is Pareto efficient. There is a clear intuition behind this case: every individual agrees on the weights to put on each utility function and these determine the social weights in the maximand used to characterize Pareto efficiency. Note that in this case the matrix B is singular: as noted in the discussion in the previous section, if everyone agrees on the weights to place on each utility function, then all have the same welfare function and we have a community of identical individuals.

Proposition 2. *With interpersonal externalities, if individuals choose according to preferences showing maximum empathy, then the competitive equilibrium is Pareto efficient.*

3.2. **Public Goods I.** Individuals are again indexed by j, $j = 1, 2, .., J$ and firms by i, $i = 1, 2, .., I$. As before, firm i produces the $i-th$ good according to the formula $Y_i = f_i\left(K_i\right)$ where K_i is the capital stock of the $i-th$ firm and Y_i the total output of good i. In this case firm i also produces a public bad X_i in an amount directly proportional to its output of consumption good: $X_i = \alpha_i Y$: think of greenhouse gases produced as a by-product of transportation or of the production of electric power. Individual $j's$ utility of consumption depends on her private consumption $C_j \in R^N$ and on the public good or bad $X = \sum_i X_i = \sum_i \alpha_i Y_i$ and is given by

$U_j(C_j, X)$ where $C_j = C_{ij,} i = 1, .., I$. As in the previous section, her overall welfare W_j depends also on the welfares of others:

$$W_j = \sum_k U_k(C_k, X)\beta_{j,k}$$

We consider a case when the individual has empathetic preferences and so seeks to maximize W_j subject to $p.C_j = Y$ where p is a price vector and W_j the individual's income. Recall that $X = \sum_i X_i = \sum_i \alpha_i C_i$ and $\sum_j C_{ij} = C_i$. We can therefore write the Lagrangian for the individual's optimization problem as

$$L_j = \sum_k \beta_{jk} U_j \left(C_j, \sum_i \alpha_i \sum_k C_{ik} \right) + \mu_j \{W_j - p.C_j\} \tag{3.9}$$

which gives as FOCs

$$\beta_{jj} \frac{\partial U_j}{\partial C_{ij}} + \alpha_i \sum_k \frac{\partial U_k}{\partial X} \beta_{jk} = p_i \mu_j,\ i = 1, 2, ..I \tag{3.10}$$

We next characterize Pareto efficiency, which is the solution to:

(3.11)

$$Max \sum_j w_j \sum_k \beta_{jk} U_k(C_k, X),\ \sum_j C_{ij} = f_i(K_i),\ \sum_i K_i = K,\ X = \sum_i \alpha_i f_i(K_i)$$

where K is the total capital available, which using the $\gamma_j s$ defined in the previous section gives the Lagrangian. From this the relevant FOCs are

$$\gamma_j \frac{\partial U_j}{\partial C_{ij}} + \underset{i}{\alpha} \sum_k \gamma_k \frac{\partial U_k}{\partial X} = \lambda_i \tag{3.12}$$

$$\lambda_i f_i^{'} = \theta \tag{3.13}$$

We can write (3.10) and (3.12) as follows:

$$\frac{\partial U_j}{\partial C_{ij}} = p_i \mu_j - \alpha_i \sum_k \frac{\partial U_k}{\partial X} \left[\frac{\beta_{jk}}{\beta_{jj}} \right] \tag{3.14}$$

and

$$\frac{\partial U_j}{\partial C_{ij}} = \lambda_i / \gamma_j - \underset{i}{\alpha} \sum_k \frac{\partial U_k}{\partial X} \left[\frac{\gamma_k}{\gamma_j} \right] \tag{3.15}$$

Clearly these equations are the same as the FOCs of the previous section: under the same assumptions as there we have the following results:

Proposition 3. *If public goods (or bads) are associated with the production of private consumption goods and consumers choose their consumption levels according to maximally empathetic preferences , then the outcome is Pareto efficient.*

3.3. **Public Goods II.** Next we consider a model in which consumers have the choice of purchasing private goods for consumption or contributing to a fund that provides public goods. Utility depends on consumption of the private good and the provision of the public good: $U_j(C_j, g)$. As before, we consider an economy in which individuals are indexed by j, $j = 1, 2, .., J$ and firms by i, $i = 1, 2, .., I$. Consumption vectors are C_j in R so there is only one consumption good, and the amount of the public good provided is g. We suppose that each person has a budget that can be divided between the regular consumption goods and contributing to the provision of the public good. The total amount of the public good provided is a function of the total amount contributed by all individuals: $g = f\left(\sum_j g_j\right)$ where g_j is $j's$ contribution to the public good and f is concave. Preferences are again empathetic, represented according the the arguments of section 2 by $W_j = \sum_k U_k(C_k, g)\beta_{jk}$. Hence the individual optimization problem is (setting the price of the private good equal to one and letting Y_k be the value of $k's$ endowment)

$$Max_{C_j,g_j} \sum_k U_k(C_k, g)\beta_{jk},\ g = f\left(\sum_k g_k\right),\ C_k = Y_k - g_k$$

In this problem the agent is optimizing over her choice of consumption of the private good and contribution to the public good, taking as given the contributions that she thinks others are making $(g_k, k \neq j)$ as in a Nash non-cooperative equilibrium, giving rise to the first order conditions

$$\frac{\partial U_j}{\partial C_j}\beta_{jj} = \lambda_j,\ \sum_k \frac{\partial U_k}{\partial g}\beta_{jk} f' = \lambda_j\ \forall j$$

from which we get

$$\sum_k \frac{\partial U_k/\partial g}{\partial U_k/\partial C_k}\frac{\beta_{jk}}{\beta_{kk}} = \frac{1}{f'} \tag{3.16}$$

Next we characterize a Pareto efficient allocation. This is a solution to the problem

$$Max_{C_j,g_j} \sum_j w_j \sum_k U_k(C_k, g)\beta_{jk},\ g = f\left(\sum_k g_k\right),\ C_k = Y_k - g_k$$

and as before consolidating the double sum gives us the first order conditions for efficiency

$$\gamma_j \frac{\partial U_j}{\partial C_j} = \mu,\ \sum_k \gamma_k \frac{\partial U_k}{\partial g} f' = \mu \tag{3.17}$$

which give the standard Bowen-Lindahl-Samuelson conditions that the sum of the marginal rates of substitution should equal the marginal rate of transformation:

$$\sum_k \frac{\partial U_k/\partial g}{\partial U_k/\partial C_k}\frac{\gamma_k}{\gamma_j} = \frac{1}{f'} \tag{3.18}$$

Comparing these conditions with (3.16) we see as in the previous two sections that if we assume that in the matrix B all elements are the same, then $\gamma_k/\gamma_j = \beta_{jk}/\beta_{jj} = \beta_k/\beta_j$ and hence

Proposition 4. *If each person has maximally empathetic preferences over private and public goods, then privately optimal choices lead to an efficient allocation of resources, in particular to the efficient provision of the public good.*

Next I present an example that illustrates this proposition: it is a simple general equilibrium model in which we can compute the equilibria analytically and compare the efficient and privately optimal levels of provision of the public good. As to be expected, the two levels of provision are the same when the levels of empathy are maximal: less obvious but also true is that as the empathy levels increase, the level of provision of the public good increases monotonically towards the efficient level.

4. Comparative Statics

4.1. **Log-Linear Preferences.** In this section we consider a special case of public good provision in which it is possible to derive analytical solutions to the FOCs. We can then show that in the case of extreme empathy, the Nash equilibrium outcome is Pareto efficient, and also that an increase in empathy leads to an increase in the provision of the public good. Let utilities be $u_j(c_j, g) = \theta log(g) + log(c_j)$. Welfare is

$$W_j = u_j(c_j, g) + \sum_{k \neq j} \alpha_{jk} W_k$$

so that $W_j = \sum_k u_k(c_k, g) \beta_{jk}$.

Let J be the total number of people, W the total endowment and W/J the share of each person. Assume the public good is produced from the private according to the linear technology $g(z) = z$ where z is the amount of the private good allocated in total to the production of the public good.

A Pareto efficient allocation is the solution to

$$Max \sum_j \sum_k \{\theta log(g) + log(c_k)\} \beta_{jk}, \quad z + \sum_j c_j = W$$

The Lagrangean is

$$L = \sum_j \sum_k \beta_{jk} \{\theta log(g) + log(c_k)\} + \lambda \left\{ W - z - \sum_j c_j \right\}$$

or

$$L = \beta\theta log(g) + \sum_j \sum_k log(c_k) \beta_{jk} + \lambda \left\{ W - \sum_k c_k - g \right\}$$

where $\beta^j = \sum_k \beta_{jk}$ is the $j - th$ column sum and $\beta = \sum_j \beta^j$. The FOCs with respect to g and c_k are

$$\beta \frac{\theta}{g} = \lambda, \; \frac{\beta^j}{c_k} = \lambda$$

from which it follows that the efficient allocation satisfies

$$c_k = \frac{\beta^j}{\beta} \frac{W}{(1+\theta)}, \quad g_e = \frac{W\theta}{1+\theta}$$

The typical individual solves the problem

$$MaxW_j = \sum_k u(c_k, g) \beta_{jk} = \sum_k \beta_{jk} \{\theta log(g) + log(c_k)\}, \quad g = z_j + \sum_{k \neq j} z_k,$$

$$c_j = \frac{W}{J} - z_j$$

Substituting into the utility function we find the FOCs are

$$\sum_k \left\{ \frac{\theta}{g} \right\} \beta_{jk} - \frac{\beta_{jj}}{\frac{W}{J} - z_j} = 0$$

or

$$c_j \theta \sum_k \beta_{jk} = \beta_{jj} g$$

and summing over j gives

$$\sum_j c_j = W - g = \frac{g}{\theta} \sum_j \frac{\beta_{jj}}{\sum_k \beta_{jk}}$$

or

$$g = \frac{W\theta}{\theta + \sum_j \frac{\beta_{jj}}{\sum_k \beta_{jk}}}$$

For simplicity let $\sigma = \sum_j \frac{\beta_{jj}}{\sum_k \beta_{jk}}$ so that $g = \frac{W\theta}{\theta+\sigma}$. This is equal to the Pareto efficient level of provision if $\sigma = 1$, and less if $\sigma > 1$. Then the following are true:

(1) $\frac{\beta_{jj}}{\sum_k \beta_{jk}}$ is the fraction of the total weight assigned by individual j to her own utility, and if she places equal weight on all - maximum empathy - it will be $1/J$. In this case $\sigma = 1$ and the private and social levels of provision are identical.
(2) If people are self-oriented in the sense of placing more weight on their utility than on that of others, then $\frac{\beta_{jj}}{\sum_k \beta_{jk}} > \frac{1}{J}$ so that $\sigma > 1$ and the provision of the public good will fall short of the efficient level.
(3) If we think of the special case in which $\beta_{jj} \geq \beta_{jk}$, then it is clear that as $\beta_{jk} \to \beta_{jj}$, $\sigma \to 1$ and the private level of provision tends monotonically from below to the efficient level.
(4) In a situation such as that in point 2 above, an increase in any off-diagonal element β_{jk} of the matrix B reduces σ and increases the amount of the public good: $\partial g / \partial \beta_{jk} > 0 \, \forall j \neq k$. An increase in an off-diagonal element corresponds to an increase in empathy. Similarly an increase in a diagonal element - the weight agent j places on her own utility - reduces the level of the public good.
(5) If each agent places weight only on her own utility - $\beta_{jj} > 0$, $\beta_{jk} = 0 \forall j \neq k$ - then $\sigma = J$ the total number of agents. In this case we have an efficient outcome only if $J = 1$.

4.2. **General Comparative Statics.** Next we generalize the results of the example of the previous subsection and establish some important results about changes in the level of empathy. We show that an increase in any off-diagonal element α_{ij} increases the welfare levels of all agents connected to agent i, meaning that if the graph is strongly connected then it increases the welfares of all agents. We also show that an increase in the weight that any agent places on the utility of another (an increase in any β_{ij}, $i \neq j$) increases the level of provision of the public good in the model of subsection 3.3, and that in that model the equilibrium allocation is a continuous function of the elements β_{ij} of the matrix B, so that if the off-diagonal elements of B converge from below to unity then the associated allocations converge to the efficient allocation. Finally we show that an increase in β_{ij}, $i \neq j$, is Pareto improving: all welfares increase or at least remain constant.

4.2.1. *Increases in α_{ij}.* An increase in any α_{ij}, $i \neq j$, increases the welfare of agent i and also that of any agent k for whom $\alpha_{ki} > 0$, because her welfare depends on that of i. Let K_i denote the set of agents for whom $\alpha_{ki} > 0$. It also increases the welfare levels of agents for whom $\alpha_{lm} > 0$ for any $m \in K_i$. In short it increases the welfare of any agent connected directly or indirectly to agent i in the digraph of A.

If the matrix is irreducible and the graph strongly connected then it increases the welfares of all agents.

4.2.2. *Increase in empathy increase public good provision.* We can use the FOC 3.16 for the individual optimization in section 3.3 as an implicit function

$$\sum_k \frac{\partial U_k\left(C_k,g\right)/\partial g}{\partial U_k\left(C_k,g\right)/\partial C_k}\frac{\beta_{jk}}{\beta_{kk}} - \frac{1}{f'\left(g\right)} = 0 \tag{4.1}$$

and from this

$$\frac{\partial g}{\partial \beta_{jk}} = -\frac{\frac{\partial U_k/\partial g}{\partial U_k/\partial C_k}}{\frac{\beta_{jk}\left\{\frac{\partial U_k}{\partial C_k}\frac{\partial^2 U_k}{\partial g^2} - \frac{\partial U_k}{\partial g}\frac{\partial^2 U_k}{\partial C_k \partial g}\right\}}{\left(\frac{\partial U_k}{\partial C_k}\right)^2}} - \frac{f''}{\left(f'\right)^2} \tag{4.2}$$

which is positive if $\frac{\partial U_k}{\partial C_k}\frac{\partial^2 U_k}{\partial g^2} - \frac{\partial U_k}{\partial g}\frac{\partial^2 U_k}{\partial C_k \partial g} < 0$: the first term here is negative by concavity of the utility function and the second will be non-positive if the cross partial is positive or zero. Its being positive implies that an increase in the public good raises the marginal utility of the private good, and it is zero if the utility function is separable in the public and private goods. As the last term on the RHS of (4.2) is positive (f'' being negative), this is a sufficient but not a necessary condition. Hence we have:

Proposition 5. *If the public good enhances (or does not decrease) the marginal utility of the private good, an increase in the weight placed by any agent on any other agent's utility leads to an increase in the provision of the public good.*

This is consistent with the analytical example in 4.1. So under these conditions more empathy leads to more of the public good.

4.2.3. *Convergence to Pareto efficiency.* Assume that $\beta_{jj} = 1\forall j$ and $\beta_{ij} < 1\forall i \neq j$. So all agents place less weight on the utilities of others than on their own, and we set the own weights at one. We can now show that if $\beta_{ij} \to 1\ \forall i,j,\ i \neq j$, then the equilibrium allocations of the public and private goods converge to the Pareto efficient allocation that arises when $\beta_{ij} = 1\ \forall i,j$. To show this we prove that the equilibrium allocations are continuous functions of the elements of the matrix $B \in R^{JxJ}$.

Consider agent j's optimization problem

$$L = \sum_k U_k\left(C_k, f\left(\sum_k g_k\right)\right)\beta_{jk} + \lambda_j\left\{Y_j - g_j - C_j\right\}$$

and let $C_j^*\left(B,Y_j\right)$ and $g_j^*\left(B,Y_j\right)$ be the solutions. Let $\hat{C}_j, \hat{g}_j$ be the values of C_j and g_j when all agents place a weight of one on every utility function: we know these to be Pareto efficient from proposition 4. We are assuming that the utility function is strictly quasi-concave and we note that the budget set is compact provided that $g_j, C_j \geq 0$. In this case we can apply Theorem 3.1 of Terazano and Matani [25] to assert that C_j^* and g_j^* are continuous functions of their arguments. Hence as $\beta_{jk} \to 1\ \forall j,k,\ C_j^*\left(B,Y_j\right), g_j^*\left(B,Y_j\right) \to \hat{C}_j, \hat{g}_j$, leading to

Proposition 6. *As every agent becomes more empathetic and the system moves toward maximum empathy, the allocation of public and private goods converges to the efficient allocation.*

Let $g(B)$ be the Nash equilibrium with the matrix B of weights on utilities, and let B^1 be the matrix of weights on utilities in which all elements are unity ($b_{ij} = 1 \, \forall i, j$). Let $\|B - B^1\|$be the distance between B and B^1, given by $\|B - B^1\| = \sum \sum_{i,j} \left(\beta_{ij} - \beta_{ij}^1\right)^2$. Then

Proposition 7. *If the public good enhances the marginal utility of the private good, there exists $\epsilon > 0$ such that whenever $\|B - B^1\| < \epsilon$, an increase in any element β_{ij}, $i \neq j$ of B is Pareto improving.*

Proof. First we prove that an increase in any β_{ij} which is less than one leads to a new Nash equilibrium at which the sum of all utilities $\sum_j U_j(C_j, g)$ is greater. At a Nash equilibrium we know from (3.16) that

$$\sum_k \frac{\partial U_k/\partial g}{\partial U_k/\partial C_k} \frac{\beta_{jk}}{\beta_{kk}} = \frac{1}{f'}$$

Here $\beta_{kk} = 1$ and $\beta_{jk} < 1$. So

$$\sum_k \frac{\partial U_k/\partial g}{\partial U_k/\partial C_k} < \frac{1}{f'} \tag{4.3}$$

□

Look at the effect of increasing g on sum of all utility functions. Suppose the public good is increased by $\Delta g > 0$ and private good decreased by $\Delta C_k < 0$. The gain in total utility from more of the public good is

$$\Delta g \sum \frac{\partial U_k}{\partial g} > 0$$

and the loss from the decrease in the private good is

$$\Delta C_k \frac{\partial U_k}{\partial C_k} < 0$$

Recall that $C_k = Y_k - g_k$ so that $g = f\left(\sum g_k\right) = f\left(\sum (Y_k - C_k)\right)$ and $dg/dC_k = -f'$ or $\Delta g = -f' \Delta C_k$

We want to know if utilities increase in total as a result of these marginal changes:

$$\Delta g \sum_k \frac{\partial U_k}{\partial g} + \Delta C_k \frac{\partial U_k}{\partial C_k} > 0 \; or \; \Delta g \sum_k \frac{\partial U_k}{\partial g} > -\Delta C_k \frac{\partial U_k}{\partial C_k}$$

Hence we need to know if

$$-f' \Delta C_k \sum_k \frac{\partial U_k}{\partial g} > -\Delta C_k \frac{\partial U_k}{\partial C_k}$$

or :

$$\frac{\sum_k \frac{\partial U_k}{\partial g}}{\frac{\partial U_k}{\partial C_k}} < \frac{1}{f'}$$

Hence from (4.3) any increase in g leads to an increase in the sum of utilities.

We know from the argument for Proposition 6 that the equilibrium is a continuous function of the matrix B. We also know from Proposition 5 that an increase in any b_{ij} leads to an increase in g. Hence it leads to an increase in the sum of utilities. If the matrix of utility weights B has all elements equal to unity, then clearly an increase in the sum of utilities leads to an increase in every welfare level and is Pareto improving. This is still true for a matrix B whose elements are one or close to one: if $\sum_j \partial U_j/\partial g > 0$ then $\sum_j b_{ij}\partial U_j/\partial g > 0\,\forall i$ if $\sum_j (1-\beta_{ij})^2 < \delta_i$ for some $\delta_i > 0$. The ϵ of the proposition is now given by $\epsilon = min_i \delta_i$.

5. Local Public Goods

Suppose that instead of benefitting everyone, a public good g benefits only a subset of people: let $L = 1, 2, ..., G$, $G < J$ be the strict subset of people who gain from a public good. Suppose also that within this group, empathy levels are strong, but are not necessarily strong towards non-members (as discussed in 2.1). Then if $b_{ij} = 1\,\forall i, j \in L$, by the arguments of section 3.3, the public good will be provided efficiently to this group. The same would be true of a second public good $\widetilde{g}$ which benefits only members of the set $\widetilde{L}$, a strict subset of the total group of people with no intersection with L. A society could therefore be divided into non-intersecting groups that provide public goods efficiently to their members, but nevertheless provide a pubic good that benefits everyone inefficiently. In this context closely-knot religious or ethnic groups come to mind as examples. The comparative statics results of propositions 5 and 6 would hold for each of these groups, so that an increase in within-group empathy would lead to greater levels of provision of the group-specific public good. In these cases the digraphs of the A matrix would not be strongly connected, but the graphs restricted to the groups such as L and $\widetilde{L}$ would be strongly connected.

6. Discussion

Empathy is a widely-recognized phenomenon in psychology, neuroscience and philosophy. Its implications for economics merit consideration. We can now see that they are quite substantial. In an economy where everyone has maximally empathetic preferences, interpersonal externalities in consumption do not cause inefficiency, and public goods are provided at a Pareto efficient level. This is a special case, but it is one that is held out as an ideal by many religions and systems of moral philosophy. For example, according to Christ, *Thou shalt love thy neighbour as thyself.* (Matthew 22:36-40, King James Version). Hinduism has a similar maxim: *Do not to others what ye do not wish done to yourself* (The Mahabarata, Brihaspati Anusasana Parva-Section CXIII Verse 8). In Buddhist texts we find *Hurt not others in ways that you yourself would find hurtful* (Udanavarga, 5:18, Tibetan Dhammapada). Confucianism echos these principles: *Do not do to others what you do not want done to yourself* (Confucius, Analects, 15:23, 6:28). And of course in Groundwork of the Metaphysic of Morals (1785) Kant stated that we should *Act only according to that maxim whereby you can, at the same time, will that it should become a universal law.*[8] All of these precepts require that we think of the wellbeing of others exactly as we think of our own wellbeing, and this is what the preference formulation (2.1) does when the weights β_{jk} are all equal to one. So in an ethically ideal society, two of the main problems of public economics - externalities and public goods - are automatically resolved. Religious precepts, if taken seriously, lead to efficiency in the public sphere. In addition an increase in empathy will lead to an increase in the provision of the public good if the public

[8]For interesting discussions of Kant's implications for economics see Laffont [19] and Roemer [23].

and private goods are complements, and may be Pareto improving. This holds for public goods that benefit all, and also for local pubic goods that benefit only a subset of the population.

Einstein [13] made a remark this is relevant here, seeing empathy and compassion as important:

> "A human being is part of a whole, called by us the "Universe," a part limited in time and space. He experiences himself, his thoughts and feelings, as something separated from the rest - a kind of optical delusion of his consciousness. This delusion is a kind of prison for us, restricting us to our personal desires and to affection for a few persons nearest to us. Our task must be to free ourselves from this prison by widening our circle of compassion to embrace all living creatures and the whole of nature in its beauty." (Einstein [13]).

The interconnection of individual welfare levels resulting from empathy makes a difference to the allocation of resources in the presence of externalities and public goods: in maximal form it leads to efficient outcomes and in weaker forms can lead to the partial internalization of external costs and benefits, and to a greater level of provision of public goods than would occur with non-empathetic preferences. In the case of public goods, an increase in empathy leads to a greater level of provision and an approach to the efficient level.

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