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ABSTRACT

I reconsider the long-standing consensus view that macroeconomic stabilization should rely on monetary policy, not fiscal policy. I use an analytically tractable heterogeneous agent New Keynesian (HANK) model that is parameterized so as to admit a bubble in public debt. In this context, I show that it is possible to stabilize either inflation or output in response to aggregate shocks by varying only fiscal policy (that is, lump-sum uniform transfers). In contrast, when the public debt bubble is large, it is impossible to stabilize either inflation or output by varying only interest rates (monetary policy).

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1 Introduction

This paper reconsiders the long-standing consensus view that macroeconomic stabilization should rely primarily on monetary policy, not fiscal policy. I use an analytically tractable heterogeneous agent New Keynesian (HANK) model that is parameterized so as to admit a bubble in public debt. This restriction is readily justified via reference to data from the prior thirty years or even before.[1] In this context, I show that it is possible to stabilize either inflation or output in the face of a wide range of aggregate shocks by varying uniform lump-sum transfers (fiscal policy) while keeping interest rates (monetary policy) unchanged. In contrast, if the public debt bubble is large, it is impossible to stabilize either inflation or output by varying monetary policy (interest rates) while keeping uniform lump-sum transfers (fiscal policy) unchanged.[2] The analysis implies that, in the presence of a large public debt bubble, adjustments to fiscal policy (specifically, transfers) are a more reliable stabilization tool than adjustments to monetary policy (interest rates).

Why does the existence of a public debt bubble enhance the effectiveness of fiscal policy? Without a bubble, an increase in current transfers needs to be financed using reductions in future spending or increases in future taxes. As a result, current stimulus can give rise to a future fiscal drag. In contrast, in the presence of a bubble, an increase in transfers can be financed solely through the issue of new debt, and does not imply a need for additional future taxes. Since the new public debt generates more asset income for its holders, current fiscal stimulus has an unambiguously positive effect on both current and future demand.

What goes wrong with monetary policy when there is a public debt bubble? Monetary policy is, in an intertemporal sense, self-defeating. Current stimulus, accomplished through a reduction in interest rates, lowers future asset income. That creates a drag on future demand which needs to be offset anew using monetary policy easing. If the size of the public debt

[1] See, for example, Blanchard (2019). Brunnermeier, Merkel, and Sannikov (2020) provide a formalization of public debt bubbles.

[2] Throughout, I impose no lower bound on the nominal interest rate. However, adding such a restriction would not affect any of these results.

is sufficiently large, then this linkage means that macroeconomic stabilization can only be accomplished via an ever-increasing - that is, unsustainable - sequence of interest rate cuts.

The results in this paper stand in stark contrast to the implications of representative agent New Keynesian (RANK) models (see, for example, Gali (2015)). In these models, fiscal policy (variations in uniform lump-sum transfers) has no effect on inflation or output because of Ricardian equivalence. Instead, monetary policy plays the lead role in stabilization, as the central bank uses interest rate cuts (increases) to offset the effects of adverse (positive) demand shocks. A fiscal policy response (via adjustment of lump-sum taxes/transfers) is necessary, but only in order to ensure that the government's intertemporal budget constraint is satisfied.

The RANK model is, at least implicitly, based on the assumption that households are fully insured against idiosyncratic shocks. As is done in this paper, this premise of full insurance has been abandoned by a fast-growing literature on monetary policy in HANK models.[3] Its focus is on how the inclusion of ex-post heterogeneity (created by incomplete asset markets) affects the impact and transmission of monetary policy. The contribution of this paper to that literature is to incorporate public debt bubbles into HANK models, and to show (as discussed above) how that feature makes fiscal policy a superior tool for stabilization policy.

There is also a literature on monetary policy in models with *private sector* bubbles.[4] It focuses primarily on how monetary policy should respond to variations in the size of the bubble. In contrast, this paper treats the size of the relevant bubble as fixed, and then studies how its existence should affect macroeconomic policy.

Finally, the analysis in this paper provides a point of contact between standard Dynamic Stochastic General Equilibrium (DSGE) macro modeling and the so-called heterodox modern monetary theory (MMT) described in, among other sources, Mitchell, Wray, and Watts (2019)

[3] See, among others, Kaplan, et al. (2018), Auclert (2019), Broer, et al. (2020), and Acharya and Dogra (2020).

[4] See, among others, Gali (2014), Allen, et al. (2018), Asiryan, et al. (2020), and Dong, Miao, and Wang (2020).

and Mankiw (2020).[5] A main starting point for MMT is the presumption is that governments can always pay off its liabilities using the issue of fresh liabilities, and so increases in spending do not give rise to a need to raise future taxes (or to cut future spending). This premise aligns with the parametric restriction in this paper that there is a public debt bubble. It is then perhaps not all that surprising that the major policy conclusion of MMT is similar to that in this paper: governments can use adjustments in taxes/transfers to stabilize the macroeconomy, without any variation in interest rates.

2 Model

In this section, I present the model, its steady-state, and a log-linearized approximation to its near-steady-state dynamics. From a technical perspective, the model essentially is an embedding of standard New Keynesian elements into a simplified version of the setup analyzed by Kocherlakota (2021).

2.1 Description

There is a unit measure of agents. Their individual states have two possible outcomes $\{H, L\}$. The states' dynamic evolution is stochastically independent across individuals and governed by a common Markov chain with transition matrix:

$$P_{HH} = (1-p) \qquad P_{HL} = p$$
$$P_{LH} = \rho \qquad P_{LL} = (1-\rho)$$

where $0 < \rho, p < 1$. The stationary density for this matrix is given by:

$$P_H = \frac{\rho}{(p+\rho)}, P_L = \frac{p}{(p+\rho)}.$$

[5] Michau (2021) provides an alternative modeling connection by assuming that a representative household derives utility from its real wealth.

I assume that in period 1, the fractions of agents in each of the two states is given by the stationary density, and so these fractions remain constant over time.

The agents live forever and maximize the expectation of a time-separable utility function:

$$\sum_{t=1}^{\infty} \beta^{t-1} u(c_t, n_t, s_t), 0 < \beta < 1.$$

Here, c_t is consumption in period t, n_t is labor in period t, and s_t is the individual's state in period t. I assume that in period t:

$$u(c_t, n_t, H) = ln(c_t) - n_t^{\psi+1}/(\psi+1)$$
$$u(c_t, n_t, L) = \bar{\nu}_t c_t$$

In state H, all agents are endowed with N^{max} units of time, and can, in period t, produce $A_t x$ units of consumption using x units of time, for any $x \in [0, N^{max}]$. Note that both A_t and $\bar{\nu}_t$ are common across all agents in state H in period t.

In what follows, I always assume that N^{max} is sufficiently high that this upper bound never binds. The logarithmic utility assumption is purely for notational convenience; all of the results in the paper generalize to the case in which marginal utility has a more general power form.

Agents in state L are not endowed with any time and so are unable to produce any goods. In this state, they experience a high urgency to consume, which is captured by linearity and high values of marginal utility. This high urgency to consume can be seen as capturing a variety of contingencies, including the impact of health shocks.

Wages are determined in a competitive labor market, and so the real wage equals the marginal rate of substitution between labor and consumption for the (productive) agents in state H. Hence, the real wage W_t satisfies the restriction:

$$W_t = \gamma(\frac{Y_t}{A_t P_H})^{\psi} C_{Ht} \tag{1}$$

where Y_t is output in period t and C_{Ht} is the consumption of agents in state H in period t. Note that the restriction (1) assumes that all agents in state H have the same consumption in period t. We shall see later that this assumption is satisfied in equilibrium.

At each date, there is a unit measure of firms. I assume that their pricing behavior is such that the inflation rate Π_t from period $(t-1)$ to period t satisfies the restriction:

$$ln(1+\Pi_t) = ln(\gamma) + \kappa ln(W_t/A_t) + \beta_F ln(1+\Pi_{t+1})$$
$$\kappa \in (0,\infty),\ \beta_F \in (0,1),\ \gamma > 0$$

This is the standard (log-linear) New Keynesian Phillips curve. I assume that all[6] firm profits[7] accrue to agents in state H.

There are two forms of macroeconomic policy in the model. In terms of monetary policy, the government commits to a sequence of one-period nominal interest rates $(R_t)_{t=1}^{\infty}$. In terms of fiscal policy, the government commits to a sequence of *real* (alternatively, indexed) transfers $(\tau_t)_{t=1}^{\infty}$ that are uniform across agents.

In period t, households can buy one-period government bonds that pay the specified (nominal) interest rate R_t. They can also borrow and lend among themselves at the given interest rate R_t. In period t, households face a short-sales/borrowing limit:

$$b_t(1+r_t) \geq -B_{t+1}^{max}$$

Here, b_t is the real (consumption) value of the household's bondholdings and includes both their holdings of public debt and net holdings of private debt. The term r_t represents the

[6]It is possible to generalize the results in the paper to allow agents in state L to receive at least some of the firms' profits. However, it would be nontrivial to include a market for firm shares (as in Kaplan, et al. (2018)) while also incorporating a public debt bubble.

[7]I do not impose any restriction on the firm discount factor β_F. However, it can be shown that, since profits only accrue to agents in state H, the firm's discount factor in a steady-state is given by $\beta(1-p)$ (that is, the discount factor applied by a household in state H in period t to its consumption in period $(t+1)$ if it is in state H). Thus, the firm discounts profits at a positive rate, even when the interest rate is negative.

real interest rate from period t to period $(t+1)$. Hence,

$$(1+r_t) = \frac{(1+R_t)}{(1+\Pi_{t+1})},$$

where Π_{t+1} is the inflation rate from period t to period $(t+1)$. (More generally, Π_{t+1} should represent inflation expectations. However, I focus on perfect foresight outcomes throughout the paper.) The borrowing limit B_{t+1}^{max} is also subject to aggregate shocks.

The per-capita real (consumption) payoff of government bonds in period t is denoted $B_t^{pub}/(1+\Pi_t)$. Hence, B_t^{pub} represents the nominal per-capita payoff in period t, normalized by the period $(t-1)$ price level. The per-capita period $(t-1)$ value of the government bonds is:

$$\frac{B_t^{pub}}{(1+\Pi_t)(1+r_{t-1})} = \frac{B_t^{pub}}{(1+R_{t-1})}$$

units of consumption. The flow government budget constraint is then given by:

$$\frac{B_{t+1}^{pub}}{(1+R_t)} = \frac{B_t^{pub}}{(1+\Pi_t)} + \tau_t$$

as the government finances its period t transfer τ_t and real bond payoff $\frac{B_t^{pub}}{(1+\Pi_t)}$ using its period t issuance of one-period bonds.

2.2 Household Consumptions

In this subsection, I provide a characterization of optimal household consumptions, under the presumption that $r_t < 0$ for all $t \geq 1$ and the borrowing limit B_t^{max} is constant over time.

Suppose that households consume a strictly positive amount in any state L (we verify this claim shortly). Their Euler equation in state H leaves them indifferent between consuming more today or saving via government debt:

$$\frac{1}{c_{Ht}} = \beta(1+r_t)\{\frac{(1-p)}{c_{H,t+1}} + p\bar{\nu}\}$$

This Euler equation implies[8] that, in period t, all households consume the same amount C^*_{Ht} in state H, regardless of their bondholdings:

$$\frac{1}{C^*_{Ht}} = p\bar{\nu}\sum_{s=0}^{\infty}\beta^{s+1}(1-p)^s\prod_{n=0}^{s}(1+r_{t+n}) \tag{2}$$

This common consumption satisfies:

$$\frac{1}{C^*_{Ht}} < \bar{\nu}$$

since $p > 0$ and $\beta(1+r_{t+s}) < \beta < 1$ for all $s \geq 1$.

Next, consider a household that enters state L in period t with bonds that pay off $b_t(\geq -B^{max}/(1+r_t))$ units of consumption. (If $b_t < 0$, then the household owes consumption to lenders.) In state L, the household prefers to borrow as much as is possible because its current marginal utility from consumption exceeds its expected future marginal utility of consumption:

$$\begin{aligned}\bar{\nu} - \beta(1+r_t)[\frac{\rho}{C^*_{H,t+1}} + (1-\rho)\bar{\nu}] \\ > \bar{\nu} - \beta(1+r_t)[\rho\bar{\nu} + (1-\rho)\bar{\nu}] \\ > 0\end{aligned}$$

where the last inequality is implied by $\beta(1+r_t) < 1$. Hence, their consumptions satisfy:

$$c_{Lt} = b_t + B^{max}/(1+r_t) + \tau_t \tag{3}$$

where b_t is the payoff of any bonds held at the beginning of the period, and τ_t is the (positive)

[8]The Euler equation is a linear difference equation that has a family of solutions:

$$\frac{1}{C_{Ht}} = \frac{1}{C^*_{Ht}} + \varepsilon_t,$$

where $\varepsilon_{t+1} = \beta^{-1}(1+r_t)^{-1}(1-p)^{-1}\varepsilon_t$ and ε_1 can take on any positive value. However, it is readily shown that, for any $\varepsilon_1 > 0$, the resulting sequence of state H bondholdings $(B_{Ht})_{t=1}^{\infty}$ grows too quickly to satisfy the household's transversality condition.

uniform real transfer. Note that, since $r_t < 0$, $c_{Lt} > 0$ for any $b_t \geq -B^{max}$.

Agents in state L consume different amounts depending on their bondholdings at the beginning of the period. But this heterogeneity is irrelevant for the determination of aggregate outcomes. For this reason, it is useful to keep track of *per-capita consumption* for agents in state L. That, in turn, depends on their per-capita bondholdings at the beginning of period t (that is, the cross-sectional average of the term b_t in (3)).

To calculate this average, note that agents in state H buy all of the public bonds and do all of the lending to agents in state L. Hence, the per-capita period t real bond payoff of agents who were in state H in period $(t-1)$ is given by:

$$\frac{B_t^{pub}}{P_H(1+\Pi_t)} + \frac{B_t^{max} P_L}{P_H}.$$

In contrast, the per capita period t real bond payoff of agents who were in state L in period $(t-1)$ is given by:

$$-B_t^{max}$$

because these agents borrow as much as possible. As a result, the per capita real bond payoff of agents in state L in period t is given by the weighted average:

$$\begin{aligned}
&\frac{pP_H}{P_L}\left(\frac{B_t^{pub}}{P_H(1+\Pi_t)} + \frac{B_t^{max} P_L}{P_H}\right) + (-B_t^{max})\frac{(P_L - P_H p)}{P_L} \\
&= \frac{pB_t^{pub}}{P_L(1+\Pi_t)} + pB_t^{max} - B_t^{max} + B_t^{max} P_H p / P_L \\
&= \frac{pB_t^{pub}}{P_L(1+\Pi_t)} - (1 - p - \rho) B_t^{max}.
\end{aligned}$$

If we plug this expression into (3), we obtain the following expression for per-capita consumption for agents in state L:

$$\bar{C}_{Lt} = \frac{pB_t^{pub}}{P_L(1+\Pi_t)} - (1-p-\rho)B_t^{max} + B_t^{max}/(1+r) + \tau_t.$$

2.3 Bubbly Stationary Equilibrium

In this subsection, I characterize, and establish the existence of, *bubbly stationary equilibria.*

Suppose that the exogenous parameters $(\nu_t, A_t, B_{t+1}^{max})$ are constant over time. Suppose too that policy is set so that the nominal interest rate equals $\bar{R}$ for all dates, and the uniform real transfer equals $\bar{\tau}$ for all dates. In this economy, a bubbly stationary equilibrium is a specification of per capita consumptions $(C_H, \bar{C}_L)$, per capita output Y, the real wage W, public debt B^{pub}, the inflation rate Π, and the real interest rate r that satisfy the seven restrictions:

$$\begin{aligned}
\frac{1}{C_H} &= \beta(1+r)(\frac{(1-p)}{C_H} + p\bar{\nu}) \\
Y &= P_H C_H + P_L \bar{C}_L \\
\bar{C}_L &= \frac{pB^{pub}/(1+\Pi)}{P_L} + B^{max}/(1+r) - B^{max}(1-p-\rho) + \bar{\tau} \\
W &= (\frac{Y}{AP_H})^{\psi} C_H \\
(1+\Pi) &= \gamma(W/A)^{\frac{\kappa}{1-\beta_F}} \\
B^{pub} &= -\frac{\bar{\tau}(1+\bar{R})}{r} \\
(1+\bar{R}) &= (1+\Pi)(1+r).
\end{aligned}$$

The following proposition establishes conditions for the existence of a bubbly stationary equilibrium.

Proposition 1. *Consider any real interest rate $r \in (-1, 0)$ and any transfer $\bar{\tau} > 0$. Assuming that N^{max} is sufficiently large, there is a nominal interest rate $\bar{R}$ such that $(C_H, \bar{C}_L, Y, W, \Pi, B^{pub}, r)$ is a bubbly stationary equilibrium given $(\bar{R}, \bar{\tau})$.*

Proof. Consider any $r \in (-1, 0)$. The consumption of agents in state H satisfies their Euler equation:

$$C_H = \frac{(1 - \beta(1+r)(1-p))}{\beta p \bar{\nu}(1+r)}$$

Then, the real debt satisfies the government's flow budget constraint:

$$\frac{B^{pub}}{(1+\Pi)} = \frac{-\bar{\tau}(1+r)}{r}$$

Then solve for $\bar{C}_L$ such that:

$$\begin{aligned}\bar{C}_L &= \frac{pB^{pub}}{P_L(1+\Pi)} + B^{max}/(1+r) - B^{max}(1-p-\rho) + \bar{\tau} \\ &= \frac{-p\bar{\tau}(1+r)}{rP_L} + B^{max}/(1+r) - B^{max}(1-p-\rho) + \bar{\tau}\end{aligned}$$

We can then solve for output Y as:

$$Y = P_H C_H + P_L C_L$$

Then, the real wage W satisfies:

$$W = (\frac{Y}{AP_H})^{\psi} C_H$$

and the households in state H set labor equal to:

$$N_H = \frac{Y}{AP_H}$$

as long as N^{max} is larger than $\frac{Y}{AP_H}$.

Next, find Π so that it satisfies the NKPC:

$$\begin{aligned}(1+\Pi)^{1-\beta_F} &= \gamma(W/A)^{\kappa}\\ &= \gamma(\frac{1}{A^{\psi+1}}(\frac{Y}{P_H})^{\psi}C_H)^{\kappa}\end{aligned}$$

Then the nominal interest $(1+\bar{R}) = (1+\Pi)(1+r)$ and the constant level of public debt $B^{pub} = -\frac{\tau(1+\bar{R})}{r}$.

Finally, check that the household's Euler inequality is satisfied in state L:

$$\begin{aligned}&\bar{\nu} - \beta(1+r)[\rho/C_H + (1-\rho)\bar{\nu}]\\ =\ &\bar{\nu} - \beta(1+r)[\rho\frac{\beta p\bar{\nu}(1+r)}{1-\beta(1+r)(1-p)} + (1-\rho)\bar{\nu}]\\ >\ &\bar{\nu} - [\rho\frac{p\bar{\nu}}{1-(1-p)} + (1-\rho)\bar{\nu}]\\ =\ &0\end{aligned}$$

where the inequality is implied by the observation that the expression is strictly decreasing in $(1+r)$ and $1/\beta > 1 > (1+r)$. $\square$

2.4 Log-Linearization

In this subsection, I provide a first-order approximation to the dynamics of the economy around a bubbly steady-state. For any $t \geq 1$, define:

$$\begin{aligned}&(\hat{c}_{Ht}, \hat{c}_{Lt}, \hat{y}_t, \hat{w}_t, \hat{b}_t^{pub}, \hat{\tau}_t)\\ =\ &(ln(\frac{C_{Ht}}{C_H}), ln(\frac{\bar{C}_{Lt}}{\bar{C}_L}), ln(\frac{Y_t}{Y}), ln(\frac{W_t}{W}), ln(\frac{B_t^{pub}}{B^{pub}}), ln(\frac{\tau_t}{\bar{\tau}}))\end{aligned}$$

to be logged deviations from steady-state and define:

$$(\hat{r}_t, \hat{\pi}_t, \hat{R}_t) = (ln(\frac{1+r_t}{1+r}), ln(\frac{1+\Pi_t}{1+\Pi}), ln(\frac{1+R_t}{1+\bar{R}})).$$

I allow the state L marginal utility ν, the borrowing limit B^{max}, and productivity A to depend on time $t \geq 1$ according to:

$$\begin{aligned} \nu_t &= exp(\hat{\nu}_t)\bar{\nu} \\ B_t^{max} &= exp(\hat{b}_t^{max})B^{max}, \hat{b}_1^{max} = 0 \\ A_t &= exp(\hat{a}_t)A \end{aligned}$$

Suppose that the policy choices $(\hat{R}_t, \hat{\tau}_t)_{t=1}^{\infty}$ and the exogenous parameters $(\hat{\nu}_t, \hat{b}_t^{max}, \hat{a}_t)_{t=1}^{\infty}$ are in a (sufficiently small) neighborhood[9] of zero. Then, the first-order Taylor series approximation to the seven time-dependent equilibrium conditions (written as functions of the hatted variables) takes the form:

$$-\hat{c}_{Ht} = \hat{r}_t + \beta(1+r)((1-p)(-\hat{c}_{H,t+1}) + p\bar{\nu}\hat{\nu}_{t+1}C_H), t \geq 1 \tag{4}$$

$$\hat{y}_t = \frac{P_H C_H}{Y}\hat{c}_{Ht} + \frac{P_L C_L}{Y}\hat{c}_{Lt}, t \geq 1 \tag{5}$$

$$\bar{C}_L\hat{c}_{Lt} = \frac{pB^{pub}}{P_L(1+\Pi)}(\hat{b}_t^{pub} - \hat{\pi}_t) - \frac{B^{max}}{(1+r)}\hat{r}_t + \bar{\tau}\hat{\tau}_t \tag{6}$$

$$+ \frac{B^{max}}{(1+r)}\hat{b}_{t+1}^{max} - B^{max}(1-p-\rho)\hat{b}_t^{max}, t \geq 1 \tag{7}$$

$$\hat{w}_t = \psi(\hat{y}_t - \hat{a}_t) + \hat{c}_{Ht}, t \geq 1 \tag{8}$$

$$\hat{\pi}_t = \kappa(\hat{w}_t - \hat{a}_t) + \beta_F\hat{\pi}_{t+1}, t \geq 1 \tag{9}$$

$$\hat{b}_{t+1}^{pub} = (1+r)(\hat{b}_t^{pub} - \hat{\pi}_t) + \hat{R}_t + \hat{\tau}_t\bar{\tau}(1+R)/B^{pub}, t \geq 1 \tag{10}$$

$$\hat{r}_t = \hat{R}_t - \hat{\pi}_{t+1}, t \geq 1 \tag{11}$$

$$\hat{b}_1^{pub} = 0 \tag{12}$$

The first equation (4) is the approximate Euler equation of agents in state H. The second equation (5) is the approximate aggregate resource constraint. The third equation (6) is the approximate consumption of agents in state L. The fourth equation (8) is the approximate

[9] As defined by the sup-norm.

labor optimality condition (for state H agents). The fifth equation (9) is the New Keynesian Phillips curve. The sixth equation (10) is the approximate law of motion of public debt. The seventh equation (11) is the approximate Fisher equation. I refer to uniformly bounded (in absolute value) solutions to these equations as being *log-linearized equilibria*.

There are at least four aspects of interest for these various restrictions.

- Because $r < 0$, given any sequence $(\hat{R}_t, \hat{\pi}_t, \hat{\tau}_t)_{t=1}^{\infty}$ that is bounded from above and below, the public debt sequence $(\hat{b}_t^{pub})_{t=1}^{\infty}$ implied by the government's flow budget constraint (10) and the initial restriction that $\hat{b}_1^{pub} = 0$ is also bounded from above and below. The presence of a public debt bubble eliminates the connection between monetary and fiscal policy imposed by an intertemporal government budget constraint.

- Equation (4) implies that (as in the RANK model) state H consumptions are functions only of the future (and present). In contrast, equation (6) implies that state L consumptions are functions only of the past (and present).

- As in McKay, Nakamura, and Steinsson (2017), the logged Euler equation (4) features a discount factor $(\beta(1+r)(1-p))$ which is absent from the RANK logged Euler equation.

- The distribution of consumption affects the location of the NKPC. Thus, suppose that $\hat{a}_t = 0$ (so that there are no productivity shocks). If we combine (8) and (9), we obtain:

$$\hat{\pi}_t = \kappa(\psi \hat{y}_t + \hat{c}_{Ht}) + \beta_F \hat{\pi}_{t+1}.$$

  Then, even if $\hat{\pi}_t = 0$ for all $t \geq 1$, it may not be true that $\hat{y}_t = 0$ for all $t \geq 1$. The consumption of state H agents acts as an endogenous shifter to the NKPC when it is written in terms of $\hat{y}_t$. Intuitively, when state H agents are consuming a lot, they are relatively unwilling to work, and the real wage is higher than would be predicted from output alone.

3 Results

In this section, I consider the following policy question. Suppose that an economy is in a steady-state and is then hit by a previously wholly unanticipated ("MIT") shock to the future time path of:

- ν (the urgency of consumption in state L).
- A (labor productivity)
- B^{max} (the borrowing limit)

Given this shock, I ask first whether it is possible to keep inflation or output at its steady-state level using fiscal policy and then whether it is possible to keep inflation or output at its steady-state level using monetary policy. The answer to the former question is yes and the answer to the latter question is no.

I provide no welfare-based justification for the pursuit of stable inflation or stable output.[10] However, it is worth noting that many central banks (including the Federal Reserve in the United States) purport to conduct policy so as to achieve these objectives.

In the proofs, I use the standard notation L to denote the lag operator on sequences and L^{-1} to denote the forward operator. I believe that any potential ambiguity between this usage and the labelling of the individual L state is resolved through context.

3.1 Effectiveness of Fiscal Policy

In this subsection, I prove through two theorems that it is possible to stabilize the macroeconomy through an appropriate choice of fiscal policy (transfers), while keeping monetary policy (interest rates) unchanged. The first theorem concerns the stabilization of inflation in response to any shock.

[10] There is a divine coincidence (Blanchard and Gali (2007)) in this model, in the sense that stable inflation ($\hat{\pi}_t = 0$ for all $t \geq 1$) is equivalent to ensuring that productivity tracks the marginal rate of substitution between consumption and leisure ($\hat{a}_t = \psi\hat{y}_t + \hat{c}_{Ht}$) for the (productive) agents in state H.

Theorem 1. *Let $(\hat{a}_t, \hat{b}^{max}_{t+1}, \hat{\nu}_{t+1})_{t=1}^{\infty}$ be any sequence of real 3-tuples that is uniformly bounded from above and below. If $\hat{R}_t = \hat{\pi}_t = 0$ for all $t \geq 1$, there exists a unique specification $(\hat{\tau}_t)_{t=1}^{\infty}$ of fiscal policy such that there is a log-linearized equilibrium.*

Proof. Let $\Delta_{t+1} = \beta(1+r)C_H p \bar{\nu} \hat{\nu}_{t+1}$ for all $t \geq 1$. Because $\hat{\pi}_t = 0$ for all $t \geq 1$, the Euler equation implies:

$$\hat{c}_{Ht} = \alpha \hat{c}_{H,t+1} - \Delta_{t+1}, t \geq 1,$$

where $\alpha = \beta(1+r)(1-p)$. Hence:

$$\hat{c}_{Ht} = -\sum_{s=1}^{\infty} \alpha^{s-1} \Delta_{t+s}, t \geq 1$$

is uniformly bounded.

Since $\hat{\pi}_t = 0$ for all $t \geq 1$, the Phillips curve implies that:

$$\begin{aligned} \hat{c}_{Ht} &= \hat{a}_t - \psi \hat{y}_t \\ &= \hat{a}_t - \psi(P_L \bar{C}_L \hat{c}_{Lt}/Y + P_H C_H \hat{c}_{Ht}/Y) \end{aligned}$$

there exists a positive constant

$$\xi' = \frac{(Y + \psi P_H C_H)}{\psi P_L} > 0$$

such that:

$$\bar{C}_L \hat{c}_{Lt} = \frac{\hat{a}_t Y}{\psi P_L} - \xi' \hat{c}_{Ht}$$

This implies that the transfers and bond supplies satisfy the state L budget constraint with $\hat{\pi}_t = 0$:

$$\hat{\tau}_t \bar{\tau} = \frac{\hat{a}_t Y}{\psi P_L} - \xi' \hat{c}_{Ht} - \frac{p B^{pub}}{P_L(1+\Pi)} \hat{b}^{pub}_t - \frac{B^{max}}{(1+r)} \hat{b}^{max}_{t+1} + B^{max}(1-p-\rho)\hat{b}^{max}_t, t \geq 1$$

and the government's flow budget constraint with $\hat{\pi}_t = 0$:

$$\hat{b}^{pub}_{t+1} = \hat{b}^{pub}_t(1 + r) + \bar{\tau}\hat{\tau}_t(1 + R)/B^{pub}.$$

(Note that $\hat{b}^{max}_1 = 0$.)

Substituting the former budget constraint into the latter, we obtain a first-order difference equation in $\hat{b}$:

$$\hat{b}^{pub}_{t+1} = \hat{b}^{pub}_t(1 + r)(1 - \frac{p}{P_L}) + \frac{(1 + R)}{B^{pub}}\phi_t, \hat{b}^{pub}_1 = 0, t \geq 1$$

where:

$$\phi_t = \frac{\hat{a}_t Y}{\psi P_L} - \xi'\hat{c}_{Ht} - (\frac{B^{max}}{(1 + r)}\hat{b}^{max}_{t+1} - B^{max}(1 - p - \rho)\hat{b}^{max}_t), t \geq 1$$

Since $|\phi_t|$ is bounded and:

$$|(1 - p/P_L)| = |(1 - \rho - p)| < 1,$$

the bond sequence implied by this difference equation is also bounded.

The transfer sequence can then be chosen using either of the budget constraints, so that:

$$\hat{\tau}_t\bar{\tau} = \frac{\hat{a}_t Y}{\psi P_L} - \xi'\hat{c}_{Ht} - \frac{pB^{pub}}{P_L(1 + \Pi)}\hat{b}^{pub}_t - (\frac{B^{max}}{(1 + r)}\hat{b}^{max}_{t+1} - B^{max}(1 - p - \rho)\hat{b}^{max}_t), t \geq 1.$$

The real wage $\hat{w}_t$ in period t is equal to $\hat{a}_t$. Output is given by:

$$\hat{y}_t = (\hat{a}_t - \hat{c}_{Ht})/\psi.$$

□

The proof shows that, in order to stabilize inflation, transfers rise in response to an increase in productivity $\hat{a}_t$, an increase in the marginal utility $\hat{\nu}_t$ of consumption in state L, or a fall in the borrowing limit $\hat{b}^{max}_t$. The higher transfers lead agents in state L to consume more, and on to an increase in marginal costs that spills into higher inflation.

Even though the specification of fiscal policy in Theorem 1 stabilizes inflation, it does not necessarily stabilize output. For example, if the productivity innovation sequence $(\hat{a}_t)_{t=1}^{\infty}$ is positive, then it is efficient for the productive agents (in state H) to generate more output through their labor. That extra output is entirely consumed by the unproductive agents (in state L) because they receive a larger transfer. If the shock to idiosyncratic risk $\hat{\nu}_1$ is high, then (without a monetary policy response) the productive agents necessarily save more and consume less. They are then more willing to work to generate output for the unproductive agents, who receive larger transfers to buy the extra consumption.

The next theorem focuses on output stabilization. (It abstracts from productivity shocks, as it does not seem sensible to keep output constant in the face of such innovations in the economy.)

Theorem 2. *Let $(\hat{b}_{t+1}^{max}, \hat{\nu}_{t+1})_{t=1}^{\infty}$ be any sequence of real 2-tuples that is uniformly bounded from above and below. If $\hat{R}_t = \hat{y}_t = 0$ for all $t \geq 1$, there exists a specification $(\hat{\tau}_t)_{t=1}^{\infty}$ of fiscal policy such that there is a log-linearized equilibrium.*

Proof. Because $\hat{y}_t = 0$, the NKPC implies that:

$$\hat{\pi}_t = \kappa \hat{c}_{Ht} + \beta_F \hat{\pi}_{t+1}, t \geq 1.$$

Hence, we can write:

$$\hat{\pi}_t = \kappa(1 - \beta_F L^{-1})^{-1} \hat{c}_{Ht}$$

Let $\Delta_{t+1} = \beta(1+r) C_H p \bar{\nu} \hat{\nu}_{t+1}$. Then, we can write the Euler equation as:

$$\hat{c}_{Ht} = \kappa(1 - \beta_F L^{-1})^{-1} \hat{c}_{Ht} + \alpha \hat{c}_{H,t+1} - \Delta_{t+1}, t \geq 1,$$

We can rewrite as:

$$((1-\beta_F L^{-1})(1-\alpha L^{-1})-\kappa)\hat{c}_{Ht} = -(1-\beta_F L^{-1})\Delta_{t+1}, t \geq 1.$$

We can factor the quadratic on the left-hand side as:

$$(1-r_1 L^{-1})(1-r_2 L^{-1})\hat{c}_{Ht} = -(1-\beta_F L^{-1})\Delta_{t+1}$$

where $0 < r_2 < \beta_F$. Hence, this expression can be rewritten as:

$$(1-r_1 L^{-1})\hat{c}_{Ht} = -(1-\beta_F L^{-1})\sum_{s=0}^{\infty} r_2^s \Delta_{t+s+1}.$$

There are then three cases. In the first case, the NKPC is sufficiently near-flat that $(1-\alpha)(1-\beta_F) > \kappa$. Under this restriction, $(1/\beta_F) > (1/r_1) > 1$ and there is a unique uniformly bounded solution for $\hat{c}_H$:

$$\hat{c}_{Ht} = (1-r_1 L^{-1})^{-1}(1-r_2 L^{-1})^{-1}(\Delta_{t+1} - \beta_F \Delta_{t+2}), t \geq 1.$$

In the second case, the NKPC is sufficiently near-vertical that $\kappa > (1+\alpha)(1+\beta_F)$. Then, $(1/r_1) < -1$. Again, there is a unique uniformly bounded solution for $\hat{c}_H$:

$$\hat{c}_{Ht} = (1-r_1 L^{-1})^{-1}(1-r_2 L^{-1})^{-1}(\Delta_{t+1} - \beta_F \Delta_{t+2}), t \geq 1.$$

Finally, in the third case, the slope of the Phillips curve is intermediate, so that $(1-\alpha)(1-\beta_F) \leq \kappa \leq (1+\alpha)(1+\beta_F)$. In this case, the root r_1 satisfies $1 \geq 1/r_1 \geq -1$. There is a family of uniformly bounded solutions for $\hat{c}_H$, indexed by $\hat{c}_{H1}$:

$$\hat{c}_{H,t+1} = (1/r_1)\hat{c}_{Ht} + (1 - \beta_F L^{-1}) \sum_{s=0}^{\infty} r_2^s \Delta_{t+s+1}/r_1, t \geq 1$$

$$\hat{c}_{H1} \text{ given.}$$

In what follows, we choose one of the possible solutions for $\hat{c}_H$ and then solve for the transfers required to stabilize output given that selection. As noted above, the inflation sequence is given by:

$$\hat{\pi}_t = \kappa \sum_{s=0}^{\infty} \beta_F^s \hat{c}_{H,t+s}$$

The consumptions in state L are given by:

$$\bar{C}_L \hat{c}_{Lt} = \frac{-\hat{c}_{Ht} C_H P_H}{P_L}.$$

This implies that the transfers and bond supplies satisfy the state L budget constraint (6) for all $t \geq 1$:

$$\begin{aligned} \hat{\tau}_t \bar{\tau} = &-\frac{P_H C_H \hat{c}_{Ht}}{P_L} - \frac{p B^{pub}}{P_L(1+\Pi)}(\hat{b}_t^{pub} - \hat{\pi}_t) - \frac{B^{max}}{(1+r)}\hat{\pi}_{t+1}, \\ &-(\frac{B^{max}}{(1+r)}\hat{b}_{t+1}^{max} - B^{max}(1-p-\rho)\hat{b}_t^{max}) \end{aligned}$$

and the government's flow budget constraint (10):

$$\hat{b}_{t+1}^{pub} = (\hat{b}_t^{pub} - \hat{\pi}_t)(1+r) + \bar{\tau}\hat{\tau}_t(1+R)/B^{pub}.$$

(Note that $\hat{b}_1^{max} = 0$.)

Substituting the former budget constraint into the latter, we obtain a first-order difference equation in $\hat{b}^{pub}$:

$$\hat{b}_{t+1}^{pub} = \hat{b}_t^{pub}(1+r)(1 - \frac{p}{P_L}) + \phi_t', \hat{b}_1^{pub} = 0,$$

where:

$$\phi'_t = -\hat{\pi}_t(1+r)(1-\frac{p}{P_L}) - \frac{(1+R)}{B^{pub}}\frac{P_H C_H \hat{c}_{Ht}}{P_L} - \frac{(1+R)}{B^{pub}}\frac{B^{max}}{(1+r)}\hat{\pi}_{t+1}$$
$$-\frac{(1+R)}{B^{pub}}(\frac{B^{max}}{(1+r)}\hat{b}^{max}_{t+1} - B^{max}(1-p-\rho)\hat{b}^{max}_t)$$

Since:

$$|(1-p/P_L)| = |(1-\rho-p)| < 1,$$

and $(\hat{c}_{Ht}, \hat{\pi}_t, \hat{b}^{max}_{t+1})_{t=1}^{\infty}$ is uniformly bounded, the bond sequence implied by this difference equation is also bounded.

The transfer sequence can then be chosen using either of the budget constraints, so that:

$$\hat{\tau}_t\bar{\tau} = -\frac{P_H C_H \hat{c}_{Ht}}{P_L} - \frac{pB^{pub}}{P_L(1+\Pi)}(\hat{b}^{pub}_t - \hat{\pi}_t) - \frac{B_{max}}{(1+r)}\hat{\pi}_{t+1}$$
$$-(\frac{B^{max}}{(1+r)}\hat{b}^{max}_{t+1} - B^{max}(1-p-\rho)\hat{b}^{max}_t), t \geq 1$$

We can complete the specification of the equilibrium by noting that $\hat{w}_t = \hat{c}_{Ht}$ and $\hat{r}_t = -\hat{\pi}_t$. $\square$

In the proof, state H consumption $\hat{c}_H$ and inflation $\hat{\pi}$ are jointly determined by the NKPC and the Euler equation as a response to $\hat{\nu}$. Then, to stabilize output, transfers are used to offset the state H consumption deviations $\hat{c}_{Ht}$, given the impact of changes in inflation $\hat{\pi}$ and the borrowing constraint $\hat{b}^{max}$ on state L consumption.

This logic is similar to that of the proof of Theorem 1. However, there is one subtlety: there is a range of values for the slope of the NKPC such that the equilibrium $\hat{c}_H$ and $\hat{\pi}$ are not uniquely pinned down by the marginal utility process $\hat{\nu}$. In this situation, in order to stabilize output, the fiscal policy must be formulated in terms of the endogenous variables $(\hat{c}_H, \hat{\pi})$, rather than the exogenous variable $\hat{\nu}$.

3.2 Ineffectiveness of Monetary Policy

In this subsection, I prove that, if r is sufficiently close to (but less than) zero, it is impossible to stabilize the macroeconomy through an appropriate choice of monetary policy (interest rates), while keeping fiscal policy (transfers) unchanged. The first theorem concerns the inability to stabilize inflation in response to any shock. In its statement, it is useful to define:

$$\hat{\theta}_{t+1}^{max} \equiv \frac{B^{max}}{(1+r)}\hat{b}_{t+1}^{max} - B^{max}(1-p-\rho)\hat{b}_t^{max}$$

to be the change in per-capita consumption that agents in state L receive from the sequence of shocks to their borrowing limit.

Theorem 3. *Let $\lambda \in (0,1)$, and suppose $\hat{a} = (\lambda^{t-1}\hat{a}_1)_{t=1}^{\infty}, \hat{\theta}^{max} = (\lambda^{t-1}\hat{\theta}_2^{max})_{t=1}^{\infty}$ and $\hat{\nu} = (\lambda^{t-1}\hat{\nu}_1)_{t=1}^{\infty}$, where exactly one element of $\{\hat{a}_1, \hat{\theta}_2^{max}, \hat{\nu}_1\}$ is non-zero. Suppose too that $\hat{\tau}_t = \hat{\pi}_t = 0$ for all $t \geq 1$. There exists a cutoff $r_{cut} < 0$ which is independent of λ such that if $r \geq r_{cut}$, the log-linearized equilibrium conditions are not satisfied for any $\hat{R} = (\hat{R}_t)_{t=1}^{\infty}$.*

Proof. Define:

$$C_H(r) = \frac{(1-\beta(1+r)(1-p))}{\beta(1+r)p\bar{\nu}}$$
$$Y(r) = P_L(\frac{p}{P_L}\frac{\bar{\tau}(1+r)}{-r} + B^{max}/(1+r) - B^{max}(1-p-\rho) + \bar{\tau}) + P_H C_H(r)$$

and:

$$\bar{C}_L(r) = \frac{(Y - P_H C_H(r))}{P_L}.$$

These are the steady-state levels of state H consumption, per-capita output, and per-capita state L consumption for any given real interest rate $r < 0$.

Since $\hat{\pi}_t = 0$ for all t, the Phillips curve implies that:

$$\hat{a}_t = \psi\hat{y}_t + \hat{c}_{Ht}, t \geq 1$$

and so:

$$\hat{c}_{Ht} = \hat{a}_t - \psi \hat{y}_t$$
$$= \hat{a}_t - \psi(P_L \bar{C}_L(r)\hat{c}_{Lt}/Y(r) + P_H C_H(r)\hat{c}_{Ht}/Y(r))$$

Hence, there exists:

$$\xi^*(r) = \frac{(Y(r) + \psi P_H C_H(r))}{\psi P_L} > 0$$

such that:

$$\bar{C}_L(r)\hat{c}_{Lt} = \frac{Y(r)}{\psi P_L}\hat{a}_t - \xi^*(r)\hat{c}_{Ht} \tag{13}$$

for all $t \geq 1$. The left-hand side can be rewritten as:

$$\bar{C}_L(r)\hat{c}_{Lt} = \frac{p}{P_L}\frac{(1+r)\bar{\tau}}{-r}\hat{b}_t^{pub} - \frac{B^{max}}{(1+r)}\hat{R}_t + \hat{\theta}_{t+1}^{max}, t \geq 1 \tag{14}$$

where I have used the restriction that:

$$\hat{\tau}_t = 0, t \geq 1.$$

Note from the government's flow budget constraint that:

$$\hat{R}_t = \hat{b}_{t+1}^{pub} - (1+r)\hat{b}_t^{pub} \tag{15}$$

given the restrictions that $\hat{\tau}_t = 0$ and $\hat{\pi}_t = 0$ for all $t \geq 1$. Define:

$$z_{t+1} = \hat{b}_{t+1}^{pub} - \lambda \hat{b}_t^{pub}$$

Since $\hat{a}_{t+1} - \lambda \hat{a}_t = \hat{\theta}_{t+2}^{max} - \lambda \hat{\theta}_{t+1}^{max} = 0$ for all $t \geq 1$, the restrictions (13), (14), and (15) imply that

$$\hat{c}_{H,t+1} - \lambda \hat{c}_{Ht} = A_1^*(r) z_{t+1} + A_2^*(r) z_{t+2}, t \geq 1$$

where:

$$A_1^*(r) = \frac{-B^{max} + \frac{p}{P_L} \frac{\bar{\tau}(1+r)}{r}}{\xi^*(r)} < 0$$
$$A_2^*(r) = \frac{B^{max}/(1+r)}{\xi^*(r)} > 0$$
$$A_1^*(r) + A_2^*(r) = \frac{\frac{p}{P_L} \frac{\bar{\tau}(1+r)}{r} - \frac{rB^{max}}{(1+r)}}{\xi^*(r)}$$

Since $\hat{\pi}_t = 0$, the log-linearized Euler equation can be written

$$\hat{c}_{Ht} = -\hat{R}_t + \beta(1+r)(1-p)(\hat{c}_{H,t+1}) - \beta(1+r)p\bar{\nu}\lambda^t \hat{\nu}_1 C_H, t \geq 1 \tag{16}$$

By quasi-differencing (16), we obtain:

$$A_1^*(r) z_{t+1} + A_2^*(r) z_{t+2} + z_{t+2} - (1+r) z_{t+1} - A_1^*(r)\alpha(r) z_{t+2} - A_2^*(r)\alpha(r) z_{t+3} = 0, t \geq 2$$

where $\alpha(r) = \beta(1+r)(1-p) < 1$. The characteristic function of this second-order difference equation is:

$$\Phi^*(L) = (A_1^*(r) - (1+r))L^2 + (A_2^*(r) - A_1^*(r)\alpha(r) + 1)L - A_2^*(r)\alpha(r).$$

We can evaluate Φ^* to find that:

$$\Phi^*(0) = -A_2^*(r)\alpha(r) < 0$$
$$\Phi^*(\alpha(r)) = \alpha(r) - (1+r)\alpha(r)^2 > 0$$
$$\Phi^*(1) = (A_1^*(r) + A_2^*(r))(1-\alpha(r)) - r$$
$$= (1-\alpha(r))\frac{\frac{p}{P_L}\frac{\bar{\tau}(1+r)}{r} - \frac{rB^{max}}{(1+r)}}{\xi^*(r)} - r$$

If r is sufficiently close to zero, then:

$$\Phi^*(1) \approx -\psi(1-\beta(1-p)) < 0$$

In this case, the roots of the characteristic equation are both positive and less than one. The only stable solution to the second-order difference equation is to set $z_{t+1} = 0$ for all $t \geq 1$. This implies that:

$$\hat{b}_{t+1}^{pub} = \lambda^{t-1}\hat{b}_1^{pub}, t \geq 1$$
$$= 0$$

But this solution leads to a contradiction. Since $\hat{b}^{pub} = 0$, $\hat{R} = 0$, and the Euler equation (16) implies that

$$\hat{c}_{Ht} = -\frac{\lambda^t\beta(1+r)p\bar{\nu}\hat{\nu}_1 C_H(r)}{(1-\alpha(r))}, t \geq 1. \tag{17}$$

At the same time, (13) and (14) imply that:

$$-\xi^*(r)\hat{c}_{Ht} = -\frac{Y(r)}{\psi P_L}\lambda^{t-1}\hat{a}_1 + \lambda^{t-1}\hat{\theta}_2^{max} \tag{18}$$

Together, (17) and (18) violate the assumption that exactly one component of $(\hat{a}_1, \hat{b}_2^{max}, \hat{\nu}_1)$ is non-zero. $\square$

The logic of the proof is most easily seen in a simplified case in which:

$$\hat{a}_t = 0, t \geq 1$$
$$B^{max} = 0$$
$$p \text{ is near } 1$$
$$\hat{\nu}_2 > 0, \text{ and } \hat{\nu}_t = 0, t > 2$$

so that there are no productivity shocks, there is no borrowing capacity for agents in state L, state H households put little weight on future state H consumption, and there is a shock to marginal utility only in period 2. In this simplified case, the government must set $\hat{R}_1 < 0$ to stabilize period 1 inflation (because there is no policy tool to influence $\hat{c}_{L1}$) and this leads $\hat{b}_2^{pub} < 0$.

Thereafter, since p is near 1 and there are no marginal utility shocks, the Euler equation implies that consumption in period $t > 1$ inversely tracks the nominal interest rate:

$$\hat{c}_{Ht} = -\hat{R}_t, t \geq 2$$

But inflation stabilization implies that consumption in state H must also inversely track the level of public debt (because consumption in state L is driven only by asset income):

$$\hat{c}_{Ht} = -h(r)\hat{b}_t^{pub}, t \geq 2$$

Here, $h(r)$ is a positive time-invariant constant for all $r < 0$ and $lim_{r \to 0^-} h(r) = \psi > 0$. It follows that, if r is near zero, the law of motion for public debt is given by:

$$\hat{b}_{t+1}^{pub} \approx (1+\psi)^{t-1}\hat{b}_2^{pub}, t \geq 2$$

Since $\hat{b}_2^{pub} < 0$ (because the central bank has set $\hat{R}_1 < 0$), the path of public debt (and consequently) nominal interest rates is unsustainable.

The following theorem extends the above logic to output stabilization in response to (demand) shocks to ν and B^{max}.

Theorem 4. *Let $\lambda \in (0,1)$, and suppose $\hat{\theta}^{max} = (\lambda^{t-1}\hat{\theta}_2^{max})_{t=1}^{\infty}$ and $\hat{\nu} = (\lambda^{t-1}\hat{\nu}_1)_{t=1}^{\infty}$, where exactly one element of $\{\hat{\theta}_2^{max}, \hat{\nu}_1\}$ is non-zero. Suppose too that $\hat{\tau}_t = \hat{\pi}_t = 0$ for all $t \geq 1$. There exists a cutoff $r'_{cut} < 0$ which is independent of λ such that if $r \geq r'_{cut}$, the log-linearized equilibrium conditions are not satisfied for any $\hat{R} = (\hat{R}_t)_{t=1}^{\infty}$.*

Proof. Let $C_H(r)$ and $\bar{C}_L(r)$ be state H and state L consumptions, in steady states indexed by the real interest rate $r < 0$ (as defined in the proof of Theorem 3). Define: $\xi(r) = \frac{P_H C_H(r)}{P_L} = \frac{P_H}{P_L}\frac{(1-\beta(1+r)(1-p))}{\beta p \bar{\nu}(1+r)} > 0$.

Since $\hat{y}_t = 0$, we know that:

$$\bar{C}_L(r)\hat{c}_{Lt} = -\xi(r)\hat{c}_{Ht}. \tag{19}$$

The left-hand side can be rewritten as:

$$\bar{C}_L(r)\hat{c}_{Lt} = \frac{p}{P_L}\frac{\bar{\tau}(1+r)}{-r}(\hat{b}_t^{pub} - \hat{\pi}_t) - \frac{B^{max}}{(1+r)}\hat{r}_t + \hat{\theta}_{t+1}^{max}, t \geq 1 \tag{20}$$

where I have used the zero restrictions on $\hat{\tau}$ and $\hat{y}$. Define:

$$x_t = (\hat{b}_t^{pub} - \hat{\pi}_t), t \geq 1$$

where $\hat{b}_1^{pub} = 0$. Then (20) implies that:

$$\hat{c}_{Ht} = A_1(r)x_t + A_2(r)x_{t+1} + A_3(r)\hat{\theta}_{t+1}^{max} \tag{21}$$

where:

$$A_1(r) = \frac{-B^{max} + \frac{p}{P_L}\frac{\bar{\tau}(1+r)}{r}}{\xi(r)} < 0$$
$$A_2(r) = \frac{B^{max}/(1+r)}{\xi(r)} > 0$$
$$A_3(r) = -\frac{1}{\xi(r)}$$

Define:

$$z_{t+1} = x_{t+1} - \lambda x_t.$$

It follows that:

$$\hat{c}_{H,t+1} - \lambda \hat{c}_{Ht} = A_1(r) z_{t+1} + A_2(r) z_{t+2}$$

The government's flow budget constraint implies that:

$$\hat{r}_t = x_{t+1} - (1+r)x_t$$

where we again use the restriction that $\hat{\tau}_t = 0$. Recall the approximate Euler equation (4):

$$\hat{c}_{Ht} = -\hat{r}_t + \beta(1+r)(1-p)(\hat{c}_{H,t+1}) - \beta(1+r)p\bar{\nu}\lambda^t \hat{\nu}_1 C_H, t \geq 1 \tag{22}$$

Then by quasi-differencing (22), we obtain

$$A_1(r)z_{t+1} + A_2(r)z_{t+2} + z_{t+2} - (1+r)z_{t+1} - A_1(r)\alpha(r)z_{t+2} - A_2(r)\alpha(r)z_{t+3} = 0, t \geq 1$$

where $\alpha(r) = \beta(1+r)(1-p) < 1$. The characteristic function of this second-order difference equation is:

$$\Phi(L) = (A_1(r) - (1+r))L^2 + (A_2(r) - A_1(r)\alpha(r) + 1)L - A_2(r)\alpha(r).$$

It is readily seen that:

$$\begin{aligned}\Phi(0) &= -A_2(r)\alpha(r) < 0 \\ \Phi(\alpha(r)) &= \alpha(r) - (1+r)\alpha(r)^2 > 0 \\ \Phi(1) &= (A_1(r) + A_2(r))(1-\alpha(r)) - r \\ &= (1-\alpha(r))\frac{\frac{p}{P_L}\frac{\bar{\tau}(1+r)}{r} - \frac{rB_{max}}{(1+r)}}{\xi(r)} - r\end{aligned}$$

The sign of $\Phi(1)$ is negative if r is sufficiently close to zero (while still being negative). In this case, the roots of the characteristic equation are both positive and less than one. The only stable solution to the second-order difference equation is to set $z_{t+1} = 0$ for all $t \geq 1$. This implies that:

$$\begin{aligned}x_{t+1} &= \lambda^t x_1 \\ &= \lambda^t(-\hat{\pi}_1), t \geq 1\end{aligned}$$

The log-linearized Phillips curve provides another connection between inflation and consumption:

$$\begin{aligned}\hat{\pi}_1 &= \kappa\hat{c}_{H1} + \beta_F\hat{\pi}_2 \\ &= \kappa\hat{c}_{H1}/(1-\beta_F\lambda) \text{ (by solving forward).}\end{aligned}$$

This expression leads to a contradiction, given that exactly one of $\hat{\theta}_2^{max}$ or $\hat{\nu}_1$ is non-zero. Suppose first that $\hat{\nu}_1$ is zero. Then the Euler equation implies:

$$\hat{c}_{H1} = \frac{-\hat{\pi}_1(1+r) + \hat{\pi}_1\lambda}{(1-\beta(1+r)(1-p)\lambda)}.$$

Suppose that r is sufficiently near zero that:

$$\frac{\kappa}{(1-\beta_F)} < \frac{(1-\beta(1+r)(1-p))}{-r}.$$

Then, for any $\lambda \in (0,1)$:

$$\frac{\kappa}{(1-\beta_F\lambda)} \frac{\lambda-(1+r)}{(1-\beta(1+r)(1-p)\lambda)} \neq 1$$

and so:

$$\begin{aligned}
\hat{\pi}_1 &= \frac{\kappa}{(1-\beta_F\lambda)}\hat{c}_{H1} \\
&= \frac{\kappa}{(1-\beta_F\lambda)} \frac{\lambda-(1+r)}{(1-\beta(1+r)(1-p)\lambda)}\hat{\pi}_1
\end{aligned}$$

implies $\hat{\pi}_1 = \hat{c}_{H1} = 0$. But this is a contradiction of (21), since $\hat{\theta}_2^{max}$ is non-zero.

Now suppose instead that $\hat{\theta}_2^{max}$ is zero. Then (21) implies that:

$$\begin{aligned}
\hat{c}_{H1} &= -A_1(r)\hat{\pi}_1 - A_2(r)\lambda\hat{\pi}_1 \\
&= (-A_2(r)\lambda - A_1(r))\frac{(1-\beta_F\lambda)}{\kappa}\hat{c}_{H1}.
\end{aligned}$$

Suppose r is sufficiently near zero that:

$$(-A_2(r) - A_1(r))\frac{(1-\beta_F)}{\kappa} > 1.$$

Then:

$$(-A_2(r)\lambda - A_1(r))\frac{(1-\beta_F\lambda)}{\kappa} \neq 1$$

for all $\lambda \in (0,1)$ and so $\hat{c}_{H1} = \hat{\pi}_1 = 0$. But this is a violation of the assumption that $\hat{\nu}_1$ is non-zero. $\square$

4 Additional Considerations

In this section, I explore two additional issues: indeterminacy and the simultaneous stabilization of output and inflation.

4.1 Indeterminacy

In the next proposition, I show that, given a specification of monetary and fiscal policy, there exists a continuum of log-linearized equilibria indexed by near-term inflation expectations.

Proposition 2. *Suppose $(\hat{R}_t, \hat{\tau}_t)_{t=1}^{\infty}$ equals zero. There exists a continuum of (log-linearized) equilibria in which inflation $(\hat{\pi}_t)_{t=1}^{\infty}$ is non-zero in all periods.*

Proof. Pick an arbitrary $\hat{\pi}_2$. We look for an equilibrium such that $\hat{\pi}_t = (1+r)^{t-1}\hat{\pi}_2, t \geq 2$. To find the equilibrium, we first solve for $(\hat{c}_{Ht}, \hat{c}_{Lt}, \hat{y}_t)_{t=2}^{\infty}$. Then, we solve for $(\hat{\pi}_1, \hat{c}_{H1}, \hat{c}_{L1}, \hat{y}_1)$ and $(\hat{b}_t^{pub})_{t=2}^{\infty}$.

We find $(\hat{c}_{Ht})_{t=2}^{\infty}$ using the Euler equation:

$$\hat{c}_{Ht} = \hat{\pi}_{t+1} + \alpha \hat{c}_{H,t+1}, t \geq 2,$$

where $\alpha = \beta(1-p)(1+r) < 1$. The Euler equation implies that:

$$\hat{c}_{Ht} = A_H \hat{\pi}_t, A_H = \frac{(1+r)}{(1-\alpha(1+r))}, t \geq 2$$

We can then use the NKPC to find $(\hat{y}_t)_{t=1}^{\infty}$:

$$\hat{\pi}_t = \kappa(\psi \hat{y}_t + \frac{(1+r)\hat{\pi}_t}{(1-\alpha(1+r))}) + (1+r)\beta_F \hat{\pi}_t, t \geq 2$$

so that:

$$\hat{y}_t = A_Y \hat{\pi}_t, A_Y = \frac{1-(1+r)\beta_F - \kappa(1+r)/(1-\alpha(1+r))}{\kappa\psi}, t \geq 2.$$

(Note A_Y could be negative.) The aggregate resource constraint then implies that:

$$\begin{aligned}\hat{c}_{Lt} &= \frac{(Y\hat{y}_t - P_H C_H \hat{c}_{Ht})}{P_L C_L} \\ &= A_L \hat{\pi}_t, A_L = (\frac{Y}{P_L C_L} A_Y - \frac{P_H C_H}{P_L C_L} A_H), t \geq 2.\end{aligned}$$

Recall the flow budget constraint for agents in state L is:

$$\bar{C}_L \hat{c}_{Lt} = \frac{pB^{pub}}{P_L(1+\Pi)}(\hat{b}_t^{pub} - \hat{\pi}_t) + \frac{B^{max}}{(1+r)}\hat{\pi}_{t+1}, t \geq 1$$

and the government flow budget constraint is:

$$\begin{aligned}(\hat{b}_{t+1}^{pub} - \hat{\pi}_{t+1}) &= (1+r)(\hat{b}_t^{pub} - \hat{\pi}_t), t \geq 1 \\ \hat{b}_1^{pub} &= 0\end{aligned}$$

Together, they imply that for all $t \geq 2$:

$$A_L \bar{C}_L (1+r)^{t-2} \hat{\pi}_2 = (1+r)^{t-1} \frac{pB^{pub}}{P_L(1+\Pi)}(-\hat{\pi}_1) + \frac{B^{max}}{(1+r)}(1+r)^{t-1}\hat{\pi}_2, t \geq 2$$

Then, we can solve for $\hat{\pi}_1$:

$$A_L \bar{C}_L \hat{\pi}_2 = (1+r)\frac{pB^{pub}}{P_L(1+\Pi)}(-\hat{\pi}_1) + \frac{B^{max}}{(1+r)}(1+r)\hat{\pi}_2$$

so that:

$$\begin{aligned}\hat{\pi}_1 &= A_\pi \hat{\pi}_2 \\ A_\pi &= \frac{(B^{max} - A_L \bar{C}_L)}{\frac{(1+r)pB^{pub}}{P_L(1+\Pi)}}\end{aligned}$$

We now look for the other period 1 components of the equilibrium. The consumption of the state H agents can again be found from the Euler equation as:

$$\hat{c}_{H1} = \hat{\pi}_2/(1 - \alpha(1 + r)).$$

Then, $\hat{y}_1$ satisfies the NKPC:

$$\hat{\pi}_1 = \kappa(\psi\hat{y}_1 + \hat{c}_{H1}) + \beta_F\hat{\pi}_2$$

which implies that:

$$\hat{y}_1 = \frac{(A_\pi - \beta_F) - \kappa A_H}{\kappa\psi}\hat{\pi}_2$$

and:

$$\hat{c}_{L1} = \frac{\hat{y}_1 - \frac{P_H C_H}{Y}\hat{c}_{H1}}{\frac{P_L C_L}{Y}}.$$

Finally, given the sequence $(\hat{\pi}_t)_{t=1}^{\infty}$, the evolution of public debt is given by:

$$\hat{b}_{t+1}^{pub} = (1 + r)(\hat{b}_t^{pub} - \hat{\pi}_t) + \hat{\pi}_{t+1}, \hat{b}_1^{pub} = 0.$$

$\square$

The source of the indeterminacy is familiar: the policy variables depend only on exogenous variables. The government can eliminate this indeterminacy using a simple fiscal policy rule, in which transfers depend on real debt. The fiscal policy rule takes the form:

$$\hat{\tau}_t\bar{\tau}(1 + R)/B = \Psi(\hat{b}_t^{pub} - \hat{\pi}_t), t \geq 2,$$

where $(\Psi + (1 + r)) > 1$. This ensures that, if $\hat{\pi}_1 \neq 0$, then the future evolution of the public debt is inconsistent with a bounded equilibrium because:

$$
\begin{aligned}
(\hat{b}^{pub}_{t+1} - \hat{\pi}_{t+1}) &= (1+r)(\hat{b}^{pub}_t - \hat{\pi}_t) + \Psi(\hat{b}^{pub}_t - \hat{\pi}_t), t \geq 2 \\
(\hat{b}^{pub}_2 - \hat{\pi}_2) &= (1+r)(-\hat{\pi}_1) \neq 0.
\end{aligned}
$$

4.2 "Conventional" Policy

As discussed in the introduction, the conventional macroeconomic policy response to shocks involves a combination of fiscal and monetary elements. The following proposition shows that, given a persistent shock to the marginal utility parameter ν, it is possible to use this more coordinated policy approach to stabilize output and inflation simultaneously.

Proposition 3. *Suppose $\hat{\nu} = (\lambda^{t-1}\hat{\nu}_1)_{t=1}^{\infty}$ for $\lambda \in (0,1)$ and $\hat{\nu}_1$ non-zero. Suppose too that $\hat{y}_t = \hat{\pi}_t = 0$ for all $t \geq 1$. Then there exists a unique specification $(\hat{R}_t, \hat{\tau}_t)_{t=1}^{\infty}$ of monetary-fiscal policy such that there is a (log-linearized) equilibrium. The specification of monetary policy satisfies the restriction:*

$$\hat{R}_t = -\beta(1+r)p\bar{\nu}\lambda^t\hat{\nu}_1 C_H, t \geq 1$$

Proof. If $\hat{y}_t = \hat{\pi}_t = 0$, then $\hat{c}_{Ht} = 0$. From the Euler equation, it follows that:

$$\hat{R}_t = -\Delta_{t+1}, t \geq 1$$

where $\Delta_{t+1} = \beta(1+r)p\bar{\nu}\hat{\nu}_{t+1}C_H$. Given this choice of interest rate policy, we can solve for transfers and debt levels to satisfy the budget constraint of agents in state L:

$$\hat{\tau}_t\bar{\tau} = -\frac{pB^{pub}}{P_L(1+\Pi)}\hat{b}^{pub}_t + \frac{B^{max}}{(1+r)}\hat{R}_t, t \geq 1$$

and the government's flow budget constraint:

$$\hat{b}^{pub}_{t+1} = (1+r)\hat{b}^{pub}_t + \hat{R}_t + \hat{\tau}_t\bar{\tau}(1+R)/B^{pub}, t \geq 1$$

By substituting the former budget constraint into the latter, we obtain:

$$\hat{b}_{t+1}^{pub} = (1+r)(1-p/P_L)\hat{b}_t^{pub} + \hat{R}_t(1 + \frac{B^{max}(1+R)}{(1+r)B^{pub}}), t \geq 1$$

$$\hat{b}_1^{pub} = 0$$

Since $|1-p/P_L| < 1$, the resulting bond sequence is bounded. We can then solve for transfers as:

$$\hat{\tau}_t\bar{\tau} = -\frac{pB^{pub}}{P_L(1+\Pi)}\hat{b}_t^{pub} + \frac{B^{max}}{(1+r)}\hat{R}_t.$$

□

A positive shock to the marginal utility parameter ν is similar to a downward shock to the natural real rate of interest in a standard New Keynesian model. The monetary policy described in Proposition 4 can thus be seen as equivalent to the standard result (Gali (2015, p. 103)) that it is optimal for the nominal interest rate to track the natural real rate of interest.

However, the economic logic underlying this characterization is different in this model from that in the standard RANK framework. In the RANK model, monetary policy is being used to select the desired point on a (policy-invariant) Phillips curve. In Proposition 3, monetary policy pins down c_{Ht} and so determines the location of the Phillips curve. Fiscal policy is then used to pick a desired point on the Phillips curve.

5 Conclusion

The conventional approach to macroeconomic stabilization is centered on monetary policy: raise interest rates to lower spending/output/inflation and reduce interest rates to spur spending/output/inflation. I show that, within a simple analytical HANK model, this approach is ineffective if the neutral real interest rate is less than, but close to, the growth rate. The problem is a dynamic one: a current interest rate cut reduces future asset income and so gives rise to a need for a future interest rate cut. When the public debt is sufficiently large (that is, the interest rate is close to the growth rate), this intertemporal linkage produces an unsustainable ever-increasing chain of monetary stimulus.

Instead, this paper finds that, in the presence of a public debt bubble, macroeconomic stabilization is best accomplished through fiscal policy, as the government can increase transfers to spur spending/output/inflation and lower transfers to reduce spending/output/inflation. A key aspect of this finding is that a debt bubble clarifies the demand effects of fiscal stimulus, because it is possible to finance increases in current transfers without any change in future taxes/transfers. The result can be seen as providing theoretical support for the temporary increases in (near-)universal base income used to confront the adverse macroeconomic conditions of 2020-21. More generally, the analysis provides a formal theoretical foundation for regularizing a fiscal approach to macroeconomic support even if the zero lower bound is no longer a relevant consideration.

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